The development of an autonomous GPS system to monitor tidal slack in estuaries

A survey of hardware and algorithm options

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Abstract

Recently, a promising method for measuring tidal slack using GPS receivers attached to buoys has been tested. The next step is to build a wireless GPS system which conducts measurements continuous and real-time. This research primarily focuses on the development of the hardware of that GPS system. Secondly it focuses on modeling the buoy behavior and calculation algorithms for determining the moment of slack.

A series of GPS measurements are conducted to test the accuracy and precision of candidate receivers. For the hardware a WaspMote-board and Meshlium router are chosen. Together they provide a complete solution for the GPS system, including GPS receiver. The solution combines adaptability with low costs. The module is made waterproof and solar powered. A buoy model is made with which data sets can be simulated to test the accuracy of the calculation algorithms. The sensitivity tests of the algorithms are performed with a Monte Carlo model.

Clear differences in performance between high-end and common receivers are observed. EGNOS improves only the performance of the high-end receiver. The buoys have different behavior, not all buoys can be modeled the same. Also a bias of up to 10 minutes is found for the calculation algorithms. In terms of sensitivity of the methods, there is a critical standard deviation for the GPS receiver for which the methods become unstable.

With all tested GPS receivers the moment of tidal slack can be calculated with a deviation up to 15 minutes. The greatest improvement can be made in reducing the bias in the calculation methods.
During January 2011 Wim Luxemburg contacted us if we were interested in developing a method for monitoring tidal slack in estuaries by determining the position of buoys.

The principle of monitoring tidal slack by determining the position of buoys had already been tested by former master student civil engineering, Mijke Lievens. Our job would be to develop the measurement principle into a system where continuous measurements are done, while all data is sent real-time to a central server on the shore. While this did not sounded much like an ordinary task for a (future) civil engineer, we decided to have a meeting with the supervisors of the project. This resulted in the decision to do this project as our bachelor thesis. During the past two months we worked with much joy on the project, partially cooperating with Martin Valk, master student Geomatics.

Unfortunately the original goal of developing the GPS system for monitoring tidal slack is only partially fulfilled. Still some interesting aspects of the development of the system are dealt with. Furthermore we believe to have contributed to a better insights in the buoy motion and data interpretation techniques. Therefore we are content with the results of our work in this thesis. Which, we realize, is part of a project with a much longer time span than the 10 weeks scheduled for our Bsc-thesis.

We are indebted to our supervisors Wim Luxemburg and Christiaan Tiberius for their support and guidance the past months. Secondly we would like to thank Martin Valk for a pleasant cooperation. Furthermore we like to thank prof. Huub Savenije for initiating and Mijke Lievens for conducting the preliminary research of this project.

To conclude this preface: we are very pleased to deliver this report as a final chapter of our bachelor civil engineering.

June 2011,

Wouter Berghuijs and Bart van Osnabrugge
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<th>Parameter</th>
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<tr>
<td>$\alpha$</td>
<td>Convergence length of the cross-Sectional Area</td>
<td>$L$</td>
</tr>
<tr>
<td>$\beta$</td>
<td>Convergence length of the stream width</td>
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<td>$\omega$</td>
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<td>Tidal amplitude</td>
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<td>Celerity number</td>
<td>$-$</td>
</tr>
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<td>$\zeta$</td>
<td>Tidal amplitude to depth ratio</td>
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<td>$\sigma_{GPS_c}$</td>
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<tr>
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<tr>
<td>$K$</td>
<td>Calculation parameter in quadrant based calculations</td>
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<td>Maximum measured X-coordinate of a buoy</td>
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<td>Wave Celerity</td>
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</tr>
<tr>
<td>$dt$</td>
<td>Time interval</td>
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<tr>
<td>$g$</td>
<td>Acceleration due to gravity</td>
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<tr>
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<td>$v$</td>
<td>Velocity</td>
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<td>$v_b$</td>
<td>Velocity of buoy in the direction of the rotated x-axis</td>
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<tr>
<td>$x$</td>
<td>Distance</td>
<td>$L$</td>
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<td>$y$</td>
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</tr>
<tr>
<td>$z$</td>
<td>Water level</td>
<td>$L$</td>
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List of terms

Alluvial estuary  Estuary with a dynamic equilibrium between deposition and erosion, whereby the deposition and erosion are influenced by the hydraulics of the estuary.

Angle Method  Calculation method for determining the moment of tidal slack, based on the position of the buoy

Celerity number  Dimensionless number expressing the ratio between the actual wave celerity and the theoretical frictionless wave celerity for long waves

Convergence length of the cross-sectional area  The ratio by which the cross-sectional area of an estuary decreases over the length of the estuary

Convergence length of the stream width  The ratio by which the stream width of an estuary decreases over the length of the estuary

Damping number  Dimensionless number expressing the relative damping of tidal range as a function of place and wave characteristic

Doppler Speed Measurement  Speed measurement with a GPS receiver based on the Doppler effect

EGNOS  European network for Wide Area Differential GPS

Estuary  Partly enclosed coastal body with one or more rivers flowing into it and a free connection to the open sea

Estuary shape number  Dimensionless number expressing the relative convergence length to the wave characteristics of the estuary

Friction number  Dimensionless number expressing the relative amount of friction

Froude number  Dimensionless number expressing the ratio of flow velocity over wave celerity

GPS  Global Positioning System

GPS module  Module of GPS receiver and sending module located on the buoy

GPS system  The network of GPS modules monitoring the location of buoys and all additional infrastructure

High water(HW)  The maximal water level during a tidal cycle

High water slack(HWS)  The moment the current in an estuary changes from an estuary inward to an estuary outward direction

Lagrangian approach  Mathematical description of a process with a moving coordinate system

Local differential GPS  GPS using a second reference receiver for increased precision

Low water(LW)  The minimum water level during a tidal cycle

Low water slack(LWS)  The moment the current in an estuary changes from an estuary outward to an estuary inward direction
Meshlium Central router for collecting the data in the GPS system

Minimum radius method Calculation method for determining the moment of tidal slack, based on the position of the buoy

Mixed wave Wave with a phase lag between 0 and $\frac{\pi}{2}$ rad

Moment of slack The moment the current in an estuary changes direction. Moment the flow velocity is zero

Monte Carlo model Mathematical model based on the repetition of stochastic processes to compute results

M2-tide Diurnal-tide with a period of 12.25 hours

NMEA Standard communication protocol and data format for GPS output

Phase lag Time span between the moment of HW and HWS, respectively LW and LWS

Progressive wave Wave with a phase lag of $\frac{\pi}{2}$ rad

RD-coordinates Coordinates in the RD reference frame

RD-reference frame National reference frame for the Netherlands (horizontal positioning)

Scheldt estuary Alluvial estuary located in the Netherlands and Belgium

Standing wave Wave with a phase lag of 0 $\pi$ rad

St. Venant’s equation Equation for the conservation of impulse for water particles

Tidal excursion Distance traveled by a water particle during a tidal cycle

Tidal range Amplitude of the tidal wave

Tidal damping The reduction of the tidal range over an estuary

Velocity method Calculation method for determining the moment of tidal slack, based on the movement of the buoy

Velocity number Dimensionless number expressing the ratio of the flow velocity divided by estuary shape characteristics and wave propagation characteristics

Waspmote Hardware system supporting a range of sensors for wireless data transmission networks

Wave celerity Propagation speed of a wave

Wave length Length of a wave which is the distance distance traveled by the wave in a period of $2\pi$ rad

WGS-84 reference frame Standard reference frame for many GPS devices, expressing the location in longitude, latitude and ellipsoidal height.
Chapter 1

Introduction

1.1 Background

The phase lag between the moment of high water and high water slack, respectively low water and low water slack, is a key parameter in tidal hydraulics which is often disregarded. Hydraulic models are most often calibrated on water levels only, a doubtful method since water levels can be easily manipulated by changing uncertain coefficients, for example the roughness coefficient. Calibration based on a combination of water levels and flow information is more reliable, but not easily executed because the lack of flow data, for example the moment of tidal slack.

