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Neuro-Adaptive Cooperative Tracking Rendezvous of Nonholonomic Mobile Robots

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***Abstract*—This paper proposes a neuro-adaptive method for the unsolved problem of cooperative tracking rendezvous of nonholonomic mobile robots (NMRs) subject to uncertain and unmodelled dynamics. A hierarchical cooperative control framework is proposed, which consists of a novel distributed estimator along with local neuro-adaptive tracking controllers. Rigorous stability analysis as well as simulation experiments illustrate the proposed method.**

***Index Terms*—Nonholonomic mobile robot, cooperative rendezvous, neuro-adaptive control, distributed estimator.**

I. Introduction

Cooperative control of nonholonomic mobile robots (NMRs) is a challenging research topic with applications in tracking, rendezvous and formation of autonomous vehicles [1]. When designing controllers to achieve such tasks, the NMR dynamics are usually assumed to be known [2]–[4]. However, more challenges arise if the NMR dynamics are uncertain. Uncertainties in NMRs arise from unmodelled dynamics, friction, resistance, velocity-controlled motor dynamics, and so on. For a single uncertain mobile robot, various adaptive and neural-adaptive designs have been proposed [5]–[7]. However, teams of uncertain NMRs have seldom been considered: research has mostly been focused on first-order [8], [9], second-order [10], or higher-order uncertain integrators [11]–[13], which do not exhibit all the challenges of nonholonomic dynamics.

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The main contribution of this paper is to address and solve the cooperative rendezvous problem for uncertain NMRs. A hierarchical cooperative control framework is proposed, consisting of a distributed estimator to asymptotically reconstruct the leader NMR state information, and local neuro-adaptive tracking controllers to approximate the uncertain nonlinearities. Although the estimator/observer idea was adopted in [14] (fractional-order integrators), [12], [15], [16] (first-order integrators), [10], [11], [17] (second-order integrators), [18]–[21] (high-order linear dynamics), the novelty of the proposed estimator is to deal with the nonholonomic dynamics. Due to the nonholonomic dynamics, the adaptive controllers, error systems and Lyapunov functions are more challenging to deal with than those in [10]–[12], [14]–[19].

Notations: $\mathbb{R}$ and $\mathbb{R}_+$ denote the sets of real and positive real numbers, respectively. For a column vector $x \in \mathbb{R}^n$, denote $||x||_1$ and $||x||$ the 1- and 2-norm, respectively. For a matrix M, define $||M||_F = \sqrt{\mathrm{tr}\{M^T M\}}$. Denote $\bar{\sigma}(M)$ and $\underline{\sigma}(M)$ the maximum and minimum singular value. $\mathbf{1}_n$ and $\mathbf{0}_n$ are the $n\times 1$ all-one vector and the $n\times 1$ all-zero vector, respectively. $\max f(\cdot)$ represents the maximum of the function $f(\cdot)$.

Consider a directed graph $\mathcal{G} = (\mathcal{V}(\mathcal{G}), \varepsilon(\mathcal{G}))$ with node set $\mathcal{V} = \{1, 2, \cdots, N\}$ and edge set $\varepsilon \subset \mathcal{V}(\mathcal{G}) \times \mathcal{V}(\mathcal{G})$. The adjacency matrix $A = (a_{ij})_{N\times N}$ of $\mathcal{G}$ is defined as: $a_{ij} \neq 0$ if $(j, i) \in \varepsilon(\mathcal{G})$ and 0 otherwise; $a_{ii} = 0$ for each $i \in \mathcal{V}(\mathcal{G})$. The Laplacian matrix $L = (l_{ij})_{N\times N}$ is defined as: $l_{ij} = -a_{ij}$, $i \neq j$, and $l_{ii} = \sum_{j\in\mathcal{N}_i} a_{ij}$ for $i = 1, 2, \cdots, N$. The set of neighbors for node i is represented by $\mathcal{N}_i = \{j \in \mathcal{V} | (j, i) \in \varepsilon(\mathcal{G})\}$. In a leader-follower graph, the leader is denoted as node 0 and the followers as nodes $\{1, \cdots, N\}$. Including the leader as node 0 results in the augmented graph $\bar{\mathcal{G}}$ [22]. The leader adjacency matrix is a diagonal matrix $B = \mathrm{diag}(b_1, \cdots, b_N)$, where $b_i > 0$ if the follower agent i has access to the leader's information (state) and $b_i = 0$ otherwise. Denote $H = L + B$.

II. Problem Statement

Consider a group of $N(\geq 2)$ NMRs described by

$$\begin{cases} \dot{x}_i(t) = (v_i(t) + f_i(x_i(t), y_i(t), \theta_i(t)) + \xi_{1_i}(t)) \cos\theta_i(t), \\ \dot{y}_i(t) = (v_i(t) + f_i(x_i(t), y_i(t), \theta_i(t)) + \xi_{1_i}(t)) \sin\theta_i(t), \\ \dot{\theta}_i(t) = \omega_i(t) + g_i(x_i(t), y_i(t), \theta_i(t)) + \xi_{2_i}(t), \end{cases} \quad (1)$$

where $i = 1, 2, \cdots, N$, $p_i(t) = (x_i(t), y_i(t))^T \in \mathbb{R}^2$ is the position of the ith NMR in the global coordinate, $\theta_i(t)$ is its orientation, the linear and angular velocity, $v_i(t) \in \mathbb{R}$ and $\omega_i(t) \in \mathbb{R}$ are control inputs to be designed; the unknown and unmodeled nonlinear functions $f_i(x_i(t), y_i(t), \theta_i(t))$ and $g_i(x_i(t), y_i(t), \theta_i(t))$ will be denoted as $f_i(t)$ and $g_i(t)$ for compactness; $\xi_{1_i}(t) \in \mathbb{R}$ and $\xi_{2_i}(t) \in \mathbb{R}$ are external bounded disturbance signals, with known upper bounds $\|\xi_1(t)\| \leq \bar{\xi}_1$ and $\|\xi_2(t)\| \leq \bar{\xi}_2$, $\forall t$, with $\xi_1(t) = (\xi_{1_1}(t), \cdots, \xi_{1_N}(t))$, $\xi_2(t) = (\xi_{2_1}(t), \cdots, \xi_{2_N}(t))$.

