verification of numerical wave propagation models with field measurements

CREDIZ verification Haringvliet

W 488 part 1b, appendices

December 1983
verification of numerical wave propagation models with field measurements

CREDIZ verification Haringvliet

M. W. Dingemans

W 488 part 1b, appendices

December 1983
CONTENTS

A  Description of the CREDIZ model. Theoretical background. .......... 138
    A.1  Introduction.................................................... 138
    A.2  Initial and boundary conditions............................... 139
    A.3  Additional physical effects.................................... 141
        A.3.1  Amplitude effect on wave celerity.......................... 141
        A.3.2  Wave breaking............................................... 142
        A.3.3  Bottom friction............................................. 143
    A.4  The parabolic approximation.................................... 144
    A.5  Summary.......................................................... 145

B  Description of the CREDIZ model. Numerical approach................. 147
    B.1  Introduction.................................................... 147
    B.2  Computation of propagation in x-direction..................... 148
    B.3  Discretization in the y-direction............................. 150
    B.4  Absorption of wave energy at internal boundaries.............. 151
    B.5  Computation of wave parameters................................ 151
    B.6  Interpolation of computed function values..................... 153
    B.7  Contour lines of amplitude and phase.......................... 155

C  The bottom geometry.................................................. 158

D  Energy dissipation due to breaking waves............................ 164

E  Energy dissipation due to bottom friction............................ 170
    E.1  Introduction.................................................... 170
    E.2  The full Rayleigh distribution................................ 171
    E.3  The clipped Rayleigh distribution.............................. 172

F  Description of the 1-D model........................................ 176
    F.1  Introduction.................................................... 176
    F.2  Refraction...................................................... 176
    F.3  Dissipation...................................................... 178
    F.4  Wave growth due to wind........................................ 179

G  A relation between α and γ.......................................... 182
APPENDIX A – Description of the CREDIZ-model. Theoretical background

A.1 Introduction

The parabolic approximation is based on the following equation for time-harmonic wave motion (cf. Booij (1981)):

\[ \nabla(\frac{c_c}{g} \nabla \phi) + 2i\omega \nabla \phi - (\omega_r^2 - \omega^2 - k^2 \frac{c_c}{g}) \phi = 0, \tag{A.1} \]

where terms with \( \nabla \hat{U} \) and \( |\hat{U}|^2 \) are neglected. In Eq. (A.1) \( \hat{U} \) is the (steady) current velocity \((U_1, U_2)\), \( \nabla \) is the horizontal gradient operator \((\partial/\partial x_1, \partial/\partial x_2)\), \( \phi(x_1, x_2) \) is the complex wave potential function, \( k \) is the wave number, \( \omega \) is the absolute angular frequency and \( \omega_r \) is the relative angular frequency. The absolute and relative frequencies are related by

\[ \omega = \omega_r + k \cdot \hat{U} \tag{A.2} \]

where \( \omega_r \) fulfills the linear dispersion relation

\[ \omega_r = \left[ gk \tanh kh \right]^{1/2} \tag{A.3} \]

with \( g \) being the gravity acceleration and \( h \) the local depth.

It is assumed that the wave amplitudes \( a \) are small (both \( a/h \ll 1 \) and \( ka \ll 1 \)) and that the current velocity and the depth vary slowly in space. (For the moment, dissipation is neglected; later on, dissipation terms will be added to the model.)

In the parabolic approximation the assumption is that the waves propagate mainly into a specific direction \( s \). Introducing the coordinate \( n \), which is orthogonal to \( s \), an operator \( M \) can be defined by:

\[ M \phi = (\omega_r^2 - \omega^2) \phi + \frac{3}{\delta n} (\beta \frac{\partial \phi}{\partial n}) + 2i\omega U_n \frac{\partial \phi}{\partial n} \tag{A.4} \]

where \( \beta = \frac{c_c}{g} \) and \( U_n \) is the current component in the direction \( n \).

