Incident, infragravity and very low frequency wave motions on an atoll reef platform
Case study Roi-Namur, Kwajalein Atoll

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INCIDENT, INFRAGRAVITY AND VERY LOW FREQUENCY WAVE MOTIONS ON AN ATOLL REEF PLATFORM

CASE STUDY ROI-NAMUR, KWAJALEIN ATOLL

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Coral reefs are hard structures that front many coasts in tropical and subtropical climates and protect them against wave attack and erosion. Despite reducing incoming wave energy by up to 98%, coral reefs are not a guarantee that mainland or island coasts are safe from being flooded. This was demonstrated by a series of wave-driven flooding events in 2008, that caused widespread damage to infrastructure and freshwater resources at islands in the western Pacific Ocean (e.g. Micronesia, the Marshall Islands, Kiribati, Papua New Guinea and the Solomon Islands). In particular atoll islands were exposed as they are only 2-5m above sea level. Atolls are typically ring shaped and enclose a lagoon in the center. The actual landmass is small and therefore offers little space for rainwater to accumulate under the subsurface and form a fresh water lens. Coastal managers are concerned that future overwash events may get more frequent due to climate change related sea levels rise and thereby cause permanent salinification of the natural fresh water sources. The United States Geological Survey (USGS), the National Oceanographic and Atmospheric Administration (NOAA) and the University of Hawaii jointly initiated an investigation on the processes that are involved in the flooding of atoll islands. Eventually, research aims at the prediction of hazardous flooding events by utilizing computer models.

Plenty of nearshore processes have been described for reef systems in general, however, few studies have been devoted to the understanding of wave run-up. For sandy beaches the latter is typically a combination of wave induced set-up, incident short wave and incident infragravity swash, (Stockdon et al., 2006). In case of coral reefs the physics behind wave run-up can get far more complex. On reefs with a steep reef face and long shallow flats, strong amplification or even resonance of low frequency harmonics have been observed, both of which are likely to increase surf beat.

During a field experiment at the Kwajalein atoll between 3-Nov-2013 and 13-Apr-2014, hydrodynamic data were collected by the USGS to study run-up at a typical atoll reef with a steep fore reef (1:18) and a long horizontal reef flat. In previous research by Quataert (2015) these data have been used to force one dimensional XBeach models and to subsequently validate the model performance with respect to set-up, short and long wave heights and wave run-up on the beach. The models performed well with exception of the reproduced wave run-up that was systematically underestimated.

The current study continued on this subject, i.e. investigating wave run-up at Kwajalein atoll. The report was divided into two parts. For the first part, subharmonic wave motions were analyzed using the available field data. For the second part wave run-up was reexamined with one dimensional (non-hydrostatic and surfbeat) XBeach models. More specifically, the purpose of the data analysis was to get a detailed description of the (long) wave hydrodynamics and to find out how major run-up events distinguish themselves from the ordinary situation. Focal points were: Generation mechanism associated with subharmonic motions across the reef site and the amplification of very low frequencies (VLF) on the reef flat. Subsequently, 1D-XBeach modelling plugged in on the open question whether run-up would eventually be predictable by XBeach.

Roi-Namur was shown to be sensitive to wave climates with long peak periods $T_p$, that generally induced strong VLF amplification across the reef flat. Concurrently, fundamental resonant periods were strikingly similar to the most energetic VLFs observed on the inner reef flat. Tidal differences also impacted the combined low frequency (LF) energy inshore as the response was larger during high tide. Instead of bound long wave release, evidence was found that free long waves were generated by a moving breakpoint. This was also confirmed by high values of the relative bedslope parameter. Further investigation of the breakpoint mechanism moreover suggested that the relative effectiveness of generating infragravity (IG) waves varied for different wave conditions, which was explained by changes in the length of the breakpoint excursion.

It was found that non-hydrostatic models used in the first study by Quataert (2015) underrated wave induced set-up because the mean sea level (MSL) was imposed 0.5 m too low on the model boundary. Taking
this water level offset into account, the performance of XBeach-Surfbeat (XB-SB) was compared to XBeach-Non-hydrostatic (XB-NH). The former captured enough physics to compete with the non-hydrostatic XBeach mode in terms of the representation of wave induced set-up, short wave heights and long wave heights. However, XB-SB fell short on the prediction of run-up, which is a crucial weakness since the models are ultimately meant to estimate just that. Furthermore, two distinct ways of boundary forcing were tried, i.e. with measured spectra or idealized JONSWAP spectra. Bulk LF energy was best reproduced by models that were forced with measured spectra. Onset of idealized JONSWAP spectra introduced erroneously high LF amplitudes at the fore reef. This problem could be improved upon by changing the value of the peak enhancement factor $\gamma$. Run-up heights were well reproduced by non-hydrostatic simulations, in contrast to simulations with XB-SB that underestimated them. It was reasoned that incident short wave swash in XB-NH was key to better run-up predictions.

Combining the findings of the data and model analyses it could be concluded that both LF waves as well as nonlinear solitary waves significantly contribute to wave run-up on the beach.
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INTRODUCTION

1.1. BACKGROUND

Coral reefs are large limestone structures that host complex ecosystems and at the same time guard coastlines against wave attack and erosion (Falter and Atkinson, 2004; Hoeke et al., 2013; Van Dongeren et al., 2013). They form as many layers as corals grow on top of each other whilst bottom layers die, leaving their limestone skeleton behind. Over the course of thousands of years, patches of coral can fuse to form a super-structure stretching hundreds of miles. Occasionally, coral reefs may rise above sea level where additional erosion by the impact of waves leads to the formation of coral islands. Since coral requires sunlight and clear, warm water, coral reefs are typically found in tropical and subtropical climates. Three main types of tropical coral reef can be distinguished: Barrier reefs are detached from the mainland by a deep lagoon whereas fringing reefs are connected to the mainland via a wide reef flat (see Figure 1.1b). Depending on the site, the latter can be (partially) submerged. The third type of coral reef are atolls an example of which is shown in Figure 1.1a. Atolls are enclosed, ring shaped islands that owe their appearance to the growth of coral on the rim of an inactive volcano. Starting of as a fringing reef, a subsequent subsidence of the volcano and sea floor leads to a barrier reef and finally to the formation of an atoll’s characteristic, ring shaped structure once the volcano is entirely submerged.

(a) Air view of South Kwajalein, Marshall Islands
(b) Air view of Ningaloo reef, Western Australia

Figure 1.1: Ring shaped appearance of an atoll (a) and fringing reef at the coast of western Australia (b)
About 30 million people inhabit coral islands and atolls, mostly a mere 2m above sea level and thus fairly exposed. They depend on the protection by coral reefs. However, recent floodings have shown that under certain conditions this protection is insufficient (Hoeke et al., 2013; Péquignet et al., 2009). In 2008, a combination of elevated sea levels and large swells caused major damage on the Marshall islands, Micronesia and other small island states. In the Western Pacific, La Nina (Hoeke et al., 2013) conditions, in combination with wave induced set-up (Vetter et al., 2010), has led to extreme water levels at the shoreline. These drivers are significant but not an unknown threat. Coastal managers are concerned that it is rather global warming that will force water levels up generally and thereby increase the present rate of flooding due to additional events per decade. These events impact infrastructure, urban areas and above all the sparse drinking water resources of atolls, which eventually could make some of the islands uninhabitable. Hence, it is existential fear that drives the Marshall islands to reduce their own carbon emissions and to pledge for global carbon cuts.\(^3\)

In prospect of rising sea levels, a joint venture of the U.S. Geological Survey (USGS), the National Oceanographic and Atmospheric Administration (NOAA) and the University of Hawaii launched an extensive investigation on the possibly heightened risk of flooding. The primary aim is to make predictions of hazardous situations. Up to this day, little is known about nearshore processes on atolls.

Kwajalein atoll is one of multiple study cases within the Strategic Environmental Research and Development Program (SERDP)\(^4\), where instruments have been deployed to collect field data. Figure 1.2 shows its remote position in the Pacific Ocean. It resembles just one of many little dots that speckle the seas of Oceania. Kwajalein atoll belongs to the Republic of the Marshall islands but is also home to a large American military base, which due to its strategic position has been important throughout war history. Locals and the military base rely on a fresh water pocket under the islands top layer. With overwashes, as in 2008 (Hoeke et al., 2013) and 2014 (Quataert, 2015) occurring more frequently in recent years, the aquifer is in danger of becoming salinated as saltwater percolates down through the top soil. A major concern is that the fresh water source might become unusable in the future and since alternatives to substitute fresh water are expensive this concern is well founded.

\(^{1}\)http://dmcaustralia.blogspot.nl/2013/03/why-travel-to-western-australia-here-is.html (last consulted on 11.07.15)
\(^{2}\)http://japanfocus.org/-Andre-Vltchek/2619 (last consulted on 11.07.15)
\(^{4}\)https://serdp-estcp.org/About-SERDP-and-ESTCP/About-SERDP (last consulted on 20.08.15)
Pioneering research by Quataert (2015) was dedicated to the reproduction of bulk parameters, e.g. significant short and long wave heights, and wave run-up using a one dimensional hydrostatic XBeach model. The rationale was to put XBeach to the test to find out if it was suitable to be nested in a chain of larger scale wave models and an infiltration model. The latter is a 3D finite element model that is presently developed at the USGS. Eventually, its goal is to predict the rate at which the islands fresh water lens salinifies. Following wave evolution across the reef, Quataert (2015) discovered strong very long frequency (VLF) amplification over the reef flat. In an attempt to mimic wave behavior, she achieved reasonable results using the surf beat model. However, simulations uncovered a crucial model weakness: Run-up was systematically underestimated. She could improve run-up results by calculating in non-hydrostatic XBeach mode, yet an additional problem emerged as water level set-up was now below expectation.

### 1.2. SCOPE AND RESEARCH OBJECTIVES

This research continues on the work done by Quataert (2015) on modeling wave transformation and run up. Problems arose when reproducing run-up from in-situ observations. The objective of this study is to address the performance of 1D hydrostatic- and non-hydrostatic Xbeach. The hydrostatic mode is also referred to as XBeach surf beat. As mentioned earlier, hydrodynamics at atoll reef environments have been scarcely described in literature and since a direct answer to model issues could not be found in the first Kwajalein study, the focus of this report lays on the system's physics. Infragravity and very low frequency waves appear to get dominant as they approach the beach, which implies strong surf beat. Hence, a more fundamental understanding of low frequency processes is the first logical step.

The following research question has been derived:

"Which physical processes are dominant on a reef platform and how do hydrostatic and non-hydrostatic XBeach modes perform in reproducing the local wave field and run-up”

Secondary research questions are:

- **Data Analysis**
  1. Is it possible to attribute certain wave conditions to extreme, very low frequency amplification on the reef flat?
  2. Is the moving break point the leading generation mechanism of subharmonic waves and how does it behave for different wave conditions?

- **1D XBeach**
  3. Why is set-up underestimated by XBeach in non-hydrostatic mode?
  4. What is the performance of XBeach for different modes (surf beat, non-hydrostatic) and different types of boundary conditions?
  5. What is the correlation between low frequency wave motions and simulated wave run-up?

### 1.3. THESIS OUTLINE

This report is divided into two major parts: A data analysis and a model part. Before discussing the data, a brief literature review will guide the reader through an array of low frequency phenomena in chapter 2. Subsequently methods applied for the data analysis are introduced in chapter 3. In chapter 4 and chapter 5 results from the data analysis and the model simulations are presented with a discussion and detailed conclusion affixed to each part separately. Finally, main conclusions of the thesis are highlighted in chapter 6 together with suggestions for further research.
2.1. INTRODUCTION

In order to draw more information from existing surface elevation data, various analyses techniques are applied in this report. Ahead of explaining their working principle in the upcoming chapter 3, some basic background information is required. The upcoming sections peruse literature pieces on reef hydrodynamics, set-up, infragravity wave motions and run-up.

2.2. REEF HYDRODYNAMICS

The geometry of atoll and fringing reefs is typically characterized by a steep fore reef slope, shoreward of which is a wide reef flat. Depending on the exact reef type and tidal range, the flats can either fall dry during certain periods of the day, as at Kwajalein atoll or stay permanently submerged, which is the case at Ningaloo Reef (Pomeroy et al., 2012). A dense coral cover translates into a rough bottom topography and confers strength to dissipate wave energy. Ferrario et al. (2014) extracted data sets from various reef studies on wave attenuation and found that reefs accounted for a total wave energy reduction of 94-98%. At various reef systems, both sea and swell environments can be encountered. For a subset of studies, Ferrario et al. (2014) investigated the reduction of energy by wave type. They concluded that both types were dissipated, although the effect appeared to be more significant for swell waves.

Wave transformation across coral reefs is a complex problem. For instance waves may refract, shoal, reflect, break or dissipate due to bottom friction. The extent to which these processes occur is strongly dependent on the water depth and the offshore wave conditions. As deep water waves approach the steep reef face they become highly nonlinear (McKee Smith, 1993). When they start to break, energy is dissipated and transferred to other wave frequencies. Previous research shows that the spectrum broadens as waves propagate over the reef and peak periods increase as periodic sea level changes are dominated by beat frequencies (Lee and Black, 1978). Recently, Péquignet et al. (2009) found that extreme water level increases during large wave events alters the nature of wave transformation over reef flats. As part of the Pacific Island Land-Ocean Typhoon (PILOT) project they analyzed the effects of tropical storm Man-Yi on Ipan-reef, Guam. Incident waves with a significant wave height of \( H_s = 4 \text{ m} \) and peak periods of \( T_p = 12 \text{ s} \) induced high wave set-up on the reef flat and an excitation of resonant modes, with an antinode at the shoreline.
2.3. **Water Level Set-up**

The surf zone starts at the transition between fore reef and reef flat. Short waves traveling towards the coast start to break here due to the sudden change in water depth. A high spatial concentration of short wave breaking translates into large local gradients in radiation stress. Released momentum causes pressure to build up in the form of cross shore water level set-up (Bosboom and Stive, 2012; Holthuijsen, 2007; Pomeroy et al., 2012). Reef flats can reach far out into the ocean and most of the wave breaking occurs far from the coastline. Hence, wave induced set-up quickly raises water levels over great distances. Many laboratory and field studies were devoted to the description of that set-up as summarized by Quataert (2015). Consistent with Seelig (1983), an important conclusion was that the role of wave set-up would increase with shallower water depths on the reef flat, from which follows that even parts of the reef flat above sea level might become submerged. Becker et al. (2014) specified this to be the consequence of a tidally dependent breaking intensity. The latter was confirmed by Quataert (2015) who moreover found a clear dependency between incident wave energy and wave induced set-up. Higher incoming waves induced higher set-up on the reef flat, again conform statements by Seelig (1983) about ponding levels observed in a scale test of Guam.

2.4. **Infragravity Waves**

Dynamics of infra gravity (IG) waves are complex and can differ between reef sites. The investigation of sub-harmonic motions in reef environments is a relatively young research field. Péquignet et al. (2009) discovered resonant behavior of IG waves on reef flats in contrast to Pomeroy et al. (2012), who described progressive and decaying infra gravity waves. Similar to Péquignet et al. (2009), Quataert (2015) found anomalous growth of low frequencies over the reef flat, but did not further explore the underlying mechanism. In conclusion, low frequency wave evolution over a reef is not predetermined.

In the following, background information on low frequency waves is briefly recapitulated.

2.4.1. **Wave Group Forced Motions**

Short period waves, \(O(0.1 \text{ Hz})\), can force long period waves, \(O(0.01 \text{ Hz})\), because they travel in wave groups. Offshore, free waves (no external forcing \(^1\)) act dispersive, implicating that longer period waves travel faster. This can readily be seen from the dispersion relationship, where for large water depths the phase speed remains only dependent on the wave period \(c \approx 1.56T\). Waves move independently (unhindered by other waves) so that eventually, after a storm, a random wave field induces an ordered wave transition in propagation direction. By virtue of dissipation effects (i.e. white capping) that act more effectively upon shorter waves, only lower frequencies survive (Holthuijsen, 2007). It should be emphasized that waves will disperse in multiple directions, but at a certain point far away from the original storm center, only the residual waves from one direction will be observed. These longer period waves, which are typically long crested and smooth in appearance, are called swell waves. Furthermore the remaining, limited bandwidth leads to a correspondingly narrow wave spectrum. Here, the phenomenon of wave grouping comes into play, as it is characterized by wave components that differ only slightly in frequency corresponding to narrow banded spectra. A wave group can most easily be described by a bichromatic wave, that consists of solely two harmonics with small difference frequency. Figure 2.1 schematically shows such a bi-chromatic wave. An expression for the surface elevation of a bichromatic wave is derived in subsection B.0.2.

For idealized conditions (assuming a horizontal bed, no friction, \(\ldots\)) in the depth integrated balance equations (see subsection B.0.3), one can show that a bound wave is created, represented by the red line in Figure 2.1. This bound wave propagates 180\(^{\circ}\) out of phase with the short wave envelope (Reniers, 2013; Van Dongeren

\(^1\) Within linear wave theory and for free waves, the dynamic boundary condition prescribes zero pressure at the free surface (Holthuijsen, 2007).
Infragravity Waves

2.4. INFRAGRAVITY WAVES

As this bound wave is a forced wave it does not comply with Airy-wave theory, one indication of which is that its phase speed is dictated by the wave group. Aside from this, the bound IG wave is no theoretical construct, but a measurable wave in the wave spectrum.

Natural sea states include more than two harmonics. Instead of bichromatic wave groups, an infinite number of frequencies in the wave field forms multichromatic wave groups. An elegant way of exploring the nature of incoming low frequency (LF) waves for a random wave field is to compute the secondary forced surface elevation $E_f(\Delta f)$ that is attributed to these multichromatic wave groups. Herbers et al. (1994) introduced a method to calculate these LF motions in two dimensional space. For practicality, the original expression was reformulated by Van Dongeren and Reniers (1999) and officially published in Van Dongeren et al. (2003):

$$E_f(\Delta f) = 2\int_{\Delta f}^{\infty} \int_{0}^{2\pi} \int_{0}^{2\pi} D^2(f + \Delta f, -f, \Delta \theta + \pi)E(f + \Delta f, \theta_1)E(f, \theta_2) \, d\theta_2 d\theta_1 df$$

(2.1)

With the difference-interaction coefficient

$$D(-f_1, f_2, \Delta \theta + \pi) = \frac{g k_1 k_2 \cos(\Delta \theta + \pi)}{8\pi^2 f_1 f_2} \frac{\cosh(k_3 h)}{\cosh(k_1 h) \cosh(k_2 h)} - \frac{g(-f_1 + f_2)}{\left|k_3 \tanh(k_3 h) - (2\pi)^2 (-f_1 + f_2)^2 f_1 f_2\right|}$$

$$\times \left\{ (-f_1 + f_2) \left[ \frac{(2\pi)^4 (f_1 f_2)^2}{g^4} - k_1 k_2 \cos(\Delta \theta + \pi) \right] - \frac{1}{2} \left[ \frac{-f_1 k_2^2}{\cosh^2(k_2 h)} + \frac{f_2 k_1^2}{\cosh^2(k_1 h)} \right] \right\}$$

where

$$k_3 = |k_1 - k_2| = \sqrt{k_1^2 + k_2^2 + 2k_1 k_2 \cos(\Delta \theta)} \quad \text{and} \quad \Delta \theta = |\theta_1 - \theta_2|$$

$E_f(\Delta f)$ is the 2-D energy density, subscripts 1 and 2 refer to the two interacting primary waves. $D$ is the difference interaction coefficient for the surface elevation energy. Other parameters are consistent with earlier equations. Evidently, not only two primary wave components determine the bound long wave (BLW) energy. Equation 2.1 is solved for the whole set of primary frequency pairs that contribute to the long wave forcing of one particular $\Delta f$. The process is repeated for all discrete low frequency values of $\Delta f$. Use of one-dimensional power spectra, implies that $\Delta \theta = 0$. Underlying assumption of Equation 2.1 is that the bound long wave is in local equilibrium with the group forcing, i.e. the LF wave is 180° out of phase with the short wave envelope.

2.4.2. BOUND LONG WAVE SHOALING

Previous findings were based on offshore conditions. With shoaling the situation gets more complicated and a couple of studies have been devoted to the understanding of the relevant processes (Battjes et al., 2004; subsection B.0.4 explains how the time dependent radiation stress can be used to get an expression for the bound long wave surface elevation in subsection B.0.5.

Airy-wave theory presupposes zero pressure at the water surface
Janssen et al., 2003; Van Dongeren et al., 2007). They found that during shoaling, a phase shift occurs between the bound long wave and the wave group. The bound long wave is somewhat dilatory. With continuously decreasing water depth, it is not able to follow the short wave envelope and consequently needs more time to adapt to the new equilibrium situation. Interestingly enough, Battjes et al. (2004) measured differences within the low frequency domain. The phase shift would be larger for relatively higher frequencies and stimulate energy transfer from primary short waves to the bound wave, enabling it to follow the shallow water limit (Equation B.16), which in Battjes et al. (2004) is actually emphasized not a shoaling limit. Compared to the relatively high frequencies, lower frequencies could continue to stay phase locked for shallower water depths, however, shoaled less intensive (Equation B.16 couldn't be followed at all).

**Potential bound long wave release**

Bound long waves are habitually perceived to be released during the short wave breaking process, (Battjes et al., 2004; Reniers, 2013; Van Dongeren et al., 2007). The concept involves the escape of bound long wave harmonics from a decaying primary wave group. When the short waves break, the water level depressions they brought about remain. This low frequency wave can now propagate freely and is therefore also called a free long wave. Theoretically, it therefore falls under Airy-wave theory since no external pressure influences wave propagation. Hence, the free wave should shoal according to Green's law (Equation B.17). If the bound long wave is not fully released in the shoaling process, this will commonly happen at the shoreline, so that the reflected IG wave is a free wave, (List, 1992). It is emphasized that bound long waves are not always released in the shoaling process. Baldock (2012) criticized the regular notion to assume that bound long waves are transformed into free long waves in the process of short wave breaking. Indeed this was not ungrounded. In an earlier flume experiment of transient wave groups over a steep slope, Baldock (2006) observed that bound long waves could survive coastline reflection. Baldock (2012) moreover fostered that bound long waves could be fully dissipated during shoaling and subsequent short wave breaking leaving no long wave residuals to be released.

**The breaking point mechanism**

Alternatively to bound long wave release, IG waves can be generated from a moving short wave breaking point, (Symonds et al., 1982). This mechanism can be significant in steep slope environments and has indeed been identified as the leading cause for surf beat at the fringing Ningaloo reef in Australia, (Pomeroy et al., 2012). Figure 2.2 illustrates how breakpoint-generated waves are created. When the highest waves of the wave-group (blue) break, they cause local set-up (red). This bulb will not immediately be noticed at the shoreline. While it travels shoreward, smaller waves move in. They protrude farther and eventually reach their breakpoint. With new wave groups following, the process repeats and a long wave pattern develops.