Using neuro-adaptive ideas [8], [10], [11], consider the approximations of the nonlinearities in (1) on a compact set $\Omega_i \in \mathbb{R}^3$, i.e.

$$f_i(\Lambda_i(t)) = \Gamma_{1_i}^T \varphi_{1_i}(\Lambda_i(t)) + \varepsilon_{1_i}(t),$$
$$g_i(\Lambda_i(t)) = \Gamma_{2_i}^T \varphi_{2_i}(\Lambda_i(t)) + \varepsilon_{2_i}(t),$$

where $\Lambda_i(t) = (x_i(t), y_i(t), \theta_i(t))^T$, $\varphi_{1_i}(\Lambda_i(t)) \in \mathbb{R}^{v_{1_i}}$ and $\varphi_{2_i}(\Lambda_i(t)) \in \mathbb{R}^{v_{2_i}}$ are basis sets for node i, $\Gamma_{1_i} \in \mathbb{R}^{v_{1_i}}$ and $\Gamma_{2_i} \in \mathbb{R}^{v_{2_i}}$ are unknown coefficients, and ε_{1_i}, ε_{2_i} are bounded approximation errors. For compactness, let us define the global network nonlinearities: $f(\Lambda(t)) = \Gamma_1^T \varphi_1(\Lambda(t)) + \varepsilon_1(t)$ and $g(\Lambda(t)) = \Gamma_2^T \varphi_2(\Lambda(t)) + \varepsilon_2(t)$, where $\Lambda(t) = (\Lambda_1^T(t), \cdots, \Lambda_N^T(t))^T$, $\varphi_1 = (\varphi_{1_1}^T, \cdots, \varphi_{1_N}^T)^T$, $\varphi_2 = (\varphi_{2_1}^T, \cdots, \varphi_{2_N}^T)^T$, $\Gamma_1^T = \text{diag}\left(\Gamma_{1_1}^T, \cdots, \Gamma_{1_N}^T\right)$, $\Gamma_2^T = \text{diag}\left(\Gamma_{2_1}^T, \cdots, \Gamma_{2_N}^T\right)$, $\varepsilon_1(t) = (\varepsilon_{1_1}(t), \cdots, \varepsilon_{1_N}(t))^T$ and $\varepsilon_2(t) = (\varepsilon_{2_1}(t), \cdots, \varepsilon_{2_N}(t))^T$. Moreover, define the unknown upper bounds $\|\Gamma_1\|_F \leq \Gamma_{1_m}$, $\|\Gamma_2\|_F \leq \Gamma_{2_m}$, $\|\varphi_1(\Lambda(t))\| \leq \phi_{1_m}$ and $\|\varphi_2(\Lambda(t))\| \leq \phi_{2_m}$.

The NMRs in (1) are required to track a leader NMR

$$\begin{cases} \dot{x}_0(t) = v_0(t) \cos \theta_0(t), & \text{(2a)} \\ \dot{y}_0(t) = v_0(t) \sin \theta_0(t), & \text{(2b)} \\ \dot{\theta}_0(t) = \omega_0(t), & \text{(2c)} \end{cases}$$

where $p_0(t) = (x_0(t), y_0(t))^T \in \mathbb{R}^2$ is the reference trajectory, $\theta_0(t)$ is the reference orientation. The reference linear and angular velocity are $v_0(t)$, and $\omega_0(t)$, satisfying $|v_0(t)| \leq \bar{v}_0$, $|\omega_0(t)| \leq \bar{\omega}_0$, $|\dot{v}_0(t)| < \rho_1$ and $|\dot{\omega}_0(t)| < \rho_2$, for known bounds $\bar{v}_0$, $\bar{\omega}_0$, ρ_1 and ρ_2.

Assumption 2.1 The augmented graph $\bar{\mathcal{G}}$ describing the interconnections of (1) and (2) is strongly connected, implying that it contains a directed spanning tree with the root node being the leader NMR.

Remark 2.1 It was proven that Assumption 2.1 guarantees that $H = (L + B)$ is nonsingular [23]. A detailed explanation for the non-singularity of matrix H is in [22].

Before moving on, the following lemma is needed.

Lemma 2.1 [24]: Suppose that $h(x(t), t)$ is essentially bounded and that the origin is a Filippov solution (denoted with $\mathbf{0}_n \in \mathcal{F}[h(\mathbf{0}_n, t)]$) of the differential system

$$\dot{x} = h(x(t), t). \tag{3}$$

Let $V(\cdot) : \mathbb{R}^n \to \mathbb{R}$ satisfy $V(\mathbf{0}_n) = 0$ and $0 < V_1(\|x(t)\|) \leq V(x(t)) \leq V_2(\|x(t)\|)$ for $x(t) \neq \mathbf{0}_n$, where $V_1(\cdot)$ and $V_2(\cdot)$ are $\mathcal{K}$-class functions. System (3) is *uniformly asymptotically stable* if there exists a $\mathcal{K}$-class function $k(\cdot)$ such that $\max \dot{\tilde{V}}(x(t)) \leq -k(x(t)) < 0$ for all $x(t) \neq \mathbf{0}_n$, where $\dot{\tilde{V}}(x(t))$ is the set-valued Lie derivative along the trajectories of (3).

