AN IMPROVED SETTLING TUBE SYSTEM

R.E. Slot

Report no. 7 - 83

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Summary

In this report a settling tube system is described for the measurement of the settling velocity distribution of sand samples with particle diameters ranging from 0.1 to 2 mm and sample weights ranging from 0.1 to 20 gf(10^{-3} to 0.2 N).

The time of arrival of the particles is detected by means of an underwater balance. The response, stability and accuracy of this balance are greatly improved by utilizing the well-known principle of feedback (Slot & Geldof, 1979). The accuracy (systematic part of the error) of the measurement of the settling velocity turns out to be better than 2%, i.e. with the exception of the error due to the concentration effect. The error due to the concentration effect is dependent on the sample weight and particle size. It is possible to choose a sample weight (for sand particles in the range from 0.1 to 2 mm) for which the error due to the concentration effect is less than 1% and the signal-noise ratio is better than 72 dB (≈ 4000). This means that the overall accuracy is better than 3%. The precision (random part of the error) of the measurements turns out to be better than 4% (95% confidence).

A rapid analysis of the samples is accomplished by connecting the settling tube to a micro computer. Results are presented by means of a hard copy yielding plots of the cumulative distribution and the calculated moments (mean, standard deviation and skewness) of the settling velocity as well as the sedimentation diameter.

Introduction

Various authors proposed settling tube systems based on different measuring principles. Three of the main principles are (Geldof & Slot, 1979)

1. Weight measurements (Odén, 1916; Doeglas, 1946; Plankeel, 1962; Bienek, Huffman & Meder, 1965; Sengupta & Veenstra, 1968; Felix, 1969; Brezina, 1972; Gibbs, 1972)
2. Pressure measurements (Ziegler, Whitney & Hayes, 1960; Schlee, 1966; Nelsen, 1976)
3. Light extinction measurements (Jordan, Fryer & Hemmen, 1971; Taira & Scholle, 1977).

As to all kinds of measurements the feasibility of the three measuring principles is mainly determined by drift and noise. The weight and pressure measurements are sensitive to an extra source of 'noise' caused by mechanical vibrations. In this respect the light extinction measurement has some advantage being insensitive to mechanical vibration. However, a disadvantage is that the settling velocity distribution is measured in terms of projected area in contrast with the weight and pressure measurements where it is done in terms of weight and volume, respectively. The problem is that the projected area is not a well defined quantity for irregular shaped particles. Furthermore, the transport equations in the field of sediment transport contain the weight (volume) of the sediment and not the projected area. In this respect weight and pressure measurements are more appropriate.

Although the authors who propose the pressure measurements do not specify drift and noise in the system it turns out that the drift in the standard differential pressure transducers is too large limiting the range of particle size to > 0.5 mm (for smaller particles the combination of maximum sample weight, determined by the error due to the concentration effect, and the settling time is unfavourable with respect to the drift).

In contrast with the pressure measurements the weight measurements have the advantage of the possibility to use the well known principle of feedback. In general feedback can greatly improve the performance of a system such as the response, stability and accuracy of a system. In this case the performance of a system is mainly determined by the components used in the feedback section which have to be accurate, linear and stable.

1. The settling tube system

In fig. 1 the complete DUST-system (Delft University Settling Tube) is shown placed on a platform with four air springs to reduce mechanical vibrations. The total height of the system from the ground to the top of the sample introduction device is 282 cm.

The settling tube itself is a perspex tube with a length of 165 cm and an inside diameter of 17 cm (volume 40 l). The housing of the balance is made of a perspex block with an inner cavity which is more or less spherical. This spherical construction is important for the reduction of mechanical vibrations.

The maximum weight on the weighing pan of the balance is limited to approximately 0.7 N (70 gf). When this limit is reached the weighing pan can be cleaned by rotating the balance. The sand particles are gathered in the funnel underneath the housing of the balance. When this funnel is full the tap at the bottom can be opened to release the sand particles (together with some water).

1.1. The underwater balance with feedback

The heart of the settling tube system is the underwater balance (fig. 2). It is composed of

1. a weighing pan with an air chamber to provide buoyancy
2. a special construction of springs only allowing axial displacements of the weighing pan
3. two inductive transducers to measure displacements
4. a solenoid for feedback and taring

This weighing system should have a fast, critically damped response. Drift and noise should be small and the relationship between weight and output signal should be linear. In general the use of feedback will greatly improve the imperfections inherent to the system. Fig. 3 shows a block diagram of the weighing system with feedback. The feedback section is composed of a solenoid and a differentiator. The latter is necessary to adjust the system for critical damping.

The transfer function $H(\omega)$ of the balance is

$$H(\omega) = (-M\omega^2 + jk\omega + C)^{-1} \quad m/N, \tag{1}$$

where M = inertial mass of the balance kg

ω = angular frequency rad/s

k = natural damping coefficient Ns/m

C = spring constant N/m

The displacement of the balance is measured with two inductive transducers and a Wheatstone-bridge amplifier. The transfer function $H'(\omega)$ of the balance with transducers and amplifier becomes

$$H'(\omega) = A(-M\omega^2 + jk\omega + C)^{-1} \quad \text{V/N} , \tag{2}$$

where A = transducer/Wheatstone-bridge amplification factor V/m.

In general the natural damping is too small (internal friction in springs and water) giving rise to an oscillatory motion of the weighing pan.

When a settling particle hits the weighing pan the force on the pan will change more or less step-wise. To test the weighing system this step-wise variation of the force was simulated by means of the driver-solenoid combination (see fig. 3) and an electrical square wave signal. The response of the weighing system without feedback is shown in fig. 4. Due to the small natural damping the output signal is oscillatory. This oscillatory motion will vanish if the damping is critical. This can be accomplished by means of feedback. The feedback section (see fig. 3) is composed of a P-D control circuit (Proportional and Differentiating) and a solenoid. Now the damping can be made critical by adjusting the time constant τ of the differentiator. In fig. 5 the response of the critically damped weighing system to a square wave signal is shown. A measure for the oscillatory motion is the quality factor Q defined as

$$Q = \frac{1}{k}\sqrt{MC} . \tag{3}$$

If $Q = 1/2$ the system is damped critically whereas for $Q > 1/2$ the system is underdamped and will oscillate.

The natural frequency of the system is

$$\omega_0 = \sqrt{\frac{C}{M}} . \tag{4}$$

When the feedback loop is used the transfer function $H_f(\omega)$ of the weighing system becomes

$$H_f(\omega) = \left[-\frac{M}{A}\omega^2 + j\left(\frac{k}{A} + G\tau\right)\omega + \left(\frac{C}{A} + G\right)\right]^{-1} , \tag{5}$$

where G = transfer function of the driver-solenoid system N/V

τ = time constant of the differentiator s

The quality factor Q_f of the weighing system with feedback becomes

$$Q_f = \frac{\sqrt{1+\frac{AG}{C}}}{1+\frac{AG\tau}{k}} Q \tag{6}$$

and for the natural frequency follows

$$\omega_{0,f} = \sqrt{1 + \frac{AG}{C}}\,\omega_0 . \tag{7}$$

As stated before the quality factor Q_f is an important parameter which indicates the measure of oscillatory motion. For $Q_f = 1/2$ the system is critically damped and this can be accomplished by adjusting the time constant τ of the differentiator. The second important parameter is the natural frequency which indicates how fast the system will respond to a variation in weight. A more appropriate parameter derived from the natural frequency is the delay time γ_0, i.e. the time lag between input and output for a linear changing input signal (see fig. 6). This delay time γ_0 for a second order system is defined as

$$\gamma_0 = \frac{1}{Q\omega_0} \tag{8}$$

For a critically damped system ($Q = 1/2$) with natural frequency $\omega_{0,f}$ this becomes

$$\gamma_0 = \frac{2}{\omega_{0,f}} . \tag{9}$$

The time delay γ_0 indicates the delay of the output signal with respect to the input signal (in this case the weight of the particles on the pan). It is important that this time delay γ_0 is small compared to the total settling time of the particles.
In order to obtain the required delay time γ_0 firstly $\omega_{0,f}$ has to be adjusted by means of G. Secondly τ has to be adjusted to make the system critically damped.
Before the adjustments of the delay time and the damping it is important to make A as large as possible (i.e. $C/A \ll G$). This will reduce the sensitivity of the system to noise, drift and non-linearity due to the imperfections in the inductive transducers, the Wheatstone-bridge amplifier and the springs of the balance.
In order to calculate this reduction the steady state transfer function (ω=0) has to be used. The steady state transfer function for the system without feedback is

$$H(0) = \frac{A}{C} \tag{10}$$

and for the system with feedback

$$H_f(0) = \frac{1}{\frac{C}{A}+G} \quad . \tag{11}$$

In the system without feedback every deviation in A and C is directly noticeable in the output signal (see eq. 10). In the system with feedback the influence of deviations in A and C will be negligible for $C/A \ll G$ since the transfer function $H_f(0)$ will mainly depend on $1/G$.
As easily follows from (10) and (11) the reduction factor f for small variations of C and A (e.g. due to temperature variations) in the feedback system is

$$f = 1 + \frac{AG}{C} \quad . \tag{12}$$

In general it can be stated that between the points A and B in the block diagram of fig. 3 the influence of sources of noise, drift and non-linearity on the output signal will be reduced by the factor f given in (12). To take full advantage of the feedback AG/C has to be made as large as possible, however, with G not too large (otherwise the output signal will be too small). The practical limitation of enlarging AG/C is reached when the system becomes unstable due to phase shift and second order system

parameters. On the other hand the influence of sources of noise, drift and non-linearity in the feedback section and external sources as well will not be reduced.
The main parts in the feedback section are the differentiator and the driver-solenoid system. The differentiator can only be a source of noise (its steady state response is zero), whereas the driver-solenoid system can also be a source of drift and non-linearity. A proper design of these relatively simple components can make the inherent imperfections small enough.

Temperature variation of the water in which the weighing pan floats and mechanical vibrations can be considered as external sources of drift and noise, respectively. A constant room (i.e. water) temperature and a quiet place may be essential although a proper construction of the housing of the balance (spherical housing !) and a platform on air springs damped in glycerine can reduce the sensitivity to mechanical vibrations for the greater part.
The characteristics of the weighing system of the DUST are:

$M = 8$ kg *	$\omega_{0,f} = 58$ rad/s
$C = 170$ N/m	$Q_f = 0.5$
$\omega_0 = 4.6$ rad/s	$A = 1.8 \quad 10^6$ V/m
$k = 5.3$ Ns/m	$G = 15 \quad 10^{-3}$ N/V
$Q = 7$	$f = 160$
	$\tau = 35 \quad 10^{-3}$ s
	$\gamma_0 = 35 \quad 10^{-3}$ s

A detailed discription of the electronics is given in appendix A.

1.2. The sample introduction device

The sand sample to be analysed is put on the introduction device shown in fig. 7. This introduction device is of the venetian blind type with rotating lamellae. The sand sample can be released by means

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*This mass is composed of the mass of the weighing pan, the magnet and the virtual mass of the acceleration in water.

of a push button which opens the lamellae by activating a solenoid. When the lamellae are open they will vibrate for an adjustable period of time (0 to 10 s) to ensure the release of all the particles. The ideal situation would be when the initial positions of all the particles would be in the same horizontal plane (homogeneously distributed) and the initial velocities are equal to their terminal velocities. In practice this ideal situation can only be approximated. The condition which has to be fulfilled is that the distances between the introduction device and the position where the particles reach their terminal velocities are small compared to the length of the settling tube.

This means for the construction of the introduction device that the width of the lamellae should be small compared to the length of the settling tube (small differences between initial vertical positions) and that the velocity induced by the device should be small compared to the terminal velocity of the particle (concave shape of the lamellae).

1.3. The platform with air springs

Mechanical vibrations can disturb the measurements with the settling tube. Since the resolution of the balance is of the order of 10 μN (1 mgf), which corresponds with displacements of the order of 10 nm, mechanical vibrations have to be reduced as much as possible. A platform placed on four air springs will reduce the vibrations induced via the ground. The cut-off frequency of the air springs is 3.5 Hz (independent of the inertial mass), hence frequencies above 3.5 Hz will be reduced with 12 dB/oct.

A disadvantage of the use of air springs is the presence of a resonance peak since air springs are underdamped ($Q \simeq 10$). This means that frequencies around 3.5 Hz will be amplified (3.5 Hz with a factor 10)*.
To make the system critically damped the platform is placed in a container filled with a viscous fluid (glycerine). By adjusting the distance between the bottoms of the platform and the container the damping can be made critical (Slot, 1977).

Since the damping of a (viscous) fluid is not ideal (the force induced on the platform does not only depend linearly on the velocity but also on the higher order terms of the velocity) the reduction of the vibrations is not optimal and frequency dependent. From measurements it appears that the reduction is about 30 dB (for the energy this means a factor 1000; for the RMS-value a factor 30).

*However, for a metal spring this resonance peak is 50 times larger than for an air spring.

2. Data acquisition

The output signal of the settling tube is the weight of the sand particles resting on the weighing pan as a function of time. Since the distribution in velocity (or particle size) is a more appropriate quantity than the distribution in settling time, the settling tube is connected to a micro computer system for the necessary conversion. By means of a programmable timer the sampling of the output signal of the settling tube is performed in equidistant velocity intervals (or equidistant size intervals; not yet implemented*).

The input parameters for the sampling program are the length L of the settling tube, the temperature t of the water, the maximum and minimum velocity v_{max} and v_{min} to be expected for the sample and the velocity sample interval Δv. The sampling is started L/v_{max} s after the particles are released from the introduction device and stopped after L/v_{min} s. The sampling takes place on interrupt request (IRQ), meanwhile the data points are plotted on the screen of the monitor giving the opportunity to check the data immediately.

Furthermore the data is stored in the memory of the micro computer and on request it can be saved on a floppy disk together with comments and labels for later use. On request a hard copy of the velocity distribution (cumulative and/or density) curve can be made in less than a minute together with the calculation and printing of the mean, standard deviation and skewness of the distribution of the velocity as well as the particle size (sedimentation diameter).

In fig. 8 the microcomputer system is shown. It is an Apple][micro computer with an A-D converter (12 bit), a printer/plotter and two disk drives, one for the programs and one for the data.

In fig. 9 an example of the output of the microcomputer is shown.

The hardware and software necessary for the data acquisition is discribed in detail in "Hardware and software for the DUST, implemented in the Apple][microcomputer" (Slot, 1983).

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*The necessity for sampling in equidistant size intervals is not so urgent since for samples with a relative small velocity range the relationship between size and velocity is almost linear.

3. Performance

The performance of the DUST, as every other measuring system, is determined by drift and noise limiting the precision of the measurements.

A limitation of the accuracy specific for settling tube systems is the concentration effect due to hindered settling and settling convection.

During the measurements the water temperature should be constant since a variation of it will cause a variation in the upward force on the weighing pan (drift !) as well as a variation in the viscosity of the water (settling velocity !).

In the next three sections the errors due to the three mentioned phenomena will be briefly discussed. For a more detailed discussion about the accuracy and precision of the DUST see "Design Aspects and Performance of a Settling Tube System" (Slót & Geldof, 1979).

The errors mentioned in the next sections are split into a systematic part (accuracy) and a random part (precision). It is always possible to make corrections for the systematic error (i.e. if the error is known) whereas this is impossible for the random error. However, the random error can be reduced by averaging a series of repetitions.

All the measurements discussed in the next sections are performed with sand samples (ρ = 2.65 g/cm^3) settling in water.

3.1. Drift and noise

The feasibility of the required precision of the weighing system is essentially limited by drift and noise. In fig. 10 a record of the long term drift is shown (Fig. 10a for a tare current zero; fig. 10b for the maximum tare current of 185 mA). The drift during the measurement has to be small compared to the total weight of the sample. During a measuring time of 4 minutes (i.e. the settling time of 100 μm particles) the drift turns out to be less than 2 μN (0.2 mgf). During the whole record of 38 minutes the drift was about 4 μN (0.4 mgf).

The drift during the measurement determines the minimum sample weight which can be measured with a certain precision. For a precision of 1% the minimum sample weight for 100 μm particles will be 200 μN (20 mgf). However, the larger the particle size the less the influence of the drift will be because of the shorter settling time.

The other limit of the required precision is the noise in the output signal due to electronical noise and mechanical vibrations. A proper design of the electronics will make the electronical noise small enough (feedback !; see section 1.1. and appendix A). Mechanical vibrations will be reduced by a platform on four air springs damped in glycerine (see section 1.3.). In general the noise has to be negligible compared to the level of the output signal that corresponds to the sample weight. An additional reduction of the noise induced by mechanical vibration may be necessary for small sample weights. This can be done by means of a low-pass electronical filter. However this will increase the delay time of the weighing system implying a decrease of the maximum particle velocity (diameter) that can be measured with a certain precision.

The delay time of the DUST is adjustable from 56 ms to 2 s. The delay time has to be negligible compared to the total settling time of a particle, say a fraction of 1/100 of it. Then the minimum settling time will be 5.6 s, corresponding to a maximum particle velocity of 30 cm/s (or a maximum sand particle diameter of 2.3 mm).
In fig. 11 the signal-noise ratio S/N versus the delay time γ_0 is shown. In general the larger the sample weight the larger the signal-noise ratio (i.e. the precision) will be. However,for larger sample weights an other phenomena known as the concentration effect will decrease the accuracy of the measurements. This will be discussed in the next section.

