Coalitional game theory for increasing the energy efficiency of cellular networks

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Coalitional game theory for increasing the energy efficiency of cellular networks

MASTER OF SCIENCE THESIS

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The telecommunication sector is characterised by a huge energy consumption that is still increasing. Mobile network operators seek for new operating techniques to minimise their energy costs and their carbon footprint. One of these energy saving techniques is the use of energy efficient collaborations among the base stations (antennas) in their cellular networks.

In this MSc thesis, a coalitional game-theoretic approach is taken to seek for the most energy-efficient collaboration among base stations in a cellular network. Three different coalitional game theory solution concepts: the Shapley value, the nucleolus, and the core are first investigated theoretically and then tested in a case study. Each of these concepts is based on different mathematical concepts such as fairness, dissatisfaction, or stability and may lead to different solutions. The concepts and corresponding solutions are compared with each other and their advantages and disadvantages are discussed. One major disadvantage that must be overcome during the case study is the computational complexity when a large number of players is involved.

The concepts are then tested on an urban heterogeneous cellular network with a dynamic traffic demand representing the urban area of Delft. Within this case study, the base stations are the players of the coalitional game and they can form collaborations with each other following the approach of the three mentioned solution concepts. To overcome the computational complexity, clustering among the base stations is applied to reduce the number of players in the game.

The case study showed that the application of clusters among base stations reduces the computational complexity enormously whereas the energy efficiency decreases slightly. Furthermore, the results of the application of the solution concepts are promising. Each concept achieves almost a 50% energy efficiency increase. The difference in results between the three concepts is small.
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S. Elderenbosch
Chapter 1

Introduction

In this chapter, the motivation and problem statement of this MSc thesis are discussed. Furthermore, this chapter gives an overview of the contributions of this MSc thesis. Finally, the structure of this MSc thesis is discussed.

1-1 Motivation

There is an increasing awareness of climate change throughout the whole world [33]. Companies in every sector become more aware of the increase of energy demand on the one hand and the energy transition on the other. Both have impact on the CO$_2$ emission. The increasing energy consumption increases the CO$_2$ emission, while the energy transition tries to decrease the CO$_2$ emission. One major energy consumer is the information and communication technology (ICT) sector. The European ICT sector is responsible for 10% of the total energy consumption in Europe. Within this sector, 8% is due to the energy consumption of telecommunication networks [39].

According to Ericsson [25], one of the leading providers of ICT to mobile network operators, global mobile data traffic has grown 18-fold in the past 5 years and it is expected to have an annual growth rate of 45% in the next 5 years [25]. In 2021, the world's monthly data traffic will be 49 exabytes (one exabyte is $10^{18}$ bytes). This explosive increase in mobile data traffic puts an enormous strain on mobile network operators and their networks. The largest part of the operational costs of mobile network operators is caused by their energy consumption. Almost 90% of these energy costs are realised by the radio access network in which the base stations (BSs) consume almost all the energy.

In 2016, the worldwide energy consumption of Vodafone was 4000 GWh [51]. If it can be assumed that Vodafone pays the average electricity price of € 0.20/kWh, the total energy bill of Vodafone is over € 800 million. If the company would have been able to save 10% of the
total energy use by implementing an energy saving algorithm for its cellular network, Vodafone could have increased its operating profit with € 80 million, which is an profit increase of almost 8% [51].

Fortunately for Vodafone, such algorithms are already being developed. In literature there are several energy saving techniques that can save over 40% of the total energy use of the cellular networks. Chapter 2 will elaborate in more depth on these energy saving algorithms.

This thesis will consider a new approach for these energy saving techniques by using game theory. Game theory is a branch in mathematics concerned with the analysis of strategies of intelligent players dealing with competitive situations where the outcome of a player’s choice of action depends critically on the actions of other players. Game theory is widely used in economics, political science, biology, physics, engineering, and many other sciences.

The goal of this thesis is to test whether coalitional game theory can be used to increase the energy efficiency of cellular networks. The energy efficiency of the cellular network is measured in the amount of data transmitted divided by the amount of energy consumed by the network [Bit/J].

1-2 Contributions

The contribution of this MSc thesis is twofold:

- **Application of three coalitional game theory solution concepts to increase the energy efficiency of cellular networks.**
  This MSc thesis models an urban cellular network as a coalition form game. In this cellular network the BSs are modelled as the players of the game. The goal of the game is to find the most energy-efficient collaboration among the players. To determine this energy-efficient collaboration three coalitional game theory solution concepts: the Shapley value, the nucleolus, and the core are used. This thesis is the first to model a cellular network as a coalition form game and to test whether these three solution concepts can contribute in increasing the energy efficiency of cellular networks.

- **Application of clusters among players to decrease computation complexity of the solution concepts**
  Especially the Shapley value and the nucleolus have as main drawback that their computational complexity increases exponentially with the number of players. To give a better insight in the computational complexity of the Shapley value and the nucleolus, for a game of 5 players 15 separate calculations have to be performed, for a game of 10 players 21147 separate calculations have to be performed, and for a game of 20 players $5.8 \cdot 10^{12}$ separate calculations have to be performed. The case study of this MSc thesis consists of 117 players. It can easily be concluded that the computation power needed for a game of 117 players would be far too much for ordinary computers. To decrease the computation power this MSc thesis applies clusters among the players. The application of clusters involves that a group of players are modelled as a single player in the game. If a cluster follows a certain strategy in the game, all the players within that cluster follow this strategy. A drawback of the application of clusters is that the number of
possible solution outcomes reduces significantly. This may involve that energy efficiency increase is also reduced due to the application of clustering. This MSc thesis showed that in this case study the computation time was drastically reduced by the application of clusters, while the energy-efficiency decrease was relatively small compared to the situation without the application of clusters.

1-3 Structure of the thesis

The rest of the thesis is structured as follows. In Chapter 2 some basic knowledge about cellular networks is provided that will be used throughout the remainder of the thesis. Chapter 3 studies coalitional game theory and more specifically three main solution concepts: the Shapley value, the nucleolus, and the core. Chapter 4 explains the cellular network model that is built for the simulations of this thesis. Chapter 5 elaborates on how the coalitional game theory solution concepts can be used to make the cellular network more energy efficient. Chapter 6 discusses the results of the simulations based on the application of the three solution concepts. In Chapter 7, conclusions and recommendations for future work are given.
In this chapter, some preliminary background is provided that will help the reader understand the rest of the thesis. The first section provides some basic information about cellular networks, what they are and how they work. Section 2-2 introduces some energy saving techniques for cellular networks. Section 2-3 will elaborate in more depth on the energy saving technique that will be used in this MSc thesis. Finally, the chapter is completed with some conclusions.

2-1 Cellular networks

As mentioned earlier in the introduction, mobile data traffic is expected to grow with an astonishing rate of 45% in the next five years [25]. This increase in data traffic puts a lot of extra pressure on current cellular networks and their energy use. This section elaborates on what a cellular network is, how it works, and the methods used to increase the energy efficiency of these networks.

A cellular network is a network of transceivers, serving an area with radio signal coverage. These transceivers, called base stations (BSs), can receive and transmit radio signals. Each area is divided into multiple hexagonal adjacent cells and each cell is served with at least one base station. These BSs serve a wide variety of mobile devices, such as mobile phones, tablets, and laptops, and provide them with a connection for data exchange. This exchange of data occurs by way of radio signals. Each BS uses a specific frequency range to serve the users within its cell. Adjacent BSs use a different range of frequencies to prevent radio interference [21].

Typically these cells differ in size in order to optimise the Quality of Service (QoS) and to reduce the total energy use of the mobile network operator. Large rural areas with a limited number of users can be covered by large cells. The base stations in these cells are called macro base stations and can cover areas with a radius of 50-70 km, while small urban areas are served by small BSs such as pico base stations, with a radius of around 200m. Cellular
Cellular networks

networks where all the BSs are of the same type and cover the same area size are called homogeneous networks, while cellular networks where different types of BSs cover different cell areas are called heterogeneous networks. The larger the cell, the more energy is needed to provide the whole area with a sufficient QoS. Table 2-1 gives an overview of the energy use of different types of BS. This table shows three different types of BSs in three different modes. The first mode when the BS uses its full capacity, i.e. it uses its full transmit power; the second mode occurs when the BS is on but does not transmit data to any user; the final mode is the sleep mode and will it be explained in more depth in Chapter 4.

<table>
<thead>
<tr>
<th>Type of BS</th>
<th>Full load</th>
<th>No load</th>
<th>Sleep</th>
</tr>
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<tr>
<td>Macro (2-70km)</td>
<td>740</td>
<td>140</td>
<td>5</td>
</tr>
<tr>
<td>Small (&lt;2km)</td>
<td>35</td>
<td>32</td>
<td>2</td>
</tr>
<tr>
<td>Pico (&lt;200m)</td>
<td>7</td>
<td>6</td>
<td>0</td>
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Table 2-1: Energy consumption [W] per type of BS in different modes [12] and [26]

Since most of the users of cellular networks are mobile users, it can occur that a user moves from one cell to another adjacent cell. If this is the case, a so called handover occurs. A handover is the process where an ongoing call or data transmission is transferred from one BS to another BS. This process is developed in such a way that the user is not aware of the handover [43].

The cellular networks are operated by so called mobile network operators. In the Netherlands KPN, Vodafone, and T-Mobile are the three major mobile network operators. Most major mobile network operators have their own cellular network to serve their clients. In the majority of cases, urban areas are covered by three different cellular networks, while in some cases only one is needed.

2-2 Energy saving strategies for cellular networks

This section gives a broad introduction to the four main energy saving strategies in cellular networks. There are numerous surveys written about these energy saving techniques [11, 22, 47]. The four main energy saving strategies are:

- **Base station optimisation.** Within a BS there are lots of hardware components that consume energy. Most of the energy consumption within a BS is consumed by the power amplifier, around 65%. The other major energy consumers are the air conditioning (17.5%), the digital signal processors (10%), and the AC/DC converters (7.5%) [9]. Since almost 80% of the total energy consumption of a mobile network operator is consumed by the BSs, it can be a very interesting investment to increase the energy efficiency of the hardware within a BS.

- **Energy-optimal base station deployment.** Due to the enormous increase in mobile data traffic, mobile network operators need to upgrade their cellular networks, since the old macro BSs do not have enough capacity for the new increased data demand to cover their cells. To solve this, mobile network operators deploy new small BSs, as micro
base stations and pico base stations, in existing cells to cover the cell with the capacity needed. However, this increased number of BSs goes hand in hand with an increase in power consumption. Therefore, extensive research has been undertaken in the strategic deployment of small base stations in order to minimise the power consumption increase and to increase the energy efficiency [10, 17].

- **Base station sleeping.** This energy saving strategy will be used in this thesis, since it can be applied with the use of game theory. The principle of BS sleeping works as follows: in a simplified heterogeneous cellular network, there are two types of BSs, small BSs and macro BSs. The macro BSs serve a large area and the small BSs are deployed in crowded areas within the service area of the macro BS to lighten the burden of data demand on the macro BS. However, each type of BS has a different distinctive power consumption profile. The macro BS power consumption increases with the increase of data demand, while the power consumption of the small BS remains almost the same when the data demand increases. Therefore, it can be more energy efficient to set small BSs in sleeping mode, when they are underutilised, and to let their users be served by the macro BS. Section 2-3 will give a small summary of the research done in recent years on this topic.

- **Network sharing between mobile network operators.** Multiple mobile network operators can be operational in the same operating area. Thus within the same area there can be multiple cellular networks, one for each mobile network operator. These networks are configured in such a way that they can deliver perfect QoS during peak traffic hours. However, during low traffic hours the networks can be underutilised. In these situations mobile network operators could collaborate by setting several BSs of one mobile network operator in sleeping mode and by roaming the users to the BSs of the other mobile network operator. Operating costs are then paid to the particular mobile network operator. In this way they can reduce the total energy consumption and all profit from the reduced energy costs. Figure 2-1 illustrates this process of one mobile network operator roaming its users to another mobile network operator. A number of studies on this topic have been carried out, several of them include game theory in their solutions [2, 34].

## 2-3 Related work on base station sleeping strategies

This section will give a broad overview on what research has already been undertaken on the topic of BS sleeping strategies. In general, there are three different strategies for BSs sleeping: **random sleep strategies, distance-aware sleep strategies, and load-aware sleep strategies.**

The random sleep approach is probably the most simple approach among existing sleep strategies. In general, it works as follows: each BS fully operates with a probability \( q_{\text{on}} \) and sleeps with a probability \( q_{\text{sleep}} \), where \( q_{\text{on}} + q_{\text{sleep}} = 1 \). The authors of [46] have optimised these probabilities for a set of small BSs for a periodic traffic demand. The authors achieve an increase in energy efficiency of 25% during low traffic hours.

In [36] two extra modes are introduced, the *off mode* and the *stand-by mode*. A BS in the...
off mode is assumed to use no energy at all, but has a wake-up time of 30 seconds. A BS in stand-by mode is assumed to use 50% of the total power consumption and has a wake-up time of 0.5 seconds. The sleep mode has a power consumption of 15% of the total power consumption and a wake-up time of 10 seconds. So in [36] \( q_{on} + q_{stand-by} + q_{sleep} + q_{off} = 1 \). The authors achieve a 30% increase in energy efficiency when they optimise these probabilities.

