Swell propagation in a natural coastal channel in the SWAN model
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Preface

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Abstract

Compare the 1D spectrum computed in SWAN (Svasek, 2007) with it measured by buoy OS4 in the Oosterschelde Estuary, serious underestimation of wave energy can be found on low-frequency band. Investigation has been performed to find the possible reasons for the missing energy on low frequency part. In this study, diffraction is hypothesized as the reason owing to 1) the model constructed by Svasek has no diffraction; 2) diffraction does re-distribute the wave energy from areas with rapid spatial variation in amplitude to areas with low amplitudes.

Before the diffraction is added to the Oosterschelde Estuary in the SWAN model, two sub-tests have been studied beforehand.

1. A ray tracing model (REFRAC model) is applied to the Oosterschelde Estuary for two purposes: 1) to find the existence of diffraction in the Oosterschelde Estuary; 2) to validate the refraction effects in the SWAN model. A parallel case called Canyon case is applied to the REFRAC model first to provide a reference. The results show that areas with rapid variations in amplitudes exist in the Oosterschelde Estuary. It is necessary to take diffraction into consideration. The effects of refraction in the SWAN model work well.

2. Three academic cases have been performed in advance to validate the diffraction implementation in the SWAN model. They are the Semi-infinite Breakwater Case, the Gap in Infinite Breakwater Case and the Ridge Case, respectively.

The results show that diffraction has apparent effects in the SWAN model. The underestimation of wave energy in the lee of the breakwaters has been improved. Diffraction in SWAN compensates the refraction over irregular bottom albeit it is not accurate enough. Model in SWAN with diffraction, under certain spatial resolution and with smaller number of smoothing steps is suggested.

By performing the Oosterschelde Estuary in the SWAN model with diffraction, the results show that diffraction has apparent effects when the incoming waves at the boundaries are unidirectional. However, when the incoming waves have broader directional spreading, diffraction cannot solve the underestimation which can be resulted from the fact that diffraction effects of the wave components may cancel each other.

Therefore, further investigations with respect to local wind sea and the ambient currents are suggested as future work since these two processes induce the wave energy on low-frequency band flow. In addition, the effects of the directional spreading of the incoming waves at the boundaries are suggested to be further studied since the value of the directional spreading is critical to the diffraction implementation in the SWAN model.
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Figure C122: ODC01 – Comparison of 1D spectra from SWAN (source terms × ambient currents × tail ×) without diffraction (red), from Svasek (black), and measured data (blue) at buoy OS4............................................................A92
Figure C123: ODC02 – Comparison of 1D spectra from SWAN (source terms × ambient currents × tail ×) with diffraction (red), from Svasek (black), and measured data (blue) at buoy OS4............................................................A92
1. Introduction

The Oosterschelde estuary (Eastern Scheldt, see Figure 1) is located in the Netherlands, in the province of Zeeland and between Schouwen-Duiveland on the north and Noord-Beveland on the south. It is a part of Dutch Delta system where the Rhine River discharges into North Sea.

It is an important region because of its economical and geographical position. The hindcast studies of the SWAN (Simulating Waves Nearshore) model for the outer Delta of the Oosterschelde estuary showed that low-frequency wave energy in the tidal inlet was underestimated. The missing wave energy on the lower-frequency band indicates that the SWAN model has difficulties to re-produce the swell wave energy penetrating into tidal inlets. It may cause the underestimated wave heights along the coastlines. Then serious consequences to the local flood control systems may be brought about. The aim of this research is to identify one of the possible reasons for the underestimation of wave energy on the low-frequency band. The focus will be placed on the effects of diffraction (turning of waves towards areas with lower amplitudes due to amplitude changes along the wave crest).

Figure 1: Oosterschelde Estuary, The Netherlands

The hindcast studies of SWAN in the Oosterschelde Estuary have been carried out in Svasek Hydraulics (2007). The obtained spectrum (Figure 2) showed that the serious underestimation of wave energy on low-frequency band existed at the point OS4 (the position of OS4 is shown in Figure 1). One year later, further investigations were conducted by Alkyon (2008) [1] in order to explore the possible reasons for the underestimation. This study was mainly focused on the effects of refraction, bottom friction, triads and deep water physics (deep water physics in SWAN indicates the chosen mode of wind input, quadruplet interactions and whitecapping) in the SWAN model. The results show that they do not play a major role in the low-frequency wave underestimation and cannot be regarded as one of the possible reasons for the underestimation of the wave energy on low-frequency band.
In the present study, the effects of diffraction are hypothesized as a possible reason for the underestimation of the wave energy on the low-frequency band. For the Oosterschelde Case performed in Svasek Hydraulics (2007), the phase-decoupled diffraction in SWAN (Holthuijzen et al., 2003) [2] was not involved. However, without diffraction, it may bring about inaccuracy in areas where variations of the wave amplitudes are rapid along the wave crests. It is meaningful and important to inspect the variation of the amplitudes in the Oosterschelde Estuary. Thereby diffraction has been taken into account in the SWAN model. It aims to determine whether it is a reason to cause the underestimation of the wave energy on the low-frequency band.

The ray tracing model REFRAC (Booij, 1994), based on the geometrical optics refraction approximation, is applied to the Oosterschelde Estuary as well to explore the areas where diffraction effects may play an important role. In addition, the distribution of the significant wave heights obtained in the REFRAC model is compared against those computed in the SWAN model. It aims to examine the characteristics of refraction effects in the SWAN model. Parallelly, another nearshore estuary case called Canyon Case (Scripps and La Jolla submarine canyons in Southern California) is performed in the REFRAC model and the SWAN model. The purpose is to provide a reference to the Oosterschelde Case. For compatibility in the comparison to the REFRAC model, diffraction effects, and all the other source terms (wind, triads, bottom friction, whitecapping, depth-induced breaking and quadruplet interactions) in SWAN have not been taken into account in these tests.

It is found that the results obtained from the SWAN model are in fairly good agreement with those from the REFRAC model in both Oosterschelde Case and Canyon case. Therefore it can be concluded that the pure wave propagation processes are well simulated in the SWAN model. The results computed by the REFRAC model also show the presence of the caustics in the Oosterschelde Estuary. It suggests that the consideration of diffraction effects may be necessary because the wave propagation is no longer independent of the wave amplitude in areas with caustics.

Three academic cases are performed in SWAN with diffraction. The analytical results and the measured data are available for these academic cases. So they can be used to be compared with the numerical results. They are performed before the Oosterschelde Case in the SWAN model with diffraction. It aims to validate the implementation and investigate the characteristics of the diffraction functionality in the SWAN model. These three cases are the Semi-infinite Breakwater Case, the Gap in Infinite

![Figure 2: Comparison of computed and measured frequency spectra at point OS4 in Oosterschelde Estuary at 2: 30 a.m., 23 Dec, 2003](image-url)
Breakwater Case and the Ridge Case respectively. In each case, a series of sensitivity control tests are done. They are concerning on the following several aspects: 1) spatial resolution, 2) numerical technique, 3) diffraction and 4) smoothing technique.

When phase-decoupled diffraction is included in the SWAN model, the application of the SWAN model becomes sensitive to the spatial resolution. As a third-generation wave model, the implicit scheme means the SWAN model is robust with different spatial resolution. However, when diffraction effects decoupled in the SWAN model, the spatial resolution has to be chosen within a certain range. It is recommended within the range of 1/5 ~ 1/10 of the dominant wave length by the SWAN Group. The purpose is to avoid unexpected inaccuracy resulted from coarse grids or unacceptably long duration for fine grids to fulfill the stopping criteria. Besides, proper application of the numerical technique (controlled by the under-relaxation parameter $\alpha$) and the diffraction smoothing technique (controlled by the filter coefficient and smoothing steps) in the SWAN model are also critical to obtain the stable results within acceptable duration.

After the diffraction is validated in SWAN through the three academic cases, it is applied to the Oosterschelde Estuary. The spectrum at the buoy OS4 obtained from the phase-decoupled diffraction SWAN model is compared with the measured data to determine whether the diffraction is the main reason for the underestimation of the wave energy on the low-frequency band. The results illustrated that diffraction effects are not responsible for the underestimation of the wave energy on the low-frequency band. Some other possibilities are suggested as a future investigation.

In chapter 2, the background knowledge is introduced. The measured spectrum combined with the SWAN computed spectrum obtained by Svasek (2007) is shown. The investigation made by Alkyon (2008) and the corresponding results are discussed. Further studies relating to the wave propagation processes (refraction) and diffraction effects in current studies are introduced. In chapter 3, the wave propagation tests performed in both SWAN and the REFRAC model and the corresponding results are given. In chapter 4, three academic cases are performed to validate the phase-decoupled diffraction effect in SWAN. In chapter 5, the Oosterschelde Estuary is applied in SWAN with diffraction. The conclusions of this study and the suggestions for future work are given in chapter 6.

Also, the introductions to SWAN model and the REFRAC model containing the mathematical and numerical principles are given in Appendix A and Appendix B. The raw data including the SWAN command files and all output results can be found in Appendix C.
2. Background Knowledge

2.1 Results from Measurements

There are three wave buoys placed in front of the Oosterschelde Estuary (Figure 3). They are LEG, SCHB, and OS4 respectively. One-dimensional frequency spectra are measured at these three locations. Measured data at wave buoy LEG and SCHB are used to provide the boundary conditions and data at wave buoy OS4 are provided as a reference to be compared with the results obtained in the SWAN model. Detailed bathymetry in Oosterschelde Estuary is shown in Figure 4.

Six cases in the outer delta of the Oosterschelde under different storm events were hindcasted in SWAN by Svasek Hydraulics (2007). These six events occurred in December 2001 and December 2003. Their conditions are summarized in Table 1. Discrepancies between the measured data and the SWAN results obtained were found in the hindcast studies. Therefore, further investigations into the sources of the discrepancies between numerical results and measured data were conducted in Delft Hydraulics (Van Vledder et al., 2008) [1] in terms of the under-prediction of low-frequency spectral components, inaccurate prediction of wave parameters in current fields, and the underestimation of depth-limited wind-wave growth. The interest in this study lay in the under-prediction of the wave energy on the low-frequency band.

Table 1: Summary of the 6 hindcast events in Oosterschelde

<table>
<thead>
<tr>
<th>No.</th>
<th>Date</th>
<th>Time</th>
<th>$U_{10}$ (m/s)</th>
<th>Wind Dir (°N)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>26 Dec. 2001</td>
<td>09:00</td>
<td>16</td>
<td>310</td>
</tr>
<tr>
<td>2</td>
<td>26 Dec. 2001</td>
<td>12:00</td>
<td>13</td>
<td>315</td>
</tr>
<tr>
<td>3</td>
<td>29 Dec. 2001</td>
<td>15:00</td>
<td>9</td>
<td>280</td>
</tr>
<tr>
<td>4</td>
<td>21 Dec. 2003</td>
<td>13:30</td>
<td>18</td>
<td>317</td>
</tr>
<tr>
<td>5</td>
<td>21 Dec. 2003</td>
<td>16:00</td>
<td>17</td>
<td>300</td>
</tr>
<tr>
<td>6</td>
<td>23 Dec. 2003</td>
<td>02:30</td>
<td>9</td>
<td>295</td>
</tr>
</tbody>
</table>

The data measured at the buoy SCHB and LEG are only used to provide the boundary conditions, so the computed data at these two locations will not be analyzed further.
At the buoy OS4, wave energy on the low-frequency band is greatly underestimated in the SWAN model (Figure 2), so that a series of runs will be performed in SWAN to explore the possible reasons. Moreover, only the storm event No.6 is selected for further analysis. Because it has a low wind speed which gives a representative set of environment conditions.

Figure 2: Bathymetry information in Oosterschelde Estuary

As shown in Figure 2, the blue solid line represents the measured frequency spectrum at the wave buoy OS4 under the conditions of No.6 event. The corresponding wind speed was around 9 meters per second, and the wind direction was 295 °N (°N denotes the nautical convention, i.e. the direction where the wind or the waves come from, measured clockwise from geographic North). All these data were recorded at 2:30 a.m., on 23rd December, 2003.

In Figure 2, it is clear that the swell waves can be found at the frequency of 0.15 Hz or so. They may be travelled from North Sea on the North-West side of the buoy OS4. The local wind sea can also be read from the high-frequency band of the spectrum. The wind was relatively weak at that moment owing to lower distribution of the wave energy on high-frequency part.

The wave conditions measured at wave buoys LEG and SCHB are set as boundary conditions. The measured frequency-directional wave energy density distribution at wave buoy LEG and SCHB are shown in Figure 5 and 6 where the dominant direction
of the swell waves is mainly around 340°N, i.e. north-north-west (NNW) and the local wind sea shifts from NNW (340°N) for low frequencies to W (280°N) for high frequencies.

Figure 5: Directional wave spectrum at LEG buoy at 2:30 on 23-12-2003

Figure 6: Directional wave spectrum at SCHB buoy at 2:30 on 23-12-2003
2.2 Results from SWAN

2.2.1 SWAN
Shallow water wave model, SWAN (Simulating WAVes Nearshore), is a freely available, open source computer model that is used widely by scientists and engineers for research and consultancy practice to obtain the realistic estimates of wave parameters. It is based on an Eulerian discrete spectral balance of action density formulation which accounts for generation, propagation and dissipation of the waves in small scale but otherwise arbitrary wind fields, bathymetry and current field. Using the linear wave theory and the conservation of wave crests, the wave propagation velocities in spatial within Cartesian framework and spectral space are described by the kinematics of a wave train (Whitham, 1974; Mei, 1983). The processes of generation, dissipation and nonlinear wave-wave interactions are represented explicitly with state-of-art formulations. This also makes the SWAN model (SWAN for short) a so called third-generation model.

In contrast to the other third-generation wave models, the SWAN model is based on implicit propagation schemes (an iterative, forward marching, four-weep technique). For the propagation in geographic space (rectilinear propagation and shoaling), an iterative, up-wind, implicit scheme with third-order diffusion is used in SWAN (the SORDUP-scheme; Rogers et al., 2002). For propagation in spectral space (which accounts for refraction), an implicit, second-order upwind scheme, which is supplemented with a central scheme, is used (Booij et al., 1999). The implicit propagation schemes are always numerically stable, independently of the values of time step and spatial resolution, which implies that for shallow water, the computations in SWAN are one to two orders more economic than in the other models. An additional advantage of this implicit scheme is that the model is very robust in practical coastal applications. Moreover, the implicit schemes allow a treatment of wave blocking and wave reflection against an opposing current that is consistent with the linear theory for surface gravity waves without any ad hoc assumption. [7] The brief mathematical and numerical principles of SWAN will be introduced in Appendix A.

2.2.2 Oosterschelde Model Description
The hincast model in Oosterschelde Estuary is set up by Svasek Hydraulics (2007). As illustrated in Figure 7, the Oosterschelde model in SWAN includes a set of three nested computational grids (the largest grid K, the intermediate grid B and the smallest grid F) in the Oosterschelde estuary. The bottom contours at NAP -5, -10 and -20 in the Oosterschelde Estuary are also shown in Figure 7 for reference.

The computation domain is located on the coast of the North Sea. It covers the entire outer delta and tidal inlet of the Oosterschelde Estuary. The direction of the largest grid K is set as 41º (measured counterclockwise from the positive x-axis of the system) that makes it mostly parallel to the coastline to count more wet area in. Meanwhile, the wind in North Sea mainly blows from the north-west, the direction of the grid K allows the wind mostly pass through the computing domain perpendicularly.

The spatial resolution is critical in the SWAN model. Generally speaking, the finer the grids, the more detailed the results are. However, the domain with higher spatial
resolution needs more computing time than the model with lower spatial resolution. Since the scale of the Oosterschelde Estuary is large, it is significant to find the proper spatial resolution to balance the discrepancies between the required time and the reliable accuracy. One option is the nested grid. For the entire computing domain with large spatial scale, the relative low spatial resolution is chosen in terms of saving computing time. For the concerned area with small spatial scale, a nested SWAN run with finer grids can be carried out in terms of the more detailed result. The boundary conditions of the nested small fine grid are provided directly by the results obtained from the large coarse grid.

In the Oosterschelde Estuary, the grid K is applied to the lowest spatial resolution since it has the largest spatial scale and covers the entire domain. The grid B is the nested grid with intermediate spatial scale and resolution. The boundary conditions of grid B are provided by the results of grid K. The grid F is the grid with the smallest spatial scale but with highest spatial resolution in which the concerned point OS4 is located. The boundary conditions in Grid F are provided by the results of grid B. The detailed characteristics with respect to the spatial scale, direction of the computation grid and the spatial resolution of the three grids are summarized in Table 2.

Table 2: Characteristics of computational grids in Oosterschelde

<table>
<thead>
<tr>
<th>Grid</th>
<th>Computational Domain Information</th>
<th>Spatial Resolution</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>xlen (Lx)</td>
<td>ylen (Ly)</td>
</tr>
<tr>
<td>K</td>
<td>94000</td>
<td>28800</td>
</tr>
<tr>
<td>B</td>
<td>15000</td>
<td>20000</td>
</tr>
<tr>
<td>F</td>
<td>4300</td>
<td>6000</td>
</tr>
</tbody>
</table>

Figure 7: Outline of computational grids K, D, B and F in Oosterschelde and the location of the wave buoys LEG, SCHB, BG2, DORA and OS4.
Notes:
- \( x(y)lenc \) – the length of the computational grid in \( x(y) \)-direction (m);
- \( \text{alpc} \) – the direction of the positive \( x \)-axis of the computational grid (°);
- \( \Delta x \) – the length of each mesh, i.e. spatial resolution in \( x \)-direction (m);
- \( \Delta y \) – the length of each mesh, i.e. spatial resolution in \( y \)-direction (m).

### 2.2.3 Input Parameters
The input information including current conditions, wave levels fields and the measured data at buoy LEG and SCHB (shown in Figure 5 and 6) are provided by Rijkswaterstaat. The Oosterschelde Estuary Case was originally simulated in the SWAN model in Svasek Hydraulics under the following numerical and physical conditions:

1. third-generation WESTH mode, i.e. nonlinear saturation-based whitecapping combined with wind input of YAN (1987);
2. wind input of YAN (1987), velocity = 10 m/s, direction = 295°N at 10m elevation;
3. default quadruplet, i.e., fully explicit computation of the nonlinear transfer with DIA (Discrete-interaction approximation) (Hasselmann) per sweep;
4. default triad wave-wave interactions using the LTA (Lumped-triad approximation) (Eldeberky, 1996);
5. default depth-induced wave breaking in shallow water;
6. default bottom friction, i.e., semi-empirical expression derived from the JOSNWAP results for bottom friction dissipation (Hasselmann et al., 1973);
7. spatial resolution: Grid K (\( \Delta x = \Delta y = 200m \)), Grid B (\( \Delta x = \Delta y = 50m \)), Grid F (\( \Delta x = \Delta y = 20m \));
8. directional resolution: \( \Delta \theta = 10° \);
9. lowest discrete frequency: 0.03 Hz, highest discrete frequency: 1.0 Hz;
10. number of frequencies used in calculation: 38;
11. resolution in sigma-space: \( df / f = 0.0994 \);

### 2.2.4 Results
The measured 1D frequency spectrum at buoy OS4 and the SWAN computed spectrum are shown in Figure 2 together. The blue solid line represents the measured spectrum and the red dashed line represents the results obtained in the SWAN model without diffraction (Svasek, 2007) under the conditions of no. 6 (Table 1, section 2.1).

The numerical results show that the local wind sea is dominant in the Oosterschelde Estuary. Higher wave energy distributed on the high-frequency band can be found than on the low-frequency band in Figure 2. It indicates that few swell waves travelled from the North Sea to point OS4 successfully.

### 2.3 Comparison

#### 2.3.1 Measured VS. SWAN
Compare the SWAN computed spectrum with the measured spectrum, it shows that large amount of the wave energy disappears on low-frequency band (0.03 ~ 0.2 Hz) in results of SWAN. It indicates that most swell waves failed to penetrate the estuary. They cannot reach the buoy OS4. On the high-frequency band (0.2 ~ 1.0 Hz), the results obtained in the SWAN model mainly agree with the measured data.
In order to correct the unsatisfactory performance of SWAN, to explore the possible reasons for the discrepancies between model and measured data, some of the possible processes have been investigated in Delft Hydraulics (Van Vledder et al. 2007). These processes included propagation, dissipation, and the effect of non-linear interactions (triad). The main results and findings are briefly discussed in the following sections.

2.3.2 Propagation
The propagation of low-frequency wave energy over the complex tidal inlet bathymetry has been investigated by Van Vledder et al. by performing sensitivity runs with a reduced amount of refraction (reduced by 10%). A special version of SWAN was made in which the spectral velocity for refraction $C_{\theta}$ (propagation velocity of wave energy in $\theta$-space) was reduced by 10%.

The results as illustrated in Figure 8 shows that reducing the strength of the refraction term has a small effect on the amount of low-frequency wave energy and an incomplete treatment of refraction cannot be regarded as the cause of the underestimation of the wave energy on low-frequency band in the Oosterschelde Estuary.

![Figure 8: Comparison of frequency spectra from SWAN, from reduced refraction SWAN (R90) and the measured data at OS4](image)

2.3.3 Dissipation
For dissipation, both bottom friction and deep water physics (deep water physics in SWAN indicates the chosen mode of wind input, quadruplet interactions and whitecapping) have been investigated by performing a set of sensitivity runs with alternative model physics.

![Figure 9: Comparison of frequency spectra from SWAN, from SWAN with Madsen bottom friction (MAD) and the measured data at OS4](image)
First, the default JONSWAP bottom friction formulation was replaced with the one of Madsen et al (1988). Equivalent roughness length scale of the bottom (i.e., value of \( k_n \)) was set as 0.5m (default).

The corresponding spectrum (compared with the measured and the SWAN computed spectrum) at wave buoy OS4 is shown in Figure 9. It indicates that the main effect of replacing the JONSWAP bottom friction with the Madsen bottom friction is to reduce the wave energy on the low-frequency more seriously. It means that using the Madsen bottom friction formulation cannot resolve the problem of the underestimation of the wave energy on low-frequency band either.

Second, the use of different deep water physics was investigated by applying the Komen third generation physics (Komen whitecapping and Snyder wind input) instead of the WESTH third generation physics (nonlinear saturation-based whitecapping and YANG wind).

As shown in Figure 10, applying the Komen third generation physics gives much different results from applying the Westhuysen physics. The swell wave was further underestimated. Besides, the wind sea was strongly overestimated. The whole spectrum is shifted towards higher frequencies. However it is still a strong indication that applying the Komen mode can be considered as an improvement in respect that the peak value of the wave energy computed by SWAN is approximately the same as the measure data.

![Figure 10: Comparison of frequency spectra from SWAN, from SWAN with Komen deep water physics (KOM) and the measured data at OS4](image1)

![Figure 11: Comparison of frequency spectra from SWAN, from SWAN without triad (NOT) and the measured data at OS4](image2)
2.3.4 Non-linear Interaction
The under-prediction of low frequency energy may be resulted from the incorrect removal of energy at these frequencies by triad interactions in the Oosterschelde Estuary. This assumption has been tested by repeating the test without activating triad interactions’ term. The results (shown in Figure 11) indicate a slight increase in spectral level at the first spectral peak. The increase is too tiny so that triad interactions, irrespective of their crude approximation in the SWAN model, are not the main reason for the removal of low frequency wave energy at the wave buoy OS4.

2.4 Discussion
SWAN runs with different physical settings show that refraction, bottom friction or non-linear wave interactions is not the main reason for the underestimation of the wave energy on low-frequency band. Applying KOMEN mode of deep water physics can partly solve the underestimation, however, the whole spectrum shifts towards higher frequencies. Thereby, studies relating to the source terms are laid aside.

On the other hand, some other hypotheses and assumptions have not been taken into profound and sound investigation yet. They may also play parts for the underestimation of the wave energy on low-frequency band. One of the possible reasons is that the effects of diffraction are not taken into consideration.

Figure 12: Occurrence of caustics and focal points in Oosterschelde (monochromatic, unidirectional incoming waves)
Consider a domain with long-crested waves. Because of the irregular bottom topography, initially neighboring wave rays may diverge strongly (some rays can even turn back) and others intersect in such a manner that locally a common envelope exists, in which a caustic is said to be present. The variation of the wave amplitudes on the caustics is so rapid that diffraction effects start to work on turning the wave waves towards areas with lower amplitudes. Therefore, the effects of diffraction cannot be ignored under such wave conditions. Another instance is the focal point of the adjacent wave rays (shown in Figure 12) where diffraction cannot be omitted due to the presence of rapid variation of the wave amplitudes, either.

Moreover, the presences of the caustics or the focal points will cause the sensitivity of the wave energy to small variations of buoy location because of the rapid change of the wave amplitudes.

Therefore, before the studies of the diffraction effects, it is meaningful to investigate the wave rays’ pattern. It aims to explore the occurrence of areas with rapid variation of the amplitudes in the domain. Besides, the characteristics of the refraction effects in the SWAN model can be examined by comparing the distribution of the significant wave heights obtained from SWAN with it obtained from the REFRAC model. Afterwards, the SWAN model involving the effects of diffraction can be validated through academic cases and applied to the Oosterschelde Estuary in the end.
3. Wave Propagation Tests

3.1 Outline

The REFRAC model (Booij, 1994) is applied to both Canyon Case (Scripps & La Jolla submarine canyons in Southern California) and the Oosterschelde Case to obtain the wave rays’ patterns and the distributions of the significant wave heights.

The wave rays are plotted in the REFRAC mode. They help to discover the areas with rapid variants of the wave amplitudes (i.e., caustics or focal points) in the domain. Since these areas indicate that the effects of diffraction may play roles there. The rays’ pattern also helps to show the sensitivity of the wave energy to small variations of a specific location (buoy OS4 in the Oosterschelde Estuary) in such areas. The distance between two adjacent wave rays at the start boundary is set as 20 meters when used to plot the wave rays.

The distributions of the significant wave heights are computed in the REFRAC model to validate the functionality of refraction in the SWAN model. They are used to be compared with those computed in the SWAN model under the same physical and numerical conditions. The distance between two adjacent wave rays at the start boundary is set as 2 meters when used to give the distributions of the significant wave heights in the REFRAC model. Special attention needs to be noticed that since the source terms and diffraction effects are not included in the REFRAC model, all source terms including the bottom friction, wave breaking processes and non-linear wave-wave interactions in the SWAN model are turned off for compatibility in the comparison.

3.2 The REFRAC Model

3.2.1 Principle

The program of the REFRAC model was originally written in Fortran 77 by N. Booij in 1994. It is a pure refraction model, which follows the geometrical optics approximation. The refraction approximation is based on the assumption that the length scale of the amplitude variations is large compared to the wavelength. A necessary condition for this is that the length scale of the depth variations to which the waves react is large compared to the wavelength. [9] The brief introduction to the REFRAC model is given in Appendix B.

