Application of Satellite Altimetry for Global Ocean Tide Modeling
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Printed in the Netherlands.
To A. Th. S. and J. W. D.

Teach me,
guide me,
love me.
The altimeter data used in this thesis have been processed in the period August 1992 - September 1996 and concern three years of TOPEX/POSEIDON altimetry, two years of ERS-1 altimetry from the 35-day repeat mission (Multidisciplinary Phase), and the first two years of GEOSAT altimetry from the 17-day Exact Repeat Mission. With TOPEX/POSEIDON, a 3-year analysis period was selected because all dominant tides as observed by this satellite decorrelate within three years. With GEOSAT, the first two years of the Exact Repeat Mission include all data of this mission that were not affected by the GEOSAT attitude problem. Data from the GEOSAT Geodetic Mission were not declassified until 1997 and therefore are not included in the analyses. With ERS-1, the two years include all data from the Multidisciplinary Phase. The altimeter data from the Ice Phases were not considered useful because of the coarse spacing of the satellite groundtracks imposed by the 3-day repeat orbit. The data of the Geodetic Phase (168-day repeat period), for which orbits with the same accuracy as those of the Multidisciplinary Phase became available in mid 1996, have not been included in the analyses. These data may, however, be useful for tidal analysis because of the existence of a 37-day subcycle. Still, the GEOSAT and ERS-1 orbits on which the analyses in this thesis are based have errors that are considerably larger than those of TOPEX/POSEIDON. This means that the normal equations of GEOSAT and ERS-1 have to be significantly downweighed with respect to the TOPEX/POSEIDON normal equations so that omitting the 168-day repeat data from the analyses will have no effect of importance on the results in this thesis. From ERS-2, i.e. the successor of ERS-1, merely a few months of altimeter data for which orbits with subdecimeter accuracy had been computed were available until September 1996. Hence, these data are also not included in the analyses.
Application of Satellite Altimetry for Global Ocean Tide Modeling

Until the beginning of the 1990s, predictions of the global ocean tide could be given with an accuracy of about 5-10 cm in the deep oceans. In oceanography and geophysics, there is a clear need for more accurate ocean tide predictions. The aim of this thesis is to develop a global ocean tide model with an accuracy of better than 5 cm in the deep oceans from about seven years of multi-satellite altimetry provided by GEOSAT, ERS-1, and TOPEX/POSEIDON. In addition, it aims at identifying specific problems associated with the estimation of the oceanic tide from altimeter observations, as well as at finding feasible solutions for these problems. The response as well as the harmonic method have been used to analyze the altimeter observations. The most accurate tidal solutions are obtained with the response method, which offers the possibility to infer a number of smaller tides from the dominant tides. The harmonic solutions have mainly been used to perform covariance analyses in order to assess the magnitudes of the correlations between the tides as observed by each of the three satellites. All tidal models developed in this thesis are empirical, which means that they fit a representation of the ocean tide through the altimeter observations. This in contrast with hydrodynamic models, which aim to solve the hydrodynamic equations.

The major problems with estimating the ocean tide from satellite altimetry are the relatively large background noise level, which mainly consists of orbit errors, and aliasing of the diurnal and semi-diurnal tides to periods of typically sixty days to one year. As a result of the relatively small signal to noise ratio, only the dominant $M_2$, $S_2$, $N_2$, $K_1$, and $O_1$ tides can reliably be obtained from the altimeter observations. If tides alias to nearly the same period, they become correlated and many years of observations may be needed to separate them. For three major satellite altimetry missions until now, i.e. GEOSAT, ERS-1, and TOPEX/POSEIDON, the alias periods as well as the Rayleigh periods over which the aliased tides decorrelate have been identified. Because of its favorable orbit design to observe the ocean tide, the only tidal correlation problem of TOPEX/POSEIDON occurs between the $K_1$ tide and the semi-annual seasonal cycle of ocean variability beyond latitudes of about 55°. In case of GEOSAT and ERS-1, more severe correlation problems arise among
the tides and among the tides and the seasonal cycles of ocean variability. However, these correlation problems can largely be solved by the differences of the tidal phase advances on crossing satellite groundtracks, as could be demonstrated by covariance analyses of the single-satellite harmonic tidal solutions. Hence, the altimeter observations of GEOSAT and ERS-1 contain useful tidal information, provided that the noise level of the ERS-1 and GEOSAT altimetry is sufficiently small.

From the admittance of a response tidal solution based on three years of TOPEX/POSEIDON altimetry, 23 tidal lines have been derived, which constitute a tidal model of relatively coarse $3^\circ \times 3^\circ$ resolution with a global accuracy of about 3 cm in the deep oceans, according to the differences between the model and independent data, i.e. the harmonic constants of a reference network of 84 tide gauges.

Two years of ERS-1 altimetry from the 35-day Multidisciplinary Phase, and the first two years of GEOSAT altimetry from the 17-day Exact Repeat Mission that were not plagued by loss of data, were added to the three years of TOPEX/POSEIDON altimeter observations. The reason to add the GEOSAT and ERS-1 altimetry is to try to improve the TOPEX/POSEIDON response solution by improving its resolution. Unfortunately, it could be shown that the orbit errors in the GEOSAT and ERS-1 altimeter observations are too large and that part of the orbit errors is likely aliased to periods comparable with the tidal alias periods. This means that the effect of orbit errors on the tidal estimates will not average out sufficiently, whereas it also means that correlated tides are difficult to separate, in spite of the decorrelation offered by the tidal phase advance differences on crossing groundtracks. Based on the fact that the alias periods of GEOSAT and ERS-1 are about twice as large as those of TOPEX/POSEIDON, it is expected that at least six years of GEOSAT and six years of ERS-1 altimetry are needed to improve the TOPEX/POSEIDON tidal solution. This means that in addition to the two years of the ERS-1 35-day repeat mission and the first two years of the GEOSAT 17-day Exact Repeat Mission, four years of ERS-2 altimetry as well as four years of altimetry from the GEOSAT Follow-On would have to be gathered and processed.

Altimeter data from the ERS-1 168-day repeat mission may also be useful because of the existence of a 37-day subcycle, i.e. if we adopt the $1^\circ$ resolution of the 35-day repeat mission. However, the 37-day subcycle induces specific alias and Rayleigh periods, so that it may not be regarded as an extension of the 35-day repeat mission.

Whether altimetry from the GEOSAT Geodetic Mission are useful for tidal analysis will require further study because of the drift orbit during this part of the mission, which leads to a non-constant satellite repeat period.

Improvement of the TOPEX/POSEIDON tidal solution may also come from the follow-up to this satellite, i.e. JASON, to be launched around
2000. From a tidal point of view, it would be interesting to fly the JASON mission in two different types of orbits. Firstly, as TOPEX/POSEIDON is expected to deliver observations beyond 2000, TOPEX/POSEIDON and JASON could fly in tandem along the same groundtrack with a lag of about twelve hours. From this tandem mission, the $K_1/S_2$ correlation problem at higher latitudes can be solved within probably one year. In the second part of the mission, JASON could be shifted to fly in between the TOPEX/POSEIDON groundtracks. These observations can be used to compute a tidal model with a resolution of 1°5, which improves the resolution of the existing TOPEX/POSEIDON tidal models by a factor two.

Arthur James Edward SMITH
Samenvatting

Het Gebruik van Satelliet Radar Hoogtemetingen voor het Modelleren van het Oceaan Getijde op Wereldwijde Schaal

Tot aan het begin van de jaren '90 kon het oceaan getijde worden voorspeld met een nauwkeurigheid van 5-10 cm in de open oceanen. In de oceanografie en de geofysica is er echter behoefte aan nauwkeuriger voorspellingen van het oceaan getijde. De doelstelling van dit proefschrift is het ontwikkelen van een wereldwijd oceaan getijde model uit zeven jaar aan satelliet radar hoogtemetingen (altimetrie) van GEOSAT, ERS-1, en TOPEX/POSEIDON. De nauwkeurigheid van het model moet beter zijn dan 5 cm in de open oceanen. Bovendien heeft het onderzoek beschreven in dit proefschrift tot doel om specifieke problemen met het modelleren van het oceaan getijde uit satelliet altimetrie te identificeren, alsmede om oplossingen voor deze problemen te vinden. Voor de getijde analyse van de radar hoogtemetingen zijn zowel de harmonische als de responsie methode toegepast. De meest nauwkeurige getijde oplossingen worden verkregen met behulp van de responsie methode die de mogelijkheid biedt om met behulp van de hoofdgetijden een aantal zwakkere getijden af te leiden. De harmonische getijde oplossingen zijn vooral gebruikt voor het uitvoeren van covariantie analyses die een maat geven voor de grootte van de correlaties tussen de getijden zoals waargenomen door elk van de satellieten. De modellen die in dit proefschrift zijn ontwikkeld zijn empirisch omdat er een aantal model parameters zodanig wordt geschat dat het model het beste bij het getijde signaal in de altimetrie metingen past. Dit in tegenstelling tot de zogenaamde hydrodynamische modellen die tot doel hebben om de hydrodynamische vergelijkingen op te lossen.

De belangrijkste problemen met het modelleren van de dagelijkse en halfdagelijkse oceaan getijden uit satelliet radar hoogtemetingen zijn de relatief hoge ruis in de metingen en het zogenaamde aliasing van deze getijden tot perioden van 60 dagen tot een jaar. Als gevolg van de relatief lage signaal-ruis verhouding kunnen alleen de dominante $M_2$, $S_2$, $N_2$, $K_1$, en $O_1$ getijden uit de altimetrie data worden verkregen. Indien aliasing van getijden optreedt tot nagenoeg dezelfde periode dan leidt dit tot correlaties tussen getijden en kunnen er vele jaren aan metingen nodig zijn om de getijden van elkaar te scheiden. Voor drie altimetrie missies waarvan voldoende data beschikbaar is, GEOSAT, ERS-1, en
TOPEX/POSEIDON, zijn de alias perioden bepaald alsmede de zogenaamde Rayleigh perioden die benodigd zijn om gecorreleerde getijden van elkaar te kunnen scheiden. In tegenstelling tot TOPEX/POSEIDON zijn de satelliet banen van GEOSAT en ERS-1 niet optimaal voor het waarnemen van oceaan getijden. Dit heeft tot gevolg dat getijde correlaties een veel grotere rol spelen voor de GEOSAT en ERS-1 satellieten dan voor TOPEX/POSEIDON. Echter, vanwege de getijde fase verschillen zoals waargenomen op locaties aan het aardoppervlak waar de satelliet groundtracks elkaar kruisen, kunnen de getijde correlatie problemen van GEOSAT en ERS-1 grotendeels worden opgelost zoals kon worden aangetoond aan de hand van covariantie analyses van de harmonische getijde oplossingen. Dit betekent dat de altimeter metingen van GEOSAT en ERS-1 gebruikt kunnen worden voor getijde analyses, mits de ruis van deze metingen niet te groot is.

Gebruikmakend van de respons methode is uit drie jaar TOPEX/POSEIDON altimeter metingen een getijde model met $3^\circ \times 3^\circ$ resolutie verkregen bestaande uit 23 spectrale lijnen. Dit model heeft in de open oceanen een nauwkeurigheid van ongeveer 3 cm volgens een referentie netwerk van 84 getijde stations.

Er is geprobeerd om twee jaar aan GEOSAT en twee jaar aan ERS-1 altimeter metingen toe te voegen aan de drie jaar TOPEX/POSEIDON altimeter om zo het TOPEX/POSEIDON getijde model te verbeteren. De altimeter metingen van ERS-1 zijn afkomstig uit de zogenaamde Multidisciplinaire Fase waarin het tijdsinterval tussen twee opeenvolgende altimeter metingen van het oceaan getijde op een vaste plaats op aarde 35 dagen bedraagt (35-dagen repeat missie). De metingen van GEOSAT zijn afkomstig van de zogenaamde Exacte Repeat Missie waarvoor het bovengenoemd tijdsinterval 17 dagen bedraagt (17-dagen repeat missie). De verbetering van het TOPEX/POSEIDON getijde model zou moeten liggen in het feit dat de groundtracks van zowel GEOSAT als ERS-1 dichter bij elkaar liggen dan die van TOPEX/POSEIDON, zodat een getijde model met hogere resolutie kan worden verkregen. Er kon echter worden aangetoond dat de baanfouten in de altimeter metingen van GEOSAT en ERS-1 te groot zijn en dat het waarschijnlijk is dat een deel van de baanfouten wordt gealiaseerd tot perioden die vergelijkbaar zijn met de alias perioden van de getijden. Dit heeft tot gevolg dat het effect van baanfouten op de geschatte getijden niet zal uitmidden. Het heeft ook tot gevolg dat gecorreleerde getijden moeilijk van elkaar te scheiden zijn ondanks de fase verschillen op kruisende groundtracks. Omdat de alias perioden van GEOSAT en ERS-1 ongeveer twee keer zo groot zijn als die van TOPEX/POSEIDON, moet worden aangenomen dat er minstens zes jaar aan GEOSAT en zes jaar aan ERS-1 altimeter nodig zijn om een verbetering van het TOPEX/POSEIDON getijde model te verkrijgen. Dit betekent dat er vier jaar aan ERS-2 waarnemingen en vier jaar aan waarnemingen van de GEO-
SAT Follow-On zullen moeten worden toegevoegd aan de hoeveelheid GEOSAT en ERS-1 data die in dit proefschrift is gebruikt, dat wil zeggen twee jaar aan GEOSAT data uit de 17-dagen repeat missie en twee jaar aan ERS-1 data uit de 35-dagen repeat missie.

Altimetrie data uit de 168-dagen repeat missie van ERS-1 is waarschijnlijk ook te gebruiken voor getijde analyses. Dit komt doordat deze missie een herhalingspatroon bevat van 37 dagen tussen opeenvolgende altimetrie metingen in een resolutie cel, mits de resolutie van het getijde model niet beter behoeft te zijn dan 1°.

Altimetrie data uit de Geodetische Missie van GEOSAT is misschien ook geschikt voor getijde analyses. Vanwege de drift orbit in deze missie is het tijdsinterval tussen twee opeenvolgende metingen van het oceaan getijde op een vaste plaats op aarde echter niet constant zodat verdere studie nodig is naar het gebruik van deze data.

Een verbetering van het TOPEX/POSEIDON getijde model moet ook kunnen worden verkregen met behulp van altimetrie data van de opvolger van deze satelliet genaamd JASON. De lancering van de JASON satelliet is gepland omstreeks 2000. Vanuit het oogpunt van getijde onderzoek zou het interessant zijn om de JASON missie in twee fasen te verdelen. Omdat wordt verwacht dat TOPEX/POSEIDON tot na 2000 operationeel zal blijven, bestaat er de mogelijkheid om TOPEX/POSEIDON en JASON in tandem te laten vliegen, dat wil zeggen langs dezelfde groundtrack. Een tijdsverschil van ongeveer twaalf uur tussen de altimetrie metingen van beide satellieten gedurende dit deel van de missie zou betekenen dat het bij TOPEX/POSEIDON optredende correlatie probleem van $K_1$ met de halfjaarlijkse seizoens cyclus in de oceanen kan worden opgelost. Wellicht is ongeveer een jaar aan data uit de tandem missie hiervoor toereikend. In het tweede deel van de missie zou de satelliet baan van JASON kunnen worden veranderd zodat de groundtracks van JASON tussen die van TOPEX/POSEIDON in komen te liggen. Hiermee kan een getijde model worden verkregen met een resolutie van 1°5, hetgeen de resolutie van de huidige TOPEX/POSEIDON getijde modellen met een factor twee verbetert.

Arthur James Edward SMITH
Acknowledgment

I sincerely wish to thank my promotor Prof. ir. K. F. Wakker for offering me the opportunity to do five years of research on a subject that has attracted my personal interest. Thanks also to Prof. ir. B. A. C. Ambrosius and my many colleagues at the Delft Institute for Earth-Oriented Space Research for their helpful advice and enthusiastic discussions. Special thanks to Ir. R. Scharroo, Dr. ir. E. J. O. Schrama, and Dr. ir. P. N. A. M. Visser, for their much appreciated comments on earlier versions of the manuscript. Finally, a special word of gratitude goes to my parents and grandparents for probably several thousands of good reasons.
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<th>Abbreviation</th>
<th>Full Form</th>
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<tr>
<td>cpr</td>
<td>cycle per revolution</td>
</tr>
<tr>
<td>CTE</td>
<td>Cartwright, Tayler &amp; Edden</td>
</tr>
<tr>
<td>DEOS</td>
<td>Delft Institute for Earth-Oriented Space Research</td>
</tr>
<tr>
<td>DORIS</td>
<td>Doppler Orbitography and Radio-positioning Integrated by Satellite</td>
</tr>
<tr>
<td>ECMWF</td>
<td>European Center for Medium-range Weather Forecasting</td>
</tr>
<tr>
<td>ENSO</td>
<td>El Niño Southern Oscillation</td>
</tr>
<tr>
<td>ERM</td>
<td>Exact Repeat Mission</td>
</tr>
<tr>
<td>ERS</td>
<td>European Remote Sensing Satellite</td>
</tr>
<tr>
<td>ESA</td>
<td>European Space Agency</td>
</tr>
<tr>
<td>ESRIN</td>
<td>European Space Research Institute</td>
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<tr>
<td>ETOPO</td>
<td>Earth Topography</td>
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<tr>
<td>FES</td>
<td>Finite Element Solution</td>
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<tr>
<td>FMO</td>
<td>French Meteorological Office</td>
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<tr>
<td>GDR</td>
<td>Geophysical Data Record</td>
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<tr>
<td>GEOSAT</td>
<td>Geodetic Satellite</td>
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<td>GFO</td>
<td>GEOSAT Follow-On</td>
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<td>GM</td>
<td>Geodetic Mission</td>
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<tr>
<td>IB</td>
<td>Inverse Barometer</td>
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<tr>
<td>NASA</td>
<td>National Aeronautics and Space Administration</td>
</tr>
<tr>
<td>OPR</td>
<td>Ocean Product</td>
</tr>
<tr>
<td>rms</td>
<td>root-mean-square</td>
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<tr>
<td>rss</td>
<td>root-sum-square</td>
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<tr>
<td>ssh</td>
<td>sea surface height</td>
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<tr>
<td>SWH</td>
<td>Significant Wave Height</td>
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<td>SWT</td>
<td>Science Working Team</td>
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<tr>
<td>TOPEX/POSEIDON (T/P)</td>
<td>Ocean Topography Experiment/POSEIDON</td>
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<tr>
<td>UT</td>
<td>Universal Time</td>
</tr>
<tr>
<td>UT/CSR</td>
<td>University of Texas at Austin/Center for Space Research</td>
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The tidal elevations of the earth’s surface as generated by the gravitational attraction of the sun and the moon include the solid-earth tide, the ocean tide and the ocean tidally-induced load tide. Table 1.1, of which the error estimates may be found in Ray [1993], clearly illustrates the importance of ocean tidal corrections in oceanography and geophysics. Not only is the ocean tide significantly larger than the tide of the solid earth, it also is the least accurately known of the two. Because the resonance periods of the solid earth are an order of magnitude smaller than the dominantly diurnal and semi-diurnal forcing periods of the sun and the moon, the solid-earth tide may be assumed to follow an equilibrium response, i.e. the deformations of the solid earth are instantaneous with regard to the periods of tidal forcing. Hence, the only uncertainty in this tide is due to inaccuracies of the Love numbers, which translate into an error of less than 1 cm [e.g., Ray, 1993]. The oceans, on the other hand, have resonance periods that are comparable to the diurnal and semi-diurnal forcing periods. Therefore, near-resonance conditions cause a dynamic response to this forcing and amplitudes of the ocean tides that are larger than predicted by equilibrium considerations. Because of this dynamic behavior, the ocean tide is much more difficult to model than the solid-earth tide, which explains the larger error in Table 1.1, as well as the error of the induced load tide.

Until the beginning of the 1990s, predictions of the global ocean tide

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<th>tide</th>
<th>$rms$</th>
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<tr>
<td>ocean</td>
<td>30</td>
<td>5-10</td>
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<tr>
<td>earth</td>
<td>14</td>
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<td>load</td>
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Table 1.1 Global $rms$ (cm) of the ocean tide, the solid-earth tide, and the load tide, and the estimated accuracies with which these tides can be predicted. With the ocean and load tides, the numbers apply to the deep oceans, i.e. depths larger than 200 m. The error estimates of the ocean tide and the ocean tidally-induced load tide are valid for TOPEX/POSEIDON pre-launch (1992) ocean tide models.
were given by the models of Schwiderski [1980] and Cartwright and Ray [1990a], Cartwright and Ray [1991], which both claimed an accuracy of about 5-10 cm in the deep oceans (see Table 1.1). To obtain more accurate ocean tide models, it seems that tidalists have followed two different methods, i.e. hydrodynamic modeling and empirical modeling. The hydrodynamic models aim to numerically solve the tidal equations of motion of the water mass, whereas the empirical models fit a representation of the ocean tide through the tidal observations, which nowadays are almost exclusively delivered by radar altimeter satellites. At present, both methods seem to have found each other as evidenced by the method of tidal data assimilation. It may be expected that this method will eventually produce the most accurate tidal models, as it combines knowledge of tidal hydrodynamics with tidal observations. Still, several intercomparison studies involving numerous tidal models [e.g., Andersen et al., 1995; Shum et al., 1997] have shown that the assimilation procedure has not yet resulted in a superior accuracy compared to the empirical solutions. Likely, errors in the bathymetry model underlying the hydrodynamic solutions are a limiting factor [Le Provost et al., 1994; Smith and Andersen, 1997], as is also an insufficient knowledge of physical parameters like, e.g., bottom friction [Schwiderski, 1980; Le Provost et al., 1994; Kantha, 1995]. Hence, now that three major altimetry missions, i.e. GEOSAT, ERS-1, and TOPEX/POSEIDON, have all together gathered about a decade of altimeter observations, the time seems right to use these observations in an empirical global multi-satellite ocean tidal solution.

It should be realized that although a decade of observations may seem an abundance, this is definitely not true. With altimetry, the diurnal and semi-diurnal ocean tides are aliased to periods as large as one year whereas the alias periods may also be nearly the same. Consequently, many years of accurate altimeter observations are needed to reliably solve and separate the oceanic tides. Only in case of TOPEX/POSEIDON, the orbit was especially designed to keep the alias periods of the dominant tides sufficiently small, i.e. of the order of sixty days, and to avoid correlations among the dominant tides and among the dominant tides and the seasonal cycles in the oceans [Parke et al., 1987]. In conjunction with the orbit design, the very precise altimeter observations of this satellite have led to a dramatic breakthrough in ocean tidal modeling, reducing errors to less than 5 cm. Still, because of the relatively coarse spacing of the TOPEX/POSEIDON groundtracks of 3° in longitude direction, the altimetry from this satellite are not very well suited to observe smaller wavelength tidal features. The spacings of the GEOSAT and ERS-1 groundtracks of about 2° and 1°, respectively, are more suited to observe the smaller features of the ocean tides, which is why we want to include the GEOSAT and ERS-1 altimetry in the tidal solution.

In this thesis, efforts are described to derive an empirical multi-satellite
tidal solution in the deep oceans (ocean depths larger than 200 m) from three years of TOPEX/POSEIDON altimetry, and two years of GEOSAT and two years of ERS-1 altimeter observations.

The purpose of Chapter 2 is to discuss how the tide-generating potentials of the sun and the moon are developed into a series of tidal harmonics or, stated more simply, tides.

In Chapter 3, the frequencies and amplitudes of these tidal harmonics are used to derive the linear formulations of the two classical methods of tidal analysis, i.e. the harmonic and the response analysis.

In Chapter 4 it is discussed that the tidal correlations induced by aliasing of the diurnal and semi-diurnal tides cause major problems with estimating the ocean tide from satellite altimetry. Moreover, this chapter introduces the concept of tidal decorrelation from phase advance differences on crossing satellite groundtracks, which plays a key role in analyzing the oceanic tide from altimeter observations.

In Chapter 5, a description and analysis of the altimeter observation correction models used for GEOSAT, ERS-1, and TOPEX/POSEIDON are presented. These models are used to obtain the sea surface height residuals that contain the tidal sea level variations. This chapter also explains the gridding procedure of the sea surface height residuals and discusses the tidal parameters that are estimated.

In Chapter 6, covariance analyses of the single-satellite harmonic tidal solutions are performed to confirm the tidal decorrelation patterns of the phase advance differences on crossing groundtracks. It is shown that these phase advance differences reduce the tidal correlations to a level that the GEOSAT and ERS-1 altimeter observations contain useful information on the ocean tides. Hence, the GEOSAT and ERS-1 altimeter observations may improve the TOPEX/POSEIDON tidal solution if the noise level of these observations is small enough.

The purpose of Chapter 7 is to discuss the results of experiments with the response formalism. These experiments were conducted to investigate which number of response weights and which value of the lag interval on the equilibrium tide, adequately describe the diurnal and semi-diurnal admittances.

In Chapter 8, the single-satellite harmonic and response tidal solutions are developed. The differences between these solutions and the harmonic constants of a network of 84 globally distributed tide gauges are used as a measure of what may be expected of the altimetry of each satellite in a multi-satellite tidal solution. The TOPEX/POSEIDON response tidal solution that is developed in this chapter will serve as the "baseline" model, i.e. the tidal model of which we want to improve the resolution by means of the GEOSAT and ERS-1 altimeter observations.

In Chapter 9, the multi-satellite response tidal solution is developed by adding the weighted GEOSAT and ERS-1 normal equations to those of TOPEX/POSEIDON. The weights are chosen such that they minimize
the differences between the multi-satellite tidal solution and the harmonic constants of the 84 globally distributed tide gauges. The performance of the multi-satellite response tidal solution as measured by the reduction of the \( \text{rms} \) of GEOSAT, ERS-1, and TOPEX/POSEIDON crossover differences, is compared with the performance of the TOPEX/POSEIDON response tidal solution. Also, the differences between the TOPEX/POSEIDON and multi-satellite response tidal solutions are computed to evaluate the effect of GEOSAT and ERS-1 orbit errors on the multi-satellite tidal solution.

In Chapter 10, it is discussed what improvements of the results in this thesis may be expected from more accurate GEOSAT and ERS-1 orbits and from an extension of their time series of sea surface height observations by the GEOSAT Follow-On and ERS-2, respectively.

In Chapter 11, conclusions are drawn and recommendations are made to improve the TOPEX/POSEIDON tidal solution by means of ERS, GEOSAT-GFO, and JASON altimetry.
Chapter 2

The tide-generating force

2.1 Introduction

In this chapter it is discussed how the gravitational forces of the sun and the moon cause periodic deformations of the surface of the earth which are known as tides. Moreover, it is discussed how the tide-generating potential from which these forces may be derived can be developed into a series of tidal harmonics. The procedures to derive the amplitudes of the sea surface elevations associated with the tidal harmonics are discussed for the developments of the tide-generating potential of Doodson [1921] and Cartwright and Tayler [1971], Cartwright and Edden [1973]. Both procedures assume an equilibrium response of the oceans to the tide-generating forces, i.e. the deformations of the sea surface are assumed to instantaneously follow the tidal forcing. Hence, the amplitudes that are derived are equilibrium amplitudes, which constitute the equilibrium tide. Because the equilibrium assumption is not correct for the ocean tide, the equilibrium tide may not be used to model the ocean tide. The equilibrium tide does, however, serve as an important reference in tidal analysis.

2.2 The tide-generating potential

According to Newton’s law of gravitation, the attraction of the tide-generating or disturbing body \( D \) on a particle \( P \) on the earth’s surface (Figure 2.1) is given by:

\[
\vec{F}_g(P) = G M_d M_p \frac{\vec{R}_{dp}}{R_p^3}
\]  

(2.1)

where the vector \( \vec{R}_p \) is directed from \( P \) to \( D \) (\( R_p \) is the magnitude of \( \vec{R}_p \)), \( G \) is the universal gravitational constant, and \( M_p \) and \( M_d \) denote the mass of \( P \) and \( D \), respectively. Integrating (2.1) over all particles \( P \) of the earth gives the attraction of the body \( D \) on the earth as a whole, which will be directed along the vector \( \vec{R}_d \). This attraction, which can be found by
Figure 2.1 The tide-generating force $\vec{F}_t$ on the earth due to a tide-generating body $D$. Denoted by $R_e$ is the earth's mean radius. The vertical and horizontal directions are denoted by $\vec{e}_r$ and $\vec{e}_g$, respectively.

imagining the entire mass of the earth to be concentrated at the earth’s center $O$, can be equally divided over all particles of the earth. Hence, the contribution of each particle to this attraction is:

$$\vec{F}_g(O) = G M_d M_p \frac{\vec{R}_d}{R_d^3}$$  \hspace{1cm} (2.2)

The difference between the attraction at $P$ (2.1) and the contribution of $P$ (2.2) to the attraction of the earth as a whole, deforms the earth and produces the tide-generating force $\vec{F}_t$ at $P$ (Figure 2.1):

$$\vec{F}_t(P) = \vec{F}_g(P) - \vec{F}_g(O) = G M_d M_p \left(\frac{\vec{R}_g}{R_p^3} - \frac{\vec{R}_d}{R_d^3}\right)$$  \hspace{1cm} (2.3)

For many practical computations it is preferred to work in terms of tidal acceleration (force per unit of mass), i.e. $\vec{a}_t = \vec{F}_t/M_p$, instead of the tide-generating force $\vec{F}_t$. The tidal acceleration can be derived from a potential function $U$ (tide-generating potential) by:

$$\vec{a}_t = \nabla U$$  \hspace{1cm} (2.4)

where $\nabla$ is the gradient operator. In its components along the vertical and horizontal axes denoted by $\vec{e}_r$ and $\vec{e}_g$, respectively, in Figure 2.1, the above equation reads:
As can be easily derived [e.g., Melchior, 1978; Pugh, 1987], the tide-generating potential \( U \) on the earth’s surface (mean radius \( R_e \)) as caused by the disturbing body \( D \) is given by:

\[
U = \frac{G M_d}{R_d} \sum_{l=2}^{\infty} \left( \frac{R_e}{R_d} \right)^l P_l(\cos \theta)
\]  

(2.6)

where \( \theta \) is the zenith distance of the body \( D \) (Figure 2.1). In the above equation, \( P_l(x) \) are the Legendre polynomials of degree \( l \). The first three of these polynomials are defined as: \( P_0(x) = 1 \), \( P_1(x) = x \), and \( P_2(x) = \frac{1}{2}(3x^2 - 1) \). Notice that the \( l = 0 \) term is absent from (2.6) as it represents the constant \( G M_d/R_d \), which produces no force according to (2.4). Also absent from the series (2.6) is the \( l = 1 \) term for the reason that it corresponds to a uniform force field with strength \( F_1 = G M_d M_p/R_d^2 \) and direction along the vector \( \vec{R}_d \). Integrated over all particles \( M_p \), this force field gives the gravitational attraction of \( D \) on the earth as a whole, and therefore does not cause a tidal deformation of the earth’s surface.

From (2.5) and (2.6) it can be seen that the tide-generating potential \( U \) and the tidal acceleration \( \vec{a} \) depend on the parameter \( M_d/R_d^{l+1} \) of the disturbing body. Hence, only the sun (due to its mass) and the moon (because of its relatively small distance to the earth) cause tide-generating forces of importance on the earth’s surface. In the series (2.6), the ratio \( R_e/R_d \) expresses the sine of the lunar or solar parallax [Taff, 1985]. Assuming for the moment that the orbit of the moon and the apparent orbit of the sun (motion of the sun as seen from the earth) are circular so that \( R_d \) remains constant, the sine parallax is merely \( 1.7 \cdot 10^{-2} \) for the moon and \( 4.3 \cdot 10^{-3} \) for the sun [Taff, 1985]. This means that terms in (2.6) with \( l \) larger than two can safely be omitted, although sometimes the \( l = 3 \) term of the moon is taken into account, e.g. in Doddson [1921], and in Cartwright and Tayler [1971], Cartwright and Edden [1973]. Substituting the relevant expression for \( P_2(\cos \theta) \), we obtain for the main term \( (l = 2) \) of the tide-generating potential (2.6):

\[
U_2 = \frac{3G M_d R_e^2}{4R_d^3} \left( \cos 2\theta + \frac{1}{3} \right)
\]  

(2.7)

and for the tidal acceleration on the earth’s surface (2.5):

\[
a_r = \frac{3G M_d R_e}{2R_d^3} \left( \cos 2\theta + \frac{1}{3} \right)
\]

\[
a_\theta = \frac{3G M_d R_e}{2R_d^3} \sin 2\theta
\]  

(2.8)
From (2.8) it can be seen that the components of the tide-generating force have a maximum value relative to the earth's mean gravity, $g = G M_e / R_e^2$, of:

$$\begin{align*}
\frac{a_r}{g} &= \frac{4}{3} \frac{3 M_d}{2 M_e} \left( \frac{R_e}{R_d} \right)^3 \\
\frac{a_\theta}{g} &= \frac{3 M_d}{2 M_e} \left( \frac{R_e}{R_d} \right)^3
\end{align*}$$

(2.9)

Substituting for the sun: $M_d = 1.99 \cdot 10^{30}$ kg, $R_d = 149.5 \cdot 10^6$ km, and for the moon: $M_d = 7.35 \cdot 10^{22}$ kg, $R_d = 384.4 \cdot 10^3$ km, we notice that the moon exerts a tidal force on the earth that is 2.2 times as large as that of the sun. Hence, although the sun is much more massive than the moon, the cube of its much larger distance to the earth significantly reduces its tidal effect on the earth. Moreover, with a mass ratio $M_e/M_d$ of 81 for the moon [Roy, 1965; Seidelmann, 1992] and $1.7 \cdot 10^{-2}$ for its sine parallax, it is noticed that the lunar tide-generating force is about $10^7$ times smaller than the earth's own gravity. In case of the sun, this force is yet another factor 0.46 smaller. Hence, it is important to realize that the vertical component of the tide-generating force plays no role in the generation of the tidal elevations, i.e. the vertical displacements of the earth's surface. The horizontal component on the other hand, although of equal magnitude as the vertical component, is of the same order of magnitude as other horizontal forces in the oceans, like bottom friction and the Coriolis force. Therefore, it is this component, usually called the tractive force, that causes the movement of
2.3 The tidal species

The water masses resulting in the tidal displacements of the earth’s surface [Franco, 1981; Pugh, 1987].

According to (2.8), the tractive force associated with \( a_\theta \) becomes zero at \( \theta = 0^\circ \mod 90^\circ \), while it reaches a maximum for \( \theta = 45^\circ \mod 90^\circ \). Thus, if in Figure 2.1, the tide-generating body would move along the earth’s equator such that \( \theta \) would equal the observer’s latitude, the water would flow away from the poles, where it would create a local decrease in water level, towards the equator, where the water level would increase (Figure 2.2). Because all points \( P \) on the earth with the same distance \( R_p \) to \( D \) experience an equal tractive force, the resulting shape of the earth would be obtained by rotating Figure 2.2 about the \( OD \) axis, thus producing an ellipsoid with its two bulges towards and away from \( D \). As the earth rotates around its polar axis in one day, a point on the earth would experience two high tides and two low tides per day (still assuming that \( D \) would move along the equator). Hence, an observer rotating with the earth would see the tidal elevation field as a wave progressing over the earth’s surface with the wave’s crest (high tide) passing him twice each day.

2.3 The tidal species

Because the sun and the moon do not move along the equator but in orbital planes that are inclined to the equatorial plane of the earth, the direction of the tidal bulge will change according to the declination of the tide-generating body in its orbit. The varying declination of the tide-generating body causes a distinct separation of the potential (2.7) in three different types of tides known as the tidal species. Apart from the “twice a day” tide as discussed in the previous section, we will now also find tides with a period of one day and periods much larger than a day.

From Figure 2.3, which shows the earth-fixed reference frame \( XYZ \) (the \( X \)-axis in the equatorial plane crossing the upper branch of the Greenwich meridian), one obtains from spherical geometry in the position triangle \( NPD \) for the zenith distance \( \theta \) of \( D \) [e.g., Melchior, 1966]:

\[
\cos \theta = \sin \phi \sin \delta + \cos \phi \cos \delta \cos H
\]  

(2.10)

where \( \lambda \) and \( \phi \) are geocentric (east) longitude and latitude, respectively, where \( \delta \) is the declination of \( D \), and where \( \lambda - \lambda_d \) defines the hour angle \( H \) of the tide-generating body \( D \) (Figure 2.3):

\[
H = \lambda - \lambda_d
\]  

(2.11)

The hour angle is measured from the upper branch of the observer’s meridian and positive to the west of the observer. With expression (2.10), the addition formula for \( P_l(\cos \theta) \) can be derived [e.g., Hobson, 1965]:

\[
P_l(\cos \theta) = \sum_{m=0}^{l} (2 - \delta_{0m}) \frac{(l - m)!}{(l + m)!} P_{lm}(\sin \phi) P_{m}(\sin \delta) \cos mH
\]  

(2.12)
The tide-generating force

Figure 2.3 Geocentric longitude $\lambda$ and latitude $\phi$ of observer $P$, geocentric longitude $\lambda_d$ and declination $\delta$ of the tide-generating body $D$, and the zenith distance $\theta$.

which, after substitution in (2.6), results in the following form of the tide-generating potential:

$$U = \frac{GM_d}{R_d} \sum_{l=2}^{\infty} \left( \frac{R_e}{R_d} \right)^l \sum_{m=0}^{l} \frac{(2-\delta_{0m})(l-m)!}{(l+m)!} P_m(\sin \phi) P_m(\sin \delta) \cos mH \quad (2.13)$$

In (2.12) and (2.13), the $P_m(x)$ are the associated Legendre functions of degree $l$ and order $m$ and $\delta_{0m}$ is the Kronecker delta ($\delta_{0m} = 1$ for $m = 0$, $\delta_{0m} = 0$ for $m \neq 0$). Substituting in (2.13) for $P_{2m}(\sin \phi)$ (with $P_{20} = P_2$) from e.g. Hobson [1965]:

$$P_{20} = \frac{1}{2} (3 \sin^2 \phi - 1)$$
$$P_{21} = \frac{3}{2} \sin 2\phi$$
$$P_{22} = 3 \cos^2 \phi$$

we get for the principal part of the tide-generating potential:

$$U_2 = \sum_{m=0}^{2} U_{2m} = \frac{3GM_dR_e^2}{4R_d^3} \left\{ \cos^2 \phi \cos^2 \delta \cos 2H + \sin 2\phi \sin 2\delta \cos H \\
+ (1 - 3 \sin^2 \phi) \left( \frac{1}{3} - \sin^2 \delta \right) \right\} \quad (2.15)$$

Obviously, the same result (2.15) can be found by substituting (2.10) in (2.7).
2.3 The tidal species

Because the earth rotates around its polar axis in one day, while the sun and the moon complete one orbit around the earth in respectively one year and one month, the term $U_{22}$ containing $2H$ will show variations of approximately 12 hours, i.e. twice a day. Hence, this term is called the semi-diurnal species. Similarly, the term $U_{21}$ containing $H$ will show once a day variations and therefore is called the diurnal species. The diurnal variations can be explained with the aid of Figure 2.4. Due to the offset of the bulge relative to the equator, an observer $P_1$ facing the tide-generating body will experience a higher tide than observer $P_2$ on opposite side of their meridian. Half a day later, when the earth has rotated through 180°, positions are reversed, so that any observer $P$ rotating with the earth will notice a diurnal inequality in the tide, which is expressed by the diurnal species in (2.15). The term $U_{20}$ from which $H$ is absent shows variations with the square of the sine of the tide-generating body's declination and thus has a period of half a year for the sun and half a month in case of the moon assuming constant $R_d$ (effects of variations in $R_d$ are discussed in Sections 2.6 and 2.7). Hence, it is called the long-period species. The periods of half a month and half a year can be explained by the fact that the same configuration will be reached when the sun and the moon are at their maximum and minimum declinations of $\delta = \pm 23^\circ$ and $\delta = 23^\circ \pm 5^\circ$, respectively.

Notice that the declination $\delta$ will cause the magnitudes of the three types of tides (2.15) to vary with the positions of the sun and the moon in their orbits. An increasing value of the declination, either in a positive
or a negative sense, amplifies the diurnal species at the expense of the semi-diurnal and long-period species, which both become smaller. Notice also that if the orbit of the moon and the apparent orbit of the sun were to lie in the earth's equatorial plane (zero declination), the tide all over the globe would be the sum of a periodic semi-diurnal tide (varying with latitude according to $\cos^2 \phi$) superimposed on a permanent tide (varying with latitude according to $\frac{1}{3} (1 - 3 \sin^2 \phi)$). The permanent tide is due to the asymmetry of the tidal bulge, i.e. at a certain latitude, high tide and low tide are not of equal magnitude as expressed by the constant term $\frac{1}{3}$ in (2.7).

2.4 The equilibrium theory of tides

With the equilibrium theory, as well as with the dynamic theory to be discussed in Section 3.3, we seek an expression for the vertical displacement of the sea surface as caused by the tractive forces, i.e. the ocean tide. In the equilibrium theory as originally established by Sir Isaac Newton in 1678, the hypothesis is that the earth is formed of a rigid spherical core completely covered with a thin layer of water without either viscosity or inertia [e.g., Franco, 1981; Pugh, 1987]. For such an earth, the sea surface would respond instantaneously to the tractive forces adapting itself to an equipotential surface [e.g., Pond and Pickard, 1983]. On this surface, all acting forces are in equilibrium (Figure 2.5). In the vertical direction we have hydrostatic equilibrium while in the horizontal direction, the tractive force is being balanced by the horizontal pressure gradient force resulting from the slope of the water surface [e.g., Pond and Pickard, 1983]. The deformed sea surface in Figure 2.5 will thus be normal to the combined force of gravity and the tractive force.

The vertical displacement $\gamma$ of a water particle initially at rest at the sea surface may be found by applying the work theorem [e.g., Schureman, 1971]. This theorem states that to rise from its rest position (undeformed surface in Figure 2.5), where the potential (in our case the earth's gravity potential) is unperturbed, to the position where the potential has been perturbed by addition of the tide-generating potential $U$ (deformed surface), a water particle has to move a vertical distance $\gamma$ given by:

$$\gamma = \frac{U}{g}$$ (2.16)

where $g$ is the earth's mean gravity. Equation (2.16) is known as Bruns' formula [Heiskanen and Moritz, 1967; Moritz and Müller, 1988] in which the tide-generating potential acts as the disturbing potential.

For several reasons it is unrealistic to assume that the ocean tide will follow an equilibrium response (2.16). First of all, because the tide-generating force is not the result of a central force field originating from the center of the earth, the movement of the water particles is not strictly
vertical but also includes the horizontal flow of tidal currents [e.g., Schureman, 1971; Dietrich et al., 1975]. However, if we are not interested in the actual motion of the water particles but only in the envelope of the displacement field, i.e. the resulting shape of the earth’s surface, the equilibrium theory does provide an order of magnitude approximation of the actual ocean tide [Lambeck, 1988]. Secondly, the earth is not entirely covered with water and the propagation of the tidal waves (each harmonic term in the spectrum of U or \( \gamma \) is called a tidal harmonic or tidal wave) in an east-west direction is impeded by the continental boundaries [Pugh, 1987]. Thirdly, the solid earth underneath the oceans is not at all rigid and so deforms under the tide-generating forces. Because the ocean tide by definition is the vertical displacement of the sea level above the moving ocean floor, (2.16) has to be corrected for the effects of the bottom tide, i.e. the tidal motion of the ocean floor, as will be discussed in Section 3.3. Fourthly, the equilibrium state in Figure 2.5 will only be achieved if the forcing periods of the sun and the moon are much larger than the resonance periods of the oceans [e.g., Munk and MacDonald, 1975; Lambeck, 1988]. In that case, the tidal elevations are effectively instantaneous and are given by static (i.e. equilibrium) considerations. On the other hand, if the forcing periods are comparable to the resonance periods, the ocean’s response to the tidal forcing will be dynamic. Analyses of tide gauge records seem to indicate that the oceans as a whole are at near resonance at diurnal and semi-diurnal periods [Pugh, 1987]. This means that the amplitudes of the diurnal and semi-diurnal tidal harmonics will, in general, be larger than their predicted equilibrium values, while we also have to account for a phase lag of these harmonics on their forcing terms in the tide-generating poten-
tial due to the inertia of the oceans. The long-period tides, of which the most important have a period of half a month or longer, may be expected to closely follow the equilibrium theory [e.g., Lambeck, 1988], although definite departures from equilibrium have been observed [e.g., Cartwright and Ray, 1990a]. Fifthly, the various ocean basins each have their individual shape and depth and therefore their individual resonance periods [Pond and Pickard, 1983]. Each ocean basin, therefore, responds differently to a harmonic term of the tidal forcing, so that the generated tide has an amplitude and phase lag that will depend on the location [Pond and Pickard, 1983; Pugh, 1987]. For the above reasons it is obvious that the actual ocean tide will bear little resemblance to the equilibrium tide. Still, in spite of its limited ability to model the ocean tide, the equilibrium tide does serve as an important reference in tidal analysis, e.g. as input values for the response method (Section 3.5).

2.5 The sun-earth-moon system

To facilitate the harmonic development of the tide-generating potential in Section 2.6, it is useful to discuss here the orbit of the moon around the earth and the apparent orbit of the sun. In the discussion that follows, Figure 2.6 is used as a reference. The positions of the sun and the moon are both described with respect to the ecliptic, i.e. the plane of the sun’s apparent orbit around the earth, in terms of ecliptic longitude, ecliptic latitude, and the distance earth-sun or earth-moon [Roy, 1965; Seidelmann, 1992]. Longitude in the ecliptic is measured from the vernal equinox \( \Upsilon \), which is the point on the equinox line (intersection of the earth’s equator and the ecliptic) when the sun crosses the equatorial plane on 21 March. Latitude is measured with respect to the ecliptic plane.

The motion of the sun (apparent orbit) takes place in the ecliptic. The ecliptic is inclined to the equator at an angle of about 23°43, known as the obliquity of the ecliptic \( \epsilon \). The mean eccentricity of the solar orbit is 0.017 and the average sun-earth distance is 149.5 \( \cdot 10^6 \) km [Roy, 1965]. The ascending node of the sun makes one revolution in the ecliptic plane in about 26,000 years (luni-solar precession), while the line of apses has a period of approximately 21,000 years due to planetary perturbations [Roy, 1965].

In case of the moon, the orbit has a mean eccentricity of 0.055 and a mean distance to the earth of 384,400 km [Roy, 1965]. Mainly due to solar perturbations, the eccentricity of the lunar orbit varies between 0.044 and 0.067, while its inclination \( i \) relative to the ecliptic (Figure 2.6) oscillates between 5° and 5°3 (mean inclination is 5°15). In contrast with artificial earth satellites, the solar perturbations dominate the earth’s perturbing effect on the lunar orbit. As a consequence, the motion of the ascending lunar node \( \Omega \) takes primarily place in the ecliptic and
2.5 The sun-earth-moon system

not in the equatorial plane [Seidelmann, 1992]. The motion of the lunar node is due to the differential force between the attraction of the sun on the earth-moon system as a whole and the attraction of the sun on the moon [e.g., Seidelmann, 1992]. This differential force, its action being much similar to the concept of the tide-generating force, causes the lunar node to undergo a practically uniform change in time along the ecliptic in a westward direction (regression). As a consequence, the inclination of the moon’s orbit $I$ relative to the equator changes with the position of the moon’s node from approximately 28°5, when the ascending node coincides with the vernal equinox, to about 18°5, when the ascending node coincides with the autumnal equinox (23 September) opposite to T. The time between two passages of the lunar node with the vernal equinox is 18.61 years. The solar perturbations also cause a practically uniform motion of the moon’s line of apses, which takes place in the plane of the lunar orbit with a period of 8.85 years. The inclination of the lunar orbit relative to the ecliptic and the lunar eccentricity do not suffer a secular motion and may be assumed constant (usually mean values are adopted).