The parabolic approximation to Eq. (A.1) is then given by:
\[
\frac{i \omega U}{b} \left( \frac{\partial}{\partial s} + \frac{3}{9 s} \right) \left[ \sqrt{\beta k} \phi + \frac{P_1}{k \sqrt{\beta k}} M \phi \right] - i k \sqrt{\beta k} \phi - i \frac{P_2}{\sqrt{\beta k}} M \phi = 0 ,
\]
(A.5)

where the coefficients \( P_1 \) and \( P_2 \) are related by

\[
P_2 = P_1 + \frac{1}{2} , \quad 0 \leq P_1 \leq \frac{1}{2} .
\]
(A.6)

(These coefficients appear in the derivation of Eq. (A.5), where pseudo operators have to be approximated by differential operators. For a simple case the derivation of these coefficients is given in Section A.4.)

As the wave number vector \( \mathbf{k} \) is not exactly known beforehand, the relative frequency \( \omega_r \) in (A.2) is approximated by

\[
\omega_r = \omega - r_f k U_s ,
\]
(A.7)
in which \( r_f \) is a reduction factor expressing the fact that the waves do not exactly follow the s-direction \( (0 \leq r_f \leq 1) \).

Equation (A.5) will be used as the basis for the numerical model. For the coefficients \( P_1, P_2 \) and \( r_f \) the following standard values are chosen:

\[
\begin{align*}
P_1 &= 1/4 \\
P_2 &= 3/4 \\
r_f &= 0.9 .
\end{align*}
\]

A.2 Initial and boundary conditions

The solution of (A.5) requires the availability of initial and boundary conditions. The initial values can be derived from the incoming wave field, which may be (weakly) non-uniform with regard to amplitude and direction. The boundary conditions are more difficult to establish, as the wave field is, generally, not known along the lateral boundaries: one has to deal with the presence of artificial open boundaries, requiring that
- waves approaching the boundary from inside are not reflected there,
- waves do not enter the computational region through the boundary.

A simple boundary condition is given by
\[
\cos x \frac{\partial \phi}{\partial s} + \sin x \frac{\partial \phi}{\partial n} = i k \phi .
\]

(A.8)

This condition absorbs waves with local wave number \( k \) approaching under an angle \( x \) exactly, and waves in other directions partially. As standard value is taken \( x = 20^\circ \).

Another method which works perfectly for all incident angles has been proposed by W.D. Smith (1974). With this approach, the computation is done twice for each absorbing (straight) boundary; once with Dirichlet boundary conditions, and once with Neumann boundary conditions. Because these two boundary conditions produce reflections which are opposite in sign, the sum of the two cases will cancel the reflections.

In order to use Smith’s method correctly for the parabolic equation model, further assumptions must be made:
- the lateral boundaries should be taken sufficiently far away from each other in order to avoid multiple reflections. Then only two solutions need to be averaged for eliminating artificial reflections on both boundaries;
- a constant-coefficient linear wave equation is required along the boundary, which implies uniform depth and current along the open boundary.

The second requirement often cannot be met; however, when the depth (and consequently, the wave number) is slowly varying along the boundary, and the current can be neglected there, the method is approximately valid.

The Neumann condition gives:

\[
\frac{\partial \phi_N}{\partial n} = 0 .
\]

(A.9a)

The Dirichlet condition gives:

\[
\phi_D = \exp(i / k \, ds) \quad \text{or} \quad \frac{\partial \phi_D}{\partial s} = i k \phi_D .
\]

(A.9b)

When the boundary conditions give a poor representation of reality, there will be a zone along each boundary in which the wave field is disturbed.

We thus have three types of boundary conditions in CREDIZ:
- condition (A.8)
- the Neumann condition (A.9a)
- the combination of Smith.
For Smith's method to be applicable the computations have to be strictly linear.

A.3 Additional physical effects

Three additional physical effects are included in the model equation, which is an extension of Eq. (A.5). The three effects are 1) dissipation due to wave breaking, 2) dissipation due to bottom friction and 3) the influence of the wave amplitude on the wave celerity. It is noticed that the growth of the waves due to the wind is an effect which is not built in the model because the wind generates waves of many frequencies and directions; this effect is therefore in contradiction with the assumptions of the model.

Wave breaking and bottom friction are taken into account through the introduction of energy dissipation terms: in case of Eq. (A.1) a term $-i\omega \psi$ is added to the left-hand side; the coefficient $W$ stands for the relative energy dissipation per unit of time, and is the sum of a coefficient due to breaking and one due to bottom friction: $W = W_b + W_f$.

