A METHOD FOR INTERMODULATION NOISE CALCULATIONS IN A CABLE TELEVISION NETWORK WITH HD-MAC, PAL AND FM RADIO SIGNALS.

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ABSTRACT

A statistical method for calculation of intermodulation noise is developed for CATV systems whose traffic can be characterized as a sum of a large number of independent signals. The Central Limit Theorem leads to a method of calculating the intermodulation noise power spectrum, given the degree of non-linearity of the transmission system. The method is demonstrated by calculating the signal to intermodulation noise ratio in the various channels of a prototype cable network supporting HD-MAC, PAL and FM radio signals. The spectra of these individual signals are simplified in order to perform part of the calculation analytically. As expected, the signal to intermodulation noise ratio decreases when the total signal power is increased. This method can be used to calculate the necessary back-off of a cable network for a given frequency plan, and thus to determine suitable plans for loading and controlling CATV networks.

I. INTRODUCTION

In December 1991, the ministers for telecommunications in the European Community (EC) decided that all satellite television broadcasting after 1994 should be in the MAC (Multiplexed Analogue Components) transmission standard. This was an important step for the introduction of high definition television (HDTV) in Europe [1].

The two existing European TV-emission standards PAL and SECAM are not compatible with the MAC-family transmission standards; therefore the new HDTV system will be introduced in two steps. The first step has been the introduction of D2-MAC television in 1992. The aspect ratio (the picture length to picture height ratio) of the D2-MAC receivers can already equal that of HD-MAC receivers, namely 16:9, but the picture resolution will remain the same as in the conventional European television, namely 625 scan lines. Introduction of HD-MAC receivers will follow in 1994, with resolution doubled to 1250 scan lines. The transition from D2-MAC to HD-MAC will not cause many problems for owners of a D2-MAC receiver, since this can extract a D2-MAC signal from the HD-MAC transmission.

The transition from the PAL transmission standard to the family of MAC transmission standards is less easy. Since the MAC receivers will obviously not penetrate the whole market at once, the MAC and PAL systems will coexist for several years after the introduction of the MAC system. For the Netherlands, where over 90 percent of all viewers is now connected to cable television networks, the cable operators will therefore have to cater for both the conventional PAL receivers and the new MAC receivers. This means that some programs will be transmitted in both standards (simulcast). As a result, the frequency space on the cable network must be reallocated in order to create space for the extra MAC channels. When a frequency plan is changed, the intermodulation due to non-linear effects in the channels also changes. Hence calculation of intermodulation noise is required for every channel of any potential new frequency plans. Using conventional computational methods this can be a formidable task.

This paper presents a statistical method developed for calculating the intermodulation noise in a CATV system whose input consists of a sum of many independent signals. The method is applied to a prototype cable network supporting both HD-MAC and PAL signals. The paper is organised as follows. The intermodulation problem is described in general in section II and is followed by a description of the basic theory in section III. The non-linear transfer characteristic and traffic load of the prototype cable network used to demonstrate this novel method are described in section IV. The computational results are given in section V, followed by our conclusions in section VI.
II. INTERMODULATION NOISE

Intermodulation noise is generated in the non-linear components of a system loaded with several signals. A non-linear memoryless component can not be described by a classical network transfer function, but is usually described by a transfer characteristic relating the output time function \( Y(t) \) to the instantaneous input \( X(t) \): \( Y(t) = T[X(t)] \). A non-linear, frequency-independent component can be described by a polynomial function:

\[
Y(t) = \sum_{n=1}^{\infty} a_n X^n(t) \tag{1}
\]

The consequences of the non-linear transfer characteristic can be visualised by the following example. Let us assume that two carriers at the frequencies \( f_1 \) and \( f_2 \) pass a non-linear component with characteristic function:

\[
Y(t) = a_1 X(t) + a_2 X^2(t) + a_3 X^3(t) \tag{2}
\]