3.2. The concentration effect

The settling velocity of the particles in a sample will be different from the settling velocity of the free falling particles due to settling convection and hindered settling (concentration effects). A series of five measurements (for reproducibility) was performed for various sample weights and for various sieve fractions. The mean settling velocity $\bar{v}$ of each sample, the average $\bar{v}_5$ of the series of five samples and the standard deviation σ_5 (as a measure of the precision of $\bar{v}$) were calculated and are shown in table 1. The precision of the mean sample velocity $\bar{v}$ turns out to be better than 4% at a confidence limit of 95% i.e. k = 2.776 times the standard deviation σ_5; Student's t-distribution).

In fig. 12 $\bar{v}_5$ and $k\sigma_5/\sqrt{5}$ (as a measure of the precision of $\bar{v}_5$) versus the sample weight W are shown for three sieve fractions. Extrapolation to W=0 will give the settling velocity of the sample without the concentration effect. Due to the scatter in the particle size in a sieve fraction this settling velocity has to be interpreted as the average of the settling velocities measured for each individual free falling particle (i.e. "free falling" in a settling tube with finite dimensions). The scatter in particle size within a sieve fraction is relatively small, hence the relationship between settling velocity and particle diameter can be assumed linear for this small region. In this case the settling velocity found by extrapolation can also be interpreted as the settling velocity of a free falling particle with a diameter equal to the mean diameter of the particles in the sieve fraction.

The relative error ε in the mean sample velocity due to the concentration effect versus the sample weight W and the concentration C is shown in fig. 13. The concentration C is defined as the quotient of the sample volume and the water volume in the settling tube. From the preliminary measurements shown in fig. 13 it follows that the smaller the particle size the larger the error ε (for equal sample weights) which is in accordance with literature (Gibbs, 1972; Taira & Scholle, 1977) and some measurements in a test version of DUST (Geldof & Slot, 1979)). However, for a more generalized judgement about this concentration effect a more comprehensive series of measurements is being analysed.

The sample weight to be used for the analysis has to be a compromise between the error ε due to the concentration effect and the signal-noise ratio S/N. The relationship between the sample weight W and the particle diameter d is shown in fig. 14 for a constant error ε due to the concentration effect (ε = 1% and ε = 5%) and for a constant signal-noise ratio (S/N = 72 dB and S/N = 60 dB) measured for a delay time γ_0 of $^1/_{100}$ of the total settling time T.

Having a sample with a given mean diameter one has to make a choice for the maximum acceptable error due to the concentration effect and for the minimum tolerable signal-noise ratio. Then the sample weight (maximum 20 gf) has to be chosen in the region below the error curve of the concentration effect and above the signal-noise ratio curve. As follows from fig. 14 for particle diameter > 0.1 mm the error due to the concentration effect can be chosen less than 1% with the signal-noise ratio better than 72 dB. Except the systematic error due to the concentration effect there are also systematic errors caused by the delay time, the introduction device and the non-ideal linearity of the weighing system. The systematic error due to the chosen delay time (1% for γ_0 = $^1/_{100}$ T) is partly compensated by the impact of the particles on the weighing pan (see Slot & Geldof, 1979). The total systematic error in the DUST is about 2% (see Slot & Geldof, 1979). Including the systematic error due to the concentration effect the overall accuracy is about 3%.

The error due to drift is larger for small particle sizes (i.e. long settling times) as well as for small sample weights as discussed in section 3.1. Since the drift is about 0.2 mgf (see section 3.1.) the recommanded sample weight for 0.1 mm particles is 0.15 gf (see fig. 14: ε = 1% & S/N = 72 dB) giving rise to an error of 0.13% caused by drift. So even for the most unfavourable combination of small particle size (0.1 mm) and small sample weight (0.15 gf) the error due to drift is very small.

3.3. The influence of the water temperature

During a measurement the water temperature should be as constant as possible since variation in the water temperature will cause a variation of the upward force on the weighing pan as well as a variation of the settling velocity of the particles.

The variation in the upward force is caused by the variation in the volume of the weighing pan as well as the variation in the density of the water. It appears that there is an optimum value for the water temperature (about 23°C) at which the variation in the upward force is zero (see appendix B). A few degrees below and above this optimum temperature the variation in upward force is of the order of + 10 mgf/$^{\circ}$C and − 10 mgf/$^{\circ}$C, respectively. In the daytime without precaution the water temperature can vary a few degrees, say 2°C in 8 hours. This implies a variation of 0.02°C during a measuring time of 4 minutes, i.e. the settling time of 100 µm particles. During this measuring time of 4 minutes the upward force can vary 0.2 mgf, limiting the minimum sample weight to 20 mgf (see section 3.1).

The variation in the settling velocity of the particles is caused by the variation in the density of the water as well as the variation in the viscosity of the water. The influence of the density on the settling velocity is negligible since it is of the order of 0.01%/$^{\circ}$C. However, the influence of the viscosity on the settling velocity is about 1%/$^{\circ}$C (0.5%/$^{\circ}$C for 2 mm particles and 2%/$^{\circ}$C for 0.1 mm particles at a water temperature of 20°C). In appendix C a formula is given for the settling velocity of spheres as a function of the viscosity and all the other appropriate parameters. A explanationof this formula is given in "Terminal Velocity Formula for Spheres in a Viscous Fluid" (Slot, 1983).

In fig. 15 the settling velocity of spheres versus the water temperature is shown for various particle diameters calculated by means of the formula given in appendix C (viscosity $\nu = \frac{40}{20+t} 10^{-6}$ m^2/s and particle density $\rho = 2.65$ g/cm^3; $\Delta = 1.65$). As follows from fig. 15 there is a rather strong dependence of the settling velocity on the water temperature. As stated above during a measurement the variation in the water temperature can be of the order of $0.01^{\circ}C$ causing a change in the settling velocity of 0.01% which is, however, of no significance. More important is a gradient in the water temperature. Without precaution this gradient can run to $1^{\circ}C$ over the total length of the settling tube giving rise to an uncertainty of the order of 1% in the settling velocity.

Another important quantity in the field of sedimentation is the size of the particle. The basic concept of 'size' of a sediment particle is best expressed in terms of volume or nominal diameter defined as the diameter of the sphere of the same volume as the particle. An other commonly used measure of size is the sedimentation diameter defined as the diameter of a sphere that has the same density and has the same terminal settling velocity as the given particle in the same sedimentation fluid. However, the found sedimentation diameter will be dependent on the used sedimentation fluid and its temperature as well as the shape of the particle and its specific weight (spherical particles are the only exception). It is convenient to have a general and accurate analytic expression for the conversion from settling velocity to either nominal diameter or sedimentation diameter. The expressions found in the literature are not of a general nature and/or their valadity is limited to a small region of Reynolds numbers. In appendix C a formula (eq. C.5) is given for the sedimentation diameter of a particle with an accuracy of 2% for Reynolds numbers up to 2000 and also a formula (eq. C.8) for the nominal diameter of a particle with an accuracy of 5% for Reynolds numbers up to 8000. If the shape factor of the particle is not known the first formula can be used to calculate the sedimentation diameter which is however a dependent quantity. If the shape factor of the particle is known the latter formula can be used to calculate the nominal diameter which is an independent quantity. Nevertheless, if the particle is spherical both formulae should give equal results within the mentioned accuracies. It is obvious that for direct comparison of experimental results the nominal diameter is the most appropriate one because it is an independent quantity. However, since the shape factor is not always known the sedimentation diameter can be used but it will be more or less meaningless without specifying the specific weight of the particle, the used sedimentation fluid and the temperature of it. Furthermore, the found sedimentation diameter is also dependent on the shape of the particle, so direct comparison of experimental results of particles with different shape factors is meaningless too. Even if the shape factor and the nominal diameter are both unknown it is at least in principle possible to calculate them if the settling velocities are known at two different temperatures (i.e. for two different values of the viscosity). This calculation requires the solution of two non-linear equations. However, the results appear to be very sensitive to errors in the (measured) settling velocities. Even for an error of a few percent in the settling velocities the error in the found shape factor can be as large as a few hundred percent (if there is any solution at all). The error in the found nominal diameter is of the same order of magnitude as the error in the settling velocities if the values of the viscosities, at which the measurements are performed, differ by a factor 5 or more (however, this is not possible for water by varying the temperature). In general, the errors in the calculated shape factor and nominal diameter are larger for smaller values of the actual shape factor. So the practical application of the calculation of the shape factor and the nominal diameter is limited by measuring errors as well as the error of 5% in the used formula.

As an example the settling velocity distribution of a sieve fraction (250 - 300 μm) of natural worn sand was measured in water at three different temperatures (15.4°C, 19°C and 24°C). The mean settling velocities and the sedimentation diameters (eq. C.5) were calculated and are shown in fig.16. The nominal diameter and the shape factor were calculated for the velocity combination measured at (15.4°C, 19°C), (15.4°C, 24°C) and (19°C, 24°C). The results are

$(15.4^{\circ}C,\ 19^{\circ}C) \rightarrow d_N = 0.322$ mm $\quad$ sf = 0.63

$(15.4^{\circ}C,\ 24^{\circ}C) \rightarrow d_N = 0.296$ mm $\quad$ sf = 0.86

$(19^{\circ}C,\ 24^{\circ}C) \rightarrow d_N = 0.278$ mm $\quad$ sf = 1.00

It is obvious that the results are very diverse and are of little practical significance.

4. Conclusions

A settling tube system with an underwater balance utilizing the principle of feedback turns out to be a suitable measuring device for the settling velocity distribution of sand samples with particle sizes ranging from 0.1 to 2 mm. The accuracy of the measurement of the settling velocity is better than 2%, i.e. with the exception of the error due to the concentration effect. The sample weight has to be chosen such that the error due to the concentration effect and the signal-noise ratio are tolerable. This is more critical for small particles. But even for small particles (100 μm) a sample weight can be chosen (0.15 gf) such that the error due to the concentration effect is 1% and the signal-noise ratio is 72 dB. This means that the overall accuracy (systematic error) is better than 3%.

The precision (random part of the error) turns out to be better than 4% (95% confidence) calculated from a series of five repetitions performed for various particle sizes and sample weights. Of course the precision can be made better by averaging a series of repetitions.

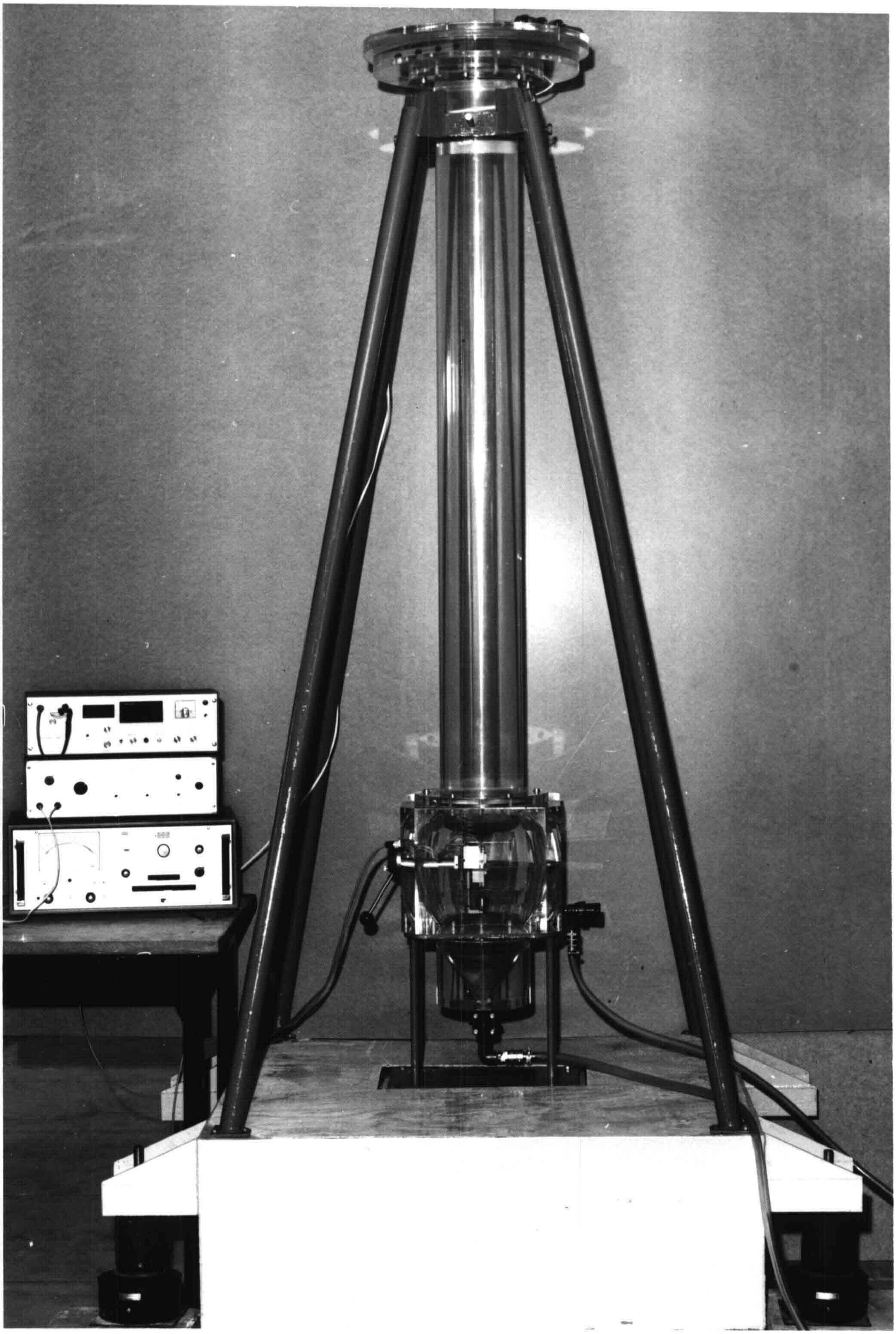


Fig. 1. Delft University Settling Tube (DUST)

