WIND TUNNEL TESTS OF THE AEROELASTIC
STABILITY OF THE HEER-AGIMONT BRIDGE

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The purpose of the present research is to investigate in a wind tunnel, the aeroelastic stability of the suspension bridge erected at Heer-Agimont (Belgium). The tests have been carried out on a sectional model of the bridge deck at a scale of 1/50 and on a model of the full bridge at a scale of 1/100. In both cases, the dynamic characteristics (mass distribution, stiffness and degrees of freedom) of the bridge have been simulated with the appropriate scaling rules. The experiments were made in the open test section of the 3 m diameter L-1 wind tunnel. The wind velocity was varied until critical conditions, corresponding to self sustained bridge oscillation, were reached. Wind velocity, frequency of oscillations, mode and amplitude of the oscillations were recorded and used to define the stability characteristics of the bridge.
1. INTRODUCTION

A moving airstream exerts, on a fixed body, a force which is a function of the shape of the body and of the dynamic pressure (defined as $\frac{1}{2} \rho V^2$) of the airstream.

In the most common cases, such a force is assumed to be steady when the air velocity is constant. Thus, most of the buildings and structures exposed to the wind are submitted to aerodynamic mean steady forces which must be taken into account as external loads. These static forces are not acting alone on the structure: the natural wind velocity is normally fluctuating with time, and this leads to unsteady solicitation of the structures. Apart from the long period wind fluctuations which may be considered as time dependent changes of the static loads, the higher frequency oscillations produce superimposed random dynamic loading of the structure. This could be expected to become more critical for the excitation frequencies which are close to the natural frequency of the structure because of resonance phenomena.

There are, however, some situations, more important because of the high amplitude oscillation which can be generated, where a steady airstream may produce time dependent aerodynamic forces. The origin of such forces may be attributed to two completely different mechanisms. In the first one, known as vortex excitation, alternate vortices are formed in the wake behind a bluff structure.

This generates an alternate distribution of pressure on the two sides of the structure and thus an alternate force, the frequency
of which is proportional to the wind speed and function of the body shape. This force is oriented normally to the wind direction and its characteristic is to be present even on a rigid body. If the structure to which this force is applied is flexible an oscillatory motion will result which will assume critical proportion at resonance, that is, when the fundamental frequency of the structure and that of the wind excitation coincide. This will happen at a particular wind speed, often referred to as the critical wind speed.

In the second case, self-generated oscillations are caused by the dependence of the aerodynamic forces on the motion of the body itself. If the structure is flexible, this means that a change in position (which leads to a change of angle of attack) will develop a destabilizing aerodynamic force. Sinusoidal oscillations will be generated at the natural frequency of the structure which are limited only by the structural damping. A particular situation is that of coupled oscillations in a direction normal to the wind and in torsion (flutter). This becomes extremely severe in the case of structures in which the frequencies corresponding to the two mode of oscillation coincide or are very close.

In each case, the resulting motion of the structure generates inertial forces such that the final loading conditions may be very critical and lead to structural damage. Typical examples of structures for which the oscillation generated by a steady wind may play an important role are tall buildings, chimneys and masts, and, last but not least, suspension bridges. It is therefore very important to check, when dealing with such structures, their aerodynamic stability, as function of the wind velocity, and possibly the amplitude of the resulting wind excited oscillations. It is in general very difficult to evaluate these characteristics analytically, thus resort has to be made to wind tunnel tests on small scale models. To obtain correct results, the model should be scaled down not only geometrically but also dynamically, i.e., the mass distribution, stiffness,
and degrees of freedom of the real structure should be simulated. It is the purpose of the present note to derive and analyze the similarity parameters required for such a simulation, describe the possible simulation techniques and discuss their application to the particular case under consideration.

2. SIMILARITY PARAMETERS

One of the purposes of model tests in a wind tunnel is to reproduce on a scaled down version the behaviour of a structure under aeroelastic loads. For this reason, the model itself should be in dynamic similarity with the full structure: this means that the relative effects of the inertial and wind forces should be the same on the model than at full scale. Mathematically speaking, this requires an equality between the ratios of the coefficients of the equation for the oscillatory motion for the model and the full scale structure.

For the complete bridge structure these equations are relatively complex. However, with a suitable choice of coordinates they can be reduced to the basic equation of a mass-spring-damper system with two degrees of freedom exposed to an exciting force. The resulting equations can be considered to describe the bending and torsional motions of a characteristic part of the structure, e.g., the central part of the bridge deck.

With the coordinates shown in the figure, they can be written as

\[ m\ddot{y} + b_f \dot{y} + k_f y = f_f(v) \]
\[ J\ddot{\theta} + b_\theta \dot{\theta} + k_\theta \theta = f_\theta(v) \]

where $m$ = mass
$J$ = moment of inertia
$k$ = stiffness factor
$b$ = damping factor
$f(v)$ = wind generated force.

the subscripts $f$ and $\theta$ referring respectively to bending and torsion.
For a complex structure \( m \) represents the distributed mass; \( k \)
is a function of the elasticity modulus \( E \) of the material used for the construction, the moment of inertia \( I \) of the structure, the elasticity at the cables and so on; \( b \) is a function of the distributed structural damping of the structure.

The function \( f(v) \) can be expressed in terms of the relevant parameters as

\[
f_f(v) = c_f \frac{k^2}{\rho} \frac{1}{2} \rho u^2 \sin(\omega_1 t)
\]

\[
f_\theta(v) = c_\theta \frac{k^3}{\rho} \frac{1}{2} \rho u^2 \sin(\omega_2 t),
\]

where it is assumed that the excitation is essentially sinusoidal.

