Event triggered non-cooperative game theory for demand side management in smart grids

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Abstract

Over the past decade, a new concept for the power grid called the smart grid emerged, to enable two way energy transport and communication. With this new smart grid architecture many opportunities for control methods arise. While this can improve the efficiency of the power grid, all these methods require communication, which is a limited resource. In this thesis I focus on one particular concept of the smart grid called demand side management. One of the possible ways to apply demand side management is by use of game theory. By molding the game into a specific kind of game called potential game, similarities between the convergence of these potential games and the methods used in event triggered control, which is a method to reduce communication in networked control systems, can be observed. By using these similarities the principles of event triggered control can be applied to potential games. This way I create an event triggered potential game, which reduces the amount of communication required to reach the Nash equilibrium. To accomplish this I design two algorithms for event triggered potential games. One in which the decision on whether or not to communicate is made in a centralized way and one in which this occurs in a decentralized way, where all players decide for themselves when to communicate.

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Preface

The topic of smart grids has always intrigued me, ever since I completed my minor in sustainable energy technologies. By applying the concepts of event triggered control provided by dr. Manuel mazo jr. to the smart grid, I was able to learn more about smart grids, which is something I wanted going in.

Acknowledgements

I would like to thank my supervisor prof.dr.ir.Manuel Mazo jr. for his assistance during the writing of this thesis.

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“In the future, airplanes will be flown by a dog and a pilot. And the dog’s job will be to make sure that if the pilot tries to touch any of the buttons, the dog bites him.”

— *Scott Adams*

Chapter 1

Introduction

Ever since the electrical power grid was created over a hundred years ago, people have been reliant on electricity for their day to day lives. Over time technology has advanced, which led to an increasing demand for electrical power. To match this demand, the energy generation has been increased and the electrical power grid has been expanded. While the grid has expanded to match the modern needs, its structure has remained the same over all this time. In the traditional power grid structure, bulk generation takes place in centralized locations. From there the generated power is then distributed down the power grid to the end users. This structure is no longer sufficient for the modern society for two reasons: Firstly the rise of distributed renewable energy resources. The traditional fossil fuels used to generate electricity are depleting. Furthermore, this method of producing energy is harmful to the environment. To solve this problem, renewable energy resources like hydro, solar and wind power are implemented. This renewable energy generation does not only occur at large plants, but can occur everywhere on a small scale, for instance by residentially owned solar panels. This does not fit into the traditional power grid structure which only has one way power transport. Secondly the absence of information to end users. The current smart grid only possesses few sensors. Furthermore these sensors only provide information for the purpose of power grid maintenance and consumer behaviour analysis. Since modern consumers will have smart appliances, which can communicate with other smart devices and plan energy consumption, and since modern consumers will also sell energy, they will require more information so they can make better decisions with regard to their energy consumption behaviour. With the limited amount sensors available in the current power grid, this will not be possible. Because of these limitations of the current power grid architecture, the public consensus is that a new power grid structure is needed. This new structure is called the smart grid (SG) [3, 5, 15, 22]. The concept of the SG was created around 2005. Table 1-1 shows some key differences between the current power grid and the proposed SG. By implementing sensors throughout the entire power grid, adding a vast communication network and providing the infrastructure for two way power distribution, the smart grid can tend to the needs for the modern power grid.

Table 1-1: Smart grid compared to traditional grid [6]

Traditional grid	Smart grid
Electromechanical	Digital
One-Way Communication	Two-Way Communication
Centralized generation	Distributed generation
Hierarchical	Network
Few Sensors	Sensors Throughout
Blind	Self-Monitoring
Manual Restoration	Self Healing
Failures and Blackouts	Adaptive and Islanding
Manual Check/Test	Remote Check/Test
Limited Control	Pervasive Control
Few Customer Choices	Many Customer Choices

Because the SG architecture includes many different sensors and a vast communication network, there are many opportunities to apply control. An example would be the automation of energy distribution or home energy management. The authors of [7] have listed a number of these smart grid applications along with the communication requirements to apply them. This is shown in Table 1-2.

Table 1-2: Communication requirements for different smart grid (SG) applications [7]

Application	Data rate	Data size	Latency	Reliability	Security
AMI	10-100 kbps/node, 500 kbps for backhaul	100 B-several MBs	2-15 sec	99-99.99%	High
AM	56 kbps	25 B	2000 ms	99%	High
DR	14-100 kbps per node/device	100 B	500 ms-several minutes	99-99.99%	High
Distribution automation (DA)	9.6-100 kbps	25-1000 B	20-200 ms	99-99.99%	High
DERs and storage	9.6-56 kbps	25-1000 B	20 ms-15 sec	99-99.99%	High
DGMA	9.6-100 kbps	25-1000 B	100 ms-2 sec	99-99.999%	High
Electric transportation	9.6-56 kbps, 100 kbps is good	100-255 B	2 sec-5 min	99-99.99%	Relatively high
Home energy management (HEM)	9.6-56 kbps	10-100 B	300-2000 ms	99-99.99%	High
Meter data management	56 kbps	25-200 B	2000 ms	99%	High
OM	56 kbps	25 B	2000 ms	99%	High
Overhead transmission line monitoring	9.6-56 kbps	25 B	15-200 ms	99-99.99%	High
Substation automation	9.6-100 kbps	25 B	15-200 ms	99-99.99%	High
WASA	600-1500 kbps	More than 52 B	20-200 ms	99.999-99.9999%	High

While all these applications are beneficial for the SG they all require communication. To facilitate this, many communication technologies exist. Some are better for short distance communication, while others excel at long range communication. For all of them however, the available bandwidth can be an issue. Since there are many smart grid applications, which will mostly occur on a very large scale, the sheer amount of communication is enormous. Communication is generally not considered a resource but in this context with large amounts of information exchange, the available bandwidth might be insufficient. Therefore reducing the amount of communication required to use the applications in Table 1-2 is of high importance. In this thesis I will focus on the reduction of communication within one of these applications, namely demand side management (DSM) or demand response (DR). Traditionally the amount of energy generated is determined by the power demand. Due to the nature of human behaviour, this will lead to an inconsistent energy profile with high energy demand during the evening and lower demand throughout the rest of the day. These peaks in generation will lead to high costs [12]. To reduce these costs DSM focuses on changing the power demand instead of changing the power generation.

In this thesis I will contribute to the need for reduced communication in smart grid applications by reducing the required communication for one of the techniques to apply DSM called game theory. By fitting the game into the potential game framework defined in [18], a link can be established with the event triggered control (ETC) methods presented in [10]. These methods are designed to reduce communication in large-scale resource-constrained wireless embedded control systems. By applying ETC methods , the amount of communication required to reach the optimal state of a game can be reduced.

This thesis will be structured as follows: First chapter 2 introduces the notation used in this thesis as well as revisit the concepts of DSM, potential game theory and event triggered control. Chapter 3 then proposes a method to apply the ETC techniques to potential games in order to reduce the communication in these games. In chapter 4 this method is then applied to DSM, which will be followed by a numerical example in chapter 5. The thesis ends with some final remarks in chapter 6.

Chapter 2

Notation and Preliminaries

2-1 Notation

In a game, S_i is the strategy chosen by player i and S_{-i} is the set of strategies chosen by all players except player i. Furthermore S is the set of strategies chosen by all players. T_i is the updated strategy of player i. In a dynamic game $S(k)$ is the set of strategies at iteration k. $\hat{S}$ is the last communicated set of strategies. $\mathbf{1}$ is a row vector of all ones. A function $f(t)$ is a class $\mathcal{K}_\infty$ function if $f(0) = 0$ and $f(t) \rightarrow \infty$ as $t \rightarrow \infty$. In a demand side management (DSM) game where players can choose their energy consumption during different time slots throughout the day, $l_{t,n}$ refers to the load consumed by player n during time slot t. $L_{t,-n}$ then denotes the vector of energy consumption of all players except player n during time slot t. In this same DSM game setting $l_{t,n,desired}$ is the desired energy consumption of player n during time slot t. L_t^{avg} denotes the average energy consumption during a time slot.

2-2 Demand side management

In the traditional power grid the power matching problem already existed. Power demand fluctuations are unavoidable since not all loads are continuous. Traditionally when the demand increased, the generation would be increased to match it. The smart grid (SG) architecture gives more possibilities than the traditional power grid, but the same methodology is still being applied. DSM takes a different approach. Instead of controlling the power generation, DSM controls the power demand side. As mentioned previously, high peaks in demand will cause high costs. Reducing these peaks to keep down costs is the main goal of DSM. An important metric to assess this is called the peak to average ratio (PAR). This metric is defined in 2-1.

$$PAR = \frac{l_{peak}}{l_{avg}} \tag{2-1}$$

The lower the PAR the lower the overall costs. The concept of DSM is visually represented in Figure 2-1. During the day there will be peaks in demands and lows or valleys. While the overall reduction of energy consumption is desirable, peaks will always remain. To reduce these peaks power demand needs to be shifted to be more spread out throughout the day. This called peak shaving or valley filling.