In the parabolic approximation, the addition of the dissipation term leads to the addition of the following terms to the left-hand side of Eq. (A.5):

$$
\frac{1}{b} \left( \frac{2}{3} \right) s \psi_1 + \frac{1}{k\sqrt{bk}} \psi + \psi \frac{\psi}{\sqrt{bk}} \cdot
$$

(A.10)

A.3.1 Amplitude effect on wave celerity

The influence of the wave amplitude on the propagation velocity is taken into account by taking for the local depth $h$ in the dispersion relation (A.3)

$$
h = d + p_h a
$$

(A.11)

where $d$ = actual depth

$a$ = local wave amplitude

$p_h$ = adjustable parameter (standard value $p_h = 1$).
By means of the "effective" depth $h = d + p_y a$ some amplitude-effects are modelled for quantities which are derived from the dispersion relation, thus $k$, $c$ and $c_g$. We note that, with $p_y = 1$, one obtains in the shallow water limit $(kh \to 0)$ for $c$ the expression $c = \sqrt{g(d+H/2)}$, where $H = 2a$ is the wave height; this corresponds with the solitary wave expression.

A.3.2 Wave breaking

The dissipation function $W_b$ due to wave breaking is computed by means of the method of Battjes and Janssen (1978). A full discussion of this method is given in Appendix D. Here we note that in the modelling of the wave breaking the essential choice is made in considering the wave height $H = 2a$ to be the significant wave height $H_s$ when an interpretation to nature (in which random waves are present) is to be made. The breaker height $H_m$ is given by

$$H_m = \frac{2\pi \gamma_d}{k} \tanh \left[ \frac{\gamma_s}{2\pi \gamma_d} kd \right], \quad (A.12)$$

where $\gamma_d = \text{coefficient breaker height in deep water}$
   (standard value $\gamma_d = 0.14$; $2\pi \gamma_d = 0.88$)
$\gamma_s = \text{coefficient breaker height in shallow water}$
   (standard value $\gamma_s = 0.80$).

Introducing

$$H_{rms} = H/\sqrt{2}, \quad (A.13)$$

the fraction of breaking or broken waves, $Q_b$, is given by the implicit expression

$$Q_b = \exp[-(1-Q_b)/b^2], \quad b = H_{rms}/H_m \quad (A.14)$$

The approximation for the computation of $Q_b$ is given in Appendix D.
We have in CREDIZ

\[ W_b = \frac{D_b}{E} \quad \text{(A.15)} \]

where \( E \) is the energy (per unit area) and the expression for \( D_b \) is given in (D.10). Because CREDIZ is a mathematical model for propagation of regular waves, we have to use now

\[ E = \frac{1}{8} \rho g H^2 \quad \text{(A.16)} \]

and for \( W_b \) is obtained:

\[ W_b = \frac{\alpha}{\pi} \frac{\omega_r}{Q_b} \frac{H^2}{H_{\infty}^2} \quad \text{for} \quad Q_b \leq 1 \quad \text{(A.17)} \]

The coefficient \( \alpha \) is an adjustable constant with standard value \( \alpha = 1 \).

**A.3.3 Bottom friction**

For bottom friction the dissipation coefficients \( W_f \) is computed as follows:

Define

\[ s_\circ = \frac{\omega_r}{\sinh(kd)}. \quad \text{(A.18)} \]

Then

\[ W_f = \frac{s_\circ^2}{2g} \frac{8}{3\pi} f_w s_\circ H + 2f_s \left\{ \frac{H}{U} \right\} \quad \text{(A.19)} \]

where \( f_w = \) coefficient shear stress, wave induced

\( f_s = \) coefficient shear stress, wave-current induced.

We have as standard values \( f_w = 0.01, f_s = 0.005 \).

It is noted that in (A.18) and also in (A.12) the local depth \( d \) is used, not the "effective" depth \( h = d + p_ya \). The wave-number \( k \), however, is obtained from the dispersion relation in which the effective depth \( h = d + p_ya \) is used.
A.4 The parabolic approximation

In this subsection a derivation is given for the parabolic approximation giving coefficients $p_1$ and $p_2$ which are optimal in some sense. This is done for the case of constant depth $d$ and no current, in which case one obtains from (A.1) the Helmholtz equation

$$
\nabla^2 \phi + k^2 \phi = 0 ,
$$

or,

$$
\frac{\partial^2 \phi}{\partial x^2} = -k^2 \left(1 + \frac{1}{k^2} \frac{\partial^2}{\partial y^2}\right) \phi .
$$