\( k \) is a typical length, \( u \) the wind velocity, \( \rho \) the air density, \( \omega_1 \) and \( \omega_2 \) the frequencies of the aerodynamic excitation in bending and torsion (not necessarily equal); \( c_f \) and \( c_\theta \) are aerodynamic coefficients depending on the shape of the structure. In general, they are dependent on the Reynolds number of the flow, however, this dependence becomes almost negligible when a critical value of the Reynolds number is exceeded. For non streamlined shapes (such as is the case for the bridge) this critical value is very low, so that the aerodynamic coefficients are practically equal on the real bridge and on the model even if the corresponding Reynolds numbers are different.

The frequencies \( \omega_1 \) and \( \omega_2 \) depend on the shape of the structure and on the wind velocity. Writing

\[
n_1 = \frac{\omega_1}{2\pi} \quad n_2 = \frac{\omega_2}{2\pi}
\]

one has

\[
n_1 = \frac{St_f u}{k} \quad n_1 = \frac{St_\theta u}{k}
\]
where \( St \) is the Strouhal number, dependent on the shape of the structure and on the value of the Reynolds number. Again, for prismatic structures, the dependence on \( Re \) becomes negligible above a fairly low critical value. Thus, \( St \) has the same value for the model and the full scale structure.

The amplitude of the oscillations described by eq. 1 becomes important at resonance, (see figure below), that is when the frequency of the aerodynamic phenomenon coincides with the natural frequency of the structure:

\[
\omega = \sqrt{\frac{k}{m}}.
\]

For this condition, taking into account the sinusoidal nature of the oscillations, eq. 1 can be written as

\[
\begin{align*}
-m \omega_1^2 a \cos(\omega_1 t) - b_f \omega_1 a \sin(\omega_1 t) + k_f a \cos(\omega_1 t) &= \frac{1}{2} c_f \rho u^2 \sin(\omega_1 t) \\
-J \omega_2^2 b \cos(\omega_2 t) - b_\theta \omega_2 b \sin(\omega_2 t) + k_\theta b \cos(\omega_2 t) &= \frac{1}{2} c_\theta \rho u^2 \sin(\omega_2 t)
\end{align*}
\]

The phenomenon becomes really severe when \( \omega_1 = \omega_2 \) which is the condition for coupled oscillations between the bending and torsional modes.

At resonance, \( \omega_1 \) and \( \omega_2 \) are the fundamental frequencies of the structure so that a corresponding "critical" wind velocity can be defined as:

\[
V_{f_{\text{crit}}} = \frac{n_1 \lambda}{St_f} \quad V_{\theta_{\text{crit}}} = \frac{n_2 \lambda}{St_\theta}
\]
Similarity laws may be derived from the above equations by equating the ratios of their coefficients when applied to the model and to the full scale structure.

Thus, using capital letters for the full scale structure, one obtains, from the equation for bending oscillations (2), for example:

- Firstly, for the mass similarity parameter

\[
\frac{a_m}{A_m} = \frac{n_1^2}{L} \frac{\rho}{N_1} \frac{u^2}{L^2} = \frac{\rho}{L} \frac{u^2}{L^2}
\]

But, with equal Strouhal numbers:

\[
\frac{n_1 L}{u} = \frac{N_1 L}{U} \quad \text{or} \quad \frac{u}{U} = \frac{n_1 L}{N_1 L}
\]

Thus

\[
\left( \frac{M}{m} \right) = \left( \frac{L}{L} \right) \frac{A}{A} = \left( \frac{L}{L} \right)^3
\]

(4)

if, for the geometrical similarity it is required \( \frac{a}{L} = \frac{A}{L} \)

- Secondly, for the stiffness similarity parameter

\[
\frac{k_f}{K_f} = \frac{m}{M} \left( \frac{n_1}{N_1} \right)^2 = \frac{\rho}{L^3} \left( \frac{n_1}{N_1} \right)^2
\]

(5)

- Thirdly, for the damping similarity

\[
\frac{b_f}{B_f} = \frac{\rho}{L^4} \frac{a}{N_1 A} = \frac{\rho}{L^3} \frac{n_1}{N_1}
\]

or introducing the damping factor
\[
\xi = \frac{b}{b_{\text{crit}}}
\]
where \(b_{\text{crit}} = 2\omega_m\)

\[
\frac{\xi_{m}}{E_{f}M} = \frac{\xi}{L^3} \quad \frac{\xi_{f}}{E_{f}} = 1
\]

which requires the equality of the damping factor on model and on full scale.

For practical reasons, the damping factor is often replaced by the logarithmic decrement \(\delta\). For sinusoidal oscillation and pure viscous damping, with a good approximation

\[
\xi = 2\delta
\]

and

\[
\delta = \frac{0.110 \, 2\pi}{c}
\]

where \(c\) is the number of oscillations required for the amplitude to decrease by a factor of 2.