Then the output signal also includes unwanted harmonics and intermodulation products at \( f_1 + f_2, f_1 - f_2, 2f_1, 2f_2, f_1 + 2f_2, f_1 - 2f_2, 2f_1 + f_2, 2f_2 + f_1, 3f_1, 3f_2 \). When the number of carriers that pass the non-linearity increases, the number of intermodulation products also increases drastically. The number of intermodulation products caused by the second order terms equals \( N^2/2 \) and the number of products caused by the third order term equals \( N^3(N-1)/2 + N \). For \( N = 30 \) the number of intermodulation products is 13530. Thus it is not practical to determine the amount of intermodulation in practical cable networks, where there can be over 40 carriers, by calculating the strength of each intermodulation product separately. A statistical method of calculating the intermodulation, when the number of input signals passing is large, is described in the next sections.

III. BASIC THEORY

The method developed in this report is based on the theory used to measure the degree and order of the non-linearity of a system [2-5]. This theory is reviewed in the following.

When a Gaussian signal with zero mean passes a nonlinear system with the transfer characteristic (1), an expression for the autocorrelation function of the output signal \( Y \) can be derived using Price's theorem:

\[
\frac{\partial^2 R_Y(t)}{\partial R_X(t)^2} = \frac{\partial^2 R_X(t)}{\partial R_X(t)} \frac{\partial^2 R_X(t)}{\partial R_X(t)} \tag{3}
\]

where \( f(t) \) is the polynomial function determined by (1), i.e.,

\[
f(t) = \sum_{n=1}^{\infty} a_n t^n \tag{4}
\]

and the bar represents the statistical expectation. If \( k = s \) then

\[
\frac{\partial^2 R_X(t)}{\partial R_X(t)} = \sigma^2 a_s \text{ and } \frac{\partial^2 R_Y(t)}{\partial R_X(t)} = (\sigma^2 a_s)^2 \tag{5}
\]

The autocorrelation function of the output, \( R_Y \), can be presented as a function of the autocorrelation of the input signal, \( R_X \), by integrating the second part of eq.(3) \( s \) times. The first integration of eq.(3) is given by:

\[
\frac{\partial R_Y(t)}{\partial R_X(t)} = C_s \cdot (\sigma^2 a_s) R_X(t) \tag{6}
\]

where the integration constant \( C_s \) is determined by:

\[
C_s = \frac{\partial R_X(t)}{\partial R_X(t)} \bigg|_{t=0} \tag{7}
\]

\( C_s \) can be calculated using equation (5) and the fact that the expectation of a zero mean Gaussian variable, \( x \), is given by [6]:

\[
\mathbb{E}[x^m] = \begin{cases} 0, & \text{m odd} \\ \frac{(m-1)!R_X(0)^{m/2}}{2^{(m-2)/2}(m/2-1)!}, & \text{m even.} \end{cases} \tag{8}
\]

Repeated integration in this manner leads to an expression in which the output autocorrelation function is given in term of the input autocorrelation function:

\[
R_Y(t) = \sum_{n=0}^{\infty} A_n R_X^n(t) \tag{9}
\]

where

\[
A_n = \frac{1}{r} \left[ \frac{1}{r} \sum_{m=0}^{l} (n+2m)!a_{n+2m} \frac{R_X^m(0)}{2^{m/ml}} \right] \tag{10}
\]

with

\[
L = \begin{cases} \frac{1}{2}(s-n), & \text{for } s-n \text{ even} \\ \frac{1}{2}(s-n-1), & \text{for } s-n \text{ odd} \end{cases} \tag{11}
\]
The power spectrum of the output signal is obtained by Fourier-transforming equation (9):

\[ S_y(t) = \int_{-\infty}^{\infty} R_y(t) \exp^{-2\pi ft} dt = \sum_{n=1}^{N} A_n S_{x_n}^e(t) + A_n S_{x_n}^i(t). \]  

(12)

Here \( S_{x_n}^e \) indicates the repeated convolution of the input power spectrum. A power spectrum calculated by (12) contains the sum of the signal power and the inter-modulation noise power. Thus the inter-modulation noise power spectrum is given by:

\[ S_y(t) = \sum_{n=1}^{N} A_n S_{x_n}^e(t) - A_n S_{x_n}^i(t) \]  

(13)

Thus given a Gaussian input signal to the non-linear system, equation (13) can be used to calculate the resulting inter-modulation noise power.

