"Travel time errors in shortest route predictions and all-or-nothing assignments: theoretical analysis and simulation findings"

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Abstract: Travel time has been proven to be the most important criterion in car travel route choice. Because of this, most route prediction models applied in transportation planning only use travel time as criterion variable. In this paper the sensitivity of shortest-time route predictions is analysed in relation to errors in the link travel time input.

Firstly, a theoretical analysis is given, based on probability theory. Expressions are derived which enable the effects to be studied and quantitatively assessed. These refer to:
- the probability that a specific route alternative will be the shortest route;
- the travel time estimate of the shortest route;
- the probability of finding the true shortest route.

Especially, the typical problem of correlation between alternatives is dealt with.

On the basis of these theoretical findings expectations are formulated as to the effects of travel time errors upon all-or-nothing assignment outcomes. Secondly, results are presented from simulations: these are repeated shortest route predictions and traffic assignments in a randomized network. The urban road network of the city of Eindhoven (The Netherlands) is the empirical basis for this simulation. Most interesting from this analysis is the result that because of the minimizing of the shortest route principle the introduction of random, unbiased input errors leads to systematic deviations in the output. Whereas the shortest travel time is only slightly affected, the route pattern as well as other traffic estimates derived from this, are much more sensitive to travel time input changes.

The findings of this study can help interpreting the inaccuracies in traffic assignment predictions. In addition, given the desired accuracy of particular travel predictions the necessary quality of travel time measurements of the network can be derived.
1. INTRODUCTION.

The shortest time paths in transportation networks play an important role in spatial research and planning. These paths, which are determined on the basis of the minimum travel time route choice criterion, are used in route choice modelling and are crucial in each transportation analysis. In addition, they give the estimates of the shortest travel times between points in space necessary as input in, for example, interaction modelling and accessibility analysis.

In some types of studies the shortest travel path is so important that all movements are assumed to use these paths. With for example so-called all-or-nothing traffic assignments all trips between an origin and a destination are assigned to the shortest path and estimates of link loads are derived from these path loads. Even if the shortest but one path is only one second longer it will not be considered in the analysis and will for example get no single trip in traffic assignment. It may be expected that results of such analyses, especially the computed travel paths themselves, will be very sensitive to measurement errors in the link travel time input.

The measurement or estimation of travel times (however defined) in a transport network is a difficult task and will always be subject to errors relative to some true value. Such deviations may stem from various sources, such as inaccurate performance of time or speed measurements, insufficient number of observations (sampling), inaccurate definitions and computation rules.

This paper deals with the consequences of such data errors for route choice prediction, shortest travel time estimation and traffic assignments. As far as possible the analysis is performed via a theoretical derivation which is fully based on probability theory. This approach follows the same line of thought as the theory of discrete choice models, and is partly similar to analyses done in the context of the development of stochastic assignment models [8]. In addition, the effects of travel time errors are studied experimentally using simulations. These repeated route choice calculations and all-or-nothing traffic assignments refer to evening peak car travel in the city of Eindhoven (The Netherlands).
2. TRAVEL TIME AS A RANDOM VARIABLE.

2.1. Definitions.

The travel time of a route in a network is the sum of the travel times of the constituent links. The measured travel time on a particular link \( i \) can be considered as a random variable defined as follows (random variables are underlined):

\[
\begin{align*}
\mathbf{t}_i &= \mathbf{t}^0_i + e_i \\
\mathbf{t}_i &= \text{measured link travel time} \\
\mathbf{t}^0_i &= \text{objective or true link travel time} \\
e_i &= \text{travel time error} \\
i &= \text{link index}
\end{align*}
\]

The travel time error \( e_i \) obeys a probability distribution with expectation \( E(e_i) \), assumed to be zero, and variance \( V(e_i) = \sigma_i^2 \). Consequently, the travel time on a particular route \( j \) is also a random variable having the following characteristics:

\[
\begin{align*}
\mathbf{T}_j &= \sum_{i \in J} \mathbf{t}_i = \mathbf{T}^0_j + \varepsilon_j \\
\mathbf{T}^0_j &= \text{objective or true route travel time} \\
\varepsilon_j &= \text{route travel time error} \\
J &= \text{set of links forming route } j \\
j &= \text{route index}
\end{align*}
\]

The travel time error \( \varepsilon_j \) obeys a probability distribution with expectation \( E(\varepsilon_j) \) and variance \( V(\varepsilon_j) = \sigma_j^2 \).

The summation of link travel times implies:

\[
\begin{align*}
\mathbf{T}^0_j &= \sum_{i \in J} \mathbf{t}^0_i \\
E(\varepsilon_j) &= \sum_{i \in J} E(e_i) \quad \text{assumed to be zero} \\
V(\varepsilon_j) &= \sum_{i \in J} V(e_i) + \sum_{i, i \in J} \text{Cov}(e_i, e_i) \quad i \neq l
\end{align*}
\]
2.2. Correlation.

The covariance term \( \Sigma \text{Cov}(\varepsilon_i, \varepsilon_j) \) represents the effect of possible correlation (positive or negative) between the errors of different links of the same route. In the following this type of correlation has been assumed to be negligible.

Thus, it can be written:

\[
\sigma_j^2 = \sum_{i \in J} \nu_i^2
\]

A more serious form of dependency is that between routes: travel times of different routes are not always independent because of overlapping route parts. The common links lead to a positive correlation between travel time errors.

If the overlapping part of two routes \( k \) and \( j \) between the same origin and destination is denoted with \( \Delta \) than the covariance of these errors is:

\[
\text{Cov}(\varepsilon_k, \varepsilon_j) = \text{E}(\varepsilon_k - \Delta \varepsilon_j + \varepsilon_{- \Delta}) = \text{E}(\varepsilon_{- \Delta}) = \nu(\varepsilon_{- \Delta})
\]

The error covariance of two routes thus equals the error variance of the common part. The coefficient of correlation of the travel time errors, a measure that will be used in the sequel, can now be expressed as:

\[
\rho_{k,j} = \frac{\text{Cov}(\varepsilon_k, \varepsilon_j)}{\nu(\varepsilon_k) \cdot \nu(\varepsilon_j)} = \frac{\nu(\varepsilon_{- \Delta})}{\nu(\varepsilon_k) \cdot \nu(\varepsilon_j)} = \rho_{\Delta}
\]

Normally, the error variance is proportional to the route length, that is:

\[
\nu(\varepsilon) = c \cdot T^*
\]

If, in addition, the alternative routes are nearly equal, i.e. \( T_k^* = T_j^* \)

then it is: \( \rho_{k,j} = \frac{T_k^*}{T_j^*} \)

In other words, the correlation between two routes can be approximated as the proportion of their overlap to the total route length.
2.3. Error distributions.

Concerning the type of error distributions to be used in our analysis a difference is made between the theoretical and the experimental part. In the following probability analysis we directly start from route travel time distributions without concern about the constituting link distributions. According to the statistical theory of extremes, which is basic to our problem of minimizing route travel time, the relevant distributions can be classified into three classes [1].

The first class consists of functions of the exponential type, such as the normal and the double-exponential distribution. In the second class there are the so-called Cauchy-type distributions which have much longer tails. The third group are bounded distributions, such as the lognormal and Weibull-functions.

Since we deal with distributions of measurement errors we can confine ourselves to the first and third class.