The energy consumption of a macro BS depends on the data demand of a user and its distance to the macro BS. Therefore, the authors of [5] modelled a cellular network with a macro BS in the middle of the cell, surrounded by small BSs. To increase the energy efficiency, they first set the small BS closest to the macro BS in sleep mode followed by small BSs located further away from the macro BS. With this approach they achieve an increase in energy efficiency of 38%.

In [4] the sleep strategy is based on the average distance of the users to their BS. This distance is compared with the average distance of the users of the adjacent cell to their BS. When these distances are small, the BSs are on. However, when both of these averages are relatively large, the authors of [4] optimise the energy efficiency by setting one BS in sleep mode and letting the other BS zoom out to cover the users. This approach achieves a 29% energy efficiency increase in the simulations, compared with the situation when no sleep strategy is applied.

Both the distance of a user to the macro BS and the data demand of a user affect the energy consumption of the macro BS. The authors of [1] created a strategy where a BS goes into sleep mode when the traffic load is below a certain threshold for a certain period. The threshold for switching on the BS is a bit higher than the threshold for going into sleep mode, to avoid frequent on and off switching. The simulated results of this strategy increase the energy efficiency with 33% during medium traffic periods and even up to 67% during low traffic periods, compared when no sleeping strategy is applied.

The authors of [19] propose a more adaptive approach based on dynamic traffic monitoring. They devised two strategies, one in which complete information about the users is constantly available and a second strategy with incomplete information. Both methods increase the
energy efficiency compared to the situation when no sleep strategy is applied; The complete situation with 48% and the incomplete method with 36%.

2-4 Conclusions

This chapter provided some basic information about cellular networks that will help the reader to better understand the rest of this MSc thesis. Cellular networks are networks of BSs transmitting data to their mobile users. A cellular network consists of multiple types of BSs. Each type of BS has its own active range and power consumption profile. The cellular networks are operated by MNOs.

There are four main energy saving techniques MNOs can use to decrease their energy consumption: First they can update the hardware of their cellular networks. Second, MNOs can strategically deploy new types of BSs in their network, to meet the increasing data demand and increase their energy efficiency. Third, due to the different power consumption profiles of the different types of BSs, some BSs can be underutilised. In these cases it can be more energy efficient to put these underutilised BSs into sleeping mode and serve their users by other BSs. This technique is used in the rest of this thesis. Finally, an MNO could collaborate with the network of another MNO, by sharing their networks during low traffic hours. In this way they can both profit from the reduced energy consumption.

Finally, this chapter gave a broad overview on what research already has been undertaken on BS sleeping strategies. There are several different sleeping strategies, all using another approach for switching BSs on and off. The results of these different sleeping techniques also differ. They increase the energy efficiency of cellular networks between 25% up to 67% during low traffic hours.
Chapter 3

Coalitional game theory

In order to increase the energy efficiency of the simulation model in the case study, the base stations (BSs) need to collaborate with each other. The goal of this thesis is to test whether coalitional game theory can be applied in order to create energy saving collaborations among BSs in a cellular network.

This chapter is an in-depth study about coalitional game theory and three main solution concepts within coalitional game theory. This chapter forms the theoretical backbone of this thesis, investigating the energy saving approaches that will be tested on the simulation model.

The chapter is structured in the following way: the first section introduces coalitional game theory and shows what coalition form games look like. Next the three main solution concepts within coalitional game theory are studied in more depth: the Shapley value, the nucleolus and the core. Followed by, the comparison of the three different solutions concepts and the chapter will be completed with some conclusions.

3-1 Coalition form games

Cooperative games, unlike the non-cooperative games, do not model individual players acting on their own and taking actions by them selves. Instead, they consider groups of players acting together in order to get higher payoffs.

Within cooperative game theory, there are two main branches: coalition form games and bargaining games. The latter describes the process of bargaining among a set of players that aim to agree on the terms of cooperation. The former investigates the formation of cooperating groups, called coalitions that can increase the personal utilities of the players [23]. For this thesis only coalition form games are investigated.

A coalition form game consists of a group of players denoted by $N$. These players seek to form coalitions of individual players in order to increase their personal utility $u_i$. Any coalition $S \subseteq N$ acts as a single entity in the game. Without loss of generality it is assumed that
Coalitional game theory

$N = \{1, 2, \ldots, n\}$. Each $S \subseteq N$ has a specific value, denoted by $v(S)$, which can be divided among the individual players within that specific coalition [23].

Thus, a coalition form game can be written as $\Gamma_{\text{coal}}(N, v)$ where:

- $N$ is the finite set of players and
- $v$ associates with every coalition $S \subseteq N$ a real-valued payoff $v(S)$ that the coalition’s members can divide among themselves. It is assumed that $v(\emptyset) = 0$. The payoff function is simply a way to represent a preference relationship among players to form a coalition [27]. It can assign positive and negative values. Positive when a coalition is beneficial to the players and negative when the coalition is worse off collaborating with each other.

Coalitional game theory makes use of the superadditivity assumption

$$v(S \cup T) \geq v(S) + v(T), \quad \forall S, T \subset N, S \cap T = \emptyset$$

(3-1)

This implies that given any two disjoint coalitions, $S$ and $T$, the coalition between them has at least as high a payoff as if they acted separately and their payoffs were added together [8].

Coalitional game theory is used in order to answer two main questions:

- Which coalition will form?
- How should that coalition divide its payoff among its members?

In relation to the first question, the superadditivity assumption implies that the grand coalition will form, since it is the coalition with the highest payoff. However, this is not always the case. For example, there can be specific restrictions in a game or other specific exceptions such that superadditivity does not apply to the game or that the grand coalition cannot form.

To answer the two questions above, coalitional game theory uses two main definitions: individual rationality and group rationality:

- A payoff vector $x \in \mathbb{R}^{|N|}$ is individual rational iff $x_i \geq v(\{i\})$, $\forall i \in N$. This implies that a payoff vector is individual rational if every player can obtain an individual payoff at least as much as it could have acted on its own [23, 32].
- A payoff vector $x \in \mathbb{R}^{|N|}$ is group rational iff $\sum_{i \in S} x_i \geq v(S)$ $\forall S \in N$. This implies that the sum of all the individual payoffs to every player $i \in S$ should equal at least the value of the coalition $S$.

When a payoff vector $x \in \mathbb{R}^{|N|}$ is both individual and group rational, the payoff vector is called an imputation. Throughout the coming sections imputations play an important part.

An example of a coalition form game is the musicians’ game.
Musicians’ game [7]: Three amateur musicians are thinking of working in evening venues. One of them is a guitarist (G), another a drummer (D), and the remaining one a singer (S). The amounts of money different combinations of the musicians can earn are as follows:

\[
\begin{align*}
    v(\{G\}) &= 50, \quad v(\{D\}) = 0, \quad v(\{S\}) = 20 \\
    v(\{D,S\}) &= 25, \quad v(\{G,S\}) = 80, \quad v(\{G,D\}) = 70 \\
    v(\{G,D,S\}) &= 100
\end{align*}
\]

To answer the two main questions within coalitional game theory, which coalition will form and how they will divide their payoff, there are several solution concepts. These concepts use different mathematical properties to find a solution to the coalition form games. Therefore, the solutions provided by these concepts will not always coincide. The three main solution concepts the Shapely value, the core, and the nucleolus [7] are discussed in Sections 3-2, 3-3 and 3-4 respectively.

### 3-2 Shapley value

In 1953 Nobel price winner Lloyd Shapley came up with the so called Shapley value. Lloyd Shapley started with the idea that a solution to a coalition form game always had to be fair. Therefore, he combined four different game-theoretical properties. If a solution to a coalition form game satisfied all these properties, it is assumed to be a fair solution. The four properties he combined were [13, 38]:

- **Efficiency**: \( \sum_{i \in N} \psi_i(N, v) = v(N) \), where \( \psi_i(N, v) \) is the payoff of player \( i \). This property shows similarities with group rationality as was mentioned in Section 3-1, but it is not the same, since the efficiency property only covers the grand coalition.

- **Symmetry**, which states that interchangeable players, who always contribute the same amount to every coalition of other players, should receive the same payoff. So if \( \forall S \setminus \{i, j\}, v(S \cup \{i\}) = v(S \cup \{j\}) \) then \( \psi_i(N, v) = \psi_j(N, v) \).

- **The dummy player property**, which states that a player \( i \) who never contributes to any coalition should receive nothing. So \( \forall S \setminus \{i\} : v(S \cup \{i\}) = v(S) \) then \( \psi_i(N, v) = 0 \).

- **Linearity**, where the payoff of the sum of two games is equal to the sum of the payoff of both games. If a game is defined as \( (N, v_1 + v_2) \) where \( (v_1 + v_2)(S) = v_1(S) + v_2(S) \) \( \forall S \) then \( \psi_i(N, v_1 + v_2) = \psi_i(N, v_1) + \psi_i(N, v_2) \) [23, 45].

Shapley came up with a solution that always would satisfy these properties. His solution was that every player in the final coalition would get what his average marginal contribution is, to that all the possible coalitions he can join. This value that indicates the average marginal contribution of a player \( i \), was later named the Shapley value and can be found by:
In Formula 3-2, \( v(S \cup \{i\}) - v(S) \) is the marginal contribution of player \( i \) to coalition \( S \). The Shapley value makes use of ordered sets. The term \(|S|!\) is the number of different possibilities, coalition \( S \) could have been ordered in a unique way, while \((N - |S| - 1)!\) is the number of ways all the players outside coalition \( S \) and player \( i \) can be ordered and complete the whole set \( \{N\} \). This is summed up for all the possible coalitions \( S \subseteq N \setminus \{i\} \). Then to finally take the average of all these possible marginal contributions of player \( i \) this is divided by \(|N|!\), which is the number of every possible line-up of all the players. So intuitively this can be interpreted as follows. Suppose all players are lined up in some order, with any order equally likely, the Shapley value is the expected marginal contribution over all orders of player \( i \) to the set of players who precede him in that specific order. It must be said that in coalitional game theory the order of a coalition does not matter for the value of the coalition. However, these orderings in the approach for the Shapley value are used, to get the right average over each coalition \( S \in N \).

The Shapley value makes use of the superadditivity property and thus it assures that the grand coalition will form. The way that the worth of the grand coalition should be divided is for every player \( i \) to receive its Shapley value \( \sigma_i \).

To calculate the Shapley value of the musicians’ game, the players need to be aligned in every possible order. In the case of the Musicians game this is \( 3! = 6 \) different orders. In the table below the Shapley value for the Musicians game is derived.

<table>
<thead>
<tr>
<th>Ordered sets</th>
<th>( v(S \cup {G}) - v({S}) )</th>
<th>( v(S \cup {S}) - v({S}) )</th>
<th>( v(S \cup {D}) - v({S}) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>G,S,D</td>
<td>( v({G}) - v(0) = 50 )</td>
<td>( v({G,S}) - v({G}) = 30 )</td>
<td>( v({G,S,D}) - v({G,S}) = 20 )</td>
</tr>
<tr>
<td>G,D,S</td>
<td>( v({G}) - v(0) = 50 )</td>
<td>( v({G,D}) - v({G,D}) = 30 )</td>
<td>( v({G,D,S}) - v({G,D}) = 20 )</td>
</tr>
<tr>
<td>S,G,D</td>
<td>( v({S,G}) - v({S}) = 60 )</td>
<td>( v({S}) - v(0) = 20 )</td>
<td>( v({S,G,D}) - v({S,G}) = 20 )</td>
</tr>
<tr>
<td>S,D,G</td>
<td>( v({S,D}) - v({S,D}) = 50 )</td>
<td>( v({S}) - v(0) = 20 )</td>
<td>( v({S,D}) - v({S}) = 5 )</td>
</tr>
<tr>
<td>D,G,S</td>
<td>( v({D,G}) - v({D}) = 70 )</td>
<td>( v({D,G,S}) - v({D,G}) = 30 )</td>
<td>( v({D}) - v(0) = 0 )</td>
</tr>
<tr>
<td>D,S,G</td>
<td>( v({D,S}) - v({D,S}) = 75 )</td>
<td>( v({D,S}) - v({D}) = 25 )</td>
<td>( v({D}) - v(0) = 0 )</td>
</tr>
</tbody>
</table>

| Shapley value | \( \sigma_G = 63 \frac{2}{6} \) | \( \sigma_S = 25 \frac{5}{6} \) | \( \sigma_D = 10 \frac{5}{6} \) |

**Table 3-1:** Calculation of the Shapley value of the musician’s game with, guitarist (G), singer (S) and drummer (D) in the Musicians game

According to Lloyd Shapley the grand coalition will be the final coalition of this game and the players will divide the grand coalitions’ payoff according to the Shapley values, as can be found in the last row in Table 3-1.

Since the Shapley value of player \( i \) is an unique average contribution over all the possible coalitions \( S \subseteq N \setminus \{i\} \), it is a unique solution to a coalition form game. Furthermore, since the Shapley value is a value that is calculated only with the number of players \(|N|\) and the vector \( v \) with the worth of every coalition \( S \) which both are real positive numbers, the Shapley value always exists.
So when applying the Shapley value to a coalition form game, an unique fair outcome to the game is assured. This characteristic makes the Shapley value one of the most used solution theorems within coalitional game theory.

The Shapley value is often used in politics as a power index (also known as the Shapley-Shubik index [16]), which assigns the power a vote has in a parliament. However, there is one main drawback to the Shapley value. The computational complexity of calculating the Shapley value increases significantly with an increasing number of players. For a game of only 5 players 32 different calculations have to be performed. Chapter 5 elaborates in more depth how this computational complexity can be simplified.

