Space-Plane Analysis

A trajectory generation and sensitivity analysis

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Title: Space-Plane Analysis: A trajectory generation and sensitivity analysis

Author(s): A.G.M. Marée, E. Mooij and B.T.C. Zandbergen

Abstract: In February 1993 the Advanced Earth to Orbit Launcher Upgrade Studies (AEOLUS) have been started. The aim of AEOLUS is to develop the technological level in the field of advanced launchers so that The Netherlands will be able to participate in advanced-launcher projects. Within this project, studies at the highest level of the launchers, or space plane, are conducted by Faculty of Aeronautical Engineering, Delft University of Technology. This space-plane analysis has been performed within the workpackage 212. In this report, AEOLUS document code AE-DUT-TN-9403 (1), a sensitivity analysis has been performed to get insight in the relation between the important trajectory parameters and the performance of a space plane (like the payload mass and the fuel/propellant mass). For the required trajectory calculations, the trajectory simulation tool ASCENT has been used. With ASCENT trajectories can be composed by hand. The sensitivity analysis has been performed with the Taguchi method. This method is an efficient approach for determining near optimum design parameters. To perform the sensitivity analysis, firstly, a Two Stage To Orbit space plane data set has been composed with the help of data of the Winged Cone Configuration and data of the HORUS 2B. For this space plane a Sänger-like trajectory is generated with the help of the simulation program ASCENT. Next, thirteen trajectory parameters have been selected for the sensitivity analysis. In 36 different simulations the trajectory parameters are varied ± 10% around their reference value. The configuration and mass values of the first stage and second stage are kept the same for all 36 simulations. The payload mass has been corrected in relation to the required amount of propellant mass for the second stage. For the conditions investigated in this report it has been found that the resulting variation in delivered payload mass was about ± 3.5 ton around the mean value of 15 ton of the simulations. The propellant mass varied about ± 23 ton around the mean value of 161 ton. The relations of these performance variables with the most important trajectory parameters have been expressed in two polynomials. Mainly the separation conditions and the trajectory of the second stage influence the delivered payload mass. The best separation conditions are for the highest possible values for the separation altitude and Mach number. The influence of the flight-path angle at separation appeared to be small. To decrease the fuel mass consumed during the first stage, the dynamic pressure and the flight-path angle should be as large as possible. The first part of the trajectory after take-off can have much influence on the required fuel mass. The sensitivity analysis results also clearly showed that trajectory optimization is a necessary tool for space plane analysis. Furthermore, because of the large variations in propellant mass, it will be necessary to have a space plane design tool to anticipate on these changes by corrections in the space plane configuration.

Keyword(s): AEOLUS, space plane, inlet, flight simulation, trajectory optimization, ASCENT, Taguchi method

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Notations

Roman

\begin{align*}
C_D & \text{ drag coefficient} \\
C_L & \text{ lift coefficient} \\
C_S & \text{ side force coefficient} \\
C_T & \text{ thrust coefficient} \\
D & \text{ drag} \\
f & \text{ degrees of freedom} \\
f_{sc} & \text{ scale factor} \\
g & \text{ gravitational acceleration} \quad \text{m/s}^2 \\
h & \text{ height} \quad \text{m} \\
l_{sp} & \text{ specific impulse} \\
L & \text{ lift} \\
m & \text{ mass} \quad \text{kg} \\
m & \text{ mean value} \\
\dot{m} & \text{ fuel/propellant mass flow} \quad \text{kg/s} \\
M & \text{ mach number} \\
n_x & \text{ axial acceleration (in Earth g)} \\
n_z & \text{ normal acceleration (in Earth g)} \\
q_{dyn} & \text{ dynamic pressure} \quad \text{N/m}^2 \\
R_N & \text{ nose radius} \quad \text{m} \\
S & \text{ side force} \quad \text{N} \\
S_{ref} & \text{ reference area} \quad \text{m}^2 \\
S_t & \text{ total variation} \\
S_A & \text{ variation resulting from parameter A} \\
T & \text{ thrust} \quad \text{N} \\
t & \text{ time} \quad \text{s} \\
V & \text{ variance} \\
V & \text{ velocity} \quad \text{m/s} \\
V_c & \text{ circular velocity} \quad \text{m/s}
\end{align*}
Greek

<table>
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<tr>
<th>Symbol</th>
<th>Description</th>
<th>Unit</th>
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<tr>
<td>$\alpha$</td>
<td>angle of attack</td>
<td>rad</td>
</tr>
<tr>
<td>$\beta$</td>
<td>angle of sideslip</td>
<td>rad</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>flight-path angle</td>
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</tr>
<tr>
<td>$\delta_T$</td>
<td>throttle setting</td>
<td></td>
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<tr>
<td>$\psi_T$</td>
<td>thrust-vector angle in plane of symmetry of space plane</td>
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</tr>
<tr>
<td>$\rho$</td>
<td>atmospheric density</td>
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Subscripts

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Abbreviations

AEOLUS  Advanced Earth to Orbit Launcher Upgrade Studies

dof     degrees of freedom

DUT     Delft University of Technology

FAE     Faculty of Aerospace Engineering

G&C     Guidance and Control

HL      Horizontal Landing

HSV     Hypersonic air-breathing vehicle

HTO     Horizontal Take-Off

LEO     Low Earth Orbit

SSTO    Single-Stage To Orbit

TSTO    Two-Stage To Orbit

TVC     Thrust-Vector Control

WCC     Winged Cone Configuration

WLC     Winged Launchers Configurations

wp      work package
1. Introduction

1.1 The AEOLUS project

In February 1993, an Advanced Earth to Orbit Launcher Upgrade Studies (AEOLUS) project has been started as a follow-up to the Airbreathing Propulsion project, which started in 1988. AEOLUS is a cooperative project wherein several Dutch industries and research institutes including TNO-Prins Maurits Laboratory, NLR, Fokker Space & Systems BV., Stork Product Engineering BV., and the Faculty of Aerospace Engineering of the Delft University of Technology (TU-Delft/LR) take part. The goal of AEOLUS is both to develop the technological and engineering level in the field of advanced space launchers as well as to market these skills in a way that The Netherlands will be able to participate in (international) advanced space launcher projects. For this, the chosen approach is a broad technological and engineering covering of the inlet (sub)system of a winged airbreathing launch vehicle (AEOLUS 1992).

As such, the inlet (sub)system not only offers challenges in the fields of aerothermodynamics, thermal protection systems, and light-weight structures, but also with respect to its profound interactions with higher order systems, like the propulsion system and even the total launcher (Staudacher 1993, Schwab 1992, Künkler 1990). It is because of these profound interactions, that AEOLUS also gives substantial attention to propulsion and space plane system analysis. In addition, space plane analysis also supports the development of larger systems than the inlet system for small hypersonic test vehicles, like the German HYTEX (Sacher, 1993), and it facilitates identification of high-ranking (with respect to pay-off) technologies (Arrington, 1985).

Within AEOLUS, Delft University of Technology, Faculty of aerospace engineering is responsible for space plane analysis.

1.2 Space plane analysis

In the context of AEOLUS, space plane analysis is performed within the work package 212 and seeks to analyze the intricate relations between space plane performance on the one hand and the vehicle design and the mission on the other. This analysis, then serves to identify the most important design parameters and technological challenges.
Space plane analysis, basically constitutes two activities, being trajectory analysis and global space plane design, which are closely related to each other. This is because, according to Schotttle (1989), the vehicle design is extremely sensitive to propulsion system weight, propellant consumption and aero-thermodynamic loads encountered during the flight, indicating the necessity to reconfigure/-design the space plane space when the space plane trajectory changes. Another reason is because also the performance (e.g. payload into orbit) of the vehicle is very sensitive to the trajectory that is flown. Hattis (1991) says about this: The relative merits of alternative vehicle configurations, subsystem technologies, and G&C design concepts can only be assessed in a valid manner if the comparative performance standard assures consideration of efficient trajectories for each case. To accomplish this, a generic tool that will generate near-minimum-fuel-to-orbit trajectory solutions for a wide variety of HSVs (Hypersonic air-breathing vehicles) is required.

As a first step in the space plane analysis, the following two objectives of the work package have been defined:

- Review of space-plane concepts.
- Definition of flight conditions along trajectory.

Based also on the space plane requirements document (Herwaarden, 1993) and the specific needs of 'System engineering', these two objectives have resulted in:

- the definition of a vehicle representative for a Sänger-like horizontal take-off and landing, Two Stage To Orbit (TSTO) winged space launchers, with hypersonic stage separation and a payload mass into Low Earth Orbit (LEO) of 7 ton, and
- the calculation of a representative trajectory for the defined vehicle using the in-house developed trajectory computation code ASCENT.

In addition, to investigate the need for trajectory optimization, a sensitivity analysis has been performed using a system of (experimental) design method developed by Taguchi. This Taguchi method is an efficient approach for determining near optimum design parameters (Stanley, 1992).

Finally, during this sensitivity analysis, also additionally needed analysis tools have been identified.

1.3 The report structure

The earlier mentioned approach, can also be clearly recognized in the structure of this report. In chapter 2, the vehicle data set, needed for the trajectoryCalculation tool ASCENT, is identified after
which a complete data set for a Sänger-like TSTO vehicle is composed based on available data from open literature. Next, using this vehicle data set, a Sänger-like trajectory is generated with ASCENT. In chapter 4, the generated trajectory is evaluated.

The trajectory sensitivity analysis using the Taguchi method is reported in chapter 5 including identification of the significant trajectory parameters and their influence on vehicle performance/design. In chapter 6, an explanation of these results is presented, whereas the consequences of these results for space plane analysis are discussed.

Finally, the conclusions and recommendations resulting from this study are given in chapter 7.
2. Vehicle modelling

At DUT-FAE, two complete vehicle data sets, with aerodynamic, propulsion, geometric and mass data, are available:

- a data set for a simulation model of an airbreathing Single Stage To Orbit (SSTO) configuration: the Winged Cone Configuration (WCC) from Shaughnessy et al. (1990).
- a data set for a simulation model of a rocket propelled second stage of a TSTO configuration: the HORUS 2B from MBB (1988a).

The space-plane concept in AEOLUS is a Sänger-like TSTO vehicle. Because no other vehicle data, which can be used for trajectory simulation, are available, these data sets are used to compose a Sänger-like TSTO vehicle through a simple scaling process. In section 2.4 will be shown how this is done.

The Sänger-like vehicle is composed so that it can be implemented in the trajectory simulation program ASCENT. Some features, however, which are required for space plane trajectory simulation, are not implemented. This is because of the vehicle data sets and simulation program used. These features are discussed in section 2.1. Thereafter the vehicle data, required for trajectory calculations with ASCENT, will be shown in section 2.2. In section 2.3 the original data sets of the two mentioned vehicles are be presented.

2.1 Remarks to vehicle modelling

The influence of the angle of attack on the thrust and specific impulse is neglected in the propulsion model, which will be proposed in section 2.2. In literature, however, it is already shown that the angle of attack has a considerable influence on the thrust. In the article of Furniss and Walters (1990) it is stated, that it is essential to have a complete map of engine data (...) covering an appropriate range of altitude, Mach number and incidence (levels of intake pre-compression) to optimize an ascent trajectory of a space plane. In figure 2.1 the influence of $\alpha$ is shown.

In this figure the relation is shown between the ramjet net thrust and the vehicle incidence, for three Mach numbers at an altitude of 25 km. The vehicle incidence is the angle between the free flow and the forebody of the vehicle. The change of the vehicle incidence is equal to the change of the angle of attack. As a result of an increase of this vehicle incidence the incoming mass flow of the inlet increases (precompression). The increase of the air mass flow is the main cause of the increase of
thrust. The air mass flow is a performance parameter of the inlet.

![Graph showing Ramjet net thrust variation with vehicle incidence.](image)

Fig. 2.1 Ramjet net thrust variation with vehicle incidence (Furniss, 1990).

The thrust level, in his turn, can have much influence on the trajectory. In figure 2.2 two trajectories are shown, one calculated with $\alpha$-dependent propulsion data and one with $\alpha$-independent propulsion data (Furniss, 1990). The figure clearly shows the change of the $h-M$ figure. From these examples it can be seen that it is important to use full angle of attack/Mach number dependent propulsion data for trajectory calculations. It is recommended, therefore, to implement this in the trajectory simulation tool ASCENT.

![Graph showing two trajectory examples.](image)

Fig. 2.2 Two optimal trajectories; one with incidence dependent propulsion data and one with non incidence dependent data (Furniss, 1990).
The trim drag is another topic, which can be important w.r.t. the accuracy of the determination of the flight conditions. The aerodynamic data, which are used for simulation, are the lift and drag coefficients at untrimmed situations. When the space plane is trimmed along the derived trajectory, the angle of attack will change. Also the drag will change because the trim drag is added. Usually these increments in $\alpha$ and D are so small that they can be neglected during the trajectory determination. In the article of Powell, however, has already been stated that, for space planes, the drag losses due to aerodynamic trim can require a significant fraction of the total energy required to achieve orbit (Powell 1990). So when trim is considered the attitude can change but also the trajectory itself because of another L/D ratio. In this report it is showed that the performance of a space plane is very sensitive for trajectory variations. It is, therefore, necessary to study whether trim has to be taken into account in the trajectory optimization.

In section 2.3 a vehicle data set will be composed for which a trajectory will be calculated in chapter 3. The configuration of this vehicle is fixed throughout this report. The design of a space plane, however, should be done with the trajectory calculation as an integral part of the design process. In the article of Furniss and Walters has been stated, for example, that the optimum trajectory can only be derived when its impact on propulsion system sizing and mass, vehicle control requirements ..., wing aerodynamics, wing design, and fuselage shape have all been addressed ... . It must combine minimum propulsion mass, optimum fuselage and wing shapes, with minimum ascent propellant (Furniss, 1990). To calculate the impact of the trajectory on the vehicle design, however, is not possible yet because no design tool is available.

When this fact is considered with respect to the AEOLUS project, it can be seen that it is important to know whether this project concentrates on a certain given, fixed space plane design, or whether it concentrates on different space plane configurations. When variations in propulsion characteristics ($T$, $I_{sp}$), aerodynamic characteristics, mass and geometric data are studied, it can already be concluded, based on literature, that it will be necessary to develop a trajectory optimization tool, and a space plane design or (at least) sizing tool.

### 2.2 Vehicle data necessary for a trajectory simulation with ASCENT

To be able to generate a Sänger-like trajectory for a TSTO space plane, a vehicle has to be modelled. For this vehicle the data, which are needed for a simulation with the trajectory computation tool ASCENT, have to be available. These data include aerodynamic, mass, propulsion and geometry data.
The trajectory computation tool ASCENT simulates the space plane as a mass point (for all basic assumptions of ASCENT see appendix A). Such simulations are called 3 degrees-of-freedom simulations, because only the three translational motions of the vehicle are simulated. The rotational motions are not considered. All external forces are considered to act on this mass point. In this investigation the side force is neglected.

Aerodynamic properties

The aerodynamic properties of the vehicle are represented by the aerodynamic coefficients, mostly given as a function of the angle of attack \( \alpha \), and the Mach number \( M \). Since only the translational motions are considered (3 DOF), only the aerodynamic lift, and drag force are needed. The aerodynamic moments can be omitted. The aerodynamic forces can easily be determined according to:

\[
L = C_L q_{dyn} S_{ref} \\
D = C_D q_{dyn} S_{ref}
\]

where \( C_L \) and \( C_D \) are respectively the aerodynamic lift and drag coefficient, \( S_{ref} \) is the reference area of the vehicle, and \( q_{dyn} \) is the dynamic pressure, given by:

\[
q_{dyn} = \frac{1}{2} \rho V^2
\]

with \( \rho \) as the atmospheric density, and \( V \) as the velocity w.r.t. the rotating earth.

So the vehicle data needed to determine the aerodynamic forces on the vehicle are:

- \( C_L(M,\alpha) \), \( C_D(M,\alpha) \), which are respectively the lift coefficient, and the drag coefficient as a function of Mach and angle of attack.
- \( S_{ref} \), the reference area of the vehicle

Mass properties

As mentioned before, the mass is concentrated in one mass point. However, due to the fuel consumption, this mass is not constant. The change of fuel/propellant mass is given by the following differential equation:

\[
m_{hp} = \frac{T}{g_0 \rho_{\text{sp}}}
\]

where

- \( m_{hp} \) = fuel/propellant mass flow rate
- \( T \) = thrust
\[ g_0 = \text{gravitational acceleration at sea level} \]
\[ l_{sp} = \text{fuel/propellant specific impulse} \]

However, to calculate the mass of the vehicle also the take-off mass, or the total mass, has to be known. The take-off mass is the sum of the dry mass, the fuel mass and the payload mass. These mass values need to be known in order to check whether the vehicle does not violate its flight capabilities (by consuming more fuel than available for example).