Analytical relations exists between the estuary topography, tidal damping, wave celerity and the phase lag. An accurate measurement of the phase lag can shed more light on the validity and use of these equations and numerical hydraulic models.

Besides that, many estuaries are hosting both commercial and non-commercial shipping activities. Better information on the hydraulics in estuaries will be valuable for the shipping industry.

Current measuring methods to identify the moment of tidal slack are very inconvenient, because fixed objects in the estuary are required.

Recently, a promising method using GPS receivers attached to buoys has been tested, with partially satisfying results. The run time of the system was limited to just over a day, data collection had to be done manually and calculations a-posteriori. In addition, different calculation algorithms on the GPS data did not yield an unambiguous solution for the moment of tidal slack.

For the reason that there are no sufficient measuring methods available, information about the current in estuaries is not detailed and the prediction of the moment of tidal slack often has a deviation of more than half an hour compared to reality. This despite the fact that information on the moment of tidal slack is valuable for many.

1.2 Research objective

As said, monitoring tidal slack in estuaries is something for which no sufficient measuring techniques have been developed. Recently, a promising method using GPS receivers attached to buoys has been tested, but a couple of practical issues have to be addressed before large scale implementation; The run time is limited to just over a day and data collection had to be done manually and a-posteriori.

Therefore the goal of this research is to eliminate these constraining aspects and thereby create an affordable autonomous operating GPS system to determine the moment of tidal slack in estuaries. The basic principle of the system is presented in figure 1.1.

This research primarily focuses on the development of the GPS system concerning the hardware for the individual modules. Secondly it focuses on modeling the buoy behavior and calculation algorithms for determining the moment of tidal slack.
Figure 1.1: This figure gives a global overview of the GPS system that will be developed. The main principle of the system will be that GPS modules are located on buoys in the estuary. These GPS modules are equipped with GPS receivers and monitor the position of the buoys. The data is transmitted in real-time to a local database. Because the system has to operate autonomously, the GPS modules will be solar powered. By interpreting the GPS positions of the buoys in a smart way, the moment of tidal slack can be calculated.
2.1 Preliminary research

The master thesis ‘Observing tidal slack in the Scheldt estuary’ \(^{[\text{Lievens, 2010}]}\) shows the potential of a hand held GPS receiver to monitor tidal slack in estuaries. The principle is tested during three measurement campaigns in the Scheldt estuary.

During the first measurement campaign, GPS receivers were attached both directly to the buoys as placed in floating baskets which were connected with a cable to the buoys. In this campaign it is shown that the movement of both the GPS receiver on the buoy as the GPS in the floating basket show a very similar movement pattern. Therefore the GPS-receiver in the floating basket is presumed to have no additional value compared to the GPS receiver attached to the buoy. Another relevant aspect is that the measurement campaign measured during a period of 7 hours. Therefore the campaign only partially measured a full tidal cycle.

During the second and third measurement campaign more data is gathered in measurements taking place over a period with a maximum of more than 24 hours. The measurements of these two campaigns are compared and analyzed.

For the data analysis three different calculation methods for determining the moment of tidal slack are used: the ‘angle method’, the ‘minimum radius method’ and the ‘velocity method’. These methods will be explained later in this section.

An aspect that stands out in this analysis is the fact that the phase lag near the estuary mouth is very long. Some speculation is done that this is caused by 3D-effects or by circular movement of the water. To confirm these speculations more research has to be done.

All methods indicate that in general the phase lag is shorter upstream in the estuary, although in the most upstream parts the phase lag increases again.

In the study, wind effects are considered to be negligible. With wind speeds reaching 6.8 m/s no substantial influences in the pattern of the buoy were observed. The only influence is a small shift in the path the buoy. More data is needed to validate the assumption that wind has a negligible influence.

For the comparison of different data interpretation techniques the third measurement campaign is used. The comparison of these results leads to the conclusion that not all techniques give a similar phase lag. The maximal phase lag is computed by the minimum radius method, while the velocity and angle method show quite similar phase lags. Because the velocity method relies on visual data inspection the conclusion is made that the angle method is the most reliable method.

Overall the conclusion is made that observing the moment of tidal slack by the use of GPS receivers is a method that seems to works. Besides that, it is stated to be affordable and relatively easy.

The author mentions some improvements that have to be made on the GPS system. The runtime of the GPS system has to be increased, data transmission must be done wireless and real-time, and the calculation methods for the moment of tidal slack have to be validated.

The three different calculation methods for determining the moment of tidal slack, as proposed by Lievens, are explained in the next sections.
Figure 2.1: Visualization of the angle and quadrants of the buoy. The angle and quadrants of the buoy are used in the angle method for calculating the moment of tidal slack.

2.1.1 The angle method

The basic principle of the angle method is that the angle the buoy has, compared to the x-axis that goes through the center of the path traveled by the buoy, determines when the current has changed direction (see figure 2.1).

The center of the traveled path is determined by averaging the minimum and the maximum X and Y-coordinates measured for the buoy:

\[
X_{center} = \frac{MinX_{buoy} + MaxX_{buoy}}{2} \quad (2.1)
\]

\[
Y_{center} = \frac{MinY_{buoy} + MaxY_{buoy}}{2} \quad (2.2)
\]

where,

\(X = X\)-coordinate of the buoy

\(Y = Y\)-coordinate of the buoy

Now the angle of the buoy relative to the x-axis can be calculated. Note that for the following formula \(x\) and \(y\) have to be calculated from \(X\) and \(Y\). This is not explicitly explained in [Lievens, 2010].

\[
\arctan\left(\frac{y}{x}\right) + K = \alpha \quad (2.3)
\]

where,

\(x, y\) = the distances of the buoy to the center[m]

\(K = 0\) for quadrant 1

\(K = \pi\) for quadrant 2 and 3

\(K = 2\pi\) for quadrant 4

The moment that \(\left| \frac{d\alpha}{dt} \right| \) starts increasing, is the moment that the buoy starts floating back to the center. In the angle method this is interpreted as the moment of tidal slack.

2.1.2 The minimum radius method

Like the angle method the minimum radius method is also based on the position of the buoy compared to the center of the path that is traveled by the buoy. The moment the radius is at its minimum, is the moment that the buoy is closest to the center. This can be interpreted as the moment when the current is at its minimum: the moment of tidal slack.

For this analysis the same center point is used that is calculated for the ‘angle method. The radius of the buoy compared to the center is calculated by using Pythagoras:

\[
\sqrt{x^2 + y^2} = r \quad (2.4)
\]
where,

\( r \) = radius[m]

\( x \) = the distance of the buoy to the center in the direction of the x-axis[m]

\( y \) = the distance of the buoy to the center in the direction of the y-axis[m]

### 2.1.3 The velocity method

The idea is that by looking how the buoy moves, one tries to find the moment before the buoy starts moving faster to the other side, considering the direction of the current. This should then be the moment the current has changed direction, which is the moment after tidal slack.

For the method the axis are rotated so that the X-axis is parallel to the direction of the flow. In order to have a X-axis parallel to the direction of the flow, the following transformation formulas for the X and Y coordinates of the buoy are used:

\[
(X - Y \tan \alpha) \cos \alpha = X_1; \tag{2.5}
\]

\[
\frac{Y}{\cos \alpha} + X_1 \tan \alpha = Y_1; \tag{2.6}
\]

where,

\( X_1, Y_1 \) = coordinates of the buoy in the rotated reference frame

\( X, Y \) = coordinates of the buoy in a non rotated reference frame

\( \alpha \) = angle of the path of the buoy

Note that these transformation formulas can be used to calculate \( x \) and \( y \) in the ‘angle’ method. The x-axis is now the axis in the direction of the current. Now the velocity in the main direction can be calculated with equation 2.7.

\[
\frac{dX_1}{dt} = v_b; \tag{2.7}
\]

where,

\( dX_1 \) = distance traveled by the buoy, parallel with the rotated x-axis[m]

\( dt \) = time interval[s]

\( v_b \) = velocity of the buoy parallel with the rotated x-axis[m/s]

### 2.2 Estuaries and wave theory

In this section some basic principles concerning estuaries and wave theory, relevant for this thesis, are explained. Furthermore analytical equations for determining the phase lag and other relevant phenomena in estuaries are given.