According to the central limit theorem [7] this method can therefore be used to calculate the inter-modulation noise power in systems whose input is the sum of a sufficiently large number of independent signals.

IV. THE PROTOTYPE CABLE NETWORK

During the transmission of radio and television signals through a cable network, they are subject to many sorts of disturbances: thermal noise, reflections and intermodulation. The basic theory described in the preceding section can be applied to calculate the inter-modulation noise generated in a cable network. The developed method can thus be used to calculate the change in signal to intermodulation noise ratio (S/I) in each television channel when the HDTV is introduced on the cable system.

The intermodulation noise is generated in the non-linear components of a system. In the case of a CATV network, the intermodulation noise is generated in the repeaters, frequency conversion networks and even in the coaxial cables (due to corrosion and other environmental changes). In the following, the transfer characteristic of the prototype cable network used to illustrate the developed method and the signals that pass the prototype network is given.

A. The transfer characteristic

A CATV network is a complex system consisting of several types of cascaded amplifiers, transmission lines and frequency converters. An exact transfer function of this network cannot be theoretically derived but should be measured in the actual situation. Typically, the calculations must be split into two parts i.e., for the trunk and the local network, because of the different frequencies used in each part. Both parts may consist of several types of amplifiers and transmission lines. In the calculations the trunk and sub network will be viewed as two black boxes (fig 1).

![Figure 1: A simplified model for the cable network.](image)

The transfer characteristic from point A to B is taken to be:

\[ B(t) = \sum_{n=1}^{N} I_n A_n \xi(n) \]  

(14)

and the transfer characteristic from point C to D by:

\[ D(t) = \sum_{n=1}^{N} W_n C_n \xi(t) \]  

(15)

The values of the coefficients \( I_n \) and \( W_n \) and the summation limits \( s_n \) and \( s_i \) of the two sub-systems have to be experimentally determined, by measuring the system output for various values of the system input, using a simple test signal. For illustrative purposes only, we will assume a power transfer characteristic based on the three first terms of the Taylor expansion of a sinusoid:

\[ Y(t) = a_1 X(t) - a_2 X^2(t) - a_3 X^3(t). \]  

(16)

The coefficients \( a_i \) of this characteristic are determined by normalising the sinusoid by the total power:

\[ a_1 = \frac{1}{1 + \left( \frac{\pi}{2} \right)^2} \frac{1}{A_{\text{max}}^{\pi/2}} \]  

(17)

where \( A_{\text{max}} \) is the average value of the maximum input signal amplitude. Thus the voltage transfer characteristic assumed in the following calculations can be described by:

\[ Y(t) = 1.57 X(t) - 0.645 X^2(t) + 0.0797 X^3(t). \]  

(18)
Figure 2 shows a plot of the corresponding prototype single-tone power transfer characteristic.

![Graph showing the transfer characteristic](image)

**Figure 2:** The transfer characteristic assumed for the cable network, relative to saturation.

B. The joint input signal

We next describe the input signal of the cable network. This signal contains a large number of RF-carriers modulated by video and/or audio signals. Note that each carrier is generated independently of all other RF-carriers. Hence the input signal is the sum of a number of statistically independent signals. According to the central limit theorem, this leads to the hypothesis that the input signal is a Gaussian signal, if the number of input signals is sufficiently large to avoid dominance of any one carrier.