This equation permits wave propagation in all directions. The propagation in positive $x$-direction can be described by means of pseudo-differential operators as follows:

$$
\frac{\partial \phi}{\partial x} = i k (1 + \frac{1}{k^2} \frac{\partial^2}{\partial y^2})^{\frac{1}{2}} \phi .
$$

The parabolic approximation consists now of approximating the square root operator. A Padé approximant is used in the following way:

$$
f(z) \equiv (1+z)^{\frac{1}{2}} = \frac{1 + p_2 z}{1 + p_1 z}
$$

This approximation is exact for $z = 0$. The approximation for $f'(0)$ is exact under the condition $p_2 = p_1 + 1/2$. The approximation for $f''(0)$ is exact when, moreover, is taken $p_1 = 1/4$. Therefore, an "optimal" parabolic approximation is

$$
\sqrt{1+z} = \frac{1 + \frac{1}{4} z}{1 + \frac{1}{4} z}.
$$

For this case (the Helmholtz Eq.) the parabolic approximation is then:
\[(1 + \frac{1}{4k^2} \frac{\partial^2}{\partial y^2}) \frac{\partial \phi}{\partial x} = i k (1 + \frac{3}{4k^2} \frac{\partial^2}{\partial y^2}) \phi \).

Booij (1981) derived the optimal result \( p_2 = p_1 + 1/2, \ p_1 = 1/4 \) in a different way.

### A.5 Summary

In this section the results are summarized and all adjustable coefficients with their standard values are given.

The model equation to be solved is (A.5) together with (A.10):

\[
\left( \frac{1}{\rho} + \frac{3}{\beta \xi} \right) \left( \sqrt{\beta k} \ \phi + \frac{p_1}{\kappa \beta \xi} \ (M \phi + i \omega W \phi) \right) - i \kappa \sqrt{\beta k} \ \phi + \frac{p_2}{\sqrt{\beta k}} (-i M \phi + i \omega W \phi) = 0,
\]

(A.20)

where

\[\beta = \frac{c c_e}{g} \]

\[M = (\omega - \omega_r^2) + \frac{3}{\beta (\beta \xi)} + 2 i \omega U \frac{\partial}{\partial n} \]

\[\omega_r = \left[ g k \tanh kh \right]^{\frac{1}{2}} \]

\[h = d + p_v a \]

\[\omega_r = \omega - r \kappa U \]

\[W = W_b + W_f \]

\[W_f = \frac{s_o}{2g} \left( \frac{3}{\beta \xi} f_s, s_H + 2 f_s, |\tilde{U}| \right), \ s_o = \omega_r / \sinh kd \]

\[W_b = \frac{\alpha}{\pi} \omega_r Q_o b_n H_m^2 / H_r^2 \] if \( Q_b \leq 1 \)

\[W_b = \frac{\alpha}{\pi} \sqrt{2} \omega_r H_m^2 / H_r^2 \] if \( Q_b > 1 \)

\[Q_b = \exp[-(1-Q_b^2)] \quad , \quad b = H_r \sin H_m \].
The standard values and description of the adjustable coefficients are:

- $p_1 = 0.25$, $p_2 = 0.75$; $0 \leq p_1 \leq \frac{1}{2}$, $p_2 = p_1 + \frac{1}{2}$; coefficients for the parabolic approximation
- $p_v = 1$ amplitude effect on velocity of propagation
- $r_f = 0.9$ reduction factor needed in estimating $\omega_r$
- $\gamma_d = 0.14$ breaking on steepness ($2\pi \gamma_d = 0.88$)
- $\gamma_s = 0.80$ breaking on depth-limitation
- $f_w = 0.01$ friction coefficient wave-induced shear stress
- $f_s = 0.005$ friction coefficient wave-current induced shear stress
- $\alpha = 1$ coefficient for magnitude $W_b$. 
APPENDIX B - Description of the CREDIZ-model. Numerical approach

B.1 Introduction

The partial differential equation (A.5), which is of parabolic type, will be used as the basis for the numerical model. A discretization with finite differences on a rectangular grid (Δx, Δy) will be chosen. For large areas (distance of wave propagation more than (about) 100 wave lengths) the computational effort may be still considerable.