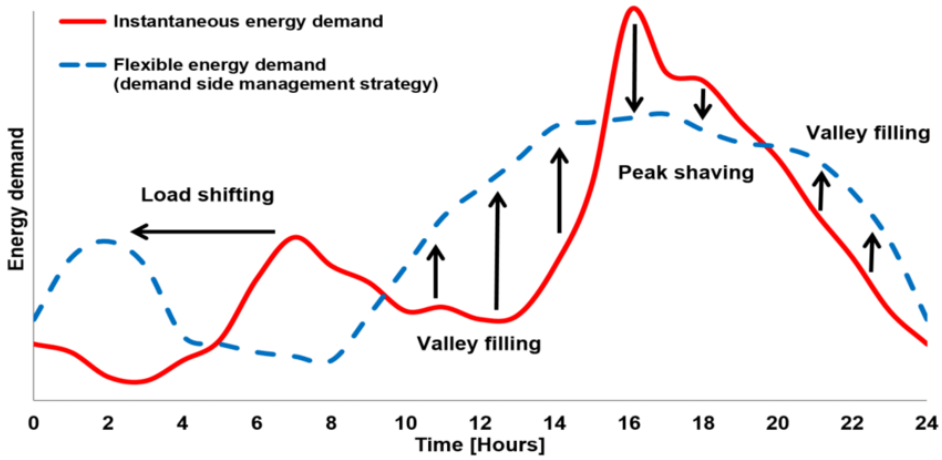

Figure 2-1: DSM principle

Several methods to apply this concept in smart grids exist. First direct load control (DLC) was proposed [4, 19, 20]. In DLC the energy generation side has full control over the consumption side, which allows the generators to reach cost optimality. Secondly, the authors of [14] apply a smart pricing approach to control the demand. Instead of applying direct control, the price of energy is used to persuade consumers to shift their energy consumption to different times to avoid consumption peaks and thus reduce the costs. Finally, [12] and [17] use a game theory framework in which all energy consumers are considered players in a game who aspire to reach a state in which the generation cost is minimized. This optimal state can be reached in a distributed way by allowing players to change their own consumption profile in an iterative manner. Furthermore, energy consumers can achieve their personal goals by changing their consumption for selfish benefits. This approach however requires a large amount of communication between energy consumers because of its distributed and iterative nature. Therefore this thesis will aim to reduce this.

2-3 Non cooperative potential game theory

Game theory is a decision making process in which a set of players try to reach a game state that maximizes their goals. This can occur in a static way in which each player makes a single decision, or dynamically in which decisions are made in an iterative manner. The core of non-cooperative game theory is that players in the game have a partial or complete conflict of interest [8]. A non-cooperative is defined as follows:

Definition 2-3.1 (Non-cooperative game)**.** *A non-cooperative game G is defined by the following three elements:*

1. *N : a finite set of players* $[n_1 \dots N]$
2. *S_i: a set of all strategies available to player* i *N*
3. *U_i: utility or pay-o function for player* i *N , which is a function of all the strategies* $[S_1 \dots S_N]$ *leading to utility set* $[U_1 \dots U_N]$

Furthermore a game is continuous if the strategy sets S_i are continuous intervals of R. A classic example of game theory is the prisoners dilemma. This example poses 2 criminals on trial for a crime. Each criminal has 2 options: stay silent or blame the other criminal. If both criminals stay silent, they are each sentenced to one year in prison. If one stays silent but the other blames the other criminal, the silent one gets 6 years in prison while the other goes free. Finally when both criminals blame the other they will both get 4 years in prison. The key sets of non-cooperative game theory are present. There is a player set of 2 players, there is an action set: stay silent or blame and each player has a utility function, their prison time, which value is dependent on both players strategies. Furthermore they can only make one move and do so in a myopic way(without knowledge of the other players strategy). This game can be represented in payoff table 2-1.

Table 2-1: Payoff table for prisoners dilemma

	1S	1B
2S	-1,-1	0,-6
2B	-6,0	-4,-4

Table 2-1 shows the payoffs for each player for every strategy combination. The optimal strategy for both players can be determined from this table. Option S has the possible outcomes one or six years, while option B has the possible outcomes zero or four years. This leads to both players choosing B and thus will both end up with four years. Oddly enough both their payoffs would have increased from this if they had both switched strategies. The point $-4, -4$ is called the Nash equilibrium (NE). The difference in total payoff between the global optimal and the NE is called the price of anarchy. A NE is defined as follows:

Definition 2-3.2 (Nash equilibrium [16])**.** *A strategy profile S is a Nash equilibrium if and only if*

$$U_i(S_i\ , S_{-i}) \geq U_i(S_i, S_{-i}) \quad S_i \quad \mathsf{S}_i \quad i \quad N \tag{2-2}$$

Once this state has been reached, the optimal solution has been found, thus ending the game. The existence of such a NE however is not guaranteed, nor are there guarantees that if such a NE exists the game will converge to it. Furthermore it is also possible for multiple NE to exist in one game. To investigate the existence of and convergence to a NE, the potential game framework of [18] can be used.

Potential games are a subclass of non-cooperative games. Besides the three fundamental sets a potential game also has a potential function $F(S)$, which like the utility functions is dependent on all the strategies $[S_1 \dots S_N]$, but is a global function that is not linked to a specific player. This function $F(S)$ maps the strategy space S to the space of real numbers

R. In this thesis I consider two types of potential games: exact potential games and ordinal potential games, which are defined as follows:

Definition 2-3.3 (Exact potential game [16]). *A non-cooperative game is an exact potential game if and only if a potential function $F(S)$: S → R exists such that i N:*

$$U_i\left(T_i, S_{-i}\right) - U_i\left(S_i, S_{-i}\right) = F\left(T_i, S_{-i}\right) - F\left(S_i, S_{-i}\right) \\ S_i, T_i \quad \mathsf{S}_i; \;\; S_{-i} \quad \mathsf{S}_{-i} \tag{2-3}$$

Definition 2-3.4 (Ordinal potential game [16]). *A non-cooperative game is an ordinal potential game if and only if a potential function $F(S)$: S → R exists such that i N:*

$$U_i\left(T_i, S_{-i}\right) - U_i\left(S_i, S_{-i}\right) \geq 0 \Leftrightarrow \\ F\left(T_i, S_{-i}\right) - F\left(S_i, S_{-i}\right) \geq 0 \\ S_i, T_i \quad \mathsf{S}_i; \;\; S_{-i} \quad \mathsf{S}_{-i} \tag{2-4}$$

The identification of these potential games can be difficult. Since the potential function is not known a priori, relations 2-3.3 and 2-3.4 can not be used to verify whether a particular game is a potential game. For both ordinal and exact potential games, a method exist for identification. To identify an ordinal potential game the existence of so-called 'weak improvement cycles' is essential [23]. A cycle is defined as any strategy evolution $[S^1 S^2 \dots S^k]$ for which $S^k = S^1$. A weak improvement cycle is defined as follows

Definition 2-3.5 (Weak improvement cycle [16]). *A cycle $[S^1 \dots S^K]$ is a weak improvement cycle if and only if $U_{i(k)}(S^k) \leq U_{i(k)}(S^{k+1})$, k where $i(k)$ is the deviating player in step k*

The deviation player is the player changing its strategy at the current iteration. The absence of weak improvement cycles is key to proving a game is an ordinal potential game[16]. The downside of this method is that for games with a large strategy space, this process can be long and computationally expensive. Fortunately, for exact potential games a simpler method exists.

Theorem 2-3.1 (Exact continuous potential game identification [18]). *The game G is a continuous exact potential game with potential function F if and only if*

$$\frac{\partial^2 U_i}{\partial S_i \partial S_j} = \frac{\partial^2 U_j}{\partial S_i \partial S_j}, \qquad i, j \quad \mathsf{N} \tag{2-5}$$

When a game has been identified as a potential game, the following can be used to conclude Nash equilibrium existence and convergence.

Definition 2-3.6 (ϵ-improvement path[16]). *A path $\rho = (S^0, S^1, S^2, ...)$ is an ϵ-improvement path if in each step k ,the deviating player $i(k)$ experiences $U_{i(k)}(S^{k+1}) \geq U_{i(k)}(S^k) + \epsilon$, for some ϵ R₊*

This definition allows for the introduction of an ϵ-equilibrium

Definition 2-3.7 (ϵ-equilibrium [16]). *The strategy profile $\tilde{S}$ S is an ϵ-equilibrium if and only if ϵ R₊ such that, i N :*

$$U_i(\tilde{S}_i, \tilde{S}_{-i}) \geq U_i(S_i, \tilde{S}_{-i}) - \epsilon, \quad S_i \quad \mathsf{S}_i \tag{2-6}$$

The ϵ-equilibrium is a more relaxed version of the nash equilibrium which can be more practical in games with high computational complexity or continuous games with asymptotic convergence.

Theorem 2-3.2 (ϵ-equilibrium convergence [16])**.** *For continuous ordinal potential games, every better-response sequence that is compatible with ϵ-improvement converges to an ϵ-equilibrium in a finite number of steps.*

Corollary 2-3.1. *All continuous ordinal potential games will converge to an ϵ-equilibrium in a finite number of steps.*

Proof. For any potential game it is known that:

$$U_i(T_i, S_{-i}) \geq U_i(S_i, S_{-i}) \quad i \quad N \tag{2-7}$$

Thus the game will follow a better response sequence. For continuous ordinal potential games, if this sequence is compatible with an ϵ-improvement path, theorem 2-3.2 states that the game converges to an ϵ-equilibrium in a finite number of steps. If the following holds:

$$U_i(T_i, S_{-i}) \geq U_i(S_i, S_{-i}) + \epsilon \quad i \quad N \tag{2-8}$$

Then this improvement path is ϵ-improvement and thus will converge to the Nash equilibrium in a finite amount of steps. If this only the case for some players then the players who cannot not improve more than ϵ will no longer change strategies. This will eventually lead to the following:

$$U_i(T_i, S_{-i}) \leq U_i(S_i, S_{-i}) + \epsilon \quad i \quad N \tag{2-9}$$

Since no player can improve its utility function by more than ϵ, the game is at an ϵ-equilibrium(2-3.7). Thus it can be concluded that every continuous ordinal potential game converges to the ϵ-equilibrium in a finite amount of steps. □

2-4 Event triggered control

In traditional control the state x is sampled continuously and the control input u is updated and communicated continuously. This is designed under the assumption that there is unlimited communication bandwidth and processor capacity available. However in practice this is not the case. To reduce the amount of communication event triggered control (ETC) is introduced. In ETC the control input is not continuously updated but only when the controller deems it necessary. These moments are called triggering instances. This schematic can be seen in figure 2-2

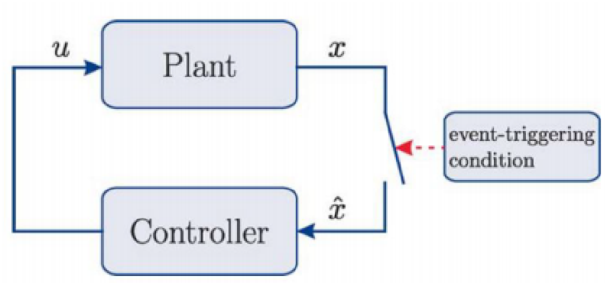

Figure 2-2: Event triggered controller schematic as presented by [9]

The figure shows that the controller gives a control input based on the last communicated state $\hat{x}$. The actual state x is only communicated once the event triggering condition is activated. The authors of [9] present the ETC framework in continuous time, but since I want to use these techniques in combination with game theory, which is played out in discrete time, I will instead show the discrete time ETC approach presented in [11].