The hypothesis that a typical input signal is a Gaussian random variable was verified by computer simulation. The joint input signal for the simulation consists of 37 PAL carriers set at 0 dB relative power, 2 HD-MAC carriers at -3 dB, 16 FM stereo broadcast carriers at -6 dB, 3 FM mono carriers at -16 dB and 2 Pilots at -16 dB, all relative to the PAL carrier. A computer program was written to sum these carriers; their phases were generated by the Pascal random generator. Then 5000 samples of the sum of these carriers were taken and the corresponding probability density was plotted and compared to the Gaussian pdf:

\[
P(X) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left(-\frac{x^2}{2\sigma^2}\right) \tag{19}\]

The total signal power \( P = \sum A_i^2 \) = 19.35, when the amplitude of the PAL carrier is taken to be 1. Figure 3 indicates that the probability density of the joint input signal appears to be well approximated by this simple Gaussian pdf.

![Graph showing probability density](image)

**Figure 3:** Comparison of joint input (a) and Gaussian (b) pdf.

Having established that the joint input signal is a Gaussian variable with zero mean and that our system transfer characteristic can be described by equation (1), we still need to describe the power spectrum of this input signal for the calculation, to this end, the power spectra of the individual signals in the trunk of our prototype network are described in the following. We will start with the two television signals, i.e., the PAL and HD-MAC signals, followed by the radio broadcast signals, i.e., the FM mono and stereo signals.

In the near future, various kinds of signals will be transmitted via Dutch cable networks: PAL signals, HD-MAC signals, FM radio signals and DSR (digital radio) signals. A general description of the spectrum allocation to these signals in the trunk network is given in table I [8]. The detailed frequency allocation to FM radio signals in band II is given in table II [8]. In addition, we have assumed insertion of two HD-MAC channels in the range of 448 - 460 MHz and 461 - 473 MHz.

In order to calculate the intermodulation noise the spectra of the TV signals are simplified. These simplified spectra and their corresponding autocorrelation functions are described below.

![Graph showing simplified spectrum](image)

**Figure 4:** The simplified version of the two-sided PAL spectrum
Table I: Frequency allocation assumed in the prototype trunk network.