As mentioned in, e.g., Doodson [1921] and Valorge [1995], the position of the moon may be obtained from E.W. Brown’s lunar theory (1905),
The tide-generating force

<table>
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<tr>
<th>mean longitude of</th>
<th>symbol</th>
<th>period</th>
</tr>
</thead>
<tbody>
<tr>
<td>moon</td>
<td>$s$</td>
<td>27.32 mean solar days</td>
</tr>
<tr>
<td>sun</td>
<td>$h$</td>
<td>365.24 mean solar days</td>
</tr>
<tr>
<td>lunar perigee</td>
<td>$p$</td>
<td>8.85 tropical years</td>
</tr>
<tr>
<td>lunar node</td>
<td>$N$</td>
<td>18.61 tropical years</td>
</tr>
<tr>
<td>solar perigee</td>
<td>$p_s$</td>
<td>21,000 tropical years</td>
</tr>
</tbody>
</table>

Table 2.1 Fundamental angles that describe the motion of the sun and the moon in the sun-earth-moon system. For the definitions of the mean solar day and the tropical year, the text may be referred to.

which includes the perturbing effects of the sun, the earth, and the other planets on the lunar orbit. With this theory, the position of the moon is given in ecliptic longitude, ecliptic latitude, and the dimensionless distance $R_d/R_d$, where the actual lunar distance is substituted for $R_d$ and the mean lunar distance for $R_d$. For the sun’s apparent position, expressions for its ecliptic longitude and dimensionless distance are provided by S. Newcomb’s solar theory (1895). With this theory, the sun’s ecliptic latitude is assumed to be zero while the eccentricity of the sun’s apparent orbit is kept fixed [e.g., Doodson, 1921; Valorge, 1995]. For the sun as well as for the moon, each of the ecliptic coordinates can be expressed in terms of five fundamental angles that by good approximation proceed linearly with time. These fundamental angles describe the long-term motion of the sun and the moon, i.e. with periodic variations of motion removed. In Table 2.1, the fundamental angles with their symbols and periods are given. All of these angles are longitudinal angles and therefore are measured in the ecliptic with respect to the vernal equinox. The angles belong to five fictive bodies that move along the ecliptic with practically uniform speed and at a constant distance from the earth. For instance, the mean longitude of the moon $s$ measures the longitude of a fictive moon traveling in the ecliptic at an angular speed of 360° per 27.32 mean solar days. The exact definition of the mean solar day will be given below.

From Newcomb’s theory for the solar elements $h$ and $p_s$, and similar expressions obtained by Brown in 1919 for the lunar elements $s$, $p$ and $N$, the following expressions are given for the five fundamental angles [Doodson, 1921; Melchior, 1966; Casotto, 1989]:

\[
\begin{align*}
    s &= 270^\circ43659 + 481267^\circ89057 T + 0^\circ00198 T^2 + 0^\circ000002 T^3 \\
    h &= 279^\circ69668 + 36000^\circ76892 T + 0^\circ00030 T^2 \\
    p &= 334^\circ32956 + 4069^\circ03403 T - 0^\circ01032 T^2 - 0^\circ000010 T^3 \\
    N &= 259^\circ18328 - 1934^\circ14201 T + 0^\circ00208 T^2 + 0^\circ000002 T^3 \\
    p_s &= 281^\circ22083 + 1^\circ71902 T + 0^\circ00045 T^2 + 0^\circ000003 T^3
\end{align*}
\]

(2.17)

where $T$ is in Julian centuries (a Julian century consists of 36,525 mean solar days) from 31 December 1899, 12 hr. Universal Time (UT). Notice that besides the linear terms, the above expressions also contain higher-
order terms in $T$. This is because the theories of Newcomb and Brown describe the long-term motions of the sun and the moon. Because these long-term motions do not only suffer a linear change, terms of higher order than $T$ arise as well. The higher-order terms are mainly due to the planetary perturbations on the eccentricity of the moon's orbit and on the eccentricity of the sun's apparent orbit [Roy, 1965].

It may be interesting to note that civil time, i.e. UT, is based on the motion of the so-called mean sun (Figure 2.7). The mean sun is defined to move along the equator at a constant speed, which equals that of the fictive sun, i.e. $h$, at a certain reference time [Franco, 1981]. Hence, over a period of many centuries (according to (2.17), $h$ varies with $0.0006^\circ$ century per century), the right ascension (i.e. longitude measured along the equator from the vernal equinox) of the mean sun may be taken equal to the mean longitude of the fictive sun $h$ in the ecliptic, if we use $\Upsilon$ as a common
reference point [Schureman, 1971; Franco, 1981]. In a similar way, we may
define the mean moon (Figure 2.7), which moves along the equator with
the same speed as the fictive moon at a reference time. Hence, at any
instant, the right ascension of the mean moon (in the equatorial plane) will
be equal to the mean longitude of the fictive moon \( s \) (in the ecliptic). The
time elapsed between two passages of the mean sun with the Greenwich
meridian, i.e. the earth’s sidereal rotation relative to the motion of the
mean sun, is called a mean solar day (one sidereal day equals 0.997 mean
solar days). Similarly, the time between two passages of the mean moon
with the Greenwich meridian is called a mean lunar day, which equals
1.035 mean solar days. The time between two passages of the mean sun
with the vernal equinox is called the tropical year, which equals 365.24
mean solar days. The mean solar day and the tropical year are the units of
time that we are accustomed with in civil life.

In Figure 2.7, the angle \( t \) between the lower branch of the Greenwich
meridian and the mean sun denotes Greenwich mean solar time or Uni-
cessary Time UT (notice that \( t + h \) defines the sidereal day). The angle of the
mean moon counted from the lower branch of the Greenwich meridian is
Greenwich mean lunar time \( \tau \). Figure 2.7 is perhaps easiest interpreted by
imagining the observer \( P \) to be fixed in space. Then, denoting the earth’s
sidereal rotation by \( \Omega_e \), the vernal equinox \( \tau \), the mean sun \( \bar{S} \), and the
mean moon \( \bar{M} \), move in a clockwise direction with respective speeds of
\( \Omega_e, \Omega_e - \bar{h} \), and \( \Omega_e - \bar{s} \). Notice from Figure 2.7 that local mean solar time
and local mean lunar time of an observer \( P \) are obtained by adding the ob-
server’s longitude \( \lambda \) to \( t \) and \( \tau \), respectively. Also notice from Figure 2.7
that the relation:

\[
\tau = t - \bar{s} + h
\]  

between mean solar time and mean lunar time must hold.

2.6 Harmonic development of the tide-generating
potential

In the previous section it was shown that the orbital motions of the sun
and the moon around the earth can be described by five angles that by
good approximation proceed linearly with time. This implies that the tide-
generating potential can be developed into a series of harmonic functions.
The amplitudes of these harmonics may be considered as constant over a
period of centuries, while their arguments are linear combinations of the
five fundamental angles and therefore vary at an almost linear rate. In
1867, Sir G.H. Darwin was the first to actually carry out such a de-
velopment. Although, Darwin’s expansion was limited to include 39 terms
[Munk and Cartwright, 1966], the names he assigned to these terms are still
widely used in tidal analysis. One of the most comprehensive develop-
ments of the tide-generating potential is due to Doodson [1921]. The Doo-
2.6 Harmonic development of the tide-generating potential

dson expansion, which contains over 400 terms, is carried out over all five fundamental angles \((s, h, p, N, p_s)\). As a sixth angle, Doodson used Greenwich mean lunar time \(\tau\) (Section 2.5) to relate the orbital motions of the sun and the moon described in inertial space to the earth-fixed reference frame of the observer [Doodson, 1921].

Important to realize is that when using the harmonic method for the analysis of tidal data, a problem arises that cannot be corrected for by a complete expansion like Doodson’s. For most tide gauge stations in the open oceans, records are available for periods of a few years or less. Therefore, it is impractical to try to resolve tidal harmonics from the data that differ by less than one cycle per year in frequency. This is especially true with altimetry where ocean tides may alias to nearly the same period (Section 4.4). As the period of the mean longitude of the sun \(h\) is one year, the data are analyzed with harmonics of which the arguments in the expansion differ in \((\tau, s, h)\), i.e. harmonics that differ in \((p, N, p_s)\) are treated as one single term [e.g., Munk and Cartwright, 1966; Schureman, 1971]. The dominant effect of the angles \((p, N, p_s)\) is the varying inclination of the lunar orbit due to the regression of its node \(N\). This effect can be accounted for with slow varying functions of \(N\) in the coefficients and arguments of the dominant harmonics [e.g., Melchior, 1966; Schureman, 1971]. Hence, the brief explanation in this section of how the tide-generating potential is developed into a series of harmonics, will only extend to a splitting of the arguments in terms of \(\tau, s\) and \(h\).

To expand the tide-generating potential into its harmonics, (2.15) for the sun or the moon is written as:

\[
U_2 = \sum_{m=0}^{2} U_{2m} = \left(\frac{\bar{R}_d}{R_d}\right)^2 \left\{ G_d^{(2)}(\phi) \cos^2 \delta \cos 2H + G_d^{(1)}(\phi) \sin 2\delta \cos H \right. \\
+ G_d^{(0)}(\phi)(\frac{2}{3} - 2 \sin^2 \delta) \right\}
\]

(2.19)

where \(\bar{R}_d\) is the disturber’s mean distance to the earth and the superscript on the \(G_d^{(m)}\) pertains to the tidal species \(m\) of \(U_2\). For the sake of brevity, some definitions according to Doodson [1921] have been introduced. Denoted by \(G_d\) will be the disturber’s general coefficient, which is defined as:

\[
G_d = \frac{3GM_dR_e^2}{4R_d^3}
\]

(2.20)

For the moon and the sun, the general coefficients are usually given the symbols \(G\) and \(G_s\), respectively, where the former has also become known as Doodson’s tidal constant [e.g., Melchior, 1966]. The general coefficients appear in the terms \(G_d^{(m)}(\phi)\) in (2.19). These terms, which involve functions of the observer’s latitude \(\phi\) multiplied by \(G_d\), are called Doodson’s geodetic coefficients (of degree \(l = 2\)) [Doodson, 1921]. In case of the moon,
they will be denoted by \( G^{(m)}(\phi) \), and in case of the sun by \( G_s^{(m)}(\phi) \). For the moon, the geodetic coefficients for \( l = 2 \) read:

\[
\begin{align*}
G^{(0)}(\phi) &= \frac{1}{2} G (1 - 3 \sin^2 \phi) \\
G^{(1)}(\phi) &= G \sin 2\phi \\
G^{(2)}(\phi) &= G \cos^2 \phi
\end{align*}
\]  

(2.21)

The expressions for the geodetic coefficients of the sun are the same as those for the moon except that \( G \) is replaced by \( G_s \). Notice that a factor \( \frac{1}{2} \) is brought into \( G^{(0)} \) so that all geodetic coefficients of the moon have \( G \) as a maximum. Because the aim of the harmonic development is to expand the functions in (2.19) that depend on \( R_a/R_d, \delta, \) and \( H \) into a series of harmonics, this means that all coefficients of the harmonic development will be normalized with respect to one and the same quantity \( G \); if we write the geodetic coefficients of the sun as \( G_s/G \) times those of the moon and absorb the factor \( G_s/G \) in the coefficients of the solar harmonics. In this way, the values of the harmonic coefficients give their relative importance [Doodson, 1921]. To proceed, each species \( U_{2m} \) of the tide-generating potential of the moon and the sun is written to a form:

\[
U_{2m} = G^{(m)}(\phi) \sum_k |\eta_k| \cos(\Theta_k + \chi_k + m\lambda)
\]  

(2.22)

where \( G^{(m)}(\phi) \) are the geodetic coefficients of the moon. The \( \eta_k \) are the coefficients of the harmonic development (containing \( G_s/G \) in case of the solar harmonics), while the summation over \( k \) is over all harmonic terms of the species. The arguments \( \Theta_k \) are the astronomical arguments or Doodson arguments at Greenwich because the angle \( \tau \) in these arguments is mean lunar time at Greenwich (\( \Theta_k + m\lambda \) is called the local argument because it is related to local mean lunar time \( \tau + \lambda \) in Figure 2.7):

\[
\Theta_k(t) = Am + Bs + Ch + Dp + EN'' + Fp
\]  

(2.23)

where \( A \) to \( F \) are integer numbers. By convenience, Doodson defined the negative of \( N \) as \( N' \), so that all angles in (2.23) increase towards the east [Doodson, 1921]. The \( \chi_k \) are so-called additive phase corrections, which are multiples of 90°. These phase corrections are needed to achieve uniformity in the harmonic series, i.e. to obtain a series (2.22) of all positive coefficients \( |\eta_k| \) and cosine functions only [Schureman, 1971; Cartwright, 1993; Casotto, 1989]. This procedure is known as the Doodson & Warburg convention, which is detailed in e.g. Casotto [1989].

For the most important tides, the outcome of the harmonic development \textit{up to a yearly splitting in frequency}, i.e. no two arguments \( \Theta_k \) in the harmonic development are the same with regard to \( \tau, s, \) and \( h \), may be shown to give for the long-period species \( U_{20} \), the diurnal species \( U_{21} \), and
the semi-diurnal species $U_{22}$ [Doodson, 1921; Schureman, 1971]:

$$U_{20} = \frac{G^{(0)}}{M_0 (L)} \times \left\{ \left( \frac{2}{3} - \sin^2 I \right) \left[ 1 + 3\epsilon \cos(s - p) \right] + \sin^2 I \cos(2s - 2\xi) + \frac{G_1}{G} \left( \frac{2}{3} - \sin^2 \epsilon \right) \left[ 1 + 3\epsilon_1 \cos(h - p_s) \right] + \frac{G_1}{G} \sin^2 \epsilon \cos 2h \right\} \right.$$ 

$$U_{21} = \frac{G^{(1)}}{O_1 (L)} \times \left\{ \sin I \cos^2 \frac{L}{2} \left[ \cos(\tau - s - 90^\circ - 2\xi - \nu + \lambda) + \frac{7}{2} e \cos(\tau - 2s + p - 90^\circ + 2\xi - \nu + \lambda) \right] + \frac{1}{2} \sin 2I \cos(\tau + s + 90^\circ - \nu + \lambda) + \frac{G_1}{G} \sin \epsilon \cos^2 \frac{L}{2} \cos(\tau - s + 2h - 90^\circ + \lambda) + \frac{G_1}{G} \frac{1}{2} \sin 2\epsilon \cos(\tau + s + 90^\circ + \lambda) \right\} \right.$$ 

$$U_{22} = \frac{G^{(2)}}{M_2 (L)} \times \left\{ \cos^4 \frac{L}{2} \left[ \cos(2\tau + 2\xi - 2\nu + 2\lambda) + \frac{7}{2} e \cos(2\tau - s + p + 2\xi - 2\nu + 2\lambda) \right] + \frac{1}{2} \sin^2 I \left[ \cos(2\tau + 2s - 2\nu + 2\lambda) + \frac{3}{2} e \cos(2\tau + s + p - 2\nu + 2\lambda) \right] + \frac{G_2}{G} \cos^4 \frac{L}{2} \left[ \cos(2\tau + 2s - 2h + 2\lambda) + \frac{7}{2} \epsilon_2 \cos(2\tau + 2s - 3h + p_s + 2\lambda) \right] + \frac{G_2}{G} \frac{1}{2} \sin^2 \epsilon \cos(2\tau + 2s + 2\lambda) \right\} \right.$$ 

(2.24) 

(2.25) 

(2.26)

where $\xi$ and $\nu$ are angles that depend on the lunar node as explained in Schureman [1971]. Denoted by $\epsilon$ and $\epsilon_1$ are the mean eccentricity of the lunar orbit and the apparent solar orbit, respectively. In the above equations, all terms of the order of $e^2$ and $\epsilon_1^2$ have been neglected. Each tide in the above equations is referred to by its Darwin name and by its origin, i.e. (L)unar or (S)olar. A brief explanation of each of the terms in (2.24) to (2.26) will be given in the next section.
2.7 The Doodson classification of tides

Because the names Darwin used to classify the tides are confined to a limited number, a more systematic notation was introduced by Doodson. Each tide is denoted by a 6-digit number, known as the Doodson number, \( k = k_1k_2k_3k_4k_5k_6 \). The Doodson number is obtained by adding 055.555 to the argument number \( ABC.DEF \) in (2.23):

\[
k_1k_2k_3k_4k_5k_6 = A(B + 5)(C + 5)(D + 5)(E + 5)(F + 5)
\]  

(2.27)

Because the integers \( A \) to \( F \) are rarely outside the range -4 to 4, adding 055.555 to the argument number makes that the Doodson numbers are always positive [Doodson, 1921]. With (2.27), the Greenwich astronomical argument (2.23) may also be written as:

\[
\Theta_k(t) = k_1 \tau + (k_2 - 5)s + (k_3 - 5)h + (k_4 - 5)p + (k_5 - 5)N' + (k_6 - 5)p_s
\]  

(2.28)

or equivalently:

\[
\dot{\Theta}_k(t) = \dot{\Theta}_k + \Theta_k(t_0)
\]  

(2.29)

where \( \dot{\Theta}_k \) is called the tidal frequency and \( \Theta_k(t_0) \) is the astronomical argument at epoch \( t_0 \), i.e. at the origin of time \( t \):

\[
\dot{\Theta}_k = k_1 \dot{\tau} + (k_2 - 5)\dot{s} + (k_3 - 5)\dot{h} + (k_4 - 5)\dot{p} + (k_5 - 5)\dot{N}' + (k_6 - 5)\dot{p}_s
\]  

(2.30)

\[
\Theta_k(t_0) = k_1 \tau(t_0) + (k_2 - 5)s(t_0) + (k_3 - 5)h(t_0) + (k_4 - 5)p(t_0) + (k_5 - 5)N'(t_0) + (k_6 - 5)p_s(t_0)
\]  

(2.31)

In the above equations, the chief index \( k_1 \) pertains to \( \tau \), which changes fastest of all six angles. Hence, \( k_1 \) corresponds to the index \( m \) of the species \( U_{2m} \).

Notice that the Doodson number (2.27) is divided into two parts for the reason that two harmonics that differ by \( (p, N', p_s) \) are difficult to separate from a few years of data. According to the definition of Doodson [1921], all tidal harmonics with equal \( k_1k_2 \) form groups, while all harmonics with equal \( k_1k_2k_3 \) form constituents. As an example, of the semi-diurnal species \( k_1 = 2 \), all harmonics with Doodson numbers 25k_3.k_4.k_5.k_6 are said to form the \( M_2 \) group while all harmonics with numbers 255.k_4.k_5.k_6 can be merged into the \( M_2 \) constituent. Notice, therefore, that the lunar terms in (2.24) to (2.26) are tidal constituents and not tidal harmonics. The harmonics themselves can be obtained by expanding the functions of \( I \) in the constituent coefficients and the arcs \( \xi \) and \( \nu \) in the constituent arguments in terms of the lunar node \( N \) [Melchior, 1966; Schureman, 1971]. However, in practice, the dependence of the constituent coefficients and constituent arguments on the lunar node is expressed by so-called nodal parameters, i.e. the nodal
factor $f_k$ and the nodal angle $u_k$. Hence, each constituent $U_{k_1 k_2 k_3}$ in (2.24) to (2.26) is written as:

$$U_{k_1 k_2 k_3} = G^{(m)}(\phi) f_k(t) |\eta_k| \cos(\Theta_k + \chi_k + u_k(t) + m\lambda)$$

(2.32)

where $k_1$ equals $m$. The amplitude $|\eta_k|$ pertains to the main spectral line (main harmonic) around which the side lines (smaller harmonics) due to the modulation of the lunar node are centered. For the major constituents, the Doodson arguments of their main lines appear in (2.24) to (2.26), e.g., the line 255.555 in case of the $M_2$ constituent and the line 273.555 in case of the $S_2$ constituent. The nodal factors $f_k$ and nodal angles $u_k$ depend on the position of the lunar node and hence vary slowly with time throughout an 18.6-year cycle [e.g., Schureman, 1971]. Notice, therefore, that the nodal parameters strictly apply to the lunar constituents. Considering $t$ as a constant, the solar constituents all have unity as a nodal factor, whereas their nodal angles are zero by definition. Actually, this means that the solar constituents in (2.24) to (2.26) are the tidal harmonics themselves. It should be mentioned that in case a constituent has a lunar and a solar contribution, like $K_1$ in (2.25) or $K_2$ in (2.26), the nodal parameters give the combined effect of the lunar and solar parts. Such is easily accomplished by adding the lunar and solar constituents and writing the combined constituent to the form (2.32) by which the nodal parameters appear as $f_k$ and $u_k$ of the lunar tide slightly modified by the ratio $G_s/G$ [Schureman, 1971].

In order to avoid confusion between tidal harmonics and tidal constituents in this thesis, the term “tide” in the context of e.g. “the $M_2$ tide” will be used to denote tidal harmonics. Hence, when we speak of the $M_2$ tide, the main spectral line 255.555 of the $M_2$ constituent is understood. In Table 2.2, the harmonic coefficients and associated equilibrium amplitudes of the Doodson expansion are listed for the main tidal lines in the constituents (2.24) to (2.26). The CTE equilibrium amplitudes pertain to the expansion of Cartwright, Tayler & Edend, which will be discussed in Section 2.8. The Doodson harmonic coefficients $|\eta_k|$ (dimensionless numbers) were taken from Doodson [1921]. Notice that in case of the solar tides, the harmonic coefficients contain the factor $G_s/G$ so that the equilibrium amplitudes of all tides are obtained by multiplying the $|\eta_k|$ with $G/g = 0.2688$ m [Cartwright and Tayler, 1971]. Also notice that the equilibrium amplitudes give the extreme values of the equilibrium tides, which occur at latitudes where the corresponding geodetic coefficients (2.21) become unity. Hence, the semi-diurnal amplitudes are found at the equator where $G^{(2)}(\phi)$ becomes 1, while the diurnal amplitudes are found at latitudes of 45° and $-45°$ where $G^{(1)}(\phi)$ becomes 1 and -1, respectively. In case of the long-period tides, the equilibrium amplitudes occur at both the poles, i.e., at latitudes of 90° and $-90°$.

In Table 2.2, the $M_0$ and $S_0$ tides are the lunar and solar permanent tide, respectively. They are the result of the asymmetry of the tidal bulge (Sec-
The tide-generating force

| Darwin name | Doddson number | Doddson argument | harmonic coeff. \(|\eta_k|\) | eq. amp. \(|\eta_k|G/g\) (m) | CTE amp. \(|B_k|\) (m) | frequency (°/hr.) | period (days) | origin |
|-------------|----------------|------------------|----------------------------|----------------------------|----------------|-----------------|--------------|--------|
| \(M_0\)     | 055,555        | –                | 0.5046                     | 0.1356                     | 0.0000         | \(-L\) permanent |
| \(S_0\)     | 055,555        | –                | 0.2341                     | 0.0629                     | 0.0000         | \(-S\) permanent |
| \(M_0 + S_0\)|               |                  |                            |                            |                |                 |              |        |
| \(P_s\)     | 056,554        | \(h - p_s\)     | 0.0116                     | 0.0031                     | 0.0049         | 0.0411          | 365.260       | \(S\) elliptic |
| \(S_{sa}\)  | 057,555        | \(2h\)           | 0.0730                     | 0.0196                     | 0.0310         | 0.0821          | 182.621       | \(S\) declination |
| \(M_m\)     | 065,455        | \(s - p\)        | 0.0825                     | 0.0222                     | 0.0352         | 0.0444          | 27.555        | \(L\) elliptic |
| \(M_f\)     | 075,555        | \(2s\)           | 0.1564                     | 0.0420                     | 0.0666         | 1.0980          | 13.661        | \(L\) declination |
|              |                |                  |                            |                            |                |                 |              |        |
| Q_1         | 135,655        | \(\tau - 2s + p\) | 0.0722                     | 0.0194                     | 0.0502         | 13.3987         | 1.210         | \(L\) elliptic of \(O_1\) |
| O_1         | 145,555        | \(\tau - s\)     | 0.3769                     | 0.1013                     | 0.2622         | 13.9430         | 1.076         | \(L\) principal |
| P_1         | 163,555        | \(\tau + s - 2h\)| 0.1758                     | 0.0473                     | 0.1222         | 14.9589         | 1.003         | \(S\) principal |
| K^L_1       | 165,555        | \(\tau + s\)     | 0.3623                     | 0.0974                     | 0.0502         | 15.0411         | 0.997         | \(L\) declination |
| K^S_1       | 165,555        | \(\tau + s\)     | 0.1682                     | 0.0452                     | 0.3687         | 15.0411         | 0.997         | \(S\) declination |

\[K^L_1 + K^S_1\] \(\text{semi-diurnal}\)

| N_2         | 245,655        | \(2\tau - s + p\) | 0.1739                     | 0.0467                     | 0.1210         | 28.4397         | 0.527         | \(L\) major elliptic of \(M_2\) |
| M_2         | 285,585        | \(2\tau\)         | 0.9081                     | 0.2441                     | 0.6320         | 28.9841         | 0.518         | \(L\) principal |
| L_2         | 295,455        | \(2\tau + s - p\) | 0.0257                     | 0.0069                     | 0.0179         | 29.5283         | 0.508         | \(L\) minor elliptic of \(M_2\) |
| T_2         | 265,655        | \(2\tau + s + p\) | 0.0624                     | 0.0187                     | 0.0495         | 29.5378         | 0.508         | \(L\) elliptic of \(K_2\) |
| S_2         | 273,555        | \(2\tau + 2s - 2h\)| 0.4326                     | 0.1138                     | 0.2940         | 30.0000         | 0.500         | \(S\) principal |
| R_2         | 274,554        | \(2\tau + 2s - h - p_s\)| 0.0035                     | 0.0009                     | 0.0025         | 30.0411         | 0.499         | \(S\) minor elliptic of \(S_2\) |
| K^L_2       | 275,555        | \(2\tau + 2s\)    | 0.0876                     | 0.0211                     | 0.0821         | 30.0821         | 0.499         | \(L\) declination |
| K^S_2       | 275,555        | \(2\tau + 2s\)    | 0.0365                     | 0.0098                     | 0.0799         | 30.0821         | 0.499         | \(S\) declination |

\[K^L_2 + K^S_2\] \(\text{semi-diurnal}\)

Table 2.2 Major tidal harmonics. The harmonic coefficients \(|\eta_k|\) pertain to the Doddson expansion with associated equilibrium amplitudes \(|\eta_k|G/g\) \((G/g=0.2688\) m). The CTE equilibrium amplitudes \(|B_k|\) pertain to the CTE expansion to be discussed in Section 2.8 and refer to the epoch of 2000.

The above table presents the major tidal harmonics and their coefficients, which are essential for understanding the tide-generating force. The coefficients are given in terms of the Doddson and CTE expansions, providing a comprehensive view of the tidal variations. The table includes various harmonics such as \(M_0\), \(S_0\), \(M_0 + S_0\), \(P_s\), \(S_{sa}\), \(M_m\), \(M_f\), \(Q_1\), \(O_1\), \(P_1\), \(K^L_1\), \(K^S_1\), \(N_2\), \(M_2\), \(L_2\), \(T_2\), \(S_2\), \(R_2\), \(K^L_2\), \(K^S_2\), and \(K^L_2 + K^S_2\), each with its respective amplitude in terms of \(|\eta_k|G/g\) and \(|B_k|\). The table also includes the frequency in degrees per hour and the period in days, along with the origin of each harmonic.

Assuming that the Earth consists of a deformable core covered with a thin layer of water, the resulting sea level (in the absence of currents) defines the geoid, i.e., the Earth’s static figure. As will be explained in Section 3.3, the effect of the permanent tide of the solid Earth and the ocean is to raise sea level by an amount of \((1 + k_2)U_{055.55}/g\) cm, where \(k_2\) is the second-degree Love number introduced by A. E. H. Love [e.g., Melchior, 1966; Munk and MacDonald, 1975]. For the Earth, a good estimate of \(k_2\) is 0.3, assuming that the Love numbers are largely frequency independent [e.g., Melchior, 1966; Munk and MacDonald, 1975; Moritz and Müller, 1988]. Hence, multiplying the sum of the \(M_0\) and \(S_0\) harmonic coefficients by \(1.3G^{(0)}(\phi)/g\) shows that the geometric shape of the permanent tide is 12.8(\(1 - 3\sin^2 \phi\)) cm so that it tends to increase the flattening of the Earth’s static figure. Usually, however, the geoid is not adjusted for this effect and the permanent tide is either modeled with the above expression or treated as part of the dynamic topography [e.g., Marsh et al., 1989; Rapp et al., 1991]. A detailed discussion on several treatments of the permanent tide with associated definitions of the geoid may be found in Nerem et al. [1990]. The \(M_f\) and \(S_{sa}\) tides result from the disturber’s declination and hence are called declinational tides. The variation of the declination of the moon corresponding to \(2s\) is 13.661 days and causes the declinational tide.
$M_f$ (moon, fortnightly). For the sun, the period of $2h$ is 182.621 days and its declinational tide is called $S_{sa}$ (sun, semi-annual). Due to the ellipticity of the lunar orbit and the apparent solar orbit, elliptic tides occur, which are named $M_m$ (moon, monthly) and $S_a$ (solar, annual). They are proportional to the eccentricities $e$ and $e_s$, respectively. Their periods are respectively governed by the mean longitude of the moon relative to lunar perigee, i.e. $s - p$, and the mean longitude of the sun relative to solar perigee, i.e. $h - p_s$. The periods of $M_m$ and $S_a$ are 27.555 days and 365.260 days, respectively.

The diurnal tides all arise directly from the declination of the sun and the moon [e.g., Pugh, 1987]. The two main lunar declinational tides are called $K^{l}_1$ and $O_1$. Their arguments are $\tau + s$ and $\tau - s$, respectively, so that they have periods of 0.997 days and 1.076 days. Because $O_1$ is the larger of the two, it is termed the lunar diurnal principal tide. The two major declinational tides of the sun are termed $P_1$ and $K^{s}_1$. The argument of $P_1$ is $\tau + s - 2h$ with a period of 1.003 days. For the same reason as the $O_1$ tide of the moon, $P_1$ is called the solar diurnal principal tide. The $K_1$ tides of the sun and the moon have the same argument $\tau + s$ (of which the rate equals the earth's sidereal rotation). Hence, they are inseparable and one may speak of the luni-solar $K_1$ tide. Due to the eccentricity of the lunar orbit and of the apparent solar orbit, each of the declinational tides $O_1$, $P_1$, and $K_1$, is accompanied by a series of elliptic tides that are proportional to $e$ or $e_s$. The largest elliptic tide is that of $O_1$. It is called $Q_1$ and has argument $\tau - s - (s - p)$, while its period is 1.120 days.

With the semi-diurnal tides, the principal lunar tide is called $M_2$ (moon, twice per day). With argument $2\tau$, the period of the $M_2$ tide equals 0.518 days. The $M_2$ tide is accompanied by a series of elliptic tides of which the two most important show a symmetry in argument about $M_2$ and therefore are denoted by letters in the alphabet on each side of $M$, i.e. $L_2$ and $N_2$. The $N_2$ tide is of argument $2\tau - (s - p)$ and has a period of 0.527 days while $L_2$ has argument $2\tau + (s - p)$ and a period of 0.508 days. The main solar semi-diurnal tide is called $S_2$ (sun, twice per day). It has argument $2\tau + 2s - 2h$ and a period of exactly half a mean solar day. The two elliptic tides accompanying $S_2$ are called $R_2$ and $T_2$ with respective arguments of $2\tau + 2s - 2h + (h - p_s)$ and $2\tau + 2s - 2h - (h - p_s)$ and periods of 0.499 days and 0.501 days. Denoted by $K^{l}_2$ is the semi-diurnal lunar declinational tide. It has argument $2\tau + 2s$ and period 0.499 days. $K^{l}_2$ has a series of elliptic tides. One of them has argument $2\tau + s + p$, which differs from the $L_2$ elliptic tide of $M_2$ by just $2p$. Hence it is also called $L_2$. The semi-diurnal solar declinational tide is termed $K^{s}_2$. It is inseparable from its lunar equivalent $K^{l}_2$ and hence has same argument and period. Due to their smallness, the elliptic tides of $K^{s}_2$ have been neglected in (2.26) and Table 2.2. In agreement with the diurnal declinational tide $K_1$, one may speak of the luni-solar $K_2$ tide.

A convenient property of the series (2.22) is that to compute the
tide-generating potential at a desired place \((\lambda, \phi)\) and at a desired time \(t\), it is not necessary to evaluate this potential all over the world. Instead, only the potential at the Greenwich meridian has to be computed for that time. For this reason, tables and computations are almost exclusively concerned with the Greenwich potential or the Greenwich equilibrium tide. The latter is obviously found by dividing the Greenwich potential by the earth's mean gravity. To illustrate the above property, we write a species \(U_{2m}\) from (2.22) as:

\[
U_{2m} = \text{Re} \left[ \frac{G^{(m)}(\phi)}{G} \cdot \sum_k G|\eta_k|e^{i(\theta_k + \chi_k)} \right]
\]

where \(\text{Re}[ ]\) denotes the real part of [ ] and \(i = \sqrt{-1}\). The term \(\sum_k G|\eta_k|e^{i(\theta_k + \chi_k)}\) is the complex Greenwich potential (of species \(m\) and degree 2) with real part \(\sum_k G|\eta_k|\cos(\theta_k + \chi_k)\) and imaginary part \(\sum_k G|\eta_k|\sin(\theta_k + \chi_k)\). Notice that the potential at a place \((\lambda, \phi)\) is obtained by multiplying the real part of the Greenwich potential by \((G^{(m)}(\phi)/G)\cos m\lambda\) and the imaginary part by \(-(G^{(m)}(\phi)/G)\sin m\lambda\). Also notice that for the computation of the Greenwich potential at an instant \(t\), mean lunar time \(\tau\) in the Doodson arguments will have to be evaluated by (2.18).

### 2.8 Development of the tide-generating potential according to CTE

Doodson's harmonic development has for long been the most complete and precise computation of the tide-generating potential. To verify Doodson's results, Cartwright and Taylor [1971], Cartwright and Edden [1973], hereafter referred to as CTE, recalculated the potential by means of computer instead of by hand like Doodson did. With their expansion, the tide-generating potential is expressed as:

\[
U = \text{Re} \left[ g \sum_{l=2}^{\infty} \sum_{m=0}^{l} c_{lm}^*\delta(t)W_{lm}(\lambda, \phi) \right]
\]

where the asterisk denotes the complex conjugate. The \(c_{lm}^*\) correspond to the Greenwich equilibrium tide of degree \(l\) and order \(m\) and hence are in units of meters:

\[
c_{lm}^*(t) = a_{lm}(t) - ib_{lm}(t)
\]

where \(a_{lm}\) and \(b_{lm}\) are the real and imaginary part of \(c_{lm}\), respectively, which implicitly contain the nodal parameters [Munk and Cartwright, 1966]. The \(W_{lm}\) are complex spherical harmonics:

\[
W_{lm}(\lambda, \phi) = \tilde{P}_{lm}(\sin \phi)e^{im\lambda}
\]
with the normalization factor $N_{lm}$ of the associated Legendre functions $P_{lm}$ ($P_{lm} = N_{lm} P_{lm}$) according to:

$$P_{lm}(\sin \phi) = (-1)^m \sqrt{\frac{2l + 1}{4\pi} \frac{(l - m)!}{(l + m)!}} P_{ln}(\sin \phi)$$  (2.37)

In contrast with Doodson, who developed the Greenwich potential in harmonic functions, CTE preferred to directly compute the lunar and solar ephemerides $\lambda_d$, $\delta$, and $R_d/R_d$ [Cartwright, 1968]. With regard to Doodson’s expansion, CTE used a more precise calculation of the lunar and solar ephemerides as a result of which their harmonic coefficients vary slowly with time [Cartwright and Tayler, 1971; Cartwright and Edden, 1973]. Most importantly, they accounted for a decrease in the obliquity of the ecliptic $\epsilon$ due to planetary perturbations of 50” per century, which leads to a secular trend of about 0.01% a century in the larger harmonic coefficients [Cartwright and Tayler, 1971; Cartwright and Edden, 1973].

In CTE, a filtering technique is applied to separate the tide-generating potential into its harmonics for three epochs (1870, 1924, and 1960). Coefficient values were interpolated to 1900 to verify their results with those of Doodson who used the epoch of 1900, 1 January, 0 hr. UT, for the value of the obliquity of the ecliptic [Doodson, 1921]. They found that their results were in very good agreement with those of Doodson, and in general showed discrepancies of the order of 0.1% [Cartwright and Tayler, 1971; Cartwright and Edden, 1973].

It is important to realize that apart from the fact that the harmonic coefficients of CTE vary with time while those of Doodson remain constant, the only essential difference between the two expansions is a different normalization of these coefficients. With the expansion of CTE, the equilibrium amplitudes that can be derived from the $c_{lm}$ are normalized relative to the spherical harmonic $W_{lm}$. Hence, for a certain degree $l$, the normalization factor of the equilibrium amplitudes depends on the tidal species $m$, which is clearly different from the Doodson procedure, where all equilibrium amplitudes are normalized with respect to $G/g$. To obtain the relation between the equilibrium amplitudes of the CTE expansion and those of Doodson, we compare one line of the species $U_{2m}$ from (2.33) with the corresponding line in (2.34) using $l = 2$:

$$G^{(m)}(\phi)e^{im\lambda} \eta_k e^{i(\theta_k + \chi_k)} = g N_{2m} P_{2m}(\sin \phi)e^{im\lambda}(-1)^{m+\delta_{0m}} |B_k| e^{i(\theta_k + \chi_k)}$$

(2.38)

The $B_k$ are the equilibrium amplitudes as tabulated in Cartwright and Tayler [1971], Cartwright and Edden [1973]. The term $(-1)^{m+\delta_{0m}}$ is introduced to obtain the same sign for $N_{2m} P_{2m}(\sin \phi)(-1)^{m+\delta_{0m}}$ as with $G^{(m)}(\phi)$, i.e. to obtain harmonics in the CTE expansion with all positive amplitudes $|B_k|$ and cosine arguments like in Doodson’s expansion (2.33):

$$c_{2m}^* = (-1)^{m+\delta_{0m}} \sum_k |B_k| e^{i(\theta_k + \chi_k)}$$

(2.39)
<table>
<thead>
<tr>
<th>degree</th>
<th>species $m$</th>
<th>ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>0</td>
<td>$\sqrt{\frac{4\pi}{5}}$</td>
</tr>
<tr>
<td>2</td>
<td>1</td>
<td>$\sqrt{\frac{32\pi}{15}}$</td>
</tr>
<tr>
<td>2</td>
<td>2</td>
<td>$\sqrt{\frac{32\pi}{15}}$</td>
</tr>
</tbody>
</table>

Table 2.3 Ratio of the equilibrium amplitudes of Doodson's expansion to those of CTE for the main term ($l = 2$) of the tide-generating potential. The ratio is defined according to CTE/Doodson.

From (2.38), the following ratio of the CTE equilibrium amplitudes to those of Doodson is found for $l = 2$:

$$\frac{|B_k|}{|\eta_k| \cdot \frac{G}{\varrho}} = \frac{(-1)^{m+\delta_m} G^{(m)}(\phi)}{G N_{2m} P_{2m}(\sin \phi)}$$  \hspace{1cm} (2.40)

With $P_{2m}(\sin \phi)$ from (2.14), $G^{(m)}(\phi)$ from (2.21), and $N_{2m}$ from (2.37), the ratio (2.40) is detailed in Table 2.3.

In Table 2.2 of Section 2.7, the equilibrium amplitudes $|B_k|$ of the CTE expansion are listed for the major tides. Notice that with the amplitudes of the permanent tide, the $K_1$ tide, and the $K_2$ tide, the lunar and solar parts are added because the tables in CTE list the amplitudes according to frequency without mention of origin, i.e. lunar or solar. The equilibrium amplitudes in Table 2.2 were taken from Cartwright [1993] and refer to the epoch of 2000. Hence, when computing the ratio of the CTE amplitudes to the Doodson amplitudes (which refer to the epoch of 1900) in Table 2.2, small deviations from the ratios in Table 2.3 are found. These deviations are, however, not significant.
Chapter 3

Tidal analysis

3.1 Introduction

As will be shown in this chapter, the tidal equations, which prescribe the horizontal currents and associated tidal elevations of the water mass as a result of the tide-generating forces, behave linearly in the deep oceans, i.e. depths larger than about 200 m [e.g., Cartwright, 1993]. As a consequence, the ocean tide in the deep oceans may be analyzed with the same frequencies that are present in the tide-generating potential. This chapter, therefore, introduces the linear formulations of the harmonic and response tidal analyses, which will be used to analyze the ocean tides from TOPEX/POSEIDON, ERS-1, and GEOSAT altimetry in the deep oceans.

The aim of the harmonic method is to estimate the harmonic constants of a number of tides that are included in the analysis, whereas the response method [Munk and Cartwright, 1966] aims at estimating a weight function on the equilibrium tide in each tidal band through a number of so-called response weights. The Fourier transform of this weight function, which is known as the admittance [Munk and Cartwright, 1966], gives the ratio of the locally observed ocean tide to the equilibrium tide as a function of tidal frequency. If we assume a certain degree of smoothness of the admittance, i.e. if we adopt the “credo of smoothness” as posed by Munk and Cartwright [1966], then the response method offers some clear advantages over the harmonic method.

The first and most important advantage is that besides the major tides, a number of smaller tides can be derived from the admittance, provided that the admittance function is well defined by the dominant tides. These smaller tides can be hard to obtain from a harmonic analysis, where we directly have to deal with the tidal signal to noise problem [e.g., Andersen, 1995]. Secondly, as the response weights define most of the tidal spectrum, the smaller tides are obtained without estimating additional parameters. This is in contrast with the harmonic method, where we would have to estimate a number of additional harmonic constants. Thus, the response method can model a larger number of tides with a smaller number of parameters, which stabilizes the tidal solution [Munk and Cartwright, 1966].
Thirdly, by posing a smooth admittance, the response method eases (but
not solves) the problem of separating two tides with nearly the same fre-
quency. If there is a significant correlation between two tides, obtaining
reliable estimates of their harmonic constants by a harmonic analysis can
be very difficult.

A disadvantage of the response method is that the credo of smoothness
will, to a certain extent, remove the details from the admittance function.
This means that the minor tides inferred from the admittance may become
dependent on the dominant tides, i.e. they become too much of an inter-
polation. Still, unless we can separate the smaller tides from the dominant
tides, e.g. from long records of tide gauges readings, admittance interpo-
lation is probably the best we can do. This is especially true with altimetry,
from which no reliable estimates of the smaller tides may be expected be-
cause of the relatively large noise level.

3.2 Global ocean tide models

Because of the shortcomings of the equilibrium theory (Section 2.4), a rea-
listic global ocean tide model must be based either on precise and global
tide observations, or on a numerical solution of the Laplace tidal equa-
tions. The first type of models are called empirical because they fit a rep-
resentation of the ocean tide through tidal observations, mostly from al-
timeter satellites. The ocean tide models developed in this thesis belong
to this type and their solution method will be discussed in detail in Chap-
ter 5. The second type of models are called hydrodynamic models because
they aim to solve the oceanic tide from the hydrodynamic equations driven
by the total tide-generating potential, which includes, in addition to the
potential (2.15), the potentials of the solid-earth tide, the ocean’s self at-
traction, and the load tide, as will be explained in Section 3.3. Examples
of such models can be found in Schwiderski [1980], Schwiderski [1983], and
Le Provost et al. [1994]. Apart from the empirical and hydrodynamic mod-
els, a hybrid type of model exists based on the concept of assimilating tidal
data into an existing hydrodynamic model. An example of this kind of
models is FES95.2.1 (see Appendix B). Other examples are given in Zahel
[1991] and Eghert et al. [1994].

Each of the three types of models has its specific advantages and dis-
advantages. Comparing the hydrodynamic models (with or without data
assimilation) with the empirical models, a clear advantage of the former is
that they are truly global. Empirical models necessarily rely on the avail-
ability of tidal data. Even in case of altimeter observations, the data are
non-global as they are confined to the extreme latitudes that the satellites
can reach. However, because most of the oceans are between latitudes of
$-60^\circ$ and $60^\circ$, this is usually not a real handicap of the empirical mod-
els. Another advantage of the hydrodynamic models is the high resolu-
tion that they can obtain. This is important to model short-wavelength
tidal features (of the order of 10 km), which result from the interaction of the tide with the bottom topography and which are common in shallow seas [e.g., Pugh, 1987]. With the empirical models based on satellite altimetry, the resolution that can be obtained is usually much less. This is because most altimeter satellites have repeat periods of about a month or less, which leads to a relatively coarse spacing of the satellite ground-tracks. Consequently, the purely empirical models, i.e., models that fully rely on the data, have to be masked in shallow seas, i.e., for depths less than about 200 m [Cartwright, 1993]. Empirical models that take a hydrodynamic model as a-priori solution to the estimated parameters may give tidal predictions in shallow seas. However, the accuracy of these predictions depends on the errors in the a-priori model and there is a risk of absorbing these errors into the solution, as will be shown for the CSR3.0 model (see Appendix B) in Section 8.2.4. Moreover, the higher resolution of the hydrodynamic models does not necessarily have to lead to more accurate tidal predictions in shallow seas, because of a lack of accurate bottom topography models and insufficient knowledge of bottom friction modeling [Schwiderski, 1980; Le Provost et al., 1994; Kantha, 1995]. Especially bottom topography errors are a limiting factor, and not only in shallow seas, but also in the deeper parts of the oceans [e.g., Le Provost et al., 1994; Smith and Andersen, 1997]. Also, because of the computational constraint, global high-resolution hydrodynamic models are usually computed for separate basins [e.g., Le Provost et al., 1994]. The global model is obtained by specifying boundary conditions on the open ocean boundaries such that the basin solutions will optimally match [Le Provost et al., 1994]. This procedure is known to introduce errors along the basin boundaries which may be difficult to remove [Andersen et al., 1995; Smith and Andersen, 1997]. Because of the above difficulties to obtain a global hydrodynamic solution, it is therefore worth to try the purely empirical concept, especially from multi-satellite altimetry as pursued in this thesis.