#### 2.2.1 The Phase lag

Water levels in estuaries are time and place dependent. An introduction is given in i.e. [Battjes, 2002]. In each cycle of the tidal water motion, the moment the water level reaches its maximum is referred to as high water(HW). The same applies for the lowest water level in the cycle, referred to as low water(LW).

Besides the water level, the state of a water body is described by a flow direction and a flow velocity. The moment the flow direction changes, that is when the flow velocity equals zero, is referred to as the moment of slack. For the moment of slack after high water this is referred to as high water slack(HWS), and for the moment of slack after low water this is referred as low water slack(LWS).

The phase lag is then defined as the time span between the moment of HW and HWS, respectively LW and LWS.

In the case of a standing wave the moment of HW and HWS, respectively LW and LWS occur at the same moment. The principle of a standing wave is visualized in figure 2.2.
Figure 2.2: Visualization of a standing wave. The water level and flow velocity in a situation where the phase lag equals zero. This situation is referred to as a standing wave. A phase lag of zero means the moment of tidal slack, i.e. the moment when the velocity of the current is zero, coincides the moment the moment of HW, respectively LW. A standing waves requires a semi-enclosed body with no significant river discharge, where the tidal wave will be fully reflected.

Figure 2.3: Visualization of a progressive wave. In the case of a maximal phase lag, i.e. the moment of LW and LWS slack, respectively HW and HWS differ a period of $\frac{\pi}{2}$ radians. This wave type is referred to as a progressive wave. In this situation the moment of maximum velocity coincides with the maximum water level, and the moment minimum velocity coincides with the minimum water level. This so called progressive wave only occurs in the situation of a prismatic flow channel with constant cross-section and infinitive length.
Figure 2.4: Visualization of a mixed wave. In the case of a mixed wave the phase lag, i.e. the moment of LW and LWS slack, respectively HW and HWS differ a period of anything between 0 and $\frac{\pi}{2}$ radian. This mixed wave is the type of tidal wave that is observed in estuaries.

Another possible situation is whenever the phase lag is maximal. In this situation a progressive wave occurs with a phase lag of $\frac{\pi}{2}$ radians, see figure 2.3.

The third possible situation is the so called mixed wave. In the case of a mixed wave the moment of tidal slack occurs after the moment HW or LW and before the mean tidal level. This principle is visualized in figure 2.4.

2.2.2 Analytical equations for alluvial estuaries

In 1992 an analytical solution using St. Venant’s equation (eq. 2.8) for tidal flow in alluvial estuaries was presented using a Lagrangian approach [Savenije, 1992].

$$\frac{\delta v}{\delta t} + v \frac{\delta v}{\delta x} + g \frac{\delta h}{\delta x} + g I + g \frac{v|v|}{C^2 h} = 0$$ (2.8)

where, $v$ is the velocity, $t$ is the time, $x$ is the distance, $g$ is the acceleration due to gravity, $h$ is the water level, $I$ is the bottom slope, and $C$ is a friction coefficient.

The presented solution thereby is not exact, because the solution is based on several assumptions, but compared earlier significant analytical solutions [Prandle and Rahman, 1980] [McDowell and O’Conner, 1977] all based on the earlier work of [Ippen, 1966], the amount of assumptions in Savenije’s work are few. Moreover, Savenije’s solution [Savenije, 1992] does not impose serious restrictions on the application to real estuaries.

Exponentially varying cross section

The method of Savenije [Savenije, 1992] makes use of an exponentially varying cross section. This cross-section corresponds to the observation of many estuaries [Ippen, 1966], and is shown to be an acceptable assumption in earlier work for the modeling of salt intrusion by Savenije. Hence:

$$B(x) = B_0 \exp\left(-\frac{x}{b}\right)$$ (2.9)

where, $B =$ the tidal mean width, $B_0 =$ the tidal mean width at the estuary mouth, $b =$ the convergence length, and $x =$ the distance measured upstream from the estuary mouth.
Combining assumptions

The combination of the St. Venants equation (2.8), the formula for the exponential cross section of the estuary (2.9), the principle of conservation of mass together and assumptions regarding the movement of water particles yields new equations.

These equations are expressions for the water level and the average flow velocity as function of the distance traveled by a water particle and give a relation between the tidal range and the tidal excursion.

The tidal damping however, was still dealt with empirically.

Phase lag equation

Later [Savenije, 1993] derived an equation for phase between the moment of high water and high water slack, respectively low water and low water slack:

\[ \epsilon = \arctan \left[ \frac{\omega b}{(1 - \alpha_c b)c} \right] \] (2.10)

where, \( \epsilon \) is the phase lag, \( \omega \) is the tidal frequency, \( c \) is the wave celerity, \( b \) is the convergence length of the stream width and \( \alpha_c \) is the convergence length of the cross sectional area.

Tidal damping

A few years after that [Savenije, 1998] presented an analytical expression for tidal damping making use of St. Venants equations, the equation for conservation of mass and the equation for conservation of momentum, which was not done in earlier mentioned equations.

Combining those aspects, the damping can analytically be described as:

\[ \frac{dH}{dx} = \frac{H}{h} \left( \frac{\frac{H}{h} - f \frac{g}{c^2} F \sin(\epsilon)}{1 + \frac{H}{h} \frac{g}{2F \sin c}} \right) \] (2.11)

where, \( H \) is the tidal range, \( x \) the distance, \( h \) is the stream depth, \( b \) is the convergence length, \( f \) is the friction number, \( c \) is the wave celerity, \( F \) is the Froude number, \( \epsilon \) is the phase lag, \( g \) = acceleration due to gravity and \( C \) is the friction coefficient.

In this equation the damping is primarily depending on the shape of the estuary compared to the amount of friction. When for example the friction term is dominant, there is tidal damping. When the converging term dominates there is tidal amplification.

Compared with results of numerical models the equation is quite good for a relative large range of \( H/h \). A drawback of the method is that it is limited to estuaries with a negligible bottom slope, which is often not the case in the upstream part of estuaries.

New equations for tidal damping

In a later stage [Savenije, 2000] came with a new equation for tidal damping. Inspired by the principle that virtually all physical processes become chaotic when we look at them in finer detail, whereby at a larger scale those processes tend to become predictable and many researchers whom tried to enhance their understanding of physical processes by going in to more detail ended up only becoming disappointed, Savenije developed an equation for tidal damping of a very simple form:

\[ y = \frac{H}{H_0} = 1 + \frac{F_{\text{tidal}}}{D} x \] (2.12)

where, \( y \) is the dimensionless tidal range, \( H \) the tidal range, \( H_0 \) the tidal range at the estuary mouth, \( F_{\text{tidal}} \) the tidal Froude number, \( D \) the Tidal damping scale and \( x \) the distance.

In the same paper Savenije also states the relative importance of the phase lag \( \epsilon \). In estuary hydraulics tidal waves are always of a mixed type. The phase \( \epsilon \) determines the character of the wave. Because the phase lag \( \epsilon \) is seldom systematically measured this requires an adjustment of the measurement protocols. [Savenije, 2000]

When the theory is compared with real estuary data, though results are satisfying, they are based on the very rough data for the phase lag \( \epsilon \).
River discharge

In derivation of the previous equations the influence of river discharge has been disregarded. Measurements in the Scheldt estuary have shown that as a result these equations are only valid in the lower part of the estuary.

This is not hard to imagine. In the downstream part of the estuary the cross section is large compared to that of the river cross section, dividing the river discharge over a large surface while in the higher part of the estuary the cross sectional area approaches the rivers cross-sectional area. This means the influence of the river discharge will dominate the tidal movement of the water.

The validity of the assumption that the effect of the river discharge can be neglected is analyzed and an expended equation is presented that accounts for the effect of river discharge is formulated.

Two aspects are considered: the influence of the river discharge on the phase lag $\epsilon$ and the influence of the river discharge on the tidal range.

The river will shift the moment of both HWS and LWS. Therefore two symbols are introduced: $\epsilon_L$, the phase lag between HW and HWS and $\epsilon_H$ for the phase lag between LW and LWS. Under the influence of the river $\epsilon_H$ will decrease and and epsilon $\epsilon_L$ increases.