Furthermore, although the Shapley value is a fair solution, this does not imply that the solution has to be stable. As a result there are games, in which it can be more profitable for a certain group of players $S \subseteq N$ to deviate from the division of payoffs according to the Shapley value from their own coalition to get higher personal payoffs. Section 3-5 will discuss this unfavourable characteristic in more depth.

3-3 Nucleolus

The second solution concept that will be tested was introduced in 1969 by David Schmeidler [48], an Israeli mathematician and economic theorist. The solution he came up with was based on an approach to find an allocation of payoffs that minimises the dissatisfaction of all the players in a coalition, which he named the nucleolus. The nucleolus aims to find an imputation $x \in \mathbb{R}^{|N|}$ that minimises the worst dissatisfaction among the players within the coalition.

The measure for the dissatisfaction for a certain coalition $S$ with an allocation $x \in \mathbb{R}^{|N|}$ in a game $(N,v)$ is called the excess and is defined as [18]:

$$e(x, S) = v(S) - \sum_{i \in S} x_i$$

(3-3)

The excess measures the size of the inequity that coalition $S$ falls short of its potential payoff $v(S)$ when the imputation $x$ is the final payoff to the coalition.

To define the nucleolus, first a new method of ordering vectors has to be defined, namely the lexicographic order. If there are two vectors $x = [x_1, x_2, ... x_k]^T$ and $y = [y_1, y_2, ... y_k]^T$ both arranged in decreasing order, then vector $x$ is said to be lexicographically less than $y$, if for some index $k$ it holds that $x_i = y_i, \forall i < k$ and $x_k < y_k$. When $x$ is lexicographically less than $y$, it can be written as, $x \prec_{\text{lex}} y$.

To finally compute the nucleolus of the game, all the excesses of an imputation $x$ of the game $(N,v)$ are arranged in a vector $O(x)$ in a decreasing order (without the grand coalition excess since this should be 0 according to the definition of an imputation). So the nucleolus is the payoff $x$ that minimises the excesses in a non-increasing order starting with the maximum excess.

This gives us the definition of the nucleolus as: vector $w \in X$ is the nucleolus of game $(N,v)$ if $\forall x \in X$, $O(w) \prec_{\text{lex}} O(x)$, where $X = \{x : \sum_{i=1}^{|N|} x_i = v(N)\}$ is the set of all imputations of game $(N,v)$ [23, 44].
To make the process of finding the nucleolus more clear, the nucleolus of the musicians’ game of Section 3-1 is derived as follows:

To find the nucleolus of this game, let \( x = [x_G, x_S, x_D]^T \) be an imputation. First an arbitrary imputation of the game has to be considered. Let us start with \( x = (55, 30, 15) \).

As seen in Table 3-2, the vector of excesses of the division of the Shapley value is \( e = [-5, -10, -15, -5, 0, -20]^T \). The largest of these numbers is 0 corresponding to coalition \( \{G,D\} \). The perspective of this coalition is that every other coalition has better payoffs than the payoff of their coalition. So either the payoff of G or D or both have to be increased, to decrease the maximum excess. This involves that the payoff of S has to be decreased. First the payoff of G is increased and the new allocation becomes \( x = (60, 25, 15)^T \). The new vector of excesses becomes \( e = [-10, -5, -15, -5, -15, -5]^T \), the largest excess of coalition \( \{G,D\} \) has decreased to -5. Now there are three coalitions with an maximum excess of -5, \( \{S\} \), \( \{G,S\} \), and \( \{G,D\} \). The only option to decrease one of these excesses without increasing one other is to increase the payoff of G and decrease the payoff of D. The new allocation becomes \( (65, 25, 10)^T \), which makes the vector of excesses \( e = [-15, -5, -10, -10, -5, -10]^T \). The maximum excess corresponds to coalitions \( \{S\} \) and \( \{G,D\} \). It is clear that no adjustment to \( x \) can be made to decrease the excess of those coalitions without increasing the other. So the nucleolus of the musicians’ game is \( (65, 25, 10)^T \). Compared to the solution of the Shapley value, the nucleolus of the musicians’ game is a lot more favourable to the guitarist (G) than to the singer (S) and the drummer (D).

<table>
<thead>
<tr>
<th>Dissatisfaction</th>
<th>Coalsions S</th>
<th>( v(S) )</th>
<th>( e(x,S) )</th>
<th>( (55,30,15) )</th>
<th>( (60,25,15) )</th>
<th>( (65,25,10) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( {G} )</td>
<td>50</td>
<td>50 - ( x_G )</td>
<td>-5</td>
<td>-10</td>
<td>-15</td>
<td></td>
</tr>
<tr>
<td>( {S} )</td>
<td>20</td>
<td>20 - ( x_S )</td>
<td>-10</td>
<td>-5</td>
<td>-5</td>
<td></td>
</tr>
<tr>
<td>( {D} )</td>
<td>0</td>
<td>-( x_D )</td>
<td>-15</td>
<td>-15</td>
<td>-10</td>
<td></td>
</tr>
<tr>
<td>( {G,S} )</td>
<td>80</td>
<td>80 - ( x_G ) - ( x_S )</td>
<td>-5</td>
<td>-5</td>
<td>-10</td>
<td></td>
</tr>
<tr>
<td>( {G,D} )</td>
<td>70</td>
<td>70 - ( x_G ) - ( x_D )</td>
<td>0</td>
<td>-5</td>
<td>-5</td>
<td></td>
</tr>
<tr>
<td>( {S,D} )</td>
<td>25</td>
<td>25 - ( x_S ) - ( x_D )</td>
<td>-20</td>
<td>-15</td>
<td>-10</td>
<td></td>
</tr>
</tbody>
</table>

Table 3-2: Calculation of the nucleolus of the musicians’ game with, guitarist (G), singer (S) and drummer (D)

The nucleolus has similar characteristics compared to the Shapley value, including the property that it always exists and that it is unique. The nucleolus satisfies the first three properties of the Shapley value, but it does only not always satisfy the linearity property. The uniqueness and the existence of the nucleolus make it an interesting solution to coalitional games. Whereas the Shapley value can be derived by filling in a formula, the nucleolus is found by minimising the maximum of a set of linear functions. This minimisation problem can be solved by a simplex method [14]. Just as with the Shapley value, the computational complexity of computing the nucleolus increases significantly with the number of players within a game. Therefore, it can be less suitable for games with large numbers of players.
The final solution concept that will be investigated and tested in this thesis is the core. While the Shapley value was based on fairness, and the nucleolus on minimising the dissatisfaction, the core is based on stability. This stability property involves an allocation of payoffs is in the core if there is no sub-coalition that wants to deviate from this allocation of payoffs in order to get a higher payoff.

The core only consists of imputations, discussed in Section 3-1, satisfying individual rationality:

\[ x_i \geq v(\{i\}) \]

and group rationality:

\[ \sum_{i \in S} x_i \geq v(S), \quad \forall S \in N \]

The stability property combined with the individual and group rationality lead to the following definition of the core [41]:

“The core of a coalitional game \( \Gamma_{coal}(N, v) \) is the set of feasible payoff vectors \( x \in \mathbb{R}^{\vert N \vert} \) for which there is no coalition \( S \) with feasible payoff vector \( y \in \mathbb{R}^{\vert N \vert} \) for which \( y_i > x_i \forall i \in S \).”

In a more mathematical notation [23]: A payoff vector \( x \in \mathbb{R}^{\vert N \vert} \) of game \( \Gamma_{coal}(N, v) \) is in the core iff

\[ \forall S \subseteq N, \sum_{i \in S} x_i \geq v(S). \tag{3-4} \]

Intuitively the core guarantees that the sum of the payoffs to all of the players is at least as much as they could earn if they would split up in any other sub-coalition. The core provides a stable outcome to all the players in the core.

Now the core of the musicians’ game can be derived, but first the game is normalised. This normalisation will make it easier to test nonemptiness of the core, later on. Furthermore, the normalisation gives a better insight in what part of the grand coalitions’ surplus belongs to what player. To normalise a coalitional game, the utilities of the single players are set to zero and from all the possible coalitions, the utility of every single player is subtracted. The remaining value of the grand coalition is set to 1. The remaining utilities of the sub-coalitions are set as the fraction between the remaining value of the grand coalition and the remaining value of the sub-coalitions.

In the case of the Musicians’ game, it becomes:

\[ v(\{G\}) = 0, \; v(\{D\}) = 0, \; v(\{S\}) = 0 \]

\[ v(\{D,S\}) = \frac{1}{6}, \; v(\{G,S\}) = \frac{1}{3}, \; v(\{G,D\}) = \frac{2}{3} \]

\[ v(\{G,D,S\}) = 1 \]
The representation shown above is called the *Zero-One Normalisation* of the original coalitional game, since the values of the coalitions of individual players is set to zero and the surplus of the grand coalition is set to one. The possible outcome of this representation has to satisfy the following set of inequality constraints:

\[
\begin{align*}
  u_G &\geq 0, \quad u_D \geq 0, \quad u_S \geq 0 \\
  u_D + u_S &\geq \frac{1}{6}, \quad u_S + u_G \geq \frac{1}{3}, \quad u_G + u_D \geq \frac{2}{3} \\
  u_G + u_D + u_S &= 1.
\end{align*}
\]

In this equation, \( u \) is the utility function, which indicates the extra payoff a coalition should at least have added to the initial values of the coalitions of single players. The core of the musicians’ game is the set of imputations that satisfy these linear inequalities, and is shown in the grey shaded plane in Figure 3-1.

![Figure 3-1: The core of the zero-one normalisation of the Musicians’ game [50]](image-url)

When this result is converted back to the original form of the coalitional game, the core is the set of payoff vectors such that:

\[
\begin{align*}
  50 \leq x_G \leq 75, \quad 20 \leq x_S \leq 30, \quad 0 \leq x_D \leq 20. \\
  x_G + x_S + x_D &= 100.
\end{align*}
\]

Two aspects about the core have not yet been mentioned:

- The core can be *empty*, i.e. there may be no stable allocation of payoffs possible in the coalition form game.
- When the core is nonempty, it can be *non-unique*, i.e. there are multiple allocations of payoffs within the core, and the concept of the core does not clarify what allocation of payoffs is better.
Nonemptiness of the core means that the core suggests at least one possible allocation of payoffs. One may say that a game with a nonempty core has less potential for a social conflict between the players than one with an empty core, since the demand of every group can be granted with at least one payoff vector. If the core is empty, the players may have to impose some restrictions on the coalitions, or choose another solution concept.

Since the number of linear constraints grows exponentially with the number of players, it can be wise to first check whether the core is nonempty before trying to find the core. One way to check whether the core is nonempty, with only checking a finite number of conditions, is by applying the Bondareva’s theorem. To do so first a balanced collection of weights has to be assigned to the game. Let $N$ be the set of players and $P$ be the set of coalitions of $N$ excluding the grand coalition. A list of $(\lambda_S)_{S \in P}$ of non-negative numbers, one for each coalition $S$, is called a balanced system of weights, if for each player $i$ the sum of $\lambda_S$’s over all coalitions $S$ to which player $i$ belongs is 1 [7].

To make the concept of a balanced system of weights a bit more concrete Table 3-3, shows some possible balanced systems of weights for the musicians’ game with players $\{G, S, D\}$, where $r$ is any number between 0 and 1.

<table>
<thead>
<tr>
<th>Coalitions</th>
<th>Balanced system of weights</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1</td>
</tr>
<tr>
<td>G</td>
<td>1</td>
</tr>
<tr>
<td>S</td>
<td>1</td>
</tr>
<tr>
<td>D</td>
<td>1</td>
</tr>
<tr>
<td>GS</td>
<td>0</td>
</tr>
<tr>
<td>GD</td>
<td>0</td>
</tr>
<tr>
<td>SD</td>
<td>0</td>
</tr>
</tbody>
</table>

Table 3-3: Some examples of balanced systems of weights in the musicians’ game, guitarist (G), singer (S) and drummer (D), where $r$ is any number between 0 and 1

A subset of $P$ is called a balanced family if there exists a balanced system of weights such that the weights are positive for coalitions in the family and zero for coalitions outside the family. In Table 3-3 the balanced families in columns 1,..., 5 are $\{G, S, D\}$, $\{G, SD\}$, $\{GS, GD, SD\}$, $\{G, S, D, SD\}$, $\{G, S, D, GS, GD, SD\}$.

A balanced family is called a minimal balanced family if there is no smaller subset of the family that is also balanced. For the musicians game the minimal balanced families are: $\{G, S, D\}$, $\{G, SD\}$, $\{S, GD\}$, $\{D, GS\}$, $\{GS, GD, SD\}$. For a minimal balanced family, there is a unique balanced system of weights. Such a unique system of weights for a minimal balanced family is called a minimal balanced system of weights. This finally leads to the Bondareva’s theorem [40]:

The core is nonempty if and only if for every minimal balanced system of weights $\{\lambda_S\}_{S \in P}$, one has

$$\sum_{S \in P} \lambda_S v(S) \leq 1. \tag{3-5}$$
This is for zero-one normalised coalition form games. Since the number of minimal balanced families is finite, only a finite number of minimal balanced systems have to be checked to check the nonemptiness condition of the core. For games with large numbers of players, the number of minimal balanced families is less than the number of linear inequalities the core has to satisfy [7]. Therefore the computational complexity of the Bondareva’s theorem is smaller than checking if all the linear inequalities allow a solution. In Appendix A Bondareva’s theorem is applied to the Musicians’ game to get a better insight in how the theorem is applied.