<table>
<thead>
<tr>
<th>CHANNEL</th>
<th>FREQUENCY (MHz)</th>
<th>VISION CARRIER (MHz)</th>
<th>SOUND CARRIER (MHz)</th>
</tr>
</thead>
<tbody>
<tr>
<td>BAND I</td>
<td>2</td>
<td>47 - 54</td>
<td>48.25</td>
</tr>
<tr>
<td></td>
<td>4</td>
<td>61 - 68</td>
<td>62.25</td>
</tr>
<tr>
<td></td>
<td>Pilot 1</td>
<td>90.5</td>
<td></td>
</tr>
<tr>
<td>BAND II</td>
<td>FM radio</td>
<td></td>
<td></td>
</tr>
<tr>
<td>MEDIUM</td>
<td>114 - 121</td>
<td>115.25</td>
<td>120.75</td>
</tr>
<tr>
<td></td>
<td>122 - 129</td>
<td>123.25</td>
<td>128.75</td>
</tr>
<tr>
<td></td>
<td>130 - 137</td>
<td>131.25</td>
<td>136.75</td>
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<tr>
<td></td>
<td>138 - 145</td>
<td>139.25</td>
<td>144.75</td>
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<tr>
<td></td>
<td>146 - 153</td>
<td>147.25</td>
<td>152.75</td>
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<tr>
<td></td>
<td>154 - 151</td>
<td>155.25</td>
<td>160.75</td>
</tr>
<tr>
<td></td>
<td>161 - 169</td>
<td>163.25</td>
<td>169.75</td>
</tr>
<tr>
<td>BAND III</td>
<td>5</td>
<td>174 - 181</td>
<td>175.25</td>
</tr>
<tr>
<td></td>
<td>7</td>
<td>188 - 195</td>
<td>189.25</td>
</tr>
<tr>
<td></td>
<td>9</td>
<td>202 - 209</td>
<td>204.25</td>
</tr>
<tr>
<td></td>
<td>11</td>
<td>216 - 223</td>
<td>217.25</td>
</tr>
<tr>
<td>UPPER</td>
<td>B1</td>
<td>230 - 237</td>
<td>231.25</td>
</tr>
<tr>
<td>BAND</td>
<td>Pilot 2</td>
<td>245.25</td>
<td>246.25</td>
</tr>
<tr>
<td></td>
<td>254 - 261</td>
<td>255.25</td>
<td>260.75</td>
</tr>
<tr>
<td></td>
<td>262 - 269</td>
<td>263.25</td>
<td>268.75</td>
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<tr>
<td></td>
<td>270 - 277</td>
<td>271.25</td>
<td>276.75</td>
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<tr>
<td></td>
<td>278 - 285</td>
<td>279.25</td>
<td>284.75</td>
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<td></td>
<td>286 - 293</td>
<td>287.25</td>
<td>292.75</td>
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<td></td>
<td>294 - 301</td>
<td>295.25</td>
<td>300.75</td>
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<td></td>
<td>302 - 312</td>
<td>303.25</td>
<td>308.75</td>
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<td>310 - 320</td>
<td>311.25</td>
<td>316.75</td>
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<td>318 - 328</td>
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<td>324.75</td>
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<td>326 - 336</td>
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<td>334 - 344</td>
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<td>342 - 354</td>
<td>343.25</td>
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<td></td>
<td>350 - 362</td>
<td>351.25</td>
<td>356.75</td>
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<tr>
<td></td>
<td>358 - 370</td>
<td>359.25</td>
<td>364.75</td>
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<tr>
<td></td>
<td>366 - 378</td>
<td>367.25</td>
<td>372.75</td>
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<tr>
<td></td>
<td>374 - 386</td>
<td>375.25</td>
<td>380.75</td>
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<tr>
<td></td>
<td>382 - 393</td>
<td>383.25</td>
<td>388.75</td>
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<tr>
<td></td>
<td>390 - 401</td>
<td>391.25</td>
<td>396.75</td>
</tr>
<tr>
<td></td>
<td>400 - 414</td>
<td>401.25</td>
<td>406.75</td>
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<tr>
<td></td>
<td>412 - 429</td>
<td>413.25</td>
<td>418.75</td>
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<tr>
<td></td>
<td>420 - 437</td>
<td>421.25</td>
<td>426.75</td>
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<td></td>
<td>430 - 447</td>
<td>431.25</td>
<td>436.75</td>
</tr>
<tr>
<td></td>
<td>440 - 457</td>
<td>441.25</td>
<td>446.75</td>
</tr>
</tbody>
</table>

Table II: FM radio carrier frequency allocation assumed in the prototype trunk network.

<table>
<thead>
<tr>
<th>FM radio</th>
<th>spectrum</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mono/Stereo</td>
<td>Frequency (MHz)</td>
</tr>
<tr>
<td>M</td>
<td>87.6</td>
</tr>
<tr>
<td>S</td>
<td>88.9</td>
</tr>
<tr>
<td>M</td>
<td>89.5</td>
</tr>
<tr>
<td>M</td>
<td>89.9</td>
</tr>
<tr>
<td>S</td>
<td>90.5</td>
</tr>
<tr>
<td>S</td>
<td>91.1</td>
</tr>
<tr>
<td>S</td>
<td>91.9</td>
</tr>
<tr>
<td>S</td>
<td>92.6</td>
</tr>
<tr>
<td>S</td>
<td>94.1</td>
</tr>
<tr>
<td>S</td>
<td>94.9</td>
</tr>
<tr>
<td>S</td>
<td>95.5</td>
</tr>
</tbody>
</table>
A typical PAL spectrum (fig. 4) is assumed to be the sum of delta and rectangular functions:

\[ S_{\text{PAL}}(\omega) = \delta(t+f_1) + \delta(t-f_1) + C \prod \left( \frac{t-t_i}{2W} \right) \prod \left( \frac{t-t_f}{2W} \right) \]  

(20)

where \( W = \frac{5.5}{2} \) [MHz] and \( f_1 = f_c - 1.25 + W \) [MHz].