From the viewpoint of their extremal properties the functions belonging to the same class have many important properties in common: if you know one, you know them all. As a representative of the first class we take the double-exponential function, which looks very much like the normal and has the same extremal properties in the asymptotic case. It is preferred because of its easy mathematical tractability pertinent to our problem which results in easy interpretable closed-form formulae. The double-exponential probability distribution function is defined as follows [2]:

\[
F(x) = 1 - \exp\left[-\exp(x - a)/b\right] \quad -\infty < x < +\infty \\
\quad b > 0
\]

\[
E(x) = a - \Psi \cdot b \\
\Psi = \text{Euler's constant} = 0.577
\]

\[
V(x) = \sigma^2 = \frac{b^2 \cdot \pi^2}{6}
\]

The constants \( a \) and \( b \) are the location and dispersion parameters respectively.

On the same grounds as before we choose the Weibull function as a representative of the third class in favor of e.g. the lognormal function. The Weibull probability distribution function is defined as follows [2]:
\[ F(x) = 1 - \exp\left(-\left(\frac{x}{c}\right)^d\right) \quad \alpha < x < \infty \]
\[ E(x) = c\Gamma(1 + \frac{1}{d}) \quad c, d > 0 \]
\[ V(x) = \sigma^2 = c^2\left[\Gamma(1 + \frac{2}{d}) - (1 + \frac{1}{d})^2\right] = \left\{ E(x) \cdot d^* \right\}^2 \]

Typically, the dispersion parameter \( d \) determines the location of the expectation and vice versa, which means that the standard deviation is proportional to the expectation.

In our simulations we only need to define link error functions. Here, the normal distribution was chosen, with a maximum error of 2 standard deviations in order to assure positive travel time values. As long as route travel times are not too small the corresponding route function is sufficiently close to the two functions assumed in the theoretical analysis in order to enable comparisons of the theoretical and experimental findings.

3. PROBABILITY DISTRIBUTION OF SHORTEST TIME.

3.1. General.

Consider an origin-destination pair with a finite number of \( N \) alternative routes between them (Fig. 1). The measured travel time on path \( j \) is a stochastic variate \( T_j \) with expectation \( E(T_j) = T_j^* \) and variance \( \sigma_j^2 \).

![Fig. 1: Choice situation of alternative routes between 0 and D](image)

The question to be analysed here is what shortest time may be expected given a set of stochastic route travel times \( T_j \) (Fig. 2). This shortest travel time is defined as follows:
\[ T_{\min} = \min_j \{ T_j \} \quad j = 1, N \]

The minimum of a set of random variables is again a random variable. The main concern of this section is to establish the probability distribution of this variable \( T_{\min} \).

Concerning the expectation of \( T_{\min} \) a very general result can be derived (which holds also in case of correlations, etc.) on the basis of Jensen's inequality [3]:

\[ E[\min(T_j)] \leq \min_j \{ E(T_j) \} \]

that is:

\[ E[T_{\min}] \leq T^* \quad T_{\min} = \text{true shortest time} \]

![Fig. 2: Probability distributions of route travel times and their minimum.](image)

Thus, the expectation of the minimum of the inaccurate travel times \( T_j \) is always smaller than (or at most equal to) the true shortest time \( T^*_{\min} \).

If considering a large set of origin-destination pairs in a network with inaccurate travel times then this finding means that on the average in these cases the minimum route travel time will be less than the true shortest time.

We can formulate this important finding as follows: random deviations in link travel times will normally lead to systematic deviations in the esti-
mated shortest travel times.
As Figure 2 illustrates this underestimation is only absent if the error
variances tend to zero or if there are large differences in travel time
between the true shortest route and the other routes.
In the following sections the shortest time distribution is analysed more
closely for a number of relevant cases. The exact shortest time distribu-
tion will be derived and the magnitude of the underestimation will be
related to the travel time measurement error and other factors.
In the first case we assume the travel time errors of the various route
alternatives to be independently and identically distributed according to
a double-exponential function. The second case also assumes independency
between alternatives, but now the stochastic travel times follow a Weibull
function. Finally, attention will be devoted to the situation with corre-
lation between the alternatives.

3.2. Independent and identical error distributions.

Let us assume the following route choice situation:
- N statistically independent route alternatives
- additive stochastic error term $\varepsilon$
- double-exponential error distribution [see section 2.3.], equal for each
alternative, defined as follows:

$$ T_j = T_j^* + \varepsilon $$

$$ F(x) = \Pr[\varepsilon < x] = 1 - \exp[-\exp(x-a)/b] $$

$$ \begin{align*}
E(\varepsilon) &= 0 \\
E(\varepsilon) &= \sigma^2
\end{align*} \Rightarrow \begin{align*}
\sigma &= 0.577 b \\
\sigma &= \frac{\sqrt{6}}{\pi}
\end{align*} \quad \forall_j $$

$a = \text{location parameter}$

$b = \text{dispersion parameter}$

The probability distribution of the measured route travel time is then [1]:

$$ F(T_j < x) = 1 - \exp[-\exp(x - a - T_j^0)/b] $$

$$ \begin{align*}
E(T_j) &= T_j^0 \\
V(T_j) &= \sigma^2
\end{align*} \quad \forall_j $$
The corresponding distribution function of the minimum travel time now is [1]:

\[ G(T_{\text{min}}) = \Pr(T_{\text{min}} < x) = 1 - \prod_{j=1}^{N} \left[ 1 - F(x - T^*_j) \right] \]

\[ = 1 - \exp \left\{ -\exp \left[ x - a + b \log \sum_{j=1}^{N} \exp \left( -T^*_j / b \right) \right] / b \right\} \]

This result reveals that the minimum of a set of independent and identical double-exponential distributions is again a double-exponential distribution with a shifted location parameter but same dispersion parameter. Expectation and variance of this minimum-time distribution are thus (see also [4]):

\[ E(T_{\text{min}}) = -b \log \sum_{j=1}^{N} \exp \left( -T^*_j / b \right) \]

\[ V(T_{\text{min}}) = \sigma^2 = \frac{b^2 \pi^2}{6} \]

For easy interpretation the expectation can also be written as:

\[ E(T_{\text{min}}) = T^*_{\text{min}} - b \log \sum_{j=1}^{N} \exp \left\{ (T^*_{\text{min}} - T^*_j) / b \right\} \]

The characteristics of the minimum-time distribution are completely defined by those of the time distributions of the various route alternatives:

- the resulting bias \( E(T_{\text{min}}) - T^0_{\text{min}} \) depends on the error variance and on the differences in objective travel times between the shortest and the other alternatives.
- the variance in minimum time is equal to the error variance.
- Typically, the bias depends on the number of alternatives \( N \), whereas the variance of the minimum time is independent of this number.

Let us examine the systematic shift in the minimum time values more closely for the following two cases.

1. If it is assumed that all alternatives have the same expected (objective) travel time \( T^*_j = T^*_{\text{min}} \), then the expected minimum time is:

\[ E(T_{\text{min}}) = T^*_{\text{min}} - b \log N \]
The expectation of the bias is in this case at maximum, namely \(-b \log N\).

2. On the other hand, by assuming that the objective travel times of the alternatives are equally spaced in the so-called choice region, then we have:

\[
E(T_{\min}) = T^*_\min - b \log \sum_{j=1}^{N} \exp\left(-\frac{j-1}{N-1} \cdot \frac{4\pi}{\sqrt{6}}\right)
\]

The expected bias will be much smaller now. The choice region is defined as the subset of all those route alternatives that are competitive with the objectively shortest alternative under the given error distributions. The travel time range of this region is the difference between the smallest objective time \(T^*_\min\) and the largest objective travel time \(T^*_\max\) of the alternative that still can be chosen. This range is roughly 4 times the standard deviation (see Figure 3).

![Diagram showing equal spacing of competitive alternatives in the 'choice region'.](image)

Fig.3: Equal spacing of competitive alternatives in the 'choice region'.

