Failure mechanism of cutting submerged frozen clay in an arctic trenching process

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Repository version 15-10-2013
Failure mechanism of cutting submerged frozen clay in an arctic trenching process

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Abstract

This version of the thesis only covers the literature study. The results analysis and conclusion of the thesis are confidential.

Pipelines in arctic waters are at risk of being damaged by gouging ice masses. To protect these pipelines, they can be buried in trenches. When trenching in arctic clayey soil, subsea permafrost can be encountered. The main objective of this research is to study the failure mechanism of frozen clay as encountered in permafrost regions. Knowledge of the cutting mechanism leads to the ability to calculate cutting forces and the specific energy required to excavate material.

To find the failure mechanism and answer the research question, a cutting setup was designed and built and cutting experiments were conducted. The setup was designed, based on requirements that result from studied literature on frozen clay and cutting theories. A series of cutting experiments were conducted where a slab of frozen clay was pressed against a transparent wall and the top layer was cut off while filming the process with a high speed camera. The results are measurements of the cutting forces and observations of the failure mechanism.
Table of contents

Introduction .................................................................................................................. 1
  1.1 Context of this research ...................................................................................... 1
  1.2 Problem description ........................................................................................... 1
  1.3 Problem definition ............................................................................................... 1
  1.4 Research approach .............................................................................................. 1

2 Trenching in the arctic .......................................................................................... 3
  2.1 Subsea permafrost ............................................................................................... 3
  2.2 Seabed gouging .................................................................................................... 5
  2.3 Trenching .............................................................................................................. 5
  2.4 Properties of arctic clay obtained by sampling .................................................... 6
  2.5 Conclusion ............................................................................................................ 6

3 Clay, frozen clay and ice ......................................................................................... 7
  3.1 Clay ....................................................................................................................... 7
  3.2 Mechanical properties frozen clay ...................................................................... 11
  3.3 Ice ......................................................................................................................... 15
  3.4 Conclusion ............................................................................................................ 17

4 Cutting theories ....................................................................................................... 19
  4.1 Soil mechanics ..................................................................................................... 19
  4.2 Cutting theories ................................................................................................... 22
  4.3 High strain rates .................................................................................................. 31
  4.4 Effect of discontinuities in material on the cutting process ................................. 32
  4.5 Calculating the specific cutting energy ............................................................... 32
  4.6 Conclusion ............................................................................................................ 32

References ................................................................................................................ 33
## Nomenclature

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>Adhesive force</td>
<td>[N]</td>
</tr>
<tr>
<td>a</td>
<td>External undrained shear strength</td>
<td>[Pa]</td>
</tr>
<tr>
<td>C</td>
<td>Cohesive force</td>
<td>[N]</td>
</tr>
<tr>
<td>c</td>
<td>Internal undrained shear strength</td>
<td>[Pa]</td>
</tr>
<tr>
<td>c'</td>
<td>Pseudo cohesion</td>
<td>[Pa]</td>
</tr>
<tr>
<td>E_sp</td>
<td>Specific cutting energy</td>
<td>[Pa]</td>
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<tr>
<td>F_A</td>
<td>Adhesive force</td>
<td>[N]</td>
</tr>
<tr>
<td>F_AVG</td>
<td>Average cutting force</td>
<td>[N]</td>
</tr>
<tr>
<td>F_H</td>
<td>Horizontal cutting force</td>
<td>[N]</td>
</tr>
<tr>
<td>F_MAX</td>
<td>Maximum cutting force</td>
<td>[N]</td>
</tr>
<tr>
<td>F_N</td>
<td>Normal force</td>
<td>[N]</td>
</tr>
<tr>
<td>F_S</td>
<td>Shear force</td>
<td>[N]</td>
</tr>
<tr>
<td>F_T</td>
<td>Tension force</td>
<td>[N]</td>
</tr>
<tr>
<td>F_V</td>
<td>Vertical cutting force</td>
<td>[N]</td>
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<tr>
<td>F_{friction}</td>
<td>Horizontal friction force</td>
<td>[N]</td>
</tr>
<tr>
<td>h_c</td>
<td>Cutting depth</td>
<td>[m]</td>
</tr>
<tr>
<td>h_k</td>
<td>Height cutting tool</td>
<td>[m]</td>
</tr>
<tr>
<td>l_f</td>
<td>Length of the failure plane</td>
<td>[m]</td>
</tr>
<tr>
<td>l</td>
<td>Length of the cut</td>
<td>[m]</td>
</tr>
<tr>
<td>l_a</td>
<td>Length of the surface between chip and cutting tool</td>
<td>[m]</td>
</tr>
<tr>
<td>m_wc</td>
<td>Weight water content</td>
<td>[%]</td>
</tr>
<tr>
<td>m_s</td>
<td>Weight of the solids</td>
<td>[kg/m\textsuperscript{3}]</td>
</tr>
<tr>
<td>m_l</td>
<td>Weight of the liquid</td>
<td>[kg/m\textsuperscript{3}]</td>
</tr>
<tr>
<td>N_1</td>
<td>Normal force on the cutting tool</td>
<td>[N]</td>
</tr>
<tr>
<td>N_2</td>
<td>Normal force on the shear layer</td>
<td>[N]</td>
</tr>
<tr>
<td>P</td>
<td>Cutting power</td>
<td>[Nm/s]</td>
</tr>
<tr>
<td>Q</td>
<td>Volume flow rate</td>
<td>[m\textsuperscript{3}/s]</td>
</tr>
<tr>
<td>r_F</td>
<td>Cutting force ratio</td>
<td>[-]</td>
</tr>
<tr>
<td>S_1</td>
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<td>[N]</td>
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<tr>
<td>S_2</td>
<td>Shear force on the shear plane</td>
<td>[N]</td>
</tr>
<tr>
<td>s</td>
<td>Saturation</td>
<td>[%]</td>
</tr>
<tr>
<td>T</td>
<td>Temperature</td>
<td>[°C]</td>
</tr>
<tr>
<td>T_0</td>
<td>Reference temperature of -1°C</td>
<td>[°C]</td>
</tr>
<tr>
<td>V_{pore}</td>
<td>Pore space volume</td>
<td>[m\textsuperscript{3}]</td>
</tr>
<tr>
<td>V_{cutting}</td>
<td>Cutting velocity</td>
<td>[m/s]</td>
</tr>
</tbody>
</table>
\( w \) \hspace{1cm} Cutting width \hspace{1cm} [m]
\( w_f \) \hspace{1cm} Width of the failure plane \hspace{1cm} [m]
\( W_2 \) \hspace{1cm} Water pressure force on the cutting tool \hspace{1cm} [N]
\( W_c \) \hspace{1cm} Work \hspace{1cm} [Nm]

\( \alpha \) \hspace{1cm} Blade angle \hspace{1cm} [°]
\( \beta \) \hspace{1cm} Shear angle \hspace{1cm} [°]
\( \gamma \) \hspace{1cm} Orientation of the shear plane \hspace{1cm} [°]
\( \delta \) \hspace{1cm} External friction angle \hspace{1cm} [°]
\( \varepsilon \) \hspace{1cm} Strain rate \hspace{1cm} \([s^{-1}]\)
\( \dot{\varepsilon} \) \hspace{1cm} Reference strain rate of 1 s\(^{-1} \) \hspace{1cm} \([s^{-1}]\)
\( \dot{\varepsilon}_c \) \hspace{1cm} Strain rate for cohesion \hspace{1cm} \([s^{-1}]\)
\( \dot{\varepsilon}_a \) \hspace{1cm} Strain rate for adhesion \hspace{1cm} \([s^{-1}]\)
\( \eta \) \hspace{1cm} Angle of cutting force \hspace{1cm} [°]
\( \lambda_c \) \hspace{1cm} Strengthening factor cohesion \hspace{1cm} [-]
\( \lambda_a \) \hspace{1cm} Strengthening factor adhesion \hspace{1cm} [-]
\( \rho_d \) \hspace{1cm} Density \hspace{1cm} \([\text{kg}\cdot\text{m}^{-3}]\)
\( \sigma \) \hspace{1cm} Normal stress \hspace{1cm} [Pa]
\( \sigma' \) \hspace{1cm} Effective stress \hspace{1cm} [Pa]
\( \sigma_0 \) \hspace{1cm} Reference tensile strength \hspace{1cm} [Pa]
\( \sigma_1 \) \hspace{1cm} Major principal stress \hspace{1cm} [Pa]
\( \sigma_2 \) \hspace{1cm} Minor principal stress \hspace{1cm} [Pa]
\( \sigma_m \) \hspace{1cm} Compressive strength in uniaxial test \hspace{1cm} [MPa]
\( \sigma_T \) \hspace{1cm} Tensile strength \hspace{1cm} [Pa]
\( \sigma_x \) \hspace{1cm} Inclined stress \hspace{1cm} [Pa]
\( \tau \) \hspace{1cm} Shear stress \hspace{1cm} [Pa]
\( \varphi \) \hspace{1cm} Internal friction angle \hspace{1cm} [°]
List of figures