3.2.2 Smoothing Technique

In the REFRAC model, each computed ray reacts to the local depth variations in isolation from other rays so that arbitrarily large variations can occur between them with the increase of the propagation distance. In areas with caustics or focal points, amplitude estimates, which based on a local ray separation, display unrealistically rapid and large spatial variations. Furthermore, the lateral positions of the rays, in case of long propagation distances, are sensitive to the frequency and direction of the incident waves, to the bottom schematization, and to the procedure of numerical integration. For these reasons, little meaning can be attached to the location of each ray individually. [9]
In addition, in the REFRAC model, the significant wave height on each grid is calculated by the integration of the wave energy package carried by all rays passing through that grid. Due to the irregular bottom topography, the wave rays usually distribute unevenly. There can be a lot of rays crossing some grids while no ray passing through some others. When the grid under computation is located in areas where several wave rays intersect each other (such as caustics and focal points), large amount of wave energy is accumulated on it. Consequently, the computed value of the significant height on the task grid (the grid which is under computation) becomes unrealistically high. Moreover the assumption of the linear wave theory is no longer fulfilled under such situations (i.e., the waves characteristics are no longer independent of the wave amplitudes). Thereby the simple superposition of the wave heights resulting from all rays becomes incorrect in those grids. For those grids through which no wave ray passes, the wave energy can be used to calculate the significant wave height on the task grid is zero so the computed significant height is also zero while the reality is not.

Moreover, inaccuracy may be resulted from the spatial resolution even if the wave rays distribute evenly. As shown in Figure 13 (the red lines represent the evenly distributed wave rays), there is one ray crossing the small pink mesh while no wave ray passing through the small blue mesh. Since the rays used to compute the significant wave heights are not the same, the REFRAC model gives two different values on these two meshes whereas they should be the same. It indicates the possible inaccuracy in the REFRAC model.

These problems can be partially solved by decreasing the distance between two adjacent rays at the boundary. However it will increase the computing time. The application of the REFRAC model will become expensive.

An alternative is to use the so-called smoothing technique provided in the REFRAC model. The main idea is to count more rays to calculate the significant wave height on each grid. By activating the smoothing technique in the REFRAC model, the wave rays crossing the task grid, together with the wave rays crossing the neighboring grids, are both used to calculate the significant wave height on the task grid.

Refer to Figure 13 again, after activating the smoothing technique, the significant wave height on the small pink mesh is determined by all the rays passing through the large pink frame instead of the small pink mesh. The infinite high values resulting from the intersection of rays, the inaccuracies resulting from the spatial resolution will be diminished to some extent.

The smoothing technique is controlled by the AVG parameter which denotes the number of the meshes will be expanded on each side of the task grid in the REFRAC model (as shown in Figure 13). It has to be noticed that in the corner area of the computing domain, when the number of the rest meshes on any side of the task grid is
smaller than the value of AVG, the re-constructed computing domain is only expanded to the boundary (two examples are shown in Figure 13, the small blue/purple grid denotes the task grid and the corresponding big blue/purple frame denotes the actual grid used in computing). The value of AVG has to be chosen with special caution. If the AVG value is too high, the details of the local wave characteristics will be smoothed out and the local wave conditions will become to be global. If the AVG value is too low, the expanding area will not be large enough to solve those problems. The selection of the AVG value mainly depends on the numbers of meshes contained in the whole domain. In other words, the spatial resolution is the most critical point to determine the AVG value. The values of AVG parameter used in Canyon Case and Oosterschelde Case are introduced in the next section.

3.2.3 Some Parameters

Some numerical parameters combined with their values used in the following runs are briefly introduced in this section in order to provide a background reference.

**Numerical Accuracy \[\text{st}\]:**
In the REFRAC model, the numerical accuracy is controlled by the integration step for the computation of rays, i.e. the fraction of the wave length. In the following tests, the integration step will be set as 0.05 of the wave length, i.e. \(\Delta \lambda / \lambda = \text{[st]} = 0.05\).

**Plot Resolution \[mk\]:**
Parameter \([mk]\) in the REFRAC model is to control the number of points given per wave length along the wave ray. It indicates the multiplication factor for the distance between two adjacent points that will be given. In the following runs, it is set as 0.1, \([mk] = 0.1\). It means that the distance is equal to one tenth of one wavelength, in other words, ten points between two adjacent wave crests will be given.

**Distance between two adjacent rays:**
It is controlled by the parameter \([\text{dist}]\) which indicates the distance of two adjacent rays at the starting line, i.e., at the boundary. It is set as 2 meters when the distributions of the significant wave heights are required. It is set as 20 meters when the wave rays’ patterns are required.

**Smoothing Technique \[AVG\]:**
It is controlled by the parameter AVG. As mentioned in last section, the AVG value is based on the total number of meshes contained in the domain. According to the spatial resolution in Canyon Case (198×168 as high resolution, and 79×67 as low resolution, refer to table 3 in section 3.3.2 for details), the AVG value is set as 2 to smooth the results. For the Oosterschelde Case, it is set as 2 when the low spatial resolution (86×120, refer to table 4 in section 3.4.1 for details) is applied. It is set as 5 when the high spatial resolution (215×300, see Table 4 for details) is applied.

3.3 Canyon Case

3.3.1 Introduction

The nearshore estuary case, the Canyon Case (Scripps and La Jolla submarine canyons in Southern California) studied here is to provide a parallel reference to the Wave Propagation Tests in the Oosterschelde Case. As shown in Figure 14, Scripps
Canyon and La Jolla Canyon are two narrow underwater gorges located in the Pacific Ocean off the coast of southern California, U.S.A. Because the bottom in Canyon Case is quite steep, it produces dramatic changes in wave energy over alongshore distances of only a few hundred meters, resulting in complex nearshore circulation and morphological changes. In addition, the Nearshore Canyon EXperiments (http://science.whoi.edu/users/elgar/NCEX/) have been performed, following a number of experiments carried out at the U.S. Army Corps of Engineers Field Research Facility. Thereby sufficient information including the bottom topography and measured data is available to support the further applications of the Canyon Case to other models. It leads to the Canyon Case is selected as a reference case to be compared with the Oosterschelde Case in the propagation tests.

![Bathymetry around La Jolla and Scripps Canyons](image)

Figure 14: Bathymetry around La Jolla and Scripps Canyons

The computational domain combined with the bottom bathymetry used in the REFRAC model is shown in Figure 14 in which the colored part represents the wet area and the white part represents the dry land. It is 3950 meters long in x-direction and 3350 meters long in y-direction. The length scale of each mesh used in the bottom topography was 4.6342 meters long in x-direction ($\Delta x = 4.6342$ m) and 4.6266 meters long in y-direction ($\Delta y = 4.6266$ m) so the spatial resolution of bottom topography in Canyon is $847 \times 721$. The data are provided by the U.S. Army Corps of Engineer Field Research Facility (created on July 2002). [5]

3.3.2 Methodology

According to the dominant directions of the long swell in the domain of the Canyon Case, four groups of the tests with respect to four specific directions are performed in the propagation tests. They are 272°N, 240°N, 300°N, and 270°N respectively. In the first group, the specific direction of the incoming waves is selected as 272°N, because the obtained rays in the REFRAC model can be compared against the rays in
Magen’s model. [5] It aims to validate the implementation of the REFRAC model. Long swell from the west was observed on 30th November, 2003, in the absence of significant local winds, wave rays were integrated forward from parallel directions and equally spaced positions (Magen et al., 2008) [5]. The resultant wave rays’ pattern obtained by Magen is illustrated in Figure 15. It was computed under the following west boundary conditions: monochromatic waves with 15.4s as period, 1m as significant wave height and unidirection of 272°N as incident direction. These boundary conditions are applied to the REFRAC model without any change. The resultant wave rays are used to be compared against the rays shown in the pink frame of Figure 15. This indicates the computation domain used in the SWAN model and the REFRAC model.

The swell waves do not only propagate from one specific direction but within certain range. Thereby in the second and third groups of the runs, the incident directions of incoming waves are turned anti-clockwise and clockwise by 30°, respectively. In other words, the boundary conditions are set as waves with incident directions of 240 °N and 300 °N in the third and fourth groups. The other wave conditions in these two groups are the same as those in the first group (272 °N).

In the fourth group, the specific incident direction of the incoming waves is set as 270 °N, i.e. swell waves propagated due east. The purpose is to compare the SWAN computed distributions of the significant wave heights with they obtained by Gerosthathis (2008) [6].

Offshore wave conditions in Canyon area typically consist of Western-North Western swell systems with peak periods ranging from 10 to 22 seconds.

A test case, in which a western swell characterized by mean wave (peak) period of Tp = 15S and the significant wave height of Hs=1m was performed in the SWAN model (Gerosthathis et al., 2005) [6]. The corresponding distributions of the significant wave heights are shown in Figure 16. In Gerosthathis’ research, the offshore directional wave spectrum was reconstructed from integrated wave parameters by using a standard JONSWAP frequency spectrum, in conjunction with a hyperbolic cosine-type directional spreading function (Donelan et al 1985), defined as follows:

\[ D(\theta; \Theta) = \frac{1}{2} \beta \cosh^{-2} \left[ \beta (\theta - \Theta) \right], \]

where
\[
\beta = \begin{cases} 
2.61 \left( \frac{\omega}{\omega_p} \right)^{1.3}, & \text{for } 0.562 < \frac{\omega}{\omega_p} < 0.95 \\
2.28 \left( \frac{\omega}{\omega_p} \right)^{1.3}, & \text{for } 0.95 \leq \frac{\omega}{\omega_p} < 1.60 \\
1.24, & \text{otherwise}
\end{cases}
\]

where \( \omega_p = \frac{2\pi}{T_p} \) and \( \Theta \) stands for the mean wave direction (Massel 1996).

Therefore the boundary conditions of the last test are set as the incoming waves with JONSWAP shape of frequency spectrum, 15 seconds as period and 270°N as the incident direction of the incoming waves at the boundaries in SWAN. The resultant distributions of the significant wave heights are compared with the results in the pink frame of Figure 16. This frame also indicates the computing domain in the SWAN model and the REFRAC model.

Figure 16: Distribution of the significant wave heights in the NCEX area, obtained in SWAN without source terms and without diffraction (Gerosthathis et al., 2005) [6]

By far four incident directions of incoming waves in four groups have been selected. In each group of the tests, two runs with different spatial resolution are performed in both the REFRAC model and the SWAN model. The detailed information of the spatial resolution can be found in Table 3. In the REFRAC model, not only are the wave rays’ patterns plotted, but also the distributions of the significant wave heights are given. In the SWAN model, the results are only shown in the form of the
distributions of the significant wave heights. The distributions of the significant wave
heights obtained in SWAN model are compared against they are obtained in the
REFRAC model to validate the implementation of refraction in the SWAN
model. Besides, the obtained rays in the first group (272°N) are compared against rays
plotted in Magen’s model in Figure 15 (Magen et al., 2008). [5] In addition,
distributions of the significant wave heights computed in SWAN model are compared
against Gerosthathis’ distributions of the significant wave heights in Figure 16
(Gerosthathis et al 2005 [6]).

The boundary conditions in the REFRAC model can only be set as unidirectional
and monochromatic incoming waves. Thereby the incoming waves are modeled as waves
with spectrum shape of BIN (energy is located in one frequency bin) in the first three
sets in SWAN model for the compatibility in comparison. The directional spreading is
set as 2° and the directional resolution is set as 0.5° as suggested in the SWAN USER
MANNUAL. [3] In the fourth group (270°N), the spectrum shape of the incoming
waves is set as JONSWAP for the compatibility in comparison with the results
obtained by Gerosthathis. The corresponding directional spreading is set as around
31.5° (m=2, refer to Page 91-92 of SWAN User MANNUAL [3] for details) and the
directional resolution is set as 5°.

The codes of all runs and the corresponding input setting are summarized in Table 3.
In the codes used in the Canyon Case, C denotes the Canyon Case, R denotes the run
is performed in the REFRAC model, S denotes the SWAN model is used and the
suffixes _f and _c denote the fine and coarse grids respectively. The results of all runs
in table 3 will be given in Appendix C as a reference for those who have further
interest.

Table 3: Summary of the tests and corresponding codes in Wave Propagation
Test of Canyon Case

<table>
<thead>
<tr>
<th>DIR (°N)</th>
<th>Resolution</th>
<th>T (s)</th>
<th>REFRAC</th>
<th>SWAN</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Nx × Ny</td>
<td>∆x × ∆y</td>
<td>∆θ</td>
<td>Rays</td>
</tr>
<tr>
<td>272</td>
<td>198×168</td>
<td>20×20</td>
<td>0.5°</td>
<td>15.4</td>
</tr>
<tr>
<td></td>
<td>79×67</td>
<td>50×50</td>
<td>0.5°</td>
<td>15.4</td>
</tr>
<tr>
<td>240</td>
<td>198×168</td>
<td>20×20</td>
<td>0.5°</td>
<td>15.4</td>
</tr>
<tr>
<td></td>
<td>79×67</td>
<td>50×50</td>
<td>0.5°</td>
<td>15.4</td>
</tr>
<tr>
<td>300</td>
<td>198×168</td>
<td>20×20</td>
<td>0.5°</td>
<td>15.4</td>
</tr>
<tr>
<td></td>
<td>79×67</td>
<td>50×50</td>
<td>0.5°</td>
<td>15.4</td>
</tr>
<tr>
<td>270</td>
<td>198×168</td>
<td>20×20</td>
<td>5°</td>
<td>15</td>
</tr>
<tr>
<td></td>
<td>79×67</td>
<td>50×50</td>
<td>5°</td>
<td>15</td>
</tr>
</tbody>
</table>

Notes:
- DIR – incoming wave direction;
- °N – a nautical convention, i.e. the direction where the wind or the waves
  come from, measured clockwise from geographic North;
- Nx (Ny) – the number of grids used in computation in x (y) – direction;
- ∆x (∆y) – spatial resolution in x (y) – direction (m);
- ∆θ – directional resolution;
- BOU – the spectrum shape of the incoming waves at the boundary;
- BIN – energy is located in one frequency bin;
- JONs – JONSWAP frequency spectrum of the incoming waves at the boundary;

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3.3.3 Results and Discussion

The wave rays’ pattern with 272 °N as incident direction of the incoming waves obtained in the REFRAC model is shown in Figure 17. The dashed lines represent the contours of bottom depth, the brown area represents the dry land and the red lines represent the computed wave rays.

Compared with Figure 15, the same rays trends are given by the REFRAC mode as those obtained in Mgaen’s model (forward ray tracing model, Magen et al, 2008). It indicates that the waves propagating over the irregular bottom topography can be well re-produced in the REFRAC model.

In the first group (272 °N), the distributions of the significant wave heights are computed in the REFRAC model and the SWAN model. The results from the REFRAC model are shown in Figure 18 and those from the SWAN model are shown in Figure 19. They are both computed under the high spatial resolution (198×168, \(\Delta x = \Delta y = 20\)m). The white area represents the dry land and the colored area represents the distributions of the significant wave heights.
Comparing the SWAN (Figure 19) results with the REFRAC results (Figure 18), small differences which are indicated as purple circles can be found in Figure 18 and 19. At the positions of number 1, 2 and 3, the SWAN model gives slightly higher wave energy than the REFRAC model, while in the purple circle of no.4, wave energy computed in the SWAN model is more evenly distributed than it computed in the REFRAC model. It may be caused by the different methods used to obtain the wave heights in the two models. In the REFRAC model, the significant wave heights are directly calculated by the rays while in the SWAN model, it is based on the spectral action balance equation. In spite of the slight differences, the results obtained in SWAN can are considered as being about equal to those obtained in the REFRAC model on the whole. Wave heights in the REFRAC model is only an indication.
Figure 19: Distribution of significant wave heights in Canyon Case obtained in the SWAN model – CS01_f

Figure 20: Distribution of significant wave heights in Canyon Case obtained in the REFRAC model – CR01_c
Under the low spatial resolution (79×67, $\Delta x=\Delta y=50\text{m}$), the distributions of the significant wave heights obtained in the REFRAC model are shown in figure 20 and those obtained in the SWAN model are shown in Figure 21. With the increasing of the mesh size from 20 meters to 50 meters, it is expected that many details of the results disappear. In spite of the fact that accuracy is prerequisites to the model consuming, the required time is also critical. In large scale model, balance has to be found between the accuracy and the required time. As indicated in circles 1, 2 and 3, the SWAN model (Figure 21) gives higher wave heights than the REFRAC model (Figure 20). It is similar with the model under the high spatial resolution. The variation of the significant wave height obtained in the REFRAC model is not as great as it in the SWAN model in those areas. It may be caused by the smoothing technique which is sensitive to the spatial resolution. When the number of all grids contained in the domain is too low and the AVG value is too high, the average effect of the smoothing technique will become too strong. It will be hard to keep the local details of the results. The wave height on one specific grid obtained in the REFRAC model will become to be global. It also indicates that the wave height is more sensitive to the location in the SWAN model than in the REFRAC model because the variation of amplitudes is stronger in the SWAN model than in the REFRAC model.
The distributions of the significant wave heights obtained in the SWAN model are shown in Figure 22. Comparing it with Figure 16 (Gerosthathis et al., 2005) [6], they show fairly good agreements with each other in general except small differences. A slight increase of the wave energy can be found in the pink circle. It may be resulted from the fact that the directional distribution is set as a hyperbolic cosine-type directional spreading function (Donelan et al. 1985) in results of Gerosthathis (Figure 16). However, in SWAN model, the directional distribution is given by

$$D(\theta) = A_m \cos^m(\theta),$$

in which $m$ determines the width of the directional distribution of wave energy (in Figure 22, it was set as $m=2$ or 31.5°) and $A_p$ is a function of $p$ such that the integral of $D(\theta)$ in the directional sector of the incoming wave components at the up-wave boundary is unity. [7]

The conclusion can be drawn that the implementation of the REFRAC model shows good function to plot the wave rays. It is reliable to be applied to the Oosterschelde Estuary. The application is aimed to explore the sensitive areas with rapid spatial variations of the amplitudes (such as caustics and focal points). The effects of refraction in the SWAN model are validated by comparing the wave energy distributions obtained in the SWAN model with they are obtained in the REFRAC model in the Canyon Case. It shows the refraction of the waves is well simulated in the SWAN model.
3.4 Oosterschelde Case

3.4.1 Methodology

In the applications of the Oosterschelde Case in Wave Propagation Tests, four groups of runs with different incoming wave directions (280ºN, 300ºN, 330ºN and 350ºN) are selected to be studied. These four directions are four representative conditions of the swell waves in the Oosterschelde Estuary under conditions of no.6 storm event (Table 1 for details) because the directions of the swell waves at the boundaries are mainly distributed in the range of 280 ºN and 350 ºN, as shown in Figure 5 and Figure 6. Similar to the Canyon Case, in each group, two runs with different spatial resolutions are performed in the REFRAC model and the SWAN model.

In the REFRAC model, the wave rays combined with the location of the buoy OS4 are plotted. It aims to investigate whether the buoy OS4 is located in areas with rapid spatial variation of amplitudes. The significant wave heights are also computed in the REFRAC model to provide a reference to those obtained in the SWAN model.

In the SWAN model, in order to simulate the unidirectional waves, the incoming waves at the boundaries in all runs are modeled as waves with BIN shape of the frequency spectrum for the compatibility of comparison. The period is set as 12 seconds. The directional spreading of the incoming waves is set as 2º and the directional resolution $\Delta \theta$ is set as 0.5º. The results computed in the SWAN model are only shown in the form of the distributions of the significant wave heights.

The codes of all runs and the main input settings are summarized in Table 4, in which O denotes the Oosterschelde Case, R denotes it is performed in the REFRAC model, S denotes it is performed in the SWAN model, suffixes _f and _c denote the fine grids (high resolution) and the coarse grids (low resolution). used in the test. The results of all runs in Table 4 can be found in Appendix C.

<table>
<thead>
<tr>
<th>DIR (ºN)</th>
<th>Resolution</th>
<th>T (s)</th>
<th>REFRAC</th>
<th>SWAN</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Nx × Ny</td>
<td>$\Delta x \times \Delta y$</td>
<td>$\Delta \theta$</td>
<td>Rays</td>
</tr>
<tr>
<td>280 ºN</td>
<td>215×300</td>
<td>20×20</td>
<td>0.5º</td>
<td>12</td>
</tr>
<tr>
<td></td>
<td>86×120</td>
<td>50×50</td>
<td>0.5º</td>
<td>12</td>
</tr>
<tr>
<td>300 ºN</td>
<td>215×300</td>
<td>20×20</td>
<td>0.5º</td>
<td>12</td>
</tr>
<tr>
<td></td>
<td>86×120</td>
<td>50×50</td>
<td>0.5º</td>
<td>12</td>
</tr>
<tr>
<td>330 ºN</td>
<td>215×300</td>
<td>20×20</td>
<td>0.5º</td>
<td>12</td>
</tr>
<tr>
<td></td>
<td>86×120</td>
<td>50×50</td>
<td>0.5º</td>
<td>12</td>
</tr>
<tr>
<td>350 ºN</td>
<td>215×300</td>
<td>20×20</td>
<td>0.5º</td>
<td>12</td>
</tr>
<tr>
<td></td>
<td>86×120</td>
<td>50×50</td>
<td>0.5º</td>
<td>12</td>
</tr>
</tbody>
</table>

Note: The meaning of the symbols and the denotations is the same as it in Table 3.
3.4.2 Results and Discussion

The wave rays’ patterns that used 300 °N and 350 °N as incoming wave directions are shown in Figure 12 and Figure 23 respectively. The blue areas represent the contours of bottom depth, the green areas represent the dry land, the red lines represent the computed way rays and the black points represent the buoy OS4.

As can be seen in Figure 12 and 24, the wave rays’ pattern with 300 °N as incident direction of incoming waves is quite different from it with 350 °N. It shows that the variation of amplitudes in certain area is sensitive to the incident directions of the waves at the boundary.

In Figure 12, the wave buoy OS4 happens to be located on the envelope of several adjacent wave rays, in other words, the position of the OS4 coincided with the caustics. It indicates that the significant wave height at wave buoy OS4 may be influenced by the diffraction effects because it is located in areas with rapid variation of amplitudes. The wave energy becomes extremely sensitive to the location so that even small variation of the location will lead to different significant wave heights in these areas.

From the rays’ pattern of the whole domain, several areas with caustics and focal points can be found. It means that the rapid variation of the wave amplitudes in those areas perhaps influence the wave energy distribution in the Oosterschelde Estuary. Therefore it is necessary to investigate the effects of diffraction.
In Figure 23, the location of OS4 is not located on either caustics or focal points. Thereby the wave energy at buoy OS4 is relative low. Compare it with Figure 12 (280 °N), the expected diffraction effects may be minor when because the wave rays do not intersect too much in the domain in Figure 23 (350 °N). It indicates the variation of the amplitudes is not so rapid. However the wave energy at buoy OS4 may increase if involving the diffraction effects. Since OS4 is located on areas with low amplitudes, the energy in neighboring areas with rapid amplitudes variation may be turned toward to areas with low amplitudes such as OS4 by the effects of diffraction. The consideration of the diffraction effects is therefore necessary.

The significant wave heights are computed in the REFRAC model and the SWAN model under two spatial resolutions. The results in the group with 300 °N as incident direction are shown in Figure 24, 25 and 26. They are the results obtained in the REFRAC model under high spatial resolution (Figure 24), in the SWAN model under high spatial resolution (Figure 25) and in the SWAN model under low spatial resolution (Figure 26), respectively.

![Figure 24: Distribution of significant wave heights in Oosterschelde Case obtained in the REFRAC model – OR02_f](image)
Figure 25: Distribution of significant wave heights in Oosterschelde Case obtained in the SWAN model – OS02_f

Figure 26: Distribution of significant wave heights in Oosterschelde Case obtained in the SWAN model – OS02_c
Comparing Figure 25 with Figure 24, the results obtained in the SWAN model are generally the same as those obtained in the REFRAC model in spite of slight differences. As indicated as purple and yellow circles in Figure 24 and 25, the significant wave heights computed in the SWAN model are lower in purple circles and higher in the yellow circle no.3 than those obtained in the REFRAC model. It indicates that the wave energy computed in SWAN model distributes more evenly than it computed in the REFRAC model. Probably it is caused by the insufficient smoothing technique (AVG value is too low). It can also be resulted from the effects of directional dispersion because the directional spreading used in the SWAN model is different from it used in the REFRAC model.

By decreasing the spatial resolution from 198×168 (Δx=Δy=20m) to 79×67 (Δx=Δy=50m), the results obtained in the REFRAC model and the SWAN model are also acceptably accurate. However, in the areas indicated in purple circles 1 and 2 in Figure 26, the significant wave heights obtained under the low spatial resolution are slightly lower than they are under the high resolution in the SWAN model. The spatial resolution can be regarded as an influence aspect to be further investigated.

3.5 Conclusion

Comparing the rays plotted in the REFRAC model in the Canyon Case with those plotted by Magen’s model (Figure 5, Magen et al., 2008) [5], they show fairly good agreements with each other. Since the effects of refraction have been validated in Magen’s model, it shows that the REFRAC model has good implementation to simulate the effects of refraction in wave propagation process as well. Therefore the resultant wave rays can be used to explore the areas with rapid variation of the amplitudes. In the Oosterschelde Estuary, both caustics and focal points have been found according to the wave rays’ pattern obtained in the REFRAC model. It suggests that it may be necessary to take the effects of diffraction into consideration in the hindcast studies of the Oosterschelde Estuary.

Moreover, by comparing the significant wave heights computed in the SWAN model with those computed in the REFRAC model, the implementation of the refraction effects in the SWAN model has been validated. The results of the Canyon Case and the Oosterschelde Case show that the effects of refraction in SWAN are fairly well simulated.
4. Diffraction Tests

4.1 Outline
The numerical computation of a wave field for practical purposes is by no means a simple matter. It even gets more difficult when both refraction and diffraction play roles in the domain. A set of simple but classical academic cases are therefore chosen to be performed in this study to test the diffraction implementation in SWAN. These cases (such as waves pass through the breakwaters, propagates along the channel, gully or over an undersea slope) are with outstanding diffraction phenomena and can be treated easily thereby the analytical solutions and the measured data from laboratory are available to be compared with the results obtained in SWAN. Then the characteristics and implementation of the diffraction functionality in SWAN can be studied and validated. Then the application of SWAN with diffraction to more complicated case will become easier owing to the sufficient experience gained in the academic cases studies.

In this chapter, the introduction to diffraction, phase-decoupled diffraction in SWAN model and some other functionality relevant to diffraction implementation in SWAN such as smoothing technique, under-relaxation technique and stopping criteria will be briefly discussed first. Next, three academic cases will be performed in SWAN to validate the implementation of diffraction. They are Semi-infinite Breakwater Case, Gap in Infinite Breakwater Case and the Ridge Case respectively. In each case, several sensitive control tests concerning on spatial resolution, numerical technique (under-relaxation technique with control parameter \( \alpha \)), diffraction (smoothing technique) and different versions of SWAN (SWAN 4041.AB vs. SWAN 4072) will be performed. The purpose of these three academic cases is aimed to validate the diffraction implementation in SWAN and investigate the relation between those functionalities and diffraction in SWAN.