So the vehicle data which are needed are:
- \( m_{dry}, m_{fuel}, m_{pay} \) which are respectively the dry mass, the fuel mass and the payload mass.
  The total mass is: \( m_{tot} = m_{dry} + m_{fuel} + m_{pay} \)
- \( m_f \), which is the fuel/propellant-mass flow rate

**Propulsion properties**

The properties which describe the propulsion system are the thrust \( T \) and the specific impulse \( l_{sp} \). In general, these properties will depend on the Mach number \( M \) (airbreathing propulsion), the altitude \( h \), the angle of attack \( \alpha \) and the equivalence ratio \( ER \). The \( ER \) is a measure for the fuel to air (in case of airbreathing propulsion) or oxidizer to fuel mass ratio (in case of propellants).

In the program ASCENT, however, \( T \) and \( l_{sp} \) are considered as a function of \( M \) and \( h \) (or \( q_{syn} \)) only. The effect of \( \alpha \) on \( T \) and \( l_{sp} \) is not accounted for, although this influence can be considerable (Furniss, 1990), as has already been stated in section 2.1. The thrust data are assumed to be the thrust at \( ER \) equal to one.

The thrust level is not controlled by changing the \( ER \), but through some theoretical throttle setting. In this respect the throttle setting \( \delta_r \) (0 \( \leq \delta_r \leq 1 \)) is nothing more than a scaling factor, with which the actual thrust can be computed according to

\[ T = \delta_r \cdot T_{max} \quad (2.3) \]

The actual thrust \( T \) is used in Eq. (2.1) to calculate the fuel mass flow.

So the propulsion data which are needed are:
- \( T(M, h/q_{syn}) \), which is the thrust at \( ER = 1 \)
- \( l_{sp}(M, h/q_{syn}) \), which is the specific impulse at \( ER = 1 \)
Geometric properties

Because the space plane is considered as a mass point, no inertia tensor is needed. This means that no information of the geometry is needed except the reference area $S_{ref}$ and the nose radius of curvature $R_n$. The reference area is used to calculate the aerodynamic forces (see also section 2.1). The nose radius of curvature is used to calculate the heat flux.

So the geometric data which are needed are:
- $S_{ref}$, which is the reference area
- $R_n$, which is the nose radius of curvature

Operational constraints

A space plane usually has some constraints on the flight possibilities during ascent and descent. These constraints have to be known in order to generate a feasible trajectory which can be flown by the vehicle in reality. The constraints can be divided into two categories, namely the control constraints and the trajectory constraints.

Control constraints

In ASCENT, the angle of attack $\alpha$, the throttle setting $\delta_T$, and the thrust vector angle $\epsilon_T$ (in plane of symmetry) are available as control variables. These variables are usually constrained; the maximum and minimum value depend on the vehicle considered. These values have to be known during the simulation. For the program ASCENT the values at least must be within the following ranges:

$$
\delta_T : 0 \leq \delta_T \leq 1
$$
$$
\alpha : -90^\circ \leq \alpha < 90^\circ
$$
$$
\epsilon_T : -90^\circ \leq \epsilon_T < 90^\circ
$$

Also the maximum allowable rate of change of these control variables has to be known.

Trajectory constraints

During flight, usually some physical parameters are not allowed to exceed certain maximum values. In ASCENT the dynamic pressure and the axial acceleration can be constrained.

So the operational constraints which are needed are:
- The maximum and minimum value of the control variables $\delta_T$, $\alpha$, and $\epsilon_T$, and their maximum allowable rate of change.
- The maximum dynamic pressure $q_{\text{dyn}}$ and the maximum axial acceleration $n_r$

The data required for trajectory simulation, given in this section, are shortly summarized in Table 2.1. In this table also the maximum size of the data arrays for the aerodynamic and propulsion data is given.

<table>
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<th>$C_L(M, \alpha)$, $C_D(M, \alpha)$ maximum of 15*15 data points per coefficient</th>
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<tr>
<td>Mass properties</td>
<td>$m$, $m_{\text{true}}$, $m_{\text{payl}}$</td>
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<tr>
<td>Propulsion properties</td>
<td>$l_{\text{sp}}(M, q_{\text{dyn}})$, $T_{\text{max}}(M, q_{\text{dyn}})$ or $l_{\text{sp}}(M, h)$, $T_{\text{max}}(M, h)$ maximum of 30*30 data points per variable</td>
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<td>Geometry</td>
<td>$S_{\text{ref}}$, $R_N$</td>
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<td>Operational constraints</td>
<td>- control: $\alpha$, $\delta_r$, $\epsilon_r$</td>
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<tr>
<td></td>
<td>- trajectory: $q_{\text{dyn}}$, $n_r$</td>
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Table 2.1 Model data needed for ASCENT trajectory simulation

2.3 The unscaled data of the Winged Cone Configuration and the HORUS 2B

Data of HORUS 2B

The data of the HORUS 2B are taken from MBB (1988a).

- The reference area is: $S = 110$ m²
- The propulsion properties are as follows (in vacuo):
  motor type is: rocket
  $T = 1055$ kN
  $l_{\text{sp}} = 467$ s
- The aerodynamic data:

In appendix B, two tables are listed with the values of the $C_L$ and $C_D$ coefficients of the HORUS 2B vehicle. The listed lift and drag coefficients are the untrimmed (clean) lift and drag coefficients as a function of $\alpha$ and Mach. The coefficient increments due to control surface deflections are assumed negligible. The assumption that these increments can be neglected with respect to the clean values should be validated.
The mass-properties of the HORUS 2B are as follows:

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<td>Residual Propellant</td>
<td>305</td>
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<tr>
<td>Payload (100%)</td>
<td>7000</td>
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<td>Reserve Propellant</td>
<td>330</td>
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<tr>
<td>Landing Mass</td>
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<tr>
<td>Propellants</td>
<td>30167</td>
</tr>
<tr>
<td>Gross Mass</td>
<td>56196</td>
</tr>
</tbody>
</table>

Table 2.2 Mass properties of the HORUS 2B (MBB 1988a).

Data of the Winged Cone Configuration (WCC)

The data of the Winged Cone Configuration are taken from Shaughnessy (1990).

- The mass of the WCC: \( m = 136,054 \text{ kg} \) (300,000 lb)
- The reference area is: \( S = 334.7 \text{ m}^2 \)
- The propulsion data are tabulated in appendix C (Korswagen, 1993). These propulsion data are taken from figures from Shaughnessy (1990) and are valid for an equivalence ratio equal to one.
- The aerodynamic data are tabulated in appendix D (Korswagen, 1993). These data are taken from figures from Shaughnessy (1990). Also for the WCC only the clean aerodynamic coefficients are used.

2.4 Scaling of the Winged Cone Configuration and the HORUS 2B

The WCC, being a SSTO-vehicle is not designed to carry a second stage like the HORUS 2B. Therefore, the size and thrust of this vehicle have to be scaled so that the total launch system is more like a Sänger-type TSTO vehicle.

Also the HORUS 2B has to be scaled. This is due to the fact that the HORUS 2B is a modified HORUS model, intended to be launched on the Ariane V, and used for re-entry simulations by
MBB. Therefore, the required amount of fuel is much less than for the HORUS launched on the Sänger configuration.

HORUS 2B

The mass properties of the HORUS 2B differ from the mass properties for other general second stages which are found in literature. From literature three examples are presented below:

- Staufenbiel (1990)
  total mass of second stage \( m_{tot} \): 100,000 kg
  burnout mass of second stage \( m_b \): 26,160 kg / 25,960 kg / 24,750 kg (3 simulations with different guidance algorithms)
  the ratio of burnout mass to total mass is: \( m_b / m_{tot} = 0.26 / 0.26 / 0.25 \)

- Sachs (1991)
  \( m_{tot} \) = 92,700 kg
  \( m_{ empt } \) = 24,200 kg
  \( m_{ empt } / m_{tot} = 0.26 \)
  (It is assumed that the empty mass is the burnout mass.)

- BMFT (1993)
  \( m_{tot} \) = 115,000 kg
  \( m_b \) = 31,000 kg
  \( m_b / m_{tot} = 0.27 \)

The second stage from Staufenbiel (1990) flies in a direct thrusted trajectory to a circular orbit at an altitude of 100 km. The staging conditions are at an altitude of 27.6 km with a velocity of 1951 m/s (w.r.t. rotating earth). The vehicle has to achieve an increase of the total specific energy of 28.5 \( 10^3 \) kJ/kg.

The second stage from Sachs is a HORUS type vehicle. It separates at an altitude of 37 km with a velocity of 2056 m/s (w.r.t. rotating earth). It has a direct thrusted flight to an altitude of 75 km, with a velocity of 7500 m/s. Thereafter it coasts to an altitude of 450 km. At \( h = 450 \) km the vehicle needs a velocity increment of about 115 m/s to circularize the orbit. The total specific energy increase during the direct, thrusted, trajectory is 28.9 \( 10^3 \) kJ/kg. This is of the same order as the increase required for the second stage of Staufenbiel (1990).
The second stage presented in BMFT (1993) is the latest HORUS version. The separation conditions are not exactly given. The HORUS can deliver 7500 kg into the space-station orbit, which is an orbit of about 400 km altitude. It can be expected therefore that the required increase of total specific energy is of the same order as the increase required for the HORUS version in Sachs (1991).

The ratio $m_b / m_{tot}$ of the HORUS 2B is: $26,029 / 56,196 = 0.46$. Based on the data from the other three second stage vehicles, it can be concluded that the mass of the propellants of the HORUS 2B is not realistic. The burnout mass, however ($m_b = 26029$ kg), is of the same order as for the three other second stages. Therefore, a mass ratio of 0.26 is used:

$$m_{tot} = (1/0.26) \times 26,029 \text{ kg} = 100,111 \text{ kg} = 100 \text{ Mg}$$

The burnout mass of the HORUS 2B is of the same order as the burnout mass of the other three vehicles, so it seems that the HORUS 2B is just like HORUS but with a reduced propellant loading. Therefore, the reference area is not scaled. It is assumed that the HORUS 2B is large enough to carry 74971 kg propellant.

**Winged Cone Configuration**

Based on the total mass of the second stage, the mass of the total launch system can be estimated. From literature (BMFT, 1993) it follows that, for Sänger, the ratio of the total mass of the second stage and the total mass of the total launch system (second stage plus first stage) is:

$$m_{sec} / m_{tot} = 115 \text{ Mg} / 410 \text{ Mg} = 0.28$$

If the same ratio is used for the reference launch system, then the mass will be:

$$m_{tot} = (1/0.28) \times 100 \text{ Mg} = 357,143 \text{ kg} = 360 \text{ Mg}$$

The mass of the WCC has to be scaled so that the total launch system mass is 360 Mg. This can be done in two ways. The mass of the WCC can be scaled to the mass of the first stage of the launch system ($m_{first} = 360 \text{ Mg} - 100 \text{ Mg} = 260 \text{ Mg}$), or it can be scaled to the mass of the total launch system ($m = 360 \text{ Mg}$). In both cases the reference area $S$ and the thrust $T$ have to be scaled in relation to the mass scaling factor.

In the first case the scaled WCC has to fly with the scaled HORUS 2B on his back until separation. In the second case the ascent trajectory until separation has to be flown by the scaled WCC only, because this vehicle already includes the mass of the second stage. After separation the trajectory
will be flown with the scaled HORUS 2B.

For both cases an aerodynamic data base is required. No aerodynamic database exists for the combination of the scaled WCC and the scaled HORUS 2B together. If the first option was chosen, it would be very difficult to determine a reliable aerodynamic database. When the WCC is scaled to the total launch system, the aerodynamic database should, of course, also be adjusted, but the original shape is not changed, only enlarged. When in this case the same aerodynamic coefficients are used, with only a scaled reference area, the aerodynamic database will be more reliable then in case these coefficients are used when the first option is chosen.

Secondly, the WCC is originally a SSTO. This means that it is a total launch system. It is, therefore, not designed to carry more weight than its own weight. By scaling the WCC to the mass of the first stage, the reference area and the thrust have to be scaled in relation with this mass scaling factor. With the HORUS 2B on top of it, the performance of this scaled vehicle will be much worse than for the original WCC.

For these two reasons the mass of the WCC is scaled to the total take off mass. The mass scaling factor for the WCC is now:

\[ f_m = 357,539 \text{ kg} / 136,054 \text{ kg} = 2.63 \]

Besides the scaling of the mass, also the reference area and the propulsion system have to be scaled. The aerodynamic coefficients are assumed not to change. The scaling factors which are used for these properties have to be related to the derived mass scaling factor. In this report it is assumed that the factors \( L/W \) and \( T/W \) of the WCC and the scaled WCC are the same. This means that it is assumed that the reference area \( S \) and the thrust increase linearly with the mass. Therefore, these parameters become:

\[ S = 2.63 \times 334.7 \text{ m}^2 = 880 \text{ m}^2 \]
\[ T = 2.63 \times T_o, \quad \text{where } T_o \text{ is the unscaled thrust} \]

The chosen scaling factors for the reference area and the thrust coefficient are of course not the only and/or the best choice that can be made. It is possible that other choices would be better. However, due to the lack of knowledge in this field, these simple assumptions are made.
3. Trajectory modelling

3.1 Modelling possibilities in the program ASCENT

The program ASCENT allows for a calculation of the ascent trajectory. This trajectory can be divided into several flight segments. During the ascent flight, the dynamic pressure is treated as a constraint. α is used as control variable to control the different flight segments (with a prescribed flight-path angle), while the throttle setting δT is used to satisfy the trajectory constraints.

In total, six control laws are available for the ascent, each of them based on a prescribed flight-path angle or flight-path angle rate, or a constant normal load factor. The flight-path angle or normal load is realized by the angle of attack α. The throttle setting is in principle always 100%. However, when a trajectory constraint is met, the throttle setting is adjusted, so that the trajectory constraints are not violated. In this way both control variables (α and δT) are always defined.

The six control laws result in a flight with a
- constant normal load factor n2
- constant flight-path angle γ
- constant flight-path angle γ at constant speed
- constant rate of change of γ
- constant rate of change of γ at constant speed
- γ as function of the altitude h

Using the first 5 control laws, it is possible to construct most of the airbreathing ascent profiles which can be found in literature (Korswagen, 1993, Schöttle, 1991, Nguyen, 1991). The take-off phase can be flown with a constant normal load factor (Schöttle, 1991). During this phase, γ rapidly increases as a result of the constant normal load. Then, γ decreases to a low value at which the remainder of the airbreathing ascent is flown. At M = 4.5 Sänger usually performs a cruise phase. The last control law is used for the rocket powered ascent of the second-stage vehicle or the second phase for a SSTO vehicle. This control law describes γ as a second-order function of h (Staufenbiel 1990).

However, during the ascent, constraints are imposed on both the trajectory and the control variables. These constraints will influence the trajectory. As trajectory constraints the dynamic
pressure \( q_{\text{dyn}} \) and the axial acceleration \( n_z \) can be considered. Also the control variables \( \alpha \) and the throttle setting can be constrained with respect to their maximum and minimum value (see also chapter 2 and appendix A).

For the descent of the first stage 5 control laws are available. These control laws result in a flight with a
- constant flight-path angle \( \gamma \)
- constant flight-path angle \( \gamma \) at constant speed
- constant rate of change of \( \gamma \)
- constant rate of change of \( \gamma \) at constant speed
- descent at constant dynamic pressure \( q_{\text{dyn}} \)

During descent the throttle setting is 0\%, except at the flight segments with a constant speed. The flight-path angle is realized by adjusting the angle of attack.

3.2 The modelled Sänger-like trajectory

The trajectory which is flown with the composed vehicle consists of 16 flight segments. The ascent trajectory until separation has 9 flight segments, the descent trajectory of the first stage has 7 flight segments and the ascent trajectory of the second stage has 1 flight segment. In this section first the trajectory flight segments are presented with their parameter values and trajectory and control constraints. After that, it is explained what is expected from these flight segments and why some flight segments and parameter values are chosen.