Besides the Bondareva’s theorem, there is another theorem that for some specific coalition form games can indicate that the core is nonempty, namely when a coalition form game is convex it has a nonempty core [38].

A coalition form game is convex iff

\[ \forall S, T \in N, \ v(S \cup T) = v(S) + v(T) - v(S \cap T). \]  

(Conv-3-6)

Convexity is a stronger condition than superadditivity, since superadditivity assumes that \( S \) and \( T \) have an empty intersection, while convexity allows an intersection between both coalitions. This theorem is as might be expected considerably faster compared to the Bondareva’s theorem. However, not all coalition form games are convex.

When the core is found to be nonempty, this does not mean that the best solution to the coalition form game is found. After all, the core can be non-unique and allow multiple different allocation of payoffs to the coalition form game. When this is the case other solutions have to be applied, possibly the solutions as discussed in Sections 3-2, and 3-3, for finding the most optimal outcome of the coalition form game.

In general, the core is found by using linear programming:

\[
\begin{align*}
\min_x & \sum_{i \in N} x_i \\
\text{s.t. } & \sum_{i \in S} x_i \geq v(S), \ \forall S \subseteq N
\end{align*}
\]  

(Conv-3-7)

### 3-5 Comparison between the coalitional game solution concepts

In this section, the three solution concepts, the **Shapley value**, the **nucleolus**, and the **core** are compared to each other and all their advantages and disadvantages are weighed against each other.

**Shapley value**

The Shapley value has three main positive characteristics:

- The Shapley value *always exists* for every coalition form game.
• The Shapley value is a unique solution for a coalition form game.
• The Shapley value is a fair solution for a coalition form game.

Especially the first two are really favourable. With the Shapley value as solution concept, one will always find a unique solution to the coalition form game. Furthermore, the solution is fair as it divides the payoffs to the players averaging their marginal contribution to all possible coalitions.

However, the Shapley value has also two main drawbacks. First, the solution found by the Shapley value does not necessarily lie in the core, and therefore, the Shapley value is not always a stable solution. This can cause some sub-coalitions to deviate from the Shapley value and form other coalitions. However, when the Shapley value lies in the core, the solution can be very favourable since it is a fair and stable solution. An interesting result in coalitional game theory, is that for convex games the Shapley value is always in the core [42].

The second main disadvantage of the Shapley value is the computational complexity increases exponentially with the number of players. Therefore, the Shapley value solution concept becomes much harder to apply to games with large numbers of players.

**Nucleolus**

The nucleolus’ main benefits are:

• The nucleolus always exists for every coalition form game.
• The nucleolus value is a unique solution for a coalition form game.
• The nucleolus is based on a concept that minimises the dissatisfaction of the players within the final coalition.

The nucleolus shares the two most favourable characteristics of the Shapley value, namely it does always exist and it is unique. Furthermore, instead of being a fair solution, it is a solution that minimises the dissatisfaction of each player. Above all, in contrast to the Shapley value, the nucleolus will be in the core when the core is nonempty. This means that when there is at least one stable solution in a coalition form game, the nucleolus is one of them [18].

However, the nucleolus shares the same drawback as the Shapley value: its computational complexity increases exponentially with the number of players [41]. In large games this can cause a very time-consuming computation process.

**Core**

The core has two main positive characteristics:
• If the core is nonempty, the payoff vectors in the core are stable solutions for the coalition
form game.

• The computation of the core is easier in games with a large number of players compared
to the nucleolus and the Shapley value.

When it is assumed that the players in the game are rational and intelligent, it can be
concluded that when the core is nonempty, the final division of payoffs will be one of the
allocations of payoffs within the core. However, to test whether the core is nonempty can still
costs some time and computation efforts. Hence, it could occur that after a long calculation
process, it is found that the core is empty and that one is left empty handed.
Furthermore, when the game is checked for nonemptiness and the core is found, it can be the
case that the core is still a large collection of possible allocations. In most cases, the core is
non-unique and the optimal allocation of payoffs is still unknown.

It can occur that an allocation in the core is stable but unfair to one or more players in the
final coalition. The core is based on the assumption that it is the set of feasible allocations
that cannot be improved by any sub-coalition. It only satisfies the individual and group
rationality; it does not necessarily satisfies the fairness property as the Shapley value does.

At last, if the core is found to be nonempty, finding it is relatively easy and can be done by
implementing a linear programming solver such as the simplex method.

Comparison of solution concepts in the musicians’ game

When the outcomes of the different solution concepts of the musicians’ game are compared, it
cannot be concluded which concept is better. In this case the core is nonempty, and thus the
nucleolus lies in the core. Furthermore, the Shapley value is in the core. Therefore, all the
possible solutions concepts are stable. In Table 3-4, the three outcomes the different solution
concepts are found. When the Shapley value \((63\frac{2}{6}, 25\frac{5}{6}, 10\frac{5}{6})\) is compared to the nucleolus
\((65, 25, 10)\), it can be concluded that the nucleolus is more favourable to the guitarist. The
singer and drummer would prefer the Shapley value over the nucleolus. In game theory it
is assumed that players only strive to maximise their own utility, and that they do not care
about the utility of others.

<table>
<thead>
<tr>
<th>Player</th>
<th>Individual value</th>
<th>Shapley value</th>
<th>nucleolus</th>
<th>core</th>
</tr>
</thead>
<tbody>
<tr>
<td>G</td>
<td>50</td>
<td>63\frac{2}{6}</td>
<td>65</td>
<td>50 ≤ u_G ≤ 75</td>
</tr>
<tr>
<td>S</td>
<td>20</td>
<td>25\frac{5}{6}</td>
<td>25</td>
<td>20 ≤ u_S ≤ 30</td>
</tr>
<tr>
<td>D</td>
<td>0</td>
<td>10\frac{5}{6}</td>
<td>10</td>
<td>0 ≤ u_D ≤ 20</td>
</tr>
</tbody>
</table>

Table 3-4: Different solutions to the musician’s game, guitarist (G), singer (S) and drummer
(D).

What this example shows, is that it cannot be concluded what solution concept is better in
this case. It differs per type of game, per amount of players and type of players. Are the
players competitive or do they care about their fellow players? What solution concept is
better in what case, depends on which properties are more important, stability, fairness, or
dissatisfaction. It can therefore be concluded that for each particular game an analysis should
be done on what solution concept would be best for that specific game.

S. Elderenbosch

Master of Science Thesis
3-6 Conclusions

This chapter studies three solution concepts within coalitional game theory. First, the two fundamental questions within game theory were introduced: which coalition will form and how will that coalition divide the payoff among the players? Furthermore, two important coalitional game theory concepts were introduced, *individual rationality* and *group rationality*. When an allocation of payoffs satisfies both, it is called an *imputation*. The first of the three solution concepts that has been investigated, is the Shapley value. The Shapley value is a solution concept that is based on four different mathematical properties: efficiency, symmetry, the dummy player property and linearity. When a payoff allocation satisfies these four properties it is called a fair solution. The Shapley value always satisfies these four properties and is therefore a fair solution to the coalition form game. The Shapley value is the average marginal contribution a single player has to every possible coalition among the players. The Shapley value always exists and is an unique solution.

The second solution concept is the nucleolus. The nucleolus solves a coalition form game by minimising the largest excess of an imputation. The excess of a coalition is defined by the value of that coalition minus the sum of payoffs of all the individual players within that coalition. Similar to the Shapley value, the nucleolus always exists and is a unique solution. Furthermore, when the core is nonempty, it lies in the core and is therefore a stable solution. The final solution concept that was investigated is the core. The core is a solution concept based on stability. This implies that when an allocation of payoffs is in the core, there is no sub-coalition that has the incentive to deviate from this allocation, since there is no other sub-coalition that gives a higher payoff. In contrast to the Shapley value and the nucleolus, the core does not always exist (i.e. it can be empty) and when it exists, it is not always unique. Finally, the three solution concepts were compared with each other and their advantages and disadvantages were weighed against each other. Which solution concept would be better to the game depends on the type of game, the number of players, and the types of players.
Chapter 4

System model

This chapter elaborates on the system model of the urban cellular network that is used in the case study. This system model consists of a network of one macro base station (BS) and multiple small base stations (BSs), a traffic model of simulated users with time-varying locations and data demand, and a power consumption model. This model is used to test the hypothesis that coalitional game theory can help to improve the energy efficiency of cellular networks.

This chapter is structured as follows: First, the network infrastructure is introduced, what types of BSs are used and where they are located; then the traffic model of users and their data demand profile is discussed; afterwards the chapter elaborates on the power consumption models of the different types of BSs; and the chapter is completed with some conclusions.

4-1 Cellular network infrastructure

The simulated network is considered to be a heterogeneous one-macro-cell network in an urban area, operated by a single MNO (mobile network operator). This involves one macro BS, which is located in the middle of the simulated cell, with a set $S = \{1, 2, ..., s\}$ of small BSs strategically positioned over the simulated cell area. All the simulated BSs belong to the same single MNO.

The simulated area is considered to be a square geographical area of $A \text{ km}^2$, located in the urban area. To make the model more tangible, it was decided to replicate the urban area of Delft. Thus, within the simulated area there are crowded areas such as shopping malls or schools and less crowded areas such as parks and residential areas. An urban area was chosen for this thesis because it has a higher population density, which results in a higher demand of data traffic. To provide this high data demand profile, numerous BSs are needed to meet the quality of service (QoS) standards of the MNO. These two conditions, high data demand profile and multiple BSs, are essential for the research question of this thesis.

In this thesis the QoS requirement is defined by the maximum outage probability $\varepsilon \ll 1$, 

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which is the probability that the achieved data rate of user \( i \) is smaller than \( b \) bits/sec. So the QoS of the provider is met when the outage probability is smaller than \( \varepsilon \) \([5]\).

To provide the entire simulated area with mobile data, the two types of BSs are strategically placed over the area. The macro BS, with active radius \( R_m \), is positioned in the middle of the area and has an active radius as large as the whole area. The positioning of the small BSs, each with active radius \( R_s \), is based at peak traffic conditions. This is done in such a way that the QoS requirement of the MNO is met. The number of small BSs was increased so that all the users were served by the cellular network at peak traffic time, without exceeding the radiation power of the macro BS and meeting the outage probability \( \varepsilon \). This strategic placement involves that most of the small BSs are located in the crowded areas to lighten the heavy burden of data demand profile of these areas on the macro BS. Each small BS \( s \in S \), has two operation modes, \( \theta_s \in \{0, 1\} \). Operation mode \( \theta_s = 1 \) indicates that the small BS is in active mode and serves all the users within its active radius. Operation mode \( \theta_s = 0 \) indicates that the small BS is in sleeping mode and that its users are provided with data by the macro BS. The vector \( \theta = [\theta_1, \theta_2, ..., \theta_s]^T \) represents the operation modes of all the small BSs. The macro BS is always in operation mode to provide seamless data coverage and to avoid any service failure \([53]\).

Figure 4-1 shows a heterogeneous cellular network where one small BS is in operating mode and the other is in sleeping mode. Furthermore, the figure shows the power consumption profiles of the two types of BSs. Section 4-3 will explain in more details these profiles.

![Heterogeneous cellular network with two types of BSs, one macro BS and two small BSs](image.png)

**Figure 4-1:** Heterogeneous cellular network with two types of BSs, one macro BS and two small BSs \([50]\).

To avoid signal interference between the small BSs and the macro BS, the two types of BSs operate over a different spectrum band, also known as separate carriers. Separate carriers have been used widely in the industry and have proven to provide seamless data to users,
when moving from one BS to another [15, 49, 52]. Therefore, it is assumed that when an small BS goes into sleeping mode and the macro BS takes over the users of that respective small BS, this process is seamless and will not be noticed by the users. Such a process of taking over a mobile user from one BS to another is called a handover [43]. In general small BSs operate over 7 different spectrum bands, the middle small BS surrounded by 6 other small BSs, all operating in a different spectrum band, in order to avoid signal interference. In literature extensive research has been done to properly control the cell interference between small BS, with schemes that coordinate the control of a network of small BS efficiently [28, 54, 55]. Again, it is assumed that there is no cell interference between the small BSs and that QoS of the MNO is assured. In Figure 4-2 the layout of the cellular network infrastructure can be found. It consists of 1 macro BS and 117 small BS.

![Figure 4-2: Simulated cellular network infrastructure](image)

### 4-2 Traffic model

The input of the cellular network model is the data demand profile of the simulated users. In order to simulate a realistic data traffic demand profile, the EARTH statistics were used.
as a guideline for the simulation model [20]. EARTH stands for Energy Aware Radio and netWork tecHnologies. EARTH is a major European research project with the main technical objective is to map the overall energy consumption of mobile broadband networks and to seek opportunities for energy reductions. The research was commissioned by the European Union.

As mentioned earlier, the simulated area should represent the urban area of Delft, which is a densely populated urban area. Therefore, the population density $p$ is chosen to be of a dense urban area, found in [20]. In order to create a more realistic model of users with crowded and less crowded areas, half of the simulated people were randomly distributed over the whole simulated area. The other half was randomly distributed over pre-selected more crowded areas. According to [20, 25] the average mobile data demand per person in Europe is equal to 2.1GB per month, which is implemented.

The traffic model is dynamic. Each hour, each user moves with a probability of 50% with a random distance in a random direction, or stays at the same spot with a probability of 50%. In this way, the model mimics a realistic dynamic behaviour of the simulated mobile users. To make the traffic model even more representative of reality, a periodic traffic demand profile is implemented, since the people’s usage of mobile phone varies during the day. During daytime there is a higher data traffic demand, while during night-time this demand drops significantly. In the EARTH studies, this fluctuation in data demand was investigated. The results can be found in Figure 4-3 [20].