![time-varying breakpoint](image)

**Figure 2.2: Idealized illustration of the breaking point mechanism.**

Free breakpoint forced wave harmonics have a linear relationship with their associated parent wave group,
2.4. INFRAGRavity WAVES

(Baldock et al., 2000; Symonds et al., 1982). Since wave groups approach the time-mean breakpoint location in different lengths and sizes, they accordingly transform into free long waves of various frequencies. This array of low frequency components propagates into opposite directions. Waves traveling shoreward fully reflected if the beach is sufficiently inclined around MSL. For an instant, the incoming and reflected part of the wave create a standing pattern that disappears as soon as the entire wave has reflected. If wave groups with identical properties enter the breaking region rhythmically, the standing waves becomes persistent. The phenomenon is therefore more likely for regular swell motions.

In the case of strong shoreline reflection, the breakpoint mechanism is most powerful if the breakpoint forcing region is small compared to the wave length of a free standing wave, (Baldock et al., 2000). The explanation lies in the nodal structure of a standing wave. The breakpoint mechanism is most effective if the time varying breakpoint concentrates around an antinode or node of the free standing wave. Nodes and antinodes are separated from each other by intervals of $\lambda_{LF}/4$ where $\lambda_{LF}$ resembles the length of the breakpoint generated long wave. Consequently, the breakpoint excursion $\Delta x$ has to be smaller than $\lambda_{LF}/4$ as otherwise the breakpoint forcing encompasses more than one node or antinode. To give this theory some leeway, the criterium is relaxed to $\Delta x < \alpha \lambda_{LF}$ where $\alpha = 0.2 - 0.3$.

For engineering purposes, Baldock and Huntley (2002) approximated $\Delta x$ by $G H/h_x$. $G$ resembles the groupiness factor, which is conform Haller and Dalrymple (1995); List (1991) computed from the time series of the group envelope with standard deviation $\sigma_A$ and burst-averaged amplitude $\langle A \rangle$. $H$ denotes the root mean squared short wave height ($H_{rms,SW}$) seaward of the breakpoint and $h_x$ the bottom slope in the breaking region. Analogous to (Baldock et al., 2000), $\lambda_{LF}$ may be estimated as $\sqrt{g H/f_{LF}}$ under the assumption that the depth at the breakpoint is equal to the short wave height (the breaker index $\gamma = 1$). $f_{LF}$ resembles the low frequency harmonic associated with $\lambda_{LF}$. Combining previous simplifications, an effective breakpoint mechanism can be verified by means of Equation 2.2.

$$G \sqrt{H_{rms,SW}} f_{LF} < \alpha \quad \text{where} \quad G \equiv \frac{\sqrt{2} \sigma_A}{\langle A \rangle}$$

OTHER IG WAVE GENERATION MECHANISMS

Yet a third explanation for free long wave generation can be found in literature. Indeed, it is highly theoretical. Mei and Benmoussa (1984) give a mathematical derivation of free long waves that are created in case of a sudden step in the bathymetry. At the location of the step, they emanate into both cross-shore directions. They are a mathematical tool, used to assure that the sudden change of the bound long wave shape (in shallower water on the step) can be realized.

SURF BEAT GENERATION INDICATORS

Recent studies Battjes et al. (2004); Janssen et al. (2003); Symonds et al. (1982); Van Dongeren et al. (2007) relate the surf beat generation mechanism to some form of relative bed slope parameter. Its currently most used form as $\beta$ is presented by Equation 2.3. subsection B.0.6 goes further into detail on this matter.

$$\beta \equiv \frac{h_x}{\omega} \sqrt{\frac{g}{h_b}}$$

The definition of the dimensionless, normalized bed slope includes the regular bed slope $h_x$, the radial frequency of the LF waves $\omega$, the gravitational acceleration $g$ and the characteristic breaking depth $h_b$. In literature $\beta$ appears in slightly different shapes, but is generally analogous to the Iribarren number, $\xi = \frac{h_x}{\sqrt{H_{rms}}}$.

For sloping beaches, Van Dongeren et al. (2007) amongst other literature found that $\beta$ governs dissipation
and reflection at the shoreline and indeed similar to the irribarren number $\xi$ for short waves. The connection between the nature of IG shoaling and the relative bed slope was also affirmed by Battjes et al. (2004). A primary feature of $\beta$ is to forecast whether a beach profile incites bound long wave release or the breakpoint mechanism. Battjes et al. (2004) proposes a threshold value of $\beta = 0.3$. Larger values indicate a steep slope regime and the breakpoint mechanism, whereas lower values suggest relatively mild slopes and bound long wave release.

In addition to $\beta$, Baldock (2012) recently introduced a surfbeat similarity parameter $\xi_{surf\,beat}$, see Equation 2.4. He stresses that a combination of the relative beach slope together with the steepness of incoming waves determines the IG generation mechanism, which makes the definition of $\xi_{surf\,beat}$ a logical choice.

$$\xi_{surf\,beat} = \frac{\beta}{\omega_{IG} \sqrt{\frac{g}{h}}} \left( \frac{H_{os}}{L_{os}} \right)$$

Easily recognizable are the normalized bedslope ($\beta$) and wave steepness term. The absolute beach slope and angular frequency of an IG wave are given by $h_x$ and $\omega_{IG}$ respectively. $g$ is the gravitational acceleration and $h$ is a representative shoaling depth that has no unique definition in the original paper (Battjes et al., 2004) and is occasionally commuted to the short wave breaking depth (Van Dongeren et al., 2007). $H_{os}$ and $L_{os}$ on their turn resemble offshore short wave height and -length.

$\xi_{surf\,beat}$ is not only a measure for surfbeat strength but also gives hints whether it originates form progressive release of the incident bound long waves. Judging from the magnitudes presented by Baldock (2012), $\xi_{surf\,beat} \geq 0.08$ when the breakpoint mechanism is the predominant forcing causing surf beat. Numbers falling in between these bounds suggest surf beat to be minimal. An important factor at this point is the location where short wave breaking commences. If shallow water conditions $H_o/L_o \leq \gamma 0.016$ are met before short wave breaking, bound long waves are sufficiently released to cause surf beat (Baldock, 2006, 2012; Baldock and Huntley, 2002), emphasized on the left hand side of Figure A.1. $\gamma$ symbolizes the wave height to depth ratio at breaking and is assumed $\gamma = 0.8$. In case of steep waves and a mild slope, $\xi_{surf\,beat}$ is in an intermediate range and the aforementioned shallow water condition is not fulfilled. This implies on the one hand a strong reduction of the incident bound long waves and on the other hand a weak breakpoint forcing due to a wide breaking region in comparison to the long wave length, (Baldock and Huntley, 2002). A mix of a steep slope with low steepness short waves is also tagged with medium range values of $\xi_{surf\,beat}$, again surfbeat is weak.

### 2.4.3. IG Wave Dissipation

A diversity of dissipation agents are listed in literature i.e. breaking, bottom friction and interactions between sea and swell. This section clarifies the process of IG wave energy dissipation for particularly coral reefs with a steep fore-reef slope.

Deep water bound harmonics approaching a highly inclined reef face have insufficient time to be released (Baldock, 2012) and are expected to be destroyed during the short wave breaking process. Subsequently, (free) IG waves are brought back into existence by the time varying breakpoint. Pomeroy et al. (2012) observed that these breakpoint generated IG waves decayed while progressing over the reef flat and determined bottom friction as the leading cause. They argued that high coral cover returns large friction coefficients that can act over the whole reef-flat, resulting in a gradual decay of the long wave energy. Yet another dissipation process might act on a reef flat. Analyzing IG wave transformation for a gradually sloping beach profile, Van Dongeren et al. (2007) demonstrated that IG waves could dissipate by means of long wave breaking in the vicinity of the shoreline. Self-self interactions were accounted for long wave breaking but occurred solely at relatively mild profile slopes. Reef flats are nearly horizontal and thus might stimulate IG wave breaking.
2.5. RUN-UP

Stockdon et al. (2006) define run-up as the sum of two components, namely time-averaged wave setup and total swash excursion. Swash on its turn can be decomposed into incident and infragravity swash. Based on data collected from sandy beaches, Stockdon et al. (2006) concluded that dissipative beaches are dominated by infra gravity (IG) swash, whereas steep slopes are mostly attacked by incident swash. Coral reefs have large flats and an application of empirical formulations by Stockdon et al. (2006) (see also subsection B.0.1 in the appendix) for predictive purposes is troublesome since they are originally forged from regression lines of gradually sloping beach data. Nevertheless a general parameterization in terms of a run-up maximum $R_{\text{max}}$ and the statistical 2% exceedence value $R_{2\%}$ is well fit for an assessment on run-up at coral atolls, (Quataert, 2015).

For Kwajalein atoll, Quataert (2015) concluded that surf beat contributes significantly to wave runup on the beach. In contrast to Pomeroy et al. (2012), long wave heights don’t decrease, but rather increase on their path towards the shoreline which explains the IG swash. From a qualitative comparison of run up simulated with hydrostatic and non-hydrostatic models, Quataert (2015) suggested that the contribution of incident swash is important, too.
3

Methodology

3.1. Introduction

The ultimate goal of the Kwajalein research project is to predict the return period and intensity of wave driven flooding events and to estimate their impact on the fresh water lens under the atoll. By analyzing wave transformation over the reef structure, this study sharpens the view on reef hydrodynamics at Kwajalein atoll. It thereby aims to provide a valuable contribution to a physics based verification of XBeach as a flooding model for atolls. The aforementioned analysis forms the first part of this report and requires a thorough preparation of the available field data. Subsequently, research continues with a numerical performance study that balances the pros and cons of a hydrostatic versus a non-hydrostatic modeling procedure. It forms the second part of the report and its key purpose is to tell whether it is worth to solve to the short wave scale.

Section 3.2 introduces the reef site and provides a brief description of the field experiment. Consecutively, the data preparation is presented in section 3.3. The data analysis is split into a part that explains the applied analysis techniques in section 3.4 and a part that presents the results. Note that the description of applied XBeach modes and their results will first be treated in later chapters.
3.2. SITE DESCRIPTION AND INSTRUMENT DEPLOYMENT

Kwajalein atoll is located in the western Pacific and belongs to the Republic of the Marshall islands (its location within Oceania is depicted in Figure 1.2). The contour line of Kwajalein is formed by a conglomerate of island strips that surround one of the world’s largest lagoons, measuring about 110 km on the largest diagonal. On average, the land mass reaches 2 m out of the water. The wave climate is dominated by long period swells with average periods of 10 s and the tidal range varies between 1.5 – 1.8 m.

The atoll section in focus is highlighted by a solid white circle at the top end of Kwajalein in Figure 3.1. This particular part of Kwajalein is called Roi-Namur and originally consisted of four separate islets, amongst which Roi and Namur, that were joined during World War II by means of sand fills.

The left hand side of Figure 3.2 specifies the study site at Roi-Namur. The right hand subfigure locates the positions of the instruments during the field experiment from 3-Nov-2013 to 13-Apr-2014. Along each of the transects four pressure sensors were deployed (type RBR virtuoso). Three on the reef flat and one on the fore reef. Unfortunately, the fore reef wave gauge on the eastern transect could not be recovered. A Nortek

\(^1\text{http://glovis.usgs.gov} \text{ (last consulted on 28.07.15)}\)
Acoustic Wave and Current profiler (AWAC) was installed offshore to provide data on the deep water wave conditions. Besides measuring the water surface movement across the reef with wave gauges, cameras were placed near the transects to record wave run-up on the beach, indicated by white crosses in the right hand subfigure of Figure 3.2.

The reef flat bathymetry was mapped before the start of the field experiment. Since wave conditions on the fore reef were too violent to measure its entire profile, solely one depth sample was taken at the outer end of the fore reef. This initially constrained the fore reef lay out to a straight line. In the meantime, the depth profile of the entire fore reef is at disposal. The underlying data have been acquired from NOAA in aftermath of the survey and were consecutively merged by the USGS with existing data of the reef flat topography. As opposed to the first Kwajalein study (Quataert, 2015) therefore a realistic representation of the fore reef can be handled in this report (see also Figure 5.2).

As in Quataert (2015), the eastern transect is neglected in the current study, as fore reef data are crucial for a visualization of the wave transformation across the reef, thus focus lies on the western transect. Figure 3.3 sketches the western transect with its main dimensions and points out the positions of the pressure gauges.

Starting offshore, the atoll reef face is almost vertical. The offshore AWAC measured pressure variations at a depth of approximately \(-20\) m below the mid reef crest level. A grey tone marks out its lateral dislocation, i.e. the AWAC is not deployed in line with the rest of the wave gauges. At about 350 m from the shoreline, the bathymetry changes over to a milder slope (approx. 1:18) and heralds the beginning of the so called fore reef. Close to the seaward edge, the fore reef RBR sampled pressure variations at a depth of about 7.5 m below the reef flat. The surrounding area is densely covered with corals and is accordingly rough, as delineated with dashes. Moving 130 m further inward, depth attenuation causes incoming waves to shoal and eventually break at the transition to the outer reef flat. One of the pressure sensors is situated next to this transition and captured parts of the breaking process on the outer reef flat. Water levels are barely 70 cm deep at this point. Although illustrated as a horizontal line, the reef flat that commences here has a gentle slope (approx. 1:700). Hence, water levels get even shallower further shoreward, causing the greater part of the reef platform to fall dry during low tide. Corals can’t get a foothold under such circumstances, giving the reef flat a relatively smooth surface that beds only small amounts of coral rubble. The flats stretch out 270 m across and connect to a relatively steep, sandy beach (approx. 1:6). Besides the outer reef flat, pressure fluctuations were recorded at the mid reef and in the vicinity of the coastline, i.e. in a region defined as the inner reef flat.

All pressure sensors including the offshore AWAC were deployed between 3-Nov-2013 and 13-Apr-2014. Every hour, a data stream of 2048 s was collected at a sampling rate of 1 Hz for the AWAC and 2 Hz for the RBR virtuoso’s. Wave run-up was recorded with digital time-lapse cameras that took snapshots every 30 min. Recordings stopped prematurely on 17-Dec-2013 when the cameras got exposed to rough wave conditions.
3.3. **DATA PREPARATION**

Ahead of the ultimate cross correlation of wave signals the original, raw pressure data had to be reexamined. Preparing the data exposed an underestimation of the wave-heights in the first study (Quataert, 2015), which therefore had to be reevaluated. Mean sea level is also higher than initially assumed, which has consequences for the implementation of boundary conditions in the models. Subsequent sections elaborate stepwise on the data treatment, yet large parts are standard procedure and are hence kept short.

3.3.1. **WAVE SPECTRA**

Pressure data, collected by the Nortek AWAC offshore and the RBR virtuoso's deployed on the fore reef and across the reef flat (Figure 3.3) are converted from decibars [dbar] to pressure head [m] \(^2\), taking into account the latitude at which the respective wave gauge is located, (Fotonoff and Millard Jr., 1983). After demeaning and detrending the resulting time series, a preliminary smooth power spectrum is determined for each burst at each instrument location separately. This is done consistent with earlier studies (Pomeroy et al., 2012; Quataert, 2015) by Welch averaging with 50% overlap. Hanning windows allow the Fast Fourier Transform to better attach to each segment and thereby optimize the transformation from the time into the frequency domain. The number of segments used for smoothening put limitations on the lowest resolvable frequencies. Research on low frequency (LF) energy, down to 0.001 Hz, is therefore based on minimally smoothed spectra, by splitting the time series into two segments, whereas higher or peak frequency quantities were estimated with a super smoothed spectrum created from eight time segments. Note that Welch averaging with 50% overlap translates to an effective use of three and fifteen windows, yet keeping a high frequency resolution. Next, a correction of the (hydrostatic) power spectrum is necessary to account for vertical accelerations induced by the waves, which especially within deeper water influence pressure measurements. A correction factor \( K_p \), referred to as the pressure response factor, is used to take these wave induced pressure fluctuations into account, (Dean and Dalrymple, 1991; Kirby, 1993; Péquignet et al., 2009). It is directly construed from the wave pressure relationship of linear wave theory, (Holthuijsen, 2007).

\[
S_{\eta \eta, f} = \frac{S_{pp, f}}{K_{p, f}^2} \quad \text{with} \quad K_{p, f} = \frac{\cosh(k_f z)}{\cosh(k_f h)}
\]  

(3.1)

To the left of Equation 3.1, the pressure response factor \( K_{p, f} \) [-] is squared to acquire a realistic variance density spectrum \( S_{\eta \eta, f} \) [m\(^2\)] from the rather indicative pressure head spectrum \( S_{pp, f} \) [m\(^2\)] that only serves to estimate the wave numbers \( k_f \) at hand. Not squaring \( K_p \) has undervalued the wave heights in the precedent study, (Quataert, 2015). \( z \) and \( h \) represent wave gauge height above reference level and water depth respectively. Subscript \( f \) denotes that Equation 3.1 is solved stepwise for each discrete frequency of the power spectrum \( S_{pp} \). Wave number values are drawn straight from the linear dispersion relationship, where each discrete frequency is associated with a unique wave number. Note that the presence of currents is neglected and that Airy wave theory is based on the assumption of free waves. With forced, bound waves at the research site, both factors have the potential to make the true wave number values deviate from the ones used throughout the present analysis.

3.3.2. **DEFINITION OF FREQUENCY RANGES**

After averaging all offshore power spectra a peak frequency of \( f_p = 0.075 \text{ Hz} \) is found compliant with earlier research and thereby justifies the use of a split frequency \( f_{\text{split}} = 0.04 \text{ Hz} \), being half the peak frequency, (Roelvink and Stive, 1989). Nyquist frequencies \( (0.5 \times \text{sampling rate}) \) are at 0.5 Hz for the offshore AWAC and 1 Hz for the remaining instruments, but don’t bind the spectra that are ultimately used, because the cut-off frequency of 0.2 Hz for the high frequency tail is far below those limits.

\(^2\)Besides, data are cleared of the pressure contribution by the atmosphere
3.3. DATA PREPARATION

Figure 3.4: Relative deviation of the raw power spectrum $S_{\eta\eta,\text{raw}}$ from the smoothed one $S_{\eta\eta,\text{smoothed}}$. Convention is that $\langle \cdots \rangle$ represents time-averaging, i.e., an average of the results for all the measured bursts. Large deviations at higher frequencies are an indication of great randomness in the signal.

Also the cut-off frequency is checked to ensure no artificial energy from white noise is included in the power spectra. This is accomplished by subtracting the smoothed power spectrum at each location from its raw counterpart and dividing again by the smooth spectrum, which, averaged over all bursts, should give an indication of the relative deviation of amplitudes around their expected value. If values become extraordinarily high above certain frequencies it is hypothesized to be largely due to white noise. Indeed, time-averaging the results over all bursts, i.e., wave climates, suggests white noise to become more significant above 0.2 Hz for the offshore wave gauge, see Figure 3.4.

Table 3.1 delineates the frequency ranges, defined analogous to Quataert (2015).

<table>
<thead>
<tr>
<th>Frequency Band</th>
<th>Definition in [Hz]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Short waves (SW)</td>
<td>0.04 - 0.2</td>
</tr>
<tr>
<td>Infra gravity waves (IG)</td>
<td>0.004 - 0.04</td>
</tr>
<tr>
<td>Very low frequency waves (VLF)</td>
<td>0.001 - 0.004</td>
</tr>
<tr>
<td>Total spectrum band</td>
<td>0.001 - 0.2</td>
</tr>
</tbody>
</table>

Table 3.1: Frequency band widths. IG- and VLF waves together define the low frequency (LF) domain.

3.3.3. MEAN SEA LEVEL

Bathymetrical elevations were only at disposal relative to an arbitrarily chosen reference 0 m point on the mid reef flat, i.e., the measured ellipsoid of the mid-reef pressure sensor was given as preliminary reference point for the tidal datum. For an assessment on wave induced setup, Quataert (2015) calculated the hypothetical water level in absence of waves conform an approach used by Van Dongeren et al. (2013); Vetter et al. (2010) and set it to a value of 33 cm above mid reef zero. Without any wave action and under the assumption that mean currents are negligible, no pressure gradient builds up across the reef flat and water levels strictly follow the tidal modulation. Hence, the so called water level offset of 33 cm represents mean sea level. Since the conversion from pressure to water surface elevations was flawed, this water level offset is recalculated.
Figure 3.5 shows a definition sketch of the required parameters.

![Figure 3.5: Definition sketch of wave set-up and offset of MSL with respect to mid reef reference point](image)

Starting in the middle of Figure 3.5, the parameter $WC$ denotes the measured water column in meters above a randomly placed pressure sensor (RBR) on the reef flat and is valued either positive or zero. $b_{mid}$ resembles the vertical displacement of the RBR with respect to the mid reef zero meter reference point (mid 0m) and is therefore tagged with the subscript mid. Note that only the vertical positions of the reef flat pressure sensors are accurate enough to be used for the determination of MSL. $\eta_{mid,s}$ indicates the water set-up above mid 0m and hence shares the subscript mid. On the left hand side, $\eta_{MSL,s}$ denotes the water level measured at the location of the RBR with respect to the unknown position of MSL, that in the sketch is assumed to be below mid 0m. It is a quantity that yet has to be determined, similar to the wave induced set-up $\eta_s$ that is also an unknown. In contrast, the tidal excursion $\eta_{MSL,tide}$ can be drawn from pressure data collected by the offshore AWAC. Demeaning them over the entire survey period gives the variation around MSL. Finally, $h_{MSL,0}$ on the right hand side represents the desired offset of MSL from mid 0 m.

Some parameters can be expressed in terms of each other, resulting in a set of equations.

\[
\begin{align*}
\eta_{mid,s} &= WC - b_{mid} \\
\eta_{MSL,s} &= \eta_{mid,s} + h_{MSL,0} \\
\eta_s &= \eta_{MSL,s} - \eta_{MSL,tide}
\end{align*}
\]

which combined yield

\[
\eta_s - h_{MSL,0} = WC - b_{mid} - \eta_{MSL,tide}
\]

The terms to the left of Equation 3.2 represent the remaining two unknowns. Wave induced set-up $\eta_s$ varies between bursts, whereas the offset $h_{MSL,0}$ is a constant. As a consequence, a single situation where $\eta_s$ is known suffices to solve the equation and quantify the offset. Indeed, under the condition of a wave free environment, wave induced set-up is naturally zero. However, this situation is never encountered at Kwajalein. The artifice is to evaluate the right hand side of Equation 3.2 for every burst and map the outcome against the associated offshore wave power $H_2^2T_{m01}$. Consecutively fitting a linear least squares regression line to the scatter extrapolates the graph to a point where in reality a completely wave-free environment would be encountered, i.e. $H_2^2T_{m01} = 0$ and consequently $\eta_s = 0$. The resulting Figure 3.6 for the Kwajalein field experiment represents the outcomes for the outer reef flat RBR. Note that previous procedure is also admissible for the mid and inner reef flat RBRs. Their results are consistent with the offset found for the outer reef flat sensor (not shown).