The mathematical expressions derived so far give rise to the following interesting conclusions:

a. minimum time estimates are directly affected by errors in the travel times of the various route alternatives:
   - the systematic underestimation \(E(T_{\min}) - T^*_\min\) of the true shortest time is nearly proportional to the standard error (parameter \(b\));
   - the variance in minimum time estimates is equal to the error variance and independent of the number of alternatives;
b. the more the true travel times of the alternatives are alike the larger
the bias will be;

c. the larger the number of relevant alternatives the larger the bias will
be.

To give an impression of the magnitude of the bias we take the average
route travel time in Eindhoven, that is 7 minutes, and assume a relative
standard error of 10%. The dispersion parameter \( b \) is then about 0.5.
In the case of ten nearly like alternatives the expected shortest travel
time will be:

\[
E(T_{\min}) = 7 - 0.5 \log 10 = 5.85 \text{ min}
\]

This is an average underestimation of about 16%.
If we assume ten evenly distributed alternatives in the choice region this
expectation is:

\[
E(T_{\min}) = 7 - 0.5 \log \prod_{j=1}^{10} \exp[-0.57(j-1)] = 6.6 \text{ min}.
\]

Thus, the average underestimation is now about 6%.

3.3. Independent equal relative variance error distribution.

Another interesting case is to use the so-called Weibull function (see
section 2.3.) as the probability distribution of travel time estimates of
a particular route [5]. From an empirical point of view this seems to be
more satisfactory since travel times are now always positive and errors
are proportional to the true travel time. Now a multiplicative error term
is assumed:

\[
\begin{align*}
T_j &= T_j^0 \cdot \xi \\
F(\xi < x) &= 1 - \exp\left[-\left(\frac{x}{c}\right)^d\right] \\
E(\xi) &= 1 + \frac{1}{\Gamma(1 + \frac{1}{d})} \\
V(\xi) &= \sigma^2
\end{align*}
\]

\[
\forall j \quad \left\{ \begin{array}{l}
c = 1/\Gamma(1 + \frac{1}{d}) \\
d = f(\sigma)
\end{array} \right.
\]
The probability distribution function of the travel time of route \( j \) is then:

\[
F(T_j < x) = 1 - \exp\left\{-\left(\frac{x}{T_j^o \cdot c}\right)^d\right\} \\
E(T_j) = T_j^o \\
V(T_j) = (T_j^o \cdot \sigma)^2
\]

\( \forall j \)

In the same way as before the distribution function of the minimum time can be derived:

\[
G(T_{\min}) = 1 - \exp\left\{-\left(\frac{x}{c \cdot T_j^o}\right)^d \cdot \left(\frac{1}{T_j^o}\right)^d\right\}
\]

Also in this case the type of distribution remains the same under the minimization operation.

Whereas the location parameter changes from \( c \cdot T_j^o \) to \( \frac{c}{(E\left(\frac{1}{T_j^o}\right))^d}^{1/d} \), the relative variance remains the same.

The expectation and variance are thus:

\[
E(T_{\min}) = \left\{ \frac{1}{T_j^o} \left(\frac{1}{T_j^o}\right)^d \right\}^{1/d}
\]

\[
V(T_{\min}) = \left\{ E(T_{\min}) \cdot \sigma \right\}^2
\]

For easy interpretation the expectation of the minimum route time can also be formulated as:

\[
E(T_{\min}) = \frac{T_{\min}^o}{d \sqrt{T_j^o \left(\frac{T_{\min}^o}{T_j^o}\right)^d}}
\]

This shows that: \( 0 < E(T_{\min}) < T_{\min}^o \)

In the special case that the objective travel times of all alternatives are very alike, the relative underestimation becomes:

\[
\frac{E(T_{\min})}{T_{\min}^o} = \frac{1}{\sqrt{N}}
\]
All these expressions show that the same conclusions can be drawn regarding the effects of travel time input error, number of alternatives etc. upon the resulting minimum time as with the double-exponential distribution. The only difference is that travel time ratios instead of differences between alternatives are now the relevant quantities. If we apply the Weibull-formulae in the computational examples roughly the same estimates of the bias result.

3.4. Correlation between alternatives.

An important aspect of stochastic route-choice analysis is the interdependency between error distributions of alternatives since routes may use common links. Such an overlap always leads to positive correlations between route characteristics which influence the choice probability significantly [7]. Only in very simple situations it is possible to derive closed-form analytical expressions describing the minimum time distribution in the case of presence of correlation. Intuitively, however, it is to be expected that the form and variance of this distribution are not affected by correlation but that the resulting bias will be smaller than in the case of independent alternatives. This can be exemplified for the case of 3 alternatives where two of them are mutually correlated (Fig. 4).

![Fig. 4: Correlated route alternatives due to overlap.](image)

If these alternatives have identical double-exponential error distributions the expectation of the minimum route time is [6]:

\[
E(T_{\text{min}}) = -b \log\left\{ \exp \frac{-T_1^*}{(1-\rho)b} + \exp \frac{-T_2^*}{(1-\rho)b} + \exp \frac{-T_3^*}{b} \right\} \quad \text{if} \quad \rho < 1
\]

With increasing correlation coefficient \( \rho \) (see also Section 2.2.) this expression increases and the bias thus diminishes. As the formula shows,
correlation damps the effect of measurement errors upon minimum time estimation. This implies that overlap between alternative routes is an agreeable circumstance when estimating minimum route travel time from inaccurate link travel times.

3.5. Conclusion.

In the case of minimum route prediction the presence of random measurement errors in the criterion variable leads to a number of peculiar phenomena regarding the minimum time estimates:
- these estimates always show an underestimation bias, which is roughly proportional to the measurement error,
- the larger the number of alternatives the larger this bias,
- correlation between alternatives leads to a smaller bias,
- the variance of the minimum time estimate roughly equals that of the measurement error.

4. PROBABILITY THAT ALTERNATIVE \( k \) IS THE SHORTEST.

4.1. General.

In this section we will investigate in what way and to what extent travel time errors will influence the probability that a particular alternative will be chosen as the shortest one. For this purpose we analyse the same cases as in the previous section. Let us again assume an origin-destination pair and a finite number of \( N \) alternative routes between them. Then, the probability of route \( k \) being the shortest is defined as:

\[
p_k = \Pr \{ T_k < T_j ; \forall \ k \neq j \}
\]

A simple method to derive this probability is, for example, to use the following partial derivative [3]:

\[
p_k = \frac{\delta E(T_{\text{min}})}{\delta T_k}
\]
4.2. **Independent and identical error distributions.**

Assume again for each of the \( N \) alternatives a statistically independent and identical double-exponential distribution of the travel time error \( \varepsilon \) with zero expectation and variance \( \sigma^2 \) (see Section 3.2.). Under these assumptions the probability \( p_k \) that alternative \( k \) will be the shortest route can be expressed by the well-known logit formula [4]:

\[
p_k = \frac{\exp(-T_k^*/b)}{\sum_j \exp(-T_j^*/b)} \quad \forall k, \quad j = 1, N
\]

where the error variance parameter \( b \) equals \( \sigma \sqrt{6/\pi} \).

For easy interpretation of the effects of the error upon this probability this formula will be rewritten: first, in terms of travel time differences between the examined and the other alternatives, secondly, in terms of travel time differences between the objectively shortest and each alternative:

\[
p_k = \frac{1}{1 + \sum_j \exp \left\{ (T_k^* - T_j^*)/b \right\}} = \frac{\exp \left\{ (T_k^* - T_j^*)/b \right\}}{\sum_j \exp \left\{ (T_k^* - T_j^*)/b \right\}}
\]

These expressions show that the larger the errors (i.e. increasing parameter \( b \)) the more each alternative is likely to be chosen as the shortest. If the error variance becomes infinitely large all alternative routes are equally likely to be the shortest, irrespective of their differences in objective travel time: \( \lim_{b \to \infty} p_k = \frac{1}{N} \forall k \).