The flight segments are as follows:

ascent until separation:
1) constant normal load factor until \( \gamma = 25^\circ, n_z = 1.5 \)
2) constant rate of change of \( \gamma \) until \( \gamma = 1.5^\circ, \dot{\gamma} = -0.25^\circ/s \)
3) constant flight-path angle \( \gamma \) until \( M = 4.5, \gamma = 1.5^\circ \)
4) constant flight-path angle \( \gamma \) at constant speed until \( h = 25400 \text{ m}, \gamma = 1.5^\circ \)
5) constant rate of change of \( \gamma \) at constant speed until \( \gamma = 0^\circ, \dot{\gamma} = -0.05^\circ/s \)
6) constant flight-path angle \( \gamma \) at constant speed until \( t = 1400 \text{ s}, \gamma = 0^\circ \)
7) constant rate of change of \( \gamma \) until \( \gamma = 1.5^\circ, \dot{\gamma} = 0.02^\circ/s \)
8) constant flight-path angle \( \gamma \) until \( M = 6.8, \gamma = 1.5^\circ \)
9) constant normal load factor until $\gamma = 5^\circ$, $n_z = 1.5$

descent first stage:
1) constant rate of change of $\gamma$ until $\gamma = -0.25^\circ$, $\dot{\gamma} = -0.2^\circ$/s
2) constant flight-path angle $\gamma$ until $t = 350$ s, $\gamma = -0.25^\circ$
3) constant rate of change of $\gamma$ at constant speed until $\gamma = 0^\circ$, $\dot{\gamma} = 0.1^\circ$/s
4) constant flight-path angle $\gamma$ at constant speed until $t = 800$ s, $\gamma = 0^\circ$
5) descent at constant dynamic pressure $q_{dyn}$ until $t = 1566$ s
6) constant flight-path angle $\gamma$ until $t = 1692$ s
7) constant normal load factor until $h = 0$, $n_z = 1.05$

ascent second stage:
1) $\gamma$ as function of the altitude $h$. Final altitude is 120 km, $\gamma_{max}$ is $12^\circ$.

In the definition of the descent of the first stage, $t = 0$ at the time of separation.

During the ascent and descent, two trajectory-constraints are active:
1) $q_{dyn} = 50$ kPa
2) $n_z = 3 (g_0)$

Also 4 steering constraints are active:
1) $-1^\circ \leq \alpha \leq 12^\circ$
2) $-3^\circ$/s $\leq \dot{\alpha} \leq 3^\circ$/s
3) $0\% \leq \delta_T \leq 100\%$
4) $-10^\circ \leq \varepsilon_T \leq 10^\circ$

Hereafter, we will explain what is expected from these flight segments and why some flight segments and parameter values are chosen.

The constraint $q_{dyn}$ is chosen the same as for Sänger (Sachs, 1991). The maximum axial acceleration of $3 \ g_0$ is generally used for crew comfort (Schöttele, 1991, Nguyen, 1991). The constraints for $\alpha$ are given by the data base used (Shaughnessy, 1990). The maximum rate of change of $\alpha$ applies to a vehicle with the size and mass of a Boeing 747 (Torenbeek, 1988).

The take off with a normal load of 1.5 until a flight-path angle of $25^\circ$ is reached, is used based on
the trajectory in Korswagen (1993). When $\gamma = 1.5^\circ$ is reached, the space plane will climb with this angle until $M = 4.5$.

The cruise Mach number of 4.5 is chosen based on figure 3.1 (BMFT, 1993). Based on the same figure, a cruise altitude of 26 km is chosen. In reality a cruise flight is performed to reach the right latitude for separation. When the right latitude is almost reached the space plane turns due east. The modelled trajectory, however, takes place in one vertical plane, and the latitude is kept $0^\circ$. The cruise flight is not really needed in this case. It is only implemented to simulate the duration and the fuel consumption which can be expected for such a space plane when it would be launched from a site somewhere in Europe. In the Winged-Launcher Configuration study (WLC-study, ESTEC, 1991) the cruise flight takes about 6 minutes. To get about the same cruise flight time the end of the cruise flight is set at 1400 s. The cruise flight starts at about $t = 1000$ s.

![Fig. 3.1 The altitude-Mach figure of the Sänger trajectory (BMFT, 1993).](image)

From the above definition of flight segments, it can be seen that the climb with a constant flight-path angle at a constant speed, stops at an altitude of 25400 m. This seems to be strange because it has just been discussed that the cruise altitude should be 26 km. In the next flight segment, however, the cruise flight is not really started yet. In this flight segment the flight-path angle is brought back to zero. So during this flight segment the space plane will still climb, and therefore the segment has to be started before the altitude of 26 km is reached in order to perform a cruise flight at 26 km.
After the cruise flight the space plane has to accelerate and climb again, to reach the pull-up manoeuvre conditions. In figure 3.1 can be seen that during this phase the maximum dynamic pressure is not reached. In the altitude versus Mach number figure the trajectory makes a curve from cruise to pull up conditions. In order to get such a curve, the flight-path angle has to increase during this flight phase until a maximum value has been reached. Due to the increase in speed and in flight-path angle, a curve will develop. To get the right curve, the flight-path angle rate is set at 0.02 °/s, and the maximum flight-path angle at 1.5°. These values are chosen based on experience, which is gained during trajectory computation exercises with ASCENT.

When $M = 6.8$ is reached the pull-up manoeuvre is started. This pull-up manoeuvre is performed with a normal load of 1.5. When $\gamma = 5^\circ$ is reached, the separation conditions are reached. The value of 5° for $\gamma$ is chosen based on Staufenbiet (1990), who states that the flight-path angle at separation has to be at least 5°. Because it is not known what flight path angle at separation is the best value, the minimum required value is chosen.

After separation the flight-path angle will be lowered until it reaches a negative value of -0.25°. In this way, it is prevented that the trajectories of the first and second stage will intersect. The flight-path angle of the second stage, namely, will immediately increase after separation. After separation also the engines of the first stage are shut down. This is done based on information from the Winged Launcher Configuration study (ESTEC, 1991). When an altitude of about 30 km is reached, a cruise flight is performed, just like in figure 3.1. In the flight phase before the cruise flight, the flight-path angle is increased to zero degrees. During this flight phase the engines are restarted.

In the definition of the descent trajectory, it can be seen that the boundary of the second flight phase is defined in elapsed time after the moment of separation. A more logical boundary would be the altitude, because the next flight segments have to start at a certain altitude (cruise flight segment should start at $h = 30$ km). In ASCENT, however, variables like $h$, $M$, $V$, and $\alpha_{dys}$ can only be used as an upper boundary for flight segments. So when a flight segment starts at an altitude of 35 km, and has to stop at an altitude of 30 km, the altitude cannot be used as a stop criterion for that flight segment. The only exception is the flight-path angle. Therefore, the time is used as boundary (which is always an upper boundary). The determination of the value of such a time boundary is done by trial and error. Here a useful extension of the programm is identified.

After the cruise flight, the space plane descends along a constant dynamic pressure trajectory. In a study of MBB, it is stated that the best gliding range will be performed roughly at constant dynamic pressure (MBB 1988b; TN-4, "Terminal Area Energy Management"). At an altitude of about 3 km a
descent with a constant flight-path angle is started. The descent continues until an altitude of about 300 m is reached. Then a pull-up manoeuvre is performed, to lower the eventual landing speed. The normal load during this pull-up manoeuvre must not be too large, because otherwise the space plane will start to climb again. Therefore, a normal load of 1.05 is chosen.

During the second-stage trajectory, $\gamma$ is described as a second-order function of $h$ (Staufenbiel 1990). Important parameters for this flight segment are the initial $\gamma$, the maximum $\gamma$ during flight and the final altitude. The final altitude is set at 120 km. The initial $\gamma$ is a result of the ascent trajectory of the first and second stage combination, and is about $5^\circ$. The final velocity of the second stage can be controlled with the maximum flight path angle.

The selection of the maximum flight path angle during the ascent of the second stage is done by trial and error. $\gamma_{\text{max}}$ is varied with a step of 0.5$^\circ$. It appeared that the selection $\gamma_{\text{max}} = 12^\circ$ resulted in the best (closest to the required final values) final values, with $h = 119.85$ km, $V_f = 7359$ m/s, $\gamma = 0.008^\circ$. These values are considered accurate enough for orbit injection. Therefore this value for $\gamma_{\text{max}}$ is used.

The final altitude $h = 120$ km is not the same as the final altitude of Sänger ($h = 400$ km). The total specific energy of this orbit with $h = 120$ km, however, does not differ much with the total specific energy of the orbit of the HORUS from Sachs (1991), after its thrusted trajectory ($E = -30.7 \ 10^3$ kJ/kg, resp. $E = -30.0 \ 10^3$ kJ/kg). So, by using a circular orbit at $h = 120$ km, a good impression can be gained of the capacity of the second stage. The additional propellant mass which is required to get in an circular orbit of, for example, 400 km is about 900 kg, when a Hohman trajectory is used. This is about 1 to 2% of the total propellant mass of the second stage.

At the start of the simulation of the ascent of the reference space plane, the state vector needs to be given an initial value. The initial velocity $V_o$ and angle of attack $\alpha_o$ are such, that the generated lift is at least equal to the weight. For the first flight segment the normal load is 1.5, so the lift has to be 1.5 times the weight. Using the maximum available $\alpha$ as the initial value ($\alpha_o = 12^\circ$) results in a required initial velocity of $V_o = 170$ m/s. This take-off velocity corresponds with the initial condition presented by Lu (1991). The flight-path angle at take-off is assumed to be zero ($\gamma_o = 0^\circ$) and the heading is due East ($\chi_o = 90^\circ$) to make maximum use of the rotational motion of the Earth.

Take-off is assumed to take place from sea-level ($h = 0$ m) at the equator ($\delta_o = 0^\circ$). The longitude is arbitrarily set equal to zero ($\tau_o = 0^\circ$). The initial throttle setting is maximal ($\delta_T = 100\%$) for maximum acceleration. The initial state vector and steering variables are summarized below. The initial state
vector is

\[ V_0 = 170 \text{ m/s} \]
\[ \gamma_0 = 0^\circ \]
\[ \chi_0 = 90^\circ \]
\[ m_0 = 360,000 \text{ kg} \]
\[ \tau_0 = 0^\circ \]
\[ \delta_0 = 0^\circ \]
\[ h_0 = 0 \text{ m} \]

and the initial steering variables are

\[ \alpha_0 = 12^\circ \]
\[ \delta_{\tau_0} = 100\% \]
4. The trajectory results

In this Chapter, the trajectory, resulting from the flight segments presented in Chapter 3, will be discussed. In this discussion reference is made to the figures found at the end of this chapter. The discussion is concentrated on the parameters of the ascent until separation and the descent of the first stage, because these phases are most important with respect to airbreathing propulsion.

In section 4.1 the trajectory results itself are shortly presented. It is verified whether the results are as we could expect, according to the modelled flight segments in chapter 3. In section 4.2 these results are compared with the trajectory of Sänger (BMFT, 1993) and the WLC trajectory (ESTEC, 1991).

4.1 General description of the trajectory

In figure 4.1 the altitude versus the velocity is given for the ascent of the total vehicle, the descent of the first stage, and the ascent of the second stage.

As can be seen the staging conditions are reached at an altitude of 32.2 km and a velocity of 2083.5 m/s. At the final altitude of 120 km the velocity has reached the value of 7359 m/s (w.r.t. rotating earth). The circular velocity for an altitude of 120 km is 7358.2 m/s, so the orbit insertion can be considered successful.

The final mass of the second stage is 26.3 ton. The burnout mass of the second stage is about 26 ton. This means that about 300 kg propellant is left of the propellant mass planned for the ascent after separation. So instead of using this mass for fuel, it could be used for payload. In that way the payload mass which is brought into orbit becomes 7.3 ton. This amount of propellant, however, can also be used to bring the second stage in an orbit with a higher altitude.

In figure 4.2 the $h-V$ figure of the ascent and descent trajectory of the first stage is given in more detail. In this figure, it can be seen that a great part of the ascent trajectory runs along the maximum dynamic pressure of 50 kPa, being one of the trajectory constraints. So this shows that the maximum dynamic pressure is very important for the trajectory.

The cruise flight is performed at $M = 4.5$ and $h = 26$ km. After the cruise-flight segment the space
plane accelerates and climbs at the same time. In the $h-V$ figure this results in a smooth curve, just as planned (see chapter 3).

During the pull-up manoeuvre, the acceleration is almost reduced to zero. After separation the space plane starts to decelerate and $\gamma$ is reduced to $0^\circ$. The altitude, however, still increases up to almost 35 km, because $\gamma$ is not instantaneously $0^\circ$. Thereafter, both altitude and velocity decrease.

The cruise flight during descent is performed at $M = 4.5$ and $h = 30.4$ km, which is about 400 m above the planned altitude. This little inaccuracy is due to the applied control implemented in the tool ASCENT.

After the cruise flight segment, the space plane descends along a constant dynamic pressure trajectory of 14.8 kPa. At an altitude of about 3 km and a velocity of 180 m/s, the space plane leaves the dynamic pressure trajectory ($\gamma$ becomes constant). Then, at an altitude of 250 m and a velocity of 150 m/s the last pull-up manoeuvre is started to lower the sink speed. Finally, the space plane lands with a velocity of 131 m/s and a sink speed of 6 m/s.

In figure 4.3 the altitude of the first and second stage, just after separation, is shown as a function of time. It can be seen that after separation the first and second stage are never at the same altitude at the same time. Hence, a collision between the two stages will not occur. We have to be aware, however, of the fact that the rotations of the vehicles are not considered here. So the fact that the trajectories of both stages do not interfere with each other, does not guarantee that the separation will be successful in this way.

During cruise flight the altitude and velocity are planned to remain constant (see chapter 3). In figure 4.4, it can be seen, however, that the cruise altitude during descent increases somewhat (~300 m). The reason for this, is that in these cruise flights, the flight-path angle is not exactly zero. Therefore a small climb speed or sink speed remains. The deviation of $\gamma$ from zero, is due to an inaccuracy of the control laws which are implemented in ASCENT. The effect of these deviation on the altitude, however, is rather small.

The cruise flight in ascent starts once the cruise altitude is reached, at $t = 958$ s. Because the end of the cruise flight was defined at $t = 1400$ s, the cruise flights takes 442 s, which is about 7 minutes. With a speed of 1337 m/s the space plane covers about 591 km during this flight segment.
In figure 4.5 the velocity is given as a function of time. In this figure it can be seen that the velocity stays very well constant during the two cruise flights and during the climb to the cruise altitude. The Mach number decreases somewhat during this climb. The decrease of the Mach number, however, is very small, as can be seen in figure 4.6. The reason for the decrease of the Mach number is the increase of the velocity of sound due to the increasing temperature with altitude. As a result the cruise flight is performed at $M = 4.47$ instead of at $M = 4.5$.

In figure 4.7 the flight-path angle is given as a function of time. In most flight segments, the flight path angle, or flight path angle rate, is prescribed to be constant. The $\gamma$-t figure, therefore, consists of straight lines for almost the whole trajectory. The only exception is the descent along a constant dynamic pressure trajectory, which lies between $t = 2350$ s and $t = 3116$ s. Because of these straight lines, all flight segments can be clearly recognized. Together with the imposed trajectory constraints, the prescribed values of $\gamma$ and $\dot{\gamma}$ determine the trajectory to a great extent.

Figure 4.9 shows the mass of the space plane as a function of time. In this figure the cruise flights can be recognized through the lower fuel mass rate. At the separation point, the mass suddenly decreases 100 ton. This is because the second stage (=100 ton) has separated. This separation has been assumed to occur instantaneously, so a discontinuity appears in the mass curve. In table 4.1 the fuel/propellant mass consumption per flight segment is given, and the total amount of fuel/propellant mass. This tables shows that during the climb along the constant dynamic pressure takes the largest amount of fuel.

<table>
<thead>
<tr>
<th>flight segment</th>
<th>fuel/propellant mass (ton)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1) $n_r = 1.5$</td>
<td>2.8</td>
</tr>
<tr>
<td>2) $\dot{\gamma}/dt = -0.25^o/s$</td>
<td>9.0</td>
</tr>
<tr>
<td>3) $\gamma = 1.5^o$</td>
<td>44.9</td>
</tr>
<tr>
<td>4) climb at $M = 4.5$</td>
<td>2.3</td>
</tr>
<tr>
<td>5) $\dot{\gamma}/dt = -0.05^o/s$</td>
<td>0.8</td>
</tr>
<tr>
<td>6) cruise</td>
<td>11.8</td>
</tr>
<tr>
<td>7) $\dot{\gamma}/dt = 0.02^o/s$</td>
<td>6.5</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>flight segment</th>
<th>fuel/propellant mass (ton)</th>
</tr>
</thead>
<tbody>
<tr>
<td>8) $\gamma = 1.5^o$</td>
<td>4.2</td>
</tr>
<tr>
<td>9) $n_r = 1.5$</td>
<td>1.2</td>
</tr>
<tr>
<td>total first phase</td>
<td>83.5</td>
</tr>
<tr>
<td>descent (cruise)</td>
<td>7.0</td>
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<tr>
<td>Total 1st stage</td>
<td>90.5</td>
</tr>
<tr>
<td>ascent 2nd stage</td>
<td>73.7</td>
</tr>
<tr>
<td>total</td>
<td>164.2</td>
</tr>
</tbody>
</table>

Table 4.1 Fuel/propellant mass consumption per flight segment

The controls used to generate the trajectory are the thrust and the angle of attack. The thrust is throttled to meet the trajectory constraints $q_{\text{dyn}}$ and $n_r$. The angle of attack is used to control the
flight path angle.

