STATISTICS OF STREAMLINE SEGMENTS IN A TURBULENT CHANNEL FLOW WITH A WAVY WALL

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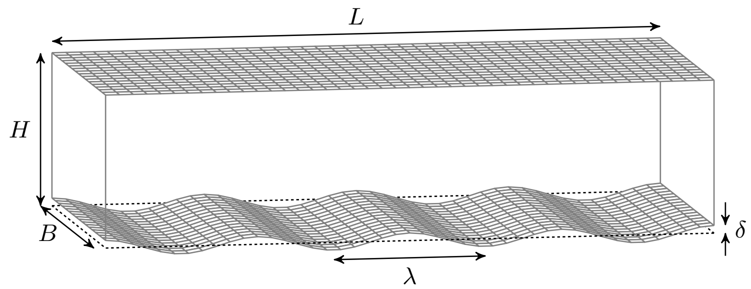
Abstract We investigate the statistical properties of so called streamline segments in a turbulent channel flow with one plane and one wavy wall. We give a short overview on the concept of streamline segments and recent results in description and modelling in this field. Finally, we show the length distribution and conditional moments of streamline segments in the wavy channel flow and compare the statistics to those in homogeneous isotropic turbulence.



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The statistical structure of turbulence is often described by means of the two-point correlation function and its transport equation (cf. von Karman [3]). However, one may ask whether the more or less arbitrary Cartesian frame, in which such theories are developed can be replaced by a more natural frame based on the flow field itself.

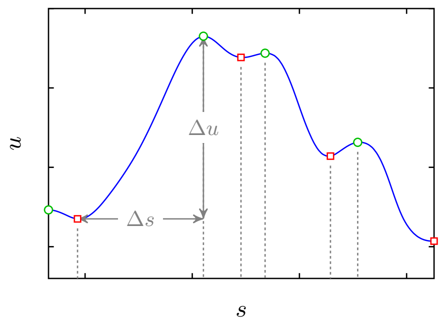

Figure 1. Velocity magnitude u (——) as function of the streamline coordinate s with local minima (□) and maxima (○).

In this spirit Wang [4] proposed to study streamlines as natural geometries of the turbulent flow field and introduced a technique called streamline segment analysis: Starting from each grid point, a streamline segment is defined as the part of its streamline bounded by the two adjacent extremal points (□ and ○ in figure 1) of the velocity magnitude projected onto the streamline (—— in figure 1).

Following Wang [4] and Schaefer [2] two parameters are chosen to statistically describe streamline segments, namely the arclength distance Δs between and the velocity difference Δu at the two extreme points which bound streamline segments. Then, most of the statistical properties are captured by the jpdf $P(\Delta s, \Delta u)$. The choice of these two parameters is not arbitrary but rather inspired by the approach to study turbulent flow fields by means of two-point statistics. However for streamline segment analysis the correlation coordinate is chosen along the streamline, i.e. it represents the local flow structure.

The normalized jpdf (see figure 2, left) has two distinct wings that correspond to positive and negative streamline segments. The lower wing, corresponding to the negative streamline segments is shorter than the upper wing. From that it can be concluded, that the length of positive streamline segments is on average larger than of negative ones. This finding has already been remarked by Wang [4] and Schaefer [2, 1] (in homogeneous isotropic and shear turbulence) and related to the kinematic difference of positive and negative streamline segments: Due to the velocity difference at the end points, Δu, negative streamline segments will on average be compressed while positive ones will on average be stretched.