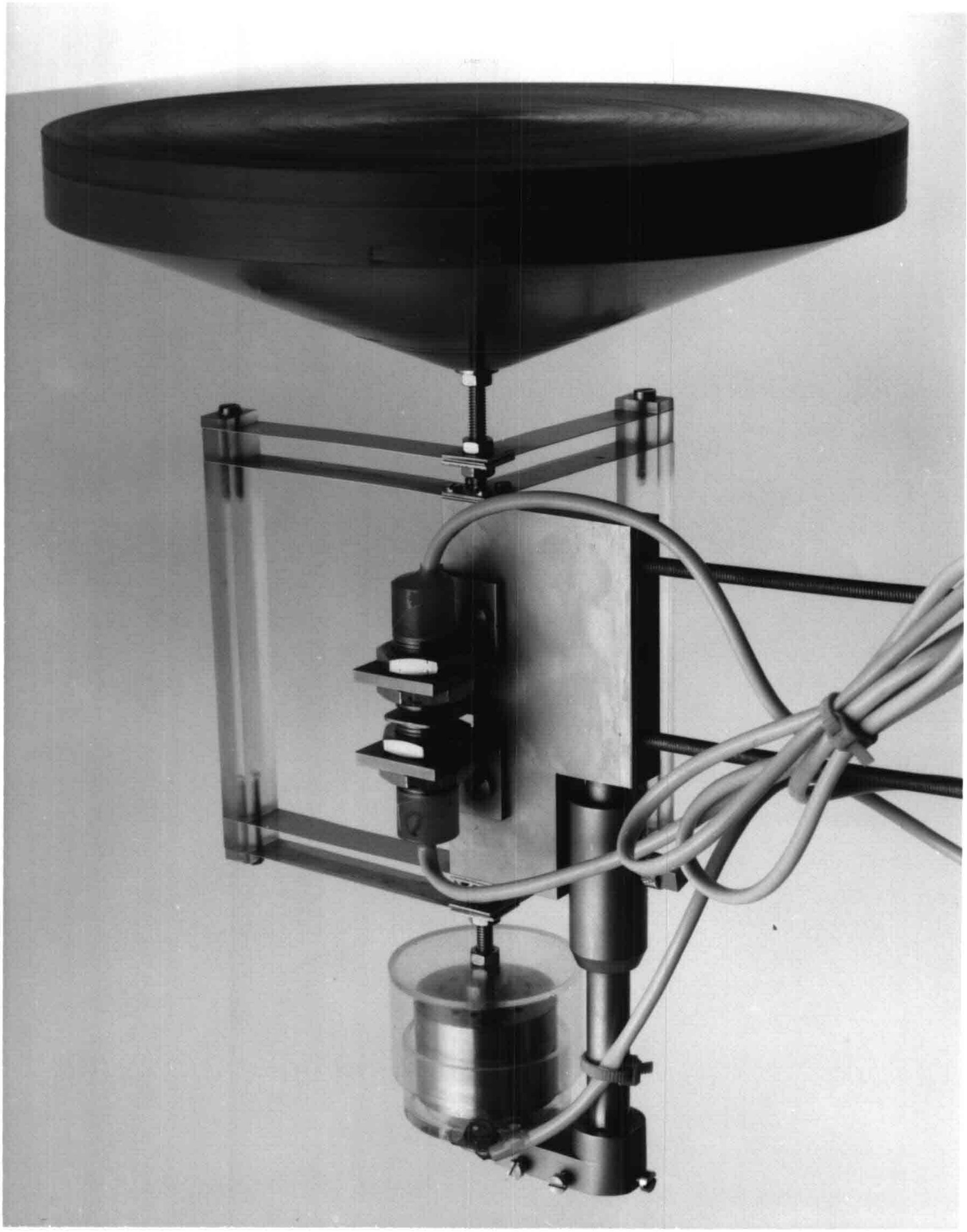


Fig. 2. The underwater balance

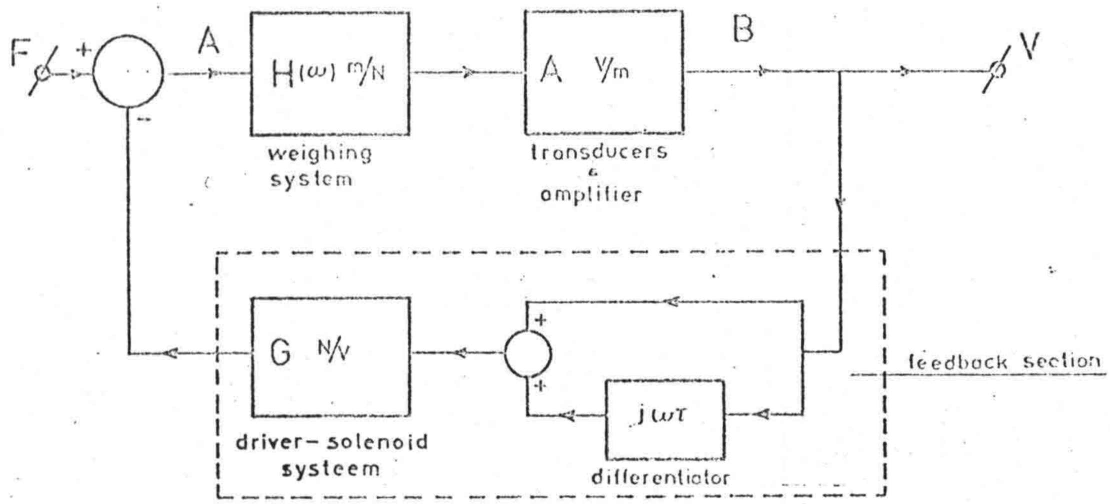


Fig. 3. Block diagram of the weighing system with feedback

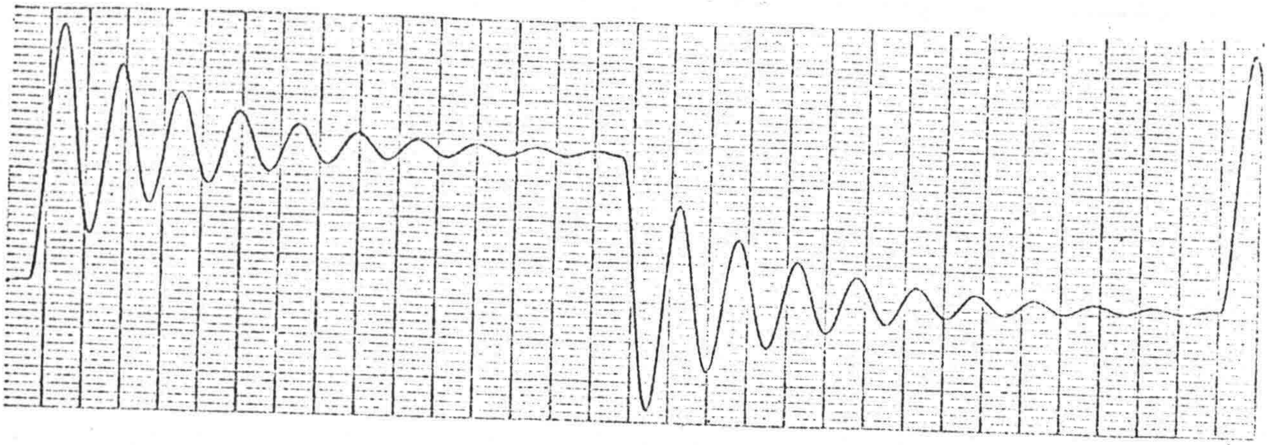


Fig. 4. Oscillatory response of the weighing system to a square wave signal

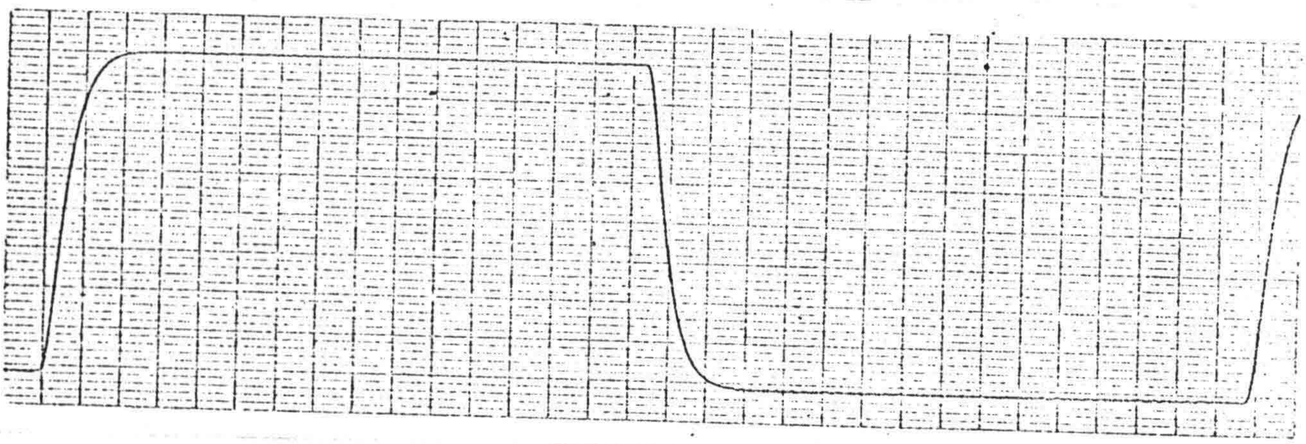


Fig. 5. Response of the critically damped weighing system to a square wave signa

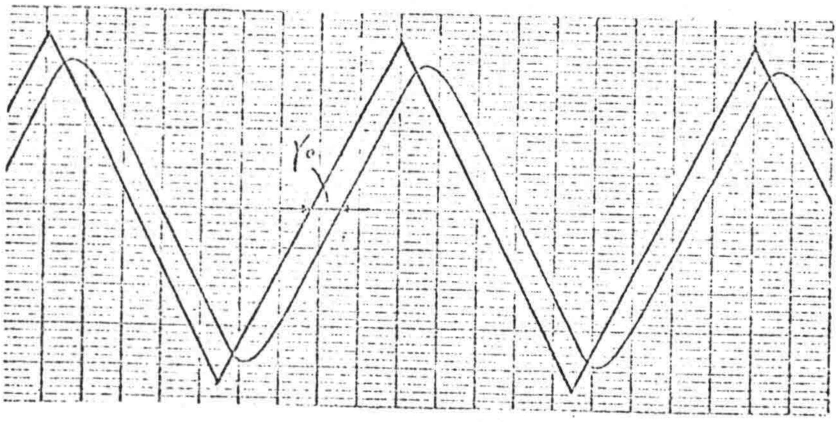


Fig. 6. Delay time γ_0 of the critically damped system

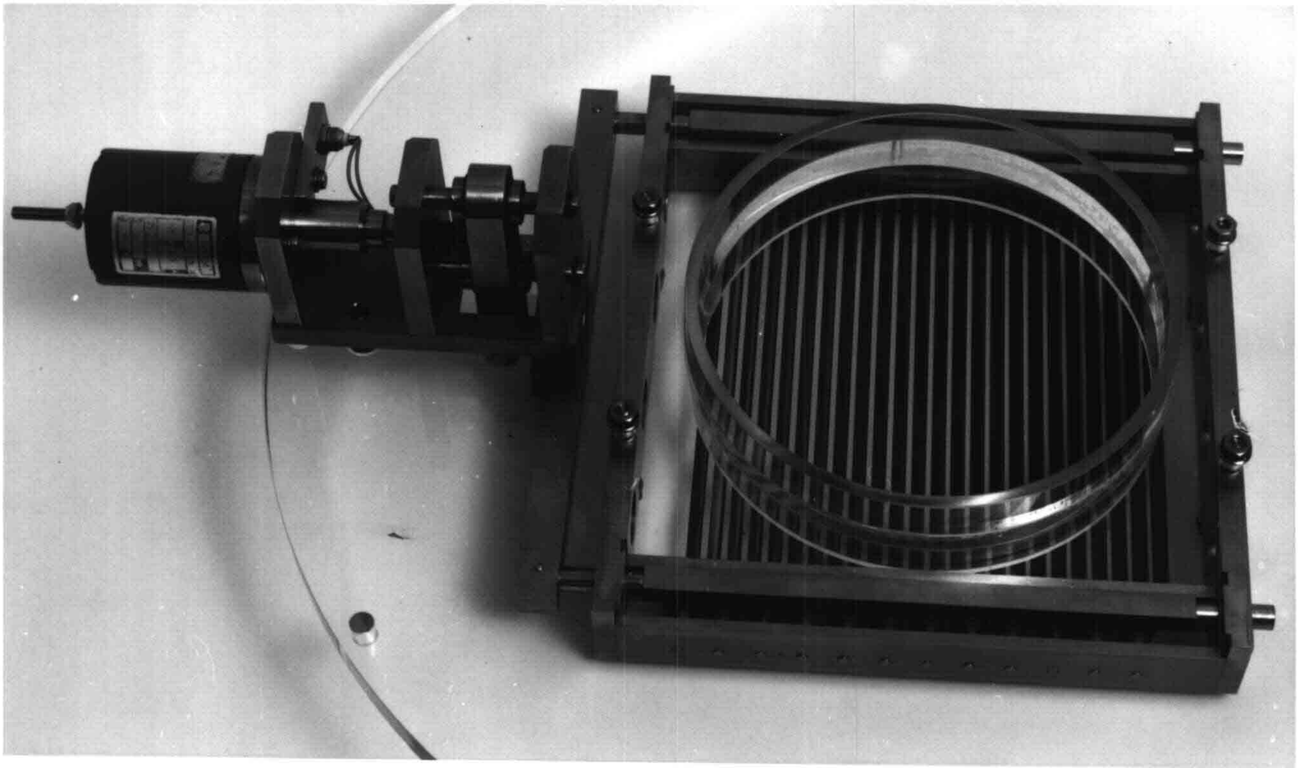


Fig. 7. The sample introduction device

Fig. 8. The Apple || microcomputer

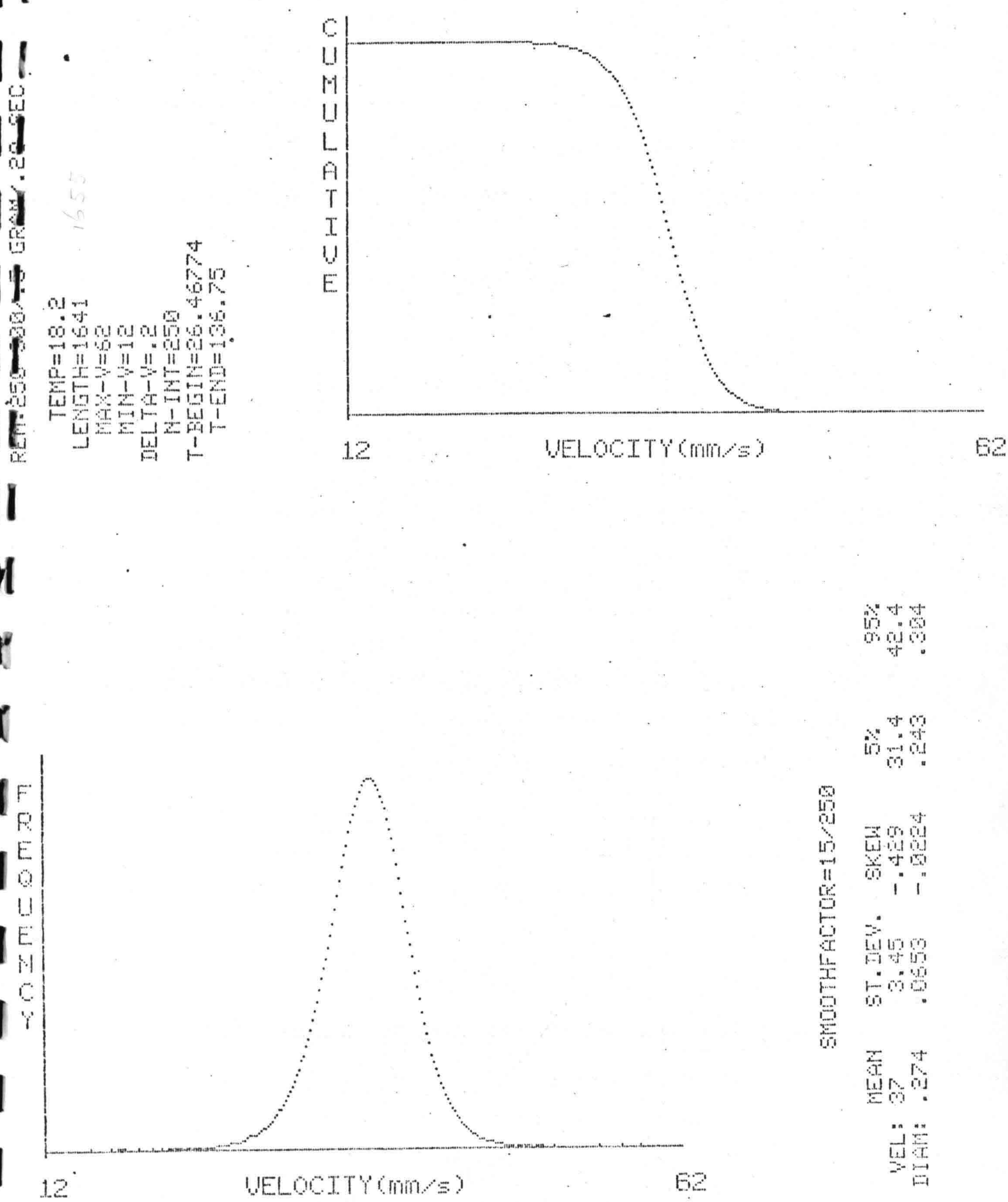
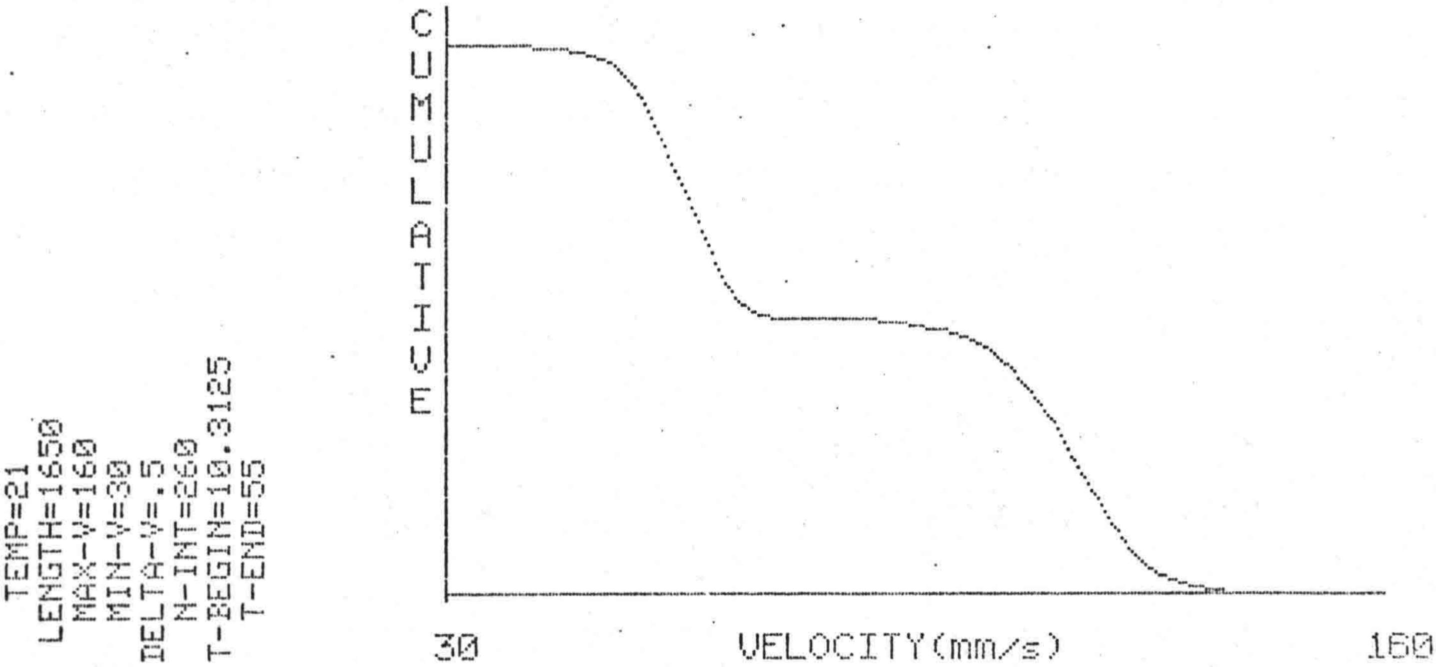