4.2 Background Knowledge

4.2.1 Diffraction
According to the definition, the phenomenon is called diffraction when the rapid spatial variation in amplitude causes the waves to turn into the areas with lower amplitude. If the waves propagate in shallow water over a non-horizontal bottom, with rapid spatial variations in wave amplitude, then both refraction and diffraction need to be accounted for (Holthuijsen, 2007). [10] When the depth-induced changes in amplitude and direction are sufficiently slow (small over the distance of one wave length), the linear wave theory for waters with a horizontal bottom can be used locally. However, occasionally the variations in amplitude are not so slow that the diffraction can not be ignored. Therefore the linear wave theory needs to be expanded. In fluid dynamics, the mild-slope equation describes the combined effects of diffraction and refraction for water waves propagating over irregular bottom and due to lateral boundaries — like breakwaters and coastlines.

Summarized by Holthuijsen et al., 2003: ‘The effects of diffraction are traditionally computed with phase-resolving models, like Boussinesq (e.g., Peregrine, 1966; Madsen and Sørensen 1992; Li and Zhan 2001; Borsboom et al., 2001) or mild-slope models (e.g., Battjes, 1968; Schönfeld, 1971; Holthuijsen, 1971; Berkhoof, 1972;
Radder, 1979; Booij, 1981; Kirby, 1986). However, this type of models does not, or only to a limited degree, account for the generation, dissipation and wave-wave interactions of the waves. For phase-averaged models, they are usually used to compute the wave conditions in the coastal zone where diffraction does not play a role since this type of models does not account for diffraction. Thereby add diffraction to a phase-average model (spectral model) or add the generation, dissipation and wave-wave interactions of waves to a phase-resolving model are two attempts. As concluded by Holthuijsen et al. 2003, adding diffraction to a spectral model has the advantage that (a) large-scale computations remain perfectly feasible (as long as the require high spatial resolution in the diffraction regions are retained, e.g., by nesting or by using a variable – resolution grid; in contrast to this, phase-resolving models require a prohibitively high spatial resolution over the entire computation region), (b) the random, short-crested character of the waves is inherent (in contrast to this, mild-slope models would require multiple runs) and (c) the formulations for all processes of generation and dissipation and wave-wave interactions are included (in contrast to this, phase-resolving models do not include all of these). Therefore, several attempts have been made to include the effects of diffraction in spectral wave models. One approach is to mimic diffraction with spatial or spectral diffusion (e.g., Resio, 1988; Booij et al., 1997; Mase et al., 2001). This approach has been shown to simulate some of the diffraction effects, but either the effects are limited (e.g., the diffraction induced turning of the wave direction is not simulated) or the numerical schemes are not stable. Another approach is to add the diffraction-induced turning rate of the waves (obtained from the mild-slope equation) to a spectral model. This was suggested by Booij et al. (1997) and Rivero et al. (1997). The SWAN model was one of them.

To accommodate diffraction in SWAN simulations, a phase-decoupled refraction-diffraction approximation is suggested (Holthuijsen et al, 2003) [2]. It is expressed in terms of the directional turning rate of the individual wave components in the 2D wave spectrum obtained from the mild-slope equation. The approximation is based on the mild-slope equation for refraction and diffraction, omitting phase information. It does therefore not permit coherent wave fields in the computational domain.

The SWAN model uses the action balance to compute the evolution of the wave field in time and space. This equation is given by:

\[
\frac{\partial N}{\partial t} + \frac{\partial (c_x N)}{\partial x} + \frac{\partial (c_y N)}{\partial y} + \frac{\partial (c_\sigma N)}{\partial \sigma} + \frac{\partial (c_\theta N)}{\partial \theta} = \frac{S}{\sigma} \tag{4.1}
\]

where the action density is defined as \( N = N(\sigma, \theta)/\sigma \), \( \sigma \) is the relative frequency. The velocity \( c_\sigma \) is the propagation speed in frequency space and \( c_\theta \) is the propagation speed due to refraction. The term \( S \) is the source term describing all the physical processes of growth, decay and redistribution of wave energy. In the case without diffraction the propagation velocities in x-y and spectral space are given by (Holthuijsen et al., 2003):

\[
\vec{c} = (c_x, c_y) = \frac{\vec{\kappa}}{\kappa} \frac{\partial \sigma}{\partial k} + \vec{U} \tag{4.2}
\]
\[ c_\sigma = \frac{\partial \sigma}{\partial d} \left[ \frac{\partial d}{\partial t} + \vec{U} \cdot \nabla d \right] - c_0 \frac{\partial U}{\partial s} \]  
(4.3)

\[ c_\theta = - \left[ c_0 \left( \frac{1}{\kappa} \frac{\partial \kappa}{\partial m} + \frac{\kappa}{\kappa} \frac{\partial \vec{U}}{\partial m} \right) \right] \]  
(4.4)

In these equations \( \vec{U} \) is the ambient current vector, \( d \) is depth and \( m \) is the local coordinate perpendicular to the direction of wave propagation. In the absence of diffraction the separation parameter \( \kappa \) is equal to the wave number \( k \).

If diffraction is accounted for, a correction term \( \delta_E \) is added to the above propagation velocities. The equations then become:

\[ \tilde{C}_\kappa = \tilde{c}_\kappa \left( 1 + \delta_E \right)^{1/2} + \vec{U} \]  
(4.5)

\[ C_\sigma = \frac{\partial \sigma}{\partial d} \left[ \frac{\partial d}{\partial t} + \vec{U} \cdot \nabla d \right] - c_0 \frac{\partial U}{\partial s} \left( 1 + \delta_E \right)^{1/2} \]  
(4.6)

\[ C_\theta = - \left[ c_0 \left( 1 + \delta_E \right)^{1/2} \left( \frac{1}{\kappa} \frac{\partial \kappa}{\partial m} + \frac{1}{2 \left( 1 + \delta_E \right)} \frac{\partial \delta_E}{\partial m} \right) \right] \frac{\kappa}{\kappa} \frac{\partial \vec{U}}{\partial m} \]  
(4.7)

The diffraction parameter \( \delta_E \) is based on a spatial average of properties of the wave field according to:

\[ \delta_E = \frac{\nabla \cdot \left( c_c \nabla \sqrt{E} \right)}{\kappa^2 c_c \sqrt{E}} \]  
(4.8)

where \( c \) and \( c_g \) are the phase and group velocity, respectively. The diffraction parameter \( \delta_E \) adds a second order spatial derivative to the system of equations.

Detailed introduction to phase-decoupled diffraction and derivation of the diffraction parameter \( \delta_E \) will be given in Appendix A.

### 4.2.2 Smoothing Technique

The SWAN model has a number of switches to control diffraction. Apart from a general switch for turning diffraction on or off, there are two parameters that control the evaluation of the diffraction parameter \( \delta_E \): SMPAR and SMNUM. To avoid numerical instabilities in the evaluation of the diffraction parameter, its evaluation is based on a smoothed field of wave energy between neighboring spatial grid points. The SMPAR parameter controls the amount of smoothing between neighboring grid points. The parameter SMNUM controls the number of times that a smoothing operation is performed.

Smoothing is performed on the basis of the directionally integrated energy \( E(\sigma) \) in neighboring spatial grid points according to:
\[ E_{i,j}^n = E_{i,j}^{n-1} - S\text{MPAR} \times \left[ E_{i-1,j} + E_{i,j-1} + E_{i+1,j} + E_{i,j+1} \right]^{n-1} \]  

(4.9)

where \( i, j \) is a grid point, the superscript \( n \) indicates iteration number of the convolution. The width of this filter (standard deviation) in x-direction \( \varepsilon_x \), when applied \( n \) times is:

\[ \varepsilon_x \approx \frac{1}{2} \sqrt{3n \Delta x} \]  

(4.10)

As suggested in SWAN USER MANUAL, \( n = 6 \) is found to be an optimum value (when the spatial resolution is around 1/5 ~ 1/10 of the dominant wavelength), so that \( \varepsilon_x \approx 2 \Delta x \). For the y-direction, the expressions are identical, with \( y \) replacing \( x \).

Note that the smoothing of wave fields is used for the computation of the diffraction parameter \( \delta_E \). For all other computations the wave field is not smoothed. [11]

The investigation in terms of the implementation of the phase-decoupled diffraction and the effects of the smoothing technique in SWAN will be performed by means of series of runs with different number of smoothing steps (\( n \) value) in the following three academic cases.

### 4.2.3 Frequency-dependent Under-relaxation Technique

Of practical interest is the convergence behavior of the SWAN model. Irregular convergence behavior is frequently encountered when computations of wave spectra are performed in SWAN. The finite difference scheme in SWAN is not suited to approximate discontinuities and singularities in the diffraction-induced turning rate of the waves at the tips of the breakwaters or other obstacles. This can be dealt by applying frequency-dependent under-relaxation (Zijlema et al., 2004). [16]

The key idea of the under-relaxation approach is to link the extent of updating to the wave frequency – the larger the frequency, the smaller the updates. Therefore it underlies a rout to steady state. This method complies with the principle of decreasing time scales at higher frequencies, which is inherent to the evolution of wind waves. As a result, the improved SWAN model is free from numerical restrictions to spectral shape in the non-equilibrium range. (Zijlema et al., 2004)

The frequency-dependent under-relaxation technique is controlled by the parameter [alpha] which is defined as proportionality constant used in the frequency-dependent under-relaxation technique. In the following three academic tests, the under-relaxation technique will be studied by means of several sensitive control runs with different \( \alpha \) value. Thereby, the relation to the convergence behavior and the characteristics of the under-relaxation technique in SWAN will be investigated.

### 4.2.4 Stopping Criteria

In general, the iterative method should be stopped if the approximation solution is accurate enough. A good termination criterion is very important, because if the criterion is too weak the solution obtained may be useless, whereas if the criterion is too severe the iteration process may never stop or may cost too much work.
In SWAN, there are two options of the stopping criteria can be chosen to influence some of the numerical properties. They are command of ‘ACCUR [drel] [dhoval] [dtoval] [npnts]’ and command of ‘STOPC [dabs] [drel] [curvat] [npnts]’ respectively.

As guided in SWAN UNSER MANNUAL, when with the one so-called ACCUR, SWAN stops the iteration if:

a) the change in the local significant wave height (Hs) from one iteration to the next is less than
   1) fraction [drel] of that height or
   2) fraction [dhoval] of the average significant wave height (average over all wet grid points)

and

b) The change in the local mean wave period \( T_{m0} \) from one iteration to the next is less than
   1) fraction [drel] of that period or
   2) fraction [dtoval] of the average mean wave period (average over all wet grid points)

and

c) conditions a) and b) are fulfilled in more than fraction [npnts]% of all wet points.

However, as concluded in SWAN TECHNICAL DOCUMENTATION [11], it has become apparent that the quantity \( T_{m0} \) is not an effective measure of convergence. In addition, the research purpose in this study is mainly focused on the wave energy distribution which is more related to the wave heights instead of the wave period, thereby the command ACCUR which is partially based on the significant wave heights and partially based on the wave period will not be used.

An alternative way to evaluate the level of numerical convergence is to consider the second derivative or curvature of the iteration curve of the significant wave height. It leads to the so-called STOPC stopping criterion which is solely based on the significant wave height.

The curvature of the iteration curve of \( H_{m0} \) is expressed in the discrete sense as:

\[
\Delta \left( \Delta \tilde{H}_{m0}^s \right)^{ij} = \tilde{H}_{m0}^s - 2 \tilde{H}_{m0}^{s-1} + \tilde{H}_{m0}^{s-2}
\]  

(4.11)

where \( \tilde{H}_{m0}^s \) is some measure of the significant wave height at iteration level \( H_{m0}^{s-3} \). To eliminate the effect of small amplitude oscillations on the curvature measure, we define \( \tilde{H}_{m0}^s = (H_{m0}^s + H_{m0}^{s-1}) / 2 \). The resulting curvature-based termination criterion at grid point \((i, j)\) is then:

\[
\left| \frac{H_{m0}^s(i, j) - (H_{m0}^{s-1}(i, j) + H_{m0}^{s-2}(i, j))}{2H_{m0}^s(i, j)} \right| < \varepsilon_c, \ s = 3, 4, ..., \]  

(4.12)

In the command of ‘STOPC [dabs] [drel] [curvat] [npnts]’, as the solution of a simulation approaches full convergence the curvature of the iteration curve will tend
to zero. SWAN stops the process if the absolute change in the local significant wave height (Hs) from one iteration to the next is less than \( [dabs] \) or the relative change in the local significant wave height from one iteration to the next is less than \( [drel] \) and the curvature of the iteration curve of \( H_s \) normalized with \( H_s \) is less than \( [curvat] \).

[3] In current studies, the second option, i.e. STOPC command, will be used to determine the numerical convergence. The corresponding parameters used in all academic cases were set as the following values:

\[
[dabs] = 0 \quad \quad [drel] = 0.01 \quad \quad [curvat] = 0.001 \quad \quad [npnts] = 100
\]

Meanwhile, parameter called \( [mxitst] \) in SWAN decides the number of iterations performed in computation directly. It denotes the maximum number of iterations for stationary computations. The computation stops when this number is exceeded. In current studies, it will set as 200 in all runs which meant that the test stops even it does not converge after 200 iterations.

### 4.3 Semi-infinite Breakwater Case

#### 4.3.1 Model Description

In most cases harbors are located along a shallow coast where both refraction and diffraction affect the penetration of waves into a harbor. Information on the penetration of waves behind breakwaters is important for the design of harbors and to assess the safety of mooring systems. The classical analytical solution which based on potential flow and linear wave assumptions was developed by Sommerfeld (1896) for wave diffraction around a semi-infinite breakwater. (Enet et al., 2005) [14] It can be provided as a reference to be compared against the numerical results. Therefore, a simple situation that is the semi-infinite breakwater in water of constant depth and omitting the growth by wind and dissipation by other effects becomes the first step to verify the implementation of diffraction in SWAN. The prototype of the case including physical and numerical parameters performed here is mainly based on the model performed by Holthuijsen et al. (2003, section 5.3.1 of [2]) Since the diffraction approximation in SWAN does not properly handle diffraction in front of reflecting obstacles, the effects of reflection will not be considered in present study.

Assume a situation with a semi-infinite long, infinitely thin, vertical, rigid, impermeable straight breakwater in an infinite body of water with constant depth. Unidirectional and monochromatic waves approach the breakwater perpendicularly from one side. Therefore a rectangular domain with four equal sides of 15 m over a flat bottom with constant water depth of 0.72m is modeled in SWAN (shown in Figure 27) and the conditions of the incoming waves are set as 0.055m as significant wave height and 1.3s as wave period. Thereby the corresponding wave length is 2.5m \((L=2.5m)\) according to the dispersion relation.

**Figure 27: Sketch of the domain in Semi-infinite Breakwater Case**
The semi-infinite breakwater is modeled as an obstacle object with zero reflection and zero transmission according to the physical situation. The location of the breakwater is shown in Figure 27. Along all up-wave boundaries (south and west sides of the domain) an incoming wave is prescribed. It is specified by a narrow BIN-shaped frequency spectrum centered on the period (T) of 1.3 s, with 0.5385 Hz as lowest discrete frequency and 1 Hz as highest discrete frequency, and with 7 as the number of frequencies, i.e., grid resolution in frequency-space. The directional spreading ($\sigma_\theta$) is set as 1.5° and the directional resolution ($\Delta \theta$) is set as 0.25°. The growth by wind, quadruplet wave-wave interaction and the dissipation by triad wave-wave interaction, bottom friction, depth-induced breaking and whitecapping are omitted in present case.

### 4.3.2 Methodology

The performance of diffraction in SWAN is tested in two groups in Semi-infinite Breakwater Case. They are the uniform rectangular computational spatial grid and the curvi-linear computational spatial grid respectively.

In the uniform spatial resolution group, there are two groups of runs with different spatial resolution which are 0.25 m ($\Delta x = \Delta y = L / 10 = 0.25m$) and 0.5m ($\Delta x = \Delta y = L / 5 = 0.5m$) respectively. It aims to test the influence of different spatial resolution to the diffraction implementation in SWAN. In both high and low spatial resolution groups, it always starts with a computation without diffraction to check whether the results are dependable. Afterwards, the concentration is focused on the implementation of diffraction in SWAN. The influence of the under-relaxation technique and the convolution filter to diffraction implementation are tested respectively. The under-relaxation in SWAN is tested by performing the model under the same spatial resolution and smoothing technique but with different under-relaxation parameters (\(\alpha = 0.01\) vs. \(\alpha = 0.1\)). The effects of smoothing technique are tested by finding an optimal number of smoothing steps, thereby a series of simulations are conducted in which the number of smoothing steps increases systematically. Basically, the simulations with zero smoothing step (without smoothing technique), with smallest number of smoothing steps required to obtain a stable solution within 200 iterations, with 6 smoothing steps (recommended as optimal in SWAN USER MANUAL) and with a relative large number of smoothing steps (corresponding results are used to be compared against the other results) are recorded.

The details of the SWAN input settings and the convergence behaviors of all runs are summarized in Table 5, where S denotes the Semi-infinite Breakwater Case, U denotes the Uniform grid, H denotes the High spatial resolution and L denotes the Low spatial resolution. The results are shown in the form of the distribution of the wave directions combined with the contours of the normalized wave heights (local height / incident wave height) in the whole domain. In addition, the normalized wave heights along the circle-section at 3.0 times wavelength L (circle-section is shown in Figure 27) are also given. The results of all runs will be given in Appendix C as a reference for those who have further interest (all results of the other two cases performed in this study can also be found in Appendix C).
Table 5: Summary of codes, input settings and corresponding convergence behaviors of all runs in the uniform grid in Semi-infinite Breakwater Case

<table>
<thead>
<tr>
<th>Codes</th>
<th>GRID</th>
<th>Diffr</th>
<th>α</th>
<th>Mxitst</th>
<th>Real</th>
<th>Accuracy</th>
<th>Fig.</th>
</tr>
</thead>
<tbody>
<tr>
<td>SUH01</td>
<td>60x60</td>
<td>0.25m</td>
<td>Off</td>
<td>/</td>
<td>0</td>
<td>200</td>
<td>4</td>
</tr>
<tr>
<td>SUH03</td>
<td>60x60</td>
<td>0.25m</td>
<td>On</td>
<td>0.01</td>
<td>200</td>
<td>200</td>
<td>72.48%</td>
</tr>
<tr>
<td>SUH05</td>
<td>60x60</td>
<td>0.25m</td>
<td>On</td>
<td>0.01</td>
<td>200</td>
<td>200</td>
<td>96.99%</td>
</tr>
<tr>
<td>SUH06</td>
<td>60x60</td>
<td>0.25m</td>
<td>On</td>
<td>0.01</td>
<td>200</td>
<td>200</td>
<td>3.57%</td>
</tr>
<tr>
<td>SUH07</td>
<td>60x60</td>
<td>0.25m</td>
<td>On</td>
<td>0.01</td>
<td>200</td>
<td>200</td>
<td>93.34%</td>
</tr>
<tr>
<td>SUH08</td>
<td>60x60</td>
<td>0.25m</td>
<td>On</td>
<td>0.01</td>
<td>200</td>
<td>200</td>
<td>100%</td>
</tr>
<tr>
<td>SUH03~</td>
<td>60x60</td>
<td>0.25m</td>
<td>On</td>
<td>0.1</td>
<td>200</td>
<td>200</td>
<td>70.46%</td>
</tr>
<tr>
<td>SUH05~</td>
<td>60x60</td>
<td>0.25m</td>
<td>On</td>
<td>0.1</td>
<td>200</td>
<td>200</td>
<td>92.07%</td>
</tr>
<tr>
<td>SUH06~</td>
<td>60x60</td>
<td>0.25m</td>
<td>On</td>
<td>0.1</td>
<td>200</td>
<td>200</td>
<td>95.22%</td>
</tr>
<tr>
<td>SUH07~</td>
<td>60x60</td>
<td>0.25m</td>
<td>On</td>
<td>0.1</td>
<td>200</td>
<td>24</td>
<td>100%</td>
</tr>
<tr>
<td>SUH08~</td>
<td>60x60</td>
<td>0.25m</td>
<td>On</td>
<td>0.1</td>
<td>200</td>
<td>19</td>
<td>100%</td>
</tr>
<tr>
<td>SUL01</td>
<td>30x30</td>
<td>0.5m</td>
<td>Off</td>
<td>/</td>
<td>0</td>
<td>200</td>
<td>4</td>
</tr>
<tr>
<td>SUL03</td>
<td>30x30</td>
<td>0.5m</td>
<td>On</td>
<td>0.01</td>
<td>200</td>
<td>200</td>
<td>60.87%</td>
</tr>
<tr>
<td>SUL05</td>
<td>30x30</td>
<td>0.5m</td>
<td>On</td>
<td>0.01</td>
<td>200</td>
<td>200</td>
<td>85.54%</td>
</tr>
<tr>
<td>SUL06</td>
<td>30x30</td>
<td>0.5m</td>
<td>On</td>
<td>0.01</td>
<td>200</td>
<td>35</td>
<td>100%</td>
</tr>
<tr>
<td>SUL07</td>
<td>30x30</td>
<td>0.5m</td>
<td>On</td>
<td>0.01</td>
<td>200</td>
<td>14</td>
<td>100%</td>
</tr>
<tr>
<td>SUL08</td>
<td>30x30</td>
<td>0.5m</td>
<td>On</td>
<td>0.01</td>
<td>200</td>
<td>9</td>
<td>100%</td>
</tr>
<tr>
<td>SUL03~</td>
<td>30x30</td>
<td>0.5m</td>
<td>On</td>
<td>0.1</td>
<td>200</td>
<td>200</td>
<td>35.69%</td>
</tr>
<tr>
<td>SUL05~</td>
<td>30x30</td>
<td>0.5m</td>
<td>On</td>
<td>0.1</td>
<td>200</td>
<td>200</td>
<td>80.33%</td>
</tr>
<tr>
<td>SUL06~</td>
<td>30x30</td>
<td>0.5m</td>
<td>On</td>
<td>0.1</td>
<td>200</td>
<td>17</td>
<td>100%</td>
</tr>
<tr>
<td>SUL07~</td>
<td>30x30</td>
<td>0.5m</td>
<td>On</td>
<td>0.1</td>
<td>200</td>
<td>17</td>
<td>100%</td>
</tr>
<tr>
<td>SUL08~</td>
<td>30x30</td>
<td>0.5m</td>
<td>On</td>
<td>0.1</td>
<td>200</td>
<td>17</td>
<td>100%</td>
</tr>
</tbody>
</table>

Notes:
- **Nx (Ny)** – the number of grids used in computation in x-direction (y-direction);
- **Δx (Δy)** – spatial resolution in x-direction (y-direction) (m);
- **DIR** – direction of the incoming wave (ºN);
- **σ₀** – directional width of the incoming wave (º);
- **Δθ** – directional resolution of the incoming wave (º);
- **State** – whether the diffraction was taken into consideration;
- **n** – whether the smoothing technique was activated in SWAN; n=0 → without smoothing technique; n≠0 → the number of smoothing steps used in SWAN;
- **α** – proportionality constant used in the frequency-dependent under-relaxation;
- **Mxitst** – the maximum number of iterations, the computation stops when this number is exceeded;
- **Real** – is the real number of iterations in computation to fulfill the stopping criteria;
- **Accuracy** – final fraction of all wet grid points fulfilled stopping criteria at 200 iteration cycles;
- **Fig.** – √ the results are convergent and the Figures of results are given; × the results do not converge and no result is available;
It always requires more time when diffraction effects counted in the computation in SWAN. One of the reasons is that the required spatial resolution is rigid. However the application of the curvi-linear grids can solve the problem to some extent by means of a variable-resolution grid in one domain. The advantage of a variable-resolution grid is that the model can be based on a high resolution where diffraction requires this and on a coarser resolution where such high resolution is not necessary. Thereby the Semi-infinite Breakwater Case is further tested under the curvi-linear grid.

Diffraction is particularly strong along the geometric shadow line of the breakwater and the wave conditions are sensitive at the tips of the breakwater. Therefore high spatial resolution is usually applied to those areas. Connected with the so-called buffer areas where intermediate spatial resolution is applied to prevent sudden change of the grid size, the relative low spatial resolution is applied to the area where diffraction does not play important role. Based on these principles, three different variable-resolution grids in the codes of NOR (Figure 28), DOU (Figure 29) and GEN (Figure 30) performed in the curvi-linear group respectively. They are with different spatial resolution and it aims to investigate the diffraction implementation under the curvi-linear grid and the influence of different mesh sizes to the diffraction implementation in SWAN.

![Curvi-linear Grid: NOR](image)

**Figure 28: Sketch of the curvi-linear grid ‘NOR’ in Semi-infinite Breakwater Case**

In the so-called NOR gird, the spatial resolution is set as 0.5m (1/5 L) around the tip of the breakwater and as 2m (4/5 L) in the insensitive area in both x- and y-
directions. Two 1-m-long meshes are set in between as the buffer area. The sketch of the grid NOR is shown in Figure 28. The big black frame indicates the domain used in computation, the small blue frame indicates the area where the results will be shown and the red line indicates the location of the breakwater.

In grid of DOU, all the meshes used in NOR are split, in other words, the size of each mesh in DOU is as half long as it in NOR grid so the number of the grids is then doubled. The detailed spatial resolution is as following: \( \Delta x = \Delta y = 0.25m = L/10 \) around the tip of the breakwater, \( \Delta x = \Delta y = 1m = 2L/5 \) in the insensitive area. Four 0.5-meter-long meshes are set in between as the buffer area (\( \Delta x = \Delta y = 0.5m = L/5 \)). The sketch of the grid DOU is shown in Figure 29. The other icons in Figure 29 have the same meaning as they were in Figure 28.

![Curvi-linear Grid: DOU](image)

*Figure 29: Sketch of the curvi-linear grid ‘DOU’ in Semi-infinite Breakwater Case*

In the last GEN grid, the buffer area between the coarse grids and the fine grids is expanded. The size of the mesh increases much more gently in grid GEN than in the other two grids. The sketch of the grid GEN is indicated in Figure 30. The detailed spatial resolution is as following: \( \Delta x = \Delta y = 0.5m = L/5 \) around the tip of the breakwater, \( \Delta x = \Delta y = 2m = 4L/5 \) in the insensitive area. Two meshes of 0.75m as length, two meshes of 1m as length, two meshes of 1.25m as length and two meshes of 1.5m as length are set as the buffer area in both x- and y- direction. The other icons in Figure 30 have the same meaning as they were in Figure 28 and 29.
Similar to the uniform grid, in the curvi-linear grid group, it always starts with a computation without diffraction to check whether the results are dependable under all three grids.

Afterwards, the concentration is solely focused on the implementation of diffraction in SWAN with respect to the smoothing technique. The effects of under-relaxation are not investigated further thereby the parameter $\alpha$ will be set as 0.01 in all runs in the curvi-linear grid group.