Consider a nonlinear closed loop discrete system of the form

$$\begin{aligned} x(k+1) &= f(x(k), u(k)) \\ u(k) &= p(x(k)) \end{aligned} \tag{2-10}$$

A Lyapunov theory based method is used to design such a triggering condition. For the system described in equation (2-10) to be stable there should exist a Lyapunov function V and class K_∞ functions $\alpha_1, \alpha_2, \alpha_3$ such that

$$\begin{aligned} \alpha_1 ||x(k)|| \leq V(x(k)) &\leq \alpha_2 ||x(k)|| \\ V(x(k+1)) - V(x(k)) &\leq -\alpha_3 ||x(k)|| \quad x \quad \mathsf{R}^n \end{aligned} \tag{2-11}$$

Now consider the event triggered version, with triggering instances k_i. Since the control input is only updated at the triggering instances, control input u can be written as:

$$u(k) = f(x(k_i)) \quad k \quad [k_i \dots k_{i+1}) \tag{2-12}$$

For the system to remain stable equation (2-11) still has to hold for the new control input so

$$\begin{aligned} V(f(x(k), u(k_i)) - V(x(k)) + \alpha_3 ||x(k)|| \leq 0 \\ x \quad \mathsf{R}^n, \quad k \quad [k_i, k_{i+1}) \end{aligned} \tag{2-13}$$

With this relation, a triggering condition can be designed in the following form:

$$\begin{aligned} k_{i+1} = \inf\{k > k_i : V(f(x(k), f(x(k_i))) - V(x(k)) + \alpha_3 ||x(k)|| \\ \geq 0\} \end{aligned} \tag{2-14}$$

As long as inequality (2-13) remains true, equation (2-11) will hold. Therefore choosing the triggering instance as the moment it becomes positive, the controller will have access to the current state of the system at those critical points. If we assume that with knowledge of the current state, the controller can produce a control input that satisfies (2-11), the closed loop system is guaranteed to be stable. Furthermore zeno behaviour cannot occur since this is a discrete time system.

Chapter 3

Event triggered potential games

3-1 Introduction

As mentioned in chapter 2, DSM is one of the concepts of the smart grid that can be applied due to the improved communication infrastructure. A technique to apply DSM is by means of game theory. While this a good method, it requires a lot of communication between players to reach the Nash equilibrium. Because of the large amount of communication required in the smart grid, reducing the amount of communication is pivotal. This leads to the following problem statement:

Problem statement. Given ordinal potential game G, create a communication triggering condition that reduces the amount of strategy communication instances between players, while guaranteeing Nash equilibrium convergence.

The first step in designing this triggering condition, is to evaluate how communication takes place in regular potential games. Figure 3-1 shows the network topology for communication in potential games. It can be seen that players do not communicate directly with each other, but communicate via a central node. This way less communication lines are required. Furthermore for some games, players do not need to know the individual strategies of all the players, but only require the aggregate or combined strategies of the other players. By using this network topology, players can send their individual strategies to the central node, which can then send the aggregate/combined strategy profile to all players. For the games considered in this thesis it is assumed that the combined strategy profile is sufficient for players to evaluate their utility function and update their strategy.

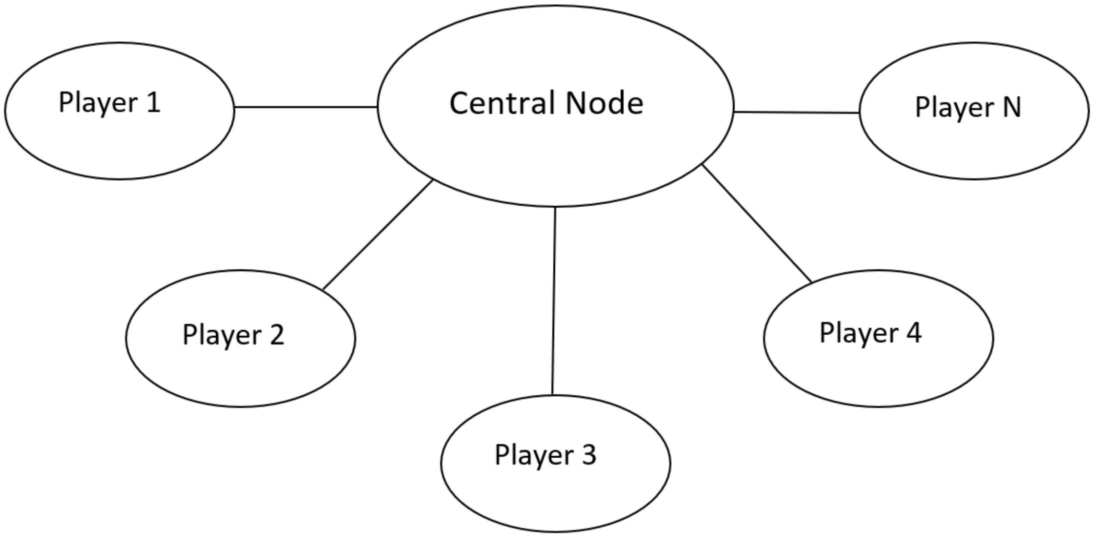

Figure 3-1: Network schematic for potential game

Regular potential games are played out sequentially. This means that only one player changes its strategy at the same time. This player will then send this changed strategy to the the central node, which then communicates the new combined strategy to all the other players. After that, the next player changes its strategy. This continues until no player can improve and thus the Nash equilibrium is reached.

Consider a non-cooperative ordinal potential game G with key elements:

$$\begin{aligned} &\mathsf{N} = [n_1 \dots N] \\ &\mathsf{S} = [\mathsf{S}_1 \dots \mathsf{S}_N]^T \\ &\mathsf{U} = [U_1 \dots U_N] \\ &F(S) \end{aligned}$$

This game is played out using algorithm 1. In this potential game algorithm, communication between the central node and all players is required after every strategy change. To reduce this, this chapter covers two event triggered potential game algorithms. The first method uses a centralized triggering mechanism in which the central node decides when communication is necessary, while the second method uses a decentralized triggering mechanism in which all players check individually whether communication is required. Afterwards in chapter 5, simulations of a potential game are performed with both methods and their performance will be analysed based on metrics like computational intensity, duration, total amount of strategy communication and number of times communication is triggered.

Algorithm 1: Non-cooperative potential game

Result: Nash equilibrium
initialization;
All players start with a strategy S_j
$a = 0$
while $a < N$ **do**
  for $j \quad N$ **do**
    $S_j(k+1) = \underset{S_j}{\text{argmax}}\ U_j(S_j, S_{-j}(k))$
    $S_{-j}(k+1) = S_{-j}(k)$
    Player j Communicates its new strategy $S_j(k+1)$ to the central node which then communicates the new combined strategy $\mathbf{1} \cdot S(k+1)$ to all other players.
    if $S_j(k+1) = S_j(k)$ **then**
      $a = a + 1$
    else
      $a = 0$
    end
    $k = k + 1$
  end
end

3-2 Centralized triggering

Consider again ordinal potential game G with potential function $F(S)$. Instead of solving this game using Algorithm 1, I propose using an event triggered game as shown in algorithm 2. In this event triggered game, players again communicate via the central node. Similar to the principles of ETC this central node will then check whether a triggering condition is violated. Only if this condition is violated, will the controller communicate the latest combined strategy to all the players, thus saving communication at all iterations where the triggering condition is not violated. To design a triggering condition such that the game converges to the Nash equilibrium with algorithm 2, I apply the approach used in ETC. In ETC the system converges to the equilibrium as long as the Lyapunov function decreases. To ensure this, a triggering condition is designed that forces communication if this Lyapunov function no longer decreases. For ordinal potential games, convergence to the Nash equilibrium is guaranteed if the potential function always increases. Therefore I design a triggering condition that forces communication if the potential function decreases.

