
Comparison of Regional and Global GRACE Gravity Field Models at High Latitudes

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Abstract

In this study we address the question of whether regional gravity field modeling techniques of GRACE data can offer improved resolution over traditional global spherical harmonic solutions. Earlier studies into large, equatorial river basins such as the Amazon, Zambezi and others showed no obvious distinction between regional and global techniques, but this may have been limited by the fact that these equatorial regions are at the latitudes where GRACE errors are known to be largest (due to the sparse groundtrack coverage). This study will focus on regions of higher latitude, specifically Greenland and Antarctica, where the density of GRACE measurements is much higher. The regional modeling technique employed made use of spherical radial basis functions (SRBF), complete with an optimal filtering algorithm. Comparisons of these regional solutions were made to a range of other publicly available global spherical harmonic solutions, and validated using ICESat laser altimetry. The timeframe considered was a 3 year period spanning from October 2003 to September 2006.

21.1 Introduction

The launch of the Gravity Recovery and Climate Experiment (GRACE) in 2002 started a new wave of research into the Earth's mass transport processes. The measurements from the mission's twin satellites have enabled the multi-year tracking of many large scale processes, such as continental hydrology and ice mass changes in the cryosphere. While these first studies have produced some truly excellent results, there is always the desire to push the boundaries of what

GRACE can observe, in terms of spatial and temporal resolution. Previous studies have demonstrated that the current processing standards of GRACE data provide mass change accuracies on the order of 2 cm of equivalent water height (EWH) over spatial scales of 400 km and time intervals of 1 month (Klees et al. 2008a). This analysis was done by comparing the performance of a range of different GRACE processing strategies, including both regional and global methods, over selected river basins and other “dry” regions where little to no hydrological signal is expected. The global methods tested primarily involved traditional spherical harmonic solutions from various processing centers (CSR, GFZ, JPL, CNES, DEOS), but with various spatial filters applied. The regional solutions examined included the “mascon” approach (Luthcke et al. 2006) as well as

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solutions computed using spherical radial basis functions. In short, the overall conclusion of this earlier study was that there was no clear advantage to using regional techniques over global methods for the river basins studied. In fact, it turned out that the choice of spatial filter was the most important aspect in the comparisons; however, one of the limitations of this particular study was that most of the regions studied were at relatively low latitudes, where the density of GRACE measurements is the lowest. For higher latitude regions, it is possible that the increased data density might offer a higher signal-to-noise ratio that regional techniques might be able to better exploit. As a result, a follow-on study was conducted (Stolk 2009) to perform a similar analysis over regions such as Greenland and Antarctica, to see if the conclusions would be the same. This paper will provide an overview of the methods and conclusions of this follow-on study.

21.2 Spherical Radial Basis Functions

The focus of the regional techniques for the high-latitude regions involved the application of spherical radial basis functions (SRBF). The general concept behind this approach is to use a distribution of space-localizing functions to represent any complex spherical shape, such as the Earth's gravity field. The functions can be constructed using a number of different methods, although the kernel adopted for the current study makes use of Poisson wavelets of order three (Holschneider et al. 2003; Wittwer 2009). The shape and spatial distribution of the SRBFs are determined by the depth (i.e., bandwidth) and the level (i.e., spacing on a Reuter grid) assigned to each SRBF, as illustrated in Fig. 21.1.

As with spherical harmonic solutions, SRBF solutions suffer from north-south error patterns (i.e., “stripes”), which require the application of a suitable filter. The anisotropic, non-symmetric (ANS) filter developed by Klees et al. (2008b) offers a number of benefits over other traditional filtering techniques, such as destriping or Gaussian smoothing, primarily because use is made of the full statistical information of the solution (i.e., signal and noise variance-covariance matrices are used). For example, if spherical harmonics are used to parameterize the time-variable gravity field, and we let $\mathbf{N}\hat{\mathbf{x}} = \mathbf{b}$ represent the normal

equations for a monthly GRACE solution, the ANS filter \mathbf{W} can be applied as follows:

$$\hat{\mathbf{x}}_w = \mathbf{W}\hat{\mathbf{x}} = (\mathbf{N} + \mathbf{D}^{-1})^{-1}\mathbf{b} \quad (21.1)$$

