Design of a particle filter for robust target tracking in object-induced clutter
Design of a particle filter
for robust target tracking in object-induced clutter

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Abstract

State of the art tracking techniques are often based upon a linear filtering scheme that consists of several building blocks which take so called plots as an input. Plots typically consist of range, bearing and possibly elevation and Doppler measurements. However, many real-world tracking scenarios cannot be described properly by a linear system. Tracking a target that induces clutter itself is an example of such a generic nonlinear real world scenario. Target-induced clutter can be present in many forms, from a bow wave or wake that is created by a vessel moving through water, to a car creating a cloud of dust while driving through the desert. Another example could be a vehicle driving over a road covered in water and inducing a water spray around itself. In general, these scenarios describe the inducement of nonlinear clutter by the target, which will hinder a conventional tracker in robust target tracking. Indeed, these scenarios often lead to unsteady track behavior, multiple tracks on target or even losing a track while the filter continues the track on the clutter instead of the target. Unreliable track behavior is highly undesirable. Think for instance of military tracking systems or tracking and navigation systems in autonomous vehicles used in order to safely control or drive these vehicles. Unreliable track behavior in real world scenarios could lead to safety hazards. Therefore, a target tracking technique that performs robust in target-induced clutter is required.

Consequently, in this thesis a track before detect particle filter is designed that should perform robust in object-induced clutter. Track before detect is a technique where tracks are directly produced based on raw sensor measurements, without intermediate processing and decision making. This type of framework has the advantage over classical tracking techniques that nonlinear clutter can be modeled very well. Another benefit is the fact that the full received sensor data is integrated over time, which will lead to an improved tracking performance especially in the case of weak targets.

During the design of this filter a step-by-step design process has been worked out and intermediate results are described. Answering the research question starts with the design of a one dimensional (range only) particle filter suitable for tracking a single target. During the following design steps the particle filter will be extended by introducing the following extensions: i) introducing a second clutter reflector; ii) expand to the two-dimensional (range and Doppler) measurement domain, and finally; iii) expand to target-induced clutter by introducing dependencies between the target and the clutter.

Throughout this research it has been shown that a particle filter is a suitable framework for robust target tracking in object-induced clutter. Besides, the filter has a generic setup which makes it generically applicable under various target, sensor and environmental conditions. The focus of this research is on the design of a robust tracking filter; therefore, the target and clutter modeling has been performed in a simple manner. This field leaves room for improvement in future work.
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It has been more than twelve years ago that my career in the Royal Netherlands Navy (RNLN) started at the Royal Netherlands Naval College in Den Helder with the naval officers training. A bachelor of science program was part of this training. After having finished this training I sailed for a couple of years on board of HNLMS Rotterdam as engineering officer. After this assignment, I started a new challenge as project manager for the Defence Materiel Organisation (DMO) in The Hague, where the majority of my work was assigned to the navy. However, in general I was responsible for the installation of Electro Optical (EO) systems on board of new ships or as a part of modernization programs. The first ideas about this thesis arose during this job.

During several system qualification and acceptance tests I noticed that state of the art target tracking systems did not always perform well in target-induced clutter scenarios. Target-induced clutter can for instance be the bow wave that is created by a vessel while moving through the water. From this moment on I started thinking of possible solutions on how to tackle this problem. As a part of my career within the RNLN, I started the Master of Science (MSc) Electrical Engineering at Delft University of Technology in autumn 2014. Via my work I came in contact with Hans Driessen and we brainstormed about possible thesis topics for addressing this problem. Eventually, the subject was chosen to be the design of a particle filter with the goal to perform as a robust target tracking technique in object-induced clutter.

I would like to thank my employer for giving me the opportunity to obtain my MSc degree in Electrical Engineering. Besides, I want to thank Hans Driessen for being my supervisor and providing me with great support and guidance during my thesis work.

At the moment, I am finalizing this thesis report and looking back at a challenging, enlightening and versatile period of my life. In the autumn of 2015 my son Tycho was born, which gave me the great privilege of becoming a father (while being a student at the same time).

To conclude, I want to thank my wife Marjon for her great support during this study which kept me on track.

Wilco Tempert
Hoorn, October 2016
Chapter 1

Introduction

This chapter will introduce the main research themes and provides an overview of the report. The main research theme is the design of a particle filter for robust target tracking in object-induced clutter. A particle filter is a specific type of a target tracking technique. Therefore, this topic can be placed in the broader context of target tracking. Consequently, the structure of this introduction chapter has been set up as follows.

The first section will describe the evolution on target tracking from a historical perspective. Secondly, the focus is shifted towards the current requirements of target tracking techniques. The requirements will reveal shortcomings of state of the art conventional techniques, which leads to the problem description in the third section. The fourth section will describe the research objective, scope and methodology. The research question will be posed in the succeeding fifth section, while the sixth section outlines the scientific motivation of this research. Concluding, the seventh section will outline the structure of the complete report.

1-1 Target tracking evolution

Target tracking has been used over many decades, but the applications and development of different tracking techniques has evolved enormously over the past thirty years. Target tracking was first carried out by Carl Friedrich Gauss in 1801, when he determined the orbit of the asteroid Ceres based upon only three measurements [1] [2]. Later in the nineteenth century some important milestones and discoveries were done [3]. In 1865 James Clerk Maxwell presented his theory of the Electro Magnetic (EM) field, successively Heinrich Rudolf Hertz proves Maxwell’s theory by discovering EM waves in 1886. A few years later, in 1897, Guglielmo Marconi accomplished the first long distance transmission of EM waves. In 1900 Nicola Tesla proposed that the reflection of EM waves could be used for the detecting of moving metallic objects. The collection of these inventions can be interpreted as the foundation of Radio Detection And Ranging (RADAR), commonly referred to as radar. Radar systems have formed an important basis for the development of target tracking techniques. Especially, the introduction of radar during World War II brought a boost to the technological area of target tracking, since numerous countries were working in secrecy in order to develop their own radar system which should be capable of providing a target range and angular location with respect to the radar antenna [4] [5]. A more detailed description of radar, within the context of this thesis will be provided in section 2-1 Radar.
After the war, the mainly military user domain of radar, rapidly extended to other areas of application, like: Air Traffic Control (ATC), maritime navigation, meteorology, medical devices, home automation, security, automotive, traffic monitoring, collision avoidance, level measurement, door openers, Video Tracking Modules (VTM) and sports [6]. In parallel with radar development, target tracking also evolved over the past decades from specific military tracking purposes towards various techniques which are suitable for tracking different kinds of targets depending on the application area. For instance, the target tracker in a VTM can often be slaved to a primary tracker such as a radar. In this case, the VTM is primary designed to maintain the track of a single target based on Electro Optical (EO) measurements for instance. On the contrary, the target tracking technique of an autonomous car should be capable of tracking various kinds of targets and multiple targets simultaneously, like motorized vehicles, but also cyclers or pedestrians. With respect to target recognition the latter application is far more multifunctional.

Another aspect which has impact on the evolution of tracking is the rapid changing world and the diversity in customer requirements. Until a few decades ago, the majority of research and development in this field was driven and mainly funded by Defense authorities. Nowadays, smart tracking techniques are applied in numerous of systems spread over various application domains. This shift and broadening of user domains resulted in more widely scientific interest and other research perspectives since different users and applications demand diverse requirements and a variety of implementations.

In addition to the extension of the user domain, the development and wide application of computers during the last decades have changed the focus on target tracking from a hardware point of view to a software en signal processing approach.

### 1-2 Target tracking requirements

Following on the historical perspective provided in the previous section, this section focuses on current target tracking requirements. In addition, an overview of conventional Multi Target Tracking (MTT) is provided in appendix A.

The general objective of target tracking is to collect sensor measurement data from a Field Of View (FOV) containing one or more potential targets of interest, followed by dividing the collected data in clusters of observations (i.e. tracks) which are produced by the same target [7]. In other words, the objective is to estimate the state of the targets of interest followed by initiating tracks on these targets and maintaining them as long as they are present. Besides, the goal is to bring adaptivity in order to be able to tackle the continuous changing target and environmental conditions [8]. After all, no single sensor can perform optimal under all conditions. Therefore, the inherent focus lies on designing an adaptive and knowledge based form of processing that can perform robust under various conditions.

Since the performance of surveillance sensors – like radar, EO and sonar – has been improved significantly over the last decades, the main field of future growth for target tracking techniques lies in the design of smart tracking algorithms. Obviously, smart hardware selection and improvement is still of great importance, however majority of the uninvestigated opportunities can be found within the software domain. This could bring significant operational and performance benefits.

Nowadays autonomous driving is a popular research topic which involves a key technical challenge for the next 20 years [9]. At this moment, numerous countries have announced their forecasts regarding driverless cars and it is the expectation that fully autonomous cars will be
reality within a few decades. Well, to make this happen the field of target tracking is of great importance in order to guarantee safe navigation and obstacle avoidance. In order to accomplish this forecast, cars will have to be equipped with different kinds of sensors – for instance, radar, laser, ultrasound and electro optical – in order to obtain a complete Situational Awareness (SA). The collected measurements of these sensors have to be processed automatically in order to carry out proper target tracking, identification, decision making and in the end, safe navigation of the vehicle. Obviously, smart hardware selection is important, but even more are the design of smart target tracking techniques in order to turn autonomous driving into a success.

In addition to the automotive field there are numerous other areas of application – ATC, Intelligence, Surveillance, and Reconnaissance (ISR), space applications (like tracking space debris), oceanography, robotics, remote sensing, computer vision, (bio)medical research – where target tracking plays an important role [10].

In general, all the areas of application have one requirement in common: there is a need for more accurate, robust and knowledge based target tracking techniques in order to automatically and dynamically track different kind of targets under various circumstances.

1-3 Problem description

Following on the target tracking requirements as described in the previous section, this section will define the problem in more detail and reveal why the requirements will not always be met in real world target tracking scenarios. This description will be the basis for the succeeding sections towards the formulation of the main research question.

1-3-1 Conventional track filtering drawbacks

The majority of the currently used tracking systems is based on the concept of conventional target tracking techniques. A conventional target tracking scheme consists of several building blocks with sequential processes based on linear system modeling.

The Kalman filter is often applied in conventional tracking systems. Meanwhile it has been more than half a century ago that the Kalman filter was invented by Rudolf E. Kalman in 1960 [11]. Kalman filters are relative easy to design and provide an accurate estimation in most cases. In contrast, the performance can be worse for some practical reasons, comprising [12]:

I. nonlinearity in the system physics equations;
II. ill-conditioned covariance matrix as a consequence of linearizing a nonlinear phenomenon or an increasing imbalance between estimation errors;
III. incorrect or lacking models regarding the underlying physics.
IV. The assumption is made that false alarms are independent of the existing tracks. This does not hold in case of target-induced clutter because there is a nonlinear dependence between clutter (false alarms) and the target. In conventional tracking, an additional feedback would be required in extraction. See figure 1-1.

As already mentioned, Kalman filters are optimal for linear Gaussian problems. However, most real world application scenarios are nonlinear and non-Gaussian. In order to fill this gap, engineers tried to make linear approximations in order to model nonlinear problems.

For surveillance systems, which are built to track relatively small (naval surface) targets, it is known that conventional Kalman-based trackers do not perform well in object-induced clutter
scenarios. These deficiencies cannot be completely attributed to the Kalman filter, because the complete conventional linear tracking chain also contributes to these drawbacks. This will often lead to unstable tracking behavior, lost tracks, or multiple tracks on the target-induced clutter. Clutter often manifests itself in various forms, though it will change the target's profile which often leads to an increase of the false alarm rate.

A more detailed description – within the scope of this thesis – of the Kalman filter and the related Multi Hypothesis Tracking (MHT) is provided in appendix B. In addition, paragraph 1-4-3 about the methodology, will further explain why conventional tracking techniques and MHT are not ideal in case of object-induced clutter.

1-3-2 Particle filter

Since the introduction of the Kalman filter, a lot of research has been done on the development of nonlinear filters in order to increase estimation accuracy [12]. One of the drawbacks of nonlinear filters at the time when Kalman invented his approach was the limited computational processing capability. After all, nonlinear filters demand a higher real time computational complexity compared to classical filters in order to achieve the required accuracy. Though, after the huge and universal increase of computational power over the last decades, those real time computational demands are nowadays far less limiting. This has paved the way for new research steps in order to improve current state of the art target tracking systems by investigating whether a nonlinear filter implementation can provide a significant performance improvement.

The Particle Filter (PF) belongs to a relatively new class of nonlinear filters, which were presented in the beginning of the nineties of the last century in [13]. This introduction resulted in a new flow of research papers on this topic. Just as is the case with the Kalman filter, “the” particle filter does not exist. Indeed, there is a broad variety in different types and implementations of particle filters. Besides, the fact that the particle filter concept provides a framework in order to estimate real world nonlinear estimation problems, it has the advantage that specific and high level stochastic calculus is not a requirement for a designer to have. A PF algorithm will approximate the whole non-Gaussian probability density of the state vector, conditioned on the observations (measurements).

1-3-3 Actual problem description

Given the previous subsection on the drawbacks of conventional Kalman-based tracking system, the following question can be posed:

*What is the actual problem?*

This question is answered as follows. Robust tracking of an object which induces clutter itself by using a conventional Kalman-based tracking filter is hard to accomplish, because this linear model cannot properly distinguish between target and clutter. In general, clutter is not always easy to model, because it is often a nonlinear or non-Gaussian dynamic phenomena. Especially, in the case of object-induced clutter there is an additional dependency related to the objects state vector, which defines its kinematics (velocity, etc.). These dependencies will often introduce nonlinearities.
Why is this a problem?

Well, since the target and the target-induced clutter cannot be distinguished properly, it is not possible to create a robust and steady track over time. Indeed, the track on the target will be temporary lost, or the clutter will be interpreted as the target of interest which leads to an estimation error, false alarms or missed detections. Depending on the application, such errors can cause serious problems or even accidents. Specifically, this might have the following effect on detections: missed detections on the target body or additional detections on the clutter. Tracks might be effected as follows: false tracks, short-lived tracks, erroneous target maneuvers or lost tracks.

Having the actual problem described, it will be illustrated with two examples in the succeeding paragraph.

1-3-4 Example problem scenarios

Example I: naval surface target tracking with radar

An example scenario could be tracking of a jet ski with a radar system, where the jet ski is maneuvering rapidly. As a consequence, the target induces background clutter like bow waves and wake. In addition, the waterjet itself can produce echoes with a distinct speed, different from the target speed. A radar system often measures range as well as Doppler velocity. When the target is of a fixed form, like the jet ski, it can be assumed that the object as a whole reflects the same radial Doppler velocity towards the radar. The clutter however, will likely produce a different Doppler velocity pattern which disturbs the tracking process. This can lead to inaccurate or even lost tracks which is highly undesirable for mission critical systems.

Example II: navigation of an autonomous car based on several sensors

An autonomous car is navigating and driving itself by using several different sensors. The vehicle is driving in an urban environment in heavy rainy conditions. A couple of hundred meters in front of it a motorcycle is driving in the same direction. Because of the heavy rain fall, the road is covered in water and as a result the vehicles driving over it induce spray around them. In order to detect traffic ahead, the car is using a radar which transmits a Radio Frequency (RF) signal. Part of the RF energy is reflected back by encountered objects and received at the antenna. The motorcycle driving in front of the car is one of these objects that reflects part of the signal back to the radar receiver. However, the spray – which is induced by the motorcycle – also causes echo’s. So, in order to perform safe navigation and obstacle avoidance it is of crucial importance that the target tracking scheme is able to accurately detect and track the motorcycle (target of interest), instead of being misled by false detections and tracks based on the induced clutter. Confusion between target and clutter echoes should be avoided at all times as this could lead to collisions and accidents.
1-4 Research objective, scope and methodology

The preceding problem description forms the basis for the research objective which will be formulated in this section. In addition, the scope of this research will be determined. Lastly, the methodology which is used to perform the research and its set up will be outlined.

1-4-1 Research objective

The research objective of this thesis is to fill the gap between the state of the art Kalman-based tracking systems – which do not perform optimal in target-induced clutter – and the user requirements for a robust target tracking technique in object-induced clutter. In the end, the user expects proper target tracking in all possible environmental conditions.

The objective is to fill this gap by designing a particle filter that performs robust in target-induced clutter. This research in this thesis aims to demonstrate that a particle filter is a suitable tracking technique that can cope with multiple clutter reflectors, even induced by the target itself.

Hence, the main objective is to find a way to suppress and eliminate the existence of target-induced clutter.