A series of simulations are performed to find an optimal number of smoothing steps, in which the number of smoothing steps increases systematically. However, only several numbers of the smoothing steps are selected and recorded in Table 6. Similar to the uniform grid, these numbers of smoothing steps are generally the simulation with zero number of smoothing step (without smoothing technique), with smallest number of smoothing steps required to obtain a stable solution within 200 iterations, with 6 smoothing steps and with a relative large number of smoothing steps. Details of the SWAN input settings and corresponding numerical convergence behaviors of all tests in the curvi-linear group are summarized in Table 6, where C denotes the Curvi-linear grid. All the other notations are with the same meaning as they were in Table 5.
### Table 6: Summary of codes, input settings and corresponding convergence behaviors of all runs in the curvi-linear grid in Semi-infinite Breakwater Case

<table>
<thead>
<tr>
<th>Codes</th>
<th>Nx×Ny</th>
<th>Diffraction State</th>
<th>α</th>
<th>Mxitst</th>
<th>Real</th>
<th>Accuracy</th>
<th>Fig.</th>
</tr>
</thead>
<tbody>
<tr>
<td>SC-NOR-01</td>
<td>16×17</td>
<td>Off  / 0</td>
<td>0.01</td>
<td>200</td>
<td>4</td>
<td>100%</td>
<td>√</td>
</tr>
<tr>
<td>SC-NOR-02</td>
<td>16×17</td>
<td>On   0</td>
<td>0.01</td>
<td>200</td>
<td>200</td>
<td>21.24%</td>
<td>×</td>
</tr>
<tr>
<td>SC-NOR-03</td>
<td>16×17</td>
<td>On   1</td>
<td>0.01</td>
<td>200</td>
<td>200</td>
<td>13.73%</td>
<td>×</td>
</tr>
<tr>
<td>SC-NOR-04</td>
<td>16×17</td>
<td>On   3</td>
<td>0.01</td>
<td>200</td>
<td>40</td>
<td>100%</td>
<td>√</td>
</tr>
<tr>
<td>SC-NOR-05</td>
<td>16×17</td>
<td>On   6</td>
<td>0.01</td>
<td>200</td>
<td>20</td>
<td>100%</td>
<td>√</td>
</tr>
<tr>
<td>SC-DOU-01</td>
<td>36×31</td>
<td>Off / 0.01</td>
<td>200</td>
<td>4</td>
<td>100%</td>
<td></td>
<td>√</td>
</tr>
<tr>
<td>SC-DOU-02</td>
<td>36×31</td>
<td>On   0</td>
<td>0.01</td>
<td>200</td>
<td>200</td>
<td>69.34%</td>
<td>×</td>
</tr>
<tr>
<td>SC-DOU-03</td>
<td>36×31</td>
<td>On   6</td>
<td>0.01</td>
<td>200</td>
<td>200</td>
<td>15.54%</td>
<td>×</td>
</tr>
<tr>
<td>SC-DOU-04</td>
<td>36×31</td>
<td>On   18</td>
<td>0.01</td>
<td>200</td>
<td>85</td>
<td>100%</td>
<td>√</td>
</tr>
<tr>
<td>SC-DOU-05</td>
<td>36×31</td>
<td>On   20</td>
<td>0.01</td>
<td>200</td>
<td>25</td>
<td>100%</td>
<td>√</td>
</tr>
<tr>
<td>SC-GEN-01</td>
<td>22×19</td>
<td>Off / 0.01</td>
<td>200</td>
<td>4</td>
<td>100%</td>
<td></td>
<td>√</td>
</tr>
<tr>
<td>SC-GEN-02</td>
<td>22×19</td>
<td>On   0</td>
<td>0.01</td>
<td>200</td>
<td>200</td>
<td>21.09%</td>
<td>×</td>
</tr>
<tr>
<td>SC-GEN-03</td>
<td>22×19</td>
<td>On   1</td>
<td>0.01</td>
<td>200</td>
<td>200</td>
<td>24.57%</td>
<td>×</td>
</tr>
<tr>
<td>SC-GEN-04</td>
<td>22×19</td>
<td>On   3</td>
<td>0.01</td>
<td>200</td>
<td>27</td>
<td>100%</td>
<td>√</td>
</tr>
<tr>
<td>SC-GEN-05</td>
<td>22×19</td>
<td>On   6</td>
<td>0.01</td>
<td>200</td>
<td>17</td>
<td>100%</td>
<td>√</td>
</tr>
</tbody>
</table>

Note: The meaning of the symbols and the denotations in current table is the same as it in Table 5.

### 4.3.3 Results and Discussion

When the uniform spatial resolution is applied, it is obvious from the SWAN results (Figure 31) that without diffraction, only a small fraction of the energy penetrates in the geometric shadow area behind the breakwater. Compared with the Sommerfeld solution, the simulated significant wave heights are seriously underestimated and the corresponding normalized significant wave heights are extremely small (min ≈ 10^{-12} m) in the lee of the breakwater. This is essentially due to some numerical diffusion, illustrating the relatively small numerical diffusion of the propagation schemes in SWAN. Obviously, the wave direction shown in that region is no long the directions of the waves but the (spectral) direction of this diffused energy. The significant wave heights are also overestimated in the range of 80-90 degrees along the circular-section of 3L (shown in Figure 33, upper right panel). Therefore it can be concluded that SWAN does not properly reproduce the Sommerfeld solution without diffraction effects.

With the phase-decoupled refraction-diffraction approximation included, SWAN gives a higher estimation of the wave energy in the lee area of the breakwater (right panel of Figure 32). The wave directions represent the direction of the waves even deep inside the shadow area since the significant wave heights are not small enough to be considered as noisy. So SWAN, with diffraction, reproduces the Sommerfeld solution reasonably well near the shadow line and in the exposed region and is certainly much better than SWAN without diffraction. Even the overshoot on the exposed side of the shadow line is reproduced to some extent.
Figure 31: SUL01 – Uniform & Low spatial resolution ($\Delta x=\Delta y=0.5m$) & No diffraction. Upper left panel: distribution of wave directions and the contours of the normalized $H_s$, upper right panel: $H_s$ along circular section at 3.0 times wavelength $L$, including comparison with analytical solution of Sommerfeld, lower panel: distribution of logarithm.

By comparing the normalized significant wave heights along the circular-section at 3 times of the wave lengths of all runs with each other and with the Sommerfeld solution shown in Figure 33 and by comparing the convergence behaviors of all runs summarized in Table 5, the conclusion of the behaviors with regard to the implementation of the frequency-dependent under-relaxation technique, the spatial resolution and the diffraction smoothing technique can be drawn.
For the under-relaxation, as shown in Table 5, it is obvious that the under-relaxation significantly influences the required number of iterations to satisfy the stopping criterion. Under the same number of smoothing steps (n=15) in the finer spatial grid ($\Delta x = \Delta y = 0.25m = L/10$), higher $\alpha$ value needs smaller number of iterations to reach 100% prescribed accuracy while lower $\alpha$ value needs larger number of iterations (SUH08, $\alpha=0.1$, iterations=19 vs. SUH08, $\alpha=0.01$, iterations=27). However, in the coarser spatial grid ($\Delta x = \Delta y = 0.5m = L/5$), under the same number of smoothing steps (n=6), higher $\alpha$ value needs larger number of iterations to reach 100% prescribed accuracy while lower $\alpha$ value needs smaller number of iterations (SUL08, $\alpha=0.1$, iterations=17 vs. SUL08, $\alpha=0.01$, iterations=14). The relation between the required number of iterations and the under-relaxation parameter $\alpha$ is not straightforward and cannot be determined directly. Thereby, the value of $\alpha$ is case-dependent and needs to be chosen with appropriate caution.

In the high spatial resolution, the higher $\alpha$ value (0.1) leads to smaller number of smoothing steps needed for a stable solution (SUH07, $\alpha=0.1$, n=10) within 200 iterations, while the lower $\alpha$ value (0.01) leads to larger number of smoothing steps needed for a stable solution (SUH08, $\alpha=0.01$, n=15) within 200 iterations. Thereby implementation of higher $\alpha$ value reduces the number of smoothing steps needed for a stable solution.

The models with different $\alpha$ value but the same other SWAN input settings give exact the same results. Thereby it can be concluded that the predicted wave conditions are not influenced by $\alpha$. Therefore under-relaxation technique will not be further investigated in the following cases since it will not solve the problem of missing wave energy on low-frequency band in Oosterschelde Estuary.
Figure 33: Comparison of normalized $H_s$ distributed along circular-section at 3L to Sommerfeld solution. Left panel: comparison of different spatial resolution (with diffraction, $n=15$), right panel: comparison of smoothing technique (with same spatial resolution: $\Delta x=\Delta y=0.5m$).

For the effects of spatial resolution, the advantages of increasing the grid size ($\Delta x \uparrow \Delta y \uparrow$, i.e., lowering the spatial resolution) is as following:

1) Increasing the grid size leads to smaller number of iterations (Table 5) needed to fulfill the stopping criteria.

2) Increasing the grid size induces the reduction of the number of smoothing steps needed for a stable solution (see Table 5).

The disadvantages of increasing the grid size ($\Delta x \uparrow \Delta y \uparrow$) are as following:

1) Under the same diffraction conditions ($n=15$), increasing the grid size ($\Delta x \uparrow \Delta y \uparrow$) brings about more serious underestimation of the wave energy in the lee area of the breakwater (see left panel in Figure 33).

2) Under the fine grid (high spatial resolution, $\Delta x = \Delta y =0.25m$, blue line in left panel of Figure 33) the overshoot on the exposed side is even reproduced but the overshoot disappears by increasing the grid size ($\Delta x \uparrow \Delta y \uparrow$).

For the effects of the smoothing technique, it is clearly shown in the right panel of Figure 33 that increasing the number of smoothing steps worsens the predictions of the significant wave height in the lee area but reduces the required number of iterations to fulfill the stopping criteria. Compared with the Sommerfeld solution, the results (right panel in Figure 33) with higher number of smoothing steps ($n=15$, yellow lines) are underestimated more seriously than the results with lower smoothing steps ($n=2$, blue lines) in the lee area of the breakwater. As shown in Table 5, for a given grid spacing and the other SWAN input settings, the simulations of diffraction without smoothing technique require larger number of iterations to achieve stability than the simulations of diffraction with smoothing technique. In addition, smaller number of iterations is required to obtain a stable solution when with larger number of smoothing steps than with smaller number of smoothing steps.

In the group of curvi-linear grids in Semi-infinite Breakwater Case, the required time to complete one four-sweep iteration cycle is shortened because the numbers of the
meshes containing in the domain decreases as shown in Table 6. It can also be found that fewer numbers of iterations are required to achieve stability in grid NOR and GEN than in grid DOU. It is essentially resulted from the effects of the spatial resolution since meshes with smaller size are used in grid DOU but relative large meshes are used in grid NOR and GEN. Meanwhile, the required number of smoothing steps to achieve a stable solution within 200 iterations is much larger in grid DOU (n=18) than it in grid NOR and GEN (n=3).

![Normalized Significant Wave Height along the Circle](image)

**Figure 34: Comparison of normalized Hs distributed along circular-section at 3L of all runs in curvi-linear group to Sommerfeld solution**

As shown in Figure 34, the significant wave heights are seriously underestimated in the lee area of the breakwater in SWAN without diffraction under all three grids by comparing to the Sommerfeld Solution (red dashed dot line → NOR, without diffraction, blue dashed dot line → DOU, without diffraction, green dashed dot line → GEN, without diffraction, black dashed dot line → Sommerfeld Solution). By taking the phase-decoupled diffraction into consideration, the situation is improved. The overshoots on the exposed side can be found in all the three grids when involving the diffraction effects. They decrease slightly by enlarging the number of the smoothing steps (solid lines vs. dashed lines). However, with larger numbers of smoothing steps (dashed lines in Figure 34), the computed significant wave heights are underestimated more seriously than they are with smaller numbers of smoothing steps (solid lines in Figure 34).

### 4.3.4 Conclusion

In this section the practical applicability of diffraction in SWAN in the Semi-infinite Breakwater Case (monochromatic, unidirectional waves) has been evaluated therefore the following conclusions can be drawn:
1) Diffraction is computed reasonably well in SWAN in Semi-infinite Breakwater Case.

2) Spatial Resolution:
   √ Larger number of iterations required in high spatial resolution grid to obtain a stable solution;
   √ Larger number of smoothing steps required in high spatial resolution grid to obtain a stable solution within 200 iterations;
   √ Overshoot on exposed side is well reproduced in high spatial resolution grid;
   √ Less underestimation of significant wave heights in the lee area of the breakwater in high spatial resolution (compared with Sommerfeld Solution).

3) Under-Relaxation
   √ Strong relation between the under-relaxation technique and the number of iterations required to achieve stability;
   √ The relation between the under-relaxation technique and the number of iterations needed to achieve stability is case-dependable and has to be treated carefully;
   √ Higher $\alpha$ value reduces the number of smoothing steps needed for a stable solution;
   √ The under-relaxation parameter $\alpha$ does not affect the results when the prescribed stopping criterion is achieved.

4) Smoothing Technique (Convolution Filter)
   √ Larger number of smoothing steps leads to smaller number of iterations required to achieve stable solutions;
   √ Larger number of smoothing steps degrades the results since the diffraction parameter becomes meaningless. The smoothing technique is not suggested when the required number of iterations to fulfill stopping criteria is not too large.

4.4 Gap in Infinite Breakwater Case

4.4.1 Model Description
The Gap in the Infinite Breakwater Case (Gap Case for short) is performed in this study as the second academic case. It aims to validate the diffraction implementation in SWAN for three reasons:
First, detached breakwaters, compared to more traditional shoreline structures, have advantages that they decrease the height of incoming waves and reduce offshore sediment losses. Thereby it is meaningful to investigate the wave conditions in the gap of one infinite-long breakwater for practical applications.
Second, both refraction and diffraction effects play important parts in the processes of waves passing through the detached breakwaters thereby it is meaningful to study the Gap case for the refraction-diffraction research.
Thirdly, Yu et al. (2000) observed the wave field under the quasi situation (a gap in an infinitely long, straight breakwater) in a laboratory wave tank thereby the observed data were available to be compared with the numerical results.

The model performed in SWAN is taken from the model in the laboratory (Yu et al., 2000). It is a rectangular domain (shown in Figure 35) which is 24m long in width and 27m long in length with constant depth of 1m. Two breakwaters are vertical walls with rounded tips. The up-wave sides of the breakwaters are absorbing. The gap was
7.85m in length and 0.35m in width. Thereby, the detached breakwaters are modeled as four obstacle objects in SWAN. Two of them are set as fully reflection and zero transmission (black lines in Figure 35) to stand for the detached breakwaters while the other two are set as the wave absorbers with zero reflection and zero transmission (green lines in Figure 35) to stand for the absorbing sides of the breakwaters. The location of the breakwaters is shown in Figure 35. The observation data (significant wave heights) measured in laboratory by Yu are provided at the red crosses in Figure 35 and they will be used to be compared with the SWAN computed data given along eight transect lines of the domain (shown as eight blue lines in Figure 35).

![Figure 35: Sketch of the domain in Gap In Infinite Breakwater Case](image)

Along the up-wave boundary (south side of the domain) an incoming wave is prescribed. It is specified by a JONSWAP-shaped frequency spectrum with the period (T) of 1.2 s, the significant wave height (Hs) of 0.05 m, the directional spreading (\(\sigma_\theta\)) of around 6° (MS = 500, the definition of MS can be found in SWAN USER MANUAL, Page 92) and the directional resolution (\(\Delta \theta\)) of 3°. The domain is over a flat bottom of constant depth (1m). Thereby the wave length is around 2.23m. The growth by wind, quadruplet wave-wave interaction, and the dissipation by triad wave-wave interaction, bottom friction, depth-induced breaking and whitecapping are omitted in present case.

4.4.2 Methodology
For all the other runs in this study, they are all performed in SWAN 4072. A relative older version of SWAN – SWAN 4041.AB is applied to the Gap Case as well and the resultant significant wave heights distributed along eight transect lines are compared.
with the results from SWAN 4072 and Yu’s data. It aims to investigate the improvement of the diffraction implementation in the new version of SWAN (4072). The main difference between the versions 40.72AB and 40.41AB with respect to the implementation of diffraction is a small correction in the discretisation of the diffraction parameter. Specifically, the computed derivatives boiled down to very large values if the grid sizes are very small. The correction is the minimization of these grid sizes to 1cm.

Similar to the Semi-infinite Breakwater Case, it is started with a test without diffraction in SWAN 4072 to check whether the results are reliable. Afterwards, the concentration is focused on the implementation of diffraction in SWAN with respect to the effect of spatial resolution and the effect of the smoothing technique. The parameter $\alpha$ is set as 0.01 in all runs in the Gap Case (the effect of under-relaxation technique is not investigated further in Gap Case).

There are two different spatial resolutions to be performed, one is with the mesh of 0.24m ($\approx 1/9L$) wide in x-direction and 0.25m ($\approx 1/9L$) wide in y-direction, the other is 0.48m ($\approx 1/5L$) in x-direction and 0.49m ($\approx 1/5L$) in y-direction. Under each spatial resolution, several runs with different numbers of smoothing steps are tested to investigate the effects of the smoothing techniques and the implementation of diffraction in SWAN. The simulations with 0 smoothing step (without smoothing technique), with smallest number smoothing steps required to obtain a stable solution within 200 iterations, with 6 smoothing steps and with a relative large number of smoothing steps are recorded in Table 7. Details of the SWAN input settings and the numerical convergence behaviors of all tests in Gap Case are given in Table 7 in which G represents the GAP Case, H and L represent the High and Low spatial resolutions and the suffix 72 represents the test is performed in SWAN 4072.

All runs performed in SWAN 4072 will be performed in SWAN 4041.AB again with the same input settings. The corresponding codes of all runs combined with their input settings and numerical convergence behaviors are summarized in Table 7, where suffix 41 represents the test is performed in SWAN 4041.AB.

The results are shown as the distribution of wave directions combined with the contours of the normalized wave heights. Besides, the normalized wave heights distributed along eight transect lines (shown as blue lines in Figure 35) combined with the measured significant wave height (also normalized) provided at the red crosses in Figure 35 (Yu et al., 2000) are also shown. It has to be noticed that all data are only shown in one half of the domain (pink frame in Figure 35) since the case is symmetrical around the center line of the gap.

### Table 7: Summary of codes, input settings and corresponding convergence behaviors of all runs in the GAP Case

<table>
<thead>
<tr>
<th>Codes</th>
<th>Spatial Resolution</th>
<th>Diffraction</th>
<th>Mxitst</th>
<th>Real</th>
<th>Accuracy</th>
<th>Fig.</th>
</tr>
</thead>
<tbody>
<tr>
<td>GH01_72</td>
<td>100×110 0.24 0.25</td>
<td>Off /</td>
<td>200 6</td>
<td>100%</td>
<td>√</td>
<td></td>
</tr>
<tr>
<td>GH02_72</td>
<td>100×110 0.24 0.25</td>
<td>On 0</td>
<td>200 200</td>
<td>91.93%</td>
<td>×</td>
<td></td>
</tr>
<tr>
<td>GH03_72</td>
<td>100×110 0.24 0.25</td>
<td>On 3</td>
<td>200 200</td>
<td>91.29%</td>
<td>×</td>
<td></td>
</tr>
<tr>
<td>GH04_72</td>
<td>100×110 0.24 0.25</td>
<td>On 4</td>
<td>200 30</td>
<td>100%</td>
<td>√</td>
<td></td>
</tr>
<tr>
<td>GH05_72</td>
<td>100×110 0.24 0.25</td>
<td>On 6</td>
<td>200 19</td>
<td>100%</td>
<td>√</td>
<td></td>
</tr>
</tbody>
</table>
Table 5: Comparison of normalized Hs distributed along transect line of \(y=3.0\,\text{L}(6\,\text{m})\) of all runs, the observation data and the Sommerfeld solution in GAP Case, SWAN 4072

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Model Type</th>
<th>Resolution</th>
<th>Friction</th>
<th>Turbulence</th>
<th>Stability</th>
<th>Iterations</th>
<th>Percent Agreement</th>
</tr>
</thead>
<tbody>
<tr>
<td>GL01_72</td>
<td>50×55</td>
<td>0.48</td>
<td>0.49</td>
<td>Off</td>
<td>/</td>
<td>200</td>
<td>5</td>
</tr>
<tr>
<td>GL02_72</td>
<td>50×55</td>
<td>0.48</td>
<td>0.49</td>
<td>On</td>
<td>0</td>
<td>200</td>
<td>26</td>
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<tr>
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<td>0.48</td>
<td>0.49</td>
<td>On</td>
<td>1</td>
<td>200</td>
<td>15</td>
</tr>
<tr>
<td>GL04_72</td>
<td>50×55</td>
<td>0.48</td>
<td>0.49</td>
<td>On</td>
<td>6</td>
<td>200</td>
<td>12</td>
</tr>
<tr>
<td>GH01_41</td>
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<td>0.25</td>
<td>Off</td>
<td>/</td>
<td>200</td>
<td>44</td>
</tr>
<tr>
<td>GH02_41</td>
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<td>0.24</td>
<td>0.25</td>
<td>On</td>
<td>0</td>
<td>200</td>
<td>200</td>
</tr>
<tr>
<td>GH03_41</td>
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<td>0.24</td>
<td>0.25</td>
<td>On</td>
<td>1</td>
<td>200</td>
<td>200</td>
</tr>
<tr>
<td>GH04_41</td>
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<td>0.25</td>
<td>On</td>
<td>4</td>
<td>200</td>
<td>70</td>
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<tr>
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<td>69</td>
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<tr>
<td>GL01_41</td>
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<td>0.48</td>
<td>0.49</td>
<td>Off</td>
<td>/</td>
<td>200</td>
<td>4</td>
</tr>
<tr>
<td>GL02_41</td>
<td>50×55</td>
<td>0.48</td>
<td>0.49</td>
<td>On</td>
<td>0</td>
<td>200</td>
<td>55</td>
</tr>
<tr>
<td>GL03_41</td>
<td>50×55</td>
<td>0.48</td>
<td>0.49</td>
<td>On</td>
<td>1</td>
<td>200</td>
<td>55</td>
</tr>
<tr>
<td>GL04_41</td>
<td>50×55</td>
<td>0.48</td>
<td>0.49</td>
<td>On</td>
<td>6</td>
<td>200</td>
<td>55</td>
</tr>
<tr>
<td>GL05_41</td>
<td>50×55</td>
<td>0.48</td>
<td>0.49</td>
<td>On</td>
<td>20</td>
<td>200</td>
<td>55</td>
</tr>
</tbody>
</table>

Note: The meaning of the symbols and the denotations in current table is the same as it in Table 5.

4.4.3 Results and Discussion

The conclusions relating to the number of iterations required to achieve stability in SWAN drawn in Semi-infinite Breakwater Case are mostly validated in Gap Case except that, as shown in Table 7, the number of iterations required in SWAN 4041.AB with low spatial resolution does not decrease with the increase of the number of smoothing steps. It does not agree with the conclusion obtained in Semi-infinite Breakwater Case which may be resulted from the improvement of the stopping criteria has been made in the new version of SWAN 4072.

![Figure 36: Comparison of normalized Hs distributed along transect line of \(y=3.0\,\text{L}(6\,\text{m})\) of all runs, the observation data and the Sommerfeld solution in GAP Case, SWAN 4072](image)

M. Sc. Thesis
The significant wave heights along transect line of y=3L (6m) are seriously underestimated when diffraction effects are not taken into account (Figure 36, SWAN 4072). The underestimation of the wave energy decreases by involving the effects of diffraction. Comparing the pink line with the green line in Figure 36, the results obtained from high spatial resolution grid (fine grid, Δx≈0.24m, Δy≈0.25m, pink line) are more in line with the observed data than those obtained from low spatial resolution grid (coarse grid, Δx≈0.48m, Δy≈0.49m, green line) under the same smoothing steps (n=6). The results obtained from SWAN 4072 with diffraction but without smoothing technique are shown as black solid line in Figure 36. They show fairly good agreements with Yu’s observation data in the lee area whereas slight overestimation exists in the exposed region. Under the same spatial resolution, results with smaller number of smoothing steps are closer to the Yu’s observation data than with larger number of smoothing steps.

Figure 37, GL01-72 – Low spatial resolution (Δx≈0.48m, Δy≈0.49m) & No diffraction & SWAN 4072. Comparison of normalized Hs distributed along eight transect lines and the measured data (Yu et al., 2000).

The normalized significant wave heights distributed along eight transect lines of the domain are shown in Figure 37. They are computed in SWAN 4072 without diffraction under the low spatial resolution. It can be seen from Figure 37 that the simulated wave heights are considerably overestimated along the cross section of y=0m. When the waves propagate further behind the breakwater, the overestimation of the significant wave heights along transect line of y=0 in the exposed region becomes weak. After propagation of 9m (4.5L), overestimation barely disappears. But after propagation of 12m (6L), overestimation re-appears in the exposed region. In
the lee area of the breakwaters, the significant wave heights are overestimated between transect lines of $y=0$m and $y=3$m ($1.5L$). They start to be strongly underestimated when crossing transect line of $y=3$m. By performing the same run in SWAN 4072 with diffraction approximation, the results (shown in Figure 38) are close to the Yu’s observation data except slight overestimation exists in the expose region.

![Figure 38, GL02-72 – Low spatial resolution ($\Delta x\approx 0.48$m, $\Delta y\approx 0.49$m) & With diffraction ($n=0$) & SWAN 4072. Comparison of normalized $H_s$ distributed along eight transect lines and the measured data (Yu et al., 2000).](image)

In SWAN 4041.AB, the normalized significant wave heights distributed along transect line of $y=6$m ($3L$) of all runs are shown in Figure 39. It shows that the implementation of the smoothing technique and the effect of the spatial resolution are generally the same as they behaved in SWAN 4072.

The normalized wave heights along eight transect lines are shown in Figure 40. They are obtained in SWAN 4041.AB with coarse grid ($\Delta x\approx 0.48$m, $\Delta y\approx 0.49$m) without diffraction. Compared it with Figure 37, overestimation of the wave heights along transect line of $y=0$m disappears. However slight underestimation of the significant wave heights can be found along transect line of $y=9$m and slight overestimation of the wave energy exists along the other 6 transect lines.
Figure 39: Comparison of normalized $H_s$ distributed along transect line of $Y=6m(3.0L)$ of all runs, the measured data (Yu et al., 2000) and Sommerfeld solution in GAP Case, SWAN 4041.AB

Figure 40: GL01-41 – Low spatial resolution ($\Delta x\approx 0.48m$, $\Delta y\approx 0.49m$) & No diffraction & SWAN 4041.AB. Comparison of normalized $H_s$ distributed along eight transect lines and the measured data (Yu et al., 2000).
By taking the effects of diffraction into consideration, the results (shown as red lines in Figure 41) show fairly good agreements with the observation data along transect line of y=9m (3L). The significant wave heights are slightly overestimated in the exposed areas along the other 7 transect lines. For the lee areas of the breakwaters, the significant wave heights are slightly underestimated between transect line of y=0m and y=6m (3L).