In figure 4.10 it can be seen that the engines are throttled back almost immediately after take off, when the maximum dynamic pressure is reached. During this climb along a constant $q_{\text{dyn}}$, the throttle setting is lowered to 60-40%. During cruise the throttle setting is even lowered to about 35% in both ascent and descent. It is questionable whether such a level of throttle setting is acceptable. In section 4.2 this will be discussed in more detail.

Until $t = 400$ s the throttle decreases. After that the throttle increases again until the climb at constant speed to the cruise altitude is started. This increase in throttle setting is a result of the fact that after $t = 400$ s, the maximum available thrust decreases (figure 4.11). The required thrust decreases slower during climb, so the throttle has to increase.

At the transition of an accelerated flight segment to a non-accelerated flight segment, or vice versa, the throttle setting changes quickly. Because the rate of change of the throttle is not constrained, these changes have become discontinuities. In reality, these changes cannot be discontinue, so the rate of change of the throttle setting should be constrained. It is not known, however, what the limit should be.

After the cruise flight the throttle setting is 100% until separation. This could be expected because after the cruise flight the dynamic pressure constraint is not met any more (fig 4.2). During descent the engines are only used during cruise.

Through the angle of attack, the flight-path angle is controlled. Due to the discontinuous transitions of the rate of change of $\gamma$, the angle of attack has to change very quickly. As a result, sharp peaks appear at most flight segment transitions (figure 4.12). Although the rate of change of $\alpha$ is limited to maximal $\pm 3^\circ$/s, better control laws, which can be expected in reality, are needed to reduce these peaks.

The trajectory constraint $n_z$ is not met during the ascent and descent of the first stage (see fig. 4.13). The available thrust is too low to violate this constraint. The constraint will be met, however, during the ascent of the second stage. Because the ascent of the second stage is of less interest in this report, this will not be shown here. Since this constraint is not met during the airbreathing phases, it must not be seen as a design requirement for the first stage. From figure 4.13 it can be seen that such a design criterium can be taken much lower.
In the curve of the normal load the pull-up manoeuvres of 1.5 g are very well recognizable. For the remaining trajectory the normal load lies mainly between 0.8 and 1.

The range which is covered during ascent and descent is shown in figure 4.14. The range covered during descent is reckoned from separation. Both ascent and descent range are about 1600 km. The range of the descent trajectory is somewhat larger than the range of the ascent trajectory. It was, however, not the intention to fly exactly a certain range from launch side to separation point and back. The ranges presented in figure 4.14 are a result of the chosen flight path angles and cruise flight durations. The cruise flights were implemented to get a realistic picture of the flight duration and required fuel mass.

In figure 4.20 two lines are given for the heat flux, both calculated for a radius of 0.1 m. Both heat fluxes are proportional to $r^{0.4}v^3$. In Appendix A the exact formulas are given. As it can be seen in figure 4.20, the two models give different results. It is not exactly known why this difference exists, but it can be assumed that the heat flux will lie in between these values (see Appendix A). Because of the large values of the heat flux in figure 4.20, it is recommended to investigate the maximum allowable heat flux for a TSTO space plane. When the heat flux has to be considered as a trajectory constraint during the ascent until separation, the resulting trajectory can change considerably.

4.2 Evaluation of the trajectory results

In this section, the Sänger-like trajectory of the scaled WCC is compared with the trajectory of Sänger (BMFT, 1993) and the WLC trajectory (ESTEC, 1991). The WLC trajectory study has been added for comparison because for this trajectory more detailed information is available compared to the trajectory of Sänger. The WLC trajectory shows much resemblance with the Sänger trajectory, and therefore can be very well used for comparison.

When figure 4.2 ($h\cdot V$ fig.) is compared with figure 4.15, it can be seen that the two figures are much alike, apart from the take off phase (until $M = 1.5$). This can be expected, of course, because the trajectory of the scaled WCC is composed with the trajectory of Sänger as a reference, apart from the take-off phase and the descent (chapter 3). Other variables, however, like $\alpha$, $T$, and $\gamma$ can still be different for the two trajectories. Unfortunately these variables are not known for the Sänger trajectory. One variable, however, which can be compared, is the fuel mass.

Based on the Sänger data the total amount of fuel for the scaled Winged Cone Configuration
(WCC) was set on 105.5 ton. For the ascent and descent trajectory, however, only 90.5 ton fuel is used. So 15 ton fuel is not used, 14% of the total carried amount of fuel.

We can think of several reasons why the fuel consumption of the scaled WCC differs from the consumption of Sänger. The reasons can be divided into two categories. On one side it can be the result of the fact that the flown trajectory is different. Secondly the difference could be the result of a deviant model of the space plane with respect to the Sänger space plane model. Both options are possible because from figure 4.2 and 4.15 can already be seen that there are still differences between the two trajectories of Sänger and the scaled WCC, and secondly one can be sure that the scaled WCC model is not exactly the same as the Sänger vehicle.

With respect to the modelled trajectory, the following items could be a reason for the difference in fuel mass consumption:

- The modelled return flight is not the same as the return flight of Sänger. During this return flight the scaled WCC uses 7 ton fuel. It is not known how much fuel Sänger uses for its return flight but the WLC uses 9.8 ton. This is a small difference with the 7 ton of the scaled WCC. Therefore it does not seem obvious that this is the main reason.

- In trajectories of space planes which start at a launch site from Europe, a turn flight segment is included (see also ESTEC, 1991). This turn is not modelled in the trajectory of the scaled WCC. It is however difficult to say how much extra fuel mass this turn will cost.

- For the first 10 km the trajectories of the scaled WCC and Sänger are different. It could be very well possible that this part of the trajectory causes a different fuel mass consumption for the two vehicles. The trajectory of Sänger will probably take more time and therefore also more fuel.

With respect to the space-plane model used, the following can be said:

- The take-off mass of the scaled WCC is smaller than the take-off mass of Sänger. It could be possible that the fuel mass to take-off mass ratio decreases with decreasing take-off mass. When the WLC-study is considered, it can be seen that the take-off mass is about 86 ton smaller than $m_{te}$ of Sänger. The ratio is also smaller: 0.23 instead of 0.29. When a linear relation is assumed between the take-off mass and the $m_t/m_{te}$ ratio, then, based on the data of Sänger and the WLC-study, the $m_t/m_{te}$ for the scaled WCC should be 0.255. This is close to the obtained value of 90.5/360 = 0.251.
- It is not known, of course, whether the aerodynamic data of the scaled WCC are much alike the aerodynamic data of Sänger. When these data would differ much, a difference in fuel mass consumption can be expected.

- The engine data used for the WCC can differ from the data from Sänger. When figure 4.10 is studied it is striking how much the available thrust is throttled. During climb throttle setting values of even less than 40% occur. This makes one suspect that the propulsion system is far too strong compared to other TSTO vehicles. From Veraar (1993) is known that at $M = 6.8$ the thrust for Sänger is 800 kN. The thrust of the scaled WCC is 1451 kN, at the same flight conditions, which is much higher than 800 kN.

- The propulsion data used for the WCC are at an ER equal to 1. The equivalence ratio of the Sänger can increase to 2 during ascent. This can cause a significant difference in required fuel mass.

From this enumeration of reasons, it can be seen that it is difficult to point out exactly what causes the difference in fuel mass consumption. However, because the $\dot{m}_i/\dot{m}_{\text{a}}$ ratio does not deviate much from the ratios of Sänger and the WLC-study, it can be concluded that the fuel consumption is at least not unrealistic.

Still, the issue with the propulsion system needs further investigation. From the figure of the throttle setting and from the fact that at $M = 6.8$ the thrust has to be 800 kN ($ER = 2$), it can be concluded that the propulsion system is far too strong. From figure 4.17 can be seen, however, that the variation of the thrust with the Mach number of the WCC does not deviate much with the variation of thrust with the Mach number for a Sänger-like propulsion system as calculated at TNO (Veraar, 1993). In this figure the thrust is given at 5 points along a trajectory which was selected by TNO (Veraar, 1993). These points are given in table 4.2. The thrust calculated at TNO is a thrust at $ER = 1$ with limited expansion. This thrust is compared with the scaled thrust of the scaled WCC. The thrust of the WCC is not scaled with a factor 2.63 this time, as was the case for the trajectory generated in chapter 3. In figure 4.17 the thrust of WCC is scaled, so that the thrust of TNO and the scaled thrust of the WCC are equal at $M = 6.8$ ($T = 660$ kN). The variation of the thrust level for both models is almost the same.
<table>
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<tr>
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<td>50</td>
<td>29.5</td>
<td>50</td>
<td>33</td>
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</tbody>
</table>

Table 4.2: Trajectory points used for thrust calculation in figure 4.17

From the fact that the thrust calculated by Veraar (1993) and the thrust from the WCC model are almost the same at the selected points, the confidence in the propulsion model of the WCC increases. When the maximum thrust is scaled down so that the maximum thrust at $M = 6.8$ is equal to the maximum thrust of Sänger, the space plane model of this study will be more realistic. The maximum thrust at $M = 6.8$ of Sänger is 800 kN. This is about 55% of the maximum thrust of the scaled WCC. When the maximum throttle setting is set at 55%, the propulsion system should be more like Sänger.

When the ascent trajectory, which is modelled in chapter 3, is flown with a maximum throttle setting of 55%, the fuel mass consumption indeed increases to 103.7 ton. When it is considered that the total amount of fuel is 105.5 ton, and that about 8 ton fuel is needed for the descent trajectory, it can be concluded that the amount of fuel of 103.7 ton for the ascent trajectory only is rather large. However, when the trajectory is improved with respect to the fuel consumption, with the help of knowledge which will be presented in the next chapter, the required amount of fuel is 94.4 ton. Obviously the fuel mass consumption of the scaled WCC, using the new scaled propulsion model, is more like the Sänger fuel-mass consumption. It is therefore advised to use this last propulsion model for the generation of a TSTO trajectory. The influence of an ER larger than 1 is still not accounted for, however, by lowering the thrust to 55%. When a variable ER would be used, it can be expected that the amount of fuel will increase more.

In figure 4.18 the two trajectories, generated with a maximum thrust of 55% of the original maximum thrust, are shown in a $h$-$V$ figure. The most striking of this figure is the first part of the ascent trajectory. Due to the lower thrust the space plane does not immediately accelerate to the maximum dynamic pressure, but it first climbs at a constant speed, after which it accelerates to the maximum dynamic pressure at an almost constant altitude. The trajectory with the smallest amount of fuel required, starts to accelerate at a higher altitude.
The first part of these trajectories shows much more resemblance with the first part of the Sänger trajectory than the original trajectory. This first part of the trajectory was already marked as a possible reason for the difference in fuel-mass consumption. Evidently, this part of the trajectory is closely related to the applied propulsion system.

Apart from the altitude and the Mach number (or velocity), also the angle of attack and the flight time are important parameters, especially when one is interested in loads on certain subsystems of a space plane, like an inlet, flaps or elevons. It would be useful, therefore, to compare the results of the trajectory with Sänger results, in order to see whether the results are realistic. These variables, however, are not known for the Sänger trajectory. Therefore, these variables are compared with the results of the WLC-study.

The ascent trajectory of the scaled WCC takes about 26 minutes. The ascent trajectory of the WLC takes about 29 minutes. These values are of the same order of magnitude. Because the trajectory of the WLC-study is also slightly different (different $q_{dyn}$) this result is satisfactory. The difference in time, needed for the descent trajectory, is larger. The descent in the WLC-study needs about 36 minutes and the descent of the scaled WCC needs about 29 minutes. This difference, however, is less important with respect to the loads because the loads are the largest during the ascent trajectory. This can be seen from the figures 4.19 and 4.20. In these figures the dynamic pressure and the heat flux are given as a function of time respectively. Both figures clearly show that these loads are strongest during the ascent phase.

The range of the trajectory is closely related to the flight time. The range of the trajectory of the scaled WCC is about 1600 km (see section 4.1). The trajectory from the WLC study (ESTEC, 1991) has a range of 2282 km. The space plane takes off from Istress and goes to an orbit with an inclination of 28.5°. The range between Istress and a latitude of 28.5° is 1670 km (ESTEC, 1991). The range of the TSTO vehicle is much larger, however, because of the turn east after the cruise flight.

In figure 4.12 the angle of attack is shown (see also section 4.1). During ascent this variable varies from 3.5° at the start of the first climb up to about 6.5° during the cruise flight. The angle of attack of the WLC-study is known for the ascent only. During this ascent $\alpha$ varies from 2.5° at the start of the climb up to 6.5° during the turn manoeuvre. The angle of attack during cruise is about 5.3°, but the cruise flight of the WLC-study is performed at an altitude of 24 km, so a lower angle of attack than in the cruise flight of the scaled WCC can be expected. Therefore, also the angle-of-attack results are realistic.
From this evaluation it can be concluded that the trajectory results which are generated are realistic for a TSTO vehicle, while comparing with results of Sänger and the WLC-study. The propulsion system, however, can better be replaced by a propulsion system with a lower thrust (approx. 55% of the originally used value).
Fig. 4.18 Two trajectories flown with a maximum thrust value of 55% of the old maximum thrust value.

Fig. 4.19 The dynamic pressure as a function of time for both ascend and descent of the first stage.

Fig. 4.20 The heat flux as a function of time for both ascent and descent of the first stage, calculated for a nose radius of curvature of 0.1 m.

Fig. 4.17 The thrust of the scaled WCC and the thrust calculated by TNO, along the same trajectory (see also Verhaar, 1993).
5. Sensitivity analysis with the help of the Taguchi Method

In the previous chapters, a Sänger-like trajectory has been generated. By flying along this trajectory, our Sänger-like space plane, is able to deliver about 7.3 ton in a circular orbit at 120 km. However, it is interesting and important to know whether and how this performance can be improved.

To investigate this subject one can select several important trajectory parameters and vary these parameters in order to study their influence on the performance. The Taguchi method of experimental design is a very welcome and useful method to solve this problem (Taguchi, 1987), since it limits the required amount of computer experiments.

In the next section a short explanation of the Taguchi method and its use will be given. In the second section of this chapter, the set up of the computer experiments, used for the sensitivity study, are given. The results of these experiments are analyzed with the help of a variance analysis in section 5.3. With these analysis results the Taguchi method make it possible to develop some polynomials which describe the relation of the space plane performance and the selected trajectory parameters. These polynomials, and their confidence limits, are derived in section 5.4. In section 5.5. the optimal values for the trajectory parameters are determined, with the help of the derived polynomials.

At the end of this chapter, it will be clear which parameters influence the payload mass and fuel-propellant mass, and how much influence they have. It is a very interesting question then, whether it can be understood why these parameters appear to be important. This question will be dealt with in chapter 6.

5.1 The Taguchi Method

Assume one is interested in a certain output of a process, like for example the delivered payload of a space plane. This output, or objective, will be depending on several parameters. Now assume that one wants to know what combination of parameter values gives the best value for the objective. When the relation between the parameters and the objective is known, this combination can mostly be obtained rather easily. When this relation is not known, however, or very complicated, then it is needed to do experiments. One option to do these experiments is to vary all the parameters over two or more levels, and to try all combinations. However, when the number of parameters and
levels grows, the total amount of combinations grows considerably. The Taguchi method makes it possible to obtain the same information from a far smaller amount of experiments, since the Taguchi method is a method which changes the parameters in an 'all-at-the-same-time' approach rather than the traditional 'one-variable-per-time' trade study approach.

The advantage of changing more parameters at the same time is that it becomes possible to find the main effect of a parameter. This means that, when the influence of A1 and A2 on the experimental values holds consistently even if the conditions of the other factors change, this influence will appear great, but when the effect of the difference between A1 and A2 reverses or changes greatly accompanying changes in the conditions of the other factors, the effect will appear small (Taguchi, 1987). In this quotation, A1 and A2 are two levels of one parameter.