where \mathbf{N} is the normal matrix, \mathbf{D} the signal covariance matrix (i.e., the auto-covariance matrix of the vector $\hat{\mathbf{x}}$), $\hat{\mathbf{x}}$ the estimated parameter vector, and \mathbf{b} the right-side vector. The matrix \mathbf{N} is determined from the partial derivatives of the system dynamics; however, the auto-covariance matrix, \mathbf{D} , must be determined empirically. This is done through an iterative process whereby the (time-independent) variances of the signal from the actual time series of monthly solutions (e.g., 36 months for this study) are computed at the nodes of an equal-angular grid and then transformed back to the spherical harmonic domain to form \mathbf{D} . This signal covariance information has the effect of suppressing spurious noise in regions that typically do not have much mass variations (e.g., oceans and deserts), while also allowing the solution to adjust more freely in areas where the mass change signal has larger variations (e.g., river basins). Since this signal covariance matrix is computed from the time series of GRACE solutions, it is particular to the solution technique.

A straightforward generalization of this concept to a SRBF parameterization is obtained when the relationship between spherical harmonic coefficients, \mathbf{x} , and SRBF coefficients, $\boldsymbol{\alpha}$, is exploited. This relationship can be written as

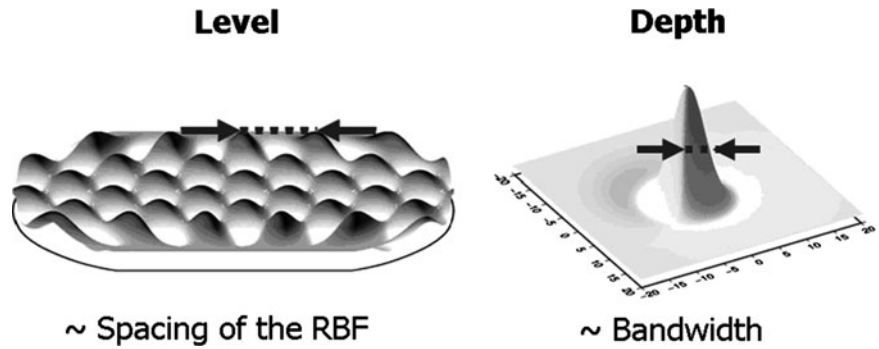
$$\mathbf{x} = \mathbf{Q}\boldsymbol{\alpha} \quad (21.2)$$

Hence, given the auto-covariance matrix in the spherical harmonic domain, \mathbf{D} , we can obtain the corresponding auto-covariance matrix in the SRBF domain according to

$$\boldsymbol{\alpha} = \mathbf{Q}^+\mathbf{x} \Rightarrow \mathbf{D}_\alpha = \mathbf{Q}^+\mathbf{D}(\mathbf{Q}^+)^T \quad (21.3)$$

where $\mathbf{Q}^+ = (\mathbf{Q}^T\mathbf{Q})^{-1}\mathbf{Q}^T$ is the pseudo-inverse of the \mathbf{Q} matrix. This approach, however, fails because the spectrum of a given SRBF parameterization comprises spherical harmonic degrees, which may exceed the maximum degree of a given GRACE monthly solution (the number of harmonic coefficients in \mathbf{x} is often much larger than the number of SRBF coefficients in $\boldsymbol{\alpha}$).

Fig. 21.1 Example spherical radial basis functions



Therefore, the optimal filter needs to be designed directly in the SRBF domain. If \mathbf{f} is the time-variable gravity signal in terms of equivalent water heights, and $\boldsymbol{\alpha}$ comprises the SRBF coefficients, we write the SRBF synthesis as

$$\mathbf{f} = \mathbf{B}\boldsymbol{\alpha} \quad (21.4)$$

Using the pseudo-inverse of \mathbf{B} , $\mathbf{B}^+ = (\mathbf{B}^T\mathbf{B})^{-1}\mathbf{B}^T$, we write

$$\boldsymbol{\alpha} = \mathbf{B}^+\mathbf{f} \quad (21.5)$$

and obtain the auto-covariance matrix in the SRBF domain, \mathbf{D}_α , as

$$\mathbf{D}_\alpha = \mathbf{B}^+\mathbf{D}(\mathbf{B}^+)^T \quad (21.6)$$

Hence, if $\mathbf{N}_\alpha \boldsymbol{\alpha} = \mathbf{b}_\alpha$ is the system of normal equations in terms of SRBFs, the equivalent expression of (21.1) is

$$\alpha_W = \mathbf{W}_\alpha \boldsymbol{\alpha} = (\mathbf{N}_\alpha + \mathbf{D}_\alpha)^{-1} \mathbf{b}_\alpha \quad (21.7)$$

