Creating a pricing mechanism for real-time balancing in sustainable electricity markets

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Creating a pricing mechanism for real-time balancing in sustainable electricity markets

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Abstract

This thesis considers the problem of designing a mechanism that efficiently regulates imbalances on the energy grid. Brokers aggregate supply and demand on the energy market by buying and selling power ahead of time. Mispredictions create discrepancies between supply and demand and need to be resolved. With the increasing usage of sustainable but often less predictable energy sources it is important that these imbalances are solved efficiently. Brokers can engage in load curtailment to create efficient ways of solving imbalances. In this thesis we design market-based mechanisms that make use of these capabilities. We define the desirable properties that a balancing mechanism should have and present several mechanisms that satisfy these. We also show that if brokers can actively participate themselves in the regulation it is impossible to create an efficient mechanism with all the desired properties as the problem becomes equivalent to a multi-unit double auction. Finally we identify an open problem in auction theory and propose a 2-item multi-unit auction.

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Preface

This thesis marks the end of my Masters in Computer Science. A question I have often heard is what my research topic have to do with computers. Well, I am not sure either but I do know that I find the more the theoretical parts of Computer Science fascinating.

First I would like to thank Mathijs de Weerdt for supervising me during my research. His knowledge of game theory and the construction of proofs have helped me greatly in writing this thesis.

Also I would like to thank my parents for their continuous moral support throughout my whole career as a student and my friends and colleagues at the University at Delft.

I hope that you enjoy the contents of this thesis.

Niels Egberts
Delft, the Netherlands
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Chapter 1

Introduction

In energy markets the balance between supply and demand has to be regulated continuously. Brokers that buy and sell energy on the energy market try to meet the demand of their customers, however it is not possible to perfectly predict the supply and demand. This thesis looks into the problem of how we can regulate imbalances in a cheaper and more sustainable way while still incentivizing brokers to accurately act upon their expected imbalances.

The contributions of renewable power resources such as hydro, wind and solar energy are becoming more important in our energy portfolio. The International Energy Agency has predicted that in 50 years most of the energy production will be from solar power generators. Renewable power resources are better for the environment but have the drawback that their energy output is not constant and may not be as predictable as say, a fossil-fuel power station. Especially the energy output of wind and solar power can change drastically and unpredictably even on the timescale of minutes.

Currently the regulation of imbalances on the grid is often done by a single party. Brokers that create shortages are penalized by this party to incentivize a better balance between supply and demand. With the advent of more and more renewable energy sources a paradigm shift is needed in the way grids are operated to keep costs and penalties within limits.

Smart grids make are a big research topic that makes it possible to gather and act on information received from the grid in real time. Brokers can then use load curtailment on the demand as well as on the supply side to regulate their own imbalances. Washing machines may be turned off temporarily or businesses may want to profit from cheap energy and consume more when the broker asks them to. Resolving imbalances with these controllable loads is a cheaper and more sustainable way of regulating energy.

This thesis focuses on the problem (or opportunity!) of balancing energy with controllable loads. The goal is to make the controllable loads of the brokers available to the central party that is responsible for regulating the total imbalance, so the whole imbalance can be solved as efficiently as possible. However, to get the brokers to cooperate and share the availability of their controllable loads, correct incentives have to in place. In this thesis we will explore a wide range of different incentives and assumptions to design a market that is efficient and limits the height of the payments.


In Chapter 2 we give an brief overview of some mechanisms and terminology found in the auction theory and mechanism design literature. In Chapter 3 we present the Energy Grid Balancing Problem and a mechanism proposed by de Weerdt et al. [19]. We show that his mechanism does not give a satisfying solution to the problem.

In Chapter 4 we propose 6 novel mechanisms that are incentive compatible, individually rational, weakly budget balanced and efficient. We identify that the revenue made by the DU is largely dependent on how brokers are incentivized to resolve imbalances on the day-ahead market. We design the mechanisms such the revenue made by the central agent is as little as possible. At the end of the chapter we do some experiments with the mechanisms.

In Chapter 5 we consider the same problem only then with the addition of controllable capacities. If brokers are able to actively participate in balancing themselves and this has consequences on the strategies that brokers employ. We show that the connection with double auction theory and what is needed to solve this extended problem. Finally in Section 6 highlight the contributions of this thesis and pose some research topics that can be solved in future work.
The goal of this thesis is to find a satisfactory mechanism to the grid balancing problem that is introduced in the next chapter. That is why in this chapter we briefly introduce some of the terminology used in the field of mechanism design. It happens to be that the decisions made in grid balancing are very much alike the decisions made in auction mechanisms. Therefore we also introduce some simple auction mechanisms. We utilize the information in this chapter to construct and analyze potential mechanisms for the grid balancing problem. The main sources of the information in this Chapter are from the dissertation of Cavallo [6], the book by Shoham and Leythom Brown [26], a paper by Rasmusson [24] and some presentation slides from Leyton-Brown [18] unless specified otherwise.

First an introduction is given of auctions in section 2.1. Then the different components of auctions are outlined, namely the agents in section 2.2, some examples of auction mechanisms in Section 2.3, then some mechanism properties in Section 2.4 and finally some generic mechanisms that can be deployed onto any theoretical game.

2.1 Mechanisms and auctions

Game theory is the study of decision making. It assumes intelligent and rational agents and it gives us a tool to analyze situations where these agents have to act and react to each other. A game consists of the participating agents (the players), the set of actions/strategies and a specification that maps actions to payoffs. A mechanism is a type of game. A mechanism is a game where a single agent acts as a central entity and the other agents may have information that they know can send to and receive messages from the central agent. After exchanging information the central agent chooses a payoff structure. How the central agent makes its decisions is defined in the mechanism.

Mechanism design is often called reverse game theory. It works from the perspective of the central agent and assumes that the central agent is interested in the game’s outcome. For example he may prefer that he earns as much as possible or he would like the other agents
to work together as best as possible, even though the agents are selfish. Mechanism design tries to construct mechanisms so that it reaches the best outcome possible.

Auctions are a way to sell goods or resources, which we refer to as items, to potential buyers. How this is done is described in an auction mechanism. Common auction mechanisms are the English and Dutch auctions. Auction mechanisms can be constructed such that the central agent receives as much as possible but they can also (for example) be constructed such that the preferences of the buyers are honored as much as possible if the central agent prefers that.

Auctions can be held in a lot of different ways. Some mechanisms require bids to be made public, other demand that the bids have to be made before a certain time, etc. As the answer to question to what an auction is can be very wide, Krishna [17] has defined two common aspects:

1. They elicit information, in the form of bids, from potential buyers regarding their willingness to pay the outcome, and who wins and how much one pays is determined solely on the basis of the received information.

2. The identities of the bidders play no role in determining who wins the object.

These common aspects are implying that auctions are universal (they may be used to sell any good) and anonymous (if two bidders would switch bids, the winning bid and resulting payment should still be the same).

In Chapter 5 we will also look at double auctions. Double auctions are auctions where the items are not solely provided by the auction holder but also by other agents, the sellers. Buyers bid on the amount they are willing to pay.

2.2 Properties of agents

To analyze the properties of a mechanism, we need to have a model of the participating agents. Here we briefly discuss what properties agents can have (among others), and how this influences the properties of a mechanism.

**Definition 2.2.1 (Private values):**
In the setting of an auction each buyer knows how much they like the item that is up for auction. This is almost always denominated in some type of currency, but it can be anything. In most auctions this valuation is private to this agent, hence private values. Auctions are used precisely because these values are private. The auction holder does not know the evaluations of the agents, and therefore if the auction holder wants to give an item to the agent that values it the most it has to employ some tricks to make the agents share some information about their private values with him. If we assumed the central agent does know these values precisely, then the auction holder would simply offer the object to the bidder with the highest valuation for a price slightly less than what the bidder was willing to pay, to make the maximum amount of profit.

Important is that the private value of the object cannot be influenced by any means during the execution of the mechanism. If for example an agent wants to buy an item only
for the sake of reselling it later, and if he notices that the other agents are bidding much less than his own private value it may influence his evaluation on the object. These are called interdependent values and we leave these out of scope.

**Definition 2.2.2 (Self-interested and utility functions):**
A natural assumption about the agents is often that they are self-interested. Each broker has his own description of the states the world can be in and knows which states it likes more than others. The function that maps the states of the world to a one-dimensional quantity that defines a degree of preference is called the utility function. The degree of preference is based on the expected value an agent receives when the world is in that state. The existence of such utility function for agents with relatively modest axioms is proven by von Neumann & Morgenstern [29].

Self-interested means that the actions the agent makes are purely based on his own utility function. This does not mean that it does not want to harm others, or that he only cares about things that directly benefits him. It only assumes that the actions an agent makes are rational and maximizes his own utility function. We will assume this for all the material discussed in this thesis.

**Definition 2.2.3 (Quasilinear utility functions):**
Utility functions can describe any ordering of all the states in the world. However assuming that an agent could have any one of all possible utility functions may not be necessary and assuming only a subset of all possible utility function is often very reasonable. Quasilinear utility functions is such a subset.

When we use quasilinear utility functions we assume that the central agent decides on a set of payments apart from the outcome \( o \). A quasilinear utility function means that the expected utility is only dependent on the value an agent is expecting to extract out of the state of the world (excluding payments) and the payment the broker needs to pay. A quasilinear utility function for an agent with the type \( \theta_i \) has the following form where \( v_i(o, \theta_i) \) is the utility gained by the resulting outcome \( o \) and \( p_i(o, \theta_i) \) is the payment that agent \( i \) has to pay:

\[
 u_i(o, \theta_i) = v_i(o, \theta_i) - p_i(o, \theta_i).
\]

The agent is not concerned with the distribution of the items over the other agents or their revenue as long as his payment and the items received remain the same.

This assumption over the agents in a game is a natural one and is present in almost any published literature in the field of game theory. Therefore we also assume quasilinear utility functions in this whole document. Having only such utility functions will later prove to be important when we get to the theorem of Gibbard-Satterthwaite 1965[25].
2.3 Example of auction mechanisms

2.3.1 Single unit auction mechanisms

One of the most simple forms of auctions is the single unit auction. In a single auction the auction holder has a single item (can be a good or resource) to sell and potential buyers can bid on the item. Each buyer $i$ of the $n$ buyers has a private value $v_i$ that describes how much utility the buyer gains when he acquires the item. We shortly discuss two mechanisms, the first-price (strongly equivalent to a Dutch auction) auction and the second-price (equivalent to an English auction) auction.

First-price auction

An example of a mechanism for such auction is the first-price sealed bid auction where each agent $i$ reports his bid $b_i$ privately to the auction holder. The auction holder then rewards the item to the buyer with the highest bid for the price that he bid.

If an agent bids truthfully, so $b_i = v_i$ his utility will always be 0. Losing the auction gives him an utility of 0 but winning also gives him 0 utility, as he pays exactly his own private value.

Example 2.3.1:

Suppose we have four bidders with private values of respectively 2, 4, 5, and 8. If they all bid their own private value the 4'th bidder wins with a bid of 8. However the winner has to pay his own bid so his utility is $8 - 8 = 0$. If he had bid 7, he would still have won the auction and his utility would be $8 - 7 = 1$.

We can see it is better for an agent to bid slightly less than his private value. If an agent wins the item and $b_i < v_i$ his utility is $v_i - b_i$ which is positive. However bidding too low is also not a good idea as the chance that agent wins diminishes.

Assuming the private values of all $n$ brokers are uniformly distributed over the interval $[0, 1]$, the equilibrium strategy for every agent is to bid [17]:

$$b_i = \frac{n - 1}{n} v_i$$ (2.1)

For other distributions of private values similar a analysis can be made.

This mechanism is strongly equivalent to the Dutch auction. In a Dutch auction the auction holder starts at an ask price that is higher than any of the bidders is willing to pay. Then the ask price is lowered until there is a bidder that is willing to pay the ask price, upon which he also has to pay this price.

Second-price auction

A mechanism that is incentive compatible is the second-price sealed auction. This mechanism is the same as the first-price sealed auction but the price that has to be paid for the item is equal to the second-highest bid. Only assuming private values, bidding truthfully is the dominant strategy. Bidding less than the private value does not lower the price to be
Mechanism design and auction theory  

2.3 Example of auction mechanisms

paid upon winning the item; an agent only risks to not win the item. Bidding more than the private value only results in the chance of having to pay more than the private value.

Example 2.3.2:
We use the same example as in section 2.3.1. 4 bidders with respectively private values of 2, 4, 5 and 8. If all agents bid their private values the 4th agent wins the auction. He then pays the second-highest bid which is 5. His utility is now $8 - 5 = 3$. The 4th agent cannot influence the price payed by the winner without giving up the item. Bidding more does nothing to the price he pays, and bidding less does not decrease the price until he pays less than 5, upon which he loses the auction.

Theorem 2.3.3. Bidding truthfully is a weakly dominant strategy in a second-price auction.

Proof. This is a paraphrased proof originally publised by Vickrey [28]
Consider the bidder $i$ with a valuation $v_i$ and a bid $b_i = v_i$. Suppose $b_j = \max_{i \neq j}$ is the highest competing bid. If $v_i > v_j$ then is utility is $v_i - v_j$, if $v_i < v_j$ his utility is 0.

If instead of bidding $v_i$ he bids an amount $x$ where $x < v_i$ then he would still win and gain the same utility in the case of $v_j < x < v_i$, however if $x < v_j < v_i$ he loses the auction where he would have won $v_j - v_i$ in the case he was truthful. In the case of $x < v_i < v_j$ he still loses and expects the same utility.

If he bids $x$ where $x > v_i$ he would still win and gain $v_j - v_j$ in the case where $v_j < v_i < x$. He would also still lose if $v_i < x < v_j$. In the case that $v_i < v_j < x$ he would win the auction where he would have lost it if he had bid $v_i$. However his utility is now $v_i - v_j$ and since $v_j > v_i$ he would have been better off bidding truthful.

Bidding something else than $v_i$ cannot improve his utility but can only lower it. So bidding truthful is a weakly dominant strategy.

The second-price auction is weakly equivalent to an English auction. In an English auction the auction holder starts at a low price and raises this price until there is only a single bidder left that is willing to pay the ask price. The price he pays is thus the last ask price that the other brokers were willing to pay. The difference between an English auction and a second-price auction and the reason it is not strongly equivalent is that the English auction reveals more information about the privately known information of the other agents. Agents can observe what prices other brokers were not willing to pay anymore for the item. When the game is only played a single time and with private values this information is not of any use, but when it is played multiple times or with interdependent values this can make a difference.

In terms of the expected revenue for the auction holder, the first-price and the second-price auction are the same. At first sight the first price auction generates a higher revenue as the payment is seemingly higher. This can also be found in older literature such as Cassady 1967 [4, p. 260]. However this notion is wrong, when all agents execute the dominant strategy (which is expected from them as they are rational) the revenue is the same as in the second-price auction.
2.4 Properties of mechanisms

Mechanisms can be evaluated by certain criteria. The criteria can be categorized in incentives, participation, budget and efficiency.

2.4.1 Incentives

Mechanisms that reward truth-telling are often desirable. For the agents it is much easier, as they do not have to make complex computations to get the most out of the auction, but also the auction holder can often make better decisions more efficient. We know agents are self-interested and the evaluations about the objects are private, rewarding truth-telling is in some instances not an easy task or even impossible.

If in an auction mechanism truth-telling, so bidding your private value, is the optimal strategy, we call the mechanism incentive compatible. Some mechanisms have a stronger sense of incentive compatibility than others. We discuss two flavors of incentive compatibility.

**Definition 2.4.1 (Strategyproofness):**
A mechanism is strategyproof if and only if truthfulness is a dominant strategy for every agent. It is never possible for an agent to deviate from the truthful strategy and to benefit from it. A good example of a strategy proof mechanism is the second price auction. It does not matter what other agents do, bidding your true private value is giving you the highest reward.

Ex post incentive compatibility is equivalent to strategy-proofness when characterized by private values [6].

**Definition 2.4.2 (Incentive compatibility in Bayes-Nash equilibrium):**
A mechanism is incentive compatible in the Nash equilibrium if truth telling is the weakly optimal strategy given that all the other agents also truthfully report their values. This is weaker than strategyproof as it means that it is possible for an agent to deviate from truthful to improve his utility, but this requires another untruthful agent.

It is not a requirement that the Nash equilibrium should be the only equilibrium in the mechanism, although that would be desirable property for the mechanism to have.

**Tactics**

If a mechanism is not incentive compatible agents can benefit from deviating from the truthful strategy by applying untruthful strategies. Most of the time the dominant strategy is one or a combination of bid shading and supply/demand reduction. Also some formally proven incentive compatible mechanisms are still vulnerable to manipulation in practice if the assumptions about an auction does not reflect the real world well enough. Tactics like false bidding and collusion can often be employed.
Bid shading
Bid shading is the act of untruthfully reporting one’s private values. It can be utilized to increase or decrease directly, or used to hide one’s true private value which may prove to be beneficial later in the mechanism.

Supply and demand reduction
In some multi unit auction mechanisms agents can employ demand and supply reduction. The idea is to sell or buy less items than you are able to, to increase or decrease the price such that your utility is more than the case you would have sold or bought more items.

If for example a single agent has 3 items to sell which he has no use for and there are 4 agents that are willing to buy a single item for respectively 1, 4, 5 and 6. We assume that the selling agent wants to earn as much as possible. We also assume that the mechanism describes that the price that has to be payed by the brokers is the \( n + 1 \)’th highest bid where \( n \) is the amount of items sold. Now bidding truthful is a weakly dominant strategy for all the bidders.

In this example the selling agent has 3 items to sell. The highest bids are 4, 5 and 6 so the 3 buying agents pay the fourth highest bid which is 1. The selling agent receives a total payment of 1 \( \times \) 3 = 3.

However if the selling agent reduces his supply and reports that he has only 2 items to sell, the \( n+1 \)’th bid is 4. His received payment and therefore utility is now 4 \( \times \) 2 = 8.

Demand reduction works the same way only the manipulation is done by a buyer instead of a seller. Demand reduction hurts the efficiency of an auction. In the example the sum of the utility of all agents is 15 without supply reduction and with demand reduction only 11.

False name bids and collusion
False name bidding is the act of one agent pretending to be multiple distinct agents. If for example a mechanism limits the amount of items a broker can receive he could report multiple bids under false names to increase his utility. Collusion is the act of two or more agents exchanging information outside of the mechanism to give themselves a tactical advantage in the mechanism.

2.4.2 Participation
In mechanism design we work from the perspective of the central agent/auction holder. This central agent can lay down rules and force payments upon the other agents. What the central agent often cannot control is if the agents are willing to participate at all. Given that the other agents know the rules of the game before they participate they have to be willing to join the mechanism, or else the preferences of the central agent will never be satisfied. If a mechanism incentivizes participation the mechanism is individually rational. The literature distinguishes three types of individual rationality, and they all depend on at what moment agents rather want to leave the mechanism.

Here we use \( u_i : O \times \Theta \rightarrow \mathbb{R} \) where \( u_i(o, \theta_i) \) denotes the utility that agent \( i \) gained from an outcome \( o \). We also denote the function \( d : \Theta \rightarrow O \) where \( d(\theta) \) denotes the decision that the central agent decides upon given the types \( \theta \). Likewise we use the function \( p : \Theta \rightarrow \mathbb{R} \),
2.4 Properties of mechanisms

$p_i(\theta)$ denotes the payment agent $i$ has to make using the decisions and rules set by the central agent.

**Definition 2.4.3** (Ex post individual rationality):
This is the strongest form of individual rationality. Ex post individual rationality is that when all the information is revealed, so after the outcome and payments are known, no agent wants to walk away from the mechanism regardless of their type $\theta_i$. So it holds when:

$$u_i(d(\theta), \theta_i) + p_i(\theta) \geq 0$$

**Definition 2.4.4** (Interim individual rationality):
A weaker form of individual rationality is interim individual rationality. If this property holds then no agent is willing to walk away when each agent knows his own type $\theta_i$ and has expectations of the types of the other agents and how the decisions are made by the central agent. In a mechanism that is interim individually rational but not ex post individual rational it is possible that agents end up worse, as long as the expectation was positive. The property holds when:

$$E[u_i(d(\theta), \theta_i) + p_i(\theta) \mid \theta_i] \geq 0$$

**Definition 2.4.5** (Ex ante individual rationality):
Finally ex ante individual rationality is the weakest form of individual rationality where no agent is willing to walk away from the mechanism before they know their own type $\theta_i$ and only have expectations of the types of the other agents and how the decisions are made by the central agent. This property holds when:

$$E[u_i(d(\theta), \theta_i) + p_i(\theta) \mid \theta_i] \geq 0$$

2.4.3 Budget

Regardless of the intentions of the central agent it almost never has unlimited amount of money. Therefore a requirement that often needs to hold is budget balancedness. When the balance of the central agent is exactly zero, so when the payments from all agents add up to exactly zero, the mechanism is strongly budget balanced. If the balance is guaranteed to be zero or higher the mechanism is weakly budget balanced. Some mechanisms in literature are weakly budget balanced with small subsidies where the amount of money that needs to be added by the central agent is provably limited.

2.4.4 Efficiency

If a mechanism is efficient it maximizes the total social welfare. It is different from budget balancedness as the payments between the agents do not impact the total social welfare as the central agent is included in the social welfare. For an auction to be efficient the items need to be distributed to those agents that value the items the most. Asymptotically efficient means that although the mechanism does not always maximize social welfare, it
achieves efficiency in the limit of, in most cases but possibly another metric, the amount of participating agents.

2.5 Generic mechanisms in literature

In this thesis we build upon existing mechanisms found in literature. We first discuss the Groves class of mechanisms which is important for efficiency and individual rationality. Then we present the VCG mechanism and how it achieves individual rationality and weakly budget balancedness. The AGV mechanism circumvents an important impossibility and is strongly budget balanced. Finally we briefly discuss a mechanism proposed by Cavallo that returns as much revenue as possible to the participating agents.

2.5.1 Groves class

Mechanisms that are part of the Groves class, proposed by Vickrey [27], Clarke[7] and Groves [13] achieve some of the previously mentioned properties. The Groves class is a class of mechanisms and contains an infinite number of mechanisms. Mechanisms that are part of the Groves class are are always efficient and strategyproof. The definition according to Cavallo [6]:

Definition 2.5.1 (Groves class of mechanisms):

A direct mechanism \((f, P)\) is a Groves mechanism if and only if:

1. \(\forall \theta \in \Theta, f(\theta) \in \arg \max_{o \in O} v(\theta, o)\) (i.e. it executes \(f^*\))

2. \(\forall i \in I, \) there is a function \(h_i : \Theta_{-i} \rightarrow \mathbb{R}\) such that \(\forall \theta \in \Theta\)

\[ P_i(\theta) = v_i(\theta_{-i}, f^*(\theta)) - h_i(\theta_{-i}) \] (2.2)

Every Groves mechanism chooses an outcome such that it maximizes the social welfare (1) and it pays every agent \(i\) the value that the other agents reported for the selected outcome minus some payment that is independent of the values that \(i\) reported. Mechanisms within the Groves class differ from each other in the way the function \(h_i\) is defined.

Green and Laffont [11] have proven that for an unrestricted type space, a direct mechanism \((f, P)\) is truthful and efficient in dominant strategies if and only if it is a Groves mechanism. Later Holmstrom [15] even proved that it not only holds for mechanisms with an unrestricted type space but also for smoothly connected type spaces. Because smoothly connected type space virtually encompasses all real world valuation function the search for a mechanism that is both strategyproof and efficient is restricted greatly. "For all practical purposes" one is bound to a mechanism from the Groves class. So the only thing left for the mechanism designer is to define the charge function \(h_i\) for each agent \(i\).

Definition 2.5.2 (Smoothly connected type space):

A type space is smoothly connected if and only if for any two valuations \(v_i\) and \(v'_i\) in the type space can be differentiably deformed into the other. This property holds for all practical purposes [15].
2.5 Generic mechanisms in literature

2.5.2 VCG mechanism

After finding that all the Groves mechanisms are strategyproof and efficient we can move on to budget balancedness and individual rationality. However Satterthwaite [23] has provided us with a very negative result:

Theorem 2.5.3 (Myerson-Satterthwaite). For an unrestricted type space, there exists no mechanism that is truthful and efficient in Bayes-Nash equilibrium, interim individual rational, and weakly budget balanced.

This means that we can stop searching for a mechanism that has all of the four properties which works on an unrestricted type space. Luckily we can restrict this type space to circumvent this result. The following restriction states that if an agents values are not taken into account, the selected outcome based on the maximization of social welfare would not harm the agent.

Definition 2.5.4 (No negative externalities): The no negative externalities property holds when, \( \forall i \in I, \theta \in \Theta, v_i(\theta_i, f^*(\theta_{-i})) \geq 0. \)

We also introduce choice-set monotonicity, removing any agent weakly decreases the mechanisms set of possible choices \( X. \)

Definition 2.5.5 (Choice-set monotonicity): An environment exhibits choice-set monotonicity if \( \forall i, X_{-i} \subset X. \)

The VCG mechanism named after Vickrey, Clarke and Groves is a mechanism in the Groves class that has very desirable properties when the no negative externalities property and choice-set monotonicity hold.

Definition 2.5.6 (VCG mechanism): The VCG mechanism is a direct mechanism \( (f^*, P) \) where, \( \forall i \in I \) and \( \theta \in \Theta: \)

\[
P_i(\theta) = v_{-i}(\theta_{-i}, f^*(\theta)) - v_{-i}(\theta_{-i}, f^*(\theta_{-i})).
\]  

(2.3)

The VCG mechanism is a Groves mechanism since the charge function \( v_{-i}(\theta_{-i}, f^*(\theta_{-i})) \) for each agent \( i \) is independent on that agents reported type. From the fact that the VCG mechanism is a Groves mechanism follows that it is truthful and efficient in dominant strategies.

The VCG mechanism is also ex post individual rational when the no negative externalities and the choice-set monotonicity properties hold.

VCG is weakly budget balanced if the no single-agent effect holds. The no single-agent effect is the property that the removal of an agent makes the remaining agents better off. In auction theory this is as removing an agent only reduces the amount of competition, and thus reduces the prices.

Definition 2.5.7 (No single-agent effect): An environment exhibits no single-agent effect if \( \forall i, \forall v_{-i}, \forall x \in \arg \max_y \sum_j v_j(y) \) there exists a choice \( x' \) that is feasible without \( i \) and that has \( \sum_{j \neq i} v_j(x') \geq \sum_{j \neq i} v_j(x). \)


But what makes the VCG the most famous mechanism of the Groves class is that for any smoothly connected 0-value admitting type space the VCG mechanism is revenue maximizing among all mechanisms that are truthful and efficient in dominant strategies and ex post individual rational.

**Definition 2.5.8 (0-Value admitting):**
A type space \( \Theta \) is 0-value admitting if and only if \( \forall o \in O, \forall i \in [n] \), there is a \( \theta_i \in \Theta_i \) such that \( v_i(\theta_i, o) = 0 \).

### 2.5.3 AGV mechanism

The VCG mechanism is weakly budget balanced but even maximizes the revenue generated by the mechanism. In a lot of cases this is not preferred and you would like to minimize the revenue obtained by the mechanism, preferably to create a strongly budget balanced mechanism with the same properties. However Green and Laffont [12] have proven that this is not possible:

**Theorem 2.5.9 (Green-Laffont).** For an unrestricted type space, there exists no strongly budget balanced mechanism that implements an efficient choice function in dominant strategies.

Unfortunately moving to a Bayes-Nash incentive-compatible mechanism does also not solve this. Myerson-Satterthwaite have proven that:

**Theorem 2.5.10 (Meyerson-Satterthwaite).** No Bayes-Nash incentive-compatible mechanism is always simultaneously efficient, weakly budget balanced and ex-interim individual rational, even if agents are restricted to quasilinear utility functions.

For strongly budget balancedness to be reached again some property has to be weakened thereby expanding our solution space. The AVG mechanism proposed by Arrow [1] and d’Aspremont & Gerard-Varet [9] is such mechanism.

The AGV mechanism is strongly budget balanced, however it is only ex ante individual rational instead of individual rational and requires common prior over agent types.

Common prior over agent types is the assumption that the differences in beliefs agents have can be completely explained by the differences in information the agents have about the world. In the AGV mechanism each agent starts with identical prior beliefs, the common prior, and if agents diverge from this common prior it can be completely attributed to the fact that they received new information.

The mechanism chooses the most efficient outcome according to the agent reports and then each agent pays an "expected externality” that it has imposed upon other agents minus some constant.

**Definition 2.5.11 (AGV mechanism):**
The AGV mechanism is a direct mechanism \((f^*, P)\) where, \( \forall i \in I \) and \( \theta \in \Theta \), given common prior beliefs \( b(\theta_{-i}) \) about the types of agents other than \( i \):
2.5 Generic mechanisms in literature

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\[ P_i(\theta) = ESW_{-i}(\theta_i) - \frac{1}{n-1} \sum_{j \in I \setminus \{i\}} ESW_{-j}(\theta_j), \text{ where} \]

\[ \forall i \in I, \theta_i \in \Theta_i, ESW_{-i}(\theta_i) = E_{\theta(\theta_{-i})}[v_{-i}(\bar{\theta}_{-i}, f^*(\theta, \bar{\theta}_{-i}))]. \]

The payment each agent \( i \) makes is the expected welfare the others obtain, given a type report \( i \) of that agent, then the next term is defined such that it balances the budget.

The AVG mechanism is truthful and efficient in Bayes-Nash equilibrium, ex ante individual rational, and strongly budget balanced.

2.5.4 Redistribution mechanism RM

The VCG mechanism has very good properties but transfers a lot of the gained utility gets transferred to the entity that organizes the mechanism and AGV needs a common prior property for it to work. The redistribution mechanism RM by Cavallo [5] is based on VCG but returns some the payments back to the agents.