Fig. 9a. Examples of the results of an analysis



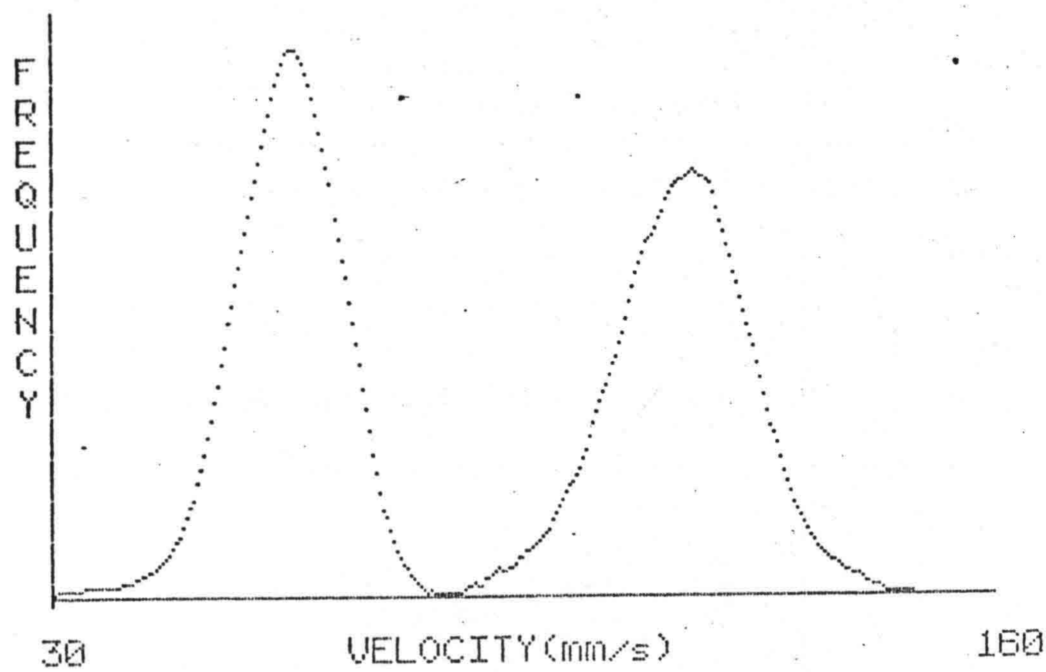
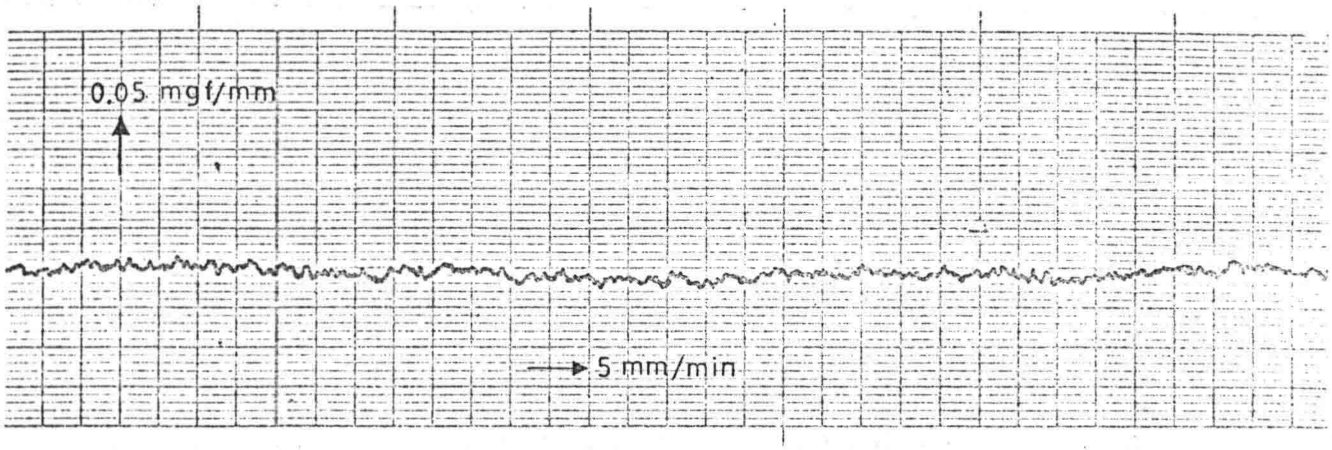
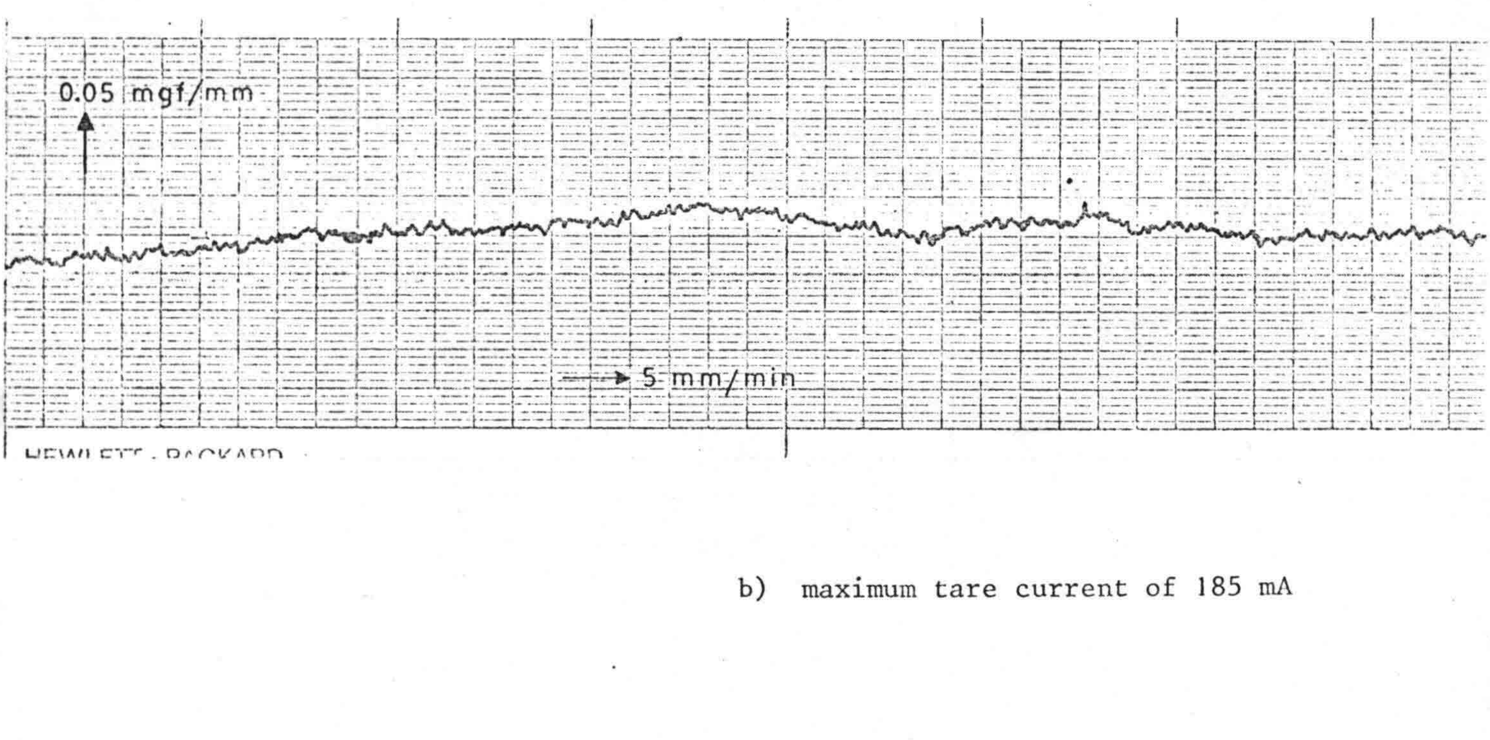