The set up of the (computer) experiments can be done with the help of orthogonal arrays, which are derived by Taguchi (1987). In Appendix E will shortly be explained how these arrays look like. When these experiments are performed, the result will be a data set with varying values of the objective. The variations of the objective are caused, of course, by the parameters which are varied, but can also be caused by variations in other sources (like interactions between two parameters) which have not been monitored. It is, therefore, important to know what part of the variation of the objective is caused by the parameters of interest. Only when this question is answered the relation between the selected parameters and the objective can be determined. In essence, three main questions have to be answered (Taguchi, 1987):

1) To what degree can it be said that the objective is influenced by the variation of the selected parameters?

2) What are the curves which represent the relationship of the objective with the parameters, and what are the confidence limits of these curves?

3) What are the parameter values that result in the best value of the objective?

These questions can be answered with the help of a variance analysis (see section 5.3).
5.2 The set up of the computer experiments

In the sensitivity analysis only the ascent of the first and second stage is considered, and not the
descent of these stages, because of the complexity of the problem. When the trajectory parameters
and the trajectory constraints are considered, fifteen parameters can be selected for the sensitivity
study. The trajectory constraint $n_x$, however, is not selected because this constraint is not met
during the first stage trajectory, which is the trajectory of most interest. Furthermore, this trajectory
constraint will probably not change in practice, because it is used for crew comfort. Also the rate of
change of $\gamma$ of flight segment 5 (see section 3.2) is not varied. This parameter is expected to have
very little influence on the performance of the space plane. So now thirteen parameters have been
selected for the sensitivity study. The idea is to vary all these parameters $+10\%$ and $-10\%$ with
respect to the reference value of the earlier derived trajectory.

The selected parameters and their reference values are:

1) The normal load factor at take off, $n_{t} = 1.5$
2) The maximum flight-path angle during take off, $\gamma_{\text{max}} = 25^\circ$
3) The change of flight-path angle after normal load segment, $d\gamma/dt = -0.25^\circ/s$
4) The flight-path angle during first climb, $\gamma_{cl} = 1.5^\circ$
5) The maximum dynamic pressure, $q_{\text{dyn}} = 50$ kPa
6) The cruise Mach number, $M_{c} = 4.5$
7) The cruise altitude, $h_{c} = 26$ km
8) The change of flight-path angle after cruise, $d\gamma/dt = 0.02^\circ/s$
9) The flight-path angle during second climb, $\gamma_{cl} = 1.5^\circ$
10) The Mach number at the start of the pull up manoeuvre, $M_{\text{pull}} = 6.8$
11) The normal load factor at the start of the pull up manoeuvre, $n_{\text{max}} = 1.5$
12) The flight-path angle at the end of the pull up manoeuvre, $\gamma_{\text{pull}} = 5^\circ$
13) The maximum flight-path angle during ascent second stage, $\gamma_{\text{max}} = 11^\circ$

For setting up the computer experiments, the orthogonal Taguchi matrix $L_{36}$ can be used to
determine the level combination for each experiment (see table 5.1).

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A flight-path angle of $11^\circ$ has been selected, while in chapter 3, $\gamma = 12^\circ$. This is due to an error. Although a slightly
different second-stage reference trajectory results, this does not have much consequences for the sensitivity analysis,
because the interest is mainly focused on the sensitivity of the performance of a space plane to the selected trajectory
parameters. The originally trajectory and the now proposed trajectory are so much alike, that it can be expected that
the results of the investigation will not differ in essence.
Table 5.1 The orthogonal matrix L36 (Taguchi, 1987)

For the sensitivity analysis in this chapter, the original propulsion model of the scaled WCC is used. In chapter 4 is stated that the propulsion model would be more realistic when the maximum thrust is decreased to 55% of the maximum thrust of the original model. Nevertheless, the more powerful propulsion system is used, because with this model a wider range of possible trajectories can be flown. As a result, the risk that certain trajectories cannot be flown is minimized. When several trajectories would not be fulfilled, namely, the sensitivity analysis could not be performed correctly.

In order to compare the results of these simulations, two assumptions have been made. First of all, it is assumed that, if the range during cruise is kept constant, the first stage covers the same range from take off to separation for every simulation. The range should be the same for all simulations to compare the performances. With the tool ASCENT, however, it would be very time consuming to adapt the cruise flight segment in such a way, so that this requirement is fulfilled. Because a great part of the range is covered during the cruise flight, the cruise flight range is kept constant for all 36 simulations.
Secondly, all simulations should have the same final conditions, i.e. the same final altitude ($h_f$), velocity ($V_f$), flight-path angle ($\gamma_f$), and inclination. It is, however, very difficult to get the final conditions of all 36 simulations at the same final values with the simulation tool ASCENT. For the second stage one flight segment can be selected, of which the maximum flight-path angle can be changed. The final altitude, at which the simulation stops, can be selected. The velocity which is reached, however, will differ more or less from the required orbital velocity as a function of the selected $\gamma_{max}$. Therefore the velocity reached, is corrected after the calculations of ASCENT. The required propellant mass for these corrections is calculated with the help of the Tsiolkovsky equation:

$$\Delta V = I_{sp}g_0\ln\left(\frac{M_f}{M_c}\right) \Rightarrow M_c = \frac{1}{e^{(\alpha V \omega_s)}} \times M_f$$

(5.1)

where

- $V_f$ = final velocity after ASCENT calculations
- $V_c$ = required circular velocity ($V_c = 7358.2$ m/s)
- $\Delta V = \text{abs}(V_c - V_f)$
- $M_f$ = final vehicle mass after ASCENT calculations
- $M_c$ = final vehicle mass after correction
- $g_0$ = gravitational acceleration at sea level, $g_0 = 9.798$ m/s²
- $I_{sp}$ = specific impulse of rocket motor, $I_{sp} = 467$ s

The resulting payload can be calculated as follows. The landing mass of the second stage (mass without propellants) is 26029 kg. The available propellant is 73971 kg. When more propellant is needed for the simulation the extra amount of propellant mass will be subtracted from the payload mass. So the payload mass will be:

$$m_{payload} = 7000 - (26029 - M_c)$$

(5.2)

In principle, the 36 experiments, according to the orthogonal array L36 (table 5.2), can be performed now. It appeared not possible to simulate all these 36 experiments, however, when all parameters are varied $\pm 10\%$. This is due to several reasons:

- When the space plane starts to climb from the highest cruise altitude ($h_c = 28.6$ km) with the highest change of flight-path angle ($\frac{dy}{dt} = 0.022$°/s), to the highest climb flight-path angle ($\gamma = 1.65$°), it reaches very low dynamic pressures. It is not capable to fulfil its mission any more. This problem also occurred when not all of the above mentioned parameters where at their highest level.
- At the highest pull-up Mach number ($M_{pull} = 7.48$) the power of the propulsion system is very low. The space plane is not capable to perform the pull-up manoeuvre any more.

- In preliminary calculations it appeared that the delivered payload and the final conditions of the second stage are very sensitive to the maximum flight-path angle during the ascent of the second stage. For example:
  
  $\gamma = 10.5^\circ$ ; $V_f = 8019$ m/s, $M_f = 22902$ kg
  $\gamma = 11^\circ$ ; $V_f = 7359$ m/s, $M_f = 26312$ kg
  $\gamma = 11.5^\circ$ ; $V_f = 7508$ m/s, $M_f = 25382$ kg

All these conditions are at $h_f = 119985$ m. When the velocities are corrected to the required circular velocity, the payloads become:

$\gamma = 10.5^\circ$ ; $m_{\text{pay}} = 793$ kg
$\gamma = 11^\circ$ ; $m_{\text{pay}} = 7279$ kg
$\gamma = 11.5^\circ$ ; $m_{\text{pay}} = 5534$ kg

To cope with these problems first the reference trajectory for the sensitivity analysis is adjusted. After the cruise flight segment the space plane will accelerate horizontally until the dynamic pressure constraint is reached, instead of accelerating and climbing at the same time. After that the flight-path angle will increase with a rate of change of 0.05°/s until the second climb flight-path angle is reached. Then, the space plane will climb and accelerate until the pull-up Mach number is reached.

As a second measure, the ranges of the following parameters have been changed:

- The cruise altitude: $h_c = 25$ km, 26 km, 27 km (±3.85%)
- The change of flight-path angle after cruise: $\gamma/d\gamma = 0.04^\circ$/s, 0.05°/s, 0.06°/s (±20%)
- The Mach number at the start of the pull up manoeuvre: $M_{pull} = 6.7, 6.8, 6.9$ (±1.47%)
- The maximum flight-path angle during ascent second stage: $\gamma_{\text{max}} = 10.5^\circ, 11^\circ, 11.5^\circ$ (±4.55%)

In figure 5.1, the new trajectory is shown. Only the ascent trajectories of the first and second stage are shown because these are of main interest in the sensitivity analysis. In figure 5.1 all the parameters have their reference value. The main difference with the original reference trajectory is the part after the cruise flight. In the original trajectory the trajectory makes a smooth curve from the cruise altitude and velocity to the pull-up conditions. The maximum dynamic pressure is not reached. In figure 5.1, however, the space plane accelerates to the maximum dynamic pressure
and thereafter follows this trajectory constraint until the pull-up Mach number is reached.

Fig. 5.1 The reference trajectory used for the Taguchi sensitivity analysis.

The performance of this reference trajectory is (after correction of the velocity):

- $m_{\text{prop}} = 79.8$ ton
- $m_{\text{pay}} = 1.2$ ton

It appears that the newly generated trajectory results in a much worse performance than the original trajectory, which delivered a payload of about 7.3 ton. However, because the sensitivity analysis is done to find important parameters and their relations more than to find an optimal trajectory, the worse performance is not a problem for the sensitivity analysis.

With the newly defined trajectory the 36 experiments, according to the orthogonal array L36, are performed. The parameter combinations of the 36 experiments are shown in appendix F.

In the figures 5.2 and 5.3 the payload mass and the fuel-propellant mass (= fuel mass first stage + propellant mass second stage) of the 36 simulations are shown. From these figures it can be seen that the variation of the resulting payload mass is large. The difference between the highest and lowest payload mass is about 7.7 ton. Also the total propellant mass varies a lot. The difference between the highest and lowest propellant mass is about 45 ton.

In figure 5.2 can be seen that about one third of the experiments results in a negative payload mass. This means that the required extra amount of fuel is more than 7000 kg, and that in theory a part of the dry mass is burned. In practice this is not possible, of course, so these missions have failed.
5.3 Variance analysis of simulation results

As a result of the performed simulations, 36 values of payload mass and fuel-propellant mass are known now. From these data the information has to be extracted which parameters are important with respect to the performance of the space plane. This can be done with the help of the variance analysis, which is presented by Taguchi (1987). When the important parameters are identified, polynomial coefficients can be determined. These coefficients are also based on the simulation results. The way to determine these coefficients is taken from Taguchi, (1987), chapter 16.

For the variance analysis, firstly the total variation of the simulation results will be determined. The total variation can be calculated as follows:

\[ S_t = (y_1 - m)^2 + (y_2 - m)^2 + \ldots + (y_{36} - m)^2 + (y_{36} - m)^2 \]

where,

\[ m = \frac{(y_1 + y_2 + \ldots + y_{36} + y_{36})}{36} = \text{mean value} \]  \hspace{1cm} (5.3)

\[ y_i = i^{th} \text{ simulation result, } i = 1,36 \]

Now, the variation of the results per parameter will be determined. The variation per parameter gives the amount of influence of that parameter on the objective. The sum of all the variations per parameter gives the amount of influence of all selected parameters on the objective. The difference between this sum and the total variation, therefore, indicates the amount of influence of other
sources which are not varied intentionally: the error variation. The parameters which have a very small influence can be neglected. The contribution of these parameters will be put into the error variation.

Because the parameters are varied over three levels, a quadratic polynomial can be composed. To check whether it is required to compose a quadratic polynomial, the variations per parameter can be divided into two terms, a linear term and a quadratic term. By expressing these terms in percents of the total variation, it can be seen what contribution each term has in the total variation. When the quadratic term appears to have a very small contribution, the quadratic contribution of this parameter can be neglected and only the linear polynomial coefficients have to be calculated.

For the calculation of the variation per parameter, first some supplementary tables have to be set up. In table 5.2 an example of such a supplementary table is given. In this table the payload mass is chosen as objective for the variance analysis. For each parameter the sum of the payload mass of each simulation with one specific level of that parameter (1, 2, or 3) is given. In this way three values are generated for each parameter corresponding to level one, two, and three. With these values the variation per parameter can be calculated. The sum of the three values has to be equal, of course, to the sum of the payload masses of all simulations. To check this, the sum of the three values is also given in table 5.2.

<table>
<thead>
<tr>
<th>traj. phase</th>
<th>Take off</th>
<th>first climb</th>
<th>cruise</th>
</tr>
</thead>
<tbody>
<tr>
<td>variable</td>
<td>$n_s$</td>
<td>$\gamma_{\text{max}}$</td>
<td>$\gamma_d$</td>
</tr>
<tr>
<td>level</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>0.180E+05</td>
<td>0.184E+05</td>
<td>0.181E+05</td>
</tr>
<tr>
<td>2</td>
<td>0.191E+05</td>
<td>0.175E+05</td>
<td>0.183E+05</td>
</tr>
<tr>
<td>3</td>
<td>0.174E+05</td>
<td>0.186E+05</td>
<td>0.181E+05</td>
</tr>
<tr>
<td>total</td>
<td>0.545E+05</td>
<td>0.545E+05</td>
<td>0.545E+05</td>
</tr>
</tbody>
</table>

Table 5.2 Total of payload mass at each level for the first 7 parameters

If the three values from the supplementary table for a variable $A$ are called $y(A_1)$, $y(A_2)$, $y(A_3)$, corresponding to the three levels of $A$, then the linear and quadratic term of the variation resulting from parameter $A$ can be calculated as follows:

$$S_A = \frac{(-y(A_1) + y(A_3))^2}{2r}$$  \hspace{1cm} (5.5)
\[ S_A = \frac{(y(A_1) - 2y(A_2) + y(A_3))^2}{6r} \]  

(5.6)

where \( r \) is the number of values of which the sums \( y(A_1), y(A_2), \) and \( y(A_3) \) consist. In this case \( r = 12 \) because for each parameter 12 simulations are performed at each level. The variation per parameter is:

\[ S_A = \frac{(y(A_1) - r \cdot \bar{m})^2 + (y(A_2) - r \cdot \bar{m})^2 + (y(A_3) - r \cdot \bar{m})^2}{r} = S_A + S_{A_1} \]  

(5.7)

These calculations have first been performed with the payload mass as objective. Secondly, also the total fuel-propellant mass is used as objective. The tables with the calculation results are presented in Appendix G. In figures 5.4 and 5.5 the linear variation terms and quadratic variation terms are shown respectively. They are given in percents of the total variation. The parameters are represented in these figures by the numbers 1 to 13. These numbers correspond with the parameters in sequence as they are given before. So number 1 corresponds with \( n_x \) at take off and number 13 corresponds with \( \gamma_{\text{max}} \) of the second stage.

![Graph 5.4](image1.png)

**Fig. 5.4** The contributions of the linear terms of variation per parameter, in percents of the total variation.

![Graph 5.5](image2.png)

**Fig. 5.5** The contributions of the quadratic terms of variation per parameter in percents of the total variation.

For both the payload mass and the total fuel-propellant mass as objective, it can be seen that the quadratic terms of the variation per parameter are all smaller than 1% of the total variation (fig. 5.5). All together the contribution of the quadratic terms is about 0.45% for the payload mass and about 1.02% for the fuel-propellant mass. Therefore, all quadratic terms are neglected in the determination of the polynomials.
As a result of the analysis with the payload mass as objective, it can be seen that four parameters mainly determine the payload mass. These parameters are (see fig. 5.4):

- The maximum dynamic pressure (no. 5)
- The Mach number at the start of the pull up manoeuvre (no. 10)
- The normal load at the start of the pull up manoeuvre (no. 11)
- The maximum flight-path angle during ascent second stage (no. 13)

Of these four parameters, the maximum flight-path angle has the greatest influence. Almost 90% of the total variation is caused by this parameter.

When the total fuel-propellant mass is chosen as objective, seven parameters have to be taken into account (see fig 5.4):

- The normal load at take off (no. 1)
- The maximum flight-path angle during take off (no. 2)
- The change of flight-path angle after normal load segment (no. 3)
- The flight-path angle during first climb (no. 4)
- The maximum dynamic pressure (no. 5)
- The cruise Mach number (no. 6)
- The maximum flight-path angle during ascent second stage (no. 13)

Of these parameters the maximum flight-path angle during take off has the largest influence. Almost 65% of the total variation is caused by this parameter.