1-4-2 Scope

In order to elaborate on the scope of this thesis it is important to mention that this research and design work can be applied generically with the focus on robustness in target tracking. Hence, it is not limited to a specific field of application, like the military domain. Indeed, the application of nonlinear filters – and particle filters in particular – is suitable for various technological areas.

The focus will mainly be on relatively small targets, in order to fit the main problem. This does not mean that the chosen filtering techniques cannot handle larger or extended targets. As previously mentioned in section 1-2, the inherent focus lies on designing an adaptive and knowledge based form of processing that can perform robust under various conditions.

The following assumptions are made for the modeling and design process. First, the target induced clutter is assumed to be nonlinear. The actual filter design and models which have been developed in this thesis are based on the measurements of a radar system. However, this tracking filter is not limited to radar applications only. Moreover, this novel technique is generic applicable to several sensors and application domains.

1-4-3 Methodology

A Track Before Detect (TBD) particle filter is designed which should perform robust in object-induced clutter scenarios. TBD is a technique where tracks are directly produced based on raw sensor measurements, e.g. power or In-phase and Quadrature (IQ) data, without intermediate processing and decision making. A schematic overview is provided in figure 1-1.
Classical Kalman-based filters are designed to deal with linear problems. The advantage of a TBD particle filter over classical tracking techniques is that nonlinear clutter can be modeled very well in such a framework. The concept is that the induced clutter can be characterized in terms of object properties like size, shape, orientation and speed. This is difficult to include in a plot based classical tracker. A tracking technique that operates directly on raw data measurements cannot be built by using a Kalman filter. A Kalman filter as such calculates the state vector, while MHT is responsible for the start-up of potential false tracks based on (target-induced) clutter. Situations like these happen if the input of the tracker is not matching to what is modeled. Clarifying, the assumption is made that false alarms are independent of the existing tracks which does not hold for target-induced clutter like explained in paragraph 1-3-1. For instance, in the vicinity of certain targets, the presence of target-induced clutter (that leads to false alarms) is expected. However, this knowledge cannot be modeled properly in a classical filter. In theory, MHT could process these false alarms (and misses) correctly. Though, in order to suppress them, estimates of the states would be necessary. In conventional tracking, an additional feedback would be required in the extraction process.

Another benefit of TBD is the fact that the full received sensor data is integrated over time, which will lead to a better tracking performance especially in the case of weak targets.

Throughout this thesis a set of measurements or observations collected at the same time will be referred to as a scan.

1-5 Research question

In order to fill the knowledge gap as mentioned in the previous section, the main research question can be deduced from the research objective and reads as follows:

*Given the drawbacks of conventional linear model-based target tracking techniques, is it possible to design and demonstrate a particle filter which is able to perform robust in object-induced clutter?*

1-6 Scientific motivation

The subject of this thesis addresses the field of particle filter development which is currently a hot research topic within a broad field of application. Research on particle filters started in the 90’s of the last century and still possesses a lot of unexploited areas and applications [15].

The great potential of the research on particle filters is the fact that it is suitable for nonlinear real world scenarios in contrast with popular and widely used conventional tracking techniques as previously described in this chapter.
1-7 Thesis structure

This thesis report is structured as follows. The first chapter provides an introduction to the topic by starting with the evolution of target tracking in the first section, followed by the current requirements in the second section and a detailed problem description illustrated with example scenarios in the third section. The problem description is succeeded by the research objective, scope and methodology in the fourth section. From the research objective, the main research question is deduced in the fifth section. The closing two sections describe the scientific motivation and the thesis structure (current section).

The second chapter provides a description of the background fundamentals, starting with a section on radar since this sensor is used to provide the target measurements. The second section is dedicated to nonlinear filtering. The general characterization is handled in a separate paragraph, as well as the four common modeling steps. Concluding the Sequential Importance Resampling (SIR) filter is defined in the last paragraph.

The third chapter provides a detailed overview of the design phases that were determined; intermediate results are also provided. The following step-by-step design approach is chosen for target and model simulation with increasing degree of difficulty, which also corresponds to the four sections of this chapter:

I. Single target measured in one dimension (1-D), i.e. range only;
II. Extension of step (I.) with a 2nd clutter reflector;
III. Extension of step (II.) to a two-dimensional (2-D) measurement model, i.e. range and Doppler;
IV. Extension of step (III.) by introducing target-induced clutter, which is the final objective.

Each design phase consists of several processing steps which have been described in the underlying paragraphs of each design phase.

Concluding, the fourth chapter contains the conclusions and recommendations. The conclusions are mainly drawn based on the results presented in chapter three.
Chapter 2

Background fundamentals

This chapter will describe relevant background information and research within the scope of this thesis. The first section of this chapter will provide background knowledge on the topic of radar. After all, this sensor will be used to simulate target measurement data that will serve as input for the TBD particle filter that will be designed later on in this research. The second section will elaborate on the fundamentals of nonlinear filtering, and particle filtering in particular.

2-1 Radar

This section describes the concept and fundamentals of radar relevant for the application in this thesis. In order to remain within the scope of this research, the underlying physics are not described in this report because this would distract from the actual focus of this research as determined in paragraph 1-4-2. Hence, this section will outline how (complex) radar measurement data is modeled and defined.

In general, a radar measures the distance to a target by transmitting a pulse of RF energy with a certain pulse length $r$. When the pulse reaches the target, it will be reflected by the target which produces an echo. Depending on the Radar Cross Section (RCS) of the target, this echo will contain a certain portion of the transmitted energy. The energy will be reflected back over several and sometimes widespread angles that contribute to the fact that only part of the reflected energy is received by the radar. By measuring the delay (or round-trip travel time) between the transmission of the pulse and reception of the reflection it is possible to determine the distance or range $r$.

Besides range measurements, a radar typically measures Doppler $d$ and bearing angles $b$. At discrete intervals ($k$) the radar receives those measurements with noise on top of it. Each measurement $z_k$ consists of $N_r \times N_d \times N_b$ reflected power measurements $z_k^{i,j,l}$, where $N_r$, $N_d$, $N_b$ correspond to the number of range, Doppler and bearing cells respectively. Each resolution cell $(i,j,l)$ is centered in [16]:

$$r_{\text{min}} + \left( i - \frac{1}{2} \right) \Delta_r, \quad d_{\text{min}} + \left( j - \frac{1}{2} \right) \Delta_d, \quad b_{\text{min}} + \left( l - \frac{1}{2} \right) \Delta_b,$$

where $\Delta_r$, $\Delta_d$, $\Delta_b$ respectively correspond to the range, Doppler and bearing resolutions of the radar. $r_{\text{min}}$, $d_{\text{min}}$, $b_{\text{min}}$ are respectively the minimum measurable range, Doppler and bearing angle of the radar.
The complex measurement data of the target is defined as follows

\[ z_{A,k} = A_k h(s_k, t_k) + n(t_k), \quad k \in \mathbb{N} \]  

(2-1)

Where \( t_k \in \mathbb{R} \) is time and

\[ A_k = \tilde{A}_k e^{i\phi_k}, \quad \phi_k \in (0, 2\pi) \]  

(2-2)

Is the complex amplitude of the target and \( \tilde{A}_k \) is the unknown modulus of \( A_k \). Furthermore, \( \phi_k \) is the phase and \( h(s_k, t_k) \) is the reflection form, or alternatively called the Point Spread Function (PSF), which is defined for each range-Doppler-bearing cell by [14] [16]

\[ h^{ijl}(s_k, t_k) = \exp \left\{ -\frac{(r_i-r_k)^2}{2\hat{R}} - \frac{(d_i-d_k)^2}{2D} - \frac{(b_i-b_k)^2}{2B} \right\} \]  

(2-3)

\[ i = 1 \ldots N_r, \quad j = 1 \ldots N_d, \quad l = 1 \ldots N_b \text{ and } k \in \mathbb{N} \]

Where the measurement space can be related to the target space by the following three parameters:

**Apparent target range**

\[ r = \sqrt{x^2 + y^2} \]  

(2-4)

**Apparent target Doppler**

\[ d = \dot{r} = \frac{xx + yy}{\sqrt{x^2 + y^2}} \]  

(2-5)

**Target bearing**

\[ b = \arctan \left( \frac{y}{x} \right) \]  

(2-6)

Where \( x \) and \( y \) describe the positions in a two-dimensional Cartesian coordinate system and \( \dot{x} \) and \( \dot{y} \) describe the velocities in the same system.

The reflection form describes the target signal amplitude in the cells surrounding the target. \( \hat{R}, D \) and \( B \) are constants related to the size of a range, Doppler and bearing cell.

The noise \( n(t_k) \) is defined as

\[ n(t_k) = n_i(t_k) + in_q(t_k) \]  

(2-7)

Which is complex Gaussian noise, where \( n_i(t_k) \) and \( n_q(t_k) \) are independent, zero-mean white Gaussian with variance \( \sigma_n^2 \).

For plotting purposes, later on in this thesis, the power measurement for each range-Doppler-bearing cell is defined as:

\[ z_k^{i,j,l} = |z_{A,k}^{i,j,l}|^2, \quad k \in \mathbb{N} \]  

(2-8)

At this point it has been outlined how a target is detected by a radar from a noisy environment and how the pulse form \( h(s_k, t_k) \) is defined as a part of the noisy measurement. The noisy measurement \( z_{A,k} \) of equation (2-1) will serve as an input for the nonlinear filter.
2-2 Nonlinear filtering

After having specified the necessary information regarding the radar measurements, this section describes the background of nonlinear filtering and paves the way for a more detailed description of the specific techniques used in this thesis. This section will be subdivided in five paragraphs, starting with a generic characterization in the first paragraph. The following paragraphs will describe the dynamic model, measurement model, prediction stage and update stage respectively. The concluding paragraph will define the algorithm for the SIR filter.

2-2-1 Generic characterization

As stated in the first chapter, the emphasis of this thesis lies within the field of nonlinear filtering, in particular a particle filtering will be implemented later on. Within nonlinear filtering, the goal is to successively estimate the state of a dynamic system by processing series of noisy measurements that describe the system of interest [17]. A state-space method is used to model the system dynamics and measurement process, which means that difference equations are used to model system evolution over time and measurements become available at discrete time slots.

In order to apply the state-space model, the state vector is used to describe the system at predefined discrete times. This vector consists of all important information necessary to define the future behavior of the system. For instance, in case of the tracking problems relevant for this thesis, the information in this vector is related to the target kinematic properties, like speed and relative position with respect to the sensor. On the other hand, the measurement vector contains noisy observations which are related to the state vector.

At least two models are required in order to draw conclusion from a dynamic system: i) the dynamic or system model which describes the state evolution over time and ii) the measurements model that provides a relation between the noise measurements and the corresponding states. Both models are assumed to be probabilistic. The combination of a probabilistic state-space approach and the constraint that information is updated upon arrival of new measurements defines the recursive Bayesian approach [17].

In order to dynamically estimate the state within the Bayesian approach it is crucial to construct the posterior Probability Density Function (PDF) of the state, based on the received set of measurements. When either one of the models (system of measurement) happens to be nonlinear, the posterior PDF will be non-Gaussian. The PDF may be considered the complete solution to the estimation problem, since it contains all available statistical information. Therefore, it may provide an optimal estimate of the state, as well as a measure of the estimates accuracy. In the recursive filtering approach the received measurements are processed for each sequence individually instead of as a batch. The advantage of this method is that it is not necessary to store the entire data set or reprocess current data as new observations come available. So, essentially a filter like this consists of two main steps, namely a prediction and update stage. The prediction stage uses the system model in order to predict the PDF of the state ahead, from one observation timeslot to the next.

In general, the state will suffer from unknown disturbances – to be simulated as random noise – which usually distorts and widens the state PDF. The most recent measurements are used in the update step with the intention of modifying (normally squeeze) the prediction PDF. This last step is performed using Bayes theorem, which acts as the tool for updating information regarding the targets state and thereby taking into account additional information from new measured data.
2-2-2 Dynamic model

In order to outline the nonlinear filtering problem, the target state vector \( s_k \in \mathbb{R}^{n_x} \) is introduced, where \( n_x \) denotes the dimension of the vector, \( \mathbb{R} \) is a set of real numbers, \( k \in \mathbb{N} \) is the time step and \( \mathbb{N} \) describes the set of natural numbers. Since this concerns a discrete time estimation problem, the state vector (i.e. the targets state) is assumed to evolve according the following stochastic discrete-time system model [13] [17]:

\[
 s_k = f_{k-1}(s_{k-1}, v_{k-1}) 
\]  

(2-9)

Where \( f_{k-1} \) is the system dynamics or transition function [13] [14]. This function is possibly nonlinear and parameterized by the target state \( s_{k-1} \) and the process noise \( v_{k-1} \) which is assumed to be standard white Gaussian noise independent of previous and current states. The process noise accommodates modeling errors or any unexpected disturbances within the system or target dynamics model, its PDF is assumed to be known.

2-2-3 Measurement model

As mentioned in paragraph 2-2-1, measurements become available at discrete times. For instance, in the case of using a radar as the sensor for target tracking, it will provide a set of measurements during each scan. The framework for processing these batches of measurements is captured within the measurement model, which is leading to the aim of nonlinear filtering: recursively estimate \( s_k \) from the measurements \( z_k \in \mathbb{R}^{n_z} \). The measurements (or observations) collected by the sensor are related to the target state by the following equation:

\[
 z_k = h_k(s_k, m_k) 
\]  

(2-10)

where \( h_k \) is called the measurement function and \( m_k \) the measurement noise which is assumed to be another zero mean, white-noise sequence with known PDF, independent of previous, current and future states and the process noise. The complete available information at time step \( k \) is defined as the total set of measurements

\[
 Z_k = \{ z_1, ..., z_k \} 
\]  

(2-11)

Furthermore, it is assumed that the initial target state vector has a known PDF \( p(x_0) \) and it is independent of both \( v_{k-1} \) and \( m_k \).

Next, it is essential to find estimates of the current state \( s_k \), based on \( Z_k \). According to the Bayesian approach, the objective is to construct the PDF of \( s_k \) based on the complete set of measurements \( Z_k \). Specifically, it is required to construct the posterior PDF \( p(s_k | Z_k) \), which can be achieved recursively in two steps: prediction and update. These two steps will be described in the following two paragraphs.
2-2-4 Prediction stage

Suppose that the required PDF at time step $k – 1$ is available: $p(s_{k-1}|Z_{k-1})$. In this case the prediction stage means applying the system model (2-9) and find an approximation of the prediction PDF of the state a time instance $k$ – often referred to as the (dynamic) prior PDF or prediction density – by using the Chapman-Kolmogorov equation [17] [16]:

$$p(s_k|Z_{k-1}) = \int p(s_k|s_{k-1}) p(s_{k-1}|Z_{k-1}) ds_{k-1} \quad (2-12)$$

Following, the derivation of equation (2-12) will be provided. Therefore, Bayes’ rule is used. Usually, this rule is written in a form with two events A and B, like the multiplication rule [18]:

$$P(A \cap B) = P(A|B) \cdot P(B) \quad (2-13)$$

However, for this application the additional event C is introduced, which leads to [19]:

$$P(A \cap B|C) = P(A|B, C) \cdot P(B|C) \quad (2-14)$$

The last step is to rewrite equation (2-14) in such a way that only two (instead of three) operators remain on the left-hand side of the equation:

$$P(A|C) = \sum_B P(A \cap B|C) \quad (2-15)$$

The form of (2-15) is suitable to start from and eventually attain the prediction density, i.e. $p(s_k|Z_{k-1})$. The left-hand side of (2-15) is replaced by the prediction density and the summation on the right-hand side is transformed into an integral, which leads to the following expression:

$$p(s_k|Z_{k-1}) = \int p(s_k, s_{k-1}|Z_{k-1}) ds_{k-1} \quad (2-16)$$

Subsequent, equation (2-14) is applied, since the right-hand side of this equation is equivalent to the integrand. The prediction density can then also be written as:

$$p(s_k|Z_{k-1}) = \int p(s_k|s_{k-1}, Z_{k-1}) p(s_{k-1}|Z_{k-1}) ds_{k-1} \quad (2-17)$$

Since the system model of (2-9) is applied for finding the prior density in (2-12), this means that the following property is used

$$p(s_k|s_{k-1}, Z_{k-1}) = p(s_k|s_{k-1}) \quad (2-18)$$

This property is valid because the system model concerns a Markov process of order one, which implicates that each following state only depends on the directly preceding state [20]. In other words: the dynamics do not depend on the measurement.