In a similar way one can derive similarity parameters for the oscillations in torsion, to obtain:

- for the inertia

\[
\frac{J}{J_{m}} = \left(\frac{L}{L}\right)^{5}
\]

- for the torsional stiffness

\[
\frac{k_{\theta}}{K_{\theta}} = \left(\frac{n_2}{N_2}\right)^{2} \left(\frac{L}{L}\right)^{5}
\]

- and for the damping

\[
\frac{\xi_{\theta}}{E_{\theta}} = 1
\]
It should be noted that the similarity conditions may be somewhat relaxed if one considers that at resonance the acting forces are balanced only by the damping. Under such conditions, the ratio of the relevant terms and the assumption of equality of the relative amplitude of oscillations leads to

\[ \frac{\delta f^m}{L^3} = \frac{\delta f^M}{L^3} \]

for the bending mode \( (10) \)

\[ \frac{u}{U} = \frac{n_1 L}{N_1 L} \]

and to

\[ \frac{\delta \theta^j}{L^5} = \frac{\delta \theta^J}{L^5} \]

for the torsional mode \( (11) \)

\[ \frac{u}{U} = \frac{n_2 L}{N_2 L} \]

The laws \( (10) \) and \( (11) \) can hence be assumed to be simplified simulation laws to be used instead of laws \( (3) \) to \( (9) \).

Coming back to the complete laws, in order to simulate simultaneously the bending and torsional oscillations, i.e., for the correct reproduction of the aerodynamic stability with coupled motion, eq. \( (3) \) to \( (9) \) should be satisfied simultaneously thus

\[ \frac{n_2}{N_2} = \frac{n_1}{N_1} \] \( (12) \)

and

\[ \frac{k_\theta}{k_f} = \left( \frac{L}{L} \right)^2 \frac{k_\theta}{k_f} \] \( (13) \)

The dynamics of the model is representative of the dynamics of the full scale structure if conditions \( (3) \) to \( (9) \) and \( (12) \) are
satisfied. The wind velocity at which oscillations take place in the model is related to the wind velocity under full scale conditions by relation (3).

These relations show that for a given scale, there is only one remaining free parameter which can be chosen arbitrarily. Usually this is the ratio of the frequencies of model to full scale (which will condition the value of the stiffness).

Because at resonant frequency the only reaction to the aerodynamic unsteady forces is the damping force, the amplitude of the oscillations and the structural forces are given by:

- in the bending mode

\[ a = \frac{c_r b_f^2 \frac{1}{2} \rho u^2}{w_f b_f} = F_{\text{aerodyn}} \frac{2\xi_f k_f}{2} \]

\[ F_s = k_f a = \frac{F_{\text{aerodyn}}}{2\xi_f} \]  

(14)

- in the torsion mode

\[ b = \frac{M_{\text{aerodyn}}}{2\xi_\theta k_\theta} \]

\[ M_s = \frac{M_{\text{aerodyn}}}{2\xi_\theta} \]  

(15)

That is, the amplitude of the oscillations and the structural stresses are inversely proportional to the damping factor.

As already mentioned, eq. 1 and following are written for a complex system in generalized coordinates: this means that the terms referred to as mass, stiffness, damping are in fact a function of the mass distribution, stiffness distribution, damping distribution weighted by shape of the deformation curve. This requires as an additional constraint that for a complex structure the deformation on the model and on the prototype must be similar under the same arbitrary load conditions.
3. SIMULATION TECHNIQUES

Two methods are available for the simulation in a wind tunnel of the dynamic behaviour of a suspension bridge.

The first method which consists in using a "sectional model", is a particularly simple way of determining the aeroelastic stability of the bridge deck. It has been developed by the National Physical Laboratory (N.P.L., U.K.) and can be summarized as follows. For a suspension bridge, at least in the fundamental mode of oscillation, the largest amplitudes, both in bending and torsion, occur at the center of the structure. Furthermore, if the structure is very long, it can be assumed that the proper deformation of an elementary portion of the deck is negligible in comparison to the total deformation. Therefore, the motion of a short element situated at the center of the bridge consists essentially of a vertical displacement and a rigid rotation along the longitudinal axis. It is thus possible to separate such an element from the rest of the bridge and connect it to some rigid supports through elastic connections, which reproduce the elasticity of the remaining part of the structure. It could be expected that its motion, when exposed to the wind, will reproduce the motion of the central part of the deck. In particular the air velocities at which oscillations will take place should correspond to the critical velocities of the bridge.

Such a simulation technique has important drawbacks: first the element should be sufficiently long to ensure a two dimensional flow, which is typical of the wind flow on the bridge. Thus for the simulation to be correct the length of the bridge itself must be large in comparison to its width. Furthermore, the wind excitation is not representative of the complete state of excitation of the bridge: the effects of a non-uniform displacement of the deck are neglected, as well as the effects of all the other parts of the structure, such as pylons, cables, etc. Another important limitation is the impossibility of reproducing the higher modes of oscillation of the structure. Within these limitations, it can, however, be considered as a useful tool to obtain preli-
minary information on the aeroelastic stability of the deck and on the first, and most dangerous, critical velocities.

Many of these limitations can be overcome, at the expense of more complexity, by using the second method of simulation. In this case, a model of the complete bridge is reproduced in dynamic similarity and exposed to the wind.

If all the similarity parameters are satisfied, the wind excited oscillations of the model will be fully similar to those of the real structure.

An analysis of the similarity conditions reveals that it is generally impossible to satisfy simultaneously the mass and stiffness requirements by using the same raw material for the model than at full scale. It is necessary either to use a material with different ratio of density to modulus of elasticity, or to use a structure such that mass and density are simulated by two independent material. The last approach is by far the most widely employed.

For the present investigation both simulation methods, the sectional model and the complete model, have been used.
4. THE MODELS

4.1 Sectional model

For the wind tunnel tests, the central part of the bridge deck was reproduced as a solid model suspended by connections having the required degrees of freedom and flexibility.