The value of $\epsilon_H$ can even become negative: HWS occurs before HW. This phenomena is both observed in real measurements in estuary as in numerical models. Though the phenomena is described and modeled in [Horrevoets et al., 2004], no analytical equations are provided.

Analytical formulas for the mixed wave

In [Savenije et al., 2008] the full range of equations available for tidal movement in estuaries is provided.

The equations are presented in implicit form, in a governing dimensionless form and an explicit form. For the explicit form, families of solutions are presented using methods from [Toffolon et al., 2006] to write them to a non-iterative form. The non-iterative family of solutions for a mixed wave reads:

$$m = \sqrt[3]{27\chi + (9 - \gamma^2)\gamma + 3\sqrt{3}\sqrt{27\chi^2 + 2(9 - \gamma^2)\gamma\chi + 8 - \gamma^2}}; \quad (2.13)$$

$$\mu_m = \sqrt{\frac{1}{3\chi}(m - \gamma + \frac{\gamma^2 - 6}{m})}; \quad (2.14)$$

$$\lambda_m = \sqrt{\frac{\chi^2\mu_m^4 - \gamma^2}{4} + 1}; \quad (2.15)$$

$$\delta_m = \frac{\gamma - \chi\mu_m^2}{2}; \quad (2.16)$$

$$\epsilon = \arctan\left(\frac{\lambda_m}{\gamma - \delta_m}\right); \quad (2.17)$$

where, $m$ is meaningless parameter helpful for the calculation, $\chi$ is the friction number, $\gamma$ is the estuary shape number, $\mu_m$ is the velocity number for a mixed wave, $\delta_m$ is the damping number for a mixed wave, and $\lambda_m$ is the celerity number of a mixed wave.

2.2.3 The phase lag of the Scheldt estuary

The phase lag of the Scheldt estuary is approximated both in an analytical form, conform [Savenije et al., 2008] as by a numerical 1D model by [Horrevoets et al., 2004]. The analytical approximation is presented in figure 2.5.

The results of the analytical solution are rather different when compared with the 1D hydraulic model approximating the phase lag in figure 2.6.
Figure 2.5: Analytical approximation of the phase lag in the Scheldt estuary

Figure 2.6: Modeled phase lag along an estuary similar to the Scheldt estuary, computed with a 1D hydraulic model with river discharge of 100 m$^3$s$^{-1}$ [Horrevoets, 2002].
2.3 GPS

GPS satellites orbit around the earth twice a day while transmitting information in the form of radio signals to earth. GPS receivers receive these signals to determine the distance (pseudo range) to the satellites. The information received from several satellites is combined to determine the position of the GPS receiver. With the information of four satellites one can determine the 3D position (latitude, longitude, altitude) of the receiver. For a more precise explanation of the GPS one refers to relevant literature i.e. [Husti, 2000]. While the basic principle of location determination by GPS receivers is based on measuring pseudo ranges (distances) to referential points of satellites, several modes for this location determination are known. For this research relevant modes are referred to as Stand Alone GPS, EGNOS and DGPS. This section only intends to give an indication of the relevant methods and their basic principle of location determination.

2.3.1 Stand Alone GPS

Stand alone GPS is based on location determination by measuring pseudo ranges to the satellites transmitting high frequency radio waves. If the pseudo ranges to several satellites are known with the addition of the characteristics of the satellite, the location of the GPS receiver can be determined. Because of a number of influences, such as the ionosphere, the troposphere, clock errors, etc, the precision of the location determined by the GPS receiver is not very accurate. The deviations are very often in the order of 5-9 meter [Tiberius, 2003].

2.3.2 Local Differential GPS

Local differential GPS is a system whereby the receiver is positioned to a nearby reference station at a given location. Pseudorange measurements for both the receiver with the arbitrary location and the receiver with the given location are combined. This combination of information increases the precision of the system since multiple errors, like satellite position error and atmospheric delays, are the similar for both the reference and the user station. Local differential GPS precision is in the order of a few decimeters, while using high-end GPS receivers a precision of 1-2 dm can be achieved [Tiberius, 2003].

2.3.3 Wide Area Differential GPS

Wide area differential GPS uses the principal of differential GPS, with a network of reference stations. A network of 40 ground stations combined with three geostationary satellites transmitting radio signals are implemented throughout Europe. This system is referred to as ‘EGNOS’. These additional pseudo ranges to ground stations make it possible to determine one’s position within the precision of 1.5 meters. EGNOS is only operating in Europe. A similar system, named ‘WAAS’ is operational in North America.

Because the differential corrections are sent via the geostationary satellites, no additional hardware or infrastructure is needed. This makes EGNOS and WAAS easy to use and free of charge.

2.3.4 Doppler speed measurement

It is also possible to directly measure speeds by using the phenomena of the Doppler effect on the received carrier waves. These Doppler speed measurements are, compared to GPS position measurements, insensitive to atmospheric disturbances, and is the most convenient method of measuring speed. It is shown that a 10-second average speed can be measured with accuracy better than 5 cm/s [Chalko, 2009].

2.3.5 Data extraction

NMEA

GPS receivers compute navigation solutions. The extraction of these computed navigation solutions occur through the NMEA protocol. A brief description of the NMEA protocol can be found in [Langley, 1995].
A lot of low end GPS receivers do give NMEA sentences as output. High end receiver often also support raw data as output. While NMEA is very convenient in terms of use, it makes a number of post-processing techniques to increase the accuracy of found navigation solutions impossible.

2.3.6 WGS-84 reference frame

The World Geodetic System 1984 (WGS84) is the standard reference system for many GPS devices. Coordinates of the WGS84 reference system are expressed in a longitude and latitude and ellipsoidal height.

2.3.7 RD reference frame

RD-coordinates are coordinates on a national level for the Netherlands. The coordinates are used for a geographical positioning of objects, giving the objects both a X and Y coordinate. For an indication of height using RD-coordinates the height in NAP can be added. The X and Y coordinates are points relative to a place 120 km south of Paris. This makes that for the Netherlands all X and Y coordinates have positive values, whereby the X values are between 0 and 300 km, and the y values are between 300 and 620 km. The validity of the RD-coordinates is defined between the values for X of -7 and +300 km, and the values for Y between 298 and 629 km. [de Bruine et al., 2006]

2.3.8 WGS-84 to RD conversion

The conversion relevant for this project is the conversion from WGS84 ellipsoidal coordinates to RD-coordinates: Formulas for the conversion of coordinates are given in the article [Schreutelkamp and Strang van Hees, 2001].

The accuracy of the transformations are limited by the fact that the transformation can not always be described well by an simple transformation formula. The ascertained deviation are within a few decimeters [Schreutelkamp and Strang van Hees, 2001].
Chapter 3

Methods

3.1 GPS tests

This chapter contains a description of the GPS tests conducted for determining a suitable GPS receiver and mode for the GPS module for measuring the moment of tidal slack in estuaries. The tests using this set-up are conducted in a co-operation with Martin Valk [Valk 2011], who also did the data processing of the measurements.

The results of the static GPS measurements can be used as an indication for the precision with which the position of the buoys in the estuary will be known. The precision of the system is both relevant for the performance of the system as in determining and validating a proper calculation techniques for determining the moment of tidal slack.

3.1.1 GPS Set-up description

The set-up used to determine the performance of the different GPS systems is located on top of the NMI building in Delft. On top of the building three different GPS devices were placed: a Waspmote with a Vincotech A1084 receiver, a handheld Garmin GPS 76CSx with an external antenna, and a Septentrio AsteRx1 PRO 1.4 with an external antenna. See figure 3.1.

The location of the NMI building is chosen for several reasons. First, the roof of the NMI building has a clear view of the sky, so that satellites can be tracked from horizon to horizon. Secondly, the coordinates of NMI building pillars are known very precisely (order of millimeters). This makes it possible to measure both the precision as the accuracy of the GPS device used.

All measurement campaigns are conducted during a period of 24 hours. The tests have been performed with both Stand Alone GPS and Wide Area Differential GPS(EGNOS).