III. The proposed Distributed Estimator

Since some of the follower NMRs cannot directly obtain the leader information necessary to solve the tracking rendezvous, we propose the following novel distributed estimator to estimate the position $(x_0(t), y_0(t))^T$ and the orientation $\theta_0(t)$ of the leader NMR by the ith follower, $i = 1, ..., N$.

$$\begin{cases} \dot{\hat{x}}_i(t) = \hat{v}_i(t) \cos \hat{\theta}_i(t) + b_i(x_0(t) - \hat{x}_i(t)) + \sum\limits_{j \in \mathcal{N}_i} (\hat{x}_j(t) - \hat{x}_i(t)), \\ \dot{\hat{y}}_i(t) = \hat{v}_i(t) \sin \hat{\theta}_i(t) + b_i(y_0(t) - \hat{y}_i(t)) + \sum\limits_{j \in \mathcal{N}_i} (\hat{y}_j(t) - \hat{y}_i(t)), \\ \dot{\hat{\theta}}_i(t) = \hat{\omega}_i(t) + b_i\left(\theta_0(t) - \hat{\theta}_i(t)\right) + \sum\limits_{j \in \mathcal{N}_i} \left(\hat{\theta}_j(t) - \hat{\theta}_i(t)\right), \\ \dot{\hat{v}}_i(t) = \alpha_1 \Big(b_i(v_0(t) - \hat{v}_i(t)) + \sum\limits_{j \in \mathcal{N}_i} (\hat{v}_j(t) - \hat{v}_i(t)) \Big) \\ \qquad + \beta_1 \text{sgn}\Big(b_i(v_0(t) - \hat{v}_i(t)) + \sum\limits_{j \in \mathcal{N}_i} (\hat{v}_j(t) - \hat{v}_i(t)) \Big), \\ \dot{\hat{\omega}}_i(t) = \alpha_2 \Big(b_i(\omega_0(t) - \hat{\omega}_i(t)) + \sum\limits_{j \in \mathcal{N}_i} (\hat{\omega}_j(t) - \hat{\omega}_i(t)) \Big) \\ \qquad + \beta_2 \text{sgn}\Big(b_i(\omega_0(t) - \hat{\omega}_i(t)) + \sum\limits_{j \in \mathcal{N}_i} (\hat{\omega}_j(t) - \hat{\omega}_i(t)) \Big). \end{cases} \tag{4}$$

Lemma 2.2 Consider the NMRs given by (1) and (2). Under Assumption 2.1, if the gains β_1 and β_2 are chosen such that $\beta_1 > \rho_1, \beta_2 > \rho_2$, then, the distributed estimators (4) converge asymptotically to the state of the leader NMR.

Proof. Define $\tilde{x}(t) := \hat{x}(t) - x_0(t)\mathbf{1}_N$, $\tilde{y}(t) := \hat{y}(t) - y_0(t)\mathbf{1}_N$, $\tilde{\theta}(t) := \hat{\theta}(t) - \theta_0(t)\mathbf{1}_N$, $\tilde{v}(t) := \hat{v}(t) - v_0(t)\mathbf{1}_N$ and $\tilde{\omega}(t) := \hat{\omega}(t) - \omega_0(t)\mathbf{1}_N$, with $\hat{x}(t) = (\hat{x}_1(t), \cdots, \hat{x}_N(t))^T$, $\hat{y}(t) = (\hat{y}_1(t), \cdots, \hat{y}_N(t))^T$, $\hat{\theta}(t) = (\hat{\theta}_1(t), \cdots, \hat{\theta}_N(t))^T$, $\hat{v}(t) = (\hat{v}_1(t), \cdots, \hat{v}_N(t))^T$ and $\hat{\omega}(t) = (\hat{\omega}_1(t), \cdots, \hat{\omega}_N(t))^T$. Let $\xi(t) := H\tilde{v}(t)$ and $\Xi(t) := H\tilde{\omega}(t)$.

Consider the Lyapunov function

$$V_1(\Xi(t)) = \Xi^T(t) K \Xi(t), \tag{5}$$

where $K = \text{diag}\,(k_i) \equiv \text{diag}\,(1/q_i)$, with $q = (q_1, \cdots, q_N)^T = (L + B)^{-1}\mathbf{1}_N$. One can thus get the set-valued derivative of $V_1(\Xi(t))$ along $\dot{\Xi}(t)$ as

$$\begin{aligned} \dot{\tilde{V}}_1(\Xi(t)) = & -\alpha_2 \Xi^T(t) Q \Xi(t) - \beta_2 \mathcal{F}\left[\text{sgn}(\Xi(t))^T H^T K \Xi(t)\right] \\ & - \beta_2 \mathcal{F}\left[\Xi^T(t) K H \text{sgn}(\Xi(t))\right] - \dot{\omega}_0(t) \mathbf{1}_N^T H^T K \Xi(t) \\ & - \dot{\omega}_0(t) \Xi^T(t) K H \mathbf{1}_N. \end{aligned} \tag{6}$$

Meanwhile, one has

$$\begin{aligned} & - \beta_2 \mathcal{F}\left[\text{sgn}(\Xi(t))^T H^T K \Xi(t)\right] \\ = & - \beta_2 \mathcal{F}\left[\text{sgn}(\Xi(t))^T (L + \text{diag}(b_1, \cdots, b_N))^T K \Xi(t)\right] \\ = & \beta_2 \sum_{i=1} k_i \sum_{j=1, i \neq j} l_{ij} \mathcal{F}[\|\Xi_i(t)\|_1 - \text{sgn}(\Xi_j(t)) \Xi_i(t)] \\ & - \beta_2 \sum_{i=1} b_i k_i \|\Xi_i(t)\|_1. \end{aligned} \tag{7}$$

According to the property of 1-norm, it is trival to show that

$$-\beta_2 \mathcal{F}\left[\text{sgn}(\Xi(t))^T H^T K \Xi(t)\right] \leq -\beta_2 \sum_{i=1} b_i k_i \|\Xi_i(t)\|_1. \tag{8}$$