3.3 The Laplace tidal equations

The Laplace tidal equations, as originally established by Marquis P.S. Laplace in 1775, prescribe the motion of the water mass (horizontal currents and associated tidal elevation) as a result of the tidal forces. In rotating, i.e., earth-fixed coordinates ($\lambda, \phi$), these equations read for the deep oceans [e.g., Hendershott, 1981; Gill, 1982]:

$$\frac{\partial u}{\partial t} - 2\Omega_e v \sin \phi = -\frac{1}{R_e \cos \phi} \frac{\partial}{\partial \lambda} [g \zeta_e - \Gamma]$$

$$\frac{\partial v}{\partial t} + 2\Omega_e u \sin \phi = -\frac{1}{R_e} \frac{\partial}{\partial \phi} [g \zeta_e - \Gamma]$$

$$\frac{\partial \zeta}{\partial t} + \frac{1}{R_e \cos \phi} \left[ \frac{\partial}{\partial \lambda} (H u) + \frac{\partial}{\partial \phi} (H v \cos \phi) \right] = 0 \quad (3.1)$$
Figure 3.1 Geometry of the geocentric tide $\zeta_c$ and the bottom tide $\zeta_b$. The ocean tide $\zeta$ is defined as: $\zeta = \zeta_c - \zeta_b$. Denoted by $h_b$ is the bathymetry, whereas $H$ is defined as: $H = h_b + \zeta_c - \zeta_b$.

with:

$$H(\lambda, \phi, t) = h_b(\lambda, \phi) + \zeta_c(\lambda, \phi, t) - \zeta_b(\lambda, \phi, t)$$  \hspace{1cm} (3.2)

In shallow seas, (3.1) may not be applied because of several effects that are related to the complex interaction of the tide with the bottom topography. These effects will be briefly discussed at the end of this section. In the above equations, $R_e$ is the earth's mean radius, $\Omega_e$ is the earth's sidereal rotation, $2\Omega_e \sin \phi$ is the Coriolis parameter, $u(\lambda, \phi, t)$ and $v(\lambda, \phi, t)$ are the east component and north component, respectively, of the depth-mean velocity (i.e. the tidal currents), $\Gamma$ is a short-hand notation for the total tide-generating potential or driving potential at the earth’s surface, $H(\lambda, \phi, t)$ is the ocean depth, $\zeta_c$ is the ocean surface’s tidal elevation (geocentric tide as measured by a satellite altimeter), $\zeta_b$ is the tidal elevation of the deformable ocean floor (bottom tide) over its time-averaged value ($z = -h_b$ in Figure 3.1 where $h_b$ denotes the bathymetry), and $\zeta$ is the tidal elevation of the ocean surface relative to the deformable sea floor (ocean tide as measured by a tide gauge) also over its time-averaged value ($z = 0$, i.e. the mean sea level in Figure 3.1). Notice that between $\zeta_c$, $\zeta$, and $\zeta_b$, the following relation must hold:

$$\zeta_c = \zeta + \zeta_b$$  \hspace{1cm} (3.3)

The first two equations of (3.1) are the horizontal momentum equations. They give the horizontal acceleration of the water mass on a rotating earth as a consequence of the tidal disturbance $\Gamma$ [e.g., Hendershott, 1981; Gill, 1982]. The vertical acceleration due to this disturbance can be neglected with respect to gravity as explained in Section 2.2. The third equation of (3.1) is the mass continuity equation, which shows that a net flux of water into or out of an area of height $H$ must be balanced by a corresponding
3.3 The Laplace tidal equations

change in water level [e.g., Pugh, 1987].

Equations (3.1) are numerically solved for \((u, v, \zeta)\) as driven by the dominant tides in the tide-generating potential. The solution of the tidal currents has to satisfy the condition that there can be no fluid flow across land boundaries whereas tidal data (usually harmonic constants derived from tide gauge measurements or from existing models) are used to specify conditions for the ocean tide solution at open ocean boundaries. Because the tidal currents in (3.1) are assumed to be independent of depth, which is valid except in very shallow waters [Gill, 1982], the solution for \(\zeta\) will give the barotropic ocean tide. The less important and spatially more irregular baroclinic or internal tide with amplitudes of about 2 cm for the dominant \(M_2\) tide [Ray and Mitchum, 1996] and length scales of tens of kilometers [Pugh, 1987] is not included in the solution. This tide is generated when the barotropic tide is impeded by large topographic features such as shelves or seamounts [Gill, 1982; Ray and Mitchum, 1996].

An expression for \(g\zeta_c - \Gamma\) to be inserted in (3.1) can be obtained as follows. As explained in Chapter 2, the tide-generating potential of the sun or the moon at the earth's surface is \(U_2\). For the earth's crust, the effect of \(U_2\) is that it is raised by an amount of \(\zeta_c\), the solid-earth or body tide. Assuming an elastic earth, which is a reasonable assumption because the resonance periods of the earth are an order of magnitude smaller than the tidal forcing periods so that the deformations are effectively instantaneous [e.g., Melchior, 1966; Munk and MacDonald, 1975; Lambeck, 1988], the equilibrium deformation of the solid earth is:

\[
\zeta_c = \frac{h_2 U_2}{g} 
\]  
(3.4)

The displacement \(\zeta_c\) causes a redistribution of mass under the crust and so an additional potential at the earth's surface of \(\Delta U_2\), the solid-earth potential:

\[
\Delta U_2 = k_2 U_2 
\]  
(3.5)

In (3.4) and (3.5), \(h_2\) and \(k_2\) are the second-degree Love numbers, which may be assumed independent of frequency [e.g., Melchior, 1966; Munk and MacDonald, 1975; Moritz and Müller, 1988].

Apart from the deformation and additional potential of the solid earth, there is the effect from elastic loading of the crust by the ocean tide itself. Integrated over the oceans, the effect of the ocean tidal elevations above mean sea level is to raise the potential at a location \((\lambda, \phi)\) by \(U_\zeta\), the ocean tide potential (also called ocean self attraction):

\[
U_\zeta = g \sum_l \alpha_l \zeta_l 
\]  
(3.6)

where \(\zeta_l\) is the \(l\)th-degree spherical harmonic of the ocean tide (because the harmonic constants of a tide are a function of \((\lambda, \phi)\) they can be decom-
posed into spherical harmonics). The \( \alpha_l \) are defined as:

\[
\alpha_l = \frac{3}{2l+1} \rho_o
\]

where \( \rho_o \) and \( \rho_e \) are the mean densities of the ocean and the earth, respectively [e.g., Hendershott, 1981]. A summation over the same tidal frequencies that are input to \( U_2 \) yields \( \zeta_l \). The combined effect of the potential \( U_\zeta \) and the tidal column of weight at \((\lambda, \phi)\) is to cause a deformation of the solid earth \( \zeta_{ol} \), the load tide:

\[
\zeta_{ol} = \sum_l h'_l \alpha_l \zeta_l
\]

(3.7)

and an additional potential arising from this displacement called the loading potential:

\[
\Delta U_\zeta = g \sum_l k'_l \alpha_l \zeta_l
\]

(3.8)

The primed numbers \( k'_l \) and \( h'_l \) are the loading Love numbers of degree \( l \). They differ from \( k_l \) and \( h_l \) in the sense that they do not only account for the uplift of the crust in \( \zeta_{ol} \) due to the ocean's self attraction \( U_\zeta \), but also for the depression of the crust caused by the tidal column of weight at \((\lambda, \phi)\). With (3.5), (3.6), and (3.8), the total tide-generating potential \( \Gamma \) becomes:

\[
\Gamma = U_2 + \Delta U_2 + U_\zeta + \Delta U_\zeta
\]

\[
= (1 + k_2) U_2 + g \sum_l (1 + k'_l) \alpha_l \zeta_l
\]

(3.9)

whereas the bottom tide becomes with (3.4) and (3.7):

\[
\zeta_b = \zeta_e + \zeta_{ol}
\]

\[
= \frac{h_2 U_2}{g} + \sum_l h'_l \alpha_l \zeta_l
\]

(3.10)

Hence, the term \( g \zeta_e - \Gamma \) in (3.1) may be written as:

\[
g \zeta_e - \Gamma = g \zeta + (g \zeta_e - U_2 - \Delta U_2) + (g \zeta_{ol} - U_\zeta - \Delta U_\zeta)
\]

\[
= g \zeta - (1 + k_2 - h_2) U_2 - g \sum_l (1 + k'_l - h'_l) \alpha_l \zeta_l
\]

(3.11)

Interesting to notice from (3.11) is that the expression \( g \zeta_{ol} - (U_\zeta + \Delta U_\zeta) \) depends on the solution of the ocean tide \( \zeta \). Hence, Equations (3.1) have to be solved iteratively. As described in Estes [1980], the procedure is to first generate a zeroth-order solution for \( \zeta \) from (3.1) by neglecting the effects of ocean loading and self attraction, i.e. \( g \zeta_{ol} - (U_\zeta + \Delta U_\zeta) \), in (3.11). Then, \( g \zeta_{ol} - (U_\zeta + \Delta U_\zeta) \) is calculated for the zeroth-order solution of \( \zeta \). A first-order solution for \( \zeta \) is then obtained from (3.1) using \( g \zeta_{ol} - (U_\zeta + \Delta U_\zeta) \), and so on. Because of the slow convergence of the series in (3.11), the series expansion has to be carried out to a quite high degree, typically up to 180 [Estes, 1980; Ray and Sanchez, 1989]. For this reason, alternative (although less accurate) procedures to solve (3.1) have been followed. One possibility is to derive global maps, in terms of tidal harmonic constants, for \( U_\zeta + \Delta U_\zeta \) and \( g \zeta_{ol} \) using the spherical harmonics decomposition of an
existing ocean tide model [e.g., Francis and Mazzega, 1990]. Such a procedure was e.g. followed by Le Provost et al. [1994] for the FES94.1 model. Another possibility is to assume that the ocean tide and load tide are in phase, which results in [e.g., Francis and Mazzega, 1990]:

$$U_\zeta + \Delta U_\zeta - g\zeta_{ol} = 0.1g\zeta$$

(3.12)

with which the ocean tide $\zeta$ becomes the only unknown in (3.1). Equation (3.12) was originally suggested by C. L. Pekeris in 1977 [Schwiderski, 1980], while Schwiderski [1983] proposed a similar expression for the load tide itself:

$$\zeta_{ol} = -0.07\zeta$$

(3.13)

Equation (3.13) has become known as the 7% rule.

An important observation in (3.1) is that for the deep oceans, the ocean tide $\zeta = \zeta_o - \zeta_b$ in the mass continuity equation can be neglected with respect to the bathymetry $h_b$, whereas bottom friction and advection, which are both not included in (3.1), may also be ignored [e.g., Cartwright, 1993]. As a consequence, the tidal equations become linear in $u$, $v$, and $\zeta$, which means that the solution $(u, v, \zeta)$ will have the same frequencies that are present in the tide-generating potential $U_2$. In shallow waters, the tidal equations take their non-linear form, mainly due to the finite water depth, bottom friction, and advection [e.g., Gill, 1982; Pugh, 1987]. These non-linear effects cannot be modeled with one single harmonic and the solution has to permit non-linear terms, which results in higher tidal harmonics (and currents) with frequencies that are multiples, sums, or differences of the frequencies in the tide-generating potential [e.g., Pugh, 1987].

Whenever the tidal elevation $\zeta$ in the continuity equation cannot be neglected with respect to the bathymetry, the solution $(u, v, \zeta)$ becomes non-linear, which is caused by the terms $Hu$ and $Hv$ with $H =ay + \zeta$ [e.g., Pugh, 1987].

Bottom friction is accounted for by adding friction terms to the momentum equations, which usually are non-linear in $u$ and $v$ and inversely proportional with $H$ [e.g., Cartwright, 1993]. An important aspect of friction is that it removes energy from the tidal wave by resisting the currents and reducing the amplitudes. This dissipation of energy is the reason why tidal amplitudes cannot become infinitely large, which is part of the physical justification of posing the concept of a smooth admittance (Section 3.5).

Advection expresses the turbulence in the tidal currents and may be regarded as perturbations on the velocity field $(u, v)$ [e.g., Pond and Pickard, 1983]. In the deep oceans, advection has a second-order effect on the solution $(u, v, \zeta)$ so that it is usually neglected [e.g., Lisitzin, 1974; Cartwright, 1993]. In shallow seas, advection may be of importance and can be accounted for by adding non-linear terms to (3.1) that are products of the
velocity field and the derivatives of this field with respect to $\lambda$ and $\phi$ [e.g., Pond and Pickard, 1983; Apel, 1987].

3.4 Harmonic analysis

Because of the dominant linearity of the Laplace tidal equations, the observed ocean tide in the deep oceans may be analyzed according to [e.g., Schwiderski, 1980, 1983; Pugh, 1987]:

$$\zeta = \sum_k f_k H_k(\lambda, \phi) \cos(\Theta_k + \chi_k + u_k - G_k(\lambda, \phi))$$  (3.14)

where the summation over $k$ includes all the tides that we want to estimate. The $H_k$ and $G_k$ represent, respectively, an unknown amplitude and phase lag, which both vary with position $(\lambda, \phi)$. The other symbols in (3.14) have been introduced earlier. The aim of the harmonic analysis is to determine the harmonic constants $H_k$ and $G_k$ through a least-squares estimation procedure. For this purpose, (3.14) is written as:

$$\zeta = \sum_k f_k C_k(\lambda, \phi) \cos(\Theta_k + \chi_k + u_k) + f_k S_k(\lambda, \phi) \sin(\Theta_k + \chi_k + u_k)$$  (3.15)

from which the harmonic cosine and sine constants, $C_k$ and $S_k$ respectively, are estimated. Their relation to the amplitude and phase lag is given by:

$$H_k = \sqrt{C_k^2 + S_k^2}$$
$$G_k = \arctan \left( \frac{S_k}{C_k} \right)$$  (3.16)

Notice that because $\Theta_k$ in (3.14) is the astronomical argument at Greenwich, $G_k$ is a phase lag on the Greenwich equilibrium tide, i.e. the Greenwich phase lag. The Greenwich phase lag expresses the time interval between high water of the observed tide of frequency $\Theta_k$ at a place $(\lambda, \phi)$ and high water of the Greenwich equilibrium tide [e.g., Schureman, 1971]. For instance, in case of $S_2$, one hour corresponds to 30°. Hence, at those places where $G_{S_2} = 30^\circ$, it takes one hour until high tide after the $S_2$ equilibrium tide crests over Greenwich.

3.5 Response analysis

With the response analysis method, it is assumed that the ocean tide can be expressed by the convolution of the equilibrium tide and a weight function [Munk and Cartwright, 1966]. Considering only the main tide ($l = 2$), the relation between the ocean tide $\zeta$, the equilibrium tide $c_{zm}$, and the weight
3.5 Response analysis

function $w_{2m}$ is given by the convolution [Munk and Cartwright, 1966]:

$$\zeta = Re \left[ \sum_{m=0}^{2} c_{2m}^*(t) * w_{2m}(\lambda, \phi, t) \right] = Re \left[ \sum_{m=0}^{2} \int_{-\infty}^{\infty} c_{2m}^*(t - \tau) w_{2m}(\lambda, \phi, \tau) d\tau \right]$$

(3.17)

The convolution (3.17) defines the linear regime of the tide [Munk and Cartwright, 1966] and hence should only be applied in the deep oceans. In shallow waters, non-linear terms should be added to (3.17) as described in Munk and Cartwright [1966]. These terms will not be discussed here.

For each tidal band $m$, the weight function $w_{2m}$ in (3.17) is defined as:

$$w_{2m}(\lambda, \phi, t) = \sum_{s=-S}^{S} w_{2ms}(\lambda, \phi) \delta(t - s\Delta T)$$

(3.18)

where $\Delta T$ is a suitably chosen lag interval on the equilibrium tide [Munk and Cartwright, 1966], and $\delta(t)$ is the unit impulse. The weights $w_{2ms}$ are called response weights [Munk and Cartwright, 1966] and must be obtained through least-squares estimation. Like the harmonic constants, the estimated weights $w_{2ms}$ depend on position $(\lambda, \phi)$. The physical meaning of the response weights is that they represent the remaining effect at time $t$ of the ocean’s response to a unit impulse at time $t - s\Delta T$, which explains the name “response” analysis [Munk and Cartwright, 1966]. The operational definition of the unit impulse is that it sifts the value of an arbitrary signal at time $t$, i.e. $x(t)$, under an integral of time, i.e. $x(t) = \int_{-\infty}^{\infty} x(\tau) \delta(t - \tau) d\tau$ [e.g., Oppenheim et al., 1983]. For the response method, this means that the ocean tide can be expressed as a weighted sum of past and future values of the equilibrium tide, which can be seen by substituting (3.18) in (3.17):

$$\zeta = Re \left[ \sum_{m=0}^{2} \sum_{s=-S}^{S} w_{2ms}(\lambda, \phi) c_{2m}^*(t - s\Delta T) \right]$$

(3.19)

The use of future values of the equilibrium tide (negative $s$) cannot be physically justified and only has some mathematical advantage, e.g. in the concept of orthotides [Groves and Reynolds, 1975], which seems to be the preferred formulation of the response analysis [e.g., Cartwright and Ray, 1990b; Ma et al., 1994; Desai and Wahr, 1995]. However, the ordinary response formulation (3.19) gives exactly the same results for the ocean tide as the orthotides formulation [Alecock and Cartwright, 1978] as will be explained in detail in Section 7.6. For this reason, (3.19) will be used throughout this thesis. With:

$$w_{2ms}(\lambda, \phi) = u_{2ms}(\lambda, \phi) + iv_{2ms}(\lambda, \phi)$$

(3.20)

(3.19) may also be written as:

$$\zeta = \sum_{m=0}^{2} \sum_{s=-S}^{S} u_{2ms}(\lambda, \phi) a_{2m}(t - s\Delta T) + v_{2ms}(\lambda, \phi) b_{2m}(t - s\Delta T)$$

(3.21)
which gives the linear least-squares formulation from which the complex weights \( w_{2m} \) are estimated. Nodal parameters do not appear in the response formulation (3.21) as they are implicitly contained in the \( c_{2m} \).

The Fourier transform of the weight function (3.18) is called the admittance \( Z_{2m}(\hat{\Theta}_k) \) [Munk and Cartwright, 1966]:

\[
Z_{2m}(\hat{\Theta}_k) = X_{2m}(\hat{\Theta}_k) + iY_{2m}(\hat{\Theta}_k) = \int_{-\infty}^{\infty} w_{2m}(t) e^{-i\hat{\Theta}_k t} dt
\]

(3.22)

Substituting \( w_{2m} \) from (3.18) in (3.22), it follows for the real and imaginary parts of the admittance, i.e. \( X_{2m} \) and \( Y_{2m} \), respectively:

\[
X_{2m}(\hat{\Theta}_k) = \sum_{s=-S}^{S} v_{2ms} \cos(\hat{\Theta}_k s \Delta T) + v_{2ms} \sin(\hat{\Theta}_k s \Delta T)
\]

\[
Y_{2m}(\hat{\Theta}_k) = \sum_{s=-S}^{S} v_{2ms} \cos(\hat{\Theta}_k s \Delta T) - v_{2ms} \sin(\hat{\Theta}_k s \Delta T)
\]

(3.23)

For each tidal band \( m \) one finds one complex admittance defined by the set of weights (3.20) of that species. Hence, we speak of the long-period admittance, the diurnal admittance, and the semi-diurnal admittance, \( Z_{20} \), \( Z_{21} \), and \( Z_{22} \), respectively. Similar to the harmonic analysis, the potential \( c_{2m} \) is evaluated at Greenwich, wherefore the admittances (3.23) are called Greenwich admittances [Cartwright and Ray, 1991], which can be directly related to the harmonic constants (3.16) as will be discussed in the next section.

From (3.21) it is observed that the response formalism depends on the choice of the lag interval \( \Delta T \) and the number of lags (index \( s \)), which will be denoted by \( L \). Once it has been decided what values these parameters should be assigned, the admittance in each tidal band is determined. The selected values of \( \Delta T \) and \( L \) that are used in this thesis are mentioned and discussed in Chapter 7. With the number of lags, the selected value has to fulfill the so-called "credo of smoothness", which states that the admittance must display a certain degree of smoothness [Munk and Cartwright, 1966]. The smoothness imposed on the oceanic response is justified by the observation that responses to tidal forcing at adjacent frequencies within a tidal band are nearly the same. This is especially true in the deep oceans where the main diurnal and semi-diurnal tides exhibit correlations of more than 30% for a grid size of about 10° [Mazzega, 1988]. In shallow seas, where near-resonance conditions may play a role, the tidal amplitudes cannot grow too large due to the effect of bottom friction [Munk and Cartwright, 1966; Pugh, 1987]. It should be mentioned that the idea of a smooth admittance strictly applies to the luni-solar gravitational tidal forcing of the ocean [Munk and Cartwright, 1966]. However, the ocean is also being forced by atmospheric pressure changes, mainly at the \( S_2 \) frequency [Chapman and Lindzen, 1970]. The sea level variations due to the
S2 atmospheric component are usually referred to as the radiational tide [Munk and Cartwright, 1966]. Because of the radiational tide, the admittance will contain an anomaly at the S2 frequency [Munk and Cartwright, 1966; Cartwright and Ray, 1994]. This anomaly shows up as a discontinuity in the otherwise smooth admittance (i.e. smooth from gravitational forcing alone), which has been frequently observed in the harmonic constants of surface tide gauges [Zetler, 1971; Cartwright and Ray, 1994]. The harmonic constants of bottom pressure gauges do not contain the radiational tide as they measure absolute pressure on the sea bed [Cartwright and Ray, 1994; Shum et al., 1997]. The response solutions developed in this thesis, however, will not show the S2 discontinuity. First of all, to detect this discontinuity requires a very accurate estimate of K2, which obviously may not be expected from altimetry because of the relatively large noise level in the altimeter observations compared to the magnitude of this tide. Secondly, because the response formalism (3.21) enforces a smooth admittance, it will remove any sharp discontinuity at the S2 frequency to the expense of an error in the neighboring weaker tides, mainly K2 [Ray, 1996]. In Cartwright and Ray [1994], it is demonstrated that this error in K2 leads to an error in sea level predictions with an rms of only 3 mm. A modified response analysis that accounts for the S2 anomaly as proposed by Cartwright and Ray [1994] has therefore not been adopted.

3.6 Relation between harmonic constants and admittance parameters

Although the admittance (3.23) and the harmonic constants (3.16) give a completely equivalent description of the ocean tide of frequency Θk, i.e. ζk, the latter are the traditional representation of the tide. Hence, it is desired to obtain an expression that transforms the admittance to harmonic constants.

From Section 2.8, one harmonic term c_k^* in (2.39) may be written as:

\[
    c_k^* = |B_k| e^{i(\Theta_k + \chi_k + [m+\delta_m] \pi)}
\]

(3.24)

where the \((-1)^{m+\delta_m}\) has been included in the exponent. The relation between the harmonic constants and admittance parameters at a certain frequency \(\dot{\Theta}_k\) may be found by substituting (3.24) in (3.19) from which we easily derive with \(\Theta_k(t - s\Delta T) = \Theta_k(t) - \dot{\Theta}_k s \Delta T\) (see (2.29)):

\[
    \zeta_k = \Re \left[ |B_k| e^{i(\Theta_k + \chi_k + [m+\delta_m] \pi)} \left\{ \left( \sum_{s=-S}^{S} u_{2ms} \cos(\dot{\Theta}_k s \Delta T) + v_{2ms} \sin(\dot{\Theta}_k s \Delta T) \right) \right. \right.
\]

\[
    + \left. \left. i \left( \sum_{s=-S}^{S} v_{2ms} \cos(\dot{\Theta}_k s \Delta T) - u_{2ms} \sin(\dot{\Theta}_k s \Delta T) \right) \right) \right] \right]
\]

(3.25)
Comparing (3.25) with (3.23), it is noticed that the term between curled brackets is the complex admittance \( Z_{2m}(\hat{\Theta}_k) \) so that:

\[
\zeta_k = \text{Re} \left[ |B_k| e^{i(\Theta_k + \chi_k + [m + \delta_0m]n)} \cdot Z_{2m}(\hat{\Theta}_k) \right]
\]  

(3.26)

If we write \( Z_{2m} = |Z_{2m}| e^{i\Omega_{2m}} \) with:

\[
|Z_{2m}| = \sqrt{X_{2m}^2 + Y_{2m}^2}
\]

\[
\Omega_{2m} = \arctan \left( \frac{Y_{2m}}{X_{2m}} \right)
\]

(3.27)

then (3.26) turns into:

\[
\zeta_k = \text{Re}[|B_k| \cdot |Z_{2m}| e^{i(\Theta_k + \chi_k + [m + \delta_0m]n + \Omega_{2m})}]
\]

(3.28)

Writing one tidal term (i.e. no nodal parameters applied) from the harmonic analysis (3.14) as:

\[
\zeta_k = \text{Re}[H_k e^{i(\Theta_k + \chi_k - G_k)}]
\]

(3.29)

and comparing (3.28) and (3.29), the relation between the constants of the harmonic analysis and the admittance parameters of the response analysis is found to be (see also Cartwright and Ray [1990b]):

\[
H_k = |B_k| \cdot |Z_{2m}(\hat{\Theta}_k)|
\]

\[
G_k = [m + \delta_0m]n - \Omega_{2m}(\hat{\Theta}_k)
\]

(3.30)

where \([m + \delta_0m]\) times \(2\pi\) have been added to the phase lag. In terms of the cosine and sine constants \(C_k\) and \(S_k\) in (3.16), the above relation yields:

\[
C_k = (-1)^{m + \delta_0m} |B_k| X_{2m}(\hat{\Theta}_k)
\]

\[
S_k = (-1)^{m + \delta_0m} |B_k| Y_{2m}(\hat{\Theta}_k)
\]

(3.31)

which may be verified with Cartwright and Ray [1990b], Cartwright and Ray [1991], and Desai and Wahr [1995]. Equation (3.31) illustrates what was mentioned in Section 3.1, namely that the admittance is the ratio of the observed tide to the equilibrium tide. Hence, representing a tide by its harmonic constants or admittance parameters is fully equivalent because the \((C_k, S_k)\) and \((X_{2m}(\hat{\Theta}_k), Y_{2m}(\hat{\Theta}_k))\) are related by a simple scaling factor \(\pm(-1)^{m + \delta_0m} |B_k|\).
Chapter 4

Satellite altimetry in ocean tide modeling

4.1 Introduction

Because of the sparseness of global tide gauge networks, satellites have been used for many years to measure the global ocean tide. The tidal measurements of these satellites can be both direct and indirect. The first type of measurements are provided by altimeter satellites which measure the sea level variations of the geocentric tide. The harmonic constants or response weights of the ocean+load tide, i.e. the vector sum of the ocean and load tides, can be obtained by subtracting the tide of the solid earth from the measurements. The second type of measurements concern the tidal perturbations of the orbits of low-earth satellites as measured by several tracking systems, such as satellite laser ranging or radio-frequency tracking [Casotto, 1989; Colombo, 1984; Christodoulidis, 1978]. Because the motion of the satellite is affected by the ocean+load tide and the solid-earth tide through the potentials $U_\zeta + \Delta U_\zeta$ and $\Delta U_2$, respectively (Section 3.3), we may model $\Delta U_2$ and estimate the spherical harmonic coefficients of the ocean tides (see (3.6) and (3.8)). For each tide, the spherical harmonic coefficients may then be transformed to the harmonic constants $H_k$ and $G_k$ as explained in Ray and Sanchez [1989]. However, obtaining an ocean tide model from satellite orbit perturbations is quite complicated, mainly because of a strong correlation between the harmonic coefficients of the ocean tides and those of the earth’s gravity field for the lower degrees [Christodoulidis et al., 1988; Nerem et al., 1994a]. Hence, much better results have been obtained so far with the direct measurement of the tides through satellite altimetry. Ideally, improved estimates of the tidal harmonic constants should be used to improve the dynamic force modeling of the altimeter satellite’s orbit and so to reduce the orbit error and thus the altimeter measurement background noise. These orbits may then be used for a better estimate of the ocean+load tide as observed by the altimeter. Hence, the significantly improved TOPEX/POSEIDON post-launch tidal models should serve as a first step to iteratively obtain more accurate so-
olutions of the ocean tide.

Geophysical satellites carrying altimeters are often launched into so-called repeat orbits. For these orbits, the satellite's groundtrack repeats itself after a certain time interval, which is called the repeat period. With a repeat orbit, tidal samples at a fixed place in the ocean are obtained once every repeat period. Because the repeat periods of most altimeter satellites are of the order of ten days to a month (see Appendix A for a brief description of the GEOSAT, ERS-1, and TOPEX/POSEIDON satellite missions), the diurnal and semi-diurnal ocean tides as observed by these satellites are aliased to periods much longer than a day or half a day. Of main concern is the question of how to separate two tides that alias to nearly the same period. Besides on the constant sampling interval, the possibility to separate such tides also depends on the change of tidal phase on adjacent and crossing satellite groundtracks around the place where a tidal solution is sought. In Section 4.5, it will be discussed that tidal decorrelation from adjacent and crossing groundtracks is very important if we have less than a few years of ERS-1 and GEOSAT altimetry. Therefore, a great deal of this chapter is devoted to an investigation of the ability of especially crossing tracks to decorrelate two tides with nearly the same alias period. These investigations will concentrate on the five dominant tides, $M_2$, $S_2$, $N_2$, $K_1$, and $O_1$, because these largely determine the diurnal and semi-diurnal admittances, as will be discussed in Chapter 7.

### 4.2 The sea surface height measurement

The purpose of a satellite altimeter is to measure the height between the satellite and the sea level. This is done by measuring the travel time of a radar pulse transmitted by the altimeter antenna and reflected by the ocean surface. Dividing this travel time by two and using the speed of light in vacuum, the sea level as defined by the altimeter is obtained. Notice that the height as measured by the altimeter is actually not a single height measurement but the average over an area underneath the satellite, the so-called altimeter footprint, which has a radius of several kilometers. Also, a number of typically ten or twenty of these area-averaged heights are obtained per second, which are then time-averaged into one-per-second altimeter heights. Usually, only these one-per-second altimeter heights are provided to the user community. In a simplified form, the altimeter measurement concept is depicted on the left side of Figure 4.1. The symbols referring to the tide gauge on the right will be treated in Section 4.3.

For ocean studies, the quantity of interest is the sea surface height $ssh$ above the geoid. The sea surface height contains a number of components caused by ocean currents, by the bottom and ocean tides, and by the sea surface response to atmospheric pressure forcing. Hence, the $ssh$ measurements may be used to derive a model for each of these components by
Figure 4.1 Concepts of tidal measurements from satellite altimetry and tide gauge stations. Denoted by $\zeta_b$ and $\zeta_c$ are the bottom tide and the geocentric tide, respectively. For an explanation of the other symbols, the text may be referred to.
supplying reference models for the others. If a reference model is supplied to remove one of the contributions to the sea surface heights, it is appropriate to speak of a sea surface height residual $\Delta ssh$. For example, in this thesis, reference models have been applied to remove the solid-earth tide $\zeta_e$, the pole tide $\zeta_p$, and the atmospheric pressure effects (assuming an inverse barometer (IB) response) from the ssh measurements:

$$\Delta ssh = h_{orb} - h_{alt} - h_g - \zeta_e - \zeta_p - IB = \zeta + \zeta_{ol} + \zeta_{sa} + \zeta_{sa} + h_0 + \Delta h_g + \varepsilon \quad (4.1)$$

Besides the ocean tide $\zeta$ and the load tide $\zeta_{ol}$, we notice from Figure 4.1 that $h_{orb}$ is the orbital height above a reference ellipsoid, $h_{alt}$ is the measured altimeter height, $h_g$ is the geoid height, IB is the inverse barometer correction, $\zeta_{sa}$ and $\zeta_{sa}$ are the annual and semi-annual cycles of ocean variability, $h_0$ is the quasi-stationary dynamic topography, $\Delta h_g$ denotes geoid errors, and $\varepsilon$ denotes the altimeter background noise, which consists of unmodeled effects, e.g. remaining ocean variability, as well as orbit errors and altimeter height measurement correction errors, e.g. atmospheric path delay correction errors, and errors from making an IB assumption. Following below is a *brief* description of each of the terms in (4.1) with exception of the tidal terms $\zeta_e$, $\zeta_p$, and $\zeta_{ol}$, which have been discussed in detail in the previous chapters, and of the geoid, which has been introduced in Section 2.7. A *detailed* description of the models that were used to derive the terms on the left side of (4.1) can be found in Section 5.2, whereas a description of the parameters that were estimated to model the right side of (4.1) can be found in Section 5.4.

In Figure 4.1, the orbital height $h_{orb}$ has to follow from a precise orbit computation above a reference ellipsoid. The orbital height must be corrected for the offset of the altimeter antenna with respect to the spacecraft’s center of mass.

The altimeter measurements $h_{alt}$ are assumed to be corrected for atmospheric path delays in the troposphere and ionosphere. Also, certain biases must be taken into account that cause the sea level as measured by the altimeter to differ from the actual sea level. These biases include the sea state bias and the altimeter bias.

The sea state bias represents the effect of ocean waves on the measured altimeter height. Thus, it can be modeled in terms of the significant wave height (SWH) in the altimeter footprint, and the wind speed at the ocean surface [Gaspar et al., 1994]. Sometimes, more simply, the sea state bias is given as a percentage of the SWH. In that case, the percentage is of the order of 1-5% [Gaspar et al., 1994]. Notice that the algorithm to compute the sea state bias is altimeter-specific. Hence, even the TOPEX and POSEIDON altimeters, although onboard of the same satellite, have their own sea state bias algorithm. With the sea state bias, the convention is that a positive value means that the measured sea level is below the actual sea level (Figure 4.2).
4.2 The sea surface height measurement

![Diagram showing measured sea level, oscillator drift, actual sea level, sea state bias, altimeter bias & drift, and measured sea level.]

Figure 4.2 Sign convention of the sea state bias, altimeter bias, and oscillator drift. Arrows indicate the positive direction of the bias and drift.

Apart from the sea state bias, the altimeter instrument itself may introduce a difference between the actual and the measured sea level. To calibrate the altimeter instrument, the time delay of the transmitted pulse in the altimeter electronics is measured on ground. When expressed as a range, this internal delay can amount to several kilometers and hence is applied as an a-priori correction to the measured range before the altimeter data are released. However, on-ground calibration of the altimeter instrument is usually not very accurate and a substantial error can be made. This error is called the altimeter bias, which can be resolved from an in-orbit calibration to about 2-5 cm precision [Francis, 1992; Christensen et al., 1994; Ménard et al., 1994]. By convention, a positive value of the altimeter bias indicates that the altimeter measures too large a range, i.e. the measured sea level is below the actual sea level (Figure 4.2). Due to degradation of the instrument, the altimeter bias may show a drift from its value as obtained from the in-orbit calibration. However, most altimeter instruments monitor their bias drift and correct for it by an internal calibration loop so that it does not affect the altimeter measurements.

In addition to the sea state bias and the altimeter bias, a drift in the onboard oscillator may cause the measured altimeter range to become shorter with time [Hancock III and Hayne, 1996]. Hence, if no correction would be made for the oscillator drift, the sea level would show an artificial rise with time (Figure 4.2).

Ocean currents cause an upset of oceanic water above the geoid [e.g., Apel, 1987]. The long-term average of this upset is called the (quasi-stationary) dynamic topography $h_o$, while the sum of the geoid and the dynamic topography is usually referred to as the mean sea level. Changes in ocean temperature and winds cause sea level variations about the mean sea level known as ocean variability [Pugh, 1987; Apel, 1987]. Some interesting examples of ocean variability are the seasonal cycles, which mainly
occur at the annual and semi-annual frequency, i.e. $\zeta_{S_a}$ and $\zeta_{S_{sa}}$, respectively. Although not strictly seasonal, these cycles in the equatorial Pacific regularly lead to the development of the ocean-atmospheric El Niño Southern Oscillation (ENSO) event, which is in some vital way related to the transfer of heat between the oceans and the atmosphere [Diaz and Markgraf, 1992]. Notice that the seasonal cycles cannot be distinguished from the much smaller $S_a$ and $S_{sa}$ tides, which occur at the same frequencies [e.g., Tsimplis and Woodworth, 1994].

As an effect of slight variations in the orientation of the earth's rotation axis relative to the crust, the pole tide $\zeta_p$ is raised in the oceans, which can be modeled as a function of the position of the earth's instantaneous rotation axis in the earth-fixed reference frame [Munk and Cartwright, 1966; Lambeck, 1988]. The pole tide has a dominant 14-month period (Chandler wobble) and thus may be assumed to follow an equilibrium response. The amplitude of the pole tide is quite small compared to that of the ocean tide and the solid-earth tide, i.e. approximately 0.5 cm [Munk and Cartwright, 1966; Lambeck, 1988].

The sea surface response to atmospheric forcing is not yet completely understood so that it usually is assumed to follow an inverse barometer response. Hence, it is supposed that a local increase of atmospheric pressure of 1 mbar causes a depression of the ocean surface of approximately 1 cm.

4.3 Satellite altimetry vs. tide gauges

The right side of Figure 4.1 shows the classical tide gauge, a tide pole mounted on the sea bed or, more usually, attached to a vertical structure in the sea bed. Although nowadays most of these poles have been replaced by more modern bottom pressure or acoustic measuring systems, the concept of any tide gauge is still in many ways the same as that of the tide pole. In Figure 4.1, the zero marker of the tide gauge gives the height of the local mean sea level while the sea level variations with respect to this marker are due to the tide, ocean variability, atmospheric forcing of the sea surface, and surface waves. As the tide gauge is attached to the sea bed it will only observe the tidal displacements of sea level with respect to the moving ocean floor, i.e. the pure oceanic tide $\zeta$. Because the ocean tidal signal has a predictable rhythm, the larger tides can be easily distinguished from sea level changes associated with ocean variability. The effect of surface waves on the readings are usually removed by some kind of damping procedure although some gauges give both the ocean tide and the wave height [Pugh, 1987]. The treatment of sea level variations due to changes in atmospheric pressure is more troublesome. Of main concern are pressure variations at the $S_2$ frequency, which cause sea level variations known as the radiational tide [Cartwright and Ray, 1990b, 1994] (see also Section 3.5). Because pressure variations at the
4.3 Satellite altimetry vs. tide gauges

$S_2$ frequency can be modeled quite accurately by an empirical formula [Chapman and Lindzen, 1970], one might think of removing their effect on the sea level by assuming an inverse barometer response. This would lead to an $S_2$ radiational tide with an amplitude of about 1 cm at moderate latitudes and even smaller at higher latitudes. However, it is expected that the ocean does not follow an instantaneous response to pressure changes with a period of less than approximately two days, a period at which the IB assumption might begin to be partially satisfied [Ray, 1994; Fu and Pihos, 1994]. Also, the existence of a dynamic $S_2$ ocean tide denies an IB response as it would mean that the $S_2$ ocean tide can be described as an equilibrium tide. Hence, the IB approach to remove the $S_2$ radiational tide is not recommended and the $S_2$ radiational and ocean tide are usually considered as one component [Ray, 1993]. The way the $S_2$ radiational tide is treated in the altimeter observations of TOPEX/POSEIDON, ERS-1, and GEOSAT, will be discussed in Section 5.2.

Modern tide gauges can measure sea level variations with an accuracy of a few mm [Pugh, 1987]. The great majority of tide gauge stations over the world, of which in total there are over 4000 [Pugh, 1987], are part of flood warning systems and hence occupy permanent sites near the coast. In the open oceans, relatively few tide gauges are deployed. This is explained by the relatively small number of locations in the open oceans to install these gauges, e.g. oil platforms, whereas the operation of deep-sea pressure gauges is quite expensive, mainly because of the maintenance costs to operate the vessels for deployment and recovery of the gauges [Pugh, 1987]. For this reason, the idea of using satellite altimeters as moving tide gauge stations has become very appealing. As the earth rotates underneath the satellite orbit, its surface is rapidly covered with a maze of groundtracks along which the tides are sampled. Each repeat period, the satellite lays a maze of tidal samples over the world’s oceans, so that the tide at a fixed point on the earth is sampled once every repeat period. In Figure 4.3, a typical global tide gauge network [Le Provost, 1994] is compared with the groundtrack patterns of TOPEX/POSEIDON, ERS-1, and GEOSAT in the North Atlantic. This figure clearly shows the superior global coverage of satellite altimetry over tide gauges. Each of the dots on the groundtracks corresponds to one altimeter observation and thus to one fixed location where the tide is sampled. In longitude direction, the spatial resolution with which the satellite samples the sea surface is dictated by the number of revolutions that the satellite performs in one repeat period (see Appendix A). For TOPEX/POSEIDON, GEOSAT, and ERS-1 in its Multidisciplinary Phase, the spacing of groundtracks is about 2°38, 1°48, and 0°72 of longitude, respectively. Notice that in latitude direction, the resolution increases towards the poles due to the convergence of the groundtrack pattern. For a tide gauge network to obtain a comparable resolution by moving the
Satellite altimetry in ocean tide modeling

Figure 4.3 Comparison of coverage and density of tidal samples from tide gauge stations and from satellite altimetry.

Gauges would obviously take many decades and would cost much more than the development and operations of an altimeter satellite.

An important difference between satellite altimetry and tide gauge stations is that an altimeter satellite provides information on the geocentric tide $\zeta_c$ rather than the pure oceanic tide $\zeta$ (see Figure 4.1). For this reason, the geocentric tide is sometimes referred to as the altimetric tide [e.g., Cartwright, 1993]. Hence, supplying a model for one or more of the components of $\zeta_c$ we may estimate for what is left of the geocentric tide. With altimetry, the common procedure is to remove the solid-earth tide $\zeta_{se}$ from the data and estimate the vector sum of the ocean tide and the load tide, i.e. $\zeta + \zeta_{se}$. In some literature, this vector sum is called the elastic ocean tide [e.g., AVISO, 1994; Andersen, 1995], but more often it is simply referred to as the ocean tide. Whether the elastic or purely oceanic tide is meant should then follow from the context.

Another difference between satellite altimetry and tide gauges is the rate with which the tide at a fixed point on the earth is sampled. With tide gauges, the sampling rate is usually one hour or less. With regard to the noise level on the tide gauge readings of a few mm, this means that a relatively large number of tides can be resolved and separated from each other if recordings are made over typically a few months. In case of altimetry,
the sea level measurements have a noise level (noise as used here means altimeter background noise $\varepsilon$ as appearing in (4.1)) of about 5-10 cm (see e.g. Table 5.3) so that only the dominant tides can be resolved. Because of the sub-Nyquist sampling of the sea surface, the tides may alias to nearly the same period and therefore become difficult to separate in the presence of a 5-10 cm noise level. Because the tidal correlations induced by aliasing cause major problems with extracting the ocean tides from satellite altimetry, they will be addressed in detail in the next two sections.

### 4.4 Tidal aliasing

For an altimeter satellite in a repeat orbit with a period of $P$ days, the altimeter samples the tide at a given point on the groundtrack once every $P$ days. From the sampling theorem [e.g., Oppenheim et al., 1983], it is known that a time-continuous signal of period $T_\omega$ can be fully reconstructed from its sampled values if these samples are taken over at least $T_\omega$ days at an interval of less than $T_\omega/2$. If the sampling interval exceeds $T_\omega/2$, the signal of frequency $\omega$ becomes aliased, i.e. it takes the identity of a signal with a much longer period, the alias period $T_a$. Because all altimeter satellites have repeat periods of a few days or more, aliasing of the diurnal and semi-diurnal tides is inherent to satellite altimetry. This means that the diurnal and semi-diurnal tidal signals at a fixed point on the groundtrack do not appear in the altimetry as such but as long-period signals. The long-period tides only become aliased if the repeat period is sufficiently large. As a consequence of aliasing, we now have to wait $T_a$ days instead of $T_\omega$ days to gather all information necessary to reconstruct the tidal signal of frequency $\omega$. In practice, however, data noise and data gaps make that tidal observations have to be collected over more than just one alias period $T_a$.

To obtain an expression with which the tidal alias periods can be computed as a function of the repeat period $P$, consider Figure 4.4. The sinusoid in this figure gives the tidal variations of sea level at a fixed point on earth, i.e. the sea level that is going up and down as a function of time with frequency $\omega$. At a certain time $t_1$, the satellite moves over the place and samples the tidal wave of frequency $\omega$. At time $t_2$, one repeat period later ($t_2 - t_1 = P$), the satellite moves over the same place again and takes a second sample of the tidal wave. In $P$ days, the phase advance $\Delta \phi_\omega$ of the tide has become: $\Delta \phi_\omega = \omega \cdot P$. Because the tidal phase changes over $2\pi$ in one period $T_\omega$, this may also be written as: $\Delta \phi_\omega = 2\pi P/T_\omega$. Reducing the tidal phase $\Delta \phi_\omega$ by the integer number of cycles traversed in $P$ days, i.e. $2\pi n$, where $n$ is a positive integer, gives the phase advance $\Delta \phi_a$ of the aliased signal, which is in the range $[-\pi, \pi]$:

$$\Delta \phi_a(P) = 2\pi \frac{P}{T_\omega} \quad [-\pi, \pi] \quad (4.2)$$
A positive value of $\Delta \phi_a$ means that the sample at $t_2$ is on the right of the point $nT_\omega$ away from $t_1$, in case of Figure 4.4 ($n = 2$) the crest at $t_1 + 2T_\omega$. A negative value of $\Delta \phi_a$ means that sample $t_2$ is taken on the left side of $t_1 + 2T_\omega$. Drawing an analogy with the motion of a wheel, a negative value of $\Delta \phi_a$ means that the spokes of the wheel seem to move backwards. As the aliased signal traverses $\Delta \phi_a$ in $P$ days, the alias period in which it will move through a full cycle is found as:

$$T_a = \frac{2\pi}{|\Delta \phi_a|} P \quad (4.3)$$

Apart from the problem of aliasing, there is another demand on the required amount of data. This demand has to do with the separability of tides that have nearly the same frequency. If the frequencies of two tides are close to each other, whether these are the actual frequencies as they appear in tide gauge data (no aliasing due to hourly sampling) or the aliased frequencies that show up in the altimetry, they become difficult to separate. The usual criterion for the tides to be separated is that they should at least differ in phase by one cycle over the amount of data analyzed [e.g., Godin, 1972]:

$$T_r |\omega_{a1} - \omega_{a2}| \geq 2\pi \quad (4.4)$$

In the above equation, $\omega_{a1}$ and $\omega_{a2}$ are the frequencies of the aliased signals (assuming that aliasing occurs) and $T_r$ is the minimum required time span over which the data should be analyzed. Because (4.4) is known as the Rayleigh criterion [Godin, 1972], the period $T_r$ is called the Rayleigh period. Obvious manipulations with (4.4) show that the Rayleigh period is conveniently expressed in terms of the alias periods of the two tides by:

$$\frac{1}{T_r} = \left| \frac{1}{T_{a1}} - \frac{1}{T_{a2}} \right| \quad (4.5)$$
4.4 Tidal aliasing

In Table 4.1, the alias and Rayleigh periods of the main tides in the semi-diurnal, the diurnal, and the long-period bands are given for the TOPEX/POSEIDON, ERS-1, and GEOSAT satellites. The entries of the $S_a$ and $S_{sa}$ tides also apply to the annual and semi-annual cycle, respectively, which have the same frequency, and also will be used in this context. The diagonal elements in Table 4.1 contain the alias periods, while the Rayleigh periods are expressed by the off-diagonal elements. Table 4.1 is a clear illustration of the difference between estimating the ocean tide from altimetry and from tide gauge data. Where we only need a few months of tide gauge data, the effect of aliasing with satellite altimetry is that we may need as much as a few years of altimeter observations to resolve and separate the tides. Hence, extracting the ocean tide from altimetry requires the processing of huge amounts of data.

From Table 4.1, it can be seen that with regard to the long-period tides, aliasing of $M_f$ occurs with all three satellites as they all have repeat periods that exceed seven days, which is half the period of $M_f$. In case of $M_{sn}$ only the repeat periods of GEOSAT and ERS-1 are large enough to cause aliasing of this tide, while none of the satellites aliases the $S_{sa}$ and $S_a$ tides. From Table 4.1, it can also be seen that for TOPEX/POSEIDON, all major tides are resolved and separated after the nominal mission lifetime of 3 years due to its favorable orbit design [Parke et al., 1987]. An exception must be made for the $K_1$ tide and the seasonal $S_{sa}$ cycle, which in theory (see discussion below) can only be separated after nine years. The same exception must be made for the minor $K_2$ and $P_1$ tides. Correlations between such minor tides are, however, less important because of the smallness of these tides and because of the fact that their estimates will be largely dictated by the admittance of the nearby dominant tides (in case of $K_2$ and $P_1$, the dominant $S_2$ and $K_1$ tides, respectively). Of major interest, therefore, are the alias and Rayleigh periods of the five dominant tides, $M_2$, $S_2$, $N_2$, $K_1$, and $O_1$, which largely define the diurnal and semi-diurnal admittances (see Chapter 7).