An alternative formulation in terms of the complex phase $\Theta$ can be useful:

$$\phi = \exp(\Theta) .$$  \hspace{1cm} (B.1)

It may be expected that the function $\Theta$ is essential smoother than the wave potential $\phi$, which oscillates rapidly. However, the transition (B.1) from the wave potential to the complex phase function leads to a non-linear model, in which instabilities may develop. Such instabilities are damped by adding an artificial numerical diffusion term to the model, thereby reducing its accuracy. It turns out that even with a fully implicit linearized numerical scheme in some cases instabilities may develop, and iterative refinement of the scheme is necessary, which enhances the computational effort. The nonlinear model (B.1) will, therefore, not further be considered here.

In the following sections, an implicit numerical scheme will be described for the linear model (A.15). In Section B.2 the discretization in $x$-direction is considered, in Section B.3 the one in $y$-direction. Section B.4 deals with the absorption of wave energy at internal boundaries, and in Section B.5 the computation of some wave parameters is described. In the last two sections procedures are given for the presentation of computed results: a two-dimensional interpolation method (Section B.6) and a contouring technique (Section B.7).
B.2 Computation of propagation in x-direction

The discretization in the coordinate x can be realized with a rather coarse grid, when the wave-like character of the solution is incorporated a priori into the expression for the complex wave function $\phi(x,y)$:

$$\phi(x,y) = \psi(x,y) \exp\{i \ S(x,y)\} ,$$

(B.2)

with $\psi$ a complex function, and where the phase function $S$ should be chosen as a suitable adaption to the variation of the local wave number.

Equation (A.15), with $x \equiv s$, $y \equiv n$, can be written as:

$$\frac{\partial}{\partial x} (P\phi) + Q\phi = 0 ,$$

(B.3)

where $P$ and $Q$ represent operators which still contain the $y$-derivatives. Let

$$R \equiv i \ S'P + Q ,$$

where $S' \equiv \partial S/\partial x$, then (B.3) becomes:

$$\frac{\partial}{\partial x} (P\phi) = i \ S'(P\phi) - (R\phi) .$$

(B.4)

This equation can be solved for the function $P\phi$. The solution on a grid with mesh-size $\Delta x$, is, with $x_B = x_A + \Delta x$:

$$(P\phi)_B = (P\phi)_A \exp\{i \ \int_{x_A}^{x_B} S'dx\} +$$

$$- \exp\{i \ \int_{x_A}^{x_B} S'dx\} \int_{x_A}^{x_B} (R\phi) \exp[-i \ \int_{x_A}^{x_B} S'dt]dx .$$

(B.5)

Assuming that an approximation for the local wave number $S'$ is known, we apply the trapezoidal rule to the integrals in (B.5):

$$\int_{x_A}^{x_B} S'dx \approx \frac{1}{2} (S_A' + S_B') \Delta x = \overline{S'} \Delta x$$

(B.6)
The \((R\phi)\)-integral in (8.5) is like-wise approximated by the trapezoidal rule, and we have:

\[
(P\phi)_B = (P\phi)_A e^{iS'Ax} - \frac{\Delta x}{2} [(R\phi)_A e^{iS'Ax} + (R\phi)_B]
\]  

(B.7)

The discretization with respect to \(y\) is done in such a way that only three nodes in that direction are needed (see next section). The scheme (B.7) then leads to the well-known Crank–Nicholson (CN) scheme. Unfortunately, the CN-scheme can lead to unrealistic, oscillating solutions when the forward step \(\Delta x\) is large; see Patankar and Baliga (1978), Lawson and Morris (1978) and Pert (1981).

In the method of Patankar and Baliga a slightly more implicit scheme is applied which possesses the accuracy of the CN-method for small time steps, the physical realism of the fully implicit method for large time steps, and the stability of both. However, the method cannot be applied directly to parabolic equations with an imaginary diffusion coefficient. Therefore we proceed by analogy and introduce a weighting factor \(\theta\):

\[
\theta = \frac{1}{1 - \exp(-\lambda)} - \frac{1}{\lambda}, \text{ with } \lambda = \frac{2p_2 \Delta x}{\kappa(\Delta y)^2}.
\]  

(B.8a)

This weighting factor has the desired property:

\[
\frac{1}{2} < \theta < 1
\]

if \(\lambda \to 0\) then \(\theta \to \frac{1}{2}\) (CN-method)

if \(\lambda \to \infty\) then \(\theta \to 1\) (fully implicit method)

