Green’s function retrieval with Marchenko equations: a sensitivity analysis.
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SUMMARY
Recent research showed that the Marchenko equation can be used to construct the Green’s function for a virtual source position in the subsurface. The method requires the reflection response at the surface and an estimate of the direct arrival of the wavefield, traveling from the virtual source location to the acquisition surface. In this paper, we investigate the sensitivity of this method. We demonstrate its robustness with respect to significant amplitude and phase errors in the direct arrival. The erroneous operators introduce low amplitude artefacts. The main reflections and internal multiples are still present and disturbing ghost events are not introduced. In case the reflection data is modeled in a medium with losses, the influence is analyzed in the estimate of the direct arrival, and the influence of incorrect data at the surface has to be free of free-surface multiples. The method requires the reflection response at the surface from the virtual position in the subsurface. The measured data at the surface, an estimate of the direct arrival at the surface, and the influence of intrinsic losses in the medium. The influence is analyzed in the retrieved up- and downgoing wavefields at the virtual receiver position. The following numerical studies are carried out:

- amplitude and phase errors of the direct arrival due to a velocity error in the model,
- amplitude errors of the direct arrival,
- with a reflection response for a medium with losses.

These sensitivity studies are a first step towards applying the method on real data. Using real data we expect to have estimates of direct arrivals, which can deviate significantly from a correct operator. By having an idea what the influence is of an erroneous direct arrival, we expect to take these effects into account in further processing steps.

Theory
In the brief theoretical background given in this section, we follow Wapenaar et al. (2012a) and present the final outcome of the scheme for 2-dimensional media. The iterative scheme starts with an estimate of the direct arrival between the surface \((x_0)\) and the virtual receiver position \(x_F = (x_F, z_F)\) in the subsurface. The time-reversal of this direct arrival is the downgoing \(p_0^d(x_0, x_F, -t)\). With reflected with the reflection response of the medium \(R(x_0, x, t)\), this results in an upgoing field at the surface:

\[ p_0^u(x_0, x_F, t) = \int_x \int_t R(x_0, x, t-t') p_0^d(x, x_F, t') dx' dx \] (1)

where \(R(x_0, x, t)\) is the measured reflection response without surface-related multiples and deconvolved for the source wavelet. In the iterative scheme, the downgoing field is updated with a time-reversed and windowed version of \(p_{k-1}^d(x_0, x_F, t)\), according to:

\[ p_k^d(x_0, x_F, t) = p_0^d(x_0, x_F, t) - w(x_0, t)p_{k-1}^d(x_0, x_F, -t) \] (2)

where \(w(x_0, t)\) is a window function as explained in Wapenaar et al. (2012a) and \(k\) the iteration number. After several iterations (in the examples shown in this paper, 10-15 iterations are sufficient), using construction equation (1) and (with subscript 0 replaced by \(k\)) update equation (2), the wavefield converges. The summation of the time-reversed final downgoing field and the final upgoing field is proportional to the Green’s function at a virtual receiver at position \(x_F\)

\[ p_k^u(x_0, x_F, t) = p_0^u(x_0, x_F, t) + w(x_0, t)p_{k-1}^u(x_0, x_F, -t) \] (3)

Based on the same equations, but with a change in the sign, a slightly different scheme is carried out as well. The update equation in this scheme is

\[ q_k^u(x_0, x_F, t) = p_0^u(x_0, x_F, t) + w(x_0, t)q_{k-1}^u(x_0, x_F, -t) \] (4)

where \(q_k^u(x_0, x_F, t)\) is the so-called symmetric counterpart of equation (2). The final outcome of equations (2) and (4) are combined to create down- and upgoing Green’s functions:

\[ G^{\pm, +}(x_F, x, t) = \frac{1}{2} \left\{ p^\pm(x_0, x_F, t) + p^\mp(x_0, x_F, -t) \right\} \]

\[ G^{\pm, -}(x_F, x, t) = \frac{1}{2} \left\{ p^\pm(x_0, x_F, t) + p^\mp(x_0, x_F, -t) \right\} \]

\[ G^{-, \mp}(x_F, x, t) = \frac{1}{2} \left\{ p^\mp(x_0, x_F, t) + p^\pm(x_0, x_F, -t) \right\} \]

\[ G^{-, \pm}(x_F, x, t) = \frac{1}{2} \left\{ p^\pm(x_0, x_F, t) + p^\mp(x_0, x_F, -t) \right\} \]

In the examples below, these up- and downgoing wavefields are computed for different approximations of the initial downgoing wavefield \(p_0^u(x_0, x_F, t)\), and with a reflection response...
Green’s function retrieval with Marchenko equations: a sensitivity analysis

$R(x_0, x, t)$ modeled in a medium with losses. The obtained up- and downgoing Green’s functions can be the input for a migration scheme (Wapenaar et al., 2012b; van der Neut et al., 2013), where internal multiples are included.