The mechanism lets the agents pay a VCG payment and then returns to every agent \( i \) a fraction of the amount that the DU would have earned anyway, independent on the reported values of \( i \).

**Definition 2.5.12 (Revenue-guarantee \( G(\Theta_i, \theta_{-i}) \))**:

The lower bound on VCG revenue that would result, computed over all possible reported types \( \theta_i \in \Theta_i \) fro agent \( i \), given the type profile \( \theta_i \) reported by the other agents, i.e.,

\[ G_i(\Theta_i, \theta_{-i}) = \min_{\theta_i \in \Theta_i} \sum_{j \in I} \left[ v_{-j}(\bar{\theta}_{-j}, f^*(\theta_{-j})) - v_{-j}(\theta_{-j}, f^*(\theta)) \right] \]  

(2.4)

**Definition 2.5.13 (Redistribution mechanism RM)**:

RM is a direct mechanism \((f^*, P)\) where, \( \forall i \in I \) and \( \theta \in \Theta \):

\[ P_i(\theta) = v_{-i}(\theta_{-i}, f^*(\theta)) - v_{-i}(\theta_{-i}, f^*(\theta_{-i})) + \frac{G_i(\Theta_i, \theta_{-i})}{n} \]  

(2.5)

RM is truthful and efficient in dominant strategies, ex post individual rational if the no negative externalities condition holds, no-deficit and redistribution anonymous.

Furthermore for smoothly connected type spaces, RM is an optimal redistribution mechanism among all redistribution mechanisms that are truthful and efficient in dominant strategies, no deficit and redistribution-anonymous.

**Definition 2.5.14 (Redistribution-anonymous[5])**:

A redistribution mechanism is redistribution-anonymous if and only if it maps agent-specific revenue-guarantees to redistribution payments according to a single deterministic function that is invariant to domain information that does not apply identically to every agent.

Guo et al. [14] have shown that without this property there exists a mechanism that shows improvements upon this result in the worst-case.
Chapter 3

Problem definition and previous work

3.1 Introduction

In "A theoretical analysis of pricing mechanisms and broker's decisions for real-time balancing in sustainable regional electricity market", de Weerdt, Ketter and Collins [19] proposed a balancing mechanism that was intended to be part of the next annual Power TAC competition (Ketter et al., 2010). The mechanism focuses on the process by which the Distribution Utility (DU) achieves a real-time balance on the grid.

In this chapter we introduce the Efficient Grid Balancing Problem and the mechanism proposed by de Weerdt et al. [19]. Then we show that the proposed mechanism does not fully satisfy the required properties. The things we learn from the proposed mechanism are the foundation on which we build new mechanisms in the next chapters.

3.2 Problem definition

In Power TAC agents act as retail brokers that purchase power from a day-ahead market and can also have contracts with homes and businesses to sell or acquire power. Research teams around the world are challenged to write autonomous or semi-autonomous agents that can operate profitably in competition with each other. The mechanism that is focused on here is to enable the market to balance the supply and demand on the grid efficiently.

The simulation in Power TAC is divided into discrete time slots. Within each time slot it is assumed that the consumption and production remains the same. Before the next time slot arrives, the trading of energy in the day-ahead market for that time slot is closed. Brokers trade on the day-ahead market to buy and/or sell energy so they have enough energy to satisfy the demand of their customers. Between closing the day-ahead market and the start of the new time slot a discrepancy between the anticipated consumption production and the real consumption is to be expected.

Normally an Independent Systems Operator (ISO) or a Distribution Utility (DU) are responsible for resolving imbalances once the time slot has started. This central agent,
3.2 Problem definition

Problem definition and previous work

which from here refer to as the DU, is able to measure and resolve imbalances but has
to incur costs to do so. These cost must be paid by the brokers so it motivates them to
continuously balance their supply and demand such that as much energy as possible is
resolved in the most efficient manner. If an imbalance, for example a spike or dip in power
consumption, is spotted, it is much cheaper to resolve this on the day-ahead market than to
balance it at the last possible moment without prior warning.

In this thesis we seek to find a more efficient mechanism than to always use the capacity
of the DU to resolve imbalances. One thing we want to exploit is the capability of brokers to
engage in load curtailment with their customers, so all or some imbalance can be resolved
through these contracts. For example some businesses could use less energy when the
broker encounters a shortage in exchange for a reduction of their energy bill, or private
individuals could be connected to a smart grid which starts their washing machine remotely
when their broker offers a reduction in price. All of the possibilities of influencing the
energy imbalance are referred to as controllable loads. This includes consuming more or
less energy but also the capability of generating more energy or temporarily storing it.

Controllable loads are assumed to be often much cheaper than the solutions available to
the DU. When faced with a shortage of energy the DU has to increase his power production
while a brokers could temporarily regulate some power usage through their clients. Of
course brokers are not going to report their controllable loads to the DU without a proper
incentive so a mechanism that manages all incentives has to be created. We call this problem
of designing a mechanism for the market the Energy Grid Balancing Problem (EGBP).

De Weerdt, Ketter and Collins (2011) have published a problem description of the grid
balancing problem as seen in Power TAC which is explained formally in the next section. It
only considers a single time slot. In section 3.2.2 we propose some slight modifications of
the problem definition.

3.2.1 Problem definition EGBP

There are \( n \) brokers denoted by 1,2,...,\( n \) and the central distribution utility denoted by 0.
The controllable loads of each agent can be represented as a capacity range \([c_i^-, c_i^+]\) and
a function \( c_i : [c_i^-, c_i^+] \rightarrow \mathbb{R} \) that represents the profit/cost of controlling energy per unit
within that range. For balancing the imbalance downwards it denotes the profit made from
balancing and for balancing upwards the cost. This price function may not be arbitrary but
is subject to some requirements.

This function is monotonically increasing between \([c_i^-, 0]\), monotonically increasing
between \((0, c_i^+])\) and \(c_i(0) = 0\). A monotonically increasing function is chosen because it
can represent all contracts that include controllable loads with different prices and loads
where the cheapest options are used first. As brokers do not make any operational cost
when they do not have to regulate any imbalance the function is 0 at \( c_i(0) \). An example of
such function can be seen in Figure 3.1.

When the current time slot starts the DU is responsible for resolving the total imbalance.
Therefore we assume that the DU has the capability to resolve an unlimited amount of
energy \([c_0^-, c_0^+] = [\{-\infty, \infty\}]\). The DU can also tell brokers to enable their controllable loads
if they have chosen to report them to the DU before the time slot starts.
Problem definition and previous work

3.2 Problem definition

Each broker also has an (potentially negative) energy surplus $x_i \in R$. This surplus of energy is assumed to be known by the DU at the start of the time slot. We refer to this imbalance as a broker’s personal imbalance. The broker cannot lie about his imbalance when the time slot has arrived, but is of course able to influence it by buying or selling more energy in advance on the day-ahead market. The total amount of imbalance is referred to the total imbalance and is denoted by $\sum_{i=1}^{N} x_i = X$.

One thing we left out of the problem description that de Weerdt et al. have included is the reporting of the expected personal imbalance by the brokers. In the problem description of de Weerdt brokers also send a prediction of their personal imbalance to the DU prior to the start of the time slot. The brokers are incentivized to report a correct prediction as they have to pay a penalty inversely proportional to the quality of their prediction.

The price functions of the brokers are assumed to be private values. We only consider integer imbalances, although everything in this thesis can be easily adapted to fractions of imbalances. Because we only consider integer imbalances, the monotonically increasing cost functions can be represented as stepwise function. Often it is easy to not reason about the whole function $c_i$ but to divide the whole function into many controllable loads with unit size. A controllable load of size 35 kWh can be represented as 35 controllable loads of 1 kWh. In this thesis, when we refer to a controllable load we refer to a controllable load with unit size except when specified otherwise.

Controllable loads that can regulate the energy upwards, so consume less or produce more, are called interruptible loads. The name is based on loads that can temporarily be interrupted but it can also mean the deployment of a generator. The controllable loads that

![Figure 3.1: A monotonically increasing stepwise price function of a set of controllable loads](image)

Figure 3.1: A monotonically increasing stepwise price function of a set of controllable loads
regulate downwards are called *optional loads*. The broker obtains these loads through load curtailment.

**Definition 3.2.1** (Interruptible load):
Interruptible loads are, unless specified otherwise, of unit size and can regulate an imbalance upwards, so they can consume less or produce more energy. Using these controllable loads costs money in almost all cases.

**Definition 3.2.2** (Optional load):
Optional loads are, unless specified otherwise, of unit size and can regulate an imbalance downwards, so they can consume more energy when enabled. These controllable loads often generate money when enabled, but it is not unusual that they don’t. We assume that none of the optional loads can generate more money per unit than that any of the interruptible loads cost.

An important aspect of the problem is that brokers cannot use their controllable loads themselves. They choose to report all, none, or some controllable loads to the DU but they cannot enable them themselves in the current time slot. If brokers are able to use their controllable loads without intervention from the DU the problem changes quite substantially and that is discussed in Chapter 5.

The actual problem is to design a mechanism that resolves the imbalance efficiently through the use of controllable loads and guarantees incentive compatibility, individual rationality, budget balancedness and efficiency. We go through these requirements in section 3.2.3.

### 3.2.2 Slight modification

Before giving an example and describing the mechanism of de Weerdt, we introduce some modifications to the problem description. These modifications do not change the nature of the problem, but makes it easier to reason about it.

First, we leave the prediction of the expected imbalance out of scope. Because the imbalances are known to the DU when the time slot arrives, and the DU has no way of acting on the prediction it is not useful to enforce the prediction in a single-time slot settings. Furthermore if a correct prediction of the expected imbalance is a requirement, the solution proposed in the paper to incentivize truthful reporting can be applied to any mechanism. Predictions could be useful in a setting where multiple time slots are considered, but in this thesis we only consider a single time slot.

The second modification is the representation of the price function. In the description of de Weerdt the price function is required to be a monotonically increasing price function over the interval \([-c_i^- , 0]\). The cost of downwards controllable loads represents the reward that using the load would bring while the cost of the upwards controllable loads represents the cost that the use of the load would bring.

It is more natural to assume a monotonically decreasing function for \([c_0^- , 0]\) and a monotonically increasing function for \((0, c_0^+ ]\). This way the cost of balancing an imbalance \(x\) by
Problem definition and previous work

3.2 Problem definition

Figure 3.2: The new stepwise price function of a set of controllable loads

broker $i$ is $c_i(x)$, and would be equal to the amount the broker needs to receive to uphold individual rationality (we come back to IR later in this chapter). An example of such function is shown in Figure 3.2.

Except for the change of sign the problem is unaltered. And of course it still holds that the downwards controllable loads are assumed to give a reward less than the cost of the upwards controllable loads.

We consider $O$ the set of all possible outcomes $o$. An outcome is a description of which controllable loads are used to resolve the total imbalance. The payments are, as almost always in the literature of mechanism design, not part of the outcome $o$.

We denote $\Theta$ as the type of all agents and $\theta_i$ as the type of agent $i$. The type of an agent consists of the private values (costs of the controllable imbalance) but does contain the imbalance of a broker. This is because when proving that a mechanism is incentive compatible, we only consider the truthfulness of reporting the controllable loads and not the imbalance. All mechanisms that we discuss are truthful in reporting the cost of the controllable loads, regardless of the imbalance that a broker ends up with at the start of the time slot. We denote $\hat{\theta}_i$ as the possibly untruthful reported type.

We denote $C$ as a set of controllable loads and use the function $f^* : X \times C \rightarrow C$ as the allocation function where $f^*(X,C)$ are the controllable loads that are used to resolve the total imbalance $X$.

Example 3.2.3:
Before defining the requirements that a mechanism needs to have in order to be useful we
first introduce an example of a situation that a mechanism may encounter. In Table 3.1 a situation is shown.

In this example there are three brokers 1, 2 and 3 plus the DU denoted by 0. The three brokers have an energy surplus of respectively $-2$, $0$ and $1$ units. The total surplus $X$ is therefore $-1$, a shortage of a single unit of energy. There are two types of controllable loads, optional loads and interruptible loads. Each number represents a single controllable load of unit size (able to regulate a single unit of energy). We can see that the DU is able to balance downwards at a price of 0 per unit and is able to generate an extra unit of energy for 8 per unit.

Broker 1 is able to generate a small profit of 1 from extracting a unit of energy from the grid at this time slot, while broker 3 is willing to pay up to 3 for this privilege. Resolving positive imbalances can have a negative or positive cost as the excess of energy can be used directly or stored for later use. Balancing energy upwards can be very costly. The first broker has the capability to enable 2 interruptible loads to regulate 2 units, he can regulate the first unit for 7 and the second for 8. The other brokers only have a capacity of 1 and can regulate a single unit 6 and 5.

In this example the most cost efficient allocation is to ask broker 3 to use his upwards balancing controllable load to resolve the total imbalance of $-1$.

### 3.2.3 Mechanism properties

A solution (mechanism) for the EGBP is expected to have some properties to make it a useful mechanism in the real world. There are five important properties that a mechanism should have (albeit in a weakened form):

1. The imbalance needs to be resolved efficiently
2. Brokers should be incentivized to truthfully report the cost and capacity of their controllable loads
3. The mechanism should not require external subsidies
4. Brokers should not be worse off when joining this mechanism
5. Imbalances should be resolved in the day-ahead market as much as possible
The first two properties are very much related. Without truthful reporting of the real cost of the controllable loads the DU cannot select the most efficient controllable loads. We also make a clear distinction between efficiency and the incentive to be resolved in the day-ahead market. While resolving in the day-ahead market can be seen as a more efficient way of resolving imbalances, when we talk about the efficiency of a mechanism we only look at how efficient a mechanism resolves the imbalance given that imbalance.

**Resolving efficiently**

The mechanism has to be efficient, in that it has to resolve the imbalance such that it maximize social welfare. It must use the cheapest controllable loads available.

When we refer to efficiency it only means how efficient the total imbalance is solved given that imbalance. It does not refer to how well the mechanism discourages imbalances. All the mechanisms discussed in this and the next chapter resolve the total imbalance in the most efficient way.

**Incentive compatibility in reporting controllable loads**

The mechanism should reward truthful reporting of the cost and capacity of the controllable loads. Truthful reporting the prices of the controllable loads should be the the dominant strategy or it should be incentive compatible in the equilibrium of strategies where brokers truthfully report their controllable loads. In the first case a broker is always just as well or better off when being truthful about the prices and capacity of his controllable loads, while in the second case the broker is only just as well or better off when all the other brokers also report their controllable loads truthfully.

**Budget balancedness**

The mechanism should be at least budget balanced. The brokers should pay for the cost of balancing their imbalances themselves so no subsidizing is needed from an external party. Mechanisms can be strongly budget balanced, where the DU ends up with neither a profit nor deficit, or weakly budget balanced where the DU may run a profit.

**Individual rationality**

If the mechanism has to be deployed in a free market, it has to be individually rational. Brokers have to better off by joining the mechanism or else it cannot do his job.

However before we can check if brokers are better off the alternative mechanism has to be defined. In most literature in mechanism design the alternative comes down to 'doing nothing'. Like in the case of auctions, the buyer does not buy anything and the seller does not sell. However doing nothing and thus not balancing the grid has never been an option. Below are three alternatives we discuss in this thesis. They are listed in the order from weak to a very strong sense of individual rationality:

1. **Predetermined penalty** Here we compare the mechanisms to the case where the DU gives a predetermined penalty per unit of imbalance (so independent of the current
3.2 Problem definition

Problem definition and previous work

total imbalance) to the brokers that is guaranteed to be enough to resolve the total imbalance. If for example the DU has an unlimited supply of interruptible loads of price \( p \) per unit, then the penalty for a negative imbalance is at least:

\[ |x_i| \cdot p \]

2. **Penalty based on loads DU** Each broker that contributes to the total imbalance pays his share of the total cost the DU makes to resolve the total imbalance.

\[ p_i = \frac{c_0(X)}{x_i} \]

3. **The usage of own controllable loads** The alternative where each broker resolves his own imbalance himself as much as possible with his own controllable loads, and the broker pays a penalty to the DU for the rest of the imbalance. We go further into this in Chapter 5.

The last one assumes that brokers have the option to measure and regulate the imbalance on the grid themselves and enable their own controllable loads at the correct time. For now, and in Chapter 4 we assume brokers do not have that capability. In Chapter 5 we research what implications it has if brokers do have that capability.

**Resolving in day-ahead market**

We assume that resolving imbalances in the day-ahead market is cheaper than enabling controllable loads at the last moment as some more efficient machines have a high startup time. In practice this may not always be the case. If agents know of an imbalance ahead of time, they should be incentivized to resolve this in the day-ahead market. Likewise they should not be able to profit from artificially introducing a larger imbalance.

In the rare case that resolving in the day-ahead market is more expensive than using controllable loads, we assume that this was a very unpredictable event as if the brokers did predict it they would have sold their controllable loads in the day-ahead market, making the price drop. So in those rare cases a broker could theoretically profit by not resolving in the day-ahead market because of our assumption, we assume he was not able to do so because of the unpredictability of such event.

The requirement to resolve in the day-ahead market comes in three types:

1. Make striving for a low absolute personal imbalance a dominant strategy.
2. Make striving for a low absolute total imbalance the optimal strategy.
3. Make striving for a low absolute personal imbalance be a Nash-equilibrium.

With the first requirement brokers are guaranteed to be worse of by not solving their imbalance in the day ahead market if they knew about their imbalance before the end of the day-ahead market deadline. When using the second requirement it could be that brokers
Problem definition and previous work

3.3 Mechanism 0

earn a profit by inflating their personal imbalance with the day-ahead market if that makes the absolute total imbalance less.

An unwanted outcome of the first requirement could be that one broker did not foresee his imbalance while others did, in that case the other brokers do not fix this imbalance by buying on the day-ahead market and thus the imbalance is not be solved in the most cost-effective way.

However an unwanted outcome of the second mechanism could be that because all brokers are trying to resolve the total imbalance independently, they overcompensate each other and create even larger imbalances. Although no research is done in regards to which requirement is the best in regards to total social welfare, it would very much depend on how much the brokers know about themselves and about others.

The last mechanisms that are proposed in the next chapter uses the last type, brokers can reach a higher utility by inflating their own imbalance or total imbalance but do not do so in the Nash-equilibrium. This makes it possible to reduce the payments to the DU.

3.3 Mechanism 0

The mechanism proposed in ”A theoretical analysis of pricing mechanisms and broker’s decisions for real-time balancing in sustainable regional electricity market” is referred in this thesis as mechanism 0. The paper first proposes a mechanism where the controllable loads of the brokers are not used and then extends this mechanism to a mechanism where the controllable loads of the brokers are taken into account.

3.3.1 Mechanism 0 with no controllable loads

The mechanism works as follows. It starts with a penalty $p_1$ to ensure that the mechanism is incentive compatible in the equilibrium of declaring the true imbalance. Because we assume the imbalance is known at the start of the time slot and the reasons explained earlier, we won’t go further into this.

Secondly, the agents are penalized with a penalty $p_2$ of $P^+$ and $P^-$ per unit if they have respectively a negative or a positive imbalance.

$P^+$ denotes the maximum price of energy for this time slot in the day ahead market over all day-ahead trade periods. So brokers that have a shortage of energy have to pay more than what they would have paid in the day ahead market, aimed to give an incentive to not have a negative imbalance.

Likewise $P^-$ denotes the minimum price of energy for this time slot in the day ahead market over all day-ahead trade periods. So if an agent has a surplus, he would have been better of selling it in the day-ahead market if he had known about his imbalance in advance. The penalty $p_1$ is aimed to give the brokers an incentive to strive for an imbalance of 0. A graphical representation of $P^+$ and $P^-$ can be seen in Figure 3.3.

**Definition 3.3.1 ($P^+$):**

$P^+$ denotes the maximum price paid in the day-ahead market for a unit of energy for the current time slot.
3.3 Mechanism 0

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Figure 3.3: The values $P^+$ and $P^-$ are based on the extremes of the price paid in the day-ahead market for a unit of energy in this time slot

**Definition 3.3.2 ($P^+$):**

$P^+$ denotes the minimum price paid in the day-ahead market for a unit of energy for the current time slot.

\[
\begin{align*}
p_{2,i} &= -x_i \cdot P^+ \text{ if } x_i \leq 0 \\
p_{2,i} &= -x_i \cdot P^- \text{ if } x_i > 0
\end{align*}
\] (3.1)

The third payment is meant to make the mechanism budget balanced, as the DU still has to resolve the total imbalance. The costs the DU makes to resolve the total imbalance is $c_0(X)$. This operational cost (which can be negative in case of a surplus of energy) can together with the penalties be used to calculate the amount $D$ the DU still needs from the brokers to balance everything and not run a deficit:

\[
D = \sum_{i \text{ if } x_i < 0} (x_i \cdot P^+) + \sum_{i \text{ if } x_i > 0} (x_i \cdot P^-) + c_0X
\] (3.2)

This amount $D$ is being paid by the brokers through the penalty $p_3$. The authors of the mechanism propose three ways of collecting this money:

1. **Equal cost sharing.** The cost $D$ is being equally shared among all brokers.

\[
p_{3,i} = \frac{1}{n} \cdot D
\] (3.3)
Problem definition and previous work

3.3 Mechanism

The cost $D$ are shared proportionally to the imbalance of each broker.

$$p_{3,i} = \frac{|x_i|}{\sum_{j=1}^{N} |x_j|} \cdot D$$

3. One-sided proportional cost sharing. Cost are shared proportionally among brokers being on the wrong side of the imbalance. So only brokers that have contributed to the imbalance are penalized.

$$p_{3,i} = \frac{|x_i|}{\sum_{\{j=1|x_jx_i>0\}} |x_j|} \cdot D$$

This makes the mechanism (without controllable loads owned by the brokers) strongly budget balanced, as any cost or profit that the DU makes is shared among the brokers.

Before we discuss how the DU selects and pays the owners of controllable loads we look at an example of this mechanism.

Example 3.3.3:
We consider the example depicted in Table 3.2, it is different from the situation described in Example 3.2.3. In this example $P^+$ and $P^-$ are equal to 3. The total imbalance is $X = 2 - 4 + 0 = -2$.

The first penalty $p_1$ is not relevant as we assume the brokers cannot lie about their imbalance. The penalty $p_2$ is based on their real imbalance. For example broker 1 pays $p_{2,1} = -x_1 \cdot P^+ = -2 \cdot 3 = -6$. Likewise with the brokers 2 and 3, they pay 12 and 0 respectively.

For the third payment we look at the total balance of the DU. The DU earned a total of $-6 + 12 + 0 = 6$ through penalties so far and to balancing the total imbalance of $X = -2$ the DU would make 12 in operational costs. So $D = 12 - 6 = 6$. With equal cost sharing each broker has a payment $p_{3,i} = \frac{6}{3} = 2$.

The result of the mechanism is shown in Table 3.3, $\sum p$ is the sum of payment $p_2$ and $p_3$ and $u_i$ is the resulting utility of each broker.

<table>
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<tr>
<th>$i$</th>
<th>$x_i$</th>
<th>optional loads</th>
<th>interruptible loads.</th>
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<tbody>
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</table>

Table 3.2: The problem instance of Example 3.3.3

3.3.2 Mechanism with controllable loads
When the the controllable loads of the brokers are cheaper than the options the DU has, the DU chooses the cheapest controllable loads to resolve the total imbalance. In the previous
Example the cheapest controllable loads are those of broker 2 and 3. They both can resolve the imbalance for the amount of 4 for a single unit.

To give the brokers an incentive to report their prices truthfully the DU employs a VCG payment on the controllable loads. Each broker receives the difference between the utility of the other agents in the case the broker does and does not participate. Although this payment has not been assigned a number in the original document it is denoted here as $p_4$. This payment is a reward in the case of a negative total imbalance but may be a penalty in the case of a negative imbalance.

Because these loads are cheaper than the controllable loads the DU originally wanted to deploy, the money the DU earns through payments $p_2$ and $p_3$ is enough to cover the payment $p_4$.

**Example 3.3.4:**

We consider again the same example as in Example 3.3.3. If we consider broker 3 then we can see that in the most efficient allocation the cost for the other brokers to resolve the imbalance is 4 (the cost broker 2 makes). If broker 3 would not have been there, the cost of balancing the imbalance of $+2$ would have been done by broker 1 and 2 for a cost of 9. So broker 3 has to pay a penalty $p_{4,3} = 4 - 9 = -5$. Broker 3 receives 5 for the use of his controllable load that costs him 4 so he makes a profit of 1.

This is done for each of the brokers. Here follows an overview of the payments under the different cost sharing proposals.

**Equal cost sharing:**

<table>
<thead>
<tr>
<th>$i$</th>
<th>$p_2$</th>
<th>$p_3$</th>
<th>$\sum p$</th>
<th>$-u_i$</th>
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<tbody>
<tr>
<td>1</td>
<td>-6</td>
<td>2</td>
<td>-4</td>
<td>4</td>
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<tr>
<td>2</td>
<td>12</td>
<td>2</td>
<td>14</td>
<td>-14</td>
</tr>
<tr>
<td>3</td>
<td>0</td>
<td>2</td>
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**Proportional cost sharing:**

<table>
<thead>
<tr>
<th>$i$</th>
<th>$p_2$</th>
<th>$p_3$</th>
<th>$p_4$</th>
<th>$\sum p$</th>
<th>$-u_i$</th>
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<tbody>
<tr>
<td>1</td>
<td>-6</td>
<td>2</td>
<td>0</td>
<td>-4</td>
<td>-4</td>
</tr>
<tr>
<td>2</td>
<td>12</td>
<td>2</td>
<td>-5</td>
<td>9</td>
<td>13</td>
</tr>
<tr>
<td>3</td>
<td>0</td>
<td>2</td>
<td>-5</td>
<td>-3</td>
<td>1</td>
</tr>
</tbody>
</table>

Table 3.3: The result of Example 3.3.3
3.3 Mechanism

One-sided proportional cost sharing:

\[
\begin{array}{cccccc}
  i & p_2 & p_3 & p_4 & \sum p & -u_i \\
  1 & -6 & 0 & 0 & -6 & -6 \\
  2 & 12 & 6 & -5 & 13 & 17 \\
  3 & 0 & 0 & -5 & -5 & -1 \\
\end{array}
\]

3.3.3 Discussion

The payments that are part of the mechanism are there to ensure the requirements given in section 3.2.3. However the payments attack each one of the requirements separately and because there exists a strong tension between the requirements, which is discussed in the next section, the mechanism as a whole can fail some of the requirements in some situations.

For example the payments \(p_2\) are such that it gives an incentive to have a low imbalance but this penalty is being softened by the redistribution of this same payment through \(p_3\). In the following subsections we go into some of the pitfalls.

Individual rationality

The following example illustrates that individual rationality does not uphold.

If the DU uses equal cost sharing to fix his deficit after receiving the penalties, brokers with a low imbalance may also be required to pay a hefty fine for the acts of others. When they also have no cheap controllable loads that are being used it can overthrow incentive compatibility. The broker may be better off by not participating and just paying the cost \(c_0(x_i)\) that the DU needs for each imbalance.

Consider the following example again with \(P^+ = P^- = 3\):

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<td>-2</td>
<td>4</td>
</tr>
<tr>
<td>3</td>
<td>0</td>
<td>-3</td>
<td>8</td>
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</table>

In this example there is a total imbalance of \(X = -1\) and broker 2 has the cheapest controllable load to resolve this. Broker 1 pays a penalty of \(p_{2,1} = -3\) and broker 2 \(p_{2,2} = 6\). So the DU still comes \(D = 3\) short to solve the imbalance. When this cost is shared by all brokers the third broker has to pay 1 while he did not have any imbalance, and thus no cost if he would not have participated.

It is easy to see that equal cost sharing is unfair for brokers that are perfectly balanced, but proportional cost sharing, which seems fair, is also not individually rational. Consider the following example again with \(P^+ = P^- = 3\):

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The sum of payments $p_1$ the DU receives is $\sum p_2 = 6$. The DU has to incur a cost of 8 to resolve the imbalance so the remainder of the cost $D = 2$ is proportionally distributed over the agents through payment $p_3$. The third broker thus has to pay $-4 \cdot \frac{2}{3} = -\frac{8}{3}$. The third broker thus still receives something for his controllable loads, but not as much as he would have received in the alternative mechanism.

**Forcing the use of own controllable loads**

Having an incentive to truthfully report the cost of the controllable loads is good, but it may interfere with reducing the total imbalance. In this example a broker knows he has an optional load at a very good price. By introducing a positive imbalance he makes sure his controllable load is being used so he can receive the reward.

Consider the following case where $P^+ = 4$ and $P^- = 3$:

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In this case there is no total imbalance, so the third broker pays nothing. However if the third broker is smart, he would have bought extra energy unit ahead of time to make his energy surplus 1. He is able to do this at cost of 4 as that is the maximum price in the day ahead market.

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Now he receives only 3 (the first payment). However because he has the best controllable load, a client of his is willing to pay 3 for an energy unit now, the DU selects his controllable load and lets him pay 1 for this right.

The broker now has paid 4 for the energy, received 3 as part of the first payment, received 3 from his client, and paid 1 to the DU. Now the broker has 1 left.

The DU received in total 1 from the first payments, and he also receives 1 from the third broker. So the DU ends up with 2, so regardless of what the cost sharing strategy is, the broker makes a profit of this scheme.

**No incentive to resolve in day-ahead market**

In this example a broker has an imbalance but payment $p_2$ fails to give him an incentive to resolve it in the day-ahead market, even though he does not have any attractive controllable loads that are being used.

We assume $P^- = 3$ and $P^+ = 6$. 

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Broker 3 has an imbalance, if he knew that he could have bought extra energy, this energy would have cost at maximum $P^+ = 6$ in the day ahead market. For the mechanism to be efficient the broker needs an incentive to buy the energy in the day-ahead market at 6 as that is cheaper than any of the controllable loads.