At the intersection of the regression line with the vertical axis, waves vanish from the mean water surface and wave induced set-up logically gets nonexistent $\eta_s = 0$. As a result, $0 - h_{MSL,0} = 0.5$ or $h_{MSL,0} = -0.5$ m. Having
Figure 3.6: Waterlevel offset at outer reef flat location. The scatter is established by evaluating Equation 3.2 for all measured bursts (vertical axis) and coupling the result to the incident offshore wave power (horizontal axis). Low tide bursts are represented by blue colors whereas red colors show high tide events.

In conclusion, mean sea level is 0.5 m above the preliminary reference point on the mid reef flat and 17 cm above the premise of the first Kwajalein study, (Quataert, 2015). Bottom levels are referenced to mid 0m, which for model simulations means that demeaned offshore water levels are artificially raised by 0.5 m.

### 3.4. DATA ANALYSIS TECHNIQUES

This section yields a list of techniques that are used to strip the mixed wave field down to its subharmonic core behavior. Most famous amongst those techniques is probably the cross correlation analysis (XCA). It is practical and can serve multiple purposes. Other than that ways are found to picture dominant frequencies, magnitude and significance of bound- and free long waves and their generation mechanism. All the applied analysis techniques are with their inherent theories presented in this section.

#### 3.4.1. CROSS CORRELATION ANALYSIS (XCA)

In the ideal case, quantitative information about incoming and outgoing waves is available to identify significant LF wave processes. For this purpose, velocity records are indispensable. Incidentally, two distinct methods exist to do a directional decomposition. One extracts phase information from signals measured by two or three collocated wave gauges and even enables operation of a least squares technique to suppress errors caused by white noise, (Bakkenes, 2002). The second method requires additional velocity records to the wave height measurements. In the case of e.g.
however, no velocity measurements exist at Roi-Namur, which unfortunately makes it impossible to extract directional information from the data. Indeed, even if wave speeds were registered by all instruments the application of the directional separation method by Battjes et al. (2004), assuming linear wave theory, could be inappropriate (Baldock, 2006) in prospect of solitons arising on the Roi-Namur reef flat, (Seelig, 1983). Instead, time domain cross correlation of the original time series comes straightforward and is meant to give a qualitative picture of the IG forcing mechanisms.\footnote{Ningaloo Reef, (Pomeroy et al., 2012), these were present. Neither is possible within this study, due to the large distances between the wave gauges and the lack of velocimiters across the measurement transects.}

Assuming ergodic wave signals, a XCA can reveal the generation mechanism of LF waves and give hints about wave propagation. Standing wave patterns would imply dominance of wave reflection over dissipation, whereas signs of a progressive wave character indicate the opposite.

First the pressure head time series are divided into high frequency (HF) and LF constituents by band-pass filtering in frequency space using the limits shown in Table 3.1. Subsequently, the procedure described in subsection 3.3.1 is followed to correct the SW and LF power spectra for growing depths. Additionally, representative surface elevation time-series $\eta_{SW}$ and $\eta_{LF}$ are built.\footnote{Note that XCAs do provide directional information if transient wave groups are investigated that are well separated in time, (Baldock, 2006), however, present data are continuous by nature.} They form the centerpiece of the cross correlation analysis, since the phase relation between the envelope of the SW signal offshore and the observed LF waves at instrument locations inner-offshore has the primary purpose of uncovering the generation mechanism responsible for surf beat.

\[
|A(t)| = \sqrt{\eta_{SW}^2(t) + i \Gamma \{ \eta_{SW}^2(t) \}^{LF}} \tag{3.3}
\]

The burst spectra are considered to be narrow banded, therefore Equation 3.3 can be used to approximate the envelope $A(t|m)$ of the short wave group. $\Gamma\{}$ denotes the Hilbert transform operator\footnote{With aid of the Inverse Discrete Fourier Transform} and $\{\}$ the amplitude of the complex function. Initial concerns regarding the reliability of the low-pass filter, connoted with superscript $LF$ in Equation 3.3, were cleared up when tests consistently showed stronger correlations for an active low-pass filter.

Finally, the cross-correlation function $R_{xy}(\tau)$ itself, is formulated analogous to Baldock and Huntley (2002); Janssen et al. (2003); Lara et al. (2011) as:

\[
R_{xy}(\tau) = \frac{1}{N} \hat{R}_{xy} \quad \text{with} \quad \hat{R}_{xy}(\tau) = \begin{cases} 
\sum_{n=0}^{N-\tau-1} x_{n+\tau} y_{n} & \tau \geq 0 \\
\hat{R}_{xy}^*(-\tau) & \tau < 0
\end{cases} \tag{3.4}
\]

Where $x$ and $y$ are two jointly stationary random processes. For a certain, discrete length $N$, the term $\frac{1}{N} \hat{R}_{xy}$ in Equation 3.4, resembles the time-averaged, raw correlation between the two signals. The latter is normalized by means of the product of the standard deviations $\sigma_x \sigma_y$, making sure that $-1 < R_{xy} < 1$. Time-shifting signal $x$ against signal $y$ with varying lags $\tau$, reveals their relation in time. Note that the asterisks denote the complex conjugate, but have only been added for the sake of completeness, since $x$ and $y$ describe instantaneous surface elevations in $[m]$ and are thus solely real valued throughout the analysis.

### 3.4.2. Relation between Frequencies and Forcing Mechanism

**Method I**

The evolution of LF energy over the reef flat gives an impression of dissipation and maybe energy transfer between HF and LF components but is insufficient to couple low frequencies to their origin, i.e. genera-
tion mechanism. Inspection of the wave height growth $H \propto h^{-\alpha}$ during shoaling is one way to determine whether LF waves are free or bound (Battjes et al., 2004; Van Dongeren et al., 2007) and can thereby help to tie frequency ranges to a generation mechanism, for details see subsection B.0.6. The flaw of this technique is the necessity to know wave directions, which mentioned earlier is not an option here. Baldock and Huntley (2002) found a suitable way to investigate the generation mechanism by yielding power spectra of the mixed wave field. Their analysis is rooted in the fact that bound long wave energy depends quadratically on the offshore short wave power $H_{SW}^2$, whereas the whole theory around breakpoint forced, free waves leans on a linear relationship, (Symonds et al., 1982). In their lab experiments Baldock and Huntley (2002) create comparable wave groups for three energy levels $H_{rms}$. Examination of the energy of a specific low frequency normalized by the squared target wave height $H_{rms}^2$ then reveals its bound or free nature. This analysis technique is adopted in the current study for an exploration on the mechanism behind dominant wave frequencies. The challenge lies in the selection of more and less energetic power spectra that share the same wave group properties. Supplementary to the presentation of the results in chapter 4 further details will be provided on (amongst others) the composition of the filter.

**Method II**

Symonds et al. (1982) state that the amplitude of the forced wave is proportional to the groupiness $G$, also called modulation, of the incident waves (see also Equation 2.2). In other words, the amplitude of a breakpoint forced wave can be coupled to the amplitude of the short wave group envelope. The Hilbert transformation of the burst signal, Equation 3.3, has earlier been explained to admeasure the short wave envelopes for a given burst. The established envelope time series can be analyzed from a frequency point of view by preparing a spectrum of which every element denotes a sine that fits one type of wave group. For current purpose not the power spectrum, but the amplitude spectrum is used as it makes physically more sense. Moreover it is readily fit to compare with amplitude changes of lower frequencies across the reef.

### 3.4.3. Surfbeat classification of ROI-Namur

Findings from chapter 4 are compared against suggestions by the normalized bed slope $\beta$ and the surf beat similarity parameter $\xi$. The indicators brand subharmonic waves with a certain stereotype, e.g. originating from the breakpoint mechanism or bound long wave release.

Though it is straightforward to distinguish the apparent generation mechanism with such parameters, the exact hydrodynamics are largely unknown. The governing forcing frequencies, the amount of reflection and dissipation have to be clarified separately. Furthermore, phenomena are difficult to be singled out from the puzzle of possibilities mentioned in literature and no general manual exists that gives aid in doing so. In order to get an organized picture, key elements that are likely involved in surfbeat generation have been mapped by means of the deductive flow chart presented by Figure A.1 in the appendix. Its purpose is to stepwise resolve into immediate and necessary events that lead to surfbeat until basic causes are identified. Logic symbols are according to convention. Important literature sources are affixed to the event boxes.

---

7 Note, that the Hilbert transform operator represents the frequency shifted variance density spectrum of the original wave signal by definition (Kirby, 1993) and its properties are in Equation 3.3 transformed into the time domain, which shows that current procedure actually takes a step back.

8 [http://www.hq.nasa.gov/office/codeq/risk/docs/ftacourse.pdf](http://www.hq.nasa.gov/office/codeq/risk/docs/ftacourse.pdf) (last consulted on 18.08.15)
Based on Quataert (2015), underestimated run-up heights in 1D-XBeach simulations of Roi-Namur in combination with lacking knowledge on wave hydrodynamics encourage a deeper analysis on the wave physics. This chapter presents the results of several analysis techniques that have been introduced in section 3.4.

Briefly summarizing the present state of research the following can be concluded:

- The growth of LF amplitudes was stated to be enhanced by a smooth reef flat, due to reduced energy dissipation by bottom friction. Yet, the conditions or mechanisms that lead to the growth itself have not been investigated. Discovery of the event-chain preceding ultimate wave run-up could be the key to understand model issues.

- What is more, wave reflection and resonance at Kwajalein atoll have not been considered until now. Indeed, resonance is one of the focal points since it could explain long wave amplification on the reef flat. It has been recognized before by Péquignet et al. (2009) in aftermath of storm Man-Yi at Ipan Reef, Guam.

A final remark is made concerning the used data. The entire analysis again purely relies on measurements at one transect, which is the Western transect depicted in Figure 3.2 and Figure 3.3. It was also used in the previous study of (Quataert, 2015). Fore reef data are crucial for this research and were not accessible for the Eastern transect, as that pressure sensor was lost during the field experiment. A single analysis utilizes AWAC data and is the first to be treated in section 4.1. It investigates the amount of bound long wave energy offshore to study long wave reflection. Subsequently, bursts classes are defined in section 4.2 as preparation for the cross correlation of wave signals in section 4.3. The latter can be presented graphically, which aids in tracing low frequency transformation across the reef. A closer analysis on the IG wave generation mechanism and its effectiveness follows in section 4.4. The chapter is finalized with a discussion and conclusion in section 4.5 and section 4.6.
4.1. **BOUND VERSUS FREE LONG OFFSHORE WAVES**

AWAC data are used to unravel the mix of bound and free long waves offshore in order to see which type is dominant. Under the assumption that incoming low frequency waves are bound, the presence of free waves would imply that long waves reflect at the coastline or emanate from a moving breaking point.

![November event](image-a) ![March event](image-b)

**Figure 4.1:** Top photographs are taken during the events in November 2013 and March 2014. Below figures show theoretical BLW energy (red stars) and measured LF spectra (black crosses) on logarithmic scale.

Figure 4.1 exhibits the relation between the measured low frequency spectrum and the theoretical offshore bound long wave (BLW) energy computed using Equation 2.1 (section 2.4). Subfigures 4.1c and 4.1d show the five hour average around the peak wave heights encountered on 17-Nov-2013, photo 4.1a and 02-March-2014, photo 4.1b. The advanced method by Van Dongeren et al. (2003) approaches the real spectrum with errors of about one order (November) or even less (March). Discrepancies in this range have been noted earlier by Herbers et al. (1994) and are not necessarily linked to a poor performance of the formula, but rather to the fact that bound long waves could be submerged in a background of free long waves. This is indeed considered likely for Roi-Namur, too, i.e. offshore directed free long waves dominate over shoreward directed bound long waves. In the discussion of section 4.5 it will be shown that by looking at the inclination of the fore reef the breakpoint mechanism can be anticipated. The latter provokes free long waves into both cross shore directions. Furthermore the steep slope gives reason to consider reflection to being an important source for free waves.

The calculation procedure is not optimal. The basic assumption of a 180° phase shift between SW envelope and IG waves might not be entirely true for the location of the AWAC at 20 m depth. Furthermore, directional spread in the incident wave spectrum has been neglected, compressing all the energy into one, mean wave
direction. However, this should over- instead of underestimate the offshore bound long wave energy. Since
the magnitude of computed BLW energy is still lower than the observed, integrated LF energy, the method is
accepted to give at least a rough idea about the weights of free against bound wave components. Especially
because the measured power spectrum shape is well approximated. In conclusion, these findings suggest
significant free long wave energy to be present seaward of the breakpoint.

4.2. **Classification of water level bursts**

Six months of hourly water level bursts caught various wave conditions present at Roi-Namur during that
period. The challenge is to find out what distinguishes some bursts causing severe wave run-up from others.
Assessing this problem directly with a cross correlation analysis of each separate burst would be very time-
consuming and difficult, as it is not known what signs to look for. Thus more information on hydrodynamics
is necessary to start with.

Each burst is ranked according to $H_s$, $T_p$ and tide and criteria are formulated that can be used to define burst
classes with a typical low frequency response. Finally a structured cross correlation analysis is possible, that
is based on the average of a class rather than one single time series and gives visual aid for understanding.
Figure 4.2: Representation of all samplings according to main hydrodynamic properties. Axes vary in combinations of peak period $T_p$, significant wave height $H_s$ and tide. The color scheme visualizes the LF wave height ranging from 0.1 – 0.6 m. Left hand subfigures illustrate LF heights as measured at the outer reef flat and right hand subfigures as measured on the inner reef flat.
Figure 4.2 arranges the complete sample of bursts according to significant wave height $H_{s,\text{fore}}$, peak period $T_{p,\text{fore}}$ and tidal elevation. The fore reef is chosen as reference for incident wave conditions as the pressure sensor is positioned in line of the transect. Additionally, a color bar denotes the burst ranking with respect to LF energy $H_{s,\text{LF,outer}}$ measured at the outer reef flat (left) and $H_{s,\text{LF,inner}}$ at the inner reef flat (right). Subfigures 4.2a and 4.2b give the full 3D representation and Subfigures 4.2c-4.2h the associated projections.

Considering the low frequency response at Roi-Namur, the two top figures highlight a clear transition from the outer- to the inner reef flat. On the outer reef flat, LF energy has a unique bond with the magnitude of incoming short waves, independent of tidal level and SW peak period $T_{p,\text{fore}}$. The prominent dependency with $H_{s,\text{fore}}$ becomes more lucid by viewing projections 4.2c and 4.2g. The former shows a clear vertical structure that rules out any influence of the tide on LF energy at the outer reef flat. This vertical structure is practically conserved in projection 4.2g, which demonstrates that the peak period $T_{p,\text{fore}}$ has negligible impact compared to $H_{s,\text{fore}}$. Note that high energetic wave events with peak periods below $T_p < 13$ s have not been measured, therefore the projection in Subfigure 4.2e gives the false impression of a dependency with the peak period.

Shoreward on the inner reef flat, Subfigure 4.2b, LF waves suddenly react to the offshore peak period $T_p$ and the tidal elevation. Subfigure 4.2d generally shows that long wave amplitudes grow during high tide conditions, as the red cloud moves to the top right and dissipate during low tide conditions, as the blue mass stratifies to the bottom left. The tidal dependency gradually disappears for higher $H_{s,\text{fore}}$, which is argued to be the consequence of relatively stronger wave breaking. It is conceivable that the resulting increase in wave induced set-up submerges the reef flat to a degree where low frequencies feel a high tide environment. The metamorphosis of LF energy from Subfigure 4.2e to the inner reef flat Subfigure 4.2f occurs analogous to the $H_{s,\text{fore}}$ plots, i.e. for very long periods the tide becomes less important. Subfigure 4.2h shows a top view of Subfigure 4.2b. Grouping of red colors in the region above $T_p = 12$ s, $H_s = 1.5$ m and during high tide, indicates that these conditions generate strong surf beat motion. Especially the case of very long periods above $T_p = 15$ s in combination with high short wave energy becomes an incentive for LF growth, as surf beat gets persistent, i.e. tidal influence reduces.

![Figure 4.3](image-url)

Figure 4.3: Evolution of power spectra from the fore reef (dashed red line) to the inner reef flat. The color scheme of copper colors for the reef flat spectra stepwise become lighter closer to shore. Subfigures show an average of bursts covering the whole tidal range with a tidal mean at MSL. Moreover bursts are filtered to achieve a comparable peakness and incident wave energy $H_{s,\text{fore}}$ for bursts with $T_p > 15$ s depicted by Subfigure 4.3a and bursts with $T_p < 12$ s, Subfigure 4.3b.

An unwanted hitch in the apparent dependency of LF energy to $T_p$ is an over all higher spectral peakness of events with lower peak frequencies. A couple of bursts with similar spectral peakness and incident wave energy $H_{s,\text{fore}}$ are selected for the two separate situations of peak periods $T_p < 12$ s, Subfigure 4.3a and $T_p > 15$
s, Subfigure 4.3b. Each of the figures depicts the average of the selected bursts. The dashed red line resembles the fore reef spectrum, whereas curves with copper colors represent the spectra at the outer (dark) to inner (light) reef flat. Comparison of the two subfigures reveals that substantial VLF growth is only observed during conditions with very long period swells. This can be derived from the LF peak at the inner reef flat (yellow) that only develops in Subfigure 4.3b. The observation is consistent with Subfigures 4.2b and 4.2h.

By ruling out the potential influence of the peakness $\gamma$, it is demonstrated that the incoming wave period is a standalone factor that modulates LF development over the reef flat. An additional contribution by the peakness itself is not further investigated at this point. Previous findings could imply that Roi-Namur is sensitive to relatively long incoming swell waves. Under these circumstances, Figure 4.3 suggests that particularly VLFs form the threat as they show the largest growth over the reef flat. Note that both the November and the March event are included in the dark red cloud of Figure 4.2b. Enhanced run up and overwash observed during those events thus coincide with surf beat motions that are generally strong for the burst class with high peak periods and significant wave heights above $H_s > 1.5$ m.

At this point, different wave classes can be formulated that will be focused on in the XCA. The first class CAT1 includes bursts with $T_p < 12$ s and incoming wave heights $1.75 \, m < H_{s,\text{fore}} < 2.75 \, m$. In comparison, the only parameter that is adjusted in the second class CAT2 is the peak period $T_p > 15$ s, which induces a stronger LF response on the reef flat.

Figure 4.4a illustrates the classes, where CAT1 is yellow and CAT2 is orange. The upper wave height restriction is chosen in order to minimize correlation enhancement by extreme events, that have only been recorded for $T_p > 15$ s. Still, in general solely higher energetic events are investigated. Research on CAT1 and CAT2 is furthermore subdivided into: High and low tide situations and the frequency band that is considered. In that context, distinguishing VLF and IG frequencies might not be optimal for the XCA. Based on overall trends in the experimental data, an alternative split frequency is chosen.

Figure 4.4b marks in green, situations with prominent wind-sea in the signal.

Figure 4.5 illustrates the increase of LF energy over the reef flat, averaged over the entire survey period. The net growth starting at the outer reef is visualized by subtraction of the fore reef spectrum from the power spectra belonging to the outer to inner reef flat. E.g. the net LF increase from fore to mid is quantified by means of $\Delta E_{l,f,\text{mid}} = E_{l,f,\text{mid}} - E_{l,f,\text{fore}}$. Higher order regression lines are fitted to the curves to highlight LF amplification as waves propagate shoreward. Persistently, a process exists that feeds the lowest frequencies with energy at a growth rate that varies between burst classes (Figure 4.3). Judging from Figure 4.5, it is
better to distinguish the frequency bands $f_{ll} = 0.001 - 0.01$ Hz and $f_{lh} = 0.01 - 0.04$ Hz instead of VLF and IG waves. The decision is based on the fact that net energy differences between fore reef and inner reef flat $\Delta E_{lf, fore-inner}$ are greatest in the range $f_{ll} = 0.001 - 0.01$ Hz. Subscripts $ll$ and $lh$ crudely stand for "low low" and "low high" respectively.

A separation into different types of waves i.e. wind sea or swell, has not been taken account of in the definition of the classes. Theoretically, the shape or steepness of waves have influence on the type and strength of surfbeat, as explained in chapter 3. Therefore several methods described in Hwang (2012), including a promising and recently proposed Spectrum Integration Method, have been tried in order to separate sea and swell in each hourly timestack. Unfortunately the relatively low cut-off frequency at 0.2 Hz hampers the performance of those methods. In the end, for each burst the spectral energy above $f_p = 0.08$ Hz is weighted against the spectral part below that value. This method presumably detects bursts with more sea in the wave field relative to others (see green cloud in Subfigure 4.4b), and shows that wind driven waves are commonly low i.e. the assumption of swell domination at Roi-Namur is justified.

### 4.3. Cross correlation results

The upcoming paragraphs distinguish between three different correlation categories. The first category covers $R_{AA}(\tau, x_i; x_r = fore)$, resembling the cross correlation $R$ at lag $\tau$, between the short wave envelope $A$ at the fore reef $x_r = fore$ (reference envelope), and the envelopes measured at the remaining gauge locations, $x_i = outer$, $mid$, $inner reef flat$. Previous notation also holds for the second category $R_{\eta\eta}$, where $\eta$ denotes the LF surface elevation. In the cross correlation analysis (XCA), the surface elevation is filtered to create two separate long wave categories. $\eta_{ll}$ resembles the filtered time series that contains frequencies $f_{ll} = 0.001 - 0.01$ Hz. $\eta_{lh}$ consists of wave harmonics in the band $f_{lh} = 0.01 - 0.04$ Hz. $R_{\eta\eta}(\tau, x_i; x_r = fore)$ also correlates a fore reef signal with the signals measured at the reef flat stations, yet never correlates wave fields that are filtered differently. The third category $R_{\eta A}(\tau, x_i; x_r = x_i)$ cross correlates the short wave group envelopes with the collocated long wave fields $\eta_{ll}$ and $\eta_{lh}$ separately.

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1 According to the Hwang (2012), the Spectral Integration Method looses accuracy below 0.5 Hz and is therefore not suitable for this study.
4. INFRAGRAVITY AND VERY LOW FREQUENCY WAVE PROCESSES AT KWAJALEIN ATOLL

(a) CAT1 high tide. \( \langle H_s, SW \rangle = 2m, \langle T_p \rangle = 10.5s, \langle \eta_{tide} \rangle = 0.65 \)

(b) CAT1 low tide. \( \langle H_s, SW \rangle = 2m, \langle T_p \rangle = 10.5s, \langle \eta_{tide} \rangle = -0.65 \)

(c) CAT2 high tide. \( \langle H_s, SW \rangle = 2m, \langle T_p \rangle = 15.5s, \langle \eta_{tide} \rangle = 0.5 \)

(d) CAT2 low tide. \( \langle H_s, SW \rangle = 2m, \langle T_p \rangle = 15.5s, \langle \eta_{tide} \rangle = -0.6 \)

Figure 4.6: Cross correlations \( R_{AA}(\tau, x_i; x_r = \text{fore}) \) of time series coinciding with CAT1 and CAT2 for high- and low tide conditions. Each subfigure contains a dashed black line, that exemplifies the theoretical group speed trajectory obtained by integration of the bathymetry profile. Next to it, a pink line is plotted that resembles a correction for nonlinear wave shapes created at the edge of the reef flat.