Modeling experiments

A simple model, shown in Figure 1, is created to illustrate the method. In this model, the density contrast between the layers is chosen very high to generate strong internal multiples. In this model, the density contrast between the layers is alternating between 1000 and 5000 kg/m$^3$.

The next experiment is carried out with a velocity error of +15% in the first layer of the model. Due to this velocity error, the operator $p^+_0$ does not focus to a band-limited point, but to a blurred focal area. The computed wavefields, shown in Figure 4a,b,c, contains the same events as in Figure 3a,b,c, and the only differences are some more pronounced artefacts and a time delay due to the velocity error.

The results in Figure 5 are based on the correct velocity model to calculate the direct arrival in $p^+_0$, but the amplitude along the first arrival time is made constant (top of Figure 5a). The effect of this amplitude error is barely visible in the downgoing $G^{+,-}$ in Figure 5a, but introduces a few errors in the upgoing $G^{-,+}$ in Figure 5b. Note that the construction equation (1) includes an integration over the (limited) receiver array at the surface. In the examples above we have not used any tapers at the edges of the acquisition domain. It is expected that tapering the edges of the acquisition domain will reduce some of the artifacts in the results (for example the indicated event in Figure 5c).

For the last experiment in this paper, the correct direct arrival time and amplitude are used for $p^+_0$, but the reflection response $R$ is modeled in a medium with losses (Q=50). In the theoretical derivation of the iterative scheme, it is assumed that the medium is lossless and it is expected that in this case the method will not give correct results. However, Figure 6a shows that the downgoing Green’s function does not have any visible extra ghost events and only in the upgoing field ghost events are introduced (indicated with an arrow in Figure 6c). We expect that these can be reduced by applying a Q compensation to the reflection data prior to running the iterative scheme.

Conclusion and discussion

The experiments in this paper indicate that the introduced scheme to calculate the up- and downgoing Green’s functions at a virtual receiver in the subsurface, from reflection data and the direct arrival is robust. The scheme does not diverge and remains stable in case the focusing operator has errors in phase and amplitude. In case the reflection data is modeled in a medium with losses, ghost events are visible in the retrieved upgoing field only, but not in the retrieved downgoing field. These ghost events can probably be reduced by applying Q-compensation.

Acknowledgements

We are thankful to research school ISES for supporting this research.
Green’s function retrieval with Marchenko equations: a sensitivity analysis

Figure 3: Up- and downgoing wavefields after 15 iterations. This is the reference result with the correct direct propagating field \( p_0^+ \) (a)) and \( R \) computed for a medium without losses. The edges of the lateral integration (over \( x \)) are not tapered and introduces small truncation artifacts visible before the first arrival in the upgoing Green’s function \( G^{-,+} \) (c).

Figure 4: Up- and downgoing wavefields after 15 iterations with an error in the velocity model to calculate \( p_0^+ \). The velocity in the top layer of the model has been increased from 2000 to 2300 m/s. In comparison with the reference results a few artefacts are introduced, which are indicated with an arrow in the upgoing field (c).
Green’s function retrieval with Marchenko equations: a sensitivity analysis

Figure 5: Up- and downgoing wavefields after 15 iterations with an amplitude error in $p_0^+$. The amplitude along the wavefield is flat for all offsets as shown in the amplitude plot in (a). In the upgoing Green’s function (c) only a few ghosts events, with a low amplitude and indicated with an arrow, are introduced.

Figure 6: Up- and downgoing wavefields after 15 iterations. The reflection data is modeled in a medium with losses ($Q=50$). This violation of the theoretical assumption of a lossless medium introduces extra ghosts events (indicated with an arrow), which are visible in the upgoing wavefield (c), but not in the downgoing wavefield (b).
Green’s function retrieval with Marchenko equations: a sensitivity analysis

REFERENCES