For ERS-1 and GEOSAT, the alias and Rayleigh periods are generally much larger than those of TOPEX/POSEIDON, which means that more data are required to obtain reliable tidal estimates from the ERS-1 and GEOSAT altimetry. In case of ERS-1, there is an additional problem due to the sun-synchronous character of the orbit. For a sun-synchronous orbit, a nodal day exactly equals a mean solar day. In case of ERS-1, this means that its 35-day repeat period exactly equals 70 times the period of $S_2$. Therefore, at a fixed point on the earth, ERS-1 will measure the $S_2$ tidal signal every repeat period with exactly the same phase so that $S_2$ appears as a constant in the ERS-1 altimetry. Notice that for ERS-1, the $M_2$ and $N_2$ tides can only be separated from one another if about 9 years of data are available. This is even worse for $K_1$ and $P_1$ of which the Rayleigh period
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<td>( S_a )</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
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<td>-</td>
<td>-</td>
<td>-</td>
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</tbody>
</table>

Table 4.1 Tidal alias periods (on the diagonals) and Rayleigh periods (off the diagonals) in days for TOPEX/POSEIDON, ERS-1, and GEOSAT, with repeat periods \( P \) of 9.9156, 35.0, and 17.0505 days, respectively.
exceeds 20,000 years. Moreover, due to the sun-synchronism of the orbit, the $K_1$ and $P_1$ tides both alias to periods of about one year and therefore cannot be distinguished from the seasonal cycle of annual period ($K_1/S_a$ and $P_1/S_a$ Rayleigh periods are of the same order of magnitude as that of $K_1/P_1$). This also applies to the semi-annual seasonal cycle and the $K_2$ tide. The $O_1/M_f$ correlation with ERS-1 is unimportant because the long-period $M_f$ tide may be assumed to follow an equilibrium response and hence can be removed from the ssh observations accordingly. In case of GEOSAT, $K_1$ and $S_2$ are inseparable within 12 years, while the alias periods of these tides are also close to that of the semi-annual cycle resulting in Rayleigh periods for $S_2/S_{sa}$ and $K_1/S_{sa}$ of 6 and 12 years, respectively. Also, GEOSAT aliases the $M_2$ tide to a period of about one year, which therefore becomes difficult to separate from the annual cycle from less than 6 years of data. The small $P_1$ tide can definitely not be resolved from the GEOSAT altimeter data but as it is one of the minor tides it is of less importance.

The smallness of the TOPEX/POSEIDON alias and Rayleigh periods largely explains the excellent results of the TOPEX/POSEIDON ocean tide models that were published [e.g., Schrama and Ray, 1994; Ma et al., 1994] after only one year of TOPEX/POSEIDON altimetry had been gathered. It also explains why most of the numerous recent tidal models that have incorporated altimeter data, are based on TOPEX/POSEIDON observations only. With ERS-1 and GEOSAT, solving tides with a large alias period and decorrelating tides with nearly the same alias period is obviously a bit more troublesome in the presence of data noise than in case of TOPEX/POSEIDON. However, it should be stressed that the situation is not as bad as Table 4.1 might suggest. The alias and Rayleigh periods in this table strictly apply to a pointwise solution of the ocean tide where we would look for a tidal solution at each single point on the groundtrack. In practice, a tidal solution is sought in a certain area or grid cell and all altimeter observations inside a cell will contribute to that particular solution. Hence, choosing the cell size sufficiently large (Section 5.3), each cell will contain several tracks, while each track will also contain more than one observation. This means that there are many more data than just one observation to contribute to the tidal solution at a place so that good results should be obtained within relatively few years. More importantly, as will be shown in the next section, the samples from additional tracks (adjacent and crossing) in a grid cell may lead to a significant decorrelation of the tidal signals so that especially the Rayleigh periods for ERS-1 and GEOSAT in Table 4.1 are somewhat pessimistic.
4.5 Tidal decorrelation from adjacent and crossing groundtracks

Additional tracks in a cell around a grid point may reduce the correlation between two tides. This happens whenever the satellite samples the tides with a significantly different phase on different tracks. Hence, tidal decorrelation may be established by adjacent tracks or crossing tracks in a grid cell. Obviously, tidal decorrelation from adjacent or crossing groundtracks becomes less important if we can enlarge the analysis periods of ERS-1 and GEOSAT to the order of the Rayleigh periods in Table 4.1, i.e. like a 3-year analysis period with TOPEX/POSEIDON concerning the $M_2/S_2$ correlation. However, with Rayleigh periods of six to twelve years, this may only be expected from a considerable extension of the ERS-1 and the GEOSAT missions (Chapter 10). Until enough data are gathered, tidal decorrelations from adjacent and crossing groundtracks will play an important role in tidal analysis from satellite altimetry.

4.5.1 Adjacent groundtracks

In case of TOPEX/POSEIDON, it may be easily shown that for any track, the closest non-repeating track occurs 2.967 days later to the east. This non-repeating or adjacent track, of course, determines the resolution of the groundtrack pattern, which is 2°83 of longitude. Hence, if we would choose the cell size of the tidal grid equal to, e.g., 3° in longitude direction, each cell would on average contain two ascending tracks spaced 2.967 days apart and two descending tracks, which will also be spaced 2.967 days apart. The importance of the 2.967-day period is that the groundtrack pattern laid over the earth until then will repeat itself but with a shift of 2°83 to the east. For this reason, the 2.967-day period is usually called the subcycle of the TOPEX/POSEIDON repeat period. In case of GEOSAT and ERS-1, the time interval $\Delta t$ between adjacent ascending or descending tracks is 3.005 and 15.998 days, respectively, where adjacent tracks are always to the east. These intervals correspond to a resolution in longitude of 1°48 and 0°72, respectively. With ERS-1, it should be noted that if we permit the groundtrack pattern to repeat itself within a distance of twice the resolution of the 35-day repeat orbit, we would find a second subcycle of 3.004 days. However, in this thesis we will only consider the main subcycle of approximately 16 days, as the ERS-1 tidal solutions are developed on a 1° × 1° grid. Notice that for adjacent tracks separated in time by more than one repeat cycle, the time interval will be $\Delta t$ plus an integer number of repeat periods $P$. The modulo $P$ term leads to the alias and Rayleigh periods in Table 4.1 so that we can restrict the discussion here to values of $\Delta t$ less than the repeat period.

With the aid of (4.2), the advance of the tidal phases between two ad-
4.5 Tidal decorrelation from adjacent and crossing groundtracks

<table>
<thead>
<tr>
<th></th>
<th>Δt</th>
<th>M₂</th>
<th>S₂</th>
<th>N₂</th>
<th>K₂</th>
<th>K₁</th>
<th>O₁</th>
<th>P₁</th>
<th>Q₁</th>
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<td>-9.0</td>
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<td>-46.5</td>
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<td>31.5</td>
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<td>-69.8</td>
<td>3.5</td>
<td>-109.1</td>
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<td>79.2</td>
<td>39.3</td>
<td>5.9</td>
<td>3.0</td>
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</tbody>
</table>

Table 4.2: Tidal phase advances (degrees) between adjacent tracks for TOPEX/POSEIDON (T/P), ERS-1, and GEOSAT. Adjacent tracks are spaced Δt days apart. Denoted by Sₙ and Sₙ₉ are the annual and semi-annual cycle, respectively.

Adjacent tracks in a grid cell may easily be computed by replacing the repeat period P by the subcycle interval Δt. For the major tides and for the seasonal cycles, the phase advances on adjacent tracks, i.e., Δφₙ(Δt), are listed in Table 4.2. Notice that a large value of the phase advance means that an adjacent track provides useful information on the tidal signal. If we take the absolute value of the phase advances, which is in the range of 0° to 180°, large may be arbitrarily defined as, e.g., between 90° and 180°. Interesting to notice from this table is that for GEOSAT, the P₁ phase advance between adjacent tracks is relatively small. This means that approximately the same phase of P₁ is measured on adjacent tracks, which thus provides little help to reduce the problem of the large alias period of twelve years of this tide (Table 4.1). The same reasoning applies to the S₂ tide in case of ERS-1. For the remaining tides, Table 4.2 shows that adjacent tracks may provide considerable information on the phases of some of the tides.

Table 4.2 is also useful to gain insight into how adjacent tracks may help to decorrelate two tides that have approximately the same alias period. This is more important than studying the phase advances of each tide in itself because for all three satellites, at least one of the five dominant tides is involved in a correlation problem, as has been discussed in Section 4.4. With the ability of adjacent tracks to decorrelate the tides, we are not so much interested in the tidal phase advances (4.2) themselves, but in the difference of the phase advances of two tides:

$$\Delta\phi_{a₁, a₂}(Δt) = \Delta\phi_{a₁}(Δt) - \Delta\phi_{a₂}(Δt)$$  \hspace{1cm} (4.6)

The decorrelating ability of adjacent tracks may be judged by the value of |Δφₙ₁ₙ₂(Δt)| expressed in the range 0° to 180°, where a value larger than 90° may be considered advantageous. From Table 4.2 it can be seen that the phase advance difference of the M₂ and S₂ tides as observed by TOPEX/POSEIDON is approximately 72°, which is not that significant. Hence, adjacent tracks inside a grid cell can do relatively little to decorrelate these tides. In case of K₁ and the semi-annual signal, TOPEX/POSEIDON measures a phase advance difference of approximately 15°, which is quite small so that adjacent tracks will offer little help to decorrelate these signals with TOPEX/POSEIDON. In case of ERS-1, decorrelating S₂ from the dynamic topography is impossible according to the very small S₂ phase advance difference on adjacent tracks of 1.4°. Also with ERS-1,
decorrelating $K_1$, $P_1$ and the annual cycle using phase information on adjacent tracks is problematic. On adjacent tracks, $K_1$ and the annual cycle as observed by ERS-1 have a phase advance difference of merely $1^\circ$ while the phase advance differences of both $K_1$ and the annual cycle with $P_1$ are approximately $32^\circ$, which is not very large. In case of the $M_2$ and $N_2$ tides, ERS-1 measures a phase advance difference of $151^\circ$, which offers some prospect to decorrelate these tides. With GEOSAT, the problem of decorrelating $K_1$, $S_2$, and the semi-annual cycle cannot be solved by adjacent tracks as the phase advance differences of these signals are less than $2^\circ$. In case of decorrelating $M_2$ from the annual cycle, GEOSAT measures a phase advance difference between these signals of $73^\circ$, which also is not very large.

4.5.2 Crossing groundtracks

In contrast with adjacent tracks, the tidal phase advances between crossing tracks are not so easily formulated. This is because the time interval between ascending and descending tracks varies with latitude in a complicated way as shown in Figure 4.5, which was computed from actual altimeter data. It is emphasized that nowhere in this thesis, crossover differences (difference between two sea surface height residuals at the same location) were actually used to estimate the tidal solutions. Hence, the tidal phase advances on crossing tracks as observed by the satellites are not the result of actually subtracting two sea surface height residuals, but simply because these residual observations are made at nearly the same place at different times. In Figure 4.5, the time interval between crossing tracks is defined as the time elapsed from an ascending pass to the intersecting descending pass, i.e. $t_{asc} - t_{desc}$. For the same reason as with the adjacent groundtracks, we will restrict ourselves to a time interval $t_{asc} - t_{desc}$ less than the duration of the repeat cycle. This means that this time interval is always between plus and minus half the repeat period $P$, where the minus sign shows up when the ascending track is tagged earlier in time than the descending track. Hence, the time interval between crossing tracks never exceeds 4.96, 17.5 and 8.53 days in case of TOPEX/POSEIDON, ERS-1, and GEOSAT, respectively. To average out statistical uncertainties due to a varying number of crossover locations per repeat cycle, Figure 4.5 was computed for each satellite from about ten repeat cycles consecutive in time. Because of rotational symmetry of the orbit with respect to the equator, all crossover differences at the same latitude have the same time interval (triangle markers in Figure 4.5) while crossover differences at opposite latitudes also have the same interval but with opposite sign. For the latter reason, the time interval between crossing tracks as displayed in Figure 4.5 is an odd function of latitude. Quite obvious from Figure 4.5 is the clustering of crossover differences towards the turnover latitudes, i.e. the maximum latitudes
that the satellite can reach, due to the convergence of the groundtracks.

Replacing the repeat period $P$ in (4.2) by the time interval $t_{asc} - t_{desc}$ of each latitude band in Figure 4.5, the changes in tidal phase from the time of the ascending pass to the time of the descending pass, i.e. $\Delta \phi(t_{asc} - t_{desc})$, may be computed as displayed in Figures 4.6 to 4.8 (absolute values of phase advances are shown). An equivalent of Figure 4.6 in tabular form may be found in Schrama and Ray [1994]. The plots of the phase advances have been grouped in pairs of tides that have approximately the same frequency, e.g. $M_2$ and $N_2$, or $K_1$ and $P_1$, and thus approximately the same phase advance on crossing tracks. Notice that because the time interval between crossing tracks in Figure 4.5 is an odd function of latitude, the absolute value of the phase advances on crossing tracks becomes an even function of latitude. Also notice that the annual and semi-annual cycles vary so slowly with time that their phase advances on crossing tracks are nearly zero except in case of ERS-1, where crossing tracks may be separated by as much as 17.5 days, which leads to a phase advance of about $17^\circ$ and $34^\circ$ at most for the annual and semi-annual cycle, respectively. It can be shown that the
Figure 4.6 Tidal phase advances (degrees) between ascending and descending tracks for TOPEX/POSEIDON. Triangle markers correspond to the solid line, circle markers to the dashed line. Denoted by $S_a$ and $S_{sa}$ are the annual and semi-annual cycle, respectively.

The time interval between crossing tracks equals approximately half a day (modulo an integer number of days) near the equator and increases to one day (modulo an integer number of days) towards the turnover latitudes. As a consequence, all three satellites observe the diurnal tides that have a period close to 1.0 day, i.e. $K_1$ and $P_1$ (see Table 2.2), with a phase advance on crossing tracks of about 180° near the equator and 0° at the turnover latitudes. Similarly, the semi-diurnal $S_2$ and $K_2$ tides with a period close to 0.5 days both have a phase advance at crossover locations close to 0° at the equator and at the turnover latitudes, while the phase advance becomes 180° at those latitudes where the time interval between crossing tracks equals three quarters of a day (modulo an integer number of days). The $O_1$ and $Q_1$ tides have periods sufficiently different from a day to destroy a regular pattern of the phase advances on crossing tracks. The same applies to $M_2$ and $N_2$ with regard to their periods of almost half a day.

Similar to the phase advance differences on adjacent tracks (4.6) we may define the phase advance difference of two tides on crossing tracks as:

$$\Delta \phi_{a_1, a_2}(t_{asc} - t_{desc}) = \Delta \phi_{a_1}(t_{asc} - t_{desc}) - \Delta \phi_{a_2}(t_{asc} - t_{desc})$$  \hspace{1cm} (4.7)
where a value of $|\Delta \phi_{a1, a2}(t_{asc} - t_{desc})|$ between 90° and 180° may be considered large and thus useful to decorrelate two tides. For TOPEX/POSEIDON, Figure 4.9 shows that the phase advance difference on crossing tracks of the $M_2$ and $S_2$ tides varies with latitude within a range of 0° to 90°. In interleaved latitude bands, a phase advance difference close to 0° or 90° is measured. Because at all latitudes, the phase advance difference on crossing tracks of $M_2$ and $S_2$ is less than 90°, an intersecting track, either ascending or descending, cannot add much information to decorrelate these tides. In case of the $K_1$ tide and the semi-annual cycle, Figure 4.9 shows that there is a considerable difference of the phase advances on crossing tracks, i.e. larger than 90° up to latitudes of 55°. Beyond these latitudes, the phase advance difference rapidly decreases as the phase advance of $K_1$ becomes less than 90° (Figure 4.6). Hence, crossing tracks at the TOPEX/POSEIDON crossover points are useful to decorrelate $K_1$ from the semi-annual cycle at latitudes below 55°. Even more, with regard to the 9-year Rayleigh period of the $K_1/S_{sa}$ pair, the phase advance differences on crossing tracks are without doubt the reason why such reliable estimates of $K_1$ could be obtained with the earlier models of e.g. Schrama and Ray [1994] and Ma et al. [1994].

In case of ERS-1, Figure 4.7 reveals that the $S_2$ phase advance on cross-
Figure 4.8 Tidal phase advances (degrees) between ascending and descending tracks for GEOSAT. Triangle markers correspond to the solid line, circle markers to the dashed line. Denoted by $S_a$ and $S_{sa}$ are the annual and semi-annual cycle, respectively.

ing tracks steadily increases with latitude. However, up to latitudes of about 66°, i.e. the TOPEX/POSEIDON turnover latitudes, an $S_2$ phase advance of less than 90° is measured so that little help may be expected from crossing tracks to decorrelate the $S_2$ tide from the dynamic topography. Hence, because sea surface height measurements separated by a repeat cycle as well as measurements from adjacent tracks also provide hardly any information on the $S_2$ tidal phase, extracting the $S_2$ tide from ERS-1 altimetry alone is an impossible task. Considering $M_2$ and $N_2$, Figure 4.10 shows that the phase advance difference of these tides at ERS-1 crossover points varies significantly with latitude. In interleaved latitude bands, this phase advance difference becomes larger than 90° so that crossing tracks are helpful to decorrelate the $M_2$ and $N_2$ tides in these latitude bands. With the triad consisting of $K_1$, $P_1$ and the annual cycle, crossing tracks help tremendously to decorrelate these two diurnal tides from the annual cycle. Figure 4.10 shows that the phase advance difference of the dominant $K_1$ tide and the annual cycle is larger than 130° up to latitudes of 66°. This is understandable because the $K_1$ (and $P_1$) phase advances of nearly 180° on crossing tracks below latitudes of 66° (Figure 4.7) significantly decorrelate these diurnal tides from a slow-varying signal like the annual cycle.
4.5 Tidal decorrelation from adjacent and crossing groundtracks

Figure 4.9 Tidal phase advance differences (degrees) between ascending and descending tracks of indicated tidal pairs as observed by TOPEX/POSEIDON. $S_{sa}$ denotes the semi-annual cycle.

Figure 4.10 Tidal phase advance differences (degrees) between ascending and descending tracks of indicated tidal pairs as observed by ERS-1. $S_a$ denotes the annual cycle.

Figure 4.11 Tidal phase advance differences (degrees) between ascending and descending tracks of indicated tidal pairs as observed by GEOSAT. $S_a$ and $S_{sa}$ denote the annual and semi-annual cycle, respectively.
However, this also means that crossing tracks of ERS-1 offer little help to
decorrelate $K_1$ from $P_1$ as both tides advance approximately $180^\circ$ in phase
from an ascending to a descending track below latitudes of $66^\circ$. As a matter
of fact, it may be shown that the phase advance difference at crossing
tracks of $K_1$ and $P_1$ is less than $30^\circ$ at all latitudes.

In case of decorrelating the $M_2$ tide and the annual cycle with GEO-
SAT, crossing tracks are useful in interleaved latitude bands as can easily
be seen in Figure 4.11. With the correlation between $S_2$, $K_1$, and the semi-
annual signal, the problem is a bit more complicated (Figure 4.11). In case
of the $S_2/S_{sa}$ pair, a phase advance difference larger than $90^\circ$ is only mea-
sured between latitudes of $55^\circ$ and $70^\circ$ so that at most latitudes, crossing
tracks can do little to decorrelate $S_2$ from the semi-annual cycle. With the
$K_1/S_{sa}$ couple, a phase advance difference of more than $90^\circ$ is measured
up to latitudes of $66^\circ$ such that their decorrelation becomes better towards
the equator. With regard to $S_2$ and $K_1$, the decorrelation also is better near
the equator and it can be seen that their phase advance difference is larger
than $90^\circ$ up to latitudes of about $66^\circ$.

4.6 Discussion

Due to the sub-Nyquist sampling of the sea surface, the diurnal and
semi-diurnal tides as observed by altimeter satellites may alias to peri-
ods as long as a year. With regard to the Rayleigh periods over which
the aliased tides decorrelate, tidal phase advance differences on crossing
satellite groundtracks were found to be very useful to decorrelate the tides.
This is of utmost importance if we analyze the altimeter observations over
a period significantly less than the Rayleigh periods of the correlated tidal
pairs.

Due to the favorable orbit design of TOPEX/POSEIDON, the domi-
nant tides are aliased to relatively small alias periods of 50 to 90 days. An
exception is the $K_1$ tide, which has an alias period of 173 days with TO-
PEX/POSEIDON and therefore becomes correlated with the semi-annual
cycle. However, because of the large phase advance differences on cross-
ing tracks of these two signals, i.e. more than $90^\circ$ up to some $55^\circ$ of latitude,
this correlation is only important beyond latitudes of $55^\circ$. In case of the $M_2$
and $S_2$ tides as observed by TOPEX/POSEIDON, full decorrelation is estab-
lished within three years.

With ERS-1, the alias periods of the dominant tides are significantly
larger than those of TOPEX/POSEIDON. Because of its sun-synchronous
orbit, ERS-1 aliases $S_2$ to a constant while $K_1$ is aliased to a year and so
becomes correlated with the annual cycle. The $S_2$ phase advances of less
than $90^\circ$ up to latitudes of $66^\circ$ cannot establish a decorrelation of $S_2$ from
the dynamic topography. However, in case of $K_1$, the phase advance dif-
fferences of more than $130^\circ$ up to $66^\circ$ of latitude will largely decorrelate
$K_1$ from the annual cycle in spite of their infinitely large Rayleigh period.
With $M_2$ and $N_2$, phase advance differences on crossing tracks larger than 90° establish a decorrelation in interleaved latitude bands.

With GEOSAT, the alias periods of most of the dominant tides are also significantly larger than those of TOPEX/POSEIDON. The $S_2$ and $K_1$ tides are both aliased to half a year and therefore become correlated with the semi-annual cycle. Because of the phase advance differences on crossing tracks, $K_1$ decorrelates better from both $S_2$ and the semi-annual cycle towards the equator. With $S_2$, its phase advance difference with the semi-annual cycle is less than 90° at all latitudes, except in a small band of 55° to 70° of latitude. Hence, the phase advance differences on crossing tracks cannot diminish the correlation of the $S_2$/$S_{sa}$ pair. With $M_2$ and the annual cycle, the phase advance differences on crossing tracks diminishes their correlation in interleaved latitude bands.

Tables 4.3 to 4.5, summarize the ability of adjacent and crossing tracks to decorrelate pairs of tides. Notice that these tables only list those tidal correlations in which one of the five dominant tides, i.e. $M_2$, $S_2$, $N_2$, $K_1$, or $O_1$ is involved. Smaller tides cannot be reliably estimated through harmonic analysis while in case of the response method, the decorrelation of smaller tides is mainly established by the assumption of a smooth admittance. The entries for $S_a$ and $S_{sa}$ denote the seasonal cycles, which cannot be distinguished from the long-period tides of the same frequencies. In case of ERS-1, the entry for $S_2$ may be regarded as decorrelating this tide from the dynamic topography. A "+" indicates that adjacent or crossing tracks significantly help in decorrelating the tides, while a "−" means that little help is expected. In the tables, the "+" and "−" symbols with the crossing tracks indicate that the decorrelation of the tidal pair increases towards the turnover latitudes or the equator, respectively. A "≡" means that the decorrelation is stronger in interleaved latitude bands. In case of adjacent tracks, there are no variations in decorrelation with latitude as the time interval between adjacent tracks is no function of latitude. From Tables 4.3 to 4.5 we conclude that the only real problem with TOPEX/POSEIDON, i.e. disentangling $K_1$ from the semi-annual cycle for latitudes larger than 55°, is not going to be solved by the GEOSAT data. This is because with GEOSAT, the $K_1$ tide and the semi-annual cycle are also intimately cor-

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<td>$-$</td>
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</tr>
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<td>$+$</td>
</tr>
</tbody>
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Table 4.3 Ability of adjacent and crossing tracks to decorrelate indicated tidal pairs for TOPEX/POSEIDON. The "+" indicates that decorrelation increases towards the equator, while the "≡" indicates that decorrelation occurs in latitude bands. Denoted by $S_{sa}$ is the semi-annual cycle.
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<table>
<thead>
<tr>
<th>$S_2$</th>
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<th>$K_1/P_1$</th>
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</tr>
<tr>
<td>crossing</td>
<td>-↓</td>
<td>+≡</td>
<td>-</td>
<td>+↑</td>
</tr>
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</table>

Table 4.4 Ability of adjacent and crossing tracks to decorrelate indicated tidal pairs for ERS-1. The "↓" indicates that decorrelation increases towards the turnover latitudes. Denoted by $S_a$ is the annual cycle.

<table>
<thead>
<tr>
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<th>$S_2/S_{sa}$</th>
<th>$K_1/S_{sa}$</th>
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<td>-</td>
<td>-</td>
</tr>
<tr>
<td>crossing</td>
<td>+≡</td>
<td>+↑</td>
<td>-↓</td>
</tr>
</tbody>
</table>

Table 4.5 Ability of adjacent and crossing tracks to decorrelate indicated tidal pairs for GEOSAT. Denoted by $S_a$ and $S_{sa}$ are the annual and semi-annual cycle, respectively.

related, while their decorrelation increases in the same direction as with TOPEX/POSEIDON. True, GEOSAT observes a $K_1/S_{sa}$ phase advance difference on crossing tracks of 90° to 110° at latitudes between 55° and 66°, but as will be shown in Section 6.5, this will not be large enough to decorrelate $K_1$ from the semi-annual cycle at higher latitudes if but a few years of GEOSAT altimetry are available ($K_1$ and $S_{sa}$ have a 12-year Rayleigh period in case of GEOSAT). With decorrelating the $K_1$ tide from the semi-annual cycle at higher latitudes, the observations of ERS-1 are more promising. Although the infinitely large Rayleigh period predicts an intimate correlation between $K_1$ and the annual cycle as observed by ERS-1, the phase advance differences on crossing tracks of more than 130° up to latitudes of 66° are expected to tremendously reduce this correlation (see Section 6.4). In Chapter 6, the decorrelation patterns of the phase advance differences on crossing tracks as given in Tables 4.3 to 4.5, will be confirmed by covariance analyses of the harmonic tidal solutions. Tidal decorrelation from adjacent groundtracks will not be discussed any further as the only tidal pair that may benefit from such decorrelation is $M_2/N_2$ as observed by ERS-1.
Chapter 5
Processing of altimeter data and estimated parameters

5.1 Introduction

In Section 5.2 of this chapter, it is discussed which models were used to reduce the $ssh$ observations of TOPEX/POSEIDON, ERS-1, and GEOSAT to the $ssh$ residuals (4.1) that contain the ocean tide including the loading component, i.e. $\zeta + \zeta_{ot}$, in which we are interested. Section 5.4 describes the parameters that were estimated from the $ssh$ residuals to model the terms on the right side of (4.1). The procedure that has been followed to grid the $ssh$ residuals onto a mesh of $3^\circ \times 3^\circ$ for TOPEX/POSEIDON, $2^\circ \times 2^\circ$ for GEOSAT, and $1^\circ \times 1^\circ$ for ERS-1, is explained in Section 5.3. The resolutions of these meshes or grids are defined by the smallest tidal wavelength that each of the satellites can observe, i.e. by the satellite groundtrack spacings. For each of these grids, the tidal normal matrices derived from the $ssh$ residuals are stored. The normal matrix grids of TOPEX/POSEIDON and GEOSAT are interpolated to $1^\circ \times 1^\circ$, i.e. to the resolution of the ERS-1 altimetry, so that tidal models of $1^\circ \times 1^\circ$ resolution are obtained from the altimetry of all satellites. The spatial interpolation of the normal matrices (instead of the tidal solutions) is done to optimally take advantage of the resolutions of the GEOSAT and ERS-1 altimetry to observe the smaller wavelength tidal features in the multi-satellite tidal solution. Hence, this procedure will be discussed in Section 9.2 of Chapter 9.

5.2 Used TOPEX/POSEIDON, ERS-1, and GEOSAT altimeter data and applied corrections

The sea surface height residuals used to compute the ocean tide models were derived from the altimeter height measurements and corrections provided on the TOPEX/POSEIDON and GEOSAT Geophysical Data Records (GDRs) and the ERS-1 Ocean Products (OPRs) (see Table 5.1).
Table 5.1 Altimeter data used for the computation of the ocean tide models.

<table>
<thead>
<tr>
<th>satellite</th>
<th>GDR/OPR version</th>
<th>data records</th>
<th>period</th>
</tr>
</thead>
<tbody>
<tr>
<td>GEOSAT</td>
<td>T2</td>
<td>GDR 1-44</td>
<td>8 Nov. 1986 – 27 Nov. 1988</td>
</tr>
</tbody>
</table>

In case of TOPEX/POSEIDON, the GDRs are those of version AVMGB [AVISO, 1994]. In case of GEOSAT and ERS-1, the data records are from GDR version T2 [Cheney et al., 1991] and OPR version ALT-OPR V3 [CERSAT, 1994], respectively. Although more precise altimeter measurement corrections have become available since 1997, e.g. for the sea state bias and for the wet troposphere, these are not expected to have significant impact on the developed tidal models. The improvements of the tidal models that may be expected from more accurate orbits that have been produced since 1997 are discussed in Chapter 10.

The geoid to which the ssh residuals are referenced is the OSU91A (Ohio State University) model complete to degree and order 360 [Rapp et al., 1991]. To save disk capacity and processing time, the fully corrected one-per-second altimeter measurements were compressed to so-called altimeter normal points. A normal point is created by fitting a third-order polynomial through 11 one-per-second measurements. For a typical altimeter satellite at an altitude of some 1000 km, 11 seconds corresponds to an along-track distance over the earth’s surface of about 0.07 or 70 km. As the spatial variation of the barotropic ocean tide in the open oceans is long-wavelength of nature with a typical length scale of 1000 kilometers, the creation of 11-second normal points retains almost all of this tidal signal. The baroclinic or internal tide, with a typical spatial length scale of several tens of kilometers [Pugh, 1987], cannot be observed by creating normal points. However, because the spacing between the groundtracks of an altimeter satellite by far exceeds the spatial length scale of the internal tide anyway, with possible exception of higher latitudes, this is of no concern. With the creation of the normal points, stringent editing was performed. If the fit of the 11 points with the polynomial exceeded a 30 cm threshold or if one of the points deviated from the polynomial by more than 3.5 times the fit (3.5σ), no normal point was created.

From Table 5.1 it can be seen that in case of TOPEX/POSEIDON, the selected data cover the entire nominal mission of three years (Appendix A), which guarantees a full decorrelation of the $M_2$ and $S_2$ tides. The first eight cycles were not considered because of mispointing errors of the altimeter [Fu et al., 1994]. For both ERS-1 and GEOSAT, about two years of altimeter data were used. More specifically, data from ERS-1 Cycles 1-18 were processed, covering its entire 35-day repeat Multidisciplinary Phase (Appendix A). For GEOSAT, Cycles 1-44 were used, which cover the first two years of its Exact Repeat Mission (Appendix A).
### Table 5.2: Applied altimeter measurement corrections, applied sea surface height corrections, and used orbital ephemerides.

<table>
<thead>
<tr>
<th></th>
<th>TOPEX/POSEIDON</th>
<th>ERS-1</th>
<th>GEOSAT</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>orbit</strong></td>
<td>JGM-3 NASA/CSR or dual freq. (TOPEX)</td>
<td>JGM-3 DEOS</td>
<td>JGM-3 UT/CSR</td>
</tr>
<tr>
<td></td>
<td>or DORIS-based (POSEIDON)</td>
<td>from TEC</td>
<td>from TEC</td>
</tr>
<tr>
<td><strong>ionosphere</strong></td>
<td></td>
<td>(Bent model)</td>
<td>(Klobuchar model)</td>
</tr>
<tr>
<td><strong>troposphere</strong></td>
<td>ECMWF press. fields (dry) and radiometer (wet)</td>
<td>FMO press. fields (dry) and radiometer (wet)</td>
<td>ECMWF press. fields (dry) and TOVS/SSMI (wet)</td>
</tr>
<tr>
<td><strong>bias:</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>-sea state</td>
<td>4-par. Gaspar model</td>
<td>5.5% of SWH</td>
<td>2.6% of SWH</td>
</tr>
<tr>
<td>-altimeter</td>
<td>-3.7 cm (TOPEX)</td>
<td>-42.0 cm</td>
<td>12.0 cm</td>
</tr>
<tr>
<td></td>
<td>-1.3 cm (POSEIDON)</td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>-timing drift:</strong></td>
<td></td>
<td>-1.1 ms</td>
<td>POD values</td>
</tr>
<tr>
<td>-altimeter</td>
<td>Wallops (TOPEX)</td>
<td>0 cm/yr</td>
<td>-</td>
</tr>
<tr>
<td>-oscillator</td>
<td>Wallops (TOPEX)</td>
<td>ESRIN</td>
<td>-</td>
</tr>
<tr>
<td>inv. barometer</td>
<td>applied</td>
<td>applied</td>
<td>applied</td>
</tr>
<tr>
<td>earth tide</td>
<td>2\textsuperscript{nd} deg. exp.</td>
<td>2\textsuperscript{nd} deg. exp.</td>
<td>2\textsuperscript{nd} deg. exp.</td>
</tr>
<tr>
<td>pole tide</td>
<td>applied</td>
<td>applied</td>
<td>applied</td>
</tr>
<tr>
<td>ocean tide</td>
<td>not applied</td>
<td>not applied</td>
<td>not applied</td>
</tr>
<tr>
<td>load tide</td>
<td>not applied</td>
<td>not applied</td>
<td>not applied</td>
</tr>
</tbody>
</table>

The last year of the ERM was neglected because the altimeter frequently fell out due to an attitude control problem [Cheney et al., 1991] (see also Section 6.2).

It should be mentioned that the values for the sea state bias of all three satellites and the orbital ephemerides of ERS-1 and GEOSAT were not taken from the data records because more precise values have become available after the records had been produced. Especially the GEOSAT orbits, which on the T2 GDRs were computed with the GEM-T2 (Goddard Earth Model) gravity field model to an estimated radial accuracy of about 25 cm [Haines et al., 1990; Cheney et al., 1991], have been much improved when they were recomputed by the University of Texas/Center for Space Research (UT/CSR) with the recent JGM-3 (Joint Gravity Model) gravity field model resulting in a 10 cm accuracy level [Chambers, 1996]. In case of ERS-1, the orbits on the OPRs have been replaced by the Delft Institute for Earth-Oriented Space Research (DEOS) orbits, which are estimated to have an accuracy of about 7-8 cm [Scharroo, 1996].

As explained in Section 4.2 (Equation (4.1)), the altimeter measurements and the ssh observations were corrected for atmospheric delays, several bias and drift effects, atmospheric pressure change (IB), and for the tide of the solid earth and the pole tide. Table 5.2, which was compiled on the basis of information taken from Fu et al. [1994], AVISO [1994], Cheney et al. [1991], and CERSAT [1994], details each of the correction terms in
(4.1). Notice that because several correction terms in Table 5.2 depend on the characteristics of the altimeter instrument, TOPEX and POSEIDON are treated separately. In Table 5.3 at the end of this section, the accuracies of the correction terms in Table 5.2 are listed. Following below is a brief discussion of each of the correction terms.

As the TOPEX altimeter operates at two frequencies, it can measure (to first order) its own ionospheric path delay. For POSEIDON, the ionospheric correction is obtained from the dual-frequency DORIS (Doppler Orbitography and Radio-positioning Integrated by Satellite) tracking system [Escudier et al., 1993]. In this case, a transformation to the altimeter raypath must be applied, which introduces some small errors (Table 5.3). ERS-1 and GEOSAT carry single-frequency altimeters [Cheney et al., 1991; ESA, 1992] and thus the ionospheric correction has to be computed from a TEC (Total Electron Content) model, which also introduces some errors.

For all three satellites, the dry component of the troposphere is derived from surface pressure fields, delivered either by the French Meteorological Office (FMO) or the European Center for Medium-range Weather Forecasting (ECMWF). In case of TOPEX/POSEIDON and ERS-1, the pressure fields are provided with a 6-hour interval, while for GEOSAT, this interval is 12 hours. The wet component, in case of TOPEX/POSEIDON and ERS-1, is derived from the water vapor content along the altimeter raypath as derived from the onboard radiometers. For GEOSAT, global fields of water vapor content as obtained by two satellites which operated during the GEOSAT mission were used. The primary source was the Special Sensor Microwave Imager (SSMI) on a US Department of Defense satellite launched in July 1987, while prior to this date, the data provided by the TIROS Operational Vertical Sounder (TOVS) were used [Cheney et al., 1991]. The vapor fields were interpolated to the GEOSAT groundtrack and converted to its altimeter pathlength, which is why the tropospheric error of GEOSAT is somewhat larger than that of the two other satellites (Table 5.3).

To model the sea state bias, a four parameter description as a function of SWH and windspeed is used for both TOPEX and POSEIDON [Gaspar et al., 1994], i.e. two different algorithms, while this bias is given as 5.5% of the SWH for ERS-1 and as 2.6% of the SWH in case of GEOSAT [Gaspar and Ogor, 1994].

A-priori altimeter biases were subtracted from the altimeter height measurements in order to refer all sea surface height measurements to the same reference level. The values of the altimeter biases of TOPEX, POSEIDON, and GEOSAT, were determined from an altimeter cross calibration as discussed in Appendix C. The outcome of this cross calibration led to an altimeter bias of approximately -4 cm for TOPEX, -1 cm for POSEIDON, and 12 cm for GEOSAT, assuming an altimeter bias for ERS-1 of -42 cm [Francis, 1992].

If possible, the time tags of the altimeter observations were corrected
5.2 Used T/P, ERS-1, and GEOSAT altimeter data and applied corrections

for the inaccuracy of the spacecraft's clock by applying a timing bias to the observations. For ERS-1, a constant timing bias of -1.1 ms (altimeter observations tagged too late) was used [Scharroo and Fioberghagen, 1996]. In case of GEOSAT, altimeter normal points and crossover differences were included in the Precise Orbit Determination (POD), so that a timing bias could be estimated for each cycle separately [Chambers, 1996]. The GEOSAT timing biases are of the order of -4 ms to -5 ms. For TOPEX and POSEIDON, a timing bias of zero is assumed.

Although an apparent rise or decline of the sea level is not likely to be mistaken for a tidal signal, the altimeter observations of the satellites were corrected for oscillator drift and for the drift of the altimeter bias if such corrections were available. For GEOSAT and POSEIDON, no values were available, so that zero oscillator drift and bias drift had to be assumed. For ERS-1, the drift of the altimeter bias should be zero as it performs an internal calibration once per minute, while the oscillator drift for each cycle was obtained from ESRIN (European Space Research Institute) tables [Loial and Benveniste, 1996]. The drift of the TOPEX altimeter bias was taken from tables in Hayne [1996], which list this drift for each cycle. The TOPEX oscillator drift was taken from tables in Hancock III and Hayne [1996]. It should be noted that these oscillator drift tables for TOPEX also correct for an error in the oscillator drift algorithm which was discovered in mid 1996 [Hancock III and Hayne, 1996]. The error caused an artificial offset of about 13 cm between the sea surface as measured by TOPEX and the actual sea surface, in the sense that sea level from TOPEX was too high. Hence, the TOPEX altimeter bias as obtained from the calibration campaign was estimated approximately 13 cm too large (in an absolute sense), i.e. -17.1 cm instead of the -3.7 cm as listed in Table C.2 of Appendix C. The algorithm error also caused an artificial sea level rise of about 1 cm/yr in the TOPEX altimeter data [Hancock III and Hayne, 1996].

To model the ocean's response to atmospheric pressure changes, a simple inverse barometer (IB) response was assumed based on the FMO and ECMWF pressure fields (via the dry tropospheric correction) and relative to a global mean of 1013.3 mbar. Of concern with the IB correction is the $S_2$ component of the pressure fields as it affects the $S_2$ solution of the ocean tide. If we conform ourselves to the tradition that the $S_2$ radiational tide should be included in the ocean tide solution [Ray, 1994], then the removal of the $S_2$ atmospheric tide introduces a small error in the estimated $S_2$ ocean tide. The magnitude of this error depends on how much the 12-hour period of $S_2$ satisfies the IB assumption, but also on the interval with which the pressure fields are updated, i.e. 6 hours for TOPEX/POSEIDON and ERS-1, and 12 hours for GEOSAT. In case of TOPEX/POSEIDON and ERS-1, the 6-hour interval is precisely the Nyquist period of $S_2$ so that the $S_2$ atmospheric tide is neither completely present nor absent from the pressure fields, depending on where the $S_2$ atmospheric tide is sampled, i.e. at its peaks, at its nodes, or most likely somewhere in between. For TO-
PEX/POSEIDON, and thus also for ERS-1, Ray [1994] has shown that the pressure fields definitely contain part of the $S_2$ atmospheric tide. In case of GEOSAT, the $S_2$ atmospheric tide cannot be observed in the 12-hourly updated pressure fields. As a matter of fact, the IB correction of GEOSAT causes an offset of the measured sea level (largest at equator) with respect to the other two satellites. Again, the magnitude of this offset depends on where the $S_2$ atmospheric tide is sampled and thus will be less than 1 cm (Section 4.3). Still, whatever the magnitude of the offset may be, it will have been absorbed by the estimated altimeter bias of GEOSAT in Table 5.2 and by the estimated dynamic topography, which is to be discussed in Section 5.4. From the above discussion we have to conclude that it is unclear which part of the $S_2$ radiational tide is contained in the tidal solutions developed in this thesis, because it is neither known which portion exactly of the $S_2$ atmospheric tide is removed from the pressure fields nor how the sea level responds to $S_2$ atmospheric forcing.

With the solid-earth tide, all three satellites essentially use a second-degree expansion of the equilibrium tide with a small number of third-degree terms added in case of TOPEX/POSEIDON and GEOSAT. The permanent component of the tidal potential has not been included in the solid-earth tide correction.

Due to an error in the computation of the pole tide [AVISO, 1996], this correction as provided on the TOPEX/POSEIDON GDRs is not correct. For this reason, the pole tide was computed as an external correction to the $ssh$ residuals for all three satellites.

Table 5.3 gives an overview of the estimated errors in the altimeter measurement corrections and sea surface height corrections of Table 5.2.

<table>
<thead>
<tr>
<th></th>
<th>TOPEX</th>
<th>POSEIDON</th>
<th>ERS-1</th>
<th>GEOSAT</th>
</tr>
</thead>
<tbody>
<tr>
<td>orbit</td>
<td>3-4</td>
<td>3-4</td>
<td>7-8</td>
<td>10</td>
</tr>
<tr>
<td>ionosphere</td>
<td>-</td>
<td>2</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>troposphere</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>sea state bias</td>
<td>2</td>
<td>2</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>earth tides</td>
<td>&lt;1</td>
<td>&lt;1</td>
<td>&lt;1</td>
<td>&lt;1</td>
</tr>
<tr>
<td>altimeter noise</td>
<td>2</td>
<td>2</td>
<td>2-3</td>
<td>2-3</td>
</tr>
<tr>
<td>rss</td>
<td>4.7</td>
<td>5.1</td>
<td>8.5</td>
<td>10.9</td>
</tr>
</tbody>
</table>

Table 5.3 Accuracy (cm) of sea surface height measurements of TOPEX/POSEIDON, ERS-1, and GEOSAT.
values listed in Table 5.3 are valid for a typical SWH range of 0-2 m. No error estimates for the ocean and load tides are listed in Table 5.3 as these are the signals that are estimated from the data. Also no estimates are given for the pole tide, for errors in the altimeter and timing biases, and for oscillator and altimeter bias drift errors, either because these errors may be neglected (pole tide and timing bias errors) or because they are not likely mistaken for tidal signals (altimeter bias and drift errors). In case of the IB correction, it is unknown at present time to what extent and with what accuracy this correction removes sea level variations due to pressure change.

In Table 5.3, it may be observed that the estimated accuracy of the TOPEX and POSEIDON sea surface height measurements is about 5 cm, while it is approximately 9 cm and 11 cm for ERS-1 and GEOSAT, respectively. Obviously, the differences in the accuracy of the ssh measurements are mainly due to the differences in radial orbit errors, which are significantly larger for GEOSAT and ERS-1 than for TOPEX/POSEIDON. An important conclusion from Table 5.3 together with Table 5.4, which lists typical rms values of tidal sea level variations for ocean depths larger than 200 m (e.g. derived from the models described in Appendix B), is that the $M_2$, $S_2$, $N_2$, $K_1$, and $O_1$ tides cause variations in sea level with an $rms$ larger than or equally large as a 5 cm error in the ssh measurements. Hence, it may be expected that these tides can be estimated from the TOPEX/POSEIDON altimeter data without orbit error reduction techniques [e.g., Tai and Kuhn, 1994; Smith et al., 1995]. In case of ERS-1, and especially in case of GEOSAT, orbit errors significantly increase the error level of the ssh measurements. Although this means that tidal signals are more difficult to extract from ERS-1 and GEOSAT altimetry, no orbit error functions were estimated along the satellite groundtracks because of their tendency to remove tidal signals [Bettadpur and Eanes, 1994; Wagner and Tai, 1994; Tai and Kuhn, 1994].

### 5.3 Gridding of sea surface height measurements

The tidal normal matrices of all three satellites were set up on grids with a constant latitude and longitude spacing between the prediction loca-
tions (grid points). The latitudes of the grids are within ±66°, because TOPEX/POSEIDON delivers no observations beyond these latitudes. The procedure to grid the ssh residuals to the prediction locations is similar to the one described in Schrama and Ray [1994]. Hence, a search area of several times the grid cell size was defined around each grid point (Figure 5.1). If a residual was found to lie within the search area of a grid point, it was allowed to contribute to the normal matrix in that point. Choosing the search area large enough, one sea surface height residual may take part in the solution of several grid points, which improves the coherence between solutions in neighboring points. Still, the main purpose of defining a search area is to take advantage of the tidal decorrelations offered by adjacent and crossing groundtracks around the grid point, as was explained in Section 4.4. The introduction of a search area, however, means that detailed tidal features are averaged out, which leads to a less accurate tidal solution. Hence, to retain these features, it is useful to assign the ssh residuals inside a search area a weight $w_{\Delta ssh}$ which depends on their distance to the grid point. For the empirical weight function, a Gaussian correlation between the ssh residuals is assumed [e.g., Zandbergen, 1990]:

$$w_{\Delta ssh} = \exp \left\{ - \left( \frac{d}{D} \right)^2 \right\}$$  \hspace{1cm} (5.1)
5.3 Gridding of sea surface height measurements

<table>
<thead>
<tr>
<th>satellite</th>
<th>initial resolution</th>
<th>search area</th>
<th>scale parameter $D$</th>
<th>final resolution</th>
</tr>
</thead>
<tbody>
<tr>
<td>T/P</td>
<td>$3^\circ \times 3^\circ$</td>
<td>$9^\circ \times 9^\circ$</td>
<td>$3^\circ$</td>
<td>$1^\circ \times 1^\circ$</td>
</tr>
<tr>
<td>GEOSAT</td>
<td>$2^\circ \times 2^\circ$</td>
<td>$6^\circ \times 6^\circ$</td>
<td>$2^\circ$</td>
<td>$1^\circ \times 1^\circ$</td>
</tr>
<tr>
<td>ERS-1</td>
<td>$1^\circ \times 1^\circ$</td>
<td>$3^\circ \times 3^\circ$</td>
<td>$1^\circ$</td>
<td>$1^\circ \times 1^\circ$</td>
</tr>
</tbody>
</table>

Table 5.5 Resolution and gridding parameters, i.e. the size of the search area and the value of the scale parameter $D$, of the single-satellite ocean tide solutions from TOPEX/POSEIDON (T/P), GEOSAT, and ERS-1 altimetry.

where $d = \sqrt{ (\delta \lambda \cos \phi)^2 + \delta \phi^2 }$ is the angular distance, i.e. in degrees, between the observation location at $(\lambda, \phi)$ and the grid point. The larger the distance $d$, the less the observation will affect the solution at the grid point. Notice that $\delta \lambda$ is weighed according to latitude to account for the fact that the angular distance in longitude direction corresponds to a smaller distance in kilometers at higher latitudes. The scale parameter $D$ prescribes how fast the weight decreases with distance. If $D$ is chosen small, then the weight rapidly decreases for points far away from the grid point and hence such a weighing scheme would result in point values. In contrast, if $D$ is selected very large, all points inside the search area are assigned about an equal weight of one and the scheme would be a simple averaging over the observations inside the search area. Some experiments showed that a search area of three times the cell size in combination with $D$ equal to the cell size ($\Delta \lambda = \Delta \phi$ in Figure 5.1) leads to good results.