Fig. 9b. Examples of the results of an analysis



a) tare current zero



b) maximum tare current of 185 mA

Fig. 10. Long term drift

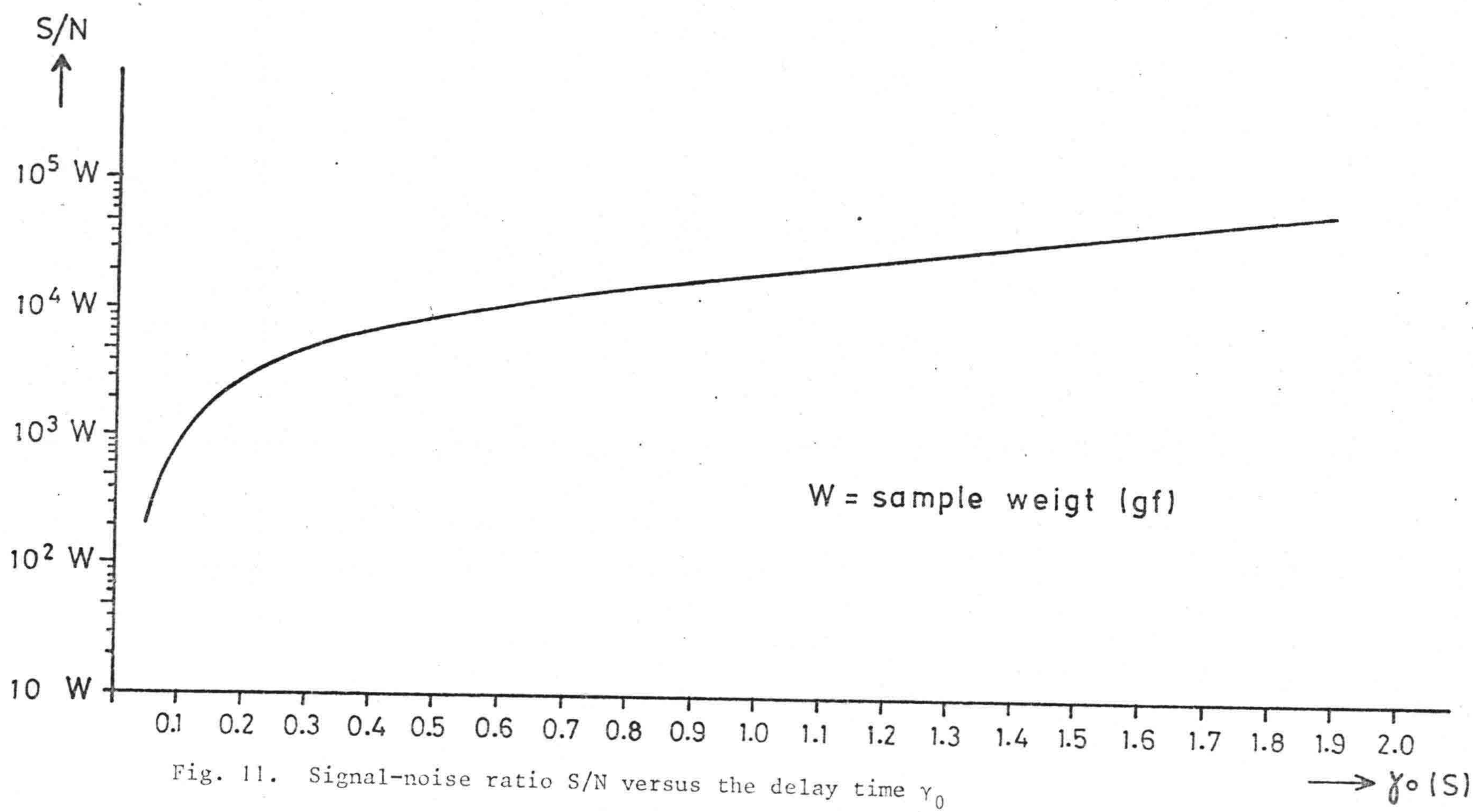


Fig. 11. Signal-noise ratio S/N versus the delay time γ_0

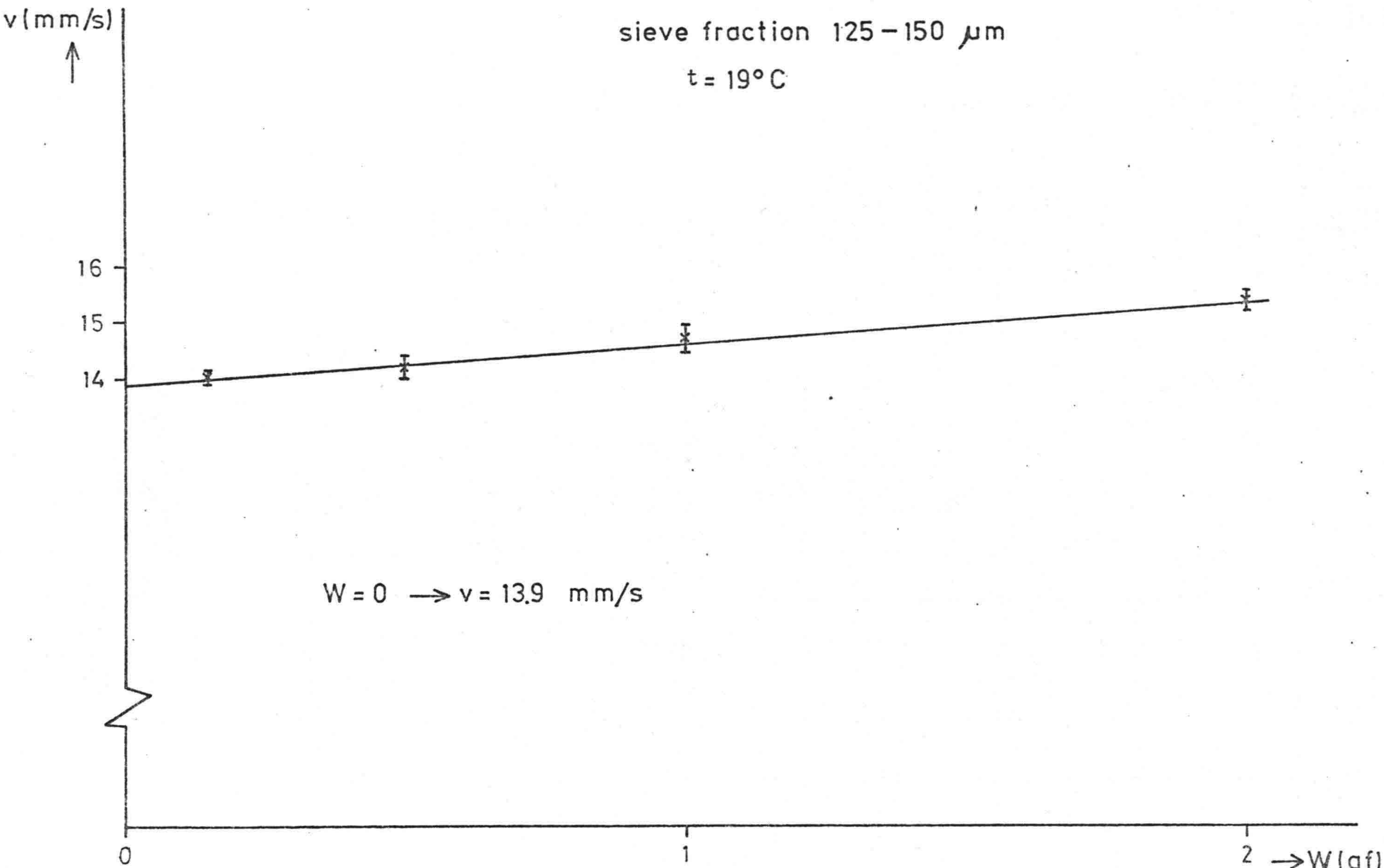


Fig. 12a Mean sample velocity versus sample weight

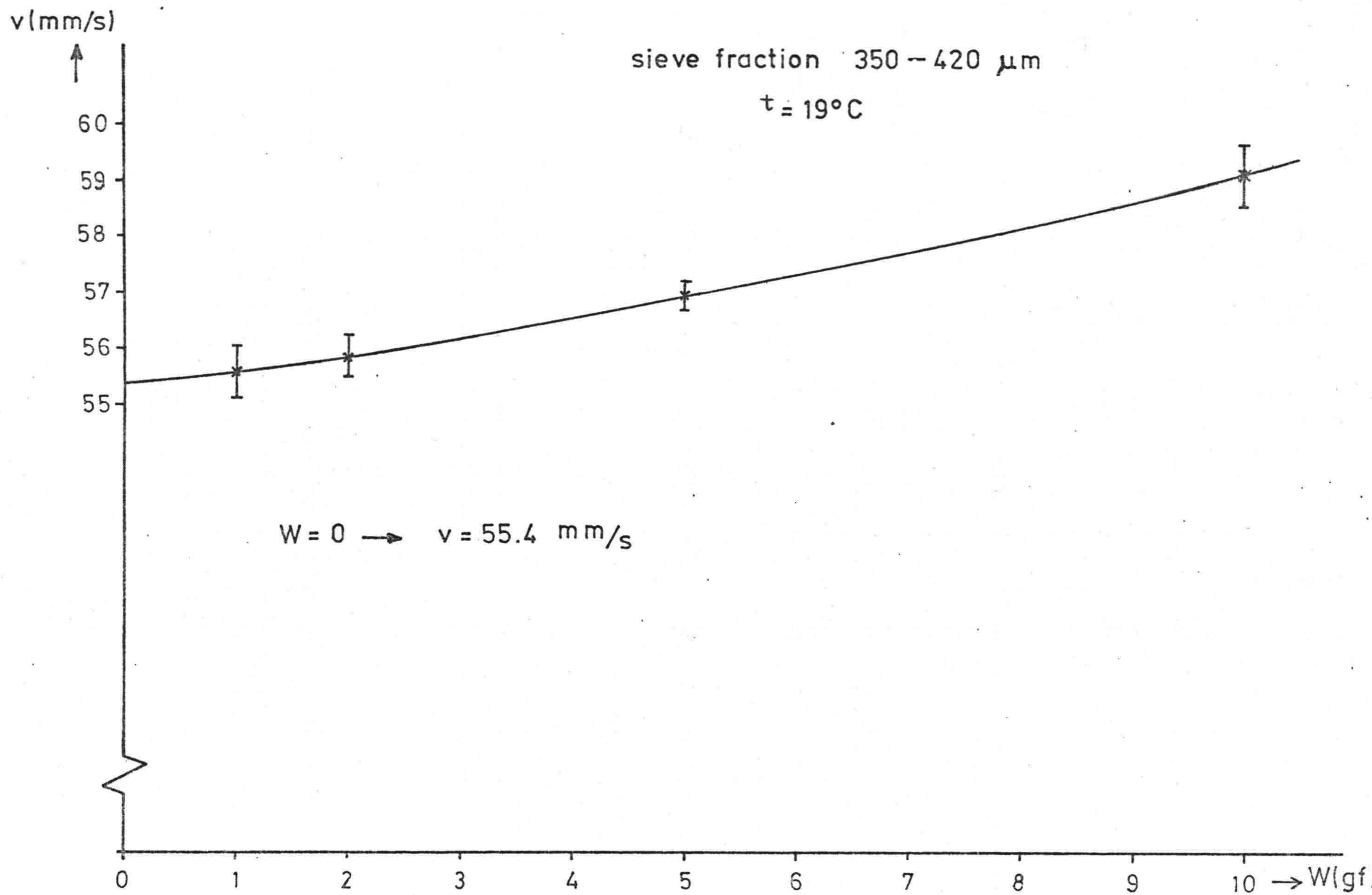


Fig. 12b. Mean sample velocity versus sample weight

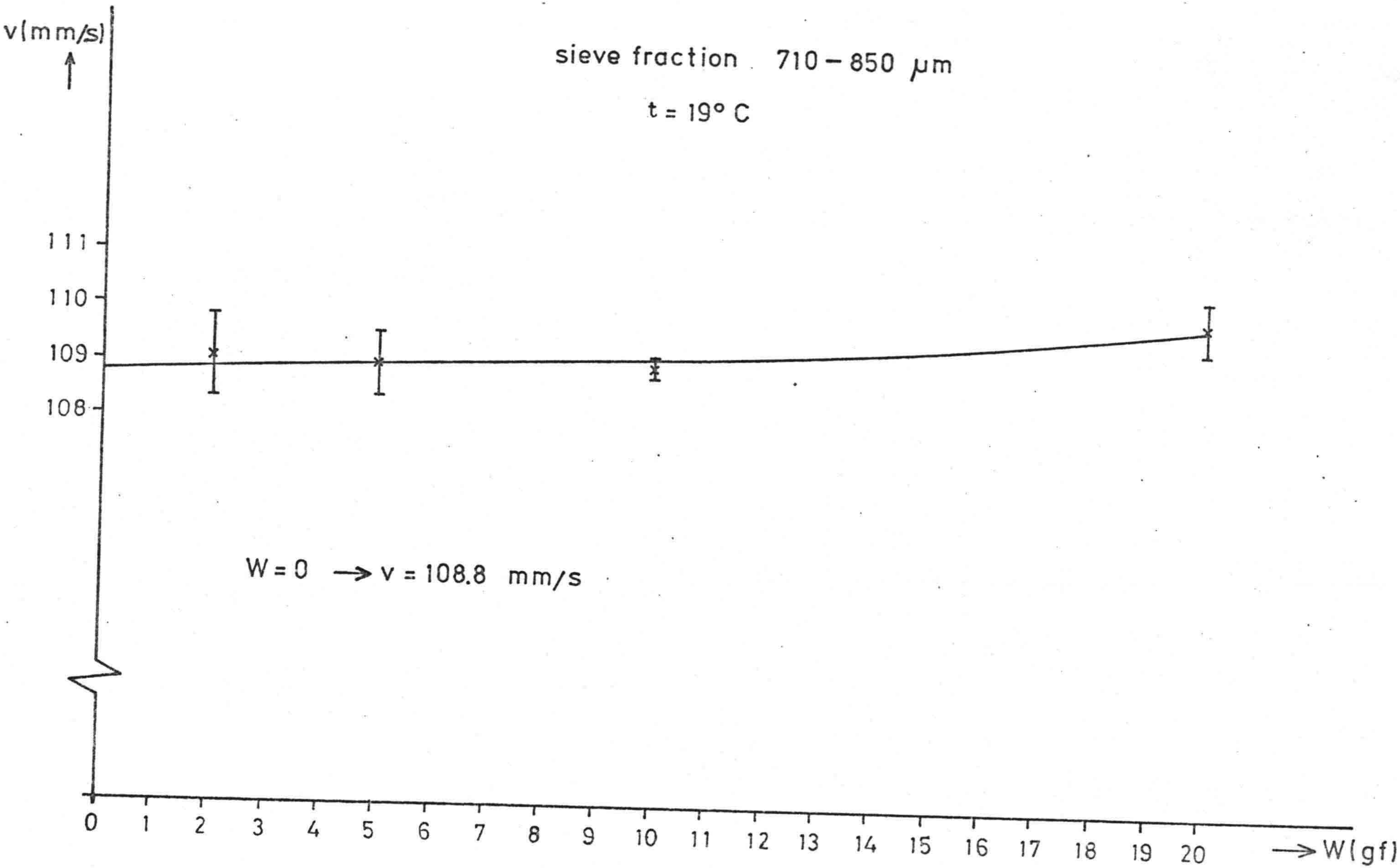


Fig. 12c. Mean sample velocity versus sample weight

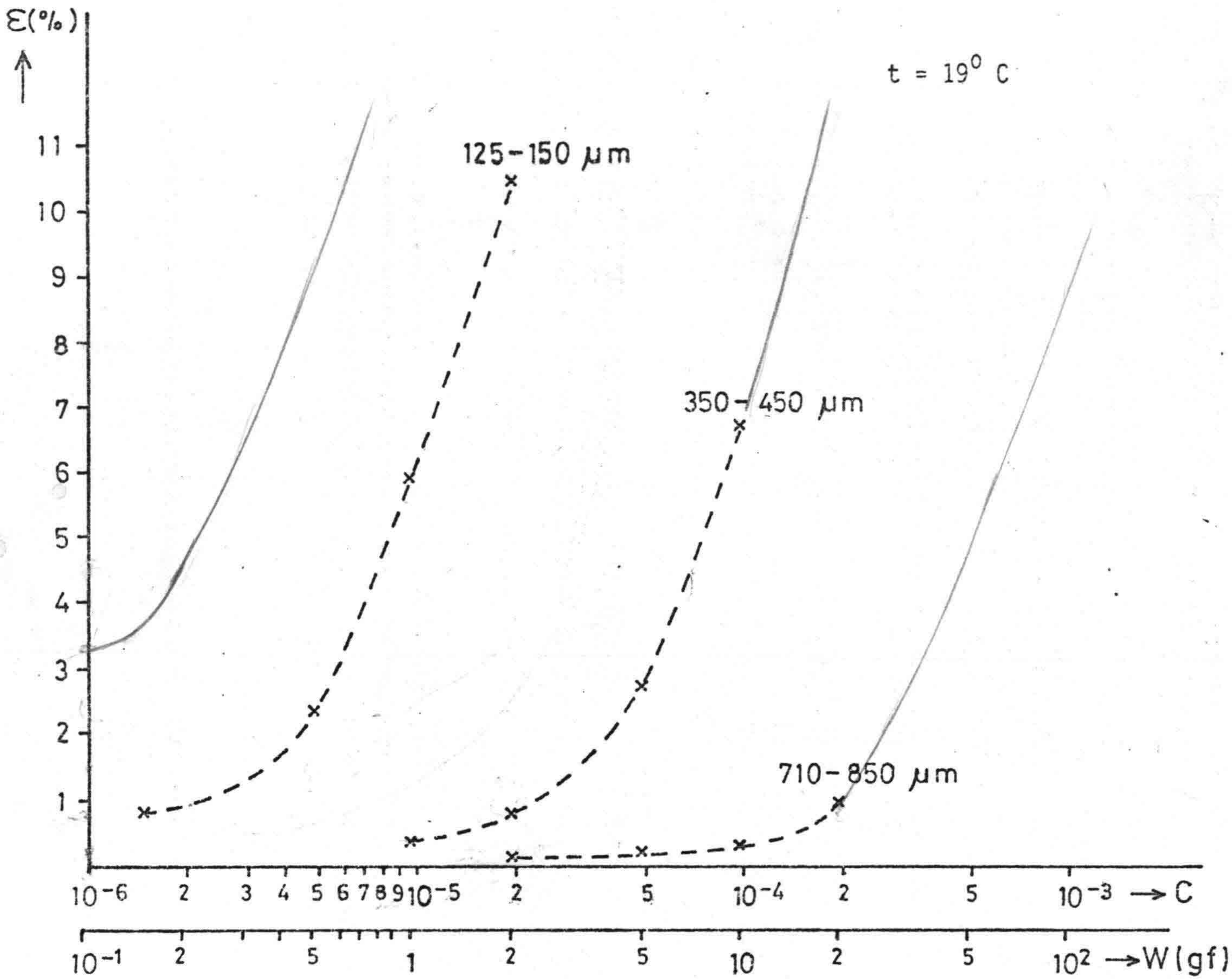


Fig. 13. Error ε due to the concentration effect versus sample weight W

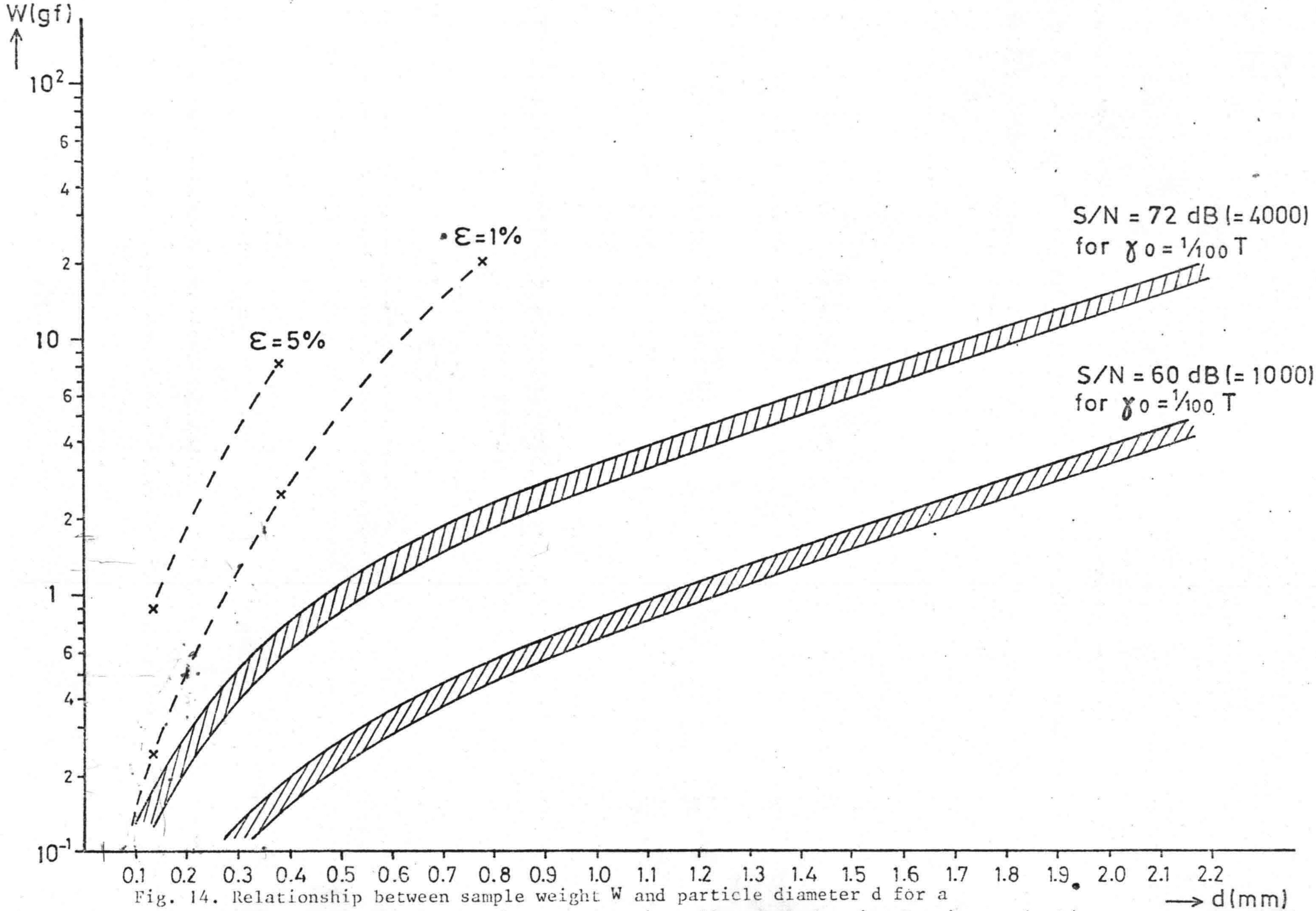


Fig. 14. Relationship between sample weight W and particle diameter d for a constant error ε due to the concentration effect and the signal-noise ratio S/N