On the other hand, if all variances tend to zero only that alternative will be chosen for which all \( (T_k^* - T_j^*) \) -values are negative. This only holds, as expected, for the objectively shortest route:

\[
b \lim_{b \to 0} p_k = \begin{cases} 1 & \text{if } k = \text{objectively shortest route} \\ 0 & \text{if } k = \text{all other routes} \end{cases}
\]

If the individual error distributions would not be identical it is plausi-
ible (also from the structure of the formulae) that the larger the error variance of a particular alternative is the larger the probability of this alternative will be (at the cost of the others), ceteris paribus. It can also be seen that the error variance and the travel time differences have opposite effects upon the shortest route probability: one might say, the presence of errors diminishes the travel time differences.

From these formulae we can learn that under the given assumptions the probabilities are governed by the objective travel time differences, and that a constant systematic error does not exert any influence.

4.3. Independent equal relative variance error distributions.

Now we assume for all alternatives statistically independent Weibull distributions of the error all having the same relative variance (coefficient of variations, see Section 3.3.). Under these assumptions the probability \( p_k \) is given by the well-known Kirchhoff law [5]:

\[
p_k = \frac{\left( \frac{1}{T^*_k} \right)^d}{\sum_{j=1}^{N} \left( \frac{1}{T^*_j} \right)^d}, \quad d > 0
\]

where the parameter \( d \) is inversely \((1)\) proportional to the error variance \( \sigma^2 \).

Also here we rewrite this expression to enable easier interpretations.

\[
p_k = \frac{1}{\sum_{j \neq k} \left( \frac{T^*_k}{T^*_j} \right)^d} = \frac{\left( \frac{T^*_k}{T^*_j} \right)^d}{\sum_{j} \left( \frac{T^*_k}{T^*_j} \right)^d}
\]

Characteristically, the probabilities are now governed by the travel time ratios. Also here, the expressions show the expected impact of the travel time errors upon the shortest time probability: the influence is the same as in the previous case. This can also be demonstrated by writing the Kirchhoff-law in logit-form:

\[
p_k = \frac{\exp(-d \log T^*_k)}{\sum_{j} \exp(-d \log T^*_j)}
\]
4.4. Correlation between alternatives.

Only in very simple situations one can derive closed-form analytical expressions for the shortest route probability which take account of correlation between alternatives. As has been explained before (Section 2.2.) this correlation is always positive. In the most simple case of two partly overlapping alternatives it is possible to analyse the effect of correlation using the probability distribution of the travel time difference. If the two alternatives have error variances of \( \sigma_1^2 \) and \( \sigma_2^2 \) respectively, the variance of the travel time difference amounts to:

\[
\sigma_{1-2}^2 = \sigma_1^2 + \sigma_2^2 - 2\rho_{12} \sigma_1 \sigma_2
\]

where \( \rho_{12} = \text{correlation coefficient} \)

\( 0 < \rho_{12} < 1 \)

The probability of each alternative is indicated in the figure 5 below:

![Figure 5: Effect of correlation on choice function for two alternatives.](image)

This shows that the stronger the correlation between the alternatives the smaller the variance of the difference, and thus the higher the probability of the objectively shortest route. Correlation exerts a variance-reducing effect and diminishes the negative impact of travel time errors upon shortest route prediction.

Some further insight can be gained from the case with three alternatives where two of them are overlapping (Fig. 4).

Assuming again identical double-exponential error distributions (see Section 3.2.) the following expression can be derived [6]:

\[
P_k = \begin{cases} 
\exp\left(-\frac{\tau}{k^2 (1-p) b}\right) \frac{\exp\left(-\frac{\tau}{(1-p) b}\right) + \exp\left(-\frac{\tau}{(1-p) b}\right)}{\exp\left(-\frac{\tau}{(1-p) b}\right) + \exp\left(-\frac{\tau}{(1-p) b}\right)} & \text{if } k = 1, 2 \\
\exp\left(-\frac{\tau}{k^2 (1-p) b}\right) & \text{if } k \neq 3
\end{cases}
\]

where \( \tau = \cdot \tau \cdot \cdot \cdot \)
One can immediately see that increasing correlation leads to a smaller denominator in the probability function which implies that the independent alternative (k=3) becomes more probable; consequently the probabilities of the two correlated alternatives both diminish. These findings can be made more clear when considering the probability ratios:

\[
\log \frac{p_1}{p_2} = \frac{T^*_{2} - T^*_{1}}{b(1 - \rho)}
\]

\[
\log \frac{p_1}{p_3} = \frac{T^*_{3} (1 - \rho)}{b(1 - \rho)} - \rho \log \prod_{i=1}^{2} \exp\left(-\frac{T^*_{i}}{b(1 - \rho)}\right)
= \frac{T^*_{3} - T^*_{1}}{b} - \rho \log 2 \quad \text{if } T^*_{1} = T^*_{2}
\]

These formulae give rise to the following conclusions regarding the effect of correlation in the case of travel time errors (ceteris paribus):
- correlation between alternatives reduces the probability of these alternatives in favor of the independent routes;
- correlation mitigates the effect of travel time errors;
- correlation increases the probability of the objectively shortest route.

In other words, if one does not take into account the presence of correlation in predicting route choice a too high probability would be assigned to overlapping routes.

4.5. **Probability of true shortest route.**

What is the probability of predicting the true shortest route in case of measurement errors in the route travel times? This is a relevant question, particularly in transportation planning, since the answer can help to determine the estimation accuracy of all kinds of traffic predictions. However, a simple answer can only be given for situations satisfying strong assumptions. The most important factor influencing this probability is the difference between the objective travel times of the truly shortest and the other alternatives.

An extreme case is the situation where all alternatives are independent and have nearly equal objective travel times $T^*$. In this case, it can be safely assumed that the corresponding error distributions are also nearly
equal. The probability of finding the true shortest route is then only a function of the number of alternatives, and is thus independent of the magnitude of the error:

\[ P_{\text{true}} = \frac{1}{N} \quad P_{\text{wrong}} = 1 - P_{\text{true}} \]

Mostly, however, the alternative routes will overlap to some extent which can lead to a still smaller but even so to a somewhat higher probability. Assuming, on the other hand, that the objective travel times of the alternatives are equally spaced in the so-called choice region (see Section 3.2.) all having nearly equal double-exponential error distributions, then the following expression results:

\[ P_{\text{true}} = \frac{1}{1 + \frac{N}{\prod_{j=2}^{N} \exp \left( -\frac{j-1}{N-1} \cdot \frac{4\pi}{\sqrt{6}} \right) }} \]

If there were an error variance such that 10 alternatives are competitive, (see the corresponding example in Section 3.2.) then the probability to find the true shortest route is about 40% in this case.

4.6. Conclusion.

In order to assess the impact of travel time errors upon route choice prediction it is necessary to distinguish three choice situations: the case with nearly equal alternatives, the case with only one real alternative (the objectively shortest) and the cases in between. It appears that in the first two cases the errors do not affect the probability of a route to be chosen as the shortest. Only in the third group of situations the predictions of route choice are sensitive to the magnitude of the travel time errors: the larger the error, the smaller the probability to find the true shortest route (ceteris paribus). Partly, this effect stems from an increase in the number of competitive alternatives. Correlation between alternatives tempers the effect of errors and favours the objectively shortest route.

Whereas the estimation of the shortest time is always affected by the presence of errors, that is in each of the three abovementioned cases (see Section 3), the impact on the choice probabilities is relatively limited.
5. EXPECTED REAL TRAVEL TIME ON PREDICTED SHORTEST ROUTES.