The tests were made in the low speed wind tunnel L-1 of the von Karman Institute. It has a free open test section of 3 m in diameter. The scale was 1/50, which resulted in a length for the model of 644 mm, with a span of 296 mm. A picture of the element is shown in Fig. 2, whereas Fig. 3 shows the model suspended to its supports by the flexible joints.

The aspect ratio was large enough for the model to be considered as two dimensional, especially when it was mounted between two vertical plates, as shown in Fig. 4. A horizontal plate placed under the model was used to simulate the water level of the river Meuse. It could be displaced vertically to simulate different water levels.

Tests were made in a uniform flow and in a flow having the same velocity gradient as the natural wind (Fig. 5). The latter test condition was obtained by using a series of cylindrical bars of variable diameter and spacing, located at the exit section of the wind tunnel contraction.

The mass and the angular inertia of the bridge model deck were determined following the rules established in the previous chapter. The bending stiffness was reproduced by connecting the model to the rigid support by means of a flexible parallelogram on each side of the model. The elasticity was obtained by using flexures on which strain gauges were mounted, calibrated to measure the bridge deflection. This allowed a vertical displacement. Elasticity in torsion was achieved by using torsion bars to connect the bridge model to the deformable
parallelogram, as shown in Fig. 6. These were positioned to coincide with the center of rotation of the deck. Again strain gauges were used to measure the angular displacement. For the measurements, the gauges were connected to a compensation bridge and the output recorded on a high speed ultraviolet photographic paper recorder.

This model could only simulate the fundamental mode of oscillations of the bridge, the frequency of which was chosen to be, for the bending mode, equal to

$$n_f = 16.5 \text{ Hz.}$$

The corresponding logarithmic damping was

$$\delta_f = 0.135$$

At the time of the tests no data was available on the torsional rigidity of the full scale bridge. Consequently, three different values were chosen. They are given in the table below, together with the corresponding logarithmic decrement.

<table>
<thead>
<tr>
<th>$n_\theta$</th>
<th>$\delta_\theta$</th>
</tr>
</thead>
<tbody>
<tr>
<td>9.36</td>
<td>0.010</td>
</tr>
<tr>
<td>17.7</td>
<td>0.020</td>
</tr>
<tr>
<td>29.05</td>
<td>0.052</td>
</tr>
</tbody>
</table>

As it may be seen the selected values of the $\delta_\theta$ were lower than $\delta_f$. It was discovered later, on the basis of measurements made on the complete model that this situation was slightly unrealistic. However, selecting a low value for the damping gave safer results. In any case results for other values of the damping can be obtained by using the relation for the amplitude of the oscillation $A$:

$$\frac{A_f}{\delta_f} = \text{const.}$$

$$\frac{A_\theta}{\delta_\theta} = \text{const.}$$
The model simulated all the structural elements of the bridge deck. The bridge deck itself was made out of balsa wood, strengthened by an aluminum plate and the structural elements were made out of brass. The mass and the angular inertia were adjusted by locating bob weights in predetermined positions. The angular inertia around the center of gravity was checked by measuring the frequency of the oscillations of the deck around axes parallel to the longitudinal axes passing through the center of gravity.

4.2 The complete model

The complete model was reproduced at a scale of 1/100, its length being limited by the dimensions of the wind tunnel. For the tests, it was placed on a round turntable simulating the surroundings of the real bridge over a radius of 150 m. The turntable enabled the rotation of the complete model around a vertical axis to simulate different horizontal wind directions. The model itself was dynamically decoupled from round table to avoid spurious vibrations of the model. Tests were made in uniform flow and with a vertical velocity gradient simulating the boundary layer of the natural wind. A picture of the model is shown in Fig. 6.

The overall bridge length was 2.040 mm for a deck span of 148 mm. It is practically impossible to construct a model in dynamic similarity using the same constructional principles as for the prototype. The solution chosen for the model was to reproduce the combined elasticity of the deck superstructure and of the longitudinal beams, by two circular cross section beams located on each side of the deck, Fig. 7. 21 elements, such as the one shown in Fig. 8, were fixed to the two beams, with a view to reproducing the correct geometry, mass distribution, and angular inertia. The scaling down was made by using the rules derived in chapter 3, and again the value of angular inertia was measured by freely oscillating each of the elements. Balsa wood, aluminum and brass were used for reproducing the structural
shape of the model, as it was done for the sectional model.

The elasticity of the two beams was computed on the basis of the data given by the designers of the prototype on the behaviour of the isolated deck.

The torsional stiffness, as in the real bridge, was mainly resulting from the antisymmetric bending of the two main beams, and on the model was adjusted by the interconnection of the deck elements with the beams.

The use of springs was required to correctly simulate the cables, as shown in Fig. 9. The pylons were made of rigid structures free to rotate at their base.

With such a model it was possible to reproduce the first mode of oscillation (symmetric bending), the second mode (antisymmetric bending), as well as higher modes of vibration. However, it was impossible to excite stable modes of oscillations higher than the second one.

The deformations were measured directly by means of 10 strain gauges symmetrically located on the two main beams and indirectly by displacement sensors installed on the ground plate and connected to the bridge by wires under tension. These tensors (shown in Fig. 10) used strain gauges and were designed for this particular application. The gauge connections could be made either to measure simultaneously the deflection at three locations, or the bending and torsion at one location. Their natural frequency was chosen to be much higher than the frequency of oscillation of the bridge.