Figure 2.1: Schematic representation of a subsea permafrost profile. 3
Figure 2.2: Approximate distribution of subsea permafrost in the arctic area. 4
Figure 2.3: Schematic figure of seabed gouging by an iceberg. 5
Figure 3.1: Differentiation of soil type based on grain size. 7
Figure 3.2: Silica layers building units, on the left the tetrahedron, on the right the octahedron. 8
Figure 3.3: From left to right an octahedral silica sheet and a tetrahedral silica sheet. 8
Figure 3.4: Different stacking configurations of clay types common in the arctic area. 9
Figure 3.5: Characteristic curve of unfrozen volumetric water content of clay. 10
Figure 3.6: Water migration during the freezing process of fine grained soil. 11
Figure 3.7: Influence of temperature on the compressive strength of frozen clay. 13
Figure 3.8: Influence of the water content on the compressive strength of frozen clay. 14
Figure 3.9: Influence of the strain rate on the compressive strength of frozen clay. 14
Figure 3.10: Tensile strength of frozen clay at different weight water contents. 15
Figure 3.11: Phase diagram water. 17
Figure 3.12: Compressive failure mode of ice at different strain rates. 17
Figure 4.1: Phases in a soil. 19
Figure 4.2: Stresses acting on a soil element. 20
Figure 4.3: The Mohr circle including cohesion and internal friction. 21
Figure 4.4: Failure mechanisms; the flow type, tear type, shear type and the curling type. 23
Figure 4.5: Forces on a layer of soil cut. 24
Figure 4.6: Resulting forces from the adhesion and cohesion on a layer cut. 25
Figure 4.7: Mohr circles describing the stress state of the tear type. 27
Figure 4.8: Phenomenological failure model by Verhoef. 28
Figure 4.9: Cutting model of Evans (1964). 29
Figure 4.10: Influence of strain rate at shear strength of clay, semi-logarithmic. 31

List of Tables

Table 3.1: Mechanical properties of frozen clay. 15
Table 3.2: Mechanical properties of ice. 16
Table 4.1: Which failure type for which type of soil? 23
Table 4.2: Cutting forces for different failure mechanisms. 30
1 Introduction

1.1 Context of this research

As the oil price rises and the demand for oil increases, drilling for oil in harsh environments becomes profitable. Vast amounts of oil and gas can be found under the continental shelves of the arctic area. As oil companies are planning to exploit the oil field in this area, new technological challenges need to be faced. One of these challenges is the installation and protection of offshore pipelines.

1.2 Problem description

Ice masses that drift through arctic waters gouge into the seabed when they float into shallower waters. Pipelines in these waters are at risk of being damaged by these ice masses. To protect them, they need to be buried into the soil of the continental shelves. One of the challenges of an arctic trenching excavation project is the excavation of submerged frozen soil.

The seabed in the arctic can consist of different layers of cohesive and cohesionless sediments mixed with pebbles and boulders. To predict the trenching process in such a soil the properties and cutting mechanisms of the different types of soil need to be understood. In the upper layers of the seabed, permafrost can most likely be encountered in clay (Osterkamp, 2001), for this reason this research is focused on cohesive soil.

1.3 Problem definition

Calculations of cutting forces in soil are based on a failure mechanism and the relevant mechanical soil parameters. In order to calculate the cutting forces and the production of an arctic trenching tool the failure mechanism of frozen soil has to be found. This leads to the questions investigated in this thesis:

- What is the failure mechanism of frozen clay in a trenching process?
- Which cutting theory can be used or altered to predict the cutting forces when cutting frozen clay?

By answering these questions the soil parameters that needs to be samples upfront of a trenching project to be able to calculate the production of the trenching tool are known. This are the parameters that are included in the frozen clay cutting model.

1.4 Research approach

To find the failure mechanism of frozen clay, linear cutting experiments were conducted. The cutting setup was built, based on requirements resulting from the literature study. The experiments resulted in cutting forces and videos of the failure mechanism. To find a cutting theory that predicts the cutting forces in frozen clay, cutting theories based on the found failure mechanism were compared to the measurements and differences are explained and adapted.
2 Trenching in the arctic

Pipelines in arctic waters are at risk of being damaged by floating ice masses. A strategy to eliminate this risk is to bury the pipeline in the seabed. This chapter answers the question: can subsea permafrost be encountered while trenching in the arctic soil, and in what types of soil is subsea permafrost most likely to be encountered? The first part of this question will be answered by comparing the depth and location of subsea permafrost to the required trench depth. The strategy to find the type of soil in which frozen layers can be encountered is by comparing the location and depth of subsea frozen layers in the seabed to the types of soil at these locations. In the first and second section subsea permafrost and seabed gouging are described. In the third and fourth section the trenching and data on subsea permafrost are discussed.

2.1 Subsea permafrost

Permafrost is defined as soil that stays frozen for two consecutive winters and the summer in between. Subsea permafrost is found beneath the seabed on the continental shelves of the arctic seas. Osterkamp (2001) extensively researched subsea permafrost. He explained how the subsea permafrost originates from the most recent ice ages, and remains frozen for times to come.

2.1.1 Formation of subsea permafrost

Subsea permafrost formed in arctic area during the most recent ice ages. The sea level dropped, exposing parts of the continental shelves to the subzero arctic temperatures. The soil froze uniaxial from the top downwards.

After the ice ages, the water level rose and the permafrost submerged. The frozen soil was exposed to water temperatures of 1°C to 4°C, and the top layer thawed. Due to the insulating properties of the thawed layer on top of the frozen soil, the permafrost survived. Nowadays the permafrost is very slowly degrading due to the subsea conditions. Figure 2.1 shows a schematic representation of a subsea permafrost profile.

![Figure 2.1: Schematic representation of a subsea permafrost profile.](image-url)
Osterkamp found a typical thawing layer thickness of 10 m to 100 m, but also thicknesses of less than 1 m. The temperatures of the permafrost under these thawed layers is between -2 °C and -6 °C. The minimal sea level in the past has been 100 m lower than nowadays. He expects to find permafrost up to these water depths. The thickness of the permafrost layer can be hundreds of meters. Fairbanks (1989) and Romanovskii (2004) suggested that ancient sea levels were 121 m lower than the present sea level, they expect to find permafrost up to these water depths. Russian researchers even mentioned sampled permafrost at water depths of 150 m. These findings have not been published.

2.1.2 Location of subsea permafrost

Osterkamp developed a map that indicates the presence of permafrost in the arctic area. The map is shown in Figure 2.2. Due to the lack of data obtained by drilling, probing and sampling, parts of the map are based on geological and geophysical models. These models use water temperature, salinity, and water depth to predict where permafrost can be expected. Based on this map permafrost can be found in the continental shelves north of Russia and Alaska.

![Figure 2.2: Approximate distribution of subsea permafrost in the arctic area.](source: Subsea permafrost - Osterkamp 2001)

2.1.3 Depth of permafrost in the seabed

Field data and modeling results conclude that impermeable layers, low temperatures, high ice contents and low geothermal heat flow from below increase the chance of survival of permafrost. Subsea permafrost relatively close to the sea bed is likelier to be found in cohesive soils (Osterkamp 2001). This leads to the important conclusion: Permafrost is most likely to be encountered in the upper 10 m of the seabed when clayey soil is excavated.
2.2 Seabed gouging

Pipelines in arctic waters are at risk of being damaged by the keels of ice bergs and ice ridges. When these ice features float into shallower water, the keel eventually touches the sea bottom. The keel will plow through the seabed as long as the driving forces, e.g. inertia of the ice mass, current or wind loads are big enough, and the keel is stronger than the seabed. This process of the keel plowing through the seabed is referred to as seabed gouging (Barette 2011, King et al. 2012).