### 5.4 Polynomial fit through variance analysis results

In this section the relations between the objectives and the parameters are expressed in polynomials and the confidence limits of these polynomials are determined. With these polynomials the best combination of parameter values can be estimated.

The polynomial can be written as (Taguchi, 1987):

\[
\hat{y} = b_0 + b_1 (A - \bar{A}) + b_2 \left( (A - \bar{A})^2 - \frac{a^2 - 1}{12} h^2 \right)
\]  

(5.8)

where,

- \(a\) = number of levels
- \(h\) = interval of spacing between two levels, \(h = A_2 - A_1 = A_2 - A_1, \bar{A} = A_1 + \frac{a - 1}{2} h\)
In this case the number of levels is 3 so:

\[ \Xi = A_1 + \frac{2}{3} \eta = A_2 \]  

(5.9)

Because only three levels are used for all parameters, the polynomial is developed until the second level. The coefficients can be calculated as follows (Taguchi, 1987):

\[ b_0 = \frac{(y_1 + y_2 + \ldots + y_{35} + y_{36})}{36} = \text{mean value} \]  

(5.10)

\[ b_1 = \frac{-(\gamma(A_1) + \gamma(A_2))}{r(\lambda S)h} \]  

(5.11)

\[ b_2 = \frac{(\gamma(A_1) - 2\gamma(A_2) + \gamma(A_3))}{r(\lambda S)h^2} \]  

(5.12)

Here \((\lambda S)\) has the value 2 (Taguchi, 1987). In section 5.3 has been shown that the quadratic terms can be neglected. So only the linear coefficients has to be calculated.

When the coefficients are calculated in the way as presented above, the polynomial for the estimation of the payload mass is as follows:

\[ m_{\text{payload}} = 1513 - 101(q_{\text{dyn}} - 50) + 4801(M_{\text{per}} - 6.8) - 3510(\eta_{\text{max}} - 1.5) + 5326(\gamma_{\text{max}} - 11) \]  

(5.13)

and the polynomial for the estimation of the fuel-propellant mass becomes:

\[ m_f = 161444 + 16944(\eta_{\text{max}} - 1.5) - 3950(\gamma_{\text{max}} - 25) - 168333(\gamma_{\text{lb}} + 0.25) - 20000(\gamma_{\text{max}} - 1.5) - 383(q_{\text{dyn}} - 50) - 5556(M_{\text{per}} - 4.5) - 6167(\gamma_{\text{max}} - 11) \]  

(5.14)

The confidence limits of the coefficients of the polynomial can be derived as follows, according to Taguchi (1987):

\[ b_i = \frac{-(\gamma(A_i) + \gamma(A_2))}{2\tau h} \pm \frac{F_XV_e}{rS_h^2} \]  

(5.15)

Here \(F\) is the value from the \(F\)-table with the numerator as 1 and the denominator as the degrees of freedom of \(V_e\). \(V_e\) is the error variance, which is the error variation \(S_e\) divided by its degrees of freedom. The degrees of freedom of \(V_e\) are the degrees of freedom of the total variance minus the degrees of freedom of the sources not included in the error variation.

The degrees of freedom of the total variation is 35 (36 simulations). The degrees of freedom of each variation per parameter is one. So in case of the payload mass, the degrees of freedom of the error variation is:
\[ f = f_s - f_{s,dyn} - f_{s,mod} - f_{s,mp} - f_{s,\text{max}} = 35 - 1 - 1 - 1 - 1 = 31 \]  

(5.16)

and the error variance becomes:

\[ V_s = \frac{S}{I} = \frac{1.6396 \times 10^6}{31} = 5.29 \times 10^4 \]  

(5.17)

The 5% value of \( F \) can be taken from the \( F \)-table (Van Soest, 1985); \( F = 4.16 \). The 5% confidence limits of the coefficients become:

\[ \hat{b}_{sw} = -101 \pm 19 \]

\[ \hat{b}_{sw} = 4801 \pm 957 \]

\[ \hat{b}_{sw} = -3510 \pm 638 \]

\[ \hat{b}_{sw} = 5326 \pm 191 \]

When these ranges are filled in, in equation (5.13), it can be seen that the payload mass lies in the range of ± 382 kg with respect to the estimated value, with a chance of 95%. Based on these data, it can be concluded that the polynomial is quite capable in predicting the order of magnitude of the payload mass.

The confidence limits of the coefficients of the polynomial for the estimation of the fuel-propellant mass, can be determined in the same way as for the coefficients of the polynomial for the payload mass estimation. However, in this case 7 parameters are selected as important parameters, so the degrees of freedom of the error variation become: \( 35 - 7 = 28 \). The error variance now becomes:

\[ V_s = \frac{S}{I} = \frac{1.02097 \times 10^8}{28} = 3.646 \times 10^6 \]  

(5.18)

The 5% value of \( F \) is 4.20 (Van Soest, 1985). The confidence limits of the coefficients are:

\[ \hat{b}_{sw} = 16944 \pm 5325 \]

\[ \hat{b}_{sw} = -3950 \pm 320 \]

\[ \hat{b}_{sw} = -168333 \pm 31953 \]

\[ \hat{b}_{sw} = -20000 \pm 5325 \]

\[ \hat{b}_{sw} = -383 \pm 160 \]

\[ \hat{b}_{sw} = -5556 \pm 1775 \]

\[ \hat{b}_{sw} = 6167 \pm 1598 \]
These ranges can be filled in in equation (5.14). It appears that the fuel-propellant mass lies in the range of \( \pm 5594 \) kg of the estimated value with a chance of 95%. This is about 3.5% of the mean fuel-propellant mass of all 36 simulations (161.4 ton). Therefore, it can be concluded that with the polynomial the amount of fuel-propellant mass can be fairly estimated.

The accuracy of the polynomials can also be tested with respect to the simulation results. All these parameter combinations can be recalculated with the polynomials. The differences between the answers of the polynomial and the simulations can be analyzed. These differences are presented in Appendix H.

The mean value of the differences \( x_i \) (i = 1,36) for the payload mass, is 0.1667 kg. So the mean values of both simulations and polynomial results are almost the same. The standard deviation of the differences is 217.7 kg. When it is assumed that the values are normal distributed, the confidence limit of 5% of the payload mass, predicted by the polynomial, is \( \pm 1.96 \sigma = \pm 427 \) kg. With this accuracy analysis the limits are 45 kg larger then with the first analysis. The order of magnitude, however, is the same so the same conclusion still holds.

The mean value of these differences \( x_i \) (i = 1,36) for the fuel-propellant mass, is 0.861 kg. So again the mean values of both simulations and polynomial results are almost the same. The standard deviation of the differences is 1715 kg. When is assumed that the values are normally distributed, the confidence limit of 5% of the fuel-propellant mass, predicted by the polynomial, is \( \pm 1.96 \sigma = \pm 3363 \) kg. With this accuracy analysis the limits are 2231 kg lower than with the first analysis, so the same conclusion as made above still holds.

5.5 'Optimal' parameter values determination with the help of polynomials

Because the polynomials consist of only linear terms, it is easy to find their maximum or minimum value. For a maximum, the highest value of a parameter has to be chosen when its coefficient is positive, and vice versa. Because the polynomial coefficients are based on parameter variations between a certain range, it is best to choose the values of the parameters in these ranges. Outside the ranges the polynomial will probably become very inaccurate.

The maximum payload mass should be obtained when the following values are chosen (see equation 5.13):

\[
\begin{align*}
q_{dyn} & = 45 \text{ kPa} \\
M_{pull} & = 6.9
\end{align*}
\]
\[ n_{\text{max}} = 1.35 \]
\[ \gamma_{\text{max}} = 11.5^\circ \]

According to the polynomial the payload mass will become 5687 kg when these values are chosen. With ASCENT a simulation has been performed, where all parameters have their reference value, except the four mentioned parameters. The resulting payload mass is 5750 kg. The difference between the estimation and the real value is 63 kg, which is about 1% of the real payload mass.

The determined payload is not higher than all payload masses resulting from the 36 simulations. The highest payload from these simulations was 5820 kg resulting from simulation 35. However, in this simulation the four important parameters have also the "optimal" values. Therefore, the difference in payload mass has to be caused by the neglected terms. These terms contributed about 0.86% to the total variation (inclusive the error variation). The difference between the two payloads is 70 kg which is about 1.2% of the both payload mass values.

In figure 5.6 the contributions of the four parameters to the payload mass are shown. Their value is expressed with respect to the mean value. From this figure it can clearly be seen that both the maximum flight-path angle of the second stage and the pull up Mach number have the largest influence on the payload mass. The influence of the pull-up normal load and the maximum dynamic pressure is almost the same. Also it can be clearly seen that the ranges for the pull-up Mach number and the maximum flight-path angle are smaller than 10% of the mean value.

---

![Graph showing the influence of four parameters on payload mass](image-url)

**Fig. 5.6** The influence of the four parameter variations on the payload mass.
Based on the previous results it seems that it is very well possible to predict the payload mass with the derived polynomial and to determine the optimal values for the trajectory parameters within a defined range.

Because it is best to use as little fuel as possible, the best values of the parameters in equation (5.14) are those which minimize the polynomial. The values of the parameters are:

- $n_{FB} = 1.35$
- $\gamma_{max,t} = 27.5^\circ$
- $\frac{dy}{dt} = -0.225^\circ/s$
- $\gamma_{cl} = 1.65^\circ$
- $q_{dyn} = 55 \text{ kPa}$
- $M_{cr} = 4.95$
- $\gamma_{max,sec,cl} = 11.5^\circ$

The predicted fuel-propellant mass is 134.3 ton. The result of the simulation with these parameters in ASCENT is 135 ton. The difference is about 700 kg which is 0.5% of the real fuel-propellant mass. This fuel-propellant mass is lower than all 36 simulation results. Of these simulations, simulation 31 comes close with a fuel-propellant mass of 137 ton. In this simulation all important parameters have their 'optimal' value except the dynamic pressure, which is 50 kPa instead of 55 kPa.

![Diagram]

*Fig. 5.7 The influence of three of the most important parameters of seven significant parameters on the fuel-propellant mass.*
In figure 5.7 and 5.8 the contributions of the seven parameters to the fuel-propellant mass are shown. Their value is expressed with respect to the mean value. In figure 5.7 the three most important parameters are shown. In figure 5.8 the remaining four parameters are shown. In this figure $\gamma_{\text{don}}$ is shown again to give an impression of the proportion of the influence with respect to each other.

From these figures it can be clearly seen that the maximum flight-path angle during take off has the greatest influence on the fuel-propellant mass variation. The maximum flight-path angle during the second stage has also much influence. This is striking, because figure 5.4 shows that, apart from the maximum flight-path angle during take off, $\frac{\text{dy}}{\text{df}}$ has the greatest influence.

This effect is due to the fact that for $\gamma_{\text{max}}$ a smaller range is used. Therefore, the absolute contribution of $\gamma_{\text{max}}$ to the total variation of the fuel-propellant mass is smaller than the contribution of $\frac{\text{dy}}{\text{df}}$. The gradient of $\gamma_{\text{max}}$, however, is larger than the gradient of $\frac{\text{dy}}{\text{df}}$, so its influence is larger.

From the previous example it can be seen that it is dangerous to use different ranges for different parameters. In such a case there is a chance that an important parameter seems not to be important, because the absolute contribution to the total variation of this parameter is small as a
result of a small range. The parameters in the experiment, for which the ranges are taken smaller than 10\%, are the cruise height \(h_c\), the pull-up Mach number \(M_{pull}\), and the maximum flight-path angle during the second stage \(\gamma_{maclen}\). For both \(M_{pull}\) and \(\gamma_{maclen}\), this had no consequence. \(\gamma_{maclen}\) is so important that it is not overlooked despite the smaller range. The influence of \(M_{pull}\) is so small (0.01\%) that it would be unimportant even if the range would be larger. The cruise height, however, was not taken into account but its contribution to the total variation is 0.54\%, which is much larger than all other small influences. In order to make sure whether the cruise height has much influence, the influence of this parameter will be studied.

The contribution of \(h_c\) to the total variation is 0.54\%. The range, however, over which \(h_c\) is changed is \(\pm 1\) km which is about \(\pm 3.8\%\). When the range would have been \(\pm 10\% = \pm 0.1^*26km\), and when it is assumed that the derived coefficient would be the same in that case, the variation of the fuel-propellant mass as a result of \(h_c\) would be \(\pm 917^*2.6 = \pm 2384\) kg. This is almost as much variation as resulting from the cruise Mach number (\(\pm 2500\) kg). So, although the influence of \(h_c\) is small for this selected range, \(h_c\) has to be regarded as an important parameter with respect to the fuel-propellant mass. According to the sign of the coefficient for \(h_c\), it can be said that the lower \(h_c\), the lower the fuel-propellant mass.
6. Discussion of sensitivity analysis results

In this chapter the results, obtained with the Taguchi sensitivity analysis, will be discussed. Firstly the reasons why certain trajectory parameters influence the performance of a space plane will be discussed. Secondly, it will be shown in section 6.2 which tools are required for space plane analysis, based on the sensitivity analysis results. It will appear that the sensitivity analysis results confirm the results found in literature (see also section 2.1). In section 6.3, the Taguchi method as an optimization method will be discussed.

6.1 Physical relations between the trajectory parameters and the objectives

In chapter 5, four of the thirteen selected trajectory parameters influence the delivered payload mass considerably. Secondly seven parameters influence the fuel-propellant mass. It appears that the payload mass and the fuel-propellant mass can vary considerably when these parameters are varied between ± 10%. With the help of the Taguchi method, it is shown that the variations of the payload mass and fuel-propellant mass can be described very well as linear functions of these important parameters.

A very interesting and important question is now: 'Can we physically understand why these parameters have so much influence on the objectives?'. The answer of this question links the results from chapter 5 with the physical background of space plane trajectories. Therefore, the relations of the payload mass with the maximum dynamic pressure, pull-up Mach number, pull-up normal load, and the maximum flight-path angle during the ascent of the second stage \( (\eta_{\text{max}}) \) will be studied. Then, also the relations will be studied of the fuel-propellant mass with the following seven parameters:

- normal load at take off \( (\eta_{\text{tno}}) \)
- maximum flight path angle during take off \( (\gamma_{\text{max}}) \)
- flight path angle rate during take off \( (\dot{\gamma}_{\text{lo}}) \)
- first climb flight path angle \( (\gamma_{\text{fcl}}) \)
- maximum dynamic pressure \( (q_{\text{dyn}}) \)
- cruise Mach number \( (M_{\text{c}}) \)
- maximum flight path angle during ascent second stage \( (\gamma_{\text{maxs}}) \)
6.1.1 The payload-mass objective

In the sensitivity analysis, the payload mass is linked with the required amount of propellant mass of the second stage. So the less propellant is used during the ascent of the second stage, the more payload mass can be delivered. The required propellant mass depends on the total trajectory of the second stage. This trajectory is determined by the separation conditions, the final altitude and \( \gamma_{\text{max separ}} \). The final altitude is not changed in the sensitivity analysis, so it can be expected that the separation conditions and \( \gamma_{\text{max separ}} \) influence the payload mass.

The separation conditions, which are especially the altitude, the velocity and the flight-path angle at separation, are determined by the trajectory of the first stage. When the parameters, \( M_{\text{pull}} \), \( q_{\text{dyn}} \), and \( n_{\text{turb}} \), are studied, it can be seen that these trajectory parameters determine the separation conditions, apart from the flight-path angle.

The pull-up Mach number is closely related, of course, to the velocity at separation. Secondly the maximum dynamic pressure determines at what altitude the pull up manoeuvre is started. After the cruise flight the space plane accelerates horizontally, until the maximum dynamic pressure is reached. Then, an altitude-velocity profile is followed, along this constant maximum dynamic pressure. So when the pull-up velocity is reached, the altitude is determined by the maximum dynamic pressure.

Also the normal load during the pull-up manoeuvre determines the separation altitude. When a high normal load is used, the space plane will reach the (prescribed) final flight-path angle very soon. As a result the time to climb after the start of the pull-up manoeuvre is shortened. So the final separation altitude will be lower.

Apparently the variation of the flight-path angle at separation was not large enough to have much influence on the delivered payload mass. This does not mean, however, that the flight-path angle at separation has no influence at all. From this study it can only be concluded that the flight-path angle at separation is not so important as the velocity and altitude at separation.