5.1. Questions.

In the preceding analyses we have looked at the prediction of minimum time routes as well as at the estimation of the minimum travel time $T_{\text{min}}$ in the case of presence of inaccurate travel time values. In assessing traffic assignment applications and other types of traffic predictions it is relevant to know the true travel time value that is to be expected on the inaccurately estimated shortest routes. A comparable question relates to the travel distance along this path compared to that along the true shortest path. By way of example, what are the true costs (in terms of travel time, distance or combination of both) of travel along signposted routes if these are determined on the basis of minimizing travel time which is subject to measurement errors? Corresponding questions refer to shortest time routes for emergency transport (fire-brigade, etc.).

In order to answer such questions we will now look at the objective (true) travel time of the predicted routes, that is that time that will be experienced in reality when using this path. Results derived for this objective variable will, to a large extent, also hold for other objective route variables if these are strongly correlated with travel time. This holds in particular for travel distance, and to a less extent to number of links of a route. The latter variable plays an important role in linking the findings to link load estimates.

5.2. Analysis.

Let us denote the real travel time of the estimated shortest time route by $T_s^\ast$. Since the shortest of a set of discrete route alternatives can be considered as the outcome of a random process the corresponding real travel time is even so a discrete random variable. Thus, $T_s^\ast$ follows a discrete probability distribution (see e.g. fig. 6):

In general, this distribution is defined by the following expression:

$$p(T_s^\ast = T_k^\ast) = p_k$$

$p_k$ = probability that alternative $k$ is the shortest (see Section 4)
Fig. 6: Real travel time probability distribution of predicted shortest routes when travel time measurement errors are present.

The expectation of the real travel time is thus:

$$E(T^*_s) = \sum_k p_k \cdot T^*_k$$

It can easily be shown that:

$$E(T^*_s) > T^*_{\text{min}}$$

This means that, on the average, the real travel time on the estimated shortest path is always larger than, or at least equal to, the true shortest time. If, for example, a system of route guiding has been set up based on shortest time estimations the experienced travel times will on the average be larger than intended and than is necessary.

Because of the strong correlations between objective travel time and other objective characteristics of routes, such as distance and number of links, this result implies that estimates of these characteristics using shortest route calculations will generally be positively biased too.

Also, traffic estimates derived from these characteristics such as vehicle mileage and link loads will therefore show, in general, an overestimation in the case of random travel time input errors.

The degree of overestimation depends on the set of alternatives at hand. If there is only one real alternative, or, if all alternatives are nearly like (in terms of objective travel time), than:

$$E(T^*_s) = T^*_{\text{min}}$$

In these cases estimates of objective shortest route characteristics are unbiased.
If we assume (see also Section 3.2.) that the objective times of the alternative routes are equally spaced in the choice region then the following relationship holds:

$$E(T^*_s) = T^*_\text{min} + \frac{4 \sigma}{N-1} \sum_{j=1}^{N-1} \frac{\exp\left(-\frac{j-1}{N-1} \cdot \frac{4 H}{\sqrt{6}}\right)}{\exp\left(-\frac{j-1}{N-1} \cdot \frac{4 H}{\sqrt{6}}\right)}$$

The so-called excess time that will be experienced in reality relative to the true shortest time, that is \(E(T^*_s) - T^*_\text{min}\), increases linearly with the travel time time input error \(\sigma\) and depends on the number of alternatives.

If we take as a computation example the same figures for \(\sigma\) and \(N\) as in Section 3.2., the expected real travel time will be:

$$E(T^*_s) = 7 + \frac{4.07}{9} \sum_{j=1}^{N-1} \exp\left(-0.57(j-1)\right)$$

This excess time is thus about 3.6%.

The expressions presented above as well as this numerical example show that the effects of random travel time input errors upon the excess time \(E(T^*_s) - T^*_\text{min}\) are relatively moderate, and much smaller than the effects upon the minimum time estimation bias \(T^*_\text{min} - E(T^*_\text{min})\).

What interests much more, however, in most cases is the difference between the estimated time value \(T^*_\text{min}\) and the really experienced time \(T^*_s\) when using the estimated shortest path. On the average, this difference is:

$$E(T^*_s - T^*_\text{min}) = E(T^*_s) - E(T^*_\text{min})$$

In the case of our first example of ten nearly like alternatives (see section 3.2.) this time difference amounts to:

$$E(T^*_s - E(T^*_\text{min})) = T^*_s - T^*_\text{min} + b \log N = 0.5 \cdot \log 10 = 1.15 \text{ min}$$
This is a difference of about 16% relative to the true shortest time. In our second example of ten evenly distributed alternatives this difference amounts to $6\% + 3.6\%$ that is about 10%.

These numerical examples show that already small measurement errors (in this case a relative standard error of only 10%) can lead to a significant bias in shortest time estimates relative to the experienced values. The relationships between predicted, true and experienced travel times are depicted in Fig. 7.

![Diagram showing relationships between predicted, true and experienced travel times.](image)

Fig. 7: Relationships between predicted, true and experienced travel times of minimum time routes based on erroneous travel times.

6. SIMULATION APPROACH OF TRAVEL TIME SENSITIVITY ON A NETWORK BASIS.


The presented theoretical analysis indicates that the results of shortest route calculations will be sensitive to errors in the input travel times: predicted shortest routes will not always follow the true paths and predicted shortest times are, on the average, underestimations of the true interzonal travel times. The analysis was based on simple situations. In addition, it referred to one single zone pair. Assessing these consequences in realistic situations and on the level of a whole network, however, is not a simple task because of the complexity of a real-world transportation network: the number of relevant alternatives, the degree of correlation, the travel time differences between alternatives will differ strongly between zone pairs.
In order to test the empirical validity of the theoretical analysis and to quantify the effects on a network scale we performed simulations in an urban road network. These simulations are repeated shortest route calculations and consecutive trip assignments to these routes. In each simulation run all link travel times were randomly varied. The network is that of the city of Eindhoven (200,000 inh.) with travel time and trip data referring to the evening peak period.

6.2. Simulation distributions.

Basic to the simulation is the generation of a set of link travel times for each network link. These random link times have the following characteristics:

a. additive error term:

\[ t_i = t_i^0 + e_i \quad \forall i \]

\[ t_i^0 = \text{simulated time} \]
\[ t_i^0 = \text{objective time} \]
\[ e_i = \text{time error} \]
\[ i = \text{link index} \]

The simulated time \( t_i \) represents the inaccurate measurements; the objective or true time \( t_i^0 \) is the input to the simulation and is the best possible measurement available.

b. Travel time errors of different links are statistically independent.

c. The error \( e_i \) follows a normal distribution with zero expectation and variance proportional to the expected time \( t_i^0 \):

\[ E(e_i) = 0 \quad \forall i \]
\[ V(e_i) = \sigma^2 \cdot t_i^0 \quad \forall i \]

Apart from the fact that this proportionality is empirically plausible it also guarantees that the variance of the resulting route travel times is independent of the network structure and number of links.

Since route travel times are a sum of link travel times the following relationships thus hold:

\[ T_j = \sum_{i \in J} t_i = \sum_{i \in J} t_i^0 + \sum_{i \in J} e_i = T_j^0 + \sum_{i \in J} e_i \]
\[ E(T_j) = T_j^o \]

\[ V(T_j) = V \left( \sum_{i \in J} T_j^c \epsilon_j \right) = \sum_{i \in J} V(\epsilon_j) = \sum_{i \in J} \epsilon_j^2 \cdot T_j^o = c \cdot T_j^o \]

The resulting inaccurate route travel times are also normally distributed with the same proportional variance. Having these characteristics the error distribution used in the simulations lies in between the double-exponential and Weibull distributions used in the theoretical analysis.