However in this case the total payments $p_1$ to the DU is $\sum_{i=1}^{n}(p(1,i)) = 18$. The total cost for the DU would make to balance the total imbalance is 6, so there is still $18 - 6 = 12$ left to divide over the brokers.

If equal cost sharing is used the third broker receives $6 - 3 = 3$, which is lower than $P^+$ so he is better off not trying to balance his energy usage in the day-ahead market.

With proportional and one-sided proportional cost sharing the problem remains. Broker 3 then pays $6 - \frac{12}{3} = 4 \frac{2}{3}$ and $6 - \frac{12}{5} = 3 \frac{3}{5}$ respectively.

In each case the mechanism does not provide enough incentive to aim for a low imbalance.

### 3.4 Conclusion

We have introduced the Energy Grid Balancing Problem (EGBP) and have discussed some properties that the solution, a mechanism, should have. De Weerdt et al. have introduced a mechanism but it does not reach individual rationality in the case of equal and proportional cost sharing and it does not always give brokers an incentive to minimize their absolute personal or total imbalance. Therefore further research was needed that the present in the next chapters.
Chapter 4

Mechanisms for grid balancing

4.1 Introduction

In this chapter we explore some useful parts of the problem space for the Energy Grid Balancing Problem (EGBP). First we discuss if EGBP can be solved by any existing mechanisms and then a total of six mechanisms and a proof of concept are presented. Every mechanism presented in this chapter is incentive compatible, individually rational and weakly budget balanced. They only differ in the way they incentivize resolving imbalances on the day-ahead market, amount of revenue collected by the DU and the how strong the individual rationality is.

The first two mechanisms, 1a and 1b, incentivize having a low personal imbalance such that resolving all of the imbalance in the day-ahead market is the dominant strategy. Then, mechanism 2 incentivizes the brokers such that the optimal strategy is to influence the imbalance such that the total imbalance is zero. The last three mechanisms, 3a, 3b and 3c, incentivizes the brokers such that having a personal imbalance of zero is a Nash equilibrium. The weaker incentive of the last 3 mechanisms makes it possible for the mechanism to be 'more' balanced, e.g. the DU profits less.

After the mechanisms and their properties are discussed the result of some experiments are shown to illustrate the behavior of the mechanisms with respect to changes in the problem parameters like the number of participating brokers. The proof of concept at the end of the chapter shows that strongly budget balancedness is possible while still requiring incentive compatibility and individual rationality, but at the expense of efficiency.

4.2 Related previous work

In Chapter 2 we have seen VCG and AGV as generic auction mechanisms which have, when applied to a simple auction, some nice properties. First we explore if the problem defined in chapter 3 can be solved by these existing mechanisms.

The AGV mechanism is the only generic mechanism found in literature that is efficient and strongly budget balanced. However it requires to have a common prior over agent types. Often the common prior does not hold in practice, (Aumann [2] and Morris [22]), rendering
4.3 Conjecture

Mechanisms for grid balancing

a lot of the claims invalid in most cases and we have no reason to assume it would hold in our setting. As the AGV mechanism does not deliver us much, we can go on to VCG.

The VCG mechanism is also a logical candidate. The VCG mechanism is ex post individually rational when the no negative externalities and the choice-set monotonicity property holds. So let us analyze these properties:

The no negative externality property states that if the preferences of an agent $i$ are not taken into account, the selected outcome based on the social welfare of others should not harm that agent $i$. However if we compare that to the situation in EGBP we can see that in the case of a total imbalance $X < 0$ the social welfare of the others is the highest if the total imbalance is regulated by agent $i$, as it costs money to generate extra energy. So that agent $i$ receives harm when his preferences are not taken into account. We thus can conclude that the no negative externality property does not hold for EGBP.

Choice-set monotonicity states that the removal of an agent $i$ should not increase the amount of possible outcomes $O$. Because the set $O$ of all possible outcomes in EGBP consists of different ways of distributing the total imbalance over the brokers, the removal of an agent $i$ with an imbalance $x_i$ and $x_i \cdot X < 0$ (so removing the agent increases the imbalance) would increase the amount of possible outcomes. So the choice-set monotonicity property also does not hold.

The VCG mechanism is also only weakly budget balanced if the no single-agent effect holds. This property states that the removal of one agent always makes the remaining agents weakly better off. However if an agent $i$ has a positive imbalance of $x_i$ and $X > 0$ then removing the broker $i$ from the system decreases the absolute total imbalance $X$ and therefore possibly negatively impact other brokers that could have earned money by using their controllable loads to regulate the imbalance of broker $i$. Thus the no-single-agent property does also not hold and VCG is not weakly budget balanced when applied to EGBP.

The only properties that VCG still has when applied to EGBP is efficiency and individual rationality which every mechanism in the Groves class achieves.

To achieve efficiency and individual rationality for an unrestricted and smoothly connected type space we are forced to look for other mechanisms in the Groves class as only mechanisms in the Groves class are efficient and individually rational.

4.3 Conjecture

So if the mechanism by de Weerdt, Ketter and Collins, AGV and VCG cannot give us a mechanism that is incentive compatible, individually rational, efficient and (weakly) budget balanced, is it possible at all?

I have searched for an adaptation or a new mechanism that is IC, IR, strongly budget balanced and efficient while giving an incentive for a low imbalance. I am now inclined to believe such mechanism cannot exist. Because I was also not able to construct a proof of the non-existence of such mechanism I want to propose the following conjecture:

Conjecture 4.3.1:
There exists no mechanism for EGBP that is efficient, incentive compatible, individually rational and strongly budget balanced that also incentivizes brokers to have a low imbalance.
The reasoning behind this is that there can be two distinct groups of brokers. Brokers that have cheap controllable loads they want to sell and brokers that have imbalances they want to see resolved. Probably a VCG payment is needed for the brokers from the first group to report their private values truthfully. The penalties made by the second group should cover these payments perfectly or else the mechanism is not strongly budget balanced. So each penalty of the second group is a function of the total amount of VCG payments made. However the brokers of the second group can influence this value by introducing a new bid just higher than the most expensive bid that is part of the most efficient allocation. This will cost the broker nothing as his controllable loads will not be executed and thus the mechanism would fail incentive compatibility. The DU cannot punish these strategies because the bids could very well have been truthful, and accidentally punishing truthful bids also threatens incentive compatibility.

4.4 Next steps

So where can we go from here? We can either adapt the problem definition such that it is still relevant and hopefully does have a mechanism with the required properties, or some properties can be removed or weakened.

Adapting the problem definition such that the VCG mechanism can be applied is not an attractive option as the relevance of such a mechanism that solves the new problem would probably not be very high. For example segregating the brokers with an imbalance and the brokers with controllable loads gives a solvable solution, but is not very exciting or useful because the original idea was that brokers could solve the imbalance themselves.

Weakening the requirements of the mechanism is more likely to result in a useful mechanism. Preferably, we only want to remove or weaken only a single requirement.

A mechanism that is not incentive compatible would probably be not hard to construct but when applied in practice the auction holder loses the ability to indirectly measure and control the efficiency of the mechanism because the private values are not known to the DU. So without incentive compatibility efficiency is also not likely to be achievable thus making it this also not a great alternative.

Replacing budget balancedness with weakly budget balancedness while still maintaining the other three requirements is a much more viable alternative. In the following sections some mechanisms are proposed that are weakly budget balanced and result in a varying amount of profits for the DU depending on how the incentive to resolve in the day-ahead market is handled.

Completely abandoning efficiency will also result in a trivial solution (having the DU solve the whole imbalance himself, like in the first part of the mechanism by de Weerdt, Ketter and Collins [19]). It may also be possible to only partially abandon efficiency while keeping strongly budget balancedness. We can briefly show that given any weakly budget balanced mechanism, a strongly budget balanced mechanism can be made at the expense of some efficiency. Given a set of agents we begin by splitting the agents randomly into two groups such that in each group there is at least a single agent. Then the weakly budget balanced mechanism is applied to both groups and the revenue is divided evenly over the
brokers in the other group. Now the mechanism is strongly budget balanced, still has all his other properties but is less efficient because brokers cannot trade with the brokers in the other group. Because the efficiency loss can be very high in the worst case this it is only meant as a proof of concept. 

In Figure 4.1 a Venn diagram is shown that summarizes the items outlined above. All the mechanisms in the Venn diagram are incentive compatible individually rational and have some type of incentive for the brokers to resolve in the day-ahead market. The mechanisms in "PROB IMP" (those that are IC, efficient and strongly budget balanced) are probably impossible to construct. The proof of concept falls into the "POC" field that is strongly budget balanced, IC but not efficient. If we do not want to compromise on IC, IR and efficiency, reducing strongly budget balancedness to weakly budget balancedness is the best solution. All the mechanisms proposed in this chapter thus fall into the "M" field. In that area we are most likely to find a useful mechanism.

Now that we have found that the most useful mechanisms are individually rational, incentive compatible, weakly budget balanced and efficient we still have to decide upon what kind of incentive we would like to give the brokers to resolve their imbalance in the day-ahead market. In Section 3.2.3 we have identified three different ways to incentivize brokers:

1. Create an incentive such that a personal imbalance of zero is the dominant strategy
2. Create an incentive such that a low total imbalance is encouraged
3. Create an incentive such that no personal imbalance is a Nash equilibrium

The first incentive is a stronger version of the third, as if resolving all imbalance in the day ahead market is the dominant strategy, than it also becomes the unique Nash equilibrium. The incentive means that brokers could profit by predicting and counterbalancing the imbalances of others. This is does not mean that a low personal imbalance is automatically a Nash-equilibrium although it is seems like a natural consequence.

In the following sections we will show that the type of incentive greatly influences the amount of revenue the DU makes. Because the amount of revenue that a mechanism makes is an important factor to take into consideration we propose mechanisms for each type of incentive. In Figure 4.2 a Venn diagram is shown with the place of the 6 proposed mechanisms in this chapter.

First we will introduce mechanisms 1a and 1b that have a very strong incentive. Then we will propose mechanism 2 that tries to minimize the total imbalance. Finally we will propose mechanisms 3a, 3b and 3c that have the weakest incentive. These last mechanisms will prove to generate very little revenue for the DU.

In Chapter 3 we have discussed three types of individual rationality. In Figure 4.3 the three types of individual rationality are depicted and what type of individual rationality they have. The strongest form of individual rationality will be used in Chapter 5.
4.5 Low absolute personal imbalance

In this section two mechanisms (1a and 1b), are presented that are incentive compatible (IC), individually rational (IR) and weakly budget balanced (WBB) and efficient regarding the reported prices of the controllable loads. Furthermore the dominant strategy in both mechanisms is to resolve all personal imbalance in the day ahead market.

The first mechanism (1a) provides us with the most simple example of a mechanism that is IC, IR, WBB and efficient. It is used as a sort of template to develop other mechanisms from. The second mechanism (1b) has the same properties as the first mechanism but makes sure the DU does not make as much revenue as in mechanism 1a. Both mechanisms also differ in how strong we can formulate individual rationality.

4.5.1 Mechanism 1a

In this mechanism the brokers pay a penalty per unit of imbalance not only high enough to give an incentive to have a low imbalance but enough to resolve the imbalance if the DU would have to balance it, no matter how large the total imbalance is.

Because the DU has received enough payments to resolve the imbalance himself he can delegate the balancing to cheaper controllable loads of the brokers to maximize social welfare and thus maximize efficiency. The mechanism does require a new restriction on the cost function of the regulating market. It assumes there is a maximum cost of resolving an extra single unit of energy by the DU.

Figure 4.3: The type of individual rationality of each mechanism in a Venn diagram
The mechanism has two payments. The first payment is to make brokers pay for their own imbalance. This payment is a fixed price $D^+$ for negative imbalances or $D^-$ for positive imbalances per unit and depends only on the price of the regulating market. $D^+$ and $D^-$ are chosen such that the total imbalance can be guaranteed to be solved by these payments, even if the brokers do not report any controllable loads. To calculate $D^+$ and $D^-$ we have to assume that those points do exist, so this mechanism does put a restriction on the cost function of the regulating market. In Figure 4.4 a graphical representation of the definitions of $D^+$ and $D^+$ are shown. It is important to remember that $D^-$ is likely to be negative (the surplus of energy can be sold) but does not have to be.

**Definition 4.5.1** $(D^+, D^-)$:

\[
D^+ = \lim_{x \to +\infty} \frac{c_0(x)}{x}
\]

\[
D^- = \lim_{x \to -\infty} \frac{c_0(x)}{x}
\]

To calculate the first payment, brokers are first divided into two groups, the contributing and the non-contributing brokers. The contributing brokers have an imbalance that is of
the same sign as the total imbalance. Any other broker is called a non-contributing broker. A broker being contributing and an imbalance of a broker being contributing are used interchangeably.

**Definition 4.5.2** (contributing):
A broker $i$ and his imbalance $x_i$ is contributing if:

$$x_i \cdot X > 0$$

A broker $i$ and his imbalance $x_i$ is non-contributing if:

$$x_i \cdot X \leq 0$$

If each agent $i$ would pay only $c_0(x_i)$ (the cost of balancing an imbalance of the size of $x_i$) it could be possible that the DU runs a deficit as balancing more energy may be more expensive per unit. For example in the case that balancing the first unit costs only one on the regulating market and every subsequent unit two, then if some brokers have only a single personal imbalance the DU is not be able to resolve the total imbalance with the received payments. So if we require from each broker that contributes to the total imbalance the following payment $p_1$, the DU is guaranteed to have enough to resolve the total imbalance:

If $x_i \leq 0$ and $X \leq 0$ then $p_{1,i} = -x_iD^+$

(4.1)

If $x_i > 0$ and $X > 0$ then $p_{1,i} = x_iD^-$

(4.2)

These payments also make sure that the contributing brokers want to make their imbalance as low as possible. Every broker with a shortage has had the chance to resolve his imbalance on the day ahead market for a price between $P^-$ and $P^+$. $P^+$ is always lower than $D^+$ because we assumed every interruptible controllable loads is more expensive than $P^+$ so resolving on the day ahead market would be the best choice. The same holds for every broker with a surplus of energy, every broker with a positive imbalance has to pay $D^-$ for each imbalance (or rather receive $-D^-$). If such a broker had sold his surplus on the day ahead market he would have received somewhere between $P^-$ and $P^+$.

The brokers that do not contribute to the total imbalance but do have an imbalance also need to pay a penalty to give them an incentive to resolve their imbalance in the day-ahead market. This payment can again be $D^+$ and $D^-$ but a payment of $P^+$ and $P^-$ per unit is also sufficient as this payment only needs to give an incentive for having a low imbalance, and is not really required for resolving the total imbalance:

If $x_i > 0$ and $X \leq 0$ then $p_{1,i} = -x_iP^-$

(4.3)

If $x_i \leq 0$ and $X > 0$ then $p_{1,i} = -x_iP^+$

(4.4)

The second payment of this mechanism is the same as the third payment in the mechanism by de Weerdt, Ketter and Collins [19]. The DU chooses the cheapest controllable loads
and gives out VCG payments for the use of these controllable loads. A broker thus receives the difference in utility of the other brokers between the broker having his controllable load, and the broker not being there with his controllable load. Because the VCG payment to a broker \( i \) only depends on the set of controllable loads \( C \) and the total imbalance \( X \), we can define this payment in terms of \( i, C \) and \( X \):

**Definition 4.5.3** (VCG\((i, C, X)\)):

The harm a broker \( i \) causes to other brokers by reporting his controllable loads with a total set of controllable loads \( C \) and with a total imbalance \( X \) is:

\[
VCG(i, C, X) = v_{-i}(\theta_{-i}, f^*(\theta_{-i})) - v_{-i}(\theta_{-i}, f^*(\theta_{-i}))
\]

The second payment is thus:

\[
p_{2,i} = VCG(i, C, X)
\]

Note that the second payment can be negative as well as positive. When the total imbalance is negative, the second payment is also negative, thereby rewarding brokers for the use of their controllable loads. If the total imbalance is positive it is possible that brokers have controllable loads that earn money, in this case \( p_2 \) is positive and can be seen as a payment to the DU for the privilege of resolving the imbalance.

**Example 4.5.4** (Mechanism 1a):

The following example is also used to illustrate the other mechanisms so the difference between the mechanisms becomes apparent.

- \( P^- = 4, P^+ = 5 \)
- The DU has optional controllable loads which cost \(-1, -1, \ldots (D^- = -1)\)
- and interruptible controllable loads which cost \(12, 13, 14, 15, 15, \ldots (D^+ = 15)\)

The example has 6 brokers and the DU. The DU has an unlimited supply of optional controllable loads that cost \(-1\); utilizing such load would increase the DU’s utility by 1. This could for example mean that there is always a neighboring region that is willing to buy energy at a price of 1. The DU has interruptible controllable loads cost 12, 13, 14
and an unlimited supply of controllable loads that cost 15. So $D^-$ and $D^+$ are $-1$ and 15 respectively, as this is the lowest price per unit the DU is able to collect while guaranteeing budget balancedness.

All the imbalances and the cost of the optional and interruptible controllable loads of each broker are shown in Table 4.1. Broker 3 is the only broker with a personal imbalance of zero and the total imbalance is $X = -3$. The three cheapest controllable loads are used to resolve the imbalance and those loads are owned by broker 4, 5 and 6 for prices of respectively 8, 7 and 6.

Broker 1 has a personal imbalance of two and is thus non-contributing. Following the mechanism it has to pay $P$ for each imbalance, which is a total of $-8$. So he receives the lowest price seen in the day-ahead market, guaranteeing that he does not make a profit with his imbalance. Broker 2 has a personal imbalance of -5, because $-5$ is a contributing imbalance he has to pay $D^+$ for each imbalance, which is $5 \cdot 15 = 75$. The same calculation holds for brokers 3-6.

Brokers 4-6 pay a penalty $p_2$ of $-9$ for the use of their controllable loads. So broker 4 effectively gains $9 - 6 = 3$ out of this transaction, as his controllable loads costs only 6. The total amount of payments and the final utility for each broker can be seen in Table 4.2.

<table>
<thead>
<tr>
<th>$i$</th>
<th>$p_{1,i}$</th>
<th>$p_{2,i}$</th>
<th>$\sum p_i$</th>
<th>$u_i$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>-8</td>
<td>0</td>
<td>-8</td>
<td>8</td>
</tr>
<tr>
<td>2</td>
<td>75</td>
<td>0</td>
<td>75</td>
<td>-75</td>
</tr>
<tr>
<td>3</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>4</td>
<td>-16</td>
<td>-9</td>
<td>-25</td>
<td>17</td>
</tr>
<tr>
<td>5</td>
<td>15</td>
<td>-9</td>
<td>6</td>
<td>-13</td>
</tr>
<tr>
<td>6</td>
<td>45</td>
<td>-9</td>
<td>36</td>
<td>-42</td>
</tr>
</tbody>
</table>

Table 4.2: The payments and utility for each broker

In this example the DU does not have to use his own controllable loads to solve the total imbalance and thus can keep the sum of all the payments which is 84 in this case. This is a significant amount, enough to balance an extra 7 units, but the next mechanism improves on this situation.

### 4.5.2 Mechanism 1b

In the previous mechanism the first payments were independent of the prices that the brokers reported for their controllable loads. Thus if a lot of brokers have much better controllable loads than those of the DU, brokers with a large imbalance would not profit from these controllable loads, only the DU would profit. So although the previous mechanism maximizes social welfare, we would much rather maximize the welfare of the brokers while still keeping the mechanism weakly budget balanced. This mechanism solves this and makes brokers pay a penalty large enough to solve the imbalance but most of the time not as large as $D^+$ and $D^-$. 
As we will see in all mechanism in this chapter the second penalty remains the same, as without it the brokers would not truthfully report the costs of their controllable loads to the DU.

\[ p_{2,i} = VCG(i, C, X) \] (4.7)

In this mechanism we want the penalty \( p_1 \) to be dependent on the prices of the controllable loads owned by the brokers instead of the price of the DU in the worst case. Therefore the function \( DU_{\text{costs}} \) is introduced. It defines the amount the DU has to spend to resolve the total imbalance.

In the case that there are not enough controllable loads available to regulate the total imbalance \( X \) the DU has to make extra cost to regulate the rest of the imbalance. We define a function \( D : \mathbb{R} \to \mathbb{R} \) where \( D(X) \) is the costs that the DU makes from using how own controllable loads. The sum of all the second payments times -1 plus the extra cost \( D(X) \) is the total cost the DU makes to balance everything. The total cost of the DU is only dependent on the cost of his own controllable loads, the set of all controllable loads of the brokers \( C \) and the total imbalance \( X \). So we can define the cost that the DU makes to resolve the total imbalance as:

**Definition 4.5.5 (DU_{\text{costs}}):**

\[ DU_{\text{costs}}(C, X) = - \sum_{i=0}^{n} VCG(i, C, X) + D(X) \]

The first payments that the brokers pay to the DU have to be at least as large as \( DU_{\text{costs}}(C, X) \) or else the DU would run a deficit. The simplest solution would be to charge each contributing broker with \( \frac{DU_{\text{costs}}(C, X)}{X} x_i \) as that would guarantee budget balancedness. However using this payment would affect incentive compatibility. When this payment is used, the first penalty is possibly dependent on all controllable loads. The first penalty should not ever depend on the cost of balancing the total imbalance, as brokers can influence this price by manipulating the price of their controllable load. The mechanism would no longer fall in the Groves class.

What contributing brokers can pay, is their share of the cost that the DU has to make to balance the total imbalance, in the case that their own controllable loads are not available. This is guaranteed to be higher than \( P^+ \), needed for the incentive to resolve in the day-ahead market, and is high enough to cover the costs of the DU. We define \( C_i \) as the collection of all controllable loads owned by broker \( i \), and \( C_{-i} \) as all the controllable loads not owned by by broker \( i \). Now we can define the first payment of our mechanism as:

For each contributing broker: \( p_{1,i} = \frac{DU_{\text{costs}}(C_{-i}, X)}{X} x_i \) (4.8)

This payment is guaranteed to be enough to resolve the total imbalance and decreases when cheaper controllable loads are reported to the DU. The \( DU_{\text{costs}} \) cannot drop when controllable loads are removed from the game. Each noncontributing broker still has to pay a penalty of \( P^+ \) and \( P^- \) to withhold them of profiting from a larger imbalance:
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Table 4.3: The payments and utility of each broker

<table>
<thead>
<tr>
<th>i</th>
<th>$p_1$</th>
<th>$p_2$</th>
<th>$\sum p_i u_i$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>-8</td>
<td>0</td>
<td>-8</td>
</tr>
<tr>
<td>2</td>
<td>45</td>
<td>0</td>
<td>45</td>
</tr>
<tr>
<td>3</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>4</td>
<td>-16</td>
<td>-9</td>
<td>-25</td>
</tr>
<tr>
<td>5</td>
<td>10</td>
<td>-9</td>
<td>1</td>
</tr>
<tr>
<td>6</td>
<td>30</td>
<td>-9</td>
<td>21</td>
</tr>
</tbody>
</table>

If $x_i > 0$ and $X \leq 0$ then $p_{1,i} = -x_i P^-$ \hspace{1cm} (4.9)

If $x_i \leq 0$ and $X > 0$ then $p_{1,i} = -x_i P^+$ \hspace{1cm} (4.10)

**Example 4.5.6 (Mechanism 1b):**

Again we use the same example as we have used for mechanism 1a. To reiterate there is a total imbalance of -3 and $P^-$, $P^+$ are respectively -4 and 5 and the imbalances and private values of the brokers can be seen in Table 4.1.

Broker 1 has a non-contributing imbalance so he only has to pay $P^-$ for each unit of imbalance, which is basically a reward for his surplus of energy but not enough to make a profit out of it. This is the same as in mechanism 1a. Broker 2 however has to pay much less than in the previous mechanism. Here he only pays the amount of VCG payments needed to resolve the imbalance without his own controllable loads divided by the total imbalance multiplied by his own imbalance. Without his controllable loads the brokers 4, 5 and 6 would have to resolve the total imbalance. The next-cheapest controllable load costs 9 and is owned by broker 3. So the brokers 4, 5 and 6 would receive a payment of 9 each to resolve the total imbalance of -3. This is 9 per unit so broker 1 has to pay $5 \cdot 9 = 45$ for his imbalance.

If we look at broker 5 we see that he pays more for his imbalance than broker 2. Without the controllable loads of broker 5 the total imbalance would have to be resolved by the brokers 3, 4 and 6. Because the next cheapest controllable load is owned by broker 2 and costs 10, each of the brokers would receive 10 in this hypothetical case. This is 10 per unit so broker 5 has to pay $1 \cdot 10 = 10$ for his total imbalance.

And like in the previous mechanism the brokers 4, 5 and 6 pay a payment $p_2$ of $-9$ for the use of their controllable loads. All the payments and utilities can be seen in Table 4.3.

The DU receives a sum of 61 because of the first payment and has to give out 27 because of the second payment. In the end he can keep 34 as he does not have to resolve any imbalance himself.

### 4.5.3 Properties

The payments for imbalances is independent on the size of the total imbalance. Therefore mechanism 1a is has the "Predetermined penalty" form of individual rationality. The
Payments of mechanism 1b scale with the size of the total imbalance and is therefore individually rational when the alternative is to let the DU solve the total imbalance and to share the cost. This is the "Penalty based on loads DU" form of individual rationality described in Section 3.2.3.

**Theorem 4.5.7.** The mechanisms 1a and 1b are both efficient and incentive compatible

*Proof.* To prove that the mechanisms are efficient and incentive compatible it is enough to see that the mechanisms are part of the Groves class. A mechanism is only part of the Groves class if it chooses an outcome such that it maximizes the social welfare. This is clearly the case as the DU selects the most efficient controllable loads to resolve the total imbalance. Also for every agent \(i\) the payment has to be such that

\[
P_i(\theta) = v_{-i}(\theta_{-i}, f^*(\theta)) - h_i(\theta_{-i})
\]

where \(h_i\) is a function independent of the values of \(i\). In both mechanisms the VCG payment \(p_{2,i}\) is used. This VCG payment is equal to \(v_{-i}(\theta_{-i}, f^*(\theta)) - v_{-i}(\theta_{-i}, f^*(\theta_{-i}))\). The first part is the same as the first part of the Groves class payment. The second part is independent of the reported values of agent \(i\), and thus can be seen as a part of the function \(h_i\). Because the payment \(p_{1,i}\) is also independent of the reported value of agent \(i\) the mechanisms are in the Groves class and thus efficient and incentive compatible.

**Theorem 4.5.8.** The mechanisms 1a and 1b are both weakly budget balanced

*Proof.* In case of a negative total imbalance with mechanism 1a, every contributing broker pays \(D^-\) per unit of imbalance and every non-contributing broker receives \(P^-\) per unit of imbalance, because \(D^- > P^-\) we know that the DU receives at least \(|X| \cdot D^-\) in payments. He can use this to resolve the imbalance himself, which costs less than \(D^-\) per unit, or use the controllable loads of the brokers if they are available, which costs him even less.

In the case of a positive total imbalance with mechanism 1a, every contributing broker pays \(D^+\) per unit of imbalance and every non-contributing broker pays \(P^+\) per unit of imbalance. Because \(P^+ > 0\) we know that the DU has at least \(X \cdot D^+\) which is possibly negative. However the controllable loads of the DU and/or those of the brokers are able to profit him at least \(D^+\). So the mechanism 1a is weakly budget balanced.

In mechanism 1b the DU has to spend an amount equal to \(DU_{costs}\) (which include the second payment) to resolve the total imbalance. Because the sum of the payments \(p_{1,i}\) of all contributing brokers is at least \(DU_{costs}\), the DU always has enough to resolve the total imbalance.

Showing that resolving in the day ahead market is the dominant strategy is shown separately for both mechanisms. The proof for mechanism 1b is considerably longer and more difficult so we start with mechanism 1a.

**Theorem 4.5.9.** In mechanism 1a the dominant strategy is to resolve all imbalances on the day-ahead market.
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Proof. For mechanism 1a we can identify 4 different cases:

If \( x_i < 0 \) and \( X \leq 0 \) then the broker \( i \) needs to buy a unit on the day ahead market to decrease his absolute total imbalance, this costs him at most \( P^+ \). However, because he also decreases the absolute total imbalance he may also increase his payment \( p_{2,i} \) (or decrease his profits from his controllable loads to put it in another way). The decrease in VCG reward would only affect a single controllable load of broker \( i \), and the decrease in value cannot be greater than \( D^+ - P^+ \) because the most expensive controllable loads that are ever going to be used costs \( D^+ \) and the controllable loads of broker \( i \) cost at least \( P^+ \). So to decrease his absolute personal imbalance it would cost him at most \( D^+ \). The payment \( p_{1,i} \) decreases by \( D^+ \) for each imbalance so it is better to have a lower imbalance.

If \( x_i > 0 \) and \( X > 0 \) then almost the same thing happens. Selling a unit on the day ahead market would profit the broker at least \( P^- \). The decrease in VCG payments is at most \( P^- - D^- \). So the broker loses at most \( D^- \) but that is exactly the decrease in \( p_{1,i} \) the broker would undergo. So it is better to have a lower imbalance.

If \( x_i > 0 \) and \( X \leq 0 \) then selling a unit would earn the broker at least \( P^- \), the payment \( p_2 \) cannot go up and payment \( p_{1,i} \) would increase by at most \( P^- \) so it is better to have a lower imbalance.

The same holds for the case that \( x_i \leq 0 \) and \( X > 0 \), buying a unit would cost at most \( P^+ \), \( p_{2,i} \) cannot go up and \( p_{1,i} \) goes up by \( P^+ \) so it is better to have a lower imbalance.

In each of the cases it is never worse to decrease the absolute total imbalance so from induction we can see that it is never worse to resolve the total imbalance in the day-ahead market.

Now follows a proof of the incentive to resolve in the day-ahead market for mechanism 1b. We show that if a broker is not completely balanced (\( x_i \neq 0 \)), he can better resolve his imbalance in the day-ahead market.