The state evolution (sometimes called the transitional property or transition kernel), $p(s_k|s_{k-1})$, is probabilistic modeled and defined by the system equation and the known statistics of the process noise $v_{k-1}$. So, finally (2-17) is equivalent to (2-12).
2-2-5 Update stage

The succeeding step after prediction is update, which will be carried out at time step $k$ when a measurement $z_k$ becomes available. This stage comprises an update of the prior density $p(s_k|Z_{k-1})$, via Bayes' theorem, that results in the posterior density [17]:

$$p(s_k|Z_k) = p(s_k|z_k, Z_{k-1}) = \frac{p(z_k|s_k, Z_{k-1})p(s_k|Z_{k-1})}{p(z_k|Z_{k-1})} = \frac{p(z_k|s_k)p(s_k|Z_{k-1})}{p(z_k|Z_{k-1})}$$

(2-19)

where the denominator is the Bayes normalizing constant

$$p(z_k|Z_{k-1}) = \int p(z_k|s_k)p(s_k|Z_{k-1})ds_k$$

(2-20)

which depends on the likelihood function $p(z_k|s_k)$. This likelihood is defined by the measurement model in paragraph 2-2-3. Within the update stage, equation (2-19), the current measurement $z_k$ is used to update the prior density $p(s_k|Z_{k-1})$, in order to obtain the essential posterior density $p(s_k|Z_k)$ that corresponds to the current state.
2-2-6 SIR filter

At this point all the necessary background information is available for describing the algorithm for the Sequential Importance Resampling (SIR) filter, introduced under the name bootstrap filter by Gordon et al. in 1993 [13].

Assume a set of random samples \( \{ s_{k-1}(i): i = 1, ..., N_p \} \) drawn from the PDF \( p(s_{k-1}|Z_{k-1}) \), where \( N_p \) is the number of particles in the filter. The SIR filter algorithm is used to propagate and update these samples in order to acquire the following set of new samples \( \{ s_k(i): i = 1, ..., N_p \} \) which are by approximation distributed as \( p(s_k|Z_k) \). Hence, the SIR filter is an approximate simulation of the prior PDF \( p(s_k|Z_{k-1}) \) as defined in equation (2-12) and the posterior density \( p(s_k|Z_k) \) as defined in equation (2-19).

2-2-6-1 Prediction step

The prediction step of the algorithm is described as follows. Each sample will pass through the system model in order to obtain samples from the prior PDF at time step \( k \):

\[
\hat{s}_k(i) = f_{k-1}[s_{k-1}(i), v_{k-1}(i)]
\]  

(2-21)

Where \( v_{k-1}(i) \) is a sample from the process noise PDF \( p(v_k) \).

2-2-6-2 Update step

Succeeding, the update step is defined as follows. When the current measurement \( z_k \) is received, the likelihood of each prior sample is evaluated. Subsequent, a normalized weight for each sample is obtained:

\[
w_i = \frac{p(z_k|\hat{s}_k(i))}{\sum_{j=1}^{N_p} p(z_k|\hat{s}_k(j))}
\]

(2-22)

Hence, determine a discrete distribution over \( \{ \hat{s}_k(i): i = 1, ..., N_p \} \) with the probability \( w_i \) assigned to element \( i \).

2-2-6-3 Resample

The final step is to resample \( N_p \) times from the discrete distribution in order to generate the samples \( \{ s_k(i): i = 1, ..., N_p \} \) so that for any \( j \) it holds that

\[
P(s_k(j) = \hat{s}_k(i)) = w_i
\]

(2-23)
Designing the particle filter starts with defining a setup for the model environment, including sensor characteristics, noise characteristics and the initiation and update of the particle cloud from scan to scan.

Since the filter which is designed for this thesis consists of a novel approach, the code and algorithm building process is started from scratch. In order to answer the main research question, it is the goal to develop a TBD PF which is able to track a target in object-induced clutter. Therefore, the following step-by-step design approach is chosen for target and model simulation with increasing degree of difficulty:

V. Single target measured in one dimension (1-D), i.e. range only;
VI. Extension of step (I.) with an additional clutter reflector;
VII. Extension of step (II.) to a two-dimensional (2-D) measurement model, i.e. range + Doppler;
VIII. Extension of step (III.) by introducing target-induced clutter, which is the final objective.

The following sections will provide a more detailed description of the above-mentioned design steps.

3-1 One dimension single target

This initial step of building the particle filter from scratch has the goal to obtain a model in which a single point target could be detected in a noisy environment, based on radar range measurements only. Therefore, a TBD PF is designed and implemented as the tracking technique.

3-1-1 Target simulation

Description of this step starts with defining the target. In this case a point target is chosen in order to simplify target characteristics and maintain the focus on achieving a functional TBD PF as a starting point. The following target properties are assigned in order to define the target:
The number of targets $N_t$, Signal-to-Noise Ratio (SNR), speed $v$ and initial position at first scan. To model the target dynamics a constant velocity model is used, which means that the target attains a constant velocity over the measurement interval. Typical target setting could be as described in table 3-1, where $x_{MT,k=1}$ is the position of the Main Target (MT) during the first scan at $k = 1$.

**Table 3-1**: Example of target settings for $N_t = 1$.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Main target</th>
</tr>
</thead>
<tbody>
<tr>
<td>$SNR$</td>
<td>13 [dB]</td>
</tr>
<tr>
<td>$v$</td>
<td>10 [m/s]</td>
</tr>
<tr>
<td>$x_{MT,k=1}$</td>
<td>5 [m]</td>
</tr>
</tbody>
</table>

### 3-1-2 Sensor characteristics

As stated before, a radar sensor is used in order to measure the target range. Therefore, the following radar characteristics are defined in order to properly model this sensor. Besides, the number of scans (or measurement intervals) $N_{sc}$ and the measurement interval (or sampling time) $T$ in seconds between two successive scans is determined. In this chapter $T = 1$. These two parameters are used in order to build the time vector $t \in \mathbb{R}^{N_{sc}}$ that contains the time at which each scan is carried out.

The parameter $N_r$ determines the number of range cells and together with the pulse length $\tau$, the range dimension vector $x_r \in \mathbb{R}^{N_r}$ is formed which contains the range cells of the radar system. The maximum element of $x$ will be denoted as $R$, the maximum measurable range of the radar. The sensor parameters are summarized in table 3-2.

**Table 3-2**: Sensor parameters for the 1-D single target design phase.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$N_{sc}$</td>
<td>Total number of scans (or measurements).</td>
</tr>
<tr>
<td>$k$</td>
<td>Discrete-time index, which indicates the current scan number.</td>
</tr>
<tr>
<td>$T$ [s]</td>
<td>Measurement interval (sampling time).</td>
</tr>
<tr>
<td>$t \in \mathbb{R}^{N_{sc}}$ [s]</td>
<td>Time vector containing the time-index corresponding to $k$.</td>
</tr>
<tr>
<td>$N_r$ [1]</td>
<td>Number or range cells.</td>
</tr>
<tr>
<td>$\tau$ [m]</td>
<td>Pulse length (or width) of the transmitted radar RF pulse.</td>
</tr>
<tr>
<td>$x \in \mathbb{R}^{N_r}$ [m]</td>
<td>Range dimension vector, which contains the range of the center of each range cell.</td>
</tr>
<tr>
<td>$R$ [m]</td>
<td>Maximum measurable range of the radar, equal to the maximum element in $x$.</td>
</tr>
</tbody>
</table>

Table 3-3 provides an example of sensor parameter settings.

**Table 3-3**: Example of sensor parameter settings for $N_t = 1$.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$N_{sc}$</td>
<td>10</td>
</tr>
<tr>
<td>$T$ [s]</td>
<td>1</td>
</tr>
<tr>
<td>$N_r$ [1]</td>
<td>100</td>
</tr>
<tr>
<td>$\tau$ [m]</td>
<td>1</td>
</tr>
<tr>
<td>$R$ [m]</td>
<td>100</td>
</tr>
</tbody>
</table>
Figure 3-1 illustrates the trajectory of a single target over an interval corresponding to the target settings in Table 3-1 and the sensor settings in Table 3-2.

**Figure 3-1**: Target trajectory for single reflector, \( N_t = 1 \), during a scan interval of 10 scans. The sensor range \( R = 100 \text{ m} \). The target is indicated by a right-pointing triangle, where the color red corresponds to the first scan, orange to the 2\(^{nd}\) scan, and so on until the 10\(^{th}\) scan. A dotted black line indicates the trajectory from scan to scan.

From this figure, it can be observed that the target is moving with a constant velocity of \( v = 10 \text{ m/s} \). Furthermore, the starting position during the first scan is \( x_{MT,k=1} = 5 \text{ m} \).

### 3-1-3 Particle cloud initialization

At this point the target as well as the sensor have been defined. The following steps consist of implementing the dynamic and measurement model as described in chapter 2. For the system dynamics, the target state vector is implemented in the PF setting as follows. In order to detect the target at the first scan, certain assumptions on the predicted target position have to be done. After all, no measurements have been carried out yet. It is valid to assume that the target range lies within the sensor range. Therefore, the initial state vector, i.e. the initial position particles, are drawn from the standard normal distribution with \( \mu_{pos} = \frac{R}{2} \) and \( \sigma_{pos} = \frac{R}{3} \). These settings assure a proper spreading of the particles over the radar range interval \( \{0 \ldots 100\} \), some particles will obviously lie outside this interval. The number of particles \( N_p \) is initially set equal to or greater than the \( N_t \).
Similarly, the initial particle velocities will be drawn from a standard normal distribution with $\mu_{vel} = 0$ and $\sigma_{vel} = 2v$. Consequently, the process starts with the following initial particle cloud

$$s_k = [p_k \ p_k^\prime]^T : \ 2 \times N_p$$

(3-1)

Where $p_k$ is the column vector with length $N_p$ containing the positions (or ranges) in meters of the particles and $p_k^\prime$ is the column vector of the same length containing the velocities in m/s of the particles.

![Initial particles, Np = 10240](image)

**Figure 3-2**: Initial particle cloud for $N_p = 10240$, $\mu_{pos} = R/2$ and $\sigma_{pos} = R/3$, $\mu_{vel} = 0$ and $\sigma_{vel} = 2v$. The blue points correspond to the particles and the red line indicates the range limitations of the radar. The black line indicates the true target speed.

Figure 3-2 shows the initial particle cloud drawn from the standard normal distribution with $\mu$ and $\sigma$ for position and velocity chosen as stated in the beginning of this paragraph. From this figure, it can be observed that along the target speed of 10 m/s the particles cover the complete range interval. Obviously, there is a tradeoff between setting increasing $\sigma$ which will relatively put more particles towards the range limits. However, the downside of this would be that the absolute number of particles within the range interval will decrease, since more particles will lie outside the limits.
3-1-4 Measurement model

The process described hereafter is carried out in a loop for all scans at time instance \( k \). The first step is to compute the pulse form of equation (2-3) for this 1-D, i.e. range only, case for each radar scan at instance \( k \):

\[
h(s_k, t_k) = \exp \left\{ -\frac{(x - x_{MT,k})^2}{2\tau} \right\}: \quad N_r \times 1
\] (3-2)

Where \( x \in \mathbb{R}^{N_r} \) is the range dimension vector as mentioned earlier, \( x_{MT,k} \) is the target true range at time instance \( k \) and \( \tau \) is the pulse length. The pulse form \( h(s_k, t_k) \) is a vector with \( N_r \) elements. Figure 3-3 shows the PSF \( h(s_k, t_k) \) at the first scan (i.e. \( k = 1 \)). From this figure, it can be observed that the target is present at a range of 5 m.

![PSF in range @ k = 1](image)

**Figure 3-3:** PSF in range at \( k = 1 \), corresponding to the target properties as provided in table 3-1.

In order to obtain the noisy measurement \( z_{Ak} \), i.e. equation (2-1), two additional terms are needed besides the already obtained pulse form. These terms are the complex random amplitude \( A_k \) – equation (2-2) – and additive complex Gaussian noise \( n(t_k) \), see equation (2-7). Generating the complex amplitude starts with determining its standard deviation

\[
\sigma_A = \sqrt{\sigma_n^2 \cdot 10^{SNR/10}}
\] (3-3)

Where \( \sigma_n^2 \) is the variance of the noise and the \( SNR \) is a target parameter setting. The next step is to compute the complete complex amplitude of the pulse form

\[
A_k = \tilde{A}_k e^{i\phi_k} = \frac{\sigma_A}{\sqrt{2}} \cdot \{n_r(t_k) + in_q(t_k)\}
\] (3-4)

The last step in order to obtain the noisy measurement is adding the complex Gaussian noise sequence

\[

\]
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\[ n(t_k) = \sqrt{\sigma_n^2} \left\{ n_i(t_k) + i n_q(t_k) \right\} \]  

(3-5)

Where \( n(t_k) \) is a vector with length \( N_r \). Finally, the noisy measurement equals

\[ z_{A,k} = A_k h(s_k, t_k) + n(t_k) : \quad N_r \times 1 \]  

(3-6)

Which is a column vector containing \( N_r \) complex entries. Similar to the previous plot of the PSF, figure 3-4 shows a plot of the noisy measurement \( |z_{A,k}|^2 \) in dB.

**Figure 3-4**: Noisy measurement in range at \( k = 1 \). From this plot the target can be observed at \( x_{MT,k=1} = 5 \text{ m} \). However, the peak is less obvious compared to the noiseless PSF.

From this plot the target can still be observed as present at \( x_{MT,k=1} = 5 \text{ m} \), however the peak is obviously less clear compared to the noiseless PSF.

**3-1-5 TBD PF**

The succeeding step after the measurement process is to update the initial particle cloud and map them conditioned on the measurement. The goal of this step is to find the likelihood of the previous described measurement, given the particles. As stated before the number of particles \( N_p \) is set equal to or larger than \( N_r \). In order to calculate the likelihood, the following computations are carried out \( N_p \) times in a loop.

The first step is to calculate the pulse form again, but this time given the particles, instead of given the true target ranges. So, the pulse form becomes
3-1 One dimension single target

\[ h_p(s_k, t_k) = \exp \left( -\frac{(x - p^i_k)}{2\tau}^2 \right) : \ N_r \times 1 \] (3-7)

Where \( p^i_k : i = 1, \ldots, N_p \) is the possible target range extracted from each particle, \( \tau \) is the pulse length and \( k \) is a discrete time index that corresponds to the current radar scan. Note that the subscript \( p \) in \( h_p \) means that this PSF is calculated for each particle separately. Besides, the vector \( h_p \) has the same dimensions as \( h(s_k, t_k) \).

The following stage is to compute the covariance of the measurement conditioned on the particles. The covariance matrix is given by

\[ Q_z = \sigma_{Ak}^2 h_p h_p^H + \sigma_n^2 I : \ N_r \times N_r \] (3-8)

The next phase is to compute the log-likelihood of the measurement given the target range, which is defined by

\[ \loglik^i_k = \ln \left( \frac{1}{\det(\pi Q_z)} \right) - z_{Ak}^H Q_z^{-1} z_{Ak} \] (3-9)

As stated before, the \( \loglik^i_k : i = 1, \ldots, N_p \) computation is carried out \( N_p \) times, therefore the collection of all log-likelihoods for a single scan at time instance \( k \) will be collected in a row vector with \( N_p \) entries as

\[ \loglik_k = \left[ \loglik^1_k \ldots \loglik^{N_p}_k \right] : 1 \times N_p \] (3-10)

Succeeding, the obtained likelihoods will be normalized

\[ w_k = \frac{\loglik_k}{\text{sum}(\loglik_k)} : 1 \times N_p \] (3-11)

3-1-6 Resampling

After having obtained the normalized likelihoods, the next step is to resample the target states \( s_k \) and their corresponding weights \( w_k \). The objective of resampling is to modify the weighted approximate density of the target states to an unweighted density by eliminating particles having low importance weights and by multiplying particles having high importance weights [21]. The algorithm for resampling can be found in appendix C. After resampling, the posterior cloud (resampled particles) together with their corresponding resampled weights are obtained, which are respectively \( s_{k,r} \) and \( w_{k,r} \) with the same dimensions as the original data before resampling.

Figure 3-5 shows the resampled version of the initial particle cloud presented in figure 3-2. Obviously, the TBD PF step is carried out before the resampled particles were calculated. From this plot, it can be observed that the particles are concentrated over a far more smaller area compared to the initial cloud which is spread over the complete radar range interval and even a bit outside. The resampled particles are concentrated around the target position, besides the number of particles seems to be a lot less than \( N_p \), however this is misleading since a number of particles have obtained similar position and velocity values. This results in the fact that various particles are plotted on top of each other which can be a result of resampling.
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Figure 3-5: Resampled version of the particle cloud of figure 3-2 presented during the first scan at \( k = 1 \). The blue points correspond to the resampled particles. The red right-pointing triangle indicates the true position and velocity of the target. The red asterisk indicates the mean of the particle cloud which is the estimated position and velocity of the target.