Figure 2.3 shows a schematization of seabed gouging. Barrette (2011) distinguishes three different zones in the gouged seabed. In the first zone the soil is displaced to the front or in lateral direction leaving an open space. Soil in the second zone is dragged forward and deformed plastically. Soil in the third zone is deformed elastic.

The required burial depth of an arctic offshore pipeline is specific for each pipeline route and depends on many parameters, of which the most important is the expected maximum gouge depth within a certain return period. Also the thickness of zone 2 and 3 play an important role. The thickness depends on the type of soil, the gouging speed and the shape of the keel. Pipelines are buried beneath or in the third zone, depending on the amount of allowable stress on the pipeline.

![Figure 2.3: Schematic figure of seabed gouging by an iceberg.](image)

Gouges can be several kilometers long and up to 5 m deep (Barrette, 2011, Barrette et al., 2012). Blasco et al. (1998) reported an extreme event in the Beaufort sea with a gouging depth of 8.5 m. The maximum water depth of gouges depend on ice mass dimensions. Ice bergs in the arctic area can have keel depths up to 230 m (Barker et al., 2004).

2.3 Trenching

In high risk areas, pipelines are buried in trenches to isolate them from external hazards. The trenches can be dug before the pipe is installed; pre-trenching, or after the pipe is installed; post-trenching. Pre-trenching is preferred when the soil is very inconsistent or unpredictable (Frederiks, 2011). The
continental shelves are dotted with large boulders, making pre-trenching most probably the preferred installation technique in these waters.

2.3.1 Trench dimensions and shape

Due to seabed gouging, relatively deep trenches are required in the arctic area. King et al (2012) and Jukes et al (2011) found required trench depths in excess of 9 m. When the seabed is frozen, the walls of the cut can remain stable as long as they are frozen. Possibly the parts of the trench in frozen soil can be constructed with vertical slopes. This type of trench is called a box cut. The advantage of a box cut is a lower excavated volume. Whether frozen vertical slopes are stable, and how fast melting makes them unstable is outside the scope of this research.

2.4 Properties of arctic clay obtained by sampling

Miller and Bruggers (1980) researched drilling samples from the Alaskan Beaufort sea. In 5 of the 20 samples they encountered permafrost in the first 10 m. These samples all where characterized by a top layer of clay with an undrained shear strength of 100 kPa in unfrozen condition. The weight water content is near the plastic limit, around 18 %. The liquid limit of the clay was typically 35 %. The temperature in the upper layer of the permafrost is near the thawing temperature, -6 °C to -2 °C.

Viscosi-Shirley et al. (2003) investigated surface sediments in the Siberian-Arctic shelf. Concluding that illite, chlorite and smectite are the most common clay minerals. Naidu et al. (1982) researched soil samples and concluded that in the East Siberian, Chukchi and western Beaufort seas, illite is the most abundant clay mineral.

2.5 Conclusion

Subsea permafrost can be found in the continental shelves of the arctic seas at water depths up to 150 m. From above, the permafrost is isolated by a thawed layer. This layer has a thicknesses between 1 m and 100 m. Clay has better isolating properties than coarser soils. Consequently the more shallow subsea permafrost, which is located in the top 10 m of the seabed, is encountered in clayey soils. Seabed gouging by ice masses in the arctic area will demand for trenches over 9 m deep. This leads to the conclusion that permafrost in arctic areas can be encountered while excavating clay in a trenching process.
3 Clay, frozen clay and ice

The previous chapter concluded that frozen clay can be encountered while trenching in the arctic seas. This chapter describes clay and frozen clay in order to get an understanding of the material and the freezing process and to be able to interpret observations made during the experiments. In the first section the mineral structure and the freezing process of clay are described. Then the literature on the mechanical properties of frozen clay is summarized and compared in the second section. In the third section ice is described.

3.1 Clay

Clay is a type of soil that consists of very small particles. These particles can be of different types, all with specific properties. In literature, clay particles are commonly defined only by their particle size, as shown in Figure 3.1. A more complete definition is formulated by Mitchell (2005), who defines clay by its four most important properties:

- Small particle size.
- Negative electrical charge.
- Plastic behavior when mixed with water.
- High weathering resistance.

Furthermore clay has a low permeability, and cohesive strength.

![Figure 3.1: Differentiation of soil type based on grain size.](image)

3.1.1 Structure of clay minerals

Clayey soil consists typically of a mixture of several kinds of clay minerals, these clay minerals consist of different silica layers stacked in a specific configuration. The silica layers are built from tetrahedron or octahedrons units. These units are composed of silicon, oxygen and different type of cations, like aluminum and magnesium ions.

**Tetrahedrons and octahedrons**

The building blocks of clay minerals are tetrahedrons and octahedrons. These basic units consist of specific atoms in a defined configuration. The tetrahedron consist of four oxygen cations, which are positive charged ions, and one silica atom. The octahedron is built from a cation, usually an aluminum or magnesium ion, and six oxygens or hydroxyls. Figure 3.2 shows the two unit cells.
**Tetrahedral and octahedral sheet**

By positioning the basic units in a specific configuration a silica sheet is formed. The tetrahedrons form the tetrahedral silica sheet by sharing three of their four vertices, the octahedrons form the octahedral silica sheet is formed by sharing all their vertices. The pattern are shown in Figure 3.3.

![Diagram of silica layers building units](image)

**Figure 3.2:** silica layers building units, on the left the tetrahedron, on the right the octahedron.

![Diagram of tetrahedral and octahedral sheet](image)

**Figure 3.3:** From left to right an octahedral silica sheet and a tetrahedral silica sheet.

### 3.1.2 Types of clay

By stacking the silica sheets, the crystal structure of a clay mineral is formed. Two different types of stacking configurations are distinguished. The 1:1 type, where a tetrahedral sheet is bonded to an octahedral sheet. And the 2:1 type, where one octahedral sheet is sandwiched between two tetrahedral sheets. By stacking in these different patterns, different subgroups of clay are formed. Figure 3.4 shows the different stacking configurations of minerals that are common in the arctic area. Different types of clay are built from sheets containing specific types of cations in the octahedrons. Viscosi-Shirley et al. (2003) stated that Illite, smectite and chlorite are the most common mineral groups in arctic clay. A map of the appearance of these clay minerals in the Siberian arctic shelf is shown in Appendix F.

#### 3.1.2.1 Illite

The illite mineral consists of a stacking of two octahedral and one tetrahedral sheets. An interlayer of potassium is bonded between two tetrahedral sheets, shown in Figure 3.4. Only the outside of a illite mineral has the ability to bond to water, making it a clay mineral that does not show a lot of swell. illite minerals are flaky, have usually an hexagonal outline and are smaller than 1 μm.

#### 3.1.2.2 Smectite

Smectite minerals consist of an octahedral sheet that is sandwiched between two tetrahedral sheets, shown in Figure 3.4. The bonds between the successive layers are cationic bonds and van der Waals bonds. These bonds are easily broken when water is absorbed. Up to three layers of water molecules...
can be bonded to the interlayer sheets of smectite, making it a swelling clay mineral. Smectite minerals appear in very thin flakes.

### 3.1.2.3 Chlorite

The chlorite mineral consist of continuous stacking of tetrahedral and octahedral sheets, shown in Figure 3.4. The octahedral sheet has alternately magnesium or aluminum cations. Like smectite, chlorite does not have the ability to absorb water in the interlayer sheets. The particles are very small and platy.

![Figure 3.4: Different stacking configurations of clay types common in the arctic area.](image)

### 3.1.3 Unfrozen water in frozen clay

There are different types of bonds which all influence the characteristics of a clay mineral. The building units of the minerals are interatomic bonded by ionic or covalent bonds. The sheets are bonded by a combination of the ionic bonds and weaker van der Waals bonds.

Meunier (2005) described how water molecules can be bonded to clay. They can be placed in between the pore spaces, be bonded to the outside of clay minerals and be bonded to the cations in between the sheets of a mineral.