No direct information was available on the frequencies of oscillation of the full scale bridge. Thus, all the data for the model was computed on the basis of theoretical information on the mass distribution and on the elasticity of the different elements, as given by the designers (Soc. GIREC). No information at all was available on the structural damping of the full scale bridge.
For the final set up of the model, the cables were prestressed so as to reproduce the static deflection of the bridge over its full length. The computed stiffnesses were checked by measuring the deflections in bending and torsion, for concentrated and distributed loads. These deflections were compared with those measured on the real bridge in the course of the preliminary loading tests. A good agreement was obtained as shown in Fig. 11.

The frequencies of oscillation of the bridge model could have been computed by using the Stodola-Vianello method once the static deformation was known. However, it was decided to use the simplified approach described here. The deformation of the central part of the bridge stiffened by the cables was assumed to be close to that of an equivalent beam clamped at both ends and of different stiffness. This gives

\[
\omega = \frac{\lambda^2}{\xi^2} \sqrt{\frac{E I}{\mu}}
\]

with
\[
\lambda = \text{length} \\
\mu = \text{mass per unit length}
\]

\[
\lambda = 4.73 \text{ for the fundamental mode} \\
= 7.85 \text{ for the second mode.}
\]

If the deflections \( \delta \) at the center of the equivalent beam and of the bridge are matched for a concentrated central load, one obtains

\[
E I \delta = \frac{1}{192} P l^3
\]

or

\[
\omega = \frac{\lambda^2}{\xi^2} \sqrt{\frac{1}{192} \frac{Pl^3}{\delta \mu}}
\]

With the parameters involved this leads to

\[
n_f = 7.67 \text{ Hz for the fundamental mode} \\
n_f = 21.16 \text{ Hz for the second mode.}
\]
No attempt was made to compute the torsional frequencies. The values measured on the model were

\[ n_f = 8.77 \text{ Hz} \]
\[ n_\theta = 14.5 \text{ Hz} \]

with a corresponding damping of

\[ \delta_f = 0.057 \]
\[ \delta_\theta = 0.34 \]

The obtained ratio of torsional to bending damping was expected to be similar to that in full scale, because of the difficulty met in trying to excite the torsional oscillations both on the model and in full scale. The other available information was that the torsional frequency of the bridge was higher than the bending one.

With the simulation rules used (ratio of the stiffness), the ratio of model to full scale frequencies would be

\[ \frac{N}{n} = \frac{1}{10} \quad \text{thus} \quad \frac{N_f}{n_f} = 0.057 / 8.77 = 0.006 \]
\[ \frac{N_\theta}{n_\theta} = 0.34 / 14.5 = 0.023 \]

In absence of other information these values will be used for the extrapolation of the results obtained on the model to compute the real critical wind speeds. No unique extrapolation to full scale could be made for the amplitude of oscillations in absence of any reliable value for the damping of the structure.

As explained in section 3, the deflections of the model and of the prototype must be similar. This has been checked under static conditions for two load distributions and the results are shown in Fig. 11. A picture of the bridge installed in the wind tunnel is shown in Fig. 12.
5. PRESENTATION AND DISCUSSION OF RESULTS

5.1 Sectional model

A first series of tests has been made in a wind with uniform velocity and for angles of attack of the bridge deck (defined in Fig. 13) of 0°, -5° and +5° to simulate a possible effect of the surroundings. Because of the construction of the model, it was impossible to modify the yaw angle, which was 0° for all the tests.

As mentioned in section 4, the model only simulate the fundamental mode of vibration in bending and torsion. Thus, only two critical velocities exist, which correspond to a maximum amplitude of oscillations either in bending or in torsion.

A third mode of vibration may exist, in which coupled oscillations in torsion and bending are excited simultaneously. It represents the most critical loading of the bridge. This is possible either when the two fundamental frequencies (or their harmonics) are equal or when the damping is small at the lowest frequency.

The measured critical wind velocities are presented in table 1 for the bending and the torsional oscillation modes, in function of the different parameters tested.

For some specific values of these parameters a coupling between the two modes of oscillation has been observed. An example of the variation of the amplitude of oscillation versus the wind velocity is shown in Fig. 14. In this case there was no coupling, i.e., the two peaks in amplitude were independent and occurred at different wind speeds. From the analysis of the behaviour of the complete model it turned out that the parameters given in this figure are quite representative of the full scale bridge.
An example of oscillations with coupling is shown in Fig. 15. In this case, the stiffness in torsion was so low that a simultaneous excitation in bending and torsion appeared at the critical wind velocity for pure bending oscillations. The oscillations persisted when the wind speed was increased above the critical value. They were maintained with very large amplitudes by the action of a new phenomenon independent of wind velocity, known as aerodynamic instability.

The amplitudes shown in the figures are those measured on the model. They cannot be extrapolated directly to full scale because of the imperfect simulation obtained with the sectional model and of the unknown value of the damping of the real structure.

An example of the effect of the damping on the amplitude of oscillations is shown in Fig. 16. The measurements are compared with the theoretical trend.

The dimensionless parameter which characterizes the unsteady aerodynamics of the bridge deck is the Strouhal number, defined by

\[ St = \frac{n \cdot \ell}{U_c} \]

where \( n \) = frequency of oscillations  
\( \ell \) = typical length (chosen in this case as the height of the bridge deck = 45 mm)  
\( U_c \) = critical velocity of the wind

From the results obtained at 0° angle of attack:

\[ St_f = 0.1 \quad \text{for the bending mode, and} \]

\[ St = 0.081 \times 0.1 \quad \text{for the torsional mode}. \]
These values characterize the full scale structure. With the estimated bending frequency of the bridge (see section 4) of 0.87 Hz, the first critical velocity $U_c$ would be:

$$\frac{NL}{U_c} = St = 0.1$$

$$U_c = \frac{0.87 \cdot 2.25}{0.1} = 19.6 \text{ m/sec} = 70.4 \text{ km/h}.$$  

This value of the wind velocity is sufficiently low to represent a possible practical situation. The corresponding stresses on the bridge structure can be determined by the relations of section 4.1, when the damping factor if the real structure is known.