With these relationships, the signal covariance matrix now can be computed, and the ANS filter applied to the SRBF coefficients. Note that since the computation of the signal covariance matrix is done iteratively, an initial set of values must first be chosen. The standard deviations chosen for this initial signal variance covariance matrix are essentially arbitrary, although proper choices might reduce the number of iterations needed. For the current study, the initial standard deviations were set to 50 mm globally. This initial standard deviation is propagated from the spatial domain to the frequency domain using (21.6), then a new signal variance matrix is created from the

filtered solution. Iteration is halted when the difference in equivalent water height between two consecutive iterations for each grid point is less than 35 mm (chosen experimentally to balance convergence speed and the determination of accurate signal variability).

The determination of the optimal values for the level and depth of the SRBF solutions depends on the spatial variations, and noise content, of the data involved. Placing a dense grid of functions at a relatively shallow depth (i.e., small bandwidth) may result in noisy solutions, especially for GRACE data. The general approach used here was to employ a level high enough to represent what was believed to be the signal content in the data, and to place these functions as deep as possible in an attempt to smooth out the noise in the data. Many combinations of level and depth were evaluated, with the determination that a level 90 (i.e., ~220 km Reuter grid spacing), depth 900 km parameterization offers the highest quality solutions for Greenland and Antarctica.

21.3 Comparisons

Having finalized the optimal parameterization and filtering of the SRBF solutions, the next step was to compare the results of the mass change estimates derived from these solutions to those derived from other techniques, over Greenland and Antarctica. Since the goal of the study was simply to compare global versus regional techniques, only a limited set of spherical harmonic solutions were involved, and included those from the Center for Space Research (CSR) and the Delft Institute of Earth Observation and Space Systems (DEOS), who now produce a set of publically available monthly gravity solutions called the DEOS Mass Transport (DMT-1) models

Table 21.1 Descriptions of the various global and regional solutions used for comparison

Model name	Solution type	Description
CSR DS400	Global	Spherical harmonic solution to 60x60 derived from CSR RL04 data (see http://podaac.jpl.nasa.gov/grace) ; destriped; 400km Gaussian smoothing applied; SLR C20 values; degree 1 coefficients taken from Swenson et al. (2008)
CSR DS0	Global	Similar to above, except without Gaussian smoothing applied
DMT-1	Global	DEOS Mass Transport models (see http://www.lr.tudelft.nl/psg/grace); spherical harmonic solutions to 120x120 generated from KBR L1B data using the range-combination approach (Liu et al. 2009); anisotropic non-symmetric (ANS) filter applied (Klees et al. 2008b)
SRBF global	Regional	Spherical radial basis function approach using Poisson wavelets, in which a global distribution of nodes with a Reuter grid spacing of level 90 and depth of 900km is used. Low level data derived from the same KBR L1B data as the DMT-1 solution, with a similar anisotropic non-symmetric filter applied (adapted for use with SRBF's)
SRBF regional	Regional	Similar to the SRBF global approach, but using only regional data (i.e., within a 30° extended boundary from the target region)

(Liu et al. 2009). A summary of the solutions used in the comparisons is provided in Table 21.1. In short, the CSR solutions used both un-filtered and Gaussian filtered solutions, with an additional destriping filter applied similar to that of Swenson et al. (2008) (see Gunter et al. (2009) for further details of the CSR solution processing). The DMT-1 solutions are global spherical harmonic solutions computed using the acceleration approach (Ditmar and van Eck van der Sluijs 2004; Liu 2008), and with the ANS filter applied. Two types of SRBF solutions were tested, one using a global distribution of functions (SRBF global), and one using a more regional distribution (SRBF regional) in which a latitudinal buffer of 30° was used to reduce edge effects. For each solution, both the long-term linear trends (with bias and annual/semi-annual terms included) and monthly variations in the signals were examined. The timeframe considered was a 3 year period spanning from October 2003 to September 2006.