The amount of energy needed to separate and freeze water molecules varies per type of bond. This means that at temperatures just below 0 °C the water in the pore space freezes first, while the water strongly bonded in between the clay minerals is still unfrozen. The lower the temperature the more water particles will freeze. Figure 3.5 shows a characteristic trend of the unfrozen water content in clay at different temperatures (Bronfenbrener, 2009).

![Figure 3.5](image)

The unfrozen water content in frozen clay depends on the temperature, the type of clay minerals, the specific surface area of the clay minerals and the concentration of impurities like salt in the soil. Typically the volumetric unfrozen water content in frozen clay at a temperature of -20 °C is between 5% and 13% (Liu, 2012, Akagawa and Nishisato, 2009). Only when the temperature drops under -110°C all the water in clay freezes (Bourbonnais and Ladanyi, 1985). This leads to the conclusion that frozen clay maintains some of its plastic behavior at temperatures characteristic for arctic subsea permafrost.
3.1.4 Freezing process of clay

Ice lenses, laminations of frozen water, form when fine grained soils freezes. The soil cracks and water from the surrounding soil migrates to the crack, and freezes to form an ice lens. The ice lenses form stiff and strong boundaries in the weaker clay, making the material inhomogeneous.

3.1.4.1 Ice lens formation and growth

For an ice lens to form the soil has to crack. Akagawa et al. (2006) explains that the pore pressure in the soils increases when pore water freezes and expands. Initiation of the cracks occurs when the pore pressure exceeds the sum of the overburden pressure and the tensile strength of the frozen soil. When the soil has cracked, water has to migrate from the surrounding unfrozen soil to the crack to enable a crack to grow. Talamucci (2003) states that if the freezing rate is too fast water cannot migrate from an unfrozen area to a frozen area and lenses will not form.

Orientation of ice lenses.

The formation of ice lenses causes frozen soil to heave when it freezes. Azmatch et al. (2012) performed experiments to determine the frost heave of clayey soil. They froze remolded consolidated clay uniaxial. The lens orientation observed was normal to the freezing direction. In nature, ice lenses seem to have a more random orientation. Miller and Bruggers (1980, p. 329) sampled subsea permafrost in the arctic Beaufort sea. They described their observations:

“The visible ice generally consisted of thin layers and laminations and individual crystals and ice coatings. The ice layers and laminations were predominantly 1.5 mm to 12 mm in thickness and were all associated with fine-grained deposits. The laminations and layers of ice, although occasionally horizontal, frequently had an orientation that was steeply inclined or even vertical. The only ice layer thicker than 12 mm was an 450 mm thick layer found in boring 13.”

The ice lens orientation found by sampling differs from the ice lens configuration obtained when clay is remolded and consolidated. Due to the ice lens orientation frozen clay will be anisotropic.
This leads to two important conclusions:
- As sample material, natural clay is preferred above remolded clay regarding the ice lens configuration.
- The ice lenses in frozen clay make the material inhomogeneous and anisotropic.

![Diagram of frozen soil layers](image)

Figure 3.6: Water migration during the freezing process of fine grained soil.

### 3.2 Mechanical properties frozen clay

Dredging and trenching in subsea permafrost has yet to be developed. For this reason there is no literature on frozen clay published by dredging engineers. Civil engineers already faced challenges with frozen soils for decades. Research was performed on the bearing capacities of frozen soil and clay. The influence of temperature, water content, salinity and low strain rates on the compressive strength of frozen soil and clay was investigated. Their findings are described in this section. Additionally the influence of the water content and the strain rate on the tensile strength of frozen clay has been studied.

The data shown in this section is determined at quasi static conditions. Researchers used different techniques of sample preparation and different types of soil. Although the data is not useful as an input for cutting force calculations, it can be used to find trends of mechanical properties for different conditions and provide an indication of the magnitude of variables.

#### 3.2.1 History of research on frozen soil

In the 1930’s Tsytovich was the first to investigate mechanical properties of frozen soil by performing uniaxial compression tests. He varied the temperature and strain rate, and concluded that the compressive strength of the soil increased with increasing strain rate and decreasing temperature.

In the 1980’s, artificial ground freezing in construction was introduced. Zhu, Sayles (1988) and many more investigated frozen soil in the temperature range of -10°C and -30°C. They studied the physical nature of frozen soils, the strength strain characteristics and creep behavior.

Aas (1981) was one of first to research the strength properties of frozen clay. With regard to the artificial ground freezing for tunnel constructions he researched frozen salt marine clay for its long term bearing capacity. Aas concluded that the shear strength of frozen clay depends on the temperature.

Nowadays civil engineers face the challenge of construction on permafrost. Chen (1988), Chen (1992), Li et al. (2004), Xiao et al. (2009) and many others study the bearing capacity of frozen clay near thawing temperatures. Their findings are summarized below.
3.2.2 Compressive strength of frozen clay

Several researchers have tested the uniaxial compressive strength of frozen clay at temperatures of 0 to -20. They performed Unconfined Compressive Strength test, referred to as UCS test. During this test low strain rates in the order of $10^{-6}$ s$^{-1}$ to $10^{-4}$ s$^{-1}$ are used.

Frozen matter melts when exposed to high pressures. For example during ice skating the blades of the skates have low drag because the ice located under the blades melts because of the weight of the skater.

The questions arises whether the clay tested in an UCS test fails brittle, or melts and fails ductile. When the clay fails ductile, brittle failure apparently would result in higher stresses and the compressive strength found can be interpreted as a lower limit.

Li et al. (2004) performed uniaxial compressive tests on saturated frozen clay to find the influence of temperature, strain rate and dry density on the compressive strength. They concluded that the compressive strength of saturated frozen clay increases with increasing dry density, strain rate and decreasing temperature. The data is shown in Figure 3.7, Figure 3.8, and Figure 3.9. By use of a regression analysis en formula was derived linking the compressive strength; $\sigma_c$, to the dry density, temperature and strain rate:

$$\sigma_c = (2.677 - 0.840\rho_d) \left(\frac{T}{T_0}\right) \left(\frac{\dot{\varepsilon}}{\dot{\varepsilon}_0}\right)^{(0.470 - 0.212\rho_d)}$$  \hspace{1cm} (3.1)

Where $T$ is the temperature [°C], $T_0$ is the reference temperature of -1 [°C], $\dot{\varepsilon}$ is the strain rate [s$^{-1}$], $\dot{\varepsilon}_0$ is the reference strain rate of 1 [s$^{-1}$] and $\rho_d$ is the dry density of the clay [g/cm$^3$].

During the experiments saturated soil samples with a dry density of 1.38 g/cm$^3$, 1.58 g/cm$^3$ and 1.88 g/cm$^3$ were used. The samples with the lower dry density had more pore space resulting in a higher weight water content. Equation (3.1) concludes that for a given strain rate and temperature a higher dry density results in a lower compressive strength. The decrease of the compressive strength for increasing weight water content is discussed in section 3.2.2.2.

Xiao et al (2009) determined the influence of temperature on the Young’s modulus and compressive strength of saturated frozen clay. They concluded that the Young’s modulus and compressive strength of frozen clay increases with decreasing temperature. The data is shown in Figure 3.7.

Li et al. (2009) studied frozen clay at temperatures of -0.5°C to -2°C. They state that due to the many micro defects randomly distributed throughout frozen clay, it is more useful to study the mechanical parameters of the frozen clay by a stochastic method, instead of a deterministic method. From their data can be deducted that the strength of ice increases with dropping. The data is shown in Figure 3.7.

Chen et al. (2011) investigated frozen clay from the Shanghai area. They tested the compressive strength, at different temperatures and strain rates. The tensile failure they observe is brittle and the tensile strength is much smaller than the compressive strength. The tensile strength increases with decreasing temperature and increasing strain rate. The data is shown in Figure 3.7, Figure 3.8, and Figure 3.9.

3.2.2.1 Influence of temperature on the compressive strength of frozen clay

Temperature has a direct and indirect effect on the compressive strength of frozen clay. The direct effect is that the compressive strength of ice increases with decreasing temperature, this effect is also seen in frozen clay. The indirect effect is that at decreasing temperature more water freezes. This higher ice content increases the compressive strength of the frozen clay.
Li et al. (2004), Li et al. (2009), Xiai et al. (2009) and Chen et al. (2011) experimentally determined the compressive strength of frozen clays. They all concluded that the compressive strength of saturated frozen clay increases with decreasing temperature. Figure 3.7 shows their findings.