Comparing the results obtained in SWAN 4041.AB with the results obtained in SWAN 4072 (Figure 41), it shows that the underestimation of the significant wave heights in the lee area of the breakwaters along transect line of y=1m and y=3m (1.5L) exists in SWAN 4041.AB. The underestimation is obviously improved in SWAN 4072. However, the significant wave heights in the central exposed area along transect line of y=0m are slightly overestimated in SWAN 4072.

According to the stopping criteria (refer to section 4.2.4 for detailed introduction), SWAN stops the process if the conditions of the absolute change in the local significant wave height from one iteration to the next is less than 0 or the relative change in the local significant wave height from one iteration to the next is less than 0.01 and the conditions of the curvature of the iteration curve of $H_s$ normalized with $H_i$ is less than 0.001 are fulfilled in more than 100% fraction of all wet grid. Since the required duration (time) of computation is not paid much attention in small scales.
academic case with research purpose, the required fraction was set as 100% for accuracy. However, for large scale case, value less than 100% (such as 98% or 99%) will be chosen as an alternative for cost reason. Then special caution is required since the difference of the $H_{m01}$ between the successive iterates can be small enough to meet the convergence criteria, causing the iteration process to stop, even though the converged solution has not yet been found. In particular, this happens when convergence is non-monotonic such that the process is terminated at local maxima or minima that may not coincide with the converged solution (partly quoted from SWAN technical documentation). [11]

In run GH02_72, the achieved accuracy is 98.93% within 200 iterations. In some large scale case, 98% sometimes will be prescribed as the required fraction to stop the computing process for the sack of economic problem. However, results from GH02_72 (Figure 42) go to infinite high and do not make sense for practical application.

In run GH03_72, the achieved accuracy is 91.29% within 200 iterations. It is a relative low prescribed fraction which is required of wet grids fulfilling the stopping criteria, however, results from GH03_72 (Figure 43) are acceptably reasonable by comparing them to the Yu’s observation data.

![Figure 42: GH02-72 – High spatial resolution ($\Delta x \approx 0.24m, \Delta y \approx 0.25m$) & With diffraction (n=0). Comparison of normalized $H_s$ distributed along eight transect lines and the measured data (Yu et al., 2000).](image)

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Therefore, selection of stopping criteria is not enough, it is also critical to choose an appropriate fraction parameter [npnts] to determine when SWAN will stop computing.

![Figure 43: GH03-72 – High spatial resolution (Δx≈0.24m, Δy≈0.25m) & With diffraction (n=3). Comparison of normalized Hs distributed along eight transect lines and the measured data (Yu et al., 2000)](image)

4.4.4 Conclusion

In this section the implementation of diffraction in SWAN has been tested in the Gap Case (monochromatic and unidirectional waves). Compared with the data observed in laboratory, the waves computed in SWAN with diffraction in the Gap case mainly agreed with Yu’s observation data. The conclusions relating to the effect of the spatial resolution and the smoothing technique drawn in last section have been further validated. Besides, the results were computed considerably well in both SWAN 4072 and SWAN 4041.AB except slight underestimation in lee areas and slight overestimation in exposed areas. In addition, the comparison between the results from SWAN 4041.AB and SWAN 4072 showed that the diffraction implementation in SWAN 4041.AB had been improved in SWAN 4072. The underestimation in lee area in results of SWAN 4041.AB disappeared in results of SWAN 4072. Last but not least, not only the error but also the fraction part needs to be paid special attention for the stopping criteria problem.
4.5 Ridge Case

4.5.1 Model Description
This section deals with wave propagation parallel to the depth contours. The case considered in this section, is a prismatic wave channel of finite width, bounded by vertical, fully reflected side-walls, over a ridge bottom. These are parallel to the depth contours. This wave channel is an interesting study object mainly because a continual balance between refraction and diffraction exists. The refraction effects tend to transfer the wave energy towards the shallower part of the channel, while the diffraction effects tend to smooth the energy distribution. This case was modeled by means of integrating over the vertical by Booij in 1981 (Booij, 1981) [12]. The effect of integrating over the vertical has already been verified and Booij’s model gave reliable results. Thereby those results can be used to validate the diffraction implementation of SWAN in case with ridge bottom. Consequently the so-called Ridge Case is chosen as the third academic case to test the diffraction implementation.

A rectangular domain based on the prototype of Booij’s model is 4 meters wide and 10 meters long (Figure 44) over a ridge bottom (Figure 45). It is symmetrical around the centre line of the domain (shown as dashed line). Two obstacle objects with zero transmission and fully reflection are set at west and east sides of the domain to stand for the fully reflected vertical side-walls (green lines in Figure 44). Along the whole south boundary the incoming waves are specified by a narrow BIN-shaped frequency spectrum centered on the period (T) of 1.3 s (monochromatic wave), with a width of around 0.42 Hz. Hence the wave length is between 1.7m (d=0.2m) and 2.4m (d=0.6m). The directional spreading (σθ) is set as 2º (unidirectional) and the directional resolution (Δθ) is set as 0.25º. The wave heights along the entire south boundary are given by the results of the model constructed by Booij in 1981.

Wave conditions on east boundary and west boundary are set as the same period and wave heights as they are on the eastern side and western side of the south boundary. The spatial resolution is set as Δx = Δy = 0.2m and the under-relaxation parameter α is set as 0.01. The irrelevant source terms including growth by wind, quadruplet wave-wave interaction, and the dissipation by triad wave-wave interaction, bottom friction, depth-induced breaking and whitecapping are not taken into consideration in Ridge Case.
4.5.2 Methodology

As stated previously, the results of Booij’s model are directly set as the input wave heights along the south boundary in SWAN (the results are shown in Figure 46). It aims to investigate whether the phase-decouple diffraction wave spectral model can find the balance between the effects of the refraction and diffraction i.e., to see whether SWAN can keep the solution intact down wave. The expectation of the computed results is supposed to keep the distribution of the input wave heights the same along the propagation of waves.

Two test runs with different directional spreading (2º and 1º) but with same prescribed wave conditions at boundary are first performed over a flat bottom with constant depth of 0.4m in SWAN without diffraction effects. It aims to investigate the diffusion of wave energy dispersion which can be resulted from numerical aspect (numerical diffusion) or physical aspect (such as the non-infinite-narrow directional spreading). Afterwards, the case is performed over the ridge bottom in SWAN. The run without diffraction effects is always performed beforehand to test if the results are dependable. It is followed by the runs containing diffraction effects in SWAN. The performance of diffraction in SWAN is tested in terms of smoothing technique. All the codes of the tests combined with the SWAN input settings and numerical convergence behaviors are summarized in Table 8.
Table 8: Summary of codes, input settings and corresponding convergence behaviors of all runs in the Ridge Case

<table>
<thead>
<tr>
<th>Codes</th>
<th>Δx=Δy</th>
<th>σθ</th>
<th>Bottom</th>
<th>Diffraction State</th>
<th>n</th>
<th>Mxitst</th>
<th>Real</th>
<th>Accuracy</th>
<th>Fig.</th>
</tr>
</thead>
<tbody>
<tr>
<td>R01</td>
<td>0.2</td>
<td>2º</td>
<td>Flat</td>
<td>Off</td>
<td>/</td>
<td>200</td>
<td>6</td>
<td>100%</td>
<td>√</td>
</tr>
<tr>
<td>R02</td>
<td>0.2</td>
<td>1º</td>
<td>Flat</td>
<td>Off</td>
<td>/</td>
<td>200</td>
<td>6</td>
<td>100%</td>
<td>√</td>
</tr>
<tr>
<td>R03</td>
<td>0.2</td>
<td>2º</td>
<td>Ridge</td>
<td>Off</td>
<td>/</td>
<td>200</td>
<td>12</td>
<td>100%</td>
<td>√</td>
</tr>
<tr>
<td>R04</td>
<td>0.2</td>
<td>2º</td>
<td>Ridge</td>
<td>On</td>
<td>0</td>
<td>200</td>
<td>200</td>
<td>49.77%</td>
<td>x</td>
</tr>
<tr>
<td>R05</td>
<td>0.2</td>
<td>2º</td>
<td>Ridge</td>
<td>On</td>
<td>6</td>
<td>200</td>
<td>200</td>
<td>9.90%</td>
<td>x</td>
</tr>
<tr>
<td>R06</td>
<td>0.2</td>
<td>2º</td>
<td>Ridge</td>
<td>On</td>
<td>10</td>
<td>200</td>
<td>34</td>
<td>100%</td>
<td>√</td>
</tr>
<tr>
<td>R07</td>
<td>0.2</td>
<td>2º</td>
<td>Ridge</td>
<td>On</td>
<td>15</td>
<td>200</td>
<td>28</td>
<td>100%</td>
<td>√</td>
</tr>
</tbody>
</table>

Note: The meaning of the symbols and the denotations in current table is the same as it in Table 5.

4.5.3 Results and Discussion

As seen from Figure 47, the significant wave heights decrease in the shallower part of the domain (middle area, range of 1.5m < x < 3m roughly) while they increase in the deeper part of the domain with the propagation of the waves when the directional spreading is set as 2º (Figure 47, σθ=2º). By decreasing the directional spreading of the incoming waves from 2º to 1º (Figure 48, σθ=1º), the decrease of the significant wave heights in the shallower area still exists but becomes smaller. Thereby the decrease of the wave energy in the shallower water may be resulted from the effects of the directional spreading. Though the incoming waves are considered as unidirectional waves and with an extreme narrow directional width of 1º or 2º, the dispersion effects are still visible and work on transferring the wave energy towards two sides. The wave energy is kept being smoothed out of the middle part when the wave propagation distance is increasing. Therefore further studies in terms of the directional spreading are suggested.

Figure 47: R01 – Flat Bottom & Δx=Δy=0.2m & σθ=2º & No diffraction.
The results of the Ridge Case obtained in SWAN without diffraction are shown as dashed lines in Figure 49. The dashed green line in Figure 49 shows that after the waves propagate for 2 meters along the channel, the wave energy is mainly pushed to the shallower area (central of the domain) by the refraction effect. It may be resulted from the disregard of diffraction effects so that the refraction effect can not be compensated. Eight dashed lines in different color in Figure 49 represent the significant wave heights distributed along eight transect lines. They reveal that with the waves propagating along the channel, the wave energy decreases in the shallower area (around the center line of the domain) and increases in the deeper area (two sides of the domain). It is partially resulted from the dispersion effects caused by the directional spreading of the incoming waves. It can also be resulted from the numerical diffusion. Making use of other numerical scheme (such as BSBT scheme) with different numerical diffusion from default SORDUP scheme is suggested in the future.

By involving the effects of diffraction in SWAN, the significant wave heights (shown as solid green line in Figure 49) are smoothed out from the shallower area (central area) to the deeper area (two sides of the domain). Obviously, the shifted wave energy is resulted from the diffraction approximation that is added in the computations. With the increase of the waves propagation distance, the smooth effects of the directional spreading are also visible but turn to be weak. It may be resulted from the fact that local variations of different spectral components compensated each other and led to the smooth of the wave heights distribution.
4.5.4 Conclusion

In this section the implementation of diffraction in SWAN has been tested in the Ridge Case. It is the case that monochromatic and unidirectional incoming waves propagate parallel to the depth contours over a ridge bottom. By setting the significant wave heights of the incoming waves as the results obtained from the model constructed by Booij (integrating over the vertical), the significant wave heights obtained in SWAN with diffraction did not successfully keep the distribution of the wave heights with the propagation of waves. However, by comparing the results obtained in SWAN without diffraction, the effects of diffraction are clearly visible to compensate the effects of refraction. The directional spreading of the incoming waves affects the wave propagation process to some extent. It is suggested to be further investigated in future work.

4.6 Conclusion

In this chapter, the performance of the phase-decoupled refraction-diffraction spectral wave model has been assessed in terms of the ability to predict the wave conditions including the significant wave height and the wave direction. The validation of the model in Semi-infinite Breakwater Case and Gap Case showed that the inclusion of diffraction in the model improved the estimation of wave heights in the shadow area behind the breakwaters. In the Ridge Case, the results from SWAN did not give good agreements with results obtained from the other validated models. However it was clearly visible that the diffraction approximation in SWAN tried to offset the refraction effect and smooth the wave energy distribution to some extent. Meanwhile the effects of the directional spreading of the input waves tried to smooth the wave energy from the shallower area to the deeper area as well. It was suggested to be investigated in the future.
Series runs with different spatial resolutions, \( \alpha \) values (under-relaxation parameter) and \( n \) values (smoothing technique parameter of diffraction) were performed in three academic cases. The following conclusions are drawn from these runs.

**Spatial Resolution:**

Higher spatial resolution grid (finer grids) → Larger number of iterations required to fulfill the stopping criteria;

→ Larger number of smoothing steps required to achieve stability within 200 iterations;

→ Overshoot on exposed side is well reproduced;

→ Less underestimation of significant wave heights in the lee area of the breakwaters.

**Under Relaxation:**

→ The frequency-dependent under-relaxation technique in SWAN influences the required number of iterations to achieve stability;

→ Higher value of \( \alpha \) reduces the number of smoothing steps needed for a stable solution;

→ The frequency-dependent under-relaxation technique is case-dependable and the parameter \( \alpha \) has to be treated carefully;

→ It does not affect the results when the required stopping criteria is achieved.

**Diffraction Smoothing Technique (Convolution Filter):**

Larger number of smoothing steps (n value) → Smaller number of iterations required to fulfill stopping criteria;

→ Large number of iterations makes the diffraction parameter becomes meaningless and degrades the results since.

The comparison of the diffraction implementation in SWAN 4072 to it in the old version of SWAN 4041.AB shows that the underestimation of the wave energy in the lee area of the breakwaters in SWAN 4041.AB (relative older version) is shrunk in SWAN 4072. It reveals that the changes made in diffraction part in SWAN 4072 improve the results.

Special caution is needed to choose appropriate stopping criteria. The absolute and relative change from one iteration to the next in local significant wave heights is important. The fraction needed to be fulfilled before SWAN stopping computing is also critical to the accuracy of the results.

Therefore in the version of SWAN 4072, diffraction with spatial resolution of one tenth to one fifth of one wave length (L/10 ~ L/5), without smoothing technique (n=0) or with smallest number of smoothing steps, with under-relaxation technique (\( \alpha \neq 0 \)) and with proper stopping criteria ([npnts] = 100), is suggested as the input settings when diffraction has to be involved in computation.
5. Oosterschelde Estuary

5.1 Methodology
The implementation of diffraction in SWAN has been validated and the corresponding characteristics have been investigated by means of three academic cases in last chapter. In present chapter, the SWAN with diffraction is applied to the Oosterschelde Estuary. Several groups of tests with different input settings of source terms, ambient currents and boundary conditions are performed in SWAN. The computed results are shown in the form of one-dimensional frequency spectrum at wave buoy OS4. They are compared with the spectrum computed in Svasek Hydraulics (2007) and the observation data. It aims to investigate whether the lack of diffraction effects is the main reason for the underestimation of the wave energy on low-frequency band.

The application of Oosterschelde Case has already been performed (Section 2.2) in SWAN without diffraction. The investigations performed here are based on this model which originally constructed by Svasek Hydraulics (2007) (Refer to Section 2.2.2 for detailed description of Oosterschelde Estuary Model). The effects of diffraction are taken into consideration in this study whereas they are not involved in Svasek’s model. Five groups of the tests are performed in SWAN with different settings of source terms, ambient currents and boundary conditions. In each group, a test performance without diffraction is always the first step. Afterwards, the effects of diffraction are taken into consideration in the smallest grid F. The SWAN simulations in larger grids K and B do not count diffraction effects in because the diffraction effects are not necessary in areas far away from sensitive point (OS4). According to the conclusions drawn in chapter 3 (Academic Cases), the simulation in SWAN with diffraction is performed without smoothing technique, with under-relaxation technique ($\alpha=0.01$) in grid F under the same spatial resolution used in Svasek Hydraulics ($\Delta x = \Delta y = 20m \approx 1/7$ average L).

In the 0th group, i.e., the origin group, the Svasek’s model is performed again. It is performed in SWAN without diffraction first and in SWAN with diffraction later. In the 1st group (code: ODa), the model originally performed in Svasek Hydraulics containing the source terms, now is performed in SWAN without source terms (including the growth by wind, quadruplet wave-wave interaction and the dissipation by triad wave-wave interaction, bottom friction, depth-induced breaking and whitecapping). It aims to investigate the amount of wave energy dissipated by the source terms. In the 2nd group, based on the model in the 1st group without source terms, the ambient currents are taken away from the model as well since it can be a possibility that the missing energy on low frequency part at OS4 is resulted from the incorrect energy remove from ambient current. Therefore the Oosterschelde Case is performed in model without source terms and without ambient current in SWAN without diffraction first and in SWAN with diffraction later in group 2. In the 3rd group, the irrelevant tails (here tail indicates the wave energy located on high-frequency band ranging from 0.2 Hz to 1.0 Hz, the tail of the wave energy generally represents energy of the local wind sea) of the incoming wave spectra at buoy SCHB and LEG are removed. Since the purpose of the present research object is the missing wave energy of low-frequency. The corresponding tail-off wave conditions are set as new boundary conditions of grid K. The resultant SWAN simulations (with no source terms or ambient currents) are performed as group 3. In the 4th group, the boundary
condition of grid F is not provided by the results obtained in larger grid K and B. It is prescribed with monochromatic and unidirectional waves (modeled as BIN-shape). The results of the REFRAC model (section 3.4.2) showed: when the incoming wave direction was 300°N, the buoy OS4 was located on the caustics where diffraction might have relative important effect, while when the incoming wave direction was 350°N, the buoy OS4 was not located on caustics. It was in the area where diffraction might have less significant effect. Therefore these two specific directions are further applied in SWAN. It always starts with a test without diffraction in SWAN and followed by the model performed in SWAN with diffraction effects. The obtained results are compared with each other to investigate the diffraction effects in the SWAN model.

The codes, the input settings and the numerical convergence behaviors of the tests in all these five groups are summarized in Table 9, where O represents the Oosterschelde Estuary Case and D represents that Diffraction effects are taken into consideration.

Table 9: Summary of codes, input settings and corresponding convergence behaviors of all runs in grid F of Oosterschelde in SWAN with diffraction

<table>
<thead>
<tr>
<th>Group</th>
<th>Boundary Conditions of Grid F</th>
<th>Codes</th>
<th>Diffraction state</th>
<th>Accuracy</th>
</tr>
</thead>
<tbody>
<tr>
<td>0. OD</td>
<td>Results of Grid B broader spectrum</td>
<td>OD01 × /</td>
<td>100%</td>
<td></td>
</tr>
<tr>
<td></td>
<td>OD02 y 6</td>
<td>100%</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1. ODa</td>
<td>Results of Grid B broader spectrum</td>
<td>ODa01 × /</td>
<td>100%</td>
<td></td>
</tr>
<tr>
<td></td>
<td>ODa02 y 6</td>
<td>100%</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2. ODb</td>
<td>Results of Grid B broader spectrum</td>
<td>ODb01 × /</td>
<td>100%</td>
<td></td>
</tr>
<tr>
<td></td>
<td>ODb02 y 6</td>
<td>100%</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3. ODC</td>
<td>Results of Grid B broader spectrum</td>
<td>ODC01 × /</td>
<td>100%</td>
<td></td>
</tr>
<tr>
<td></td>
<td>ODC02 y 6</td>
<td>100%</td>
<td></td>
<td></td>
</tr>
<tr>
<td>4. ODD</td>
<td>Monochromatic, unidirectional waves defined on boundaries of Grid F directly</td>
<td>ODD01 × /</td>
<td>100%</td>
<td></td>
</tr>
<tr>
<td></td>
<td>ODD02 √ 6</td>
<td>100%</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>ODD03 × /</td>
<td>100%</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>ODD04 √ 6</td>
<td>100%</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Notes:
- Provided By – the source of the boundary conditions of grid F;
- Tails – indicates whether the input wave spectra on boundaries contained the wave energy of local wind sea located on high-frequency band (0.2195Hz ≤ f ≤ 1.0Hz);
- √(×) – indicates the specific item is (not) contained in model;
- T – wave period (s);
- DIR – direction of the incoming wave (°N);
- σθ – directional width of the incoming wave (°);
- Δθ – directional resolution of the incoming wave (°);
- State – denotes whether the diffraction was taken into consideration;
- n – whether the smoothing technique was activated in SWAN; n=/ no diffraction effects; n=0 → no smoothing technique; n≠0 → the smoothing steps used in SWAN.
5.2 Results and Discussion

The results of group 0 are shown in Figure 50. Serious underestimation can be found in the one-dimensional frequency spectrum (black solid line) on low-frequency band at location of buoy OS4. It is obtained from the model constructed by Svasek Hydraulics (2007). Since this model does not include diffraction, the model with the same input settings is performed in SWAN with diffraction in present study. The obtained 1D frequency spectrum at OS4 is shown as green dashed line in Figure 50. By comparing the spectrum obtained in SWAN with diffraction with it in SWAN without diffraction, only tiny increase of the wave energy can be found on low-frequency band. The energy on high-frequency band decreases slightly. It indicates that diffraction does not play the key role in wave propagation processes in the Oosterschelde Estuary when the incoming waves are non-unidirectional in model containing the source terms and ambient currents. Since the effects of diffraction are approximated in SWAN. In addition the effects of the source terms, the ambient currents and the boundary conditions (with tail or without, multi-directional or unidirectional, etc.) may have more influence to the missing energy on low-frequency band than diffraction. Thereby it is too early to conclude that the assumption of omitting the effects of diffraction in SWAN inducing the underestimation of the wave energy on low-frequency band is wrong. The emphasis of the research is subsequently concerned on these aspects in the following tests.

In group 1, the source terms contained in the model constructed in Svasek are removed. The resultant model is performed in SWAN without diffraction (red line in Figure 51) and in SWAN with diffraction (green line in Figure 51). By removing the source terms, the wave energy increases significantly on low-frequency band. It is expected since there is no dissipation of wave energy resulted from the source terms. The wave energy on high-frequency band also decreases slightly. It indicates that the effects of the source terms not only dissipate the wave energy out of the domain but also transfer the wave energy from the low-frequency to the high-frequency band.
When the effects of diffraction are taken into consideration, the wave energy obtained in SWAN with diffraction is only lower than they are obtained in SWAN without diffraction at OS4. However, the difference between results from SWAN with diffraction and without diffraction is so small that the diffraction can not be regarded as one of the main reasons for the underestimation of the wave energy on low-frequency band at OS4 in model without source terms.

![Figure 51: Comparison of 1D spectra from SWAN (source terms \times ambient currents \times \text{tail}) without diffraction (red), with diffraction (green), from Svasek (black) and measured data (blue) at buoy OS4](image)

Afterwards, the ambient currents in the model of Svasek are also removed, i.e., Oosterschelde Estuary is applied to the model with no source terms, no ambient currents in SWAN. The resultant 1D frequency spectrum computed in SWAN without diffraction is shown as red line in Figure 52. The green line in Figure 52 represents the spectrum computed in SWAN with diffraction.

![Figure 52: Comparison of 1D spectra from SWAN (source terms \times ambient currents \times \text{tail}) without diffraction (red), with diffraction (green), from Svasek (black) and measured data (blue) at buoy OS4](image)

It shows that the peak wave energy (≈1.8 m$^2$/Hz) of low-frequency in model without currents is much higher than it is (≈1.3 m$^2$/Hz) in model with currents. It indicates that the ambient currents work on transferring wave energy out of the low-frequency band as well. Meanwhile, two minor energy peaks can be found at 0.1997Hz and 0.3526Hz which may be resulted from the numerical fluctuations. Compared with the
wave energy obtained in SWAN without diffraction, the increase of the wave energy obtained in SWAN with diffraction is extraordinarily small on the first peak. However, decrease, albeit slight, can also be found on the second and third wave peaks when it performed in SWAN with diffraction. Thereby the effects of diffraction can not be regarded as the major responsibility for the underestimation of the wave energy on low-frequency band at wave buoy OS4 in model without source terms, without ambient currents and with broad-directional incoming waves of broad-frequency spectrum.

Since the concerning part in this study is the missing energy on low-frequency band, it closely relates to the propagation of the swell waves. Moreover the wave energy of local wind sea distributed on the high-frequency bands, i.e., the tails of the input wave spectra at the boundaries of grid K may bring about the unexpected influence to the swell wave energy prediction in SWAN. The irrelevant wave energy of local wind sea (energy located on high-frequency band: $0.2195\,\text{Hz} \leq f \leq 1.0\,\text{Hz}$) are thereby removed from the 2D frequency-direction wave energy spectra of buoy LEG and buoy SCHB. They are set as the wave boundary conditions of grid K. The corresponding tail-off 2D spectrum are shown in Figure 53 (refer to Figure 5 and Figure 6 for comparison). The resultant wave spectra (with no source terms or ambient currents) are shown in Figure 54 where the red line represents the wave energy obtained in SWAN without diffraction and the green line represents the wave energy obtained in SWAN with diffraction.

![Tail-off directional wave spectrum at wave buoy LEG & SCHB](image_url)

*Figure 53: Tail-off directional wave spectrum at wave buoy LEG & SCHB*
By cutting the wave energy of local wind sea, the wave energy located on low-frequency band decreases (Figure 54). The wave energy peak located on 0.1243Hz reduces from around 1.8 m²/Hz (Figure 52, with tails) to about 0.9 m²/Hz (Figure 53, tail-off). It indicates that part of the wave energy on low-frequency band is transferred from energy of local wind sea (high-frequency) so the computation of wave energy resulting from local wind sea is also significant to the swell wave energy.

The second wave energy peak appears at 0.1977 Hz. It appeared in the spectrum obtained in group 2 (code: ODd, model with tail, without source terms and without ambient currents, Figure 52) as well. However there was no wave energy fluctuation on low-frequency band in the spectrum of group 1 (code: ODa, model with tail, without source terms but with ambient currents, Figure 51). It indicates that the numerical fluctuations are caused by the removal of the ambient currents.

By taking the effects of diffraction into consideration, the wave energy of the two peaks even decreases slightly, so that the diffraction cannot be regarded as the main reason for the underestimation of swell wave energy on low-frequency band.

The results of the group 4 which contains four runs with two different incoming wave directions (300 °N and 350 °N respectively) performed in SWAN without diffraction and SWAN with diffraction are summarized in Table 10.