The proofs use the variables \( c_a, c_b, c_c \) and \( c_d \). These are the prices of certain controllable loads that are important in the proof. For each agent \( i \), each variable may point to other controllable loads. They are only relevant if the total imbalance is nonzero.

Some equations are boxed. These are used in the proof, however can be seen relatively easily so they do not have the status of lemmas.

Definitions for negative total imbalance

In Figure 4.5 the relevant controllable loads and variables needed are shown for the case that the total imbalance is negative. If the total imbalance is negative only the interruptible controllable loads are relevant. All the interruptible controllable loads of all brokers are sorted in Figure 4.5 from expensive (top) to cheap (bottom) and some (but not all) are part of one of the three groups. The groups may be different, and so \( c_a, \ldots, c_d \) may point to different loads for each broker. In this proof we only look at a single broker \( i \).

The first group in Figure is the group of the most efficient controllable loads. These are the cheapest controllable loads reported by all the agents. There are exactly \( |X| \) controllable loads in this group. None, some, or all controllable loads in this group may be owned
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Figure 4.5: A graphical representation of the variables $c_a, c_b, c_c$ and $c_d$ and their order in the case of a negative total imbalance

by broker $i$. This group is the same for every broker. The price of the most expensive controllable load in this group is referred as $c_a$.

Brokers receive VCG payments for the use of their controllable loads. The height of this payment for broker $i$ depends on the marginal benefit of his controllable loads. If broker $i$ owns $q$ controllable loads in the first group, the VCG payment is calculated by summing the prices of the next $q$ controllable loads from the top of the first group. These $q$ controllable loads are part of the second group as they are the controllable loads on which the VCG payment of broker $i$ are based on. The controllable loads in this group are all controllable loads not owned by broker $i$. $c_b$ is the price of the most expensive controllable load in this group. If $q = 0$ it follows from the definition that $c_b = c_a$.

Brokers also pay for their imbalance through payment $p_1$. This payment is based on the $DU_{costs}$ (so VCG payments that the DU has to give out to brokers) in the case where the controllable loads of broker $i$ are not available. Because broker $i$ has $q$ controllable loads among the most efficient controllable loads, without those $q$ controllable loads, $q$ controllable loads extra have to be used, all of the controllable loads in the second group. The cost the DU makes depends on the price of the next $|X|$ controllable loads after the second group. In the best case where each controllable load is owned by a different broker the DU only has to pay the price of the next cheapest controllable load after the second group, which we refer to as $c_c$, times the absolute total imbalance. However in the worst case where all controllable loads are owned by a single broker the DU has to pay the sum of the next $|X|$ controllable load from group 2 not owned by $i$ and that one broker. We define these $|X|$ controllable loads as third group. The price of the cheapest controllable load is $c_c$.
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Figure 4.6: A graphical representation of the variables $c_a, c_b, c_c$ and $c_d$ and their order in the case of a positive total imbalance

and the most expensive one is $c_d$. In the case of $X = -1$ one can see that $c_c = c_d$.

Definitions for positive total imbalance

If the total imbalance is positive the same variables $c_a, c_b, c_c$ and $c_d$ are used in the proof. But instead of referring to the prices of interruptible controllable loads they point to optional controllable loads.

In Figure 4.6 the same groups can be identified for the case of a positive imbalance. The controllable loads are again sorted from expensive to cheap (however these controllable loads can be so cheap there price may be negative).

Note that the same variables $c_a, c_b, c_c$ and $c_d$ are pointing to different controllable loads depending on the imbalance being negative or positive. This is necessary to keep the proof concise.

Order of variables

After having identified the four relevant prices for the proof we can note some observations. Because every interruptible controllable load is more expensive than the price of the day-ahead market we can see that:

\[
\text{if } X < 0 \text{ then } 0 \leq P^- \leq P^+ \leq c_d \leq c_b \leq c_c \leq c_a
\]  

(4.11)

And because the optional controllable loads cannot earn more than $P^+$:
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If \( X > 0 \) then \( 0 \leq -P^+ \leq -P^- \leq c_d \leq c_b \leq c_e \leq c_c \) (4.12)

Zero payments

In a few cases the broker does not have to pay one or both of the payments. These lemmas are very straightforward.

If the total imbalance is 0, then the payment \( p_{2,i} \) and the operating cost is zero for every broker.

\[
\text{If } X = 0 \text{ then } \forall i, p_{2,i} = 0, O_i = 0 \quad (4.13)
\]

If a broker does not have an imbalance, his payment \( p_{1,i} \) is zero.

\[
\text{If } x_i = 0 \text{ then } p_{1,i} = 0 \quad (4.14)
\]

Lower imbalance

We use \( u_i \) to denote the utility a broker receives when he has an imbalance \( x_i \) and reports all his controllable loads truthfully. From here on \( x'_i \) denotes the imbalance in the case broker \( i \) would reduce his absolute personal imbalance by a single unit by buying or selling a unit on the day ahead market.

\[
x'_i = x_i - \frac{x_i}{|x_i|} \quad (4.15)
\]

Likewise \( X \) denotes the total imbalance when an agent reports \( x_i \) and \( X' \) denotes the total imbalance in case the broker has an imbalance of \( x'_i \). The same holds for the payments \( p_{1,i} \) and \( p_{2,i} \) which become \( p'_{1,i} \) and \( p'_{2,i} \) in the case of \( x'_i \). The operating cost changes from \( O \) to \( O' \). The total utility in both cases is now:

\[
\begin{align*}
  u_i &= -p_{1,i} - p_{2,i} - O_i \quad (4.16) \\
  u'_i &= -p'_{1,i} - p'_{2,i} - O'_i - B_i \quad (4.17)
\end{align*}
\]

Where \( B_i \) is the payment broker \( i \) has to make to balance the single unit in the day ahead market to reach the imbalance of \( x'_i \) instead of \( x_i \).

Lemma 4.5.10 (Maximum payment to resolve in the day ahead market):
The payment \( B_i \) broker \( i \) pays to have a single unit less absolute personal imbalance is always less than \( c_d \).

\textbf{Proof}: In the case of \( x_i < 0 \) the payment to resolve a unit in the day ahead market is maximum \( P^+ \), which is less than \( c_d \). In the case of \( x_i > 0 \) the payment to resolve a single unit is at maximum \( -P^- \), which is less than \( c_d \). \qed
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Profit from controllable loads

If an agent reduces the absolute total imbalance by a single unit, the profits that broker receives from his controllable load cannot increase. Less imbalance means a lower marginal price for the remaining controllable loads. So in the case the imbalance $X$ is reduced to an imbalance $X'$ it holds that:

$$\text{If } x_i \cdot X > 0 \text{ then } VCG(i, C, X) + O_i - VCG(i, C, X') - O'_i \leq 0$$

(4.18)

The following lemma is an improvement on this notion. The difference in profits between the two situations with total imbalances $X'$ and $X'$ is exactly the difference between the most expensive chosen controllable load and the marginal price of the chosen controllable loads:

**Lemma 4.5.11 (Profit reduction from imbalance reduction):**

$$\text{If } x_i \cdot X > 0 \text{ then } VCG(i, C, X) + O_i - VCG(i, C, X') - O'_i = -c_b + c_a$$

(4.19)

**Proof.** If an agent reduces his absolute personal imbalance by 1 and thereby reduces the total imbalance, it is possible that his payment $p_{2,i}$ increases (reward decreases). We show that this decrease in utility is bounded by $-c_b + c_a$. If this decrease was not bounded it would give brokers an incentive to not decrease their absolute total imbalance. We assume the broker has at least a single controllable load that is part of the first group, if this is not the case the decrease is trivial as the broker does not experience any decrease in utility.

If an agent reduces his imbalance when $x_i \cdot X > 0$ then the size of the first group drops by 1, as there is 1 less absolute total imbalance. Then there are two cases. In the first case controllable load $c_a$ is not owned by broker $i$. Then the amount of controllable loads $q$ in the second group stays the same. Because controllable load $c_a$ now enters the group, $c_b$ leaves the group. Because the payment $p_{2,i}$ is the sum of all controllable loads in the second group, the difference is $-c_b + c_a$.

In the second case the controllable load $c_a$ is owned by broker $i$. The controllable load $c_a$ gets pushed out of the first group but not into the second (as it is owned by broke $i$). The amount of controllable loads $q$ in group 2 does drop by 1 so $c_b$ is being pushed out of group 2. In the end broker $i$ loses the reward $c_b$ but also does not have to pay the operating cost of controllable load $c_a$.

So if broker $i$ reduces his total personal imbalance $x_i$ by 1 to $x'_i$, and if he thereby decreases the total imbalance the decrease in utility is at most $-c_b + c_a$. 


**Imbalance penalties**

If an agent reduces his absolute personal imbalance and thereby reduced the absolute total imbalance, the payment $p_{1,i}$ he has to pay to the DU becomes less. This difference is at least the size of the marginal price of the chosen controllable loads.
Lemma 4.5.12 (Profit from having a lower imbalance):

If $x_i \cdot X > 0$ then

$$\frac{DU_{\text{costs}}(C_{-i}, X)}{X} x_i - \frac{DU_{\text{costs}}(C_{-i}, X')}{X'} x'_i \geq c_b$$

Proof. Because we have defined the third box as the controllable loads on which $DU_{\text{costs}}(C_{-i}, X)$ is based on we can make an upper and lower bound on the difference in penalty $p_{1,i}$ in the case the agent reports $x_i$ or $x'_i$. First some propositions need to be introduced.

Broker $i$ has to pay a fraction of the $DU_{\text{costs}}$ depending on the size of his own imbalance. The payment he pays therefore cannot exceed $c_d$ or be lower than $c_c$ per unit of imbalance:

Proposition 4.5.13:

If $x_i < 0$ and $X < 0$ then $c_c \leq - \frac{DU_{\text{costs}}(C_{-i}, X)}{X} \leq c_d$

Proposition 4.5.14:

If $x_i > 0$ and $X > 0$ then $c_c \leq \frac{DU_{\text{costs}}(C_{-i}, X)}{X} \leq c_d$

In the case of a shortage of energy a minus sign is needed, as it is later multiplied by $x_i$, which is negative.

Also, if the absolute personal imbalance of the broker is decreased by one (so the absolute total imbalance is also decreased by one) we have already said that $c_b$ would be kicked out of the second group possibly into the third. In that case $c_b$ is the cheapest (or smaller than the cheapest) of the third group making the payment for each imbalance at least $c_b$. And because lowering the total imbalance can only decrease the payment per imbalance the payment is still at most $c_d$ per unit of imbalance:

Proposition 4.5.15:

If $x < 0$ and $X < -1$ then $c_b \leq - \frac{DU_{\text{costs}}(C_{-i}, X + 1)}{X + 1} \leq c_d$

Proposition 4.5.16:

If $x > 0$ and $X > 1$ then $c_b \leq \frac{DU_{\text{costs}}(C_{-i}, X - 1)}{X - 1} \leq c_d$

With these propositions we show that the theorem holds:

$$\frac{DU_{\text{costs}}(C_{-i}, X)}{X} x_i - \frac{DU_{\text{costs}}(C_{-i}, X')}{X'} x'_i \geq -c_c x_i - \frac{DU_{\text{costs}}(C_{-i}, X')}{X'} x'_i x_i$$

$$\geq -c_c x_i - c_b x'_i$$

$$\geq -c_b x_i - c_b x'_i$$

$$= c_b$$
Incentive to resolve in the day-ahead market

**Theorem 4.5.17** (Incentive for zero imbalance). In mechanism 1b for any broker i, it is the dominant strategy to resolve all imbalance on the day-ahead market

Before we prove the above theorem, we will first prove the following lemma, after which we can prove the above by induction:

**Lemma 4.5.18** (Incentive for low imbalance): In mechanism 1b for any broker i that has an imbalance not equal to 0, it is never worse for that broker to resolve a single unit on the day-ahead market so that he has an imbalance of \( x'_i \) when reporting truthfully.

**Proof.** The payments \( p_{1,i} \) depend on the broker being a contributing or non-contributing broker. Therefore the theorem will be proven for each of the following mutually exclusive exhaustive list of cases:

1. \( x_i \leq -1, X < -1 \) or \( x_i \geq 1, X > 1 \)
2. \( x_i = -1, X = -1 \) or \( x_i = 1, X = 1 \)
3. \( x_i < -1, X = -1 \)
4. \( x_i > 1, X = 1 \)
5. \( x_i < 0, X \geq 0 \)
6. \( x_i > 0, X \leq 0 \)
1. For \( x_i \leq -1 \) and \( X < -1 \) or \( x_i \geq 1 \) and \( X > 1 \) it holds that \( u_i \leq u_i' \)

\[
u_i = -p_{1,i} - p_{2,i} - O_i = -\frac{DU_{\text{costs}}(C_{-i}, X)}{X} x_i - p_{2,i} - O_i \\
\leq -c_b - \frac{DU_{\text{costs}}(C_{-i}, X')}{X'} x_i' - p_{2,i} - O_i \tag{Lemma 4.5.12} \\
= -c_b - \frac{DU_{\text{costs}}(C_{-i}, X')}{X'} x_i' - VCG(i, C, X) - O_i \\
= -c_b - \frac{DU_{\text{costs}}(C_{-i}, X')}{X'} x_i' - VCG(i, C, X') - O_i' + c_b - c_a \tag{Lemma 4.5.11} \\
= -\frac{DU_{\text{costs}}(C_{-i}, X')}{X'} x_i' - VCG(i, C, X') - O_i' - c_a \\
\text{if } |x_i| = 1 \text{ then } x_i' = 0 \text{ and } p_{1,i}' = 0 \text{ or substitute definition of } p_{1,i}' \\
= -p_{1,i}' - p_{2,i}' - O_i' - c_a \\
\leq -p_{1,i}' - p_{2,i}' - O_i' - B_i \tag{Lemma 4.5.10} \\
= u_i'
\]

2. For \( x_i = -1 \) and \( X = -1 \), or \( x_i = 1 \) and \( X = 1 \) it holds that \( u_i \leq u_i' \)

\[
u_i = -p_{1,i} - p_{2,i} - O_i = -\frac{DU_{\text{costs}}(C_{-i}, X)}{X} x_i - p_{2,i} - O_i \\
\leq c_b - p_{2,i} - O_i \tag{prop 4.5.15 or 4.5.13} \\
= c_b - VCG(i, C, X) - O_i \\
= c_b + c_b - c_a \\
= -c_a \\
\leq -B_i \tag{Lemma 4.5.10} \\
= -p_{1,i}' - p_{2,i}' - B_i \\
= u_i'
\]
3. For $x_i < -1$ and $X = -1$ it holds that $u_i \leq u'_i$

$$u_i = -p_{1,i} - p_{2,i} - O_i$$
$$= - \frac{DU_{\text{costs}}(C_{-1},X)X_i - p_{2,i} - O_i}{X}$$
$$\leq c_d x_i - p_{2,i} - O_i$$  \hspace{1cm} \text{prop 4.5.13}$$
$$= c_d x_i + c_b - c_a$$
$$= c_d x'_i - c_d + c_b - c_a$$
$$= c_d x'_i - c_d - p'_{2,i} - O'_i + c_b - c_a$$  \hspace{1cm} \text{Lemma 4.5.11}$$
$$\leq c_d x'_i - p'_{2,i} - O'_i - c_a$$
$$\leq P^+ x'_i - p'_{2,i} - O'_i - c_a$$
$$= -p'_{1,i} - p'_{2,i} - O'_i - B_i$$  \hspace{1cm} \text{Lemma 4.5.10}$$
$$= u'_i$$

4. For $x_i > 1$ and $X = 1$ it holds that $u_i \leq u'_i$

$$u_i = -p_{1,i} - p_{2,i} - O_i$$
$$= - \frac{DU_{\text{costs}}(C_{-1},X)X_i - p_{2,i} - O_i}{X}$$
$$\leq c_d x_i - p_{2,i} - O_i$$  \hspace{1cm} \text{prop 4.5.13}$$
$$= c_d x_i - \text{VCG}(i,C,X) - O_i$$
$$= c_d x'_i - p'_{2,i} - O'_i + c_b - c_a$$  \hspace{1cm} \text{Lemma 4.5.11}$$
$$\leq c_d x'_i - p'_{2,i} - O'_i - c_a$$
$$\leq P^- x_i - p'_{2,i} - O'_i - c_a$$
$$\leq -p'_{1,i} - p'_{2,i} - O'_i - B_i$$  \hspace{1cm} \text{Lemma 4.5.10}$$
$$= u'_i$$
5. For $x_i < 0$ and $X \geq 0$ it holds that $u_i \leq u_i'$

$$
u_i = -p_{1,i} - p_{2,i} - O_i = P^+ x_i - p_{2,i} - O_i = P^+ x_i' + P^+ - p_{2,i} - O_i = P^+ x_i' - p_{2,i}' - O_i' + P^+ \leq P^+ x_i' - p_{2,i}' - O_i' + B_i \quad \text{Lemma 4.5.10}$$

6. For $x_i > 0$ and $X \leq 0$ it holds that $u_i \leq u_i'$

$$
u_i = -p_{1,i} - p_{2,i} - O_i = P^- x_i - p_{2,i} - O_i = P^- x_i' + P^- - p_{2,i} - O_i = P^- x_i' - p_{2,i}' - O_i' + P^- = -p_{1,i}' - p_{2,i}' - O_i' + P^- \leq -p_{1,i}' - p_{2,i}' - O_i' + B_i \quad \text{Lemma 4.5.10}$$

\[\square\]

**Incentive for zero imbalance.** Now that we have proven for each case that it is better to have less absolute imbalance if $x_i \neq 0$, and because the cases encompass all possible cases where $x_i \neq 0$ it follows from induction that it is always better to resolve the whole imbalance in the day-ahead market. Thus we have proven Theorem 4.5.17 true. \[\square\]
4.6 Low absolute total imbalance

In this section mechanism 2 is proposed that can achieve a lower profit for the DU by returning a lot of payments from the contributing brokers to the non-contributing brokers. Also, instead of incentivizing brokers to have an imbalance of zero, brokers are incentivized to have a high non-contributing imbalance, thus stabilizing the grid as a whole in the day-ahead market.

In mechanism 2 non-contributing brokers are rewarded for their imbalance instead of being penalized. Consider the case where the total imbalance is positive but some brokers have a negative imbalance. Because without those brokers the imbalance would be even greater, and thus cost more, those brokers are entitled to the money that they saved the others.

Also, in a setting where some brokers can make better predictions about other brokers than those brokers themselves this mechanism can be more efficient, as in that it results in more imbalance resolved in the day-ahead market. For example consider the case of 2 brokers. Broker 1 owns a large solar energy farm and predicts that the energy output of his farm is enough to match the consumption of his costumers. However broker 2 has access to a much more accurate weather model and predicts that the solar energy farm of broker 1 will produce 2 units of energy less than predicted with the more basic model of broker 1. Broker 2 anticipates this and buys 2 extra units of energy in the day-ahead market creating a total imbalance of 0.

In the case of mechanism 1a and 1b broker 2 would be penalized for his imbalance (or rather be rewarded with ‘only’ \(-P^-\), guaranteed less than he had paid on the day-ahead market) resulting in a total imbalance of \(-2\), because broker 2 would only be interested in maintaining a low personal imbalance. By rewarding non-contributing brokers the brokers resolve more in the day-ahead market.

It is possible that more brokers saw the shortage of broker 1 coming, and if they all make the same decision as broker 2 a huge overcompensation would occur, thereby hurting the efficiency. This is not seen as a problem because every overcompensating broker is now a contributing broker, and thus automatically being penalized for this behavior. The brokers thus have enough incentive to watch out for overcompensation and take into account the action of others.

Also, it may be possible for the mechanism to reach less profit for the DU in comparison to mechanisms 1a and 1b. For example in the case when there is a small total imbalance but each personal imbalance is very large, the sum of all payments in the mechanisms 1a and 1b are very large. To incentivize brokers to resolve their imbalance on the day ahead market, at least \(P^+\) and \(P^-\) have to be charged, even when there is no total imbalance to resolve. Less profit can only be made possible when that incentive is weakened, like

4.6.1 Mechanism 2

The mechanism is very simple. It is like mechanism 1, but instead of charging \(P^+\) or \(P^-\) for non-contributing brokers, we reward them with the same amount per unit the contributing brokers have paid.
Mechanisms for grid balancing

4.6 Low absolute total imbalance

<table>
<thead>
<tr>
<th>i</th>
<th>( p_1 )</th>
<th>( p_2 )</th>
<th>( \sum p )</th>
<th>( u_i )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>-30</td>
<td>0</td>
<td>-30</td>
<td>-30</td>
</tr>
<tr>
<td>2</td>
<td>75</td>
<td>0</td>
<td>75</td>
<td>-75</td>
</tr>
<tr>
<td>3</td>
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<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>4</td>
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<td>-9</td>
<td>-69</td>
<td>61</td>
</tr>
<tr>
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<td>-9</td>
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<td>-13</td>
</tr>
<tr>
<td>6</td>
<td>45</td>
<td>-9</td>
<td>36</td>
<td>-42</td>
</tr>
</tbody>
</table>

Table 4.4: The imbalances and the cost of controllable loads for each broker in example 4.6.1.

\[
\text{If } X \leq 0 \text{ then } p_{1,i} = -x_i D^+ \tag{4.20}
\]

\[
\text{If } X > 0 \text{ then } p_{1,i} = x_i D^- \tag{4.21}
\]

Payment \( p_{2,i} \) still remains the same to incentivize truth-telling:

\[
p_{2,i} = \text{VCG}(i,C,X) \tag{4.22}
\]

**Example 4.6.1 (Mechanism 2):**

We again consider the same example as used for mechanism 1a (example 4.5.4) and 1b (example 4.5.6) to illustrate this mechanism:

- \( P^- = -4 \), \( P^+ = 5 \)
- Du has downwards controllable loads of \(-1, -1, \ldots (D^- = -1)\)
- and upwards controllable loads of \(12, 13, 14, 15, 15, \ldots (D^+ = 15)\)

<table>
<thead>
<tr>
<th>i</th>
<th>( x_i )</th>
<th>( c_i(-1) )</th>
<th>( c_i(1) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2</td>
<td>-1</td>
<td>11</td>
</tr>
<tr>
<td>2</td>
<td>-5</td>
<td>-2</td>
<td>10</td>
</tr>
<tr>
<td>3</td>
<td>0</td>
<td>-2</td>
<td>9</td>
</tr>
<tr>
<td>4</td>
<td>4</td>
<td>-3</td>
<td>8</td>
</tr>
<tr>
<td>5</td>
<td>-1</td>
<td>-3</td>
<td>7</td>
</tr>
<tr>
<td>6</td>
<td>-3</td>
<td>-3</td>
<td>6</td>
</tr>
</tbody>
</table>

Just like in the example of mechanism 1a and 1b, broker 1 has an imbalance of 2 but instead of him paying \( P^- \) for each imbalance like in mechanisms 1a and 1b he receives \( D^+ \) for each imbalance. \( D^+ \) is 15 so he receives 30. The same for broker 2 but because his imbalance is negative he has to pay 15 for each unit of imbalance and has to pay 75 in total.

Just like in the other mechanisms the brokers 4, 5 and 6 receive a payment of 9 for the use of their controllable loads. The result can be seen in Table 4.4.

The DU keeps a total amount of payments of 45.
4.6 Low absolute total imbalance

4.6.2 Properties

In case of an imbalance the DU earns way more than it needs to resolve the imbalance so the mechanism is far from strongly budget balanced. Only in the case of $X = 0$ the mechanism is strongly budget balanced, even in the case that the brokers have large personal imbalances, which was not the case with mechanisms 1a and 1b.

Like the other mechanisms the mechanism is part of the Groves class of mechanisms, which means that the mechanism is efficient and incentive compatible.

**Theorem 4.6.2 (Efficiency and IC).** Mechanism 2 is efficient and incentive compatible

**Proof.** We can prove that mechanism 2 is efficient and incentive compatible in the same way we have proven that mechanisms 1a and 1b are. In short, a part of the payment $p_{2,i}$ consists of the Groves payment, all the other payments are independent of the agents reported private values. Furthermore the mechanism selects the cheapest controllable loads to resolve the total imbalance. Therefore the mechanism is in the Groves class and is efficient and incentive compatible.

**Theorem 4.6.3 (WBB).** Mechanism 2 is weakly budget balanced

In the case of a negative total imbalance, each broker pays $-x_i \cdot D^+$ which comes down to a total payment of $|X|D^+$ to the DU. The DU can then resolve the imbalance $X$ with his own controllable loads, or the controllable loads of the other brokers, which costs him at most $D^+$. The same holds for a positive total imbalance, then the brokers pay $D^- x_i$ which can be negative but the controllable loads of the DU generate enough revenue to pay out these penalties. In the case the total imbalance is zero, the sum of all payments to the DU is zero. Therefore the mechanism is weakly budget balanced.

**Theorem 4.6.4.** If a broker $i$ anticipates that the absolute total imbalance $X$ is greater than one he should resolve a single unit on the day ahead market so the new absolute imbalance is $|X'| = |X| - 1$.

**Proof.** If the total imbalance is less than zero then broker $i$ has to pay $D^+$ for each imbalance if he has a shortage. He would always be better of to resolve a unit on the day ahead market as the cost of resolving on the day ahead market plus any profits that he would make with his controllable load cannot exceed $D^+$. If he has a surplus he actually receives $D^+$ for each unit, so increasing his absolute personal imbalance only improves his utility.

If the broker has a positive imbalance then he has to pay $-D^-$ for each unit. So actually he has to pay at most $P^-$ for each imbalance but can possibly even receive a reward for each unit. Therefore if he sells another unit his absolute personal imbalance increases but he makes a profit as selling a unit makes him at least $P^-$. If the broker has a positive imbalance then he has to pay $D^-$ (or receive $-D^-$) for each imbalance. He is better off resolving it on the day ahead market, because he then receives at least $P^-$ for that unit, which is always more.

**Theorem 4.6.5.** Mechanism 2 has a Nash equilibrium where each broker has resolved all imbalance in the day ahead market
If all brokers have a personal imbalance of zero then all brokers pay a penalty of zero. If broker $i$ deviates from this and introduces a negative imbalance of $x_i$ then that broker has to pay $-x_i \cdot D^+$ which is guaranteed to be more than what he would have received on the day ahead market for his energy. If broker $i$ deviates from zero and introduces a positive imbalance of $x_i$ then he has to pay $x_i \cdot D^i$. This may be negative (a reward) but he had to pay at least $P^-$ for each unit and because $D^+ - P^-$ broker $i$ cannot profit from this. Because deviating from the current strategy is never better, we have shown that it is a Nash equilibrium.
4.7 Low absolute personal imbalance in the Nash equilibrium

The previous mechanisms proposed in this chapter still leave a lot to be desired in terms of the amount of revenue the DU makes. The mechanisms proposed in this section make less revenue, but take a weaker stance on the incentive to balance in the day-ahead market. Each of the three mechanisms (3a, 3b and 3c) have the same properties but only differ in the amount of revenue the DU makes. The first mechanism 3a is the most simple and is already a big improvement on the mechanisms proposed earlier. The second and third mechanisms are slightly more complex but each dominates the former in terms of revenue made by the DU.

In the previous mechanisms 1a and 1b, the DU earns a lot when the personal imbalances of the brokers are very high, but the total imbalance is very small. In that case the DU still had to collected money from the brokers to keep the incentive for balancing in the day-ahead market. This money cannot be returned without affecting this incentive.

The second mechanism (mechanism 2) did not have this problem but introduced \( D^+ \) and \( D^- \) again. The DU is likely to receive far too much payments when the real cost of balancing is well below \( D^+ \) or \( D^- \) per unit.

For the following mechanisms 3a, 3b and 3c we weaken the notion of giving an incentive for a low imbalance. Instead of making the strategy of having low imbalances, or low total imbalance dominant, we make it the dominant strategy in the Nash equilibrium. In these mechanisms, brokers are able to profit from their imbalance if and only if they do not contribute to the total imbalance, e.g. if the personal imbalance is of the opposite sign of the total imbalance. Just like in mechanism 2 we allow brokers to profit from a non-contributing imbalance, but unlike mechanism 2 there is no incentive for the non-contributing brokers to make the total imbalance lower. But still every non-contributing is never worse off by decreasing his and consequently the total imbalance.

This weakening makes it possible that we distribute more payments from the contributing brokers to the non-contributing brokers while still incentivizing having a low imbalance for those on the wrong side of the imbalance.

Idea and rationale

The mechanism works approximately the same as the mechanism earlier discussed in Section 4.5.1 with payments \( p_{1,i} \) and \( p_{2,i} \), but requires a \( p_{1,i} \) which is on average smaller than in previous mechanisms. In this mechanism the sum of the payments \( p_{1,i} \) is designed to be as close as possible to the real cost the DU makes to resolve the total imbalance.

Because we will see that \( p_{1,i} \) is dependent on \( p_{2,i} \) just like in mechanism 1b, payment two is discussed first. Again \( p_{2} \) is a VCG payment/reward to brokers for making their controllable loads available and \( p_{1} \) is to deter brokers from having a large imbalance. By using a VCG payment we are sure that brokers cannot successfully influence their utility by lying about the cost or capacity of their controllable loads. This of course only holds if manipulating the price and capacity does not affect their utility in other ways, but we will come back to that later.
So the \( p_2 \) is defined as:

\[
p_{2,i} = VCG(i, C, X)
\]  

(4.23)

Just like in mechanism 1b we want the penalty \( p_{1,i} \) to be dependent on the real cost the DU has to make to resolve the total imbalance. And this time we do not only want to use it to lower the penalty to contributing brokers, but also to lower the penalty (or increase reward) to the non-contributing brokers.