For a more detailed description of the set-up see appendix A

3.2 Building the GPS system

The idea of the GPS system can be fulfilled with different systems. In the search for the most suitable system, several systems passed in review. Important is the fact that the system should be affordable, robust and suitable for setting up a wireless network of GPS receivers.

A system called ‘Waspmote’ is considered very suitable and therefore chosen to be used.

This system is based on a wireless network using Waspmote and Meshlium. Waspmote is a microcontroller supporting a whole range of sensor devices and enables communication between different waspmote boards. All data is centrally collected by a Meshlium router. Waspmote supports different communication protocols, respectively: 802.14.4, ZigBee-Pro and RF. The module consists of several Waspmote boards, all equipped with a GPS receiver, a Xbee data sending module, a rechargeable battery and a solar panel. All data is centrally collected by a Meshlium router. When the data is centrally collected several storage options are possible. Waspmote and Meshlium are products of Libelium, see(www.libelium.com). The basic principle is presented in figure 3.2.

There are several reasons this system is considered suitable.
Figure 3.1: A photograph of the GPS set-up on the roof of the NMI building

Figure 3.2: This figure gives an overview of the GPS system and some of its components. While the measurements are conducted by the solar powered GPS receivers, data is transmitted in real time using a RF (868 MHz) protocol for transmitting data to the central router. This data transmission can be done in many configurations while both tree topologies, star topologies and a combination of both are possible. Whenever the data is collected at the central router, this data will be send to the World Wide Web by either a Ethernet cable of a Wi-Fi connection. Thereby data interpretation in university is possible.
The system is offered as a package where all components can relatively easy be matched both on hardware as software level. The system uses radio waves for data transmission for distances up to 12 km. The radio waves are broadcasted in a frequency band which is permit free in most of Europe. Therefore it can be used without any extra operation costs for the data transmission. The Waspmote comes with support for many different network topologies. By developing a smart topology a robust network for the Scheldt estuary can be created. And the system is relative cheap, when compared to alternatives.

3.3 Buoy model

A model to simulate the behavior of the buoy is made. The goal of the buoy model is twofold. In the first place, the model is a mean to get a rough insight into the buoy motion. Later, the model can be used to simulate data to test new tidal slack calculation algorithms.

3.3.1 Manual prediction of behavior

To build the model and to check the model afterwards, the behavior of the buoy is assessed manually in a couple of critical points. A schematic is given in figure 3.3.

Origin (1) When the buoy position is in the same vertical line as the anchor the buoy is drifting freely. The anchor chain does not offer significant resistance. The anchor chain hangs straight to the ground underneath the buoy. Acceleration occurs in the direction of the flow. At this point, the flow velocity is always increasing.

Free roaming (2) When the buoy is between the origin point and the point of maximum divergence the chain force increases slowly. Still this force remains so small that the buoy is not pulled back to it’s origin position. It is a passive resistance force. Acceleration is always in the direction of the flow.

Maximum divergence (2) At this point the buoy has drifted far enough for the anchor chain to become active. A larger part of the anchor chain is lifted from the ground. The chain counteracts the force exerted on the buoy and a point of maximum divergence is reached. This is the most interesting phase of the buoy movement, because it is difficult to predict on forehand and it is around here that the moment of slack should occur.

Total behavior (3) In short, the buoy is expected to move through the origin quite fast, while accelerating. This continues till the chain becomes active and an equilibrium is formed between the chain and the flow forces. When the flow velocity decreases, the buoy is pulled back by the chain to the point where the chain became active. Following this reasoning, this should be the point to look for in the data. Then the flow velocity increases in the opposite direction and takes the buoy through the origin towards it’s other equilibrium point.

3.3.2 Mathematical description

In general, the buoy movement is described as a 2nd-order differential equation in accordance with the general (dampened) mass-spring system.

The buoy is assumed to be a point mass, moving in just one dimension in the direction of the flow, with coordinate x. For the one dimension movement of the buoy only long wave types are taken into account. The buoy is subject to forces from the flow, \( F_{\text{flow}} \). The flow and therefore \( F_{\text{flow}} \) does not depend on location over the buoy path, only on time, namely the tidal cycle. \( F_{\text{flow}} \) is the driving force of the system.

Next, the buoy is hold in place by a chain, anchored to the sea-bottom. The chain more or less counteracts the motion of the buoy, by the weight and sheer of the chain, \( F_{\text{chain}} \). We model this \( F_{\text{chain}} \) as the spring in the system with spring ‘constant’ \( k \), but \( k \) is \( f(x) \).

The set is complete with the introduction of resistance factor \( \chi \) which depends on the relative buoy speed compared to the flow velocity.
Figure 3.3: The figure shows the different behaviors of the anchor chain. The anchor chain as vertical dead weight (1), as vertical dead weight and horizontal passive frictional force (1-2) and as active force when the chain is raised from the ground (2-3). At (2) the chain becomes ‘active’. The blue arrows depict the flow direction. The double arrow at (3) indicates that the flow velocity decreases, but does not has changed direction. The double arrow at (2) is to show the assumption that this is the position of the buoy while the tidal slacks occurs.

The general equation of this mass-spring system is then given by:

\[ m \ddot{x} + \chi(x) \dot{x} + k(x)x = F_{flow}(t) \]  

(3.1)

**Driving force**

As said, the driving force in the system is the force of the water on the buoy. This \( F_{flow} \) is proportional to the squared water flow velocity, mass of the water, effective surface of contact of the buoy to the water and draft coefficient. In formula form the equation reads:

\[ F_{flow} = \frac{1}{2} A_b c \rho v^2 \]  

(3.2)

In this equation it is assumed the \( A_b \) and \( c \) are dependent on the buoy shape only [McCormick, 2010]. The density of the water, \( \rho \), is dependent on the salinity of the water and may differ between 1000 – 1025 kg/m\(^3\) for fresh and salt water respectively. With the tide, the salinity of the water fluctuates between those two bounds. In this model however, the salinity is presumed constant at 1000 kg/m\(^3\).

The flow velocity for the tidal motion is assumed to be a nice harmonic function. On top of this, the flow induced by the river discharge \( v_q \) is superposed.

\[ v_{tidal} = \dot{v} \cos \omega t + v_q \]  

(3.3)

Besides this primal tidal motion and river discharge, higher harmonics and seiches can also influence the observed flow velocity. In the model, these aspects can be easily implemented by adjusting equation 3.3. Adding more temporary effects to equation 3.3 increases the accuracy of the description of the flow speed, but reduces the time span for which the equation is valid.

**Chain force**

The chain force is difficult to describe. The actual force depends on the length, weight, and the form of the chain over the depth. The problem is that this force is badly approximated with a linear spring constant \( k \). In fact, the chain has different behaviors, as already indicated in figure 3.3. Multiple mathematical descriptions can be given to model the different behaviors.
Linear behavior  The most simple representation of the chain is still to model the spring linear. The question is then what should be used as $k$. If the chain behaves linear, then $k$ is given by:

$$k_{lin} = \frac{mg}{d}$$  \hspace{1cm} (3.4)

Where $k_{lin}$ is the linear chain constant, $m$ the mass of the chain and $d$ the depth of the estuary.

Higher order behavior  Succeeding the linear approximation, higher order approximations can be used. These higher order approximations come closer to the behavior of the chain as sketched in ??.

$$k = |f(x^p)|$$  \hspace{1cm} (3.5)

Where $p$ is the order of the spring. Note that $k \geq 0$ for all $x$. The most extreme case of a higher order behavior is when the chain is described exponentially.

Discrete behavior  The discrete behavior of the chain can be represented with the use of if-else and min-max conditional functions. Besides the mathematical limitations this brings, it does not necessarily yield a better approximation than the options above. Regarding the mechanics of the chain and the buoy movement, there should be a transitional phase as well between every set of behaviors, which is not included.

The exception is when the buoy is suddenly pulled back by tension in the chain, and not by it’s weight. Under those circumstances the chain will directly counter all the forces on the buoy.

Noise-simulation

Interference parameters are included to simulate GPS data. The governing interference parameter is the GPS precision. GPS precisions are obtained by performing the tests as in ??