3-1-7 Target tracking

The succeeding computations in order to perform the actual target tracking, i.e. determining target range per scan, are performed based on the resampled particle cloud \( s_{k,r} \). The remaining computation basically consists of finding the mean of \( s_{k,r} \) calculated over the rows, which will provide a mean for target range and velocity respectively.

An example of target tracking based on the resampled particles of the first scan is shown in figure 3-5. The red asterisk corresponds to the mean of the particle cloud, that is the mean of all the particle positions and the mean of all particle velocities, plotted against each other.

3-1-8 PF prediction

In order to model the target dynamics, the nearly constant velocity model is used [22] [23]. This is a widely-used model to describe the position and velocity in a Cartesian coordinate system. Besides, a random generator is used to model the unknown target complex amplitude \( \tilde{A}_k \), see equation (2-2). Furthermore, the model has an additive process noise term \( v_k \) which is assumed to be standard white Gaussian noise with covariance \( G \). Given those assumptions the state-space model for the corresponding updated target state vector is
\[ s_{k+1} = F s_k + G v_k =: 2 \times N_p \]  
(3-12)

Where \( F \) is a system transition matrix with constant measurement interval \( T \)

\[ F = \begin{bmatrix} 1 & T \\ 0 & 1 \end{bmatrix} \]  
(3-13)

And \( G \) is given by

\[ GG^T = \begin{bmatrix} \frac{T^3}{3} & \frac{T^2}{2} \\ \frac{T^2}{2} & T \end{bmatrix} \]  
(3-14)

In fact, \( s_{k+1} \) is the updated particle cloud which is used at the beginning of the next loop during the next scan at time instance \( k + 1 \). From this point the processes beginning at equation (3-2) will be repeated until all scans have been finished.

Figure 3-6 illustrates the resampled particle cloud from scan to scan. From this plot, it can be observed that the target is steady in track over all scans. The spread of the particles over the velocity is significantly larger for the first scan, which can be explained by the fact that during this modeling step only range is measured by the radar and not velocity.

![Particles during trajectory. N_t = 1, N_p = 10240, scan 10 | 10.](image)

**Figure 3-6:** Resampled particles are plotted for each scan at time instance \( k \). The particles are plotted as points, with a different color for each scan, beginning with red for scan one, orange for scan two, etc. The right-pointing triangles indicate the target true position and velocity, again with the same distinct color per scan. The dotted black line represents the target trajectory between two successive scans.

Figure 3-7 shows a plot similar to figure 3-5, but this time the plot corresponds to scan ten at time index \( k = 10 \) (instead of scan one). From this plot, it can be observed that the target is tracked properly, since the error in range is less than 0.4 m and the error in velocity is about 0.6 m/s. Exact errors, for both position and velocity are listed in table 3-4.
Figure 3-7: Resampled particles during the tenth scan at $k = 10$, corresponding to the rightmost cloud in figure 3-6. The blue points correspond to the resampled particles. The red right-pointing triangle indicates the true position and velocity of the target. The red asterisk indicates the mean of the particle cloud which is the estimated position and velocity of the target.

Figure 3-8 shows a graph of the position estimation error over the complete measurement interval with ten successive scans. From this plot, it can be observed as well that the target is tracked properly, since the errors are relatively small. At the end of this paragraph, additional comments on the errors will be provided.

Figure 3-8: Position estimation error [m] plotted for each time step $k$. The target estimate (asterisk) correspond to the mean of the particle cloud for each scan.
Figure 3-9 provides a similar graph as the previous plot, but now the velocity error is plotted over the same measurement interval. Again, it can be observed that the errors are relatively small.

![Velocity estimation error (average: 1.2965 m/s)](image)

**Figure 3-9:** Velocity error \([m/s]\) plotted for each time step \(k\). The target estimate (asterisk) correspond to the mean of the particle cloud for each scan.

The table below contains the estimation errors for both position and velocity during each scan. From this table, it can be concluded that the TBD PF is capable of successfully tracking a single target based on a 1-D, i.e. range only, measurement environment since the errors are relatively small and steady. The velocity error during the first scan is significantly larger compared to the succeeding velocity errors. This can be explained by the fact that the measurement is only carried out by using a point spread function in range. During the first scan, there is no velocity information available. During the succeeding scans the particle cloud has been updated using the likelihood ratios of the previous tracking results which results in a more accurate velocity error calculation.

**Table 3-4:** Absolute position and velocity estimation errors for ten successive scans corresponding to the 1-D single target case with the parameters presented in paragraph 3-1-1, rounded to two decimals.

<table>
<thead>
<tr>
<th>Scan (time index (k))</th>
<th>Position error [m]</th>
<th>Velocity error [m/s]</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.08</td>
<td>9.88</td>
</tr>
<tr>
<td>2</td>
<td>0.06</td>
<td>0.19</td>
</tr>
<tr>
<td>3</td>
<td>0.25</td>
<td>0.30</td>
</tr>
<tr>
<td>4</td>
<td>0.00</td>
<td>0.67</td>
</tr>
<tr>
<td>5</td>
<td>0.04</td>
<td>0.07</td>
</tr>
<tr>
<td>6</td>
<td>0.03</td>
<td>0.02</td>
</tr>
<tr>
<td>7</td>
<td>0.07</td>
<td>0.11</td>
</tr>
<tr>
<td>8</td>
<td>0.38</td>
<td>0.66</td>
</tr>
<tr>
<td>9</td>
<td>0.20</td>
<td>0.47</td>
</tr>
<tr>
<td>10</td>
<td>0.37</td>
<td>0.60</td>
</tr>
<tr>
<td><strong>Average</strong></td>
<td><strong>0.15</strong></td>
<td><strong>1.30</strong></td>
</tr>
</tbody>
</table>
3-2 Extension with additional clutter reflector

The previous section has shown a TBD PF implementation for tracking a single target based on 1-D measurements, i.e. range only. This section will extend the above steps by adding an additional reflector that moves independent from the main target. Since not all stages (which correspond to the paragraphs of the previous section) will change, only the extended stages will be mentioned in this section.

3-2-1 Target simulation

The target simulation will be extended with an additional reflector, typically this could be employed as in table 3-5.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Main target</th>
<th>2nd reflector</th>
</tr>
</thead>
<tbody>
<tr>
<td>SNR</td>
<td>13 [dB]</td>
<td>12 [dB]</td>
</tr>
<tr>
<td>v</td>
<td>10 [m/s]</td>
<td>10 [m/s]</td>
</tr>
<tr>
<td>$x_{r^#,k=1}$</td>
<td>10 [m]</td>
<td>5 [m]</td>
</tr>
</tbody>
</table>

In this table $x_{r^\#,k=1}$ indicates the position of the main target and the additional reflector $r2$ at the first scan when $k = 1$. In general, $r#$ will refer to the reflector number in this report, which will indicate that a certain parameter or vector has to be computed for each selected reflector including the main target to start with.

From this table, it obvious that the secondary target has a relative range of $-5 \, m$ with respect to the main target. This property is used later on.

Figure 3-10 illustrates the trajectory of a target accompanied by an additional clutter reflector over an interval corresponding to the target settings in table 3-5 and the sensor settings in table 3-3. From this plot, it can be observed that both targets have the same and constant speed over the complete scanning period. Besides, the relative range between the clutter reflector and the target is constant during all scans and equal to $-5 \, m$. 
Figure 3-10: Trajectory of the target accompanied with an additional clutter reflector. In this setting, $N_t = 2$, the trajectory covers an interval of 10 scans. The sensor range $R = 100 \text{ m}$. The target is indicated by a right-pointing triangle, where the color red corresponds to the first scan, orange to the 2nd scan, and so on until the 10th scan. A dotted black line indicates the trajectory from scan to scan. A circle indicated the additional clutter reflector, again the color corresponds to the scan similar to the target color.

3-2-2 Measurement model

Since there will be no modifications for the sensor characteristics and particle cloud initialization, the measurements model will be the succeeding phase subject to changes.

The pulse form of equation (3-2) is again computed for every scan at instance $k$. However, this time there is a pulse form for each reflector

$$h_{MT}(s_k, t_k) = \exp\left\{-\frac{(x - x_{MT,k})^2}{2\tau}\right\}: N_r \times 1 \quad (3-15)$$

Where $x \in \mathbb{R}^{N_r}$ is the range dimension vector as mentioned earlier, $\tau$ is the pulse length and $x_{MT,k}$ is the main targets true range at time instance $k$. Similarly, for the secondary reflector

$$h_{R2}(s_k, t_k) = \exp\left\{-\frac{(x - x_{R2,k})^2}{2\tau}\right\}: N_r \times 1 \quad (3-16)$$

Where $x_{R2,k}$ is the true range of the secondary reflector at time instance $k$. The total PSF is equal to the sum to the PSF for each reflector

$$h(s_k, t_k) = h_{MT}(s_k, t_k) + h_{R2}(s_k, t_k): N_r \times 1 \quad (3-17)$$
Figure 3-11 provides a plot of $h(s_k, t_k)$ at scan two when $k = 2$. From this plot, it can be observed that two reflectors are present, the main target at 15 m and the second reflector at 10 m (relative range of −5 m). These results correspond to the target setting provided in table 3-5.

In order to obtain the noisy measurement, $A_k$ (complex random amplitude) has to be determined for both reflectors separately based on the assumptions that the reflections of two reflectors are independent of each other. Generating $A_k$ starts again with determining its standard deviation

$$\sigma_{A_{r\#,k}} = \sqrt{\sigma_n^2 \cdot 10^{SNR_{r\#}/10}}$$  \hspace{1cm} (3-18)

Where $\sigma_n^2$ is the variance of the noise and the $SNR_{r\#}$ is the signal-to-noise ratio for each reflector, which will lead to a $\sigma_{AMT,k}$ and $\sigma_{AMT,k}$ for the main target and secondary reflector respectively.

The next step is to compute the complete complex amplitude of the pulse form for both reflectors as well

$$A_{r\#,k} = \frac{\sigma_{A_{r\#,k}}}{\sqrt{2}} \left\{ n_{I,r\#}(t_k) + i n_{Q,r\#}(t_k) \right\}$$  \hspace{1cm} (3-19)

The last step in order to obtain the noisy measurement is adding the complex Gaussian noise sequence which is determined as in paragraph 3-1-4.

$$n(t_k) = \frac{\sqrt{\sigma_n^2}}{\sqrt{2}} \left\{ n_I(t_k) + i n_Q(t_k) \right\} : \ N_r \times 1$$  \hspace{1cm} (3-20)
Finally, the noisy measurement equals

\[ z_{A,k} = A_{MT,k} h_{MT}(s_k, t_k) + A_{r2,k} h_{r2}(s_k, t_k) + n(t_k) : N_r \times 1 \]  

(3-21)

Which is again a column vector containing \( N_r \) complex entries.

Similar to the previous plot of the PSF, figure 3-12 shows a plot of the noisy measurement \( |z_{A,k}|^2 \) in dB. From this plot the target can still be observed as present at \( x_{MT,k=2} = 15 \text{ m} \), however the peak is obviously less clear compared to the noiseless PSF. The second reflector can also be observed, but very poor during this scan because of noise interference.

![Noisy measurement \( |Z|^2 \) in range @ k = 2, \( N_t = 2 \)]

**Figure 3-12:** Noisy measurement in range at \( k = 2 \). From this plot the target can be observed at \( x_{MT,k=2} = 15 \text{ m} \). However, the peak is less obvious compared to the noiseless PSF. The additional clutter reflector is present at \( x_{r2,k=2} = 10 \text{ m} \), but less obviously because of the noise interference.

As stated above the presence of the reflectors is not always very obvious from observing the noisy measurement. This is caused by the presence and interference of random additive white Gaussian noise, which intensity will by the definition of randomness vary from scan to scan. This is indicated by the graphs in figure 3-13 since both reflectors can be clearly observed above noise level during scan 7.
Design and results

Figure 3-13: Left plot: similar graph as in figure 3-11, but this time at \( k = 7 \). Right plot: same plot as in figure 3-12, but this time at \( k = 7 \). From the left plot, the target presence can be clearly observer at 65 m and the clutter reflector at 60 m. During this scan the noise is less influential since both reflectors can be observed clearly in the right plot of the noise measurement.

3-2-3 TBD PF

In this stage – finding the likelihoods of the measurement – similar changes have to be adopted as in the measurement modeling stage. Thus, the pulse form has to be calculated for each reflector individually

\[
h_{p,r#}(s_k, t_k) = \exp \left\{ - \frac{(x - p^i_k - x_{rel,r#k})^2}{2\tau} \right\} \quad N_p \times 1 \quad (3-22)
\]

Where \( p^i_k \): \( i = 1, \ldots, N_p \) is the possible range for each reflector extracted from each particle. In addition, the term \( x_{rel,r#k} \) has been introduced, which is the relative range of the current reflector with respect to the main target during a scan at time instance \( k \). This can be illustrated by going back to table 3-5. For these settings \( x_{rel,r2,k} \) (corresponding to the additional reflector) will be \(-5 \text{ m}\) for every \( k \), since both targets have the same speed. Obviously, for the main target will \( x_{rel,MT,k} \) always be zero.

However, if the speed of the additional clutter reflector will be different than the main target speed, as illustrated in table 3-6, the relative ranges for the clutter reflector will be different from scan to scan.

Table 3-6: Example of target and secondary reflector settings with different reflector speeds.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Main target</th>
<th>2nd reflector</th>
</tr>
</thead>
<tbody>
<tr>
<td>( SNR )</td>
<td>13 [dB]</td>
<td>12 [dB]</td>
</tr>
<tr>
<td>( v )</td>
<td>9 [m/s]</td>
<td>10 [m/s]</td>
</tr>
<tr>
<td>( x_{r#k=1} )</td>
<td>10 [m]</td>
<td>5 [m]</td>
</tr>
</tbody>
</table>

Table 3-7 provides the absolute range for both reflectors during a measurement period of five successive scans according to the settings in table 3-6.
Table 3-7: Absolute range for two reflectors during ten consecutive scans at time instance $k$, corresponding to the settings in table 3-6 where $T = 1$.

<table>
<thead>
<tr>
<th>Scan (time index $k$)</th>
<th>Main target range [m]</th>
<th>2nd Reflector range [m]</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>10</td>
<td>5</td>
</tr>
<tr>
<td>2</td>
<td>19</td>
<td>15</td>
</tr>
<tr>
<td>3</td>
<td>28</td>
<td>25</td>
</tr>
<tr>
<td>4</td>
<td>37</td>
<td>35</td>
</tr>
<tr>
<td>5</td>
<td>46</td>
<td>45</td>
</tr>
<tr>
<td>6</td>
<td>55</td>
<td>55</td>
</tr>
<tr>
<td>7</td>
<td>64</td>
<td>65</td>
</tr>
<tr>
<td>8</td>
<td>73</td>
<td>75</td>
</tr>
<tr>
<td>9</td>
<td>82</td>
<td>85</td>
</tr>
<tr>
<td>10</td>
<td>91</td>
<td>95</td>
</tr>
</tbody>
</table>

Figure 3-14 illustrates the trajectory of both reflectors, i.e. the target and the clutter reflector. It can be observed from this figure that the clutter reflector is 5 m closer (relative range is $-5$ m) to the radar compared to the target, during the first scan. Since the speed of the clutter reflector is higher, both reflectors have the same range during the 6th scan. From the seventh scan the clutter reflector has overtaken the main target, up to 4 m during the 10th scan.

![Target trajectory. $N_t = 2$, $R = 100$ m, scan 10](image)

Figure 3-14: Trajectory of the target accompanied with a clutter reflector. In this setting, $N_t = 2$, the trajectory covers an interval of 10 scans. The sensor range $R = 100$ m. The target is indicated by a right-pointing triangle, where the color red corresponds to the first scan, orange to the 2nd scan, and so on until the 10th scan. A dotted black line indicates the trajectory from scan to scan. A circle indicates the clutter reflector, again the color corresponds to the scan similar to the target color.
The table below provides the relative reflector range corresponding to the absolute ranges as depicted in table 3-7.

**Table 3-8**: Reflector range relative to range of main target for ten consecutive scans at time instance $k$, corresponding to the settings in table 3-6 where $T = 1$.

<table>
<thead>
<tr>
<th>Scan (time index $k$)</th>
<th>$x_{rel,MT,k}$ [m]</th>
<th>$x_{rel,R2,k}$ [m]</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0</td>
<td>-5</td>
</tr>
<tr>
<td>2</td>
<td>0</td>
<td>-4</td>
</tr>
<tr>
<td>3</td>
<td>0</td>
<td>-3</td>
</tr>
<tr>
<td>4</td>
<td>0</td>
<td>-2</td>
</tr>
<tr>
<td>5</td>
<td>0</td>
<td>-1</td>
</tr>
<tr>
<td>6</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>7</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>8</td>
<td>0</td>
<td>2</td>
</tr>
<tr>
<td>9</td>
<td>0</td>
<td>3</td>
</tr>
<tr>
<td>10</td>
<td>0</td>
<td>4</td>
</tr>
</tbody>
</table>

So, in fact $x_{rel} : N_t \times N_{sc}$ is a matrix and $x_{rel,r\#k}$ is an entry of it corresponding to the reflector and current time index $k$.