### 3.2.2.2 Influence of the water content on the compressive strength of frozen clay

Li et al. (2004) and Chen et al. (2011) investigated the influence of the weight water content on the compressive strength of frozen clay on a rather small range of the water content. Both experimented with samples at -10°C and a strain rate of $4 \times 10^{-4}$ s$^{-1}$. They concluded that the higher the water content the lower the compressive strength. Figure 3.8 presents the data. The trend that the compressive strength increases for decreasing weight water content is trivial for unfrozen clay. For frozen clay one could assume that a higher ice content would result in a higher compressive strength because the compressive strength of clay is lower than the compressive strength of ice. This is not the case. This leads to the conclusion that in frozen clay the compressive strength of the clay grains in between the ice lenses determine the compressive strength.

### 3.2.2.3 Influence of the strain rate on the compressive strength of frozen clay

Li et al. (2004) and Chen et al. (2011) published data on the influence of the strain rate on the compressive strength on saturated frozen clay, shown in Figure 3.9. The data shows that an increase of strain rate results in an increase of the compressive strength. The strain rates are multiple orders lower than the strain rates involved in a dredging process.

![Figure 3.7: Influence of temperature on the compressive strength of frozen clay.](image-url)
3.2.3 Tensile strength of frozen clay

Akagawa and Kohei (2009) investigated the tensile strength of frozen soil near thawing temperatures. They state that the tensile strength of a frozen silty clay depends on the strain rate and temperature, but they did not recognize a trend.

Chen et al. (2011) investigated frozen clay from the Shanghai area. They tested the tensile strength at different temperatures and strain rates. The tensile failure observed is brittle and the tensile strength is in the order of 1 MPa. The tensile strength increases with decreasing temperature and increasing strain rate.

Akagawa and Nishisato (2009) measured the tensile strength of frozen clay at temperatures between -0.3 °C and -1.3 °C. They found a tensile strength of about 400 kPa.

Anagnostopoulos and Grammatikopoulos (2005) conducted splitting tests on frozen silty clay. They concluded that the tensile strength increases for higher water content. frozen clay at a temperature of -10 °C and strain rates of $1 \times 10^{-5}$ s$^{-1}$ were used. Their findings are presented in Figure 3.10.
3.2.4 Internal friction angle

Chen (1988) conducted triaxial tests on unsaturated frozen clay obtained from mining shafts. He concluded that the frozen clay had an internal friction angles of 3% to 7%.

3.2.5 Salt in frozen clay

Ogata et al. (1983) investigated the influence of salinity on the compressive strength of frozen soils concluding that an increase of salinity leads to a decrease of the compressive strength. Saylces (1988) published on the mechanical properties of frozen soil. He stated that salt in soil lowers the freezing temperature. His conclusion was that the strength of the soil with salt in the pore fluid is comparable to the strength of the same soil without salt in the pore fluid with the same unfrozen water content, this means at slightly higher temperatures.

3.2.6 Conclusion mechanical parameters frozen clay

<table>
<thead>
<tr>
<th>Compressive strength frozen clay</th>
<th>2 - 8 [MPa]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tensile strength frozen clay</td>
<td>0.4 - 1 [MPa]</td>
</tr>
<tr>
<td>Internal friction angle frozen clay</td>
<td>3 - 7 [°]</td>
</tr>
</tbody>
</table>

3.3 Ice

Ice in permafrost formed at temperatures of -15°C to -20°C under pressures in the range of 1 - 10 MPa. As can be seen in the phase diagram of water (Figure 3.11) this is in the range of 1h ice, referred to as hexagonal ice. This type, that forms under ordinary conditions, has a hexagonal crystal structure and the protons are disordered. Occasionally type eleven ice is found in arctic ice that dates from the ice ages. Type eleven ice is proton ordered and forms out of hexagonal ice under very high pressures. When the pressure is released the
type eleven ice transforms endothermic to hexagonal ice. If type eleven ice is encountered during a cutting process, the endothermic reaction could possibly freeze the surrounding water.

According to the ISO19906 the strength of ice increases with decreasing temperature, salinity, porosity and grain size (crystallography). A decrease of temperature results in an increase of the density, which increases the strength. An increase of salinity or porosity increases the number of brine or air pockets trapped in the ice matrix. Stress concentrations around the pockets decrease the strength of ice. An increase in the grain size leads to a less random orientation of the crystals making it anisotropic.

### 3.3.1 Tensile failure and tensile strength

When loaded tensile, ice fails ductile, at low strain rates, or brittle, at high strain rates. At very low strain rates ductile behavior is observed. Mohamed and Farzaneh (2011) investigated the influence of the strain rate on the tensile strength in the regime of \(10^{-5} – 10^{-3}\) sec\(^{-1}\), they found no dependency of the tensile strength on the strain rate and a tensile strength of 1 - 1.6 MPa. Schulson and Kuehl (1993) found tensile strengths of 0.6 – 1 MPa for pure ice and 0.2 – 0.3 MPa for saline ice near thawing temperatures.

### 3.3.2 Compressive failure and compressive strength

When compressed, ice fails ductile, at low strain rates, or brittle, at high strain rates. Figure 3.12 shows representations of stress-strain plots of ice failing under compression. The transition zone between ductile to brittle is at a strain rates of \(10^{-3}\) sec\(^{-1}\) (Schulson, 1990, Batto et al. 1993), far below strain rates involved in a dredging process.

Shazly et al. (2009) determined the compressive strength of polycrystalline freshwater ice at very high strain rates of \(10^2 – 10^3\) sec\(^{-1}\). They conducted experiments at -10 °C and found compressive strengths of 9 - 33 MPa. The external friction angle between ice and steel is 0.1° – 0.6° (Liang et al, 2003).

<table>
<thead>
<tr>
<th>Property</th>
<th>Value</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Compressive strength ice</td>
<td>9 - 33</td>
<td>[MPa]</td>
</tr>
<tr>
<td>Tensile strength ice</td>
<td>200 - 1600</td>
<td>[kPa]</td>
</tr>
<tr>
<td>External friction angle (steel - ice)</td>
<td>0.1 – 0.6</td>
<td>[°]</td>
</tr>
</tbody>
</table>

Table 3.2: Mechanical properties of ice
3.4 Conclusion

Frozen clay can best be described as an inhomogeneous, anisotropic material with a tensile and compressive strength comparable to soft rock. It has a small internal friction angle. Li et al (2009) suggest that due to the inhomogeneous character induced by micro cracks and ice lenses distributed throughout frozen clay, the mechanical properties of frozen clay can also be studied as a stochastic model.

Frozen clay has compressive and tensile strengths comparable to soft rock, some plasticity in frozen clay remains due to its unfrozen water content. From the ice in the clay it is known that it fails brittle when failed at the strain rates of a dredging process. For the frozen clay it is hard to deduct from literature whether frozen clay will fail brittle or ductile in a dredging process.

In remolded clay, ice lenses form with a consistent configuration normal to the freezing direction. Samples from the arctic show varying ice lens configurations. For this reason natural clay has to be used.
4 Cutting theories

Knowing whether the tensile, compressive or shear strength of a material limits the cutting force is essential when performing cutting calculations. Which of these mechanical parameters limits the cutting force depends on the failure type of the cutting process. Cutting theories are based on a failure mechanism and describe the cutting forces. In this chapter the cutting theories for clay and rock are studied with the purpose to match the failure mechanism observed in the cutting experiments to the theory. The first section defines the basic parameters of a soil system. In the second section the cutting theories are summarized. In the third and fourth section the influence of cutting speed and inhomogeneous characteristics of the material on the cutting forces are discussed.

4.1 Soil mechanics

To basic parameters of a soil system are defined. Then the Mohr Coulomb failure criterion is explained.

4.1.1 Soil phases

Pore spaces in grained soils are filled with liquids or gasses which makes soil a multiphase system. Figure 4.1 defines the units used to express the weight of solids, liquids and gasses.