6.3. Simulation data.

The value of the variance parameter \( c \) was generally set equal to 0.09 minutes. This value had, however, to be adapted for short links in order to guarantee positive time values. In addition, the maximum possible error was set equal to two standard deviations. As a result, the percentual root-mean square error for all links of the network (N=2490) appeared to be 36%. This may be considered as a moderate measurement error in the case of car travel times on individual links.

On a network-wide basis the average \( c \)-value can be computed to be about 0.045. This value will be used in assessing the route travel time variances.

Five simulation runs have been performed, that means, for each link five travel times have been randomly selected from the link-specific travel time distribution.

Accordingly, five sets of shortest routes are determined between all origin-destination pairs (168 zones). Finally, the set of origin-destination trips is assigned to each set of shortest paths in order to assess the effects of travel time errors upon traffic estimates.

Below we give an impression of the meaning of the assumed link time error in relation to the route travel times.

The average route travel time of a car trip in Eindhoven is about 7 minutes. The percentual standard error in this case is then about \( \sqrt{0.045/7} = 8\% \). The smallest possible random route time is then 7-2.0.08.7 = 5.9 minutes, and the largest 8.1 minutes.

If the objectively shortest time is \( T_{min}^o \), how long might the objective time of an alternative route maximally be in order to be competitive?
If we denote the maximum with $T^*_a$ the following equation holds:

$$T^*_a + 2 \sqrt{c \cdot T^*_a} = T^*_a - 2 \sqrt{c \cdot T^*_a}$$

This maximum time is approximately:

$$T^*_a = T^*_a + 4 \sqrt{c \cdot T^*_a} = T^*_a + 0.85 \sqrt{T^*_a} \quad c = 0.045$$

If we take the average route $T^*_\text{min} = 7 \text{ min.}$, then the longest competitive alternative will have about 9.5 minutes objective travel time. All routes having an objective time within this range of four standard deviations is a potential shortest route, and will be called a relevant alternative.

In the following sections on simulation results we confine the presentation to those items directly related to the theoretical results derived in the preceding sections.

7. ROUTE SIMULATION RESULTS.

7.1. Estimated shortest travel time.

The analysis of the random shortest route time is confined to a random sample of 488 internal zone pairs. In order to facilitate the analysis the variable of interest is normalized into the travel time ratio of the simulated and the objective shortest route: $T_{\text{min}} / T^*_\text{min}$, thus expressing all relevant quantities in unit travel time which can be interpreted as percentages. The results of the five simulations form together a frequency distribution of time ratios with average 0.95 and standard deviation 0.10 (Fig. 8). The ratios found range from about 0.5 to about 1.5.

![Fig. 8: Simulation results of predicted and true minimum travel times.](image)
Averaged over the whole network the inaccurate shortest route time is 95% of the corresponding true shortest time. As could be expected from the theoretical analysis (see Section 3), the objective shortest time between origin and destination is underestimated on the average, in this case by about 5%. The size of this estimation error is within the same range as the theoretical expectations. The percentual standard deviation of the estimations of the shortest time is about 10% on a network-wide scale.

Interesting insights can be gained when stratifying the normalized shortest times into time classes (Table 1). Because of the chosen proportional error the (percentual) standard deviation of the predicted shortest times (Col. 3) decreases with increasing route travel time. The size of these standard deviations prove to be in good agreement with what might be expected on the basis of the theoretical relationships established before (see Section 3), namely that the variance of the minimum time estimates is about equal to the variance of the input travel time errors:

$$\sigma_{T_{\text{min}}} = \sigma_T = \sqrt{c \cdot T_{\text{min}}^D}$$

Table 1: Classification of shortest route predictions according to time classes (internal zone pairs, N= 488).

<table>
<thead>
<tr>
<th>Objective route travel time [min]</th>
<th>Travel time ratio $T_{\text{min}} / T'_{\text{min}}$ average</th>
<th>Number of observations No.</th>
<th>Pred. of true route %</th>
<th>Identity of pred. routes %</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>5</td>
</tr>
<tr>
<td>0-3</td>
<td>0.97</td>
<td>0.18</td>
<td>265</td>
<td>69</td>
</tr>
<tr>
<td>3-6</td>
<td>0.95</td>
<td>0.10</td>
<td>705</td>
<td>44</td>
</tr>
<tr>
<td>6-9</td>
<td>0.95</td>
<td>0.07</td>
<td>780</td>
<td>32</td>
</tr>
<tr>
<td>9-12</td>
<td>0.96</td>
<td>0.06</td>
<td>550</td>
<td>24</td>
</tr>
<tr>
<td>&gt; 12</td>
<td>0.95</td>
<td>0.05</td>
<td>140</td>
<td>13</td>
</tr>
<tr>
<td>Total</td>
<td>0.95</td>
<td>0.10</td>
<td>2440</td>
<td>39</td>
</tr>
</tbody>
</table>
Surprisingly, the percentual bias (Col. 2) is nearly constant (5%) across the time classes. On the basis of the formulae developed before a decreasing percentual bias was expected with increasing route lengths. One of the possible explanations is that the structure of the choice situation changes with the interzonal distance, that is, ceteris paribus: more relevant alternatives and less correlation between alternatives with increasing distance.

7.2. Shortest route choice.

A comparison between the paths of the simulated and the true shortest routes gives figures about the choice probability of the true shortest route (Col. 5) in case of measurement errors. At the network-wide level and for internal pairs nearly 40% of the randomly generated routes appeared to have exactly the same spatial pattern as the objective shortest route (these identical paths may, however, have different simulated travel times!).

As Table 1 shows, the percentage of correct predictions of the shortest paths diminishes strongly with increasing interzonal distance. This decrease can only partly be explained as a consequence of the increasing input error variance. Thus, these figures also indicate changes in the choice situation. Presumably, the number of relevant alternatives increases with increasing distance.

The same phenomenon can be observed when looking at the identity of simulated shortest routes with each other (Table 1, Col. 6). The number of fully identical simulated paths is lower than in the comparison with the true route, but the dependency from the interzonal travel time is the same (see Fig. 9).

![Diagram](image)

**Fig. 9**: Distance dependency of error sensitivity of predicted shortest routes.
The spatial pattern of the simulated routes has been investigated in more
detail using a special sample of 232 origin-destination pairs. The sample
includes internal and external zone pairs. First, it has been analysed to
what extent simulated routes use the same links as the objectively short-
est path. The degree of overlap is fairly high: on the average, a link
from the true shortest path is also present in 3,6 of the maximally five
simulated paths. In other words, the probability that a link from the true
shortest path will be selected in a simulated path between the same origin
and destination is about 71%. It appears that this high percentage is
strongly influenced by that of the external relations. In this category
there is nearly no choice at the cordon end of the route even if there
were large changes in travel times: in many cases four or even all(five)
of the simulated paths are identical with the true shortest path (the
travel times, however, will be different).

It will be clear from this that the mutual overlap between simulated paths
will be large too: a link of a simulated path will, on the average, also
be present in 1,4 of the four other simulated paths. This is a probabili-
ty of 35% that a link selected from one random path will also be part of
another random path. If we would restrict this analysis to internal zone
pairs only, these figures would be significantly smaller.

From these detailed route-by-route comparisons as well as other analyses
(see following section on load predictions) it appears that the composi-
tion of the simulated paths according to link type is significantly
different from the true shortest route. The estimated paths use much more
secondary roads and often take a short cut.

7.3. Real travel time on estimated routes.

Using again the sample of 488 internal zone pairs it appears that the real
travel time on the predicted shortest routes is generally higher than the
true shortest time, namely by about 5%. This figure indicates the per-
centual excess time that will be experienced in reality when using the
predicted route in case of travel time measurement errors (see Section 5).