The constant \( C \) is the ratio of the maximum signal amplitude and the carrier amplitude. By inverse Fourier transformation of the power spectrum the corresponding autocorrelation is obtained, i.e.

\[ R_{\text{PAL}}(t) = \cos(2\pi f_c t) + 2\cos(2\pi f_c t) C 2W \sin(2Wt) \]  

(21)

The shape of the HD-MAC spectrum (fig. 5) is similar to the PAL spectrum; only the values of \( f_c \) and \( W \) differ, namely \( W = \frac{10.875}{2} \) [MHz] and \( f_1 = f_c 0.75 + W \) [MHz]. The autocorrelation function of the HD-MAC signal is thus given by:

\[ R_{\text{HD-MAC}}(t) = K_1 \cos(2\pi f_c t) + 2\cos(2\pi f_c t) K_2 2W \sin(2Wt) \]  

(22)

where \( K_1 \) is the ratio of the PAL and HD-MAC carrier, and \( K_2 \) is the ratio of the maximum signal amplitude and the PAL carrier in the CATV network.

\[ \text{Figure 5: The simplified version of the two-sided HD-MAC spectrum.} \]

\[ \text{Figure 6: The simplified two-sided spectra of the FM radio signals.} \]

The spectrum of the FM stereo and mono signals (fig. 6) is represented by their carriers:

\[ R_{\text{FM}}(t) = 2A \cos(2\pi f_c t) \]  

(23)

where \( A \) is the ratio of the PAL carrier and radio carrier in the CATV network. The spectrum of the joint input signal is the sum of all these individual spectra, with the carrier frequencies allocated according to table I and II and appropriate choices for \( C, K_1, K_2 \) and \( A \), the relative settings of the carrier and signal powers.

V. THE COMPUTATION OF THE SIGNAL TO INTERMODULATION NOISE RATIO

Some general formulas to calculate the signal to intermodulation noise ratio at a particular frequency \( f \): \( S(f)/S_n(f) \) in a transmission channel of the cable network, are given in the following. Instead of using equation (13) directly, we will calculate the convolutions of the input spectrum with the help of Fast Fourier Transforms. The inverse Fourier transform of equation (13) yields:

\[ R_s(t) = \sum_{n=2}^{N} A_n R_n(t)^n \]  

(24)

Therefore the intermodulation noise power can be calculated by:

\[ S_n(t) = \text{FFT} \left[ \sum_{n=2}^{N} A_n R_n(t)^n \right] \]  

(25)

where FFT is the Fast Fourier Transform operator.

The autocorrelation of the input, \( R_x \), is attained by:

\[ R_x(t) = \sum_{n=1}^{N_{\text{MN}}} \text{IFFT}\left[ S_{x_{\text{MN}}}(f) \right] \cdot \sum_{n=1}^{N_{\text{FM}}} \text{IFFT}\left[ S_{x_{\text{FM}}}(f) \right] \cdot \sum_{n=1}^{N_{\text{PAL}}} \text{IFFT}\left[ S_{x_{\text{PAL}}}(f) \right] \]  

(26)

where IFFT is the inverse Fast Fourier Transform operator.

Now we will compute the intermodulation noise in the trunk network of the prototype cable network with the prototype voltage transfer characteristic:

\[ Y(t) = 1.57 X(t) - 0.645 \delta(t) + 0.0797 X^2(t). \]  

(27)

A study of eq.(13) shows that the computation of intermodulation noise can be split in two independent parts:

- the calculation of the coefficients \( A_n \) which depend on the non-linear transfer characteristics (27) and the total input power of
the joint input signal, but not on the frequency plan.
the calculation of the n\textsuperscript{th} convolution of the input power
spectrum, \( S_\text{n}^\text{th}(\omega) \), which depends on the frequency plan and
the relative power of the joint input signal but not on the
transfer characteristic.