To explain the rationale behind the payment \( p_1 \) let us assume a situation where there is a negative total imbalance (a shortage of energy). Some brokers may have a shortage of energy and others may have a surplus. This mechanism assumes that because the brokers with the non-contributing positive imbalance have resolved some imbalance on the day ahead market and thus contributed to a more efficient solution. They are thus entitled to a reward roughly the size they have saved the others by their presence.

A solution could be to see for how much the brokers with the allocated controllable loads are compensated and to use that to reward the non-contributing brokers. Then all brokers that help solving the imbalance, being it in the day-ahead market or last minute, would be rewarded the same, which seems fair.

However \( p_1 \) should not be directly based on the sum of VCG payments (the second payments) as non-contributing brokers could influence this number up and down by manipulating the prices of their controllable loads. We can however base it on the sum of VCG payments in the hypothetical case where the non-contributing broker did not report any controllable loads, just like we did with mechanism 1b:

\[
p_{1,i} = \frac{DU_{\text{costs}}(C - i, X)}{X} x_i
\]  

(4.24)

With this payment a non-contributing broker receives the same price per unit as the brokers with the most efficient controllable loads got in the case the controllable load of his were not available.

This reward is on average equal or larger per unit than the original VCG rewards \( p_{2,i} \). So the price per unit the contributing brokers have to pay have to be even larger than this.

By asking what that DU would have to spend on VCG values without the controllable loads of that broker and without the controllable loads of brokers that have a non-contributing imbalance it is guaranteed that this payment is larger per unit than the reward of the non-contributing have received. This payment can be seen in equation 4.25.

\[
p_{1,i} = \frac{DU_{\text{costs}}(C - i \setminus B_n, X)}{X} x_i
\]  

(4.25)

Where \( B_n \) is the set of all controllable loads owned by brokers with a non-contributing imbalance.

This way, the penalty \( p_{1,i} \) a contributing broker pays is not dependent on his own bids bids but only dependent on the bids of others. Furthermore it guarantees that a large part of the payments go back to the brokers.

This is the idea behind the mechanism. However the following description of the mechanism also includes extra equation for the special case where \( X = 0 \). The other 2 mechanisms, 3b and 3c, build further upon this concept to return more payments to the brokers.
4.7.1 Mechanism 3a

If \( X \neq 0 \), then for each broker \( i \) where for which equation 4.26 holds we apply equation 4.27 else equation 4.28.

\[
\begin{align*}
    x_i \cdot X & \leq 0 & (4.26) \\
    p_{1,i} &= \frac{D_{\text{costs}}(C_{-i} \setminus B_n),X)}{X} x_i & (4.27) \\
    p_{(1,i)} &= \frac{D_{\text{costs}}(C_{-i},X)}{X} x_i & (4.28)
\end{align*}
\]

Where \( B_n \) is the set of all controllable loads owned by non-contributing brokers.

If \( X = 0 \) every broker pays a penalty \( p_1 \) of:

\[
p_{1,i} = \frac{1}{2}(P^+ + P^-) x_i & (4.29)
\]

For each broker the payment \( p_2 \) is equal to:

\[
p_{2,i} = VCG(i,C,X) & (4.30)
\]

Example 4.7.1 (Mechanism 3a):

Below, just like with the previous mechanisms, the same example 4.5.4 is used consisting of 6 brokers:

- \( P^- = -4, P^+ = 5 \)
- Du has downwards controllable loads of \(-1, -1, \ldots (D^- = -1)\)
- and upwards controllable loads of \(12, 13, 14, 15, 15, \ldots (D^+ = 15)\)

The total imbalance is -3. Broker 1 is a non-contributing broker and thus pays the amount needed per unit to solve the imbalance per unit without his controllable loads. Without his upwards controllable load that costs 11 the imbalance would be resolved by the brokers 4, 5 and 6 and they would each receive 9. Thus broker 1 has to pay \( -9 \cdot 2 = -18 \). The penalties of broker 3 and 4 are calculated likewise.

Broker 5 has a shortage of 1. Thus he is a contributing broker and equation 4.28 applies to him. Without his controllable load and the controllable loads of all brokers with an
imbalance of greater than 0 the imbalance would have to be resolved by the brokers 6, 3 and 2. Each of them would receive a payment of 12. So broker 5 has to pay $1 \cdot 12 = 12$ for his imbalance. Likewise for the brokers 2 and 6.

After calculating the $p_2$ payments which are $-9$ for the brokers 4, 5 and 6 we end up with these payments:

<table>
<thead>
<tr>
<th>$i$</th>
<th>$p_1$</th>
<th>$p_2$</th>
<th>$\sum_p$</th>
<th>$u_i$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>-18</td>
<td>0</td>
<td>-18</td>
<td>18</td>
</tr>
<tr>
<td>2</td>
<td>60</td>
<td>0</td>
<td>60</td>
<td>-60</td>
</tr>
<tr>
<td>3</td>
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<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>4</td>
<td>-40</td>
<td>-9</td>
<td>-49</td>
<td>41</td>
</tr>
<tr>
<td>5</td>
<td>12</td>
<td>-9</td>
<td>3</td>
<td>-10</td>
</tr>
<tr>
<td>6</td>
<td>36</td>
<td>-9</td>
<td>27</td>
<td>-33</td>
</tr>
</tbody>
</table>

The DU runs a profit of 23.

### 4.7.2 Mechanism 3b

The previous mechanism has almost all the properties we desire and guarantees less revenue for the DU. But it is possible to let the DU make even less profit without having to further weaken some of the properties. Mechanisms 3b and 3c are such that mechanism 3b dominates 3a and mechanism 3c dominates 3b.

This mechanism is the same as mechanism 3a, except for the penalty for contributing brokers. Instead of discarding all controllable loads $C_k$ reported by the non-contributing brokers in the calculation, we can discard only the controllable loads of a single non-contributing broker that maximizes the penalty for that broker $i$.

If $X \neq 0$, for each broker $i$ where for equation 4.31 holds we apply equation 4.32 else equation 4.33.

\[
x_i * X \leq 0 \tag{4.31}
\]

\[
p_{1,i} = \begin{cases} 
\max(DU_{\text{costs}}(C - i \setminus C_k), X) : \forall k \in B_n & \text{if } B_n \neq 0 \\
DU_{\text{costs}}(C - i, X) \cdot x_i & \text{if } B_n = 0 
\end{cases} \tag{4.32}
\]

\[
p_{1,i} = \frac{DU_{\text{costs}}(C - i, X)}{X} \cdot x_i \tag{4.33}
\]

If $X = 0$ every broker pays a penalty $p_1$ of:

\[
p_{1,1,i} = \frac{1}{2} p^- + p^+ \tag{4.34}
\]

For each broker the payment $p_2$ is equal to:

\[
p_{2,i} = VCG(i, C, X) \tag{4.35}
\]
Example 4.7.2 (Mechanism 3b):
The penalties $p_{2,i}$ and the penalties $p_{1,i}$ for non-contributing brokers remain the same. We again consider the same example as for the previous mechanism(s).

We are now going over the calculation of $p_{1,i}$ for broker 2. Broker 2 has a shortage of 5 and therefore has to pay a payment $p_{1,i}$ defined by equation 4.32. We calculate the cost the DU makes without broker 1 and 4 and choose the one that maximizes that cost.

Without the controllable loads of broker 2 and 1 the imbalance has to be resolved by the brokers 6, 5 and 4. Each of them receives 9 and thus the cost the DU makes to resolve the total imbalance in this case is 27.

Without the controllable loads of broker 2 and 4 the imbalance has to be resolved by the brokers 6, 5 and 3. Each of these three brokers would receive a payment of 11. So the total cost for the DU would be 33 in this case.

If we fill this into equation 4.32 we get: $\max(27,33) \cdot -5 = 55$ so that is the amount broker 2 has to pay. The payments for the other brokers look like this:

<table>
<thead>
<tr>
<th>i</th>
<th>$p_{1,i}$</th>
<th>$p_{2,i}$</th>
<th>$\sum p_{1,i}$</th>
<th>$u_{i}$</th>
</tr>
</thead>
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<tr>
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<tr>
<td>6</td>
<td>33</td>
<td>-9</td>
<td>24</td>
<td>-30</td>
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</table>

The DU makes a profit of 14. This is 9 less than mechanism 3a.

4.7.3 Mechanism 3c

In mechanism 3a the contributing brokers pay per unit the total cost the DU makes (including VCG payments) divided by their personal imbalance, given that the controllable loads of that broker, and the controllable loads of all non-contributing brokers are not available. In mechanism 3b it was weakened by only removing the controllable loads of a single other non-contributing imbalance. In mechanism 3c we introduce another step to lower the payments.

Here we let go of the requirement that the payments contributing brokers make per unit of imbalance need to be larger than the rewards the non-contributing brokers get. Instead, we only only make sure the average payment is at least as big as the average reward.

If $X \neq 0$, for each broker $i$ where for which equation 4.36 holds we apply equation 4.37 else equation 4.38.

$$x_{i} \cdot X \leq 0 \quad (4.36)$$
Mechanisms for grid balancing

4.7 Low imbalance in Nash equilibrium

\[ p_{1,i} = \begin{cases} \sum_{k \in B_n} (DU_{costs}(C_{-i} \setminus C_k, X) \cdot x_k) \\ \frac{X \cdot \sum_{j \in B_n} x_j}{DU_{costs}(C_{-i}, X)} \cdot x_i \end{cases} \begin{array}{c} \text{if } B_n \neq \emptyset \\ \text{if } B_n = \emptyset \end{array} \]  
(4.37)

\[ p_{1,i} = \frac{DU_{costs}(C_{-i}, X)}{X} \cdot x_i \]  
(4.38)

If \( X = 0 \) every broker pays a penalty \( p_1 \) of:

\[ p_{1,i} = \frac{1}{2} P^- + P^+ \]  
(4.39)

The payments of this mechanism can also be made independent of \( P^- \) and \( P^+ \) by using the price of the cheapest interruptible or optional controllable load not owned by broker \( i \).

For each broker the payment \( p_2 \) is equal to:

\[ p_{2,i} = VCG(i, C, X) \]  
(4.40)

Although mechanism 3c is not defined as a redistribution mechanism the intuition has some parallels with the Cavallo’s redistribution mechanism. The mechanism of Cavallo is based around the intuition that we can calculate the revenue that the central agent would make anyway, given any type \( \theta_i \) for broker \( i \) and then returns his share of this revenue. In this mechanism we cannot use this but we calculate the maximum cost that the DU needs to make to resolve the total imbalance given any type \( \theta_i \), and then let the broker pay his share. And because having no controllable loads always maximizes the cost that the DU makes, we can leave them out.

**Example 4.7.3 (Mechanism 3c):**

The example used to illustrate the other mechanisms is also used to illustrate this mechanism. The payments \( p_2 \) and the payments \( p_1 \) for non-contributing brokers remain the same.

The calculation of the payment \( p_1 \) of broker 2 is as follows. The broker has a contributing imbalance so equation 4.37 is used. The set of brokers \( B_n \) contains the brokers 1, 3 and 4. The \( DU_{costs} \) without the brokers 1 and 4 have already been calculated in the example for mechanism 2b. The \( DU + cost \) without broker 3 is 33. So we can fill it in directly:

\[ p_{1,2} = \frac{\sum_{k \in B_n} (DU_{costs}(C_{-i} \setminus C_k, X) \cdot x_k)}{X \cdot \sum_{j \in B_n} x_j} \cdot x_i = \frac{27 \cdot 2 + 33 \cdot 0 + 33 \cdot 4}{-3 \cdot (2 + 4)} \cdot -5 = 51 \frac{2}{3} \]  
(4.41)

Thus broker pays a penalty of \( 51 \frac{2}{3} \) for his imbalance.
4.7 Low imbalance in Nash equilibrium

<table>
<thead>
<tr>
<th>i</th>
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<th>$\sum p$</th>
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<td>6</td>
<td>32</td>
<td>-9</td>
<td>23</td>
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</table>

The DU gets to keep $9\frac{1}{3}$ profit, which is less than the 23 profit of mechanism 1a and the 14 profit of 1b.

4.7.4 Properties

The 3 mechanisms proposed in this document are all efficient, budget balanced and incentive compatible. Furthermore, the mechanism gives an incentive to have a low imbalance in the Nash equilibrium. Each property is discussed in the next subsections.

**Theorem 4.7.4.** Mechanism 3a, 3b and 3c are efficient and incentive compatible in reporting the cost of the controllable loads

**Proof.** An all three mechanisms the cheapest controllable loads are part of the Groves class and thus efficient and incentive compatible. We can see this by observing that the mechanism maximizes social welfare by choosing the most efficient controllable loads to solve the imbalance. Payment $p_{2,i}$ is a VCG payment and therefore satisfies the first part of the Groves payment. The other payments are independent on the reported type and therefore constitutes of the second part of the Groves payment.

Mechanism 3b accumulates equal or less profit than mechanism 3b and mechanism 3c in turn accumulates equal or less profit than mechanism 3b. Therefore proving budget-balancedness only for mechanism 3c is sufficient.

**Lemma 4.7.5:**

In mechanism 3c, the contributing brokers (combined) pay at least as much per unit of imbalance than the contributing brokers (combined):

$$\frac{\sum_{i \in B_c} p_{1,i}}{\sum_{i \in B_c} x_i} \geq \frac{\sum_{k \in B_n} p_{1,i}}{\sum_{k \in B_n} x_k}$$

**Proof.** The idea is that the non-contributing receive per unit the costs he DU has if their own controllable loads were not available. The contributing brokers have to pay per unit average amount the non-contributing brokers would have received if their own controllable loads were not available. Because it also removes their own controllable load, on top of the controllable loads removed of the non-contributing brokers, therefore the lemma is true.

The above can also be deduced from the definitions of the payments:
Mechanisms for grid balancing

4.7 Low imbalance in Nash equilibrium

\[ \sum_{i \in B} c_{p1,i} \sum_{i \in B} x_i = \sum_{i \in B} \sum_{k \in B_n} (DU_{costs}(C \setminus C_k, X) \cdot x_k) \cdot x_i \]

substitution of Definition \( p_{1,i} \)

\[ \sum_{i \in B} x_i \sum_{i \in B} \sum_{k \in B_n} (DU_{costs}(C \setminus C_k, X) \cdot x_k) \cdot x_i \geq \sum_{i \in B} x_i \sum_{k \in B_n} (DU_{costs}(C \setminus C_k, X) \cdot x_k) \]

adding controllable loads \( B_c \)

\[ = \sum_{k \in B_n} \frac{X \cdot \sum_{k \in B_n} x_k}{X} \cdot x_k \]

summation over \( B_c \) now superfluous

\[ = \sum_{k \in B_n} \frac{p_{1,i}}{\sum_{k \in B_n} x_k} \]

substituting \( p_{2,i} \)

\[ \square \]

**Theorem 4.7.6.** Mechanism 3c is weakly budget balanced

**Proof.** To prove the theorem we have to show the following:

\[ \sum_{i=1}^{n} (p_{1,i} + p_{2,i}) - D \geq 0 \quad (4.42) \]

We know that each contributing broker pays as least as much per unit for their imbalance as each non-contributing broker receives per unit from Lemma 4.7.5. And because we know that each non-contributing broker pays at least enough per unit to resolve the total imbalance, it means that the sum of the payments \( p_{1,i} \) is large enough to cover \( \sum_{i=1}^{n} p_{2,i} - B \).

**Theorem 4.7.7.** When every broker is perfectly balanced, mechanism 3c is in a Nash equilibrium

If every broker is perfectly imbalanced every broker would pay nothing. Let us consider a broker \( i \). If he sells one or more units in the day-ahead market at a price \( P^+ \) he ends up with a positive imbalance. Now the total imbalance \( X \) is equal to \( x_i \). Broker \( i \) is now the...
only broker with an imbalance and has to pay at least the full cost the DU makes to balance everything. Even if broker $i$ has perfect controllable loads that could balance at a price of $P^+$ this would not net him any profit. The same holds for buying an extra unit on the day-ahead market. Therefore it is a Nash equilibrium.

**Theorem 4.7.8.** Any broker that has a contributing imbalance is weakly better off by reducing his absolute total imbalance by one. If the price of each controllable load is unique, each contributing agent is better off by reducing his total imbalance by one.

**Proof.** Each broker that has a contributing positive personal imbalance has to pay at least the cost for the DU to resolve the total imbalance per unit. If that broker has any controllable loads, these would net him at most the cost that the DU has to make per unit plus $P^-$. Because the could have sold the surplus of energy on the day ahead market of at least $P^-$, the broker would never be worse of by doing this.

Likewise for a broker that has a contributing negative personal imbalance. In that case the contributing broker has to pay for each unit at least the costs that the DU has to make to resolve the imbalance per unit. Any controllable loads that the broker owns would net him at most the costs that the DU makes per unit minus $P^+$ as $P^+$ are the cheapest controllable loads that we allow. In the day-ahead market the broker would have to pay at maximum $P^+$ to resolve a single unit. Therefore resolving a unit of energy is always weakly optimal in case of a contributing imbalance.
4.8 Experiments

The mechanisms described in this chapter all have the properties that we require from the mechanisms. However they all differ in terms of the amount the DU gets to keep at the end of each mechanism. In the following sections the difference in profit of each mechanism will be demonstrated and how the mechanisms react in changes in instance parameters.

4.8.1 Setup experiment

The instances on which the mechanisms were run are randomly generated. Unless specified otherwise the instances consist of 20 brokers each with a uniformly distributed imbalance between $-5$ and 5 (including). Each broker has two upwards controllable loads with a price between 50 and 100 and 2 downwards controllable loads with a price between -50 and 0. The DU has an unlimited supply of up- and downwards controllable loads which cost 100 and 0 respectively. Furthermore $P^+ = P^- = 50$.

The mechanisms are scored by the amount the DU profits in comparison with the actual balancing cost. The summation of the real cost over all instances is divided by the summation of the profits the DU makes. This way very different instances in respect to balancing cost can still be compared. The rationale is that the exact amount the DU makes is not very important as long as it remains a small overhead. The profits that the DU makes can be seen as a tax to achieve an efficient mechanism.
4.8 Experiments

Mechanisms for grid balancing

4.8.2 Variating the number of brokers

The most important test is to see how well the mechanisms scale with the introduction of new controllable loads. For this test 2000 random instances were run varying the amount of brokers (1, 2, 3, 4, 5, 10, 20, 30, 40 and 50 brokers) while keeping the rest of the variables the same. The results can be seen in Figure 4.8.2

![Figure 4.7: The mechanism are run on the same set of instances with an increasing amount of brokers (and thus available controllable loads). Each point represents the average of 200 instances.](image)

It is evident that the first mechanism (1a) performs very badly, as can be expected, as it does not take into account the controllable loads reported by brokers in the calculation of $p_{1,i}$. When there are 50 controllable loads the profit of the DU is on average more than 4 times greater than the actual cost of resolving the total imbalance.

Notice that the mechanisms 3a, 3b and 3c score the same for 3 brokers or less. Mechanism 3a is the same as mechanism 3b for 2 brokers or less as with 2 or less brokers there can only be 1 non-contributing broker and thus the improvement of 2b does not help. That 3a, 3b and 3c score the same for 3 brokers is due to the way the experiment is set up. The guarantee that each broker has the same amount of controllable loads and each controllable load is guaranteed to be cheaper than the ones that the DU has makes it that they score the same.

The mechanisms 3b and 3c are the only ones where the DU clearly profits less in the limit of the amount of controllable loads. For mechanism 3a the new brokers with their controllable is not enough to keep the ratio down.

As expected, mechanism 3c scores by far the best when there are a lot of brokers, and thus cheap controllable loads. At first, with only a few brokers the DU can profit a lot, up to 25% of the real balancing cost. This is because if there are few controllable loads the
removal of controllable loads in the calculation of $p_{1,i}$ have a larger effect. For 50 brokers
the average cost of balancing the instances is 887.52 and the DU makes on average a profit
of 21.5 (2.4%).

Interesting is that mechanism 2, which penalizes brokers only with $D^-$ and $D^+$ works
better in terms of profit when there are 5 or less brokers, but after mechanisms 3b and 3c can
profit from cheaper controllable loads that new brokers bring they are significantly better.
4.8 Experiments

4.8.3 Constant number of controllable loads, increasing number of brokers

For this experiment the amount of brokers is varied but unlike the previous experiment the amount of controllable loads is kept the same. 400 controllable loads are randomly divided among the brokers.

![Graphs showing profit of DU with increasing number of brokers with 400 loads](image)

One thing important to remember is that the introduction of new brokers also has an effect on the total imbalance. With more brokers, the total imbalance gets bigger on average. Because of this greater imbalance, some mechanisms perform worse. Mechanism 3b and 3c clearly profit from having more brokers. This is probably because the removal of the controllable loads of a subset of brokers has a much larger impact if the controllable loads are distributed among a smaller group of brokers.
4.8.4 Distribution of controllable loads

This experiment tests what the effect is of the distribution of the controllable load. Here the distinction between the mechanisms that do not take into account the cost of the controllable loads and those that do are very clear.

![Figure 4.9](image)

Figure 4.9: The mechanisms are run on the same set of instances but the controllable loads are owned by an increasing fraction of the 20 brokers. 0.05 represents that all controllable loads are owned by only a single broker and 1.00 represents that all controllable loads are distributed evenly over the brokers. Each point represents the average of 600 instances.

Interestingly, the mechanisms 2 and 3c seem to have the opposite behavior. Mechanism 2 performs less when the controllable loads are evenly distributed, while mechanism 3c performs better. Mechanism 2 performs worse because the payments to the DU remain the same, while the costs of balancing is cheaper with evenly distributed controllable loads. The VCG payments the DU has to pay are only based on the controllable loads of others, so the cost for the DU get larger when less brokers have controllable loads. The result of this is that the mechanism is percentage-wise better with few controllable load owners. Mechanism 3c also suffers from this, but can at the same time lower the payments of the first payment enough to counter this effect.
4.8.5 Maximum imbalance

In this experiment the maximum imbalance of each broker was raised from 1 to 200. Mechanism 3b first rises, as the increasing total imbalance makes the gap between $p_1$ and $p_2$ larger. When the total imbalance gets on average too high to be solved by the controllable loads of the brokers, the DU resolves the most imbalance. Because the DU does not have to make a profit on his controllable load, the profit of the DU almost reaches zero.

If the DU solves most of the imbalance mechanism 2 scores the same as mechanism 3a and 3b, as $D^+$ and $D^-$ come more and more close to the real cost of balancing.

Figure 4.10: The mechanisms are run on instances where the 20 brokers have an increasing maximum imbalance. All the brokers have imbalances normally distributed over (-maximum,maximum). Each point represents the average of 24 instances.
4.8.6 Increasing number of controllable loads

In this experiment the influence of the amount of controllable loads available is tested. Like we have seen in the first experiment mechanism 2 behaves very well at the start because most of the imbalance is being balanced at the price of $D^+$ and $D^-$, as soon as more controllable loads are made available the price drops.

![Profit of DU with increasing number of controllable loads](image)

Figure 4.11: The mechanisms are run on instances where the 20 brokers have an increasing number of controllable loads. The controllable loads are evenly distributed over the brokers. Each point represents the average of 600 instances.

4.9 Conclusion

In this chapter it has been shown that it is possible to create a mechanism for EGBP that is incentive compatible, individually rational, weakly budget balanced and efficient. Furthermore, we have shown that by adjusting the incentive to balance in the day-ahead market it is possible to make mechanisms that return more payments back to the brokers.

In mechanisms 1a and 1b, the dominant strategy is to resolve all imbalance on the day-ahead market. In mechanism 2, it is profitable to help resolving the total imbalance in the day ahead market. And finally in mechanisms 3a, 3b and 3c we have shown that even more payments could be returned if we only require balancing in the day-ahead market to be optimal in the Nash equilibrium. The most important result is that mechanism 3c is a mechanism that results in very little revenue for the DU, especially with an increasing number of brokers, controllable loads or imbalances.

Finally some experiments are shown that give us better insight on the amount of returned payments and behavior of the proposed mechanisms.
Chapter 5

Dealing with economic loads

5.1 Introduction

In this chapter we introduce a modified version of the Energy Grid Balancing Problem (EGBP) where brokers are able to use their loads as economic capacities which they can enable themselves. This new problem, the Energy Grid Balancing Problem with Economic Capacities (EGBPEC) proves to be much harder than EGBP. We show that EGBPEC is equivalent to a multi-unit auction mechanism by providing reductions from and to a multi-unit double auction.

Then we explore what solutions for EGBPEC are possible and practical and which are not. We discuss some mechanisms found in literature and what improvements we still need to get an adequate solution to EGBPEC. We propose a 2-item double auction mechanism where the dominant strategy is to bid/ask truthfully for the first item and never bid less or ask more for the second item.

To anticipate on the possibility that a completely satisfactory solution of EGBPEC may not exist, we propose a mechanism that solves EGBPEC given that each broker only has a single price for each type of controllable loads and given that there exists a satisfactory solution to a multi-unit auction where agents only have a single reservation price.

5.2 EGBPEC

To reiterate the Energy Grid Balancing Problem (EGBP), there is an imbalance on the grid and it is the task of the DU to resolve this imbalance with the use of controllable loads. The DU owns some controllable loads himself but can also use controllable loads reported by the brokers. The task of the DU to resolve the imbalance as efficiently as possible so it has to use a mechanism that makes the brokers report their controllable loads truthfully and incentivize them to resolve their imbalance in the day ahead market.

However, until now we have assumed the brokers themselves cannot decide for themselves when to use their controllable loads. In the setting of Power TAC these are called economic capacities, but essentially these are just controllable loads that can be enabled by the brokers. We will refer to them as economic loads. Economic loads are always more ex-
5.2 EGBPEC Dealing with economic loads

pensive than resolving in the day-ahead market but brokers can use them to resolve balance unpredicted imbalances in the current time slot.

We show that if brokers are able to decide for themselves when to enable a controllable load, the problem of finding a satisfactory mechanism becomes more difficult. In the mechanisms described in previous chapters it was possible that brokers needed to pay more than the cost of their own controllable loads, while their loads were not used to resolve the total imbalance. In those cases a broker could profit by enabling his own controllable load (incurring an efficiency loss because it was not deemed to be among the cheapest) and pay less in the end. This has four implications:

1. If a broker decides to use his controllable load even though it was not deemed to be among the cheapest, there is an efficiency loss. This efficiency loss may not be confined to the controllable loads of this single broker, because if other brokers have anticipated on this strategy they themselves may have to use their controllable loads to not be worse off. This waterfall effect may result in a situation where none of the cheapest controllable loads are used to resolve the total imbalance.

2. When brokers use their controllable loads themselves, the efficiency of the system cannot be measured anymore by the DU. The brokers are not and cannot be enforced to truthfully report their used controllable loads, so the DU has no information as to how well the mechanism is doing.

3. The reported prices of the other controllable loads are also affected if no other incentives are created. For example one broker may decide to drop the price of his controllable load to stop the waterfall from happening and hurting him.

4. Even if a mechanism is constructed where brokers only use their controllable loads if they believe it to be among the cheapest, it could still incur an efficiency loss. Brokers work with incomplete information while the DU has, in an incentive compatible mechanism, complete information about the total imbalance and the prices of the controllable loads.

In this chapter we look at the Energy Grid Balancing Problem with Economic Controls (EGBPEC). It considers EGBP but then also gives brokers the capability to enable a set of controllable loads they own themselves, just before reporting their remaining controllable loads to the DU.

In EGBP at the start of the time-slot 1) the DU measures the imbalance of each broker, 2) each broker reports his controllable loads to the DU and 3) the DU enables the cheapest controllable loads and hands out penalties. In EGBPEC 1) the DU measures the imbalance of each broker and each broker measures his own imbalance, 2) each broker enables a subset of his own controllable loads, 3) each broker reports his remaining controllable loads and 4) the DU enables the cheapest controllable loads and hands out penalties.

In Section 3.2.3 we have described three forms of individual rationality: predetermined penalty, penalty based on loads DU, and the usage of own controllable loads. Previously we have only considered the first two types, but in this chapter we consider only the last one.
Because we are already very limited with the introduction of economic loads we only look at mechanisms that have an incentive to resolve the imbalance in the Nash-equilibrium. We do not touch this subject further because this incentive comes naturally if the mechanism has a strong sense of individual rationality and weakly budget balanced as it implies that the ones that introduce an imbalance pay for the imbalance as no one can be forced to pay more than what he would have if he had not participated.

5.3 Reducing EGBPEC to a double auction

A mechanism that could work is one where the brokers have to solve their own imbalance with their own controllable loads that are cheaper than those of the DU, and if this is not sufficient, the DU could solve the rest. Then no broke would have to pay more than the cost of their controllable loads. However the mechanism would not be very efficient. Brokers with a low imbalance and a large array of cheap controllable loads will be under-utilized.

A way to solve this is to, at least conceptually, let brokers be responsible for their own imbalance instead of the DU, but to create a market for trading controllable loads. Then when one broker has a large imbalance and another can provide cheaper controllable loads, a trade can happen between the two parties to achieve a more efficient solution where both parties would profit. Likewise, if one broker has a positive imbalance and another a negative imbalance. A similar trade could be made so that both parties would profit.

We can organize this market as a multi-unit double auction. The DU still receives the controllable loads from the agents and it simulates a double auction. Each broker represents an agent on the double auction and has bids and asks on the double auction depending on the brokers personal imbalance and available controllable loads.

5.3.1 Reduction to a double auction

In a double auction agents can submit bids or asks for a certain commodity. An ask means that an agent is willing to sell the commodity for the price of the ask, but not for less. Likewise a bid means an agent is willing to buy an item at that price, but not any higher. In our case a surplus of energy will be the commodity that is being traded.