The GPS measurement error in two dimensions is added to the one-dimensional solution of the mass-spring system:

$$X = x + \Delta s, \quad Y = \Delta s$$  \hspace{1cm} (3.6)

$s$ is a normally distributed variable with $\mu = 0$ and $\sigma = \sigma_{GPS}$. $s$ is sampled individually for both $X$ and $Y$. Note that GPS the noise $\Delta s$ of $X$ and $Y$ have a certain covariance in reality, though this is not modeled.

3.4 Data interpretation methods

This section presents several aspects of the data interpretation methods. In the case of an autonomous operating system the GPS measurements have to be processed automatically and real-time by a computer. As a first setup a matlab script is written that performs the calculations a-posteriori for all three methods when measured with the developed GPS system.

A short description of the steps for automation of the calculation methods is given and a few adaptations of the methods are explained.

The scripts are found in appendix D.

3.4.1 Data conversion

Before the calculation methods can be applied the data has to be imported in Matlab and a series of conversions are done.

Data import  First the matlab script reads the data strings from the stored data txt-file and extracts the time, date, longitude, latitude, altitude, instantaneous velocity and direction of the velocity. This information is stored in one matrix for later calculation.
Conversion of reference system  After the matrix has stored all the information the WGS84-coordinates of the GPS measurement are recalculated to Rijksdriehoeknet coordinates (RD-coordinates), conform [Schreutelkamp and Strang van Hees, 2001]. The RD coordinates present the coordinates in a 2D reference frame, which is suitable for the calculation methods.

Translation of the axis  First the center point of the buoy motion is found using equation 2.1 and 2.2. Then the axis is translated such that the found center point has coordinate (0,0).

Rotation of the axis  After the translation, a rotation of the axis is done such that the x-axis is in the main direction of the buoy motion which is also the direction of flow. This flow direction is determined by fitting a first order polynomial function over the translated \( X_{RD} \) and \( Y_{RD} \) coordinates. For the first order polynomial two constant are determined:

\[
Y_{RD} = p_1 X_{RD} + p_2 
\]

where, \( p_1 \) and \( p_2 \) are constants determined by the function fit.

The constant \( p_1 \) is used for calculating the direction of flow. The angle \( \alpha \) of the flow is approximated by the following formula:

\[
\alpha = \arctan p_1 
\]

After the calculation of \( \alpha \), formula 2.5 and 2.6 are used to calculate the new coordinates for the rotated axis.

Result of the conversion  The translation and the rotation of the axis give satisfying results. Figure 3.4 shows the relative X and Y coordinates after translation and rotation of the axis.

The conversion method is verified here, because it is used in the validation of the buoy model, which part precedes the calculation section in the chapter results.

3.4.2 Automation of calculation methods  

With the conversions done, the calculation methods of [Lievens, 2010] can be implemented. In some cases the methods can be simplified, resulting in the ‘new radius’ and ‘new angle’ method.
The new radius method

In the radius method the moment on which the buoy passes through the origin is found by using Pythagoras on the x and y values (sec 2.1.2). This does not always yield unambiguous solutions. With the performed axis transformation, one has to look at the x-values only. Thereby the ‘new radius’ method searches simply for minimum x-values.

The new angle method

The angle method (sec 2.1.1) focuses on finding the position where the buoy clearly changes direction by having a fast decreasing or increasing angle. Just like the ‘radius method’ this method does not yield unambiguous solutions. The reason for the ambiguity is that the call ‘fast increasing’ is arbitrary.

Therefore another way of interpreting the angle of the buoy for determining the moment of tidal slack is introduced. This so called’maximum angle method’ determines the moment of tidal slack by looking for the maximum angle with the transformed axis. The formula for the angle in this method is:

$$\alpha = \arctan \frac{y}{x} K$$

where,

- $x$ = the distance of the buoy to the center in the direction of the x-axis [m]
- $y$ = the distance of the buoy to the center in the direction of the y-axis [m]
- $K = 1$ for quadrant 1 and 3 and $K = -1$ for quadrant 2 and 4.

By looking for the maximum angle the moment the buoy passes the origin is found. Note that the ‘maximum angle’ method searches for the same point as the ‘new radius’ method, while the original angle method searches for points around the maximum deviation of the buoy. Though this method is very similar to the minimum radius method, it uses more information in the form of both $x$ and $y$ values.

Velocity method

The velocity method (sec 2.1.3) is easy implementable once the axis transformation is done. However, the method only gives a graph of the velocity. From this, however, the moment of tidal slack is still not easily found. Again the phrase ‘when the velocity starts increasing’ is too arbitrary.

3.5 Sensitivity analysis of calculation methods

The different methods for determining the moment of tidal slack all have their own way of interpreting the GPS measurements. Besides that, different GPS receivers all have their own measurement precision. The measurement precision of the GPS receiver and the chosen calculation method influence the calculated moment of tidal slack.

To observe the influence of the precision of the GPS measurement on the precision of the calculation methods, simulations are made with different noise levels as described in ?? . The response of the different calculation methods on increasing $\sigma_{GPS}$ is investigated with a Monte Carlo approach as described below.

3.5.1 Monte Carlo model

The simulation for noise levels as described in section 3.3.2 is systematically repeated for many different $\sigma_{GPS}$. For each simulation the moment of tidal slack is calculated, using the different calculation methods. Because the process is repeated many times for each $\sigma_{GPS}$, standard deviations for the precision of the calculation methods can be determined.

The matlab script for this sensitivity analysis is found in appendix E.
Chapter 4

Results

4.1 GPS test results

In this section the results of the GPS measurements are presented in a bar plot, see figure 4.1. The plot shows the standard deviation in the upward direction and bias in the downward direction, expressing both accuracy and precision.

The bias expresses accuracy of the mean measurement compared to the actual position of the GPS receiver. The actual position of the measuring points is known in the order of [mm], but due to placement of the GPS receivers, it can be said that the actual position of the GPS receivers is known in the order of [cm].

The precision of the measurement is expressed as the value of the standard deviation compared to the mean measurement from the GPS receiver. Both the precision as the accuracy are expressed in three directions: North, East, and Height. Positional graphs from which the data is extracted can be found in appendix A.

4.1.1 Comparison of measurement results

The results of the precision and accuracy of the measurement give a clear difference in performance between the different GPS receivers. The high-end Serpentrio receiver performs the best, especially when EGNOS is enabled.

EGNOS does not seem to make a difference in case of the (mid-range) Garmin receiver, with similar scores for stand alone and EGNOS enabled.

The low-end Vincotech receiver performs the worst. It has the largest standard deviation of all and shows a very strange bias of no less than 4 m in the north direction.

The found results were expected beforehand, as the high-end receiver scores bests and the low-end receiver worst. Unexpected is, however, that EGNOS has no effect on the Garmin measurements and the very large bias of the Vincotech receiver.

4.2 Progress on the GPS system

For the GPS module it was chosen to use equipment from Libelium. Once the equipment was received, it became clear that a lot of fundamental programming was still needed to get the network operational.

Some progress is made till the point the module is able to transmit GPS data directly to a computer.

A more severe set back came in the form of a crash of the Meslium router. After a quick and unsuccessful test with a direct Wasmote-computer connection over a long distance along the Schie river, it was decided that measurements in the Scheldt river would no longer be possible in the time span of this thesis.

More progress was booked on the enclosure for the modules, which are ready for use.

Because of the unfinished nature it is decided to go into more detail about the GPS modules in appendix B.
Figure 4.1: GPS test results. The standard deviation and bias of the different receivers and modes are presented in a bar plot. For the bias absolute values are used.

4.3 Buoy model

In this section the results from the buoy model are presented. First the behavior of the buoy for different types of modeling of the chain is shown. Then the generated simulated GPS data is presented and compared with two sets of buoy motion data from [Lievens, 2010].

4.3.1 Buoy behavior

The buoy behavior is expressed in placement and speed of the buoy in time. The different chain modeling options as described in 3.3.2 are clearly visible.