The following stage is to compute the covariance of the measurement conditioned on the particles. The covariance matrix is in the case of two reflectors given by

$$Q_z = \sigma_{AMT,k}^2 h_{p,MT}^H p_{,MT} + \sigma_{A_{r2,k}}^2 h_{p,r2}^H p_{,r2} + \sigma_n^2 I : N_r \times N_r$$ \hspace{1cm} (3-23)

In principle, the computation of $Q_z$ did not change, however an additional outer product of $h h^H : N_r \times N_r$ has to be computed for the clutter reflector which extends the processing time a little.

The next phase is to compute the log-likelihood of the measurement given the reflector ranges. These computations have not changed from the one-target-case.

**3-2-4 Target tracking**

Actually, the target tracking stage does not change. However, this section is used to show the tracking results based on the target settings presented in table 3-6. For these simulations, the number of particles was set to $N_p = 500$ in order to show that the TBD PF can also function with a different and lower number of particles.

Figure 3-15 it can be seen that the particle filter is able to track the main target, despite the fact that an additional clutter reflector is present in the measurement.

Figure 3-16 and figure 3-17 respectively show the position and velocity estimation error from scan to scan. The position error is steady over the interval and on average relative small, which shows that the PF is able to successfully track the main target despite the fact that it is overtaken by the clutter reflector during the scanning period. The second plot, regarding the velocity error also shows a steady pattern, except during the first scan where the filter is not able to properly detect the target speed. This is caused by the fact that no velocity likelihood can be calculated during the first scan, since velocity estimation during this scan in only based upon the initial particle cloud with random velocities.
Figure 3-15: Resampled particles are plotted for each scan at time instance $k$. The particles are plotted as points, with a different color for each scan, beginning with red for scan 1, orange for scan two, etc. The right-pointing triangles indicate the target true position and velocity, again with the same distinct color per scan. The dotted black line represents the target trajectory between two successive scans. The circle, with the same color index as the triangle, represents the clutter reflector.

Figure 3-16: Position estimation error [m] plotted for each time step $k$. The target estimate (asterisk) correspond to the mean of the particle cloud for each scan.
3-2-5 Synopsys

To conclude this section a short synopsis is of the changes with respect to the single target case is provided in table 3-9.

Table 3-9: Synopsis of changes from extension of 1-D single target case to the addition of an additional clutter reflector, specified for each processing stage.

<table>
<thead>
<tr>
<th>Processing stage</th>
<th>Extension</th>
</tr>
</thead>
<tbody>
<tr>
<td>I. Target simulation</td>
<td>From 1 target to additional reflector.</td>
</tr>
<tr>
<td>II. Sensor characteristics</td>
<td>None.</td>
</tr>
<tr>
<td>III. Particle cloud initialization</td>
<td>None.</td>
</tr>
<tr>
<td>IV. Measurement model</td>
<td>From single PSF to two PSFs with independent random complex amplitude.</td>
</tr>
<tr>
<td>VI. TBD PF</td>
<td>From single PSF to two PSFs with independent random complex amplitude. Besides, the PSF is in the TBD PF is now compensated for the relative range of the clutter reflector with respect to the main target.</td>
</tr>
<tr>
<td>VII. Resampling</td>
<td>None.</td>
</tr>
<tr>
<td>VIII. Target tracking</td>
<td>None.</td>
</tr>
<tr>
<td>IX. PF prediction</td>
<td>None.</td>
</tr>
</tbody>
</table>
3-3 Extension to 2-D measurement model

In this section the extension from the 1-D (range only) measurement model with more than one reflector towards the 2-D measurements model will be described. Specifically, 2-D means the extension from range only to range and Doppler measurements. Once more, only the relevant (i.e. modified) processing stages will be mentioned in separate paragraphs of this section. Concluding, there will be presented a synopsis with a similar table as in the previous section.

The first stage, i.e. target simulation, is left unchanged. Therefore, the next paragraph will describe the first extended stage, which concerns the sensor characteristics.

3-3-1 Target simulation

Target simulation is unchanged. In this modeling stage the same settings are used as in table 3-6, which will lead to the same target trajectory as visualized in figure 3-14.

![Target trajectory. N_t = 2, R = 100 m, scan 10 | 10.](image)

Figure 3-18: Duplicate of figure 3-14. See the latter one for a description.
3-3-2 Sensor characteristics

Table 3-10 provides the amendments for the sensor characteristics which are marked in grey.

Table 3-10: Sensor parameters for the 1-D single target modulation step.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$N_{sc}$ [1]</td>
<td>Total number of scans (or measurements).</td>
</tr>
<tr>
<td>$k$ [1]</td>
<td>Discrete-time index, which indicates the current scan number.</td>
</tr>
<tr>
<td>$T$ [s]</td>
<td>Measurement interval (sampling time).</td>
</tr>
<tr>
<td>$t \in \mathbb{R}^{N_{sc}}$ [s]</td>
<td>Time vector containing the time in seconds corresponding to $k$.</td>
</tr>
<tr>
<td>$N_r$ [1]</td>
<td>Number of range cells.</td>
</tr>
<tr>
<td>$N_d$ [1]</td>
<td>Number of Doppler cells.</td>
</tr>
<tr>
<td>$N_{rd}$ [1]</td>
<td>Number of range-Doppler cells, i.e. $N_r \times N_d$.</td>
</tr>
<tr>
<td>$\tau$ [m]</td>
<td>Pulse length (or width) of the transmitted radar RF pulse.</td>
</tr>
<tr>
<td>$x \in \mathbb{R}^{N_r}$ [m]</td>
<td>Range dimension vector, which contains the range of the center of each range cell.</td>
</tr>
<tr>
<td>$R$ [m]</td>
<td>Maximum measurable range of the radar, equal to the maximum element in $x$.</td>
</tr>
<tr>
<td>$</td>
<td>D_{max}</td>
</tr>
<tr>
<td>$D$ [m/s]</td>
<td>Size of Doppler cell, which is set to $D = 2D_{max}/N_d$.</td>
</tr>
<tr>
<td>$d \in \mathbb{R}^{N_d}$ [m/s]</td>
<td>Doppler vector which is $d = [(-D_{max} + D) (-D_{max} + 2D) \ldots D_{max}]^T : N_d \times 1$. The vector elements correspond to the center of a Doppler cell.</td>
</tr>
</tbody>
</table>

From Table 3-10 it can be concluded that additional parameters have been added in order to extend the sensor with Doppler measurement capabilities which lead to introducing a 2-D sensor space. This will have a number of consequences for the succeeding stages that will be described in the following paragraphs. The system dynamics stage will not be affected, so the first altered stage will be the measurement model, described hereafter.

Table 3-11: Example of sensor settings for $N_t = 2$ in a 2-D measurement environment, that is based on range and Doppler measurements.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$N_{sc}$ [1]</td>
<td>10</td>
</tr>
<tr>
<td>$T$ [s]</td>
<td>1</td>
</tr>
<tr>
<td>$N_r$ [1]</td>
<td>100</td>
</tr>
<tr>
<td>$\tau$ [m]</td>
<td>1</td>
</tr>
<tr>
<td>$R$ [m]</td>
<td>100</td>
</tr>
<tr>
<td>$N_d$ [1]</td>
<td>100</td>
</tr>
<tr>
<td>$</td>
<td>D_{max}</td>
</tr>
</tbody>
</table>

3-3-3 Particle cloud initialization

Initialization of the particle cloud has been changed during this step, besides there has been an important addition in the sensor characteristics, i.e. extension to the Doppler domain. This extension will be taken into account when drawing the initial particles from a random generation. In this modeling step the initialization of the initial particle cloud has been updated in such a way that no particles will be lost, since they will all be positioned within the range-Doppler interval of the radar. Instead of using the standard normal distribution, this time the
uniform distribution will be used. The interval for the uniform distribution for the position particles will be specified from \(\{0, \ldots, R\}\) and for the velocities the Doppler interval range \(\{-D_{max}, \ldots, D_{max}\}\) will be used.

This time \(N_p = 5000\) which generates the initial particle cloud as shown in figure 3-19. This plot shows that the particles are uniform distributed within the range-Doppler box. When this plot is compared to the initial particle cloud in figure 3-2 it can be seen that no particles are positioned outside the range-Doppler box, which would have been the case if a normal distribution would have been used. A detailed description of the particle cloud initialization algorithm using the uniform distribution can be found in Appendix D.

![Initial particles, Np = 5000](image)

**Figure 3-19:** Initial particle cloud for \(N_p = 5000\). A random uniform distribution is used to draw the positions and velocities. \(R\) and \(D_{max}\) are used in order to set the intervals of the distributions for position and velocity respectively. The blue points correspond to the particles and the red line indicates the range-Doppler limitations of the radar.

### 3-3-4 Measurement model

Due to the extension to the Doppler domain, the PSF is separated in a dedicated range and Doppler PSF in order to build a 2-D space. Besides, both PSFs will be calculated in a loop over the number of reflectors in such a way that the PSFs will be stacked as separate column vectors into a matrix where each column corresponds to the reflector number. The first column will correspond to the main target and the second to the additional reflector, and so on in case of more than two reflectors. The range PSF is now defined as

\[
h_{r_{\text{range}}}(s_k, t_k) = \exp \left\{ -\frac{(x - x_{r\#k})^2}{2\tau} \right\} : \quad N_r \times 1
\]  

(3-24)
Where $x \in \mathbb{R}^{N_r}$ is the range dimension vector as mentioned earlier, $\tau$ is the pulse length and $x_{r\#k}$ is true range for the current reflector at time instance $k$. Similarly, the Doppler PSF is defined as

$$h_{Dop,r\#}(s_k, t_k) = \exp\left(-\frac{(d - v_{r\#k})^2}{2D}\right): N_d \times 1$$  \hspace{1cm} (3-25)

Where $v_{r\#k}$ is the true velocity of the secondary reflector at time instance $k$ and $D$ is the size of a Doppler cell. The matrices containing the range and Doppler PSFs, will contain the column vector PSFs which are stacked next to each other. For range

$$H_{rng} = \begin{bmatrix}
  h_{rng,r1} & h_{rng,N_t} \\
  \vdots & \vdots \\
  \vdots & \vdots \\
\end{bmatrix}: N_r \times N_t$$  \hspace{1cm} (3-26)

Where the $r1$ corresponds to the first reflector (i.e. the main target) and $N_t$ corresponds to the last target. Similar for the Doppler PSFs

$$H_{Dop} = \begin{bmatrix}
  h_{Dop,r1} & h_{Dop,N_t} \\
  \vdots & \vdots \\
  \vdots & \vdots \\
\end{bmatrix}: N_d \times N_t$$  \hspace{1cm} (3-27)

Where $r1$ corresponds to the first reflector (i.e. the main target) and $N_t$ corresponds to the last reflector.

With the aim of building a 2-D range-Doppler space – and accordingly a 2-D PSF – the PSF vectors for range and Doppler have to be multiplied for each target separately

$$H_{rD,r\#} = H_{rng,r\#} H_{Dop, r\#}^T: N_r \times N_d$$  \hspace{1cm} (3-28)

Where $H_{rD,r\#}$ is the matrix containing the 2-D PSF in range and Doppler for the reflector number $r\#$.

Figure 3-20 provides a plot of the 2-D PSF during the complete scanning interval of ten scans. From this plot, it can be observed that the clutter reflector is closer to the radar at the beginning of the measurement interval, but has a higher speed and attains the same range as the main target during the sixth scan (marked by the light blue indicators). This results in a higher peak for the pulse form. From the seventh scan, the secondary reflector has overtaken the main target and has a larger range.
Figure 3-20: Two dimensional PSF in range and Doppler plotted for the complete scanning period of ten scans, corresponding to the reflector properties as provided in table 3-6 and the sensor properties of table 3-11. From this plot, it can be observed that besides the main target, a secondary clutter reflector is present. Similar to the trajectory plot in figure 3-14 the right-pointed triangles mark the main target and the circles mark the clutter reflector. Every color corresponds to a scan, with red indicating the first scan, orange the second, and so on.

Figure 3-21 shows four plots of the range-Doppler PSF during scan three under four different angles. Theoretically the peaks of the targets should be equally in height, since the target strength, i.e. $SNR$, has not taken into account yet. However, the main target peak is lower because of a rounding issue which is caused by the target speed of $9 \text{ m/s}$. Since the second reflector has a speed of $10 \text{ m/s}$, this provides an exact match of the peak height with the center of a range-Doppler cell. Therefore, this peak does result in a height of exactly one.
**Figure 3-21**: Two dimensional PSF in range and Doppler during scan three at time-index $k = 3$. The PSF is plotted in four different angles. The main target corresponds to the peak with the largest range (28 m) and the lowest Doppler (9 m/s).

However, a 2-D space is desired for plotting purposes as in a range-Doppler map, this matrix is not suitable for computing the noisy measurement since a vector is desired. Therefore, the matrix $H_{rD,r\#}$ is reshaped by extracting row by row and stacking these in a column vector:

$$h_{rD,r\#}: N_{rd} \times 1$$

(3-29)

Where $h_{rD,r\#}$ is a vector of $N_{rd}$ elements, containing the 2-D range-Doppler PSF for each reflector $r\#$. The complex random amplitude for each reflector is still computed in the same way as this was done in the previous steps, see equation (3-18) and (3-19). In this step the complex amplitude $A_{r\#,k}$ is multiplied with $h_{rD,r\#}$ and subsequently stacked in a matrix, which consists of a preliminary version of $z_{A,k}$, since the complex Gaussian noise $n(t_k)$ will be added later on. This preliminary noiseless measurement is defined as follows:

$$Z_{i,j,r\#,k,NF}^{ij} = A_{r\#,k} h_{rD,r\#} : N_t \times N_{rd}$$

(3-30)

Where $NF$ means noise free and $i = \{1 ... N_t\}$ and $j = \{1 ... N_{rd}\}$ correspond respectively to the rows and columns of the matrix. The other parameters are explained earlier.
For plotting purposes, all the \( N_t \) versions of \( Z_{A,r\#k,NF} \), corresponding to each reflector, are superposed and noise is added in order to obtain the matrix

\[
Z_{A,k} = n(t_k) + \sum_{r\#=1}^{N_t} Z_{A,r\#k,NF}
\]  

(3-31)

Where

\[
n(t_k) = \frac{\sqrt{\sigma_n^2}}{\sqrt{2}} \{ n_r(t_k) + i n_q(t_k) \} \quad : \quad N_{rd} \times 1
\]  

(3-32)

Is the complex Gaussian noise sequence, which is a vector of \( N_{rd} \) elements, instead of \( N_r \) as was the case during the previous 1-D design stages.

Figure 3-22 shows a plot of the noisy measurement during the complete measurement interval. Distinction between main target and clutter reflector is less obvious compared to the noise free PSF plots. The noise is interfering with the reflectors and the level of interference can be different from scan to scan because of the randomness in the noise and target amplitude levels.

Figure 3-22: Two dimensional noisy measurement in range and Doppler plotted for the complete scanning period of ten scans, corresponding to the reflector properties as provided in table 3-6 and the sensor properties of table 3-11. From this plot, it can be observed that besides the main target, a secondary clutter reflector is present. However, this is less obvious compared to the previous plots of the noise free PSF. Similar to the trajectory plot in figure 3-14 the right-pointed triangles mark the main target and the circles mark the clutter reflector. Every color corresponds to a scan, with red indicating the first scan, orange the second, and so on.

Figure 3-23 shows two plots from the same data as in figure 3-22, however from different angles. Especially the lower plot shows that both reflectors cannot be observed properly and apart from each other during each scan. The randomness of the noise, the fluctuation noise level and the fluctuating target amplitude intensity from scan to scan cause this distorted pattern.
Design and results

Figure 3-23: Both plots illustrate the same data as in figure 3-22, however from different angles. The upper plot provides a view along the Doppler-axis, which show that the reflectors on average over all scans reach above the noise level. The bottom plot is taken along the range-axis which shows that it is not obvious to observe both reflectors above the noise level during each scan.