The coefficients are calculated using eq. (10) for various values
of \( R_s(0) \), which equals the total input power. Figure 7 shows the
coefficients \( A_1, A_2 \) and \( A_3 \) for \( 0 < R_s(0) < 1 \).

![Figure 7: The coefficients \( A_n \) for varying input power.](image)

The \( n \textsuperscript{th} \) convolution of the input-signal power spectrum of a
signal with total power \( R_s(0) \) is given by:

\[
S_\text{n}^\text{th}(f) = R_s(0) \text{FFT} \left( \frac{R_s(0)}{n} \right)
\]

(28)

where \( R_s(0) \text{norm} \) is the autocorrelation function normalised by the
total power. In order to calculate the \( n \textsuperscript{th} \) convolution of the input
power spectrum, the normalised input autocorrelation must be
Fourier transformed and multiplied by the total input power
raised to the power \( n \). The autocorrelation function of the input
signal is the sum of the inverse Fourier transform of the power
spectrum of each channel. In order to be able to calculate the
autocorrelation function analytically, the power spectra of the
various TV and radio signals have been approximated by the
simple functions in section IV.B.

Now the autocorrelation function of the input signal can be
calculated using equations (20) to (22) and tables I and II. The
autocorrelation of the signal in each channel in the tables is
chosen from equation (20) to (22), depending on the particular
kind of signal the channel contains. The autocorrelation of the
joint input signal is obtained by summing the autocorrelation
functions of all channels. Once the joint autocorrelation function
is available, the next step is to calculate the \( n \textsuperscript{th} \) power of this
function. The value of \( n \) depends on the transfer characteristic
of the system; for the transfer characteristic in equation (27) it
will be necessary to calculate \( R_s(\tau) \) and \( R_s^2(\tau) \). Fast Fourier
transformations of these \( R_s(\tau) \) give the \( n \textsuperscript{th} \) convolution of the
power spectrum of the input signal.

In order to Fourier transform these functions, it is necessary to
determine the minimum number of samples that is needed to
obtain an accurate numerical transform. In the following an
example of this calculation is given for \( n=5 \). The highest
frequency in the input signal equals 473 MHz, so the total
frequency domain is \( 2 \times 473 \times 5 \) MHz. For a resolution of 0.05
MHz, a FFT with 94600 sample points is necessary. Since the
FFT procedure from the NAG library [9] used for the calculations
is most efficient when the number of points used is a power of
two, the number of points used is \( 2^{17} \).

Finally when the necessary convolutions are calculated, each
convoluted spectrum is multiplied with the appropriate
coefficient \( A_n \) and the \( n \textsuperscript{th} \) order. The signal to intermodulation noise
eratio per channel (PAL, HD-MAC or FM) is now computed by
summing the samples of that channel, leading to

\[
S_i = \frac{\sum_{\text{channel}} A_n \text{FFT} [R_s(\tau)]}{\sum_{\text{channel}} n A_n \text{FFT} [R_s^2(\tau)]}
\]

(29)

This ratio is given in figure 8 as a function of the input power for
our prototype system, for the PAL channels 2 and B24 (table I)
and a HD-MAC channel (448-460 MHz).

![Figure 8: Signal to intermodulation-noise ratio in 2 typical PAL
channels (2 and B24) and a HD-MAC channel (448-460 MHz).](image)
In order to have a reasonable value for the signal to noise ratio, the prototype system must be driven at a much lower operating point than its saturation point. The ratio of the maximum input power (the power at saturation) and the power necessary to obtain a reasonable signal to noise ratio is known as the back-off. Amongst the requirements for the Dutch cable networks [8], a minimum acceptable level for the signal to intermodulation noise ratio per TV-channel is specified, namely, 54 dB. From figure 8 we can conclude that the necessary back-off for our prototype system would be approximately 23 dB.

