STOCHASTIC APPROACHES FOR DAMAGE EVOLUTION IN STANDARD AND NON-STANDARD CONTINUA

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Abstract

Damage evolution in heterogeneous materials is described using a continuum damage approach. It is shown that a stochastic description of the strength properties is not sufficient to remedy the ill-posedness of the boundary value problem that arises during progressive damage evolution, but that some form of non-locality must be added. On the other hand, the presence of random fields in the damage model is indispensable to realistically describe their failure mode in heterogeneous media.

Introduction

Failure in quasi-brittle, disordered materials involves localisation of deformation in narrow zones. Standard continuum theories, which assume sufficiently smooth variations of deformation, are not capable of properly describing this failure mode. Non-local or gradient theories (Pijaudier-Cabot and Bazant 1987, de Borst et al. 1993), which introduce higher-order deformation gradients, are capable of properly incorporating failure zones. The higher order deformation gradients can be seen as the outcome from a homogenisation of micro-scale phenomena like microcrack initiation, growth and coalescence. On the other hand, the localisation process can be highly dependent upon the heterogeneity of the material at a relative large scale. An appropriate approach to account for the heterogeneity of a material at the macro-scale is to consider the macro-scale material properties (e.g. strength and/or softening properties) as random fields, describing their spatial distribution and mutual correlations in space (Carmeliet and Hens, 1994).

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Non-local Damage model

The constitutive relation for the isotropic elasticity-based damage theory
\[ \sigma = (1 - D) \mathbb{C} \varepsilon \]  

introduces a damage parameter D, which accounts for degradation of the elastic stiffness. Damage growth is determined by an evolution law \[ D = F(\varepsilon^d) \], in which \( \varepsilon^d \) is the equivalent strain measure as defined by Mazars and Pijaudier-Cabot (1989). Damage growth is possible if the damage loading function \( f = \varepsilon^d - K \) vanishes. The damage parameter K initially equals the damage threshold \( K_0 \) and during damage equals the maximum value of \( \varepsilon^d \) ever reached during the loading history. The damage loading function \( f \) and the rate of damage growth \( \dot{D} \) have to satisfy the discrete Kuhn-Tucker conditions: \( f \leq 0 \), \( D \geq 0 \), \( f \, D = 0 \).

In the non-local model (Pijaudier-Cabot and Bazant 1987) the equivalent strain \( \varepsilon^d \) is replaced by an spatial averaged or non-local equivalent strain value \( \bar{\varepsilon} \), such that
\[ \bar{\varepsilon}(x) = \frac{1}{V_\tau} \int_V \varepsilon^d(x + \tau) \alpha(\tau) \, dV \quad , \quad \alpha(\tau) = \exp\left(-|\tau|^2 / 2 \ell^2 \right) \]  

with \( \tau \) the separation between two points \( x \) and \( x + \tau \), \( V_\tau \) a normalizing factor, \( \alpha \) a squared exponential weight function and \( \ell \) the so-called internal length scale.

Stochastic approach

The randomness in the damage process is introduced by considering the initial damage threshold \( K_0 \) as a random field. The random field is described by a type III extreme value cumulative distribution function \( F_{K_0} \) and an autocorrelation coefficient function \( \rho_{K_0} \) of the same form as the weight function of the nonlocal damage model:
\[ F_{K_0}(K_0) = \exp\left(-\lambda(K_0 - K_0^{\text{min}})\mu\right) \quad , \quad \rho_{K_0} = \exp\left(-|\tau|^2 / 2 \sigma^2 \right) \]  

with \( \lambda \) and \( \mu \) model parameters, \( K_0^{\text{min}} \) the lower bound of the initial damage threshold and \( \sigma \) a length parameter, which is a measure for the rate of fluctuations of the random field. For finite element analysis involving random field properties, it is necessary to discretise the continuous random field into random vector representations. In this paper, we will use the midpoint method. The random field is digitally generated according to the method of Yamazaki and Shinozuka (1988).

Tensile specimen

A fundamental question regarding application of stochastic approaches to localisation phenomena is whether a statistical description of the standard continuum resolves the deficiency of the continuum model at localisation. This question becomes imperative especially if we consider that the description of a heterogeneous continuum by correlated random variables introduces an length parameter in the form of the correlation length parameter \( d \) analogous to the introduction of the internal length scale \( l \) in non-standard continuum approaches. The analysis of a tensile specimen
with random initial damage is well suited to study this fundamental issue. Figure 1 gives the cumulative distributions of the total energy dissipation during damage, calculated from the responses of 100 using the Monte Carlo technique, for two different finite element discretisations. The results for the local damage model are clearly mesh dependent, while the results for the non-local stochastic model show a perfect agreement for both discretisations.

![Graph showing cumulative distribution of energy dissipation](image)

Figure 1. Cumulative distribution of energy dissipation during damage. Elastic modulus $E=20000$ MPa, $K_0^{\min}=0.66 \times 10^{-4}$, $\lambda=5.72 \times 10^5$, $\mu=2$, $d=5$ mm, specimen size=100x25 mm$^2$. Local damage model (a): constant softening modulus $h=-0.01$ E; non-local damage model (b): $l=5$ mm, $h=-0.1$ E.

Pull-out of an anchor bolt

The geometry and material properties of the second proposal of the round-robin analysis proposed by RILEM-committee TC90-FMA have been used in the analysis of the pull-out of a steel anchor bolt embedded in concrete (Vervuert et al. 1993). Initial damage is implemented using the simulation method as described in the example of the tensile specimen. Figure 2a shows the load versus displacement of the upper outer edge of the anchor head and figure 2b the corresponding crack patterns for three different specimens. Different maximum loads, softening curves and crack patterns are observed, showing the importance of a stochastic approach for damage processes in quasi-brittle heterogeneous materials.

Conclusions

1. It has been shown that a stochastic description does not solve the difficulties associated with the use of strain softening in a standard continuum.
2. The analysis of the pull-out of an anchor bolt indicates that the exact failure mode can be highly dependent upon the precise initial flaw distribution and that stochastic descriptions of the strength must be adopted in addition to the non-standard continuum approach.
3. In both formulations a length parameter is introduced: the internal length scale of the non-local continuum $l$ and the correlation parameter of the random field $d$. 
Figure 2. Pull-out of an anchor bolt; (a) Load-displacement diagram; (b) active cracks (dark) and non-active cracks (gray) at maximum and final loading.

References