The initial resolution of the tidal grids on which the single-satellite normal matrices were stored, i.e. before interpolation to $1^\circ \times 1^\circ$ resolution, as well as the parameters of the gridding procedure, i.e. the size of the search area and the value of the scale parameter $D$, are listed in Table 5.5. Hence, in case of e.g. TOPEX/POSEIDON, Table 5.5 shows that a tidal grid was set up with a cell dimension of $3^\circ$ in latitude and $3^\circ$ in longitude and that all observations within a search area of $9^\circ \times 9^\circ$ were gridded to the center of the $3^\circ \times 3^\circ$ cell. The initial resolutions of the tidal grids are chosen approximately equal to the groundtrack spacings of the satellites (Appendix A), which dictate the smallest tidal wavelength that each of the satellites can observe. For ERS-1 with its $1^\circ$ groundtrack spacing, which offers the best resolution of all three satellites, this wavelength equals $2^\circ$. Hence, with the relatively coarse $3^\circ \times 3^\circ$ resolution of the TOPEX/POSEIDON altimetry, we can only observe the large-scale tidal features in the deep oceans, which have wavelengths of typically $10^\circ$ [e.g., Ray, 1994]. With the $2^\circ \times 2^\circ$ and $1^\circ \times 1^\circ$ resolutions of the GEOSAT and ERS-1 altimetry, however, we can observe the smaller wavelength tidal features as well. With regard to ocean depths larger than 200 m, these smaller features exist in the transition region from shallow waters to the deeper parts of the oceans, i.e. the region of ocean depths between some 200 m and 1000 m [e.g., Shum et al.,
1997], but they may also exist in the deeper parts of the oceans over areas of steep topography such as seamounts and ridges [e.g., Kantha, 1995]. To optimally take advantage of the resolutions of the GEOSAT and ERS-1 altimetry in the multi-satellite tidal solution, the single-satellite tidal normal matrices and not the single-satellite tidal solutions must be combined (see Section 9.2). Hence, the tidal normal matrix grids of TOPEX/POSEIDON and GEOSAT are interpolated to the resolution of the ERS-1 altimetry, i.e. 1° × 1°, which defines the smallest tidal wavelength that can be observed in the multi-satellite solution. Therefore, the “final” resolution of the normal matrix grids is 1° × 1° for all satellites, as shown in Table 5.5. In Section 9.2, the validity of the interpolation of the normal matrices will be discussed.

5.4 Estimated parameters

With the 1° × 1° grids with tidal normal matrices, the vector of unknowns includes the terms on the right side of (4.1) in Section 4.2, i.e. the sum of the ocean and load tides, ζ + ζ_{ol}, the seasonal cycles of ocean variability ζ_{sa} (annual) and ζ_{sam} (semi-annual), and the time-invariant surface of the dynamic topography plus geoid errors, h_0 + Δh_g. The term Δh_g is included to model errors in the OSU91A geoid relative to which the sea surface height residuals are defined. Hence, the modeled part of the sea surface height residuals (4.1) may be formulated as:

$$\Delta ssh = \zeta + \zeta_{ol} + \zeta_{sa} + \zeta_{sam} + h_0 + \Delta h_g$$

(5.2)

The tides that we will concentrate on are the diurnal and semi-diurnal barotropic ocean+load tides. In case of the harmonic solution, the tides above a 5 cm noise level in the ssh measurements, i.e. M_2, S_2, N_2, K_1, and O_1 (see Table 5.4) were estimated according to expression (3.15). The corresponding nodal corrections were derived from expressions in Melchior [1966], Franco [1981], and Pugh [1987]. With the response solution, the diurnal and semi-diurnal admittances were estimated through the weights u_{ams} and v_{ams} in (3.21). As will be demonstrated in Chapter 7, the admittances can be adequately modeled with lags sΔT = 0, 2, 4 days. No long-period tides were estimated. Instead, these tides were removed from the data by assuming an equilibrium response.

With both the harmonic and response method, the seasonal cycles are modeled similar to the tides (3.15). Hence, in case of the annual signal, the parameterization becomes:

$$\zeta_{sa} = C_{Sa}(\lambda, \phi) \cos(\hat{\theta}_{sa} t) + S_{Sa}(\lambda, \phi) \sin(\hat{\theta}_{sa} t)$$

(5.3)

where \hat{\theta}_{sa} denotes the annual frequency. For the semi-annual cycle, a similar expression has been used. When expressed in days, the phases of the estimated seasonal cycles as derived from the harmonic sine and cosine constants refer to the beginning of the year. The purpose of estimating the
seasonal cycles is to reduce the correlation problems in which these cycles are involved (see Tables 4.3 to 4.5) so that the tidal observations of the three satellites may be optimally used in the multi-satellite tidal solution as will be explained in Section 9.2.

Because the geoid was chosen as the constant, i.e. time-invariant, surface to which the sea surface height residuals are referenced, these residuals will contain the sum of the dynamic topography and geoid errors. Although a constant surface is not likely to be mistaken for a tidal signal, the estimated tidal parameters may absorb part of this surface. Especially the estimated sine and cosine constants of the harmonic method are expected to have this potential [e.g., Ma et al., 1994]. Therefore, it was decided to simultaneously estimate a bias term for the geoid errors and the dynamic topography along with the tide and the seasonal cycles:

\[ h_0 + \Delta h_g = a_0 \]  \hspace{1cm} (5.4)

Because the OSU91A geoid does not contain any tidal deformation [Rapp et al., 1991], whereas the permanent tide of the solid earth was not removed as part of the ssh corrections, the estimated dynamic topography will contain both, the permanent tide due to the direct effect of the tide-generating potentials of the sun and the moon, i.e. \( U_{055,55}/g \), and the permanent tide induced by the solid-earth deformation, i.e. \( k_2 U_{055,55}/g \) (Section 2.7). Notice that if the mean sea surface would have been selected as the reference surface of the ocean tide by subtracting a dynamic topography model from the sea surface height residuals, the bias term (5.4) would model corrections to the dynamic topography model instead of the dynamic topography itself. Still, if no constraints are put on the estimated parameters, adding these corrections to the reference model will give the same mean sea surface solution as would be obtained by solving for this surface directly.

With regard to the above discussion, the system of normal equations \( Nx = b \) in a 1° \( \times \) 1° grid cell of the single-satellite tidal solutions may be written as:

\[
\begin{bmatrix}
N_{\zeta\zeta} & N_{\zeta\delta} & N_{\zeta\alpha_0} \\
N_{\delta\zeta} & N_{\delta\delta} & N_{\delta\alpha_0} \\
N_{\alpha_0\zeta} & N_{\alpha_0\delta} & N_{\alpha_0\alpha_0}
\end{bmatrix}
\begin{bmatrix}
x_{\zeta} \\
x_{\delta} \\
a_0
\end{bmatrix}
= 
\begin{bmatrix}
b_{\zeta} \\
b_{\delta} \\
a_0
\end{bmatrix}
\]  \hspace{1cm} (5.5)

where for the cross-term blocks \( N_{ij} \, (i \neq j) \), the relation \( N_{ij} = N_{ji} \) holds. Apart from the estimated dynamic topography \( a_0 \) and the four parameters of the seasonal cycles \( x_s \), the harmonic solution includes five tides in \( x_{\zeta} \), each modeled with two harmonic constants, while the response solution includes three complex diurnal and three complex semi-diurnal weights in \( x_{\zeta} \). Hence, with both solution methods, an equivalent number of parameters is estimated, i.e., 15 in case of the harmonic solution and 17 in case of the response solution.
Chapter 6

Covariance analyses of single-satellite harmonic tidal solutions

6.1 Introduction

In Chapter 4, it has been discussed that tidal phase advance differences on crossing groundtracks are useful to reduce the tidal correlations. In this chapter, results are presented of covariance analyses of the single-satellite harmonic solutions of TOPEX/POSEIDON, ERS-1, and GEOSAT, to quantify these correlations. The harmonic solutions themselves are not presented here but in Chapter 8. Obviously, special attention will be given here to those tides that are involved in the correlation problems listed in Table 4.4 and Table 4.5, because these correlations should be small enough to use the ERS-1 and GEOSAT altimeter observations in a multi-satellite tidal solution.

Extensive covariance analyses of the single-satellite response solutions have not been performed. Such analyses are less useful because the correlations among the estimated response weights have little meaning, whereas the correlations between the response weights and the other estimated parameters depend on a number of tidal lines in a band, instead of on a single tidal line as with the harmonic analysis and therefore are more difficult to interpret. However, it may be mentioned that correlations of the order of 50% were found among the response weights of the same band. For the correlations between the response weights of different bands, values of less than 5% were found. In general, values of the same order of magnitude, i.e. 1-5%, were also found for the correlations between the response weights and the seasonal cycles, and for the correlations between the response weights and the estimated dynamic topography. With ERS-1, large correlations were found between the semi-diurnal response weights and the estimated topography, which are obviously due to the aliasing of \( S_2 \), but also between the semi-diurnal response weights and the semi-annual cycle. These correlations will be briefly discussed in
The normal matrices of the harmonic tidal solutions that were stored on the $1^\circ \times 1^\circ$ grids (see Sections 5.1 and 5.3) were inverted to obtain the covariance matrices. For each $1^\circ \times 1^\circ$ grid cell, the covariance matrix carries the formal errors of the solution, i.e. the standard deviations and the correlations. The magnitudes of the correlations depend on the sampling of the tides along the satellite groundtrack and on possible non-orthogonalities of the tidal signals. Correlations due to non-orthogonality arise whenever the product of two tidal signals integrated over the analysis period is non-zero (notice that when we speak of tidal signals or tidal pairs, the seasonal cycles are also understood). This may happen when there are data gaps or when the alias periods of the two tidal signals do not both fit an integer times in the analysis period. Depending on the times at which the tides are sampled, these correlations may become larger or smaller. Obviously, correlations because of non-orthogonality become less important for tidal pairs that have a large number of Rayleigh periods in the analysis period. Hence, the correlations will eventually vanish if we keep adding more data. If the observations have not been weighed with the data noise, then the standard deviations depend on the number of observations and on the correlations. The more observations are used, the smaller the standard deviations become, while a large correlation means that two tides are difficult to separate, which results in larger standard deviations.

Because the formal errors directly depend on the amount of data included in the analysis, they may not be used to assess the accuracy of the tidal solutions. However, the formal errors do provide a suitable means to verify the integrity of the solutions, i.e. if the formal errors show a systematic geographic pattern, it tells us that the involved tides can be estimated more accurately in those areas where the standard deviations and correlations are small than in areas where the standard deviations and correlations are large. Hence, the formal errors are ideally suited to verify the tidal decorrelation patterns of the phase advance differences on crossing groundtracks as predicted by Tables 4.3 to 4.5.

6.2 Number of sea surface height observations

To explain the geographic pattern of the standard deviations in the following sections, Figure 6.1 shows the weighted number of TOPEX/POSEIDON, GEOSAT, and ERS-1 ssh residuals inside each search area of $9^\circ \times 9^\circ$, $6^\circ \times 6^\circ$, and $3^\circ \times 3^\circ$, respectively, around the center of a grid cell (Section 5.3). The weighted number of residuals corresponds to the $N_{wog}$ element in (5.5) of the interpolated tidal normal matrices (Section 5.1) so that the resolution of the grids in Figure 6.1 is $1^\circ \times 1^\circ$. The ssh residuals were gathered during the periods listed in Table 5.1. Notice that with increas-
Figure 6.1 Weighted number of $ssh$ residuals from the gridding procedure in $9^\circ \times 9^\circ$, $6^\circ \times 6^\circ$, and $3^\circ \times 3^\circ$ cells of TOPEX/POSEIDON (top), ERS-1 (middle), and GEOSAT (bottom) altimeter data, respectively. Notice the differences in scale.
ing repeat period, the number of ssh residuals in a grid cell becomes less. Hence, the largest number of ssh residuals is found with TOPEX/POSEIDON.

Generally, due to the convergence of the satellite groundtracks, more ssh residuals are found towards the turnover latitudes than near the equator, which is especially clear with TOPEX/POSEIDON. With ERS-1, the data outages in the North Atlantic and near 60° south are due to limitations of the tape capacity to record all altimeter data between consecutive periods of telemetering these data down to one of the ground stations once every orbit [ESA, 1992]. In case of GEOSAT, an attitude problem associated with a solar radiation torque caused off-nadir excursions of the altimeter with a maximum disturbance once every five hours [Cheney et al., 1991; Sandwell and McAdoo, 1990]. When these excursions exceeded 1°, the altimeter could not lock on to the returned pulse so that the data could not be recovered. This led to five evenly-spaced bands (a 5-hour period equals about one fifth of the earth's rotation) in which there are less altimeter observations [Cheney et al., 1991]. As solar activity increased dramatically during 1988, loss of data due to the solar radiation torque became worse towards the end of the ERM.

Near the coast and around island groups, the number of ssh residuals is seen to decrease by about 50% for all three satellites. The same is true for the Antarctic seas, i.e. from 80° west to 180° east longitude and below 60° south, due to the yearly growth and decay of Antarctic sea ice [Smith and Andersen, 1997]. Especially the Weddell Sea in the South Atlantic is affected by the yearly motion of the ice edge, which reaches latitudes as high as 60° south during winter in the southern hemisphere [Guzkowska
As an example, Figure 6.2 shows the weighted number of TOPEX/POSEIDON ssh residuals for each cycle in the $9^\circ \times 9^\circ$ grid cell centered at $-18^\circ$ longitude and $-63^\circ$ latitude. Clearly visible are the summer periods in the southern hemisphere, e.g. Cycles 9-27 (11 December 1992 - 17 June 1993) and the winter periods during which the ice cover may lead to a total absence of altimeter observations at Antarctic latitudes. Notice that Cycles 102-117 coincide with the winter period in the southern hemisphere so that Figure 6.2 does not go beyond Cycle 101.

### 6.3 Covariance analysis of TOPEX/POSEIDON harmonic solution

Table 6.1 presents the standard deviations and correlations of the estimated tides, the seasonal cycles, and of the dynamic topography of the TOPEX/POSEIDON harmonic tidal solution. In this table, the standard deviation $\sigma$ of a tide has been defined as the *rms* of the standard deviations of its harmonic cosine and sine constants, $\sigma_C$ and $\sigma_S$, respectively:

$$\sigma = \sqrt{\frac{1}{2}(\sigma_C^2 + \sigma_S^2)}$$

(6.1)

Likewise, the correlation $\rho$ between two tides has been defined as the *rms* of the correlations between their harmonic constants $\rho_{ij}$, where $i$ is one of the harmonic constants of the first tide and $j$ is one of the harmonic constants of the second tide:

$$\rho = \sqrt{\frac{1}{4}(\rho_{C1,C2}^2 + \rho_{C1,S2}^2 + \rho_{S1,C2}^2 + \rho_{S1,S2}^2)}$$

(6.2)

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Table 6.1 Standard deviations in cm (on the diagonal) and correlations (off the diagonal) as computed over the area $-180^\circ < \lambda < -150^\circ$, $-60^\circ < \phi < 50^\circ$ for the TOPEX/POSEIDON harmonic tidal solution. Boxed numbers in bold refer to Table 4.3. $S_a$ and $S_{sa}$ are the annual and semi-annual cycle, respectively, whereas $a_0$ denotes the dynamic topography.
Figure 6.3 Standard deviations of the TOPEX/POSEIDON harmonic tidal solution (cm). Denoted by $S_a$ and $S_{sa}$ are the annual and semi-annual cycle, respectively, whereas $\alpha_0$ denotes the dynamic topography. Values representative of the global $rms$ of the depicted standard deviations are given in Table 6.1. Notice the differences in scale.
Figure 6.4Correlations in which the dominant $M_2$, $S_2$, and $K_1$ tides and the semi-annual cycle $S_{sa}$ of the TOPEX/POSEIDON harmonic tidal solution are involved. Values representative of the global rms of the depicted correlations are given in Table 6.1. Notice the differences in scale.

In Figure 6.3, the global maps of the standard deviations are presented. The standard deviations are obviously related to the (weighted) number of $ssh$ residuals in Figure 6.1. Hence, large values are found near the coast and in the Antarctic seas, which means that the tidal solutions in these areas are less accurate. Along the coast, standard deviations may occasionally exceed 10 cm. The formal errors in Table 6.1 have been computed over the area $-180^\circ < \lambda < -150^\circ$, $-60^\circ < \phi < 50^\circ$, which has been selected to exclude the large standard deviations in the Antarctic seas and along the coast. Because this local area covers most latitudes, the given numbers will not differ much from their global values, excluding coastal areas and the Antarctic seas, as the number of observations and the ability of the phase advance differences on crossing tracks to decorrelate the tides are both dictated by latitude alone. Indicated in bold in Table 6.1 are the correlations (boxed numbers) and associated standard deviations of the tidal pairs of Table 4.3. In case of the $M_2$/$S_2$ pair, a small correlation of 0.01, i.e. 1%, is found. This is because the TOPEX/POSEIDON altimeter data span almost exactly three years, which equals the Rayleigh period of these tides. Hence, besides a full decorrelation over this period, the non-orthogonalities are also very small. Still, they are clearly visible as the pattern of groundtracks along which the tides are sampled in Figure 6.4, which presents the global correlation maps of $M_2$/$S_2$ and $K_1$/$S_{sa}$. As expected, the pattern of interleaved latitude bands in Table 4.3 does not show up in the $M_2$/$S_2$ correlation map as the phase advance difference on crossing tracks of $M_2$ and $S_2$ is below $90^\circ$ at all latitudes. In case of the $K_1$/$S_{sa}$ pair, a much larger correlation of 19% is found in Table 6.1 because of their much longer Rayleigh period, i.e. the $K_1$ tide and the semi-annual cycle are far from orthogonal over three years. The correlation map of $K_1$ and $S_{sa}$ in Figure 6.4 clearly shows the predicted pattern of Table 4.3, i.e. the decorrelation towards the equator increases because of an increasing phase advance difference on crossing tracks in that direction.
(Figure 4.9). As a result of their larger correlation, the standard deviations of \( K_1 \) and the semi-annual cycle are slightly larger than those of the \( M_2 \), \( S_2 \), and \( N_2 \) tides and that of the annual cycle (Table 6.1). The somewhat larger standard deviation of \( O_1 \) is found because the standard deviations of the tides are inversely proportional to their nodal factors and it may be shown that \( O_1 \) has the smallest nodal factor over the TOPEX/POSEIDON analysis period. The global maps for the standard deviations of \( K_1 \) and the semi-annual cycle in Figure 6.3 show that up to latitudes of \( 30^\circ \), the standard deviations agree with the number of observations. Beyond these latitudes, the standard deviations become larger because of the increasing correlation, \( i.e. \) \( K_1 \) and the semi-annual cycle become more difficult to separate at higher latitudes.

Notice the small correlations of the dynamic topography with the tides and with the seasonal cycles in Table 6.1. Also notice the somewhat larger correlation of \( K_1 \) and the annual cycle in Table 6.1 due to their Rayleigh period of almost a year, which is relatively long with respect to three years so that non-orthogonalities cause a slight correlation. For the same reason, a somewhat larger correlation is found for \( S_{sa}/S_a \). It may be shown that the correlation of this pair is largest in the Antarctic seas where ice cover causes a temporal absence of observations leading to large non-orthogonalities for \( S_{sa}/S_a \). Because of this correlation, the standard deviations of the seasonal cycles in the Antarctic seas are significantly increased, which means that less accurate solutions are obtained for the seasonal cycles in these seas.

### 6.4 Covariance analysis of ERS-1 harmonic solution

Table 6.2 presents the standard deviations and correlations of the har-

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Figure 6.5 Standard deviations of the ERS-1 harmonic tidal solution (cm). Denoted by $S_a$ and $S_{sa}$ are the annual and semi-annual cycle, respectively. Values representative of the global rms of the depicted standard deviations are given in Table 6.2.

monic tidal solution from ERS-1 altimetry. As explained in the previous section, the numbers in this table are for the area $-180^\circ < \lambda < -150^\circ$, $-60^\circ < \phi < 50^\circ$. Global maps of the standard deviations are presented in Figure 6.5. Obviously, the standard deviations in this figure show the same global pattern as the number of observations in Figure 6.1. From the boxed numbers in Table 6.2 we notice that a correlation of almost 100% is found for $S_2$ and the $a_0$ parameter of the dynamic topography. The correlation between $S_2$ and $a_0$ was found to increase from 98% at the turnover latitudes to 100% at the equator so that the direction of decorrelation is as predicted by Table 4.4. Because of this correlation,
huge standard deviations over 7000 cm were found for $S_2$ and $a_0$, which were also found to increase towards the equator. The standard deviations of $S_2$ and $a_0$ are not presented in Figure 6.5. In case of the $M_2/N_2$ pair, a correlation of 15% is found. In Figure 6.6, the predicted band structure of Table 4.4 is clearly noticed. In bands where the phase advance difference of $M_2$ and $N_2$ in Figure 4.10 exceeds 90°, the correlation is approximately a factor two times smaller. The interleaved band pattern can also be seen in the standard deviations of $M_2$ and $N_2$ in Figure 6.5, i.e. bands with larger correlations are accompanied by a larger standard deviation. In case of the $K_1/S_a$ pair, a surprisingly small correlation of 8% is found with regard to their infinitely large Rayleigh period (Table 4.1). This is because the phase advance difference of $K_1$ and the annual cycle is larger than 130° up to latitudes of 66° (Figure 4.10). This clearly demonstrates the enormous potential of the phase advance differences on crossing tracks to decorrelate two tides if the analysis period is less than their Rayleigh period. The predicted decorrelation pattern of $K_1/S_a$ in Table 4.4 is confirmed by Figure 6.6. Notice the small correlation of $K_1/S_a$ in Table 6.2, which should be helpful to decorrelate these signals with TOPEX/POSEIDON and GEOSAT at latitudes beyond 55°. In Table 6.2, also notice the somewhat larger correlation between the seasonal cycles of which the reason has been explained in the previous section. In the Antarctic seas, this correlation may be shown to increase up to 90% leading to standard deviations of over 50 cm for both of the seasonal cycles.

6.5 Covariance analysis of GEOSAT harmonic solution

Table 6.3 presents the formal errors of the GEOSAT harmonic tidal solution. Global maps of the standard deviations are displayed in Figure 6.7, whereas the correlation maps are given in Figure 6.8. Notice that the same
Table 6.3 Standard deviations in cm (on the diagonal) and correlations (off the diagonal) as computed over the area $-180^\circ < \lambda < -150^\circ$, $-60^\circ < \phi < 50^\circ$ for the GEOSAT harmonic tidal solution. Boxed numbers in bold refer to Table 4.5. $S_a$ and $S_{sa}$ are the annual and semi-annual cycle, respectively, whereas $a_0$ denotes the dynamic topography.

structure as in Figure 6.1 can be seen for all standard deviations in Figure 6.7. From Table 6.3, the correlation between $M_2$ and the annual cycle is found to be 17%. The predicted band structure of Table 4.5 and Figure 4.11 is confirmed by Figure 6.8 where bands with a phase advance difference larger than 90° are seen to have a correlation for $M_2/S_a$ about three times as small as latitude bands with a phase advance difference less than 90°. In Figure 6.7, the same band structure can be seen with the standard deviations of $M_2$ and the annual cycle. In case of the $S_2/K_1/S_{sa}$ triad, relatively large correlations of 37% and 59% are found in Table 6.3. These correlations, which are much larger than in case of TOPEX/POSEIDON and ERS-1, can be explained by the magnitude of the Rayleigh periods in Table 4.1 compared to the 2-year GEOSAT analysis period in case of $K_1/S_{sa}$ and $S_2/K_1$, and by the very small amount of decorrelation from the phase advance differences on crossing groundtracks in case of $S_2/S_{sa}$. The global correlation maps in Figure 6.8 show that in case of the $S_2/K_1$ pair, the decorrelation increases towards the equator due to the increasing tidal phase advance difference in Figure 4.11. For the same reason, the decorrelation of the $K_1/S_{sa}$ couple also increases towards the equator. Notice that the $K_1/S_{sa}$ correlation as observed by GEOSAT at higher latitudes is approximately 60%. As mentioned in Section 4.6, this means that a $K_1/S_{sa}$ phase advance difference on crossing tracks of 90° to 110° at latitudes between 55° and 60° is not large enough to decorrelate $K_1$ from the semi-annual cycle within an analysis period of about two years. In case of $S_2/S_{sa}$, the phase advance difference only exceeds 90° between latitudes of 55° and 70° (Figure 4.11) so that crossing tracks can do little to decorrelate $S_2$ from the semi-annual cycle as predicted by Table 4.5. Hence, for the $S_2/S_{sa}$ correlation pattern, an almost constant value of about 60% is observed in Figure 6.8. The fact that crossing tracks cannot help to decorrelate $S_2$ and $S_{sa}$ also explains why the correlation is larger than that of
Figure 6.7 Standard deviations of the GEOSAT harmonic tidal solution (cm). Denoted by $S_a$ and $S_{aa}$ are the annual and semi-annual cycle, respectively, whereas $a_0$ denotes the dynamic topography. Values representative of the global $rms$ of the depicted standard deviations are given in Table 6.3. Notice the differences in scale.
6.6 Discussion

Figure 6.8 Correlations in which the dominant $M_2$, $S_2$, and $K_1$ tides and the annual and semi-annual cycles, $S_a$ and $S_{sa}$, respectively, of the GEOSAT harmonic tidal solution are involved. Values representative of the global rms of the depicted correlations are given in Table 6.3. Notice the differences in scale.

$S_2/K_1$ and $K_1/S_{sa}$ (Table 6.3). In Figure 6.7, the $K_1$ standard deviation is seen to increase towards the turnover latitudes as its correlation with both $S_2$ and the semi-annual cycle increases in this direction. The standard deviations of $S_2$ and the semi-annual cycle are seen to be dominated by the correlation of $S_2/S_{sa}$, i.e. the correlation of $S_2/K_1$ and that of $K_1/S_{sa}$ are less strong and therefore not present in the standard deviations of $S_2$ and the semi-annual cycle. Because the $S_2/S_{sa}$ correlation is almost constant over the globe, the standard deviations of $S_2$ and the semi-annual cycle do not vary with latitude. Hence, more accurate estimates may be obtained towards the equator for $K_1$ but not for $S_2$ because of its large correlation with the semi-annual cycle. Notice the larger correlations of $S_2/S_{sa}$, $K_1/S_{sa}$, and $S_{sa}/S_a$ in Table 6.3, which all have a Rayleigh period close to a year. Their correlation patterns were found to be similar to that of the GEOSAT ssh observations so that the reduced number of observations in the five bands of Figure 6.1 must play an important role in causing these correlations.
6.6 Discussion

The standard deviations and correlations of the harmonic ocean tide solutions of TOPEX/POSEIDON, ERS-1, and GEOSAT confirm the latitude patterns of tidal decorrelations from crossing satellite groundtracks. Hence, the formal errors predict a less accurate tidal solution at latitudes where a small phase advance difference is observed. Because of the tidal phase advance differences, the correlation problems that especially arise with ERS-1 and GEOSAT can be largely solved. With the dominant tides, global correlations between 10% and 40% are found in general from approximately two years of ERS-1 and two years of GEOSAT altimetry. Hence, although observing the ocean tides was not part of their orbit design, ERS-1 and GEOSAT do give valuable tidal information in addition to TOPEX/POSEIDON. For this reason, the altimeter observations of ERS-1 and GEOSAT may improve the TOPEX/POSEIDON tidal solution, provided that the tidal signal is strong enough relative to the altimeter background noise.

With TOPEX/POSEIDON, the phase advance difference of \( M_2 \) and \( S_2 \) being less than 90° at all latitudes makes that no interleaved band pattern can be seen in the formal errors of these tides. In case of \( K_1 \) and the semiannual cycle, the formal errors increase towards the turnover latitudes because of a decrease in their phase advance difference in that direction. The correlation of \( K_1 \) and the semi-annual cycle as observed by TOPEX/POSEIDON is of importance beyond latitudes of 55°, where it has a value of 50-60%.

In case of ERS-1, the formal errors show that \( S_2 \) can definitely not be separated from the dynamic topography. In case of \( K_1 \) and the annual cycle, the phase advance difference of more than 130° up to 66° of latitude largely decorrelates these signals, i.e. in spite of their infinitely large Rayleigh period, a global average correlation of only 8% is computed from about two years of data. The \( K_1 \) estimate of ERS-1 may help to improve the \( K_1 \) estimate of TOPEX/POSEIDON at higher latitudes, provided that the noise level of the ERS-1 altimetry is small enough. With \( M_2 \) and \( N_0 \), formal errors are found to increase in latitude bands where a phase advance difference significantly less than 90° is observed.

With GEOSAT, the formal errors of \( M_2 \) and the annual cycle increase in latitude bands where a phase advance difference less than 90° is observed. With the \( S_2/K_1/S_{sa} \) triad, the formal errors predict more accurate \( K_1 \) estimates towards the equator. Towards the turnover latitudes, less accurate \( K_1 \) estimates will be obtained from GEOSAT data because of a correlation of about 60% with both \( S_2 \) and the semi-annual cycle. The predicted accuracy of \( S_2 \) does not vary with latitude as it is determined by its correlation with the semi-annual cycle, which has an almost constant global value of 60% using approximately two years of observations.
Chapter 7

Experiments with the response formalism

7.1 Introduction

In this chapter, results are presented of some experiments with the response formalism (3.21). With these experiments, it was investigated whether there are certain values of the lag interval $\Delta T$ and the number of lags $L$ which best describe the admittance curve. For both $\Delta T$ and $L$ there exists no straightforward criterion so that trial and error seems the obvious approach, bearing in mind that the number of lags should be kept sufficiently small as not to violate the credo of smoothness. Munk and Cartwright [1966] chose for a lag interval $\Delta T$ of 2 days based on observations from a Honolulu tide gauge station. Results in this chapter, however, demonstrate that their choice of 2 days may have led to an optimum solution for this specific station, but that a different value like 1 or 3 days may give better results for other stations. Moreover, it is shown that an optimum value for the lag interval that gives the most accurate response solution for a given number of weights does not exist in a global sense.

It is emphasized that this chapter is concerned with experiments with the response formalism of which the results should justify the choice of the lag interval and of the number of response weights to be used in this thesis. For this reason, only results of the four dominant diurnal tides and semi-diurnal tides that have been derived from the admittances are discussed here. With the chosen values of the lag interval and the number of response weights, the single-satellite and multi-satellite response solutions in full, which account for 23 tidal lines, are presented and discussed in Chapter 8 and Chapter 9, respectively.

All response tidal solutions in this chapter, as well as the harmonic and response solutions to be discussed in Chapters 8 and 9, are masked for an ocean depth of 200 m using the ETOPO5 bathymetry [National Geophysical Data Center, 1993] to avoid inaccurate tidal predictions in shallow waters due to non-linear tides and short-wavelength tidal features. The former are not included in the linear formulations (3.15) and (3.21). Modeling of
the latter would require a grid resolution of the order of 0\degree 1 [Le Provost et al., 1994; Kantha, 1995], which is much finer than even the 35-day repeat orbit of ERS-1 can offer. Masking of the tidal grids was found to remove approximately 10% of the 1\degree x 1\degree grid cells.

7.2 Response solution parameterizations

To investigate whether there are optimum values of the lag interval and the number of lags, five model parameterizations of (3.21) have been tried, which are listed in Table 7.1. All five response solutions solve for the diurnal and semi-diurnal admittances of the ocean-load tide from three years of TOPEX/POSEIDON altimetry. The tidal normal matrix interpolation procedure to be discussed in Section 9.2 was used to create tidal solutions with a resolution of 1\degree x 1\degree. The TOPEX/POSEIDON satellite was chosen because it can resolve and separate all major tides within three years (with slight interference of K1 and the semi-annual cycle beyond latitudes of about 55\degree), which obviously contributes to a clear interpretation of the results. Models A to C all have the same number of lags L of 3 but vary in lag interval from \Delta T = 1 day for model A to \Delta T = 3 days in case of model C, i.e. one day less and one day more than the value of \Delta T = 2 days suggested by Munk and Cartwright [1966]. Notice from the fourth column of Table 7.1 that the admittance curve is strictly modeled with positive time lags, which means that the observed tide is expressed as a weighted sum of past values of the equilibrium tide. Negative lags associated with future values of the equilibrium tide as were used in, e.g., Cartwright and Ray [1990b] were not considered realistic. In Section 7.6 it will be discussed why the admittance can be resolved from positive lags only. Also notice that for convenience, only integer values of \Delta T were considered. In Section 7.6 it is explained why non-integer lag intervals as well as a lag interval larger than three days will give the same result for the admittance. A similar conclusion was arrived at by Munk and Cartwright [1966]. Models B, D, and E, all have the same lag interval \Delta T = 2 days but differ in number of lags from L = 2 in case of model D to L = 4 in case of model E.

To investigate the accuracy of the tidal models in Table 7.1, the estimated response weights were transformed to harmonic constants at the four dominant frequencies in the diurnal and semi-diurnal bands using (3.23) and (3.31). The harmonic constants were then submitted to two tests.

First of all, the harmonic constants of the dominant tides were compared with those of 84 tide gauge stations selected from the tide gauge set of Le Provost [1994]. No distinction was made between surface tide gauges (tide pole concept), of which the harmonic constants include the radiational tide, and bottom pressure gauges, in which the radiational tide is


<table>
<thead>
<tr>
<th>model</th>
<th>$\Delta T$ (days)</th>
<th>$L$</th>
<th>$s\Delta T$ (days)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A$</td>
<td>1</td>
<td>3</td>
<td>(0, 1, 2)</td>
</tr>
<tr>
<td>$B$</td>
<td>2</td>
<td>3</td>
<td>(0, 2, 4)</td>
</tr>
<tr>
<td>$C$</td>
<td>3</td>
<td>3</td>
<td>(0, 3, 6)</td>
</tr>
<tr>
<td>$D$</td>
<td>2</td>
<td>2</td>
<td>(0, 2)</td>
</tr>
<tr>
<td>$E$</td>
<td>2</td>
<td>4</td>
<td>(0, 2, 4, 6)</td>
</tr>
</tbody>
</table>

Table 7.1 Parameter specification of the response solutions used to determine the optimum description of the admittance.

not included in the harmonic constants. To ensure compatibility with the harmonic constants from the tide gauges, the load tide derived from the FES95.2.1 ocean tide model [Le Provost et al., 1994] was subtracted from the harmonic constants of the response tidal models. Although this load tide is not consistent with the response solutions, no significant discrepancies are expected.

Secondly, the harmonic constants of the response tidal models were compared with those of the FES95.2.1 ocean+load tide solution on a grid with $1^\circ \times 1^\circ$ resolution. Although FES95.2.1 may contain bathymetry-induced errors and some regional errors associated with the basin-wise solution procedure of this model [Le Provost, 1996; Smith and Andersen, 1997], it provides a valuable reference, i.e. one that is largely independent of altimetry (see also Section 8.2.4 for details on possible errors in FES95.2.1).

The 84 stations were selected on basis of the following criteria. First of all, interpolation of harmonic constants to a given place is performed with simple bilinear interpolation (see also Section 9.2). This means that the place must be surrounded by four valid grid points. As some of the grid points of the FES95.2.1 ocean tide model are on small islands, this excluded four stations. Secondly, masking of the response tidal models for an ocean depth of 200 m excluded another eight stations. Thirdly, harmonic constants must be available for all eight considered tides to warrant a fair comparison. Five stations with an incomplete set of harmonic constants were deleted. Fourthly, two coastal stations were removed because of extremely large discrepancies between their harmonic constants and all semi-diurnal tides of the five response solutions. These stations were found to lie on the European Continental Shelf and in the Drake Passage near South Georgia Island. As the semi-diurnal tides show complex short-wavelength features in these areas, the coarse resolution of the TOPEX/POSEIDON altimetry is unable to model these features, which requires a much finer resolution, probably of the order of a few tenths of a degree. Displayed in Figure 7.1 are the remaining stations, which show a fairly well global distribution, i.e. 12 stations in the Indian Ocean, 35 in the Atlantic, and 37 in the Pacific. This reference set of 84 tide gauges will be used at various occasions throughout this thesis.
Figure 7.1 Positions of 84 selected tide gauges for the evaluation of the response tidal model parameterizations.

7.3 Varying the lag interval

For the tidal models with the varying lag interval, i.e., models A, B, and C ($L = 3, \Delta T = 1, 2, 3$ days), the results of the two tests are displayed in Figure 7.2. The top plots present the \textit{rms} tide gauge differences, i.e., the differences between the tidal models and the harmonic constants of the tide gauge stations, for the four major diurnal and semi-diurnal tides as well as the root-sum-square (\textit{rss}) over these tides. The \textit{rms} tide gauge difference is defined as:

$$
rms = \sqrt{\frac{1}{2} \cdot \frac{1}{n} \sum_{i=1}^{n} (\Delta C_i^2 + \Delta S_i^2)}
$$  \hspace{1cm} (7.1)

where $\Delta C_i$ and $\Delta S_i$ are the differences between the observed (by the gauges) and predicted (by the model) harmonic cosine and sine constants at a place $(\lambda, \phi)$, respectively, and $n$ is the number of gauges, i.e., 84 in case of the reference tide gauge set. The bottom plots in Figure 7.2 show the \textit{rms} differences of the tidal solutions relative to FES95.2.1. The \textit{rms} difference between two models is defined according to (7.1) but now $\Delta C_i$ and $\Delta S_i$ are the solution differences of the harmonic cosine and sine constants at a place $(\lambda, \phi)$, respectively, and $n$ is the number of grid cells over which the \textit{rms} is computed. Notice that all results in Figure 7.2 are given as a (percentage) change relative to the solution with $\Delta T = 1$ day, i.e.:

$$
\Delta rms = \frac{rms(\Delta T) - rms(\Delta T = 1)}{rms(\Delta T = 1)}
$$  \hspace{1cm} (7.2)
with the \textit{rms} computed according to (7.1). Hence, the results do not reflect the \textit{absolute} accuracy of the response tidal solutions but a \textit{change} in accuracy with increasing lag interval. For this reason, a different value for the lag interval may significantly alter the \textit{rms} tide gauge difference of a tide like $Q_1$, which is difficult to observe from the altimetry, whereas its \textit{rms} model difference relative to FES95.2.1 is hardly affected. As can be seen, the \textit{rms} tide gauge differences as well as the \textit{rms} differences relative to FES95.2.1 show only slight variations when the lag interval is increased from 1 to 2 or 3 days and typically stay within 6%. An exception is the $Q_1$ tide for which the \textit{rms} tide gauge difference was found to increase with 8.8\% for $\Delta T = 2$ days and with 18.4\% for $\Delta T = 3$ days. From all plots in Figure 7.2 it can be observed that the strongest tidal lines, i.e. $M_2$, $S_2$, $N_2$, $K_1$, and $O_1$, are little affected when the lag interval is increased. Obviously this indicates that the admittance of these tidal lines is well defined by their signal in the altimeter data. This becomes clear from Figures 7.3 and 7.4, which show the diurnal and semi-diurnal admittances and corresponding harmonic constants for an arbitrary place in the North.
Figure 7.3  Diurnal admittance and harmonic constants of the TOPEX/POSEIDON response solutions as a function of the lag interval for \( \lambda = -150^\circ, \phi = 30^\circ \). Denoted by a \( \star \) are the FES95.2.1 values while the \( \circ, \times, \) and \( \Box \) denote a lag interval of 1, 2, and 3 days, respectively.

Pacific, i.e. \( \lambda = -150^\circ, \phi = 30^\circ \), because all admittance curves are seen to pass through the same points at the dominant \( M_2, S_2, N_2, K_1, \) and \( O_1 \) frequencies. In Figures 7.3 and 7.4, the phases \( \chi_k \) are due to the Doodson & Warburg convention (Section 2.6) whereas the relation between the harmonic constants and admittance parameters is given by (3.30). In contrast with the dominant tides, the weaker tides like \( K_2, P_1, \) and \( Q_1 \), have a signal in the altimetry that is well below the background noise. Hence, at the frequencies of the weaker tides, there is no tidal signal strong enough on which to base the admittance, which then becomes ill defined. For this reason, the less dominant tides in Figures 7.2 to 7.4 show the largest variations when the lag interval is increased. For the same reason, the weaker tides have to rely to a considerable extent on the admittance of the dominant tides for their estimates, and a slight variation in e.g. the \( S_2 \) admittance will, therefore, have a noticeable effect on the estimate of the nearby \( K_2 \) tide. Hence, the differences in the weaker tides for different choices of \( \Delta T \) largely reflect how the admittance curve fits the dominant tides. Notice that the semi-diurnal admittance in Figure 7.4 does not show
7.3 Varying the lag interval

Figure 7.4 Semi-diurnal admittance and harmonic constants of the TOPEX/POSEIDON response solutions as a function of the lag interval for $\lambda = -150, \phi = 30$. Denoted by a $\star$ are the FES95.2.1 values while the $\circ$, $\times$, and $\square$ denote a lag interval of 1, 2, and 3 days, respectively.

a discontinuity at the $S_2$ frequency due to the radiational tide as discussed in Section 3.5. In case of FES95.2.1, this discontinuity also does not develop because no radiational forcing terms were included in the driving potential of the FES model series [Le Provost et al., 1994]. Additionally, its application of CSR2.0 as a-priori solution probably means that it has partly copied the smooth admittance of this model.

The global effect of the dominant tides on the weaker tides as a result of changing $\Delta T$ is illustrated in Figure 7.5. As an example, this figure shows the vector differences, i.e. $\sqrt{\Delta C_{1}^2 + \Delta S_{1}^2}$, between admittance solutions with a lag interval of 1 and 3 days. The geographic pattern of the vector differences largely follows the amplitude pattern of the diurnal and semi-diurnal tides, i.e. where there is a large amplitude (see Figure 8.1 in Section 8.2.1) there is a large difference. Clearly visible is a strong correlation between the $S_2$ and $K_2$ differences and between the $O_1$ and $Q_1$ differences, but also between the $M_2$ and $N_2$ differences. The correlation between the $M_2$ and $N_2$ differences indicates that although the $N_2$ tide is relatively strong, the $M_2$ tide apparently is so dominant that it notice-
Figure 7.5 Vector differences between the TOPEX/POSEIDON reponse solutions with a lag interval $\Delta T$ of 1 and 3 days and a number of lags $L$ of 3. The solution differences have a global $rms$ (mm) of: $M_2 = 0.4$, $S_2 = 0.4$, $N_2 = 1.4$, $K_2 = 1.1$, $O_1 = 1.0$, $Q_1 = 2.9$. Notice the differences in scale.

ably affects the $N_2$ estimate. The influence of $M_2$ on $N_2$ is, however, much smaller than that of $S_2$ on $K_2$ as can be judged from the speckle of the $N_2$ differences in Figure 7.5 compared to the relatively smooth $K_2$ differences. Worth mentioning is that no apparent correlation between the $K_1$ differences and the $P_1$ differences could be detected although these tides are close in frequency. The most probable cause is that the small $P_1$ tide is in between the dominant $K_1$ and $O_1$ tides (Figure 7.3). Because the admittance between $K_1$ and $O_1$ has to fit both these dominant tides, a strong correlation between $P_1$ and $K_1$ is avoided. In case of $K_2$, the strong $S_2$ tide is on its left (Figure 7.4), while on its right there are no strong tidal signals,
which clearly is to the benefit of a correlation between $K_2$ and $S_2$. In case of $Q_1$, the differences relative to FES95.2.1 were found to be relatively noisy, which shows that the $Q_1$ frequency is too far away from the dominant $O_1$ tide to be properly constrained by the smooth admittance assumption, as was also indicated by the large changes in the $rms$ tide gauge difference of this tide in Figure 7.2.

As an overall conclusion from the experiments with the varying lag interval, it may be said that for well-defined parts of the admittance curve, i.e. at frequencies where the tidal signal is well above the altimeter background noise, it makes no difference if one takes $\Delta T = 1, 2,$ or 3 days. It was also found that for the major tides in both bands ($M_2$, $S_2$, $N_2$, $K_1$, and $O_1$), the geographic pattern where a solution with $\Delta T = 1, 2,$ or 3 days is closest to the FES95.2.1 model was completely random so that no obvious systematic differences between the response solutions with various $\Delta T$ are expected. Hence, the ocean's admittance seems to have no preference for a certain lag interval smaller than 3 days, i.e. there seems to exist no optimum value of this interval, at least not in a global sense. In Section 7.6 it will be explained why different choices of $\Delta T$ result in nearly the same admittance solution. The almost legendary choice of Munk and Cartwright [1966] of $\Delta T = 2$ days based on observations from a Honolulu tide gauge station may therefore be conclusive for the "optimum" admittance of this one station but certainly not in a global sense. Would they have selected a different station, they would have probably come up with a different value for the "optimum" lag interval. Still, all response solutions in this thesis were also parameterized with a lag interval of 2 days. As the $rms$ differences between the solutions of models A to C were found to be less than a mere 1.4 mm for all tides but $Q_1$, for which an $rms$ of 2.9 mm between the solutions with $\Delta T = 1$ and 3 days was found, it is clear that a lag interval of 1 or 3 days will do just as well.

7.4 Varying the number of response weights

With the selected lag interval of 2 days, the influence of the number of lags $L$, and thus of the number of response weights, on the admittance solution can now be studied by increasing this number from 2 (model $D$ in Table 7.1) to 3 (model $B$) and further to 4 (model $E$). These three models were submitted to the same two tests as with the lag interval experiments and the results are presented in Figure 7.6. Just like the experiments with the varying lag interval, a change in the number of lags mostly affects the smaller tides while the dominant tides remain fairly unchanged. From the left plots it can be seen that both the $rms$ tide gauge differences and the $rms$ differences relative to FES95.2.1 show insignificant changes in the diurnal band when the number of lags is increased from 2 to 3. This means that the diurnal admittance can be adequately modeled with two lags, i.e. $\Delta T = 0, 2$ days, and that further increasing the number of lags
Figure 7.6 Percentage change in the $rms$ tide gauge differences (top) and percentage change in the $rms$ differences relative to FES95.2.1 (bottom) for TOPEX/POSEIDON response solutions with a lag interval $\Delta T = 2$ days and several choices of the number of lags $L$.

has little use. As may be expected, anomalous behavior is displayed by the ill-observed $Q_1$ tide of which the $rms$ difference relative to FES95.2.1 was found to increase with 18.5% for $L = 3$ and with 27.2% for $L = 4$. In contrast with the diurnal tides, the plots on the right of Figure 7.6 show that the $rss$ values of the $rms$ tide gauge differences and the $rms$ differences relative to FES95.2.1 for the semi-diurnal tides both significantly decrease when the number of lags is enlarged from 2 to 3. This indicates that two lags are not enough to model the semi-diurnal admittance and that 3 is the optimum as further enlargement to 4 has little effect. The most spectacular improvement was found in case of $N_2$, for which the $rms$ tide gauge difference decreased with 32.1% for $L = 3$, and with 31.3% for $L = 4$, while the $rms$ difference relative to FES95.2.1 decreased with 34.3% for $L = 3$ and with 33.7% for $L = 4$. The explanation of the behavior of the $N_2$ tide is given in Figure 7.7, which shows the semi-diurnal admittance for the same arbitrary place as in Figures 7.3 and 7.4. Estimating too few parameters to correctly model the semi-diurnal admittance (four parameters for $L = 2$) means that the curve becomes too flat as steep changes are smoothed out.
7.4 Varying the number of response weights

Figure 7.7 Semi-diurnal admittance and harmonic constants of the TOPEX/POSEIDON response solutions as a function of the number of lags for $\lambda = -150$, $\phi = 30$. Denoted by a * are the FES95.2.1 values while the o, x, and □ denote a number of lags of 2, 3, and 4, respectively.