Proposition 3-2.1. *Any event triggered ordinal potential game G will converge to the Nash-equilibrium using algorithm 2, with the following triggering condition:*

$$k_{i+1} = \mathit{inf}\{k > k_i : F(S(k+1)) - F(S(k)) \leq 0\} \tag{3-1}$$

Proof. Definition 2-3.2 states the following about the Nash equilibrium S :

$$U_i(S_i, S_{-i}) \geq U_i(S_i, S_{-i}) \quad S_i \quad \mathsf{S}_i \quad i \quad N \tag{3-2}$$

For ordinal potential games, because of definition 2-3.4 this also means:

$$F(S\ ) \geq F(S\ ) \quad S \quad \mathsf{S} \tag{3-3}$$

Thus it can be concluded that the Nash equilibrium lies at the maximum of the potential function. Therefore convergence to the Nash equilibrium is guaranteed if the following holds:

$$F(S(k+1)) - F(S(k)) \ \geq 0 \quad k \tag{3-4}$$

Triggering condition (3-1) triggers communication when:

$$F(S(k+1)) - F(S(k)) \leq 0 \tag{3-5}$$

After communication is triggered at the triggering instance k_i, all players know the current combined strategy profile $\mathbf{1} \cdot S(k_i)$. Since this is an ordinal potential game, with full information players will always choose a new strategy that improves their utility function and thus also increases the potential function. This means the following will always hold:

$$F(S(k_i+1)) - F(S(k_i)) \geq 0 \quad k_i \tag{3-6}$$

With this it can be concluded that by using triggering condition (3-1) in algorithm 2, the event triggered ordinal potential game will keep increasing its potential function and thus guarantees Nash equilibrium convergence. □

Algorithm 2: Event triggered non-cooperative potential game

```
Result: Nash equilibrium
initialization;
All players start with a strategy S_j
a = 0
while a < N do
    for j   N do
1       S_j(k+1) = argmax_{S_j} U_j(S_j, 1 · Ŝ_{-j})
        S_{-j}(k+1) = S_{-j}(k)
        Communicate S_j(k+1) to central node
        Central node checks triggering condition
        if triggering condition is violated then
            Central node communicates the current combined strategy 1 · S(k) to all
              players
            return to 1
        end
        if S_j(k+1) = S_j(k) then
            a = a + 1
        else
            a = 0
        end
        k = k + 1
    end
end
```

While triggering condition (3-1) guarantees convergence to the Nash-equilibrium, to apply it, knowledge of the potential function is needed. Finding the potential function is usually difficult. Thus it is necessary to create a triggering condition that guarantees the increase of the potential function without knowing what it is.

Proposition 3-2.2. *Any event triggered ordinal potential game G will converge to the Nash-equilibrium using algorithm 2 with the following triggering condition, which does not require knowledge of the potential function:*

$$\begin{aligned} k_{i+1} = \mathit{inf}\{k > k_i : U_j(S_j(k+1), S_{-j}(k+1)) \\ -U_j(S_j(k), S_{-j}(k)) \leq 0\} \end{aligned} \tag{3-7}$$

Proof. Since G is an ordinal potential game, the following implication will hold:

$$\begin{aligned} U_j(S_j(k+1), S_{-j}(k+1)) \geq U_j(S_j(k), S_{-j}(k)) \Leftrightarrow \\ F(S_j(k+1), S_{-j}(k+1)) \geq F(S_j(k), S_{-j}(k)) \end{aligned} \tag{3-8}$$

In this implication player j is the player changing its strategy. Increasing the utility function of player j will increase the global potential function. Because of proposition 3-2.1 it is known that by triggering communication when a player decreases the potential function, Nash equilibrium convergence is guaranteed. Therefore it can be concluded that applying triggering condition (3-7) to algorithm 2, guarantees Nash equilibrium convergence. □

Triggering condition (3-7) is easy to apply, since the utility functions of all the players are known. While this method decreases the amount of communication at some iterations, it is possible that the lack of communication leads to a slower convergence rate. Theoretically this means that even though communication is reduced at some iterations, the game could take more iterations to converge and thus increase the total amount of communication. Additionally this method requires the controller node to have knowledge of the utility functions of the players, which can be privacy sensitive information. For these reasons, I aim to further reduce the amount of communication by using a decentralized triggering condition.

3-3 Decentralized triggering

Algorithm 2 reduces the communication needed at some iterations. However, after every strategy change, the player needs to communicate with the central node, who then has to check the triggering condition. If players could check a triggering condition locally, the amount of communication would be further reduced. Additionally decentralized triggering has a higher degree of privacy since there is no centralized entity that needs access to personal information to evaluate the centralized triggering condition. To achieve this, a triggering condition is needed that can be evaluated with only local information. The only information available to the players is their own strategy $S_j(k)$ and the last communicated combined strategy of all players $\mathbf{1} \cdot \hat{S}(k)$. Furthermore, since in a decentralized triggering scheme communication does not take place after every strategy change, the game is no longer played out sequentially as in algorithm 2, but simultaneously. This means that all players change their strategy at the same time. Simultaneous games have different convergence guarantees than sequential games, but [21] provides the following theorem:

Theorem 3-3.1 (Simultanious potential game convergence [21])**.** *If every subgame of a potential game has a unique pure Nash equilibrium, then better-reply dynamics converge for any simultaneous schedule.*

Since no convergence guarantees exist for simultaneous games where not every subgame of a potential game has a unique pure Nash equilibrium, they will not be considered in this thesis. Better reply dynamics are defined as follows:

Definition 3-3.1. *A strategy change $S_j(k)$ to $S_j(k+1)$ is a better reply if:*

$$U_j(S_j(k+1), S_{-j}(k)) \geq U_j(S_j(k), S_{-j}(k)) \tag{3-9}$$

Because theorem 3-3.1 states that convergence is guaranteed with better reply dynamics, the goal is to design a triggering condition that ensures that all players only give better replies. First the measurement error ϵ is defined as follows:

Definition 3-3.2 (Measurement error)**.**

$$\epsilon_j(k) = S_j(k) - \hat{S}_j(k) \quad j \tag{3-10}$$

$$E(k) = [\epsilon_1(k) \dots \epsilon_N(k)]^T \tag{3-11}$$

Each player can only identify its own ϵ_j and can therefore not check if

$$U_j(S_j(k+1), \mathbf{1} \cdot (\hat{S}_{-j}(k) + E_{-j}(k))) \geq U_j(S_j(k), \mathbf{1} \cdot (\hat{S}_{-j}(k) + E_{-j}(k))) \tag{3-12}$$

To ensure that equation (3-12) holds at all times, the decentralized event triggered potential game method differs from the centralized in two key things. Firstly, the triggering conditions will not be based on the increase of a players utility function. Instead, all players will have a local triggering condition of the following form:

$$k_{i+1} = \inf\{k > k_i : |\epsilon_j(k)| \geq \epsilon_{max}(k)\} \tag{3-13}$$

Due to these triggering conditions players trigger communication when their own error $\epsilon_j(k)$ exceeds the maximum error $\epsilon_{max}(k)$. This maximum error is agreed upon beforehand by all players and thus known by everyone. Because of this agreement, players know that the following will hold even if communication has not taken place in a while:

$$|\mathbf{1} \cdot E_{-j}(k)| \leq (n-1) \cdot \epsilon_{max}(k) \quad k \tag{3-14}$$

Secondly the way players change their strategy is changed. In the centralized case players change their strategy as follows:

$$S_j(k+1) = \underset{S_j}{\text{argmax}}\ U_j(S_j, \mathbf{1} \cdot \hat{S}_{-j}) \tag{3-15}$$

When players have access to the current combined strategy profile, this is guaranteed to be a better reply. However, in the event triggered game an error is present and thus this is not guaranteed to be a better reply. As mentioned before this error cannot be evaluated locally by the players. However the players know that the errors are within an error bound because of triggering condition 3-13. They can thus check if the following holds:

$$U_j(S_j(k+1), \mathbf{1} \cdot (\hat{S}_{-j}(k) + E_{-j})) \geq U_j(S_j(k), \mathbf{1} \cdot (\hat{S}_{-j}(k) + E_{-j})) \quad E_{-j} \\ s.t. \; |\mathbf{1} \cdot E_{-j}| \leq (n-1) \cdot \epsilon_{max}(k) \tag{3-16}$$

By evaluating the increase in potential function by changing strategy to $S_j(k+1)$ for all possible errors, a player can guarantee $S_j(k+1)$ is a better reply if it is a better reply for all possible errors. If a player can not guarantee this it will not change its strategy at that iteration. An exception to this rule occurs at the triggering instances k_i. Since all players know the error is zero $S_j(k+1)$ is guaranteed to be a better reply and players do not need to check (3-16). With these 2 changes to the centralized case players will only give better replies and the game will therefore converge. However, this convergence will not necessarily be towards the Nash equilibrium. If ϵ_{max} is chosen very large then 3-16 will probably not hold for any player. This would mean that no player changes its strategy. If this occurs when the game is not yet at the Nash equilibrium, the Nash equilibrium will never be reached. To prevent this, players will communicate to the central node when they can no longer improve their strategy due to the large ϵ_{max}. If this is the case for all players, the central node will communicate this to the players. All players then decrease ϵ_{max} by the same amount, which is also agreed on before the game. This way the game will only converge to the Nash equilibrium. This procedure for the decentralized event triggered potential game is shown in algorithm 3.