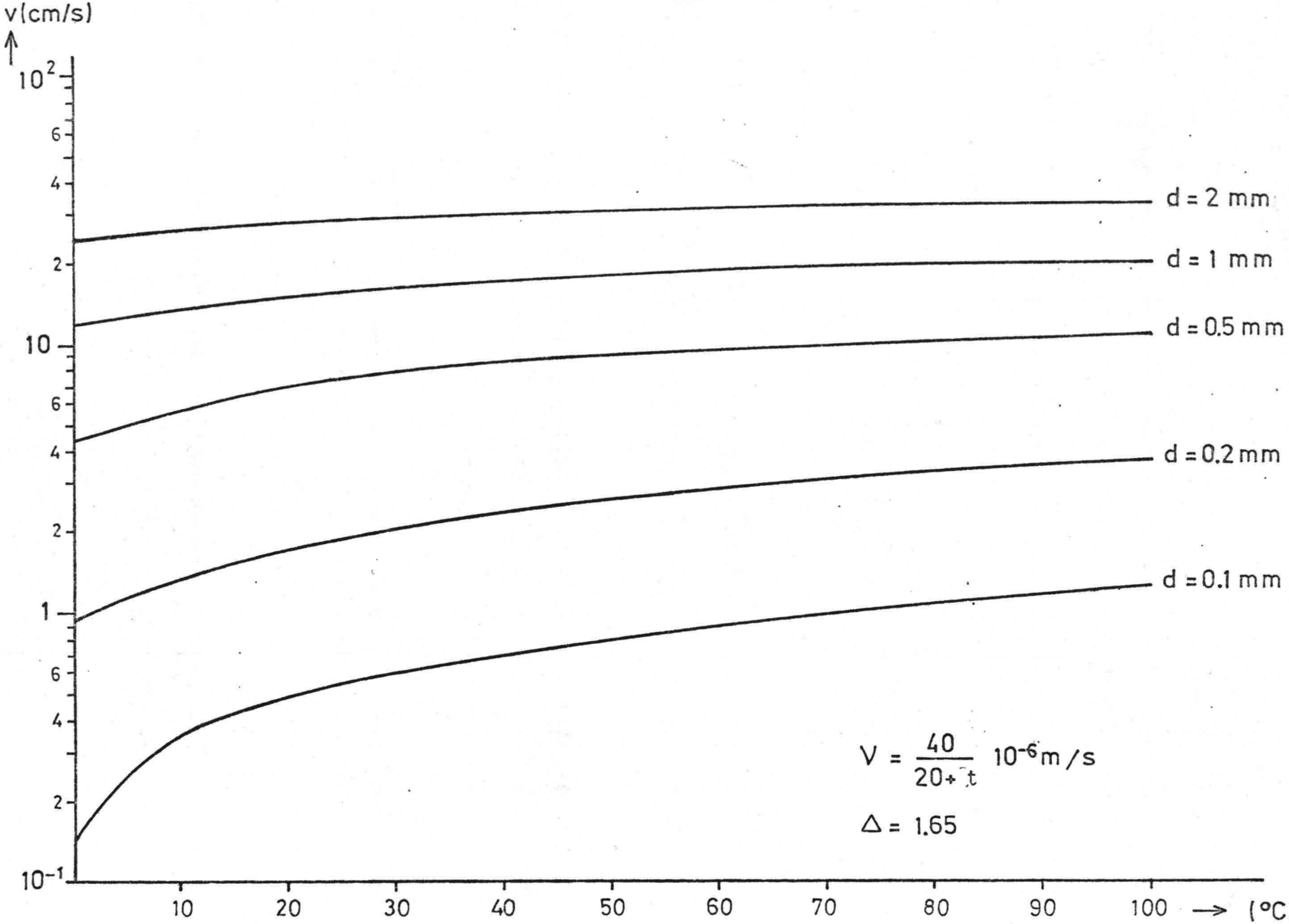


Fig. 15. Settling velocity of quartz spheres versus water temperature

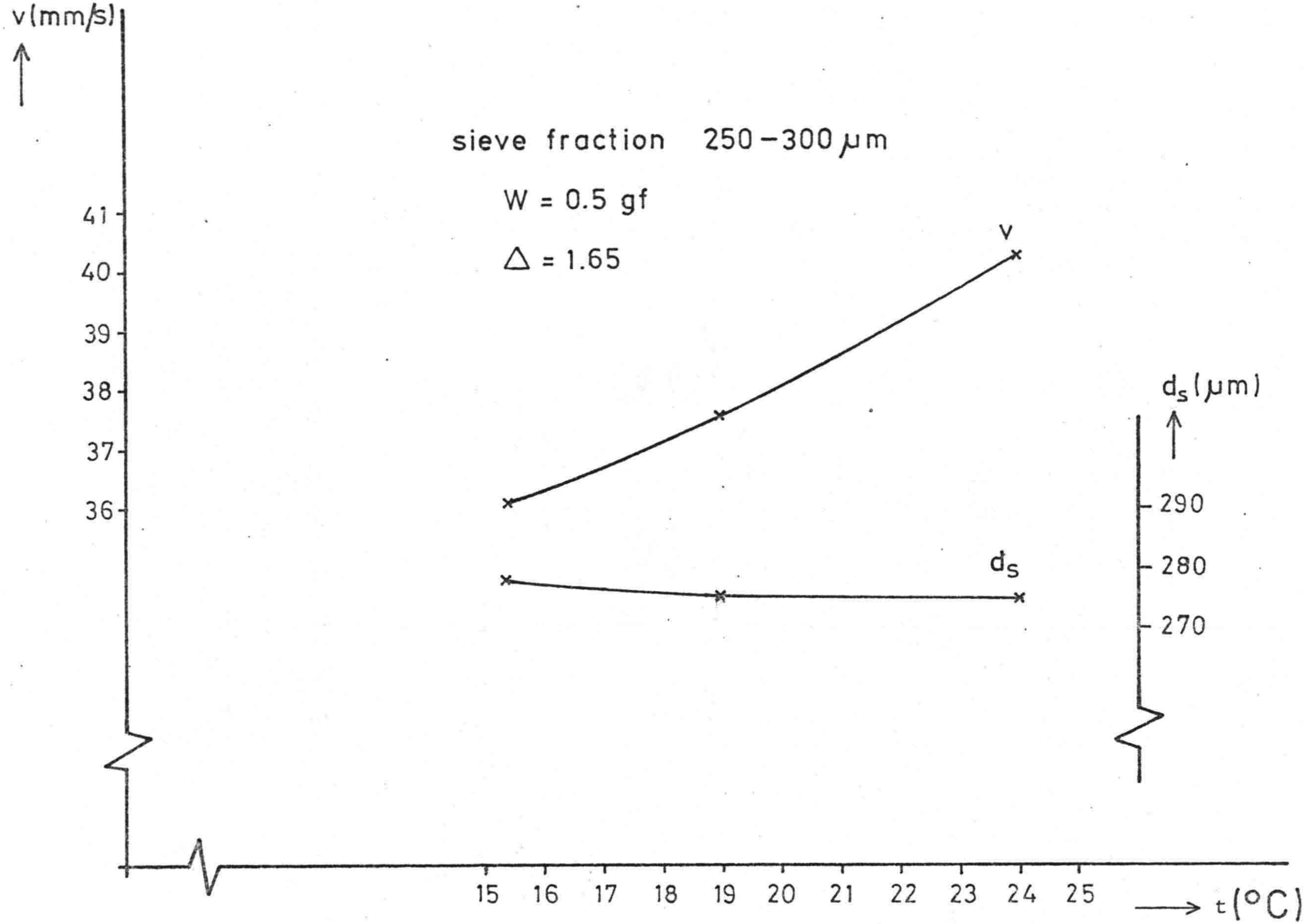


Fig. 16. Mean sample velocity and sedimentation diameter versus water temperature

Sample weight W (gf)	mean settling velocity $\bar{v}$ (mm/s) of sample nr. :					average velocity $\bar{v}_5$ (mm/s) of the series	standard deviation σ_5 (mm/s) of the series
	1	2	3	4	5		
0.15	14.08	14.10	13.94	14.04	14.04	14.04	0.06
0.5	14.25	14.04	14.38	14.04	14.41	14.23	0.18
1	14.83	14.56	14.68	14.50	15.04	14.72	0.22
2	15.41	15.21	15.31	15.45	15.37	15.35	0.10
sieve fraction 125 - 150 μm							

Sample weight W(gf)	mean settling velocity $\bar{v}$ (mm/s) of sample nr.:					average velocity $\bar{v}_5$ (mm/s) of the series	standard deviation σ_5 (mm/s) of the series
	1	2	3	4	5		
1	55.99	55.27	55.65	55.35	55.95	55.64	0.33
2	55.90	55.82	56.25	55.43	55.89	55.86	0.29
5	56.79	57.24	56.76	56.88	56.91	56.91	0.19
10	58.88	59.50	58.73	59.76	58.70	59.11	0.49
sieve fraction 350 - 420 μm							

sample weight W(gf)	mean settling velocity v (mm/s) of sample nr. :					average velocity $\bar{v}_5$ (mm/s) of the series	standard deviation σ_5 (mm/s) of the series
	1	2	3	4	5		
2	110.15	108.85	109.35	108.57	108.65	109.11	0.65
5	109.38	109.41	108.62	108.55	108.51	108.89	0.46
10	109.07	109.12	108.83	109.15	108.86	109.01	0.15
20	110.57	109.61	109.70	109.94	109.83	109.93	0.38
sieve fraction 710 - 850 μm							

Table 1. Measurements of the mean settling velocity of sand samples

Appendix A

The electronics

The electronic part of the settling tube can be split into two sections, the actual feedback section and the electronics necessary for an easy adjustment of the various parameters (zero level, full scale weight and delay time) and the record of the water temperature and the tare range (see main circuit board PC 1). The value of potentiometer P_{11} for the full scale weight adjustment (measure of the amplification factor of the KWS-signal; important for maximum resolution of the A-D converter) is continuously measured by means of a high frequent signal (5kHz) and displayed on a digital panel meter DP 1. The amplification factor of the KWS-signal is measured by setting the 5 kHz signal at the input of the amplifier section (A_8 & A_9) and by taking the quotient of the output and input signal by means of a divider. In this way a very simple oscillator can be used (no extreme stability !). Later on this 5 kHz signal is filtered out of the actual output signal of the settling tube. The range of the full scale weight can be altered by a factor 10 by means of switch S_1.

The delay time is adjusted by means of potentiometer P_{13}. The value of P_{13}, as a measure of the delay time, is measured by means of a special circuit on the main circuit board PC1. By means of a push button PB1 the potentiometer P_{13} is connected between the output and negative input pin of OPAMP A_{16} (like a feedback resistor). Now the amplification factor of OPAMP A_{16} is a measure of the delay time and is displayed on the digital panelmeter DP1.

Instead of using only the variable part of P_{13} in the filter (delay time) section the fixed part of P_{13} is used to eliminate changes in the bias current of OPAMP A_{10} due to changes in resistance. However, there is a problem if the cut-off frequency of the filter is minimum (minimum delay time). The impedance seen by the plus input of OPAMP A_{10} is very high (1MΩ) so the 5 kHz signal is very easily picked up. To eliminate this 5 kHz signal from the output signal of the settling tube the runner of P_{13} and the plus input of A_{10} is 'short circuited' by means of a condensator C_1 of 2.2 μF (impedance 15 Ω for 5 kHz).

The potentiometers P_{19} (coarse) and P_{20} (fine) for taring the balance (or adjusting the zero level of the output signal) are connected to a separated stabilized voltage. This is done to minimize the drift in the weighing system.

Repair/adjustment procedures

I Adjustment of the feedback transfer factor and tare range reading (should be done after replacing the coil and/or magnet; necessary for a correct reading of the full scale weight and tare range).

1. Feedback transfer factor

1.1 Put 5 gram of sand on the weighing pan and measure the output swing of the KWS.

1.2 Set a low frequent ($\sim$ 0.1 Hz) square wave function (preferable unipolar) at testpin 20 (PC1).

1.3 Set the amplitude of the square wave to the value which induces an output swing of the KWS equivalent to 5 gram (as measured at 1.1).

1.4 Adjust P_{17} (PC1) until the output swing of the KWS is a quarter of the full scale swing of the KWS (preferable 10% less for a safety margin). This means a swing of 1.8V (full scale swing of KWS is 8 V).

1.5 Check the damping of the output signal of the KWS (or output of the settling tube signal itself). If necessary adjust the damping to critical by means of P_{18} (PC3).

2. Tare range reading

2.1 Set the range selector of the KWS to zero position.

2.2 Adjust P_6 (PC2) until the swing displayed on the digital panelmeter (DP2) is 5 gf.

2.3 Disconnect the square wave signal.

2.4 Turn the tare (or zero position) potentiometers 'coarse' and 'fine' fully counter clock wise (maximum tared).

2.5 Adjust P_5 (PC2) until the reading of the digital panelmeter (DP2) is zero.

II Adjustment of the openloop amplification (should be done after replacing the inductive transducers) and the damping.

1. Adjust the amplification of the KWS to 10% below the minimum value for oscillation.

2. Adjust P_{18} (PC1) until damping is critical.

III Adjustment of the delay time reading

1. Turn the delay time adjustment potentiometer fully counter clock wise (minimum delay time).

2. Measure the delay time by setting a triangle wave signal at test pin 20 (PC1).

3. Adjust P_{22} (PC1) until the delay time displayed on the digital panelmeter (DP1) is correct (see 2).
4. Turn the delay time adjustment potentiometer fully clock wise (maximum delay time).
5. Measure the delay time (see 2).
6. Adjust P_{21} (PC1) until the delay time displayed on the digital panelmeter is correct (see 5).

IV Adjustment of the weight reading (should only be done if the feedback transfer factor is correctly adjusted: see I)

1. Check the offset of OPAMP's A_3 and A_6 (PC1); adjust P_2 and P_9 (PC1) if necessary.
2. Check range switch 1x/10x; adjust P_3 (PC1) if necessary.
3. Turn the weight adjustment potentiometer fully counter clock wise; adjust P_1 (PC1) until output OPAMP A_3 is -9.5 V.
4. Turn the weight adjustment potentiometer fully clock wise; adjust P_{10} (PC1) until the weight displayed on the degital panelmeter (DP1) is 19.95 gf.

V Adjustment of the output voltage swing of the settling tube and the analog panelmeter reading.

1. <u>Output voltage swing</u>

1.1 Check the range selector switch 1x/10x (S_1) for the output signal; adjust P_{14} (PC1) if necessary.

1.2 Set the weight reading to maximum ($\sim$ 19.95 gf).

1.3 Adjust P_{12} (PC1) until the output swing is 20 V (-10 V to +10 V) for a full scale swing of the KWS (-4 V to +4 V).

2. <u>Analog panelmeter reading</u>

2.1 Adjust P_{16} (PC1) until a full scale swing of the analog panelmeter is obtained for a full scale swing of the output signal (-10 V to +10 V).

2.2 With zero output voltage adjust P_{15} (PC1)until the reading of the analog panelmeter is half scale.

2.3 The adjustments are interactive; so repeat.

VI Adjustment of the temperature reading and the temperature output signal.

1. <u>Temperature reading</u>

1.1 At a temperature of $0^{o}C$ adjust P_1 (PC2) until the reading of the digital panelmeter (DP2) shows zero.

1.2 At a temperature in the range of $40^{o}C$ to $60^{o}C$ adjust P_2 (PC2) until the reading of the digital panelmeter (DP2) is correct.

2. <u>Temperature output signal</u>

2.1 Adjust P_4 (PC2) until the temperature output signal is equivalent to 100 $mV/^{o}C$.

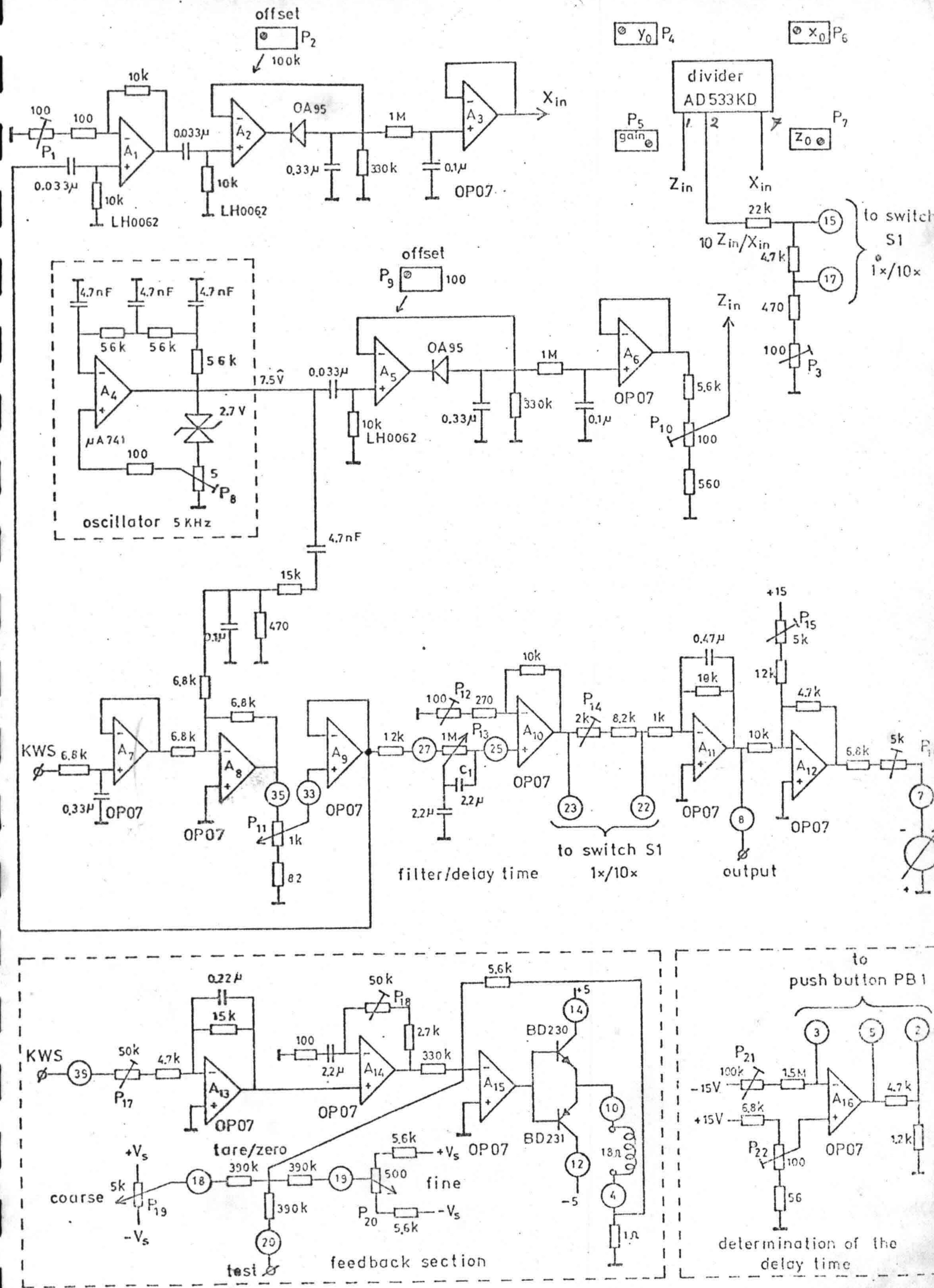
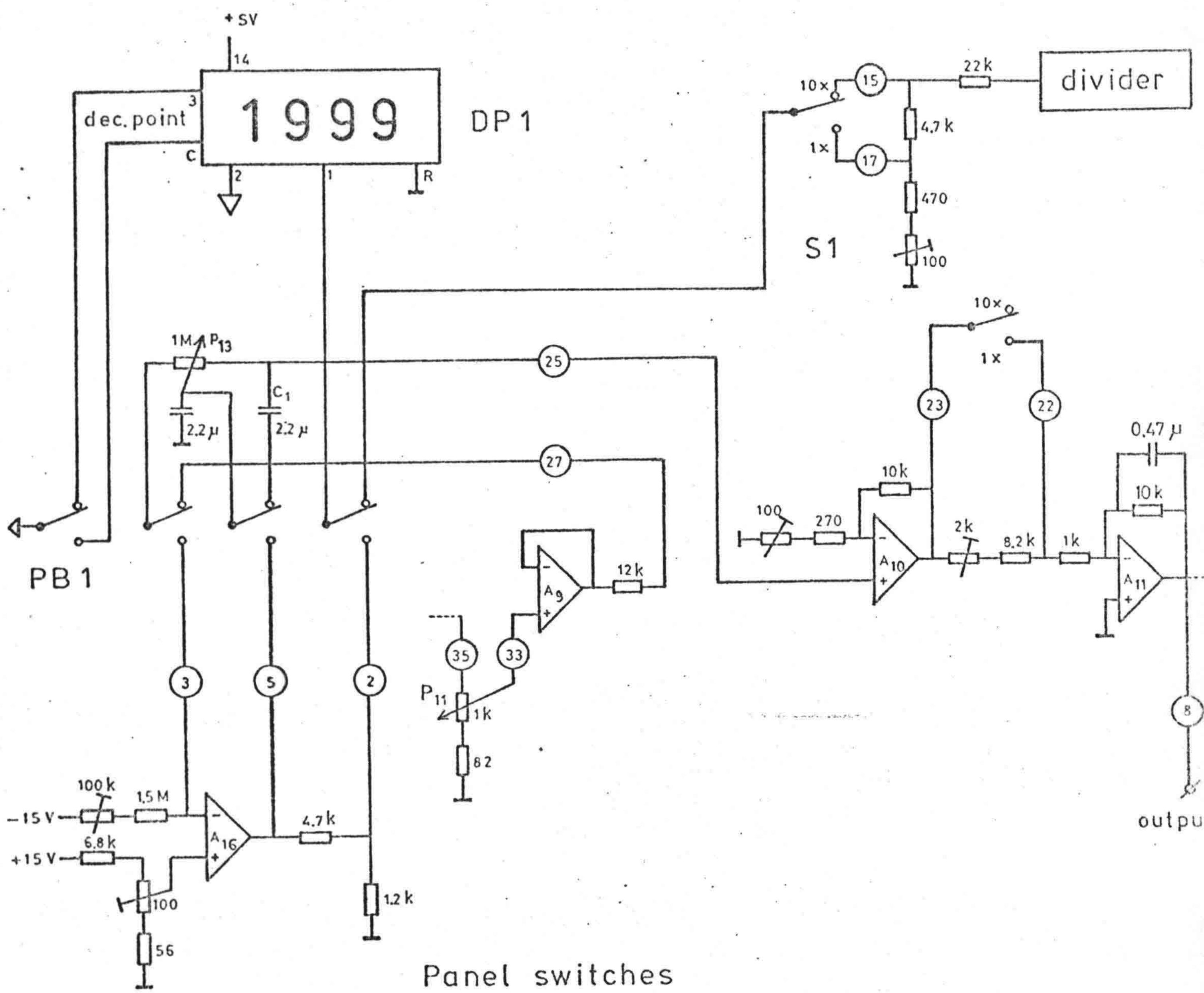
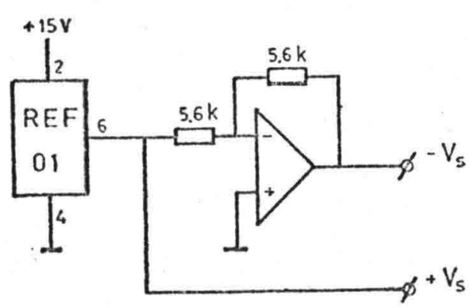