(B.8b)

Let \(\phi_A\) be the value of \(\phi\) at the point \(x_A\), \(\phi_B\) at \(x_B\), and let the matrix \(P_A\) be the discretization of the operator \(P\) for \(x = x_A\), etc. Then the new scheme can be defined from (B.7) and (B.8) as:

\[
P_B\phi_B = P_A\phi_A e^{iS'Ax} - \Delta x \cdot R[(1-\theta)\phi_A e^{iS'Ax} + \theta \phi_B],
\]

(B.9)
where \( \overline{R} = \frac{1}{2} (R_A + R_B) \).

For \( S' \) we may take

\[
S' = p_S (k - \frac{\omega}{x c c g}) \tag{B.10}
\]

where \( p_S \) is an adjustable parameter with standard value \( p_S = 1.0 \).

It appears that the scheme (B.9) gives fairly accurate solutions for values of \( \Delta x / L \lesssim 1/4 \), where \( L = 2\pi / k \) is the local wave length.

**B.3 Discretization in the y-direction**

Discretization in the y-direction is done such that only 3 nodes in the y-direction are needed. The second order term will be represented as

\[
\frac{\partial}{\partial y} \left( b \frac{\partial \phi}{\partial y} \right) \approx \frac{(b_2 + b_3)(\phi_3 - \phi_2) + (b_1 + b_2)(\phi_1 - \phi_2)}{2(\Delta y)^2}, \tag{B.11}
\]

while a term with a first-order derivative is approximated by

\[
\frac{\partial \phi}{\partial y} \approx \frac{\phi_3 - \phi_1}{2\Delta y}. \tag{B.12}
\]

For the zero-order term an alternative form is used (cf. Booij (1981)):

\[
\phi \approx p_o \phi_1 + (1 - 2p_o) \phi_2 + p_o \phi_3, \tag{B.13}
\]

where \( p_o \) is an adjustable parameter with standard value \( p_o = 0.1 \). The operators \( P_A, P_B, R_A \) and \( R_B \) in (B.9), which contain y-derivatives, now become matrices, and the resulting set of simultaneously linear equations can be solved efficiently. (When additional physical effects are induced, as in (A.10) and (A.14), the equations are linearized by taking, for the amplitude-dependent coefficients, the values at the level \( x = x_A \).)

The value of \( \Delta y \) should be chosen sufficiently small in order to obtain a reasonable accuracy. In practice, a value of \( \Delta y / L \lesssim 1/6 \) appears to give satisfactory results.
B.4 Absorption of wave energy at internal boundaries

In areas where the local depth becomes zero or negative (e.g., islands), the value of $\phi$ is set to zero. A coast-line can then be conceived as an internal boundary, with boundary condition $\phi = 0$. In some cases, however, it may be useful to add a dissipation mechanism to the model equation, in order to simulate an absorbing internal boundary. This can be achieved by replacing the second-order term (B.11) by, if, e.g., $\phi_1 = 0$ on a right internal boundary:

$$\frac{b_2+b_3}{2} \frac{\phi_3-\phi_2}{\Delta y} + i \frac{p_a b_2 k_2 \phi_2}{\Delta y}$$  \hspace{1cm} (B.14)

It amounts to a discretization of the internal boundary condition

$$\frac{\partial \phi}{\partial y} = -i p_a k \phi \hspace{1cm} (B.15)$$

where $p_a \geq 0$ is an adjustable parameter with standard value $p_a = 0.3$.

For a left internal boundary one has $\phi_3 = 0$ and the opposite sign should be used in analogous expressions for (B.14) and (B.15).

B.5 Computation of wave parameters

The solution of Eq. (A.5) requires the calculation of coefficients which are functions of the local depth, the local wave number, the current velocity, etc. The local wave number $k$ is evaluated from the dispersion relation:

$$\varepsilon(\kappa) \equiv \omega^2_r - g k \tanh[k(d+p_a)] = 0 \hspace{1cm} (B.16)$$

where $\omega_r$ is the relative frequency

$$\omega_r = \omega - r_x k U_x \hspace{1cm} (B.17)$$

and $a = H/2$ is the wave amplitude:

$$a = \omega_r |\phi|/g \hspace{1cm} (B.18)$$
Given the values of $g$, $d$, $U_x$, $\omega$ and $\phi$, the transcendental equation (B.16) is solved for $k$, using the iterative method of Newton-Raphson. To ensure the convergence of the method, a maximum value of $\omega_r/(2k)$ for the current velocity component $|U_x|$ will be held during the iteration process. The process is broken off when $\varepsilon \leq 10^{-5}$ or when a total of 50 iterations is exceeded.