(circle markers). Because $N_2$ is relatively weak compared to $M_2$ and $S_2$, the admittance curve has to go through the $M_2$ and $S_2$ admittances whereby it becomes tilted at the $N_2$ frequency, which leads to large errors at that part of the admittance.

The effect of estimating too few parameters to model the admittance should not be underestimated. If the admittance curve becomes too flat, all tides in a band become correlated. To illustrate the correlations among the tides within the semi-diurnal band when too few weights are estimated, i.e. $L = 2$, Figure 7.8 gives the vector differences between the admittance solutions with a number of 2 and 3 lags ($\Delta T = 2$ days) for the four largest semi-diurnal tides. Notice that in agreement with the experiments with the lag interval, the differences largely follow the amplitude pattern of the semi-diurnal tides. Clearly visible is that the differences of $N_2$ and $K_2$ are strongly correlated with those of $M_2$ and $S_2$, respectively. Because the differences between solutions with $L = 3$ and $L = 4$ revealed a somewhat random geographic pattern, it may be assumed that the correlation between the tides largely disappears for $L = 3$. This means that the agree-
Figure 7.8 Vector differences between the TOPEX/POSEIDON response solutions with a lag interval $\Delta T$ of 2 days and a number of lags $L$ of 2 and 3. The solution differences have a global rms (cm) of: $M_2 = 0.4$, $S_2 = 0.3$, $N_2 = 1.1$, $K_2 = 0.2$. Notice the differences in scale.

ament between, e.g., the $M_2$ and $N_2$ differences in Figure 7.8 is indeed due to a correlation between the $M_2$ and $N_2$ solutions for $L = 2$. Worth to mention is that some slight correlations could still be detected between the $S_2$ and $K_2$ differences and between the $O_1$ and $Q_1$ differences with $L = 3$ and $L = 4$, which confirms that the harmonic constants of the weaker tides are largely based on the admittance of the dominant tidal lines.

As an overall conclusion with regard to the experiments with the number of lags, it may be stated that the optimum number of lags is 2 for the diurnal admittance and 3 for the semi-diurnal admittance. "Optimum" here is to mean sufficient to model the steep variations in the admittance curve. In Zetler and Munk [1975] and in Desai and Wahr [1995], similar findings are reported from, respectively, the analysis of tide gauge records and the remaining variance of TOPEX/POSEIDON sea surface heights after tidal solutions with a different number of lags were subtracted. However, these authors have not experimented with varying the lag interval, which they kept at two days. According to Zetler and Munk [1975], the more complex shape of the semi-diurnal admittance is most likely caused by the fact that the semi-diurnal tides are stronger compared to the tides in the diurnal band. Apart from $K_1$ and $O_1$, most diurnal tides are relatively weak so that they do not come sufficiently through in the altimeter data, which
causes a relatively flat diurnal admittance.

7.5 Selected lag interval and number of response weights

It was decided to analyze all altimeter data with a response parameterization of $\Delta T = 2$ days and $L = 3$, i.e. $s\Delta T = 0, 2, 4$ days, for both bands. The number of lags $L$ is dictated by the semi-diurnal admittance in the sense that apparently three complex weights are needed to model the changes with frequency of this curve. Notice that because the ocean is assumed to respond linearly to the astronomical forcing, the diurnal and semi-diurnal tides can also be analyzed separately, in which case $L = 2$ would be used for the diurnal band and $L = 3$ for the semi-diurnal band. However, such an approach would require a considerable amount of extra computing time because when the diurnal or semi-diurnal admittance is solved, a reference model for the tides in the other band must be subtracted from the altimeter data to avoid contamination as diurnal and semi-diurnal tides may alias to the same period, e.g. $K_1$ and $S_1$ in case of GEOSAT. As no wiggles were found to show up in the admittance curves when enlarging the number of lags up to $L = 4$, analyzing the diurnal tides with $L = 3$ does not violate the credo of smoothness and the only disadvantage is some extra computing time to estimate the diurnal weights $u_{212}$ and $v_{212}$ so that $L = 3$ for both bands seems to be a good choice.

7.6 Linear dependence of the estimated response weights

The main purpose of this section is to explain why different choices of the lag interval $\Delta T$ result in nearly the same solution for the admittance, at least at those frequencies where the tidal signals are well above the altimeter background noise. Furthermore, it is discussed why the admittance can be modeled without introducing negative time lags $s\Delta T$ and without the use of the frequently encountered orthotides [Groves and Reynolds, 1975]. As a matter of fact, these three issues may all be explained by the same reason, namely that the estimated response weights of two solutions are linearly dependent.

According to the response formalism (3.21), the observed tide $\zeta$ at an instant $t$ can be approximated by a weighted sum of lagged equilibrium tide values. Hence, it may be expected that the equilibrium tide itself at time $t$ can also be approximated by a weighted sum of lagged values. If this is true, then the $a_{2m}(t - s\Delta T)$ and $b_{2m}(t - s\Delta T)$ of a response solution with lag interval $\Delta T$ (where $\Delta T$ may be either integer or non-integer) can be written as a nearly linear combination of the lagged equilibrium
tide values of another response solution, i.e. one that is modeled with a different lag interval, say $\Delta T'$. Hence, for $L = 3$ we may write:

\[
\begin{bmatrix}
  a_{2m}(t - s\Delta T) \\
  b_{2m}(t - s\Delta T)
\end{bmatrix} =
\begin{bmatrix}
  \alpha_1 & \alpha_2 & \alpha_3 & \alpha_4 & \alpha_5 & \alpha_6 \\
  \beta_1 & \beta_2 & \beta_3 & \beta_4 & \beta_5 & \beta_6
\end{bmatrix}
\begin{bmatrix}
  a_{2m}(t - \Delta T') \\
  b_{2m}(t - \Delta T') \\
  a_{2m}(t - 2\Delta T') \\
  b_{2m}(t - 2\Delta T')
\end{bmatrix}
\]

(7.3)

where $s = 0, 1, \text{or } 2$. In the above equation, the equilibrium tide values on the left and on the right side are either both diurnal or both semi-diurnal. A relation (7.3) between diurnal and semi-diurnal equilibrium tide values is not likely. This could be easily verified by covariance analyses of the response solutions from which correlations between the diurnal and semi-diurnal response weights of about 5% were found, as mentioned in Section 6.1.

To verify the existence of the relation (7.3), consider, for example, the response solutions that were modeled with a number of lags of 3 and with a lag interval of 1 and 2 days, i.e. models $A$ and $B$ in Table 7.1. Using (7.3) to express $a_{2m}(t - 4)$ and $b_{2m}(t - 4)$ of model $B$ in terms of the lagged equilibrium tide of model $A$, i.e. in terms of $a_{2m}(t - s\Delta T')$ and $b_{2m}(t - s\Delta T')$ with $s\Delta T' = 0, 1, 2$ days, the following relation between the response weights of these two models can be found from (3.21):

\[
\begin{bmatrix}
  u'_{2m0} \\
  u'_{2m1} \\
  u'_{2m2}
\end{bmatrix} =
\begin{bmatrix}
  1 & 0 & 0 & 0 & \alpha_1 & \beta_1 \\
  0 & 1 & 0 & 0 & \alpha_2 & \beta_2 \\
  0 & 0 & 0 & 0 & \alpha_3 & \beta_3 \\
  0 & 0 & 0 & 0 & \alpha_4 & \beta_4 \\
  0 & 0 & 0 & 1 & \alpha_5 & \beta_5 \\
  0 & 0 & 0 & 1 & \alpha_6 & \beta_6
\end{bmatrix}
\begin{bmatrix}
  u_{2m0} \\
  u_{2m1} \\
  u_{2m2}
\end{bmatrix}
\]

(7.4)

The weights on the left side are those of model $A$ ($\Delta T = 1$ day) while the weights on the right belong to model $B$ ($\Delta T = 2$ days). The matrix in (7.4) would give the relation between the response weights of model $A$ and model $B$ in the ideal case of an exact linearity between the $a_{2m}(t - 4)$, $b_{2m}(t - 4)$ and the $a_{2m}(t - s\Delta T')$, $b_{2m}(t - s\Delta T')$ and without the presence of noise in the sea surface height measurements. As both are not true, the actual values of the matrix elements have to be determined from a linear regression on each of the weights of model $A$ separately. For e.g., $u'_{2m0}$, this means that the observation equation:

\[
u'_{2m0} = c_1u_{2m0} + c_2u_{2m1} + c_3u_{2m2} + c_4v_{2m1} + c_5v_{2m2} + c_6v_{2m2}
\]

(7.5)

was used to solve the six coefficients $c_i$ ($i = 1$ to 6), which in the ideal case would equal the values in the first row of (7.4). The $u'_{2m0}$ and the $v_{2m1}$ and $v_{2m2}$ in the observation equation were taken from the interpolated
1° × 1° TOPEX/POSEIDON grid solutions. From the linear regression, the following matrix was found for the diurnal weights:

\[
\begin{bmatrix}
0.77 & 0.01 & 0.38 & 0.00 & 1.78 & -2.25 \\
0.01 & 0.84 & -0.05 & 0.24 & 2.07 & 1.54 \\
0.43 & 0.05 & -0.71 & -0.10 & -5.76 & 4.79 \\
-0.12 & 0.31 & 0.26 & -0.46 & -4.34 & -5.38 \\
-0.20 & -0.06 & 1.32 & 0.10 & 4.98 & -2.55 \\
0.11 & -0.15 & -0.21 & 1.22 & 2.28 & 4.84
\end{bmatrix}
\]  

(7.6)

and for the semi-diurnal weights:

\[
\begin{bmatrix}
0.95 & -0.02 & 0.00 & -0.01 & 0.19 & -2.81 \\
-0.01 & 0.97 & 0.01 & -0.03 & 2.78 & 0.16 \\
0.10 & 0.05 & 0.00 & 0.04 & -3.08 & 6.47 \\
0.00 & 0.05 & -0.02 & 0.05 & -6.39 & -3.04 \\
-0.05 & -0.03 & 1.00 & -0.02 & 3.88 & -3.66 \\
0.01 & -0.02 & 0.01 & 0.98 & 3.61 & 3.88
\end{bmatrix}
\]  

(7.7)

Comparing these matrices with the matrix in (7.4) we notice a good agreement, which demonstrates the nearly linear dependence between the weights with \(\Delta T = 1\) day and the weights with \(\Delta T = 2\) days. Especially for the semi-diurnal response weights there is a good agreement because of the strong tidal signals in the semi-diurnal band. In case of the diurnal band, the tides are not that strong relative to the background noise, resulting in a more noisy solution for the diurnal weights so that the linear relation (7.4) is somewhat distorted. Most likely, the noise level in the diurnal weights also causes the outlier of -0.71 in the coefficient solution of \(\hat{a}_{2m1}\), where zero is expected.

The same linear regression procedure has been performed on the weights of all response solutions with a number of lags of 3. Hence, in addition to the test with \(\Delta T = 1\) and 2 days, a linear relation was sought between the response weights with \(\Delta T = 1\) day and those with \(\Delta T = 3\) days, and between the response weights with \(\Delta T = 2\) days and \(\Delta T = 3\) days. Although the matrices relating the weights of these solutions are different from that in (7.4), the same level of agreement was found as with the weights of \(\Delta T = 1\) day and \(\Delta T = 2\) days. Hence, in the matrix of the semi-diurnal weights, values between 0.9 and 1.0 were found where an exact linearity and no data noise predict one, while values of the order of 0.1 to 0.2 were found where we would expect zero. As expected, in case of the diurnal weights, the agreement with the predicted matrix is a little worse, but values of 0.8 to 1.2 compared to one, and 0.1 to 0.3 compared to zero are still quite acceptable. Again, outliers in the estimated coefficients \(c_i\) of the diurnal weights were found but for different weights than \(\hat{a}_{2m1}\), which confirms that noise is the most likely cause of these outliers.

Although no test has been performed with negative lags \(s\Delta T\), there
is no reason to assume that a linear dependence (7.3) as could be shown between the weights of models A, B, and C in Table 7.1 will disappear when negative lags $s\Delta T$ (future instants) are included. Hence, nearly the same admittance should be found from a response model with, e.g., $s\Delta T = -2, 0, 2$ days, as was used in Cartwright and Ray [1990b], and the models with strictly positive lags as used in this thesis.

The linear dependence between the equilibrium tide values also explains why the use of so-called orthotides instead of the $a_{2m}(t - s\Delta T)$ and $b_{2m}(t - s\Delta T)$ would not have resulted in any different admittance solution. A known disadvantage of the response formalism (3.21) is that the pairs of functions $c_{2m}(t), c_{2m}(t - s\Delta T)$ are not orthogonal in time. As a consequence, there will be significant correlations between the response weights inside the same band, as mentioned in Section 6.1. This means that two analyses, each over a different time span, will give different weights for the same station. To obtain weights that are truly characteristic for a tidal band, Groves and Reynolds [1975] introduced the so-called orthotide functions or orthotides. For the $l = 2$ term of the tide-generating potential, they reformulated (3.21) to:

$$\zeta = \sum_{m=0}^{2} \sum_{j=0}^{J} U_j^{(2m)} P_j^{(2m)}(t) + V_j^{(2m)} Q_j^{(2m)}(t)$$  \hspace{1cm} (7.8)$$

where $P_j^{(2m)}(t)$ and $Q_j^{(2m)}(t)$ are the orthotide functions (of tidal band $m$), which are nearly orthogonal in time, and the $U_j^{(2m)}$ and $V_j^{(2m)}$ are the so-called orthoweights. Usually, an upper limit of $J = 2$ is set to the series, which covers most of the tidal signal as demonstrated in Alcock and Cartwright [1978]. For $l = 2$, the orthotides of band $m$ are computed as linear combinations of the equilibrium tidal terms $a_{2m}(t - s\Delta T)$ and $b_{2m}(t - s\Delta T)$ of that band, where Groves and Reynolds [1975] adopted the value of 2 days from Munk and Cartwright [1966] for the lag interval $\Delta T$ using positive and negative $s$. Consequently, the orthoweights may be computed as a linear combination of the ordinary response weights. Thus, although the orthoweights may have a more physical meaning in that they are truly characteristic for the tide, they give exactly the same solution for the admittance as the ordinary response weights.

7.7 Discussion

From experiments with the parameters of the response formalism, i.e. the lag interval and the number of weights, it could be shown that the dominant tides largely determine the admittance solution. The weaker tides that can be determined from the admittance reflect how the estimated response weights fit the dominant tides. For this reason, the weaker tides furthest away from the frequencies of the dominant tides, i.e. near the edges of the tidal spectrum, are most sensitive to variations of the lag
interval and the number of weights. For the same reason, varying the lag interval with response solution parameterizations that have an equal number of weights, results in differences of the weaker tides that vary random with geographic position.

Varying the value of the lag interval was found to have negligible effect on the estimates of the dominant tides and hence on the weaker tides in their vicinity. Typically, the rms differences of the four largest tides in the diurnal and semi-diurnal bands for different choices of the lag interval are less than 1.4 mm and therefore insignificant. The observation that different choices of the lag interval lead to nearly the same admittance solution, away from the edges of the tidal spectrum, could be explained by the linear dependence of the estimated response weights of different solutions. Although only lag intervals $\Delta T$ of 1, 2, and 3 days have been tested, a linear dependence of the weights is also expected for non-integer and larger values of $\Delta T$ because of the implicit assumption of the response method that the ocean tide at an instant of time can be approximated by a weighted sum of lagged equilibrium tide values. The linear dependence of the response weights also justifies the adopted approach in this thesis of not using the so-called orthoweights, which are a linear combination of the ordinary response weights.

Concerning the number of lags, it was found that two lags (i.e. four weights) are enough to model the diurnal admittance. For the semi-diurnal admittance, which is less flat than the admittance of the diurnal band, three lags are required as two lags were found to enforce correlations between neighboring tides.

From the conducted experiments, it was decided to parameterize the response tidal models with a lag interval of two days and a number of lags of three (i.e. three complex weights) for both the diurnal and semi-diurnal band.
8.1 Introduction

In this chapter, the single-satellite harmonic and response tidal solutions are developed and the $rms$ differences between these solutions and the harmonic constants of the network of 84 globally distributed tide gauges are computed. Such computations give an indication of the effect of the altimeter background noise and the tidal correlations on the tidal solutions. To explain the effects of these correlations, detailed comparisons are made between the single-satellite harmonic and response solutions. The TOPEX/POSEIDON response solution is analyzed in much detail because it is the tidal solution of which we want to improve the resolution by means of the GEOSAT and ERS-1 altimeter observations in Chapter 9. All harmonic and response tidal solutions were obtained by solving the $1^\circ \times 1^\circ$ normal matrix grids (Section 5.1) and masking the solutions for ocean depths less than 200 m (Section 7.1). All response solutions are modeled with a lag interval of two days and a number of three complex response weights in the diurnal and semi-diurnal bands, as was explained in Section 7.5.

8.2 TOPEX/POSEIDON harmonic and response solutions

This section discusses the TOPEX/POSEIDON harmonic and response tidal solutions. A TOPEX/POSEIDON response tidal solution was derived that accounts for 23 tidal lines. For the dominant tides of this solution as well as for the smaller tides inferred from the admittance, the $rms$ tide gauge differences are computed for the reference network of 84 tide gauges. Also, the statistics of TOPEX/POSEIDON, ERS-1, and GEOSAT crossover differences are computed with this response solution. The
rms tide gauge differences and the crossover statistics of all three satellites as obtained with the TOPEX/POSEIDON response solution are compared with the results of the FES95.2.1 and CSR3.0 models. Also, the differences of the TOPEX/POSEIDON response solution relative to FES95.2.1 and CSR3.0 are investigated. The comparisons with FES95.2.1 and CSR3.0 are not done to obtain a ranking of the models, but to examine how well the empirical altimetry-based solutions developed in this thesis compare with hydrodynamically consistent solutions.

8.2.1 Amplitude/phase charts

To give an impression of the TOPEX/POSEIDON tidal solutions, Figure 8.1 shows the amplitude/phase charts (amplitude and Greenwich phase lag) or cotidal charts of the dominant $M_2$ and $K_1$ tides as obtained from the TOPEX/POSEIDON harmonic solution. Because the response of the ocean to luni-solar gravitational forcing at nearby frequencies is almost the same, all semi-diurnal tides have cotidal charts similar to that of $M_2$, while the same applies to the diurnal tides and $K_1$. As will be shown in Section 8.2.3, the harmonic and response tidal solutions derived from TOPEX/POSEIDON altimetry are in excellent agreement so that the cotidal charts derived from the response solutions are almost exactly the same as those derived from the harmonic solutions. With the cotidal charts in Figure 8.1, the amplitudes are given by the color scale. The Greenwich phase lag is contoured at 30° intervals with solid lines indicating a phase lag between 0° and 180°, while dashed lines correspond to a phase lag between 180° and 360°. As the phase lines give the time of high tide (Section 3.4), the motion of the tidal waves on the rotating earth is in the direction of increasing phase. The tidal waves appear to rotate around points of zero amplitude called amphidromes. The rotation around the amphidromes is due to a combination of the Coriolis force and the boundaries of the land masses that impede the propagation of the tidal wave as explained in Pugh [1987] and Apel [1987]. Notice that due to this impediment, the tidal waves in the northern hemisphere are seen to rotate in a counterclockwise direction, while in the southern hemisphere, the rotation is in a clockwise sense. This is exactly in the opposite direction of the rotation of a free particle under influence of the Coriolis force, i.e. a particle that is not impeded by the land masses. However, amphidrome systems can be detected in Figure 8.1 that rotate in a "wrong" sense, e.g. the $M_2$ system in the South Atlantic, west of South Africa, which rotates counterclockwise due to a complicated dynamic behavior imposed upon the tidal wave by the land boundaries [Pugh, 1987].

Figure 8.2 shows the amplitudes and phases of the seasonal cycles, also from the TOPEX/POSEIDON harmonic solution. As mentioned in Section 5.4, the phases of the seasonal cycles refer to the beginning of
Figure 8.1 Cotidal charts of $M_2$ (top) and $K_1$ (bottom) from the TOPEX/POSEIDON harmonic tidal solution. Amplitudes are given by the color scale in cm. The Greenwich phase lag is contoured at 30° intervals. Solid lines indicate a phase lag between 0° and 180°, dashed lines correspond to a phase lag between 180° and 360°. The thick lines indicate a phase lag of 0°. The solutions are masked for depths less than 200 m (purple color). Notice the different scales.
Figure 8.2 Seasonal cycles from the TOPEX/POSEIDON harmonic tidal solution. Amplitudes are in cm. Phases are given in degrees with respect to the beginning of the year. The solutions are masked for depths less than 200 m. Notice the different scales.
the year. With the annual cycle in Figure 8.2, variable current systems are easily detected, e.g. the Gulfstream in the North Atlantic, the Somali Current and the South Equatorial Current in the Indian Ocean, and the Kuroshio Current in the North-West Pacific. The large amplitudes in the Antarctic seas represent no actual signal but are due to the correlation with the semi-annual cycle as explained in Section 6.3. The phase of the annual cycle shows an asymmetry of almost 180° between the two hemispheres, i.e. when anomalies are positive in the northern hemisphere, they are negative in the southern hemisphere. Obviously this is related to the annual solar heating and cooling of the sea surface. Apparently, the annual variations in the northern hemisphere adopt a maximum around October (average phase of 280°), while they become maximal around April in the southern hemisphere (average phase of 100°). In case of the semi-annual cycle, anomalies in the north Indian Ocean associated with the cycle of the monsoon winds [Tapley et al., 1994b; Apel, 1987] are visible. Especially the two “tails” in the amplitude of the semi-annual cycle are evidence of the monsoon cycle [Nerem et al., 1994b]. The phase of the semi-annual cycle shows much less coherence so that the times at which these anomalies adopt their maximum strongly depend on the location. A striking feature in the amplitude plots of both seasonal cycles is the large anomaly in the equatorial Pacific. This large-scale variation can be associated with the 1992-1993 ENSO event [e.g., Tapley et al., 1994b]. Although not strictly seasonal, it has a strong component at both the annual and the semi-annual frequency. A more detailed description of the seasonal cycles may be found in Tapley et al. [1994b], Knudsen [1994], and
Nerem et al. [1994b].

For completeness, Figure 8.3 shows the estimated dynamic topography from the TOPEX/POSEIDON harmonic solution. Because the dynamic topography is a time-invariant surface, Figure 8.3 shows the amplitude and there is no phase. The estimated topography clearly reproduces all general features of the ocean circulation. For instance, the low around the Antarctic associated with the Antarctic Circumpolar Current is evident. In the Pacific, the high connected with the Kuroshio Current can be seen, while the Gulfstream in the North Atlantic also appears.

8.2.2 Selected tidal lines of the response solution

A convenient property of the FES95.2.1 model is that its minor tides are derived from a number of tidal lines that well define the spectrum in the diurnal and semi-diurnal bands, i.e. \( M_2, S_2, N_2, K_2, 2N_2, K_1, O_1 \) and \( Q_1 \) (Appendix B). Although independent estimates of the \( K_2, 2N_2, \) and \( Q_1 \) tides may not be expected from altimetry, the same minor tides as those of the FES95.2.1 model were inferred from the TOPEX/POSEIDON response weights. When possible, the integrity of the inferred tide was verified against the reference network of 84 tide gauges, while it was also judged by the spatial coherence of its amplitude and especially its Greenwich phase lag. If the phase lag showed little coherence, the tide was rejected. Of the 26 tidal lines of the FES95.2.1 model (Figure 8.4) a number of 23 were selected. Not selected were the \( 2Q_1, \sigma_1, \) and \( OO_1 \) lines, which apparently are too far away from the dominant diurnal lines that can be observed in the altimetry to be properly constrained. The \( Q_1 \) estimate was also relatively inaccurate as was already discussed in Chapter 7. Still, judging from the coherence of its cotidal chart, its accuracy was considered acceptable. Hence, this leaves a number of 11 tidal lines in the diurnal band and 12 tidal lines in the semi-diurnal band. Notice that if the response solution is used to compute tidal predictions at a given place and a given time, nodal factors and angles are applied to the 23 selected spectral lines to obtain the corresponding constituents.

![Diagram](image-url)

**Figure 8.4** Selected 23 tidal lines of the TOPEX/POSEIDON response tidal solution. All depicted tides but \( 2Q_1, \sigma_1 \) and \( OO_1 \) are selected.
8.2.3 Comparison of harmonic and response solutions

For the reference network of 84 tide gauges, Table 8.1 shows the \textit{rms} tide gauge differences of the TOPEX/POSEIDON harmonic and response tidal solutions for the five dominant tides (load tide derived from FES95.2.1). For each tide, the harmonic and the response solution yield values for the \textit{rms} tide gauge differences that are nearly the same. Also, the fourth column in Table 8.1 shows that the number of gauges that agree better with the response solution than with the harmonic solution is approximately half the number of gauges in the network. This means that the harmonic and response solutions of TOPEX/POSEIDON are in excellent agreement, which is explained by the fact that tidal correlations are largely absent in the TOPEX/POSEIDON altimetry. In case of the $M_2$ and $S_2$ tides as observed by TOPEX/POSEIDON, the harmonic and response solution perform equally well according to Table 8.1, which is in agreement with the small correlation of $M_2/S_2$ in Table 6.1. The $K_1$ tide of the harmonic and the response solution will be discussed at the end of this section.

The closeness of the TOPEX/POSEIDON harmonic and response solutions is confirmed by Table 8.2, which shows that the \textit{rms} solution differences are less than about 0.2 cm for all tides, as well as for the seasonal cycles and the dynamic topography. All differences have been edited using a 3.5\textit{$\sigma$} criterion. Figure 8.5 shows the vector differences between the TOPEX/POSEIDON harmonic and response solutions, which are caused by a mixture of tidal magnitude, the effect of altimeter background noise on the estimated parameters, and the fact that we have used two different types of estimated parameters, \textit{i.e.} response weights versus harmonic constants. In Figure 8.5, large solution differences can be seen near the coast and in the Antarctic seas. Obviously, these are due to the smaller number of observations in these regions leading to less accurate solutions (Section 6.3). All plots in Figure 8.5 show a certain degree of trackiness, suggesting that orbit errors explain most part of the solution differences.

Orbit errors have a very rich spectrum. Most of this spectrum is accounted for by three components. The dominant component is the one cycle per revolution (1-cpr) orbit error. Besides this component, there are gravity-induced orbit errors, and errors resulting from the so-called "background" ocean tide model, \textit{i.e.} the model for $U_c + \Delta U_c$ (Section 4.1), for which all three satellites use the JGM-3 background tide model [Marshall et al., 1995; CERSAT, 1994; Chambers, 1996].

With precise orbit computations, it is customary to estimate 1-cpr acceleration terms [Tapley et al., 1994a; CERSAT, 1994; Chambers, 1996]. These terms should absorb several kinds of dynamic modeling errors in the orbit computations [\textit{e.g.}, Tapley et al., 1994a]. The acceleration terms are usually held constant over a day or even less, which means that, at a fixed location on earth, the remaining 1-cpr orbit errors will show a tendency to average out in the tidal solutions. However, part of the 1-cpr orbit errors may be aliased to larger periods and hence may have been absorbed by the esti-
Table 8.1 *Rms* tide gauge differences (cm) of the TOPEX/POSEIDON single-satellite harmonic and response tidal solutions for the reference network of 84 tide gauges. Ocean loading from FES95.2.1 is subtracted from the TOPEX/POSEIDON tidal solutions. The fourth column gives the number of gauges that agree better with the response solution than with the harmonic solution. Boxed numbers in bold refer to the correlation problems identified in Table 4.3.

<table>
<thead>
<tr>
<th>tide</th>
<th>harmonic</th>
<th>response</th>
<th>no.</th>
</tr>
</thead>
<tbody>
<tr>
<td>$M_2$</td>
<td>1.77</td>
<td>1.78</td>
<td>45</td>
</tr>
<tr>
<td>$S_2$</td>
<td>1.08</td>
<td>1.08</td>
<td>51</td>
</tr>
<tr>
<td>$N_2$</td>
<td>0.71</td>
<td>0.72</td>
<td>44</td>
</tr>
<tr>
<td>$K_1$</td>
<td>1.08</td>
<td>1.05</td>
<td>50</td>
</tr>
<tr>
<td>$O_1$</td>
<td>0.80</td>
<td>0.80</td>
<td>45</td>
</tr>
</tbody>
</table>

Table 8.2 *Rms* (cm) of the differences between the TOPEX/POSEIDON harmonic and response tidal solutions. Denoted by $S_a$ and $S_{sa}$ are the annual and semi-annual cycle, respectively, whereas $a_0$ denotes the dynamic topography. Boxed numbers in bold refer to the correlation problems identified in Table 4.3. All differences are edited using a 3.5σ criterion.

<table>
<thead>
<tr>
<th></th>
<th>$M_2$</th>
<th>$S_2$</th>
<th>$N_2$</th>
<th>$K_1$</th>
<th>$O_1$</th>
<th>$S_{sa}$</th>
<th>$S_a$</th>
<th>$a_0$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0.08</td>
<td>0.19</td>
<td>0.17</td>
<td>0.18</td>
<td>0.11</td>
<td>0.15</td>
<td>0.12</td>
<td>0.12</td>
</tr>
</tbody>
</table>

At a fixed location on earth, orbit errors resulting from the gravity field model used in the orbit computations show up as a constant. Half of the gravity-induced orbit error is geographically correlated [Rosborough, 1986], which means that this error has the same magnitude and sign on ascending and descending tracks inside a grid cell of 3° × 3° for TOPEX/POSEIDON, 2° × 2° for GEOSAT, and 1° × 1° for ERS-1. The other half of the gravity-induced orbit error is geographically anticorrelated [Rosborough, 1986], which means that it has an equal magnitude but an opposite sign on ascending and descending tracks in a grid cell. The geographically-correlated orbit error will have been captured by the estimated dynamic topography. The anti-correlated orbit error, however, will not have been captured by the topography and part of it may have been absorbed by the tidal parameters, although this is not likely because of the time-invariant character of this error.

As shown in Bettadpur and Eanes [1994], sampling of errors in the background ocean tide model with the satellite repeat period leads to alias periods that are identical to those of the ocean tides so that these background errors are inseparable from the ocean tidal signals in the altimeter observations. Hence, errors in the background ocean tide model and 1-cpr orbit errors will be responsible for the main part of the orbit...
Figure 8.5 Vector differences between the TOPEX/POSEIDON harmonic and response tidal solutions (cm). Denoted by $S_a$ and $S_{sa}$ are the annual and semi-annual cycle, respectively, whereas $a_0$ denotes the dynamic topography. The white color indicates depths less than 200 m.
errors in Figure 8.5.

Of particular interest with TOPEX/POSEIDON are the differences of the $K_1$ tide in Figure 8.5. Large $K_1$ differences are found beyond some 55° of latitude. Although part of these differences is undoubtedly related to the magnitude of $K_1$ (see Figure 8.1) as well as to a less accurate solution in the Antarctic seas (Chapter 6), the sudden increase and the similarity between the $K_1$ and $S_{sa}$ differences beyond latitudes of 55° suggest a systematic difference between the harmonic and the response tidal solutions at higher latitudes due to the correlation of $K_1$ with the semi-annual cycle. Of the fourteen gauges in the reference set that are located at latitudes larger than 50°, ten agree better with the response solution than with the harmonic solution. With these fourteen gauges, the $rms$ tide gauge difference of the response solution is 1.66 cm against 1.73 cm for the harmonic solution. It should be realized, however, that these values strongly depend on the location of the gauges, i.e. in Figure 7.1 it can be seen that half the number of these fourteen gauges are clustered along the coast of Alaska (numbers 62 to 64) and in the Drake Passage (numbers 36 and 40 to 42). Hence, whether the response solution in general performs better at higher latitudes is difficult to decide from the tide gauges because of their irregular distribution over the globe.

8.2.4 Comparisons with FES95.2.1 and CSR3.0

To verify how well the empirical TOPEX/POSEIDON harmonic and response solutions agree with hydrodynamically consistent solutions, extensive comparisons have been made with the FES95.2.1 and CSR3.0 ocean tide models. The results of these comparisons are described in detail in Smith and Andersen [1997] and are briefly summarized in this section. Basically, all comparisons are between the TOPEX/POSEIDON response tidal solution and the hydrodynamically consistent FES95.2.1 model. The FES95.2.1 model was selected because it is almost independent of altimetry. To explain some of the differences between FES95.2.1 and the TOPEX/POSEIDON response tidal solution, the CSR3.0 model has been included in the comparisons. As explained in Appendix B, the CSR3.0 model is essentially a long-wavelength correction to FES94.1, which is an earlier version of FES95.2.1. Interesting to mention is that the FES95.2.1 and CSR3.0 models were also recommended by the TOPEX/POSEIDON Science Working Team (SWT) after a careful evaluation of ten candidate tide models [Shum et al., 1997] to replace the somewhat outdated models of Schwiderski [1983] and Cartwright and Ray [1990b], Cartwright and Ray [1991] on the TOPEX/POSEIDON GDRs for reprocessing in early 1997.

From the model comparisons with FES95.2.1, it was found that all five dominant tides agree to better than 1 cm in vector magnitude in most
parts of the deep oceans. This indicates a remarkable agreement between the dominantly hydrodynamic FES95.2.1 model and the empirical TOPEX/POSEIDON tidal solution. In waters deeper than 200 m near the coast, the differences can be larger than 5 cm. These large differences can be explained by the relatively coarse spacing of the TOPEX/POSEIDON groundtracks. As a consequence, the short-wavelength tidal features in coastal areas due to the interaction of the tide with the bottom topography cannot be resolved, whereas the coastal geometry is difficult to follow with the 3° longitude resolution of the TOPEX/POSEIDON altimetry resulting in few observations in grid cells along the coast and hence in less accurate solutions than in the deeper parts of the oceans. Nevertheless, errors in the ETOPO5 model that is used to specify the bathymetry in the hydrodynamic equations of the FES model series [Le Provost et al., 1994] may also explain part of these differences. Differences larger than 1 cm were also found in the Antarctic seas, especially in the Weddell Sea where differences of 2 cm vector magnitude were found. These differences seem to appear in general between the empirical ocean tide models based on altimetry and the models that are independent or less dependent on altimetry [Smith and Andersen, 1997]. The differences are mainly due to errors in the altimetry-based tidal solutions, which are caused by the temporal absence of altimeter observations due to Antarctic ice cover [Smith and Andersen, 1997] (see also Chapter 6).

In Figure 8.6, the $M_2$ vector differences of the TOPEX/POSEIDON response tidal solution relative to FES95.2.1 and CSR3.0 are shown. If a 3.5σ editing criterion is applied, the $rms$ of the differences between the TOPEX/POSEIDON response solution and the FES95.2.1 solution for $M_2$ is 0.8 cm in the deep oceans. For the differences between the TOPEX/POSEIDON response solution and CSR3.0, an $rms$ of 0.7 cm is found for $M_2$. The $rms$ of the differences between the FES95.2.1 and CSR3.0 solutions for $M_2$ is 0.6 cm in the deep oceans. The differences in Figure 8.6 have been illuminated from the northwest. Besides in coastal areas, bathymetric features of about 1 cm magnitude can be seen in the top and middle plots. These features appear to be strongly correlated with ridge structures like the Mid Atlantic Ridge, the Walvis Ridge off the Namibian coast, and the Marianas Trench south of Japan. As these bathymetry-related differences are in the deep oceans where the interaction of the tide with the ocean bottom should be small, they may just as well be due to FES95.2.1 and CSR3.0 as due to the TOPEX/POSEIDON tide model. Possibly, the bathymetric features can be explained by errors in the ETOPO5 model. Because bathymetry errors may be as large as several kilometers [National Geophysical Data Center, 1993], the bathymetry might have polluted the FES ocean tide model series from the very outset of FES94.1. Then, as CSR2.0 (a-priori model to FES95.2.1) is based on the coarse resolution of the TOPEX/POSEIDON altimetry, it cannot help FES95.2.1 to correct these short-wavelength bathymetry-induced errors. Likewise, this would ex-
Figure 8.6 $M_2$ vector differences between FES95.2.1, CSR3.0, and the TOPEX/POSEIDON response tidal solution (cm). The differences are masked for depths less than 200 m (purple color).
plain why the bathymetry-induced differences also appear in the comparison of the TOPEX/POSEIDON response solution with CSR3.0, as they are simply copied from FES94.1 (a-priori model to CSR3.0) without being altered by the long-wavelength corrections. Another possibility, of course, is that the interaction of the tidal wave with the bottom topography is still large enough to cause short-wavelength tidal features in the deep oceans. In that case, the coarse resolution of the TOPEX/POSEIDON altimetry implies that the TOPEX/POSEIDON response tidal model would be in error. It should be noted that although both the TOPEX/POSEIDON response model and CSR3.0 estimate the dynamic topography (actually CSR3.0 estimates corrections to the dynamic topography [Ma et al., 1994]), this procedure does not introduce topographic signals into the tidal solutions as shown in Smith and Andersen [1997].

From the comparisons with FES95.2.1 and CSR3.0, bathymetry-induced differences between FES95.2.1 and the TOPEX/POSEIDON response solution and between CSR3.0 and the TOPEX/POSEIDON response solution were also discovered in $S_2$ and $N_2$. They occur mainly at the same locations as with $M_2$, i.e. at the Mid Atlantic Ridge, the Walvis Ridge, and at the Mariana Trench. However, compared to the 1 cm vector magnitude of these differences in $M_2$, their significance is much less in $S_2$, about 0.5 cm, and quite negligible in $N_2$, about 0.2 cm. In the diurnal tides, no bathymetry-induced differences could be detected in the dominant $K_1$ and $O_1$ tides, most likely because the diurnal tides are much weaker over the indicated areas of steep topography. In the smaller tides as inferred from the admittance, bathymetric differences, if present, are obscured by other differences.

To decide which of the models is responsible for the bathymetry-induced signals, i.e. FES95.2.1/CSR3.0 or the TOPEX/POSEIDON response tidal model, a test has been performed in which statistics of TOPEX/POSEIDON crossover differences were computed with each of the models over the Mid Atlantic Ridge. The crossover differences of TOPEX/POSEIDON were computed from altimeter data that were not used in any of the models (see Section 8.2.5). Using tide gauge harmonic constants as an independent data type was not possible as there are very few gauges over the Mid Atlantic Ridge, e.g. only three in the tide gauge set of Le Provost [1994]. Crossover differences were computed applying only the $M_2$ and $S_2$ tidal solutions because these contained the strongest bathymetric signals. Unfortunately, the crossover difference rms values of the three models differed so little, i.e. of the order of one millimeter, that they do not allow any conclusion to be drawn on the origin of the bathymetric signals.

In Figure 8.6, several features appear that can be explained by errors in CSR3.0 that are related to domain boundary problems in the older FES94.1 model [Smith et al., 1994; Andersen et al., 1995; Shum et al., 1997].
It is known that the basin-wise solution procedure of the FES models introduced some inconsistencies at the basin boundaries of FES94.1 [Le Provost, 1996]. A picture of the grid and the exact locations of the basin boundaries of the FES models can be found in Le Provost et al. [1994], and hence need not be reproduced here.

Because the FES94.1 solution is used as the a-priori model to CSR3.0, boundary errors will also appear in CSR3.0, as these errors are of small wavelength and therefore cannot be corrected by the CSR3.0 solution procedure. Basin boundary errors are most obvious in the differences between CSR3.0 and FES95.2.1 in Figure 8.6, but they can also be seen in the differences between CSR3.0 and the TOPEX/POSEIDON response tidal model. The most obvious boundary error is the edge-like structure, which is along the equator in the Pacific ocean turning northeast south of Hawaii towards Mexico. This feature has been noted at several occasions [e.g., Andersen et al., 1995; Shum et al., 1997]. Interesting to notice is that this feature is not present in the differences between FES95.2.1 and the TOPEX/POSEIDON response solution so that the boundary problem along the equator in the Pacific ocean of FES94.1 has apparently been solved in FES95.2.1. Other features in CSR3.0 that are apparently related to FES94.1 boundary errors are the straight lines between Africa and Brazil, and south of South Africa [Smith and Andersen, 1997]. These lines respectively follow the FES boundary between the North Atlantic and South Atlantic oceans, and the boundary between the South Atlantic and Indian oceans [Le Provost, 1994]. The characteristic features in CSR3.0 along these boundaries were not noticed in the model comparisons of Shum et al. [1997], probably because they did not illuminate the differences. Besides in \( M_2 \), errors related to the FES94.1 boundary problems were also detected in some of the other tides of CSR3.0, i.e. the same sharp features as shown in the bottom plot of Figure 8.6 were found with a vector magnitude of 1 cm in \( S_2 \), 0.5 cm in \( N_2 \), and 0.2 cm in \( K_1 \). Notice, that the numbers for \( N_2 \) and \( K_1 \) are small compared to other differences between the tidal models and that the only reason why they can so easily be detected is their characteristic shape.

In case of FES95.2.1, notice the difference of 2 cm magnitude in \( M_2 \) between FES95.2.1 and CSR3.0, and between FES95.2.1 and the TOPEX/POSEIDON response solution, below South Africa, i.e. along the boundary between the South Atlantic and Indian oceans. Large differences between FES95.2.1 and CSR3.0, and between FES95.2.1 and the TOPEX/POSEIDON response solution of 1-1.5 cm along this boundary could also be detected in \( S_2 \) and \( N_2 \) [Smith and Andersen, 1997]. Most likely, the large differences below South Africa indicate an error in FES95.2.1 in that part of the ocean (see also Section 8.2.5). Whether this difference is related to the FES94.1 boundary has not been ascertained, although it is a likely explanation. Unfortunately, there are no tide gauges in this region in the sets of Le Provost [1994] and Smithson [1992] to verify this assumption. Possibly, the straight line between Africa and Brazil in the solution differences
8.2 T/P harmonic and response solutions

of FES95.2.1 and the TOPEX/POSEIDON response model in Figure 8.6, may be related to the boundary between the North Atlantic and the South Atlantic, although this has also not been ascertained.

It may be mentioned here that comparisons with the smaller tides of FES95.2.1 and CSR3.0 did not show boundary-related differences because of their smallness, if they are present.

8.2.5 Tide gauge and crossover difference analyses

For all tides present in the reference set of 84 tide gauges, Table 8.3 lists the rms tide gauge differences of the empirical TOPEX/POSEIDON response solution. For comparison, the rms tide gauge differences of the FES95.2.1 and CSR3.0 models are listed as well. As mentioned in Section 7.2, the empirical TOPEX/POSEIDON ocean+load tide response solution is converted to the pure oceanic tide by subtracting the FES95.2.1 load tide. Although this means that the results in Table 8.3 might be biased against the TOPEX/POSEIDON tidal solution, no significantly different results for the rms tide gauge differences are expected. The most important conclusion from Table 8.3 is that all three models seem to perform equally well in a global sense, and that they can predict tidal sea level variations with an accuracy of approximately 2.7 cm in the deep oceans, assuming that the errors in the harmonic constants of the gauges can be neglected. This level of accuracy is a significant improvement over tidal models such as those of Schwiderski [1980] and Cartwright and Ray [1990b], Cartwright and Ray [1991], which have an accuracy of about 5-10 cm in the deep oceans [Ray, 1993]. To value the importance of the rms tide gauge differences, the second column of Table 8.3 lists the rms of the tidal sea level variations at the tide gauge stations. This rms has been computed with (7.1), where the harmonic cosine and sine constants at the tide gauge are substituted for $\Delta C_i$ and $\Delta S_i$, respectively. Clearly visible is that the errors in the tidal models become more important for the smaller tides. This is because all three models in Table 8.3 infer their minor tides from the dominant lines, which is always an approximation, whereas errors in the dominant lines may also affect the smaller tides as was shown in Chapter 7. Notice that the $Q_1$ solution of the TOPEX/POSEIDON response model shows a larger rms tide gauge difference compared to FES95.2.1 and CSR3.0 due to the fact that the $Q_1$ tide cannot be properly constrained by $O_1$ (Sections 7.3 and 7.4). With CSR3.0, the $Q_1$ tide is apparently constrained by the a-priori FES94.1 solution.

In Table 8.4, the reduction of the crossover difference rms by using the different tidal models is listed. For the long-period tides, all models use the same equilibrium tide model. Crossover differences are computed for all three satellites from approximately sixty days of altimetry. This includes TOPEX/POSEIDON GDRs 125-130, ERS-1 OPRs 11-12,
Table 8.3  *Rms* tide gauge differences (cm) of the TOPEX/POSEIDON (T/P) response tidal model and of the FES95.2.1 and CSR3.0 tidal models for the reference network of 84 tide gauges. Ocean loading from FES95.2.1 is subtracted from the T/P tidal model.

<table>
<thead>
<tr>
<th>tide</th>
<th><em>rms</em></th>
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<tr>
<td>$2N_2$</td>
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<td>0.27</td>
<td>0.24</td>
</tr>
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<td>0.35</td>
<td>0.34</td>
</tr>
<tr>
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<td>0.15</td>
<td>0.16</td>
<td>0.15</td>
</tr>
<tr>
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<td>0.18</td>
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<tr>
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<td>1.05</td>
<td>1.09</td>
<td>1.06</td>
</tr>
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</tr>
<tr>
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<td>0.28</td>
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<tr>
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<td>38.07</td>
<td>2.73</td>
<td>2.77</td>
<td>2.71</td>
</tr>
</tbody>
</table>

GEOSAT GDRs 1-4, which cover the periods February-March 1996, April-May 1993, and November-December 1986, respectively. The TOPEX/POSEIDON cycles were selected to compute crossover differences from altimeter data that were not used in the tidal models, *i.e.* neither in the empirical TOPEX/POSEIDON response model, nor in FES95.2.1 and CSR3.0. With GEOSAT, the first four cycles of the ERM were chosen as these are least affected by loss of altimeter data. The ERS-1 OPRs were chosen arbitrarily. The numbers in Table 8.4 refer to crossover differences separated by about half the repeat period of the satellites at most, *i.e.* 5 days for TOPEX/POSEIDON, 18 days for ERS-1, and 9 days for GEOSAT. This time interval was chosen as a compromise between keeping the effect of ocean variability within limits and getting crossover differences at all latitudes. If a time interval much larger than, *e.g.*, a month is admitted, ocean variability will make a significant contribution to the crossover difference *rms*, which makes it difficult to judge how much the tidal models can reduce this *rms*. On the other hand, selecting a maximum time interval between crossing tracks less than half the repeat period results in latitudes at which no crossover differences will be found. All crossover differences were weighed according to latitude to avoid that crossover locations at higher latitudes would dominate the statistics (see Section 5.3). The third column of Table 8.4 lists the *rms* of the crossover differences with no ocean tidal correction subtracted from the altimeter data. Uncorrected crossover differences exceeding 3.5σ were removed and therefore not included in the statistics. To the remaining crossover differences...
8.2 T/P harmonic and response solutions

<table>
<thead>
<tr>
<th>no.</th>
<th>T/P</th>
<th>ERS-1</th>
<th>GEOSAT</th>
</tr>
</thead>
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<td>GEOSAT</td>
<td>73745</td>
<td>36.2</td>
<td>10.7</td>
</tr>
</tbody>
</table>

Table 8.4 Crossover difference $rms$ (cm) of the TOPEX/POSEIDON (T/P) response tidal model and the FES95.2.1 and CSR3.0 tidal models for the deep oceans, i.e. depths larger than 200 m. The crossover data are from TOPEX/POSEIDON GDRs 125-130, ERS-1 OPRs 11-12, and GEOSAT GDRs 1-4. The uncorrected crossover differences (no ocean tidal corrections applied) are edited using a $3.5\sigma$ criterion. The maximum time interval between crossing tracks is 5 days, 18 days, and 9 days for TOPEX/POSEIDON, ERS-1, and GEOSAT, respectively.

differences, tidal corrections were applied at exactly the same crossover locations to warrant a fair comparison between the models.