Proposition 3-3.1. *Any event triggered ordinal potential game G, for which every subgame has a unique pure strategy Nash equilibrium, will converge to the Nash equilibrium using algorithm 3, with the following decentralized triggering condition:*

$$k_{i+1} = inf\{k > k_i : \epsilon_j(k) \geq \epsilon_{max}(k)\} \tag{3-17}$$

Proof. The chosen triggering condition guarantees the following:

$$\epsilon_j(k) \leq \epsilon_{max}(k) \quad j \quad N, \quad k \tag{3-18}$$

Since only the combined strategy profile is required to evaluate the utility functions of the players , the individual errors are not important. However (3-18) can be used to conclude the following:

$$|\mathbf{1} \cdot E_{-j}(k)| \leq (n-1) \cdot \epsilon_{max}(k) \quad k \tag{3-19}$$

In algorithm 3 players evaluate whether the inequality below holds:

$$U_j(S_j(k+1), \mathbf{1} \cdot (\hat{S}_{-j}(k) + E_{-j})) \geq U_j(S_j(k), \mathbf{1} \cdot (\hat{S}_{-j}(k) + E_{-j})) \quad E_{-j} \\ s.t. \; |\mathbf{1} \cdot E_{-j}| \leq (n-1) \cdot \epsilon_{max}(k) \tag{3-20}$$

If this holds for player j, then $S_j(k+1)$ will be a better reply if $|\mathbf{1} \cdot E_{-j}| \leq (n-1) \cdot \epsilon_{max}(k)$, which is guaranteed due to triggering condition 3-17. If it does not hold player j will not change its strategy:

$$S_j(k+1) = S_j(k) \tag{3-21}$$

This is also a better reply according to definition 3-3.1. Furthermore at triggering instances k_i players will not evaluate (3-16) since the error will be 0 for all players, which guarantees

$S_j(k+1)$ is a better reply because this is an ordinal potential game.
Thus it can be concluded that players only use better reply dynamics, which guarantees that the game will converge. By decreasing $\epsilon_{max}(k)$ whenever no one can improve, but the Nash equilibrium has not been reached, the game will go on until at some point $\epsilon_{max}(k) = 0$. When this point is reached the communication will no longer be event triggered but occur continuously and is thus guaranteed to converge to the Nash equilibrium. □

Algorithm 3: Decentralized event triggered non-cooperative potential game

Result: Nash equilibrium
initialization;
All players start with a strategy S_j
1 **for** j N **do**
  player j checks local triggering condition
  $S_j(k+1) = \underset{S_j}{\operatorname{argmax}}\ U_j(S_j, \hat{S}_{-j}(k))$
  Player j checks if (3-16) holds.
  if *(3-16) does not hold* **then**
    $S_j(k+1) = S_j(k)$
    Player j communicates to the central node that no improvement is possible due to large maximum error ϵ_{max}(k)
  end
  if $S_j(k) = \underset{S_j}{\operatorname{argmax}}\ U_j(S_j, \hat{S}_{-j}(k))$ **then**
    Player j communicates to the central node that $S_j(k+1) = S_j(k)$
  end
end
if *The triggering condition for any player is violated* **then**
  All players communicate their strategy to the central node and then the central node communicates the current combined strategy $\mathbf{1} \cdot S(k)$ to all players.
  Return to 1
end
if *Central node receives the information that all players can not improve due to large* ϵ_{max}*(k)* **then**
  Central nodes communicates this to all players so all players calculate $\epsilon_{max}(k+1)$ such that $\epsilon_{max}(k+1) \leq \epsilon_{max}(k)$
else
  $\epsilon_{max}(k+1) = \epsilon_{max}(k)$
end
if *Central node knows* $S_j(k+1) = S_j(k)$ j **then**
  End game
end
$k = k+1$
Return to 1

Chapter 4

Event trigggered DSM game

4-1 Smart grid network topology

Since a smart grid has such an abundant communication network, there are many different possibilities for residential users, which are the players in the DSM game, can communicate with each other. They can communicate using the concept of internet of things (IOT) in which all sorts of smart devices can communicate with each other using the internet. Another possibility is using the Neighbourhood area network (NAN) which is a wireless network based on either Wimax or cellular network technologies. Communicating via the electric network using power line communication (PLC) is also a possibility. Since this DSM game will be performed on a microgrid level the communication network for the DSM game should perform well in a microgrid. Figure 4-1 shows the architecture of a microgrid.

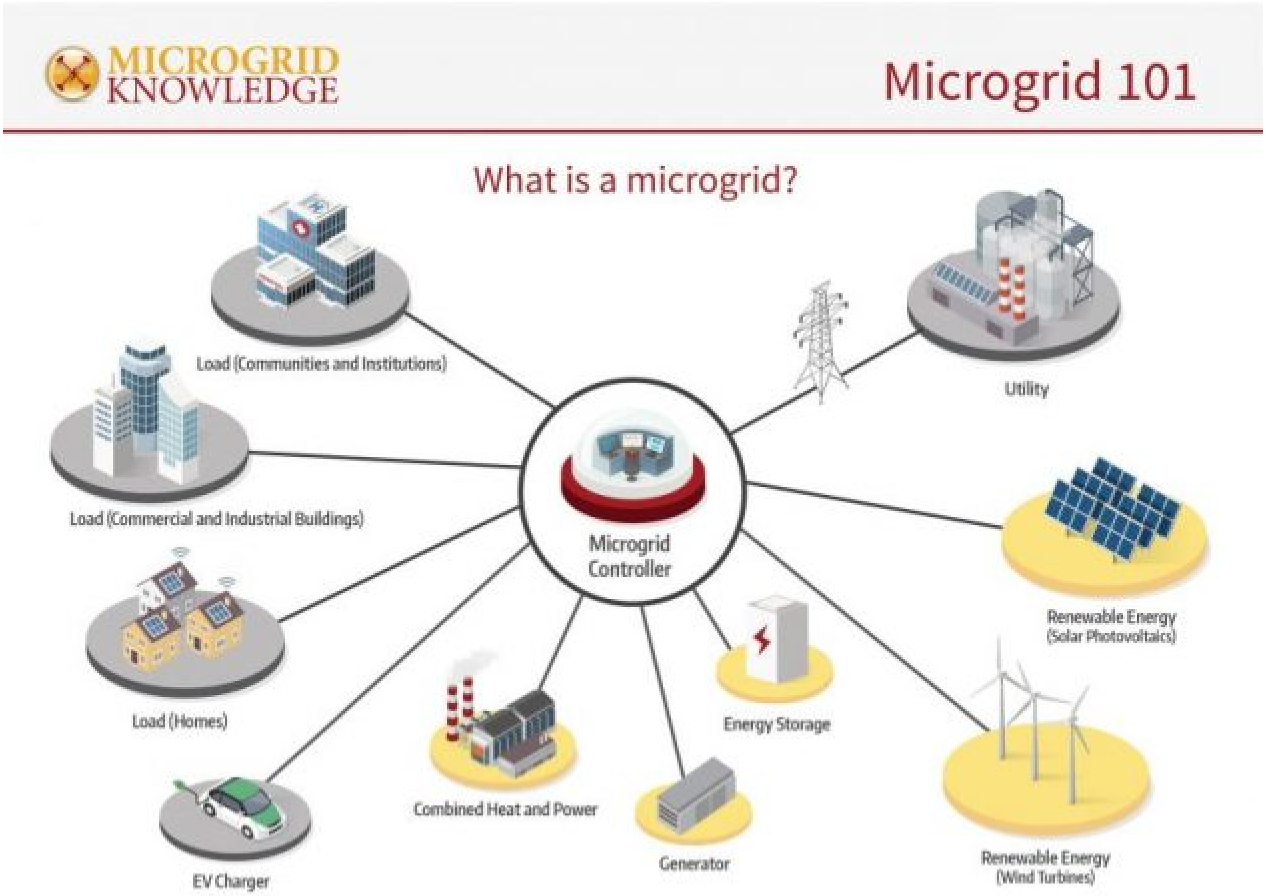

Figure 4-1: Microgrid architecture in the smart grid [1]

In this figure it can be seen that all the residential users are connected to the microgrid central controller (MGCC). Since there already exists communication between all the players in the game via this MGCC, which is the NAN network, and this mirrors the network topology discussed in chapter 3 this network is selected for the DSM game.

4-2 Game setup

To show how the techniques in chapter 3 can be applied in practice I propose a non-cooperative demand side management game. In this game the players represent residential energy consumers that wish to minimize their electricity costs as well as try to consume energy at times that is convenient to them. The game will consist of N consumers that can change their energy consumption during a 24h period. This 24h period is divided into 24 slots of 1h. In the game setup I use the following assumptions

Assumption 1. All players have only continuous loads. This means the energy consumption during a timeslot can take all values between 0 *and* P_{max}*.*

Assumption 2. The total amount of energy consumed by each player remains constant through-

out the game.
The characteristic sets of this game are as follows

1. N: $[n_1 \dots N]$
2. L_n: $[l_{1,n}, l_{2,n} \dots l_{24,n}]$
3. U : $[U_1 \dots U_N]$

The utility function of the players is:

$$U_n(L_n, L_{-n}) = -\sum_{t=t_1}^{T} \left(a(l_{t,n} + \mathbf{1} \cdot L_{t,-n}^T)^2 + b(l_{t,n} - l_{t,n,desired})^2 \right) \tag{4-1}$$

The first part of the utility function represents the energy costs. Similar to [20] these costs are modelled quadratic in the amount of energy consumed. The second part represents the consumers convenience. Each player has a preferred energy consumption profile. Deviation from this profile leads to inconvenience. To ensure both lower or higher than desired energy consumption decreases the utility function this is also modelled quadraticly. a & b are constants that indicate the importance of cost and convenience to the utility. The utility function shows that the individual consumption profiles are not required for players to evaluate their utility function, but only the combined energy consumption profile of the other players. This confirms the assumption in chapter 3 that only the combined strategy profile is sufficient.