A small dependence of $St_f$ and $St_\theta$ on the angle of attack was found to exist.

The same tests were repeated in a non uniform flow simulating the velocity gradient of the natural wind. The reference velocity was selected in this case as the velocity at the level of the bridge deck. The results are presented in table 2. There was hardly any difference between the results of these tests and those with uniform flow: the critical velocities were practically the same and the amplitudes of oscillation only slightly lower.

5.2 The complete model

5.2.1 Generalities

The critical velocities were measured in a uniform flow and in a flow simulating the velocity gradient of the atmospheric wind. The scale of the model of the complete bridge was small enough to enable the simulation of some of the turbulent properties of the wind, namely, the low frequency velocity fluctuations and the non-uniformity at the velocity fluctuations along the bridge length.
Tests were again made for angles of attack of -5°, 0° and +5° obtained by rotating the complete model with respect to the ground plate. The effect of the yaw angle was investigated in the range -30° to +30°, obtained by rotating the turntable which was supporting the model (as defined in Fig. 13).

As mentioned in section 4, the complete model may simulate higher modes of oscillation than the first one. To detect the amplitudes and the modes of oscillation, couples of strain gauge transducers were installed at 5 different locations along the two main supporting beams. They gave an indication of the local bending stresses as well as of the torsional stresses by taking the difference between the readings of the corresponding gauges on the two beams. The actual deflections both in bending and in torsion were also measured with the technique discussed in section 4, at three different positions along the bridge span. The location of the transducers is indicated in figures 17 and 18.

5.2.2 Tests in uniform wind

Self sustained oscillations were obtained for the fundamental mode and for the first harmonic both for bending and torsion. As expected from the values of the corresponding frequencies, critical velocities for the torsional mode were higher than for the bending mode. No coupling between the two modes was observed. However, for certain wind velocities, an alternance of oscillations from bending to torsion was visualized. A similar phenomenon was also observed between the bending in the fundamental mode and in the first harmonic. An example is given in the oscillograms of Fig. 19. In all cases, this happened when the amplitude of oscillations was relatively small and at wind velocities higher than the first critical wind speed for bending.

A summary of the results obtained is presented in table 3 as critical wind speeds, corresponding to a peak in the amplitude of oscillations, frequency of oscillations and amplitude of oscillations.
The Strouhal number $\text{St}_f$ for the bending mode, computed from the results was consistently equal to

$$\text{St}_f = 0.114$$

for a yaw angle equal to $0^\circ$ and for the fundamental oscillation mode.

A slight dependence on yaw angle and on angle of attack was found to exist ($0.107 < \text{St}_f < 0.116$), as shown in Fig. 20. For the first harmonic, a lower value of the Strouhal number was found:

$$\text{St}_f = 0.078 \quad \text{(first harmonic)},$$

with again a weak dependence on yaw angle and angle of attack.

These results were in good agreement with the results given by the sectional model.

The first critical speed for the full scale structure is given by

$$V_c = \frac{NL}{\text{St}_f} = \frac{2.25 \times 0.87}{0.114} = 17.1 \text{ m/sec} = 62 \text{ km/h}.$$ 

The amplitudes of the oscillations measured on the model are given in the tables. At full scale these values will be 100 times larger if the structural damping is the same as on the model. Correction for different dampings could be estimated, as a first approximation, using relations (14) and (15).

The oscillations in torsion appeared at higher wind velocities and their amplitude was relatively small in all the tests (Table 3). The Strouhal number was found to vary in the range:

$$0.1 < \text{St}_\theta < 1.25$$
which is again in good agreement with the results obtained with the sectional model.

An example of the dependence of amplitude of oscillation, both for the bending mode and the torsional mode, on the wind speed is shown in Fig. 21. The largest amplitudes are reached for the fundamental frequency, and thus for the lowest wind speeds. Furthermore, Fig. 22 indicates that these oscillations were the most stable ones: once they are triggered their amplitude remained constant, whereas for the oscillations at higher speeds the amplitude fluctuated randomly. From the analysis of the same figure, the sinusoidal nature of the oscillations in the fundamental mode is also very evident.

The oscillations at fundamental frequency critically depended on wind speed: indeed, they only existed for a value of the wind velocity contained between very narrow limits (< 10%).

5.2.3 Tests in simulated atmospheric wind

It can be expected from the previous conclusion that the effects of the gustiness of the natural wind should be important in reducing the amplitude of the oscillations at the fundamental mode and in making them more random. This was confirmed by the tests made with a simulated atmospheric wind, as shown in Fig. 23, which is a direct record of the oscillations.

As it may be seen, the wind velocity fluctuated and the oscillations of the structure had a random nature even at the fundamental frequency. This characteristic remained at higher speeds. The gradient of the wind is shown in Fig. 24.

A summary of the results obtained with the simulated wind gradient is presented in table 4. The reference wind velocity here is the velocity at the height of the bridge deck.
Similarly to the case of uniform flow, a positive angle of attack of the wind is leading to more critical results.

The critical velocities are nearly the same as for the uniform wind case, both for bending and for torsion. Their exact determination is, however, a little more difficult because of the random nature of the amplitude of the oscillations mentioned earlier.