![Figure 4.1: Phases in a soil](image_url)

4.1.2 Weight water content and saturation

The weight water content, \( m_{wc} \), is the ratio between the mass of the liquid and the total mass of the soil.

\[
m_{wc} = m_l/m
\]

(4.1)

Where \( m_{wc} \) is the weight water content, \( m_l \) is the weight of the liquid [kg] and \( m \) is the weight of the soil [kg].
the saturation, $s$, is the volume of the pore space that is not filled with water:

$$ s = \frac{V_{\text{pore}} - V_{\text{water}}}{V_{\text{pore}}} \cdot 100 \quad (4.2) $$

Where $V_{\text{pore}}$ is the volume of the pore space $[m^3]$ and $V_{\text{water}}$ is the volume of the pore fluid $[m^3]$.

### 4.1.3 Atterberg limits

The Atterberg limits are used to identify, describe and classify cohesive soils. The most common Atterberg limits are the liquid limit and the plastic limit. They correspond to a certain water content of a specific type of clay. The liquid limit describes the water content of clay when it has a shear strength of about 2 kPa. The plastic limit is the state of clay where there is just no more free water in the clay.

### 4.1.4 Mohr circle

The stress state of a point can be described by two principal stresses that act on that point under a certain angle and are perpendicular to each other. Another way to describe the stress state of this point is by two stresses with a fixed configuration and shear stress, Figure 4.2. Equation (4.3) and (4.4) show the relation between the principal stresses and stresses in a random direction.

![Figure 4.2: Stresses acting on a soil element](image)

The force equilibrium in horizontal and vertical direction of the stresses shown in Figure 4.2 leads to a description of the principal stresses:

$$ \sigma_1 = \sigma_X \cdot \cos(\gamma) + \tau \cdot \sin(\gamma) \quad (4.3) $$

$$ \sigma_2 = \sigma_X \cdot \sin(\gamma) - \tau \cdot \cos(\gamma) \quad (4.4) $$

Where $\sigma_1$ is the major principle stress [Pa], $\sigma_2$ is the minor principle stress [Pa], $\sigma_X$ is the inclined stress[Pa], $\gamma$ is the orientation of the shear plane and $\tau$ is the shear stress [Pa]

Rewriting these equations for the shear stress, $\tau$, and the normal stress, $\sigma$, results in:

$$ \sigma = \left( \frac{\sigma_1 + \sigma_2}{2} \right) + \left( \frac{\sigma_1 - \sigma_2}{2} \right) \cdot \cos(2 \cdot \gamma) \quad (4.5) $$

$$ \tau = \left( \frac{\sigma_1 - \sigma_2}{2} \right) \cdot \sin(2 \cdot \gamma) \quad (4.6) $$
4.1.5 Mohr coulomb failure criterion

Squaring Eq. (4.5) and (4.6) results in a formula for a circle that describes the stress state of a soil at any angle for a given principle stresses. This circle is called the Mohr circle and can be used to graphically describe stress states, Figure 4.3.

![Mohr circle diagram](image)

Figure 4.3: The Mohr circle including cohesion and internal friction

Adding up and rewriting Eq. (4.5) and (4.6) leads to the Mohr Coulomb failure criterion. This criterion describes the shear strength of soil based on its cohesion, stress state and internal friction angle. The cutting theory of Miedema is based on this criterion.

\[ \tau = c + \sigma \cdot \tan(\varphi) \]  \hspace{1cm} (4.7)

Where \( \sigma' \) is the effective stress [Pa], \( \varphi \) is the internal friction angle [°] and \( c \) is the internal shear strength [Pa].
4.2 Cutting theories

This section summarizes the cutting theories that are considered during this research. First the cutting theory of Miedema that includes different failure mechanisms for soil distinguished by Hatamura and Chijiiwa and Miedema are discussed. Then the rock cutting theories of Verhoef, Evans and Nishimatsu are summarized. Brittle tensile failure is expected when cutting frozen clay based on the literature study, for this reason the tear type cutting theory of Miedema will be discussed more extensive.

4.2.1 Cutting theory of Miedema

The cutting theory of Miedema (2012) is based on four different failure types for soil. The first three failure types were observed by Hatamura and Chijiiwa (1975, 1976 A-B, 1977 A-B). While cutting dry and wet sand, loam and clay they distinguished the shear type, flow type and tear type. Miedema (1992) added a forth cutting mechanism, the curling type. The four failure types are shown in Figure 4.4. Cutting forces are determined based on the equilibrium of forces on a layer or chip of soil cut. The derivation of the cutting forces is made under the assumption that the stresses on the shear plane and the knife are constant and equal to the stresses acting on these surfaces. The theory includes gravity, inertia, pore pressure, cohesion, adhesion and friction. When cutting clay the cutting forces are determined by the cohesion and adhesion, for rock the cohesion and friction are dominant.

The flow type

The flow type is based on shear failure and occurs generally in materials without an internal friction angle. A continuous chip is sheared from the material without a clear shear line, failure occurs in a shear plane. The flow type can occur when cutting clay in normal circumstances, or rock in hyperbaric conditions.

The shear type

The shear type is based on shear failure. A discontinuous chip shears from the bed with a clear shear line. This cutting mechanism occurs when cutting materials with an internal friction angle like dry sand or rock. The maximum cutting forces are modeled as the flow type, the mean forces are 25% - 50% of the maximum forces.

The tear type

The tear type is based on tensile failure. Lumps of material are torn from the soil. This is possible when negative stresses can occur. Miedema (2012) explained these negative stresses by the bending of a beam. When a piece of soil is bend, the outer radius will elongate resulting in tensile stress. Hatamura and Chijiiwa (1976A) measured these tensile stresses during cutting experiments with buried stress sensors. The tear type occurs when cutting rock under atmospheric conditions. It also might occurs during clay cutting when the blade height is very low with respect to the layer thickness or when the adhesion is small compared to the cohesion and the blade angle is small.

The curling type

The curling type is based on shear failure. It is a variation on the flow type. A continuous chip shears from the bed, after shearing the chip curls away from the blade. Clay can be cut with the curling type in normal conditions, rock in hyperbaric condition.
Figure 4.4: Failure mechanisms; the flow type, tear type, shear type and the curling type

Cutting of clay
Clay in normal dredging circumstances will be cut with the flow type. The curling type occurs when the adhesive force is big with respect to the normal force on the shear plane, when the blade height is big with respect to the layer thickness, when the adhesion is high compared with the cohesion, or when the blade angle is relatively big. The tear type can occur in stiff clays when the blade height is small with respect to the layer thickness, the adhesion is small compared to the cohesion and the blade angle is relatively small.

Table 4.1: Which failure type for which type of soil?

<table>
<thead>
<tr>
<th>Soil and cutting conditions</th>
<th>Flow type</th>
<th>Shear type</th>
<th>Curling type</th>
<th>Tear type</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dry sand</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Wet sand</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Rock</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Clay</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Rock hyperbaric</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 4.1 shows in general which failure type can occur for different types of cutting conditions and soil. In special occasions other failure types can occur.
4.2.2 The tear type

When clay fails with the tear type, the adhesion and cohesion dominate the cutting force. When rock fails with the tear type, the tensile strength dominates the cutting force. For this reason the tear type model for rock that includes the tensile strength, the friction angles and the blade angle, and the general tear type model that besides the tensile strength, the friction angles and the blade angle also includes adhesion, result in different relations for the cutting force.

The equilibrium of forces

The relations of both the general tear type and the tear type in rock are depending on the equilibrium of forces on a layer of soil cut. This equilibrium of forces for the flow type is used to determine the cutting forces for the tear type.