\( R_{AA} \) is established by lowpass filtering Equation 3.3 for the entire LF range \( f = 0.001 - 0.04 \) Hz. Red colors denote positive- and blue colors negative correlations. The lowest bar in each panel shows the autocorrelation of the fore reef wave-group envelope. Furthermore, all subfigures have been complemented with a dashed black line, that marks the theoretical path of the wave groups and by that the short wave envelope \( A \). The inverse V-shape is able to trace reflected waves, too.

A representative group velocity \( c_{g,fp} \), based on the peak frequency \( f_p \), is supposed to give a good approximation for the wave trajectory, \( \text{(Janssen et al., 2003)} \). \( f_p \) is drawn from the super smoothed power spectrum on the fore reef \( f_{p, \text{fore}} = f_{p, \text{fore}} \) and is assumed to stay constant while moving shoreward, which should be accurate as the resolved frequencies all fall under shallow water conditions on the reef flat\(^2\). Starting at the outer reef flat, wave speeds based on linear wave theory underestimate the real wave propagation. The latter is indicated by red bars of positive correlation, that are arranged in a typically progressive way. The lag could occur

\(^2\)To be certain \( c_{g,fp} \) was in a test case updated twice, with the known \( f_{p, \text{outer}} \) and \( f_{p, \text{mid}} \), however, the resulting wave trajectory was identical, as expected.
4.3. Cross correlation results

due to the fact that linear theory falls short on waves with high steepnesses that are typically nonlinear. An investigation of wave heights and wave numbers on the outer reef flat reveals strong nonlinearity associated with higher wave celerities. Therefore a correction is made to the linear wave trajectory:

At all reef flat positions Ursell numbers (a measure for non-linearity) are consistently above 40. Computation-wise this means that Cnoidal theory works best, (Fenton, 1990). Equation 4.1 forms the basis for the corrected wave path:

\[
\frac{\bar{U}}{\sqrt{gh}} = 1 + \frac{H}{h} \left( \frac{1}{2} - e \right) + \left( \frac{H}{h} \right)^2 \left( \frac{3}{20} + \frac{5}{12} e \right) + \left( \frac{H}{h} \right)^3 \left( \frac{3}{656} - \frac{19}{6000} e \right) + \left( \frac{H}{h} \right)^4 \left( -\frac{309}{5600} + \frac{3719}{21000} e \right) + \left( \frac{H}{h} \right)^5 \left( \frac{12237}{616000} - \frac{997699}{8820000} e \right)
\]

(4.1)

where

\[e(m = 1) \approx \frac{1}{3}(1 - \pi^2)\]

Equation 4.1\(^4\) solves for a correction factor \(F_{\text{soliton}} = \frac{\bar{U}}{\sqrt{gh}}\), where \(c_{\text{nonlinear}} = \bar{u}_1 + \bar{U}\). The time-mean horizontal fluid velocity at any point is denoted by \(\bar{u}_1\) and \(\bar{U}\) is the mean horizontal fluid velocity over one wave length. No current meters have been deployed on the reef flat and \(\bar{u}_1\) is simply imposed zero. Thereby the nonlinear correction factor becomes \(F_{\text{soliton}} = c_{\text{nonlinear}} / c_{\text{linear}}\). Wave heights of the solitary waves are approximated with \(H_s,SW\) and together with the water depth \(h\) updated at each reef station. Linear interpolation finally completes the calculation procedure, which yields the pink trajectories visible in Figure 4.6. They abut the data quite convincingly.

The dependency of wave group speed on varying water depths is excellently pictured by both, the theoretical wave group trajectory as well as the cross-correlation bars themselves. Above all, low tide bursts are on average correlated slightly higher. Tidally dependent wave breaking described by Quataert (2015) is suggested to intensify the creation of solitary waves during low tide conditions, which would also explain why outer reef flat correlations are stronger. Subfigures 4.6c and 4.6d provide overall stronger correlations for CAT2. It is plausible that enhancement of low frequency waves in case of longer \(T_p\) offer a better platform for solitary wave trains with some group structure. Particularly in Subfigure 4.6d dispersion of those solitary waves makes the red patches gradually swell up. Rounding off, none of the plots show noticeable signs of grouped short wave reflection.

\(R_{\eta\eta;fh} = 0.01 - 0.04\ Hz\) As described at the start of this section, \(R_{\eta\eta}\) embodies the correlation of LF waves between stations. In this case the filtered signals of the frequency band 0.01 – 0.04 Hz are correlated. Figure 4.7 supplies first visual proof of the breakpoint mechanism. The positive bar at the fore reef, see Subfigure 4.7a, seems to split up into two parts of opposite correlation. A positive, red correlation stands for a wave trough and correspondingly a blue color highlights a wave crest. The blue crest signifies dynamic set-up. While wave groups shoal intensively due to the steep bathymetry, a leading long wave crest develops as a result of asymmetry in the radiation stresses. This effect can be found in the outmost right branch of the flow chart, Figure A.1. With commencement of short wave breaking, it starts receiving energy from the rhythmic pulse of a moving breakpoint, (Baldock, 2006). While dynamic setup leads the theoretical wave trajectory, the corresponding trailing trough lags behind it. At the outer reef flat location, this lag is ascribed to the phase shift that built up during shoaling between bound long waves and primary short wave groups, (Janssen et al., 2003). It is plausible because the destruction of wave groups is yet not completed at the outer reef flat, meaning that bound long waves are still forced to some degree. Although the trailing red patch on the outer reef flat is partly a remainder of the incident bound long wave, there is probably another effect responsible for

\(^3\)(Fenton, 1990) suggests to simplify the sharply convergent elliptic functions by taking the modulus \(m = 1\), which results in Equation 4.1

\(^4\)Full description for variable modulus \(m\) can be found in Table 4-1 and equation 4.6 of Fenton (1990)
it to persist over the entire reef flat. A matured return current comes forth from shallow water depths on the reef flat. Consequently large shear stresses arise when a dynamically forced wave travels against the return current and the water surface is sucked down, creating wave troughs on both sides of the crest. Note that the periodical relief of the forcing i.e. the natural shape of the incoming long waves, will also contribute to a symmetric trio of bars on the outer reef flat, that progresses with opposite correlations.

Baldock (2006) and Lara et al. (2011) focused on an experiment with transient wave groups. No reef flat is present in the flume. They too, found a trio of troughs with a crest in the center, however solely for the reflected signal. Moreover the enclosing troughs don’t develop as an effect of strong shear stresses in the water column, but represent breakpoint generated outgoing free long waves (OFLW) for the leading- and bound long wave for the lagging depression respectively. Besides the dynamic set-up, their observations are therefore not comparable to this study.

Following the wave trajectory in Subfigure 4.7a reveals partial reflection of LF energy back to the fore reef. It is difficult to imagine that troughs forced by an opposing return current will survive when they are reflected. Ergo, pale red and yellow bars of the reflected signal might be associated with rhythmic run down in the swash
zone. (Baldock and Huntley, 2002). It would also agree with wave crests (blue) almost entirely disappearing at the shoreline. Wave energy is not reflected but rather unloaded on the beach in the form of pulsating wave run-up. Time shifted run-down is thus the only way to regain conservation of mass.

Reflection of LF energy is substantiated by the "V"-pattern that unfolds from the center of the fore reef autocorrelation band. Its left branch goes parallel with the path investigated just before. The difference is that the former results from correlation of the reflected signal at the fore reef with the reflected signals at the other stations, whereas the latter is the correlation of the ingoing pulse at the fore reef with the reflected signals at the remaining stations.5

Continuing with Subfigure 4.7b any form of correlation is absent on the mid and inner reef flat during low tide. This happens simply because the reef flat is mostly dry. CAT2 Subfigure 4.7c is affiliated with the high tide results of CAT1 and copies most of the behavior, although higher correlations close to the shoreline insinuate heavier wave attack. Bottom right Subfigure 4.7d reveals no big changes compared to its CAT1 counterpart, in spite of somewhat higher correlations that can be related to a sporadically submerged reef flat stemming from a slightly higher low-tide average.

Figure 4.8: Cross correlation of frequencies in the range $f_l = 0.001 - 0.01 \text{ Hz}$. With $R_{xy}(\tau, x_i; x_f = \text{ fore})$.

5For details one is referred to Figure 6. in Janssen et al. (2003)
$R_{\eta A}; f_{lh} = 0.001 - 0.01 Hz$  XCA results of the lowest frequencies share many of the corresponding higher frequency features described in foregoing paragraph. Consequently focus lies on the differences. Note that the bottom axis range is doubled for figures including $ll$ frequencies. Lagging depressions Subfigure 4.8a are less paramount as a consequence of the naturally long periods that induce weaker shear stresses with the return current. In the entire XCA collection of Figure 4.8 these lagging troughs often disappear at the mid reef flat. The leading trough on the other hand is greatly enhanced toward shore, which conforms a bore-like shape of the $ll$ waves. It grows so significantly that it doesn’t vanish when it reflects from the shoreline and travels in direction of the return current. This can be recognized from the paler blue color that belongs to the wave crest on the mid reef. It crosses the path of the supposedly reflected leading trough. Thus reflection also occurs for $ll$ frequencies but is less obvious due to relatively broad correlation patches. All in all, dynamic set-up in combination with high tide shows no loss of correlation as waves approach the coastline, which is a strong indication for a forcing that is able to balance or even outweigh dissipation on the reef flat.

CAT2 Subfigure 4.8c again describes clearer wave shapes with conjointly higher correlations. Besides, especially longer peak periods with $T_p > 15$ s seem to enforce standing waves on the fore reef, where a strongly barred auto-correlation pattern is visible. Low tide situations depicted by Subfigures 4.8b and 4.8d are dominated by dynamically forced set-up and troughs are weaker, as vertical space is limited. It is worth noticing that Subfigure 4.8d partially shows a checkerboard pattern. It is hypothesized that this graphical feature could be linked to standing waves.

![Figure 4.9: Cross correlation of frequencies in the range $f_{lh} = 0.01 - 0.04 Hz$.](image)

$R_{\eta A}; f_{lh} = 0.01 - 0.04 Hz$  Figure 4.9 depicts results of cross correlating local envelopes with conjoint $lh$ waves, $R_{\eta A}(\tau, x_i; x_r = x_l)$. CAT1 and CAT2 plots are almost identical, which is why solely CAT2 is treated in the follow up. A clear switch from a negative to positive relation between fore- and outer reef once more substantiates existing notion of a dominant breakpoint mechanism. The additional value of this figure is the information it provides on the phase relation between short wave envelopes and LF waves, in this case mid to higher IG frequencies. Blue bar on the for reef has a slight lag which is consistent with previous suggestion of a phase shift forced by a steep bathymetry slope during shoaling and thereby rules out the possibility of dominant local forcing typically associated with steep waves, see central brach of Figure A.1. Thus incoming wave groups are not excessively nonlinear. Furthermore displaced red patches at the mid and inner reef flat confirm the existence of dispersive solitons, however, also give reason to consider shallow water wave speeds slower than $c = \sqrt{gh}$ as a result of friction. (Pomeroy et al., 2012). High correlations of Subfigure 4.9b moreover confirm that low tide situations are defined by intensified wave breaking. Surprisingly little evidence can be found on OFLWs directly radiating from the moving breakpoint. One is inclined to think that the incoming wave crest on the trailing edge mostly cancels out with the breakpoint generated wave trough, leaving green space to the right of the blue correlation bar in 4.9a and a vaguely recognizable low in the red bar of 4.9b.
4.3. Cross correlation results

(a) CAT1 high tide. \( h_{mid} = 1.20 \) m

(b) CAT1 low tide. \( h_{mid} = 0.26 \) m

(c) CAT2 high tide. \( h_{mid} = 1.13 \) m

(d) CAT2 low tide. \( h_{mid} = 0.35 \) m

Figure 4.10: Cross correlation of frequencies in the range \( f_{ll} = 0.001 - 0.01 \) Hz. With \( R_{\eta A}(\tau, x_i; x_r = x_l) \).

**Figure 4.10 finalizes the XCA with a look at the lowest frequencies and their relation to the local short wave envelope.** Comparing Subfigures 4.10a and 4.10c a great difference in correlation jumps to the eye. Judging from Figure 4.6 and Figure 4.8, a boost in soliton creation as well as dynamic set-up combines to a more forceful appearance of CAT2 compared to CAT1. Differences are less obvious for low tide Subfigures 4.10b and 4.10d. The main information they provide is a clear lag developing at the mid reef flat. Large wave lengths and shallow water depths now provide perfect conditions for friction to act as a break, slowing down long waves and thereby increasing the phase shift with short waves, which on their turn tend to move to the bore front, (Van Dongeren et al., 2007). The slowing down of \( ll \) waves could already be observed earlier in Figure 4.8, where blue patches lag the shallow water wave trajectory. Generally, correlations of CAT2 are much stronger than CAT1 at the lowest frequencies presented here. In the preceding paragraph, differences between CAT2 and CAT1 were negligible i.e. for frequencies in the range \( f_{ll} = 0.001 - 0.01 \) Hz, which implies that changes in the incoming swell periods mainly influence VLF and long IG wave behavior.

The cross correlation analysis has shown clear signs of the breakpoint mechanism and confirms the relationship between long period swells and strong VLF response. Yet it only gives a qualitative picture of wave transformation over the reef platform. The following sections present the outcomes of additional data analyses that help to quantify some of the hydrodynamic phenomena and elicit further information on the breakpoint mechanism from the data.
4.4. Effectiveness of the breakpoint mechanism

The XCA demonstrates that the moving breakpoint is the dominant LF generation mechanism at Roi-Namur. In this section a general analysis on the behavior of the breakpoint mechanism is conducted. Moreover its effectiveness to generate IG waves at varying wave frequencies is investigated by looking at the relation between breakpoint excursion and the nodal structure of potentially standing IG components.

The approach of Figure 4.12 mimics the laboratory study conducted by Baldock and Huntley (2002) and presents a relationship between frequency and generation mechanism. The principle is described in chapter 3 as Method I. In short, fore reef bursts are collected that have comparable properties at different energy levels $H_s$. The evolution of IG waves across the reef during those bursts is analyzed. Normalizing the LF wave energy by the total incident wave energy $H_s^2$ reveals their relation. If LF waves are bound the relation is quadratic and if waves are breakpoint generated, the relation is linear, (Baldock and Huntley, 2002; Symonds et al., 1982). The former is unlikely as the breakpoint mechanism leaves its mark on the reef platform in the XCA plots (e.g. Figure 4.7 and Figure 4.8). Moreover, the comparison of theoretical bound long wave energy and measured LF energy at the location of the AWAC indicates that free waves also dominate the LF domain offshore (Figure 4.1). As a consequence, the analysis mainly shows changes in the effectiveness of the breakpoint mechanism.

Low frequency energy has been normalized by the squared SW energy on the fore reef instead of the offshore AWAC in order to exclude refraction effects from the results. The quantities show an average of 133 sets of each three bursts with $H_{s,SW} = 1$ m, $H_{s,SW} = 1.5$ m, $H_{s,SW} = 2$ m. Sets of bursts have been selected based on the former three bulk values $H_{s,SW}$ that subsequently were filtered for a comparable peak frequency $f_p$, spectrum shape, peakness and low frequency distribution. Note that the effect of tidal elevation is neglected. Figure 4.11 depicts the collection of bursts and the averages that form the basis for the analysis.

![Figure 4.11](image1.png)

Figure 4.11: Average of 133 selected sets of 3 bursts with $H_{s,SW} = 1$ m, $H_{s,SW} = 1.5$ m, $H_{s,SW} = 2$ m respectively. Colors are analogous to use in Figure 4.12.

Figure 4.12 presents the results. The vertical axis measures the normalized energy for bursts with $H_{s,SW} = 1$ m, $H_{s,SW} = 1.5$ m, $H_{s,SW} = 2$ m on logarithmic scale. Instrument positions are shown on the bottom axis. Across the transect the lines with a low central frequency of $f_c = 0.006$ Hz, Subfigure 4.12a, stay bundled. This is typical for an overall linear response to the incoming short wave energy $H_{s,SW}$, as expected. Also at the fore reef, free waves are dominant. What is more, normalized energy curves are ordered upside down on the reef flat, i.e. from high incoming wave energy (light blue) at the bottom to small $H_{s,SW}$ (dark blue) at the top. This can be interpreted as more efficient VLF and low IG wave generation for calmer wave conditions, which is a new discovery. In terms of the wave amplitude, minor changes between the outer- and mid reef flat are followed by a significant increase towards shore which has readily been noticed by Quataert (2015) and in section 4.2.
Figure 4.12: Energy around increasing frequencies $f_c \pm 0.0049$ Hz of the low frequency band, normalized by and dependent on varying $H_s$. Colors are according to legend.

Péquignet et al. (2009) mentioned the involvement of resonance. If the Roi-Namur reef flat is idealized as a step shelf with a length of 270m, fundamental resonant periods are calculated to lie between 0.0012 − 0.0034 Hz for the used burst sets and thus are completely embedded in the frequency band around $f_c = 0.006$ Hz represented by subfigure 4.12a. Given the fact that Péquignet et al. (2009) prove resonant behavior at a reef, presents a clear warning sign that Roi-Namur might be sensitive to resonant VLFs. In that respect, the reef flat resembles the triggering feature of the beach profile as e.g. Symonds et al. (1982) recognize that resonance is absent in their study case of a steeply sloped bathymetry without a reef flat. Note that earlier hypothesis that a checkerboard pattern in Subfigure 4.8d might represent standing waves is strengthened by this calculation of fundamental resonant periods.

Subfigure 4.12b for $f_c = 0.016$ Hz still displays overall linear LF response. Again, the proposition of significant free wave dynamics on the fore reef is reinforced. Nothing is known about the absolute amount of energy stored in outgoing free long waves (OFLWs) an their correlated frequencies in particular. Baldock et al. (2000) find that energy is concentrated at $f_{OFLW,\text{max}} \approx 2h_x$ which would imply $f_{OFLW,\text{max}} \approx 0.11$ Hz at the Roi-Namur fore reef, yet can't be determined with hard evidence.

Bottom Subfigures 4.12c and 4.12d representing the intermediate and upper IG range have a large spread, i.e. inverse ordering of the three energy levels gets more extreme. As mentioned earlier, the pattern translates into
a decreasing breakpoint efficiency for higher incident wave heights. Current analysis is susceptible to errors, but assuming the phenomenon has a physical background an explanation may be found in the cancellation of standing waves. In particular since it explained similar patterns in affiliated figures of Baldock and Huntley (2002). chapter 2 comprehensively describes the potential interference of the breakpoint excursion with the nodal structure of standing wave components, (Baldock and Huntley, 2002). In short: If the length of the average breakpoint excursion is larger than the distance between a node and an antinode of a free standing wave with frequency $f_{LF}$, the breakpoint mechanism is less effective at producing this particular frequency.

Equation 2.2 developed by Baldock et al. (2000), is a check to determine whether a certain harmonic could be subjected to this effect, that typically acts on higher IG frequencies and prevents the development of a standing character. The original equation is reorganized and the sign reversed to find the frequency threshold:

$$f_{th} > \frac{\alpha h_{x,fore} \sqrt{g}}{G \sqrt{H_{rms,SW,fore}}}$$ (4.2)

$f_{th}$ represents a standing wave component on the verge of being cancelled. The groupiness factor $G$ retains the definition of Equation 2.2. Other parameters are the fore reef slope $h_{x,fore} = 1:18$, gravitational acceleration $g$ and the root mean squared short wave height $H_{rms,SW}$ seaward of the breakpoint, approximated with $H_{rms,SW,fore}$. Remaining $\alpha$ is a fraction of the free long wave length and analogous to Baldock and Huntley (2002); Baldock et al. (2000) quantified by $\alpha = 0.2 - 0.3$. Details on this matter are presented in chapter 2.

![Boxplots showing theoretical threshold frequency $f_{th}$ above which the breakpoint excursion interferes with the nodal structure.](image)

Figure 4.13: Theoretical threshold frequency $f_{th}$ above which the breakpoint excursion interferes with the nodal structure. Subfigures 4.13a and 4.13b are evaluated for different $h_x$. Both boxplots show the average results for the setting of Figure 4.12 in combination with an array of $\alpha = 0.2 - 0.3$. Boxes are framed by 25th and 75th percentiles.

Subfigure 4.13a predicts the interference of the breakpoint region with frequencies too high in order to explain the energy patterns visible in Figure 4.12. Yet, as expected, the threshold depresses for bursts that are more energetic. A quick glance at Equation 2.2 reveals that the downshift is a trivial response to $\sqrt{H_{rms,SW}}$ as groupiness values $G$ remain pretty constant (not shown). Thus to serve as an explanation for decreasing breakpoint effectiveness, the imposed slope $h_x$ should be milder. Due to the presence of a reef flat at Roi-Namur, simply imposing $h_x = h_{x,fore}$ might indeed have to be reconsidered. The time-averaged breakpoint location is at the transition from a steep fore reef to an almost horizontal reef flat and by that gives good reason to decrease $h_x$. Assuming that fore reef and reef flat are equally important, e.g. halving the slope, results in Subfigure 4.13b. It is not evinced whether this approach is just. Nevertheless, a moving breakpoint that subdues standing waves above $f_{th} > 0.25$ Hz for $H_{s,SW} = 2$ m is possible.

In contrast to the lowest frequencies, amplitudes in Subfigures 4.12c and 4.12d now reach their maximum on the outer reef flat instead of the shoreline. Dissipation of breakpoint forced waves can occur due to bottom
friction, long wave breaking, unfavorable interference or a combination of those, but is not further elaborated on.

Under the name of Method II an alternative way is found to identify the IG generation mechanism. Consistent with earlier findings, it confirms a dominant moving breaking point during the November and March event and the general notion of VLF resonance is substantiated. Besides that, the referred analysis does not offer additional insight and has therefore been attached to the appendix section D.1.

Based on the above results, it can be concluded that IG waves are generated by the breakpoint mechanism and that under energetic wave conditions this mechanism is more effective in case of relatively long waves. In addition, free IG waves are found to be significant offshore of the breakpoint.
4.5. Discussion

A change in the phase relationship between interconnected wave components measured at the fore reef and outer reef flat stations of the Western Roi-Namur transect lends support to the idea of a dominant breakpoint mechanism. Visualized by means of a cross correlation analysis, the change occurs with the generation of dynamic set-up on the reef flat and is generally in line with findings for Ningaloo Reef (Pomeroy et al., 2012). For mild slope regimes, Janssen et al. (2003) found a different phasing and showed that negative correlations between short wave group envelope and IG waves persisted across the beach, characteristic of released bound long waves. Baldock (2006) conducted a cross correlation analysis on transient wave groups propagating over a highly inclined bottom topography. They described a coupled appearance of a bound long wave trough lead by a crest of dynamic set-up, which was argued to be generated by a moving breakpoint. Cross correlation results for wave fields at Kwajalein atoll show the largest resemblance with Pomeroy et al. (2012).