In addition, because $|\dot{\omega}_0(t)| \leq \rho_2$, one has

$$\begin{aligned} & - \dot{\omega}_0(t) \mathbf{1}_N^T H^T K \Xi(t) \\ = & - \dot{\omega}_0(t) \mathbf{1}_N^T \left(KL + \text{diag}(b_1 k_1, \cdots, b_N k_N)\right)^T \Xi(t) \\ \leq & |\dot{\omega}_0(t)| \sum_{i=1} b_i k_i \|\Xi_i(t)\|_1 \leq \rho_2 \sum_{i=1} b_i k_i \|\Xi_i(t)\|_1. \end{aligned} \tag{9}$$

Combining (6)-(9) and by using the condition of $\beta_2 > \rho_2$, it follows that $\max \dot{V}_1(\Xi(t)) \leq -\alpha_2 \Xi^T(t) Q \Xi(t) \leq 0$, where $Q = K(L+B) + (L+B)^T K$. This implies that $\Xi(t)$, and thus $\tilde{\omega}(t)$ converge to $\mathbf{0}_N$ asymptotically. Similarly, under the assumption of $|\dot{v}_0(t)| < \rho_1$, consider the Lyapunov function

$$V_2(\xi(t)) = \xi^T(t) K \xi(t), \tag{10}$$

one can verify that the derivative of V_2 along $\dot{\xi}(t)$ satisfies $\max \dot{V}_2(\xi(t)) \leq -\alpha_1 \xi^T(t) Q \xi(t) \leq 0$. So, $\tilde{v}(t)$ converges to $\mathbf{0}$ asymptotically as well.

The time derivative of $\tilde{\theta}(t)$ along with (2c) and the estimation dynamics is given by

$$\dot{\tilde{\theta}}(t) = -H\tilde{\theta}(t) + \hat{\omega}(t) - \omega_0(t)\mathbf{1}_N = -H\tilde{\theta}(t) + \tilde{\omega}(t).$$

It is easy to get the solution $\tilde{\theta}(t) = e^{-Ht}\tilde{\theta}(0) + \int_0^t e^{-H(t-\tau)}\tilde{\omega}(\tau)d\tau$. Since $\tilde{\omega}(t)$ converges to $\mathbf{0}_N$ asymptotically and H is a Hurwitz matrix, one has

$$\lim_{t\to\infty} \tilde{\theta}(t) = \lim_{t\to\infty} e^{-Ht}\tilde{\theta}(0) + \lim_{t\to\infty} \int_0^t e^{-H(t-\tau)}\tilde{\omega}(\tau)d\tau = \mathbf{0}_N.$$

In addition, one obtains

$$\begin{aligned}\dot{\tilde{x}}(t) =& \text{diag}\left(\cos\hat{\theta}_i(t)\right)(\hat{v}(t) - v_0(t)\mathbf{1}_N) \\ &+ v_0(t)\text{diag}\left(\cos\hat{\theta}_i(t) - \cos\theta_0(t)\right)\mathbf{1}_N - H\tilde{x}(t).\end{aligned}$$

Similarly, let $\varpi(t) := \text{diag}\left(\cos\hat{\theta}_i(t)\right)(\hat{v}(t) - v_0(t)\mathbf{1}_N) + v_0(t)\text{diag}\left(\cos\hat{\theta}_i(t) - \cos\theta_0(t)\right)\mathbf{1}_N$. Then,

$$\tilde{x}(t) = e^{-Ht}\tilde{x}(0) + \int_0^t e^{-H(t-\tau)}\varpi(\tau)d\tau.$$

Since $\tilde{\theta}(t)$ and $\tilde{v}(t)$ converge to $\mathbf{0}_N$ asymptotically, one gets

$$\begin{aligned}\lim_{t\to\infty} \varpi(t) =& \lim_{t\to\infty} \text{diag}\left(\cos\hat{\theta}_i(t)\right)(\hat{v}(t) - v_0(t)\mathbf{1}_N) \\ &+ \lim_{t\to\infty} v_0(t)\text{diag}\left(\cos\hat{\theta}_i(t) - \cos\theta_0(t)\right)\mathbf{1}_N = \mathbf{0}_N.\end{aligned}$$

Therefore, it can be verified that $\lim_{t\to\infty}(\hat{x}(t) - x_0(t)\mathbf{1}_N) = \mathbf{0}_N$. In a similar way, one can verify $\lim_{t\to\infty}(\hat{y}(t) - y_0(t)\mathbf{1}_N) = \mathbf{0}_N$.

IV. Distributed Protocol Design

A distributed neuro-adaptive controller is adopted to deal with the uncertainties in (1).

$$\begin{cases} v(t) = -\hat{f}(t) + c_1 x_e(t) + \text{diag}\left(\cos\theta_{e_i}(t)\right)\hat{v}(t), & \text{(11a)} \\ \omega(t) = \hat{\omega}(t) - \hat{g}(t) + c_2 \sin\theta_e(t) + \text{diag}\left(y_{e_i}(t)\right)\hat{v}(t), & \text{(11b)} \end{cases}$$

where $c_1, c_2 > 0$, $v(t) = (v_1(t), \cdots, v_N(t))^T$, $\omega(t) = (\omega_1(t), \cdots, \omega_N(t))^T$, $x_e(t) = (x_{e_1}(t), \cdots, x_{e_N}(t))^T$, $\sin\theta_e(t) = (\sin\theta_{e_1}(t), \cdots, \sin\theta_{e_N}(t))^T$, $\hat{f}(t) = \left(\hat{f}_1(\Lambda_1(t)), \cdots, \hat{f}_N(\Lambda_N(t))\right)^T$ and $\hat{g}(t) = (\hat{g}_1(\Lambda_1(t)), \cdots, \hat{g}_N(\Lambda_N(t)))^T$ with