Integration of the jpdf leads to the marginal pdf of Δs. In figure 2 the length distribution of SLS in the wavy channel flow is compared to results in homogeneous isotropic turbulence. Similar to the findings by Schaefer [2, 1] the curves

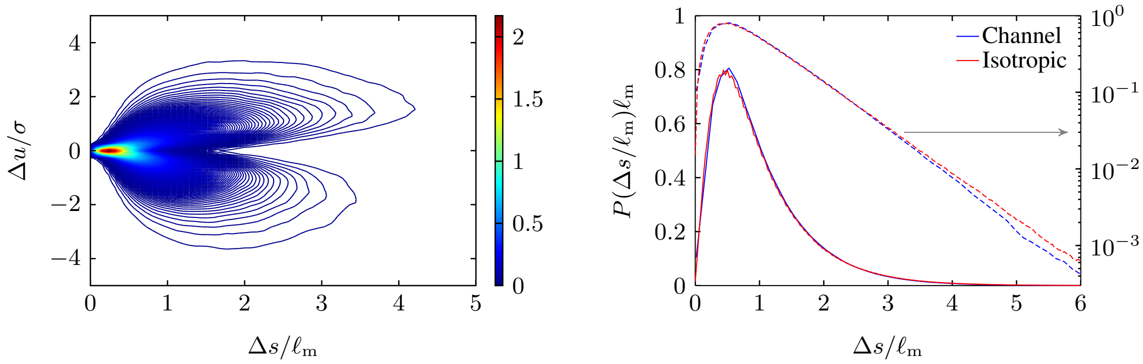

Figure 2. Left: Normalized joint pdf of the arclength and the velocity difference. Right: Normalized marginal pdfs of the arclength for different flow types.

show a linear rise at the origin for small and an exponential tail for larger segments. The choice of the mean length as normalization parameter leads to an almost perfect collapse of the curves not only in the diffusion dominated region for small elements close to the origin but also for larger elements. The latter is especially visible in the semilogarithmic plot in figure 2 which also reveals the exponential character of the tails. This good agreement is surprising, as we compare the highly anisotropic wavy channel flow to homogeneous isotropic turbulence.

CONDITIONAL MOMENTS OF STREAMLINE SEGMENTS

In the spirit of classical structure function analysis of turbulent fields, where two points are separated by a distance r, we can define the $\mathrm{n^{th}}$ order conditional moment $\langle(\Delta u)^n|\Delta s\rangle$ of streamline segments. It is obtained from the conditional pdf $P(\Delta u|\Delta s)$ by integration over Δu.

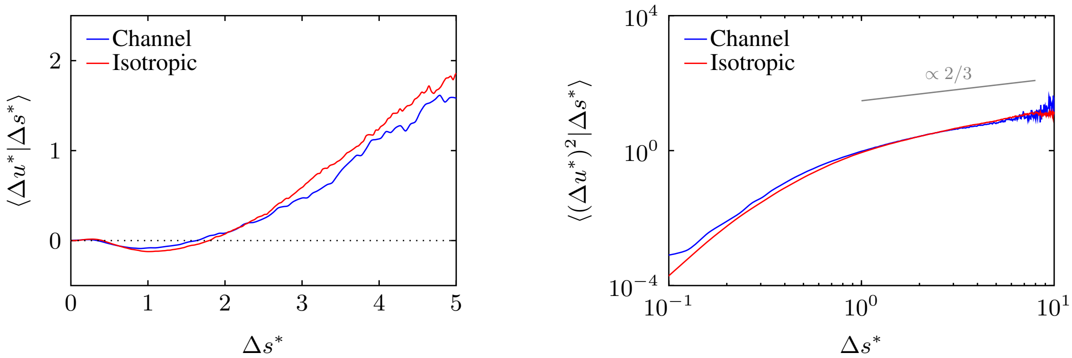

Figure 3. First and second order conditional moment of Δu^*

For the first moment (see figure 3, left), we observe a flat start at the origin, an intermediate negative section and positive values for larger elements. Physically, the velocity difference at the ending points of streamline segments thus has a zero net effect for small segments, while intermediate segments are compressed and large segments are subject to extensive strain (cf. [1]).

The second order moment (see figure 3, right) shows again a very good agreement between the two DNS cases. Additionally to the two curves the slope of the theoretical scaling according to Kolmogorov for the second order moment is shown. It turns out that this scaling is significantly different between structure function analysis and in streamline coordinates.

References

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