In contrast to Pomeroy et al. (2012), low frequency waves at Roi-Namur don’t have a purely progressive character but experience partial reflection at the coastline, which is ascribed to a much smoother reef flat. Moreover, strongly barred autocorrelations at the fore reef and fragmented checkerboard patterns on the reef platform are suggestive of standing VLFs during conditions with long incident peak periods. Contardo and Symonds (2013) attribute resonantly forced free long waves to long period swells. In their study on a barred beach, bound long waves appear to be released conform the shallow water condition formulated by Baldock (2006). At Roi-Namur, bound long waves clearly leave their signature, recognized in Rho results by their lag with the SW envelope at the fore reef. Signs of bound long wave release, however, are absent and according to Baldock (2012) indeed not expected at steeply sloped bathymetries. Hence, the question whether a relation exists between VLF resonance and long incoming peak periods \( T_p \) remains unsettled.

Unlike studies conducted by Baldock where maximum dynamic set-up is found for \( f_{dyngSetup} = 0.25 - 0.5f_p \), extreme VLF and low IG development is demonstrated for the Roi-Namur reef flat, with a distinct peak at \( f_{dyngSetup} \approx 0.05f_p \). Anomalous amplification of VLFs peaking at the shoreline has been observed before during storm conditions with high wave set-up at Ipan reef, Guam. Consistent with Contardo and Symonds (2013) also Péquignet et al. (2009) suggest resonance. If idealized as a step shelf, the fundamental eigenmode of the Roi-Namur reef flat lies around 0.003 Hz for water depths of about 1 m, which results in an immaculate fit with the generally observed energy peak at the inner reef flat. Not shown in the main text is that low tide events have generally a dampening effect on VLF creation, which could be ascribed to shallower water depths. Smaller values of \( h \) imply even longer resonant periods but at the same time offer limited room for LF amplitudes to grow.

Deeper investigation on the effectiveness of the breakpoint mechanism reinforces the proposition of bound long wave decay rather than release and hence speaks against a hydrodynamic situation as reported by Janssen et al. (2003) or Baldock (2006). An answer to varying breakpoint efficiency in the range \( f = 0.02 - 0.04 \) Hz is sought in the destructive interaction of the moving break point with the nodal structure of free standing waves. The existence of this effect was proved experimentally by Baldock and Huntley (2002) for a plane beach with a gradient of \( h_x = 1 : 10 \) where analogous to this study long wave generation at higher IG frequencies was weaker than linearly proportional to the offshore wave height. Roi-Namur has a fore reef gradient of \( h_x = 1 : 18 \) and a practically horizontal reef flat (\( h_x = 1 : 700 \)). A reasonable match with observations is achieved by interpreting the reef flat as a slope reducing element in the applied formula, Equation 4.2. It is emphasized that the actual occurrence of free wave cancellation by a relatively large breaker region is not indisputable.

Symonds et al. (1982) first mentioned that the significance of breakpoint forcing to long wave generation depends on the relative/normalized bed slope. Later work by Battjes et al. (2004); Van Dongeren et al. (2007) confirmed the proposition and redefined the normalized bed slope parameter to its presently most common form as \( \beta \), see Equation 2.3. A quick assessment on \( \beta \) shows that the Roi-Namur fore reef classifies as a steep slope regime. With a slope of \( h_x = 1 : 18 \), limiting periods of the IG band (25 s and 250 s) and a shoaling depth taken to be 5 m, the dimensionless normalized bed slope \( \beta = 0.31 - 3.1 \). It thereby spans far beyond the threshold value of 0.3 that was suggested by Battjes et al. (2004). I.e. dominance of the breakpoint mechanism is confirmed for the entire LB band. Baldock (2012) recently introduced a surfbeat similarity parameter \( \xi_{surfbeat} \) that incorporates the short wave steepness, see Equation 2.4. Roi-Namur has typical values...
of $\xi_{surf\ beat} \approx 0.29$ for the lower IG limit of 0.004 Hz and $\xi_{surf\ beat} \approx 0.029$ for the upper IG limit of 0.04 Hz. Thus bursts dominated by low IG frequencies are likely to ignite serious breakpoint forcing.

In Subfigure 4.14a, $\xi_{surf\ beat}$ is evaluated for the entire field experiment and based on average wave periods $\omega_{IG} = \frac{2\pi}{T_{m01, offshore}}$. The mean overall bursts shows $\xi_{surf\ beat} \approx 0.054$ which implies minimal surf beat for most of the time. Only 6% of the bursts are above $\xi_{surf\ beat} = 0.08$ and neither the November nor the March event are amongst those, which strongly disagrees with observations. Alternative substitution of the peak period $T_p$ instead of $T_{m01}$ does not improve the estimate and only increases the variance, see Subfigure 4.14b. In conclusion, $\beta$ ranks Roi-Namur as a typical steep slope regime with an active breakpoint mechanism. The introduction of the wave steepness alters the classification and only in cases where energy is packed at lower IG frequencies $\beta$ dominates $\xi_{surf\ beat}$ to such a degree that it too, indicates a breakpoint dominated environment.

Besides previous findings, no direct correlation is visible between $\xi_{surf\ beat}$ and $H_{0,SW}$ (not shown). Coincidentally, the shallow water limit is exceeded for about 50% of the bursts. Emphasizing the lower steepness of swell waves and for many situations leaving the option of bound long wave release if it weren’t for the steep fore reef slope that provides insufficient time for it to happen, Baldock (2012). Investigation on the excess threshold where $n = c_g/c > 0.97$ exposes the region between the fore reef and outer reef flat as shallow water barrier for incident bound long waves. Note that the shallow water limit is supposed to be surpassed for cases with steep storm waves, yet swell is the prominent wave type at Roi-Namur.

4.6. CONCLUSIONS

A field study has been conducted at the reef site of Roi-Namur on Kwajalein atoll. During half a year of data collection the reef site was exposed to various, mostly swell-dominated wave conditions. Strong enhancement of low frequencies was observed in the first Kwajalein study by Quataert (2015) and was proposed to play a major part in wave run-up on the beach. In the first part of this report focus was put on the transformation of VLF and IG waves over the Western transect at Roi-Namur.

AWAC data were solely used to assess the balance between free waves and bound waves offshore. Calculation of theoretical bound long wave spectra for two selected bursts suggested that outgoing free long waves dominate the low frequency wave field offshore.
A time stack of 3800 bursts allowed an investigation of wave hydrodynamics in terms of wave classes. Separation of bursts occurred on the basis of the low frequency response on the inner reef flat and uncovered a striking sensitivity to long incident swell motions. To investigate the difference in reef response to shorter and longer peak periods, two energetic wave classes were defined according to peak periods above a specified as \( T_p > 15 \) s and below \( T_p < 12 \) s. New frequency bands were chosen for the cross correlation analysis. One band was specified as \( ll \) and comprised VLFs and low infragravity frequencies (0.001 – 0.01 Hz). The choice was based on average energy differences between fore reef and inner reef flat that were largest in the \( ll \) range. A second frequency band contained the remaining intermediate to high IG frequencies and was called \( lh \) (0.01 – 0.04 Hz).

Within the predefined classes, cross correlation of wave sequences focussed on differences between low and high tide conditions and differences between frequency bands. Images resulting from cross correlation of short wave envelopes on the fore reef with envelopes on the reef flat, \( R_{AA} \), showed higher contrast for longer incoming peak periods (CAT2). This more structured appearance of groupiness on the reef flat is reasoned to be a consequence of stronger steepening of long period swells in the shoaling process and subsequently more violent breaking. It ultimately increases the amplitudes of both IG waves as well as solitary waves and leads to a stronger cross correlation signal. Nonlinearity of short waves on the reef platform was confirmed by high Ursell numbers. Using Cnoidal wave theory, nonlinear wave trajectories were computed and gave an accurate fit with observations. Higher concentrations of short waves were also observed during low tide conditions as a result of a tidally dependent breaking intensity. Long incident peak periods as well as low tide conditions both amplified the cross correlation signal and therefore likely enforce incident swash on the beach.

Results acquired from cross correlation of low frequency signals \( R_{O0} \) and cross correlation of low frequency signals with the conjoint short wave envelope \( R_{O4} \) showed a sudden switch in correlation between fore reef and outer reef flat characteristic of a dominant breakpoint mechanism. It was continuously observed in the figures and is therefore consistent with time averaged \( R_{OA} \) plots established for Ningaloo Reef, Australia (Pomeroy et al., 2012). In the majority of cases, troughs were shown to lag and lead the wave crests of \( ll \) as well as \( lh \) waves, shaping at least one complete wave period. Situations where peak periods are high enhanced the amplitude of the entire wave and suggest stronger surf beat. Both \( ll \) waves as well as \( lh \) waves partly reflect and radiate back to the fore reef, although for different reasons. Reflection of lagging \( lh \) troughs was argued to originate from run down in the swash zone, whereas the reflection of leading \( ll \) depressions could be an interplay of a bore like wave shape with a strong return current. Disregarding their exact generation mechanism, reflected troughs completed the evidence on a free LF environment at the fore reef.

Particularly for long incident wave periods, the results of cross correlating \( ll \) frequencies with each other or with collocated short wave envelopes showed little or no loss of correlation as waves approach the coastline, which is a strong indication for a forcing that is able to balance or even outweigh dissipation on the reef flat. On average low tide situations produced stronger signals than the high tide counterpart, which is again ascribed to stronger wave breaking. What is more, graphical features suggested standing \( ll \) waves at the fore reef and possibly on the reef flat. Generally differences between burst classes were larger for the longest subharmonics than for shorter subharmonics, meaning that increasing the incident peak period primarily affects very long wave transformation.

Finally, an analysis was performed that could further assess the effectiveness of the breakpoint mechanism and exclude significant bound long wave release. The working concept relied on a linear response of breakpoint forced waves on the offshore short wave energy as opposed to a quadratic relation of bound long waves. Three discoveries are most important for Roi-Namur. First of all the entire low frequency domain is dominated by free breakpoint forced waves. Secondly, even the fore reef showed a background of primarily OFLWs consistent with calculations of the theoretical bound long wave energy offshore. Last but not least, reversed ordering of normalized energy particularly in the range \( f = 0.02 – 0.04 \) Hz showed analogies with findings by Baldock and Huntley (2002) and suggested a higher breakpoint efficiency during mellow wave climates. This could be concluded as the ratio between local LF energy and incident offshore wave power increased with decreasing offshore wave power. A potential explanation was found in an adverse effect of a large breakpoint excursion on the nodal structure of free waves on the reef platform.
The overall goal of the thesis is to give a more complete picture of low frequency motions at Kwajalein atoll and to investigate options to improve run-up in model simulations. Low frequency dynamics on the reef platform have been studied extensively in the previous chapter. VLF and IG waves are found to be generated by a moving breakpoint and their amplification on the reef flat is determined by incident wave periods. This chapter focuses on the reproduction of recorded run-up heights with XBeach. Presently considered as model for future run-up predictions, the question is whether a less computationally expensive surf beat mode suffices or if more expensive, short wave resolving modes must be utilized. Currently three different calculation modes are implemented in XBeach that will be illuminated at the beginning of this chapter. The hydrostatic surf beat mode (XB-SB) has already been applied before as part of the ongoing research at Kwajalein atoll (Quataert, 2015) and is used as reference. XBeach-non-hydrostatic (XB-NH) and an adapted version of it referred to as XB-NH+ represent completely short wave resolving modes. Note that XB-NH+ is the most sophisticated model, however, restricted to a time series forcing only and first appears in the discussion.

For reference, conclusions from Quataert (2015) are enumerated that are relevant for this chapter:

- XB-SB accurately reproduces bulk values for short and long waves.
- XB-NH gives better approximations for wave run-up which is ascribed to the additional contribution of incident swash. A drawback is the underestimation of mean water levels on the reef flat.
- Tests with time series boundary forcing assured that observed wave phases were kept in the model. Mirroring the real situation increased run-up and lead to the hypothesis that phase information is needed for a correct prediction of run-up.
- No tests have been conducted on the shape of the low frequency spectrum on the inner reef flat. Substantial VLF growth was observed for the March event by Quataert (2015) and in chapter 4, which is most certainly an important contributor to wave run-up at Roi-Namur. Ergo, the question whether this was captured by initial simulations remains open.

The assessment of the listed bullet points is carried out with a top-down approach. Starting with short wave resolving XB-NH down to XB-SB. By doing so the strengths and weaknesses of both models are exposed and it can be determined whether the simple engineering model XB-SB captures the physics sufficiently in order to be nested (see also chapter 1).
Moreover the sensitivity to the spectral shape at the incoming boundary condition is investigated. Forcing with measured spectra is achieved by creating XBeach-readable SWAN output files from measured data. It is emphasized that SWAN is not run. Simultaneously, the usual routine of imposing a JONSWAP spectrum is tested and compared against the former. Evidently a JONSWAP spectrum is preferred as it can already be established if the significant wave height $H_s$, the peak period $T_p$ and the mean wave direction are known. It does, however, require assumptions for the parameterized directional spreading $s$ and the peak enhancement factor $\gamma$.

The first upcoming section 5.1 describes three utilized XBeach modes. In addition to XB-SB and XB-NH a pilot version of the new XBeach mode XB-NH+ is explained. The latter utilizes an extra pressure layer that improves dispersion properties, yet only a few simulations were run with this model. In succession of the model descriptions, the model set-up is treated in section 5.2 followed by the presentation of results for XB-NH section 5.3 and XB-SB in section 5.4. The chapter is finalized by a discussion in section 5.5.

5.1. XBeach modes

This section gives a compact description of the three XBeach modes that are currently available. XBeach-Surfbeat (XB-SB) and XBeach-Non-Hydrostatic (XB-NH) are mainly considered in this research. XBeach-Non-Hydrostatic+ can only be utilized for simulations that are forced with time series. Only core features are treated to clear up how the models are distinguished. For more complete descriptions on XB-SB one is referred to Roelvink et al. (2009) or Quataert (2015) for a concise summary. Other than that the open source manuals\(^1\) can be drawn in for XB-SB and XB-NH. Currently no documentation of the newest tool-kit XB-NH+ is available.

5.1.1. XB-SB

The declared surf beat model XB-SB is originally developed as a straightforward, computationally inexpensive dune erosion model. Its application within this report does not exploit the built in sediment transport formulations. Kwajalein is schematized as a hard coral structure and neither erosion nor sediment movement are considered here. XB-SB aims exclusively at the representation of IG and mean motions as they are typically responsible for severe dune retreat. For that purpose, sea and swell are separated from the IG bands. Sea and swell amplitude variations are described on the wave group scale and solved with the wave action equation. A boundary description of the field envelope is achieved by a Hilbert-transformation of the surface elevation time series, see Equation 3.3. IG and mean motions are solved with the nonlinear shallow water equations that include terms for radiation stress forcing and use the wave action as input. At the boundary, IG energy is estimated from the short wave group envelopes and computed by means of Equation 2.1, (Herbers et al., 1994).

It has been mentioned previously, that wave run-up consists of wave induced set-up plus incident swash, (Stockdon et al., 2006). Momentum that is released during wave breaking has to be balanced by a pressure gradient which leads to higher water levels on the reef flat. Hence, the way in which wave breaking is implemented in the model directly affects wave run-up. Therefore extra attention is paid to the implementation of wave breaking.

Due to the exclusion of wave overturning in XBeach, wave breaking can be treated as a sub grid process. In XB-SB, the breaking process is accounted for by a dissipation factor $D_w$ in the wave action balance. Slightly different from the breaker formulation used by Quataert (2015), here a version with a small adaptation to the breaker criterium is applied. In addition to the conditional breaker index $\gamma_1$ that triggers wave breaking, Daly et al. (2010) propose that dissipation should continue until the wave height falls under a lower threshold $\gamma_2$. The wave height to depth ratios in this study are kept at their default values $\gamma_1 = 0.55$ and $\gamma_2 = 0.3$.

Note that wave breaking is only one of three distinct energy sinks. Besides $D_w$, bed friction also plays an

\(^1\)http://oss.deltares.nl/web/xbeach/manuals (last consulted on 12.07.15)
important role. The latter is split into two components. Friction associated with mean currents and short wave friction. Both are assumed to be independent. The artificial short wave friction coefficient $f_w$ is typically one order of magnitude larger than its counterpart. For more detailed insight see also Lowe et al. (2005); Quataert (2015); Van Dongeren et al. (2013).

5.1.2. XB-NH

Non-hydrostatic models solve water disturbances on the short wave scale, which is made possible by including a dynamic pressure term in the equations of motion. Conform the hydrostatic version, XB-NH is depth averaged. The pursuit of keeping a comparably cheap model while requesting detailed information on water surface fluctuations comes at a price. By holding on to one layer, depth averaging of velocities becomes imperative. XB-NH therefore attains a limited $kd$ resolution (wavenumber × water depth), implying that dispersion qualities are worth improvement.

No depth limitation is needed for waves to break in XB-NH and XB-NH+. As suggested by Smit et al. (2013) the rate of change of the free surface supplies enough information to initiate wave breaking, which is said to occur when $\frac{\partial \zeta}{\partial t} > \alpha_1$, where $\alpha_1 = 0.6$. Once breaking has begun, $\alpha_1$ is reduced to $\alpha_2 = 0.3$ in neighboring points to make sure that the course of breaking continues. Note that sensitivity analysis on these steepness thresholds in the first study Quataert (2015) pointed out that they had negligible influence on wave induced set-up.

Inclusion of the dynamic pressure in non-hydrostatic modes does have a side effect. In contrast to XB-SB, wave dispersion now counteracts the steepening of the wave front. Thus to be able to get a vertical breaker face, the dynamic pressure term is neglected in the vicinity of a breaking wave, i.e. XBeach switches to the hydrostatic mode until $\alpha < 0.3$.

The magnitude of energy decay in the surf zone is regulated by assuring that the numerical scheme is momentum conservative, (Stelling and Duinmeijer, 2003). Presupposing mass and momentum conservation is typical in the context of hydraulic jumps and bores, and also works out for a breaking wave with comparable height. Nevertheless, it has been documented that depth averaging and the lack of vertical structure in XBeach are likely to overestimate wave heights in the surf zone. In that sense the enhanced vertical resolution of NH+, treated in the upcoming section, is expected to give a more authentic approximation.

5.1.3. XB-NH+

Accompanying the original one-layered, non-hydrostatic mode (NH) a new option (NH+) is available within XBeach. It makes use of an additional virtual pressure layer that optimizes the model capabilities to reproduce the right amount of dispersion. In general models become stronger at representing dispersion as the amount of vertical layers increases, which is due to the fact that velocities don’t have to be depth averaged. In this context the step from one layer to two layers has relatively more impact than e.g. from nine- to ten layers, which explains why the dispersive properties of NH+ outshine NH. On the flip side, the extra layer makes NH+ computationally a little more expensive.

5.1.4. Model Limitations

The validity of non-hydrostatic models for the $kd$ range, where $k$ denotes the wave number and $d$ the local water depth, is limited. Generally, the dispersion relation and group velocities are approximated well up to $kd < 1$ with error margins smaller than 5% and stay reasonable up to $kd \approx 2$. Beyond that limit, the accuracy rapidly decreases. Hence, in deep water the presentation of dispersion is limited to very long waves but with attenuating depths the model gets accurate for a larger frequency range. The coarse vertical resolution of just one layer makes non-hydrostatic discretisations sensitive to steep gradients in the bathymetry. Humps or troughs in the depth-profile quickly induce evanescent modes in the water surface, resembling numerical
instabilities that are local, stationary and typically associated with higher frequencies. A problem ignites if these distortions keep growing. XB-NH is unable to push the error forward, due to the earlier mentioned dispersion limitations it inherits from depth averaging. As a consequence, XBeach is restricted to frequencies below a certain limit. For that case \( f_{\text{limit}} \approx \sqrt{\frac{g}{h}} \) can be used as a rule of thumb. Even though the onset boundary forcing might obey this rule, strong depth variability in the spatial model domain can evoke higher harmonics beyond the frequency limit. More specifically, the effect occurs due to local deshoaling and is furthermore independent from wave breaking, Beji and Battjes (1993).

![March event](image1)

(a) Real bathymetry

![Single harmonic wave](image2)

(b) Idealized bathymetry with hump

Figure 5.1: Development of evanescent modes at sharp transitions in the bathymetry. Blue line is computed with XBeach-NH and magenta with XBeach-NH+. Subfigure 5.1a is a snapshot of the March event with original depth profile. Subfigure 5.1b highlights the local disturbances for a simplified setting with a monochromatic wave and a fabricated depth contour.

Subfigure 5.1a discloses the presence of stationary evanescent modes as encountered for the actual time series of the peak hour on 2-Mar-2014. It is not uncommon to witness amplitudes of 1 m or more. The sensitivity to sudden depth changes is also confirmed by Subfigure 5.1b. A single harmonic is forced over an obstacle on a smooth, idealized version of the Roi-Namur bathymetry. Higher harmonics emerge at the transition from offshore to fore reef as well as the hump and can’t be transported by XBeach-NH. Both figures endorse the fact that XBeach-NH+ is capable of moving higher frequencies than NH. Evanescent modes are practically absent.

5.2. MODEL SET UP

5.2.1. BATHYMETRY AND COMPUTATIONAL GRID

The fore reef lay out as a straight line that was used by Quataert (2015) is no longer handled. In the meantime, more detailed data on the fore reef bathymetry are available that have been provided by NOAA. This research only resorts to the updated bathymetry presented by the red line in Figure 5.2. Note that depths are given with respect to the reference point on the mid reef. The real bathymetry is approximated by the black line that shows the bottom profile as used in the model. It is limited to a depth of approximately MSL – 22 m.
5.2. **MODEL SET UP**

Turquoise dots indicate the computational grid for XB-NH with a maximum spacing of $\Delta x_{\text{max}} = 1.5 \text{ m}$ that gradually decreases to the minimum spacing of $\Delta x_{\text{min}} = 14 \text{ cm}$ at the fore reef. Green dots show the grid for XB-H. Independently of the XBeach mode, the grid is extended with 3 extra cells to ease the startup of the computation. Since the offshore spacing is coarser for the surfbeat model with $\Delta x_{\text{max}} = 18 \text{ m}$ ($\Delta x_{\text{min}} = 55 \text{ cm}$) the artificial extension makes the model domain about 50 m longer.

![Figure 5.2: Measured bottom profile (red) and bottom profile as used in the model (black). Grid spacing is indicated by blue/green dots. The grid has been extended with 3 extra cells to ease the startup of the computation.](image)

5.2.2. **BOUNDARY CONDITIONS**

Models are forced with a sequence of 48 hr varying bursts, starting 24 hr before peak wave heights encountered during the overwash event on 2-Mar-2014 and ending 24 hr after, which consistent with model simulations by Quataert (2015). Burst spectra originally presuppose a stationary hydrodynamic situation for 34 min of continuous sampling. Their validity is extended to one hour of simulated time to prevent gaps of 26 min in between consecutive bursts. As will be explained later, bursts are based on fore reef measurements.