$$\begin{pmatrix} x_{e_i}(t) \\ y_{e_i}(t) \\ \theta_{e_i}(t) \end{pmatrix} = \begin{pmatrix} \cos\theta_i(t) & \sin\theta_i(t) & 0 \\ -\sin\theta_i(t) & \cos\theta_i(t) & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \hat{x}_i(t) - x_i(t) \\ \hat{y}_i(t) - y_i(t) \\ \hat{\theta}_i(t) - \theta_i(t) \end{pmatrix}. \tag{12}$$

and the neuro-adaptive approximators $\hat{f}_i(\Lambda_i(t)) = \hat{\Gamma}_{1_i}^T(t)\varphi_{1_i}(\Lambda_i(t))$, $\hat{g}_i(\Lambda_i(t)) = \hat{\Gamma}_{2_i}^T(t)\varphi_{2_i}(\Lambda_i(t))$, where $\hat{\Gamma}_{1_i}(t) \in \mathbb{R}^{v_{1_i}}$ and $\hat{\Gamma}_{2_i}(t) \in \mathbb{R}^{v_{2_i}}$ are designed as

$$\begin{cases} \dot{\hat{\Gamma}}_{1_i}(t) = -F_{1_i}\varphi_{1_i}(\Lambda_i(t))x_{e_i}(t) - \kappa_1 F_{1_i}\hat{\Gamma}_{1_i}(t), \\ \dot{\hat{\Gamma}}_{2_i}(t) = -F_{2_i}\varphi_{2_i}(\Lambda_i(t))\sin\theta_{e_i}(t) - \kappa_2 F_{2_i}\hat{\Gamma}_{2_i}(t), \end{cases} \tag{13}$$

where $F_{1_i} = \rho_{1_i} I_{v_{1_i}}$, $F_{2_i} = \rho_{2_i} I_{v_{2_i}}$ and $\rho_{1_i} > 0$, $\rho_{2_i} > 0$, $\kappa_1 > 0$ and $\kappa_2 > 0$ are scalar tuning gains.

For the global network, the estimators can be written as $\hat{f}(\Lambda(t)) = \hat{\Gamma}_1^T(t)\varphi_1(\Lambda(t))$, $\hat{g}(\Lambda(t)) = \hat{\Gamma}_2^T(t)\varphi_2(\Lambda(t))$, where $\hat{\Gamma}_1^T(t) = \text{diag}\left(\hat{\Gamma}_{1_1}^T(t), \cdots, \hat{\Gamma}_{1_N}^T(t)\right)$ and $\hat{\Gamma}_2^T(t) = \text{diag}\left(\hat{\Gamma}_{2_1}^T(t), \cdots, \hat{\Gamma}_{2_N}^T(t)\right)$.

Remark 4.1 The sign function in (4) allows to reconstruct the information of the leader asymptotically (sliding mode-observer). If the sign function is replaced with a sigmoid or a saturation function, chattering will be avoided at the price of losing asymptotic reconstruction of the leader's state.

V. Main Results

Theorem 3.1 For a sufficiently large number of neurons $\bar{v}_{1_i}$ and $\bar{v}_{2_i}$, $i = 1, \cdots, N$, the distributed control protocol (11) with adaptive law (13) guarantees the overall cooperative error vectors $x_e(t)$, $y_e(t)$, $\sin\theta_e(t)$ and the parametric estimation errors $\tilde{\Gamma}(t)$ to be uniformly ultimately bounded.

Proof. Based on (12), one has

$$\begin{cases} \begin{aligned} \dot{x}_{e_i}(t) =& (\omega_i(t) + g_i(t) + \xi_{2_i}(t))y_{e_i}(t) \\ &+ \Big(\hat{v}_i(t)\cos\hat{\theta}_i(t) - (v_i(t) + f_i(t) + \xi_{1_i}(t))\cos\theta_i(t) \\ &+ b_i(x_0(t) - \hat{x}_i(t)) + \sum_{j\in\mathcal{N}_i}(\hat{x}_j(t) - \hat{x}_i(t))\Big)\cos\theta_i(t) \\ &+ \Big(\hat{v}_i(t)\sin\hat{\theta}_i(t) - (v_i(t) + f_i(t) + \xi_{1_i}(t))\sin\theta_i(t) \\ &+ b_i(y_0(t) - \hat{y}_i(t)) + \sum_{j\in\mathcal{N}_i}(\hat{y}_j(t) - \hat{y}_i(t))\Big)\sin\theta_i(t), \\ \dot{y}_{e_i}(t) =& -(\omega_i(t) + g_i(t) + \xi_{2_i}(t))x_{e_i}(t) \\ &- \Big(\hat{v}_i(t)\cos\hat{\theta}_i(t) - (v_i(t) + f_i(t) + \xi_{1_i}(t))\cos\theta_i(t) \\ &+ b_i(x_0(t) - \hat{x}_i(t)) + \sum_{j\in\mathcal{N}_i}(\hat{x}_j(t) - \hat{x}_i(t))\Big)\sin\theta_i(t) \\ &+ \Big(\hat{v}_i(t)\sin\hat{\theta}_i(t) - (v_i(t) + f_i(t) + \xi_{1_i}(t))\sin\theta_i(t) \\ &+ b_i(y_0(t) - \hat{y}_i(t)) + \sum_{j\in\mathcal{N}_i}(\hat{y}_j(t) - \hat{y}_i(t))\Big)\cos\theta_i(t), \\ \dot{\theta}_{e_i}(t) =& \hat{\omega}_i(t) + b_i\left(\theta_0(t) - \hat{\theta}_i(t)\right) - \omega_i(t) - g_i(t) - \xi_{2_i}(t) \\ &+ \sum_{j\in\mathcal{N}_i}\left(\hat{\theta}_j(t) - \hat{\theta}_i(t)\right). \end{aligned} \end{cases} \tag{14}$$