When $k$ is known, other quantities can be derived from it, such as $c = \omega_r/k$, $c_g = \omega_r/\delta k$; the wave direction is given by the direction of the energy transport, apart from a factor $\rho$:

$$ \hat{E} = \frac{\omega_r}{2g} (cc_g \nabla S + \omega_0) |\phi|^2, $$

where $S$ is the local phase:

$$ \nabla S = \text{Im}(\nabla \phi / \phi). $$

The local depth $d$ and the local current velocity components $U_x$ and $U_y$ are simply evaluated by four-point bilinear interpolation from the values specified at the grid points of a rectangular grid (cf. Abramowitz and Stegun (1968), Ch. 25.2).

The accuracy of a grid schematization of the bottom topography can be roughly estimated as follows. In shallow water shoaling obeys Green's Law: $a_d = $ constant and $k_d = $ constant. Thus, a relative error in the local depth, $(\Delta d)/d$ (being connected either with the smoothing of depth soundings or with the introduction of a grid) leads to a relative error in the local wave amplitude and wave number:

$$ \frac{\Delta a}{a} = -\frac{1}{4} \frac{\Delta d}{d}, \quad \frac{\Delta k}{k} = -\frac{1}{2} \frac{\Delta d}{d}. $$

The grid spacing of the bottom grid should be chosen in accordance with the required accuracy in the area of interest.

In many cases the following rule of thumb may be used to obtain the grid spacing for the bottom grid: the maximum change in still water depth $d$ over a grid-size should be less than 1 m.
B.6 Interpolation of computed function values

For the presentation of computed results, a two-dimensional interpolation formula is needed, which interpolates for the potential function \( \phi \) and its partial derivatives at some arbitrary point within the computational grid. It is assumed that \( \phi \) can be written in the form (B.2):

\[
\phi(x, y) = \psi(x, y) \exp[i \, S(x, y)]
\]

with \( \partial S/\partial x = S' \) given by (B.10):

\[
S' = p_s (k - U_x \frac{\omega}{c_c c_g})
\]

An interpolation formula is used which meets the following requirements:
- The function and its first and second derivatives should be interpolated continuously.
- A linear and a parabolic function should be interpolated exactly.

Let the function value \( \psi \) be known in equidistant grid points \( \bar{x}_j \) and let

\[
\bar{x}_j = j \Delta x \quad , \quad x = \bar{x}/\Delta x \quad , \quad x^* = \text{entier}(x) . \quad (B.20)
\]

As a consequence of the above requirements, the interpolation involves at least 4 grid points in one direction. The interpolation is based on a cardinal spline function \( G \) (see Ahlberg et al. (1967)):

\[
\psi(x) = \sum_{j=-1}^{2} \psi(x^* + j) \, G(x-x^*-j) . \quad (B.21)
\]

The spline function \( G(x) \) is symmetrical with respect to \( x = 0 \) and satisfies:

\[
G(0) = 1 \quad , \quad G(1) = 0 \quad , \quad G(2) = 0 . \quad (B.22)
\]

To interpolate a linear and a parabolic function exactly, a fifth degree polynomial is needed to represent \( G(x) \) in each interval, and the following conditions should apply:
\[ G'(0) = 0, \quad G'(1) = -\frac{1}{7}, \quad G'(2) = 0 \]
\[ G''(0) = -2, \quad G''(1) = 1, \quad G''(2) = 0. \]  

(B.23)

Let \( \xi = 1-x \) on the interval \( 0 \leq x < 1 \); then there can be derived from (B.22) and (B.23):

\[ G(x) \equiv g_1(\xi) = \frac{1}{2} \xi + \frac{1}{2} \xi^2 + \frac{9}{2} \xi^3 - \frac{15}{2} \xi^4 + 3\xi^5 \]  

(B.24)

Let \( \xi = 2-x \) on the interval \( 1 \leq x < 2 \). Then

\[ G(x) \equiv g_2(\xi) = -\frac{3}{2} \xi^3 + \frac{5}{2} \xi^4 - \xi^5. \]  

(B.25)

Furthermore,

\[ G(x) \equiv 0 \quad \text{on} \quad 2 \leq x < \infty. \]  

(B.26)

A sketch of \( G(x) \) is given below.