Table 10: Summary of significant wave height at buoy OS4 obtained in group ODd (Oosterschelde Case in SWAN with diffraction, monochromatic incoming waves as boundary conditions)

<table>
<thead>
<tr>
<th>DIR</th>
<th>Caustics or Focal points?</th>
<th>Diffraction (×)</th>
<th>Diffraction (✓)</th>
<th>Trend</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Code</td>
<td>Code</td>
<td>Hs (m)</td>
<td>Code</td>
</tr>
<tr>
<td>300°N</td>
<td>✓</td>
<td>ODd01</td>
<td>1.05070</td>
<td>ODd02</td>
</tr>
<tr>
<td>350°N</td>
<td>×</td>
<td>ODd03</td>
<td>0.19645</td>
<td>ODd04</td>
</tr>
</tbody>
</table>

Notes:
- DIR — direction of the incoming wave (°N);
- No Diffraction — red line; With Diffraction — green line; Observation — black line; Results in Svasek — blue line.
Diffraction (∗) = performed in SWAN without diffraction;
Diffraction (√) = performed in SWAN with diffraction;
Hs = significant wave height (m);
Trend = Effect of taking the diffraction into consideration to the
SWAN computed significant wave height at buoy OS4;

When the incoming wave direction is set as 300 °N, the location of buoy OS4 is
located on caustics (Figure 12). By taking diffraction effects into account, the SWAN
computed significant wave height at OS4 decreases slightly. It may be resulted from
the diffraction approximation in SWAN smoothed the wave energy to the area with
low amplitudes when variation of amplitudes are rapid (such as focal points and
causics). When the incoming wave direction is 350 °N, the location of buoy OS4 is
located in area where no adjacent wave ray intersects each other (Figure 23), i.e. in
areas without rapid variation of wave amplitudes. The significant wave height
computed in SWAN increased considerably (from around 0.2 to around 0.3, increases
by 50%, Table 10) by involving the effects of diffraction. The diffraction effects may
push the wave energy from the areas with rapid variation of wave amplitudes where
diffraction plays role to buoy OS4. OS4 is located in the areas with low amplitudes.
Therefore it indicates that by involving the diffraction approximation in SWAN, the
wave energy are smoothed out from area with rapid variation of amplitudes to area
with low amplitudes when the incoming waves (boundary conditions) are
monochromatic and unidirectional waves in Oosterschelde Estuary.

5.3 Conclusion

The following conclusions have been drawn based on the application of the SWAN
model in the Oosterschelde Case.

By excluding the source terms, the results show that the effects of source terms in
SWAN are to dissipate the wave energy. The effects of source terms are also to
transfer the wave energy of low-frequency to high-frequency band. Since bottom
friction, triad wave-wave interaction and the depth-induced breaking have already
been investigated that they are not responsible for the underestimation of wave energy
on low-frequency band by Van Vledder (refer to section 2.3 for details) [1], we can
draw the conclusion that any of the left three aspects of the source terms, i.e., the
inaccurate simulation of the local wind growth, quadruplet wave-wave interaction, or
whitecapping could be one of the possible reasons for the underestimation of the wave
energy on low-frequency band.

The wave energy of low-frequency increases after remove the ambient currents from
the model. It indicates that the effects of the ambient currents are to dissipate the wave
energy on low-frequency band in the Oosterschelde Estuary. Therefore the ambient
currents can be regarded as a possible reason for the underestimation of swell wave
energy at buoy OS4. Further investigations concerning on the effects of ambient
currents are suggested.

The wave energy of the low-frequency further decreases after cut the energy of local
wind sea (on high-frequency band) at the boundaries. It indicates that some part of the
wave energy of low-frequency may be transferred from energy on high-frequency
band (local wind sea). Thereby the insufficient supplement of local wind sea energy
can be considered as a possible reason for the underestimation of wave energy on low-frequency band.

The performance of diffraction in SWAN in the Oosterschelde Estuary shows that in SWAN, the effects of diffraction are barely to be found when the boundary conditions of the model are waves with broad incident directions. But when the incoming waves have small directional spreading ($\sigma_\theta=2^\circ$, quasi-unidirectional), the smooth effects of the wave energy resulting from diffraction are clearly visible in areas with rapid variation of wave amplitudes (Group 4, ODd). Since the diffraction is well approximated in the SWAN model under some conditions (such as Semi-infinite Breakwater Case and Gap Case), it may be resulted from the directional spreading reduces the importance of diffraction. Local variations of different spectral component cancel each other. The effects of directional spreading try to smooth out the typical spatial variation owing to diffraction. Therefore when the incoming waves have certain directional spreading at the boundaries, it is not suggested to take the diffraction into account in SWAN simulation. Involving diffraction will extend the required computing time but the effects of diffraction are negligible under such conditions.

Diffraction can be considered as a possible reason for the underestimation of wave energy in SWAN on low-frequency band at wave buoy OS4 when the incoming waves have narrow directional spreading. However, when the directional spreading of the incoming waves are broad, diffraction in SWAN is not able to solve the problems of underestimation of wave energy on low-frequency band.
6. Conclusions and Suggestions

6.1 Conclusions
Wave Propagation Tests of the Oosterschelde Case and Canyon Case were performed in SWAN model (without diffraction) and validated by the REFRAC model. The results show that refraction is well simulated in SWAN. The wave rays’ pattern obtained in the REFRAC model show that the area with rapid variation of wave amplitudes can be found in Oosterschelde Estuary. The effects of diffraction are therefore hypothesized as a reason for the underestimation of wave energy on low-frequency band at wave buoy OS4.

Three academic cases are performed in the SWAN model with diffraction. The computed results are compared with either the observed data or with results from the other validated model. The characteristics and implementations of the diffraction in SWAN are investigated. The spatial resolution, under-relaxation technique, smoothing technique, stopping criteria and different versions of SWAN are studied. Their relations with diffraction implementation are also studied. The following input settings are therefore suggested as optimum in SWAN containing diffraction:
1) spatial resolution: $\Delta x (\Delta y) = 1/10 \sim 1/5$ of one wave length;
2) without smoothing technique ($n=0$) or with fewest numbers of smoothing steps;
3) with under-relaxation technique ($\alpha \neq 0$)
4) with proper stopping criteria, with 100% fraction has to be fulfilled before SWAN stop computing.

In such a model, a balance can be found between the number of iterations cycles required in each run and the diffraction effects. The number of iterations cycles determines the computing time. The diffraction effects increases the accuracy of the results since they turn the wave energy towards the area with low amplitudes in those areas with rapid variation of wave amplitudes.

The Oosterschelde Case is applied to the SWAN model with diffraction. When the incoming waves have a narrow directional spreading, the results show that the diffraction has certain effects. Diffraction successfully increases the wave energy at wave buoy OS4. However when the incoming waves have a broader directional spreading, the diffraction does not show obvious effect to re-distribute the wave energy. Diffraction can only be conditionally considered a reason for the underestimation of wave energy on low-frequency band at wave buoy OS4. The specific condition is the incoming waves at the boundary are unidirectional.

6.2 Suggestions
The future work on the underestimation of wave energy on low-frequency band based on the Oosterschelde Case at buoy OS4 is suggested in the following three aspects.

The first aspect is the directional spreading of the waves at the boundaries. In SWAN, waves with a small directional spreading of around 1° to 2° usually stand for the unidirectional waves. However, when waves propagate for certain distance, the dispersion effects still play a part that can not be ignored. It can be a reason for the underestimation of the wave energy since a part of wave energy is dispersed. It is also meaningful to investigate what the extent local variations in the different spectral
components cancel each other out owing to the weakened diffraction effects. It may also induce the underestimation.

The second aspect is the simulation of local wind sea. Removal of the wind wave energy (energy on high-frequency band) induces the decrease of the swell wave energy (energy on low-frequency band). It indicates that a part of the wave energy on high-frequency band is transferred to the high-frequency band. Therefore the underestimation of the swell wave energy may be resulted from the insufficient supply of the local wind wave energy. The improvement of the simulation of the local wind waves may solve the underestimation of swell in Oosterschelde Estuary.

The third aspect is the effects of ambient currents. The swell wave energy decreases drastically after removing the ambient currents. It illustrates that the ambient currents play the role in the increase the wave energy of low-frequency. Therefore it can be regarded as a possible reason for the underestimation of wave energy at buoy OS4. Since the currents do not provide adequate wave energy to low-frequency band.

The following three suggestions are made for future development of the diffraction functionality in SWAN.

The first is directional spreading. The variations of different spectral components cancel each other out which will weaken the effects of diffraction. Thereby it becomes meaningless to involve the diffraction effects in computing in SWAN when the directional spreading increases to certain extent (say critical value). It is necessary to investigate the specific critical value of the directional spreading. When the directional spreading is lower than the critical value (narrow), it is meaningful to count diffraction in SWAN. When the directional spreading is higher than the critical value (broad), there is no need to involve the diffraction in computation. Then it will make the application of SWAN more efficient and economic.

The second suggestion is the frequency-independent under-relaxation technique. In spite of the fact that it only affects the numerical convergence behavior but it does not influence the computing results, it still has significant practical sense, since the time required for the simulation in SWAN is closely related to the under-relaxation technique. It will help to save much time and make the application of diffraction in SWAN in large scale models more feasible.

The third suggestion is to solve the numerical instability in the diffraction implementation. When the simulation in SWAN containing the diffraction effects, it always requires more iteration cycles to fulfill the stopping criteria. Sometimes, it even cannot reach the numerical instability without other techniques (such as under-relaxation technique and smoothing technique). It makes the application of diffraction in SWAN with less practical meaning. Thereby to solve the numerical instability in SWAN with diffraction will be helpful.
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Appendix A: SWAN MODEL

A.1 Governing Equations

In SWAN, wave characteristics are described in terms of two-dimensional wave action density spectrum by the spectral action balance equation. For the Cartesian coordinates (Hasselmann et al., 1973), the results are in:

\[
\frac{\partial}{\partial t} N + \frac{\partial}{\partial x} (c_x N) + \frac{\partial}{\partial y} (c_y N) + \frac{\partial}{\partial \sigma} (c_\sigma N) + \frac{\partial}{\partial \theta} (c_\theta N) = \frac{S}{\sigma}
\]  

(A.1)

\(\sigma\) – the relative frequency (as observed in a frame of reference moving with the current velocity);

\(\theta\) – the wave direction (the direction normal to the wave crest of each spectral component);

\(N\) – the wave action density that is equal to the energy density divided by the relative frequency: \(N(\sigma, \theta, x, y, t) = E(\sigma, \theta, x, y, t) / \sigma\).

The first term on left-hand side of Equation (A.1) represents the local rate of change of the wave action density in time, the second and third term represent propagation of wave action density in geographical space, with propagation velocities \(c_x\) and \(c_y\) in respectively x and y space. The fourth term represents change in wave action arising from shifting of the relative frequency due to variations in depths and currents, with propagation velocity \(c_\sigma\) in \(\sigma\) space. The fifth term represents wave action changes from depth-induced and current-induced refraction, with propagation velocity \(c_\theta\) in \(\theta\) space. The expressions for these propagation speeds are taken from linear wave theory (Whitham, 1974; Mei, 1983; Dingemans, 1997). On the right-hand side of Equation (A.1), \(S = S(\sigma, \theta, x, y, t)\) is a source term. It consists of energy density, representing the effects of generation, dissipation and non-linear wave-wave interactions.

In view of the use of SWAN at shelf, sea or oceanic scale, the user can choose to express the basic equation in spherical coordinates:

\[
\frac{\partial}{\partial t} N + \frac{\partial}{\partial \lambda} [c_x N] + \left(\cos \phi \right)^{-1} \frac{\partial}{\partial \phi} [c_\phi (\cos \phi) N] + \frac{\partial}{\partial \sigma} [c_\sigma N] + \frac{\partial}{\partial \theta} [c_\theta N] = \frac{S}{\sigma}
\]  

(A.2)

where \(\lambda\) is the longitude and \(\phi\) is the latitude.

A.2 Functionality

The following wave propagation processes are represented in SWAN:

- \(\sqrt{\text{rectilinear propagation through geographic space;}}\)
- \(\sqrt{\text{refraction due to spatial variations in bathymetry and current;}}\)
- \(\sqrt{\text{shoaling due to spatial variations in bathymetry and current;}}\)
- \(\sqrt{\text{blocking and reflections by opposing currents;}}\)
- \(\sqrt{\text{transmission through, blockage by or reflection against sub-grid obstacles;}}\)
The following wave generation and dissipation processes are represented in SWAN:

- generation by wind;
- dissipation by whitecapping;
- dissipation by depth-induced wave breaking;
- dissipation by seabed or bottom friction;
- wave-wave interactions (quadruplets and triads).

In addition the wave-induced set-up of the mean sea surface can be computed in SWAN.

Cycle III of SWAN is stationary and optionally non-stationary and formulated in Cartesian (recommended only for small scales) or spherical (small scales and large scales) coordinates. The stationary mode should be used only for waves with a relatively short residence time in the computational area under consideration that means the travel time of the waves through the region should be small compared to the time scale of the geophysical conditions (wave boundary conditions, wind, tides and storm surge). A quasi-stationary approach can be taken with stationary SWAN computations in a time-varying sequence of stationary conditions. For one-dimensional (geographical) situations SWAN can be run in one-dimensional mode.

### A.3 Source Terms

The SWAN code gives to user the option to include different physics modules. A brief summary of the formulations that are used for the various source terms in SWAN is given next, with an overview in Table A1. More details are given in Booij et al. (1999) and Ris et al. (1999).

#### Table A1: List of available options of the source terms in SWAN

<table>
<thead>
<tr>
<th>Source term</th>
<th>Theory</th>
<th>1st</th>
<th>2nd</th>
<th>3rd</th>
</tr>
</thead>
<tbody>
<tr>
<td>Linear wind growth</td>
<td>Cavalieri &amp; Malanotte-Rizzoli (1981)</td>
<td>×</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Cavalieri &amp; Malanotte-Rizzoli (1981) modified</td>
<td></td>
<td></td>
<td>×</td>
</tr>
<tr>
<td>Exponential wind growth</td>
<td>Snyder et al. (1981) modified</td>
<td>×</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Snyder et al. (1981)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Janssen (1989, 1991)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Whitecapping</td>
<td>Holthuijsen and De Boer (1988)</td>
<td>×</td>
<td>×</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Komen et al. (1984)</td>
<td></td>
<td></td>
<td>×</td>
</tr>
<tr>
<td></td>
<td>Janssen (1991), Komen et al. (1994)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Quadruplet</td>
<td>Hasselman et al. (1985)</td>
<td></td>
<td></td>
<td>×</td>
</tr>
<tr>
<td>Triad</td>
<td>Eldeberky and Battjes (1996)</td>
<td>×</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Breaking (depth)</td>
<td>Battjes and Janssen (1978)</td>
<td></td>
<td>×</td>
<td></td>
</tr>
<tr>
<td>Bottom Friction</td>
<td>Hasselman et al. (1973)</td>
<td>×</td>
<td></td>
<td>×</td>
</tr>
<tr>
<td></td>
<td>Collins (1972)</td>
<td></td>
<td>×</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Madsen et al. (1988)</td>
<td>×</td>
<td></td>
<td>×</td>
</tr>
<tr>
<td>Obstacle Transmission</td>
<td>Seelig (1979)</td>
<td>×</td>
<td></td>
<td>×</td>
</tr>
</tbody>
</table>
A.4 First, Second and Third Generation Mode

SWAN can operate in first, second and third generation mode. State-of-the-art formulations of the processes of wave generation, dissipation, and wave-wave interactions in phase-averaged models are presently third generation. In the first generation models these physical processes are not properly represented. Generation is simulated with simple empirical expressions, and dissipation (whitecapping) is simulated with an assumed universal upper limit of the spectral densities. The absence of quadruplet wave-wave interactions is compensated by enhancing the wave growth (e.g., Ewing, 1971). Second generation models try to remedy this for the local wind sea by parameterizing these interactions (e.g., Young, 1988) or by using a sea-state and wind-dependent upper limit of the spectral densities (e.g., Holthuijsen and De Boer, 1988) or by reducing the wave description to a few integral spectral parameters (Hasselmann et al., 1976). Such models are usually supplemented with freely propagating swell. In a third-generation model all relevant processes are represented explicitly without a priori restrictions on the evolution of the spectrum. An overview of the options that are available in SWAN is given in the Table A1.

A.5 Implementation of Diffraction in SWAN

Recall action balance equation:

\[
\frac{\partial}{\partial t} N + \frac{\partial}{\partial x} (c_x N) + \frac{\partial}{\partial y} (c_y N) + \frac{\partial}{\partial \sigma} (c_\sigma N) + \frac{\partial}{\partial \theta} (c_\theta N) = \frac{S}{\sigma}
\]  
(A.1)

In the case without diffraction the propagation velocities in x-y and spectral space are given by (Holthuijsen et al., 2003):

\[
\bar{c}_g = (c_x, x_y) = \frac{k}{k} \frac{\partial \sigma}{\partial k} + \bar{U}
\]
(A.3)

\[
c_\sigma = \frac{\partial}{\partial \sigma} \left[ \frac{\partial d}{\partial t} + \bar{U} \cdot \nabla d \right] - c_g \frac{k}{k} \frac{\partial \bar{U}}{\partial s}
\]
(A.4)

\[
c_\theta = - \left[ c_g \left( \frac{1}{k} \frac{\partial \kappa}{\partial m} + \frac{k}{k} \frac{\partial \bar{U}}{\partial m} \right) \right]
\]
(A.5)

In these equations \( \bar{U} \) is the ambient current vector, \( d \) is depth and \( m \) a local coordinate perpendicular to the direction of wave propagation. In the absence of diffraction parameter \( \kappa \) is equal to the wave number \( k \).

If diffraction is accounted for, a correction term \( \delta_k \) is added to the above propagation velocities. The equations then become:

\[
\bar{C}_g = \bar{c}(1 + \delta_k)^{1/2} + \bar{U}
\]
(A.6)

\[
C_\sigma = \frac{\partial \sigma}{\partial d} \left[ \frac{\partial d}{\partial t} + \bar{U} \cdot \nabla d \right] - c_g \frac{k}{k} \frac{\partial \bar{U}}{\partial s} (1 + \delta_k)^{1/2}
\]
(A.7)

\[
C_\theta = - \left[ c_g (1 + \delta_k)^{1/2} \left( \frac{1}{k} \frac{\partial \kappa}{\partial m} + \frac{1}{2(1 + \delta_k)} \frac{\partial \delta_k}{\partial m} \right) + \frac{k}{k} \frac{\partial \bar{U}}{\partial m} \right]
\]
(A.8)
The diffraction parameter $\delta_E$ is based on a spatial average of properties of the wave field according to:

$$\delta_E = \frac{\nabla \cdot (c c_g \nabla \sqrt{E})}{\kappa^2 c c_g \sqrt{E}}$$  \hspace{1cm} (A.9)

where $c$ and $c_g$ are the phase and group velocity, respectively. The diffraction parameter $\delta_E$ adds a second order spatial derivative to the system of equations. The detailed derivation of $\delta_E$ is given in section A.6.

### A.5 Numerical Implementation

The integration of the action balance equation (A.1) and (A.2) has been implemented in SWAN with finite difference schemes in all five dimensions (time, geographic space and spectral space). Time is discretized with a simple constant time step $\Delta t$ for the simultaneous integration of the propagation and the source terms. This is different from the time discretization in the WAM model (model (WAMDI group, 1988, the first third generation wave model that was developed to solve the wave transport equation explicitly without any presumptions on the shape of the wave spectrum) or the WAVEWATCH III model (another third generation wave model developed at NOAA/NCEP in the spirit of the WAM model) where the time step for propagation is different from the time step for the source terms. Geographic space is discretized with a rectangular grid with constant resolutions $\Delta x$ and $\Delta y$ in x and y direction respectively. The spectrum is discretized with a constant directional resolution $\Delta \theta$ and a constant relative frequency resolution $\Delta \sigma / \sigma$ (logarithmic frequency distribution). For reasons of economy, an option is available to compute only wave components traveling in a pre-defined directional sector $\theta_{\min} < \theta < \theta_{\max}$ that means only those components that travel shoreward within a limited directional sector.

The discrete frequencies are defined between a fixed low-frequency cut-off and a fixed high-frequency cut-off (the prognostic part of the spectrum). For these frequencies the spectral density is unconstrained. Below the low-frequency cut-off (typically $f_{\min} = 0.04$ Hz for field conditions) the spectral densities are assumed to be zero. Above the high-frequency cut-off (typically $f_{\max} = 1$ Hz for field conditions) a diagnostic $f^m$ tail is added (this tail is used to compute nonlinear wave-wave interactions at the high frequencies and to compute integral wave parameters). The reason for using a fixed high-frequency cut-off rather than a dynamic cut-off frequency that depends on the wind speed or on the mean frequency, as in WAM and WAVEWATCH III, is that in coastal regions mixed sea states with rather different characteristic frequencies may occur. For instance, a local wind may generate a very young sea behind an island, totally unrelated to (but superimposed on) a simultaneously occurring swell. In such cases a dynamic cut-off frequency may be too low to properly account for the locally generated sea state. Based on physical arguments the value of $m$ (the power in the above expression of the spectral tail) should be between 4 and 5 (Phillips, 1985). In SWAN $m=4$ if the wind input formulation of Komen et al. (1984) is used (as in WAM Cycle 3), and $m=5$ if the wind input formulation of Janssen (1991) is used (as in WAM Cycle 4).
Fully implicit numerical schemes are used in the SWAN model for propagation in both geographical space and spectral space (an iterative, forward-marching, four-sweep technique, Ris et al., 1994). This scheme is unconditionally stable in contrast with the explicit schemes of conventional spectral wave models which are only conditionally stable and which require very small solution steps in shallow water (typically 10s for 100m resolution in water depth of 10m where in SWAN model the time increment may be as large as 15 min). The formulation is basically in terms of finite differences on a regular, rectangular grid. This is inconvenient in regions with highly variable scales such as tidal inlets, tidal flats and estuaries. Nesting of grids with decreasing resolution is the conventional approach in such cases but it requires extra computations.

The formulations for generation, dissipation and quadruplet wave-wave interactions are taken from the WAM model. These are supplemented with a spectral version of the dissipation model for depth-induced breaking of Battjes and Janssen (1978), with maximum wave height to depth ratio from Nelson (1987) and a recently formulated discrete interaction approximation for triad wave-wave interactions (Eldeberky and Battjes, 1995).

As introduced before, the effect of diffraction is implemented in SWAN by adding the diffraction parameter $\delta_\theta$ to the expressions for the group velocity components $C_x$, $C_y$ and to the turning rate $C_\theta$ (i.e., the refraction term) in the propagation schemes. In each of the iterations of the original propagation scheme, the second-order derivative $\nabla \cdot \left( C_{g} \nabla \sqrt{E(\sigma, \theta)} \right)$ in the expression of $\delta_\theta$ is obtained with a simple, second-order central scheme based on the results of the previous iteration. For the $x$-dimension the estimation is:

$$\left[ \frac{\partial}{\partial x} \left( C_{g} \frac{\partial \sqrt{E}}{\partial x} \right) \right]_n \approx \frac{1}{2\Delta x} \left[ \left\{ (C_{g})_i + (C_{g})_{i-1} \right\} \sqrt{E}_{i-1} - \left\{ (C_{g})_{i-1} + 2(C_{g})_i + (C_{g})_{i+1} \right\} \sqrt{E}_i + \left\{ (C_{g})_{i+1} \right\} \sqrt{E}_{i+1} \right]^{n-1} \quad (A.10)$$

where $i$ is a grid counter in $x$-dimension and the superscript $n$ indicates iteration number. For the $y$-dimension, the expression is identical, with $y$ replacing $x$. The estimation of $\delta_\theta$ is thus based on the values of the energy density $E$ obtained from the preceding iteration in the geographic propagation scheme (the value of $\delta_\theta$ is cut off at the low side at -1 to avoid imaginary propagation speeds; there is no upper bound). For models with a variable geographic resolution (optional in SWAN), the numerical approximation of the gradients needs to account for different values of $\Delta x$ and $\Delta y$ on either side of the central grid point in the scheme. The advantage of a variable-resolution grid is that the model can be based on a high spatial resolution where diffraction requires this (near obstacles) and on a coarser resolution where such high resolution is not required. Diffraction is thus computed in the entire computation domain with high resolution only where needed. At the boundaries of the computational domain (coast, obstacle or open boundaries) first-order spatial
gradients in SWAN are set at zero. Open boundaries should therefore be chosen such that diffraction near these boundaries can be ignored, i.e., the gradients should be small there. At the coast or at an obstacle, the second-order gradients are approximated with the assumption of a horizontal first-order gradient at the coast or the obstacle.

The discretization of the gradients with the finite difference scheme of Equation (A.10), in terms of the values of $\sqrt{E(\sigma, \theta)}$ in neighboring points in the $x$- and $y$-dimension, seems trivial but the following shows that it is not. Diffraction of random, short-crested waves can also be computed as the superposition of solutions for a large number of incident monochromatic, unidirectional waves, each computed independently with the mild-slope equation. In each such computation, the spatial gradient is taken of the amplitude of the monochromatic wave. The direction of such a monochromatic wave varies geographically so that for each grid point in the computational domain, the wave energy in neighboring grid points propagates in different directions. The spatial gradient of $\sqrt{E(\sigma, \theta)}$ in the present approach (estimated from these neighboring points) should therefore also be taken at these slightly different directions. Instead, Equation (A.10) states that at these grid points the same direction is to be taken as in the central grid point.

A.6 Derivation of Diffraction Parameter $\delta_E$

A.6.1 the Mild-slope Equation

Consider a harmonic wave with complex wave function $\xi = a \exp(ia\psi)$, where $\text{Re}(\xi e^{i\omega t})$ is the surface elevation as it varies in space and time, $a = a(x, y)$ is the (stationary) amplitude and $\psi = \psi(x, y)$ is the (stationary) phase function. In the linear theory of surface gravity waves over a mildly sloping bottom, the propagation of this wave is given by the mild-slope equation (Berkhoff, 1972):

$$\nabla \cdot c_g \nabla \xi + \kappa^2 c_g \xi = 0$$

where $c = \omega / \kappa$, $c_g = \partial \omega / \partial \kappa$ and the separation parameter $\kappa$ is determined from $\omega^2 = g \kappa \tanh(\kappa d)$ (where $g$ is gravitational acceleration and $d$ is water depth). In the absence of diffraction, $c$ and $c_g$ would be the phase speed and the group velocity respectively and $\kappa$ would be equal to the wave number $k$ (the magnitude of the spatial gradient of the phase function $\psi$): $\kappa = k = |\nabla \psi|$. By multiplying the mild-slope equation with the complex conjugate of $\xi$ and then taking the imaginary part, we obtain an energy balance equation, which for a constant frequency $\omega$ (a basic assumption for the mild-slope equation), reduce to:

$$\nabla \left( \frac{k}{\kappa} c_g a^2 \right) = 0$$

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This energy balance equation shows that, in the presence of diffraction the energy propagation speed in geographic space is:

\[ C_g = \frac{k}{\kappa} c_g \]  
(A.13)

By substituting the wave function into the mild-slop equation it is readily shown that the magnitude of the gradient of the phase function in the presence of diffraction is given by:

\[ k^2 = \kappa^2 + \frac{\nabla \cdot (cc_g \nabla a)}{cc_g a} \]  
(A.14)

If diffraction is ignored (i.e., the variation in amplitude is taken to be zero), the second term on the right-hand-side of Equation (A.14) vanishes and this equation reduces to \( k = \kappa \). This second term therefore represents diffraction in the phase function \( \varphi \).