According to Table 8.4, the CSR3.0 model gives the lowest crossover difference $rms$ for all three satellites. However, the differences of the crossover statistics are so marginal that it may be concluded that all the models perform equally well, as was also indicated by the fit with the tide gauges in Table 8.3. Notice that besides a larger altimeter background noise level, the larger ERS-1 and GEOSAT crossover difference $rms$ values can be explained by the limit that was set to the time interval between crossing tracks, i.e. in case of ERS-1 and GEOSAT, the larger time interval causes ocean variability to take more effect on the crossover difference $rms$ values. Still, as mentioned earlier, these time intervals are based on a compromise between keeping the effect of ocean variability small and getting crossover differences at all latitudes.

To investigate in what regions the tidal models show the largest reduction of the crossover difference $rms$, Figures 8.7 to 8.9 show the crossover difference $rms$ obtained with FES95.2.1 and CSR3.0, relative to the crossover difference $rms$ obtained with the TOPEX/POSEIDON response solution. For all three satellites, the differences of the crossover difference $rms$ values were derived by computing the crossover difference $rms$ values of the FES95.2.1 and CSR3.0 models on a $4^\circ \times 4^\circ$ grid, and next subtracting from these values, the crossover difference $rms$ computed with the TOPEX/POSEIDON response model. The resolution of $4^\circ$ was dictated by TOPEX/POSEIDON. It may be shown that if a $3^\circ$ latitude resolution is adopted, three latitude bands centered around the equator and at latitudes of $12^\circ$ will contain no crossover differences. In the top plots of each of the Figures 8.7 to 8.9, the crossover difference $rms$ itself is shown as computed with the TOPEX/POSEIDON response tidal model. In these top plots it is easy to detect ocean variability associated with current systems like the Agulhas Current and its Return Current below South Africa, the Brazil and Malvinas Currents in their confluence region off the east coast of South America, the Gulfstream in the North Atlantic, and the
Figure 8.7 TOPEX/POSEIDON (T/P) crossover difference $rms$ (top plot). The middle and bottom plots show, respectively, the crossover difference $rms$ obtained with FES95.2.1 and CSR3.0 relative to the crossover difference $rms$ obtained with the T/P response tidal solution. All depicted values are in cm and are computed over $4^\circ \times 4^\circ$ grid cells. The admitted time interval between crossing tracks is five days. Notice the differences in scale.
Figure 8.8 ERS-1 crossover difference \( rms \) (top plot). The middle and bottom plots show, respectively, the crossover difference \( rms \) obtained with FES95.2.1 and CSR3.0 relative to the crossover difference \( rms \) obtained with the TOPEX/POSEIDON (T/P) response tidal solution. All depicted values are in cm and are computed over \( 4^\circ \times 4^\circ \) grid cells. The admitted time interval between crossing tracks is eighteen days. Notice the differences in scale.
Figure 8.9 GEOSAT crossover difference $rms$ (top plot). The middle and bottom plots show, respectively, the crossover difference $rms$ obtained with FES95.2.1 and CSR3.0 relative to the crossover difference $rms$ obtained with the TOPEX/POSEIDON (T/P) response tidal solution. All depicted values are in cm and are computed over $4^\circ \times 4^\circ$ grid cells. The admitted time interval between crossing tracks is nine days. Notice the differences in scale.
Kuroshio Current in the North-West Pacific. Judging from the crossover differences, it seems that the accuracy of the different tidal models varies strongly with location, and no model may be said to perform best globally. Notice that because the time interval between crossing tracks at a certain latitude is not the same for all satellites, tidal errors show up differently in the crossover differences of the three satellites. Therefore, judging from crossover differences in what region a tidal model performs best, to some extent depends on the considered satellite.

Interesting to notice in Figures 8.7 to 8.9 is the large bright spot below South Africa in the crossover difference rms computed with FES95.2.1 relative to this rms computed with the TOPEX/POSEIDON response tidal model. The spot is especially clear in the difference plot of the TOPEX/POSEIDON crossover rms values, i.e. the middle plot of Figure 8.7. Likely, this spot is related to the FES95.2.1 boundary problem suggested in Section 8.2.4. In case of ERS-1 and GEOSAT, i.e. in Figures 8.8 and 8.9, respectively, the bright spot is less clear than with TOPEX/POSEIDON. Likely, this is because FES95.2.1 used CSR2.0, which is based on TOPEX/POSEIDON altimetry, i.e. the TOPEX/POSEIDON crossover differences in Figure 8.7 are along the same tracks on which the tidal samples were taken that produced CSR2.0. Hence, it seems that errors in the ocean tide models appear strongest along the tracks of the satellite of which the data were used in the model. Also interesting to notice from Figure 8.7 (middle and bottom plots) is the dark spot in the Antarctic seas, especially in the Weddell Sea, due to the yearly growth of sea ice (Chapter 6) which leads to errors in the TOPEX/POSEIDON response solution. In case of ERS-1 and GEOSAT, it may be shown that the crossover differences are computed within periods of ice cover so that almost no crossover differences are found in the Weddell Sea.

8.3 ERS-1 and GEOSAT harmonic and response solutions

The cotidal charts of the ERS-1 and GEOSAT harmonic and response tidal solutions look essentially the same as those of TOPEX/POSEIDON, with the obvious exception of $S_2$ from ERS-1, and therefore are not presented. It should be mentioned, however, that the amplitudes but especially the phase lags of the ERS-1 and GEOSAT tidal solutions are less spatially coherent than those of the TOPEX/POSEIDON solution. Obviously, this is due to the larger noise level of the ERS-1 and GEOSAT altimetry and because of the larger Rayleigh periods. As a consequence, orbit errors have gone into the ERS-1 and GEOSAT tidal solutions, whereas correlated tides are more difficult to separate. Both destroy the spatial coherence of the tidal amplitudes and phase lags, which was particularly clear with the phase lag of the $S_2$ and $K_1$ tides as observed by GEOSAT, and the $K_1$ tide
as observed by ERS-1.

With the seasonal cycles as observed by ERS-1 and GEOSAT, many amplitude features were found to agree with Figure 8.2, such as the Gulfstream, the Kuroshio Current, the Somali Current, the South Equatorial Current, and the anomalies associated with the monsoon. The phase of the annual cycle also showed considerable agreement. Especially the north-south asymmetry could be clearly detected but also the complicated phase pattern in the Indian Ocean was found to be largely the same. In case of the semi-annual cycle, the phase showed less agreement, but this is most likely because it is less spatially coherent by itself. Still, besides these agreements there are also many differences, which, in general, were found to be of the order of 2 cm in vector magnitude and can mainly be explained by the correlations of the tides with the seasonal cycles as discussed in Chapter 6. However, the semi-annual cycle of the response solution of ERS-1 was found significantly different from that of TOPEX/POSEIDON and GEOSAT, and large differences of about 10 cm vector magnitude were found near the equator. This will be explained later in this section. Other large differences were found of about 5 cm vector magnitude. These differences are not caused by the tidal correlations but by the strength of the seasonal cycles, and they are mainly found in the equatorial Pacific where all satellites detect an ENSO. Because the ENSO event is not strictly annual or semi-annual, these differences are due to the different time periods of the satellite missions [e.g., Nerem et al., 1994b].

In case of the estimated dynamic topography of ERS-1, the \( a_0 \) parameter of the harmonic solution was completely ill-determined because of its correlation with \( S_2 \). With the response solution, however, a reasonable estimate for the dynamic topography could be obtained. The \( \text{rms} \) difference of this topography solution with the solution of TOPEX/POSEIDON was found to be about 10 cm. Besides the \( S_2 / a_0 \) correlation, the differences of the topography estimates from the three satellites are caused by a different spatial sampling of the sea surface and by the differences of the grid cell sizes (Table 5.5). Also, the geographically-correlated part of the orbit error (see Section 8.2.3), which will be different for each satellite and which will be absorbed by the estimated topography, will introduce differences. In case of GEOSAT, the topography estimates of the harmonic and response tidal solutions gave nearly equal results. Still, differences with the TOPEX/POSEIDON dynamic topography solution of about 10 cm \( \text{rms} \) were found, which is of the same magnitude as for ERS-1. The main part of these differences can be explained by a bad centering of the GEOSAT orbit in the earth-fixed reference frame due to the sparseness of the tracking stations [Chambers, 1996; Naeije et al., 1996] (see also Figure 9.4 in Section 9.6).

The treatment in the multi-satellite tidal solution of the different estimates of each satellite for the seasonal cycles and for the dynamic topography will be discussed in Section 9.3.
8.3 ERS-1 and GEOSAT harmonic and response solutions

<table>
<thead>
<tr>
<th>tide</th>
<th>ERS-1 harmonic response</th>
<th>no.</th>
<th>GEOSAT harmonic response</th>
<th>no.</th>
</tr>
</thead>
<tbody>
<tr>
<td>$M_2$</td>
<td>2.65</td>
<td>2.72</td>
<td>31</td>
<td>2.40</td>
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<tr>
<td>$S_2$</td>
<td>-</td>
<td>-</td>
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<td>1.70</td>
<td>46</td>
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</tr>
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</tr>
<tr>
<td>$O_1$</td>
<td>1.67</td>
<td>1.67</td>
<td>41</td>
<td>1.11</td>
</tr>
</tbody>
</table>

Table 8.5 *Rms* tide gauge differences (cm) of the ERS-1 and GEOSAT single-satellite harmonic and response tidal solutions for the reference network of 84 tide gauges. Ocean loading from FES95.2.1 is subtracted from the tidal solutions. The fourth and seventh columns give the number of gauges that agree better with the response solution than with the harmonic solution. Boxed numbers in bold refer to the correlation problems identified in Tables 4.4 and 4.5.

<table>
<thead>
<tr>
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<th>GEOSAT</th>
</tr>
</thead>
<tbody>
<tr>
<td>$M_2$</td>
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</tr>
<tr>
<td>$S_2$</td>
<td>-</td>
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<td>$O_1$</td>
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<tr>
<td>$S_{3a}$</td>
<td>7.63</td>
</tr>
<tr>
<td>$S_a$</td>
<td>1.87</td>
</tr>
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</table>

Table 8.6 *Rms* (cm) of the differences between the ERS-1 harmonic and response tidal solutions and between the GEOSAT harmonic and response tidal solutions. Denoted by $S_a$ and $S_{3a}$ are the annual and semi-annual cycle, respectively, whereas $a_0$ denotes the dynamic topography. Boxed numbers in bold refer to the correlation problems identified in Tables 4.4 and 4.5. All differences are edited using a 3.5σ criterion.

In Table 8.5, the *rms* tide gauge differences of the five dominant tides are listed for the ERS-1 and GEOSAT single-satellite harmonic and response tidal solutions (load tide derived from FES95.2.1). Table 8.6 gives the *rms* differences between the harmonic and response tidal solutions. Obvious from Tables 8.5 and 8.6 is that the tides involved in a correlation problem, most noticeably $K_1$ with ERS-1, and $S_2$ and $K_1$ with GEOSAT, have large *rms* tide gauge differences and large *rms* solution differences. Also obvious is that the response solutions of these tides agree better with the tide gauges than the harmonic solutions because the response method better separates correlated tides. This suggests that the differences in Table 8.6 are due to errors that are mainly in the harmonic tidal solutions. Still, it can be seen that if a tide is involved in a correlation problem, the *rms* tide gauge difference of the response solution also increases. Again, this can most clearly be seen for $K_1$ with ERS-1 and for $S_2$ and $K_1$ with GEOSAT. This shows that although the response method gives a better separation of correlated tides, it does not solve the correlation problems, *i.e.* the admittance at the frequencies of the correlated tides is definitely affected. In the remainder of this section, the results of Tables 8.5 and 8.6
will be discussed in more detail.

In case of ERS-1, Table 8.6 shows that for the dominant $M_2$ tide and for the $O_1$ tide, which is not involved in a correlation problem, the differences between the harmonic and response solutions after editing are of the order of 0.4 cm $rms$. In case of $N_2$, which is correlated with $M_2$ in interleaved latitude bands, an $rms$ solution difference of about 0.8 cm is found. For the $S_2$ and $K_1$ tides, much larger solution differences are computed because of their correlation with the dynamic topography and the annual cycle, respectively. The $S_2$ and $a_0$ differences are not listed because of their abnormally large value, i.e. everywhere larger than 300 cm with a global $rms$ of about 500 cm. It was found that these large differences are mainly caused by the harmonic solution of which the $S_2$ and $a_0$ estimates were found to be completely ill-determined. In case of the response solution, the $a_0$ parameter was found to reasonably represent the dynamic topography. However, the $S_2$ tide of the response solution was found ill-determined, i.e. the $M_2$ and $K_2$ signals in the admittance were unable to enforce a reliable $S_2$ estimate. As will be explained below, the reason is that the admittance nearby $S_2$ and $K_2$ is distorted because ERS-1 aliases $K_2$ to a period of half a year, i.e. the period of the semi-annual cycle. This leads to a correlation between this cycle and the admittance nearby $K_2$, which also explains the large $S_{sa}$ solution difference $rms$ of 7.63 cm with ERS-1 in Table 8.6.

Figure 8.10 shows the vector differences between the ERS-1 harmonic and response solutions. Notice that in all plots, a number of satellite tracks can be seen across the Atlantic and below Australia, which very likely indicates large orbit errors. Interesting are the differences of $K_1$ in Figure 8.10, which resemble the amplitude pattern of this tide in Figure 8.1. This indicates a globally better performance of the $K_1$ response solution, which can be seen from Table 8.5, where the response solution has an $rms$ tide gauge difference of 2.17 cm compared to 4.04 cm for the harmonic solution. Also, the number of 58 out of 84 gauges which show a better agreement with the response solution is significant. More importantly, the fact that the solution differences of $K_1$ and those of the annual cycle do not look alike demonstrates that their phase advance difference on crossing tracks of more than 130° up to latitudes of 66° leads to a global decorrelation in spite of their infinitely large Rayleigh period.

Of concern in Table 8.6 and Figure 8.10 are the large differences of the semi-annual cycle for ERS-1. It was found that the $S_{sa}$ estimate of the harmonic solution from ERS-1 is a mixture of semi-annual and semi-diurnal signals. Looking at Table 4.1, it can be seen that $K_2$ aliases to half a year, while it may be shown that the phase advance difference of $K_2$ and the semi-annual cycle is less than 90° up to 66° of latitude. Hence, the semi-diurnal signal detected in the harmonic solution of $S_{sa}$ is most likely the $K_2$ tide at places where it has a large amplitude. A detailed
8.3 ERS-1 and GEOSAT harmonic and response solutions

![Vector differences of $M_2$ (left) and $N_2$ (right)](image)

![Vector differences of $K_1$ (left) and $O_1$ (right)](image)

![Vector differences of $S_{26}$ (left) and $S_6$ (right)](image)

Figure 8.10 Vector differences between the ERS-1 harmonic and response tidal solutions (cm). Denoted by $S_6$ and $S_{26}$ are the annual and semi-annual cycle, respectively. The white color indicates depths less than 200 m. Notice the different scales.

analysis of the semi-diurnal admittance from ERS-1, mainly by comparing it with the admittance from TOPEX/POSEIDON, revealed that in the absence of $S_6$, there is no signal other than $K_2$ to determine the semi-diurnal admittance around the $S_2$ frequency. Because of the $K_2$ alias period of half a year in case of ERS-1, part of the admittance nearby $K_2$ becomes correlated with $S_{26}$, i.e. correlations of about 70% were found between the semi-diurnal response weights and the harmonic constants of the semi-annual cycle (correlations of about 50% were found between these weights and the estimated dynamic topography for obvious reasons). As a consequence of aliasing of the $K_2$ tide in case of ERS-1, the admittance nearby
$K_2$, as well as the $S_s$ solution of the response method, become distorted. This was found especially true in areas where there are less observations (Figure 6.1), which explains the speckle pattern with large $S_s$ differences around the equator in Figure 8.10. The ill-determined $S_s$ solution of the ERS-1 response solution is, therefore, almost completely responsible for the $S_s$ differences in Figure 8.10 and the large value of 7.63 cm in Table 8.6. Because of the correlation of $S_s$ with the admittance nearby $K_2$, the $S_2$ and $K_2$ tides as inferred from the ERS-1 admittance showed the same amplitude pattern as the ill-determined $S_s$ seasonal cycle, which resembles that of the $S_s$ differences in Figure 8.10.

Interesting to notice in Figure 8.10 is that the solution differences of the annual cycle look similar to the solution differences of the semi-annual cycle. This is due to the slight correlation between the seasonal cycles (Section 6.4). Hence, the distortion of the semi-diurnal admittance and the semi-annual cycle seems to affect the solution of the annual cycle as well, which largely explains the somewhat larger $S_a$ difference in Table 8.6 for ERS-1.

For GEOSAT, Table 8.5 shows that the $rms$ tide gauge differences of the dominant $M_2$ tide and of the uncorrelated $N_2$ and $O_1$ tides, are approximately the same for the harmonic and the response solutions. Table 8.6 shows that the $rms$ solution differences of these tides are of the order of 0.4 cm. For the differences of the dynamic topography solutions, an $rms$ value of 1.03 cm is found.

In Figure 8.11, the vector differences between the GEOSAT harmonic and response solutions are displayed. Like with ERS-1, notice the thickness in the solution differences due to orbit errors. With the $M_2$ differences, a band pattern can be seen that seems to resemble the predicted pattern of latitude bands due to the correlation of $M_2$ with the annual cycle in Figure 6.8. However, the bands with large $M_2$ differences do not exactly coincide with bands of large $M_2/S_s$ correlation. Possibly, this is because the GEOSAT tidal solutions are developed on a $2^\circ \times 2^\circ$ grid. Because the GEOSAT crossover locations do not exactly occur at $2^\circ$ spacing in longitude, whereas the ground track pattern becomes denser with increasing latitude, there are grid cells that contain more crossover points than others. Therefore, the pattern in Figure 8.11 may be due to a combination of decorrelation by phase advance differences on crossing tracks and the number of crossover points. In the differences of the annual cycle, no latitude bands as with $M_2$ can be seen. This is probably because $M_2$ is a much stronger signal than $S_s$, i.e., if a band pattern exists in the solution differences of the annual cycle, it will likely be obscured by altimeter background noise. The fact that the bands can be seen in the $M_2$ solution of GEOSAT and not in that of ERS-1 in Figure 8.10 (because of the $M_2/N_2$ correlation), may be due to the larger correlation in bands with a small phase advance difference, i.e., 30% for GEOSAT ($M_2/S_a$ correlation in Fig-
Figure 8.11 Vector differences between the GEOSAT harmonic and response tidal solutions (cm). Denoted by $S_a$ and $S_{sa}$ are the annual and semi-annual cycle, respectively, whereas $a_0$ denotes the dynamic topography. The white color indicates depths less than 200 m. Notice the different scales.
ure 6.8), and 20% for ERS-1 ($M_2/N_2$ correlation in Figure 6.6). Also, the $M_2/S_a$ correlation band pattern with GEOSAT is much broader so that it may appear more clearly.

For $S_2$, $K_1$, and the semi-annual cycle, which are all involved in the same correlation problem with GEOSAT, $rms$ differences of about 2 cm are observed in Table 8.6. With the $S_2/K_1/S_{sa}$ triad, the differences in Figure 8.11 look the same because of their correlations. That the $S_2/K_1/S_{sa}$ correlations mainly affect the harmonic solution is demonstrated by Table 8.5, where the $S_2$ and $K_1$ tides of the harmonic solution are seen to have a much larger $rms$ tide gauge difference than those of the response solution. The number of 65 gauges which agree better with the $S_2$ response solution, clearly shows that the response method better separates correlated tides. This could be confirmed by a covariance analysis of the GEOSAT response solution from which a correlation between the semi-diurnal response weights and the semi-annual cycle was found of about 15%, which is small compared to the correlation of 59% between $S_2$ and the semi-annual cycle in case of the harmonic solution (Table 6.3). In case of $K_1$, a better performance of the response solution at 47 of the 84 gauges seems not very convincing. However, the correlation of $K_1$ with $S_2$ and with $S_{sa}$ is largest at higher latitudes (Section 6.5) where the small number of tide gauges makes a comparison between the harmonic and the response solution very difficult.

The results presented in this section clearly show that the ERS-1 and GEOSAT tidal solutions are inferior to the TOPEX/POSEidon solution. Hence, detailed tide gauge and crossover difference analyses as described in Section 8.2.5 for the TOPEX/POSEidon response solution have not been performed for ERS-1 and GEOSAT.

8.4 Discussion

According to the $rms$ tide gauge differences, the response method gives better estimates than the harmonic method if correlations exist between the tides. If these correlations do not exist, or if the correlations are reasonably small and the signal to noise ratio is large enough like with the dominant $M_2$ tide, then both methods give nearly the same results.

In case of TOPEX/POSEidon, the $rms$ tide gauge differences of the harmonic and response solutions, and the crossover statistics obtained with the response solution, compare quite well with the results of the FES95.2.1 and CSR3.0 models. This indicates a remarkable agreement of the empirical TOPEX/POSEidon tidal solution and the hydrodynamically consistent FES95.2.1 and CSR3.0 tidal solutions in the deep oceans. Moreover, the results of the $rms$ tide gauge differences suggest an accuracy of approximately 2.7 cm for the TOPEX/POSEidon response tidal model, as well as for the FES95.2.1 and CSR3.0 models, in the deep oceans. This
is extremely encouraging and a clear tribute to the accuracy of the TOPEX/POSEIDON sea level measurements as well as to its favorable orbit design to observe the ocean tide.

Interesting are the bathymetry-induced differences of cm magnitude that were found between the empirical TOPEX/POSEIDON tidal model and the hydrodynamically consistent FES95.2.1 and CSR3.0 models. These differences are either due to errors in the bathymetry model underlying the hydrodynamic solutions of the FES model series, or due to the relatively coarse resolution of the TOPEX/POSEIDON altimeter data. Possibly, the actual cause of these differences may be revealed by using different bathymetry models in the hydrodynamic equations. Also interesting are the differences between the TOPEX/POSEIDON tidal model and FES95.2.1 of 1-2 cm vector magnitude in the Antarctic seas. These differences are because of ice cover, which causes errors in the TOPEX/POSEIDON tidal model, and in altimetry-based tidal models in general. In this respect, an interesting subject for further study may be to use the harmonic constants of tide gauges to constrain the solutions in these seas. The harmonic constants of existing models may also be used for this purpose, which then should be independent of altimeter data.

In case of ERS-1 and GEOSAT, which have larger orbit errors than ERS-1 and GEOSAT, the rms tide gauge differences are worse than those of TOPEX/POSEIDON for all five dominant tides. This is especially true for the $K_1$ tidal solution of ERS-1 and the $K_1$ and $S_2$ tidal solutions of GEOSAT. These results can be explained by orbit errors that have gone into the ERS-1 and GEOSAT tidal solutions and by the $K_1/S_a$ correlation problem of ERS-1 and the $S_2/K_1/S_a$ correlation problem of GEOSAT.

With ERS-1, a problem is that in the absence of the $S_2$ tidal signal in the ERS-1 sea surface height observations, the semi-diurnal admittance nearby the $S_2$ and $K_2$ frequencies becomes defined by the semi-annual cycle because of the aliasing of the $K_2$ tide.
Chapter 9

Multi-satellite response tidal solution

9.1 Introduction

In this chapter, the tidal normal matrices of TOPEX/POSEIDON, ERS-1, and GEOSAT are weighed and combined to a multi-satellite response tidal solution. The response solution has been chosen because it can provide a number of smaller tides by admittance interpolation, and because it better separates correlated tides, as was shown in Section 8.3. A multi-satellite harmonic solution has therefore not been developed. The weights on the normal matrices are selected such that they minimize the $\text{rss}$ of the differences between the multi-satellite response solution and the harmonic constants of the five dominant tides from the reference network of 84 tide gauges. With the selected weights, a multi-satellite response solution is created that accounts for the same 23 tidal lines that were selected for the TOPEX/POSEIDON response solution in Section 8.2.2. The performance of the multi-satellite response solution is compared with that of the response solution from TOPEX/POSEIDON data only. Whether or not the multi-satellite tidal solution has improved the TOPEX/POSEIDON solution is determined mainly from the crossover statistics of sixty days of TOPEX/POSEIDON altimetry that were not used in the response tidal solutions. The differences of the TOPEX/POSEIDON and multi-satellite response solutions are also examined to answer this question.

9.2 Combining the single-satellite tidal normal matrices

In this section, the procedure to derive the multi-satellite tidal response solution is explained. As discussed in Section 5.3, the first step of this procedure is the gridding of the $ssh$ residuals of each satellite onto a regular mesh with single-satellite tidal normal matrices. Hence, tidal normal
matrices are obtained on grids with a resolution of $3^\circ \times 3^\circ$, $2^\circ \times 2^\circ$, and $1^\circ \times 1^\circ$ for TOPEX/POSEIDON, GEOSAT, and ERS-1, respectively. The second step is to spatially interpolate the single-satellite tidal normal matrices of TOPEX/POSEIDON and GEOSAT to the resolution of ERS-1, i.e. $1^\circ \times 1^\circ$. The resolution of $1^\circ$ defines the smallest tidal wavelength that can be observed in the multi-satellite tidal solution. The validity of the spatial interpolation of normal matrices will be discussed at the end of this section. The third and final step is to combine the single-satellite tidal normal matrices to a multi-satellite tidal normal matrix, and to solve this multi-satellite normal matrix, thus obtaining a multi-satellite tidal solution with $1^\circ \times 1^\circ$ resolution.

In general, the reason to interpolate the solutions is to retain an acceptable resolution near the coast lines in case of TOPEX/POSEIDON and GEOSAT, by interpolating the solution grids to, e.g., $1^\circ \times 1^\circ$ resolution before masking them in shallow waters. However, with regard to the multi-satellite tidal solution, another and more important reason to interpolate is that we want to let the TOPEX/POSEIDON ssh observations reduce the correlation problems of ERS-1 and GEOSAT (Tables 4.4 and 4.5) to optimally take advantage of the resolutions of the GEOSAT and ERS-1 altimetry to observe smaller wavelength tidal features. This is especially useful to decorrelate the tides from the seasonal cycles. Likewise, we want the $K_1$ tide as observed by ERS-1 to reduce the $K_1/S_{sa}$ correlation problem of TOPEX/POSEIDON and GEOSAT at higher latitudes.

Because there is no significant correlation between the seasonal cycles and the diurnal and semi-diurnal ocean tides as observed by TOPEX/POSEIDON, the seasonal cycles are well defined in the TOPEX/POSEIDON normal equations, with exception of $K_1$ and the semi-annual cycle at latitudes beyond $55^\circ$. Hence, when these normal equations are combined with those of ERS-1, they will further reduce the correlation of the annual cycle with $K_1$ as observed by ERS-1 (see Table 4.4 and Section 6.4). Likewise, when the TOPEX/POSEIDON normal equations are combined with those of GEOSAT, they will reduce the correlation between the annual cycle and $M_2$ as observed by the latter satellite, as well as the correlation of both $S_2$ and $K_1$ with the semi-annual cycle (see Table 4.5 and Section 6.5). In case of ERS-1, there is no significant correlation between $K_1$ and the semi-annual cycle, whereas the phase advance differences on crossing tracks largely decorrelate $K_1$ from the annual cycle as was shown in Section 6.4. Hence, the $K_1$ observations of ERS-1 should be useful to reduce the $K_1/S_{sa}$ correlation problem of TOPEX/POSEIDON and GEOSAT at higher latitudes. Thus, to obtain an accurate multi-satellite solution, we need to interpolate and combine the normal matrices and not the tidal solutions, because if we first solve the normal equations of each satellite, then the correlations will already have deteriorated the solutions. Notice that the procedure as sketched above is based on the assumption that the seasonal cycles do not vary too much over the periods in Table 5.1.
This will be discussed in Section 9.3.

Weighing the normal equations (5.5) of ERS-1 with \( w_E \) and those of GEOSAT with \( w_G \), relative to a weight on the TOPEX/POSEIDON normal equations (5.3) of one, results in a system of normal equations \( Nx = b \) from which the multi-satellite response tidal solution \( x \) is solved on a \( 1^\circ \times 1^\circ \) grid:

\[
N = \frac{1}{9} N_{T/P} + w_{E} N_{E} + \frac{1}{4} w_{G} N_{G} \tag{9.1}
\]

\[
b = \frac{1}{9} b_{T/P} + w_{E} b_{E} + \frac{1}{4} w_{G} b_{G}
\]

In the above equation, the subscripts "\( T/P \)" "\( E \)" and "\( G \)" are used to denote the single-satellite normal equations of TOPEX/POSEIDON, ERS-1, and GEOSAT, respectively. In Section 9.4, the weights \( w_E \) and \( w_G \) will be determined empirically. The factors \( \frac{1}{9} \) and \( \frac{1}{4} \) account for the differences in grid resolution. Because the initial grid resolutions of TOPEX/POSEIDON and GEOSAT are \( 3^\circ \times 3^\circ \) and \( 2^\circ \times 2^\circ \), respectively (see Table 5.5), whereas pseudo observations were created on a grid of \( 1^\circ \times 1^\circ \), the TOPEX/POSEIDON normal equations have to be downweighted by \( (\frac{1}{3})^2 = \frac{1}{9} \) with respect to ERS-1. In case of GEOSAT, this factor is \( (\frac{1}{2})^2 = \frac{1}{4} \). Notice that storing the single-satellite tidal normal matrices on a \( 1^\circ \times 1^\circ \) grid has provided a flexible tool for trying several combinations of the weights \( w_E \) and \( w_G \) in Section 9.4 without having to go through the data reduction process anew for each combination. Also notice that the number of estimated parameters of the multi-satellite response solution does not necessarily have to be the same as with the single-satellite response solutions, i.e. 17 as explained in Section 5.4. For instance, the blocks \( N_{ij} \) in (5.5) can be arranged such that they give a multi-satellite normal matrix from which three separate sets of seasonal cycles may be solved. This could be necessary because non-periodic ocean variability, e.g. from ENSO, leads to different estimates of the seasonal cycles for the different satellite missions. Also, it may be necessary to solve for three separate dynamic topographies if large systematic differences between these surfaces are expected with the three satellites. However, as will be discussed in Section 9.3, it is preferred to estimate the same number of parameters with the multi-satellite solution as with the single-satellite solutions. Hence, the normal matrix \( N \) and the right-hand side \( b \) in the multi-satellite normal equations (9.1) have the same form as in the single-satellite normal equations (5.5).

In Figure 9.1, the concept of spatial interpolation of the tidal normal matrices is shown. This figure shows that the TOPEX/POSEIDON or GEOSAT tidal normal matrix at the desired location \( \otimes \), which will be on the \( 1^\circ \times 1^\circ \) grid, is obtained by bilinear interpolation of the normal matrices \( N_1 \) to \( N_4 \) that were computed on the grids with initial resolutions
Figure 9.1 Bilinear interpolation of normal matrices. The normal matrix $N$ at the desired location $\otimes$ is obtained by bilinear interpolation of the normal matrices $N_1$ to $N_4$. The initial resolution of the grid, i.e. before interpolation, is denoted by $\Delta \lambda$ and $\Delta \phi$. The final resolution of the grid, i.e. after interpolation, is given by the grid cell centered around $\otimes$. The desired position $\otimes$ is specified relative to the bottom-left corner of the grid cell of initial size by $\delta \lambda$ and $\delta \phi$.

$\Delta \lambda$ and $\Delta \phi$ as given in Table 5.5:

$$N = (1 - t)(1 - u)N_1 + t(1 - u)N_2 + tuN_3 + (1 - t)uN_4$$  \hspace{1cm} (9.2)

Simple bilinear interpolation was chosen because with regard to, e.g., ocean tide models, it seems to be the generally preferred method for interpolating in grid cells of small size [Andersen et al., 1995]. The parameters $t$ and $u$ are defined as (Figure 9.1):

$$t = \frac{\delta \lambda}{\Delta \lambda} \hspace{2cm} u = \frac{\delta \phi}{\Delta \phi}$$  \hspace{1cm} (9.3)

where $\delta \lambda$ and $\delta \phi$ denote the position of the desired location $\otimes$ relative to the bottom-left corner of the grid cell of initial size.

The validity of spatially interpolating the normal matrices may be demonstrated by comparing the $1^\circ \times 1^\circ$ tidal solutions as obtained by interpolation of the $3^\circ \times 3^\circ$ and $2^\circ \times 2^\circ$ normal matrix grids of TOPEX/POSEIDON and GEOSAT, respectively, with the $1^\circ \times 1^\circ$ tidal solutions obtained...
9.2 Combining the single-satellite tidal normal matrices

<table>
<thead>
<tr>
<th></th>
<th>$M_2$</th>
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<th>$N_2$</th>
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<td>0.06</td>
<td>0.06</td>
<td>0.10</td>
<td>0.16</td>
</tr>
<tr>
<td>GEOSAT</td>
<td>0.30</td>
<td>0.15</td>
<td>0.07</td>
<td>0.13</td>
<td>0.09</td>
<td>0.18</td>
<td>0.31</td>
<td>0.34</td>
</tr>
</tbody>
</table>

Table 9.1 *Rms* (cm) of the TOPEX/POSEIDON response tidal solution differences and of the GEOSAT response tidal solution differences. In case of TOPEX/POSEIDON, the differences are computed between a $1^\circ \times 1^\circ$ tidal solution obtained by interpolation of a $3^\circ \times 3^\circ$ tidal solution, and a $1^\circ \times 1^\circ$ tidal solution derived from interpolation of a $3^\circ \times 3^\circ$ normal matrix grid to a $1^\circ \times 1^\circ$ normal matrix grid. In case of GEOSAT, the differences are computed between a $1^\circ \times 1^\circ$ tidal solution obtained by interpolation of a $2^\circ \times 2^\circ$ tidal solution, and a $1^\circ \times 1^\circ$ tidal solution derived from interpolation of a $2^\circ \times 2^\circ$ normal matrix grid to a $1^\circ \times 1^\circ$ normal matrix grid. Denoted by $S_a$ and $S_{sa}$ are the annual and semi-annual cycle, respectively, whereas $a_0$ denotes the dynamic topography. All differences are edited using a 3.5σ criterion.

by first solving the $3^\circ \times 3^\circ$ and $2^\circ \times 2^\circ$ normal matrix grids of TOPEX/POSEIDON and GEOSAT, and then interpolating the solutions to $1^\circ \times 1^\circ$ resolution. Obviously, both solution types will give the same result if the normal matrices are spatially coherent, i.e. if the elements in the normal matrices of neighboring cells in the initial grid are largely the same. This may be expected because the number of ssh observations and the tidal correlations vary smoothly with geographic location (Chapter 6). The coherence may be destroyed in areas where there are relatively few observations, e.g. near the coast and in the Antarctic seas. It should be mentioned, however, that if the coherence of the normal matrices in neighboring cells is destroyed, the estimated parameters loose their intrinsic value, meaning that the errors introduced by interpolation of the normal matrices are probably just as large as the errors introduced by interpolation of the solutions.

Table 9.1 shows the *rms* of the $1^\circ \times 1^\circ$ solution differences of the five dominant tides of the TOPEX/POSEIDON response solutions and of the GEOSAT response solutions. The solution differences of the seasonal cycles and of the estimated dynamic topography are listed as well. All differences in Table 9.1 are edited using a 3.5σ criterion. It can be seen that the TOPEX/POSEIDON solution differences have a global *rms* of about 0.1-0.2 cm. In case of GEOSAT, the solution differences are of the order of 0.1-0.3 cm. This number is slightly larger than the 0.1-0.2 cm for TOPEX/POSEIDON due to the larger altimeter background noise of GEOSAT. As expected, the largest solution differences, i.e. with a vector magnitude of about 0.5 cm, were found near the coast and in parts of the Antarctic seas, for both TOPEX/POSEIDON and GEOSAT.

In conclusion, because the differences of the TOPEX/POSEIDON tidal solutions and of the GEOSAT tidal solutions in Table 9.1 have an *rms* of the order of only a few millimeters, the adopted procedure of spatially interpolating the tidal normal matrices to $1^\circ \times 1^\circ$ resolution seems rigorously
valid.

9.3 Treatment of the estimated topography and seasonal cycles

In Section 8.3, it was recognized that the different spatial samplings of the sea surface and the differences of the grid cell sizes, as well as the differences in the geographically-correlated part of the orbit error, cause different topography estimates for the single-satellite response tidal solutions. In case of GEOSAT, a bad centering of the orbits also leads to dynamic topography differences (Section 8.3). The effect of different estimates for the dynamic topography on the tides was investigated by solving for a dynamic topography for each satellite separately in a multi-satellite response solution (and one common set of seasonal cycles and $w_E$ and $w_G$ equal to one). The obtained solutions for the tides were then compared with the response solution in which one dynamic topography is estimated common to all satellites (and one common set of seasonal cycles and $w_E$ and $w_G$ equal to one). For all dominant tides of the response solutions but $S_2$, $rms$ differences of merely 0.2 cm were found. Clearly, these numbers are quite insignificant, which is the result of the very small correlations between the response weights and the dynamic topography, for which values of the order of 2% were found. In case of $S_2$, a larger $rms$ difference of about 1 cm was computed, which was found to be caused by the correlation of the semi-diurnal response weights with the dynamic topography in the normal matrix of ERS-1 (Section 8.3). Because of the decorrelating effect of the TOPEX/POSEIDON and GEOSAT altimetry in the response solution where one common topography is estimated, the $S_2$ differences are mainly due to a less reliable $S_2$ estimate of the response solution where three separate topographies were estimated. The results of this experiment, therefore, show that the single-satellite normal equations may be merged into a system of multi-satellite normal equations in which one common topography is estimated without affecting the tidal estimates. This is especially true because the ERS-1 and GEOSAT normal equations will be assigned weights that are much smaller than one in the multi-satellite tidal solution, as will be discussed in Section 9.4.

In Section 8.3, it was also recognized that at some locations, like the equatorial Pacific, non-periodic ocean variability will lead to different estimates of the seasonal cycles for the different satellite missions. Possibly, this means that decorrelating tidal pairs by including the seasonal cycles in the estimation scheme cannot be realized to complete satisfaction. Still, it could be shown that single-satellite response solutions for which the seasonal cycles were left in the data give less accurate tidal estimates for $S_2$ and $K_1$ according to the $rms$ tide gauge differences for the 84 tide gauges
9.4 Optimizing the normal matrix weights

For a number of 25 combinations of the weights \( w_F \) and \( w_G \) in (9.1), it was investigated how the \( \text{rms} \) tide gauge differences for the five dominant tides of the multi-satellite response tidal solution improved with respect to the TOPEX/POSEIDON response tidal solution (weights \( w_F \) and \( w_G \) equal to zero). From Figure 9.2, it can be seen that the combination of weights that gives the smallest \( \text{rms} \) tide gauge difference is different for each tide. Hence, the "optimum" set of weights has been defined as those weights that give the smallest \( \text{rss} \) of the \( \text{rms} \) tide gauge differences. According to this criterion, weights of \( w_F = 0.03 \) and \( w_G = 0.2 \) were chosen. The dominant tides of the multi-satellite response solution derived with these weights reduced the \( \text{rss} \) of the \( \text{rms} \) tide gauge differences of the TOPEX/POSEIDON single-satellite response solution from 2.57 cm to 2.33 cm. Whether this actually means that the multi-satellite tidal solution is more accurate than the TOPEX/POSEIDON tidal solution may not be judged from the gauges as the weights were obtained by minimizing the discrepancies between the model and tide gauge harmonic constants. The differences between the TOPEX/POSEIDON and multi-satellite tidal solutions in Section 9.6 are more suitable to answer this question, as are also the statistics of TOPEX/POSEIDON, ERS-1, and GEOSAT crossover differences, to be discussed in Section 9.5.

If the errors in the \( \text{ssh} \) residuals of each of the satellites are uncorrelated with a Gaussian distribution, then the weights \( w_F \) and \( w_G \) should be equal to the inverse square of the ratio of the sea level measurement accuracies in Table 5.3, \( i.e. \ w_F = \left( \frac{5}{\sqrt{3}} \right)^2 \approx 0.35 \) and \( w_G = \left( \frac{5}{10.\sqrt{3}} \right)^2 \approx 0.21 \) [\textit{e.g., Lindgren}, 1968]. Notice that the accuracies of the TOPEX and POSEIDON sea level measurements have been averaged to 5 cm. In case of the weight of the GEOSAT normal equations, \( w_G \), the selected value of 0.2 is indeed equal to the value based on the assumption of uncorrelated Gaussian errors. However, this is probably just a coincidence because the value of the weight \( w_G \) has been derived by minimizing the \( \text{rss} \) of the \( \text{rms} \) tide gauge differences, \( i.e. \) the weight \( w_G \) that reduces the \( \text{rms} \) tide gauge differences of the TOPEX/POSEIDON tidal solution the most is different for each of the dominant tides. With ERS-1, the selected weight \( w_F \) of 0.03 is
an order of magnitude smaller than suggested by the assumption of un-
correlated Gaussian errors. Moreover, it was found that only for a weight
\( w_E \) of less than 0.04, a slight improvement of the \( rms \) tide gauge differences relative to the TOPEX/POSEIDON response solution could be seen. Most likely, these results for GEOSAT and ERS-1 indicate that systematic errors are present in the sea level measurements. Possibly, these errors may be the result of the JGM-3 background ocean tide model used for the orbit computations and of 1-cpr orbit errors that may have become aliased (see Section 8.2.3). The fact that tracking data of TOPEX/POSEIDON and

Figure 9.2 \( rms \) tide gauge differences (cm) of the multi-satellite response tidal solution for the reference network of 84 tide gauges for different combinations of the weights \( w_E \) and \( w_G \). Ocean loading from FES95.2.1 is subtracted from the multi-satellite tidal solutions. Notice the differences in scale.
GEOSAT but not of ERS-1 were used in the development of the JGM-3 background ocean tide model [Nerem et al., 1994a; Marshall et al., 1995], might explain why systematic orbit errors could be significantly larger for ERS-1 than for TOPEX/POSEIDON and GEOSAT.

### 9.5 Tide gauge and crossover difference analyses

Table 9.2 gives the *rms* tide gauge differences of the TOPEX/POSEIDON and multi-satellite response tidal solutions (load tide derived from FES95.2.1). The values of the TOPEX/POSEIDON response solution have already been presented in Table 8.3 and are given in Table 9.2 for reference. With the multi-satellite tidal solution, the slightly smaller *rss* of 2.69 cm is seen to be caused by the dominant tides. In case of the minor tides, the *rms* tide gauge differences are the same as those of the TOPEX/POSEIDON response tidal solution. Hence, the slightly smaller value of 2.69 cm is mainly because the weights of the multi-satellite solution were obtained by minimizing the discrepancies between the model and tide gauge harmonic constants.

Table 9.3 gives the crossover difference *rms* obtained with the TOPEX/POSEIDON and multi-satellite response tidal solutions. The crossover data are computed from the same GDRs and OPRs as in Table 8.4. The values of the TOPEX/POSEIDON response solution have already been presented in Table 8.4 and are given in Table 9.3 for reference. In case of the crossover difference *rms* of GEOSAT, the multi-satellite

<table>
<thead>
<tr>
<th>model</th>
<th>$M_2$</th>
<th>$S_2$</th>
<th>$N_2$</th>
<th>$K_2$</th>
<th>$2N_2$</th>
<th>$\mu_2$</th>
<th>$\nu_2$</th>
<th>$L_2$</th>
<th>$T_2$</th>
<th>$K_1$</th>
<th>$O_1$</th>
<th>$P_1$</th>
<th>$Q_1$</th>
<th><em>rss</em></th>
</tr>
</thead>
<tbody>
<tr>
<td>T/P</td>
<td>1.78</td>
<td>1.08</td>
<td>0.72</td>
<td>0.49</td>
<td>0.27</td>
<td>0.35</td>
<td>0.15</td>
<td>0.26</td>
<td>0.17</td>
<td>1.05</td>
<td>0.80</td>
<td>0.37</td>
<td>0.38</td>
<td>2.73</td>
</tr>
<tr>
<td>multi-satellite</td>
<td>1.77</td>
<td>1.02</td>
<td>0.70</td>
<td>0.48</td>
<td>0.27</td>
<td>0.35</td>
<td>0.15</td>
<td>0.26</td>
<td>0.16</td>
<td>1.06</td>
<td>0.78</td>
<td>0.38</td>
<td>0.38</td>
<td>2.69</td>
</tr>
</tbody>
</table>

Table 9.2 *Rms* tide gauge differences (cm) of the multi-satellite response tidal solution for the reference network of 84 tide gauges. The values of the TOPEX/POSEIDON (T/P) response tidal solution have already been presented in Table 8.3 and are given here for reference. Ocean loading from FES95.2.1 is subtracted from the response tidal solutions.

<table>
<thead>
<tr>
<th>model</th>
<th>T/P</th>
<th>ERS-1</th>
<th>GEOSAT</th>
</tr>
</thead>
<tbody>
<tr>
<td>T/P</td>
<td>5.6</td>
<td>10.9</td>
<td>10.7</td>
</tr>
<tr>
<td>multi-satellite</td>
<td>6.3</td>
<td>10.9</td>
<td>10.5</td>
</tr>
</tbody>
</table>

Table 9.3 Crossover difference *rms* (cm) of the multi-satellite response tidal solution for the deep oceans, *i.e.* depths larger than 200 m. Crossover data are the same as in Table 8.4. The values of the TOPEX/POSEIDON (T/P) response tidal solution have already been presented in Table 8.4 and are given here for reference.
Figure 9.3 Crossover difference \( r_{\text{ms}} \) obtained with the multi-satellite response tidal solution minus the crossover difference \( r_{\text{ms}} \) obtained with the TOPEX/POSEIDON response tidal solution for TOPEX/POSEIDON crossover differences (top), ERS-1 crossover differences (middle), and GEOSAT crossover differences (bottom). All depicted values are in cm and are computed over \( 4^\circ \times 4^\circ \) grid cells. The admitted time interval between crossing tracks is five, eighteen, and nine days for TOPEX/POSEIDON, ERS-1, and GEOSAT crossover differences, respectively.
solution gives the lowest \( \text{rms} \) value. However, this is because the altimeter data from which the GEOSAT crossover differences have been computed are included in the multi-satellite response solution (Section 8.2.5) as all GEOSAT cycles of the ERM that were not strongly affected by the attitude control problem have been used. This is clearly demonstrated by the TOPEX/POSEIDON crossover difference \( \text{rms} \), for which the multi-satellite response tidal solution gives a significantly larger value than the TOPEX/POSEIDON response tidal solution. Because the TOPEX/POSEIDON crossover differences are derived from independent data, i.e. altimeter observations that were not used in the computation of the TOPEX/POSEIDON and multi-satellite response solutions, this result shows that the multi-satellite solution is less accurate than the response solution from TOPEX/POSEIDON data only. In case of ERS-1, nearly the same crossover difference \( \text{rms} \) is found for the TOPEX/POSEIDON and multi-satellite response solutions, because of the low weight that could be assigned to the ERS-1 normal equations.

Figure 9.3 shows the difference between the TOPEX/POSEIDON, ERS-1, and GEOSAT crossover differences \( \text{rms} \) values as obtained with the TOPEX/POSEIDON and multi-satellite response tidal solutions. Figure 9.3 is clearly in agreement with the results of Table 9.3 and shows that the TOPEX/POSEIDON response tidal solution gives a smaller \( \text{rms} \) for the TOPEX/POSEIDON crossover differences, whereas the multi-satellite response tidal solution gives a smaller \( \text{rms} \) for the GEOSAT crossover differences. Notice in the middle plot of Figure 9.3 that the multi-satellite tidal solution gives a larger \( \text{rms} \) for the ERS-1 crossover differences along an ERS-1 groundtrack below Australia near a longitude of 120\(^\circ\). Very likely, this is because a number of ERS-1 tracks contain large orbit errors that have leaked into the multi-satellite tidal solution (see also Section 8.3 and Figure 8.10). Also notice that some trackiness can be seen in the bottom plot with the difference of the GEOSAT crossover difference \( \text{rms} \) values, most clearly in the East Pacific. This indicates that GEOSAT orbit errors have been absorbed into the multi-satellite tidal solution, which therefore are removed from the GEOSAT crossover differences because the GEOSAT altimetry from which these crossover differences have been computed were included in the multi-satellite tidal solution. As a result, the multi-satellite response tidal solution gives a smaller crossover difference \( \text{rms} \) along several GEOSAT groundtracks (black color) than the TOPEX/POSEIDON response tidal solution.