The most striking result of the sensitivity analysis, is that the \( \pm 4.5\% \) variation of the maximum flight-path angle during second stage is for 90\% responsible for the variation in payload mass. Apparently \( \gamma_{\text{max separ}} \) has the largest influence compared to the separation conditions.

So, summarizing, it can be said that it is important to optimize the trajectory of the second stage.
with respect to the delivered payload mass. The trajectory of the first stage has to be optimized with respect to the separation conditions, to deliver a payload mass as large as possible. Secondly, it can be said that, according to this sensitivity analysis, much more payload mass can be gained by optimizing the trajectory of the second stage, than by optimizing the trajectory of the first stage w.r.t. the separation conditions. It has to be noted, however, that this last remark is valid with the assumptions, that the separation altitude lies between 30 and 35 km, the separation Mach number lies between about 6.5 and 7.0, and the separation flight-path angle lies between 4.5° and 5.5°, as was the case in all 36 simulations.

Optimizing the trajectory of the second stage w.r.t. the payload mass, really means minimizing the propellant mass needed to deliver the payload in its target orbit. Optimizing the trajectory of the first stage w.r.t. the separation conditions, however, does not automatically mean minimizing the fuel mass needed to reach these conditions. The maximum reachable separation conditions, namely, are much depending on the capabilities of the (airbreathing) propulsion system. Above a certain altitude and Mach number, the propulsion system will not be able to accelerate the vehicle any more. So even if enough fuel is available, the performance of the first stage is restricted. Optimizing the trajectory of the first stage, therefore, actually means that the trajectory has to be such that the maximum feasible separation conditions are reached.

6.1.2 The fuel-propellant mass objective

It seems that the fuel mass consumed by the first stage is not so important for the payload mass, as long as the right separation conditions are reached. This conclusion, however, is not quite right. In the sensitivity analysis, namely, the link between the fuel mass consumed by the first stage and the delivered payload mass has been neglected. Hence, savings in fuel mass for the first stage, do not result in added payload mass.

When the sensitivity results for the total fuel-propellant mass are considered now, it follows that the first stage trajectory has a large influence on this objective. The trajectory parameters for the first stage trajectory are (together) for 91% responsible for the variation of the fuel-propellant mass. The reference fuel mass of 83.6 ton could be brought down to 58 ton, with the help of the Taguchi analysis. So indeed much fuel mass could be saved with respect to the original first stage trajectory. The reference propellant mass of the second stage of 82.3 ton is only reduced to about 75 ton,

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2 The term 'optimizing' is placed between quotation marks because in the sensitivity analysis performed in this report cannot really be spoken of optimization. However, with the help of the sensitivity analysis, the performances of the space plane could be improved considerably, so one can speak of a sort semi optimization technique.
which is a much smaller saving in mass.

The fact that the first stage trajectory is for 91% responsible for the variation of the fuel-propellant mass in the 36 simulations, is unexpected. When one looks to the proportion of the fuel mass to the propellant mass of the reference trajectory, it can be seen that these masses are of the same order (resp. 83.6 ton and 82.3 ton). One would expect, therefore, that the influence of both first and second stage trajectory on the total fuel-propellant mass are of the same order. Or maybe one should even expect that the influence of the second stage is larger, because the second stage carries both fuel and oxidizer. When more fuel is needed, more oxidizer is needed so this will almost double the increase of mass. This effect, however, does not show from the sensitivity analysis.

The explanation for this phenomenon is that the second stage trajectory (which is taken from Staufenbiel 1990) is already a semi-optimal trajectory, whereas the first stage reference trajectory is not optimal at all. This appears from the amount of fuel mass that could be saved with the help of the Taguchi method. So the difference in influence on the variation of the total fuel-propellant mass, is mainly a result from the fact that the second stage reference trajectory was much closer to an optimal trajectory than the first stage trajectory.

The fact that the required fuel mass of the first stage could be decreased with about 25 ton, indicates that the reference vehicle, used for the simulation, was oversized. This is the result of the 'oversized' propulsion system. As has been shown in chapter 4, the required amount of fuel increases considerably (up to 94.4 ton) when a propulsion system is used with 55% thrust of the old propulsion system.

In the sensitivity analysis, seven parameters appeared to influence this required fuel and propellant mass. These seven trajectory parameters are \( \gamma_{massa} \), \( \gamma_{massa} \), \( \gamma_{vs} \), \( \gamma_{w} \), \( Q_{dyn} \), \( M_{en} \) and \( \gamma_{massen} \). The influence of \( \gamma_{massen} \) is, of course, strongly related to the influence of this parameter on the payload mass. This can be seen when the two polynomial coefficients of this parameter, for respectively the payload mass and fuel-propellant mass, are compared with each other. In the polynomial for the payload mass the coefficient has the value of 5326 kg\(^{m} \) and in the polynomial for the fuel-propellant mass the coefficient has the value of -6167 kg\(^{m} \). About the same amount of mass with which the fuel-propellant mass will decrease, the payload mass will increase with.

The other six parameters have influence on the fuel consumption during the first stage. It can be noticed that these parameters determine especially the trajectory until the end of the cruise flight.
Apparently the trajectory after the cruise flight is not so important w.r.t. the fuel mass. The six parameters can be divided into 3 groups. The first group consists of the four parameters \( n_{zr} \), \( \gamma_{max} \), \( \dot{\gamma}_{zr} \), and \( \dot{\gamma}_{cr} \). From the sensitivity analysis appeared that the lower \( n_{zr} \) and the higher the other three parameters (within the specified range), the lower the fuel mass will be. This actually means that the longer the space plane climbs with a high flight-path angle, the lower the fuel mass will be.

The second group consists only of \( q_{dyn} \). The higher the dynamic pressure, the lower the fuel mass will be. A higher dynamic pressure will result in a larger specific impulse, so the propulsion system will be more efficient. Secondly the thrust level will be higher. Therefore the space plane has more power to accelerate. In those parts where the thrust is not throttled down, the space plane will accelerate faster and as a result the flight time will decrease. This will also decrease the fuel mass.

The third group consists of \( M_{cr} \), and actually also of \( h_{cr} \). In chapter 5, it is shown that, although this parameter did not seem to be important from the analysis results, it has about as much influence as \( M_{cr} \). The fuel mass will decrease when \( M_{cr} \) increases and when \( h_{cr} \) decreases. Actually this means that the fuel mass will be lower when the cruise flight is performed at a higher dynamic pressure, because the dynamic pressure increases with increasing Mach number and decreasing altitude. So again it can be seen that a higher dynamic pressure results in less fuel consumption. This parameter group is therefore closely related to group 2.

So, summarizing, it can be said that it is very important to optimize the trajectory until separation not only w.r.t. the separation conditions, but also w.r.t. the required fuel mass. Small variations in trajectory parameters can cause large variations in fuel mass. Savings in required fuel mass can result in lower take off and empty mass, or in larger payload mass. A high dynamic pressure and an as large as possible flight path angle during climb will decrease the required fuel mass. It can also be noticed that the parameters, which determine the ascent until separation after the cruise flight, appeared to have no influence on the required fuel mass.

### 6.2 Tools required for space plane analysis

From the results of the sensitivity analysis, it follows that the performance of a space plane (like \( m_{sys} \) and \( m_{sys} \)) is very sensitive to trajectory variations. As a consequence, to predict the 'true' performance of a space plane, its trajectory has to be optimized. Only with trajectory optimization, it can be investigated what the best performance is of the design, or whether the space plane is not overdesigned w.r.t. a given (required) performance. So trajectory optimization has also to be part of
the design process.

The latter can be shown with the following example. When the propulsion characteristics $T$ and $I_{sp}$ of a space plane change, the mass properties and size of the space plane will also change due to the different fuel consumption. When the size of the space plane changes, the aerodynamic characteristics could also change. Because of all these changes in the characteristics of the space plane, the optimal trajectory of the space plane will also change.

The space plane configuration has to be corrected, to determine the influence of the change of the propulsion characteristics. However, to do so, a calculation tool with which the new geometry and mass properties can be determined is needed. Such a tool is now being developed within the Work Package 151 of the project AEOLUS (Oving, TO BE PUBLISHED).

To get some idea about these effects, an calculation example will be given. In chapter 4 already has been shown that the characteristics of the propulsion system have much influence on the required total amount of fuel. The original space plane configuration, however, was not corrected for these sort of influences. The total amount of fuel was 105 ton, and this was not changed. In the next example the trajectory with the highest payload and the lowest fuel-propellant mass (see chapter 5) is flown again. However, now the take-off mass of the first stage is decreased with 40 ton. The fuel mass which is needed for the trajectory is 64 ton, while 105 ton could be stored in the vehicle. So this difference of about 40 ton is subtracted from the total take off mass. The size of the vehicle is not changed, so strictly speaking the same space plane is going to fly with partly filled fuel tanks.

The required fuel mass for the space plane with 320 ton take-off mass now becomes about 58 ton. This is 6 ton less than the original 64 ton fuel. In figure 6.1 and 6.2 the altitude versus the velocity and the angle of attack versus the time are given.

In these figures, it can be seen that the resulting trajectory has already been changed although the values of the flight segments parameters (like maximum dynamic pressure and pull-up Mach number) have not been changed. The angle of attack has become smaller over the total trajectory, because less lift has to be generated. Also the flight time is reduced. Furthermore, the separation conditions have been changed. This is the result of the fact that the space plane, with less take-off mass, will have a larger acceleration after the cruise flight. Therefore the flight-path angle of the second climb will be reached at a higher speed. As a result the space plane has gained less altitude when the pull-up Mach number is reached.
Fig. 6.1 The altitude versus the velocity of the trajectories of two space planes with different take off mass (M_0).

Fig. 6.2 The angle of attack of the trajectories of two space planes with different take-off mass (M_0).

So apart from the fact that the required amount of fuel has decreased again, and the space plane configuration should be changed accordingly, also the trajectory has been changed. It confirms that the trajectory calculation and the vehicle design have large influence on each other.

In Chapter 2 it has been shown that it is important to know the angle of attack very accurate as a function of time, because of precompression effects, to predict the propulsion performance and the resulting trajectory. However, α is also a result of trajectory calculations. When these trajectory calculations are not optimal trajectory calculations, it will be very difficult to say how much the
trajectory parameters deviate from the optimal values. As an example $\alpha$, $h$, $M$, and $t$ are shown in figure 6.3 and 6.4 for two different trajectories calculated for the sensitivity analysis (chapter 5). One trajectory has the largest amount of fuel of all 36 simulations. The other has the best combination of delivered payload mass and required fuel mass.

![Graph of altitude versus velocity for two trajectories with different performances.](image)

**Fig. 6.3** The altitude versus the velocity for two trajectories with different performances.

![Graph of angle of attack versus time for two trajectories with different performances.](image)

**Fig. 6.4** The altitude versus the velocity for two different trajectories with different performances.

The parameters $\alpha$, $h$, $M$, and $t$ are shown because these parameters are especially important with respect to the study and design of a ramjet inlet. $\alpha$, $h$, and $M$ especially influence the performance (pressure recovery, air mass flow) and loads (pressure, temperature and heat flux loads) of an inlet.
The parameter \( t \) is important with respect to the duration of certain loads on the structure. From the figures 6.3 and 6.4, it can be seen what deviations of these important trajectory parameters can be expected, when the trajectory is not optimized. Especially the differences in \( \alpha \) and \( t \) are considerable. The difference in \( \alpha \) can reach about 3°. At the end of the pull-up manoeuvre the difference reaches even the value of 4°. The difference in flight time is about 750 s.

From the previous examples can be concluded that the important trajectory parameters are significantly influenced by changing propulsion data. Furthermore, of these trajectory parameters especially \( \alpha \) has much influence on the performance of the propulsion system. It is, therefore, very important to know what accuracy is needed for these parameters, to do serious studies on an inlet system. Based on the data shown in this report, one would expect that non optimized trajectory calculations will not give trajectory parameters with sufficient accuracy.

When this fact is considered with respect to the AEOLUS project, it can be seen that it is important to know whether this project concentrates on a certain given, fixed space plane design, or whether it concentrates on different space plane configurations. When variations in propulsion characteristics (\( T, I_\alpha \)), aerodynamic characteristics, mass and geometric data are studied, it will be necessary to develop a trajectory optimization tool, and a space plane design or (at least) sizing tool.

Apart from the previous remarks, it is still advisable to develop a trajectory optimization tool and a space plane design or sizing tool. Without these tools, namely, it will be very difficult to predict whether an improvement of the performance of the ramjet inlet, or the propulsion system, will also result in better space plane performances.

### 6.3 The Taguchi method as an optimization method

In chapter 5 the Taguchi method is used for the sensitivity analysis. It is found that the trajectory could be improved considerably with this method. So a logical question would be whether this method can be used as an optimization method. It is, however, not advisable to use this method because it has some important shortcomings.

Firstly, it is only possible to use the method on a predefined trajectory, consisting of several predefined flight segments. It is not known beforehand, however, whether the optimal trajectory will consists of those flight segments. Or better, one can bank on it that the optimal trajectory does not consists of the predefined flight segments. Buhl has already stated that an evident dissimilarity between the suboptimal trajectories based on predefined flight segments and the optimal trajectory
is the absence of any constant parameter phase in the optimal trajectory (Buhl, 1992).

Secondly, one has to determine a reference value for the parameters of interest, around which the variations will be performed. The 'optimal' values of the parameters, which can be found with the Taguchi method, will lie within the range of variation. The reference values of the parameters, therefore, have to be such that the resulting trajectory is at least suboptimal. It is, however, very difficult to determine these values such, so that a suboptimal trajectory can be found.

Thirdly, when a suboptimal trajectory has been found, it is still not known how much the resulting flight conditions and performance differ from the optimal values. So, as a result, it will not be possible to say anything about the accuracy of the results. It will be impossible, therefore, to say whether the trajectory conditions, for example, are accurate enough for propulsion system design.

So, concluding, it can be said that the Taguchi method is very useful to get insight in the relation of the performance of a space plane with the trajectory parameters. For trajectory optimization, however, it is better to use mathematical optimization techniques.
7. Conclusions and recommendations

Conclusions

In this report a vehicle data set has been composed, with which a TSTO vehicle trajectory has been calculated, with the help of the trajectory calculation tool ASCENT. For this trajectory a parameter sensitivity analysis has been performed. From this analysis, the following can be concluded:

- The payload mass and the total fuel-propellant mass of a TSTO space plane are very sensitive for trajectory parameter variations:

- Especially the shape of the second stage trajectory and the separation conditions influence the delivered payload mass. In the sensitivity study a variation of the separation altitude between 30 and 35 km, and a variation of the separation Mach number between 6.5 and 7.0, caused a variation in payload mass of about 3000 kg. The second stage variations (± 4.5% variation of maximum flight-path angle during second stage) caused a variation in payload mass of about 6000 kg.

- Especially the maximum dynamic pressure and the flight-path angle during the first stage ascent, influence the fuel mass consumption of the first stage. Of these parameters especially the maximum flight-path angle during take off has much influence. Due to a variation of ±10% of the important trajectory parameters, the fuel mass varied about ± 25 ton.

- For a correctly performed space plane analysis, at least a trajectory optimization tool and a space plane design tool are required. Only then the influence of configuration changes on the trajectory and vice versa can be studied correctly. Hence, trajectory optimization has to be an integral part of space plane design.

- Angle-of-attack-dependent propulsion data have to be used for trajectory optimization, to get reliable results. The angle of attack influences the thrust data considerably, due to precompression effects.

- It is not advisable to use the Taguchi method for the trajectory optimization, although it is
a useful method to gain insight in relations between several parameters in complex systems.

Recommendations

- It is advised use the Winged Cone Configuration (= WCC; Shaughnessy, 1990) as a reference vehicle, when it is scaled as follows:
  - \(m_0 = 2.63 \times m_0 \text{ WCC}\)
  - \(S_{\text{w}} = 2.63 \times S_{\text{w}} \text{ WCC}\)
  - \(T = 1.45 \times T \text{ WCC}\)

  With this vehicle a trajectory is generated with ASCENT, which shows good resemblance with other TSTO trajectories (like e.g. Sänger) and also requires a comparable amount of fuel mass. This trajectory, therefore, can be used as a reference trajectory for AEOLUS, until optimal trajectories can be calculated.

- It is recommended to develop or buy a trajectory optimization tool and a space plane design tool.

- It should be investigated:
  - whether pitching moment equilibrium (trim) has to be taken into account for trajectory optimization.
  - how accurate the flight conditions along the trajectory have to be known to be able to design a satisfactory propulsion system.