Combining (1) and (4), the system (14) can be written as

$$\begin{cases} \begin{aligned} \dot{x}_e(t) =& \text{diag}\left(\omega_i(t) + g_i(t) + \xi_{2_i}(t)\right)y_e(t) \\ &- (v(t) + f(t) + \xi_1(t)) + \text{diag}\left(\cos\theta_{ei}(t)\right)\hat{v}(t) \\ &- \text{diag}\left(\cos\theta_i(t)\right)H\tilde{x}(t) - \text{diag}\left(\sin\theta_i(t)\right)H\tilde{y}(t), \\ \dot{y}_e(t) =& -\text{diag}\left(\omega_i(t) + g_i(t) + \xi_{2_i}(t)\right)x_e(t) \\ &+ \text{diag}\left(\sin\theta_{e_i}(t)\right)\hat{v}(t) + \text{diag}\left(\sin\theta_i(t)\right)H\tilde{x}(t) \\ &- \text{diag}\left(\cos\theta_i(t)\right)H\tilde{y}(t), \\ \dot{\theta}_e(t) =& -H\tilde{\theta}(t) + \hat{\omega}(t) - \omega(t) - g(t) - \xi_2(t). \end{aligned} \end{cases} \tag{15}$$

Now, the function estimation error is

$$\begin{cases} \hat{f}(\Lambda(t)) - f(\Lambda(t)) = \tilde{\Gamma}_1^T(t)\varphi_1(\Lambda(t)) - \varepsilon_1, \\ \hat{g}(\Lambda(t)) - g(\Lambda(t)) = \tilde{\Gamma}_2^T(t)\varphi_2(\Lambda(t)) - \varepsilon_2, \end{cases} \tag{16}$$

where $\tilde{\Gamma}_1^T(t) = \hat{\Gamma}_1^T(t) - \Gamma_1^T$ and $\tilde{\Gamma}_2^T(t) = \hat{\Gamma}_2^T(t) - \Gamma_2^T$ are the parameter estimation errors.

Consider the Lyapunov function

$$
\begin{aligned}
V_3(t) =& \\
&\frac{1}{2}x_e^T(t)x_e(t) + \frac{1}{2}y_e^T(t)y_e(t) + \mathbf{1}_N^T(\mathbf{1}_N - \cos\theta_e(t)) \\
&+ \frac{1}{2}\mathrm{tr}\left\{\tilde{\Gamma}_1^T(t)F_1^{-1}\tilde{\Gamma}_1(t)\right\} + \frac{1}{2}\mathrm{tr}\left\{\tilde{\Gamma}_2^T(t)F_2^{-1}\tilde{\Gamma}_2(t)\right\} \\
&+ \frac{1}{2}\tilde{x}^T(t)\tilde{x}(t) + \frac{1}{2}\tilde{y}^T(t)\tilde{y}(t) + \frac{1}{2}\tilde{\theta}^T(t)\tilde{\theta}(t) + \xi^T(t)P\xi(t) \\
&+ \Xi^T(t)P\Xi(t),
\end{aligned} \tag{17}
$$

where F^{-1} is the inverse of $F = \mathrm{diag}(F_1, \cdots, F_N)$.

Using (13) and (16), one gets

$$
\begin{aligned}
&x_e^T(t)\left(\hat{f}(t) - f(t) - \xi(t)\right) + \mathrm{tr}\left\{\tilde{\Gamma}_1^T(t)F_1^{-1}\dot{\hat{\Gamma}}_1(t)\right\} \\
\leq& -\kappa_1\left\|\tilde{\Gamma}_1(t)\right\|_F^2 - \kappa_1\Gamma_{1_m}\left\|\tilde{\Gamma}_1(t)\right\|_F + \left(\varepsilon_{1_m} + \bar{\xi}_1\right)\|x_e(t)\|
\end{aligned}
$$

and

$$
\begin{aligned}
&(\sin\theta_e(t))^T\left(\hat{g}(t) - g(t) - \xi_2(t)\right) + \mathrm{tr}\left\{\tilde{\Gamma}_2^T(t)F_2^{-1}\dot{\hat{\Gamma}}_2(t)\right\} \\
\leq& -\kappa_2\left\|\tilde{\Gamma}_2(t)\right\|_F^2 - \kappa_2\Gamma_{2_m}\left\|\tilde{\Gamma}_2(t)\right\|_F + \left(\varepsilon_{2_m} + \bar{\xi}_2\right)\|\sin\theta_e(t)\|.
\end{aligned}
$$

By Lemma 2.2, it follows that $\max\dot{\tilde{V}}_1(\Xi(t)) \leq -\alpha_2\Xi^T(t)Q\Xi(t)$, $\max\dot{\tilde{V}}_2(\xi(t)) \leq -\alpha_1\xi^T(t)Q\xi(t)$, $\|\tilde{\omega}(t)\| \leq \frac{1}{\underline{\sigma}(H)}\|\Xi(t)\|$ and $\|\tilde{v}(t)\| \leq \frac{1}{\underline{\sigma}(H)}\|\xi(t)\|$. By calculating the set-valued Lie derivative of $V_3(t)$, one has