All model boundaries are closed wall boundaries, except for the ingoing boundary where a weakly reflective condition is imposed so that mass fluxes can leave the model.

Burst spectra can be imposed on the ingoing model boundary in their measured shape or be approximated by JONSWAP spectra. A third option is to force the model with original surface elevation time series. The three forcing options are treated in the subsection below.

**FORCING TYPES**

Different boundary conditions are imposed on the model to test the performance of XB-SB, XB-NH and XB-NH+ against measured data. Within XBeach, three distinct ways exist to define an offshore boundary forcing.
They are arranged according to the level of preference as:

1. JONSWAP spectrum based on $T_p$, $H_{m0}$, $\gamma$
2. Measured spectrum
3. Measured timeseries

A JONSWAP spectrum is most preferred as it implies that: First of all the loss of phase information does not have an effect on wave run-up and secondly, that a simplified spectrum shape contains sufficient spectral information. For that matter a peak enhancement function in the analytical JONSWAP formulation leaves the opportunity to rework the spectrum to a more slender and tall shape or on the contrary to make it more full-bodied. Whether it becomes one or the other depends on the peakness factor $\gamma$, not to be confused with the breaker index. A JONSWAP spectrum is quite slender and typified by a $\gamma$ of 3.3 as opposed to e.g. a more compact Pierson-Moskowitz spectrum with $\gamma = 1$. Mind the fact that the overall energy contained in the spectrum remains the same since it has to match the demanded significant wave height $H_{m01}$. In order to find the most suitable peakness for the JONSWAP spectrum, a technique developed by Mansard and Funke (1990) is applied. Amongst others it requires the peak frequency, which is drawn from super smoothed spectra built from 15 Hanning windows with 50% overlap to circumvent the problem of a double peak in some of the lesser smoothed spectra (3 Hanning windows), see also chapter 3. Simulations, where measured spectra are approximated with a JONSWAP shape use the same peakness. The average peakness of spectra measured during the March event is $\gamma = 1.96$. For convenience the value is rounded to $\gamma = 2$. For specifics about the calculation procedure, one is referred to equations 6 and 7 of the original paper (Mansard and Funke, 1990).

Measured spectra that have been smoothed with three Hanning windows of 50% overlap serve as template for XBeach readable SWAN files. Super smoothed spectra with 15 windows perform worse and therefore won’t be further elaborated. It is emphasized that SWAN is not actually run but only the file structure is used to impose measured spectra in XBeach.

Last but not least, run up can be simulated by means of an exact measured time series, where all phase information is still present. It has no predictive skill, since it can’t be constructed from a spectrum and therefore resembles the least desirable boundary forcing. Continuous simulations of several hours are not possible as consecutive timeseries are interrupted by half an hour. Times series forcing is part of the discussion section 5.5.

**Boundary location and forcing data**

In contrast to Quataert (2015), boundary conditions are based on measurements by the fore reef pressure sensor instead of the offshore AWAC. Model cases that run directly from a time-series are ideally forced from the fore reef, since the signal has been shown to correlate nicely with pressure recordings further shoreward, see chapter 4. For consistency, runs with JONSWAP or measured spectra are therefore also built from fore reef data. A drawback compared to AWAC data as basis, is the lack of information on mean wave directions and directional spread. Out of necessity the crude assumption is made that waves have refracted enough to approach perpendicularly to the coastline. In case of no refraction, waves at the fore reef would enter under a mean angle of 35°. It is emphasized that signals correlated well between fore reef and reef flat, making the perpendicular wave assumption partly justifiable.

A problem arises when the boundary itself is located exactly at the fore reef RBR. While XBeach builds random time series from either of the spectra, it assumes deep water which at depths of ca. $\text{MSL} - 9 \text{ m}$ is not true. Simply ensuring that $kh > 0.5$ guarantees that the generated wave input is tolerable. Moving the incoming boundary to a depth of $\text{MSL} - 22 \text{ m}$ meets this criterion even for wave periods of $T \sim 15 \text{ s}$. Relocation of the fore reef spectrum to larger depths requires deshoaling. Wave breaking first commences shoreward of the fore reef gauge, which allows to deshoal by conserving the energy flux $c_{g,\text{offshore}}E_{\text{offshore}} = c_{g,\text{fore}}E_{\text{fore}}$. Measured spectra are deshoaled per frequency, whereas JONSWAP spectra are deshoaled by the integral over the entire frequency band, i.e. $H_f$. 


As stated in section 5.1, XBeach invents bound long waves that suit the primary wave signal at the boundary. To circumvent a doubling with naturally present IG energy in measured spectra, frequencies up to 0.04 Hz are manually removed from the spectra before they are put on the boundary.

5.2.3. Model Calibration

Regardless of the XBeach mode, a standard combination of friction coefficients is applied. The reef site at Roi-Namur is characterized by a complex reef with high roughness on fore reef but low roughness on the reef flat. This spatial heterogeneity is mirrored by distinguishing between a fore reef roughness $C_{fore}$ and a reef flat roughness $C_{reef, flat}$ in XBeach. Standard roughnesses are determined by finding the optimum balance between $C_{fore}$ and $C_{reef, flat}$ for a non-hydrostatic simulation forced with a measured spectrum on the ingoing boundary. Analogous to Quataert (2015), 48 hr around peak wave attack encountered on 2-Mar-2014 are chosen for calibration. The model performance is tested on the basis of three parameters, consisting of water level (set-up), short wave height $H_{rms, SW}$, and low frequency energy $H_{rms, LF}$. Consistent with the approach by Quataert (2015), their bias and scatter indices tell how well they are represented by the model. Definitions of the latter are marked out in the referred thesis.

In the course of the recent alterations in the fore reef bathymetry, Chezy values of $C_{fore} = 10$ and $C_{reef, flat} = 30$ ($c_{f, fore} = 0.1; c_{f, flat} = 0.01$) suggested by Quataert (2015) are found to be suboptimal. A relatively high profile resolution now introduces a natural amount of friction and it becomes necessary to decrease the roughness. From combinations of friction coefficients in the ranges $C_{fore} = 10 - 60$ and $C_{reef, flat} = 10 - 60$ an optimal setting of $C_{fore} = 30$ and $C_{reef, flat} = 60$ ($c_{f, fore} = 0.01; c_{f, flat} = 0.003$) is found.

For hydrostatic computations with XB-SB the short wave friction coefficient is set to a default value of $f_w = 0.3$ that has been determined for the paper Quataert et al. (2015), that followed the original thesis, (Quataert, 2015). The paper took the water level offset of 0.5 m and the corrected offshore wave energy into account, which increased the initial value of $f_w = 0.15$ to $f_w = 0.3$.

5.2.4. Other

If not stated otherwise, model parameters are kept at their default value. All of the computations use a CFL condition of 0.9. The chosen Breaker criteria are mentioned in section 5.1.

Because the model needs time to reach a stationary state, the first 300 s of each simulation are neglected in the evaluation of the test criteria. It provides a small buffer on top of the visually estimated spin-up time of ca. 4 min (240 s).

One of the main questions is how well run-up is represented by the different modes of XBeach. The best estimates can be made for snapshots taken during the November event, making it the favored test case for run-up simulations.

In chapter 3 a water level offset was calculated that resembles an elevated MSL with respect to the preliminary tidal datum on the mid reef flat. Since the bottom profile uses mid reef zero as reference, water levels at the offshore boundary are manually raised by 0.5 m.

5.3. Non-hydrostatic Model Results

This section presents the results for XB-NH. Models are forced with measured spectra and idealized JONSWAP spectra. Both are treated separately.
5.3.1. Continuous forcing with measured spectra

A sequence of 48 hr varying fore reef bursts creates a continuously changing wave environment that experiences times of greater and lesser wave action. Figure 5.3 shows the envelope of the 48 wave spectra that create the two days lasting model simulation. Three of the used bursts are singled out to give an impression of changes in spectral shape and energy. Note the representation as discrete variance spectrum and not variance density spectrum.

The sequence of measured spectra produces optimal results for friction values of $C_{\text{fore}} = 30$ and $C_{\text{reef flat}} = 60$. They are presented in Figure 5.4. Subfigure 5.4a depicts the model performance with respect to measured values of burst-mean: Water level ($Set-up$), root mean squared short wave height $H_{rms,SW}$ and long wave height $H_{rms,LF}$. Copper colors elucidate the differentiation between stations across the reef. Related Subfigure 5.4b comprises the hourly performances in an 48 hour average and illustrates the cross reef evolution of wave heights and water levels.

Altogether, the real situation is well reflected by XBeach-NH. Water levels are slightly over predicted by a few cm but close to measurements, as highlighted by a small bias and little scatter. An explanation for the overestimation might lie in the fact that no mass can exit the model through the longshore boundaries. All mass flux has to return to the offshore boundary via the undertow. In reality, there is a good chance that some of the flux is released in alongshore direction, especially since the western transect is located near the end of the island. During the March event, waves fall in under a low angle with respect to the normal. Moreover the coastline is curved. Both factors ask for a broader view and a two dimensional model might prove the hypothesis. An overestimation of set-up in a 1D-XBeach simulation of a reef has been observed before. Van Dongeren et al. (2013) mentioned that at Ningaloo Reef, Australia, set-up on the reef flat was computed 10 cm above the supposed height, which is worse than the average result for the Roi-Namur reef. This is also confirmed by the orange line in Subfigure 5.4b, resembling the time-averaged water level as modelled by XBeach. It intersects with most of the orange crosses that represent the measured average at each instrument location.

Short wave heights $H_{rms,SW}$ at the fore reef are for energetic bursts above measurements. This is probably an artifact of deshaling, since the black scatter in Subfigure 5.4a appears to follow a line that is slightly more inclined than 1:1. All in all, the deshaling procedure works reasonable. The modelled blue line in Subfigure 5.4b smoothly ascends to the measured fore reef ‘x’. Discrepancies at the fore reef quickly diminish.
5.3. **Non-hydrostatic model results**

(a) Burst specific performance

![Graph showing modeled vs. measured values for set-up, $H_{rms,SW}$, and $H_{rms,LF}$](image)

Further shorewards and become non-existent as the outer reef flat is reached. As waves continue to propagate to the inner reef flat region, $H_{rms,SW}$ is again a little overrated. It is a recognizable problem that as been mentioned in chapter 3 and is ascribed to insufficient energy dissipation by wave breaking as a consequence of depth averaged flow velocities. The error may get smaller by reducing the maximum breaker steepness ($\alpha_1 = 0.6$), yet this remains subject for further research.

(b) Time averaged cross shore evolution

![Graph showing cross shore distance](image)

Similar to hydrostatic calculations by Quataert (2015), the graph of long wave heights $H_{rms,LF}$ shows more scatter. The bias, however, is close to zero for all stations, which is also fostered by the almost perfect overlap of the black line with measurements in Subfigure 5.4b. Thus, for individually simulated hours the model is less reliable, yet the average result is good.
5.3.2. CONTINUOUS FORCING WITH IDEALIZED JONSWAP SPECTRA

Figure 5.5: Spectral variation observed during 2 days (48 hours) around the March peak event. Measured spectra are approximated with JONSWAP shapes. A black line represents the 48h average of all measured spectra. The curve in magenta represents the calculated JONSWAP fit of $\gamma = 2$. Modulated example spectra from Figure 5.3 are indicated by red colors.

Figure 5.5 has the same arrangement as Figure 5.3 in the preceding subsection and shows the approximation of measured spectra with JONSWAP shapes. For the determination of a suitable peakness value, visual fitting is successfully replaced by an automated function. The method used is proposed by Mansard and Funke (1990) and produces an average of $\gamma = 2$, see magenta curve, which gives a reasonable fit to the measured average spectrum (black). The model is still forced with a sequence of hourly changing boundary spectra. Instead of adjusting the peakness value for every single burst, it is kept constant at $\gamma = 2$, which makes the example spectra of Figure 5.3 reappear in the form presented here (Figure 5.5).

The optimal calibration with $C_{norm} = 30$, $C_{refflat} = 60$ for measured spectra is kept constant for models forced with JONSWAP spectra. The peakness of the spectra is $\gamma = 2$ for each individual forcing hour. Results are presented in Figure 5.6. From comparison of Subfigure 5.6a with related graphs of Subfigure 5.4a it can be concluded that the reproduction of set-up and short waves heights using JONSWAP spectra is almost identical to a model forcing with exact, measured spectra. Spatial patterns in Subfigures 5.4b and 5.6b reinforce the statement.

Performances differ for the low frequency band. Backed by a high value of the scatter index (SCI) the fore reef estimates of $H_{rms,LF}$ clearly overshoot the real data. The peculiar low frequency response gives reason to consider the model sensitivity to the onset peakness. A representative peakness of $\gamma = 2$ was applied to all of the burst spectra. However, the spectral variation illustrated in Figure 5.3 includes spectra that are extremely narrow banded, i.e. where a description by $\gamma = 2$ is imperfect. See also the representation of example spectrum 3 between Figure 5.3 and Figure 5.5. To test whether changes in peakness affect the result, a higher energetic wave situation is simulated for a peakness value of $\gamma = 7$ that fits this particular burst much better. Indeed, the model accuracy increases, which is highlighted by the cyan colored mark that has moved down compared to the original estimate (magenta).

Still, simulations with measured spectra in the previous section showed better resemblance with the collected data. The only major difference that is left between the simulation with measured spectra and the simulation with JONSWAP spectra is the structure of the boundary files. In case of the former, frequencies are individually linked to a certain variance density and the file structure conforms a SWAN output file. In case of the latter, JONSWAP spectra are produced by XBeach itself. The energy integrals, wave directions and the peak period are identical. In both cases, moreover no directional spread is allowed. To test whether XBeach treats
5.3. NON-HYDROSTATIC MODEL RESULTS

5.3.1. Burst specific performance

(a) Burst specific performance

(b) Time averaged cross shore evolution

Figure 5.6: XB-NH: MARCH: Model performance in terms of set-up, short wave height and long wave height for continuous model forcing with JONSWAP spectra of $\gamma = 2$, $C_{f,\text{fore}} = 30$, $C_{\text{reef,flat}} = 60$. Model performance changes for different file structure or adjustment of the peakness as highlighted by magenta (default), cyan ($\gamma = 7$) and green (Swan file structure with $\gamma = 7$).

the commands internally different, the same JONSWAP spectrum as used for the variation of the peakness, is imposed in the form of a SWAN output structure and given the original peakness $\gamma = 2$. The green circle in Subfigure 5.6a shows that this has little effect on the reproduction of long wave heights. Deshoaling of the fore reef spectrum happens slightly different for the two approaches and is argued to be responsible for the small discrepancy between the green and magenta circle. The SWAN output file contains a JONSWAP spectrum that has been deshoaled per frequency instead of being deshoaled by the integrated energy in case of the commonly used JONSWAP routine.

On the reef platform, long wave heights are underestimated. The problem appears to be caused on the fore reef. While long wave amplitudes continuously increase in Subfigures 5.4b they stagnate in Subfigure 5.6b.
This could be a direct consequence of an initial overestimation at the location of the fore reef station as some of the lower frequencies might start to dissipate prematurely by breaking or bottom friction.

### 5.3.3. Reproduction of Run Up and Low Frequency Spectrum

#### (a) November event with $\gamma = 2$

![Figure 5.7: XB-NH, NOVEMBER: Optimal March calibration of $C_{fore} = 30$, $C_{reef\ flat} = 60$ applied to the November run-up event. Top figure shows the measured (black) and idealized JONSWAP spectrum with $\gamma = 2$ (red). The burst specific peakness is calculated as $\gamma = 2.6$ (orange). Bottom figures show the performance of the two spectra with respect to run-up height (left) and approximation of the inner reef flat LF spectrum.](image)

Next, the model performance with respect to wave run-up is investigated. For that purpose the camera-recorded November event is used and not the March event. The course of consideration here, is that a fairly good guess for the November-run-up can be made, whereas the amount of over wash during the March event
is impossible to quantify. A range of plausible values for the actual run-up maximum is established by comparing wet lines of the two most significant camera snapshots and ought to lie between 2.8 – 3.3 m above reference zero, i.e. 2.3 – 2.8 m above MSL, see subsection 3.3.3. For the modeling part, the last wet point represents the run-up maximum $R_{\text{max}}$. A single extreme value could be coincidental, therefore it is supplemented with $R_{2\%}$. The latter resembles the upper 2% fraction. The simulation time reduces to one single hour instead of 48 hr of (uninterrupted) simulation time for the March event.

Keeping the peakness value of $\gamma = 2$ that was originally determined for JONSWAP spectra of the March event results in the approximation depicted in Subfigure 5.7a for November. Separate calculation of the burst specific peakness (Mansard and Funke, 1990) suggests a slightly higher value of $\gamma = 2.6$. Although it may seem inappropriate, November simulations are ultimately based on the lower $\gamma = 2$ to stay consistent. The effect of varying the peakness on run-up and the low frequency spectrum is part of the discussion section 5.5.

Run-up computations visible to the left of Subfigure 5.7b are reasonable, as illustrated by crosses for maximum run up $R_{\text{max}}$ and the $R_{2\%}$ thresholds. Blue colors denote the result of the simulation with the measured spectrum and green colors the result for the JONSWAP-type forcing. XB-NH convinces with run-up estimates that are more or less close to the visually determined range of possibility. $R_{2\%}$ moreover inflicts that $R_{\text{max}}$ is not unique, but that it is indeed representative of the local wave milieu. Actually, this is not entirely correct. The right end plot of Subfigure 5.7b compares the produced LF spectra with the measured one at the location of the inner reef flat. Hence, close to wave impact. Both forcing types, the clear-cut measured spectrum as well as the smooth JONSWAP spectrum show the right tendencies towards energy accumulation around the VLF frequency $f \approx 0.003$ Hz, yet the surface areas are only about half of the real spectrum. Albeit the LF amplitudes are underestimated, run-up looks legitimate. An explanation is found by reviewing the time averaged spatial patterns of set-up, $H_{\text{rms,SW}}$ and $H_{\text{rms,LF}}$ for the November simulation, see Figure 5.8. Figures are constructed analogous to the application in previous sections. Black lines associated with bulk long wave energy substantiate the mediocre representation of the LF spectrum as they end below the measured crosses at the position of the inner reef pressure sensor. However, an underrepresentation of energy in the LF band is compensated by overrated energy in the short wave range. Thereupon incident short wave swash might keep run-up at an acceptable level.
Figure 5.8: XB-SB, NOVEMBER: Cross sections showing cross shore evolution of bulk values for measured and JONSWAP spectrum.

\[ C_{\text{forc}} = 30, C_{\text{refflat}} = 60, \gamma = 2 \text{ (JONSWAP)} \text{ and } f_w = 0.3. \] Performance in terms of set-up, short wave height and long wave height.
5.4. SURFBEAT MODEL RESULTS

Similar to the first part covering XB-NH, consecutively the results for measured spectrum forcing, subsection 5.4.1 and idealized JONSWAP forcing, subsection 5.4.2 are presented for XB-SB. By now the implementation of the fore reef spectra has been clarified and since they stay the same, Figure 5.3 and Figure 5.5 shown for XB-NH are not reposted.

5.4.1. CONTINUOUS FORCING WITH MEASURED SPECTRA

(a) Burst specific performance

(b) Time averaged cross shore evolution

Figure 5.9: XB-SB: MARCH: Optimal calibration for measured spectra where $C_{\text{fore}} = 30$, $C_{\text{reeflat}} = 60$ and $f_w = 0.3$. Performance in terms of set-up, short wave height and long wave height.

Results for the measured boundary forcing, Figure 5.9 in combination with hydrostatic surf beat mode are
certainly rewarding. Set-up is well reproduced as can be seen to the left of Subfigure 5.9a. Summed bi-
asses are now near zero. Despite closed wall boundaries and all mass flux returning to the ingoing boundary,
modelled set-up closely intersects the measurements, which is enforced by the orange line and crosses in
Subfigure 5.9b.

Short wave heights on the fore reef are a little underestimated. The error develops due to a larger spatial
model domain of XB-SB compared to XB-NH that provides more surface area for bottom friction to dissipate
wave energy. Truncating the model domain most likely solves the problem but is not further elaborated.
Having the possibility to tune the short wave friction coefficient \( f_w \) pays off for the prediction of short wave
heights on the reef flat. It compensates for the shortcomings of the breaker model, see section 5.1, causing
the scatters of the reef flat stations to neatly follow the optimal line. Note that the underestimation at the fore
reef also contributes to lower wave height values inshore.

The graph on the right hand side of Subfigure 5.9a, that comprises the hourly results for the long wave heights
closely overlaps with the outcomes for the non-hydrostatic calculation in subsection 5.3.1, besides SCIs being
somewhat higher. Similar to XB-NH, XB-SB is therefore also strong in the representation of the 48 hr average,
as elucidated by the black line in Subfigure 5.9b.

5.4.2. CONTINUOUS FORCING WITH IDEALIZED JONSWAP SPECTRA

The fourth and last model is forced by JONSWAP spectra and computes wave transformations in hydrostatic
mode. The model inherits errors that have been observed earlier from (1) an increased model length that
forms a larger surface area for bottom friction to dissipate wave energy, as well as (2) from the suboptimal
JONSWAP shape. Since these phenomena have been treated extensively in the previous sections, only some
features of this specific computation are marked out.

In Subfigure 5.10a, set-up and \( H_{rms,SW} \) are again well balanced with exception of the recently discovered
error at the fore reef. Indeed, estimates of \( H_{rms,LF} \) on the reef flat have improved compared to the related
non-hydrostatic computation. The error on the fore reef, which for XB-NH improved by adjusting the peak
enhancement factor is also present in the surf beat computation, but can most likely be improved in a similar
way. These facts are summarized by the spatial representation in Subfigure 5.10b, which is very similar to
the average of the forcing with measured spectra in the preceding subsection 5.4.1. Following next is an
elaboration on the run-up results for XB-SB.
5.4. 

SURFBEAT MODEL RESULTS

(a) Burst specific performance

(b) Time averaged cross shore evolution

Figure 5.10: XB-SB: MARCH: Model performance in terms of set-up, short wave height and long wave height for continuous model forcing with JONSWAP spectra of $\gamma = 2$. $C_{f_{\text{fore}}} = 30$, $C_{r_{\text{refflat}}} = 60$. Performance in terms of set-up, short wave height and long wave height.
5.4.3. REPRODUCTION OF RUN UP AND LOW FREQUENCY SPECTRUM

The implementation of idealized JONSWAP spectra for the November event has been elucidated with Subfigure 5.7a and is not reposted at this point, for details see referred figure.

Again the model performance in terms of run-up and low frequency spectrum is determined from a one hour lasting simulation of the November event. Moreover, the peakness of the JONSWAP spectrum is consistent with earlier simulations set to \( \gamma = 2 \).