The problem with the moment equilibrium is closely related to the flight condition data accuracy problem. Depending on the accuracy required, trim has to, or does not have to be taken into account.
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Appendix A  Basics of the trajectory simulation program ASCENT

In this appendix the basics of the program ASCENT are shortly described. For a more detailed description, one is referred to Korswagen (1993).

Equations of motion

The equations of motion, used in ASCENT, are expressed with respect to the rotating earth. For the derivation of these equation of motion the following assumptions are made:

- The space plane can be described as a mass point. Only 3 degrees of freedom are taken into account.
- There are no side winds. As a result the slip angle is zero degrees.
- The bank angle is 0°.
- The thrust vector is in the plane of symmetry.

As control variables are considered:
- The angle of attack, \( \alpha \).
- The throttle setting, \( \delta_T \).
- The thrust vector angle (in plane of symmetry), \( \epsilon_T \).

Environment

To calculate the forces acting on the space plane, environmental models are required. These models are:

The Earth

The Earth is considered as an oblate ellipsoid with mass symmetry about the rotation axis. The Earth radius is:

\[
R_\text{e} = R_\text{e}(1 - esin^2\delta) \tag{A.1}
\]

where
- \( R_\text{e} \) is Earth radius at equator, \( R_\text{e} = 6378139 \text{ m} \)
- \( e \) is Earth ellipticity, \( e = 1/298.257 \)
- $\delta$ is the latitude

The rotational rate of the Earth is assumed to be constant:

- $\omega_e = 7.292115 \cdot 10^{-5}$ rad/s

**The gravitation**

A simple model for describing the gravitational field of the Earth is given by the inverse square law which describes the radial gravity component along the position vector

$$g_r = \frac{\mu}{r^2} \quad (A.2)$$

where $\mu$ is the gravitational parameter of the Earth and $r$ is the length of the position vector $\vec{r}$. Assuming that the Earth is flattened at the Poles and mass symmetric about the rotation axis, the acceleration due to gravity can be defined as:

$$\vec{g} = \begin{bmatrix} g_r \\ 0 \\ g_\theta \end{bmatrix} \quad (A.3)$$

Here $g_r$ is directed along the position vector $r$ to the centre of the earth, and $g_\theta$ ($\delta$ is latitude) is directed to the local south. With only the second harmonic $J_2$ taken into account to describe the gravitational field, these two components can be expressed as:

$$g_r = \frac{\mu}{r^2} \left( 1 + \frac{3}{2} J_2 \left( \frac{R_e}{r} \right)^2 \left( 1 - 3 \sin^2 \delta \right) \right) \quad (A.4a)$$

$$g_\theta = 3 \frac{\mu}{r^2} J_2 \left( \frac{R_e}{r} \right)^2 \sin \delta \cos \delta \quad (A.4b)$$

When the oblateness is not considered (i.e. $J_2 = 0$), the inverse square law of Eq. (A.2) is obtained.

The values of the parameters $\mu$ and $J_2$ are:

$$\mu = 3.9860047 \cdot 10^{14} \text{ m}^3/\text{s}^2$$

$$J_2 = 1.082627 \cdot 10^{-3}$$

Due to the oblateness, the sea-level gravity depends on the lateral position. The sea level gravity is defined at zero altitude at the equator. Using Eq. (A.4) yields
\[ g_0 = \frac{\mu}{R_e^2} \]

Using the numerical values, results in

\[ g_0 = 9.798 \text{ m/s}^2 \]

The atmosphere

Two atmosphere models are available in ASCENT:
- The US standard atmosphere 1976 (US76)
- The exponential atmosphere

The US76 is implemented as default.

Heating rate

For the calculation of the heat flux, two formulas are implemented in ASCENT:

- The first one is taken from Lu (1991):
  \[ \dot{Q} = 4.919 \times 10^{-7} \rho_0^{0.6} V^3 \]  \hspace{1cm} (A.5)
  where \( \dot{Q} \) is given in kW/m², \( \rho \) in kg/m³ and \( V \) in m/s. \( V \) is the velocity with respect to the rotating Earth. The equation corresponds to reradiation equilibrium conditions on the surface of a wing leading edge 10 cm in radius. A surface emissivity coefficient \( \varepsilon = 0.8 \) is used for the calculation of the equilibrium wall temperatures.

- The other model is taken from Chapman (1959) and is represented by
  \[ \dot{Q} = \frac{C}{V} \left( \frac{\rho_0}{\rho_\infty} \right) \left( \frac{V}{V_o} \right)^m \]  \hspace{1cm} (A.6)
  which gives the heating rate in kW/m² in a hypersonic flow at the stagnation point, where \( R_w \) is the nose radius of curvature in m, and the constants \( C, n, m \) depend on the type of boundary-layer flow. \( \rho_0 \) is the sea-level density and \( \rho_\infty \) is the free-stream density, both in kg/m³. \( V \) is the velocity with respect to the rotating Earth, and \( V_o \) is the orbital velocity with respect to the rotating Earth.

Assuming a laminar flow results in \( n = \frac{1}{2} \) (Chapman, 1959). For purposes of simplicity, the value \( m = 3 \) is used (Chapman, 1959), corresponding to a gas with viscosity proportional to
the square root of the temperature. The value of \( C = 1.06584 \times 10^5 \) kW/m\(^3\) (Chapman, 1959) is used to complete the model.

Bannink (1991) demonstrates that

\[
\dot{Q} \propto \frac{\rho^{0.5} V^3}{\sqrt{R_N}} \quad \text{(A.7)}
\]

when the viscosity is proportional to the square root of the temperature, and the wall temperature is much smaller than the total temperature of the free flow: \( T_w \ll T_f \). This structure can be recognized in both models.

When Chapman's model, however, is expressed as the model of Lu, for \( R_N = 0.1 \) m and the circular velocity at sea level (\( V_c = 7905 \) m/s), the factor appears to be about 1.25 times the factor of Lu:

\[
\dot{Q} = 6.165 \times 10^{-5} \rho^{0.5} V^3
\]

It is not exactly known why these factors differ so much from each other. It is known, however, that Chapman's model is derived for descent trajectories, and that Lu's model is derived for ascent trajectories at a constant dynamic pressure of \(-95\) kPa. This will probably be the cause of the difference.

Because it is not known yet what model is best, it is recommended to use the Chapman model to indicate the order of magnitude of the heat flux, until further investigations have resulted in better models.
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## Appendix B Aerodynamic data of the HORUS 2B

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Table B1 $C_D$-clean as function of $\alpha$ and $M$ (MBB, 1988).

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<td>0.604</td>
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<td>0.425</td>
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<td>1.064</td>
<td>0.906</td>
<td>0.824</td>
<td>0.789</td>
<td>0.707</td>
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<td>1.050</td>
<td>1.016</td>
<td>0.920</td>
</tr>
</tbody>
</table>

Table B2 $C_D$-clean as function of $\alpha$ and $M$ (MBB, 1988).

**Reference**

MBB Space Communication and Propulsion Systems Division;
"Study on Re-entry Guidance and Control; Final Report-volume 2. Reference Vehicle definition and Orbit constraints";
ESA report reference: ESA CR (P) 2652;
MBB, Munich, 1988a.
Appendix C Propulsion data of the Winged Cone Configuration

These data are taken from Korswagen (1993). Korswagen has measured and converted these data from figures in Shaughnessy (1990).

<table>
<thead>
<tr>
<th>$q_{dyn}$ (N/m²)</th>
<th>0</th>
<th>7182</th>
<th>47,880</th>
<th>95,760</th>
<th>239,401</th>
</tr>
</thead>
<tbody>
<tr>
<td>$M$</td>
<td></td>
<td></td>
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<td></td>
<td></td>
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<tr>
<td>0.00</td>
<td>0</td>
<td>1,334,466</td>
<td>8,896,440</td>
<td>17,792,879</td>
<td>44,482,198</td>
</tr>
<tr>
<td>0.30</td>
<td>0</td>
<td>1,334,466</td>
<td>8,896,440</td>
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<td>44,482,198</td>
</tr>
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<td>6,672,330</td>
<td>16,680,824</td>
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Table C1 T (N) for a turbojet and ramjet engine as a function of $M$ and $q_{dyn}$ (Korswagen, 1993).
<table>
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<td>$M$</td>
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</table>

Table C2 $\lambda_p$ (s) for a turbojet and ramjet engine as a function of $M$ and $q_{syn}$ (Korswagen, 1993).

References

1) Korswagen, J.H.;
   "ASCENT Trajectory Simulation for Aerospace Planes; Development of a Simulation Program";

2) Shaughnessy, J.D., Pinckney, S.Z., McMinn, J.D.;
   "Hypersonic Vehicle Simulation Model; Winged Cone Configuration";
Appendix D Aerodynamic data of the Winged Cone Configuration

These data are taken from Korswagen (1993). Korswagen has measured and converted these data from figures in Shaughnessy (1990).

<table>
<thead>
<tr>
<th>$\alpha$ (°)</th>
<th>-1</th>
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<th>1</th>
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<th>4</th>
<th>6</th>
<th>8</th>
<th>10</th>
<th>12</th>
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<tbody>
<tr>
<td>$M$</td>
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<td></td>
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<td></td>
<td></td>
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</tr>
<tr>
<td>0.3</td>
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<td>-0.005</td>
<td>0.015</td>
<td>0.035</td>
<td>0.080</td>
<td>0.125</td>
<td>0.190</td>
<td>0.245</td>
<td>0.315</td>
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<td>0.035</td>
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<td>0.260</td>
<td>0.330</td>
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<td>0.035</td>
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<td>0.140</td>
<td>0.200</td>
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<tr>
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<td>-0.005</td>
<td>0.015</td>
<td>0.035</td>
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<td>0.150</td>
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<td>0.190</td>
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<td>-0.005</td>
<td>0.010</td>
<td>0.020</td>
<td>0.050</td>
<td>0.080</td>
<td>0.110</td>
<td>0.140</td>
<td>0.175</td>
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<td>0.010</td>
<td>0.015</td>
<td>0.035</td>
<td>0.060</td>
<td>0.085</td>
<td>0.110</td>
<td>0.145</td>
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<td>0.005</td>
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<td>0.005</td>
<td>0.015</td>
<td>0.030</td>
<td>0.050</td>
<td>0.070</td>
<td>0.095</td>
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<td>0.070</td>
<td>0.095</td>
<td>0.110</td>
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Table D1 $C_L$-clean as a function of $M$ and $\alpha$ (Korswagen, 1990).
<table>
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<th>$\alpha$ (°)</th>
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<th>6</th>
<th>8</th>
<th>10</th>
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</thead>
<tbody>
<tr>
<td>$M$</td>
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<td></td>
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<td>0.008</td>
<td>0.009</td>
<td>0.011</td>
<td>0.020</td>
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<td>0.053</td>
<td>0.077</td>
</tr>
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<td>0.007</td>
<td>0.006</td>
<td>0.006</td>
<td>0.007</td>
<td>0.011</td>
<td>0.020</td>
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<td>0.056</td>
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<td>0.006</td>
<td>0.007</td>
<td>0.013</td>
<td>0.022</td>
<td>0.037</td>
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<td>0.039</td>
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<td>0.015</td>
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<td>0.011</td>
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<td>0.006</td>
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<td>0.003</td>
<td>0.004</td>
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<td>0.019</td>
<td>0.028</td>
<td>0.040</td>
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<td>0.003</td>
<td>0.004</td>
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</tr>
<tr>
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<td>0.003</td>
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<td>0.012</td>
<td>0.016</td>
<td>0.025</td>
<td>0.039</td>
</tr>
</tbody>
</table>

Table D2 $C_p$-clean as a function of $M$ and $\alpha$ (Korswagen, 1993).

references

1) Korswagen, J.H.;
   "ASCENT Trajectory Simulation for Aerospace Planes; Development of a Simulation Program";

2) Shaughnessy, J.D., Pinckney, S.Z., McMinn, J.D.;
   "Hypersonic Vehicle Simulation Model; Winged Cone Configuration";
Appendix E Orthogonal matrices according to Taguchi

The Taguchi method of experimental design, varies parameters of interest in an all at the same time approach. In this way it becomes possible to find the main effect of each parameter on the objective, by using a far smaller amount of experiments as in the case that all possible combinations of parameter values are investigated. This reduction of number of experiments can be achieved by using orthogonal arrays (Taguchi, 1987). In an orthogonal array each column belongs to a parameter of interest which will be varied. The amount of rows of an orthogonal array corresponds to the amount of experiments which are required to obtain enough information.

In table E.1 an example of an orthogonal array is shown which can be used for the study of seven parameters. These parameters are varied at two levels. The numbers 1 and 2 in the matrix represent the lowest and highest level of each parameter respectively. When we observe two parameters, four combinations of parameter levels can occur: (1,1), (1,2), (2,1), and (2,2). The columns of the matrix in table 5.1 are called orthogonal because these four combinations (1,1), (1,2), (2,1), and (2,2) appear with the same frequency in each of two columns. With this matrix eight experiments are needed to get the wanted information. When the best combination was searched for by studying all possible combinations, than \(2^7 = 128\) experiments should have been done. This small example already shows what benefit can be gained by using the Taguchi method.

For detailed information about the derivation of orthogonal arrays one is referred to Taguchi (1987).
<table>
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<td>1</td>
<td>2</td>
</tr>
</tbody>
</table>

Table 5.1 example of an orthogonal array: the L8-table (Taguchi, 1987).

References

1) Taguchi, G.;
"System of experimental Design; Engineering Methods to Optimize Quality and Minimize Costs";
Appendix F Parameter settings for 36 simulations

<table>
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<tr>
<th>traj. phase</th>
<th>variable</th>
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<th>$\gamma_{\text{max}}$ (°)</th>
<th>$\delta_{\text{ylit}}$ (°/a)</th>
<th>$\gamma_{\text{cl}}$ (°)</th>
<th>$\rho_{\text{sim}}$ (kg/a)</th>
<th>$M_{\text{cr}}$</th>
<th>$h_{\text{cr}}$ (km)</th>
<th>$\delta_{\text{ylit}}$ (°/a)</th>
<th>$\gamma_{\text{cl}}$ (°)</th>
<th>$M_{\text{pull}}$</th>
<th>$n_{\text{zar}}$</th>
<th>$\gamma_{\text{pull}}$ (°)</th>
<th>$\gamma_{\text{max}}$ (°)</th>
</tr>
</thead>
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Table F1 The parameter settings for simulations 1-18.
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Table F2. The parameter settings for simulations 19-36.
### Appendix G Taguchi calculation results

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Table G1a: The fuel mass, propellant mass, and the payload mass resulting from simulations 1-18.
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Table G1b: The fuel mass, propellant mass, and the payload mass resulting from simulations 19-36.
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**mean value** = .1513139E+04

**total variation** ($S_T$) = .1901146E+09

**error variation** ($S_e$) = .4123059E+06

Table G2 Variance analysis data for payload mass.
<table>
<thead>
<tr>
<th>traj. phase</th>
<th>variable</th>
<th>Take off</th>
<th>first climb</th>
<th>cruise</th>
</tr>
</thead>
<tbody>
<tr>
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<td>( n_s )</td>
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<td>( \gamma_e )</td>
</tr>
<tr>
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<td>.545E+05</td>
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<table>
<thead>
<tr>
<th>traj. phase</th>
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<th>pull up manoeuvre</th>
<th>sec.stage</th>
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Table G3 Total of payload mass at each level for each parameter.

<table>
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<th>traj. phase</th>
<th>variable</th>
<th>Take off</th>
<th>first climb</th>
<th>cruise</th>
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<td>.581E+07</td>
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<table>
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<tr>
<th>traj. phase</th>
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<th>pull up manoeuvre</th>
<th>sec.stage</th>
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Table G4 Total of amount of fuel at each level for each parameter.
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<th>variation</th>
<th>perc. of total variation</th>
<th>polynomial coefficients</th>
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mean value fuel-propellant mass $= .1614444E+06$

| total variation ($S_t$) | .3704889E+10 |
| error variation ($S_e$) | .2716667E+08 |

Table G5 Variance analysis data for fuel mass
## Appendix H Polynomial calculations

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<tr>
<th>exp.no.</th>
<th>$\Delta m_{\text{error}}$ (kg)</th>
<th>$\Delta m_{\text{error}}$ (%)</th>
<th>$\Delta m_{\text{error}}$ (kg)</th>
<th>$\Delta m_{\text{error}}$ (%)</th>
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<table>
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<th>$\Delta m_{\text{error}}$ (kg)</th>
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Table H1: The differences between the results of the 36 simulations and the corresponding polynomial calculations.