Linear modeled chain

For a linear modeled chain, the largest deviation occurs as expected when the flow velocity and thus $F_{\text{flow}}$ is highest. Likewise the buoy moves through the origin when the flow velocity is zero, see figure 4.2. This means that for a linear modeled chain the inertia of the buoy is negligible. The moment of tidal slack is equal to the moment the buoy moves through the origin, as long as the inertia of the buoy is negligible. Therefore the calculation methods ‘minimum radius’ (sec: 2.1.2) and ‘maximum angle’ (sec: 3.4.2) are unbiased, because they use the moment of passing through the origin as moment for tidal slack.

Higher order approach

For higher order springs, the buoy motion becomes more like a step-function. The speed of the buoy increases dramatically around the moment of tidal slack and the inertia of the buoy gains more weight, resulting in an with the order increasing lag between the water motion and the buoy. This lag can become up to 10 minutes for an exponential modeled chain, see figure 4.3. As a result the moment of tidal slack does occur before the moment the buoy moves through the origin. The moment of tidal slack is no longer easily found. The ‘velocity method’ makes use of the highly increased speeds around the moment of tidal slack. The ‘minimum radius’ and ‘maximum angle’ methods have always a bias equal to the lag between the water and buoy motion.
Figure 4.2: Buoy behavior with linear modeled chain. The largest deviation occurs as expected when the flow velocity and thus $F_{\text{flow}}$ is highest. Likewise the buoy moves through the origin when the flow velocity is zero. The moment of tidal slack according to this model with linear modeled chain is equal to the moment the buoy moves through the origin. This is only true for cases in which the inertia of the buoy is negligible.

Figure 4.3: Buoy behavior for higher order modeled chains. The movement for a linear (–), 2nd (..), 4th (–.) and exponential (-) modeled chain are plotted. All deviations are enclosed by the deviation of the exponential chain, which shows almost step-function like behavior. Very noticeable are the high peaks in the buoy speed. The exponential variant is most extreme.
Figure 4.4: Behavior of the buoy when the chain behaves discrete. Two types of discrete behavior is modeled. The first behavior (green line) is when there is a critical $x_{\text{max}}$ for which the buoy is not able to travel further due to building tension in the chain. The second behavior (blue line) is when there is a critical $x_{\text{min}}$ under which the chain delivers no force.

**Discrete modeling**

Two different ways of discrete modeling have been tried. In the first option, there is a critical $x_{\text{max}}$ for which the buoy is unable to travel further and is pulled back by tension in the chain. In the second option, there is a critical $x_{\text{min}}$ under which the chain delivers no force on the buoy.

The discrete models are a combination of a certain order spring and the if-else conditions above. In figure 4.4 the two options are presented with a second order spring.

When the $x_{\text{max}}$ condition applies, the behavior of the buoy regarding the calculation of the moment of tidal slack is unchanged. The condition $x_{\text{min}}$ however does change the condition on which the moment of tidal slack is found. In concordance with the manual prediction (sec: 3.3.1) the moment of tidal slack finds place while the buoy is in an equilibrium position outside the origin. For linear and lower higher orders this position is easy found with the velocity method. For high orders, the discrete modeled buoy motion converges towards the motion described by the continuous modeled buoy motion.

**4.3.2 Model validation**

The different behaviors form the building blocks to come to a good approximation. The outcome of the different continuous models are compared against the data sets ‘Breskens’ and ‘85’ from measurement campaign 3 of Lievens [2010]. To make the comparison possible a series of transformations are performed on the measurement data as described in 3.4.2.

It appears all orders can be ‘calibrated’ to roughly reassemble the buoy motion. Therefore it was chosen to include a plot of the density function of the position of the buoy. This gives a more distinct view of the difference between the different models and the measurements.

Note that the density function has no real physical meaning. It only serves as a handy comparison tool. The position density function is a substitute for histograms to compare the simulated and measured data sets. The plot gives a more distinguished view than the positional plot only.
Figure 4.5: The deviation of the buoy in simulated data compared to the ‘Breskens’ data set. The blue line represents the measurement data. Note that the density function has no real physical meaning. The exponential approach is not included. The orders of the chain are: linear (), 2nd (), and 8th (). The linear approach shows aberrant behavior as it has a clear inflection point, unlike the data set. All other approaches follow roughly the behavior of the data set.

Breskens data set

The ‘Breskens’ data set was taken at the mouth of the Scheldt estuary. The maximum deviation is between 40-50m and has a rounded off character. The linear approach gives a clear inflection point, which is not found in the data. The exponential approach is beforehand unsuited for this set as deviations of this amplitude can only be reached by changing certain spring constants in the model in the order of $10^6$ and it shows generally a more step-like character. All other approaches can be ‘tuned’ to follow roughly the behavior of the data set, see figure 4.5. In the case of the ‘Breskens’ data set, the motion is bests reassembled by a second order modeled chain, see figure 4.6.

Data set 85

Data set ‘85’ is quite different than the ‘Breskens’ data set. The buoy deviation is far less, and has a very topped of character. The buoy motion is influenced by the river discharge; The buoy resides unequal times at its low and high maximum.

For the modeled data, the zero line is the ‘true’ origin point. For the data set, the ‘origin’ is calculated as the middle of the minimum and maximum deviation. As the river discharge shifts the whole buoy motion, the ‘true’ origin and the calculated origin point for the data set are different. This results in a seemingly drift of the simulation data. Higher order modeled chains are less influenced as the drift of the linear approach has a drift of more than 10m while the drift of the exponential approach is only a couple of meters.

The behavior is best described by the exponential approach, see figure 4.7.
Figure 4.6: The position density function of the buoy from simulated data compared to the ‘Breskens’ data set. The blue line represents the data set. The 2nd order approach (red line) follows the pattern of the ‘Breskens’ set bests. Again the linear approach shows completely different behavior. Higher order approaches result in a more pointy graph.

Figure 4.7: Buoy deviation of the simulated data compared to data set ‘85’. In data set ‘85’ (blue line) the influence of the river discharge is clear. The buoy resides unequal times at its low and high maximum. For the data set, the zero line is in the middle of the minimum and maximum deviation. For the modeled data, the zero line is the ‘true’ origin point. The river discharge provides not only the unequal distribution but also a drift. Because of that the measurement data should be shifted upwards.
Figure 4.8: The position density function of the buoy from simulated data compared to the ‘85’ data set. The exponential approach (black line) follows the pattern of the ‘85’ set bests. The river discharge influence results in a horizontal shift of the measurement data to the left compared to the ‘true’ zero point from the modeled data. The difference in height between the left and the right peak can also be explained by the river discharge. The shift is greatest for lower orders.

4.4 Calculation methods

The data conversion did yield two new variant methods and makes the velocity method implementable in Matlab. The results of the new methods are compared with their older counterparts. Also the velocity of the buoy according the velocity method is plotted. The calculations are performed on the ‘85’ data set.

4.4.1 The new angle method

The new angle method gives a better overview than the old angle method, see figure 4.9.

Every peak in the graph of the new angle method is a transition of the buoy from one end to the other. The maximum of each peak finds place when the buoy passes the origin.

Figure 4.9: The angle method defined by Lievens left and the redefined angle method for the same measurement right.
4.4.2 The new radius method

There is not much difference to be observed between the former and the new radius method, see figure 4.10. The downward peaks in the new radius method, however, go down all the way to zero, which makes them easier to detect automatically.

![Graph showing new radius method](image)

Figure 4.10: The radius method defined by Lievens left and the redefined angle method for the same measurement right.

4.4.3 Velocity method

The velocity of the buoy calculated with the velocity method is presented in figure 4.11. A lot of random noise is observed. The velocity peaks which should indicate the moment of slack are hard to detect.

![Graph showing velocity method](image)

Figure 4.11: The velocity of the buoy according to the velocity method.

4.5 Sensitivity analysis

The sensitivity analysis using the Monte Carlo simulation is performed for the methods for which the moment of tidal slack is unambiguously calculated. This includes the maximum angle, radius and new radius method.

The data sets 'Breskens' and '85' are used. The buoy motion is simulated as found most appropriate in 4.3.2. For both simulations the noise level is simulated with standard deviations for the GPS device, $\sigma_{\text{GPS}}$, varying from 0-20 m.