Run up heights presented on the left hand side of Figure 5.11 are lower than observed in the real situation. This contrasts with better approximations of the measured low frequency spectrum by both measured and idealized JONSWAP spectra compared to XB-NH. Particularly the simulation forced with measured spectra performs well and ultimately generates more LF energy at the inner reef flat than the non-hydrostatic simulation shown by Subfigure 5.7a in subsection 5.4.3. Thus the ability to present the low frequency spectrum more accurately does not guarantee that also run-up predictions increase. The statement is generally confirmed by the related non-hydrostatic results, that showed better run-up estimates although the low frequency spectrum was represented worse than here.

The computation lacks two elements, i.e. phase information and individual short wave motion. The latter resembles the major difference between XB-NH and XB-SB since both models don't include measured wave phases. Incident short wave swash does increase run-up and could be responsible for the differences in results between XB-NH and XB-SB. Although \( H_{rms,SW} \) is only 15 cm, nonlinear waves that likely contribute to wave run up are hidden in the bulk value. Figure 5.12 reinforces the notion of individual solitons that don’t fully dissipate as they ride on long wave crests and cause swash motion on the beach. The figure is based on a simulation with XB-NH+ forced with the original November timeseries to get the best impression of short wave shapes on the reef platform during the event.

Wave phases have been lost with the formulation of the spectrum. Quataert (2015) observed one distinct wave group that was larger than others in the surface elevation time series of the November event. It was presented as a local bulge that propagated progressively through the time series across reef stations. What is more, the time of impact was suggested to coincide with one of the run-up snapshots taken by the beach camera. Such unique wave structures only reappear in the simulation if the wave phases are preserved. This can be accomplished by forcing with timeseries and has been tested, however, the description of the boundary files needs improvement. Hence, no statements can be made concerning the role of wave phases. Associated
5.4. SURFBEAT MODEL RESULTS

Figure 5.12: XB-NH+, NOVEMBER: Snapshot of time series forcing with XB-NH+. Short waves can occasionally still be 40 cm height just seconds before impact. This simulation perfectly captures the measured bulk value $H_{rms,SW}$ on the inner reef flat (see also Figure C.7).

model results have been attached to the appendix subsection C.0.1.

Lastly, both boundary options (measured spectrum, JONSWAP) end up in good representations of the calibration parameters Set-up, $H_{rms,SW}$ and $H_{rms,LF}$ and will as such not be further investigated. The affiliated figures are attached to the appendix (Figure C.4).
5.5. DISCUSSION

5.5.1. INTRODUCTION

Picking up the open ends of the first Kwajalein study by Quataert (2015) has thus far turned out to be fruitful. So have e.g. run-up estimates improved for the hydrostatic surf beat mode of XBeach and non-hydrostatic modes appear to have sufficient predictive skill to be nested in future research. What still has to be addressed is the role of the spectral peakness. Burst specific adjustment of the peak enhancement factor $\gamma$ (for simulations that were forced with JONSWAP spectra) induced changes in bulk values of the long wave height. The importance of more or less peaked spectra on wave run-up and the low frequency spectrum in the vicinity of the swash zone has, however, not been investigated and thereby also forms a point of discussion. Bottom roughnesses were kept at their calibrated value of $C_{f,reef} = 30$ and $C_{f,reef} = 60$ throughout the report. A sensitivity analysis to the bottom roughness is conducted by examining the effects of alternative friction coefficients. Finally the consequence of an elevated MSL has not yet been demonstrated. This will be done by using the model set-up from the first Kwajalein study (Quataert, 2015). Run-up results for the case where MSL is at mid reef zero and the case where MSL is 0.5 m higher are compared.

The first upcoming section presents the effects of an increased MSL on run-up results. Subsequently, the effects of alternative friction coefficients on bulk values in non-hydrostatic calculations are presented. Finally, the significance of boundary spectrum peakness $\gamma_{f,peak}$ regarding the long wave environment on the inner reef flat is examined.

5.5.2. CONSEQUENCE OF ELEVATED MSL ON RUN-UP PREDICTION

In chapter 3 a water level offset of 0.5 m has been determined. In the first Kwajalein study this offset was set to 33 cm and embedded in the formulation of wave induced set-up, however, was not imposed on the boundary in model simulations. As a consequence, water levels at the offshore boundary are half a meter higher in current simulations. Figure 5.13 is based on the model set-up as applied in the precedent study by Quataert (2015), i.e. XB-SB in combination with JONSWAP forcing. The boundary condition relies on offshore

![Figure 5.13: XB-SB: NOVEMBER. Old model set-up used in Quataert (2015). Difference in the run-up estimate with introduction of a water level offset of $h_0 = 0.5m$. Red cross denotes maximum run-up produced without the offset and the cyan colored cross with offset.](image-url)
AWAC data instead of fore reef data, therefore spectra don’t have to be deshoaled. Furthermore, mean wave directions and directional spread are included. Bottom friction values greatly increase to $C_{\text{fore}} = 10$ and $C_{\text{reef flat}} = 30$ and the fore reef bathymetry is turned back to be a straight line interpolation. The short wave friction is reduced from $f_w = 0.3$ to $f_w = 0.15$ and the peakness is increased to the characteristic JONSWAP value of $\gamma = 3.3$.

Focus lies on the run-up maxima presented as crosses in Figure 5.13. The position of the red cross indicates the run-up without water level offset and the turquoise cross the run-up with offset. The discrepancy between the crosses is larger than the offset itself and signifies that run-up reacts disproportionally on an increased water level which is consistent with Quataert et al. (2015).

5.5.3. Re-examination of friction coefficients in XB-NH

![Graph showing run-up maxima with different friction coefficients](image)

(a) $C_{\text{fore}} = 10$, $C_{\text{reef flat}} = 60$

(b) $C_{\text{fore}} = 30$, $C_{\text{reef flat}} = 10$

Figure 5.14: XB-NH, MARCH: Optimal March calibration with $C_{\text{fore}} = 30$, $C_{\text{reef flat}} = 60$ compared against a setting with higher fore reef roughness $C_{\text{fore}} = 10$, $C_{\text{reef flat}} = 60$ and setting with higher reef flat roughness $C_{\text{fore}} = 30$, $C_{\text{reef flat}} = 10$.

The optimal friction set used in this study, was determined on the basis of the summed inaccuracies described by SCI and bias for the model XB-NH. Figure 5.14 compares the calibrated setting with a Chezy value of $C_{\text{fore}} = 30$ and $C_{\text{.flat}} = 60$ against model results for a rougher fore reef $C_{\text{fore}} = 10$ in Subfigure 5.14a or a rougher reef flat $C_{\text{reef flat}} = 10$ in Subfigure 5.14b. Test parameters are set-up (orange), root mean squared short wave height $H_{\text{rms,SW}}$ (blue) and long wave height $H_{\text{rms,LF}}$ (black). Increasing fore reef friction pushes all of the three grading parameters down, which can be expected as more energy is dissipated. The result is still reasonable, but it becomes clear why a smoother fore reef was preferred. $H_{\text{rms,LF}}$ dives below the measured data, which is deemed to affect run-up more than a minuscule increase in wave set-up of 1−2 cm. Altering the reef flat roughness might be much more valuable than adjustments to the fore reef. It mostly influences subharmonic wave power and consistent with the examination of friction coefficients in Quataert (2015) long wave heights strongly decrease.
5.5.4. Sensitivity of XB-SB to changes in peakness

Figure 5.15: XB-NH, NOVEMBER: Effect of peakness bc on low frequency spectrum at inner reef flat

Figure 5.15 discusses the influence of the spectrum peakness on run-up results and low frequency spectrum on the inner reef flat. The November event is simulated with XB-SB for a Pierson Moskowitz spectrum $\gamma = 1$ and a highly peaked JONSWAP spectrum with $\gamma = 7$ as shown by 5.15a. The presentation of run-up results and inner reef flat LF spectrum in the form presented by 5.15b is analogous to earlier applications. The results for the commonly used peakness $\gamma = 2$ is shown in green. Red represents the outcomes for a Pierson Moskowitz spectrum with $\gamma = 1$ and turquoise for a spectrum with $\gamma = 7$.

Altering the shape of the spectrum apparently has little effect on run-up. The produced results are very alike.
Furthermore no obvious patterns are visible for the spectra, that could indicate a change of characteristics with adjustment of the peakness. The plot appears messy as the low frequency peak is largest for a JONSWAP spectrum with $\gamma = 2$ instead of a $\gamma = 7$, that is supposed to give the closest resemblance with the measured spectrum. Generally, low frequency peaks increase as short wave energy is more concentrated at $f_p$, but this might as well be coincidental. Especially since it contradicts with an increasing run-up height for decreasing $\gamma$.

5.6. CONCLUSIONS

Generally, all models performed reasonably. Indeed, smaller errors that were e.g. caused by enlarged model domains that induced more bottom friction dissipation could be easily improved upon. This section briefly summarizes the most important conclusions from the model assessment. Note that the performance in terms of bulk values was tested with a simulation spanning two days around the March event, whereas the November event formed the basis for one hour simulations of wave run-up. Conclusions about the performances of different XBeach mode and boundary forcing combinations concerning the three calibration parameters are the first to be presented. Afterwards main conclusions are drawn on simulated wave run-up, finalized by brief comments on the adapted MSL height and forcing models with time-series.

Set up is very well approximated by all models employed. The bias and scatter index are small across all reef stations, which translates to a good model performance.

Short wave heights are in non hydrostatic computations slightly overestimated. On the fore reef, the reason can be attributed to small inaccuracies caused by deshoaling. On the mid and inner reef flat positions, these overestimations are an artifact of the breaker model that has been described in chapter 3. This error can be mitigated by the introduction of the additional short wave friction coefficient $f_w$ in hydrostatic surf beat mode.

Bulk LF energy is best reproduced by models that are forced with measured spectra. Onset of idealized JONSWAP spectra introduces erroneously high amplitudes at the fore reef. This problem could be improved upon by changing the standard peakness of $\gamma = 2$ to a value that fitted the burst specific spectrum shape better. What is more, describing the JONSWAP spectrum by means of a typical SWAN output format completely solved the problem, which is surprising since underlying assumptions were identical.

Run up heights were realistically reproduced in non-hydrostatic simulations, in contrast to simulations with XB-SB that underestimate them. In this context it appears paradox that low frequency spectra on the inner reef flat were much better approximated by XB-SB than by XB-NH. This especially applies to the integrated energy. Consequently, overall smaller low frequency amplitudes in XB-NH had to be overcompensated by incident swash in order to produce run-up heights that were within a realistic range. Figure 5.12 confirms that solitons ride on long wave crests and can thereby reach the beach without being fully dissipated. A good representation of infragravity waves close to the swash zone in XB-SB does not guarantee that run-up is correct. Furthermore, adjusting the peakness of JONSWAP spectra in XB-SB has no apparent influence on run-up.

Compared to Quataert (2015) the largest difference in run up predictions was caused by correctly representing MSL. Relative to the preliminary estimate for the tidal datum on the mid reef flat, a positive water level offset of 0.5 m was determined. Including this in the boundary description of the surface elevation greatly improved the run up estimate.

The influence of including original phase information was meant to be examined by forcing models with timeseries. An incorrect representation of short wave heights however prevented detailed investigation on that matter. Improvements can probably be made by revisiting the implementation of the timeseries at the model boundary.

In conclusion, XB-SB appears to capture enough physics to compete with XB-NH in terms of the represen-
tation of wave induced set-up, short wave heights and long wave heights. Currently the model performance falls short on the prediction of run-up, which is a crucial weakness since the models are ultimately meant to estimate just that. In that context, it might be best if research moves forward with XB-NH forced with spectra produced by SWAN, since measured spectrum forcing gave the best results.
CONCLUSIONS AND RECOMMENDATIONS

Within the framework of the overarching Kwajalein project, the goal is to predict wave run-up and overwash and their return period for various future scenarios. This is done in order to find out if the islands fresh water lens can salinify to a degree where fresh water has to be brought in externally. XBeach is currently considered as the run-up model that will quantify the amount of overwash. In the end it is meant to be nested within a structure that is crowned by a 3D infiltration model. Previous work by Quataert (2015) showed that XBeach was capable of reproducing bulk values as e.g. short and long wave heights across the reef platform, but issued weaknesses concerning wave set-up and run-up. This report continued on that study by examining (LF) wave processes in detail using in-situ measurements and by further optimizing the XBeach performance.

A broad research has been conducted on subharmonic processes at Kwajalein atoll. The purpose of the investigation was to get a detailed description of the hydrodynamics and to find out how major run-up events distinguish themselves from the ordinary situation. Focal points of the data analysis were: Generation mechanism associated with subharmonic motions across the reef site and the amplification of very low frequencies on the reef flat. Subsequently, 1D-XBeach modelling plugged in on the open question whether run-up would eventually be predictable by XBeach.

Subsequently the conclusions from the data analysis and the model assessment are presented. Note that a more detailed elaboration on the results has been affixed to the respective data chapter 4 and model chapter 5.

6.1. DATA

A flow chart has been established that structures modern knowledge on surf beat. It aids in the establishment of a proper research set up and the interpretation of various test results. Coming back to the research questions and putting the most important findings in a nutshell:

Is it possible to attribute certain wave conditions to extreme, very low frequency amplification on the reef flat?

Roi-Namur is shown to be sensitive to wave climates with long peak periods $T_p$, that generally induce strong VLF amplification across the reef flat. Concurrently, fundamental resonant periods are strikingly similar to the most energetic VLFs observed on the inner reef flat. Whether a connection exists between long incoming wave periods and potential VLF resonance is not clear based on this study. However, cross correlation of wave signals across reef stations confirmed a stronger response of VLF and low infragravity frequencies on the reef flat to longer incoming short wave peak periods. Tidal differences also impact the combined LF energy inshore as the response is larger during high tide. The influence gets smaller as incoming wave heights
Is the moving breakpoint the leading generation mechanism of subharmonic waves and how does it behave for different wave conditions?

Evidence is found that confirms a dominant breakpoint mechanism. One that is powerful enough to even dictate the amount of long wave energy offshore by means of both outgoing breakpoint generated waves and shoreline reflected LF waves. Investigation of the breakpoint effectiveness under various wave conditions moreover suggests that the generation of intermediate to high IG waves gets relatively weaker as offshore wave heights increase. The observation may be explained by a relatively large breakpoint excursion that interferes with the theoretical nodal structure of (partially) standing waves. An assessment on the origin and intensity of surf beat using characteristic parameters $\beta$ and the surf beat similarity parameter $\xi_{surf\ beat}$ reveals generally good predictive qualities. Nevertheless, while surf beat is omnipresent, no evidence is found that incident short waves are steep and incorporating the wave steepness term appears superfluous. Most studies indicate that the magnitude of the breakpoint excursion is only proportional to the normalized bed slope. This parameter is therefore the favored tool for surf beat prediction.

6.2. Modeling

Why is set-up underestimated by XBeach in non-hydrostatic mode?

Four different model and forcing combinations have been chosen and compared with each other. It can be concluded that underrated wave induced set-up in the first study by Quataert (2015) stemmed from a MSL that was too low at the model boundary. Indeed, a water level offset of $+0.5$ m submerges the reef flat for most of the time.

What is the performance of XBeach for different modes (surfbeat, non-hydrostatic) and different types of boundary conditions?

XBeach-Surfbeat (XB-SB) appears to capture enough physics to compete with the non-hydrostatic XBeach mode (XB-NH) in terms of the representation of wave induced set-up, short wave heights and long wave heights. Currently the model performance falls short on the prediction of run-up, which is a crucial weakness since the models are ultimately meant to estimate just that.

Imposing measured spectra on the boundary is more accurate than imposing idealized JONSWAP spectra. Key difference is the reproduction of low frequency energy at the fore reef. Bulk LF energy is best reproduced by models that are forced with measured spectra. Onset of idealized JONSWAP spectra introduces erroneously high amplitudes at the fore reef. This problem could be improved upon by changing the standard peakness of $\gamma = 2$ to a value that fitted the burst specific spectrum shape better.

What is the correlation between low frequency wave motions and simulated wave run-up?

Run up heights were well reproduced by non-hydrostatic simulations, in contrast to simulations with XB-SB that underestimated them. In this context it appears paradox that low frequency spectra on the inner reef flat were much better approximated by XB-SB than by XB-NH. This especially applies to the integrated energy. Consequently, overall smaller low frequency amplitudes in XB-NH had to be overcompensated by incident swash in order to produce run-up heights that were within a realistic range. Viewed differently, a good representation of infragravity waves close to the swash zone does in XB-SB not guarantee that run-up is correct. Furthermore, adjusting the peakness of JONSWAP spectra in XB-SB has no apparent influence on run-up. Combining the findings of the data and model analyses it can be stated that both LF waves as well as nonlinear solitary waves significantly contribute to run-up on the beach.
6.3. **RECOMMENDATIONS**

During this study a lot of insight was gained on hydrodynamic processes at Kwajalein atoll. Moreover it could be shown that one dimensional models can be capable of predicting wave run-up. Some issues were, however, more difficult to address. In order to enrich future research, some recommendations are made.

**Gather more data**  Deeper research on the available data is seen to be limited. To find more answers on e.g. interferences between distinct wave frequencies or resonant behavior, velocity records have to be established parallel with sampling the concurrent surface elevation. Splitting the wave field into outgoing and incoming wave components opens up a whole new range of possibilities. By investigating the nodal structure of IG waves on the reef platform, one could find out if there exists a link between long period swells above $T_p > 15$ s and triggered resonant motion of VLFs on the reef flat.

**Application of 1D-models**  (1) The surf beat mode of XBeach is fast and computationally inexpensive. It is presumed that incident swash is the missing piece in the puzzle of wave run-up. Simply introducing a run-up enhancement factor on the modelled run-up height maybe an option for hydrostatic computations. (2) Wave phases could still be important. The suggestion is to reexamine model forcing with time-series as it might be boosted with adaptations to the description of the boundary files. (3) Non-hydrostatic models unveil their strength through their capability to resolve down to the short wave scale. The computational effort is compared to hydrostatic calculations much larger. In this study solely very high grid resolutions were chosen. It could be fruitful to investigate how far the grid size can be increased without losing accuracy. In case the surf beat model turns out to be inadequate for nesting, a faster non-hydrostatic model might be considered. (4) To improve the accuracy of reproducing short waves on the reef platform in XB-NH, a sensitivity analysis on the maximum breaker steepness is suggested. (5) Adjusting the peakness of a JONSWAP spectrum on the model boundary to get a more realistic representation of the measured spectrum affects the model accuracy in reproducing LF energy inshore. Models forced with JONSWAP spectra showed an overestimation of LF waves at the incoming boundary that was overcompensated by dissipative effects at the fore reef. It may be argued that for even larger LF amplitudes, the aforementioned long wave dissipation commences before the fore reef station is reached and that long wave amplitudes are thereby effectively smaller behind that point. Since spectra with a higher peakness provoke larger bound long waves at the offshore boundary this might explain why in a testcase (see Subfigure 5.6b) less LF energy was present at the fore reef as the value of the peak enhancement factor was increased. To confirm the theory, it is recommended to compare fore reef spectra and their associated, deshoaled spectra as there may be other reasons.

**Coupling of model and data**  Every tool that has been utilized for the data analysis could as well decompose the model results. E.g. a cross correlation analysis of simulated data could show how compatible it is. Note that it was tried to tie frequencies to wave numbers and to investigate the redistribution of energy between different harmonics by means of a two dimensional Fourier Analysis. Although it correctly indicated that waves were more compact on the reef flat, see section D.2 the decomposition was not fit to draw fundamental conclusions from.

**Incorporating more physics**  1D-models applied for this research did not resolve lateral wave movement and it is recommended to extend the research with 2D-models. It will open the doors to an investigation on the importance of wave directions and lateral variations in the local bathymetry. Latest satellite imagery uncovered cross shore directed rims in the fore reef bottom profile. This factor could change the conception of the moving breakpoint, since it could differ in longshore direction and by that introduce new hydrodynamic phenomena.
Surfbeat potential
Defined as: Significant long wave forcing evoked by incident wave groups, disregarding dissipation effects.

Run-up/ overwash

Remaining SW

Dissipation by bottomfriction, SW breaking, interference with other IG generation mechanisms

Break point mechanism (BPM)

Dissipation by bottomfriction, SW breaking, interference (sym.)

Artificial IG in/ out

LW breaking for higher frequencies

IGin / IGout

No BLW release

Domination of BLW
and BPM at different f
→ indeed no IBLW release

Bound vs. free:
Hrms,IG^2 = cta/cHrms,SW at all cross-shore locations.

For higher IG frequencies: Agree with calculated potential phase-cancellation?

Yes, max at f = \(2^{nd}\) "n"ing

\(He/Lo \leq 0.055\)

High wave run-up

No BLW release

SW breaking commences

Dynamic set-up: Amplification due to SW breaking

Dynamic set-up: Due to asymmetry in forcing

SB/IGlarge and Hs/IGlarge large in front of group

Substantial wave group coalescence (events bunching)

SW shoaling

Surf zone

Inner surf zone

SW breaking commences

Surf zone

SW shoaling

Dissipation by bottomfriction, SW breaking, interference (sym.)

Artificial IG in/ out

LW breaking for higher frequencies

IGin / IGout

No BLW release

Domination of BLW
and BPM at different f
→ indeed no IBLW release

Bound vs. free:
Hrms,IG^2 = cta/cHrms,SW at all cross-shore locations.

For higher IG frequencies: Agree with calculated potential phase-cancellation?

Yes, max at f = \(2^{nd}\) "n"ing

\(He/Lo \leq 0.055\)

High wave run-up

No BLW release

SW breaking commences

Dynamic set-up: Amplification due to SW breaking

Dynamic set-up: Due to asymmetry in forcing

SB/IGlarge and Hs/IGlarge large in front of group

Substantial wave group coalescence (events bunching)

SW shoaling

Surf zone

Inner surf zone

SW breaking commences

Figure A.1: Flow chart highlighting the hierarchy in events and conditions that lead to surf beat. Focus lies on the conditions or criteria illustrated by oval bubbles, as they are at least partly necessary to confirm a certain event or statement. If colored blue, a test is adopted for the current study.
B.0.1. **Stockdon**

Stockdon et al. (2006) defines run-up as the sum of two different processes, namely the time-averaged wave setup and the total swash excursion.

**Definitions**

The convention is that (...) denotes time-averaging

Run-up (see also Figure B.1):

\[ R(y, t_i) \equiv \langle \eta \rangle (y) + S(y, t_i) \]  \hspace{1cm} (B.1)

in which \( \langle \eta \rangle (y) \) and \( S(y, t_i) \) are defined as

\[ \langle \eta \rangle (y) \equiv \text{time-averaged water level elevation at shoreline (= maximum set-up)} \]  \hspace{1cm} (B.2)

\[ S(y, t_i) \equiv \text{vertical fluctuations about temporal mean (= swash excursion)} \]  \hspace{1cm} (B.3)

Figure B.1: Example water-level time series, indicating individual runup maxima, \( R \), setup at the shoreline, \( \langle \eta \rangle \), and swash excursion, \( S \)

**Setup**

Setup, \( \langle \eta \rangle (y) \), the super-elevation of the mean water level, is driven by the cross-shore gradient in radiation stress that results from wave breaking. Over the past decennia, different dependencies between wave setup
and hydraulic conditions have been found, however, the main difference seems to be whether or not to include the (foreshore) bedslope $\beta$. Stockdon et al. (2006) rates the bedslope parameter as being important for accurate data fits. Only for highly dissipative beaches, where $\zeta_0 < 0.3$, the bed slope can and should be excluded to gain higher accuracy.