Proposition 4-2.1. *The proposed DSM game with Utility functions* $U_n(L_n, L_{-n}) = -\sum_{t=t_1}^{T} \left(a(l_{t,n} + \mathbf{1} \cdot L_{t,-n}^T)^2 + b(l_{t,n} - l_{t,n,desired})^2 \right)$ *is a continuous exact potential game.*

Proof. Theorem 2-3.1 can be used to identify this game as a continuous exact potential game. The second term of the utility of all the players only has one variable which is $l_{t,n}$. Thus the following is true:

$$\frac{\partial^2 b(l_{t,n} - l_{t,n,desired})^2}{\partial l_{t,n} \partial l_{t,m}} = 0 \quad n, m \quad N \tag{4-2}$$

Furthermore the first part of the utility function is equal for all players, which means:

$$\frac{\partial^2 a(l_{t,n} + \mathbf{1} \cdot L_{t,-n}^T)^2}{\partial l_{t,n} \partial l_{t,m}} = \frac{\partial^2 a(l_{t,m} + \mathbf{1} \cdot L_{t,-m}^T)^2}{\partial l_{t,n} \partial l_{t,m}} \quad n, m \quad N \tag{4-3}$$

Thus it can be concluded that for this game theorem 2-3.1 holds and this game is a continuous exact potential game. Furthermore the exact same proof will hold for any sub game and thus simultaneous gameplay will also have Nash equilibrium convergence. □

The players change their strategy according to the following dynamics:

$$l_{t,n}(k+1) = l_{t,n}(k) + \alpha(L_t^{avg} - \mathbf{1} \cdot L_t^T(k)) + \beta(l_{t,n,desired} - l_{t,n}(k)) \tag{4-4}$$

These dynamics are very intuitive. The old value is taken as a starting point and is then adjusted based on two terms that correspond to the terms present in the utility function. Due to the quadratic nature of the cost of energy, large spikes in energy consumption leads to high costs. To reduce these costs, the energy consumption during all time slots needs to remain close to the average. This is represented in the dynamics through the $\alpha(L_t^{avg} - \mathbf{1} \cdot L_t^T(k))$ term. If the total energy consumption at time slot t is lower than the average, the players energy consumption increases and vice versa. The same logic applies to the convenience. If the player consumes less energy than their desired energy consumption, the energy consumption increases and vice versa. The coefficients α & β can be tweaked to change the rate of converge for the game, but always need to adhere to the relation $a/b = \alpha/\beta$ to match the importance of costs and inconvenience of in the utility function. Furthermore, it can be proven that these dynamics are better reply dynamics, and thus guarantee nash equilibrium convergence under full information, by using lyapunov theory. This proof can be found in appendix A.

4-3 Triggering conditions

As shown in chapter 3 knowledge of the potential function is not needed to synthesize a triggering condition since the utility function can be used to create a triggering condition. For the centralized triggering condition, the MGCC will act as the centralized entity which will be checking the triggering condition. The MGCC has full information and triggering condition (3-7) can simply be applied. For this particular game it will look as follows:

$$\begin{aligned} k_{i+1} = \inf\{k > k_i : & U_n(L_n(k+1), L_{-n}(k+1)) \\ & - U_n(L_n(k), L_{-n}(k)) \leq 0\} \end{aligned} \tag{4-5}$$

Furthermore this game can be solved with a decentralized triggering condition which bounds the error ϵ as shown in equation (3-17). To apply this, the errors are defined below:

$$\epsilon_{n,t}(k) = l_{t,n}(k) - \hat{l}_{t,n}(k) \tag{4-6}$$

$$\epsilon_t(k) = \mathbf{1} \cdot L_t^T(k) - \mathbf{1} \cdot \hat{L}_t^T(k) \tag{4-7}$$

$$E(k) = [\epsilon_1(k), \ldots, \epsilon_T(k)] \tag{4-8}$$

Since all players do not just have a single strategy but a strategy profile over 24 time slots, the triggering condition will have to check whether any of the The decentralised triggering conditions thus look as follows:

$$k_{i+1} = \inf\{k > k_i : \epsilon_{n,t}(k) \geq \epsilon_{max}(k), \text{for any } t\} \tag{4-9}$$

Chapter 5

Simulation Results

5-1 Four player event triggered DSM game

To simulate this game a desired energy profile for all the players needs to be created. To closely match the reality I construct it based on the research conducted by [13]. The authors gathered information of the energy consumption by residential, commercial and industrial consumers. Since the players in this game are residential consumers, the residential profile will be mimicked. The cumulative desired energy consumption for a game with four players is shown in Figure 5-1.

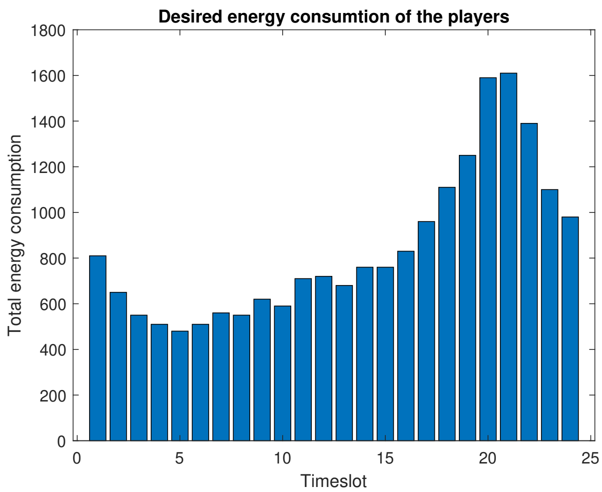

Figure 5-1: Cumulative desired energy profile

While the game converges to the NE regardless of initial condition, picking the right initial condition can save some time. Since the NE is not known a priori, it can not be used to choose an initial condition. In practice players will have no information on the consumption profiles of others. Therefore it is logical for players to initialize their own consumption profile based only on their convenience. The profile in Figure 5-1 will therefore be used as initial condition of the game. In game theory games progress until the NE is reached. However due to the nature of the chosen dynamics, the game will converge to the NE asymptotically. To prevent the game from never ending, it is stopped once the players reach the ϵ-equilibrium with an ϵ of 1. I assume that this is close enough to the NE, such that the difference is insignificant. For this example I choose $a = 1$ & $b = 0.5$ as constants with convergence parameters $\alpha = 1/N$ & $\beta = \alpha/2$ for the game with full information and $\alpha = 1/(2 \quad N)$ for the event triggered game to prevent overshoot. Additionally for the decentralized method if the maximum error ϵ_{max} needs to be reduced all players will divide it by 5. To show proof of concept, I conduct this DSM game with four players. Figure 5-2 shows the Nash equilibrium state of this 4 player game. Both the centralized and decentralized event triggered games reach this Nash equilibrium. Thus both methods succeed in reducing the peak to average ratio of the energy profile, which is the main goal of demand side management, from 1.92 to 1.10. The centralized and decentralized games reach this Nash equilibrium with 13 and 39 triggering instances respectively where as the full information cases require approximately 30 communication instances. The centralized triggering seems to perform better in the four player game than the decentralized triggering, however this result is not enough to draw any conclusions. To further investigate the performance of the methods I move on from four player games to games with higher amounts of players, which better matches the reality of DSM games in microgrids.

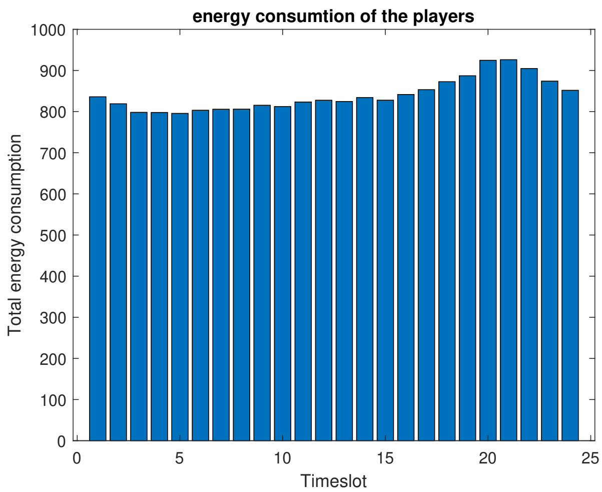

Figure 5-2: Nash Equilibrium of the demand side management game

5-2 Games with more players

The four player game example shows the validity of the event triggered approach. In reality however, DSM in microgrids will include a large amount of players. To show this method can be applied in various situations I simulate multiple games with different amount of players. Furthermore I simulate these games with different initial conditions to show the communication reduction does not depend on initial conditions. I create these differing initial conditions by taking the desired energy profile as a starting point and for every player 10 of the 24 values will be increased by 20 and 10 values will be decreased by 20. The values that are changed are chosen randomly. Tables 5-1 and 5-2 show the means and standard deviations over 10 games with differing initial conditions for the sequential game with full information, the centralized event triggered game, the simultaneous game with full information and the decentralized event triggered game. In these tables 'it.' refers to the number of iterations required to reach the ϵ-equilibrium and 'itx.' refers to the number communication triggering instances. In the sequential game it takes n iterations for everyone to change their strategy, but in the simultaneous game all players change strategies in only 1 iteration. To better compare the tables, in the full information table the iterations numbers are divided by N.