The next step is to obtain the noisy measurement in vector form. Consequently, $\mathbf{Z}_{A,k,NF}$ from equation (3-31) is summed over the columns in order to sum the PSFs for each reflector. Secondly, the complex Gaussian noise is added on top of the summation. Hence, the measurement is defined as follows:

$$\mathbf{z}_{A,k} = \sum_{j=1}^{N_{rd}} \{ \mathbf{Z}_{A,k,NF} \} + \mathbf{n}(t_k) : N_{rd} \times 1 \quad (3-33)$$

Where $\mathbf{n}(t_k)$ is explained in equation (3-20). Note that the measurement vector has now significantly increased in length, which will have consequences for the processing complexity and processing time, especially in the next stage where the covariance and its inverse have to
be computed. Typical values for \( N_r \) and \( N_d \) would be \( N_t = N_d = 100 \), which would lead to the vector \( z_{A,k} \) of length 100 in the 1-D modeling step and to a \( Z_{A,k} \) of length 10000 in this 2-D step. In this case \( Q_z \) would grow from \( 100 \times 100 \) to \( 10000 \times 10000 \) matrix which has catastrophic consequences in computational complexity when calculating its inverse \( N_t \) times over all scans. Therefore, alternative numerical methods will be implemented in order to keep the algorithm work efficiently.

### 3-3-5 TBD PF

During this stage, the extension to the 2-D domain will first affect the PSF since it has to be calculated twice, both for range and Doppler. This extension is similar to the extension for the measurement model.

The PSF for range is defined as

\[
h_{\text{rng}, p, r#}(s_k, t_k) = \exp \left\{ -\frac{(x - p^i_k - x_{\text{rel}, r#}^i, k)^2}{2\tau} \right\} ; \quad N_r \times 1 \tag{3-34}
\]

Where all the parameters have been explained after equation (3-22). The actual change consists of renaming the PSF to a dedicated range PSF. Similarly, the Doppler PSF is defined as

\[
h_{\text{Dop}, p, r#}(s_k, t_k) = \exp \left\{ -\frac{(d - p^i_k - v_{\text{rel}, r#}^i, k)^2}{2D} \right\} ; \quad N_d \times 1 \tag{3-35}
\]

Where \( \dot{x}_{k,p}^i \): \( i = 1, ..., N_p \) is the possible velocity (m/s) for each reflector extracted from each particle at time instance \( k \). The term \( v_{\text{rel}, r#} \) has been introduced, which is the relative speed of the current reflector with respect to the main target. This can be illustrated by going back to table 3-6 containing an example of target settings. For this example, the main target has a constant speed of 10 m/s and the clutter reflector has a speed of 11 m/s, which means that \( v_{\text{rel}, \text{MT}} = 0 \) for the main target and \( v_{\text{rel}, \text{cl}} = 1 \) for the additional clutter reflector. Since the reflectors are simulated using a constant velocity model, the relative speeds will be constant as well and therefore be independent of the time-index \( k \).

In order to build a 2-D range-Doppler space – and accordingly a 2-D PSF – the PSF vectors for range and Doppler have to be multiplied for each target separately

\[
H_{rD, p, r#} = h_{\text{rng}, p, r#} h_{\text{Dop}, p, r#} H ; \quad N_r \times N_d \tag{3-36}
\]

Where \( H_{rD, p, r#} \) is the matrix containing the 2-D PSF in range and Doppler for reflector number \( r# \). Note that each calculation during this stage is still calculated in a loop over all particles \( N_p \) individually in order to obtain the desired loglik for each particle. Hence, the subscript \( p \) is used to indicate that a certain parameter is calculated for each particle.

However, a 2-D space is desired for plotting purposes as in a range-Doppler map, this matrix is not suitable for computing the noise measurement since a vector is needed. Therefore, the matrix \( H_{rD, p, r#} \) is reshaped by extracting row by row and stacking these in a column vector

\[
h_{rD, p, r#} ; \quad N_{rd} \times 1 \tag{3-37}
\]

Where \( h_{rD, p, r#} \) is a vector of \( N_{rd} \) elements containing the 2-D range-Doppler PSF for each reflector \( r# \) and particle.
The succeeding step is to stack \( h_{rD,p,r#} \) in a matrix where each column corresponds to a separate reflector. The assembled PSFs for all \( r# \) (i.e. reflector numbers) are stacked in the following matrix

\[
H_{rD,p} = \begin{bmatrix}
  h_{rD,p,r1} & h_{rD,p,Nr} \\
  \vdots & \vdots \\
  \vdots & \vdots \\
  h_{rD,p,Nrd} & h_{rD,p,Nt}
\end{bmatrix}_{N_{rd} \times N_t} \tag{3-38}
\]

Where the \( r1 \) corresponds to the first reflector (i.e. the main target) and \( N_t \) corresponds to the last reflector. During this design phase \( N_t = 2 \) and therefore the last reflector is the additional clutter reflector. In the following design \( N_t \) will be extended further. Hence, a generic approach is chosen from here.

The succeeding step would now be to compute the covariance of the measurement as was done before in equation (3-23), however this will result in extensive matrix inversions for the following computations. Consequently, an alternative numerical method is selected in order to calculate the \( \text{loglik} \) for each particle

\[
\text{loglik}_k = \ln \left( \frac{1}{\det(\pi Q_z)} \right) - z_{Ak}^H Q_z^{-1} z_{Ak} \tag{3-39}
\]

The aim of this numerical solution is to avoid the \( \det(\pi Q_z) \) and in particular \( Q_z^{-1} \) since this is a limiting factor for the processing time. The following two paragraphs describe the numerical methods which are selected to provide alternative methods for calculating the determinant and the inverse of \( Q_z \) respectively.

### 3-3-5-1 Matrix determinant lemma

First, an alternative is described for \( \det(\pi Q_z) \), which is found in the Matrix Determinant Lemma (MDL) and its general form is defined as follows [24]

\[
\det(UV^H + B) = \det(B)\det(V^H B^{-1} U + I_m) \tag{3-40}
\]

Where \( B \) is an invertible matrix with dimensions \( n \times n \) and \( U, V \) are \( m \times n \) matrices. \( I_m \) is the identity matrix with size \( m \times m \).

From this general form a more specific form for this application is extracted in order to calculate \( \det(\pi Q_z) \). The covariance \( Q_z \) is of the general form

\[
Q_z = CC^H + B \tag{3-41}
\]

Where the right-hand side of this expression is substituted in the left-hand side of equation (3-42) which leads to the following MDL alternative for the covariance matrix

\[
\det(CC^H + B) = \det(B)\det(C^H B^{-1} C + I_{Nt}) \tag{3-42}
\]

Where

\[
C = H_{rD,p} \Sigma: \quad N_{rd} \times N_t \tag{3-43}
\]

In this equation \( \Sigma \) is a diagonal matrix containing the complex amplitudes \( A_{r#,k} \) from equation (3-19). In fact, the matrix \( C \) is a multiplication of the range-Doppler PSF with its corresponding complex amplitude for each target. Specifically, \( \Sigma \) is defined as

\[
\Sigma = \begin{bmatrix}
  A_{k,r1} & 0 & 0 \\
  0 & \ddots & 0 \\
  0 & 0 & A_{k,Nt}
\end{bmatrix} \tag{3-44}
\]
Furthermore, explaining of equation (3-40)

\[ \mathbf{B} = \sigma_n^2 \mathbf{I}_{N_{rd}} \]  

(3-45)

At this point all the matrices are available for further specifying the MDL

\[
\det( \mathbf{C} \mathbf{C}^H + \mathbf{B} ) = \det( \sigma_n^2 \mathbf{I}_{N_{rd}} ) \det( (\sigma_n^2 \mathbf{I}_{N_{rd}})^{-1} \mathbf{C} + \mathbf{I}_{N_t} ) \\
= (\sigma_n^2)^{N_{rd}} \det( \mathbf{C}^H \mathbf{C} / \sigma_n^2 + \mathbf{I}_{N_t} ) \\
= (\sigma_n^2)^{N_{rd}-N_t} \det( \mathbf{C}^H \mathbf{C} + \sigma_n^2 \mathbf{I}_{N_t} )
\]  

(3-46)

A last step is to substitute \( \pi \) into the previous equation, since \( \det(\pi \mathbf{Q}_z) \) has to be computed. This results in the following equation

\[
\det(\pi \mathbf{Q}_z) = \det(\pi ( \mathbf{C} \mathbf{C}^H + \mathbf{B} ) ) = \pi^{N_{rd}} (\sigma_n^2)^{N_{rd}-N_t} \det( \mathbf{C}^H \mathbf{C} + \sigma_n^2 \mathbf{I}_{N_t} )
\]  

(3-47)

Note that the following matrix determinant rule has been used for this equation [25]

\[
\det(g \mathbf{G}) = g^n \det(\mathbf{G})
\]  

(3-48)

Where \( \mathbf{G} \) is an \( n \times n \) matrix.

The advantage of the MDL implementation is the fact that the determinant in this case has to be computed of a \( N_t \times N_t \) matrix. In this modeling step with two reflectors that is a \( 2 \times 2 \) matrix. Initially – before implementing the MDL – the determinant of a \( N_{rd} \times N_{rd} \) had to be computed, which is in this example a \( 10000 \times 10000 \) matrix.

This concludes the description of the MDL application. The following paragraph will describe the application of the numerical method for implementing an alternative manner for computing the inverse of \( \mathbf{Q}_z \).

### 3.3.5.2 Woodbury matrix identity

Succeeding, this subparagraph will present a second numerical alternative computation in order to avoid the calculation of the inverse of the covariance matrix \( \mathbf{Q}_z^{-1} \). Initially, this would be a matrix inversion of a \( 10000 \times 10000 \) matrix in this example, which would lead to an excessive processing load. The Matrix Inversion Lemma (MIL) or Woodbury Matrix Identity (WMI) is applied in order to reduce the computational complexity. Specifically, it will avoid calculating the inverse of \( \mathbf{Q}_z \) at all. This will be achieved by defining an alternative for the second part of equation (3-39): \( \mathbf{z}_{A,k}^H \mathbf{Q}_z^{-1} \mathbf{z}_{A,k} \).

The Woodbury matrix identity is in general form defined as [26] [25]

\[
(\mathbf{C} \mathbf{F} \mathbf{C}^H + \mathbf{B})^{-1} = \mathbf{B}^{-1} - \mathbf{B}^{-1} \mathbf{C} (\mathbf{C} \mathbf{B}^{-1} \mathbf{C} + \mathbf{F}^{-1})^{-1} \mathbf{C}^H \mathbf{B}^{-1}
\]  

(3-49)

Where the covariance matrix is defined as

\[
\mathbf{Q}_z = \mathbf{C} \mathbf{F} \mathbf{C}^H + \mathbf{B}
\]  

(3-50)

And \( \mathbf{B}, \mathbf{C} \) and \( \mathbf{F} \) are defined as follows
\[ B = \sigma_n^2 I_{N_{rd}} \] (3-51)

\[ C = H_{rD,p} \Sigma: \quad N_{rd} \times N_t \] (3-52)

\[ F = I_{N_t} \] (3-53)

Substituting \( B \) and \( F \) in equation (3-49) leads to the following expression

\[
Q_z^{-1} = \frac{I_{N_{rd}}}{\sigma_n^2} - \frac{I_{N_{rd}}}{\sigma_n^2} C \left( \frac{C^H I_{N_{rd}} C + I_{N_t}}{\sigma_n^2} \right)^{-1} \frac{C^H I_{N_{rd}}}{\sigma_n^2} 
\] (3-54)

\[
= \frac{I_{N_{rd}}}{\sigma_n^2} - \frac{C}{\sigma_n^2} \left( \frac{C^H C + I_{N_t}}{\sigma_n^2} \right)^{-1} \frac{C^H}{\sigma_n^2} 
\]

\[
= \frac{I_{N_{rd}}}{\sigma_n^2} - C \left( C^H C + \sigma_n^2 I_{N_t} \right)^{-1} \frac{C^H}{\sigma_n^2} 
\]

Note that the following property is used in order to make to last step in equation (3-49) [27]

\[
(gG)^{-1} = g^{-1}G^{-1} 
\] (3-55)

This concludes the description of the WMI. The succeeding subparagraph will emphasize on substituting both MDL and WMI into the loglik expression.

### 3-3-5-3 Numerical loglik representation

This subparagraph will combine the results of the subparagraphs 3-3-5-1 and 3-3-5-2 with the purpose of acquiring a numerical expression for the loglik. Finally, the equations (3-47) and (3-54) corresponding to the MDL and WMI respectively have to be substituted in equation (3-39) in order to numerically calculate the loglik. These steps lead to the following expression for the loglik for each particle

\[
\text{loglik}_k^i = \ln \left[ \frac{1}{\pi^{N_r}(\sigma_n^2)^{N_{rd}-N_t}} \det(C^H C + \sigma_n^2 I_{N_t}) \right] - \frac{z_{A,k}^H C (C^H C + \sigma_n^2 I_{N_t})^{-1} z_{A,k}}{\sigma_n^2} 
\]

Where \( i = 1, ..., N_p \). From this equation, it can be observed that neither \( \det(\pi Q_z) \) nor \( Q_z^{-1} \) have to be computed which lead to a significant gain in processing speed. From this point on calculations for the next processing steps will start with combining all the loglik\(^i\) in the vector \( \text{loglik}_k : 1 \times N_p \) identical as in equation (3-10).
3-3-6 Target tracking

In principle target tracking, has not changed, therefore this paragraph is only used in order to show plots regarding this stage.

From figure 3-24 it can be observed that the particles during each scan are concentrated around the target and not around the clutter reflector which indicated that the target tracking process is working properly and that clutter is suppressed.

![Figure 3-24: Resampled particles are plotted for each scan at time instance \( k \). The particles are plotted as points, with a different color for each scan, beginning with red for scan 1, orange for scan two, etc. The right-pointing triangles indicate the target true position and velocity, again with the same distinct color per scan. The dotted black line represents the target trajectory between two successive scans. The circle, with the same color index as the triangle, represents the clutter reflector.](image)

Figure 3-25 shows that the position estimation error is relatively small over the complete scanning period. The error during the fourth scan is larger than during the other scans. This can be explained by the fact that the TBB PF is implemented as a Sequential Importance Resampling (SIR) filter which has the drawback that it is subject to random influences during the resampling stage. This can lead to not efficiently re-weighting particles.

Figure 3-26 shows the velocity estimation error over the complete scanning interval. This plot shows a steady and relatively low estimation error over the complete interval.
Figure 3-25: Position estimation error [m] plotted for each time step $k$. The target estimate (asterisk) correspond to the mean of the particle cloud for each scan.

Figure 3-26: Velocity error [m/s] plotted for each time step $k$. The target estimate (asterisk) correspond to the mean of the particle cloud for each scan.
3-3-7 Synopsis

To conclude this section a short synopsis is provided of the changes with respect to the single target case is provided in table 3-9.

Table 3-12: Synopsis of changes from the 1-D single target with an additional reflector case to the extension to the 2-D measurement domain, specified for each processing stage.

<table>
<thead>
<tr>
<th>Processing stage</th>
<th>Extension</th>
</tr>
</thead>
<tbody>
<tr>
<td>I. Target simulation</td>
<td>None.</td>
</tr>
<tr>
<td>II. Sensor characteristics</td>
<td>Extension with Doppler parameters as listed in <strong>table 3-10</strong> in order to measure in 2-D</td>
</tr>
<tr>
<td>III. Particle cloud initialization</td>
<td>Initial particles are drawn from uniform distribution and spread within range-Doppler interval</td>
</tr>
<tr>
<td>IV. Measurement model</td>
<td>From two 1-D PSFs to two 2-D PSFs (one for each reflector).</td>
</tr>
<tr>
<td>VI. TBD PF</td>
<td>Implementation of MIL and WMI in order to convert 2-D PSFs to a vector in order to speed up processing time.</td>
</tr>
<tr>
<td>VII. Resampling</td>
<td>None.</td>
</tr>
<tr>
<td>VIII. Target tracking</td>
<td>None.</td>
</tr>
<tr>
<td>IX. PF prediction</td>
<td>None.</td>
</tr>
</tbody>
</table>
3-4 Extension to target-induced clutter

This is the final design phase in which the previous phase will be extended by modeling target-induced clutter. Specifically, this means that the independent additional clutter reflector that was introduced in section 3-2 will be extended to multiple clutter reflectors that move dependent of the main target position and velocity.

Throughout this section, the relevant (i.e. modified) processing stages will be mentioned in separate paragraphs. Concluding, a synopsis of the extensions will be presented.

3-4-1 Target and clutter simulation

Instead of modeling a main target and an independent second reflector, during this stage multiple clutter reflectors will be modelled that are moving dependent on the main target in order to simulate a simplified form of target-induced clutter. The simulation of the target has remained unchanged.