We assume that in a double auction there are \( n \) agents. We denote the bids as \( \hat{B} = \{\hat{B}_1, \hat{B}_2, \ldots, \hat{B}_n\} \) where \( \hat{B}_i \) is the set of bids of agent \( i \). We denote the asks as \( \hat{A} = \{\hat{A}_1, \hat{A}_2, \ldots, \hat{A}_n\} \) where \( \hat{A}_i \) is the set of asks of broker \( i \). We denote the real private values of the agents as \( B = \{B_1, B_2, \ldots, B_n\} \) and \( A = \{A_1, A_2, \ldots, A_n\} \). A double auction mechanism receives the possibly untruthful bids and distributes some items of the sellers to the buyers. The mechanism also decides how high the payments are.

The reduction to a double auction is organized as follows. If a broker has a shortage of energy the DU puts a bid on the market at a price equal to the cost the broker would have to make to resolve it. If a broker has an imbalance of -2 and the broker has two interruptible controllable loads that cost 5 and 6, the DU puts 2 bids at 5 and 6 respectively on the double auction. This means that some other broker can deliver a unit of energy for less than 5, or two units for less than 11, a trade could be made between the two parties. If a broker does
5.3 Reducing EGBPEC to a double auction

Dealing with economic loads

Figure 5.1: An overview of how a solution to the multi-unit double auction can be used to create a mechanism for EGBPEC

not have enough controllable loads to resolve his own imbalance we assume that his cost to resolve it himself is equal to infinity, as not resolving an imbalance is unacceptable.

Likewise, a surplus of energy is put on the market as an ask equal to the amount the broker would have gained if he would have resolved it himself, as we are offering a surplus of energy. If a broker with a surplus of 1 has a downwards controllable load that costs -2 then an ask is put on the market at a price of 2. Each controllable load that still remains can also be put on the market. Each interruptible controllable load thus becomes an ask at the price of that load, and each optional controllable load is put on the market as a bid at the price of that controllable load times -1.

After deciding on the bids and asks the DU runs some multi unit auction and the multi-unit auction creates an allocation and calculates the height of the payments. The allocation can be transformed to an allocation in EGBPEC and the payments are used as-is.

Definition of reduction

The set of imbalances of all brokers is denoted as $X$ and their controllable loads as $C$. $C_i$ is the set of controllable loads owned by broker $i$. The reported loads are denoted as $\hat{C}$ and may not be truthful. $(X, C)$ is the input of the mechanism and the output is a set of controllable loads $C_u$ that are going to solve the total imbalance and a set of payments $p_c$. This can also be seen in Figure 5.1.

Definition 5.3.1 (Transformation function $t_c$):
We define a transformation function $t_c : X \times C \rightarrow A \times B$ where $t_c(x, \hat{c}) = (\hat{b}, \hat{a})$ are the bids and asks in the double auction sub-mechanism.

For each broker we create a corresponding agent. Every controllable load of unit size will be represented as a single bid or ask. Each broker with a negative imbalance puts the controllable loads he needs to resolve his own imbalance as bids on the market, so the amount it would have cost him to do it himself. If he does not have enough controllable
loads the imbalance will be put as bids at $\infty$. Likewise for each broker with a positive imbalance, except the broker gets a series of asks of the size of how much that broker would have \textbf{gained} if he would have resolved his personal imbalance himself, again an ask of $-\infty$ for each unit of imbalance that the broker cannot resolve himself. Each controllable load that still remains can also be put on the market. Each interruptible load is an ask at the \textbf{price of that load}, and each optional controllable load is put on the market as a bid at the \textbf{price of that controllable load times -1}.

\textbf{Definition 5.3.2 (Allocation function $f_d$):}
There is an allocation function $f_d : B \times A \to B \times A$ where $f_d(\hat{b}, \hat{a}) = (b_u, a_u)$ are the bids and asks that are executed in the double auction mechanism that is efficient.

\textbf{Definition 5.3.3 (Payment function $p_d$):}
There is an allocation function $f_d : B \times A \to \mathbb{R}^n$ where $p_d(\hat{b}, \hat{a}) = \ldots$ are the payments that the agents pay in a double auction mechanism that is IC, IR, WBB and efficient.

\textbf{Definition 5.3.4 (Transformation $t_g$):}
We define a transformation function $t_g : B \times A \to C$ where $t_g(b_u, a_u) = C_u$ are the controllable loads that are used to resolve the imbalance in the EGBPEC mechanism.

\[
C_u = \bigcup \left\{ c \in C_i \mid c \text{ is an interruptible load corresponding to bid } b \wedge b \notin B_i^u \right\} \\
\bigcup \left\{ c \in C_i \mid c \text{ is an optional load corresponding to bid } b \wedge b \in B_i^u \right\} \\
\bigcup \left\{ c \in C_i \mid c \text{ is an interruptible load corresponding to bid } b \wedge b \in A_i^u \right\} \\
\bigcup \left\{ c \in C_i \mid c \text{ is an optional load corresponding to bid } b \wedge b \notin A_i^u \right\}.
\]

$B_i^u$ and $A_i^u$ denote the bids and asks of agent $i$ that are executed, so for each bid in $B_i^u$ would receive an item in the double unit auction. This reduction can also be seen in Figure 5.2. On the left are the bids from high to low and on the right the bids from low to high. The horizontal line in the middle is the crossover point. All the bids and asks above the crossover point are executed but these are not the controllable loads we want to enable. We want to enable only the controllable loads in the gray area, therefore this transformation function.

Each bid that was executed is either an interruptible load, in that case the broker does \textit{not} use that load as he has ‘won’ a cheaper one, or it was an optional load, in that case the broker executes his controllable load. Likewise for the asks, each ask is either an interruptible load, in that case the broker enables this load, in the case of an optional load that broker does \textit{not} use this controllable load. For each bid and ask that is not executed the reverse holds.

So for each bid it holds that it will be executed (used) when:

\begin{itemize}
  \item it is an interruptible load and the corresponding bid was not executed, this means that no other load was found that could balance it for cheaper and the broker has to use it to resolve his personal imbalance
\end{itemize}
• it is an optional load and the corresponding ask was not executed, this means that no other load was found that could balance it for cheaper and the broker has to use it to resolve his personal imbalance

• it is an optional load and the corresponding bid was executed, that means that another cheaper controllable load was found and the broker does not have to use it to resolve his own imbalance, he will pay for this through the payments

• it is an interruptible load and the corresponding ask was executed, that means that another cheaper controllable load was found and the broker does not have to use it to resolve his own imbalance, he will pay for this through the payments

**Definition 5.3.5** (Transformation \( t_q \)):
We define a transformation function \( t_q : \mathbb{R}^n \rightarrow \mathbb{R}^n \) where \( t_q(p) \) are the payments that the brokers have to pay in EGBPEC if \( p \) are the payments made by the agents in the double auction sub-mechanism.

Each payment to an agent \( i \) is also used for the corresponding broker \( i \).

In Figure 5.1 a graphical representation of the reduction can be seen. Using the functions described above the controllable loads that are executed is given by:

\[
t_q(f_d(t_c(x, \hat{c})))
\]

And the payments that the brokers need to pay is given by:

\[
t_q(p_d(t_c(x, \hat{c})))
\]

**Definition 5.3.6** (Utility function \( u_{a,d,i} \)):
For each agent \( i \) in the double auction subproblem we define the utility function (allocation double auction utility of broker \( i \)) \( u_{a,d,i} : A \times T_i \) where \( u_{a,d,i}(B^u_i, B^w_i) \) is the utility that agent \( i \) receives from the chosen allocation.

\[
u_{a,d,i} = \sum_{b \in B^u_i} b - \sum_{a \in A^w_i} a
\]

**Definition 5.3.7** (Utility function \( u_{a,e,i} \)):
For each broker \( i \) in EGBPEC we define the utility function (allocation energy grid problem utility of broker \( i \)) \( u_{a,e,i} : C \rightarrow \mathbb{R} \) where \( u_{a,e,i}(C_u) \) is the utility that broker receives from the chosen allocation. It is dependent on which controllable loads are chosen \( (C^u) \) to resolve the total imbalance and the private values of broker \( i \).

The set of controllable loads of broker \( i \) that are used to resolve the total imbalance is denoted as \( C^u_i \). Therefore the utility that broker \( i \) receives from an allocation can be denoted as:

\[
u_{a,e,i}(C_u) = - \sum_{c \in C^u_i} c
\]
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<table>
<thead>
<tr>
<th>i</th>
<th>$x_i$</th>
<th>optional loads</th>
<th>interruptible loads</th>
</tr>
</thead>
<tbody>
<tr>
<td>DU</td>
<td>-</td>
<td>0, 0, 0, ...</td>
<td>10, 10, 10, ...</td>
</tr>
<tr>
<td>A1</td>
<td>0</td>
<td></td>
<td>4</td>
</tr>
<tr>
<td>A2</td>
<td>4</td>
<td>-1</td>
<td></td>
</tr>
<tr>
<td>A3</td>
<td>-2</td>
<td></td>
<td>2</td>
</tr>
<tr>
<td>A4</td>
<td>-4</td>
<td></td>
<td>5</td>
</tr>
</tbody>
</table>

Table 5.1: The imbalances and the cost of controllable loads for each broker for Example 5.3.8

<table>
<thead>
<tr>
<th>bids</th>
<th>asks</th>
</tr>
</thead>
<tbody>
<tr>
<td>(A3, $\infty$)</td>
<td>(A2, $-\infty$)</td>
</tr>
<tr>
<td>(A4, $\infty$)</td>
<td>(A2, $-\infty$)</td>
</tr>
<tr>
<td>(A4, $\infty$)</td>
<td>(A2, 1)</td>
</tr>
<tr>
<td>(A4, 5)</td>
<td>(A1, 4)</td>
</tr>
<tr>
<td>(A3, 2)</td>
<td>(DU, 10)</td>
</tr>
<tr>
<td>(DU, 0)</td>
<td>(DU, 10)</td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 5.2: The bids and asks of the double auction simulation that the DU internally uses for Example 5.3.8

Example 5.3.8:

We now consider the example depicted in Table 5.1. We have denoted the four brokers as A1, A2, A3 and A4. The total imbalance is here $4 - 2 - 4 = -2$ and there are 4 controllable loads (minus those owned by the DU). There are three optional loads, so loads that are able to use less or generate extra electricity, and one optional load, a load that is able to use more energy. We can see that broker A2 is willing to pay 1 for an extra unit of energy in the current time slot.

After the DU has received the costs of all loads and the imbalances on the grid he simulates a double auction mechanism like described in Section 5.3.1. The bids and asks of the resulting double auction can be seen in Table 5.2. We can see that broker A4 is able to resolve a single unit himself so he puts 3 bids at $\infty$ and one at the price of what it would have cost if he would do it himself.

Each bidder can then be matched with a seller right next to it and if bid > ask we speak of a valid transaction, meaning a trade could happen. Then either the seller or buyer may have to enable his controllable load, or if both buyer and seller had a shortage/surplus nothing happens.

Using Table 5.2 we can see that in this example there are 5 valid trades. If we then assume there is a uniform double auction mechanism that can make the first 4 trades happen at a price of $s$ we can see that the utility of broker A2 is $4 \cdot s - 1$. The brokers A3 and A4
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have to use their own controllable loads to resolve a single unit of their imbalance and buy
the rest (1 unit and 3 units for the brokers A3 and A4) from broker A2. They would have to
pay $2 + s$ and $5 + 3 \cdot s$ respectively.

The mechanism for the double auction mechanism can be any double auction mecha-
nism that has the properties we need but the bids at infinity can pose problems to some
mechanisms. While the imbalance has to be solved at any cost, having the possibility that
infinite or almost infinite payments have to be paid is not an option. A solution to this is to
give each broker an unlimited supply of controllable loads at the prices $D^+$ for interruptible
loads and a price of $D^-$ for optional loads as we can assume they can buy them at that price
from the DU, therefore limiting the the bids and asks to $D^+$ and $D^-$. The result of this
adaptation on the example can be seen in Table 5.3.

<table>
<thead>
<tr>
<th>bids</th>
<th>asks</th>
</tr>
</thead>
<tbody>
<tr>
<td>(A3, $D^+$)</td>
<td>(A2, $D^-$)</td>
</tr>
<tr>
<td>(A4, $D^+$)</td>
<td>(A2, $D^-$)</td>
</tr>
<tr>
<td>(A4, $D^+$)</td>
<td>(A2, 1)</td>
</tr>
<tr>
<td>(A4, 5)</td>
<td>(A1, 4)</td>
</tr>
<tr>
<td>(A3, 2)</td>
<td>(DU, 10)</td>
</tr>
<tr>
<td>(DU, 0)</td>
<td>(DU, 10)</td>
</tr>
<tr>
<td>:</td>
<td>:</td>
</tr>
<tr>
<td>etc.</td>
<td>etc.</td>
</tr>
</tbody>
</table>

Table 5.3: The bids and asks of the double auction simulation that the DU internally uses
where bids and asks are capped to $D^+$ and $D^-$ for Example 5.3.8

Lemma 5.3.9:

Given an IC, IR, WBB and efficient multi-unit double auction mechanism the proposed
mechanism is WBB.

Proof. The payments to the DU from the agents in the double auction sub-mechanism are
transformed using the transformation function $p_d$. We know that the double auction sub-
mechanism is weakly budget balanced, and because the function $p_d$ is the identity function
(it does not transform any payments) the mechanism is weakly budget balanced.

Lemma 5.3.10:

Given an IC, IR, WBB and efficient multi-unit double auction mechanism the proposed
mechanism is efficient.

Proof. The transformation function $t_c$ transforms the interruptible and optional controllable
loads into bids and asks. We can still refer to each bid and asks as a load, because the
transformation defines a one-to-one mapping between controllable loads and bids. Optional
loads use energy and therefore are possibly profitable and interruptible loads can use less or

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Figure 5.2: How the double auction looks like in case of a negative or positive imbalance. The bids are sorted from high (top) to low (bottom) and the asks from low to high. The gray area denotes the controllable capacities that need to be enabled if the horizontal lines is the crossover point.

create energy which generally costs money. We know that the best optional load does not generate more money than the interruptible load costs. Therefore at both the bid and the ask side there is a clear divide between the interruptible loads and the optional loads if you sort them by price.

In Figure 5.2 we have sorted the bids from high (top) to low (bottom) and the asks from low (top) to high (bottom). In case of a negative imbalance the transformation function $t_c$ creates more interruptible loads at the bidding side than optional loads at the ask side, because the brokers with negative imbalances have a greater imbalance than the brokers with a positive imbalance. In the case of a positive total imbalance it is the other way around as can also be seen in Figure 5.2. If the total imbalance is 0 then there are as many interruptible loads at the bidding size as optional loads at the ask side.

If the double auction sub-mechanism is efficient, we know that the crossover-point lies somewhere between the two divides at the bid and ask sides. The horizontal line denotes the crossover-point. Following the transformation function $t_g$ the gray area denotes the
controllable loads that will be enabled. The gray area is always large enough to control the total imbalance, as it denotes the difference of the amount of interruptible loads on the bid size and the optional loads on the ask side. On the bid side we chose the controllable loads that are cheaper than the crossover-point, and also at the ask side the cheaper controllable loads are used as they are sorted from low to high. So the gray area consists of the cheapest controllable loads that can resolve the total imbalance.

The same holds for the case where the total imbalance is positive, except the optional loads in the gray area are used. In the case of zero imbalance, the crossoverpoint is such that the mechanism selects no controllable loads which is also optimal.

Lemma 5.3.11:
Given an IC, IR, WBB and efficient multi-unit double auction mechanism the proposed mechanism is IC.

Proof. The above lemma states that:

\[(X, C) \in \arg\max_{(\hat{C})} u_{a, e, i}(t_g(f_d(t_c(X, \hat{C})))) + t_q(p_d(t_c(X, \hat{C})))\]
given \((X, C) \in \arg\max_{(\hat{C})} u_{a, d, i}(f_d(t_c(X, \hat{C}))) + p_d(t_c(X, \hat{C}))\)

Because \(t_q\) does not transform the payments, each agent \(i\) pays the same amount as each broker \(i\) it is essentially the identity function and is therefore not relevant to our analysis, we can reduce it to:

\[(X, C) \in \arg\max_{(\hat{C})} u_{a, e, i}(t_g(f_d(t_c(X, \hat{C}))))\]
given \((X, C) \in \arg\max_{(\hat{C})} u_{a, d, i}(f_d(t_c(X, \hat{C})))\)

We can now show for each bidder \(i\) that submitting the true values of the controllable loads \(C_i\) is the dominant strategy:

\[(X, C) \in \arg\max_{(\hat{C}_i)} u_{a, d, i}(f_d(t_c(x_i, \hat{C}_i))))\]
\[= \arg\max_{(\hat{C}_i)} u_{a, d, i}(B_{t_i}^u, A_{t_i}^u)\]
\[= \arg\max_{(\hat{C}_i)} \sum_{b \in B_{t_i}^u} b - \sum_{a \in A_{t_i}^u} a\]

If a bid that corresponds to an interruptible load is not executed it is a positive event for the agent. To also reflect that in the corresponding broker it needs to enable the corresponding controllable load. Each optional load is put on the market reversed \((\times - 1)\) so each ask that is sold is a negative event for the agent (we only look at the effect of the allocation, not the
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payments) and to reflect that in the broker it is not executed, etc, etc. So we can rewrite this using definition 5.3.7 to:

\[
\arg\max_{(\hat{C}_i)} \left( \bigcup \left\{ c \in C_i | c \text{ is an interruptible load corresponding to bid } b \land b \notin B^u_i \right\} \right)
= \arg\max_{(\hat{C}_i)} \left( \bigcup \left\{ c \in C_i | c \text{ is an optional load corresponding to bid } b \land b \in B^u_i \right\} \right)
\]

Which we can simplify using the definition 5.3.4 of \( t_g \)

\[
\arg\max_{(\hat{C}_i)} \left( \bigcup \left\{ c \in C_i | c \text{ is an interruptible load corresponding to ask } a \land a \notin A^u_i \right\} \right)
= \arg\max_{(\hat{C}_i)} \left( \bigcup \left\{ c \in C_i | c \text{ is an optional load corresponding to ask } a \land a \in A^u_i \right\} \right)
\]

Lemma 5.3.12:
Given an IC, IR, WBB and efficient multi-unit double auction mechanism the proposed mechanism is IR.

Proof. We know that the double auction subproblem is individually rational, the payments are not modified in the reduction. All other changes are in the transformation \( t_g \) and how the utility function \( u_{a,e,i} \) works. \( t_g \) is constructed such that each change in the allocation in the double auction mechanism reflects into the same change in utility in EGBPEC. Also each extra imbalance or each extra controllable load has the same impact. It is easy to see that when the broker does not have any imbalance and no controllable loads it has a utility of zero in both mechanisms. This provides us with a base case and can thus conclude that the utility is never worse than in the case that the broker resolved the imbalance himself.

Theorem 5.3.13. Given an IC, IR, WBB and efficient multi-unit double auction mechanism we can construct an IC, IR, WBB and efficient mechanism for EGBPEC.

Proof. The theorem follows from the Lemmas 5.3.9, 5.3.10, 5.3.12 and 5.3.11.

5.4 Impossibilities

In the previous section we have shown that EGBPEC is reducible to the multi-unit double auction mechanism. This is useful as we may be able to use existing research on double auction theory for EGBP. However the literature cannot tell us what solutions are and are not possible with only this reduction.

In the next section we will show a reduction from the multi-unit double auction to EGBPEC, and therefore show that the problems are equally hard. With hardness we refer to the (im)possibility for mechanisms to have the properties we want. We will show that given an incentive compatible (IC), individually rational (IR), weakly budget balanced (WBB)
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and (asymptotic) efficient mechanism for EGBPEC a multi-unit double auction mechanism can be constructed that has these same properties.

5.4.1 Reduction from double auction

Each agent in the double auction is represented by a broker in EGBPEC. Every bid or ask represents a controllable load with a cost equal to the bid or ask. Each bidder gets a unit of negative imbalance for each bid and all sellers have an imbalance of zero.

Then we can execute an IC, IR, WBB and (asymptotic) efficient mechanism for EGBPEC on this setting and translate the allocation of controllable loads and payments back to the setting on the double auction mechanism.

Reduction

The truthful sets of bids and asks are denoted as $B$ and $A$. The possibly untruthful reported bids and asks are denoted as $\hat{B}$ and $\hat{A}$. The bids and asks $\langle \hat{B}, \hat{A} \rangle$ are transformed to a set of imbalances $\hat{X}$ and a set of reported controllable loads $\hat{C}$. The bids and asks of a broker $i$ is denoted as $\hat{B}_i$ and $\hat{A}_i$ respectively.

We will now define all the transformation steps, a graphical representation can be found in Figure 5.3.

Definition 5.4.1 (Transformation function $t_b$):
We define a transformation function $t_b : B \times A \rightarrow X \times C$ where $t_b(\hat{B}, \hat{A}) = (\hat{X}, \hat{C})$ are the imbalances and reported controllable loads of the brokers in the EGBP subproblem.

Each bid $\hat{b} \in \hat{B}_i$ is transformed into an interruptible load (the new interruptible load is owned by the same agent as the bid) with the size of a single unit and a reported cost equal to that bid $\hat{b}$. Each ask $\hat{a} \in \hat{A}_i$ is also transformed into an interruptible load (owned by the owner of the ask) with a size of a single unit and a reported cost equal that ask $\hat{a}$. Also, each bidder has a negative imbalance equal to the amount of bids he has reported. Each seller has an imbalance of zero.

Then we assume there is an IC, IR, WBB and (asymptotically) efficient mechanism defined by the functions $f$ and $p$.

Definition 5.4.2 (Allocation function $f$):
There is an allocation function $f : X \times C \rightarrow C$ where $f(\hat{X}, \hat{C}) = C_u$ is the list of controllable loads that can resolve the total imbalance in the most efficient manner.

Definition 5.4.3 (Payment function $p$):
Because we assume there is an IC, IR, WBB and (asymptotically) efficient mechanism there is a payment function $p : X \times C \rightarrow \mathbb{R}^n$ where $p(\hat{X}, \hat{C}) = (p_{e1}, p_{e2}, \ldots) = P_e$ is the list of payments that each broker has to pay, such that the mechanism is IC, IR, WBB and (asymptotically) efficient.
The allocation of each controllable load is transferred to a list of bids and asks that are executed in the double auction through the function $t_f$ where the resulting sets $B^u$ and $A^u$ are the sets of bids and asks that are executed. $B^u_i$ and $A^u_i$ denote the bids and asks of broker $i$ that are executed. For every executed bid, that broker receives an item and each seller with an executed ask loses an item. The payments $P_e$ made by the brokers are then transferred back to payments made by the agents through the transformation function $t_p$ where the resulting payments $P_d$ is the set of payments made by the agents.

**Definition 5.4.4** (Transformation function $t_f$):
We define a transformation function $t_f : C \to B \times A$ where $t_f(C_u) = (B_u, A_u)$ is a list of bids that are executed and a list of asks that are executed

\[
B^u_i = \{ b \in B_i | b \text{ corresponds to a load } c \land c \not\in C_u \} \\
A^u_i = \{ a \in A_i | a \text{ corresponds to a load } c \land c \in C_u \}
\]

Each controllable load can be traced back to a bid or ask in the double auction. Each ask which has his corresponding controllable load in the set $C^u$ is executed. Also each bid that does not have his corresponding controllable load in the set $C^u$ is executed.

**Definition 5.4.5** (Transformation function $t_p(P_e)$):
We define a transformation function $t_p : \mathbb{R}^n \to \mathbb{R}^n$ where $t_p(P_e) = (p_{d,1}, p_{d,2}, \ldots) = P_d$ is a list of payments that will be paid by the agents in the double auction.

This transformation function does not manipulate the payments. Each payment for broker $i$ gets transferred to a payment for agent $i$.

Using the functions described above the executed trades given the bids $\hat{b}$ and asks $\hat{a}$ are given by:

\[
t_f(f(t_b(\hat{B}, \hat{A})))
\]

And the payments that the agents have to pay is given by:

\[
t_p(p(t_b(\hat{B}, \hat{A})))
\]

The reduction with the transformation functions are depicted in Figure 5.3.

**Definition 5.4.6** (Utility function $u_{a,e,i}$):
For each broker $i$ in EGBPEC we define the utility function $u_{a,e,i} : C \to \mathbb{R}$ where $u_{a,e,i}(C_u)$ is the utility that broker receives from the chosen allocation. It is dependent on which controllable loads are chosen ($C^u$) to resolve the total imbalance and the private values of broker $i$.

The set of controllable loads of broker $i$ that are used to resolve the total imbalance is denoted as $C^u_i$. Therefore the utility that broker $i$ receives from an allocation can be denoted as:

\[
u_{a,e,i}(C_u) = - \sum_{c \in C^u_i} c
\]
Definition 5.4.7 (Utility function $u_{a,d,i}$):
For each agent $i$ in the double auction we define the utility function $u_{a,d,i} : A \times T_i$ where $u_{a,d,i}((B^a_i, B^d_i))$ is the utility that agent receives from the chosen allocation. It is dependent on which bids and asks are executed and the private values of broker $i$.

If we consider a bidder, we denote the set of bids that correspond to the executed controllable loads in EGBPEC $B^i_{exec}$. The utility that a broker receives from an allocation can then be expressed using this set:

$$u_{a,d,i}(B^a_i, A^d_i) = \sum (B^a_i \setminus B^i_{exec})$$

Likewise for a seller with the set $A^i_{exec}$:

$$u_{a,d,i}(B^a_i, A^d_i) = \sum (A^i_{exec})$$

Definition 5.4.8 (Utility function $u_{d,i}$):
We denote $u_{d,i}(\hat{B}, \hat{A})$ as the utility that an agent receives in the constructed double auction mechanism. It can be calculated with the previously defined functions:

$$u_{d,i}(\hat{B}, \hat{A}) = u_{a,d,i}(f(t_b(\hat{B}, \hat{A}))) + t_p(p(t_b(\hat{B}, \hat{A})))$$

Definition 5.4.9 (Utility function $u_{e,i}$):
We denote $u_{e,i}(\hat{B}, \hat{A})$ as the utility a broker $i$ would have received in the EGBPEC sub-mechanism.

This utility can also be calculated using the previously defined functions:

$$u_{e,i}(\hat{B}, \hat{A}) = u_{a,e,i}(f(t_b(\hat{B}, \hat{A}))) + p(t_b(\hat{B}, \hat{A}))$$

Lemma 5.4.10:
Given a mechanism for EGBPEC that is IR, IC, WBB and (asymptotically) efficient the proposed double auction mechanism is also IC.
Proof. We will show that truthfully reporting the bids and asks will maximize the utility in the EGBPEC subproblem and therefore also maximize the utility in the double auction.

We know that if a broker is truthful in EGBPEC that his utility is maximized. Because each bid in the double auction is converted to a controllable load of the same price we know that the utility received in the EGBPEC subproblem is maximized when the agent is truthful in the double auction mechanism. Now we only have to prove that $u_{d,i}$ is maximized when $u_{e,i}$ is maximized to prove that the constructed double auction mechanism is truthful. We know that $u_{e,i}$ is maximum on $(B,A)$ so we have to prove that:

$$
(B,A) \in \arg \max_{(\hat{B},\hat{A})} u_{a,d,i}(t_f(f(t_b(\hat{B},\hat{A})))) + t_p(p(t_b(\hat{B},\hat{A})))
$$

(5.3)

given $(B,A) \in \arg \max_{(\hat{B},\hat{A})} u_{a,e,i}(f(t_b(\hat{B},\hat{A}))) + p(t_b(\hat{B},\hat{A}))$

(5.4)

The transformation function $t_p$ is an identity function we only have to show that $u_{d,i}$ is at its maximum when $u_{e,i}$ is at its maximum.

$$
(B,A) \in \arg \max_{(\hat{B},\hat{A})} u_{a,d,i}(t_f(f(t_b(\hat{B},\hat{A}))))
$$

(5.5)

given $(B,A) \in \arg \max_{(\hat{B},\hat{A})} u_{a,e,i}(f(t_b(\hat{B},\hat{A})))$

(5.6)

Because agents can either be a buyer or seller and the transformation $t_f$ is different for bids and asks we deal with buyers and sellers separately. First we show the lemma for a buyer $i$.

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\[(B, A) \in \arg \max_{(\hat{B}, \hat{A})} u_{a,e,i}(f(t_b(\hat{B}, \hat{A})))\]

\[C_i^u \text{ denotes the chosen controllable loads of broker } i: C_i^u = f(t_b(\hat{b}, \hat{a})):\]

\[= \arg \max_{(\hat{B}, \hat{A})} u_{a,e,i}(C_i^u)\]

\[= \arg \max_{(\hat{B}, \hat{A})} - \sum_{c \in C_i^u} c\]

\[B_i^u \text{ denotes the bids corresponding to } C_i^u:\]

\[= \arg \max_{(\hat{B}, \hat{A})} - \sum_{b \in B_i^u} b\]

\[= \arg \max_{(\hat{B}, \hat{A})} \sum_{b \in B_i \setminus B_i^u} b\]

\[= \arg \max_{(\hat{B}, \hat{A})} u_{a,d,i}(B_i \setminus B_i^u)\]

Now using the definition 5.4.4 of \(t_f:\)

\[= \arg \max_{(\hat{B}, \hat{A})} u_{a,d,i}(t_f(C_i^u))\]

\[= \arg \max_{(\hat{B}, \hat{A})} u_{a,d,i}(t_f(f(t_b(\hat{B}, \hat{A}))))\]

\[\exists (B, A)\]

For a seller \(i\) the process is much easier, as each ask is represented as a controllable load that the broker sells at the same price:

\[(B, A) \in \arg \max_{(\hat{B}, \hat{A})} u_{a,e,i}(f(t_b(\hat{B}, \hat{A})))\]

\[= \arg \max_{(\hat{B}, \hat{A})} - \sum_{c \in C_i^u} c\]

\[A_i^u \text{ denotes the asks corresponding to } C_i^u:\]

\[= \arg \max_{(\hat{B}, \hat{A})} - \sum_{a \in A_i^u} b\]

\[= \arg \max_{(\hat{B}, \hat{A})} u_{a,d,i}(A_i^u)\]

This time the transformation \(t_f\) is much more straightforward:

\[= \arg \max_{(\hat{B}, \hat{A})} u_{a,d,i}(t_f(C_i^u))\]

\[= \arg \max_{(\hat{B}, \hat{A})} u_{a,d,i}(t_f(f(t_b(\hat{B}, \hat{A}))))\]

\[\exists (B, A)\]
Dealing with economic loads

5.4 Impossibilities

Figure 5.4: Graphical representation of a double auction. The red line represents the cheapest loads that are chosen in an efficient mechanism for EGBPEC

Lemma 5.4.11:
Given a mechanism for the EGBPEC that is IR, IC, WBB and (asymptotically) efficient the proposed double auction mechanism is also (asymptotically) efficient.