Denoting the diffraction parameter \( \delta_a \) as:

\[ \delta_a = \frac{\nabla \cdot (cc_g \nabla a)}{\kappa^2 cc_g a} \]  
(A.15)

the diffraction-corrected phase speed \( C \) is:

\[ C = \omega / k = c (1 + \delta_a)^{-1/2} \]  
(A.16)

and the diffraction-corrected group velocity is:

\[ C_g = c_g (1 + \delta_a)^{1/2} \]  
(A.17)

---

Figure A1: The Directional Turning of an Iso-phase line and Corresponding Orthogonals (Wave Rays)

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The spatial rate of directional turning can be derived for a spatially varying phase speed, without assuming that the variations are induced by refraction. The derivation can be formal, using general mathematical properties of the phase function (e.g., Dingemans, 1997; Eq. (2.94)). As an alternative, the following derivation (by L. Holthuijsen, et al. 2003) is based on a more physically oriented geometric argument.

[2] Consider in an arbitrary situation, a line of equal phase (iso-phase line) of the wave, along which the phase speed varies (Figure A1). A right-turning system of orthogonal \( m, s \) coordinates is used with \( m \) oriented along the iso-phase line. Two points, A and B on the iso-phase line, separated by distance \( \Delta m \), move in a time interval \( \Delta t \) normal to the iso-phase line (along an orthogonal or wave ray) over a distances \( \Delta S_\lambda = c\Delta t \) and \( \Delta S_\beta = (c + \Delta c)\Delta t \), respectively. The corresponding directional turning of the iso-phase line is \( \Delta \theta = (\Delta S_\lambda - \Delta S_\beta)/\Delta m \). The spatial rate of directional turning \( \Delta \theta / \Delta s \) (per unit distance in the wave direction, i.e., along the wave ray) is therefore \( \Delta \theta / \Delta s = -\Delta c\Delta t / \Delta m \Delta s \). Or, since \( \Delta s = c\Delta t \), \( \Delta \theta / \Delta s = -(\Delta c / c) / \Delta m \). For infinitesimally small differences, this is:

\[
\frac{\partial \theta}{\partial s} = \frac{1}{c} \frac{\partial c}{\partial m} \quad (A.18)
\]

With \( c = \omega / k \) and \( \omega = \) constant substituted in Equation (A.18), this turning rate can also be written as:

\[
\frac{\partial \theta}{\partial s} = \frac{1}{k} \frac{\partial k}{\partial m} \quad (A.19)
\]

or in terms of the separation parameter \( \kappa \) (substituting Equation (A.14) into Equation (A.19)):

\[
\frac{\partial \theta}{\partial s} = \frac{k}{1} \frac{\partial k}{\partial m} + \frac{1}{\kappa} \frac{\partial \kappa}{\partial m} + \frac{1}{2(1 + \delta_\nu)} \frac{\partial \delta_\nu}{\partial m} \quad (A.20)
\]

If diffraction is ignored, i.e., \( \delta_\nu = 0 \), the turning rate reduces to the commonly used turning rate for refraction only:

\[
\frac{\partial \theta}{\partial s} = \frac{1}{k} \frac{\partial k}{\partial m} = \frac{1}{\kappa} \frac{\partial \kappa}{\partial m} \quad (A. 21)
\]

### A.6.2 Spectral Energy Balance

In the absence of diffraction and currents, by omitting the source terms, the wave action density balance equation becomes to be the spectral energy balance equation:

\[
\frac{\partial}{\partial t} E + \frac{\partial}{\partial x} (c_x E) + \frac{\partial}{\partial y} (c_y E) + \frac{\partial}{\partial \theta} (c_\theta N) = 0 \quad (A.22)
\]
where \( E = E(\omega, \theta) \) is the energy density of the waves as a function of frequency \( \omega \) and direction \( \theta \). The first term on the left side represents the local rate of change of energy density, the second and third term represent rectilinear propagation in the horizontal, flat plane \((x, y)\); propagation on a sphere is ignored here). The fourth term represents propagation in spectral directional space (refraction). The propagation speeds \( c_x, c_y \) are the x-, y-component of the group velocity \( c_g \) respectively and \( c_\theta \) is the rate of directional turning (propagation speed in \( \theta \)-dimension). (The dependencies on \( x, y \) and \( t \) have been ignored in the notation for the sake of brevity in this section.) These propagation speeds are taken from the linear theory of surface gravity waves without the effects of diffraction (e.g., Mei, 1983; Dingemans, 1997). To add diffraction, note that the turning rate \( c_\theta \) is the rate of directional change of a single wave component as it travels along the wave ray with the group velocity. It is readily obtained from the spatial turning rate \( \partial \theta / \partial s \). As the wave energy travels along the ray over a distance \( \Delta s \) in a time interval \( \Delta t \), it turns over an angle \( \Delta \theta \). The spatial rate of turning is then \( \Delta \theta / \Delta s \). Since the distance \( \Delta s = c_g \Delta t \), it follows that the temporal rate of turning \( \Delta \theta / \Delta t = c_g \Delta \theta / \Delta s \). Taking the infinitesimal limits, denoting the temporal rate of turning as \( c_\theta \) and substituting Equation (A.20), gives:

\[
c_\theta = c_g \left( \frac{\partial \theta}{\partial s} \right) = c_g \left( \frac{1}{k} \frac{\partial}{\partial m} \right)
\]  

(A.23)

Adding diffraction to the action balance equation involves only replacing \( c_g \) by \( C_g \) and \( c_\theta \) by \( C_\theta \) in the spectral energy balance of Equation (A.1).

\[
C_\theta = C_g \left( \frac{\partial \theta}{\partial s} \right) = C_g \left( \frac{1}{\kappa} \frac{\partial \kappa}{\partial m} + \frac{1}{2(1 + \delta_a)} \frac{\partial \delta_a}{\partial m} \right)
\]  

(A.24)

These expressions for \( C_g \) and \( C_\theta \) are formulated in terms of the diffraction parameter \( \delta_a \), which is expressed in terms of the amplitude of a harmonic wave and not in terms of the spectral density \( E(\sigma, \theta) \). Unfortunately, formulation in terms of \( E(\sigma, \theta) \) are not available but it seems reasonable to replace the normalized derivatives of amplitude \( \alpha \) in these expressions by the normalized derivatives of the square root of the energy density, \( \sqrt{E} = \sqrt{E(\sigma, \theta)} \) (the more so as both are normalized), so that \( \delta_a \) is replaced by \( \delta_e \) :

\[
\delta_e = \frac{\nabla \left( c_g \sqrt{E} \right)}{\kappa^2 c_g \sqrt{E}}
\]  

(A.25)

Including this diffraction term in the expression of Equation (A.18) for the turning rate seems to be a straightforward expansion of the energy balance equation. But it is not. The energy balance equation does not account for the phase evolution of the waves, whereas the mild-slope equation does: replacing the amplitude \( a = a(x, y) \)
with the energy density $E(\sigma, \theta; x, y)$ removes phase-information and therefore decouples the phases spatially. Therefore this approximation is referred as a phase-decoupled refraction-diffraction approximation. (Since ambient currents are not relevant for this study, the corresponding expressions for the propagation speeds in the presence of an ambient current are not given here.) The most important effect of decoupling the waves in practical applications seems to be that standing wave patterns are not accounted for (i.e., nodal wave patterns are absent). This characteristic is shared with the parabolic refraction-diffraction approximation of the mild-slope equation, albeit for other underlying reasons. In the parabolic approximation, waves cannot reflect and propagate against the incident wave direction (e.g. Radder, 1979; Booij, 1981), which implies that standing wave patterns cannot be properly represented in the parabolic approximation. In the present phase-decoupled approximation, waves can reflect and propagate against the incident wave direction but phase coupling between the incident wave and its reflection (required to represent a nodal pattern) is not possible.

With this addition of phase-decoupled diffraction, the spectral energy balance equation changes from a second-order differential equation to a fourth-order equation: the refraction term on the left-hand-side of Equation (A.22) contains a first-order spectral derivative of $c_\theta$, which itself contains a first-order spatial derivative. The diffraction parameter $\delta_\varepsilon$ adds a second-order spatial derivative of $\sqrt{E(\sigma, \theta)}$. By separating the numerical treatment of diffraction (see below) from the numerical treatment of refraction, a conventional spectral model for refraction can be expanded to include this approximation of diffraction.
Appendix B: REFRAC MODEL

The Program of the REFRAC model was originally written in Fortran 77 by N. Booij in 1994. It is a pure refraction model, which follows the geometrical optics approximation.

A ray is a characteristic of the energy balance equation. The position along the ray is described by 3 parameters: x, y and θ (ray direction). The change of these parameters with arc length along the ray is given by the following equations:

\[
\frac{dx}{ds} = \cos(\theta) \tag{B.1}
\]
\[
\frac{dy}{ds} = \sin(\theta) \tag{B.2}
\]
\[
\frac{d\theta}{ds} = \frac{1}{R} \tag{B.3}
\]

R – the curvature of the ray.

Since the distance along the wave ray expressed in the REFRAC model is not arc length but the number of wavelengths p. Besides, \(\frac{dp}{ds} = \frac{1}{\lambda}\) (λ is the wavelength). Hence, in terms of the parameter p, the equations can be re-written in:

\[
\frac{dx}{dp} = \lambda \cdot \cos(\theta) \tag{B.4}
\]
\[
\frac{dy}{dp} = \lambda \cdot \sin(\theta) \tag{B.5}
\]
\[
\frac{d\theta}{ds} = \frac{\lambda}{R} = \frac{1}{\lambda} \cdot \frac{\partial \lambda}{\partial m} = \sin \theta \cdot \frac{\partial \lambda}{\partial x} - \cos \theta \cdot \frac{\partial \lambda}{\partial y} \tag{B.6}
\]

where \(\lambda\) is a known function of the given wave frequency and depth; \(\partial \lambda/\partial m\) is the operator defines the derivation along the iso-phase line (wave crest, refer to Figure A1). The ray will be plotted from its starting point up to the boundary of the region or up to the coast.

In the REFRAC model, information of depth is given on a grid. For each grid point, the local wavelength is computed for that point. These values are used to compute the local λ and its derivatives at the point of the ray. The ray is propagated using the Runge-Kutta 4th order integration scheme.

According to Bouws and Battjes (1982), the total wave energy in the computing domain A can be found by integrating along the ray over the interval in the area A as follows:

\[
E_{\text{tot}} = \sum_i \left\{ c_{g0} E_0 \right\} \left\{ \Delta n_0 \right\} \int_{e_i}^{f_i} ds / c_g
\]

where \(c_{g0}\) is the group velocity at incidence, \(E_0\) is the energy density at incidence and \(\Delta n_0\) is the initial separation of the rays. \(e_i\) is the start point of ray i in area A and \(f_i\) is the end point of ray i in area A.
Appendix C: RAW DATA AND RESULTS

C.1 Wave Propagation Tests

C.1.1 Canyon Case

Table 3: Summary of the tests in Canyon Case (Wave Propagation Test)

<table>
<thead>
<tr>
<th>DIR (ºN)</th>
<th>Resolution</th>
<th>T (s)</th>
<th>REFRAC</th>
<th>SWAN</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>N_x × N_y</td>
<td>Δx × Δy</td>
<td>Δθ</td>
<td>Rays</td>
</tr>
<tr>
<td>272</td>
<td>198 × 168</td>
<td>20 × 20</td>
<td>0.5º</td>
<td>15.4</td>
</tr>
<tr>
<td></td>
<td>79 × 67</td>
<td>50 × 50</td>
<td>0.5º</td>
<td>15.4</td>
</tr>
<tr>
<td>240</td>
<td>198 × 168</td>
<td>20 × 20</td>
<td>0.5º</td>
<td>15.4</td>
</tr>
<tr>
<td></td>
<td>79 × 67</td>
<td>50 × 50</td>
<td>0.5º</td>
<td>15.4</td>
</tr>
<tr>
<td>300</td>
<td>198 × 168</td>
<td>20 × 20</td>
<td>0.5º</td>
<td>15.4</td>
</tr>
<tr>
<td></td>
<td>79 × 67</td>
<td>50 × 50</td>
<td>0.5º</td>
<td>15.4</td>
</tr>
<tr>
<td>270</td>
<td>198 × 168</td>
<td>20 × 20</td>
<td>5º</td>
<td>15</td>
</tr>
<tr>
<td></td>
<td>79 × 67</td>
<td>50 × 50</td>
<td>5º</td>
<td>15</td>
</tr>
</tbody>
</table>

Input file of C01 in the REFRAC model

```
INP BOTTOM 0 0 0 847 721 4.6342 4.6266
READ BOTTOM 1. 'Bcan.bot' IDLA=5 NHEDF=0 FREE
period 15.4
num 0.05
area 0 0 0 198 168 20 20
Plot 'bf.plt' 'Haringvliet' iso pen=3 bottom
BB 'h.txt' 'x.grd' 'y.grd' AVG 2 2
Mark 1 .1 file='bf3f01.tab'
Rays same parallel angle=-2 dist=20.
STOP
```

Input file of CR01_f in the SWAN model

```
INP BOTTOM 0 0 0 847 721 4.6342 4.6266
READ BOTTOM 1. 'Bcan.bot' IDLA=5 NHEDF=0 FREE
period 15.4
num 0.05
area 0 0 0 198 168 20 20
Plot 'bf.plt' 'Haringvliet' iso pen=3 bottom
BB 'h.txt' 'x.grd' 'y.grd' AVG 2 2
Mark 1 .1 file='bf3f01.tab'
Rays same parallel angle=-2 dist=2.
STOP
```
### Input file of CS01_f in the SWAN model

```
$********** HEADING ************************************************************************
PROJ 'SWAN Canyon' ' CR01_f'

$********** MODEL INPUT ************************************************************************
SET MAXERR = 3
CGRID REG 0 0 0 3950 3350 198 168 CIRCLE 720 0.03 1.00 37
INP BOTTOM REG 0 0 0 847 721 4.6635 4.6463
READ BOTTOM 1 'BF.bot' IDLA=5 NHEDF=0 FREE

$********** BOUNDARY CONDITIONS ************************************************************************
BOUNd SHAPespec BIN PEAK DSPR DEGRees
BOUNdsnpec SIDE West CON PAR 1.00 15.4 -2 2

$********** PHYSICA ************************************************************************
OFF WINDGrowth
OFF QUADrupl
OFF WCAPping
OFF BREaking

$********** NUMERIEKE PARAMETERS ************************************************************************
NUM STOPC 0.00 0.01 0.001 100 STAT mxitst=100 alfa=0.01

$ ********** OUTPUT ************************************************************************
OUTPUT OPTIONS '%' TABLE 16 BLOCK 9 1000 SPEC 8
BLOCK 'COMPGRID' NOHEAD 'bf3XP.mat' LAY-OUT 3 XP
BLOCK 'COMPGRID' NOHEAD 'bf3YP.mat' LAY-OUT 3 YP
BLOCK 'COMPGRID' NOHEAD 'bf3fHS.mat' LAY-OUT 3 HSIGN
COMPUTE
STOP
```

### Input file of CS04_f in SWAN model

```
$********** HEADING ************************************************************************
PROJ 'SWAN Canyon' ' CR04_f'

$********** MODEL INPUT ************************************************************************
SET MAXERR = 3
CGRID REG 0 0 0 3950 3350 198 168 CIRCLE 720 0.03 1.00 37
INP BOTTOM REG 0 0 0 847 721 4.6635 4.6463
READ BOTTOM 1 'BF.bot' IDLA=5 NHEDF=0 FREE

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```
**BOUNDARY CONDITIONS**

BOUnD SHApespec JONswap PEAK DSPR POWer
BOUndspec SIDE WEST CON PAR 1 15 0 2

**PHYSICA**

OFF WINDGrowth
OFF QUADrupl
OFF WCAPping
OFF BREaking

**NUMERIEKE PARAMETERS**

NUM STOPC 0.00 0.01 0.001 100 STAT mxitst=100 alfa=0.01

**OUTPUT**

OUTPUT OPTIONS '%’ TABLE 16 BLOCK 9 1000 SPEC 8

BLOCK 'COMPGRID' NOHEAD 'bf3fXP.mat' LAY-OUT 3 XP
BLOCK 'COMPGRID' NOHEAD 'bf3fYP.mat' LAY-OUT 3 YP
BLOCK 'COMPGRID' NOHEAD 'bf3fHS.mat' LAY-OUT 3 HSIGN

**COMPUTE**

STOP
Figure C2: Wave Rays’ Pattern in Canyon Case – C02

Figure C3: Wave Rays’ Pattern in Canyon Case – C03
Figure C4: Wave Rays' Pattern in Canyon Case – C04

Figure C5: Distribution of Hs in Canyon Case obtained by REFRAC – CR01_f
Figure C6: Distribution of Hs in Canyon Case obtained by REFRAC – CR01_c

Figure C7: Distribution of Hs in Canyon Case obtained by REFRAC – CR02_f
Figure C8: Distribution of Hs in Canyon Case obtained by REFRAC – CR02_c

CR02_c: Canyon, REFRAC, Dir=240°N, Coarse Grid: Δx=Δy=50m (79×67)

Figure C9: Distribution of Hs in Canyon Case obtained by REFRAC – CR03_f

CR03_f: Canyon, REFRAC, Dir=300°N, Fine Grid: Δx=Δy=20m (198×166)
Figure C10: Distribution of Hs in Canyon Case obtained by REFRAC – CR03_c

Figure C11: Distribution of Hs in Canyon Case obtained by REFRAC – CR04_f
Figure C12: Distribution of Hs in Canyon Case obtained by REFRAC – CR04_c

Figure C13: Distribution of Hs in Canyon Case obtained by SWAN – CS01_f
Figure C14: Distribution of Hs in Canyon Case obtained by SWAN – CS01_c

Figure C15: Distribution of Hs in Canyon Case obtained by SWAN – CS02_f
Figure C16: Distribution of $H_s$ in Canyon Case obtained by SWAN – CS02

Figure C17: Distribution of $H_s$ in Canyon Case obtained by SWAN – CS03
CS03 - Canyon, SWAN, Dir= 300°N, coarse grids: Δx=Δy=50m (79x67)

Figure C18: Distribution of Hs in Canyon Case obtained by SWAN – CS03

CS04 - Canyon, SWAN, Dir= 270°N, fine grids: Δx=Δy=20m (198x168)

Figure C19: Distribution of Hs in Canyon Case obtained by SWAN – CS04
C1.2 Oosterschelde Case

Table 4: Sensitive Control Test of Oosterschelde (Wave Propagation Test)

<table>
<thead>
<tr>
<th>DIR (°N)</th>
<th>Resolution</th>
<th>T (s)</th>
<th>REFRAC</th>
<th>SWAN</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$N_x \times N_y$</td>
<td>$\Delta x \times \Delta y$</td>
<td>$\Delta \theta$</td>
<td>Rays</td>
</tr>
<tr>
<td>280 °N</td>
<td>215×300</td>
<td>20×20</td>
<td>0.5°</td>
<td>12</td>
</tr>
<tr>
<td></td>
<td>86×120</td>
<td>50×50</td>
<td>0.5°</td>
<td>12</td>
</tr>
<tr>
<td>300 °N</td>
<td>215×300</td>
<td>20×20</td>
<td>0.5°</td>
<td>12</td>
</tr>
<tr>
<td></td>
<td>86×120</td>
<td>50×50</td>
<td>0.5°</td>
<td>12</td>
</tr>
<tr>
<td>330 °N</td>
<td>215×300</td>
<td>20×20</td>
<td>0.5°</td>
<td>12</td>
</tr>
<tr>
<td></td>
<td>86×120</td>
<td>50×50</td>
<td>0.5°</td>
<td>12</td>
</tr>
<tr>
<td>350 °N</td>
<td>215×300</td>
<td>20×20</td>
<td>0.5°</td>
<td>12</td>
</tr>
<tr>
<td></td>
<td>86×120</td>
<td>50×50</td>
<td>0.5°</td>
<td>12</td>
</tr>
</tbody>
</table>

Input file of O01 in the REFRAC model