Fig. A1. Main circuit board PC1



Panel switches

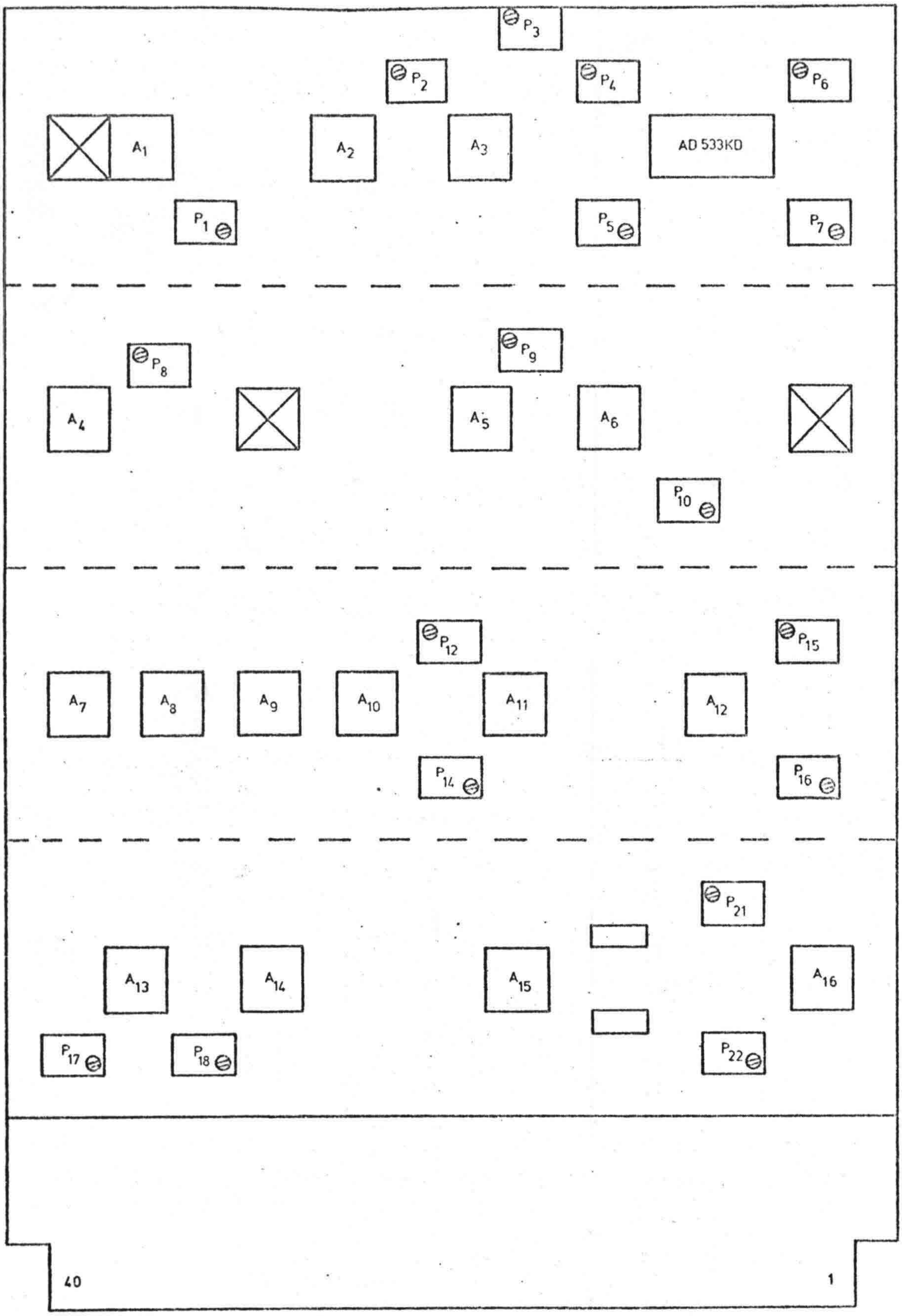


stabilized voltage for taring i.e zero level (coarse and fine)

mounted on analog panelmeter

Stabilized voltage for taring

Fig.A.2 . Panel switches and circuit for stabilized voltage for taring



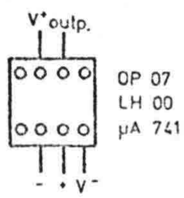
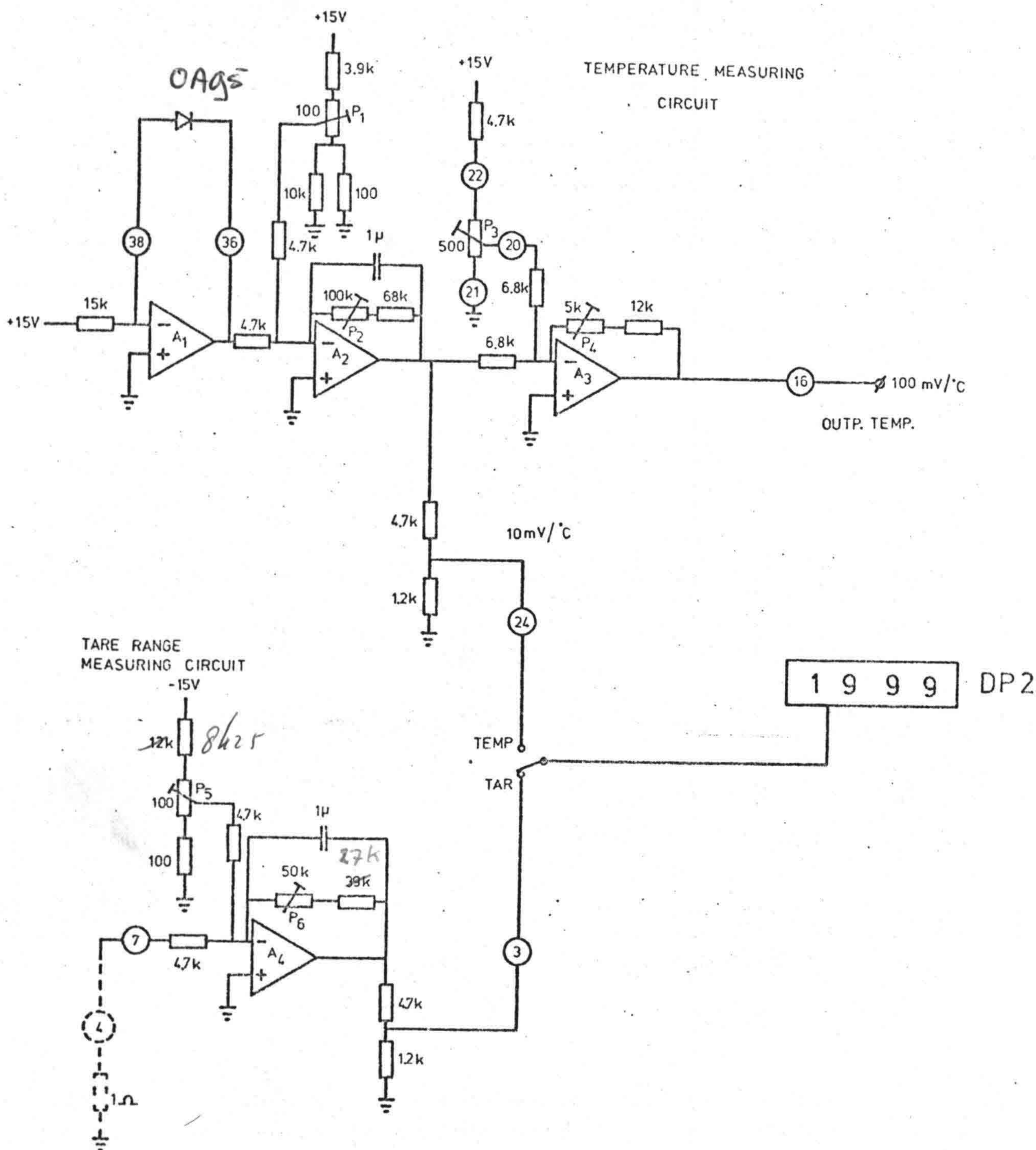


fig.A.3 Layout main circuit board PC1



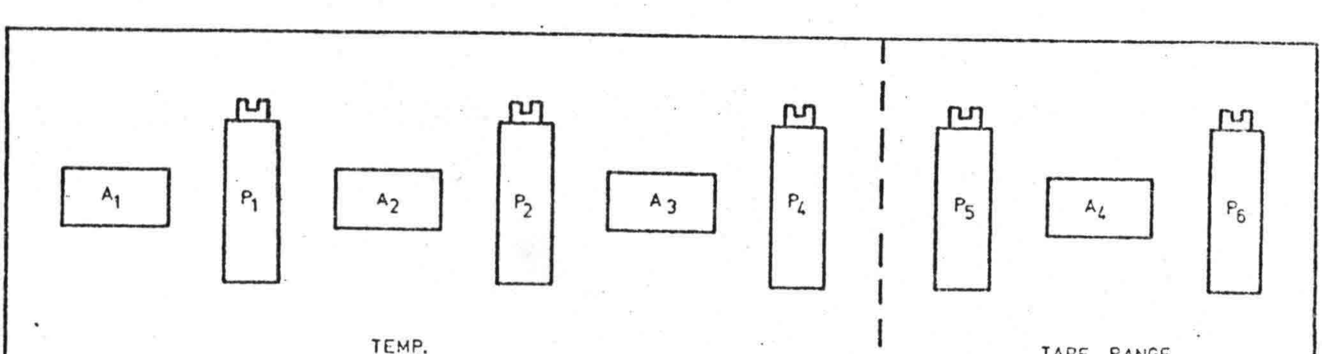


fig A.4 Temperature and tare range measurement circuit board PC 2

Appendix B

Upward force variation on weighing pan due to temperature variation

Temperature variation of the water in which the weighing pan floats will induce a variation in the upward force on the weighing pan due to variation in the volume of the weighing pan as well as variation is the density of the water.

The upward force on the weighing pan as a temperature dependent quantity can be written as

$$K(t) = \rho_w(t)\, V_0(1 + \gamma t)g \,, \tag{B.1}$$

where $K(t)$ = upward force on weighing pan at $t\,^{\circ}C$

$\rho_w(t)$ = specific mass (density) of water at $t\,^{\circ}C$

V_0 = volume of weighing pan at $0\,^{\circ}C$ ($\approx$ 1000 cm^3)

γ = cubic expansion coefficient of weighing pan ($2.4 \; 10^{-4}/^{\circ}C$)

t = temperature ($^{\circ}C$)

g = acceleration of gravity

In fig. B.1 the upward force K versus the temperature is shown. The values of K were calculated utilizing the tabulated values of the density of water shown in table B.1.
In fig. B.2 the variation dK/dt of the upward force per unit of temperature is shown. As follows from fig. B.2 the variation in upward force is zero (minimum) at about $23\,^{\circ}C$.