![Figure 4-3: Daily periodic mobile data traffic [20].](image)

The final areal traffic demand profile in Mbps/km$^2$ is

$$R(t) = \frac{p}{N_{op}}a(t)\sum_k r_k s_k$$  [20].

(4-1)

Here $p$ is the population density of the specific area, $N_{op}$ is the number of MNOs in the area, in this thesis this always assumed to be 1, $a(t)$ is the periodic traffic, as in Figure 4-3, $r_k$ is the weighted average of heavy and ordinary subscribers of device type $k$. For instance a heavy
consumer on a PC is assumed to consume 900MB/hour, while an ordinary user consumes 112.5MB/hour, \( s_k \) is the number of users of device type \( k \), where \( k \) are the different device types, such as smart phones, tablets, PCs, that can connect to the wireless network.

Each user in the simulated area is provided with mobile data meeting the QoS requirements of the MNO. When a user is in the active radius of a small BS and the operation mode of the small BS is on, i.e. \( \theta_s = 1 \), the user is served by the small BS. If the small BS is in sleeping mode, i.e. \( \theta_s = 0 \), the user is served by the macro BS. Every user located exactly on the border of the active range of an small BS is served by the macro BS. Figures 4-4 and 4-5 show the simulated users connected to their specific BSs at a specific time step \( t \).

![Figure 4-4: Simulated users within the cellular network model at time step \( t = 1 \) served by the macro BS](image1)

![Figure 4-5: Simulated users within the cellular network model at time step \( t = 1 \) served by a specific small BS](image2)

## 4-3 Power consumption model

This section elaborates on the power consumption model of the macro BS and the small BS. The power consumption is the output of the model and has to be minimised by forming coalitions among the BSs.

The first subsection will consider the power consumption of the macro BS and the second subsection will discuss the power consumption of the small BSs.

### 4-3-1 Macro base station power consumption

The power consumption \( P \) of the macro BS consists of two components, \( P_s \), which is the static power consumption component of the macro BS and \( P_t \), which is the variable transmit power component. The static power consumption is used for all the static processes within the macro BS keeping the macro BS running. In Europe macro BS typically have \( P = 712W \)
The variable transmit power consumption is mostly the power consumption of the power amplifier, used to transmit the radio signals. This increases with the macro BS data output.

To calculate the total variable power consumption $P_t$, first the transmit power per user has to be derived. To do so MNOs derive the minimum signal power that each user should receive $P_{ri}$ to meet their QoS standards,

$$P_t = \begin{cases} P_t h_i D \left( \frac{r_i}{r_0} \right)^{-\alpha}, & \text{if } r_i \geq r_0 \\ P_t h_i D, & \text{otherwise.} \end{cases}$$

Here $P_t$, is the transmit power of the macro BS to user $i$ in W, $h_i$ is the Rayleigh fading channel, $D$ is the distance dependent path loss component, $r_0$ is the reference distance of the macro BS, $r_i$ is the distance between the receiver and the macro BS, and $\alpha$ is the distance-dependent path-loss exponent. All these terms will be discussed in more depth.

To meet the desired QoS, MNOs have a threshold to derive the minimum transmit power, called the outage probability $\varepsilon$, such that each user receives the minimum signal power. The outage probability is the probability that an outage occurs, i.e. when the link between the mobile user and the macro BS cannot support a desired data demand rate $b_i$ bits/sec [37].

The transmission power per user depends on the bandwidth $W$ of the macro BS; the number of total users connected to the macro BS; the data demand profile $b_i$ of the specific user; the outage probability $\varepsilon$ of the MNO; and the distance between the user and the macro BS.

There are four main effects that negatively affect the transmission of signals from the macro BS to the user: Rayleigh fading; the distance dependent path loss; environment noise; loss of capacity due to coding and modulation [29]:

- **Rayleigh fading** is a statistical model that describes the scattering and fading of radio signals due to objects in the environment, such as buildings, trees etc., before they arrive at the receiver. In this case Rayleigh fading is indicated by $h_i$ and is assumed to follow an exponential distribution with unit mean. Furthermore, it is assumed that $h_i$ is mutually independent for all $i$’s [5, 37]

- **Distance-dependent path-loss** occurs when the distance $r_i$ between a user and the connected BS is larger than a certain reference distance $r_0$. For a macro BS with an active radius between 2-25 km, $r_0 = 200m$ [24]. The transmit power of the BS to the user experiences a fixed path loss $D > 0$ when the distance between user $i$ and the macro BS is smaller than $r_0$. This fixed path loss increases with an additional path loss exponent $\alpha > 0$ when the distance becomes larger than $r_0$. So the total effect of the distance dependant path loss can be expressed as $D \left( \frac{r_i}{r_0} \right)^\alpha$ [6, 24, 30].

- **Environment noise** is omnipresent [31]. It is caused by transport, industrial activities, recreational activities, etc. Therefore, this signal noise has to be implemented into the model to recreate a realistic cellular network. The noise is modelled as additive white Gaussian noise and is expressed as $N_0$.

- **Loss of capacity due to coding and modulation**. Adaptive modulation and coding can be used to improve the throughput of a wireless network [35]. Radio signals
sometimes have to be modulated to higher frequencies in order to reach the desired
destination. However, this can also have a negative effect on the transmission of radio
signals. Therefore, an extra term $\Gamma$ was added to the power consumption function.

As mentioned earlier, MNOs seek to maximise their QoS by minimising their outage proba-
bility. This outage probability threshold is used for deriving the transmit power per user,

$$\varepsilon = \Pr \left\{ P_t^i < \frac{\Gamma N_0 W}{K} \left(2^{\frac{K_b}{W}} - 1\right) \right\} \quad (4-3)$$

where $P_t^i$ is the same as in Formula 4-2, $W$ is the total bandwidth over which the macro BS
operates, and $K$ is the number of users connected to the macro BS. Since $h_i$ is an exponential
random variable with unit mean, this can be rewritten as:

$$\varepsilon = \begin{cases} 
1 - \exp \left( \frac{-\Gamma N_0 W}{K P_t^{i,D}} \left(2^{\frac{K_b}{W}} - 1\right) \right), & \text{if } r_i \geq r_0 \\
1 - \exp \left( \frac{-\Gamma N_0 W}{K P_t^{i,D}} \left(2^{\frac{K_b}{W}} - 1\right) \right), & \text{otherwise}. 
\end{cases} \quad (4-4)$$

When the natural logarithm of Formula 4-4 is taken, it can be rewritten, to a function that
gives the transmit power of the macro BS to a specific user in order to meet the right QoS.
Finally, the variable transmission power of the macro BS per user becomes [5, 37]:

$$P_t^i = \begin{cases} 
\frac{\Gamma N_0 W}{K} \frac{2^{\frac{K_b}{W}} - 1}{K} \left(\frac{r_i}{r_0}\right)^{\alpha}, & \text{if } r_i \geq r_0 \\
\frac{\Gamma N_0 W}{K} \frac{2^{\frac{K_b}{W}} - 1}{K}, & \text{otherwise}
\end{cases} \quad (4-5)$$

When this variable transmission power is summed up for all the users connected to the macro
BS and is added to the static power consumption $P$, this gives the final power consumption
equation.

$$P = P + \sum_{i=1}^{K} P_t^i \quad (4-6)$$

The power consumption Formulas 4-5 and 4-6, result in a linear increasing power consumption
of the macro BS with the data traffic, as can be seen in Figure 4-6.

### 4-3-2 Small base station power consumption

The power consumption model of the small BS is quite similar to that of the macro BS. According to [20, ?] the power consumption model of an small BS is:

$$p_s = \begin{cases} 
p_1 = P + p_t^i, & \text{if } \theta_s = 1 \text{ (active)} \\
p_0, & \text{if } \theta_s = 0 \text{ (sleep)}
\end{cases} \quad (4-7)$$
where $p_1$ and $p_0$ are the power consumption level when the small BS is in active or respectively sleeping mode, $P$ is the base power level and $p_t$ is the transmit power level.

The power consumption of the small BS, represented in Formula 4-7, looks similar to the power consumption of the macro BS in Formula 4-6. However, their outcome is completely different. This is due to $p_t$ of the small BS, which barely increases with the data traffic, e.g. only 0.07W increase when traffic load increases from 80% to 100%. This mild increase in power consumption is due to the short-range communication in small-cells [20].

For simplicity it is assumed that $p_1=10W$ independent of the traffic load. This value will be used for the rest of the thesis. Furthermore, for the small BS in sleeping mode $\theta_s=0$ it is assumed that $p_0=0$ [20].

### 4-3-3 Total power consumption

Combining the various components as described in this chapter, i.e. the BS infrastructure; the traffic model; and both the power consumption models, the output of the simulation model is the total power consumption of the simulated cellular network. The goal of this thesis is to minimise the total power consumption while maintaining the QoS of the MNO. In Figure 4-7 the power consumption of the cellular network without any collaborations is shown.

### 4-4 Conclusions

This chapter describes the system model that is used to test the coalitional game theory solution theorems. The model is a realistic model of a heterogeneous cellular network.
The model is based on the dense urban area of Delft and consists of two different types of BSs. One macro BS is located in the centre of the area and is surrounded by multiple small BSs. These small BSs are strategically placed in more crowded areas, to lighten the heavy burden of the data demand profile in these areas on the macro BS. Statistics from EARTH [20] were used to model a realistic time-dependent data traffic model. A dense urban population density was unequally distributed over the simulated area, in order to mimic a city with crowded and less crowded areas. Furthermore, a periodic traffic demand profile was implemented. Each user is connected to a specific BS, either a small BS, when it is within the active range of the given small BS, or the macro BS when this is not the case. Finally, the power consumption models for the macro BS and the small BS were added to the model. The power consumption of the macro BS increases linearly with the data traffic, while the power consumption of the small BS barely increases with the data traffic and therefore is assumed to be constant.

Finally, the different elements were combined to the final cellular network model. Within this model, the BS infrastructure combined with the power consumption, functions as the system. The data traffic is the input of the system and the total power consumption is the output of the system.
Chapter 5

Application of the coalitional game theory solution concepts

In Chapter 3, the three coalitional game theory solution concepts, the Shapley value, the nucleolus and the core have been introduced and investigated. This chapter explains how these solution concepts are used to determine the most energy efficient coalition using the data of the cellular network model, introduced in Chapter 4.

This chapter is structured as follows: Section 5-1 explains how the cellular network is modelled as a coalition form game, what the final coalition will look like, and how the payoff values are assigned to each coalition. Section 5-2 introduces the application of clusters among base stations (BSs) to reduce the computational complexity. Then the chapter elaborates on the application of the three different solution concepts: the Shapley value, the nucleolus, and the core. Finally, the chapter is completed with some conclusions.

5-1 Cellular network as a coalition form game

In this MSc thesis, the players of the coalition form game are the small BSs of the cellular network. First, this section introduces how the final coalition looks like. Then the payoff function assigning the values to all the coalitions is explained.

5-1-1 Energy-efficient coalition

Chapter 4 explained that since the power consumption of small BSs barely increases with an increase in data demand, the BSs can be underutilised during low-traffic hours. This may involve that the total power consumption of the cellular network can be decreased. To achieve this, the particular small BS goes into sleeping mode $\theta_S = 0$, and the macro BS takes over the users connected to that specific small BS. One may argue that finding the most energy
efficient coalition is easy by switching off all the small BSs that consume more power than the macro BS would do when taking over their users. However, the problem is not as simple as that. In view of health safety concerns for the people, strict regulations are in place for the maximum transmit power that macro BSs may use. This thesis has adopted the values of the EARTH research [20]. So the problem is to find the most energy efficient coalition among the small BSs to use the available power of the macro BS, i.e. the difference between the power used by the macro BS to serve its own users and its maximum capacity. The final coalition to solve this coalition form game will always consist of the macro BS and, when the available power of the macro BS is sufficient, a set of small BSs.

5-1-2 Assigning payoffs to all coalitions

As was introduced in Section 3-1, a coalition form game consists of two different aspects. The players of the game, in this thesis the small BSs, and the payoff function \( v(S) \) \( \forall S \in N \), assigning a payoff to every possible coalition among all the players. So far this payoff function still needs to be determined in order to model this problem as a coalition form game. First of all, it needs to be specified that the payoff of the coalitions will only be used to determine which coalition will form. Since the players in the game are the small BSs and there is no such thing as a transferable payoff that can be divided among these BSs, this MSc thesis only uses the payoff to find out which coalition will form. The payoffs assigned to all the possible coalitions, from single BSs to the grand coalition, are determined by the contribution of that specific coalition to the total power consumption of the whole cellular network. When an small BS is underutilised it can be more energy efficient to put this small BS in sleeping mode and serve its users by the macro BS. If this is the case, the specific small BS has a positive payoff, which equals the difference between the power consumption the macro BS needs to serve the users of the small BS and the power the small BS uses when it is active. In Chapter 4 the power consumption of a small BS has been decided to be 10W.

So for example, when the extra power needed for the macro BS to take over all the users of coalition \( A \) consisting of small BS \( A = \{1\} \) \( A \subset N \) is 6.5W, the payoff of this specific small BS would be \( v(\{A\}) = 10 - 6.5 = 3.5 \). This positive payoff involves that it would be beneficial for the cellular network to put this small BS into sleeping mode. The opposite holds for coalition \( B \) consisting of small BS \( B = \{2\} \) for which the extra power needed by the macro BS to take over the users is equal to 12.8W. The payoff value to this small BS would be \( v(\{B\}) = 10 - 12.8 = -2.8 \). In this case the negative payoff involves that the cellular network would consume more power when this small BS is put into sleeping mode, compared to the situation when the small BS would remain active.