For all levels the process of sampling and the calculation of the moment of tidal slack is repeated thousand times.

The sensitivity of the new angle method and the velocity method are discussed separately.
4.5.1 Monte Carlo Results

The sensitivity analysis shows the sensitivity of the different calculation methods for $\sigma_{GPS}$, see figure 4.12. The maximum angle, radius and new radius method all show very similar response. This can be explained by the fact that all three methods look for the buoy passing through the origin.

There is a far greater difference in response between the data sets than between methods. The calculations for ‘Breskens’ are less sensitive for larger $\sigma_{GPS}$, than is the case for ‘85’. The relative better performance of the buoy ‘Breskens’ in the high ranges of standard deviations is caused by the relative larger amplitude of the movement of this buoy. (This difference in amplitude is shown earlier, see figures 4.5 and 4.7.)

In general, first there is a steady increase, then there is a certain $\sigma_{GPS}$ for which the origin-methods become unstable; For ‘85’ this is for a $\sigma_{GPS}$ of around 3.5[m], for ‘Breskens’ this is not as clear, but irregular behavior seems to start around a $\sigma_{GPS}$ of 10[m] and is less severe.

For both methods this critical value for $\sigma_{GPS}$ is around $\frac{1}{4}$ of the maximum buoy deviation.

Unstable in this context is used as a term to indicate that the yielded solutions for the moment of tidal slack occur more randomly.

For very high relative values of $\sigma_{GPS}$, the behavior can be assumed to be truly random as the relation between $\sigma_{GPS}$ and $\sigma_{calculation}$ becomes linear. Put differently, for too high values the normally distributed $\sigma_{GPS}$ results in a normally distributed calculated moment of tidal slack.

In the next figure 4.13, a detail of figure 4.12 is shown. It is seen that methods perform better for buoy ‘85’ in the lower regions of $\sigma_{GPS}$. This is caused by the fact that buoy ‘85’ passes the center relatively fast.

4.5.2 The angle method

No unambiguous algorithm to calculate the moment of tidal slack by means of the angle method is written. Therefore the sensitivity of the calculation method is described by manual interpretation.

Because the angle method is based on finding the moment where the buoy clearly starts moving to the other side, the method searches for a shifting angle of the buoy. The noise of the buoy is simulated as a random distributed variable with a covariance of zero. Therefore the noise level will influence the calculated angle randomly.
Interpreting a shifting angle of the buoy over several measurement points will therefore require a more sophisticated calculation method based on multiple data points. If no sophisticated technique is used, the moment of tidal slack will be determined randomly because the shift of the angle between two measurements will mainly be caused by noise of the GPS receiver.

4.5.3 Velocity method

The velocity method is very sensitive for the noise of the GPS measurement; In the case of a relatively low noise level of the GPS measurement, for example $\sigma_{GPS}=1m$, the velocity method is not useful if no filtering methods are performed. This can be seen in figure 4.14. In this figure both the theoretical speed of the buoy without measurement noise is shown, as the speed after the measurement with measurement noise of $\sigma_c = 1m$.

In the new graph with measurement noise velocity almost all peaks are caused by the noise. This is caused by the relative low theoretical buoy speed.

![Graph showing velocity of buoy with and without noise](image)

**Figure 4.14**: The velocity of the buoy with noise (bottom), and the velocity of the buoy without noise (top). The buoy with noise ($\sigma_{GPS}=1m$) is following a visually random pattern with no distinct movement patterns.

A new approach to the velocity method

The velocity method is a method very sensitive for noise of GPS measurements. As an alternative to calculate the speed of the buoy Doppler measurements for the speed can be used.

These Doppler measurements give an instantaneous velocity of the GPS receiver. Because the instantaneous velocity of the GPS receiver is most often dominated by the turbulent movement of the buoy, the instant velocity does not seem like a proper method to determine tidal slack.

However when a couple of Doppler measurements are averaged a direction a speed which is representative for the movement of the buoy can be determined. This method is not tested, because no data is available.
Chapter 5

Conclusions

A GPS system to monitor buoy motions is in development. When this GPS system is operational, tidal slack in estuaries can be monitored and more insight in the hydraulics of the estuary is gained. This hydraulic information is directly valuable for the shipping industry and can be used to validate both proposed analytical formulas as numerical models. Providing the hydraulic information gained from the GPS system for shipping industries can also be commercially exploited. Models validated with a more complete set of hydraulic data than water levels alone have increased reliability and make stronger evidence for decision makers.

For a working GPS system a choice of hardware had to be made, and calculation algorithms have to be developed. To come to a calculation method the buoy movement has to be better understood. This thesis has provided in a choice of hardware, insight in the buoy movement and interpretation and improvement of existing calculation methods.

For the hardware a Waspmote-board and Meshlium router were chosen. Together they provide a complete solution for the GPS system, including a GPS receiver. The solution combines adaptability with relative low costs. A low cost solution increases the chance of actual large scale implementation and adaptability ensures the system to function at more locations than just the test location.

An enclosure was made to ensure a waterproof protection for the module, thereby providing in a practical step for measurements.

The Waspmote-Meshlium solution has however some unresolved practical issues, which have to be sorted out before testing can find place. Setting up the network involves a great deal of software programming, which is best handled by someone from the programming and electronics field.

Another hardware issue is the choice of GPS receiver. The performed GPS measurements suggest looking into a more expensive GPS receiver than the with Waspmote included Vincotech-A1084 receiver as this receiver shows anomalous behavior. Expensive high-end GPS equipment, however, has a great influence on the cost of the module, reducing the feasibility of the project. The importance of precise GPS measurements had to be established first. For this we look at the results from the buoy model and sensitivity analysis.

The buoys can be modeled as a mass-spring system. Different approaches in the modeling of the spring result in a set of different motional behaviors. As long as several options are open, the reliability of the buoy model is limited. The model is calibrated on two different measurements. Validation of the model on more data sets will make the model more reliable and conclusions based on the model stronger.

From two different data sets two different modeling approaches were chosen to fit bests: a second order spring for the ‘Breskens’ data set and an exponential spring for the ‘85’ data set. These approaches are used in the sensitivity analysis of the calculation methods.

Independent from which approach fits bests, a lag between buoy motion and water motion is observed for higher order springs as inertia becomes more important due to increased velocities. As result the buoy does not pass the origin on the moment of tidal slack, but up to 10 minutes later. This means that calculation methods which point to the moment the buoy passes the origin are biased. This are the minimum and new minimum radius method and the maximum angle method. More insight in this bias can be gained when the model is calibrated on more measurement data.

The angle method and the velocity method are too arbitrary in form to come to conclusions about possible bias. The methods look, however, for a point in time which is always before the moment of
slack, so an unknown negative bias is expected.

Besides that some methods are always biased, the different methods respond to the precision of the GPS measurements, $\sigma_{GPS}$. This response of the different methods is simulated with the calibrated buoy model with a Monte Carlo approach.

The origin orientated methods show similar response for increasing $\sigma_{GPS}$. There is a certain critical value for $\sigma_{GPS}$ for which the methods become unstable. This $\sigma_{GPS_c}$ is $\frac{1}{4}$th of the maximum buoy deviation.

For the velocity method very precise measurements are needed. For a $\sigma_{GPS}$ of 1m the buoy velocity is already drowned in the noise, rendering the method useless. The velocity method can still be used, but then direct Doppler speed measurements are needed, with a minimal precision in the order of cm/s.

As a result, if the moment of tidal slack has to be found with the velocity method, a high-end receiver has to be used. In case of one of the origin orientated methods $\sigma_{GPS}$ should be always smaller than $\sigma_{GPS_c}$. As no of the GPS receivers recorded standard deviations bigger than 2m all receivers pass this criteria. The according standard deviation of the found solutions is in the order of 2min for the Garmin and Vincotech receivers and 30s for the Septentrio. With a possible bias of 10 minutes for the method the moment of tidal slack can be found with an accuracy in the order of 15 minutes with all tested GPS receivers. This is already an improvement compared to the called half hour deviation available from hydraulic models.

Therefore, the GPS system to monitor tidal slack in estuaries is still a promising method.
Bibliography


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