```
INP BOTTOM 38300 406900 60 215 300 20 20
READ BOTTOM 1. 'Bf.bot' IDLA=5 NHEDF=0 FREE

period 12.
num 0.05
area 38300 406900 60 215 300 20 20
Plot 'bf.plt' 'Haringvliet' iso pen=3 bottom
BB 'h.txt' 'x.grd' 'y.grd' AVG 3 3
```
Mark 1 .1  file='bf3f01.tab'
Rays same parallel angle=-10  dist=20.
STOP

Input file of OR01_f in the REFRAC model

INP BOTTOM 38300 406900 60 215 300 20 20
READ BOTTOM 1. 'Bf.bot' IDLA=5 NHEDF=0 FREE
period 12.
num 0.05
Plot 'bf.plt' 'Haringvliet' iso pen=3 bottom
BB 'h.txt' 'x.grd' 'y.grd' AVG 4 4
Mark 1 .1  file='bf3f01.tab'
Rays same parallel angle=-10  dist=2.
STOP

Input file of OS01_f in the SWAN model

$********** HEADING*****************************************************************************
PROJ 'Hindcast Oosterscheldemoning'
$********** MODEL INPUT ************************************************************************
SET MAXERR = 3
CGRID REG 38300 406900 60 4300 6000 215 300 CIRCLE 720 0.03 1.00 37
INP BOTTOM REG 38300 406900 60 215 300 20 20
READ BOTTOM 1. 'BF.bot' IDLA=5 NHEDF=0 FREE
$********** BOUNDARY CONDITIONS **************************************************************
BOUNd SHAPespec BIN PEAK DSPR DEGRees
BOU SEGM IJ 0 300 215 300 CONstant PAR 1 12 -10 2
BOU SEGM IJ 0 0 0 300 CONstant PAR 1 12 -10 2
$
$********** PHYSICA ***********************************************************************
OFF WINDGrowth
OFF QUADrupl
OFF WCAPping
OFF BREaking
$
$********** NUMERIEKE PARAMETERS **************************************************************
NUM STOPC 0.00 0.01 0.001 98 STAT mxitst=40 alfa=0.01
$********** OUTPUT ***********************************************************************
OUTPUT OPTIONS %' TABLE 16 BLOCK 9 1000 SPEC 8
$
Figure C21: Wave Rays’ Pattern in Oosterschelde Case – O01
**Figure C22: Wave Rays’ Pattern in Oosterschelde Case – O02**

**Figure C23: Wave Rays’ Pattern in Oosterschelde Case – O03**
Figure C24: Wave Rays' Pattern in Oosterschelde Case – O04

Figure C25: Distribution of Hs in Oosterschelde Case obtained by REFRAC – OR01_f
Figure C26: Distribution of Hs in Oosterschelde Case obtained by REFRAC – OR01_c

Figure C27: Distribution of Hs in Oosterschelde Case obtained by REFRAC – OR02_f
Figure C28: Distribution of Hs in Oosterschelde Case obtained by REFRAC – OR02_c

Figure C29: Distribution of Hs in Oosterschelde Case obtained by REFRAC – OR03_f
Figure C30: Distribution of $H_s$ in Oosterschelde Case obtained by REFRAC – OR03_c

Figure C31: Distribution of $H_s$ in Oosterschelde Case obtained by REFRAC – OR04_f
Figure C32: Distribution of Hs in Oosterschelde Case obtained by REFRAC – OR04_c

Figure C33: Distribution of Hs in Oosterschelde Case obtained by SWAN – OS01_f
Figure C34: Distribution of Hs in Oosterschelde Case obtained by SWAN – OS01c

Figure C35: Distribution of Hs in Oosterschelde Case obtained by SWAN – OS02f
Figure C36: Distribution of Hs in Oosterschelde Case obtained by SWAN – OS02_o

Figure C37: Distribution of Hs in Oosterschelde Case obtained by SWAN – OS03_f
Figure C38: Distribution of $Hs$ in Oosterschelde Case obtained by SWAN – OS03_c

Figure C39: Distribution of $Hs$ in Oosterschelde Case obtained by SWAN – OS04_f
C.2 Diffraction Tests

C.2.1 Semi-infinite Breakwater Case

Table 5: Summary of SWAN model settings for all runs in the uniform grid in Semi-Infinite Long Breakwater Case

<table>
<thead>
<tr>
<th>Codes</th>
<th>GRID</th>
<th>Diffr</th>
<th>α</th>
<th>Mxitst</th>
<th>Real</th>
<th>Accuracy</th>
</tr>
</thead>
<tbody>
<tr>
<td>SUH01</td>
<td>60×60</td>
<td>0.25m</td>
<td>Off</td>
<td>0</td>
<td>200</td>
<td>100%</td>
</tr>
<tr>
<td>SUH02</td>
<td>60×60</td>
<td>0.25m</td>
<td>On</td>
<td>0</td>
<td>200</td>
<td>98.39%</td>
</tr>
<tr>
<td>SUH03</td>
<td>60×60</td>
<td>0.25m</td>
<td>On</td>
<td>0.01</td>
<td>200</td>
<td>72.48%</td>
</tr>
<tr>
<td>SUH04</td>
<td>60×60</td>
<td>0.25m</td>
<td>On</td>
<td>0.1</td>
<td>200</td>
<td>70.46%</td>
</tr>
<tr>
<td>SUH05</td>
<td>60×60</td>
<td>0.25m</td>
<td>On</td>
<td>1</td>
<td>150</td>
<td>96.99%</td>
</tr>
<tr>
<td>SUH06</td>
<td>60×60</td>
<td>0.25m</td>
<td>On</td>
<td>6</td>
<td>200</td>
<td>3.57%</td>
</tr>
<tr>
<td>SUH07</td>
<td>60×60</td>
<td>0.25m</td>
<td>On</td>
<td>10</td>
<td>200</td>
<td>93.34%</td>
</tr>
<tr>
<td>SUH08</td>
<td>60×60</td>
<td>0.25m</td>
<td>On</td>
<td>15</td>
<td>200</td>
<td>100%</td>
</tr>
<tr>
<td>SUH05~</td>
<td>60×60</td>
<td>0.25m</td>
<td>On</td>
<td>1</td>
<td>200</td>
<td>92.07%</td>
</tr>
<tr>
<td>SUH06~</td>
<td>60×60</td>
<td>0.25m</td>
<td>On</td>
<td>6</td>
<td>200</td>
<td>95.22%</td>
</tr>
<tr>
<td>SUH07~</td>
<td>60×60</td>
<td>0.25m</td>
<td>On</td>
<td>10</td>
<td>200</td>
<td>100%</td>
</tr>
<tr>
<td>SUH08~</td>
<td>60×60</td>
<td>0.25m</td>
<td>On</td>
<td>15</td>
<td>200</td>
<td>100%</td>
</tr>
<tr>
<td>SUL01</td>
<td>30×30</td>
<td>0.2m</td>
<td>Off</td>
<td>0</td>
<td>200</td>
<td>100%</td>
</tr>
<tr>
<td>SUL02</td>
<td>30×30</td>
<td>0.2m</td>
<td>On</td>
<td>0</td>
<td>200</td>
<td>4.58%</td>
</tr>
<tr>
<td>SUL03</td>
<td>30×30</td>
<td>0.2m</td>
<td>On</td>
<td>0.01</td>
<td>200</td>
<td>60.87%</td>
</tr>
<tr>
<td>SUL04</td>
<td>30×30</td>
<td>0.2m</td>
<td>On</td>
<td>0.1</td>
<td>200</td>
<td>35.69%</td>
</tr>
<tr>
<td>SUL05</td>
<td>30×30</td>
<td>0.2m</td>
<td>On</td>
<td>1</td>
<td>200</td>
<td>85.54%</td>
</tr>
</tbody>
</table>

Figure C40: Distribution of Hs in Oosterschelde Case obtained by SWAN – OS04_c
$********** HEADING ***********************************************
PROJ 'SEMI-INFINITE' 'SUH08~'
$ Hs   = 0.055 m
$ Tp   = 1.30 s

$********** MODEL INPUT************************************************
SET MAXERR = 3
CGRID REgular 0. 0. 0. 15. 15. 60 60 CIRCLE 1440 0.5385 1. 6
INPGGRID BOTTOM 0. 0. 0. 1 1 15. 15.
READINP BOTTOM 1. 'test.bot'

$********** BOUNDARY CONDITIONS ********************************
BOUn SHAPespec BIN PEAK DSPR DEGRees
BOU SIDE W CCW CON PAR 0.055 1.3 90 1.5
BOU SIDE S CCW CON PAR 0.055 1.3 90 1.5
OBSTACLE TRANSm 0. REFL 0. LINe 7.5 2.5001 17.0 2.5001
Diffraction smpar=0.2 smnum=15

$********** PHYSICA ***********************************************
OFF WINDGrowth
OFF QUADrupl
OFF WCAPping
OFF BREaking

$********** NUMERIEKE PARAMETERS ********************************
NUM STOPC 0.00 0.01 0.001 100 STAT mxitst=200 alfa=0.1

$ ********** OUTPUT ***********************************************
POINTS 'curve' FILE 'curve.loc'
TABLE 'curve' NOHEAD 'cXp.tab' XP
TABLE 'curve' NOHEAD 'cHs.tab' HS
BLOCK 'COMPGRID' NOHEAD 'Xp.mat' LAYOUT 3 XP
BLOCK 'COMPGRID' NOHEAD 'Yp.mat' LAYOUT 3 YP
BLOCK 'COMPGRID' NOHEAD 'Hs.mat' LAYOUT 3 HS
BLOCK 'COMPGRID' NOHEAD 'Dir.mat' LAYOUT 3 DIR

COMPUTE
STOP
Figure C41: SUH01 – Normalized Wave Height of unidirectional, monochromatic waves propagating around a semi-infinite, straight breakwater

Figure C42: SUH01 – Normalized significant wave heights distributed along circular-section at 3L
Figure C43: SUH08 – Normalized Wave Height of unidirectional, monochromatic waves propagating around a semi-infinite, straight breakwater

Figure C44: SUH08 – Normalized significant wave heights distributed along circular-section at 3L
**Figure C45:** SUH07\~ – Normalized Wave Height of unidirectional, monochromatic waves propagating around a semi-infinite, straight breakwater

**Figure C46:** SUH07\~ – Normalized significant wave heights distributed along circular-section at 3L
Figure C47: SUH08~ – Normalized Wave Height of unidirectional, monochromatic waves propagating around a semi-infinite, straight breakwater

Figure C48: SUH08~ – Normalized significant wave heights distributed along circular-section at 3L
Figure C49: SUL01 – Normalized Wave Height of unidirectional, monochromatic waves propagating around a semi-infinite, straight breakwater

Figure C50: SUL01 – Normalized significant wave heights distributed along circular-section at 3L
Figure C51: SUL06 – Normalized Wave Height of unidirectional, monochromatic waves propagating around a semi-infinite, straight breakwater.

Figure C52: SUL06 – Normalized significant wave heights distributed along circular-section at 3L.
Figure C53: SUL07 – Normalized Wave Height of unidirectional, monochromatic waves propagating around a semi-infinite, straight breakwater

Figure C54: SUL07 – Normalized significant wave heights distributed along circular-section at 3L
Figure C55: SUL08 – Normalized Wave Height of unidirectional, monochromatic waves propagating around a semi-infinite, straight breakwater.

Figure C56: SUL08 – Normalized significant wave heights distributed along circular-section at 3L.
Figure C57: SUL06~ – Normalized Wave Height of unidirectional, monochromatic waves propagating around a semi-infinite, straight breakwater

Figure C58: SUL06~ – Normalized significant wave heights distributed along circular-section at 3L
Figure C59: SUL07~ – Normalized Wave Height of unidirectional, monochromatic waves propagating around a semi-infinite, straight breakwater.

Figure C60: SUL07~ – Normalized significant wave heights distributed along circular-section at 3L.
Figure C61: SUL08~ – Normalized Wave Height of unidirectional, monochromatic waves propagating around a semi-infinite, straight breakwater

Figure C62: SUL08~ – Normalized significant wave heights distributed along circular-section at 3L
Table 6: Summary of SWAN model settings for all runs in the curvi-linear grid in Semi-Infinite Long Breakwater Case

<table>
<thead>
<tr>
<th>Codes</th>
<th>Diffraction State</th>
<th>Diffraction n</th>
<th>α</th>
<th>Mxistt</th>
<th>Real</th>
<th>Accuracy</th>
<th>Fig.</th>
</tr>
</thead>
<tbody>
<tr>
<td>SC-NOR-01</td>
<td>Off</td>
<td>/</td>
<td>0</td>
<td>200</td>
<td>4</td>
<td>100%</td>
<td>√</td>
</tr>
<tr>
<td>SC-NOR-02</td>
<td>On</td>
<td>0</td>
<td>0.01</td>
<td>200</td>
<td>200</td>
<td>21.24%</td>
<td>×</td>
</tr>
<tr>
<td>SC-NOR-03</td>
<td>On</td>
<td>1</td>
<td>0.01</td>
<td>200</td>
<td>200</td>
<td>13.73%</td>
<td>×</td>
</tr>
<tr>
<td>SC-NOR-04</td>
<td>On</td>
<td>3</td>
<td>0.01</td>
<td>200</td>
<td>40</td>
<td>100%</td>
<td>√</td>
</tr>
<tr>
<td>SC-NOR-05</td>
<td>On</td>
<td>6</td>
<td>0.01</td>
<td>200</td>
<td>20</td>
<td>100%</td>
<td>√</td>
</tr>
<tr>
<td>SC-DOU-01</td>
<td>Off</td>
<td>/</td>
<td>0.01</td>
<td>200</td>
<td>4</td>
<td>100%</td>
<td>√</td>
</tr>
<tr>
<td>SC-DOU-02</td>
<td>On</td>
<td>0</td>
<td>0.01</td>
<td>200</td>
<td>200</td>
<td>69.34%</td>
<td>×</td>
</tr>
<tr>
<td>SC-DOU-03</td>
<td>On</td>
<td>6</td>
<td>0.01</td>
<td>200</td>
<td>200</td>
<td>15.54%</td>
<td>×</td>
</tr>
<tr>
<td>SC-DOU-04</td>
<td>On</td>
<td>18</td>
<td>0.01</td>
<td>200</td>
<td>85</td>
<td>100%</td>
<td>√</td>
</tr>
<tr>
<td>SC-DOU-05</td>
<td>On</td>
<td>20</td>
<td>0.01</td>
<td>200</td>
<td>25</td>
<td>100%</td>
<td>√</td>
</tr>
<tr>
<td>SC-GEN-01</td>
<td>Off</td>
<td>/</td>
<td>0.01</td>
<td>200</td>
<td>4</td>
<td>100%</td>
<td>√</td>
</tr>
<tr>
<td>SC-GEN-02</td>
<td>On</td>
<td>0</td>
<td>0.01</td>
<td>200</td>
<td>200</td>
<td>21.09%</td>
<td>×</td>
</tr>
<tr>
<td>SC-GEN-03</td>
<td>On</td>
<td>1</td>
<td>0.01</td>
<td>200</td>
<td>200</td>
<td>24.57%</td>
<td>×</td>
</tr>
<tr>
<td>SC-GEN-04</td>
<td>On</td>
<td>3</td>
<td>0.01</td>
<td>200</td>
<td>27</td>
<td>100%</td>
<td>√</td>
</tr>
<tr>
<td>SC-GEN-05</td>
<td>On</td>
<td>6</td>
<td>0.01</td>
<td>200</td>
<td>17</td>
<td>100%</td>
<td>√</td>
</tr>
</tbody>
</table>

Note: The meaning of the symbols and the denotations in current table is the same as it in Table 5.

Figure C63: SC-NOR-01– Normalized Wave Height of unidirectional, monochromatic waves propagating around a semi-infinite, straight breakwater
Figure C64: SC-NOR-01 – Normalized significant wave heights distributed along circular-section at 3L

Figure C65: SC-NOR-04– Normalized Wave Height of unidirectional, monochromatic waves propagating around a semi-infinite, straight breakwater
Figure C66: SC-NOR-04 – Normalized significant wave heights distributed along circular-section at 3L

Figure C67: SC-NOR-05 – Normalized Wave Height of unidirectional, monochromatic waves propagating around a semi-infinite, straight breakwater
Figure C68: SC-NOR-05 – Normalized significant wave heights distributed along circular-section at 3L

Figure C69: SC-DOU-01– Normalized Wave Height of unidirectional, monochromatic waves propagating around a semi-infinite, straight breakwater
Figure C70: SC-DOU-01 – Normalized significant wave heights distributed along circular-section at 3L

Figure C71: SC-DOU-04 – Normalized Wave Height of unidirectional, monochromatic waves propagating around a semi-infinite, straight breakwater
Figure C72: SC-DOU-04 – Normalized significant wave heights distributed along circular-section at 3L

Figure C73: SC-DOU-05– Normalized Wave Height of unidirectional, monochromatic waves propagating around a semi-infinite, straight breakwater
Figure C74: SC-DOU-05 – normalized significant wave heights distributed along circular-section at 3L

Figure C75: SC-GEN-01– Normalized Wave Height of unidirectional, monochromatic waves propagating around a semi-infinite, straight breakwater
Figure C76: SC-GEN-01 – Normalized significant wave heights distributed along circular-section at 3L

Figure C77: SC-GEN-04– Normalized Wave Height of unidirectional, monochromatic waves propagating around a semi-infinite, straight breakwater
Figure C78: SC-GEN-04 – Normalized significant wave heights distributed along circular-section at 3L

Figure C79: SC-GEN-05 – Normalized Wave Height of unidirectional, monochromatic waves propagating around a semi-infinite, straight breakwater
Figure C80: SC-GEN-05 – Normalized significant wave heights distributed along circular-section at 3L

C2.2 Gap in Infinite Breakwater Case

Table 7: Summary of SWAN model settings for all runs in the GAP Case

<table>
<thead>
<tr>
<th>Codes</th>
<th>Spatial Resolution</th>
<th>Diffraction State</th>
<th>Mxitst</th>
<th>Real</th>
<th>Accuracy</th>
<th>Fig.</th>
</tr>
</thead>
<tbody>
<tr>
<td>GH01_72</td>
<td>100×110 0.24 0.25</td>
<td>Off /</td>
<td>200</td>
<td>6</td>
<td>100%</td>
<td>√</td>
</tr>
<tr>
<td>GH02_72</td>
<td>100×110 0.24 0.25</td>
<td>On 0</td>
<td>200</td>
<td>200</td>
<td>91.93%</td>
<td>×</td>
</tr>
<tr>
<td>GH03_72</td>
<td>100×110 0.24 0.25</td>
<td>On 3</td>
<td>200</td>
<td>200</td>
<td>91.29%</td>
<td>×</td>
</tr>
<tr>
<td>GH04_72</td>
<td>100×110 0.24 0.25</td>
<td>On 4</td>
<td>200</td>
<td>30</td>
<td>100%</td>
<td>√</td>
</tr>
<tr>
<td>GH05_72</td>
<td>100×110 0.24 0.25</td>
<td>On 6</td>
<td>200</td>
<td>19</td>
<td>100%</td>
<td>√</td>
</tr>
<tr>
<td>GL01_72</td>
<td>50×55 0.48 0.49</td>
<td>Off /</td>
<td>200</td>
<td>5</td>
<td>100%</td>
<td>√</td>
</tr>
<tr>
<td>GL02_72</td>
<td>50×55 0.48 0.49</td>
<td>On 0</td>
<td>200</td>
<td>26</td>
<td>100%</td>
<td>√</td>
</tr>
<tr>
<td>GL03_72</td>
<td>50×55 0.48 0.49</td>
<td>On 1</td>
<td>200</td>
<td>15</td>
<td>100%</td>
<td>√</td>
</tr>
<tr>
<td>GL04_72</td>
<td>50×55 0.48 0.49</td>
<td>On 6</td>
<td>200</td>
<td>12</td>
<td>100%</td>
<td>√</td>
</tr>
<tr>
<td>GH01_41</td>
<td>100×110 0.24 0.25</td>
<td>Off /</td>
<td>200</td>
<td>44</td>
<td>100%</td>
<td>√</td>
</tr>
<tr>
<td>GH02_41</td>
<td>100×110 0.24 0.25</td>
<td>On 0</td>
<td>200</td>
<td>200</td>
<td>98.90%</td>
<td>×</td>
</tr>
<tr>
<td>GH03_41</td>
<td>100×110 0.24 0.25</td>
<td>On 1</td>
<td>200</td>
<td>200</td>
<td>98.38%</td>
<td>×</td>
</tr>
<tr>
<td>GH04_41</td>
<td>100×110 0.24 0.25</td>
<td>On 4</td>
<td>200</td>
<td>70</td>
<td>100%</td>
<td>√</td>
</tr>
<tr>
<td>GH05_41</td>
<td>100×110 0.24 0.25</td>
<td>On 6</td>
<td>200</td>
<td>69</td>
<td>100%</td>
<td>√</td>
</tr>
<tr>
<td>GL01_41</td>
<td>50×55 0.48 0.49</td>
<td>Off /</td>
<td>200</td>
<td>4</td>
<td>100%</td>
<td>√</td>
</tr>
<tr>
<td>GL02_41</td>
<td>50×55 0.48 0.49</td>
<td>On 0</td>
<td>200</td>
<td>55</td>
<td>100%</td>
<td>√</td>
</tr>
<tr>
<td>GL03_41</td>
<td>50×55 0.48 0.49</td>
<td>On 1</td>
<td>200</td>
<td>55</td>
<td>100%</td>
<td>√</td>
</tr>
<tr>
<td>GL04_41</td>
<td>50×55 0.48 0.49</td>
<td>On 6</td>
<td>200</td>
<td>55</td>
<td>100%</td>
<td>√</td>
</tr>
</tbody>
</table>
Note: The meaning of the symbols and the denotations in current table is the same as it in Table 5.

Input file of GL04_72 in the SWAN model

$********** HEADING ***********************************************
PROJ 'GAP' 'GL04_72'
$ WL   = + 0.0
$ Hs   = 0.05 m
$ Tp   = 1.20 s

$********** MODEL INPUT********************** ********************
SET LEVEL 0.0 MAXERR 1
Cgrid  -12., -7., 0. xlenc=24., ylenc=27., mxc=50, myc=55  circle  120, 0.3, 2.5, 24
Input grid bottom -12., -7., 0. mxinp=1, myinp=1, 24., 27.
Read bottom 0.4, 'flat.bot'

$********** BOUNDARY CONDITIONS ********************************
Boundary South const par 0.05, 1.2, 90., 100.

$********** PHYSICA ***********************************************
OFF WINDGrowth
OFF QUADrupl
OFF WCAPping
OFF BREaking
Diffraction smpar=0.2 smnum=6

$ breakwater gap is 7.85 m, width of breakwater is 0.35 m
Obstacle transm 0.  refl 1.   &
        line -12., -0.175, -4.1, -0.175, -3.976, -0.05, -3.925, 0., &
        -3.976, 0.05, -4.1, 0.175, -12., 0.175
Obstacle transm 0.  refl 1.   &
        line  12., -0.175,  4.1, -0.175,  3.976, -0.05,  3.925, 0., &
         3.976, 0.05,  4.1,  0.175,  12.,  0.175
$ Wave absorbers:
Obstacle transm 0.  refl 0.   &
        line -12., -0.975, -4.275, -0.975, -4.275, -0.175
Obstacle transm 0.  refl 0.   &
        line  12., -0.975,  4.275, -0.975,  4.275, -0.175

$********** NUMERIEKE PARAMETERS ********************************
NUM STOPC 0.00 0.01 0.001 100 STAT mxitst=200 alfa=0.01

$ ********** OUTPUT ***********************************************
Quantity Xp   hexp=30.
Quantity Yp   hexp=30.
Quantity Dist hexp=30.
Quantity Hs   hexp=0.2

Curve 'y=0m'   -3.9, 0.,  39, 3.9, 0.
Curve 'y=1m'   -12., 1.,  100, 12., 1.
Curve 'y=3m'   -12., 3.,  100, 12., 3.
Curve 'y=6m'   -12., 6.,  100, 12., 6.
Curve 'y=9m'   -12., 9.,  100, 12., 9.
Curve 'y=12m'  -12., 12., 100, 12., 12.
Curve 'y=15m'  -12., 15., 100, 12., 15.
Curve 'y=18m'  -12., 18., 100, 12., 18.

Table 'y=0m'     head 'pwnl_14a.tab' Xp Yp Hs Difpar Dir Dspr
Table 'y=1m'     head 'pwnl_14b.tab' Xp Yp Hs Difpar Dir Dspr
Table 'y=3m'     head 'pwnl_14c.tab' Xp Yp Hs Difpar Dir Dspr
Table 'y=6m'     head 'pwnl_14d.tab' Xp Yp Hs Difpar Dir Dspr
Table 'y=9m'     head 'pwnl_14e.tab' Xp Yp Hs Difpar Dir Dspr
Table 'y=12m'    head 'pwnl_14f.tab' Xp Yp Hs Difpar Dir Dspr
Table 'y=15m'    head 'pwnl_14g.tab' Xp Yp Hs Difpar Dir Dspr
Table 'y=18m'    head 'pwnl_14h.tab' Xp Yp Hs Difpar Dir Dspr

Block 'COMPGRID' nohead 'diffrac.mat' layout 3 xp yp hsign dir difpar

COMPUTE
STOP
Figure C81: GH01-72 – Wave Direction Distribution and Contours of the Normalized Wave Height in GAP Case in SWAN 4072

Figure C82: GH01-72 – Comparison of normalized significant wave heights distributed along eight transect lines and the observation data
Figure C83: GH04-72 – Wave Direction Distribution and Contours of the Normalized Wave Height in GAP Case in SWAN 4072.

Figure C84: GH04-72 – Comparison of normalized significant wave heights distributed along eight transect lines and the observation data.
Figure C85: GH05-72—Wave Direction Distribution and Contours of the Normalized Wave Height in GAP Case in SWAN 4072

Figure C86: Comparison of normalized significant wave heights distributed along eight transect lines and the observation data
Figure C87: GL01-72– Wave Direction Distribution and Contours of the Normalized Wave Height in GAP Case in SWAN 4072

Figure C88: – Comparison of normalized significant wave heights distributed along eight transect lines and the observation data.
Figure C89: GL02-72– Wave Direction Distribution and Contours of the Normalized Wave Height in GAP Case in SWAN 4072

Figure C90: GL02-72 – Comparison of normalized significant wave heights distributed along eight transect lines and the observation data
Figure C91: GL03-72– Wave Direction Distribution and Contours of the Normalized Wave Height in GAP Case in SWAN 4072

Figure C92: GL03-72 – Comparison of normalized significant wave heights distributed along eight transect lines and the observation data
Figure C93: GL04-72– Wave Direction Distribution and Contours of the Normalized Wave Height in GAP Case in SWAN 4072

Figure C94: GL04-72 – Comparison of normalized significant wave heights distributed along eight transect lines and the observation data.
Figure C95: GH01-41 – Wave Direction Distribution and Contours of the Normalized Wave Height in GAP Case in SWAN 4041AB

Figure C96: GH01-41 – Comparison of normalized significant wave heights distributed along eight transect lines and the observation data
Figure C97: GH04-41– Wave Direction Distribution and Contours of the Normalized Wave Height in GAP Case in SWAN 4041AB

Figure C98: GH04-41 – Comparison of normalized significant wave heights distributed along eight transect lines and the observation data
Figure C99: GH05-41 - Wave Direction Distribution and Contours of the Normalized Wave Height in GAP Case in SWAN 4041AB

Figure C100: GH05-41 – Comparison of normalized significant wave heights distributed along eight transect lines and the observation data
Figure C101: GL01-41– Wave Direction Distribution and Contours of the Normalized Wave Height in GAP Case in SWAN 4041AB

Figure C102: GL01-41 – Comparison of normalized significant wave heights distributed along eight transect lines and the observation data
Figure C103: GL02-41 – Wave Direction Distribution and Contours of the Normalized Wave Height in GAP Case in SWAN 4041AB

Figure C104: GL02-41 – Comparison of normalized significant wave heights distributed along eight transect lines and the observation data
Figure C105: GL03-41 – Wave Direction Distribution and Contours of the Normalized Wave Height in GAP Case in SWAN 4041AB

Figure C106: GL03-41 – Comparison of normalized significant wave heights distributed along eight transect lines and the observation data
Figure C107: GL04-41– Wave Direction Distribution and Contours of the Normalized Wave Height in GAP Case in SWAN 4041AB

Figure C108: GL04-41 – Comparison of normalized significant wave heights distributed along eight transect lines and the observation data
Figure C109: GL05-41 – Wave Direction Distribution and Contours of the Normalized Wave Height in GAP Case in SWAN 4041AB

Figure C110: GL05-41 – Comparison of normalized significant wave heights distributed along eight transect lines and the observation data
### C.2.3 Ridge Case

**Table 8: Summary of SWAN model settings for all runs in the Ridge Case**

<table>
<thead>
<tr>
<th>Codes</th>
<th>Δx=Δy</th>
<th>θ</th>
<th>Bottom</th>
<th>Diffraction State</th>
<th>n</th>
<th>Mxitst</th>
<th>Real</th>
<th>Accuracy</th>
<th>Fig.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rr01</td>
<td>0.2</td>
<td>2°</td>
<td>Flat</td>
<td>Off</td>
<td>/</td>
<td>200</td>
<td>6</td>
<td>100%</td>
<td>√</td>
</tr>
<tr>
<td>Rr02</td>
<td>0.2</td>
<td>1°</td>
<td>Flat</td>
<td>Off</td>
<td>/</td>
<td>200</td>
<td>6</td>
<td>100%</td>
<td>√</td>
</tr>
<tr>
<td>Rr03</td>
<td>0.2</td>
<td>2°</td>
<td>Ridge</td>
<td>Off</td>
<td>/</td>
<td>200</td>
<td>12</td>
<td>100%</td>
<td>√</td>
</tr>
<tr>
<td>Rr04</td>
<td>0.2</td>
<td>2°</td>
<td>Ridge</td>
<td>On</td>
<td>0</td>
<td>200</td>
<td>200</td>
<td>49.77%</td>
<td>×</td>
</tr>
<tr>
<td>Rr05</td>
<td>0.2</td>
<td>2°</td>
<td>Ridge</td>
<td>On</td>
<td>6</td>
<td>200</td>
<td>200</td>
<td>9.90%</td>
<td>×</td>
</tr>
<tr>
<td>Rr06</td>
<td>0.2</td>
<td>2°</td>
<td>Ridge</td>
<td>On</td>
<td>10</td>
<td>200</td>
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Note: The meaning of the symbols and the denotations in current table is the same as it in Table 5.

Input file of Rr06 in the SWAN model

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$ WL   = + 0.0
$ Hs   = Variables
$ Tp   = 1.30 s

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INPGRID BOTTOM REGular 0. 0. 0. 1 1 4 10
READINP BOTTOM 1. 'flat.bot' 3 0 FREE

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Boundary SIDE West par 0.04314, 1.3, 90., 2.
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$********** NUMERIEKE PARAMETERS ****************************
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OFF WCAP
OFF QUAD
OFF BREAKING
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COMPUTE
STOP

Figure C111: R01 – Significant wave heights distributed along six transect lines in Ridge Case
Figure C112: R02 – Significant wave heights distributed along six transect lines in Ridge Case

R 02 - Flat, $\sigma_8=1^\circ$, No Diffraction

Figure C113: R03 – Significant wave heights distributed along six transect lines in Ridge Case

R 03 - Ridge, $\sigma_8=2^\circ$, No Diffraction
Figure C114: R06 – Significant wave heights distributed along six transect lines in Ridge Case

Figure C115: R07 – Significant wave heights distributed along six transect lines in Ridge Case
## C.3 Oosterschelde Estuary

**Table 9: Summary of codes, input settings and corresponding convergence behaviors of all runs in grid F of Oosterschelde in SWAN with diffraction**

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Notes:
- **Provider** – denotes the provider of the boundary conditions of grid F;
- **Tails** – denotes whether the input wave spectra on boundaries contain the wave energy of local wind sea located on high-frequency band ($0.2195\text{Hz} \leq f \leq 1.0\text{Hz}$);
- √ – indicates the model contained the specific item;
- × – indicates the model did not contain the specific item;
- **T** – wave period (s);
- **DIR** – direction of the incoming wave (ºN);
- $\sigma_\theta$ – directional width of the incoming wave (º);
- **Δθ** – directional resolution of the incoming wave (º);
- **State** – denotes whether the diffraction was taken into consideration;
- **n** – denotes whether the smoothing technique was activated in SWAN;
  - n=0 → no smoothing technique;
  - n≠0 → the smoothing steps used in SWAN;
Figure C116: OD01 –Comparison of 1D spectra from SWAN (source terms √
ambient currents √ tail √) without diffraction (Svasek 1D spectra, black)
and measured data (blue) at buoy OS4

Figure C117: OD02 –Comparison of 1D spectra from SWAN (source terms √
ambient currents √ tail √) with diffraction (red), from Svasek (black), and measured
data (blue) at buoy OS4

Figure C118: ODa01 –Comparison of 1D spectra from SWAN (source terms ×
ambient currents √ tail √) without diffraction (red), from Svasek (black), and measured
data (blue) at buoy OS4
Figure C119: ODa02 – Comparison of 1D spectra from SWAN (source terms \times ambient currents \times tail) with diffraction (red), from Svasek (black), and measured data (blue) at buoy OS4

Figure C120: ODb01 – Comparison of 1D spectra from SWAN (source terms \times ambient currents \times tail) without diffraction (red), from Svasek (black), and measured data (blue) at buoy OS4

Figure C121: ODb02 – Comparison of 1D spectra from SWAN (source terms \times ambient currents \times tail) with diffraction (red), from Svasek (black), and measured data (blue) at buoy OS4
Figure C122: ODc01 – Comparison of 1D spectra from SWAN (source terms × ambient currents × tail ×) without diffraction (red), from Svasek (black), and measured data (blue) at buoy OS4.

Figure C123: ODc02 – Comparison of 1D spectra from SWAN (source terms × ambient currents × tail ×) with diffraction (red), from Svasek (black), and measured data (blue) at buoy OS4.