Proof. In the double auction efficiency is reached if all the asks up until the crossover point are executed and all the bids up until the crossover point.

In an efficient mechanism for EGBPEC the cheapest loads are chosen to resolve the total imbalance. These are the cheapest loads that can fulfill the sum of all imbalances. Because all the bids are transformed in a controllable load and an imbalance, we can see that the mechanism chooses the cheapest bids and asks to fulfill a quantity of the sum of all bids.

In Figure 5.4 an instance of a double auction is represented. The red line indicates the chosen loads by the EGBPEC mechanism. The red line The reduction states that the chosen asks are executed and the bids that are not chosen are executed. This is clearly optimal. So if the EGBPEC mechanism is efficient then resulting mechanism for the double auction is also efficient.

If the EGBPEC mechanism is only asymptotically efficient then some efficiency could be lost. After the transformation $t_f$ this loss remains the same size. Therefore if the EGBPEC mechanism is asymptotically efficient, the mechanism for the double auction mechanism is also asymptotically efficient.

Lemma 5.4.12:
Given a mechanism for EGBPEC that is IR, IC, WBB and (asymptotically) efficient the proposed double auction mechanism is also IR.

Proof. Individual rationality in the energy market means that the utility for a broker $i$ should
not be less than in the case he would have done it himself. Because the difference between agent \(i\) and broker \(i\) is precisely that (the sum of the price of his first \(x_i\) controllable loads) the utility of an agent \(i\) cannot be lower than zero.

Lemma 5.4.13:
Given a mechanism for EGBPEC that is IR, IC, WBB and (asymptotically) efficient the proposed double auction mechanism is also WBB.

Proof. The payments from the agents to the central auction holder are the same payments from the brokers to the DU. Since the DU does not have any extra income and the auction holder does not have any extra expenses the auction holder does not run into a deficit if the DU also does not run into a deficit. Therefore the resulting mechanism for the double auction is also budget balanced.

Theorem 5.4.14. If there exists a mechanism for EGBPEC that is IR, IC, WBB and (asymptotically) efficient the proposed double auction mechanism is also IR, IC, WBB and (asymptotically) efficient.

Proof. This follows from lemmas 5.4.10, 5.4.11, 5.4.12 and 5.4.13.

Impossibilities

We show that it is impossible to create an IC, IR, WBB an efficient mechanism for EGBPEC.

Theorem 5.4.15 (Gibbard 1973 and Satterthwaite 1975). Consider an arbitrary social function \(f\) and assume that 1) agents are unrestricted. 2) there are at least 3 outcomes (\(|O| \geq 3\)) and 3) for each \(o \in O\) there is at least a \(\theta \in \Theta\) such that \(f(\theta) = o\). If \(f\) is implementable in dominant strategies then \(f\) is dictatorial.

This means that there are no social choice functions that are implementable in dominant strategies where the choice functions is not dictatorial, the outcomes are always based on the preferences of a single agent. This is a very negative result but there is room to circumvent this theorem. It is possible to look at Nash or Bayes-nash incentive compatibility instead of dominant strategies, but in this thesis we would like to use the strongest concept of incentive compatibility.

The concept that we have also exploited in the previous chapters is quasilinear utility functions. Instead of assuming unrestricted preferences we assume agents utility functions are only dependent on the outcome \(o\) and decreases or increases by the amount we charge or give them in the form of penalties. This makes it possible to circumvent Theorem 5.4.15. The Groves class of mechanisms (which the mechanisms from the previous chapter belong to) can therefore implement dominant strategies, and are the only mechanisms that are efficient and incentive compatible.

If we consider a simple exchange setting [26] where buyers and sellers with quasilinear utility functions can trade identical units and buyers. It turns out that in this simple exchange setting it is not possible to create mechanisms that are WBB and efficient, see Theorem 5.4.16 by Green and Laffont [12].
Theorem 5.4.16 (Green and Laffont). [12] No dominant-strategy incentive compatible mechanism is always both efficient and weakly budget balanced, even if agents are restricted to the simple exchange setting.

Theorem 5.4.17 by Meyerson and Satterthwaite [26] show that it is impossible to create an IC, IR, WBB and efficient mechanism even if the mechanism is only Bayes-Nash incentive compatible.

Theorem 5.4.17 (Meyerson and Satterthwaite [26]). No Bayes-Nash incentive compatible mechanism is always simultaneously efficient, weakly budget balanced, and ex interim individually rational, even if agents are restricted to quasilinear utility functions.

However as described in Section 2.5.2 the VCG mechanism circumvents this by being only individually rational when the no-negative externalities effect and choice-set monotonicity hold. Furthermore it is only weakly budget balanced when the no single-agent effect holds. We have shown in the previous chapter that the no negative externalities, choice-set monotonicity and 0-value admitting properties all do not hold in EGBP.

So how were we able to construct mechanisms that were IC, IR, WBB and efficient? In EGBP we could see the brokers with an imbalance as buyers, as they have to buy controllable loads to resolve their imbalance. However, as the decision of which controllable loads are enabled lies with the DU, so they will not reject any offer that the DU makes if it does not violate their individual rationality.

In EGBP we have seen two different types of individually rationality. One where the agents can ‘choose’ between participating into the mechanism and paying $D^+x_i$ or $D^+x_i$, and one where alternative was to pay $c_0(X) \cdot x_i$ (the costs per unit it would take for the DU to balance the total imbalance times their own imbalance). These payments could be considered fair but the brokers have thus no way of ‘rejecting’ an offer as having an imbalance on the grid is unacceptable. The types of the brokers with imbalances are thus restricted and while VCG still does not work, we can still circumvent Theorems 5.4.16 and 5.4.17.

The main difference between this simple exchange setting and EGBP is thus that buyers are fine with ending up with no item if it costs them too much in the simple exchange setting, while in EGBP the item has to be bought.

The multi-unit auction as defined in this chapter is exactly a simple exchange setting. And as we have shown that the multi-unit auction is equivalent to EGBPEC, we have no way to circumvent Theorems 5.4.16 and 5.4.17. Which leads us to our final result:

Theorem 5.4.18. There exists no dominant-strategy incentive compatible mechanism for EGBPEC that is always both efficient and weakly budget balanced.

Theorem 5.4.19. There exists no Bayes-Nash incentive compatible mechanism for EGBPEC that is always efficient, weakly budget balanced, and ex interim individually rational.

In the following sections we will discuss what we can do about this.
5.5 Double auction mechanisms

Now that we know that it is impossible to generate a mechanism that is incentive compatible, individually rational, weakly budget balanced and efficient we again come to the point that we have to compromise.

Avoiding this impossibility through further restrictions on the type of the agents is possible but may not be realistic. For example McAfee [20] has shown that the Myerson and Satterthwaite theorem can be avoided if agents are sufficiently ex ante symmetric. Symmetry may be realistic in some settings but brokers on the energy market may be very different.

Again incentive compatibility, individual rationality and weakly budget balancedness are very important properties if the mechanism is going to be used in a real world situation. The only thing left to compromise on is efficiency. If asymptotic efficiency can be guaranteed or the resulting efficiency is close enough to optimal for real-world situations that would be very useful.

A lot of research has been done on truthful auctions however as of yet there exists no incentive compatible, individually rational and weakly budget balanced that is asymptotically efficient in the case each buyer and seller may want to buy/sell multiple items, however also the impossibility of such mechanism has yet to be shown.

Open problem 5.5.1:
Does there exist a multi-unit double auction that is incentive compatible, individually rational, weakly budget balanced and asymptotically efficient mechanism where each agent may sell and buy multiple units, even if restricted to only two items per agent or only a single reservation price?

Figure 5.5 depicts a Venn diagram with individually rational mechanisms. Mechanisms that fall in the "PROV IMP" area are provably impossible to construct without further type restrictions. We do not know if there exist any mechanisms that fall into the "PROB IMP" area.

Some papers consider problems with a single buyer or seller [3][10] or is only truthful for one side of the market [30](either the buyers or sellers are truthful). We do not further discuss they are unlikely to help us with a satisfactory solution to EGBPEC.

In Figure 5.6 the asymptotically efficient multi-unit double auction problem is depicted at the bottom. Above the multi-unit double auctions are the problems that are or need to be solved first. For example if the generic multi-unit double auction can be solved, then the easier auction where each agent only has two reservation prices is also possible, but that one is also currently not known to be (im)possible. The diagram starts with a result by McAfee, a truthful multi-unit double auction mechanism where each buyer and seller only wants to trade a single item. In each step on the diagram the problem becomes more generic until it reaches the final multi-unit double auction we need to solve EGBPEC.

Huang et al. [16] have published a mechanism that works in the case where brokers and buyers are interested in multiple items but assumes a single reservation price for all items. Furthermore the mechanism is not resistant to demand reduction.

Dani et al. [8] have published a mechanism of which they claim to have the same prop-
properties mentioned as Huang, however also does not give an incentive for demand reduction. Although this mechanism is weaker than a mechanism for a true multi-unit auction it can be used to solve a subset of EGBPEC problems where everyone only has a single evaluation value as we discuss later. However as is explained in Section 5.5.2 the claim that the mechanism gives no incentive for demand reduction unfortunately does not hold.

Because the result of Dani et al. [8] does not seem to be easily fixable, we explore another route to reach a better understanding of the (im)possibility of a true multi-unit double auction mechanism. In Section 5.5.4 we propose a new mechanism for a double auction where each seller and each buyer may want to trade up to 2 items. Although the mechanism is not incentive compatible, we show that in this mechanism it is a dominant strategy to bid the true value for one item and bid higher or ask lower for the second item. To put it into other words, demand reduction never yields more utility.

### 5.5.1 Mechanism McAfee

McAfee [21] proposed the first incentive compatible, individually rational, budget balanced and almost efficient double auction mechanism. It is the most simple one and therefore we discuss it first.

The mechanism assumes there are \( m \) buyers and \( n \) sellers. Each buyer \( i \) has a private value of \( b_i \) for a single unit of the good and each seller \( j \) has a private value of \( s_j \) of the good that he owns. Each broker reports their bid \( b_i \) or ask \( s_j \) to the central auction holder.
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The auction holder ranks the bids \( b_1 \geq b_2 \geq \ldots \geq b_m \) and ranks the asks of the sellers \( s_1 \leq s_2 \leq \ldots \leq s_n \). Then the efficient trade \( k \) is found that satisfies \( b_k \geq s_k \) and \( b_{k+1} < s_{k+1} \).

The \( k-1 \) buyers with the highest bids can then trade with the \( k-1 \) sellers that have the lowest asks. Each buyer pays \( b_k \) to the auction holder and each seller receives \( s_k \). It can easily be seen that because \( b_k \leq s_k \) the mechanism is completely budget balanced, and that an agent is never worse off. Only a single trade is sacrificed so the mechanisms scales very well with respect to the amount of agents. For the proof of the incentive compatibility of the mechanism we refer to the original paper.

5.5.2 Mechanism Dani et al.

Before going to the result by Huang et al. we discuss the result by Dani et al. because the claims made in their paper make it dominate over the mechanism proposed by Huang et al. (as can be seen by the arrow in Figure 5.6. A.R. Dani, Arun K. Pujari and V.P. Gulati [8] have published a paper of a mechanism for a multi-unit auction where both buyers and sell-
Dealing with economic loads 5.5 Double auction mechanisms

ers have multiple items that is asymptotically efficient, strategy proof, individual rational, budget balanced and does not have an incentive for demand reduction. The mechanism uses a restricted type space where every agent has a single reservation price. However, contrary to the claims made in the paper, we show below that the mechanism is not strategyproof against demand reduction. We first very briefly describe only an overview of how the proposed mechanism works and secondly the counterexample and an analysis of the problem. For more detailed information on how the mechanism works see the original paper.

The mechanism sorts the bids from high to low and the asks of all the agents from low to high. Table 5.4 is an example of an auction we use for the counterexamples. Then an index $L$ (at the side of the bidders) and $K$ (at the side of the askers) is calculated to help find a crossover point. $L$ and $K$ are chosen such that some conditions are satisfied. For the conditions and other details see the original paper [8]. Because $L$ and $K$ are not based on the actual size of the demands $m$ and $n$ are used to calculate the real crossover point that is being used. Again $m$ and $n$ are chosen such that some conditions are satisfied. In summary the indexes $m$ and $n$ are chosen at the real crossover point. From the conditions it follows that either $m = L$ or $n = K$ or both. Essentially the other is made as small as possible while still meeting the demand of the other side.

Then $m - 1$ bids and $n - 1$ asks are honored and the owners of those transactions trade. The bid $m - 1$ and the ask $n - 1$ which could have been executed are not, this makes the mechanism asymptotically efficient.

If the supply and demand do not perfectly matched, the reduced demand or supply is averaged over the buyers or sellers respectively. The price each agent pays for his units is based on the bids and asks of the other agents, for the exact calculation we refer to the original paper.

Counterexamples

In the paper the Claim 5.5.2 is made for which we provide a counterexample. The following write-up is very brief, to fully understand it I recommend running through the example yourself using the original paper.

Claim 5.5.2 (Theorem 4 in the paper of Dani et al.):
If any buyer (or seller) agent decreases his demand his utility decreases but utility of others remains unaffected or improves.

These are actually two separate claims, 1) a buyer cannot gain utility from demand reductions and 2) the utilities of others can only improve improve. Both claims are covered by the following counterexample. In Table 5.4 we have the situation where everyone reports their true prices and demands. There are 5 buyers who want to buy 50 items total and 4 sellers that can sell 40 items.

In this case $L$ is chosen to be 4 and $K$ equals 3. To make demand meet supply $m = 3$ and $n = 3$. Because the last transaction is always thrown away four agents (two buyers and two sellers) trade 20 items. The broker with the bid of 12 receives ten items for a price of 9 each and broker 2 also receives 10 items but for 6.5 each. The sellers receive 4.5 and 5.5
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<table>
<thead>
<tr>
<th>Bid</th>
<th>Quantity</th>
<th>Ask</th>
<th>Quantity</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>12</td>
<td>3</td>
<td>10</td>
</tr>
<tr>
<td>2</td>
<td>11</td>
<td>4</td>
<td>10</td>
</tr>
<tr>
<td>3</td>
<td>7</td>
<td>5</td>
<td>10</td>
</tr>
<tr>
<td>4</td>
<td>6</td>
<td>20</td>
<td>10</td>
</tr>
<tr>
<td>5</td>
<td>2</td>
<td></td>
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</tbody>
</table>

Table 5.4: The bids and asks in the case where the broker with bid 120 submits his demand truthfully

<table>
<thead>
<tr>
<th>Bid</th>
<th>Quantity</th>
<th>Ask</th>
<th>Quantity</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>12</td>
<td>9</td>
<td>10</td>
</tr>
<tr>
<td>2</td>
<td>11</td>
<td>4</td>
<td>10</td>
</tr>
<tr>
<td>3</td>
<td>7</td>
<td>5</td>
<td>10</td>
</tr>
<tr>
<td>4</td>
<td>6</td>
<td>20</td>
<td>10</td>
</tr>
<tr>
<td>5</td>
<td>2</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 5.5: The bids and asks in the case where the broker with bid 120 applies demand reduction respectively for each of their items. The broker with the bid of 12 now has a total utility of $12 \cdot 10 - 9 \cdot 10 = 30$. The broker with the bid of 11 has a utility of $11 \cdot 10 - 6.5 \cdot 10 = 45$

In Table 5.5 the same situation is shown except the broker with the bid of 12 reduces his demand from 10 to 9. If Claim 5.5.2 holds then the utility of broker 1 should either stay the same or drop.

Because the reported prices have stayed the same it still holds that $L = 4$ and $K = 3$. But $m$ and $n$ now have different values from than in the previous situation. Because the first 3 bids is now no longer enough to be as big (in terms of the amount of units) as the first 3 asks, $m$ has to be 4 and $n$ remains 3. With the removal of the last buyer and seller there are 3 buyers who want to buy 29 units in total and 2 sellers that can sell 20. Now the demand and supply do not match so the demand is averaged over the 3 buyers. The first buyer with the bid of 12 receives 6 items, and the other two both receive 7 items.

The agent that applied demand reduction buys 6 items at a price of 6.7. His utility is now $120 \cdot 6 - 6.7 \cdot 6 = 31.6$, which is 1.6 higher than in the case he reported his demand truthfully. Also, the agent with the bid of 11 now needs to pay 6.65 for each unit. His utility is now $11 \cdot 7 - 6.65 \cdot 7 = 30.45$, 14.55 less than in the case where the other agent was truthful about his demand.

So both claims made in do not hold. Demand reduction can increase the utility of an agent and can change the utility of other agents (even negatively).

Analyzing the proof provided

The paper [8] includes a proof of the lack of incentive for demand reduction, which we have shown to be false. Again we do not discuss the whole proof, only the most relevant
parts. For more information see the original paper. The proof goes as follows. \( q \) is the total quantity to be traded where buyer \( i \) is truthful (a demand of \( bq_p \)) and \( q_n \) is the total quantity to be traded where the \( i \)th buyer reduces his demand to \( bnq_p \). We assume that all other brokers are truthful about their demand. \( ba_{-i} \) is the total amount traded by others. Then the paper states that in the case true demand is submitted the following holds:

\[ q = bq_{-i} + bq_i \]  

(5.7)

And in the case of the new demand:

\[ q_n = bq_{-i} + bnqp_i \]  

(5.8)

Then the paper states that because \( bnq_i < bq_i \) it has to follow that \( q > q_n \) as there is no change in demand by others. But the fact that the demand of others do not change does not mean the quantity traded by the others does not change. The reduced demand can move the crossover point and increase the demand traded by others as we can see in the counterexample. The demand traded by others \( q \) goes from 10 to \( q_n = 14 \). So the assumption that \( q > q_n \) does not hold, and therefore the conclusion that the mechanism gives sufficient incentive against demand reduction is false. In the counterexample we can see that \( q = 20 \) and \( q_n = 29 \).

### 5.5.3 Mechanism Huang et al.

Huang, Scheller-Wolf and Sycara [16] have developed a multi-unit auction mechanism that is strategy-proof with respect to reservation price, weakly budget balanced and individually rational. It also assumes a single reservation price for the agents so the properties are equivalent to the mechanism proposed by Dani et al. but does not claim to be strategy-proof against demand and supply reduction.

The proposed mechanism also first defines a crossover-point where the demand and supply meet. At the crossover point there is buyer \( K \) and seller \( L \) (be aware that the paper of Dani et al. used \( L \) to denote the buyer and \( K \) to denote the seller at the crossover point). The exact definition of \( K \) and \( L \) can be found in the relevant paper [16] but it is also based on the quantity of the demand and supply so it is somewhat equivalent to \( m \) and \( n \) in the previous mechanism by Dani et al.

The mechanism lets the first \( K - 1 \) and \( L - 1 \) sellers participate in the transaction. Because the demand of the first \( K - 1 \) buyers is likely to not be the same as the supply of the first \( L - 1 \) sellers the side with the most supply/demand have to buy or sell a lower volume to match the other side. So if the inequality \( \sum_{i=1}^{K-1} X_i \geq \sum_{i=1}^{L-1} Y_i \) holds the first \( L - 1 \) sellers sell all their volume and the buyers may each sell a volume of

\[ X_j - \frac{\sum_{i=1}^{K-1} X_i - \sum_{i=1}^{L-1} Y_j}{L - 1} \]  

(5.9)

If the inequality does not hold the demand of the sellers is divided over the sellers an each seller may sell:
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\[ Y_j - \frac{\sum_{i=1}^{L-1} Y_j - \sum_{i=1}^{K-1} X_j}{L-1} \]  \hspace{1cm} (5.10)

This gives in most cases a satisfactory allocation of items. The buyers then buy the quantity of items for the price that the \( K \)th buyer has bid, and the sellers sell for the price of the \( K \)th ask. Because the crossover is chosen such that \( b_K \geq a_L \) the mechanism is budget balanced.

It is possible that after the demand/supply matching some seller has to trade a negative amount of items if the equations above are strictly followed. In that case that buyer or seller trades nothing and the "burden left" is averaged over the remaining buyers or sellers at that side of the market.

5.5.4 2-Item double auction mechanisms

Because no-one as of yet has published a double unit auction mechanism where each buyer and each seller can buy or sell more than a single unit, possibility or impossibility of an incentive compatible, individual rational, budget balanced and polynomial efficient multi-unit auction where buyers can buy and sell multiple item again a hard choice has to be made. In the following two mechanism we explore the possibility to create a 2-unit double auction, where each buyer and each seller wants to trade up to two items. The mechanism is not fully incentive compatible but does give an incentive to not reduce demand or supply.

In this mechanism the dominant strategy for sellers is to bid truthfully for the item that they value the least and for buyers the dominant strategy is to bid truthfully for the item they value the most. So the weakening of incentive compatibility is confined to the second controllable load. One can see that this is also the case if we naively use the mechanism of McAfee on a 2-item setting and consider each ask and each bid as separate broker. In that case the best strategy is to bid truthfully for the first item and use the reported value of the second item to influence the price that you pay for the first. However the case with the mechanism of McAfee is that the optimal strategy is to always bid less or ask more than the cost of the second item. We consider the act of making the last item more unattractive a form of demand reduction.

Although it is not an objective fact, the idea that agents reduce their demand or supply to profit more does not sound very good. If you think of the energy market, it is easy to see that the DU could use the wrong controllable loads because some of the cheapest controllable loads were reported to be more expensive. The increase in cost would likely end up at the brokers that have an imbalance or the DU itself. However if the optimal strategy would be to bid slightly higher or ask slightly lower than the real value of the item, the broker that placed the untruthful bid would suffer the most when the DU 'wrongfully' chooses his load because he was reported to be very cheap. If a broker tells the DU that he can balance some portion of the imbalance for a price \( p \) the broker will have to comply and use his controllable loads if he is deeded to be among the cheapest. Because of this, we assume a mechanism that incentivizes demand reduction is worse than a mechanism where it is incentivized to bid more or ask less for the items.
Assumption 5.5.3:
Mechanism where the optimal strategy is to bid higher or ask slightly less have less chance to impact truthful brokers negatively than a mechanism where buyers and sellers reduce their demand.

This assumption is somewhat crude and may not hold any ground in some situations. Note that the assumption is not used for the construction, or the proofs of the mechanism, it is only for the justification of this mechanism.

Description
In this mechanism there are again \( n \) buyers and \( m \) sellers. Each buyer wants to buy up to two items, and each seller wants to sell up to two items. Each buyer and seller has two distinct private values for each item it wants to buy or sell. Each buyer may place up to two bids, and each seller may place up to two asks. We assume there are \( x \) bids and \( y \) asks. We define the bids as \( b_1, \ldots, b_x \) where \( b_1 \geq \cdots \geq b_x \). And the asks \( a_1, \ldots, a_y \) as \( a_1 \leq \cdots \leq a_y \).

We call the set of asks and bids \((b_j, a_j)\) for \( j = 1, \ldots, \min(x, y) \) transactions. If \( a_1 < b_1 \) there is a point \( k \) where \( a_k > b_k \) and \( a_{k+1} < b_{k+1} \). All transactions up until \((b_k, a_k)\) are called valid transactions.

All the valid transactions up until \( k - 2 \), are executed. The transactions \( k - 1 \) and \( k \) are thrown away. This leads to some inefficiency but because the 2 trades that are thrown away are a fixed amount, it reaches an asymptotic efficiency.

The buyers pay a penalty of:
- 0 if they receive 0 items through executed transactions
- \( b_{k-1} \) if they receive 1 item through executed transactions, unless the bid \( b_{k-1} \) was theirs, then they pay \( b_k \)
- \( b_{k-1} + b_k \) if they receive 2 items through executed transactions

The same for the sellers, they receive:
- 0 if they receive 0 items through executed transactions
- \( a_{k-1} \) if they receive 1 item through executed transactions, unless the ask \( a_{k-1} \) was theirs, then they pay \( a_k \)
- \( a_{k-1} + a_k \) if they sell 2 items through executed transactions

The nice property of this mechanism is that the dominant strategy is to report at least one item truthfully, and not employ demand reduction on the other item. For buyers this means that the item with the highest private value is reported truthfully and the other item is also reported truthfully, or reported to be worth more. For sellers this means that the item with the lowest private value is reported truthfully and the other item is also reported truthfully, or reported to be worth less. We prove this only for the case of the sellers, the proof for the bidders is equivalent.
To prove this, we first show that a seller agent should never ask more for his cheaper item than his more expensive item. After that we show that the dominant strategy is to ask more for the more expensive item than the private value of the cheaper item. Then from this we construct a proof that shows that truthfully reporting the cheapest item is the dominant strategy. After that we finally show that a broker is never better off when asking more than the private value of the expensive item.

We assume a seller agent \( i \) has two private values \( v_{i,1} \) and \( v_{i,2} \) where without loss of generality we assume that \( v_{i,1} \leq v_{i,2} \). The agent places two asks, \( v'_{i,1} \) for his first item and \( v'_{i,2} \) for his second item.

**Definition 5.5.4** \((e_1 \text{ and } e_2)\):
In the following lemmas we will denote \( e_1 \) and \( e_2 \) as the two asks that are not executed, respectively \( a_{k-1} \) and \( a_k \). When we want to compare the utility gained with the utility gained when using another strategy, we can use the same two loads \( e_1 \) and \( e_2 \) to express the utility gained in both cases even if \( k \) changes.

**Lemma 5.5.5:**
The weakly dominant strategy is to ask \( v'_{i,1} \leq v'_{i,2} \)

**Proof.** The asks of all agents are first sorted from low to high, then the transactions of the first \( k-2 \) asks are executed. Because of this it is impossible that a more expensive ask is being executed when a lower ask is not.

Because the real private values \( v_{i,1} \) and \( v_{i,2} \) do not influence the price an agent would much rather sell the cheaper item instead of a more expensive one. Therefore it is a dominant strategy to bid \( v'_{i,1} \leq v'_{i,2} \) if \( v_{i,1} \leq v_{i,2} \).

**Lemma 5.5.6:**
The weakly dominant strategy is to ask \( v'_{i,2} \geq v_{i,1} \)

**Proof.** Lemma 5.5.5 states that asking \( v'_{i,1} \leq v'_{i,2} \) is the dominant strategy. Therefore we can proof this lemma by proving that asking \( v'_{i,1} = v'_{i,2} = v_{i,1} \) is never worse than asking \( v'_{i,1} \leq v'_{i,2} < v_{i,1} \).

If an agent asks so that \( v'_{i,1} \leq v'_{i,2} < v_{i,1} \), the reward the broker receives is either 0, \( e_1 \), \( e_2 \), or \( e_1 + e_2 \) where \( e_1 \) and \( e_2 \) are the reported costs of the two asks in the valid transactions that will not be executed. We go over each scenario and prove that the alternative (asking \( v'_{i,1} = v'_{i,2} = v_{i,1} \) is never worse).

1. If 0 is being paid out then \( v'_{i,1} \) and \( v'_{i,1} \) are not in the transactions that are executed then bidding \( v_{i,1} \) for both items will also result in no transactions.

2. If \( e_1 \) is being paid out to our broker it means that only \( v'_{i,1} \) is part of the valid transactions that are executed. If \( v_{i,1} < e_1 \) than the broker should just have just done the alternative as he currently loses. If \( v_{i,1} \geq e_1 \) then the broker should also have done the alternative as he will then also sell his second item with profit and he will now get less for his first.
3. If \( e_2 \) is being paid out to our broker that means that \( v'_{i,2} = e_1 \) and \( v'_{i,1} \leq v'_{i,2} \leq e_2 \). If this results in a profit for our broker, it would have also resulted in at least as big profit if the alternative is used. If this results in a loss, then the broker would have been better off with asking \( v'_{i,1} = v'_{i,2} = v_{i,2} \) because then none ask of his would have been executed.

4. In the case that \( e_1 + e_2 \) is the reward for the broker the idea is the same as the previous case. If this is a profit for our broker, changing the strategy to \( v'_{i,1} = v'_{i,2} = v_{i,2} \) would not change anything. However if he would have incurred a loss, he would have been better of with asking \( v'_{i,1} = v'_{i,2} = v_{i,2} \).

In all cases it was better to ask \( v'_{i,1} = v'_{i,2} = v_{i,2} \) then to ask \( v'_{i,1} \leq v'_{i,2} < v_{i,1} \) and together with the fact that bidding \( v'_{i,1} \leq v'_{i,2} \) is weakly dominant it follows that it is weakly dominant to bid \( v'_{i,2} \geq v_{i,1} \) for every seller. \( \square \)

**Theorem 5.5.7.** The weakly dominant strategy is ask truthfully the private value of the cheapest item.

**Proof.** If \( a_1 > b_1 \) then no trades are possible as there are no valid transactions. Broker \( i \) ends up with an utility of 0. The two bids \( v'_{i,1} \) and \( v'_{i,2} \) of broker \( i \) could make at maximum 2 valid transactions (if they are not \( a_1 \) or \( a_2 \) by lowering the asks) when \( v'_{i,1} < b_1 \) and \( v'_{i,2} < b_1 \) but as the last two valid transactions will be thrown away, so still no transactions will take place and agent \( i \) has a utility of 0 either way. So bidding \( b_1 = v_{i,1} \) is weakly dominant in this case.