The expected difference between estimated shortest time $E(T_{\text{min}})$ and real
travel time $E(T^*)$ is even twice this value, namely about 10%. (see Fig.
10).
Fig. 10: Relationships between estimated and true travel times on predicted shortest routes in Eindhoven simulations.

7.4. Conclusions.

The results of the simulations confirm the findings derived from probability theory. Random errors in network travel times lead to corresponding errors in minimum interzonal travel time estimates. In addition, however, they result in an average underestimation of the true shortest time. Apart from the input error this bias depends on the network structure of the choice situation. The simulation results suggest significant differences in this respect between short and long routes. Averaged over all zone pairs in the Eindhoven network the size of the percentual bias might be approximated by the following rule-of-thumb:

\[ \text{% - minimum time bias} = 0.5 \times \text{average % - standard error in interzonal times}. \]

If the shortest paths predicted with inaccurate travel times are really used (e.g. for emergency transport or because of signposting of these routes) then the really experienced travel time is longer than estimated. As a first guess of this experienced time bias (expressed as a percentage of the true shortest time) we might derive from our limited empirical findings the following rule-of-thumb:

\[ \text{% - experienced time bias} = \text{average % - standard error in interzonal times}. \]

Apart from the inaccurate time estimates the spatial pattern of the pre-
dicted shortest routes is affected too. As will be shown, also in this respect systematic deviations result from random, unbiased input errors. Since shortest time travel data are crucial input to various analysis procedures, such as interaction modelling, modal choice prediction and traffic assignment, the estimation results of these models will be affected in some or other way.

8. TRAFFIC ASSIGNMENT OUTCOMES.


In the preceeding chapter effects of travel time errors have been studied in relation to shortest route characteristics of separate zone pairs. Now we will go a step further and deal with the sensitivity of all-or-nothing traffic assignment outcomes. This means that all zone pairs have to be considered simultaneously and that the coincidence of shortest routes from different zone pairs at the same link plays a role. In addition, the effects will be dependent upon the way the total number of assigned trips is distributed among all zone pairs. The travel time sensitivity of the assignment procedure will be confined to load outcomes. First, we concentrate on network load totals, and then deal with load estimates for individual links. This sensitivity analysis has been performed by simply assigning the same trip matrix to each of the five sets of simulated shortest routes.

8.2. Network load totals.

Total network load expressed in estimated load hours can be defined in two equivalent ways, as follows:

\[(8.2.1.) \quad Q_h = \sum R_{ij} \cdot T_{ij} \quad \text{where:} \quad Q_h = \text{total estimated load hours} \]

\[(8.2.2.) \quad = \sum R_{ij} \cdot V_{l} \cdot T_{l} \quad \text{where:} \quad R_{ij} = \text{number of trips between zones i and j} \]

\[T_{ij} = \text{shortest interzonal travel time} \]

\[V_{l} = \text{load on link l} \]

\[T_{l} = \text{travel time on link l} \]
All random variables are underlined. The random error term originally introduced in the link travel time \( t_{ij} \), first leads to random shortest interzonal travel times \( T_{ij}^* \). As a consequence of all-or-nothing trip assignment to the corresponding shortest time paths, the resulting link loads \( V_{ij} \) are also random variables.

From equation 8.2.1, it is clear that the bias inherent in the predicted interzonal travel times \( T_{ij}^* \) (see Section 3) will lead to a negative bias in the estimated load hours \( Q_{ih}^* \). This total network bias is a weighted sum of the interzonal travel time biases, since:

\[
E(Q_{ih}^*) = \frac{1}{I_j} R_{ij} \cdot E(T_{ij}^*)
\]

and

\[
E(Q_{ih}) - Q_{ih}^* = \frac{1}{I_j} R_{ij} \left\{ E(T_{ij}) - T_{ij}^* \right\}
\]

\( T_{ij}^* = \text{objective shortest interzonal travel time} \)

The resulting bias thus depends on the joint distribution of both the number of trips and the shortest time bias over all zone pairs. Table 2 presents the estimated network load hours of each of the five simulations as a percentage of the assignment outcome resulting with the true travel times. These results are classified by link type.

**Table 2**: Estimates of simulated vehicle hours as a percentage of objective all-or-nothing vehicle hours, by link type and total.

<table>
<thead>
<tr>
<th>Link type</th>
<th>Simulation run 1 (%)</th>
<th>Simulation run 2 (%)</th>
<th>Simulation run 3 (%)</th>
<th>Simulation run 4 (%)</th>
<th>Simulation run 5 (%)</th>
<th>Objective time (veh.hrs)</th>
</tr>
</thead>
<tbody>
<tr>
<td>FC 1</td>
<td>95.6</td>
<td>93.1</td>
<td>95.4</td>
<td>93.1</td>
<td>95.5</td>
<td>4159</td>
</tr>
<tr>
<td>FC 2</td>
<td>104.4</td>
<td>108.8</td>
<td>102.4</td>
<td>98.4</td>
<td>101.5</td>
<td>1899</td>
</tr>
<tr>
<td>FC 3</td>
<td>91.2</td>
<td>92.4</td>
<td>94.5</td>
<td>99.9</td>
<td>94.7</td>
<td>1110</td>
</tr>
<tr>
<td>Total</td>
<td>97.3</td>
<td>97.1</td>
<td>97.1</td>
<td>95.5</td>
<td>97.0</td>
<td>7168</td>
</tr>
</tbody>
</table>

Note: FC 1: primary roads
FC 2: secondary roads
FC 3: connectors
In general, total load hours calculated on the basis of inaccurate link travel times is underestimated by about 3%. This value seems relatively stable among the simulations. The degree of underestimation is somewhat smaller than was found with a sample of (internal) interzonal route times. (Section 7.1). This stems from the inclusion of external routes which have a much smaller time bias, as well as from weighting with the number of trips.

There appears to be a systematic difference in sensitivity between the various link types. The load hour outcomes on the primary roads show on the average, a 6% underestimation, whereas on the secondary roads an average overestimation of 3% results. Presumably, this stems from the fact that the network of secondary roads is much larger than that of the primary roads, about twice as large. Given a certain route part, there exist therefore much more alternatives over the secondary network than over primary roads.

Estimates of total load kilometers (see Table 3) show a reverse reaction:

<table>
<thead>
<tr>
<th>Link type</th>
<th>run 1 (%)</th>
<th>run 2 (%)</th>
<th>run 3 (%)</th>
<th>run 4 (%)</th>
<th>run 5 (%)</th>
<th>Objective time (veh.km)</th>
</tr>
</thead>
<tbody>
<tr>
<td>FC 1</td>
<td>97.7</td>
<td>94.9</td>
<td>97.8</td>
<td>100.1</td>
<td>99.6</td>
<td>199.139</td>
</tr>
<tr>
<td>FC 2</td>
<td>116.3</td>
<td>127.2</td>
<td>117.0</td>
<td>111.1</td>
<td>116.1</td>
<td>68.561</td>
</tr>
<tr>
<td>FC 3</td>
<td>99.9</td>
<td>101.1</td>
<td>100.8</td>
<td>101.6</td>
<td>100.6</td>
<td>33.878</td>
</tr>
<tr>
<td>Total</td>
<td>102.2</td>
<td>103.0</td>
<td>102.5</td>
<td>102.8</td>
<td>103.4</td>
<td>301.578</td>
</tr>
</tbody>
</table>

on the average, there is now an overestimation of about 3%. This estimation bias is in accordance with the theoretical expectations valid for objective route variables such as distance (see Section 5).