In general, it can be said that the turbulence of the wind, by decreasing the coherence of the excitation on the structure, decreases the aerodynamic sollicitations at the critical velocities.

Coupled oscillations (torsion and bending simultaneously over a large range of wind speed) were never detected. This can be considered as a safe feature of the structure which has been tested.
6. CONCLUSIONS

Tests were made on models of the Heer-Agimont suspension bridge. Both a sectional model at a scale 1/50 and a complete model of the prototype (scale 1/100) were used to determine the aeroelastic characteristic of the structure.

The models were subsequently exposed to a uniform wind and to a stream simulating the gradient of the natural wind. The results obtained from the two models showed a very good agreement and indicated the existence of a critical wind speed at approximately 62 km/h in uniform wind. This corresponded to bending oscillations in the fundamental mode. Torsional oscillations were of smaller amplitude and excited only at higher wind speeds.

Oscillations at higher modes, essentially the first harmonic, antisymmetric mode, were also obtained on the complete model. Oscillations were observed to be essentially sinusoidal at the fundamental mode of oscillation.

A slight effect of the yaw angle on the critical velocity was noticed.

The effect of a high turbulence of the wind was to decrease the amplitude of the oscillations and to make them more random even at the first critical velocity. This may be considered a feature increasing the safety of the structure.

Coupled oscillation in the torsional and bending mode have never been observed, which is again a safety feature.
FIG. 5  WIND GRADIENT SECTIONAL MODEL
FIG. 11 STATIC DEFLECTIONS ON MODEL AND FULL SCALE

- Concentrated load at centre:
  - + model 200 gr
  - * full scale 50 T

- Distributed load:
  - □ model 4.13 gr/cm over 120 cm
  - ○ full scale one row of 5 trucks
DEFINITION OF ANGLE OF ATTACK

DEFINITION OF YAW ANGLE

FIG. 13
**FIG. 14** EFFECT OF WIND SPEED ON AMPLITUDE OF OSCILLATIONS. SECTIONAL MODEL
FIG. 15  EFFECT OF WIND SPEED ON AMPLITUDE OF OSCILLATIONS. SECTIONAL MODEL

n_f = 16.5 Hz
n_θ = 9.36 Hz
α = 5°
FIG. 16 EFFECT OF DAMPING ON AMPLITUDE OF OSCILLATIONS

\[ A = \frac{\text{const.}}{\delta} \]
FIG. 18
AMPLITUDE MEASUREMENT POINTS

620
390
370

FIG. 19b EXAMPLE OF ENERGY TRANSFER FROM FUNDAMENTAL TO HARMONICS.
FIG. 20  EFFECT OF YAW ANGLE ON CRITICAL WIND SPEED.
FIG. 21a AMPLITUDE OF BENDING OSCILLATIONS VS. WIND SPEED - COMPLETE MODEL
FIG. 21b AMPLITUDE OF TORSIONAL OSCILLATION VS. WIND SPEED - COMPLETE MODEL
### TABLE 1

SECTIONAL MODEL - UNIFORM WIND

<table>
<thead>
<tr>
<th>( \theta )</th>
<th>( \alpha )</th>
<th>critical speed m/sec</th>
<th>bending mm</th>
<th>critical speed m/sec</th>
<th>torsion (')</th>
<th>Notes</th>
</tr>
</thead>
<tbody>
<tr>
<td>29,05</td>
<td>0°</td>
<td>7,48</td>
<td>2,85</td>
<td>13,5</td>
<td>2,14</td>
<td>small coupling at 13,5 m/sec</td>
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<td>5°</td>
<td>7,48</td>
<td>9,12</td>
<td>11,3</td>
<td>1,93</td>
<td>small coupling</td>
</tr>
<tr>
<td></td>
<td>-5°</td>
<td>7,04</td>
<td>4,16</td>
<td>11,5</td>
<td>0,728</td>
<td>small coupling</td>
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<td>2,84</td>
<td>9,88</td>
<td>1,07</td>
<td>coupling at 7,59</td>
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<tr>
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<td>5°</td>
<td>8</td>
<td>10</td>
<td>8,2</td>
<td>8,1</td>
<td>coupling&gt;8 m/sec</td>
</tr>
<tr>
<td></td>
<td>-5°</td>
<td>6,93</td>
<td>3,49</td>
<td>8,2</td>
<td>8,1</td>
<td>coupling&gt;8 m/sec</td>
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<td>2,52</td>
<td>4</td>
<td>0,45</td>
<td>coupling&gt;10 m/sec</td>
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<td>1,7</td>
<td>4,77</td>
<td>1,7</td>
<td>coupling&gt;5,5 m/sec</td>
</tr>
<tr>
<td></td>
<td>-5°</td>
<td>6,93</td>
<td>5,24</td>
<td>10,6</td>
<td>7,43</td>
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### TABLE 2

SECTIONAL MODEL - WITH WIND GRADIENT

<table>
<thead>
<tr>
<th>( \theta )</th>
<th>( \alpha )</th>
<th>critical speed m/sec</th>
<th>bending mm</th>
<th>critical speed m/sec</th>
<th>torsion (')</th>
<th>Notes</th>
</tr>
</thead>
<tbody>
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<td>7,04</td>
<td>2,39</td>
<td>5,14</td>
<td>0,76</td>
<td>coupling&gt;5 m/sec</td>
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<tr>
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<td>5°</td>
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<td></td>
<td>3,1</td>
<td>1,8</td>
<td></td>
</tr>
<tr>
<td></td>
<td>-5°</td>
<td>7,04</td>
<td>3,56</td>
<td>4,77</td>
<td>1,7</td>
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TABLE 3