If \( a_1 \leq b_1 \) then there is a \( k \) for which holds that \( a_k > b_k \) and \( a_{k+1} < b_{k+1} \). We will consider the case that agent \( i \) asks truthful for his first load, so \( v'_{i,1} = v_{i,1} \). His ask for his second load \( (v'_{i,2}) \) is greater than \( v_{i,1} \) (Lemma 5.5.6) and thus also greater than \( v'_{i,1} \). We show that if agent \( i \) reports his asks in this way, there is no incentive to deviate from this strategy in each of the 4 possible cases:

1. Both asks \( v'_{i,1} \) and \( v'_{i,2} \) belong to the valid transactions that will be executed
2. \( v'_{i,1} \) belongs to the valid transactions that will be executed and \( v'_{i,2} \) belongs to the 2 valid transactions that will not be executed
3. \( v'_{i,1} \) belongs to the valid transactions that will be executed and \( v'_{i,2} \) does not belong to a valid transaction
4. Both \( v'_{i,1} \) and \( v'_{i,2} \) are not in the transactions that will be executed

There are 2 transactions that are valid but will not be executed, we refer to them as \( e_1 \) and \( e_2 \). Likewise we refer to the last ask that does get executed as \( e_0 \).

1. Because it has to hold that the dominant strategy is to bid \( v'_{i,1} < v'_{i,2} \) (Lemma 5.5.6), \( v'_{i,1} \) will be part of the transactions that are executed regardless in the optimal strategy. The utility gained when truthful about \( v'_{i,1} \) is \( e_1 + e_2 - v_{i,1} - v_{i,2} \), deviating from truthful does not influence the utility. Truthful bidding \( v'_{i,1} = v_{i,1} \) is weakly dominant in this case.
2. In this case it holds that either \( e_1 = v'_{i,2} \) or \( e_2 = v'_{i,2} \) must hold. We prove that bidding \( v'_{i,1} = v_{i,1} \) is dominant for both of the following cases:

- If \( e_1 = v'_{i,2} \) the utility when truthful about \( v'_{i,1} \) is \( e_2 - v_{i,1} \). It is not possible to influence this gained utility by manipulating \( v'_{i,1} \) with the requirement that \( v'_{i,1} < v'_{i,2} \), as \( v'_{i,1} \) will always be lower than \( e_1 \) and thus be part of the valid transactions that are executed. Thus truthful reporting is weakly dominant.

- If \( e_2 = v'_{i,2} \) the utility when truthful about \( v'_{i,1} \) is \( e_1 - v_{i,1} \), which is positive. Decreasing \( v'_{i,1} \) does not influence the utility gained. Increasing \( v'_{i,1} \) until \( e_1 \) does also not influence the utility. If \( e_1 < v'_{i,1} < v'_{i,2} \), \( v'_{i,1} \) is now part of the valid transactions that are not executed and the utility is 0, worse than in the case of truthful reporting.

3. When truthful about \( v'_{i,1} \) the reward is \( e_1 - v_{i,1} \) which is positive. Decreasing \( v'_{i,1} \) does not influence the utility. Increasing \( v'_{i,1} \) does not influence the utility when \( v'_{i,1} < e_1 \). Past \( e_1 \) the utility is 0, as the agent is not part of any executed transaction. Thus being truthful about \( v'_{i,1} \) is also weakly dominant in this case.

4. When truthful about \( v'_{i,1} \) his utility is 0, as both valid transactions the agent is in will not be executed. Increasing \( v'_{i,1} \) does not influence this utility. Decreasing \( v'_{i,1} \) will also not influence the utility when \( v'_{i,1} > e_0 \). If \( v'_{i,1} < e_0 \), \( e_0 \) will be moved down and will be put into the transaction that does not get executed and \( v'_{i,1} \) will take a place of another ask and will be executed. The gained utility is \( e_0 - v_{i,1} \), however because \( e_0 < v_{i,1} \) this utility is negative. Being truthful about \( v'_{i,1} \) is weakly dominant in this case.

Because in all possible situations bidding truthfully for \( d_1 \) (so \( d_1 = v_{i,1} \)) is the dominant strategy, bidding truthfully for \( d_1 \) is always the weakly dominant strategy.

\[ \square \]

**Theorem 5.5.8.** The dominant strategy as a seller is to ask \( v'_{i,2} \leq v_{i,2} \)

**Proof.** We prove this by showing that it is always weakly better to bid \( v'_{i,2} = v_{i,2} \) than to bid \( v'_{i,2} \geq v_{i,2} \)

We know from Theorem 5.5.7 that it is always best to report the cheapest item truthfully. If the second item is also reported truthfully and the second item ends up in the transactions that are executed then they receive \( e_1 + e_2 \). Increasing the reported value of the second item cannot increase the utility as the broker can only push the second item out of the transactions that are executed. When that happens the payment the broker receives cannot become larger than \( e_2 \). Therefore the utility stays the same or is less.

If the second item is reported truthfully and the second item ends up in the transactions that are not executed, then increasing the reported price of the second item does not influence the utility of the agent and the utility stays the same.

So in all cases asking truthfully for the second item is weakly better than asking more for the second item.

\[ \square \]
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5.5 Double auction mechanisms

Figure 5.7: How a broker can profit from bidding untruthfully. The green transactions are the transactions that are valid but not executed

**Theorem 5.5.9.** When a broker \( i \) deviates from truthfully reporting the cost of his more expensive load and profits from this, no other agent is worse off.

**Proof.** To see that the theorem is true we first analyze how a broker can profit by being untruthful. The only way a broker can deviate from a truthful strategy and profit from it is to execute a strategy explained in the example in Figure 5.7. We assume the broker has a very good ask on the market that will be executed anyway and the broker also has a second item that he is willing to sell for 120. In the case that he is truthful the transactions with bid 120 and 108 are cut off although they are valid transactions and our broker receives 90 for his first item.

However if the broker drops the ask of his second load from 120 to 100, the decisions the mechanism makes change. The last 2 valid transactions are not the transactions with the bids 120 and 108, but now the transactions with the bid 108 and 102 are the last valid transactions. Now instead of 90 our broker receives 98 for the use of his load. But this also holds for all the other brokers, instead of receiving 90 and/or 98 they receive 98 and/or 100. Also the buyers buy at a price of 108 and/or 102 instead of 108 and/or 120.

Then for the general case we can see that the way the broker earns money is to make an extra transaction happen so it influences the prices. If a broker applies this strategy the 2 last transactions move 1 transaction down. Because the payments of everyone is based on these 2 transactions, and all transactions are sorted high to low for the buyers and low to high for the sellers, everyone is better off.

**Hypothesis 5.5.10:**
If all brokers execute their dominant strategy, the mechanism does not ’collapse’

Because the optimal strategy is to bid lower than the real cost for the most expensive controllable load, we have to research what happens if everyone start to execute their own optimal strategy. If all brokers bid lower then that means that the optimal strategy is to bid even lower. We test this hypothesis experimentally.

Tests are conducted to see what happens if all agents try to optimize their own imbalance. In the following test we have 5151 sellers and 5151 buyers. Each seller has two items and each buyer wants to buy two items. The private values for the first item ranges from 0 to 105.
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100 (inclusive) and the items for the second value also range from 0 to 100 (inclusive). Because the items are interchangeable there are only \((101 \cdot 101)/2 + (101/2) = 5151\) different sellers. The same holds for the buyers.

Then at each iteration we pick 20 buyers and 20 sellers from the sets of buyers and sellers. The brokers are not chosen uniformly over the brokers but are chosen such that each set of private values is chosen uniformly (the broker with values \((x,x)\) is less likely than a broker with \((x,y)\) because that broker also represents the broker with values \((y,x)\)). No buyer or seller is chosen twice in each iteration. Having \(2 \times 5151\) brokers in total and 40 agents in each instance makes it possible to have enough diversity in controllable load prices and is fast enough to run enough instances to let the brokers converge to their optimal strategy.

Each seller and buyer bids truthfully for his cheapest load, as this is the dominant strategy. Each buyer and seller has a current bid/ask in his head of what he is going to bid/ask for his second load. Then for each agent the simulation of the auction is run in 3 cases: the agent bids is current number, the agent bids higher than the current number or the agent bids lower than his current number. The agent registers the utility that he got with each of the 3 strategies.

Then after a agent has been played the game a certain amount of times he evaluates what the best strategy was in terms of utility. This strategy is now his new strategy and he again tries to define if he should bid higher/lower. Because it is possible that an agent moves away from his optimal strategy through sheer coincidence, the number of iterations the brokers needed to decide what the best strategy is goes up as the experiment progresses. In the end each broker on average went through 1800 (600 current, 600 up and 600 down) auctions to decide to alter his strategy or not. The initial strategy of the agents proved to be unimportant to the outcome of the experiment (the brokers converged to the same strategies), but in the test depicted in Figure 5.8 every broker started out with \(v_{i,2} = 0\).

In Figure 5.8 the outcome of all 101 sellers that have a cheapest load of \(v_{i,1} = 10\) is shown. The cheaper the first load is, the greater the incentive is for a broker to move away from truthfulness because the strategy is based on increasing the profit from the first load, and a cheap first load guarantees that it will be sold.

The Figure shows very well that the brokers that have a second load that costs between 20 and 50 clearly show that although they prefer to bid lower than truthful, they don’t alter their strategy by that much. Only at 50 and onwards the dominant strategy starts to diverge from truthfulness more steeply.

The brokers with secondary loads below 20 and above 70 seem not to converge to any strategy. This can easily be explained by looking at where the crossover-point appears. In Figure 5.9 we can see a histogram of the crossover-point. Because the bids and asks are uniformly distributed over 0-100 the mean of the crossover-point is at 50. The crossover-point happens rarely below 40 and rarely above 60, so for brokers with expensive loads it does not really matter how much they ask as long as it is above 70. Likewise for the brokers with very cheap loads.
Dealing with economic loads

5.6 Using only a single reservation price

Figure 5.8: The calculated strategy of 101 sellers which have a first item with value 10

Testing our assumption

Hypothesis 5.5.11:
In most cases the other brokers are not worse when a broker lowers his asks or increases his price

We can test the hypothesis experimentally for the uniform distribution that was also used for previous tests. For example in 5.10 the strategy of a seller with one item priced at 40 and one at 60 is depicted. The first load is being reported truthfully and the second load is being reported between 0-100 to see what strategy results in the most utility. For each reported value 10,000 random instances were run.

When the agent is truthful he receives an average utility of 4.45. However if he reduces his price from 60 to 54 he receives an average utility of 4.71. All the other brokers also profit from this, their average utility goes form 18.07 to 18.34. At least here the social welfare aligns pretty well with the utility function of our broker. Still, if the other agents could decide the agent should put is loads at an even lower price of 46 on the market.

The behavior shown here seems standard, although sometimes, and especially with other kinds of distributions, the broker does harm the other agents by executing his optimal strategy.

5.6 Using only a single reservation price

At the beginning of this chapter we have seen that EGBPEC is equivalent to a multi-unit auction. We have also seen that a multi-unit auction mechanism that is IC, IR, WBB and asymptotically efficient is not yet known. The following mechanism/reduction does not
5.6 Using only a single reservation price  

Dealing with economic loads

make use of a full multi-unit auction but uses a mechanism that is IC, IR, WBB and asymptotically efficient given that agents only have a single reservation price. So a mechanism that satisfies the claims of the proposed mechanism by Dani. et al.

A requirement for this reduction is that each broker has only two reservation prices, a reservation price for his interruptible loads and one for the optional loads. In practice this would mean that each broker offers the same price to all his customers. The DU may still have a very diverse set of controllable loads.

Reduction to single reservation price auction

If we want to use a single private value double auction mechanism to solve the problem we need to make sure that no broker can manipulate more than 1 bid and more than a single ask. There are two types of controllable loads (upwards and downwards) but we are at any time only interested in trading a single type of controllable load. If the total imbalance is positive the most efficient allocation of controllable loads does not contain any upwards controllable loads, as any negative imbalance can be resolved with the positive imbalance of another broker. Likewise with a negative imbalance, then we are only interested in upwards controllable loads.

At the start of the time-slot brokers report their private values to the DU. The DU can then translate these reported values together with the imbalances into bids and asks at a
Dealing with economic loads

5.6 Using only a single reservation price

simulated double auction. This double auction can then be solved by any double auction mechanism that is IC, IR, WBB and asymptotically efficient where each agent only has a single reservation price. The DU translates the information into a double auction in the following way:

If the total imbalance is 0 or negative ($X \leq 0$) then do for each broker:

1. For each positive imbalance place an ask of the price $-\infty$

2. For each negative imbalance, place a bid with the price of the subsequently cheapest upwards controllable load. If the owner does not have enough capacity to solve his own imbalance, place a bid for each remaining imbalance of $\infty$.

3. For every upwards controllable load left, place an ask with the price of the controllable load.

If the total imbalance is positive ($X > 0$), do for each broker:

1. For each negative imbalance place asks with a price of $-\infty$

2. For each positive imbalance, place a bid with the price of the subsequently cheapest downwards controllable load. If the owner does not have enough capacity to solve his own imbalance, place a bid for each remaining imbalance of $\infty$.

Figure 5.10: The utility of the untruthful brokers vs the average utility of the other brokers
3. For every downwards controllable load left, place an ask with the price of the controllable load.

Because each broker only has a single reservation price for each type of controllable load, each broker has at maximum 1 bid and 1 ask (a contributing broker with more controllable load than his own imbalance), 2 bids (a contributing broker with not enough controllable load to resolve his own imbalance), or 2 asks (a non-contributing broker with some relevant controllable load). Each bid and each ask may be for multiple units.

In the first image 5.11 an example is shown of such auction with 4 bids \( b_1, \ldots, b_4 \) and 5 asks \( a_1, \ldots, a_5 \). Each bid and ask may represent multiple controllable loads. The blue bid is of a broker that does not have enough controllable loads to solve that imbalance (and thus pushed onto the market for the maximum price of the DU), and the two blue asks are of brokers that have a noncontributing imbalance.

![Figure 5.11: Graphical representation of a double auction. The blue bids/asks represent bids of brokers which do not have enough controllable loads to solve that imbalance themselves](image)

Before using a single-evaluation multi-unit double auction mechanism on this problem we need to get rid of the blue bids and asks.

The first asks of brokers with a non-contributing imbalance can be removed by selling them to the buyers. The sellers sell their non-contributing imbalance for a price of \( P^- \) to the buyers in case of a total shortage of energy. The value \( P^- \) makes sure that there cannot be an incentive to not resolve the imbalance in the day ahead market. The buyers buy the imbalance for a price of \( P^+ \), incentivizing resolving imbalances in the day-ahead market.

We cannot sell this energy for a price of \( P^+ \) to a subset of buyers, as balancing for \( P^+ \) is very cheap. The solution is to fulfill every bid only partially, so every bid gets executed for the very same percentage. The red bars in Figure 5.12 indicate the part of the bids that will be executed with the blue asks. This does not incur an efficiency loss, as the blue asks are always low enough to be part of the most optimal trades.

The end result of this step can be seen in Figure 5.13

Now we can be sure that every broker only has a single ask on the market. However there are still multiple bids on the market. We remove this by letting the DU resolve this imbalance at a cost of \( D^+ \). This step does incur an efficiency loss. The remaining imbalance
Dealing with economic loads

5.6 Using only a single reservation price

Figure 5.12: Graphical representation of a double auction. The red bars indicate the part of the bids that get executed with the blue asks.

Figure 5.13: Graphical representation of a double auction. The blue bids/asks represent bids of brokers which do not have enough controllable loads to solve that imbalance themselves which the contributing brokers could not solve themselves are balanced at an expensive rate, therefore also giving them an incentive to have cheap controllable loads themselves. The result of this can be seen in 5.14.

**Example 5.6.1:**
The example we use here is depicted in the Tables 5.6 and 5.7. There are 4 brokers A1, A2, A3 and A4 with imbalances of respectively 0, 3, -2, and -4. Then suppose there are 4 upwards controllable loads (A1, 4), (A3, 5), (A4, 6, 4.5, 4) and 4 downwards controllable...
5.6 Using only a single reservation price

Dealing with economic loads

loads (A2, 1), (A3, 2), (A4, 3), (A4, 3, 5). The DU can resolve upwards at a cost of 10 and downwards at a cost of −1.

<table>
<thead>
<tr>
<th>broker</th>
<th>imbalance</th>
</tr>
</thead>
<tbody>
<tr>
<td>A1</td>
<td>0</td>
</tr>
<tr>
<td>A2</td>
<td>3</td>
</tr>
<tr>
<td>A3</td>
<td>-2</td>
</tr>
<tr>
<td>A4</td>
<td>-4</td>
</tr>
</tbody>
</table>

Table 5.6: Imbalances of brokers in Example 5.6.1

<table>
<thead>
<tr>
<th>broker</th>
<th>kwh upwards</th>
<th>price upwards</th>
<th>kwh downwards</th>
<th>price downwards</th>
</tr>
</thead>
<tbody>
<tr>
<td>A1</td>
<td>1</td>
<td>4</td>
<td></td>
<td></td>
</tr>
<tr>
<td>A2</td>
<td>1</td>
<td>6</td>
<td>1</td>
<td>-1</td>
</tr>
<tr>
<td>A3</td>
<td>1</td>
<td>5</td>
<td>1</td>
<td>-3</td>
</tr>
<tr>
<td>A4</td>
<td>2</td>
<td>3.5</td>
<td>1</td>
<td>-3</td>
</tr>
</tbody>
</table>

Table 5.7: Controllable loads of brokers in Example 5.6.1

The first broker places an ask of 4 for his only upwards controllable load. The second broker has a surplus of 3 so that translates to 3 asks at infinity. The third broker cannot resolve his own imbalance so the DU places 1 bid at 2 and another at ∞. The same goes for the fourth broker, he can resolve a single imbalance himself so 1 bid is placed at 3 and 3 bids at ∞. The bids and asks are listed in Table 5.8.
Dealing with economic loads

5.6 Using only a single reservation price

<table>
<thead>
<tr>
<th>bids</th>
<th>asks</th>
</tr>
</thead>
<tbody>
<tr>
<td>(A3, 1 kwh, $\infty$/kwh)</td>
<td>(A2, 1 kwh, $-\infty$/kwh)</td>
</tr>
<tr>
<td>(A4, 1 kwh, $\infty$/kwh)</td>
<td>(A2, 1 kwh, $-\infty$/kwh)</td>
</tr>
<tr>
<td>(A3, 1 kwh, 5/kwh)</td>
<td>(A1, 1 kwh, 4/kwh)</td>
</tr>
<tr>
<td>(A4, 2 kwh, 3.5/kwh)</td>
<td>(DU, 1 kwh, 10/kwh)</td>
</tr>
</tbody>
</table>

etc.

Table 5.8: Example

Then we sell the 2 asks at $-\infty$ to the bids (averaged) and end up with Table 5.9.

<table>
<thead>
<tr>
<th>bids</th>
<th>asks</th>
</tr>
</thead>
<tbody>
<tr>
<td>(A3, 0.5 kwh, $\infty$/kwh)</td>
<td>(A1, 1 kwh, 4/kwh)</td>
</tr>
<tr>
<td>(A4, 0.5 kwh, $\infty$/kwh)</td>
<td>(A2, 1 kwh, 6/kwh)</td>
</tr>
<tr>
<td>(A4, 0.5 kwh, $\infty$/kwh)</td>
<td>(DU, 1 kwh, 10/kwh)</td>
</tr>
<tr>
<td>(A3, 0.5 kwh, 5/kwh)</td>
<td>(DU, 1 kwh, 10/kwh)</td>
</tr>
<tr>
<td>(A4, 1.0 kwh, 3.5/kwh)</td>
<td>(DU, 1 kwh, 10/kwh)</td>
</tr>
</tbody>
</table>

etc.

Table 5.9: Example

After which we let the DU resolve the remaining bids at $\infty$ for a price of $D^+$ so Table 5.10 is the final double auction.

<table>
<thead>
<tr>
<th>bids</th>
<th>asks</th>
</tr>
</thead>
<tbody>
<tr>
<td>(A3, 0.5 kwh, 5/kwh)</td>
<td>(A1, 1 kwh, 4/kwh)</td>
</tr>
<tr>
<td>(A4, 1.0 kwh, 3.5/kwh)</td>
<td>(A2, 1 kwh, 6/kwh)</td>
</tr>
</tbody>
</table>

etc.

Table 5.10: Example

Then if this auction is being run on some hypothetical auction that is fully efficient then 3 imbalance is resolved through the imbalances of non-contributing brokers, 1.5 is resolved by the DU and the other 1.5 imbalance by the most efficient controllable loads (the loads of brokers A1 and A4)

**Efficiency**

The mechanism gets more efficient as more brokers are involved but there is no guarantee of efficiency if nothing is known about the distribution of the controllable loads over the
brokers. If all the brokers with a contributing imbalance do not have loads of their own and the non-contributing loads all have enough imbalance to resolve their imbalance the efficiency is at its worst. In that case the DU has to resolve the total imbalance himself. In all other cases the mechanism can work very well.

In the case that each broker has enough controllable loads to resolve his own imbalance, the mechanism is efficient.

5.7 Conclusion

In this chapter we have introduced EGBPEC, a grid balancing problem where brokers are able to use economic capacities. We have shown that this problem is equivalent to a multi-unit double auction and that it is therefore not possible to create a mechanism that is IC, IR, WBB and efficient. It is unknown if an asymptotic efficient mechanism is possible and we have identified it as an open problem in auction theory.

Dani et al. have proposed a mechanism that was claimed to give an incentive against demand reduction in the case that each broker has a single reservation price but we have shown that it is not the case. To better explore the search space of asymptotically efficient double auctions we have shown a 2-item each mechanism with two reservation prices that does not incentivizes demand reduction.

Finally we have shown that given a result like Dani et al. it is possible to create a mechanism for EGBPEC that is IC, IR and WBB and is asymptotically efficient if brokers have enough controllable loads to resolve their own imbalance.
Chapter 6

Conclusions and Future Work

6.1 Conclusions and Contributions

This section provides an overview of the scientific contributions of this thesis. First, we have introduced the Energy Grid Balancing Problem (EGBP) that was first described by de Weerdt, Ketter and Collins[19]. It considers how to efficiently resolve imbalances on the energy grid with the use of controllable loads owned by the brokers. We have identified the properties that a mechanism should have to be a satisfactory solution for EGBP. We have shown that the proposed mechanism by de Weerdt et al. does not hold up to all the requirements through some counterexamples and that further research is needed.

In the Chapter 4 we have shown that existing generic mechanisms are not sufficient to solve EGBP. We propose a total of 6 novel mechanisms that solve EGBP that are all incentive compatible (IC), individually rational (IR), weakly budget balanced (WBB) and efficient. Mechanism 1a is IC, IR, WBB and efficient where brokers are incentivized to resolve their personal imbalance in the day ahead market. The DU makes a lot of revenue therefore we constructed mechanism 1b that has the same properties but with less payments for imbalances.

Mechanism 2 is designed to give brokers an incentive to resolve their imbalance on the day ahead market such that the total imbalance is zero, to also incentivize reducing imbalances of other brokers.

Because we would like the revenue of the DU to be as small as possible, we have looked into a way to reduce this. An important result of this thesis is that if the incentive to resolve imbalances in the day ahead market was weakened to a Nash-equilibrium, a large part of the payments can be redistributed back to the brokers that do not contribute to the total imbalance. This is the intuition behind the mechanisms 3a, 3b and 3c. Mechanism 3b dominates 3a and 3c dominates 3b in terms of (less) revenue made by the DU while keeping the same properties. Mechanism 3a is part of new Power TAC specification. Through experiments we have shown how the mechanisms perform and that mechanism 3c performs very well with increasing number of brokers, controllable loads and imbalances.

In the fifth chapter we have proven that when brokers can actively enable controllable loads themselves, there exists no incentive compatible, individually rational, weakly bud-
get balanced and efficient mechanism. We call this new problem Energy Grid Balancing Problem with Economic Capacities (EGBPPEC). Because incentive compatibility, individual rationality and weakly budget balancedness are very important properties, we have explored the possibilities of weakening efficiently to asymptotic efficiency.

We have shown that the problem is equivalent to a multi-unit double mechanism and that given a multi-unit double auction that is IC, IR, WBB and (asymptotic) efficient we can construct a mechanism for EGBPPEC that has the same properties. An important result for the auction theory is that have found that the construction of such multi-unit double auction mechanism is an open problem in auction theory, even for the case of 2-items per agent or when each agent only has a single reservation price. We have made a taxonomy of double-auctions so see which problems needed to be solved first before we can construct such multi-unit double auction mechanism. We have shortly discussed a few double auction mechanisms from literature and have shown that although the paper of Dani et al. [8] claims to have an IC, IR, WBB and asymptotic efficient mechanism in the case that each agent has only a single reservation price, it does not discourage demand reduction.

To give more insight into multi-unit-double auction we have proposed a novel 2-item double auction mechanism, where demand and/or supply reduction is not incentivized, and where the dominant strategy is to bid truthfully for one item and bid more or ask less for the other. At last we have shown that if a mechanism with the properties as claimed by Dani et al. [8] does exist, it is possible to construct a mechanism for EGBPPEC that is IC, IR, WBB and gives asymptotic efficiency in the case that brokers have enough imbalances to resolve their own imbalance.

The advancements we have made in this thesis will make it possible to create more efficient and sustainable energy markets where brokers are motivated to align demand and supply while mispredictions are solved as efficiently as possible without enforcing massive payments.

**6.2 Future research**

We have presented new knowledge and many ideas on how to solve the Energy Grid Balancing Problem. Here we discuss some new research directions that are probably worthwhile to pursue.

*Lower bound on revenue*

Mechanism 3c is a mechanism that results in very little revenue for the DU, especially with an increasing number of brokers, controllable loads or imbalances. However we have not been able to prove that the mechanism is optimal or if there exists another mechanism that generates even less revenue for the DU while having the same properties.

*Double-unit multi auctions*

In the fifth chapter we have identified an open problem: Does there exist a multi-unit double auction that is IC, IR, WBB and asymptotically efficient where each agent may sell and buy multiple units, even if restricted to only two items per agent or only a single reservation
price? Many other fields could benefit from such mechanism or it would be very interesting to see what property makes it unsolvable.

**Multiple time slots**

In this thesis we have only considered a single time slot. In a real world situation the grid has to be balanced time slot after time slot. If each time slot would be completely independent of the others, this would not be a problem. However it is very likely that the set of possible outcomes in time slot $x$ depends very much on the choices made earlier. Some controllable loads may have a startup time longer than a single time slot, or can only be used once every 24 hours. If the DU still wants to solve the imbalances in the most efficient way possible, it needs accurate predictions of the brokers to make the right decisions. This gives all sorts of new dynamics. Brokers could for example manipulate predictions to force certain outcomes in the current time slot or they make their controllable loads only available on a subset of possible time slots to maximize their reward.

In the creation of this thesis some research was done to extend any single time slot mechanism to multiple time slots. It seems to be possible to extend mechanisms while also making it possible to have various restrictions on the controllable loads (for example, only available between 2:00 and 5:00 pm for 30 minutes at maximum 3 times a week) and incentivize good predictions. We can apply VCG mechanisms over all timeslots to incentivize truthfulness and because all imbalances are known after each timeslot, the effect of each misprediction on social welfare can be calculated for each broker and handed out as a payment. However the computational intensity of the mechanism increases rapidly in the amount of time slots and controllable loads. Also the payments are calculated at the very last time slot. More research should be done on the problem of having multiple time slots. To be more specific asymptotically efficient algorithms should be developed to limit the computational complexity and mechanisms should be able to handle an infinite amount of future time slots.

**Bayes-Nash equilibria**

In this thesis we have assumed that a broker is always worse off by having a higher absolute personal imbalance, and that therefore the broker will aim for an imbalance of zero. This is true if the game is non-Bayesian, but may not hold in a Bayesian version of the game. The payments for negative imbalances are generally higher than the money lost when having a positive imbalance. Therefore if agents do not have a single exact prediction of their future imbalance but know that their prediction, for example, is some normal distribution, agents will likely aim for a slightly positive imbalance to avoid high payments. This does not have to be bad as it could align with the total social welfare but the definition of incentivizing resolving imbalances in the day-ahead market has to be reconsidered. This also has an impact on the previously mentioned predictions that have to be reported to the DU in the case of multiple time slots. If we want to let the DU make optimal decisions, brokers should report the whole distribution function of their expected imbalance to the DU. This could make the mechanism even more computationally intensive and needs proper algorithms to handle this information.
6.2 Future research

Non-quasilinear utility functions and small brokers
In this thesis we have also assumed that brokers have quasilinear utility functions. This is quite normal in the literature of mechanism design and seems reasonable given that brokers are businesses and are only interested in generating as much revenue as possible for their stakeholders. However businesses are also known to absorb short term losses if the negative consequences are much larger for their competitors to drive them out of the market. This is not modeled and as competition is very important in the energy market, the effects have to be considered.

In the mechanism by de Weerdt et al. [19] and the mechanisms proposed in the fourth chapter we hand out VCG payments for the use of their controllable loads. This makes the mechanism incentive compatible but rewards brokers with a lot of controllable loads more than brokers with only a few controllable loads. This is because the marginal price gets larger for each controllable load added. Research has to be done on the impact of this advantage to protect smaller brokers and thus competition.

Interaction with the rest of the energy market
The Energy Grid Balancing Problem is not solved in a vacuum. When implemented it also influences and is influenced by other factors. Research like Power TAC that look at the market as a whole and takes into account the dynamics between brokers are very important. The relationship between brokers and their customers may affect how controllable loads can be used or brokers could work together to achieve mutual benefit which could impact other brokers. There may be a lot of dynamics that are hard to formulate and any market mechanism should therefore also be tested in a real world situation such as a competition where agents will actively try to play the market.
Bibliography