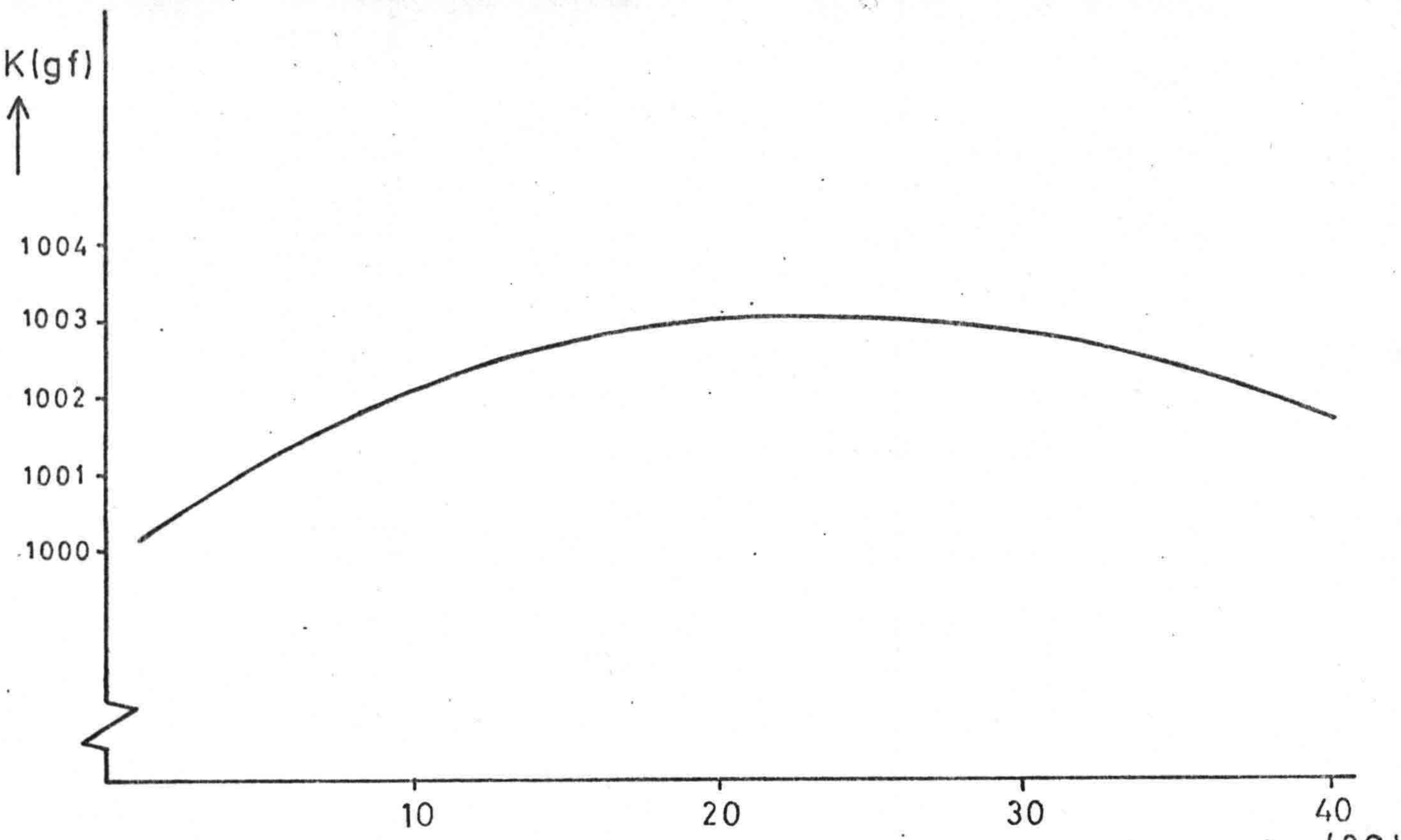


Fig. B.1. Upward force K on weighing pan versus water temperature t

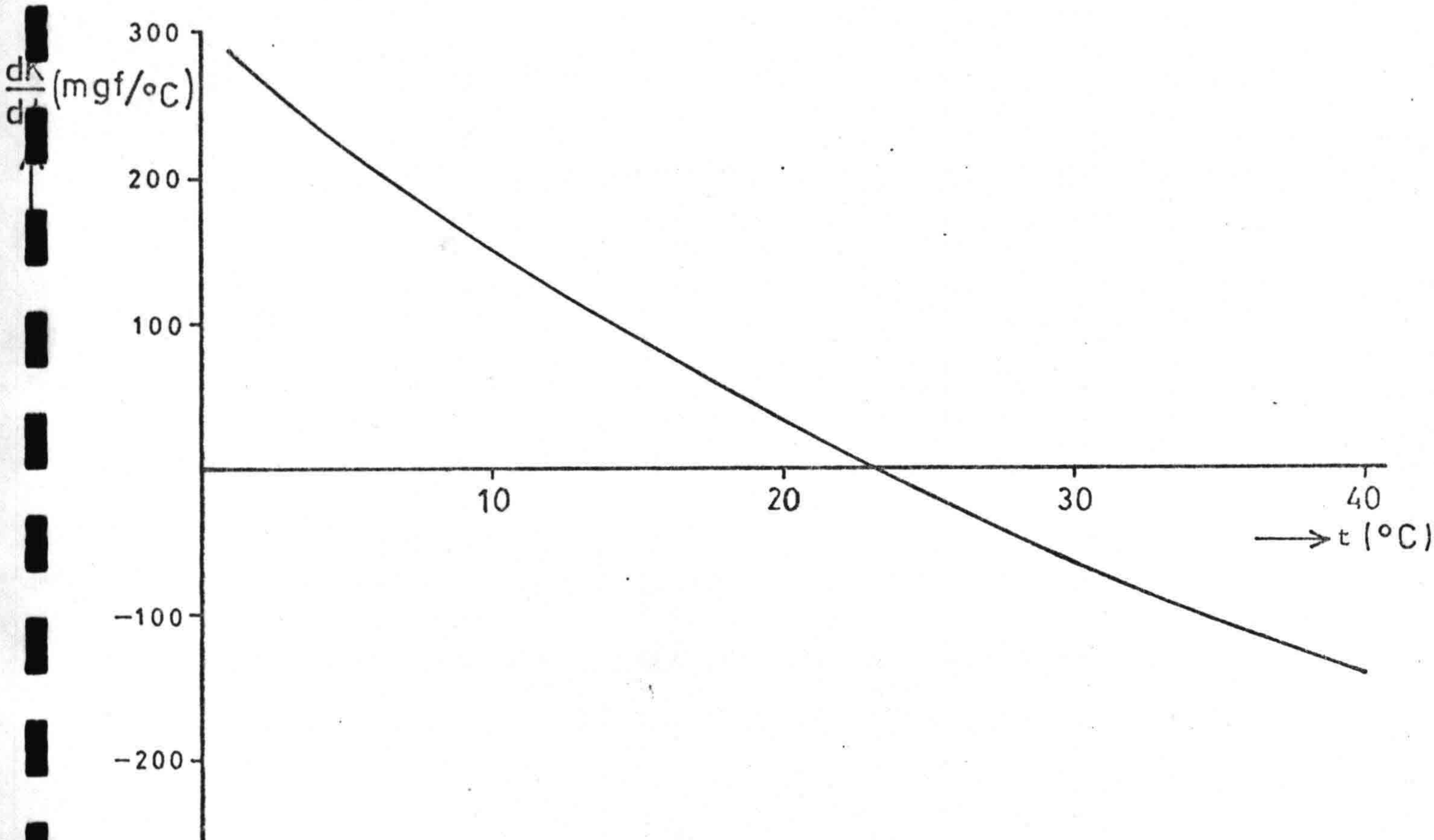


Fig. B.2. Variation in upward force per unit of temperature versus the water temperature

(°C)	ρ_w (g/cm^3)
0	0.99987
3.98	1.00000
5	0.99999
10	0.99973
15	0.99913
18	0.99862
20	0.99823
25	0.99707
30	0.99567
35	0.99406
38	0.99299
40	0.99224
45	0.99025

Table B.1. Density of water

Appendix C

Formulae for the settling velocity, the sedimentation diameter and the nominal diameter

The general formula for the velocity v of a sphere with diameter d induced by an acting force F and moving in a viscous fluid with kinematic viscosity ν has the form

$$v = \frac{\nu}{2df_\beta}\left[-1 + \sqrt{1 + \frac{4}{3\pi}\left(\frac{F}{\rho\nu^2}\right) f_\beta}\right] , \qquad \text{(C.1)}$$

where f_β is a function of the dimensionless quantity $F/(\rho\nu^2)$ (Slot, 1983). The formula holds for Reynolds numbers up to 2000 with an accuracy of 2% if a linear relationship is taken between f_β and $[F/(\rho\nu^2)]^{-1/3}$ as

$$f_\beta = C_0 + C_1 \left[\frac{F}{\rho\nu^2}\right]^{-1/3} \qquad \text{(C.2)}$$

where $C_0 = 0.0125$
$C_1 = 0.348$

For a free falling sphere the dimensionless quantity $[F/(\rho\nu^2)]^{-1/3}$ is expressed as

$$\left[\frac{F}{\rho\nu^2}\right]^{-1/3} = \frac{Q}{d} , \qquad \text{(C.3)}$$

where $Q = [6\,\nu^2/(\pi\Delta g)]^{1/3}$
$\Delta = (\rho_s - \rho)/\rho$
ρ_s = specific mass (density) of the sphere
ρ = specific mass (density) of the sedimentation fluid
g = acceleration of gravity

Substitution of (C.3) into (C.1) gives for the settling velocity of a free falling sphere

$$v_s = \frac{\nu}{2df_\beta}\left[-1 + \sqrt{1 + \frac{2\Delta g d^3}{9\nu^2} f_\beta}\right] , \tag{C.4}$$

where $f_\beta = C_0 + C_1\ Q/d$.

The above expression, solved for d, yields the expression for the sedimentation diameter

$$d_s = \frac{9\,C_0}{\Delta g}\ \nu^2\left[1 + \sqrt{1 + \frac{2\Delta g}{9C_0^2\nu^2}\ (\frac{\nu}{v} + C_1 \cdot Q)}\right]. \tag{C.5}$$

In general the particles analyzed by settling tube systems are not spherical but are irregularly shaped. An attempt to define the shape of a particle is done by the shape factor

$$sf = \frac{c}{\sqrt{ab}} , \tag{C.6}$$

where a = the longest (or major) axis of the particle
b = the intermediate axis of the particle
c = the shortest (or minor) axis of the particle, with all axis mutually perpendicular.

It appears that for large Reynolds numbers (or small values of $[F/(\rho\nu^2)]^{-1/3}$) as well as small shape factors the linear relationship between f_β and $[F/(\rho\nu^2)]^{-1/3}$ does not hold any more; f_β becomes more or less a constant of which the actual value depends on the shape factor. In fig. C.1 the relationship between f_β and $[F/(\rho\nu^2)]^{-1/3}$ is shown for various shape factors calculated from data presented in "A Study of Methods Used in Measurement and Analysis of Sediment Loads in Streams" (Report no. 12, Some fundamentals of particle size analysis, St. Anthony Fall Hydraulic Laboratory, Minneapolis, Minnesota, december 1957).
To approximate this relationship f_β will be expressed as

$$f_\beta = K_0 + K_1\left[\frac{F}{\rho\nu^2}\right]^{-1/3} + K_2\left[\frac{F}{\rho\nu^2}\right]^{1/3} = K_0 + K_1\ Q/d + K_2\ d/Q , \tag{C.7}$$

where K_0, K_1 and K_2 are functions of the shape factor. The motivation of this form is that eq. C.4 can be solved for d, yielding the expression for the nominal diameter

$$d_n \doteq \frac{K_0 v^2}{\frac{\Delta g}{9} - \frac{2K_2 v^2}{Q}} \left[1 + \sqrt{1 + 2\,\frac{\frac{\Delta g}{9} - \frac{2K_2 v^2}{Q}}{k_0^2\, v^2}\left(\frac{\nu}{v} + K_1\, Q\right)}\, \right] . \qquad \text{(C.8)}$$

Taking linear functions of the shape factor for K_0, K_1 and K_2 as

$$\begin{aligned} K_0 &= C_{00} + C_{01}\ sf \\ K_1 &= C_{10} + C_{11}\ sf \\ K_2 &= C_{20} + C_{21}\ sf \end{aligned} \quad ,$$

the formula for the nominal diameter will hold for Reynolds numbers up to 8000 with an accuracy of 5% for

$$\begin{aligned} C_{00} &= 0.109 && \text{and} \quad C_{01} = -\ 0.100 \\ C_{10} &= 0.635 && \text{and} \quad C_{11} = -\ 0.253 \\ C_{20} &= 8.95\ 10^{-5} && \text{and} \quad C_{21} = -\ 6.34\ \ 10^{-5} \end{aligned}$$

If the nominal diameter and the shape factor of a particle are known then substitution of (C.7) into (C.4) gives the settling velocity of the (irregular) shaped particle with the same accuracy of 5% for Reynolds numbers up to 8000.

In fig. C.2 the settling velocity of a sphere in water versus the sphere diameter is shown, calculated from eq. C.4 for $t = 20^\circ C$ and $\Delta = 1.65$. Furthermore the settling time versus the sphere diameter is shown for a settling tube length of 1.65 cm.

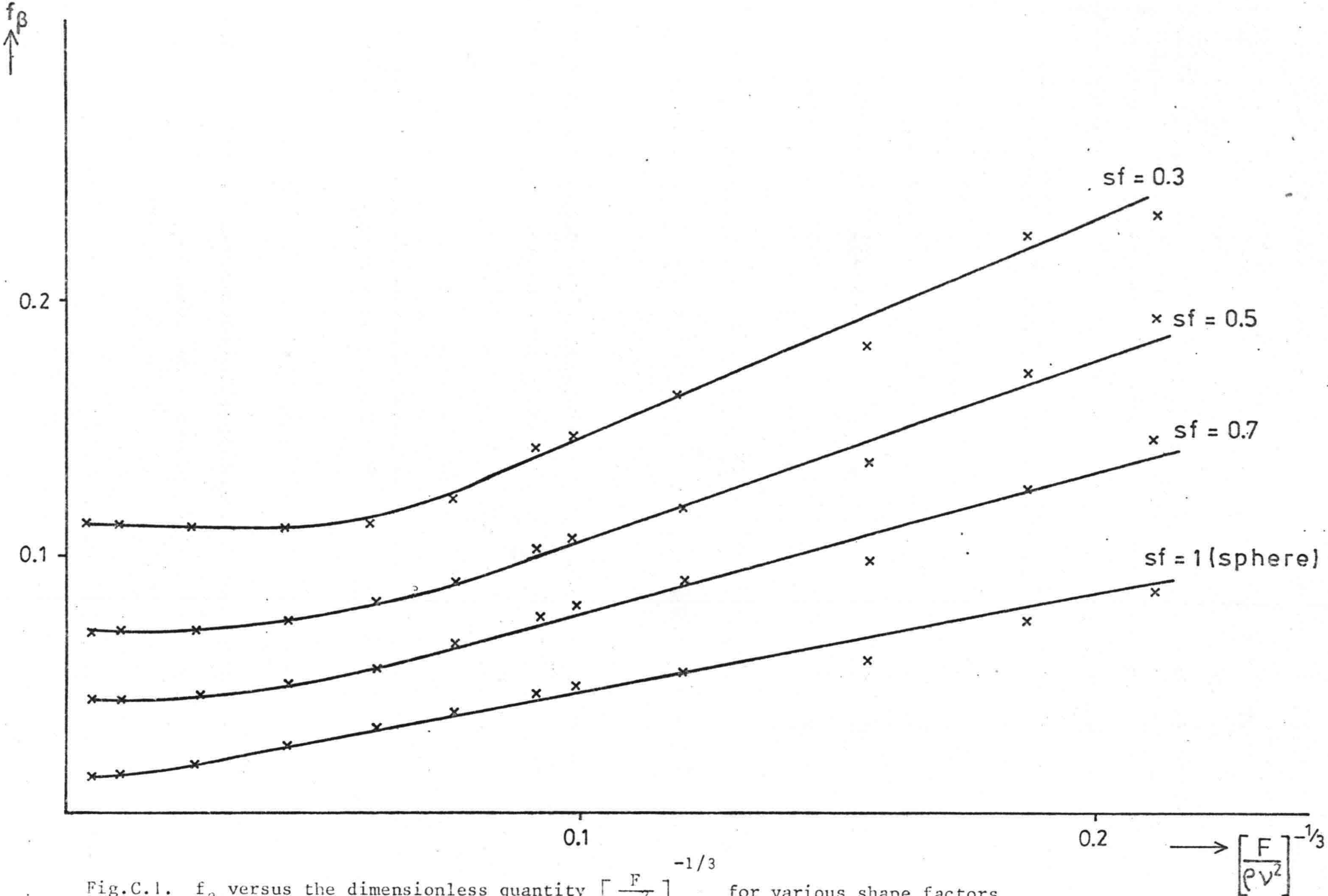


Fig.C.1. f_β versus the dimensionless quantity $\left[\frac{F}{\rho \nu^2}\right]^{-1/3}$ for various shape factors

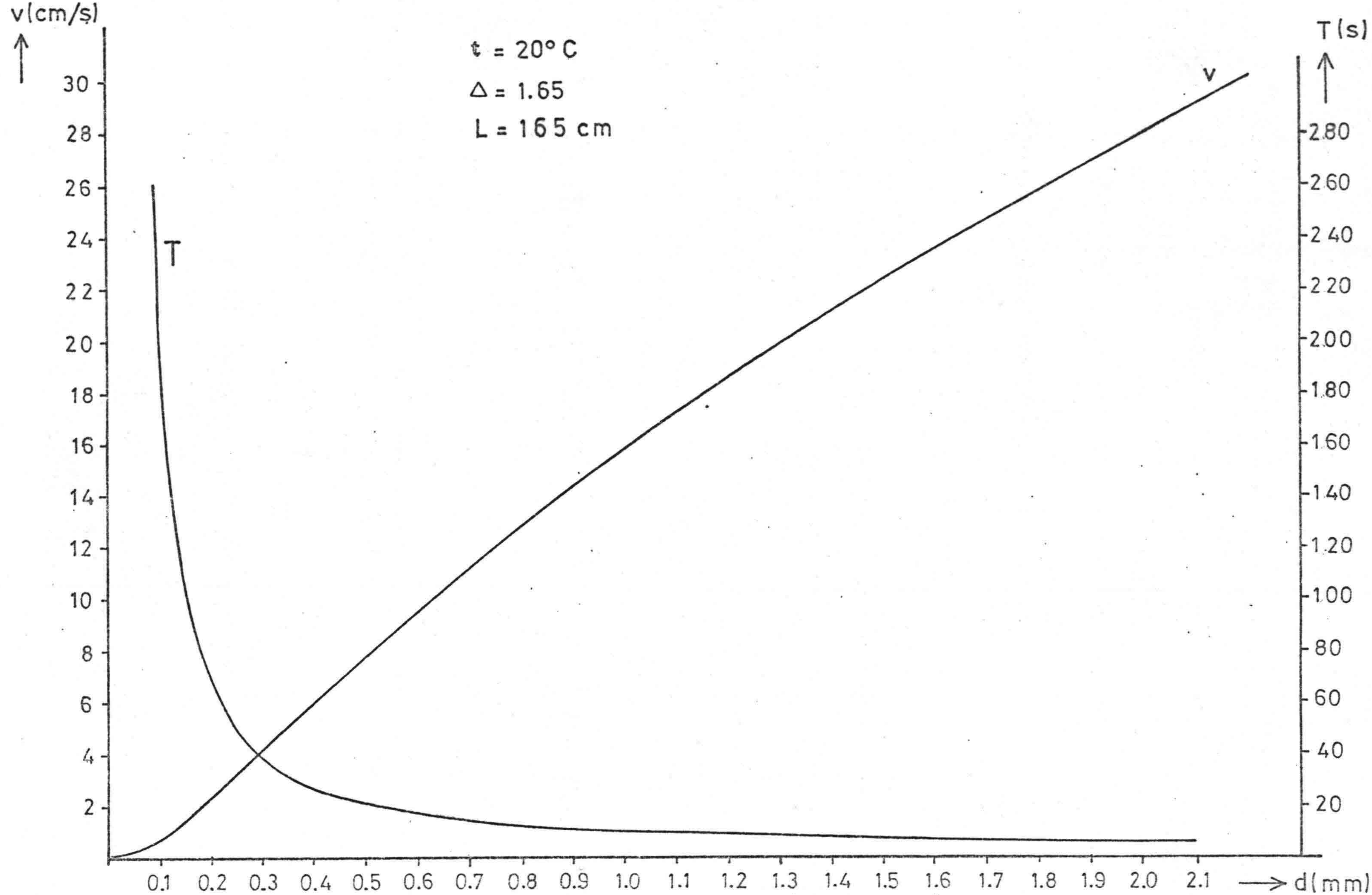


Fig. C. 2. Settling velocity and settling time of a sphere versus the sphere diameter

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