### 9.6 Differences between the TOPEX/POSEIDON and multi-satellite response solutions

In Figure 9.4, the vector differences between the TOPEX/POSEIDON and multi-satellite response tidal solutions are shown for the five major tides,
Figure 9.4 Vector differences between the TOPEX/POSEIDON and multi-satellite response tidal solutions (cm). Denoted by $S_a$ and $S_{aa}$ are the annual and semi-annual cycle, respectively, whereas $a_0$ denotes the dynamic topography. The white color indicates depths less than 200 m. Notice the different scales.
9.6 Differences between the T/P and multi-satellite response solutions

<table>
<thead>
<tr>
<th>$M_2$</th>
<th>$S_2$</th>
<th>$N_2$</th>
<th>$K_1$</th>
<th>$O_1$</th>
<th>$S_{sa}$</th>
<th>$S_a$</th>
<th>$a_0$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.18</td>
<td>0.23</td>
<td>0.08</td>
<td>0.28</td>
<td>0.15</td>
<td>0.23</td>
<td>0.29</td>
<td>2.73</td>
</tr>
</tbody>
</table>

Table 9.4 Rms (cm) of the differences between the TOPEX/POSEIDON and multi-satellite response tidal solutions. Denoted by $S_a$ and $S_{sa}$ are the annual and semi-annual cycle, respectively, whereas $a_0$ denotes the dynamic topography. All differences are edited using a 3.5σ criterion.

the seasonal cycles, and the estimated dynamic topography. The global rms values of these differences can be found in Table 9.4. This table shows that the solution differences of the tides and the seasonal cycles have an rms of less than 0.30 cm after editing on 3.5σ. For the smaller tides in the tidal spectrum (Figure 8.4), rms differences of less than 0.10 cm were found, except for the the ill-determined $O_1$ tide for which a value of 0.14 cm was computed. For the differences between the dynamic topography solutions, an rms value of 2.73 cm was found. Notice that the difference between the dynamic topographies in Figure 9.4 is systematically larger over the South Atlantic and over the North Pacific, which can be explained by a bad centering of the GEOSAT orbits in the earth-fixed reference frame [Chambers, 1996; Naeije et al., 1996]. However, no evidence has been found that the dynamic topography differences have leaked into the multi-satellite tidal solution, which is because of the small correlations between the dynamic topography and the response weights of the multi-satellite solution of about 2%. As explained in Section 9.3, these small correlations are the result of the decorrelating effect of the TOPEX/POSEIDON and GEOSAT altimetry on the correlation of the semi-diurnal response weights and the dynamic topography as observed by ERS-1, and of the small weight that could be assigned to the ERS-1 normal equations.

Figure 9.4 clearly shows that main part of the solution differences is explained by GEOSAT orbit errors that have gone into the multi-satellite response tidal solution. Especially with the $M_2$ and $S_a$ differences, the GEOSAT groundtrack pattern is clearly visible. Evidence of ERS-1 orbit errors can, in general, not be seen in the plots because of the low weight $w_E$ assigned to the normal equations of this satellite. Only in the $K_1$ differences below Australia we notice ERS-1 groundtracks (compare with Figure 8.10). With regard to the low weight $w_E$, this means that orbit errors on a number of ERS-1 tracks are significantly larger than the estimated budget of Table 5.3.

With the seasonal cycles, the differences are largely explained by the different analysis periods of TOPEX/POSEIDON and GEOSAT. Hence, the solutions of the seasonal cycles show large discrepancies in the equatorial Pacific where both satellites detect the ENSO, which has a different strength for different analysis periods. For all tides but $S_2$, Figure 9.4 shows that the differences of the seasonal cycles have not leaked into the tidal solutions. This is because of the small correlations of the seasonal
cycles and the response weights of the multi-satellite solution, for which values of 3-5% were found. One reason that these correlations are so small is that the TOPEX/POSEIDON normal equations have a decorrelating effect on the correlation of the semi-diurnal response weights and the semi-annual cycle as observed by ERS-1 and GEOSAT of 70% and 15%, respectively (Section 8.3). Another reason, of course, is that the ERS-1 normal equations have been assigned a very small weight in the multi-satellite tidal solution. In spite of their low weight, however, the ERS-1 normal equations do have some effect on the multi-satellite solution in the sense that they deteriorate the $S_2$ solution due to aliasing of the $K_2$ tide to semi-annual frequency (Section 8.3). For this reason, the same speckle pattern in the $S_2$ differences of Figure 9.4 can be seen near the equator as in the $S_{sa}$ differences of Figure 8.10. If the ERS-1 normal equations are assigned a weight of one in the multi-satellite response solution, the rms of the $S_2$ differences between the TOPEX/POSEIDON and multi-satellite response solutions was found to increase to 2 cm. Also, the solutions of the tides nearby the $S_2$ and $K_2$ frequencies, in particular $T_2$, were found to deteriorate significantly. Hence, the correlation between the semi-annual cycle and the semi-diurnal response weights as established by aliasing of $K_2$ to semi-annual period with ERS-1, makes it difficult to add ERS-1 altimetry to the multi-satellite response solution because the $K_2$ tide in the TOPEX/POSEIDON and GEOSAT altimetry is not strong enough to prevent a deterioration of the semi-diurnal admittance.

The solution differences presented in this section are clearly in favor of the TOPEX/POSEIDON response tidal solution rather than the multi-satellite response tidal solution. The differences between the TOPEX/POSEIDON and multi-satellite response solutions show that the orbit errors of ERS-1 and GEOSAT deteriorate the multi-satellite tidal solution so that the higher resolution of the ERS-1 and GEOSAT altimeter data could not be exploited to improve the TOPEX/POSEIDON tidal solution.

### 9.7 Discussion

In this chapter, a procedure has been explained to process the ssh observations of the TOPEX/POSEIDON, ERS-1, and GEOSAT satellites such that the ability of each satellite to reduce tidal correlations may be used to optimally take advantage of the better resolution of the ERS-1 and GEOSAT altimeter data in a multi-satellite response tidal solution. However, the clear presence of orbit errors in the multi-satellite tidal solution shows that orbit errors of about 7-8 cm of ERS-1 and 10 cm of GEOSAT are too large to improve the TOPEX/POSEIDON tidal solution. Very likely, a part of the 1-cpr orbit errors is aliased to periods comparable with the tidal alias periods. Hence, although the orbit errors of ERS-1 and GEOSAT are 10 cm or less, they become correlated with the tides and therefore significantly interfere with the tidal estimates. Based on the fact that the alias periods
of ERS-1 and GEOSAT are approximately twice as large as those of TOPEX/POSEIDON, this means that to decorrelate the orbit errors from the tides to the same extent as with TOPEX/POSEIDON, probably requires six years of ERS-1 altimeter observations as well as six years of GEOSAT altimeter observations. To improve the TOPEX/POSEIDON tidal solution, which is based on three years of altimetry, these ERS-1 and GEOSAT observations preferably should have an accuracy comparable to that of the TOPEX/POSEIDON ssh observations, which means that orbits will have to be computed for ERS-1 and GEOSAT with near TOPEX/POSEIDON accuracy (see Chapter 10).

Six years of altimeter data will also largely solve the tidal correlations that have been identified. A time series of six years will decorrelate $M_2/S_a$ and $S_2/S_{sa}$ with GEOSAT, as both these pairs have a Rayleigh period of six years. The $S_2/K_1$ and $K_1/S_{sa}$ pairs, which have a Rayleigh period of twelve years with GEOSAT, will largely decorrelate by the phase advance differences on crossing tracks in a fashion similar to $K_1/S_{sa}$ with TOPEX/POSEIDON.

Decorrelation from phase advance differences on crossing tracks is also expected with ERS-1 for the $M_2/N_2$ pair with its 9-year Rayleigh period and for the $K_1/S_a$ pair with an infinitely large Rayleigh period. The decorrelation of $K_1$ from the annual cycle may then provide the necessary improvement of $K_1$ at latitudes beyond $55^\circ$ that cannot be accomplished with either TOPEX/POSEIDON or GEOSAT as these satellites both observe $K_1$ with interference of the semi-annual cycle at higher latitudes.

With respect to the above 6-year solution procedure, the extensions of the ERS-1 mission by ERS-2 and of the GEOSAT mission by the GEOSAT Follow-On, seem promising to obtain a multi-satellite tidal solution that may be more accurate than the existing TOPEX/POSEIDON tidal solutions. The expected improvements of the TOPEX/POSEIDON tidal solution from such extended missions will be discussed in the next chapter.

With ERS-1, the deterioration of the semi-diurnal admittance nearby the $S_2$ and $K_2$ frequencies because of aliasing of the $K_2$ tide to semi-annual period, may be difficult to overcome by the altimetry of TOPEX/POSEIDON and GEOSAT. This might necessitate the use of an a-priori tidal model, preferably one that has not already been fitted through altimeter data. However, this should be considered with care, as there may be errors in the a-priori model that are difficult to correct by altimetry, as was shown for CSR3.0 in Section 8.2.4. A feasible alternative to specifying an a-priori solution is to solve the multi-satellite tidal solution in an iterative procedure. With this procedure, we would first estimate the diurnal and semi-diurnal admittances from the TOPEX/POSEIDON and GEOSAT altimetry. Next, we would compute tidal corrections from this solution, which are applied to the ERS-1 sea surface height observations,
so that corrections can be obtained to the admittance solutions from TOPEX/POSEIDON and GEOSAT.
Chapter 10

Outlook

10.1 Introduction

As discussed in the previous chapter, the TOPEX/POSEIDON tidal solution could not be improved by using GEOSAT and ERS-1 altimeter observations because of tidal correlations and because of ERS-1 and GEOSAT orbit errors that likely have aliased to the same periods as the tides and so have become correlated. Better results can be obtained if the orbit errors of ERS-1 and GEOSAT can be reduced, or if their time series of sea surface height observations can be extended. The latter will reduce the tidal correlations and the correlations between the orbit errors and the tides. Recently, new orbits have become available for ERS-1 and GEOSAT, which are more accurate than the orbits used in this thesis. This means that 1-cpr orbit errors have become smaller so that less of these errors will be absorbed in the tidal parameters (see Section 8.2.3). The background ocean tide models have not significantly been improved since JGM-3. Hence, the background ocean tide errors in the new orbits (Section 8.2.3) will probably not have been reduced. This might necessitate an iterative procedure (Section 4.1) to obtain a more accurate ocean tide model and a more accurate background ocean tide model that can be used in the orbit computations of ERS-1 and GEOSAT. Besides the more accurate orbits, each of the three satellites has, or will have, a successor, which allows to extend their time series of observations. In this chapter, it is discussed what improvements of the results in this thesis may be expected from the extended missions and the more accurate orbits.

10.2 GEOSAT and the GEOSAT Follow-On

In February 1998, the GEOSAT Follow-On (GFO) was launched. This satellite is an extension of the GEOSAT mission in the sense that it has the same orbit characteristics and that it flies along the same ground track as GEOSAT in its ERM. For the GFO, an orbit accuracy comparable to that of TOPEX/POSEIDON is expected, i.e. of the order of 4 cm [Lemoine, 1998;
If this level of accuracy can also be obtained for GEOSAT, which is less well tracked than the GFO, then some improvement of the results presented in this thesis may be expected. Still, because GEOSAT and the GFO have the same orbit characteristics, their alias and Rayleigh periods are exactly the same. This means that the same tidal correlations exist with the GFO as with GEOSAT. It also means that because the alias periods of most of the dominant tides as observed by the GEOSAT-GFO satellites are about equally large or twice as large as those of TOPEX/POSEIDON, probably no less than six years of GEOSAT-GFO altimetry are required to improve the TOPEX/POSEIDON tidal solution (Section 9.7).

In addition to the GEOSAT data from the ERM, there are also the data from the GM. Whether these data are useful for tidal analysis is difficult to say because of the drift orbit during this part of the GEOSAT mission which leads to a non-constant satellite repeat period. Whether alias and Rayleigh periods for such an orbit exist will, therefore, require additional analyses.

10.3 ERS-1 and ERS-2

The successor of ERS-1, i.e. ERS-2, was launched in April 1995. It has the same orbit characteristics as ERS-1 in its 35-day repeat mission and flies along the same groundtrack. For all phases of the ERS-1 mission (Appendix A), as well as for the ERS-2 mission, orbits with an accuracy of about 5 cm have become available [Scharroo and Visser, 1998], which should offer some room to improve the results in this thesis by extending the ERS-1 observation series. Still, for similar reasons as with the GEOSAT-GFO satellites, this will probably require six years of ERS altimetry.

With regard to reducing tidal correlations, an interesting part of the ERS missions is the period May 1995-June 1996, during which the two satellites have flown in tandem. The lag of ERS-2 on ERS-1 during the tandem mission was exactly 24 hours [e.g., Scharroo and Visser, 1998], i.e. one mean solar day. Hence, on each ascending or descending track, an altimeter measurement of ERS-1 is followed by an altimeter measurement of ERS-2 one day later. This may offer the possibility of using phase advance differences to decorrelate the tides similar to tidal decorrelation from adjacent tracks (Section 4.5.1). Moreover, it also offers the possibility for tidal decorrelation from phase advance differences on crossing tracks (Section 4.5.2) because of the additional crossing tracks between ERS-1 and ERS-2. Notice that the crossing tracks between ERS-1 and ERS-2 occur at the same locations as those of ERS-1 (and ERS-2) because the two satellites have flown in tandem. At each ERS crossover location we get three time intervals between crossing tracks, i.e. $t_{asc} - t_{desc}$ for crossing tracks of ERS-1 as well as for crossing tracks of ERS-2, and $(t_{asc} - t_{desc}) \pm 1$ day for crossing tracks between ERS-1 and ERS-2. The time intervals of
Table 10.1 Tidal phase advances (degrees) along the ground track of the ERS tandem mission. The time interval $\Delta t$ between altimeter measurements of ERS-1 and ERS-2 is exactly one mean solar day. Denoted by $S_a$ and $S_{sa}$ are the annual and semi-annual cycle, respectively.

<table>
<thead>
<tr>
<th>$\Delta t$</th>
<th>$M_2$</th>
<th>$S_2$</th>
<th>$N_2$</th>
<th>$K_2$</th>
<th>$K_1$</th>
<th>$S_{sa}$</th>
<th>$S_a$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.000</td>
<td>-24.4</td>
<td>0.0</td>
<td>-37.5</td>
<td>2.0</td>
<td>1.0</td>
<td>2.0</td>
<td>1.0</td>
</tr>
</tbody>
</table>

$(t_{asc} - t_{desc}) \pm 1$ day may provide extra information to decorrelate the tides, i.e. information that we do not get from ERS-1 altimetry alone or ERS-2 altimetry alone.

Table 10.1 lists the tidal phase advances as observed by the ERS tandem configuration along the ground track. Shown are the dominant tides that are involved in a correlation problem (Table 4.4) as well as the minor $K_2$ tide because it distorts the semi-diurnal admittance by aliasing to semi-annual period (Section 8.3). Obviously, because the time interval between two measurements of the tandem configuration equals one mean solar day, the phases of all diurnal and semi-diurnal tides will not advance very much within this interval. This is especially true for the $S_2$, $K_2$, and $K_1$ tides, of which the phase advances reduced by the number of cycles traversed in one day, are all less than $2^\circ$. Consequently, as can be derived from Table 10.1, the phase advance differences between the tides and between the tides and the seasonal cycles in this table are also very small for one day. Hence, the samples with a time interval of one day along the ground track of the tandem configuration will not ease the correlation problems of $M_2/N_2$, $K_1/S_a$, and of $S_2$ with the dynamic topography, whereas the tandem configuration can also not prevent the distortion of the semi-diurnal admittance because of aliasing of $K_2$ to semi-annual period.

For the same reason as explained above, the time intervals between crossing tracks of ERS-1 and ERS-2 of $(t_{asc} - t_{desc}) \pm 1$ day, will both give nearly the same phase advance patterns and phase advance difference patterns as in Figure 4.7 and Figure 4.10, respectively. Hence, the crossing tracks between ERS-1 and ERS-2 may help to further decorrelate $M_2/N_2$ and $K_1/S_a$ but they will not decorrelate $S_2$ from the dynamic topography or prevent the distortion of the semi-diurnal admittance because of the $K_2/S_{sa}$ correlation.

Also interesting are the data from the 168-day repeat orbit of the Geodetic Phase of ERS-1 because these data cover the earth with a dense network of tracks that are spaced 0.15 apart in longitude. Unfortunately, it may be shown that the 168-day repeat period leads to tidal alias periods which are larger than one year for the dominant tides if we consider a resolution of 0.15 x 0.15. However, if we adopt a resolution of 1°, then the existence of a 37-day subcycle gives repeated measurements at an interval
of 37.0 days. The alias and Rayleigh periods of this ERS-1 subcycle are given in Table 10.2 for the dominant tides, the minor $K_2$ tide, and for the seasonal cycles. Comparing this table with Table 4.1, it is noticed that the alias and Rayleigh periods of the 37-day subcycle are very different, with obvious exception of the alias periods of the $S_2$, $K_2$, and $K_1$ tides, and the Rayleigh periods between these three tides. Hence, the 37-day subcycle should not be regarded as an extension of the 35-day repeat mission but as a different mission because of a different sampling interval. Compared to the 35-day repeat mission, it is noticed that the Rayleigh period of $M_2/N_2$ has become much smaller, which should be useful to decorrelate these tides. However, although this may improve the $M_2$ estimate, it will most likely not improve that of $N_2$. The reason is that the Rayleigh periods of $N_2$ with the dominant $K_1$ tide and of $N_2$ with both of the seasonal cycles have increased, thus inducing larger correlations. Hence, the major problem with the 37-day subcycle will be to obtain an accurate estimate of the $N_2$ tide, and further study is required to examine if the phase advance differences on crossing tracks can reduce the correlations in which this tide is involved. Also, the aliasing problem of $K_2$ to semi-annual period with the 35-day repeat mission remains to exist with the 37-day subcycle.

10.4 TOPEX/POSEIDON and JASON

Around 2000, the follow-on of TOPEX/POSEIDON, called JASON, will be launched. It seems decided that JASON will fly the same orbit as TOPEX/POSEIDON. However, whether JASON should fly along the same groundtrack or in between the TOPEX/POSEIDON tracks is still a subject of discussion. Two clear possibilities to improve the TOPEX/POSEIDON tidal model with JASON altimeter data can be mentioned.

Firstly, JASON could fly in tandem with TOPEX/POSEIDON, i.e. similar to ERS-1 and ERS-2. From the observations of this tandem mission, the $K_1/S_{sa}$ correlation problem can be solved. A phasing of the two satellites
of about twelve hours ensures that a phase difference of 180° of the $K_1$ tide will be observed by the tandem configuration everywhere along the groundtrack. Additionally, a phasing of twelve hours means that the time interval between crossing TOPEX/POSEIDON-JASON tracks will equal approximately a day (modulo an integer number of days) near the equator and will decrease to half a day (modulo an integer number of days) towards the turnover latitudes. This means that exactly the opposite of the $K_1/S_{sa}$ phase advance difference pattern in Figure 4.9 will be measured by the tandem configuration, i.e. a phase advance difference of 0° at the equator, which steadily increases to 180° at the turnover latitudes. The observations from the above tandem mission will clearly solve the $K_1/S_{sa}$ correlation problem of TOPEX/POSEIDON at higher latitudes. With regard to the excellent TOPEX/POSEIDON tidal models that were published after one year of altimetry had been gathered, a tandem mission that will last for about a year will probably suffice.

Secondly, JASON could fly in between the TOPEX/POSEIDON tracks. If the TOPEX/POSEIDON level of orbit accuracy cannot be obtained for GEOSAT and ERS-1, the altimeter observations of JASON in this orbit configuration will produce a tidal model with the same accuracy as the $3° \times 3°$ TOPEX/POSEIDON tidal solution, but with a resolution of $155 \times 155$. 
Chapter 11
Conclusions and recommendations

The main problems with extracting the ocean tide from satellite altimetry are aliasing of the diurnal and semi-diurnal tides because of the sub-Nyquist sampling of the sea surface and the relatively large background noise level. For three major satellite altimetry missions until now, i.e. GEOSAT, ERS-1, and TOPEX/POSEIDON, the diurnal and semi-diurnal tides are aliased to periods of typically sixty days to one year and the Rayleigh periods over which the tides decorrelate may be as large as three to twelve years. With an accuracy of sea level measurements of 5-11 cm, which is mainly determined by orbit errors, only the dominant $M_2$, $S_2$, $N_2$, $K_1$, and $O_1$ tides can reliably be obtained from altimetry.

The diurnal and semi-diurnal tidal admittances estimated from satellite altimetry are largely determined by the above-mentioned dominant tides. The minor tides inferred from these admittances are therefore not independent of the dominant tides. Still, a number of 23 tidal lines derived from the diurnal and semi-diurnal admittances estimated from TOPEX/POSEIDON altimetry, results in an empirical tidal model that can predict tidal sea level elevations with a global accuracy of about 3 cm in the deep oceans. In this part of the oceans, such an accuracy is competitive with that of one of the most recent and most accurate hydrodynamically consistent models, i.e. FES95.2.1.

For GEOSAT, ERS-1, and TOPEX/POSEIDON, tidal correlations induced by aliasing can largely be solved for the dominant tides because of the tidal phase advance differences on crossing satellite groundtracks. Because of the decorrelating ability of these phase advance differences, the altimeter observations of ERS-1 and GEOSAT contain useful information on the ocean tides, in spite of the fact that their orbits were not designed to avoid tidal correlations like that of TOPEX/POSEIDON. It has therefore been attempted to develop a multi-satellite tidal solution by improving the TOPEX/POSEIDON tidal solution, based on three years of altimetry,
Conclusions and recommendations

with about two years of GEOSAT altimeter data from the 17-day Exact Repeat Mission and two years of ERS-1 altimeter data from the 35-day repeat mission. The improvement should obviously come from the better resolution of the ERS-1 and GEOSAT groundtrack networks compared to the relatively coarse spacing of the TOPEX/POSEIDON groundtracks. Unfortunately, it could be shown that the orbit errors in the GEOSAT and ERS-1 altimeter observations are too large and that part of the orbit errors is likely aliased to periods comparable with the tidal alias periods. This means that the effect of orbit errors on the tidal estimates will not average out sufficiently, whereas it also means that correlated tides are difficult to separate, in spite of the decorrelation offered by the tidal phase advance differences on crossing groundtracks. Hence, two years of ERS-1 and two years of GEOSAT altimetry based on orbit accuracies of 7-8 cm and 10 cm, respectively, could not improve the TOPEX/POSEIDON tidal solution.

There are, however, clear indications that the orbits of GEOSAT and ERS-1 may be computed with an accuracy comparable to that of TOPEX/POSEIDON in the near feature. Based on those new orbits, about six years of GEOSAT and six years of ERS-1 altimetry may probably improve the TOPEX/POSEIDON tidal solution. A 6-year time series of sea level measurements along the ERS-1 and GEOSAT groundtracks is possible because of their successors, i.e. ERS-2 and the GEOSAT Follow-On (GFO), respectively. ERS-2 has the same orbit characteristics and therefore the same alias and Rayleigh periods as ERS-1 in its 35-day repeat mission, and the same is valid for the GEOSAT Follow-On and GEOSAT in its Exact Repeat Mission. With the alias periods of the ERS and GEOSAT-GFO satellites being about twice as large as those of TOPEX/POSEIDON, six years of altimeter data will probably decorrelate the aliased orbit errors of the ERS and GEOSAT-GFO satellites from the tides to the same extent as with the TOPEX/POSEIDON altimeter observations. Because of the likely assumption that more than three years of TOPEX/POSEIDON altimetry will not give significantly more accurate tidal estimates, six years of ERS and six years of GEOSAT-GFO altimeter data derived from orbits with near TOPEX/POSEIDON accuracy will probably improve the TOPEX/POSEIDON tidal solution.

Moreover, six years of ERS and GEOSAT-GFO altimetry will largely solve the tidal correlations that were identified, i.e. the correlation of $K_1/S_{sa}$ with TOPEX/POSEIDON, the correlation of $M_2/N_2$ and $K_1/S_a$ with the ERS satellites, and the correlation of $M_2/S_a$ and $S_2/K_1/S_{sa}$ with the GEOSAT-GFO satellites ($S_a$ and $S_{sa}$ denote the annual and semi-annual seasonal cycle, respectively). The $M_2/S_a$ and $S_2/S_{sa}$ pairs, which both have a Rayleigh period of six years with the GEOSAT-GFO satellites, will fully decorrelate within a 6-year analysis period. The $S_2/K_1$ and $K_1/S_{sa}$ pairs, which both have a Rayleigh period of twelve years in case of GEOSAT-GFO, will largely decorrelate by the phase advance differences
on crossing groundtracks (as evidenced by $K_1/S_\alpha$ with the TOPEX/POSEIDON mission) except at latitudes larger than some $55^\circ$ (as also evidenced by the TOPEX/POSEIDON mission). In case of the ERS satellites, tidal phase advance differences will largely decorrelate $M_2/N_2$ with their 9-year Rayleigh period, and $K_1/S_\alpha$ with infinitely large Rayleigh period. The $K_1$ estimate obtained from the ERS altimetry will then improve the GEOSAT-GFO and TOPEX/POSEIDON solution of this tide beyond latitudes of $55^\circ$.

With reference to the above 6-year solution procedure, it is expected that tidal phase advance differences on crossing groundtracks will continue to play an important role in tidal analysis from satellite altimetry.

In addition to six years of altimetry from the ERS 35-day repeat missions, the data from the 168-day repeat mission flown by ERS-1 may also be useful for tidal analysis. This is because of the existence of a 37-day subcycle, if we adopt the 1° resolution of the 35-day repeat mission. The alias and Rayleigh periods of the 37-day subcycle are, however, different from those of the 35-day repeat mission, so that the 37-day subcycle should not be seen as an extension of that mission. With regard to the 35-day repeat mission, the 37-day subcycle will largely decorrelate $M_2$ and $N_2$. This may improve the $M_2$ estimate but most likely not that of $N_2$ because of additional correlations of $N_2$ with $K_1$ and with both of the seasonal cycles. Whether tidal decorrelation from crossing tracks can reduce these correlations requires additional study.

Whether the data from the GEOSAT Geodetic Mission are useful for tidal analysis also requires additional study, because of the non-constant repeat period during this part of the GEOSAT mission.

A problem that may need special attention is the loss of GEOSAT altimeter data due to the attitude control problem of this satellite. Areas over which there are less GEOSAT altimeter observations may require extra data of the GFO in order to improve the TOPEX/POSEIDON solution.

Another problem that needs to be carefully addressed is the aliasing of $K_2$ to semi-annual period with the ERS missions. In the absence of the dominant $S_2$ tide, this leads to a distortion in the semi-diurnal admittance nearby the $S_2$ and $K_2$ frequencies. The $K_2$ tide as observed by TOPEX/POSEIDON and the GEOSAT-GFO satellites will not be strong enough to solve this problem, which might necessitate the use of an a-priori solution or an iterative solution procedure. In such an iterative solution, the GEOSAT-GFO and TOPEX/POSEIDON altimeter observations are used to estimate the diurnal and semi-diurnal admittances. Next, tidal corrections derived from these admittances are subtracted from the ERS altimeter observations, and corrections to be added to the TOPEX/POSEIDON+GEOSAT-GFO admittances are estimated.
An interesting alternative to solving the $K_1/S_{sa}$ correlation problem at higher latitudes of TOPEX/POSEIDON and the GEOSAT-GFO satellites by means of ERS altimetry may be offered by the future JASON satellite. From a tidal point of view, it is recommended that this mission will be flown in two parts. During the first part of the mission, JASON should fly along the same groundtracks as TOPEX/POSEIDON. The phasing of the two satellites along their groundtrack should be approximately twelve hours. As a result, a phase advance difference between $K_1$ and the semi-annual cycle of 180° will be measured along the entire groundtrack and at the crossover locations, which will solve the $K_1/S_{sa}$ correlation problem within probably one year. During the second part of the mission, JASON should fly in between the TOPEX/POSEIDON groundtracks. These observations can be used to compute a tidal model with a resolution of 1°5, which will improve the resolution of the existing TOPEX/POSEIDON tidal models by a factor two.
Appendix A

Description of GEOSAT, ERS-1, and TOPEX/POSEIDON satellite altimeter missions

After Skylab, GEOS-3, and SEASAT, the United States (US) Navy’s GEOSAT was the fourth satellite to carry an altimeter. It was launched in March 1985 and has flown two types of orbits. The first served to meet its primary mission objective, which was to obtain a high-resolution marine geoid [Cheney et al., 1991]. Because of their military relevance, the altimeter data gathered during the first part of the mission, which lasted about 18 months and was labeled the Geodetic Mission (GM), were classified. After the Geodetic Mission, GEOSAT was maneuvered into a 17-day repeat orbit. During this second part of the mission, known as the Exact Repeat Mission (ERM), GEOSAT has delivered more than three years of unclassified altimeter observations. A tape recorder failure in October 1989 terminated the GEOSAT mission. At the end of 1997, all data from the GM were declassified and made available to the user community.

Launched in July 1991, the European Space Agency’s (ESA) ERS-1 was equipped with an altimeter as part of its payload. Other components of the ERS-1 payload included a Synthetic Aperture Radar (SAR) for making images over the oceans and land, and an Along-Track Scanning Radiometer (ATSR) for measuring sea surface and cloud top temperatures. The ATSR also measures the water vapor content along the altimeter beam from which the wet tropospheric correction is derived [ESA, 1992]. The ERS-1 satellite has flown three different kinds of orbits. In December 1991, after mission check-out and altimeter calibration, ERS-1 was maneuvered into a 3-day repeat orbit which proved to be optimal for Antarctic and sea ice experiments [ESA, 1992]. This so-called Ice Phase ended in March 1992, when the orbit was changed to produce a ground track that repeated every 35 days for its Multidisciplinary Phase in which SAR imaging the earth’s surface was the main objective. This phase lasted for about two years and ended in December 1993, when the satellite was maneuvered back into a 3-day repeat orbit for its second Ice Phase from January to April 1994. In
April 1994, the orbit was changed into a 168-day repeat orbit for a precise mapping of the marine geoid. This so-called Geodetic Phase lasted until March 1995. After the Geodetic Phase, the satellite was maneuvered back into a 35-day repeat orbit to fly in tandem with its successor ERS-2. In June 1996, ERS-1 was put into hibernation mode.

In August 1992, the joint US/French TOPEX/POSEIDON altimeter satellite was launched with the aim to study global ocean circulation. It carries two altimeters, TOPEX and POSEIDON, of which the latter serves as a proof of concept for solid-state altimeter technology [Raizionville et al., 1980]. Both altimeters use the same antenna on a time-sharing basis. The POSEIDON altimeter operates 10% of the time, mostly in the sense that it is switched on once every ten repeat cycles. One of the reasons to select a 10-day repeat period for TOPEX/POSEIDON was to avoid tidal aliasing to frequencies of ocean current variabilities, especially to annual and semi-annual frequency [Fu et al., 1994]. The selected repeat period led to alias periods typically less than ninety days for most of the largest tides (see Section 4.4). Unfortunately, aliasing of the $K_1$ tide to semi-annual frequency could not be avoided. Still, results in Chapter 6 and Chapter 8 show that this only affects the $K_1$ tidal solution at latitudes beyond 55°. The nominal lifetime of TOPEX/POSEIDON is three years but at present, the satellite continues to deliver altimeter observations with undegraded accuracy and is expected to do so beyond 2000.

Table A.1 lists the orbit characteristics of GEOSAT, ERS-1, and TOPEX/POSEIDON, for those parts of the missions of which altimeter data are used in this thesis. Denoted by $N_r$ is the number of revolutions that the satellite performs in one repeat period. $N_d$ is the number of nodal days in the repeat period, where a nodal day is defined as the period between two successive crossings of the satellite orbit’s ascending node over the same earth-fixed meridian. An important parameter that determines the resolution of the tidal models derived from altimetry is the longitude spacing of the satellite groundtracks. This spacing depends on the number of revolutions $N_r$ as $360°/N_r$ so that values of approximately 2°83, 1°48, and 0°72 are found for TOPEX/POSEIDON, GEOSAT, and ERS-1, respectively.

<table>
<thead>
<tr>
<th>satellite</th>
<th>orbital height (km)</th>
<th>inclination (°)</th>
<th>$N_r$</th>
<th>$N_d$</th>
<th>repeat period (days)</th>
</tr>
</thead>
<tbody>
<tr>
<td>GEOSAT</td>
<td>800</td>
<td>108</td>
<td>244</td>
<td>17</td>
<td>17.0505</td>
</tr>
<tr>
<td>ERS-1</td>
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<td>98</td>
<td>501</td>
<td>35</td>
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</tr>
<tr>
<td>T/P</td>
<td>1360</td>
<td>66</td>
<td>127</td>
<td>10</td>
<td>9.9156</td>
</tr>
</tbody>
</table>

Table A.1 GEOSAT, ERS-1, and TOPEX/POSEIDON (T/P) orbit characteristics. The repeat period is given in mean solar days. $N_r$ is the number of revolutions in a repeat period. $N_d$ denotes the number of nodal days in a repeat period.
Appendix B

Description of FES95.2.1 and CSR3.0 ocean tide models

This appendix provides some background information on the FES95.2.1 and CSR3.0 ocean tide models. Details on FES95.2.1 and CSR3.0 can be found in LeProvost et al. [1994] and Shum et al. [1997].

The Grenoble FES95.2.1 model is a linearized solution to the non-linear hydrodynamic equations [LeProvost et al., 1994]. FES95.2.1 stems from the earlier purely hydrodynamic FES94.1 ocean tide solution and is obtained by assimilating the empirical CSR2.0 solution based on two years of TOPEX/POSEIDON altimetry into FES94.1 for ocean depths greater than 1 km. The last decimal in the FES designation indicates a change to FES95.2 after a small error in the assimilation scheme was discovered in October 1995 [Callahan, 1996], which only affected the $M_2$ component in the Asian seas, e.g. the Arafura Sea north of Australia. The assimilation was performed over five basins separately, i.e. the North Atlantic, South Atlantic, Indian, North Pacific, and South Pacific oceans. Afterwards, a global solution is obtained by fitting the basin-scale solutions through a set of harmonic constants from tide gauges and from the model of Schwiderski [1980] on the open boundaries. Essentially, the short-wavelength tidal features are controlled in the assimilation scheme by hydrodynamics, whereas the longer wavelengths are controlled by altimetry. The model is harmonic and consists of 26 spectral lines. Eight lines that well define the diurnal and semi-diurnal admittance, i.e. $M_2$, $S_2$, $N_2$, $K_2$, $2N_2$, $K_1$, $O_1$ and $Q_1$, are directly solved using the assimilation scheme, while 18 minor tides have been deduced by admittance interpolation. Nodal corrections are applied to each of the spectral lines to obtain constituents, so that the actual number of spectral lines contained by the model is much larger. The FES95.2.1 oceanic tide, as well as the load tide derived from this solution, are distributed among the user community on a grid of 0.5 x 0.5 resolution, although the varying resolution with which the model is solved is much higher in coastal seas.

The University of Texas/Center for Space Research CSR3.0 tidal model is in essence a long-wavelength correction to FES94.1 using two and a half
years of TOPEX/POSEIDON altimetry. To convert the FES94.1 ocean tide model to the elastic ocean tide, i.e. including the loading component, ocean loading as derived from CSR2.0 was added to the four dominant tides of FES94.1 in both the diurnal and semi-diurnal band. As it was recognized from a comparison with altimetry-derived tide models, e.g. that of Schrama and Ray [1994], that FES94.1 contained some large-scale errors in $M_2$ and $S_2$, the solutions of these tides were taken from an adjusted version of FES94.1 by Andersen et al. [1995]. Next, a-priori diurnal and semi-diurnal orthoweightes were fit through the dominant tides in both bands. Corrections to the a-priori orthoweightes were then estimated from TOPEX/POSEIDON altimetry and added back to their a-priori values to obtain the new model. In the CSR3.0 model, thirty spectral lines are derived from the orthoweightes to predict the tidal heights. These lines should account for the most important part of the spectrum including the nodal side bands. The CSR3.0 model provides the ocean tide as well as the corresponding load tide on the same resolution as that of FES95.2.1, i.e. $0.5\times0.5$.

It should be mentioned that neither FES95.2.1 nor CSR3.0 solves for the long-period tides. The tides in this band are treated according to the equilibrium formulation for which both models use the same subroutine.
Appendix C

Altimeter cross calibration

To successfully merge the altimeter observations of the different satellites, the sea surface height residuals derived from these observations must define the same sea level. Hence, the altimeter observations have to be corrected for the altimeter biases of the satellites. In case of ERS-1, the altimeter bias is accurately known, i.e. -42 cm [Francis, 1992], but the altimeter bias of GEOSAT has never been calibrated while that of TOPEX was calibrated falsely due to a clock algorithm error [Hancock III and Hayne, 1996]. Hence, to avoid that altimeter bias differences enter the sea surface height residuals, we need to cross calibrate the altimeter biases of TOPEX and GEOSAT relative to the altimeter bias of ERS-1. The altimeter bias of POSEIDON is less suitable for this purpose because of the sparseness of the altimeter data on which the bias estimate from the calibration campaign was based [Ménard et al., 1994].

A straightforward approach to cross calibrate the altimeter biases is based on so-called dual-satellite crossover differences. Similar to the crossover difference of one satellite (single-satellite crossover difference), the dual-satellite crossover difference is defined as the difference between two sea surface height residuals on crossing tracks, although the tracks now belong to two different satellites, e.g. TOPEX/POSEIDON and ERS-1. A convenient property of the dual-satellite crossover differences is that the global mean of these differences reflects the altimeter bias difference of the two satellites involved. The difference on crossing tracks of the time-varying components on the right side of (4.1), as well as the effects of geographically-correlated orbit error differences, which are contained in $\varepsilon$, will largely average to zero in the global mean of the dual-satellite crossover differences. An exception is ocean variability from a secular rise in sea level and from the seasonal cycles if dual-satellite crossover differences are computed from data of significantly different time frames, e.g. a few years apart. However, because global sea level rise is in the order of only a few mm/year [e.g., Minster et al., 1995], it will make a relatively small contribution to the mean of the dual-satellite crossover differences. To minimize the effect of the dominant annual cycle, the data of the different satellites may be chosen such that there is approximately an integer
Figure C.1 Data sets used for the altimeter cross calibration of TOPEX (TOP), POSEIDON (POS), and GEOSAT, from dual-satellite crossover differences with ERS-1.

For a cross calibration of the altimeter biases, the cycles as depicted in Figure C.1 were used. The ocean and load tides were derived from the FES95.2.1 model [Le Provost et al., 1994]. Three different sets of data arcs were selected to enable a cross comparison of the bias results. To remove spurious crossover differences, a $3.5\sigma$ editing criterion was applied. The crossover differences were not weighed according to latitude because we are interested in their mean and not the rms. Latitude weighting is usually done to ensure that the crossover locations at higher latitudes do not bias the global crossover statistics (Section 8.2.5), e.g. due to regions of large ocean variability. The mean of the dual-satellite crossover differences, however, equals the altimeter bias difference, which is the same for all dual-satellite crossover differences. Notice that with each set, all data are selected within approximately one month, except for GEOSAT. Over such a time span, ocean variability from the seasonal cycles will not change very much, thus minimizing their effect on the bias estimation. In case of GEOSAT, the data are selected such that they differ approximately
<table>
<thead>
<tr>
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<th>POSEIDON</th>
<th>GEOSAT</th>
<th>ERS-1</th>
</tr>
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<tbody>
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<td>-39.4/46603</td>
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<tr>
<td><strong>Set 2</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>TOPEX</td>
<td>-0.1/52919</td>
<td>0.8/18557</td>
<td>14.8/92814</td>
<td>-39.4/131564</td>
</tr>
<tr>
<td>POSEIDON</td>
<td>-0.6/1674</td>
<td>14.0/16398</td>
<td></td>
<td>-40.2/22912</td>
</tr>
<tr>
<td>GEOSAT</td>
<td>-0.9/42442</td>
<td></td>
<td>-55.2/106352</td>
<td></td>
</tr>
<tr>
<td>ERS-1</td>
<td>-0.5/39624</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Set 3</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>TOPEX</td>
<td>0.2/42873</td>
<td>3.5/30295</td>
<td>10.8/86342</td>
<td>-38.8/117938</td>
</tr>
<tr>
<td>POSEIDON</td>
<td>0.5/5581</td>
<td>7.4/31095</td>
<td>-42.4/41676</td>
<td></td>
</tr>
<tr>
<td>GEOSAT</td>
<td>-3.6/45888</td>
<td></td>
<td>-51.9/112038</td>
<td></td>
</tr>
<tr>
<td>ERS-1</td>
<td>0.1/39421</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table C.1 Results of the altimeter cross calibration between TOPEX/POSEIDON, ERS-1, and GEOSAT. Given are the mean \( \mu \) (cm), and the number of crossover differences \( no \) listed as \( \mu/no \). All crossover differences are edited using a 3.5\( \sigma \) criterion.

an integer number of years with those of the other satellites. No limit was set to the time interval between crossing tracks (Section 4.5.2) to obtain as many crossover differences as possible for the determination of their mean.

The results of the altimeter bias cross calibration are listed in Table C.1. The entries on the diagonal contain the number of single-satellite crossover differences with their mean \( \mu \) for each of the three sets of Figure C.1. Ideally, the mean of the single-satellite crossover differences should equal zero because the altimeter bias of the satellite is removed when subtracting sea surface height residuals on crossing tracks. For all satellites, the mean of the single-satellite crossover differences can be seen to be well below 1 cm except for GEOSAT of which the mean is quite significant. The cause is a bad centering of the GEOSAT orbits in the terrestrial reference frame due to the sparseness of the tracking stations [Chambers, 1996; Naeije et al., 1996]. To constrain the orbit's Z-component, which is affected most because of an uneven distribution of ground stations in the northern and southern hemispheres, altimeter data in the form of altimeter normal points and crossover differences were added to the radio-frequency tracking data when the orbits were recomputed by UT/CSR [Chambers, 1996]. However, in order not to overconstrain the Z-component, the altimeter observations were assigned a relatively low weight, which means that there still may be some degree of freedom left. The off-diagonal entries in Table C.1 present the statistics of the dual-
### Table C.2

<table>
<thead>
<tr>
<th></th>
<th>TOPEX</th>
<th>POSEIDON</th>
<th>GEOSAT</th>
</tr>
</thead>
<tbody>
<tr>
<td>Set 1</td>
<td>-5.2</td>
<td>-2.6</td>
<td>12.8</td>
</tr>
<tr>
<td>Set 2</td>
<td>-2.6</td>
<td>-1.8</td>
<td>13.2</td>
</tr>
<tr>
<td>Set 3</td>
<td>-3.2</td>
<td>0.4</td>
<td>9.9</td>
</tr>
<tr>
<td>mean</td>
<td>-3.7 ± 1.1</td>
<td>-1.3 ± 1.3</td>
<td>12.0 ± 1.5</td>
</tr>
</tbody>
</table>

Table C.2 Altimeter bias estimates (cm) of TOPEX, POSEIDON, and GEOSAT for an altimeter bias of ERS-1 of -42 cm.

Satellite crossover differences. Notice that with the dual-satellite crossover differences, each of the two satellites can move on an ascending as well as on a descending track. As a result, there are many more dual-satellite than single-satellite crossover differences. The dual-satellite crossover differences in Table C.1 were computed as the sea surface height residual of the satellite listed in the left column minus the residual of the satellite listed at the top of the table. Hence, the bias difference, which is reflected by the mean $\mu$, should be interpreted accordingly. For instance, from Set 1, a bias difference of -36.8 cm is found if the sea surface height residuals of ERS-1 are subtracted from those of TOPEX.

In Set 2, more than half of the altimeter data were absent from POSEIDON GDR 31, which explains the smaller number of crossover differences in which POSEIDON is involved. Interesting to notice is that the numbers of single-satellite and dual-satellite crossover differences of Set 1 are significantly larger than the corresponding numbers of Set 2 and Set 3. As explained in Section 6.2, this is caused by the yearly growth and decay of Antarctic sea ice, i.e. during winter in the southern hemisphere, the Antarctic ice boundary moves northwards and so covers most of the Antarctic seas, which results in less crossover locations. Set 1 covers altimeter data mainly from April (Figure C.1), which is inside the period December-June, i.e. when the ice is withdrawing. Set 2 and Set 3, on the other hand, mainly cover the months July/August and November, respectively, which is in the period during which there are less data in the Antarctic seas (data gaps and ascending parts of the curves of Figure 6.2 in Section 6.2). As can be seen from Table C.1, the consistency between the mean of the dual-satellite crossover differences for the different sets is best in case of crossover differences between ERS-1 and TOPEX. In case that POSEIDON and GEOSAT are involved in the crossover computations, less consistent results are found due to a smaller amount of altimeter data in case of POSEIDON, and due to the bad centering of the orbits in case of GEOSAT.

With the adopted value of -42 cm for the ERS-1 bias, the (absolute) altimeter biases of TOPEX, POSEIDON, and GEOSAT as derived from the utmost right column of Table C.1 are listed in Table C.2 for all three sets. Mean values of about -4 cm for the TOPEX bias, -1 cm for the POSEIDON bias, and 12 cm for the GEOSAT bias are found, all with a standard devia-
tion of about 1-2 cm. Notice that according to the sign convention of the altimeter bias (Section 4.2), these results indicate that all altimeters are measuring short, except for GEOSAT. Comparing the result for POSEIDON with the bias estimate from the calibration campaign (1 cm according to Ménard et al. [1994], and -0.2 cm according to Christensen et al. [1994], both with an uncertainty of about 2.5 cm), there seems to be a reasonable agreement. Notice that the values in Ménard et al. [1994] and Christensen et al. [1994] were both obtained with the same sea state bias model as given in Table 5.2. For TOPEX, a rough check of the value in Table C.2 is to compute the mean of the corrections for the clock algorithm error over all cycles as given in Hancock III and Hayne [1996], and subtract that value from the bias as obtained from the calibration campaign, i.e. -17.1 cm [Christensen et al., 1994]. With a mean of the corrections for the clock algorithm error of -13.3 cm, this leads to a bias of -3.8 cm, which agrees quite well with the value in Table C.2. The values in Table C.2 were used to bring all altimeter data onto the same reference surface by adding them to the appropriate ssh residuals.


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Arthur Smith was born on 8 July, 1969, in Zeist, The Netherlands. From 1981 to 1987 he visited secondary school at R.S.G. Schoonoord in Zeist with mathematics and physics as the main courses.

After secondary school he started his study at the Faculty of Aerospace Engineering of Delft University of Technology. After two and a half years of courses in the general education program of that Faculty, the final phase of the study was dedicated to satellite orbit computations. As part of the study, a three months training period was spent at Fokker Space & Systems, Amsterdam, where he worked on the problem of reconstructing a spacecraft's state of motion from an array of linear accelerometers.

In 1992, he graduated Cum Laude on the subject of gravity field modeling from satellite-to-satellite tracking data. That year, he accepted a position for a Ph.D. study at the Faculty of Aerospace Engineering and five years were spent on modeling the global ocean tide from satellite altimeter observations. Results were presented at several international conferences and published in refereed journals.

In 1997, Arthur Smith accepted a job at TNO Physics and Electronics Laboratory in The Hague, where he currently works on applications of airborne and spaceborne Synthetic Aperture Radars (SAR).