The following scenario will illustrate the simulation of water spray, a bow wave and wake of a small vessel, which is induced by the target itself. Obviously, in this case, the vessel will be the main target. Figure 3-27, taken form [28] provides a graphical representation of an example of a water spray, bow wave and wake.

Figure 3-27: The Fast Raiding Interception and Special Forces Craft (FRISC) from the Royal Netherlands Navy, which is creating spray, a bow wave and wake while sailing. Source: Netherlands Ministry of Defence [28]. Note that a bow wave and wake are two different
phenomena’s which are captured here. The bow wave is the wave that is created by the bow of the vessel when it moves through the water [29]. The spreading of the bow wave defines the outer boundaries of the wake. Wake is the wave pattern shown on the water surface produced by a moving vessel, caused by pressure differences of the water above and below the surface and gravity [30]. Spray is the splashing water also induced by the target’s bow and hull.

In order to remain within the scope of this research, the bow wave and wake will be further explained. Besides, some assumption will be made in order to keep the simulations simple and remain the focus on the design of the tracking filter.

A ship’s wake pattern is often called a Kelvin V-wake pattern named after the mathematical explanation of this phenomena by Lord Kelvin [31]. From figure 3-28 – taken from [32] – it can be observed that the wake indeed show a V-shaped pattern when observed from above.

![Figure 3-28: Kelvin V-wake pattern generated by a small boat, taken from [32] © Edmont | Creative Commons Attribution-Share Alike 3.0 Unported license.](image)

The following assumptions have been made for simulation of spray and a bow wave with multiple reflector PSFs:

- The target moves at constant speed;
- The speed of the bow wave is related to the speed of the target;
- Currents, tides and other environmental effects are not taken into account;
- The bow wave speed is chosen to be close to the true target speed at the location on top of the bow where it is initiated;
- While spreading, and expanding, the bow wave speed will decrease;
- $SNR$ of the bow wave PSFs will decrease as the bow wave is located further behind the target;
- Width of the bow wave PSFs will increase in Doppler as the bow wave is located further behind the target;
- Fading for both $SNR$ and speed $v$ have been set to 90%, with respect to the main target parameters which are assigned as the 100% reference.

The above-mentioned assumptions have been made in order to provide a simple target-induced clutter simulation. Therefore, it is not guaranteed that all the assumptions correspond to the real-world physics of this target-induced clutter phenomena.
Table 3-13 provides an overview of the target and clutter reflector settings for the simulation of a vessel that acts as the target and induces spray and a bow wave that is represented by the clutter reflector PSFs.

**Table 3-13**: Target and target-induced clutter reflector settings for spray and bow wave simulation. \( N_t = 5 \), since the total number of reflectors is five.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Main target</th>
<th>2(^{nd}) reflector</th>
<th>3(^{rd}) reflector</th>
<th>4(^{th}) reflector</th>
<th>5(^{th}) reflector</th>
</tr>
</thead>
<tbody>
<tr>
<td>( SNR [dB] )</td>
<td>13,00</td>
<td>10,49</td>
<td>7,97</td>
<td>5,46</td>
<td>2,94</td>
</tr>
<tr>
<td>( v [m/s] )</td>
<td>8,00</td>
<td>6,20</td>
<td>4,40</td>
<td>2,60</td>
<td>0,80</td>
</tr>
<tr>
<td>( x_{k=1} [m] )</td>
<td>10,00</td>
<td>9,50</td>
<td>9,00</td>
<td>8,50</td>
<td>8,00</td>
</tr>
<tr>
<td>Width Doppler PSF [%]</td>
<td>100%</td>
<td>200%</td>
<td>300%</td>
<td>400%</td>
<td>500%</td>
</tr>
</tbody>
</table>

Figure 3-29 illustrates the trajectory of a small vessel – that represents the target – accompanied with four target-induced clutter reflectors which provides a simplistic simulation of spray and a bow wave.

![Target trajectory. \( N_t = 5 \), \( R = 100 \, m \), scan 10 | 10.](image)

**Figure 3-29**: Trajectory of the target (small vessel) and four target-induced clutter reflectors that provide a simplistic simulation of spray and a bow wave. In this setting, \( N_t = 5 \), the trajectory covers an interval of 10 scans. The sensor range \( R = 100 \, m \). The target vessel is indicated by a right-pointing triangle, where the color red corresponds to the first scan, orange to the 2\(^{nd}\) scan, and so on until the 10\(^{th}\) scan. A dotted black line indicates the trajectory from scan to scan. A circle indicates a clutter reflector, where the color corresponds to the scan similar to the target color.

From this figure, it can be observed that the vessel is moving with a constant velocity of 8 m/s during the complete trajectory. The spray and bow wave are simulated by four additional clutter reflectors that move dependent of the target speed, besides the aforementioned assumptions.
and settings from table 3-13 are implemented. It can be observed that the speed of the bow wave is decreasing as the distance to the target increases, besides the effect is constant from scan to scan since the target moves at constant speed.

The sensor characteristics and particle cloud initialization have not been changed during this modeling stage with respect to section 3-3. Hence, the next phase that will be extended in the measurement model.

3-4-2 Measurement model

The measurement model itself has not been changed, therefore only the results will be discussed in this paragraph.

Figure 3-30 shows the 2-D noiseless PSF without target strength in range and Doppler for the complete scanning period of ten scans according to the target and sensor settings as mentioned in the caption of the figure. From this plot, it can be seen that the target vessel – depicted as a right-pointed triangle – moves with a constant speed during the scanning interval. The spray and bow wave PSFs, marked with the circles are dependent of the target speed, besides it can be observed that the PSFs of the target-induced clutter reflectors are wider in Doppler. As a consequence, this widening causes more overlap between the reflectors.

Figure 3-30: Two dimensional noiseless PSF target strength in range and Doppler plotted for the complete scanning period of ten scans, corresponding to the reflector properties as provided in table 3-13 and the sensor properties of table 3-11. From this plot, it can be observed that besides the main target, four target-induced clutter reflectors are present which simulate the spray and target bow wave. Similar to the trajectory plot in figure 3-29 the right-pointed triangles mark the main target and the circles mark the clutter reflectors. Every color corresponds to a scan, with red indicating the first scan, orange the second, and so on.

Figure 3-31 shows a plot of the same data as presented in the previous figure, however shown from a different angle. The purpose of this plot is to support the reader by interpreting the results.
As mentioned during the description of the two previous figures, these plots represent the PSF without multiplication by the complex amplitude. It is not that obvious to observe, but the peaks of the clutter reflectors are a little higher compared to the main target peak in these plots. This is caused by the fact that the clutter reflector PSFs are broader in Doppler and cause therefore more overlap. This results in the superposition of the PSFs with higher maximum values. However, this is only a plotting issue since the PSFs of all reflectors are multiplied with their random complex amplitude $A_{k,r}\#$ as defined in equation (3-30).

In order to provide an improved realistic graphical representation of the PSF, figure 3-32 displays the 2-D noiseless PSF multiplied by their random complex amplitude and shows a plot of the squared modulus of it. Besides the addition of the complex amplitude, no further changes have been made to the data. Note that the random complex amplitude is calculated for each reflector separately and depends on the target SNR. Hence, this time the PSFs will be plotted in $dB$ instead of a dimensionless quantity.

In order to show the height of the PSFs, figure 3-33 contains two plots of the same data presented from different viewing angles. From this figure, it can be observed that the main target PSF – corresponding to the vessel – has a strength of $13\,dB$ and that the clutter reflectors have decreasing intensities as they are further away from the target.
Figure 3-32: Similar plot as presented in figure 3-30, however this plot shows the 2-D PSFs multiplied by their corresponding random complex amplitude $A_{k, r}$ which is based on the reflector $SNR$. Consequently, the squared modulus has been plotted on a $dB$ scale.

Figure 3-33: This figure shows to plots of the same data as shown in figure 3-32, however presented from two different angles. The left plot shows the data from Doppler perspective and the right plot displays the PSFs along the Range axis. Both plots show that the main target (small vessel) has a peak of $13 \, dB$ and the clutter reflectors are decreasing in amplitude as the drift further away from the vessel. This corresponds to the pattern of a bow wave as assumed.

Figure 3-34 again shows a similar plot from a different perspective in order to offer the reader the possibility to observe the peak height all PSFs over the complete interval.
Design and results

The next step is adding the random complex Gaussian noise $n(t_k)$ as defined in equation (2-8), which leads to the measurement as depicted in figure 3-35. This could alternatively be called the collection of noisy PSFs. As can be observed from the figure below, the PSFs and hence the corresponding reflectors cannot be detected that obvious anymore since they are suspect to the additive noise.

Figure 3-34: Plot of the same data as presented in figure 3-32, however visualized from a different perspective in order to provide an overview of the complete scanning interval while peak heights can be observed as well.

Figure 3-35: Two dimensional noisy measurement in range and Doppler plotted for the complete scanning period of ten scans, corresponding to the reflector properties as provided in table 3-13 and the sensor properties of table 3-11. From this plot, it can hardly be observed that besides the
main target (small vessel), four target-induced clutter reflectors are present. This is less obvious compared to the previous plots of the noise free PSF. Similar to the trajectory plot in figure 3-29 the right-pointed triangles mark the main target and the circles mark the clutter reflector. Every color corresponds to a scan, with red indicating the first scan, orange the second, and so on.

In order to illustrate the height of the peaks and to present another perspective on the data in the previous plot, figure 3-35 shows the same data from two different viewing angles. The goal of inserting these figures is to show the reader that the vessel as well as the spray and bow wave reflectors are visually hard to identify by looking at the range-Doppler maps. However, this does bring the measurements closer to a real-life scenario.

Figure 3-36: This plot provides a reproduction of the same data as shown in figure 3-35, however from a different perspective.
Figure 3-37: This plot provides a reproduction of the same data as shown in figure 3-35, however from a different perspective.

Since the TBD PF design has not been changed for this extension, target tracking will be the next paragraph to describe.

3.4.3 Target tracking

In principle target tracking, has not changed, therefore this paragraph is used to show plots with results regarding this stage.

From figure 3-38 it can be observed that the particles during each scan are concentrated around the target (small vessel) and not around the clutter reflectors which simulate the target-induced clutter. This indicates that the target tracking process in working properly and that target-induced bow wave and spray are suppressed properly.
Figure 3-38: Resampled particles are plotted for each scan at time instance $k$. The particles are plotted as points, with a different color for each scan, beginning with red for scan 1, orange for scan two, etc. The right-pointing triangles indicate the target (small vessel) true position and velocity, again with the same distinct color per scan. The dotted black line represents the target trajectory between two successive scans. The circle, with the same color index as the triangle, represents the target-induced clutter reflectors that simulate the spray and bow wave.

Figure 3-39 shows a plot of the resampled particle cloud during the last scan. From this plot, it can be observed that the target is tracked properly, since the error in range is relative small and less than 0,03 m and the error in velocity is about 0,04 m/s. Average errors, for both position and velocity are listed in table 3-14. A graph of the errors is provided at the end of this paragraph.

Table 3-14: Average position and velocity estimation errors for ten successive scans corresponding to the 2-D target-induced clutter case according to the parameters earlier presented in this section. Values are rounded to two decimals.

<table>
<thead>
<tr>
<th></th>
<th>Position error [m]</th>
<th>Velocity error [m/s]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Average over all scans</td>
<td>0,15</td>
<td>1,30</td>
</tr>
</tbody>
</table>
Figure 3-39: Resampled particles during the tenth scan at $k = 10$, corresponding to the rightmost cloud in figure 3-38. The blue points correspond to the resampled particles. The red right-pointing triangle indicates the true position and velocity of the target (small vessel). The red asterisk indicates the mean of the particle cloud which is the estimated position and velocity of the target.

Figure 3-40 and figure 3-41 show that the position, respectively velocity estimation errors are relatively small over the complete scanning period.

Figure 3-40: Position estimation error [m] plotted for each time step $k$. The target estimate (asterisk) correspond to the mean of the particle cloud for each scan.
3-4 Extension to target-induced clutter

**Figure 3-41:** Velocity error \( [m/s] \) plotted for each time step \( k \). The target estimate (asterisk) correspond to the mean of the particle cloud for each scan.

### 3-4-4 Synopsis

**Table 3-15:** Synopsis of changes from the 2-D single target with 2\(^{nd}\) reflector case to the 2-D case with target-induced clutter.

<table>
<thead>
<tr>
<th>Processing stage</th>
<th>Extension</th>
</tr>
</thead>
<tbody>
<tr>
<td>I. Target and clutter simulation</td>
<td>From one additional clutter reflector to multiple clutter reflectors. Introduce target-induced clutter by making the clutter reflectors dependent of the main target speed and position.</td>
</tr>
<tr>
<td>II. Sensor characteristics</td>
<td>None.</td>
</tr>
<tr>
<td>III. Particle cloud initialization</td>
<td>None.</td>
</tr>
<tr>
<td>IV.</td>
<td></td>
</tr>
<tr>
<td>V. Measurement model</td>
<td>None.</td>
</tr>
<tr>
<td>VI. TBD PF</td>
<td>None.</td>
</tr>
<tr>
<td>VII. Resampling</td>
<td>None.</td>
</tr>
<tr>
<td>VIII. Target tracking</td>
<td>None.</td>
</tr>
<tr>
<td>IX. PF prediction</td>
<td>None.</td>
</tr>
</tbody>
</table>
This chapter will finalize this thesis by providing conclusions and recommendations. The conclusions will be drawn from the results as presented in the previous chapter. While drawing these conclusions, the research question will also be answered and further explained within the scope of this research.

4-1 Conclusions

This research goal of this thesis is to design a tracking technique that is capable of overcoming the gap between state of the art Kalman-based tracking systems – which do not perform optimal in target-induced clutter – and the user requirements for a robust target tracking technique in object-induced clutter. The particle filter has been deliberately designated as the tracking technique to design for this matter.

By designing the particle filter the objective is to demonstrate that such a filter is a suitable framework to overcome the drawbacks that current state of the art tracking algorithms encounter in target-induced clutter. Concrete, the main objective is to find a way to suppress and eliminate the existence of target-induced clutter by using knowledge on the clutter and implementing this in the tracking filter. As a result, it should be demonstrated that the particle filter is able to robustly track a target which induces clutter itself.

Hence, the main research question as stated in section 1-5 has to be answered.

*Given the drawbacks of conventional linear model-based target tracking techniques, is it possible to design and demonstrate a particle filter which is able to perform robust in object-induced clutter?*

Answering this question cannot be satisfied with a just a simple yes or no.

During the chapter Design and results a step-by-step design process has been worked out and described. Answering the research question starts with the design of a one dimensional (range only) particle filter suitable for tracking a single target. Throughout the following design steps this filter has been extended by introducing the following extension:

- Extension to tracking an additional clutter reflector;
Conclusions and recommendations

- Extension to the two-dimensional (range and Doppler) measurement domain, and finally;
- Extension by introducing target-induced clutter.

During these steps, it has been shown that the particle filter is a suitable framework:

I. for a knowledge-based tracking technique, since knowledge on the target as well as on the clutter can be implemented in the filter relative simple. Obviously, the correctness of this prior knowledge is based upon the quality of modeling a certain clutter phenomenon and the assumptions that are made in simulating it;

II. for target tracking in real world scenarios, where nonlinearities in target-induced clutter often occur. The spray and bow wave that were modelled during the last design stage in chapter 3 are an example of this;

III. for tracking, various kinds of targets under various conditions with measurements of various sensors. In this thesis, the radar was chosen as the sensor, however the filter has a generic setup and can also work with the input of various other sensors. Regarding the various targets and their conditions, this framework can be extended and refined in various ways. For instance, a detailed modulation of target-induced wake and bow wave for small vessels could be implemented.

Besides it has been shown that, despite the implementation of a quite rudimental SIR particle filter, the target tracking process did also perform quit well while running the algorithm with a low number of particles. This has been shown in section 3-2. During the target-induced clutter simulation the ratio of particles versus the number of range-Doppler cells was 1:2 which means that one particle was used to cover two range cells on average during the initialization of the process.

Hence, the design and results of this thesis have demonstrated that a particle filter can perform robust target tracking in object-induced and nonlinear clutter. Generically, it can be concluded that this framework can perform robust-target tracking in scenario’s where proper knowledge on the clutter is present during the complete processing flow. This was the case during the simulations in this thesis.

4-2 Recommendations

After assessing the previous described conclusions, the following recommendations should be mentioned as well.

Throughout the design process of the particle filter, a simplistic model for target generation as well as clutter modeling is used. In order to extend and refine the filter during future research and application in practice, these modeling steps have to be refined in order to obtain a more real world approximation of the target-induced clutter.