![Figure 4.5: Forces on a layer of soil cut](image)

First the normal force on the cutting tool, \( N_2 \), and the shear force on the cutting tool, \( S_2 \), are combined to an expression for the resulting force acting on the cutting tool \( K_2 \), Eq. (4.8). Then the water pressure force on the cutting tool, the adhesion, \( A \), and the expression for the forces of the chip on the cutting tool are combined to find the horizontal and vertical forces on the cutting tool.

\[
K_2 = \sqrt{N_2^2 + S_2^2}
\]  

(4.8)

The horizontal and vertical force on the cutting tool:

\[
F_H = -W_2 \cdot \sin(\alpha) + K_2 \cdot \sin(\alpha + \delta) + A \cdot \cos(\alpha)
\]

(4.9)

\[
F_V = -W_2 \cdot \cos(\alpha) + K_2 \cdot \cos(\alpha + \delta) - A \cdot \sin(\alpha)
\]

(4.10)

Where \( F_V \) and \( F_H \) are the vertical and horizontal force on the cutting tool [N], \( W_2 \) is the water pressure force on the knife [N] and \( A \) is the adhesive force between the knife and the soil chip.
4.2.2.1 The general tear type

For the general tear type theory the adhesion, cohesion, internal and external friction angle are considered. To determine the horizontal and vertical force on the cutting tool first the cohesive force on a shear plane and the adhesive force on the cutting tool are determined, this corresponds to the cutting forces on a layer cut in the flow type. The relation for the undrained shear strength, \( C \), follows from the cohesion and the length of the shear layer. The relation for the adhesive force, \( A \), follows from the adhesion and the length of the cutting tool.

\[
C = \frac{c \cdot h_c \cdot w}{\sin(\beta)} \tag{4.11}
\]

\[
A = \frac{a \cdot h_k \cdot w}{\sin(\alpha)} \tag{4.12}
\]

Where \( a \) is the adhesion [Pa], \( c \) is the internal undrained shear strength [Pa], \( h_c \) is the length of the surface between the knife and the layer cut [m], \( h_c \) is the cutting depth [m] and \( \beta \) is the shear angle.

Figure 4.6: Resulting forces from the adhesion and cohesion on a layer cut

An expression for the horizontal and vertical cutting force is found by combining the adhesive force and the resulting forces on the cutting tool and finding the component of these forces in the horizontal and vertical direction. The expressions for the forces include the angle of the shear plane, the cohesion and adhesion. In this cutting model the water pressure is not considered, this eliminates the water pressure, \( W_2 \) form Eq. (4.11) and (4.12). Equation (4.8) and (4.12) are substituted in Eq. (4.9) and (4.10):

\[
F_H = \sqrt{N_2^2 + S_2^2 \cdot \sin(\alpha + \delta)} + \frac{a \cdot h_k \cdot w}{\sin(\alpha)} \cdot \cos(\alpha) \tag{4.13}
\]

\[
F_V = \sqrt{N_2^2 + S_2^2 \cdot \cos(\alpha + \delta)} - \frac{a \cdot h_k \cdot w}{\sin(\alpha)} \cdot \sin(\alpha) \tag{4.14}
\]

\( N_2 \) depends on the cohesion and the adhesion, \( S_2 \) depends on \( N_2 \) and the external friction angle. Substituting the cohesion and adhesion in Eq. (4.13) and (4.14) results in relations for the horizontal and vertical force depending on the geometry of the cutting process, the adhesion and undrained shear strength, the friction angles and the shear angle:
Where $c'$ is the pseudo cohesion [Pa].

The internal friction angle, external friction angle, blade angle, cutting depth and width of the cut has to be known upfront to calculate the cutting forces. The pseudo cohesion is a scaled form of the cohesion based on the Mohr circle for the tear type and described below.

**Pseudo cohesion**

The cohesion is determined from the UCS and the internal friction angle:

$$c = \frac{UCS \cdot (1 - \sin(\varphi))}{\cos(\varphi)}$$

(4.18)

The Mohr Coulomb failure criterion defines the relation of the stresses on the shear plane:

$$\tau_{S1} = c + \sigma_{N1} \cdot \tan(\varphi)$$

(4.19)

Based on the stress condition on the shear plane a Mohr circle can be drawn: Figure 4.7 Circle 1. When the minimum principle stress found in the Mohr circle is smaller than the tensile stress (tensile stresses are negative), the tear type will occur. A stress state where the minimal principle stress is smaller than the tensile strength can never occur because the material will fail when the minimal principal stress equals the tensile stress. To find the new Mohr circle that describes this stress state: Circle 2 in Figure 4.7, the first Mohr circle is scaled. The scaling factor between the old and the new circles, which is also the scaling factor between the pseudo cohesion and the cohesion, is based on the difference between the minimal principle stress and the tensile stress of the material, both with respect to the stress on the shear plane:

$$\frac{c'}{c} = \frac{\sigma_{N1} - \sigma_T}{\sigma_{N1} - \sigma_{\text{min}}}$$

(4.20)

Where $c'$ is the pseudo cohesion [Pa], $\sigma_{N1}$ is the stress state on the shear plane [Pa], $\sigma_T$ is the tensile strength [Pa] and $\sigma_{\text{min}}$ is the minimum principle stress [Pa].

The first Mohr circle and the Mohr circle describing the stress state of the tear type are shown in Figure 4.7.
The Mohr circle can be found as followed:
1. Determine the radius of the first Mohr circle.
2. Determine the minimal principle stress.
3. Equal the minimal principle stress to the tensile strength of the material. This results in an equation depending on the cohesion, the friction angles and the cutting tool angle.
4. Rewrite for the cohesion, the equation found described the pseudo cohesion used in the equations for the cutting forces Eq. (4.21).

\[ c' = \frac{\sigma_t}{r \cdot \frac{\sin(\beta) \cdot \cos(\delta)}{\sin(\alpha)} - \cos(\alpha + \beta + \delta) - \sin(\alpha + \beta + \delta + \varphi)} \cdot \left( \frac{1 - \sin(\varphi)}{\cos(\varphi)} \right) \]  

(4.21)

![Figure 4.7: Mohr circles describing the stress state of the tear type.](image)

**Shear angle**

The shear angle is determined by finding the minimal horizontal force using the minimum energy principle. This results in an equation that depend on the cutting tool angle and internal and external friction angle.

\[ \frac{\delta F_H}{\delta \beta} = 0 \]  

(4.22)

The shear angle can be found by solving the derivative of the horizontal with respect to the shear angle force for zero. When in the equation for the force the adhesion is included the derivative would be complicated and depending on several parameters. Because the adhesion is very small compared to the cohesion it is neglected and the shear angle reads:

\[ \beta = \frac{\pi}{2} - \frac{\alpha + \delta + \varphi}{2} \]  

(4.23)
4.2.2.2 Tear type in rock

Rock has no adhesion to steel, the tear type for rock is based on the tensile strength, the shear angle and the blade angle. Using the same strategy as for the general tear type a pseudo cohesion is found. Because this model does not consider adhesive forces, the resulting pseudo cohesion and relations for the cutting forces are less extensive:

\[ c' = \frac{\sigma_T}{\left( \sin\left(\frac{\alpha + \delta - \varphi}{2}\right) - 1\right) \cdot \left(1 - \sin(\varphi)\right)} \]  

(4.24)

\[ F_H = \frac{2 \cdot c' \cdot h_e \cdot w \cdot \cos(\varphi) \cdot \sin(\alpha + \delta)}{1 + \cos(\alpha + \delta + \varphi)} \]  

(4.25)

\[ F_V = \frac{2 \cdot c' \cdot h_e \cdot w \cdot \cos(\varphi) \cdot \cos(\alpha + \delta)}{1 + \cos(\alpha + \delta + \varphi)} \]  

(4.26)

4.2.3 Cutting of rock

Verhoef (1997) presented a phenomenological model based on rock cutting experiments of the Delft Hydraulic laboratory (Figure 4.3). During rock cutting, in front and under the tip of a cutting tool a crushed zone is formed. A shear crack forms as shown in Figure 4.8. This shear crack bifurcates into a tensile crack. Only by numerical simulation this model can lead to cutting forces. The rock cutting models of Evans and Miedema are based on simplified failure mechanisms. They include single formulas to calculate the cutting forces.

![Figure 4.8: Phenomenological failure model by Verhoef.](source: Verruijt (2001))
4.2.4 Evans

The brittle tensile rock cutting model of Evans (1962, 1965) is based on observations of cutting experiments on coal breakage by wedges. A crack that originates at the point of the wedge propagates upwards to the surface. The equation that determines the horizontal cutting force, $F_H$, is derived from the tensile strength of the rock and the geometry of the cutting tool and the cut. Later researchers added the influence of the external friction angle to the cutting theory of Evans resulting in Eq. (4.27):

$$F_H = 2 \cdot \sigma_t \cdot h_c \cdot w \cdot \frac{\sin(\alpha + \delta)}{1 - \sin(\alpha + \delta)}$$

Where $\sigma_t$ is the tensile strength [Pa], $h_c$ is the cutting depth [m], $w$ is the width of the cut [m], $\alpha$ is the blade angle and $\delta$ is the external friction angle.