In order to assign a payoff value to a coalition \( A \) with \(|A| > 1\), the same procedure of assigning payoff values is used. When coalition \( A \) consists of three small BSs \( A = \{1,2,3\} \), for which the power needed by the macro BS to serve their users is 3.5W, 12.8W, and 7.2W respectively, the payoff value \( v(\{A\}) = 10 - 3.5 + 10 - 12.8 + 10 - 7.2 = 6.5 \). The payoff value of a coalition thus solely depends on the difference in power consumption of the whole cellular network, between active and sleeping mode of the coalition.
This involves that the game is convex, since the combined payoff value of two coalitions that partially coincide, is the same as the sum of the payoffs of the two coalitions minus the payoff value of the intersecting part of both coalitions. For example, in the case of two coalitions $S = \{1, 2, 3\}$ and $T = \{2, 4, 5\}$ $S, T \subset N$ where the power needed by the macro BS is 3.5W, 12.8W, and 7.2W respectively for coalition $S$ and 12.8W, 5.1W, and 9.4W respectively for coalition $T$, the combined payoff value of coalition $\{S \cup T\}$ will be

$$v(\{S \cup T\}) = (10 - 3.5 + 10 - 12.8 + 10 - 7.2) + (10 - 12.8 + 10 - 5.1 + 10 - 9.4) - (10 - 12.8) = 12.$$ 

As explained previously in Chapter 3, a convex game implies that the core is nonempty and that the Shapley value lies in the core. This implies that the Shapley value of this game is a stable solution. Furthermore, the fact that the core is nonempty means that the nucleolus lies in the core.

Table 5-1 shows the payoff values of the examples mentioned above.

<table>
<thead>
<tr>
<th>Coalition</th>
<th>Power needed by macro BS [W]</th>
<th>Payoff value</th>
</tr>
</thead>
<tbody>
<tr>
<td>${1}$</td>
<td>3.5</td>
<td>6.5</td>
</tr>
<tr>
<td>${2}$</td>
<td>12.8</td>
<td>-2.8</td>
</tr>
<tr>
<td>${3}$</td>
<td>7.2</td>
<td>2.8</td>
</tr>
<tr>
<td>${4}$</td>
<td>5.1</td>
<td>4.9</td>
</tr>
<tr>
<td>${5}$</td>
<td>9.4</td>
<td>0.6</td>
</tr>
<tr>
<td>${1,2,3}$</td>
<td>23.5</td>
<td>6.5</td>
</tr>
<tr>
<td>${2,4,5}$</td>
<td>27.3</td>
<td>2.8</td>
</tr>
<tr>
<td>${1,2,3,4,5}$</td>
<td>38</td>
<td>12</td>
</tr>
</tbody>
</table>

**Table 5-1:** Assigning payoff values to arbitrary coalitions of imaginary small BSs.

## 5-2 Clustering

A recurring problem with all the solution concepts is the increasing computational complexity when the number of players increases. The number of calculations that need to be performed for the nucleolus and the Shapley value increases according to the Bell number. The Bell number is the number of ways a set of $n$ elements can be partitioned into nonempty subsets \[3\]. The first ten numbers, $n = 0, 1, ..., 10$ of the Bells’ series are: 1, 1, 2, 5, 15, 52, 203, 877, 4140, 21147. The model of the cellular network consists of 117 small BSs. It can easily be concluded that the number of calculations that has to be made to find the Shapley value for all these BSs is enormous and way too much for ordinary computers. Therefore, a method needs to be found to drastically reduce the number of calculations.

The first way of reducing the number of calculations is by excluding the small BSs that would never contribute to a more energy efficient cellular network. In Section 5-1-2, it was shown that several small BSs contribute with a negative payoff when joining a coalition. This implies that the total power consumption of the cellular network increases when these small BSs go into sleeping mode and their users are served by the macro BS. Therefore, these BSs can

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excluded from the calculation of the three solution concepts, since they will never improve the energy efficiency of the cellular network.

The second method to reduce the total number of calculations is by forming clusters among small BSs. Instead of applying the three solution concepts to the 117 single small BSs, the solution concepts can be applied to small clusters consisting of a few small BSs. These clusters of BSs are modelled as one single player in the game. This drastically decreases the total number of players within the game. For instance, when clusters of 3 small BSs are formed among the small BSs, only 39 players remain. When the number of small BSs within a cluster is increased to 5 BSs, the maximum total number of players within the game decreases to 24. Simulations with the cellular network model on an ordinary computer with an Intel Core i7-7700HQ processor at 2.8GHz with 8.00 GB of RAM, have shown that solving the solution concepts with 24 players is feasible.

When a cluster among small BSs is formed, the power needed for the macro BS to take over the small BSs is summed up for the cluster. Consequently, the same happens with the payoff value of the small BSs. A cluster of small BSs is actually the same as a coalition of small BSs. The only difference is that a cluster is not formed by using one of the three solution concepts. The clusters are formed by combining neighbouring small BSs that consume less power than the macro BS would need to serve their users. These clusters behave as a single player, so all the small BSs within the clusters go either into sleeping mode together or remain active together.

While the computational complexity decreases drastically by increasing the cluster size among small BSs, it is expected that the energy efficiency will also decrease when the cluster size is increased. This is due to the fact that with the use of clusters it is no longer possible to put single BSs in sleeping or active mode. Instead, it is decided for whole clusters of BSs to go into sleeping mode or remain active. Therefore, it could happen that some BSs remain active, while it would have been more energy efficient if these specific BSs where put into sleeping mode. For example, if there are two clusters of small BSs $S$ and $T$, both clusters consist of 5 small BSs, cluster $S = \{1, 2, 3, 4, 5\}$ and $T = \{6, 7, 8, 9, 10\}$. Assuming that for each small BSs except BSs 5 and 10, the macro BS consumes 4W to serve their users, for BS 5 and 10 the macro BS consumes 8W. So for each of the clusters, the macro BS needs 24W to serve their users. Furthermore, it is assumed that the amount of data transmitted by each of the individual small BSs is the same. The macro BS in this example only has 32W of available power left before it reaches its maximum capacity. In this example for the most optimal solution it does not matter whether cluster $S$ or cluster $T$ is set into sleeping mode, since they both contribute the same energy saving to the cellular network. However, the most energy efficient solution is to set small BSs $\{1, 2, 3, 4, 6, 7, 8\}$ in sleeping mode, using all the available power of the macro BS and leave BSs $\{5, 10\}$ active. This example shows that due to clustering the energy efficiency may decrease. In Chapter 6, this decrease in energy efficiency while increasing the cluster size will be investigated for all three solution concepts by completing several simulations with increasing cluster size.
5-3 Applying the three solution concepts

The output of the cellular network model is the dynamic power consumption of the macro BS and the power the macro BS would consume when taking over any small BSs. The three solution concepts use this output to find the most optimal coalitions among the BSs. The three solution concepts all start with the same procedure. First, all the small BSs that consume more than 10W at time step $t$ are excluded from the calculation, such that only the BSs that can contribute to an increased energy efficiency of the whole cellular network are included in the solutions. Next, the payoff values of each individual small BSs is calculated. The final general step is the forming of the clusters. All the small BSs that are left are divided into clusters of 5 small BSs. The clusters are formed among adjacent small BSs. The final payoff value of a cluster is the sum of all the individual payoffs of the small BSs within that cluster. The same holds for the energy needed by the macro BS to take over a cluster, this is the sum of the energy needed to take over all the individual small BSs within a cluster.

5-3-1 Shapley value

Among the clusters of small BSs all the possible coalitions are formed, with the exception of the grand coalition of clusters. Next, the Shapley value is derived for each cluster of small BSs according to Formula (3-2). After the Shapley value has been derived for all of the clusters, the clusters are ordered in descending order of Shapley values. The clusters are then one by one added to the final coalition starting with the cluster with the highest Shapley value, until the macro BS reaches its peak capacity. In this way the clusters with the highest Shapley value form the final coalition, while the clusters with lower Shapley values remain active. Finally, the energy efficiency of the cellular network with the sleeping coalition is derived. This process is repeated for every time step in $t$. Without loss of generality it is assumed that $t = \{1, 2, ..., 24\}$.

In this process, the Shapley value is only used to find the coalition with the clusters that have the highest average marginal contribution to the energy efficiency of the cellular network.

5-3-2 Nucleolus

The application of the nucleolus is not as straightforward as the application of the Shapley value. Chapter 3 concerned the finding of the nucleolus, which required that an imputation has to be adjusted such that it minimises the largest excess. However, this can be applied in games where the payoff can be divided in any arbitrary way, such as when the payoff is money. In the case of the cellular network model, the payoff value of a coalition solely depends on the power saving this coalition induces when going into sleeping mode. This power saving cannot be divided in any arbitrary way, since each small BSs contributes a fixed amount of energy saving. A small BSs is either in sleeping mode or active. When in sleeping mode, it contributes a fixed energy saving to the cellular network, when in active mode it consumes 10W of power. It is not possible to set half a small BS into sleeping mode such that it contributes half of its payoff value to the energy saving coalition. Therefore, the payoff allocation can only be adjusted in fixed steps of the contribution of single separate small BSs, a BS is
on or in sleeping mode. In the case when clustering is applied the payoff function can only be adjusted in the steps of the fixed energy saving contribution of a cluster switching from active mode into sleeping mode. This significantly decreases the computational complexity of finding the final allocation of payoffs that minimises the dissatisfaction of all coalitions, since only a limited number of variations to the payoff allocation is possible.

So in order to find the final coalition of small BSs with the nucleolus concept, first all the possible coalitions among the clusters are formed, with exception of the grand coalition. Next, all the coalitions of clusters that need a higher power consumption of the macro BS to take over that coalition than the available power of the macro BS, are excluded from the calculation. All the remaining coalitions are ordered in ascending order of size, as was the case in Table 3-2. Then the dissatisfaction between one specific coalition and all the other coalitions is derived. This procedure is repeated for every remaining coalition. The coalition that generates the smallest maximum dissatisfaction among all coalitions is the final coalition according to the nucleolus approach and will be set into sleeping mode. This process is repeated for every time step $t$.

**5-3-3 Core**

For the core, the same holds as with the nucleolus, that the payoff allocation cannot be divided in an arbitrary way, as was explained in Section 5-3-2. It can only be divided into parts equal to the contribution of single clusters. To find the core, first all the possible coalitions among the clusters are formed. Then the linear inequalities are formed for all the coalitions of clusters, following the same procedure as was the case with the Musicians’ game in Figure 3-1. The allocation of payoffs satisfying all these inequalities forms the core.

Since in the case of this specific game the payoffs cannot be arbitrarily divided, the only allocation that can satisfy all the linear inequalities is the allocation of payoffs to the grand coalition. However, the capacity of the macro BS is not always sufficient to take over the grand coalition. In these cases there is no feasible allocation of payoffs that can satisfy all the linear inequalities. Therefore, the core would be empty and there would have been no comparable result to compare with the Shapley value and the nucleolus. When this was the case the best alternative was chosen as solution of the core, in order to have comparable results with the Shapley value and the nucleolus. The best alternative, is the allocation of payoffs to the largest possible coalition in number of clusters that the macro BS still can take over. This is the allocation that satisfies most of the linear inequalities and thus the best alternative solution for the core.

However, there are situations in which there are multiple different coalitions with the same largest number of clusters that the macro BS can take over. In these situations the collection of allocations to all these possible largest coalitions form the core. This situation shows various analogies with a situation in which the core is non unique. When this is the case the average energy efficiency of every possible coalition in the core is taken to compare with the Shapley value and the nucleolus.
5-4 Conclusions

This chapter explained how the three coalitional game theory solution concepts can be used to find the most energy-efficient coalition of small BSs in the cellular network. First, the coalition form game is described. The small BSs are the players of the game that aim to find the most energy-efficient coalition among them. This coalition will then collaborate with the macro BS by going into sleeping mode and let their users be served by the macro BS.

Next, the payoff function is introduced. The payoff function solely depends on the contribution of a small BS to the decrease of the power consumption of the whole cellular network, when joining the coalition that goes into sleeping mode.

To overcome the problem of computational complexity with large number of players, clustering among small BSs has been introduced. Small BSs form clusters with adjacent BSs and act as a single player in the coalition form game. The clustering approach drastically reduces the computational complexity. However, a reduction in energy efficiency is also expected compared to the situation in which all the individual small BSs will be modelled as single players. Chapter 6 will concern the simulations to test this energy efficiency decrease.

Finally, the application of the three coalitional game theory solution concepts has been discussed. The application of the Shapley value is straightforward by simply filling in the formula. Finding the nucleolus and the core is a bit different requires a bit different approach since the allocation of payoffs cannot be arbitrarily divided. This is due to the fact that a small BS is either active or in sleeping mode without any state between them.
Application of the coalitional game theory solution concepts

S. Elderenbosch

Master of Science Thesis
This chapter discusses the simulations that are performed resulting from the application of the coalitional game theory solution concepts to the cellular network model. The cellular network model was built in MATLAB, as were the applications of the solution concepts.

This chapter is structured as follows: First the settings of the simulation model are discussed. Afterwards, the results of the simulation runs of the different solution concepts are presented. Next, the results are discussed and compared with each other. Finally, the chapter is completed with some conclusions.