COMPLETE MODEL - UNIFORM WIND

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<thead>
<tr>
<th>$\psi$ yaw</th>
<th>$\alpha$ incidence</th>
<th>critical speed m/sec</th>
<th>bending peak to peak point b mm nf</th>
<th>point a mm nf</th>
<th>point c mm nf</th>
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<td>1.8</td>
<td>0.40 8.7</td>
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<td>0.37 8.7</td>
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<td></td>
<td></td>
<td>4.7</td>
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<td>5.8</td>
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<td>0.87 8.8</td>
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<tr>
<td>15°</td>
<td>0°</td>
<td>1.63</td>
<td>0.54 8.7</td>
<td>1.33 8.7</td>
<td>0.47 8.7</td>
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<td></td>
<td></td>
<td>5.3</td>
<td>0.50 8.5</td>
<td>0.80 8.5</td>
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<td>0.72 8.6</td>
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<td></td>
<td>3.38</td>
<td>0.48 17.9</td>
<td>0.60 17.9</td>
<td>0.63 17.9</td>
</tr>
<tr>
<td></td>
<td></td>
<td>4.6</td>
<td>0.30 8.4</td>
<td>0.53 8.4</td>
<td>0.39</td>
</tr>
<tr>
<td></td>
<td></td>
<td>5.6</td>
<td>0.41 8.4</td>
<td>0.89 8.4</td>
<td>0.56</td>
</tr>
<tr>
<td>15°</td>
<td>5°</td>
<td>1.85</td>
<td>0.86 8.7</td>
<td>2.41 8.7</td>
<td>0.76 8.7</td>
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<td></td>
<td>3.30</td>
<td>0.30 18</td>
<td>0.40 9.0/18</td>
<td>0.39 18</td>
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</tbody>
</table>

<table>
<thead>
<tr>
<th>$\psi$ yaw</th>
<th>$\alpha$ incidence</th>
<th>critical speed m/sec</th>
<th>bending peak to peak point a mm/m nf</th>
<th>torsion peak to peak point a-a' n/a</th>
</tr>
</thead>
<tbody>
<tr>
<td>0°</td>
<td>0°</td>
<td>1.48</td>
<td>1.60 8.9</td>
<td>2.7</td>
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<tr>
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<td>1.64</td>
<td>1.48 8.8</td>
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<tr>
<td></td>
<td></td>
<td>3.03</td>
<td>0.37 9.0</td>
<td>5.0 15.5/32</td>
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<td>0.6 9.4</td>
<td>8.0 15.3/32</td>
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<tr>
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<td>1.85</td>
<td>2.92 16.4</td>
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<tr>
<td></td>
<td></td>
<td>4.90</td>
<td>11.0 15.6</td>
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</table>

* antisymmetric
<table>
<thead>
<tr>
<th>( \psi ) yaw</th>
<th>( a ) incidence</th>
<th>critical speed m/sec</th>
<th>bending peak to peak</th>
<th>torsion peak to peak ( n_\theta )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0°</td>
<td>0°</td>
<td>1.92</td>
<td>0.23 8.9</td>
<td>0.56 8.8</td>
</tr>
<tr>
<td></td>
<td></td>
<td>2.7</td>
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<td>0.51 8.2</td>
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<tr>
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<td></td>
<td>3.2</td>
<td>0.52 9.1/18</td>
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<td>4.8</td>
<td>1.09 8.9</td>
<td>2.92 8.4</td>
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<tr>
<td></td>
<td></td>
<td>5.4</td>
<td>1.18 9.08</td>
<td>2.11 8.9</td>
</tr>
</tbody>
</table>

| 15° | 0° | 1.7 | 0.26 8.8 | 0.66 8.8 | 0.24 8.8 |
| | | 2.94 | 0.32 11.2/17 | 0.56 8.90 | 0.31 11.4/ |
| | | 3.25 | 0.55 8.80 | 1.43 8.80 | 0.59 8.9 |
| | | 4.30 | 0.74 9.25 | 1.41 8.70 | 0.60 7.7/ |
| | | 5.00 | 0.92 10.9/18.1 | 1.75 9.00 | 0.78 17.7 |
| | | 5.60 | 0.86 9.00/17.4 | 1.51 8.90 | 0.78 17.7 |

<table>
<thead>
<tr>
<th>( \psi ) yaw</th>
<th>( a ) incidence</th>
<th>critical speed m/sec</th>
<th>bending peak to peak</th>
<th>torsion peak to peak ( n_\theta )</th>
</tr>
</thead>
<tbody>
<tr>
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<td>0°</td>
<td>1.64</td>
<td>0.55 8.9</td>
<td>2.4</td>
</tr>
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<td>0.92 9.0</td>
<td>7.6 15.0</td>
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<td>1.48 8.9</td>
<td>9.3 14.8</td>
</tr>
<tr>
<td></td>
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<td>13.5 14.8</td>
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<td></td>
<td>5.50</td>
<td>2.0 8.8</td>
<td>23.6 14.4</td>
</tr>
</tbody>
</table>

| 0° | 5° | 2.40 | 6.5 15.6 |
| | | 2.76 | 11.3 15.2 |
| | | 3.80 | 13.5 15.7 |
| | | 4.36 | 15.0 15.3 |
| | | 5.40 | 26.0 15.2 |

| 15° | 0° | 2.76 | 7.0 15.5 |
| | | 3.43 | 10.5 14.6 |
| | | 4.60 | 14.0 14.4 |
| | | 5.40 | 28.0 14.8 |