Some selected results from the comparisons are shown in Figs. 21.2 and 21.3. Figure 21.2 shows the geographical plot of the linear trends for the CSR400, DMT-1 and SRBF regional solutions over Greenland and Antarctica. Figure 21.3 is a plot of the maximum amplitude of the annual signal variation which is co-estimated along with the linear trend parameter. This is useful to visualize where the largest fluctuations in mass change exist.

The first observation that can be made from looking at these two figures is that the resolution for the ANS filtered solutions is much higher than those of the CSR DS400 solution, particularly for Greenland. The

DMT-1 and SRBF solutions are quite similar, but differences do exist. It is also interesting to note that the amplitude plots for the DMT-1 and SRBF regional solutions show subtle difference as well. For example, the SRBF solution shows a noticeable variation at the tip of the Antarctic Peninsula, where the DMT-1 solution does not. Similarly, in the Amundsen Sea sector (SW Antarctica), the SRBF solutions show two distinctive peaks, where the DMT-1 solutions show only one.

21.4 Validation

The determination of whether the differences seen in Figs. 21.2 and 21.3 represent genuine improvements in the signal recovered by the SRBF solutions is a difficult question to answer, and is a topic of current and future research efforts. One attempt made in this study to do this utilized surface elevation change data from the Ice, Cloud and Land Elevation Satellite (ICESat), a laser altimetry mission launched in 2003. ICESat observes the volume changes due to ice mass changes, which are naturally correlated to the mass changes observed by GRACE. The spatial resolution of ICESat is also much higher than that of GRACE, so a test was developed whereby the ICESat data was smoothed using a full-width Gaussian filter [as opposed to the traditional half-width filter normally used in geodesy, e.g., Jekeli (1981)] at intervals ranging from 0 to 2,500 km. Trend maps over the 3-year time period for both GRACE and ICESat were computed and each map was individually normalized. The normalization was needed because the ICESat