$$
\begin{aligned}
\dot{V}_3(t) \leq& \\
&-c_1\|x_e(t)\|^2 + \bar{\sigma}(H)\|x_e(t)\|\|\tilde{x}(t)\| + \bar{\sigma}(H)\|x_e(t)\|\|\tilde{y}(t)\| \\
&+ \bar{\sigma}(H)\|y_e(t)\|\|\tilde{x}(t)\| + \bar{\sigma}(H)\|y_e(t)\|\|\tilde{y}(t)\| \\
&+ \bar{\sigma}(H)\|\sin\theta_e(t)\|\left\|\tilde{\theta}(t)\right\| - c_2\|\sin\theta_e(t)\|^2 - \kappa_1\left\|\tilde{\Gamma}_1(t)\right\|_F^2 \\
&-\kappa_1\Gamma_{1_m}\left\|\tilde{\Gamma}_1(t)\right\|_F + \left(\varepsilon_{1_m} + \bar{\xi}_1\right)\|x_e(t)\| - \kappa_2\left\|\tilde{\Gamma}_2(t)\right\|_F^2 \\
&-\kappa_2\Gamma_{2_m}\left\|\tilde{\Gamma}_2(t)\right\|_F + \left(\varepsilon_{2_m} + \bar{\xi}_2\right)\|\sin\theta_e(t)\| + \frac{1}{\underline{\sigma}(H)}\|\tilde{x}(t)\|\|\xi(t)\| \\
&+ \|\tilde{x}(t)\|2\bar{v}_0\sqrt{N} - \underline{\sigma}(H)\|\tilde{x}(t)\|^2 + \frac{1}{\underline{\sigma}(H)}\|\tilde{y}(t)\|\|\xi(t)\| \\
&+ \|\tilde{y}(t)\|2\bar{v}_0\sqrt{N} - \underline{\sigma}(H)\|\tilde{y}(t)\|^2 - \underline{\sigma}(H)\left\|\tilde{\theta}(t)\right\|^2 \\
&+ \frac{1}{\underline{\sigma}(H)}\left\|\tilde{\theta}(t)\right\|\|\Xi(t)\| - \alpha_2\underline{\sigma}(Q)\|\Xi(t)\|^2 - \alpha_1\underline{\sigma}(Q)\|\xi(t)\|^2.
\end{aligned} \tag{18}
$$

Let $z(t) = \Big(\|x_e(t)\|, \|\sin\theta_e(t)\|, \left\|\tilde{\Gamma}_1(t)\right\|_F, \left\|\tilde{\Gamma}_2(t)\right\|_F, \|\tilde{x}(t)\|, \|\tilde{y}(t)\|, \left\|\tilde{\theta}(t)\right\|, \|\Xi(t)\|, \|\xi(t)\|\Big)^T$. Then, (18) can be written as

$$
\dot{V}_3(t) \leq -z^T(t)Rz(t) + r^Tz(t) + \bar{\sigma}(H)(\|\tilde{x}(t)\| + \|\tilde{y}(t)\|)\sqrt{2V_3(t)}, \tag{19}
$$

where

$$
R = \begin{pmatrix}
c_1 & 0 & 0 & 0 & c & c & 0 & 0 & 0 \\
0 & c_2 & 0 & 0 & 0 & 0 & c & 0 & 0 \\
0 & 0 & \kappa_1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & \kappa_2 & 0 & 0 & 0 & 0 & 0 \\
* & 0 & 0 & 0 & d & 0 & 0 & 0 & -\frac{1}{2\underline{\sigma}(H)} \\
* & 0 & 0 & 0 & 0 & d & 0 & 0 & -\frac{1}{2\underline{\sigma}(H)} \\
0 & * & 0 & 0 & 0 & 0 & d & -\frac{1}{2\underline{\sigma}(H)} & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & * & \alpha_2\underline{\sigma}(Q) & 0 \\
0 & 0 & 0 & 0 & * & * & 0 & 0 & \alpha_1\underline{\sigma}(Q)
\end{pmatrix}, \quad c = -\frac{1}{2}\bar{\sigma}(H),
$$

$d = \underline{\sigma}(H)$ and $r = \Big(\varepsilon_{1_m} + \bar{\xi}_1, \varepsilon_{2_m} + \bar{\xi}_2, -\kappa_1\Gamma_{1_m}, -\kappa_2\Gamma_{2_m}, 2\bar{v}_0\sqrt{N}, 2\bar{v}_0\sqrt{N}, 0, 0, 0\Big)$.

Define $V_z(z) = z^TRz + r^Tz$, which is positive definite if R is positive definite, and $\|z\| > \frac{\|r\|}{\underline{\sigma}(R)}$, i.e. far enough from the origin. According to the Sylvester criterion, R is positive definite when

$c_1 > 0$, $c_1c_2 > 0$, $c_1c_2\kappa_1 > 0$, $c_1c_2\kappa_1\kappa_2 > 0$,
$c_1c_2\kappa_1\kappa_2 d > 0$, $c_1c_2\kappa_1\kappa_2 dg > 0$, $c_1c_2\kappa_1\kappa_2 dgh > 0$,
$c_1c_2\kappa_1\kappa_2 dgh\alpha_2\underline{\sigma}(Q) > 0$,
$c_1c_2\kappa_1\kappa_2 dgh\alpha_2\underline{\sigma}(Q)\alpha_1\underline{\sigma}(Q) > 0$.

By Lemma 2.1, $\int_0^t \bar{\sigma}(H)\|\tilde{x}(t)\|ds$ and $\int_0^t \bar{\sigma}(H)\|\tilde{y}(t)\|ds$ are all bounded. Hence, $\bar{\sigma}(H)\|\tilde{x}(t)\| \in \mathcal{L}_1$ and $\bar{\sigma}(H)\|\tilde{y}(t)\| \in \mathcal{L}_1$ are nonnegative. Combining $\|y_e(t)\| \leq \sqrt{2V_3(t)}$, one obtains $\bar{\sigma}(H)\|y_e(t)\|\|\tilde{x}(t)\| + \bar{\sigma}(H)\|y_e(t)\|\|\tilde{y}(t)\| \leq \bar{\sigma}(H)(\|\tilde{x}(t)\| + \|\tilde{y}(t)\|)\sqrt{2V_3(t)}$. By using (19), one has $\dot{V}_3(t) \leq p(t)\sqrt{V_3(t)}$, where $p(t) := \sqrt{2}\bar{\sigma}(H)(\|\tilde{x}(t)\| + \|\tilde{y}(t)\|)$ and $p(t) \in \mathcal{L}_1$, implying $\frac{d\left(\sqrt{V_3(t)}\right)}{dt} \leq \frac{p(t)}{2}$, or equivalently $\sqrt{V_3(t)} \leq \sqrt{V_3(0)} + \int_0^t \frac{p(s)}{2}ds$. Since $p(t) \in \mathcal{L}_1$, $V_3(t)$ is bounded, implying the existence of a positive constant δ such that, for each $l > 0$, $\sqrt{V_3(t)} \leq \delta, \forall\sqrt{V_3(0)} \leq l$. Then, from (19), for $\sqrt{V_3(0)} \leq l$, one has $\dot{V}_3(t) \leq -V_z(z(t)) + p(t)\delta$, which implies the following nonincreasing relation $\frac{d}{dt}\left(V_3(t) - \delta\int_0^t p(s)ds\right) \leq 0$.