Also for this quantity there are noticeable differences between the link types: considerable overestimations (by about 20%) for the secondary roads, and relatively small underestimations for the primary roads (ca. 3%).
On the basis of these tables and earlier findings we can give the following explanation for the effects of travel time errors:

- The predicted shortest routes are, on the average, somewhat shorter (in measured time) than the objectively shortest route;
- In objective terms (e.g. in number of links, distance or true travel time) however, these routes are, on the average, longer than the true shortest route: they make a detour. For, shorter than the true shortest is not possible, but longer can.
- The false shortest routes will take less use of the (faster) primary roads, but will use more of the (slower) secondary roads: with increasing travel time errors secondary roads will have a relatively larger probability of being selected in the shortest routes since there are much more secondary roads.
- All other variables derived from incorrect and true route characteristics respectively, such as average trip length, total load hours, total load kilometrage, etc., will show corresponding deviations from the true values.

8.3. Individual link loads.

On the basis of the preceding results we may expect significant effects of the travel time variations upon individual link load estimates. The fact that the average estimated shortest route consists of about 5% more links, would have to lead to correspondingly higher link load estimates. In addition, however, the estimated shortest routes to a large extent do not coincide any more with the true shortest path, which may cause large differences in load predictions. For a random sample of 293 links the load differences between the inaccurate and true shortest path assignments have been established (see Table 4).

Indeed, there is a small systematic shift in link loads by about + 5%. The percentual root-mean-square-error is about 70%. This implies that most of the load differences are random and stem from random path deviations of the estimated routes with regard to the true shortest route. The original percentual error in link travel times of about 35% appears to lead to percentual load estimation errors of about twice this value.
Table 4: Average deviations in link loads between assignments with simulated times and with true travel times.

<table>
<thead>
<tr>
<th>assignment with</th>
<th>( \bar{V} ) (veh/2h)</th>
<th>( \Delta \bar{V} ) (%)</th>
<th>RMSE (veh/2h)</th>
<th>RMSE (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>run 1 times</td>
<td>769</td>
<td>+4.8</td>
<td>403</td>
<td>55</td>
</tr>
<tr>
<td>run 2 times</td>
<td>782</td>
<td>+6.4</td>
<td>547</td>
<td>76</td>
</tr>
<tr>
<td>run 3 times</td>
<td>784</td>
<td>+6.8</td>
<td>510</td>
<td>69</td>
</tr>
<tr>
<td>run 4 times</td>
<td>758</td>
<td>+3.3</td>
<td>411</td>
<td>56</td>
</tr>
<tr>
<td>run 5 times</td>
<td>783</td>
<td>+6.7</td>
<td>526</td>
<td>71</td>
</tr>
<tr>
<td>Obj. time</td>
<td>734</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Note: \( \bar{V} \) = average predicted link load (veh/2h)

\[
\begin{align*}
\Delta \bar{V} &= \frac{\bar{V}_{\text{sim}} - \bar{V}_{\text{obj}}}{\sum_{i=1}^{N} (V_{\text{sim}}^i - V_{\text{obj}}^i)^2} \\
\text{RMSE} &= \sqrt{\frac{\sum_{i=1}^{N} (V_{\text{sim}}^i - V_{\text{obj}}^i)^2}{N-1}} \\
\Delta \bar{V} &= \frac{\Delta \bar{V}}{\bar{V}_{\text{obj}}} \cdot 100 \\
\text{RMSE} [\%] &= \frac{\text{RMSE}}{\bar{V}_{\text{obj}}} \cdot 100
\end{align*}
\]

Another assessment of the effects of travel time variations is possible by comparing load predictions with traffic counts (see Table 5).

Table 5: Average deviations between predicted link loads and counts for simulated and true shortest time assignments, specified by road type.

<table>
<thead>
<tr>
<th>Assignment with</th>
<th>primary roads (N= 57)</th>
<th>secondary roads (N= 83)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( \bar{V} ) (veh/2h)</td>
<td>( \Delta \bar{V} ) (%)</td>
</tr>
<tr>
<td>run 1 times</td>
<td>1242</td>
<td>+12.2</td>
</tr>
<tr>
<td>run 2 times</td>
<td>1310</td>
<td>+18.2</td>
</tr>
<tr>
<td>run 3 times</td>
<td>1353</td>
<td>+22.1</td>
</tr>
<tr>
<td>run 4 times</td>
<td>1398</td>
<td>+26.2</td>
</tr>
<tr>
<td>run 5 times</td>
<td>1367</td>
<td>+23.4</td>
</tr>
<tr>
<td>Obj. time</td>
<td>1360</td>
<td>+22.9</td>
</tr>
<tr>
<td>Count</td>
<td>1107</td>
<td></td>
</tr>
</tbody>
</table>

Note: \( \bar{V} \) = average link load (predicted or counted) in veh/2h
\[ \Delta \bar{V} = \frac{\bar{V}_{\text{pred}} - \bar{V}_{\text{count}}}{\sqrt{\frac{\sum (V_{\text{pred}}^i - V_{\text{count}}^i)^2}{N - 1}}} \]

\[ \Delta \bar{V} \% = \frac{\Delta \bar{V}}{\bar{V}_{\text{count}}} \times 100 \]

\[ \text{RMSE} = \frac{\text{RMSE}}{\bar{V}_{\text{count}}} \times 100 \]

This comparison is done separately for primary and secondary roads. From the figures the following can be learned:

- there is a significant reduction in predictive performance when using inaccurate travel times. Compared with the 'true' all-or-nothing assignments the prediction error (RMSE) in loads for primary roads increases by about 20%, whereas for the secondary roads the error is more-or-less doubled.

- the systematic shift (overestimation) in load predictions due to the travel time errors appears to be particularly present in the secondary roads.

The relatively smaller sensitivity of the primary roads might stem from the following factors:

- since much more interzonal trips make use of primary roads there is a greater probability of cancelling out of route errors than at secondary roads.

- a considerable portion of the load on primary roads comes from external trips, the routes of which have a much smaller sensitivity to travel time errors.

- the predicted shortest routes use relatively more secondary road links than the true shortest routes.

9. SUMMARY.

An investigation has been presented dealing with the effect of random measurement errors in network travel times upon prediction of shortest times and shortest travel paths. Starting from probability theory a number of interesting theoretical results have been derived for a variety of circumstances. They can be summarized as follows:

- the probability of finding a different shortest travel path is strongly dependent upon the size of the input error in connection with the network structure of the choice situation(e.g. number of alternatives,
time differences between alternatives, overlap between alternatives);
- on the average, the predicted shortest time will be an underestimation.
The size of this bias is at least proportional to the input standard
error, while the proportionality factor depends on the network struc-
ture;
- the random error in the shortest time estimation is equal to that in
the input travel times.
- objective interzonal characteristics (e.g. distance) estimated on the
basis of shortest routes will be overestimated.

Formulæ have been derived with which the quantitative effects can be
assessed on the basis of a few assumptions.

Using simulations (repeated shortest route search and trip assignment in a
randomized network) the effects of time errors have been traced in a real-
world network. The simulation results confirm the theoretical findings
with respect to routes. Only individual link load estimates, especially
for secondary roads, are sensitive to time input errors. The other assign-
ment results, such as trip characteristics and load totals, show only
small but truly systematic deviations. Random travel time errors lead to a
shift towards higher load estimates for secondary roads.

The analysis shows that the minimization operation in shortest route
search, which is the basis of the prediction techniques considered, is the
real cause of the systematic shifts found in the route estimation and trip
assignment output. These findings can help in clarifying the inaccuracies
in traffic assignment predictions. In addition, when the desired accuracy
of particular travel predictions is given, the necessary quality of travel
time measurements of the network can be derived.

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