Table 5-1: Communication instances for centralized DSM games

	FI it. mean (std)	ET itx. mean (std)	ET it. mean (std)
N=10	76.4 (1.65)	26.4 (1.17)	108.7 (2.75)
N=50	321.7 (7.47)	100.6 (3.37)	403.5 (13.44)
N=100	577.2 (10.13)	171.2 (3.94)	692.8 (14.99)
N=200	1008.8 (15.68)	274.6 (12.65)	1109.0 (38.29)

Table 5-2: Communication instances for decentralized DSM games

	FI it. mean (std)	ET itx. mean (std)	ET it. mean (std)
N=10	60.9 (1.20)	45.4 (11.84)	107.2 (9.75)
N=50	241.2 (5.59)	240.3 (60.22)	437.3 (75.13)
N=100	428.7 (5.72)	320.9 (43.96)	706.3 (25.29)
N=200	720.4 (14.39)	337.0 (21.96)	1165.4 (45.25)

By comparing these two tables the following things can be observed: Firstly, for DSM games, which only have to be played out several times a day, time for the game to reach the ϵ-equilibrium is not important, but for games that need to be played out more often, the time required to finish the game could be important. The sequential game with centralized triggering and the simultaneous game with decentralized triggering have approximately the same amount of iterations in the tables. However, keep in mind that the iterations in Table 5-1 are divided by N. Therefore the centralized game actually takes N times longer. On the other hand the decentralized game requires all players to perform more complex calculations which means each individual iteration takes longer. Nevertheless this is not enough to make up for the extra iterations. Thus if speed is important, the decentralized approach is probably the most suited. Secondly the computational intensity can be important. Especially in the smart grid where smart home controllers have various responsibilities all processing power that is being used for the DSM game can not be used elsewhere. The iterations column in Tables 5-1 and 5-2 accurately represent how often players have to perform calculations. This is nearly equal for both methods for all game sizes, but as mentioned before in the decentralized approach players need to perform more complex calculations. Therefore the centralized approach performs better in this regard. Thirdly, the centralized method triggers significantly less for all game sizes (especially for low game sizes). This is to be expected since the centralized triggering scheme uses full information to trigger, but the decentralized triggering scheme uses only local information. However to gain this full information the centralized triggering scheme requires some communication at each iteration. To accurately assess which method requires the least communication, not only the amount of triggering instances is important but the total amount of communication. In the decentralized scheme, when communication is triggered, all players send information to the MGCC, which then responds by sending information back to all players. I define this as one communication instance. In the centralized scheme all players, regardless of triggering, send information to the MGCC every iteration. Contrary to the the decentralized scheme the MGCC does not instantly reply. Therefore every iteration in Table 5-1 there is communication equal to half a communication instance. Then when communication is triggered, players do not have to send information to the MGCC, as it already has all current information. What does occur is the MGCC sends information to all players. This is again equivalent to half a communication instance. Therefore the total communication for the centralized triggered sequential game is equal to half the amount of triggering instances plus half the amount of iterations in Table 5-1. Table 5-3 compares the average total communication of the centralized and decentralized triggering schemes.

Table 5-3: Total required communication for event triggered DSM games

	Centralized mean	Decentralized mean
N=10	67.6	45.4
N=50	252.1	240.3
N=100	432	320.9
N=200	691.8	337.0

With the information in table 5-3 it can be concluded that the decentralized triggering scheme performs best with regard to total communication reduction.

While the centralized method performs worse, it does show that the game can converge to the ϵ-equilibrium with less triggering instances than the decentralized scheme currently uses. The decentralized triggering mechanism is thus over-triggering. To analyze where exactly this over-triggering occurs, I will use a DSM game with a 100 players. Figures 5-3 and 5-4 show the amount of triggering instances per 10 iterations over the course of the game for the centralized triggering and decentralized triggering respectively.

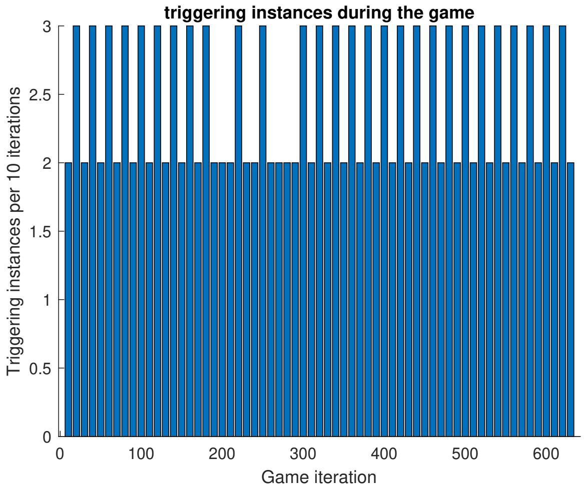

Figure 5-3: Overview of the amount triggering instances per 10 iterations during the centralized triggered game

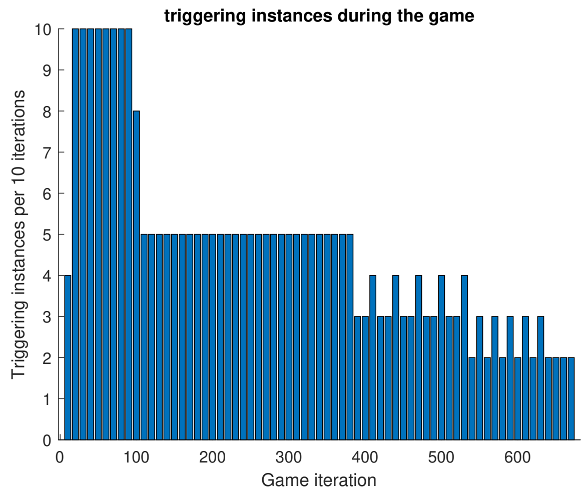

Figure 5-4: Overview of the amount triggering instances per 10 iterations during the decentralized triggered game

From these figures it can be observed that the centralized triggering consistently triggers two or three times per ten iterations throughout the entire game. However for the decentralized triggered game this is not the case. While near the ϵ-equilibrium it triggers very similarly to the centralized version, but during the early parts of the game it triggers more often. Since the centralized also converges to the ϵ-equilibrium, the decentralized triggering is over triggering during the first part of the game. Thus there might be room for improvement for the decentralized triggering scheme.

Chapter 6

Final Remarks

6-1 Conclusions

In the future, the current power network, which is designed with centralized generation and unidirectional power distribution in mind, will be replaced by the so called smart grid. This new power grid architecture will allow for distributed generation, many of which will be sustainable power sources, and two directional power distribution. Furthermore a large communication network will be present in this smart grid so that a lot of opportunities for control are present. Due to these control opportunities, which all require use of this communication network, the amount of communication available can become a constraint. Therefore finding ways to apply these control methods with reduced communication can be pivotal. One such control method where communication can be reduced is demand side management. A technique that aims to reduce the cost of energy production by changing the consumption behaviour of electricity consumers to reduce peaks and thus reduce generation costs. By molding the DSM concept into a potential game framework, the concepts of event triggered control can be applied to perform DSM with reduced communication. By realising that potential functions are to potential games as lyapunov functions are to control systems, a triggering condition can be designed that will guarantee convergence to the Nash equilibrium. In this thesis I have designed event triggered potential game algorithms that succeed in converging to the Nash equilibrium while reducing the amount of communication required compared to regular potential games. Firstly a centralized event triggered game in which a central entity is present, which in the case of a smart grid will be the microgrid central controller, through which all players communicate. In this case the game is played out sequentially with communication after every strategy change from one player to the centralized entity. This entity, which thus has full information, will decide whether or not to trigger communication based on the increase of the potential function. Notably to apply this method knowledge of the potential function, which is usually hard to find, is not required. Secondly, a decentralized event triggered game. In this game all change strategies simultaneously. Furthermore unless a triggering condition is violated no communication occurs during a game iteration. Since the centralized entity does not have full information at all times in this scenario, it will not be the one to trigger communication. Instead all players will decide for themselves whether communication is required. An advantage of this approach is that communication is not required after every strategy change. Additionally players wont have to share privacy sensitive information with the central entity, since they are triggering themselves. A downside of this method is that the player will need to trigger using only local information. This can lead to over triggering and thus more communication. Also not all games will converge to the Nash equilibrium with simultaneous game play and can thus not be applied to all potential games. After applying both methods for event triggered games to a DSM game it can be concluded that both methods succeed in significantly reducing the amount of communication required to reach the nash equilibrium of the DSM game. In terms of performance the centralized triggered game uses less triggering instances, but due to the communication after every strategy change is the total amount of communication lowest for the decentralized triggered game.

6-2 Future work

While this thesis provides two methods to reduce communication in potential games, there still are opportunities to farther explore this topic.

For instance, in the centralized triggering method the centralized entity determines whether communication is necessary. To evaluate this it needs to know the utility functions of all players. Since these potential games are usually non cooperative, the Utility function will contain private information. Thus communicating this to the centralized entity is not desirable. A possible way to circumvent this problem is the use of homomorphic encryption [2]. This is an encryption method that allows parties to perform mathematical operations on the data without knowing the encryption key. These homomorphic encryption techniques are usually designed for single user use only, but the authors of [24] have created an encryption scheme applicable to multi user systems. In game theory this approach would mean that the centralized entity would get an encrypted utility function, perform the calculation required to identify if communication is needed, then send it back to the player who can then decrypt and actually trigger if necessary. Future work could investigate whether this homomorphic encryption method is feasible for potential games.

Secondly, this thesis has only considered the total amount of communication as a metric to judge performance of the event triggered methods, but total communication alone might not be enough to judge performance. The centralized method triggers less than the decentralized method but requires a small amount of communication after every strategy change and thus has a higher total communication. However, because of this the communication of the centralized method is more spread out. This small amount of communication that occurs after every iteration might not be a problem in some circumstances. Thus future work could study the impact of this spread out communication in for instance smart grid environments.

Finally, To apply these methods to the DSM problem, this thesis focuses on communication reduction in non-cooperative potential games. However some cooperative games can also be classified as potential games. Cooperative games, like for instance coalition forming games, are played out differently than non-cooperative games, thus this work can not be directly applied to cooperative games. Future research could explore the possibilities to use the techniques shown in this thesis in a cooperative potential game setting to reduce communication.