6-1 Settings

Chapter 4 has discussed the cellular network model. Table 6-1 shows the values of the parameters that were used in the case study. The cellular network model has a dynamic power consumption output with time step $t \in \{1, 2, ..., 24\}$ where each time step corresponds with one hour.
### 6-2 Simulation results

This section shows the results of the simulations that have been performed. The first subsection shows the results of the simulations that have been performed to determine the decrease in computation time when clustering is applied. Next, the results of the implementation of the Shapley value, the nucleolus, and the core are presented.

#### 6-2-1 Clustering

The simulations to determine the decrease in computation time and energy efficiency when clustering is applied, are performed using each of the solution concepts. The simulations were performed at time step $t = 2$, since the data demand on that time step is around the average of data demand over all time steps. The process for calculating the energy efficiency is the same for every time instant $t$. Therefore, it was decided to run this simulation only for $t = 2$, to save computation time. The simulations were performed with an increase in the number of small BSs from 6 to 20. When the number of small BSs was increased further, the computational complexity simply became too big for the computer, Intel Core i7-7700HQ processor at 2.8GHz with 8.00 GB of RAM. Since for each simulation only a small random sample size of the 117 small BSs is taken, each computation was repeated 20 times with different random samples of small BSs. Finally the average of these 20 simulation is determined and presented in Figures 6-1, 6-2, and 6-3.
Shapley value (a) Energy efficiency of the Shapley value.

Shapley value (b) Relative energy efficiency increase of the Shapley value compared to when all small BSs are active.

Shapley value (c) Computation time of the Shapley value.

Figure 6-1: (a) Difference in energy efficiency when using different cluster sizes with an increasing number of small BSs. (b) Difference in relative energy efficiency increase when using different cluster sizes with an increasing number of small BSs, compared to the situation when all small BSs are active. (c) Different computation times for different cluster sizes with an increasing number of small BSs.
Simulation and results

Figure 6-2: (a) Difference in energy efficiency when using different cluster sizes with an increasing number of small BSs. (b) Difference in relative energy efficiency increase when using different cluster sizes with an increasing number of small BSs, compared to the situation when all small BSs are active. (c) Different computation times for different cluster sizes with an increasing number of small BSs.
Figure 6-3: (a) Difference in energy efficiency when using different cluster sizes with an increasing number of small BSs. (b) Difference in relative energy efficiency increase when using different cluster sizes with an increasing number of small BSs, compared to the situation when all small BSs are active. (c) Different computation times for different cluster sizes with an increasing number of small BSs.
Results of the three coalitional game theory solution concepts

This section presents the results of the simulation that was completed with the three different solution concepts for finding the final coalition. The simulation includes all the 117 small BSs and uses clusters of 5 to reduce the computation time. The simulation was repeated for every time step $t \in \{1, 2, \ldots, 24\}$. In Figure 6-4, the results of the Shapley value are shown. Figure 6-5 shows the results of the nucleolus and Figure 6-6 shows the results of the core. In Table 6-2 the numerical results of the three different solution concepts are presented.

<table>
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<tr>
<th></th>
<th>Energy efficiency [Bit/J]</th>
<th>Energy efficiency increase [%]</th>
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<tr>
<td>1</td>
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</tr>
<tr>
<td>2</td>
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</tr>
<tr>
<td>24</td>
<td>344.7</td>
<td>438.6</td>
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Table 6-2: Overview of the results of the three coalitional game theory solution concepts. These numbers are the results of simulations with all 117 small BSs. $T =$ time step, $A =$ situation in which all small BSs are active.
**Shapley value (a)** Energy efficiency when the core is applied to all small BSs.

**Shapley value (b)** Relative energy efficiency increase of the core applied to all small BSs, compared to when all small BSs are active.

**Figure 6-4:** (a) Difference in energy efficiency for each time step $t$ when the final coalition among all small BSs is found using the Shapley value. (b) Relative energy efficiency increase for each time step $t$ when the final coalition among all small BSs is found using the Shapley value.
Simulation and results

**Nucleolus (a)** Energy efficiency when the nucleolus is applied to all small BSs.

**Nucleolus (b)** Relative energy efficiency increase of the nucleolus applied to all small BSs, compared to when all small BSs are active.

**Figure 6-5:** (a) Difference in energy efficiency for each time step \( t \) when the final coalition among all small BSs is found using the nucleolus. (b) Relative energy efficiency increase for each time step \( t \) when the final coalition among all small BSs is found using the nucleolus.
Core (a) Energy efficiency when the core is applied to all small BSs.

Core (b) Relative energy efficiency increase of the core applied to all small BSs, compared to when all small BSs are active.

Figure 6-6: (a) Difference in energy efficiency for each time step $t$ when the final coalition among all small BSs is found using the core. (b) Relative energy efficiency increase for each time step $t$ when the final coalition among all small BSs is found using the core.
6-3 Discussion on results

In this section, observations and deductions from the results are discussed. First, the results of the implementation of clusters are discussed. Next, the individual results of each of the solution concepts are discussed and compared to each other. Finally, a critical note on the use of the coalitional game theory solution concepts in this particular case is made.

Clustering

When looking closer at the results presented in Section 6-2-1, several deductions can be made immediately. First, the implementation of clustering decreases the computation time drastically. The computation time for calculating the Shapley value for a set of 20 small BSs is 1600 seconds for single BSs; 0.7 seconds for clusters of 2; and 0.01 seconds for clusters of 5 BSs. The difference in computation time between single BSs and clusters of 5 BSs is a factor of 160000. This factor increases exponentially with the number of BSs. If the Shapley value was applied to 117 BSs this factor would be enormous. The energy efficiency on the other hand does not decrease with such factors. The difference in energy efficiency for 20 BSs between the calculation for single BSs and clusters of 5 BSs is only 1.5%. This difference becomes a constant difference when the number of BSs is increased. Therefore, applying clustering for the Shapley value is an excellent method to reduce the computation time, while minimising the decrease in energy efficiency.

When the results of clustering with the nucleolus are investigated, clustering is even more beneficial than for the Shapley value. The computation time for calculating the nucleolus for a set of 20 small BSs is 500 seconds for single BSs; 0.01 seconds for clusters of 2 BSs; and 0.001 seconds for clusters of 5 BSs. The difference in computation time between single BSs and clusters of 5 BSs is a factor of 500000. It can be concluded that in this case the computation time of the nucleolus is much lower than that of the Shapley value. The energy efficiency of the clusters among 5 BSs is only 1% lower compared to the situation with single BSs, when 20 BSs are included in the calculation. Just as with the Shapley value, this difference becomes constant when the number of BSs is increased. Again, clustering is an excellent approach to reduce the computation time of the nucleolus, while minimising the energy efficiency reduction.

Finally, the results of the core when clustering is implemented are investigated. The computation time for calculating the core is lower than for the nucleolus and the Shapley value. For calculating the core for a set of 20 small BSs the computation time is only 3 seconds; for clusters of 2 it is 0.006 seconds; and for clusters of 5 BSs it is 0.004 seconds. The computation time of the application of clusters of 5 is larger compared to the nucleolus, contrary to the expectations since the situation without clusters and clusters of 2 of the core have faster computation times. This difference can either be explained by inefficient coding or possibly by different running circumstances, as different background programs active during simulations. The energy efficiency of the approach with clusters of 5 BSs is only 1.5% lower compared to the approach with single BSs. Again, this difference in energy efficiency becomes constant with increasing number of BSs.
It can be thus concluded that clustering is a great method for decreasing the computation time of the coalitional game theory solution concepts. Furthermore, the decrease in energy efficiency is relatively small when clustering is applied and seems constant with an increasing number of small BSs.

**Solution concepts applied to whole cellular network**

In the previous subsection it was concluded that using clusters of 5 BSs drastically reduces the computation time while the decrease in energy efficiency is relatively small. Therefore, clusters of 5 small BSs were chosen for the computation of the energy efficiency of the whole cellular network.

The results of the simulations when determining the final coalition with each of the solution concepts are shown in Table 6-2 and Figures 6-4, 6-5, and 6-6. Each of the solution concepts has significantly increased the energy efficiency of the cellular network.

The results of the Shapley value show an increase in energy efficiency between 21-121% over time. The computation time of the simulation was 18 hours and 45 minutes. The total average energy efficiency increase of the cellular network using the Shapley value solution approach with clusters of 5 small BSs is 48.3% compared to the situation in which all the small BSs are active.

When the nucleolus solution concept is applied to determine the final coalition, the results concerning the energy efficiency are exactly the same as the results of the Shapley value. The energy efficiency increases between 21-121%. The total average energy efficiency increase of the cellular network using the nucleolus as solution with clusters of 5 small BSs is 48.3% compared to the situation in which all the small BSs are active. The only difference is the computation time. The computation time of the nucleolus is 5 hours and 27 minutes, which is significantly less than the Shapley value.

It makes sense that the final coalition found with the Shapley value is the same as with the nucleolus. The final coalition of the Shapley value is the collection of clusters with the highest Shapley values. This implies that these clusters contribute the most to every possible coalition. Therefore, this collection of clusters also minimises the maximum excess. However, it does not imply that if there would be a transferable utility, the partitioning of payoffs among the players would be the same for both solution concepts.

Finally, when the concept of the core is used to determine the final coalition among the small BSs, an energy efficiency increase between 20-121% is achieved. The total average energy efficiency increase of the cellular network is 47.5% compared to the situation when all small BSs are active. The computation time of the core is 1 hour and 24 minutes. One main drawback of the core is that it can be non-unique. The number of possible solutions of the core during these simulation ranges per time step \( t \) between 1 and 449. The results presented in Figure 6-6 and Table 6-2, show the average of all these solution for every time step \( t \). An interesting finding is that the most energy-efficient solution within the core at every time step \( t \), is the same as the Shapley value and the nucleolus. When for each time step \( t \) the solution with the lowest energy efficiency within the core is applied, the energy efficiency would increase with 47.1%.
The energy efficiency of the three solution concepts at \( t \in \{4, 5, \ldots, 10\} \) is the same. This is caused by the fact that the macro BSs has enough available capacity to serve the users of all the small BSs. Therefore, all the solution concepts have the same final coalition, which is the grand coalition of all small BSs.

When all the approaches are compared with each other, the best performing solution concept is the nucleolus. It is the best performing solution concept because together with the Shapley value it has the highest energy efficiency increase, which was the goal of this thesis. Furthermore, the nucleolus has a smaller computation time than the Shapley value. The second best performing solution concept is the Shapley value due to the longer computation time compared to the nucleolus. The core had the lowest energy efficiency increase and therefore comes third. However the core was the solution with the shortest computation time. Table 6-3 shows the key results of the simulations, comparing the three solution concepts with each other.

One note concerning the the results of the computation time needs to be made. The computation time strongly depends per computer and the active background programs. Furthermore, it can occur that one of the solution concepts is programmed in a less efficient way than the others. Therefore, the results presented in this thesis only give an idea on how the computation times of the solution concepts differ from each other.

<table>
<thead>
<tr>
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<th>Shapley value</th>
<th>Nucleolus</th>
<th>Core</th>
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<tr>
<td>Energy efficiency [bit/J]</td>
<td>355.1</td>
<td>355.1</td>
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<tr>
<td>Relative energy efficiency increase [%]</td>
<td>48.3</td>
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</tr>
<tr>
<td>Computation time [hours]</td>
<td>18.8</td>
<td>5.5</td>
<td>1.4</td>
</tr>
</tbody>
</table>

**Table 6-3:** Key results of the simulations for determining the final coalition using the three different solution concepts.

**Critical note**

While the results of each of the solution concepts is a great improvement on the energy efficiency of the cellular network, the solution concepts are not optimal to use for real cellular networks. First, the computation time of each of the solution concepts is too long. Small BSs start up in seconds from sleeping mode. Therefore, a control algorithm that has to efficiently control a network of BSs needs to work with far smaller time steps than an hour, as used in this thesis.

Furthermore, finding the most energy efficient collection of small BSs can be achieved much easier in this case study. The output of the cellular network model is the power consumption of the macro BS at every time step \( t \) and the power the macro BS would need to take over any small BSs at every time step \( t \). The power needed by the macro BS to take over a group of small BSs is simply the sum of the power needed for each individual BS. So when the energy needed to take over each individual small BSs is aligned in increasing order, the most
energy-efficient coalition would be obtained by simply adding the small BSs one by one in this order until the maximum capacity of the macro BS is reached. When this is applied to the same cellular network without the use of clusters, the overall energy efficiency of the cellular network would increase with 51.8%. The computation time to find this coalition was 0.03 seconds.

This thesis shows that coalitional game theory can significantly increase the energy efficiency of a cellular network. However, the methods used are highly inefficient due to computational complexity.

6-4 Conclusions

This chapter has studied the results obtained by the simulations with the implementation of the solution concepts. The results of the simulations when implementing clusters among the small BSs show that clustering drastically decreases the computation time of all three solution concepts, while the energy efficiency does have a slight decrease of around 1%. Therefore, it has been decided to use clusters of 5 small BSs for the simulations with the three solution concepts.

The results of the simulations with the three different solution concepts are promising. Each of the solution concepts increases the energy efficiency enormously. Each solution concept has a peak energy efficiency increase of over 120% compared to the situation when all small BSs are active. The energy efficiency improvements of the solution with the Shapley value and the nucleolus are the same with an average energy efficiency increase of 48.3%. Both yield a slightly better energy efficiency than the core. The nucleolus requires a smaller computation time than the Shapley value and is therefore the best solution concept in this case.