In order to check the performance of this tracking technique, the algorithm could be extended and refined for future work. It would be recommended to further investigate certain types of target-induced clutter and model them in a more refined way which has to be checked against the physics of these natural phenomena.
Appendix A

Multi target tracking

Multi target tracking (MTT) is an essential technique of various surveillance systems – consisting of one or more different sensors – in order to provide situational awareness in the operational environment. A typical surveillance systems might consist of a radar, infrared (IR) and or a sonar which report measurements from various kinds of sources. Besides the target(s) of interest, these sources can also consist of physical background like clutter and internal noise sources.

The objective of target tracking is to estimate the state of the targets of interest followed by initiating tracks on these targets and maintaining them as long as they are present. As soon as tracks are produced and confirmed (i.e. reduction of background clutter and false alarms) they are complemented with the computation of corresponding kinematics such as target velocity. Successively, the predicted position for the next scan is calculated. Besides, possible features in order to classify the target can be processed as well.

A surveillance system should in general be capable of tracking more than one target, therefore MTT is the most common implementation method. figure a-1, obtained from [7], provides a scheme of a generic MTT system with its elementary components. It is assumed that tracks are provided based on previous data and that a new set of measurements or observations will become available afterwards.

![Diagram](https://example.com/diagram.png)

**Figure a-1**: Basic elements of typical MTT (Multi Target Tracking) system, © 2004 IEEE [7].

The measurements collected during such a scan can either belong to an existing track, a new track or background clutter. Once a track is predicted, a gate is set around it, based on the
acceptable measurement range added with the prediction error. Only measurements within the track gate are taken into account for updating the current track. At this point problems, may arise, since closely space targets also produce closely space observations. In this case, it is likely to occur that multiple observations are located within multiple track gates, which makes it difficult to assign the observation to the correct track, see figure a-2. These situations are controlled by the Observation-to-Track Association and Track Maintenance blocks.

Figure a-2: Typical data association conflict situation, © 2004 IEEE [7]. O1, O2 and O3 represent the observation positions. P1 and P2 depict the target positions. The two circles represent the track gates for both targets. O1 would probably be assigned to P1 and O2 could be assigned to P2 and O3 could even initiate a new track.

Global Nearest Neighbor (GNN) is a conventional data association method that finds the most likely assignment of an observation to an existing track. GNN is distinguished from the outdated Nearest Neighbor (NN) method which updates a track with the closest observation, even when the measurement is already used for a different track.
Majority of the tracking systems, which are currently used, are based on the concept of conventional target tracking. A conventional target tracking scheme consists of several building blocks with sequential processes based on linear system modeling. The Kalman filter is often applied in conventional tracking systems.

In general, these systems often use a (single) Kalman filter. An improvement of these techniques, is the Interacting Multiple Model (IMM) were multiple Kalman filters – optimized for different types of targets and their maneuvering – run simultaneously [22] [33]. During this process the tracks are predicted in the future aligned within the successive scan. The GNN method (see appendix A) only takes into account the single most probable hypothesis and performs well in scenarios with large spacing between targets, precise measurements and a low false alarm rate within the track gates. Results from [22] indicate that even if the observation of the true target is present, a uniform distributed false alarm in a three-dimensional observation space results in a drop down of correct track association to about 0.85 [7]. So, one out of six track updates are likely to be based on a false alarm instead of a true target observation. In case of a more realistic scenario with multiple closely spaced targets and undetected true targets, the false track update probability will only become worse. Based on experience it can be concluded that one incorrect track update often leads to a lost track and two successive false updates will almost always lead to a lost track [7].

Already during the basic phase of tracker development, it was found that false association leads to an extra error source in a Kalman filter tracker [34] [35] [36]. Increasing the covariance matrix of the Kalman filter was one of the suggested methodologies to improve the GNN performance in order to reflect the additional error uncertainty [34] [35]. A second methodology allows a track update by a weighted sum over all observations – based on their corresponding probabilities – within the track gate [33] [36]. This method is called Joint Probabilistic Data Association (JPDA), which means that a single observation may contribute to the update of several tracks. So, for the scenario in figure a-2 (see appendix A) this could mean that O1, O2 and O3 contribute to the update of track 1, but that O2 and O3 also contributes to the update of track 2.

The above-mentioned problems resulted of improvements to the GNN method on the one hand, and the significant increase in computational power on the other hand. Both issues have led to a common acceptance of multiple or Multi Hypothesis Tracking (MHT) as the favorite data association method for nowadays systems [7]. Within MHT alternative data association
hypothesis are generated each time a conflict between observations and tracks occur as shown in figure a-2. At that moment, instead of choosing the best fitting hypothesis, or combining the two as in JPDA, both hypothesis is propagated and predicted into the future with the expectation that succeeding measurements will resolve the uncertainty.

Kalman filters are widely used and very helpful in numerous applications, like: Global Positioning System (GPS), weather forecasting, ATC, Sensor Data Fusion (SDF), inertial GPS, tracking of satellites, missiles, people and so on. Besides, Kalman filters are relative easy to design and provide an accurate estimation in most cases. In contrast, the performance can be extraordinarily worse for some practical reasons, comprising: i) nonlinearities in the system physics equations, ii) ill-conditioned covariance matrix as a consequence of a linearized nonlinear phenomena, and iii) incorrect or lacking models regarding the underlying physics [12]. As already mentioned, Kalman filter are only suited for linear Gaussian problems. However, most real world application scenarios are nonlinear and non-Gaussian. In order to fill this gap, engineers tried to make linear approximations in order to model nonlinear problems.
Appendix C

Resampling algorithm

function \([x_r, w_r] = \text{resample}(x, w, N)\)

% \([x_r, w_r] = \text{resample}(x, w, N)\) resamples the states \(x\) and their associated weights \(w\) using random sampling
% \([x_r, w_r] = \text{resample}(x, w, N)\) returns a resampled distribution of size \(N\)
% =========================================================================
% check if number of inputs is smaller than 3:
if nargin < 3
    N = length(w);
    \% if so: determine \(N\) based on the number of weights
end

u = sort(rand(1,N));  \% generate a vector of uniform ordered numbers: 1x\(N\)
wc = cumsum(w);       \% generate cumulative weights (ordered): 1x\(N\)
ind = zeros(1,N);     \% indices of the selected states
k = 1;                \% start with \(k = 1\)

for i = 1:N          \% loop over all particles
    while(wc(k)<u(i)) \% determine interval
        k = k+1;    \% \(k\) is raised with 1 as long as \(wc(k) < u(i)\)
    end

    \% index of selected state is set to last known \(k\), i.e. the index of \(wc\)
    \% at which the cumulative weight of \(w\) is still smaller than the \(u(i)\)
    ind(i) = k;
end

\[x_r = x(:, \text{ind});\]  \% determine new states
\[w_r = \text{ones}(1,N) ./ N;\]  \% determine corresponding weights
Particle cloud initialization

```matlab
function [x00,y00] = RDbox(R,Dmax,Np)

% The function RDbox generates the initial particle cloud, containing Np
% particles which are placed within the range-Doppler box (i.e.
% range-Doppler interval) of the radar. A uniform distribution is used in
% order to determine the positions (x00) and the velocities (y00) of the
% initial particles.

% INPUTS:
% R     = Range of the radar
% Dmax  = Maximum measurable unambiguous Doppler
% Np    = Number of particles

% OUTPUTS:
% x00   = Positions of initial particles
% y00   = Velocities of initial particles

% =========================================================================
% Determine positions of initial particles by drawing Np times from the
% uniform distribution and multiplying the samples with R:
% x00 = R*rand(1,Np);

% Determine velocities of initial particles by drawing Np times from the
% uniform distribution and multiplying the samples with the length of the
% Doppler interval (max-min) and shifting it by adding the lower bound,
% such that the center lies between -Dmax and Dmax:
min = -Dmax;    % maximum value of a sample from uniform distribution
max =  Dmax;    % maximum value of a sample from uniform distribution
y00 =  (max-min).*rand(1,Np) + min;

dend
```


[38] A. v. Bruggen, "Copyright, all rights reserved. Fast Raiding Interception & Special Forces Craft (FIRSC)," 20 June 2014. [Online]. Available:
https://www.flickr.com/photos/60191572@N02/14479722464. [Accessed 27 September 2016].
### List of Acronyms

<table>
<thead>
<tr>
<th>Acronym</th>
<th>Description</th>
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<tbody>
<tr>
<td>ATC</td>
<td>Air Traffic Control</td>
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<tr>
<td>DMO</td>
<td>Defence Materiel Organisation</td>
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<tr>
<td>EM</td>
<td>Electro Magnetic</td>
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<tr>
<td>EO</td>
<td>Electro Optical</td>
</tr>
<tr>
<td>FOV</td>
<td>Field Of View</td>
</tr>
<tr>
<td>FRISC</td>
<td>Fast Raiding Interception and Special Forces Craft</td>
</tr>
<tr>
<td>GNN</td>
<td>Global Nearest Neighbor</td>
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<tr>
<td>GPS</td>
<td>Global Positioning System</td>
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<tr>
<td>IEEE</td>
<td>Institute of Electrical and Electronics Engineers</td>
</tr>
<tr>
<td>IMM</td>
<td>Interacting Multiple Model</td>
</tr>
<tr>
<td>IQ</td>
<td>In-phase and Quadrature</td>
</tr>
<tr>
<td>IR</td>
<td>Infrared</td>
</tr>
<tr>
<td>ISR</td>
<td>Intelligence, Surveillance, and Reconnaissance</td>
</tr>
<tr>
<td>JPDA</td>
<td>Joint Probabilistic Data Association</td>
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<tr>
<td>MDL</td>
<td>Matrix Determinant Lemma</td>
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<tr>
<td>MHT</td>
<td>Multi Hypothesis Tracking</td>
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<tr>
<td>MIL</td>
<td>Matrix Inversion Lemma</td>
</tr>
<tr>
<td>MT</td>
<td>Main Target</td>
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<tr>
<td>MTT</td>
<td>Multi Target Tracking</td>
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<tr>
<td>NN</td>
<td>Nearest Neighbor</td>
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<tr>
<td>PDF</td>
<td>Probability Density function</td>
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<td>PF</td>
<td>Particle Filter</td>
</tr>
<tr>
<td>PSF</td>
<td>Point Spread Function</td>
</tr>
<tr>
<td>RADAR</td>
<td>RAdio Detection And Ranging</td>
</tr>
<tr>
<td>Abbreviation</td>
<td>Description</td>
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<tr>
<td>RCS</td>
<td>Radar Cross Section</td>
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<tr>
<td>RF</td>
<td>Radio Frequency</td>
</tr>
<tr>
<td>RNLN</td>
<td>Royal Netherlands Navy</td>
</tr>
<tr>
<td>SA</td>
<td>Situational Awareness</td>
</tr>
<tr>
<td>SDF</td>
<td>Sensor Data Fusion</td>
</tr>
<tr>
<td>SIR</td>
<td>Sequential Importance Resampling (or Bootstrap filter)</td>
</tr>
<tr>
<td>SNR</td>
<td>Signal-to-Noise Ratio</td>
</tr>
<tr>
<td>TBD</td>
<td>Track Before Detect</td>
</tr>
<tr>
<td>TU</td>
<td>Technische Universiteit</td>
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<tr>
<td>VTM</td>
<td>Video Tracking Modules</td>
</tr>
<tr>
<td>WMI</td>
<td>Woodbury Matrix Identity</td>
</tr>
</tbody>
</table>
**List of Symbols**

- $r$: Apparent target range
- $D$: Apparent target Doppler
- $b$: Target bearing
- $k \in \mathbb{N}$: Discrete time step
- $z_k$: Measurement (vector)
- $N_r$: Number of range cells
- $N_d$: Number of Doppler cells
- $N_b$: Number of bearing cells
- $z_{kijl}$: Power measurements for each resolution cell $(i, j, l)$, where $i = 1 \ldots N_r, j = 1 \ldots N_d, l = 1 \ldots N_b$
- $\Delta_r$: Range resolution
- $\Delta_d$: Doppler resolution
- $\Delta_b$: Bearing resolution
- $r_{\text{min}}$: Minimum measurable range
- $d_{\text{min}}$: Minimum measurable Doppler
- $b_{\text{min}}$: Minimum measurable bearing
- $z_{Ak}$: Complex measurement data
- $t_k \in \mathbb{R}$: Time
- $A_k$: Complex amplitude of the target
- $\tilde{A}_k$: Modulus of $A_k$
- $\sigma_{Ak}$: Standard deviation of the complex amplitude $A_k$
- $h(s_k, t_k)$: Reflection form or Point Spread Function (PSF)
- $h^{ijl}(s_k, t_k)$: PSF for each range-Doppler-bearing cell
- $\hat{R}$: Size of a range cell
- $D$: Size of a Doppler cell
- $B$: Size of a bearing cell
- $n(t_k)$: Complex Gaussian noise
- $n_i(t_k)$: In-phase component of $n(t_k)$
- $n_q(t_k)$: Quadrature component of $n(t_k)$
- $\phi_k \in (0, 2\pi)$: Phase of complex signal
- $x$: $x$-Positions in a Cartesian coordinate system
- $y$: $y$-Positions in a Cartesian coordinate system
- $\dot{x}$: Velocity in $x$-direction in a Cartesian coordinate system
- $\dot{y}$: Velocity in $y$-direction in a Cartesian coordinate system
\( \sigma_n^2 \) Noise variance

\( s_k \in \mathbb{R}^{n_x} \) Target state vector at time index \( k \)

\( n_x \) Dimension of the target state vector \( s_k \)

\( \mathbb{R} \) Set of real numbers

\( \mathbb{N} \) Set of natural numbers

\( f_{k-1} \) System dynamics or transition function

\( v_{k-1} \) Process noise

\( h_k \) Measurement function

\( m_k \) Measurement noise

\( Z_k \) Total set of measurements available at time index \( k \)

\( p(s_k|Z_k) \) Posterior PDF (alternatively: posterior density)

\( p(s_k|Z_{k-1}) \) (Dynamic) prior PDF (alternatively: prediction density or prior density)

\( A, B, C \) Events or sets of outcomes of an experiment

\( P(A \cap B) \) Joint probability of observing the events \( A \) and \( B \)

\( P(A|B, C) \) Conditional probability of observing event \( A \), given events \( B \) and \( C \) are true

\( p(s_k|s_{k-1}) \) State evolution (alternatively called: transitional property or transition kernel)

\( \hat{s}_k(i) \) Sample from the prior PDF at time step \( k \), where \( i = 1, ..., N_p \)

\( w_i \) Normalized weight corresponding to each sample \( \hat{s}_k(i) \)

\( N_t \) Number of targets (i.e. number of reflectors)

\( SNR \) Signal-to-Noise Ratio

\( v \) Target or reflector speed

\( x_{MR,k=1} \) (Main) target position at during first scan at time index \( k = 1 \)

\( N_{sc} \) Number of scans or measurement intervals

\( T \) Measurement interval (alternatively: sampling time)

\( t \in \mathbb{R}^{N_{sc}} \) Time vector (contains the time instances at which each scan is carried out)

\( N_r \) Number of range cells

\( \tau \) Pulse length

\( x \in \mathbb{R}^{N_r} \) Range dimension vector (contains the range cells of the radar system)

\( R \) Maximum measurable range of the radar

\( \mu_{pos} \) Mean of initial particle positions

\( \sigma_{pos} \) Standard deviation of initial particle positions

\( \mu_{vel} \) Mean of initial particle velocities

\( \sigma_{vel} \) Standard deviation of initial particle velocities

\( \hat{s}_k \) Initial particle cloud containing the initial positions and velocities of the particles

\( p_k \) Vector containing the particle positions

\( \hat{p}_k \) Vector containing the particle velocities
Possible target range extracted from each particle, where \( i = 1, \ldots, N_p \)

PSF calculated for each particle

Covariance matrix

Row vector containing the log-likelihoods for a single scan at time instance \( k \)

Row vector containing the normalized weights at time instance \( k \)

Resampled particles at time instance \( k \)

Resampled weights corresponding to \( s_{k,r} \) at time instance \( k \)

Covariance matrix of the process noise

System transition matrix

Assignment of reflector number, will be used as a subscript of several symbols

Number of Doppler cells

Number of range-Doppler cells, i.e. \( N_r \times N_d \)

Maximum measurable unambiguous Doppler

Size of Doppler cell, which is set to \( D = 2D_{max}/N_d \)

Doppler vector which is \( d = [(-D_{max} + D) (-D_{max} + 2D) \ldots D_{max}]^T \)

Subscript that assigns a certain parameter to the range domain

Subscript that assigns a certain parameter to the Doppler domain

Subscript that indicates that a certain parameter has to be computed for each particle