![Figure 9: Signal to intermodulation noise ratio in two typical PAL channels (2 and B24), and a HDMAC channel, respectively.](image)

Figure 9: Signal to intermodulation noise ratio in two typical PAL channels (2 and B24), and a HDMAC channel, respectively.

We will continue with a comparison of the intermodulation noise in the HD-MAC and PAL channels. In figure 9 a part of figure 8, namely the area around 54 dB, is enlarged for this purpose. The signal to intermodulation noise ratio in a HD-MAC channel is seen to be lower than in a PAL channel. The input signal power in a PAL channel therefore has to be approximately 2.1 dB higher to achieve the same S/I as the HD-MAC channel.

Another aspect examined is the influence of the fifth order term in the transfer characteristic. This is accomplished by comparing the value of S/I when the fifth order intermodulation is considered and the S/I when the fifth order intermodulation is neglected. Thus S/I is calculated by

\[
\left( \frac{S}{I} \right)_5 = \frac{A_3 S_x^3}{A_5 S_x^5} \tag{30}
\]

and

\[
\left( \frac{S}{I} \right)_3 = \frac{A_1 S_x}{A_3 S_x^3} \tag{31}
\]

Figure 10 shows almost no difference between these two values of S/I. The only visible difference is in the vicinity of the saturation point; this is not important in practice as the back-off required is -23 dB.

![Figure 10: comparison between S/I calculated with eq.(43) and (44).](image)

Finally we investigated the influence of the signal power on the intermodulation noise. The signal to intermodulation noise ratio was calculated assuming that the signal only consisted of pure carriers. Figure 11 shows that if the carrier modulator is not considered, the signal to intermodulation ratio in the PAL channels is higher than the S/I in the HD-MAC channels.

![Figure 11: S/I when the input signal consists of unmodulated carriers only.](image)

Thus a sufficiently accurate modelling of the individual spectra of the television signals is needed for a useful calculations of the S/I. The actual values probably lie between those given in figures 8 and 11.
VI. CONCLUSIONS

The possibility of computing the signal to intermodulation noise ratio of a CATV network with an input signal consisting of a large number of independent signals is demonstrated. A statistical method is used for this purpose. This method can be applied for intermodulation calculation in a CATV network with PAL, HD-MAC and FM signals. The method splits the calculation of intermodulation noise in two independent parts, namely the convolutions of the joint input spectrum, and the transfer coefficients $A_n$ which depend on the system characteristic. The method can be used for determining both the total system back-off and for planning of the individual channel loads. The aim of frequency planning is to allocate channel capacity as efficiently as possible. One of the factors that have to be considered is that the intermodulation noise should be below a certain specified limit. For this purpose the S/I should be calculated for different input spectra while the total input power is held constant. The frequency plan with the best signal to noise ratio is chosen. This procedure requires that spectrum convolutions are computed for each possible frequency scheme, and is therefore quite time demanding. The necessary back-off can be determined by computing the worst-case S/I for various values of the input signal. The joint input power at which this S/I has the specified value is then chosen and the corresponding back-off can be calculated. In this application, the time- and capacity-demanding computer convolutions of the input spectrum need to be performed only once.

The calculations were made with input signals in two extreme cases: either the power of the baseband video and audio signals has a maximum value (fig.9) or only the unmodulated carriers are considered (fig.11). The difference between the outcome of these calculations show that the information about the modulation and load aspects is important. To obtain realistic results, the value of the coefficients $A_1$ and $A_2$ in the transfer characteristic of the system are also important. These can be obtained by measurements with simple unmodulated inputs (one- or two-tone characteristics). The method described in this paper can be applied to investigate the upgrading of existing CATV systems with new traffic types, such as HDTV. To determine the total traffic capacity of a system, the calculations of the necessary back-off in this paper should be expanded to consider the thermal noise as well as the intermodulation noise at the customers' receivers.


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