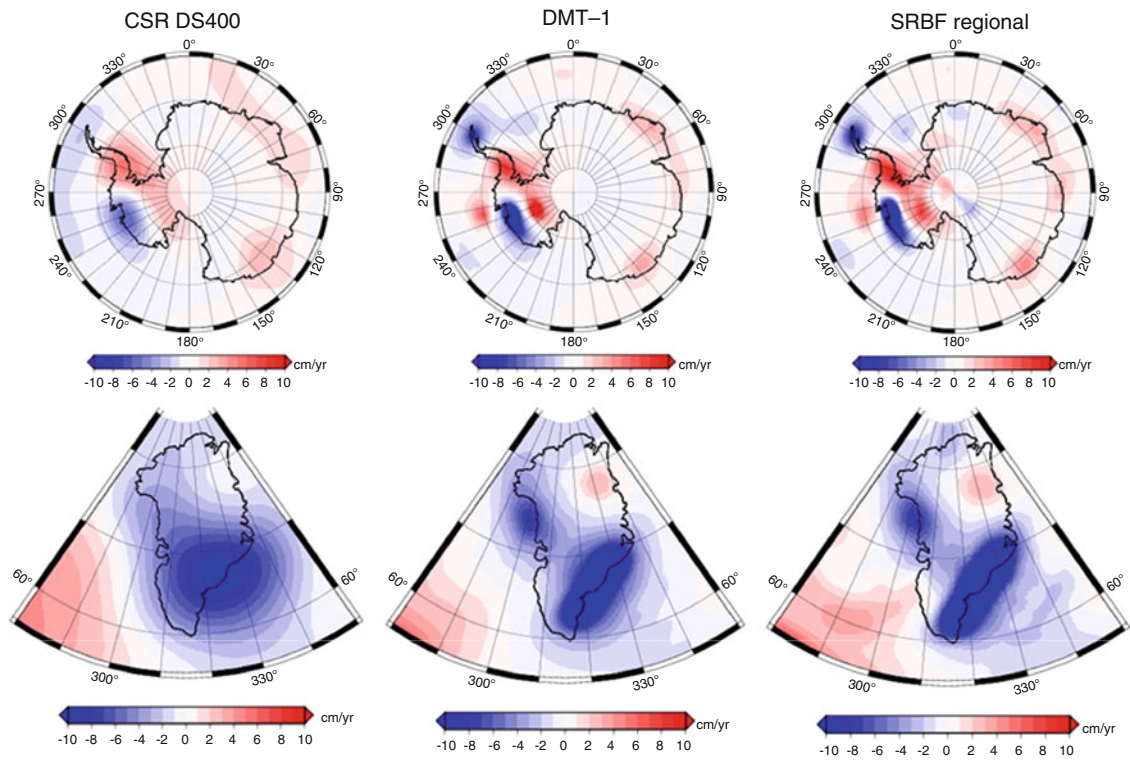


Fig. 21.2 Geographical plot of the 3-year trend computed from selected global and regional solutions, in units of equivalent water height

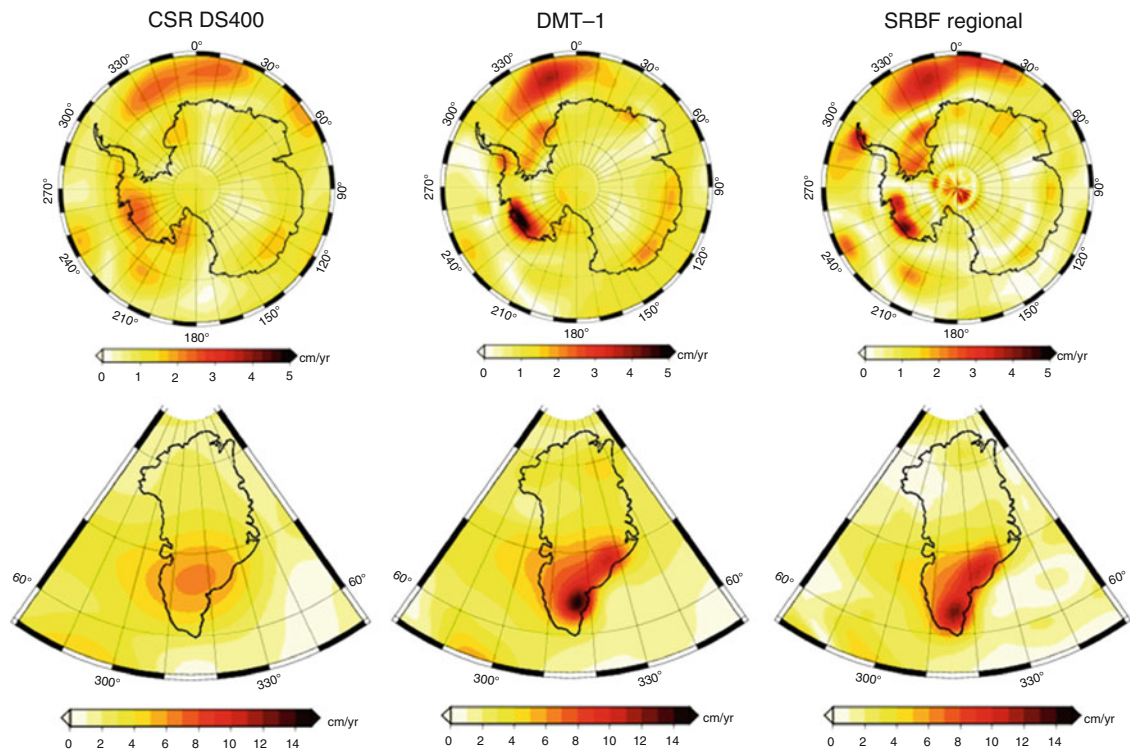


Fig. 21.3 Geographical plot of the estimated annual amplitude variations computed from selected global and regional solutions, in units of equivalent water height

map represents physical height changes (dh/dt , in cm/yr), whereas GRACE maps represent annual changes in EWH (also in cm/yr). As these are not the same quantities, the normalization allows a more direct comparison of the two data types under the assumption that a strong change in volume directly corresponds to a strong change in mass (and vice-versa). For each smoothing increment, correlations were computed between the smoothed ICESat map and the corresponding GRACE map. A peak in the resulting correlation curve would give an indication of the spatial resolution of the GRACE solution tested. The results of this test for all of the GRACE solutions mentioned in Table 21.1 are provided in Fig. 21.4.