The results of the simulations show that coalitional game theory can increase the energy efficiency of cellular networks significantly. However, due to the computational complexity of the solution concepts they are not the best solutions for finding the most energy efficient collaboration among small BSs. Another approach for finding this collaboration in this case study was found with an energy efficiency increase of 51.8% with a far smaller computation time than the coalitional game theory solution concepts.
Chapter 7

Conclusions and future work

This chapter discusses the conclusion of this master thesis. Furthermore, it gives some suggestions for future work concerning this research.

7-1 Conclusions

The research question of this thesis is: Can coalitional game theory increase the energy efficiency of cellular networks? The goal of this thesis is to answer this question by means of a case study. The energy efficiency is measured in the amount of data transmitted divided by the amount of energy consumed \([\text{Bit/J}]\). An urban cellular network was modelled with two types of base stations (BSs): small BSs and a macro BS. The different power consumption profiles of the two types of BSs make it possible to increase the energy efficiency of the cellular network. The macro BS has a linearly increasing power consumption with increasing data demand, while the small BS has an almost constant power consumption with increasing data demand. Therefore, some of the small BSs can be underutilised, i.e. they use only a little of their maximum transmission capacity while consuming their peak power consumption. In these cases, it can be more energy efficient to put these BSs in sleeping mode and serve their users by the macro BS. However, due to safety regulations the macro BS has a limited transmission capacity. Therefore, the key is to find the most energy-efficient coalition of small BSs.

To find this optimal coalition of small BSs, coalitional game theory is applied. Three solution concepts within coalitional game theory have been investigated: the Shapley value, the nucleolus, and the core. Each of these solution concepts has its own approach and is based on different mathematical concepts. Because of these differences, the solutions of the three concepts do not always coincide. Furthermore, each concept has its own benefits and drawbacks. The Shapley value is a solution based on fairness. It is the average marginal contribution a player has to every possible coalition among the players. The solution concept makes use of the superadditivity assumption and therefore implies that the grand coalition will form. Each
player will get his Shapley value as partition of the payoff of the grand coalition.

The nucleolus is based on dissatisfaction, which is measured by the excess of a coalition, where the excess of a coalition is the value of a coalition minus the sum of individual payoffs to all the players within that coalition. The nucleolus is the imputation that minimises all the excesses of all the possible coalitions except for the grand coalition in a non-increasing order.

The core is based on stability. A stable solution implies that no player or group of players wants to deviate from this solution. The solution of the core is the collection of imputations in which there are no sub-coalitions that can more profitably deviate from these imputations. One of the drawbacks of the core is that it can be non-unique.

An urban cellular network is built to test the hypothesis whether coalitional game theory can be used to increase the energy efficiency of cellular networks. An urban network was chosen, since it guarantees the two requirements for this test: a large fluctuating data demand profile and a big set of BSs. The case study network consists of one macro BS and 117 small BSs, providing the simulated area with mobile data. Within the simulated area there are crowded and less crowded areas with mobile users. The users have a dynamic data demand profile. The output of the model is the power consumption of the macro BS and the power consumption the macro BS would need to take over any small BSs at any time step $t$.

The coalitional game theory solution concepts use this output to test whether they can increase the energy efficiency of the cellular network. But first, one drawback all of the solution concepts have must be overcome. The computational complexity with a large number of players within a game. To overcome this problem clusters of small BSs can be formed. A cluster of small BSs then acts as one single player in the game: If the cluster goes into sleeping mode, all the small BSs within that cluster go into sleeping mode. The case study showed that clustering drastically decreases the computation time of the solution concepts. However, in general it could also decrease the energy efficiency, compared to the situation when all the small BSs are considered as single players. The results of the case study showed that there is merely an 1% energy efficiency decrease between the situation with clusters of 5 small BSs and with single small BSs considered as individual players. Therefore, the case study continued with clusters of 5 small BSs.

The three solution concepts are then used to determine the most energy efficient coalition among the 117 small BSs. Each of the three solution concepts induces an enormous increase in energy efficiency. The Shapley value and the nucleolus yield the same results in terms of energy efficiency. They increase the energy efficiency with 48.3% compared to the situation when all the small BSs are active. The energy efficiency increase induced by the solution of the core is 47.5%. The computation time of the nucleolus is 5.5 hours, which is around one third of the computation time of the Shapley value. Therefore, the nucleolus is considered as the best solution concept.

Although the solution concepts used in this thesis have shown to enormously increase the energy efficiency of cellular networks, due to their computational complexity they are not suited to be implemented in real cellular networks. There are simpler and quicker methods to find the most energy efficient coalitions of small BSs.
7-2 Future work

This section proposes several directions for further research.

Extension of the cellular network model

The cellular network model can be extended in multiple ways. The three key points in which the model can be extended are:

- **Cellular network infrastructure.** At this moment the cellular network consists of two types of BSs: the macro BS and the small BS. Due to the increasing demand of data, mobile network operators (MNOs) are extending their network by implementing new types of BSs within their current cellular network. Therefore, current heterogeneous cellular networks have around 4-5 different types of BSs, each with their own active range and power consumption profile. The model could be extended with more types of BSs.

  Furthermore, the deployment of the BSs can be improved. Extensive research has been undertaken in the strategic placement of BSs within a heterogeneous network. The strategic placement of new BSs can significantly increase the energy efficiency of the network [10, 17]. In the cellular network model used in this thesis, the small BSs are contiguously placed in the extra dense areas of the model, without paying attention to the energy efficiency. Therefore, it could be that the energy efficiency of the network can be increased when the small BSs would be strategically rearranged.

- **Including imperfect information.** At this moment the cellular network model gives precise information on the location and data demand of the users at any time step \( t \). With this information the power needed by the macro BS to take over any small BSs can be determined exactly. However, state-of-the-art distance estimation between users and BSs has an error of around 4% [56]. Therefore, the power needed by the macro BS also suffers from uncertainty. This situation of imperfect information of the location of each user could be implemented in the model. Within game theory, imperfect information is often used when modelling games. Therefore, this would be an interesting addition to the cellular network model.

- **Improve the dynamic data demand profile.** The current data demand profile consists of the modelled users in the simulated area and their data demand. For this thesis the EARTH statistics were used [20] to model the data demand. However, the EARTH report was published in 2010 and predicted an average mobile data consumption of 2.1Gb per mobile user in Western Europe in 2015. At the end of 2016 Ericsson [25] published that the average data use in Western Europe in 2016 was 2.7Gb. This is an increase of 28% compared to the prediction of the EARTHs report in one year. Furthermore, Ericsson indicated that the compound average growth rate of mobile data between 2016-2022 will be 45% annually [25]. This increase in data consumption will have a huge impact on the energy consumption of the cellular network and should therefore be updated in a new cellular network model.

  Furthermore, the model of simulated users can be improved. At this moment the users are spread over the simulated area with crowded and less crowded areas. The overall
Conclusions and future work

population density equals the population density of a dense urban area. However, in these crowded and less crowded areas the population density does not change during the simulated day. The model could be improved if the population density in these areas changes during the day. For instance, during the day a higher population density in an office district could be considered, while during the evening the residential areas would be more crowded.

Change the optimisation goal

The goal of this thesis is to find the most energy efficient coalition of small BSs. Therefore the payoff function solely depends on the energy savings each coalition has to the whole cellular network. Furthermore, the power consumption profile of the macro BS increases linearly with the data demand. The combination of these two dynamics make the game in this thesis a convex game. This convexity makes the solutions of the game theory solution concepts non distinctive, since they give the same answer. It would be interesting if the solutions of the three concepts would be different. To achieve this, the optimisation goal can be changed in a way that the utility function is not convex anymore. Potential ways to achieve this are to give some small BSs extra priority to put them both combined into sleeping mode; or add restrictions that some BSs cannot be put into sleeping mode at the same time because of coverage problems; or to give extra priority to large adjacent groups of BSs to go into sleeping mode, because this situation could needs less control. These dynamics can make the utility function non-convex and therefore the solutions could be different. This could cause situations in which the solution of the Shapley value is the most energy efficient solution, but since it is not in the core this would be an unstable solution.

Network sharing among MNOs

Another interesting follow up on this thesis would be to expand the cellular network model to a network with multiple MNOs. In such a network two or more MNOs are active, each with their own cellular network. In this situation the MNOs could collaborate with each other by sharing their networks in order to decrease their overall energy costs. This can be achieved in several ways.

The first method has various similarities with the method used in this thesis. In this method two or more MNOs (for simplicity let us assume MNO 1 and 2) look for the most energy-efficient collaboration of small BSs of all their small BSs combined. This coalition is then served with data by the macro BS of MNO 1 and the coalition of small BSs goes into sleeping mode. The MNO 2 of which the users are now served by MNO 1, pays roaming costs to MNO 1. The roaming costs have to be calculated in such a way that both MNOs profit from the energy saving costs.

Another method is that the small BSs of the one MNO take over the users of the other MNO. In this situation an underutilised small BS of the one MNO can take over the users of a small BSs of the other MNO, when they are active in the same area. Similar to the previous method, the MNO that sets its small BS in sleeping mode pays roaming costs to the other. A combination of the two methods mentioned above is also possible. This situation of network sharing among MNOs would be very interesting to model and solve with game theory. It has multiple extra dimensions compared to the situation with one MNO. In this case, money
could be considered as transferable utility so that not only the final coalition is of interest, but also the division of payoffs. Furthermore, players can hold back some crucial information to strengthen their position in the game. Moreover, the order of how the game is played is important. An MNO can either decide to first create the most energy efficient coalition within its own network before contacting the other MNO, but this might reduce the total benefit. Alternatively if the MNO first approaches the other MNO it could give away unnecessary important information to the other MNO that might negatively affect the given MNO. All these different decision possibilities and uncertainties are common situations that can be included in a game. This situation lends itself better to apply game theory to than the situation in this thesis, since it models actual people (MNOs) who are in conflict with each other aiming for the highest payoff, in this case money. It is expected that the different solution methods used in this thesis have more distinctive solutions when they are applied to the multiple MNO situation.
Appendix A

Bondareva’s theorem applied to the musicians’ game

In this appendix Bondareva’s theorem is applied to the musicians’ game to give a better insight in how the theorem is applied to a game.

**Bondareva’s theorem** [7] (1962,1963). The core is nonempty if and only if for every minimal balanced systems of weights \( \{\lambda_S\}_{S \in \mathcal{P}} \), one has

\[
\sum_{S \in \mathcal{P}} \lambda_S v(S) \leq 1
\]

where \( \mathcal{P} \) is the set of all the coalitions among the players \( N \) except for the grand coalition.

The minimal balanced families of the Musicians’ game are: 1.\{G, S, D\}, 2.\{G, SD\}, 3.\{S, GD\}, 4.\{D, GS\}, 5.\{GS, SD, GD\} as was introduced in Section 3-4. The corresponding minimal balanced systems of weights are given in Table A-1:

<table>
<thead>
<tr>
<th>Balanced system of weights (\lambda_S)</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>G</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>S</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>D</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>GS</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>1/2</td>
</tr>
<tr>
<td>GD</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>1/2</td>
</tr>
<tr>
<td>SD</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>1/2</td>
</tr>
</tbody>
</table>

**Table A-1:** The minimal balanced systems of weights of the musicians’ game with a guitarist (G), singer (S) and drummer (D).
Now assume that for this zero-one normalised three-person game \(v(\{\text{GS}\}) = a, v(\{\text{GD}\}) = b, v(\{\text{SD}\}) = c\), where \(0 \leq a, b, c \leq 1\). Furthermore, the zero-one normalisation implies that \(v(\{\text{G}\})=v(\{\text{S}\})=v(\{\text{D}\})=0\). According to Bondareva’s theorem the following conditions have to be satisfied to prove that the core is nonempty:

\[
\lambda_{\text{G}} v(\{\text{G}\}) + \lambda_{\text{S}} v(\{\text{S}\}) + \lambda_{\text{D}} v(\{\text{D}\}) \leq 1
\]

\[
1 \cdot 0 + 1 \cdot 0 + 1 \cdot 0 \leq 1
\]

\[
\lambda_{\text{G}} v(\{\text{G}\}) + \lambda_{\text{SD}} v(\{\text{SD}\}) \leq 1
\]

\[
1 \cdot 0 + 1 \cdot a \leq 1
\]

\[
\lambda_{\text{S}} v(\{\text{S}\}) + \lambda_{\text{GD}} v(\{\text{GD}\}) \leq 1
\]

\[
1 \cdot 0 + 1 \cdot b \leq 1
\]

\[
\lambda_{\text{D}} v(\{\text{D}\}) + \lambda_{\text{GS}} v(\{\text{GS}\}) \leq 1
\]

\[
1 \cdot 0 + 1 \cdot c \leq 1
\]

\[
\lambda_{\text{GS}} v(\{\text{GS}\}) + \lambda_{\text{SD}} v(\{\text{SD}\}) + \lambda_{\text{GD}} v(\{\text{GD}\}) \leq 1
\]

\[
\frac{1}{2} a + \frac{1}{2} b + \frac{1}{2} c \leq 1
\]

Due to the zero-one normalisation only the last inequality has to be checked to test whether the core has a nonempty core. In other words, the inequality

\[
a + b + c \leq 2
\]

is not only a necessary but also sufficient condition for existence of a nonempty core.


S. Elderenbosch Master of Science Thesis


[38] Y. Shoham M. O. Jackson, K. Leyton-Brown. Game theory, online course on coursera, 2016.