![Cutting model of Evans (1964)](source, Evans (1964))

Figure 4.9: Cutting model of Evans (1964)

4.2.5 Nishimatsu

Nishimatsu (1971) derived a model based on brittle shear failure. The model is based on the model of Merchant that included internal and external friction and shear strength. Nishimatsu added a distribution of the resulting stress on the failure line that is proportional $n^{th}$ power of the distance from the cutting tool.

Table 4.2 summarizes the relations for the cutting forces for different failure mechanisms. These are the general equations, they might differ for specific soil types.
Table 4.2: Cutting forces for different failure mechanisms

<table>
<thead>
<tr>
<th>Type</th>
<th>Horizontal cutting force</th>
<th>Vertical cutting force</th>
</tr>
</thead>
<tbody>
<tr>
<td>Miedema Flow type</td>
<td>$F_h = w \cdot \frac{c \cdot h_c \cdot \sin(\alpha) + a \cdot h_k \cdot \sin(\beta)}{\sin(\alpha + \beta)}$</td>
<td>$F_v = w \cdot \frac{c \cdot h_c \cdot \cos(\alpha) - a \cdot h_k \cdot \cos(\beta)}{\sin(\alpha + \beta)}$</td>
</tr>
<tr>
<td>Miedema Shear type</td>
<td>$F_{h_{\max}} = w \cdot \frac{c \cdot h_c \cdot \sin(\beta) \cdot \sin(\alpha) + a \cdot h_k \cdot \sin(\beta)}{\sin(\alpha + \beta)}$</td>
<td>$F_{v_{\max}} = w \cdot \frac{c \cdot h_c \cdot \sin(\beta) \cdot \cos(\alpha) - a \cdot h_k \cdot \cos(\beta)}{\sin(\alpha + \beta)}$</td>
</tr>
<tr>
<td>Miedema Curling type</td>
<td>$F_h = c \cdot h_c \cdot w \cdot \frac{\cos(\alpha)}{\sin(\beta)}$</td>
<td>$F_v = -c \cdot h_c \cdot w \cdot \frac{\sin(\alpha)}{\cos(\alpha + \beta)}$</td>
</tr>
<tr>
<td>Miedema Tear type</td>
<td>$F_h = c' \cdot h_c \cdot w \cdot \frac{\sin(\alpha + \beta) \cdot \cos(\delta) + r \cdot \sin(\beta + \phi) \cdot \cos(\delta)}{\sin(\alpha + \beta + \delta + \phi)}$</td>
<td>$F_v = c' \cdot h_c \cdot w \cdot \frac{\cos(\alpha + \beta) \cdot \cos(\phi) - r \cdot \cos(\delta + \beta) \cdot \cos(\delta)}{\sin(\alpha + \beta + \delta + \phi)}$</td>
</tr>
<tr>
<td>Miedema Tear type</td>
<td>$c' = \frac{\sigma_T}{\sin(\alpha + \beta) \cdot \cos(\delta) + r \cdot \sin(\beta + \phi) \cdot \cos(\delta)} \cdot \left( \frac{1 - \sin(\phi)}{\cos(\phi)} \right)$</td>
<td></td>
</tr>
<tr>
<td>Rock</td>
<td>$F_H = 2 \cdot c' \cdot h_c \cdot w \cdot \cos(\phi) \cdot \sin(\alpha + \delta) \cdot \frac{1}{1 + \cos(\alpha + \delta + \phi)}$</td>
<td>$F_V = 2 \cdot c' \cdot h_c \cdot w \cdot \cos(\phi) \cdot \cos(\alpha + \delta) \cdot \frac{1}{1 + \cos(\alpha + \delta + \phi)}$</td>
</tr>
<tr>
<td>Rock</td>
<td>$c' = \frac{\sigma_T}{\sin(\alpha + \delta - \phi) \cdot \frac{1}{\cos(\alpha + \delta - \phi)}} \cdot \left( \frac{1 - \sin(\phi)}{\cos(\phi)} \right)$</td>
<td></td>
</tr>
<tr>
<td>Evans Rock</td>
<td>$F_H = 2 \cdot \sigma_T \cdot h \cdot w \cdot \frac{\sin(\alpha + \delta)}{1 - \sin(\alpha + \delta)}$</td>
<td></td>
</tr>
<tr>
<td>Rock</td>
<td>$F_H = \frac{1}{(n + 1)} \cdot 2 \cdot c \cdot h \cdot w \cdot \cos(\phi) \cdot \sin(\alpha + \phi) \cdot \frac{1}{1 + \cos(\alpha + \delta + \phi)}$</td>
<td>$F_V = \frac{1}{(n + 1)} \cdot 2 \cdot c \cdot h \cdot w \cdot \cos(\phi) \cdot \cos(\alpha + \phi) \cdot \frac{1}{1 + \cos(\alpha + \delta + \phi)}$</td>
</tr>
</tbody>
</table>
4.3 High strain rates

Miedema (1992) developed the rate process theory. This theory predicts the strengthening factor of the cohesion and adhesion of clay for increasing strain rate. The Boltzmann distribution is used to adapt the rate process theory to be valid for pure cohesion and adhesion. The theory is verified by data of Hatamura and Chijiiwa (1977-A), Figure 4.10.

Equation (4.28) and (4.29) can be used to determine the strain rate that can be used for the adhesion, the strain rate on the blade, and cohesion, the strain rate on the shear line or plane, in a cutting process:

\[
\dot{\varepsilon}_c = 1.4 \cdot \frac{v_c}{h_i} \cdot \frac{\sin(\alpha)}{\sin(\alpha + \beta)}
\]

\[
\dot{\varepsilon}_a = 1.4 \cdot \frac{v_c}{h_i} \cdot \frac{\sin(\beta)}{\sin(\alpha + \beta)}
\]

(4.28)

(4.29)

Where \(\dot{\varepsilon}_c\) is the strain rate for cohesion and \(\dot{\varepsilon}_a\) is the strain rate for Adhesion.

The strengthening factor for cohesion and adhesion reads:

\[
\lambda_c = 1 + \frac{\tau_0}{\tau_y} \cdot \ln \left( 1 + \frac{\dot{\varepsilon}_c}{\dot{\varepsilon}_0} \right)
\]

\[
\lambda_a = 1 + \frac{\tau_0}{\tau_y} \cdot \ln \left( 1 + \frac{\dot{\varepsilon}_a}{\dot{\varepsilon}_0} \right)
\]

(4.30)

(4.31)

Where \(\lambda_c\) is the strengthening factor for cohesion and \(\lambda_a\) is the strengthening factor for adhesion. The ratio \(\tau/\tau_0\) is 0.1428 and \(\dot{\varepsilon}_0\) is 0.03 s⁻¹.

Figure 4.10: Influence of strain rate at shear strength of clay, semi-logarithmic
4.4 Effect of discontinuities in material on the cutting process

Verhoef stated that the limitations of the simplified cutting theories are that they all consider a homogeneous material, neglecting cracks or other anisotropic impurities. Ice lenses in frozen clay make the material highly anisotropic. Li et al. (2009) suggest that due to this anisotropy, the strength of frozen clay should be studied with a stochastic model.

4.5 Calculating the specific cutting energy

The Specific Cutting Energy, $E_{sp}$, is the energy required to cut one cubic meter of material. $E_{sp}$ is defined as the cutting power divided by the volume flow rate. The cutting power depends on the force in the direction of the cutting process, for this reason only the mean horizontal cutting force has a contribution to the $E_{sp}$:

$$ E_{sp} = \frac{P_c}{Q_c} = \frac{F_{H-mean} \cdot v_c}{h \cdot w} = \frac{F_{H-mean}}{h \cdot w} $$

(4.32)

Where $F_{H-mean}$ is the mean horizontal cutting force, $P$ is the cutting power [Nm/s] and $Q$ is the volume flow rate [m$^3$/s]

4.6 Conclusion

Chapter 3 concluded that the frozen clay can behave ductile while the strength is comparable to soft rock, for this reason it is hard to predict the failure type, brittle or ductile, based on literature. The failure type and failure mechanism will be determined in the cutting experiments explained in chapter 5.

If the frozen clay fails ductile the cutting forces can be compared to the forces predicted by the clay cutting theory of Miedema. For brittle failure the forces can be compared to the outcome of the rock cutting models of Miedema, Evans and Nishimatsu.
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