For Greenland, the correlations with ICESat for the ANS filtered solutions (DMT-1 and the SRBF solutions) peak at around 1,300 km (full-width)

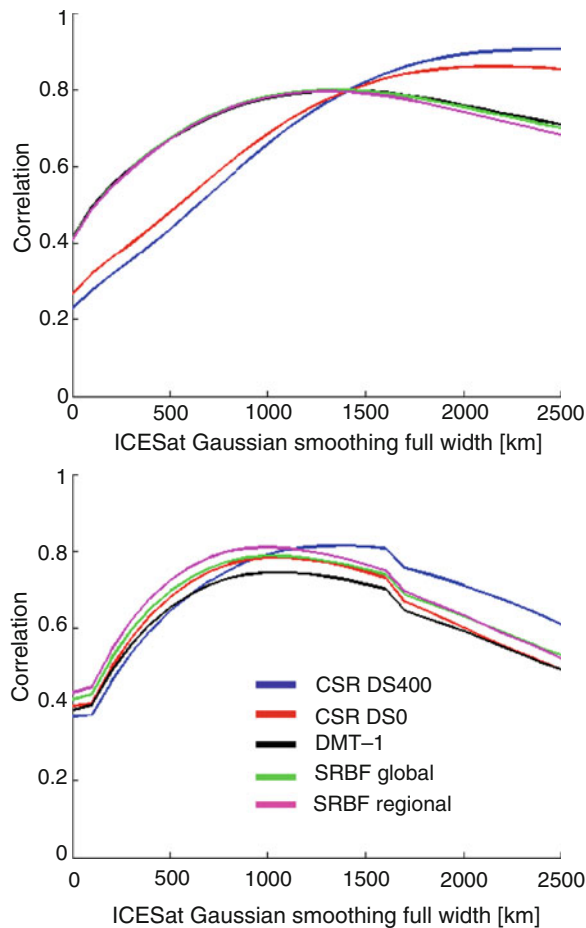


Fig. 21.4 Results of the spatial correlation test to ICESat data for Greenland (*top*) and Antarctica (*bottom*), for the various GRACE solutions

Gaussian smoothing, where the CSR solutions peak in the 2,200–2,500 km range. This implies that the ANS filter is the driving force for the accuracy levels in Greenland, and not necessarily the solution technique. For Antarctica, the situation is slightly different. Here, the correlation peak of the SRBF regional solution is approximately 5–10% higher than the SRBF global solution and the unfiltered CSR solution. This would suggest that the SRBF regional approach is achieving slightly better spatial resolution than the other global approaches.

Conclusions

The results of the analysis for this study supports the earlier conclusions by Klees et al. (2008a) that the choice of the spatial filter used in the GRACE processing has the largest impact on the comparisons. When compared to the standard de-striping and Gaussian filter approach (i.e., DS400), the anisotropic, non-symmetric (ANS) filter offers many benefits in terms of improved spatial resolution. That said, there were other indications that the choice of solution method may also offer some improvements, although to a much smaller degree. For Antarctica, the SRBF regional solution had the best spatial correlation when compared to the corresponding height change data from ICESat (Fig. 21.4), and was the only solution to observe annual variations in the Antarctic Peninsula (Fig. 21.3). For Greenland, all ANS filtered solutions (global and regional) performed essentially the same, with all of them offering substantial improvements over the corresponding CSR fields (DS400 and DS0). This is primarily due to the fact that the CSR fields have inherently lower resolution (with maximum degree and order 60), and because a Gaussian filter was applied (equivalent ANS filtered CSR solutions were not possible since the monthly noise covariance matrices are not publicly available). Regardless, the results suggest that, at a minimum, the regional SRBF techniques are equivalent to other global spherical harmonic solutions (i.e., DMT-1), but that there is also the possibility that a 5–10% improvement might be gained, depending on the region. Future studies will attempt to verify these results with more extensive comparisons with independent data sets, such as in-situ glaciological measurements or other satellite measurements.

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