Appendix A

Appendix A

This appendix covers mathematical proofs

A-1 Dynamics convergence

For the dynamics to be better reply dynamics the following needs to hold:

$$U_n(k+1) - U_n(k) \geq 0 \quad i, k \tag{A-1}$$

$$U_n(k) = -\sum_{t=t_1}^{T} a(l_{t,n}(k) + \mathbf{1} \cdot L_{t,-n}^{T}(k))^2 - b(l_{t,n}(k) - l_{t,n,desired})^2 \tag{A-2}$$

This utility function can be used as a lyapunov function for the proposed dynamics.
The proposed discrete dynamics:

$$l_{t,n}(k+1) - l_{t,n}(k) = \alpha(L^{avg} - (l_{t,n}(k) + \mathbf{1} \cdot L_{t,-n}^{T}(k))) + \beta(l_{t,n,desired} - l_{t,n}(k)) \tag{A-3}$$

note: $\alpha/\beta = a/b$. For short hand notation the timestep indicator (k) will be omitted on the right hand side.
lyapunov equation:

$$V(k+1) - V(k) = \sum_{t=t_1}^{T} \Big(-2a(l_{t,n} + \mathbf{1} \cdot L_{t,-n}^{T}) - 2b(l_{t,n} - l_{t,n,desired}) \Big) \Big(\alpha(L^{avg} - (l_{t,n} + \mathbf{1} \cdot L_{t,-n}^{T})) + \beta(l_{t,n,desired} - l_{t,n}) \Big)$$

$$
\begin{aligned}
V(k+1) - V(k) = \sum_{t=t_1}^{T} & - 2a\alpha((l_{t,n} + \mathbf{1} \cdot L_{t,-n}^T)(L^{avg} - (l_{t,n} + \mathbf{1} \cdot L_{t,-n}^T)) \\
& - 2a\beta((l_{t,n} + \mathbf{1} \cdot L_{t,-n}^T)(l_{t,n,desired} - l_{t,n})) \\
& - 2b\alpha((l_{t,n} - l_{t,n,desired})((L^{avg} - (l_{t,n} + \mathbf{1} \cdot L_{t,-n}^T)) \\
& - 2b\beta((l_{t,n} - l_{t,n,desired})(l_{t,n,desired} - l_{t,n}))
\end{aligned}
$$

$$
\begin{aligned}
V(k+1) - V(k) = \sum_{t=t_1}^{T} & - 2a\alpha((l_{t,n} + \mathbf{1} \cdot L_{t,-n}^T)(L^{avg} - (l_{t,n} + \mathbf{1} \cdot L_{t,-n}^T)) \\
& - 2a\beta((l_{t,n} + \mathbf{1} \cdot L_{t,-n}^T)(l_{t,n,desired} - l_{t,n})) \\
& - 2b\alpha((l_{t,n} - l_{t,n,desired})((L^{avg} - (l_{t,n} + \mathbf{1} \cdot L_{t,-n}^T)) \\
& + 2b\beta(l_{t,n} - l_{t,n,desired})^2
\end{aligned}
$$

$a\beta = b\alpha$, thus this can be written as

$$
\begin{aligned}
V(k+1) - V(k) = \sum_{t=t_1}^{T} & - 2a\alpha((l_{t,n} + \mathbf{1} \cdot L_{t,-n}^T)(L^{avg} - (l_{t,n} + \mathbf{1} \cdot L_{t,-n}^T)) \\
& - 2b\alpha((l_{t,n} - l_{t,n,desired})(L^{avg} - 2(l_{t,n} + \mathbf{1} \cdot L_{t,-n}^T)) \\
& + 2b\beta(l_{t,n} - l_{t,n,desired})^2
\end{aligned}
$$

$\sum_{t=t_1}^{T}(l_{t,n} - l_{t,n,desired}) = 0$. Since L^{avg} is a constant it can be taken out of that part of the equation to get:

$$
\begin{aligned}
V(k+1) - V(k) = \sum_{t=t_1}^{T} & + 2a\alpha((l_{t,n} + \mathbf{1} \cdot L_{t,-n}^T)((l_{t,n} + \mathbf{1} \cdot L_{t,-n}^T) - L^{avg}) \\
& + 2b\alpha(2(l_{t,n} + \mathbf{1} \cdot L_{t,-n}^T)(l_{t,n} - l_{t,n,desired})) \\
& + 2b\beta(l_{t,n} - l_{t,n,desired})^2
\end{aligned}
$$

The top and bottom line will always be positive while the middle row can be both positive and negative. To prove that the difference equation is always positive or zero, the worst case will be considered. This will be when $(l_{t,n} - l_{t,n,desired})$ is at its lowest when $(l_{t,n} + \mathbf{1} \cdot L_{t,-n})$ is at its highest and vice versa at another time slot. Both will be zero elsewhere. Since they are zero elsewhere the following relations will hold.

$$
\begin{aligned}
((l_{t,n} + \mathbf{1} \cdot L_{t,-n}^T)_{max} - L^{avg}) &= -((l_{t,n} + \mathbf{1} \cdot L_{t,-n}^T)_{min} - L^{avg}) \\
(l_{t,n} - l_{t,n,desired})_{max} &= -(l_{t,n} - l_{t,n,desired})_{min} \\
(l_{t,n} + \mathbf{1} \cdot L_{t,-n}^T)_{max} - (l_{t,n} + \mathbf{1} \cdot L_{t,-n}^T)_{min} &= 2((l_{t,n} + \mathbf{1} \cdot L_{t,-n}^T)_{max} - L^{avg})
\end{aligned}
$$

With this information the difference equation can be rewritten as

$$
\begin{aligned}
V(k+1) - V(k) = &+ 2a\alpha\Big((l_{t,n} + \mathbf{1}\cdot L^T_{t,-n})_{max}((l_{t,n} + \mathbf{1}\cdot L^T_{t,-n})_{max} - L^{avg}) \\
&+ (l_{t,n} + \mathbf{1}\cdot L^T_{t,-n})_{min}((l_{t,n} + \mathbf{1}\cdot L^T_{t,-n})_{min} - L^{avg})\Big) \\
&+ 2b\alpha\Big(2(l_{t,n} + \mathbf{1}\cdot L^T_{t,-n})_{max}(l_{t,n} - l_{t,n,desired})_{min} \\
&+ 2(l_{t,n} + \mathbf{1}\cdot L^T_{t,-n})_{min}(l_{t,n} - l_{t,n,desired})_{max}\Big) \\
&+ 2b\beta(l_{t,n} - l_{t,n,desired})^2_{max} + (l_{t,n} - l_{t,n,desired})^2_{min}
\end{aligned}
$$

$$
\begin{aligned}
V(k+1) - V(k) = &+ 2a\alpha\Big(((l_{t,n} + \mathbf{1}\cdot L^T_{t,-n})_{max} - (l_{t,n} + \mathbf{1}\cdot L^T_{t,-n})_{min})((l_{t,n} + \mathbf{1}\cdot L^T_{t,-n})_{max} - L^{avg})\Big) \\
&- 4b\alpha\Big(((l_{t,n} + \mathbf{1}\cdot L^T_{t,-n})_{max} - (l_{t,n} + \mathbf{1}\cdot L^T_{t,-n})_{min})(l_{t,n} - l_{t,n,desired})_{max}\Big) \\
&+ 4b\beta(l_{t,n} - l_{t,n,desired})^2_{max}
\end{aligned}
$$

$$
\begin{aligned}
V(k+1) - V(k) = &+ 4a\alpha((l_{t,n} + \mathbf{1}\cdot L^T_{t,-n})_{max} - L^{avg})^2 \\
&- 8b\alpha((l_{t,n} + \mathbf{1}\cdot L^T_{t,-n})_{max} - L^{avg})(l_{t,n} - l_{t,n,desired})_{max}) \\
&+ 4b\beta(l_{t,n} - l_{t,n,desired})^2_{max}
\end{aligned}
$$

$$
\begin{aligned}
V(k+1) - V(k) = &+ a\alpha((l_{t,n} + \mathbf{1}\cdot L^T_{t,-n})_{max} - L^{avg})^2 \\
&- 2b\alpha((l_{t,n} + \mathbf{1}\cdot L^T_{t,-n})_{max} - L^{avg})(l_{t,n} - l_{t,n,desired})_{max}) \\
&+ b\beta(l_{t,n} - l_{t,n,desired})^2_{max}
\end{aligned}
$$

Multiply with b/β

$$
\begin{aligned}
&\frac{a\alpha b}{\beta}((l_{t,n} + L_{t,-n})_{max} - L^{avg})^2 \\
&- \frac{2b\alpha b}{\beta}((l_{t,n} + \mathbf{1}\cdot L^T_{t,-n})_{max} - L^{avg})(l_{t,n} - l_{t,n,desired})_{max}) \\
&+ b^2(l_{t,n} - l_{t,n,desired})^2_{max}
\end{aligned}
$$

$\alpha b/\beta = a$ so

$$
\begin{aligned}
&a^2((l_{t,n} + \mathbf{1}\cdot L^T_{t,-n})_{max} - L^{avg})^2 \\
&- 2ab((l_{t,n} + \mathbf{1}\cdot L^T_{t,-n})_{max} - L^{avg})(l_{t,n} - l_{t,n,desired})_{max}) \\
&+ b^2(l_{t,n} - l_{t,n,desired})^2_{max}
\end{aligned}
$$

This has the form of $a^2 + b^2 - 2ab$ which is always ≥ 0 and thus

$$V(k+1) - V(k) \geq 0$$

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Glossary

List of Acronyms

DSM	demand side management
NE	Nash equilibrium
SG	smart grid
ETC	event triggered control
PAR	peak to average ratio
DLC	direct load control