Since $V_3(t)$ is bounded from below by zero, $V_3(t)$ converges to some nonnegative constants.

According to (17), $\|x_e(t)\|$,$\|y_e(t)\|$,$\|\tilde{x}(t)\|$,$\|\tilde{y}(t)\|$ and $\left\|\tilde{\theta}(t)\right\|$ are all bounded and their same upper bound are $\sqrt{2}\delta$. Besides, $\left\|\tilde{\Gamma}_1(t)\right\|_F$ and $\left\|\tilde{\Gamma}_2(t)\right\|_F$ have the same upper bound $\delta\sqrt{2\Pi_{1_{\max}}}$. Also, $\|\xi(t)\|$ and $\|\Xi(t)\|$ can obtain their same upper bound $\delta\sqrt{\frac{1}{\underline{\sigma}(P)}}$.

VI. Numerical simulation

The communication network of 4 NMRs is depicted in Fig. 1. The linear velocity and the angular velocity of the leader NMR are set as $v_0 = 100|\omega_0|$ and $\omega_0 = -\frac{\sin t}{1+(\cos t)^2}$, respectively. The initial states of the leader NMR are $(x_0, y_0, \theta_0)^T = (10, 30, 900/\pi)^T$, and the initial states of the follower NMRs are $(x_1, y_1, \theta_1, x_2, y_2, \theta_2, x_3, y_3, \theta_3)^T = (15, -10, -360/\pi, 10, -20, 1440/\pi, 4, -6, -900/\pi)^T$. The parameters are chosen to be $\alpha_1 = 100$, $\beta_1 = 0.5$, $\alpha_2 = 30$, $\beta_2 = 1$ and $b_1 = 35$.

Let $\Lambda_i = (x_i, y_i, \theta_i)^T$, $f_i(\Lambda_i) = \frac{1-\exp^{-\Lambda_i}}{1+\exp^{-\Lambda_i}}$ and ξ_i be uniformly distributed noise. The local NMR NN tuning laws are given by (13), with $\varphi_i(\Lambda_i) = \exp\left(\frac{-\|\Lambda_i - \pi_i\|^2}{\eta_i^2}\right)$, where $\pi_i = (\pi_{i_1}, \pi_{i_2}, \cdots, \pi_{i_q})^T$ and η_i are the centers of the receptive fields and the widths of the Gaussian function, respectively. Set $c_1 = 40$, $c_2 = 40$, $\kappa = 0.01$, $F_1 = 1500$, $F_2 = 1500$, $F_3 = 1500$, $\pi_1 = (30, 30, 30)^T$, $\pi_2 = (25, 25, 25)^T$, $\pi_3 = (20, 20, 20)^T$, $\eta_1 = 8$, $\eta_2 = 8$ and $\eta_3 = 8$. The position and orientation observer tracking errors between the observers and the leader NMR are in Figs. 2-4 showing asymptotic reconstruction of the leader's state. Fig. 5 shows that cooperative tracking rendezvous is achieved successfully,

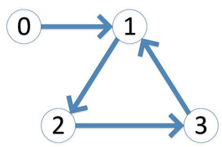

Fig.1: Communication topology with three follower NMRs and one leader NMR.

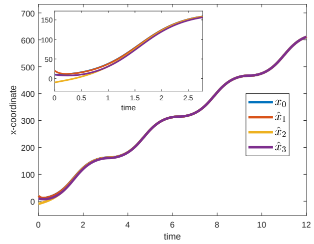

Fig.2: Evolution of the leader NMR and the three observers in the x coordinates.

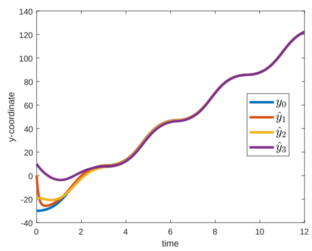

Fig.3: Evolution of the leader NMR and the three observers in the y coordinates.

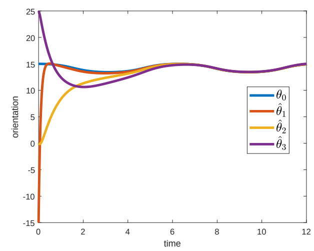

Fig.4: Evolution of the leader NMR and the three observers about the angle.

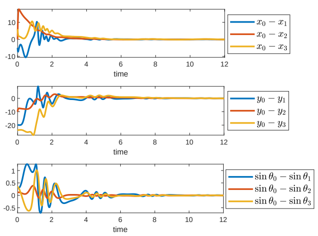

Fig.5: Evolution of the tracking errors.

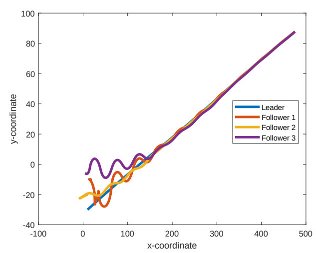

Fig.6: 2D trajectories of the leader robot and the three follower robots.

i.e. all followers meet and follow the leader. This is further shown in the $x-y$ plane of Fig. 6.

VII. Conclusions

The problem of tracking rendezvous for multiple NMRs with unmodelled uncertain nonlinearities and disturbances has been addressed and solved in this work. The proposed protocol relies on distributed estimation of the leader's state and neuro-adaptive local controllers. Future research could include the study of switching topologies and second-order nonholonomic dynamics.

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