**Swash**

Swash can be decomposed into the incident and infragravity frequency bands, both contributing to the total swash excursion, Equation B.5. The former, $S_{inc}$, demands a $\beta$ for good results, whereas the latter, $S_{IG}$, shows no statistically significant, linear dependence on either foreshore- or surf-zone slope, in other words, $\beta$ is neglected. Again, highly dissipative beaches change the situation, because friction becomes much more important. For that case, $\beta$ should also be removed from the parameterization of $S_{inc}$.

**General Expressions**

Run up with 2% exceedence value

$$ R_{2\%} = 1.1 \times \left( \langle \eta \rangle + \frac{S}{2} \right) \quad (B.4) $$

where

$$ S \equiv \sqrt{S_{IG}^2 + S_{inc}^2} \quad (B.5) $$

$\langle \eta \rangle, S_{IG}, S_{inc} = f(H_0, T_0, \beta_f)$. $S$ was calculated from $S = 4\sigma (\sigma = \sqrt{swash~variance})$ and $S_{IG}(f < 0.05Hz)$ and $S_{inc}(f > 0.05Hz)$ summed over their frequency bands. Natural swash is slightly non-Gaussian, which is indicated by the regression slope of 1.1. Set up and swash are parameterized according to best-fits with lowest bias.

Set up

$$ \langle \eta \rangle = 0.35 \beta f \sqrt{H_0 L_0} \quad or \quad = 0.35 \zeta_0 H_0 \quad (B.6) $$

Swash

$$ S_{inc} = 0.75 \beta f \sqrt{H_0 L_0} \quad (B.7) $$

$$ S_{IG} = 0.06 \sqrt{H_0 L_0} \quad (B.8) $$

**B.0.2. Surface Elevation of a Bi-Chromatic Wave**

If the surface elevation, $\eta$, of a bi-chromatic wave is described by two regular waves with same amplitude $a$ and small difference frequency $(\omega_1 - \omega_2 \ll \omega_1)$

$$ \eta = a \sin(\omega_1 t - k_1 x) + a \sin(\omega_2 t - k_2 x) \quad (B.9) $$
then

\[
\eta = 2a \sin \left( \frac{\omega_1 + \omega_2}{2} t - \frac{k_1 + k_2}{2} x \right) \times \cos \left( \frac{\omega_1 - \omega_2}{2} t - \frac{k_1 - k_2}{2} x \right)
\]

which can be rewritten to

\[
\eta = 2a \cos(\psi_{\text{long}}) \times \sin(\psi_{\text{short}})
\]

or simply

\[
\eta = A(x,t) \times \sin(\psi_{\text{short}})
\]  \hspace{1cm} (B.10)

B.0.3. WAVE GROUP INDUCED OSCILLATIONS OF THE MEAN SURFACE

With \( h = d + \eta \)

Depth integrated Continuity

\[
\frac{\partial h}{\partial t} + \frac{\partial hU}{\partial x} + \frac{\partial hV}{\partial y} = 0
\]

Momentum

\[
\frac{\partial U}{\partial t} + U \frac{\partial U}{\partial x} + V \frac{\partial U}{\partial y} = -\frac{1}{\rho h} \left( \frac{\partial^2 S_{xx}}{\partial x^2} + \frac{\partial^2 S_{xy}}{\partial y^2} \right) - g \frac{\partial \eta}{\partial x} + u_t \left( \frac{\partial^2 U}{\partial x^2} + \frac{\partial^2 U}{\partial y^2} \right) - \frac{\tau_x}{\rho h}
\]

A relatively simple expression for the bound long wave surface elevation can be found by assuming that:

1. waves are normally incident and the the beach is alongshore uniform
2. turbulent mixing is neglected (internal shear stresses)
3. bottom friction is neglected (external shear stress)
4. advection by the mean motions is negligible
5. horizontal bed

\[
\frac{\partial^2 \eta}{\partial t^2} - g \frac{\partial^2 \eta}{\partial x^2} = \frac{1}{\rho} \frac{\partial^2 S_{xx}}{\partial x^2}
\]  \hspace{1cm} (B.11)

B.0.4. WAVE GROUP FORCING

The time averaged expression for the radiation stress caused by a single harmonic wave can according to airy-wave theory be expressed as

\[
S_{xx} = (n-1)E + 0.5E + \nabla \cdot \nE
\]

\[
= (2n - 0.5)E
\]  \hspace{1cm} (B.12)
normally incident waves are assumed.

However, in order to see the local effect of a wave group on the mean surface elevation, the radiation stress has to be described dependent on space and time. This implicates that spatial variability of the wave energy has to be taken into account. The (joint) amplitude of a bi-chromatic wave as found in ??, is now used to describe wave-group energy.

\[
E(x, t) = \frac{1}{2} \rho g A(x, t)^2 = \rho g a^2 (1 + \cos(\Delta \omega t - \Delta kx)) = \bar{E} + \hat{E} \cos(\Delta \omega t - \Delta kx)
\]

or simply

\[
E(x, t) = \bar{E} + \hat{E}
\]

and the radiation stress becomes

\[
S_{xx}(x, t) = (2n - 0.5) E(x, t) = (2n - 0.5) \left[ \bar{E} + \hat{E} \cos(\Delta \omega t - \Delta kx) \right]
\]

### B.0.5. **BOUND LONG WAVE SURFACE ELEVATION**

Combination of Equation B.11 and Equation B.14 and trying \( \eta_{\text{long, bound}} = \hat{\eta}_{\text{long, bound}} \cos(\Delta \omega t - \Delta kx) \) as solution to the differential equation yields

\[
\eta_{l,b}(x, t) = \frac{(2n - 0.5) \hat{E}(x, t)}{\rho \left( \frac{\Delta \omega^2}{\Delta k^2} - g d \right) c_g^2}
\]

### B.0.6. **SHOALING OF BOUND- AND FREE LONG WAVES**

Assuming shallow water and applying a Taylor expansion for the group velocity \( c_g \) for small \( kh \) one can show that the amplitude of the surface elevation \( \hat{\eta}_{l,b} \propto \bar{E} h^{-2} \). As the bound long waves are by definition bound, only the process of shoaling is considered. Meaning that the dissipation of short wave energy is neglected as it first occurs during breaking. As a result, the energy balance under shallow water conditions reduces to

\[
\frac{\partial \hat{E}_{l,b}}{\partial x} \approx \frac{\partial \sqrt{g \bar{h}}}{\partial x} = 0 \text{ indicating that } \bar{E} \sim h^{-1/2}.
\]

Combination of the aforementioned relations yields the, so called, shallow water limit, Equation B.16.

Bound long wave:

\[
\hat{\eta}_{l,b} \propto h^{-5/2}
\]

After the free long wave is released from the wave group, which often happens first during reflection at the shoreline, the energy it carries is defined by a certain amplitude, \( \hat{\eta}_{l,f} \), that can be interpreted as the remainder of \( \hat{\eta}_{l,b} \) after breaking of the primary, short waves. Thus, \( E_{l,f} = 0.5 \rho g \hat{\eta}_{l,f}^2 \). Again, only the process of shoaling is considered and the same energy balance as for bound long waves holds. The relation of free long wave amplitude to water depth becomes
Free long wave:

\[ \eta_{l,f} \propto h^{-1/4} \]  \hspace{1cm} (B.17)

Equation B.17 above is commonly also known as Green’s Law for conservative shoaling. The growth rate \( \alpha \) of a shoaling incoming long wave generally lies between the two theoretically devised bounds presented above, that is 0.25 and 2.5 for \( \eta_{l} \propto h^{-\alpha} \). Field data and experiments show that \( \alpha \) depends on the (normalized) bed slope. To account for it, the dimensionless normalized bed slope parameter \( \beta_b \) is introduced Van Dongeren et al. (2007).

\[ \beta_b \equiv \frac{h_x}{\omega} \sqrt{\frac{g}{h_b}} \]  \hspace{1cm} (B.18)

where \( h_x \) is the bed slope, \( \omega \) is the radial frequency of the LF waves, \( g \) is the gravitational acceleration and \( h_b \) the characteristic breaking depth. In literature \( \beta \) appears in slightly different shapes, but is generally analogous to the Iribarren number, \( \xi \). Van Dongeren et al. (2007) amongst other literature found that \( \beta \) governs dissipation and reflection at the shoreline (similar to \( \xi \) for short waves).

Relative to the general trend in Figure B.2a, Van Dongeren et al. (2007) found that firstly, variations of short-wave modulation and the short-wave amplitude had little effect and secondly, that for small offshore depth the short waves themselves feel the bottom already (the forcing of the IG waves is not off-resonant) and IG waves shoal according to Green’s law. Moreover the idea of \( \beta \) dictating the amount of long wave dissipation could be substantiated. This was done by assessing the amount of reflection at the shoreline, defined as \( R = H_{l,\text{incoming}} / H_{l,\text{outgoing}} \). Where \( H_{l,\text{incoming}} \) was primarily determined by bound IG waves and \( H_{l,\text{outgoing}} \) by free IG waves. In Figure B.2b \( R \) values determined from the experiments (dots) are plotted against Equation B.19

\[ \beta_{H,\text{long}} \equiv \frac{h_x}{\omega} \sqrt{\frac{g}{H_{\text{long}}}} \]  \hspace{1cm} (B.19)

in which \( H_{\text{long}} \) is the wave height of the incoming long wave near the shoreline. (\( \beta_H \) is defined slightly different from \( \beta_b \)). The scatters seem to follow the line \( R = 0.12 \Omega^2 = 0.2 \pi \beta_H^2 \) rather well, which represents an earlier found relationship for short waves. Indeed LF’s reflect in a similar manner. At values above \( \beta_{H,\text{long}} > 1.25 \) IG waves will fully reflect with \( R \approx 0.88 \) (reflection coefficient is “unity”), specified as the steep-slope regime.
Figure C.1: The fresh water lens under a coral atoll; Courtesy USGS
Figure C.2: Illustration of more severe run-up due to water level rise under climate change; Courtesy USGS

Figure C.3: On 2 March 2014, due to 5 m-high waves with 15 s-periods striking the island chain during spring high tides, the largest overwash event occurred in the Republic of the Marshall Islands (RMI, including Kwajalein Atoll and Roi- Namur Island) since the devastating 18 December 2008 event; Courtesy USGS
C.0.1. FORCING WITH TIME SERIES

What about the effect of the wave phase on run-up? For that purpose, a few models have been forced with original time-series recorded during the November run-up event. Simulated time series form an exception in terms of simulated hours, as they can only be onset for isolated sequences of 34 minutes each. To circumvent the computational effort of separately running all 48 time series, a selection of four bursts is made, consisting of the first two high and two low tide events of the two days lasting March reference case.

Finally also XB-NH+ can be utilized. Since their results are alike, only the scatterplot of XB-NH+ is interpreted. Figure C.8 and Figure C.9 associated with respectively XB-SB and XB-NH are shown but without being elaborated. Details on the creation and implementation of the time series are affixed to the graphs. The procedures are identical in case of XB-NH and XB-NH+.

Results from time series are somewhat disappointing. The collective of bursts makes an overall mediocre appearance in Figure 4.1. The SCI and bias in set-up are reasonable, but for $H_{rms,SW}$ they express a strong underestimation at the fore reef. Furthermore, long wave energy $H_{rms,LF}$ seems very scattered, albeit the inaccuracy is mainly connected to the fore reef.

Next, run-up of the November event is considered. Figure C.6 illustrates the model performance conform the earlier approach. Low frequency spectra are massively underestimated by all three XBeach modes. Indeed, the black line of Figure C.7 elucidates that $H_{rms,LF}$ is underrated across the entire reef. This is most likely a consequence of short wave heights that are too low already at the boundary. Since the moving breakpoint is the dominant generation mechanism of infragravity waves, $H_{rms,SW}$ offshore and $H_{rms,LF}$ on the reef flat are
directly correlated. This results in less infragravity wave energy being produced at the breakpoint if $H_{rms,SW}$ is too small.

Results for XB-SB and XB-NH and boundary descriptions:

When XBeach is commanded to run a spectrum in hydrostatic mode, it first builds representative time series and then reconstructs them into time variations of the short wave field envelope by means of a Hilbert transformation, Van Dongeren et al. (2013). Requesting a specific time series, requires foregoing procedure to be carried out manually. In that instance, envelope variations are described by an unfiltered Hilbert transformation and thereby depart from their low-pass filtered utilization in the cross correlation analysis. A minor drawback of this approach is the presence of artificial energy in the envelope sequence, due to sporadic overshoot of the approximated envelope shape over the actual short wave group. On the upside, water surface movement is caught in greater detail, which is deemed more important. The conversion from spatial envelope changes $A(t)$ into a variation of energy $E(t)$ is achieved using $E(t) = \frac{1}{2} \rho g A(t)^2$. For the definition of $A(t)$ the reader is referred to Equation 3.3. The water density is taken $\rho = 1025 \frac{kg}{m^3}$ and the gravitational acceleration $g = 9.81 \frac{m}{s^2}$.

Pairing a time series with either of the non hydrostatic XBeach options, is incomparable to earlier implementation in XBeach-hydrostatic. There is no need to fuse short waves to the wave group scale. Instead, the model asks for depth averaged velocities, to draw a link between water surface elevation and the propagation speed. These have to be generated artificially as no velocity records exist. On that account, first the wave number associated with $T_{m01}$ is estimated after which the velocity profile is calculated by means of Equation C.1.

$$u(z) = \frac{2\pi}{T_{m01} \eta} \frac{\cosh k(h + \eta)}{\sinh kh}$$  (C.1)
\[ \eta \text{ is the instantaneous surface elevation and } h \text{ the burst mean water depth at the fore reef. Subsequently taking the depth average } \frac{1}{\eta h} \int_{-h}^{\eta} u(z) \text{ completes the procedure.} \]
Figure C.7: XB-NH+: Times Series. Performance in terms of set-up, short wave height and long wave height.

Figure C.8: XB-SB, MARCH: Optimal March calibration in combination with time series forcing.

Figure C.9: XB-NH, MARCH: Optimal March calibration in combination with time series forcing.
D.1. **ALTERNATIVE WAY OF PROVING THE BREAKPOINT MECHANISM**

D.1.1. **RESULTS OF METHOD II**

In opposition to the method used in section 4.4, current technique allows more detailed inferences about the frequency-generation relationship. It is however restricted to a smaller number of bursts.

![Figure D.1: Three hour average around peak run-up events on 17-Nov-13 and 02-Mar-14, of low frequency growth over the reef and fore reef envelope amplitude.](image)

Figure D.1 compares the fore reef amplitude spectrum of the group envelopes with the net amplitude growth of LF waves across the reef flat, i.e. the fore reef LF spectrum is subtracted from the spectra at reef flat stations. The November and March event are investigated. Applied analysis technique is introduced in subsection 3.4.2. Envelope signals are low pass filtered for the entire LF band, 0.001 - 0.04Hz i.e. including VLFs.

At first glance, the two wave climates appear to provoke a different hydrodynamic response on the reef. Nevertheless, both figures strongly encourage the assumption by Symonds et al. (1982) of linear proportionality between wave group amplitude and the subsequently breakpoint forced long wave. This is elucidated by the parallelism of solid black- and dashed red line, respectively representing netto LF amplitude growth be-
between fore reef and outer reef flat $\Delta a_{lf, fore-out}$, and the group amplitude on the fore reef $A_{fore}$. Both show the same trends. Looking more closely at the exact shapes it is worthwhile to mention a good match of the two lines, especially for the November event, Figure D.1a. It demonstrates that regardless of the frequency, just a multiplication constant needs to be determined in order to get almost perfect proportionality, i.e. $a_{lf}(f) = constant \times A(f)$. The observations generally state that the breakpoint mechanism is prominent and that it happens close to the outer reef flat, although big differences exist between November and March with respect to the energy redistribution. Initially, both events extract roughly the same amount of energy from their respective wave group environment (not shown). While traveling shoreward, however, LF wave amplitudes measured on 17-Nov-13, Figure D.1a, increase significantly, whereas on 02-Mar-14, Figure D.1b, the transformation stagnates. Furthermore, since the magnitudes of the November and March peak are comparable this could mean that VLF amplification is limited. Regarding the possibility of resonance, amplitudes can’t grow larger than the actual water depth on the reef flat which means that $h_{reef}$ is the theoretical VLF-amplitude maximum.

Reaching the location of the mid reef flat, VLF energy in the left Figure D.1a indeed surpasses what is potentially offered by the wave groups within that frequency band. It suggests that resonance might be involved. The question arises whether the breakpoint mechanism can be accountable for VLF growth at all. Baldock (2006) and Lara et al. (2011) demonstrate significant growth of dynamic set-up by short wave breaking, depicted by the right branch of the flow chart of Figure A.1, yet judging from their power spectra that happens for frequencies of about $f_{dynSet}$ = 0.25 – 0.5 $f_p$ unlike $f_{dynSet} \approx 0.05 f_p$, Roi as would have to be the case for Roi-Namur. All in all, the results are also consistent with theoretical bound long wave calculations presented before in Figure 4.1. That is, relatively more OFLWs are present during the November event, Figure 4.1c, which makes the discrepancy between theoretical bound long wave spectrum and measured spectrum larger than for the March event seen in Figure 4.1d. Besides, a larger proportionality constant for the March event compared to the one in November is consistent with Figure 4.13, that indicates lower breakpoint efficiency for higher incident waves. Phase cancellation does not leave an imprint on the outer reef flat, yet it might become effective further shoreward, as on the mid and inner reef flat a decrease of IG energy can be noticed.

D.2. Reflection on Data Analysis

D.2.1. Dispersion relationship and energy redistribution using a 2D-Fourier transform

A very mild slope of the flats $\sim 1:700$ opens the doors to a special analysis technique. Under the assumption of constant depths, a two dimensional Fourier transform allies wave frequency $f$ and wave number $k$ through temporal and spatial decomposition. The wave field is described in terms of individual harmonics equivalent to a one dimensional Fourier transform, with the difference that their length doesn’t have to be estimated from the dispersion relation of Airy wave theory, i.e. it unlocks the actual $f - k$ relationship. Applying the concept to a real situation does have flaws. Wave breaking and bottom friction deform a wave while it progresses, which the 2D-Fourier transform corrects by introducing artificial wave components. Signals might thus be stationary in time, but in space that homogeneity lacks.

Limitations to the wave number resolution are recognizable from the usual truncations in frequency space. E.g. Sampling in space is discrete, too. Hence, where the Nyquist-frequency is given by $f_{Ny} = \frac{1}{2\Delta t}$ the Nyquist-wave-length analogously becomes $\lambda_{Ny} = \frac{1}{2\Delta l}$, with $\Delta l[m]$ representing the distance between wave gauges. Within the data analysis current method was not an option, since the RBRs are located too far apart. In that sense, models are less restricted. Information can be extracted at arbitrary positions and in large quantities. Longest resolvable waves are dictated by the length of the spatial domain, e.g. a reef flat of 300m length and the requirement of at least two full wave periods translates to $\lambda_{max} = 150 m$ or $k_{max} = 0.04 \text{ rad/m}$.
Results

There is good reason to contemplate what simulation the analysis should be founded on. Actual time series are most tempting, as they ought to come closest to reality. However, considering low frequency spectra and run-up, it is advisable to give priority to a non-hydrostatic XBeach simulation forced with a compact JONSWAP spectrum of peakness $\gamma = 2$. Taking the November event as basis, furthermore guarantees a good model performance with respect to run-up and LF-spectrum, see Figure 5.11.

Virtual wave gauges are placed from fore reef to reef flat at 1 m distance to establish a continuous $f-k$ image. On top of that, color-interpolation between pixels gives a smooth finish.

As a first tryout, a 2D-Fourier analysis of the entire reef flat, Subfigure D.2b, is straightforward.

![Figure D.2](image)

Figure D.2: Two dimensional Fourier transform in time and space. Left hand Subfigure D.2a shows energy concentrations, complemented by a black curve that outlines the linear dispersion relationship (AWT: Airy Wave Theory). It is furthermore cropped to the frequency range of interest. The sphere of action is the full reef flat width, elucidated by Subfigure D.2b.

It jumps to the eye that energy concentrations in Subfigure D.2a are displaced relative to the black line resembling the theoretical dispersion relationship. Waves are generally shorter, $k_{\text{real}} > k_{\text{AWT}}$ and consequently non-linear. Recollecting knowledge from section 4.3, the presence of solitary waves on the reef flat has been exposed before during cross correlation of wave signals. It thereby fosters the models capability of capturing veritable wave physics. Broad patches of red cover frequencies around 0.06 Hz and 0.01 Hz. The former is close to the measured peak frequency of $f_p = 0.066 Hz$. Violent wave breaking partly extends over the reef flat, meaning that the 2D-Fourier transform has to interpret complicated and discontinuous wave patterns. Suspicion is that loads of artificial harmonics are being created as a reaction to this problem. Thereupon it seems legit, that a wide wave number distribution is mainly visible at the most energetic frequencies of the (fore reef-) spectrum.

One way to prove the hypothesis is to look at changes that happen between the outer reef and inner reef flat as depicted by Figure D.3. By reducing the operation length the integrity of the transformation increases, since wave modifications induced by friction and depth attenuation are less matured.

As suspected, a large black area can be observed at the fore reef peak frequency $f_p$ and thereby substantiates a large amount of artificial waves in the related Subfigure D.2a.
Figure D.3: Two dimensional Fourier transform in time and space. Left hand Subfigure D.2a shows the energy redistribution between the outer and the inner reef flat. In dark areas energy is lost and in brighter areas gained. Consistent with earlier color labeling, the dispersion relationships at the outer reef flat and the inner reef flat have dark and light copper colors respectively.

The red arrow in Subfigure D.3a gives the impression of an energy shift to more compact wave shapes at the inner reef flat. It contradicts with decreasing values of the nonlinear correction factor $F_{\text{soliton}}$ in section 4.3 and the snapshot of Subfigure 5.1a, that both speak of sustained solitary wave dissipation. Yet, the shift is defensible. Shallower water on the inner reef flat declines the dispersion curve, which is subsequently exaggerated by the coarse wave number resolution.

Summarizing the findings from the two dimensional Fourier analysis on the model results of non hydrostatic XBeach, nonlinear short waves on the reef flat are indeed reproduced by the model. The method itself performs adequately but is not perfect. Spatial discontinuity is counteracted with virtual wave creation. Moreover a lack of precision prevents a sound investigation on energy transfers between wave components.