In order to interpolate in two dimensions, the described procedure is done twice: once for the \( x \)-direction, and once for the \( y \)-direction. Outside the computational grid, \( \phi \) is taken zero; consequently, if the interpolation point is situated less than one grid spacing from the boundary of the computational grid, there will be some loss of accuracy.
B.7 Contour lines of amplitude and phase

An effective means for presenting the distribution of amplitude and phase in the area of interest is the construction of contour lines (iso-lines). A rectangular grid is defined within the contouring region where the function to be contoured is assumed to be given as an array of smoothed function values. The GPCP contouring program is then used to generate the contour plots (see GPCP-II (1972)).

There remains the problem of supplying smooth function values on the contouring grid. For that purpose, the amplitude and phase are computed at points of the computational grid; for each of these points a contribution of weighted function values to the 4 surrounding points of the contouring grid is calculated.

The smoothed function value, for instance at the contouring grid point Q is found as

$$
\bar{f}_Q = \frac{\sum w_i f_i}{\sum w_i},
$$

(B.27)

where the weighting factors $w_i$ are computed by

$$
w_i = (1 - \frac{r_i^2}{4R^2}),
$$

and $r_i$ is the distance between Q and the point of the computational grid, corresponding with the function value $f_i$, and $R$ is the smallest grid spacing.
The amplitude $a$ is given by (B.18) and the phase $F$ may be evaluated, making use of

$$F = \text{Im}(\log \phi),$$  \hspace{1cm} (B.28)

where $\log$ denotes the principal value of the natural logarithmic function. Unfortunately, the phase $F$ is a multiple-valued function in the vicinity of points where $\phi = 0$ (amphidromic points), and a special procedure must be applied in order to obtain a smooth contour plot for the phase (wave fronts). This procedure consists of the following steps.

- define the smallest rectangle $C$ within the computational grid which contains the contouring grid (output rectangle $R$)
- the value of the phase $F_m$ at a grid point $Q_m$ will be evaluated, in general, from (B.28), using the phase $F_0$ at the preceding grid point $Q_0$:

$$F_m = F + 2\pi N_m$$

where $N_m = \text{entier} \left\{ \frac{F_0 - F}{2\pi} + 1 \right\}$.  \hspace{1cm} (B.29)

- on each grid line $x = \text{constant}$, we look for a point at which the amplitude has a maximum, and define the phase $F_m$ according to (B.29). At point $Q_1$, adjacent to $Q_m$, the phase is calculated in two ways: firstly, according to (B.29): $F_1 = F + 2\pi N_1$; alternatively, as
\[ F_a = F + 2\pi N_a, \]

where \( N_a = \text{entier} \left\{ \frac{m}{2\pi} + 0.5 \right\} \).  \hspace{1cm} (B.30)

If \( N_a = N_i \), then \( F_a = F_i = F(Q_{i}) \) is taken as the phase at \( Q_i \); if \( N_a \neq N_i \), then a phase-jump has occurred, and a smoothed value is taken as the phase at \( Q_i \):

\[ F(Q_i) = \frac{1}{2}(F_i + F_m). \] \hspace{1cm} (B.31)

- In this way, starting from the grid point which has maximum amplitude, we work to the left and right boundary respectively, and compute the phase according to (B.30) or (B.31).
- On the grid line \( x = 0 \) only (B.30) applies; at the maximum amplitude grid point, the phase should be given some initial value.

This procedure being finished, we are able to compute smoothed phase values on the contouring grid, according to (B.27). As it is required by the interpolation technique of the GPCP-program, the contouring grid should be enlarged by a border, one grid spacing in width, and function values should be supplied accordingly. Consequently, if a part of the grid is situated close to the boundary of the computational grid, the contour lines may be disturbed there.
APPENDIX C - The bottom geometry

In this Appendix some details are given concerning the chosen region and the location of the wave sensors. The region for which waterdepths are given is sketched in Figure 1. This region ABCD has dimensions of 21.75 by 29 km and the waterdepths are given on a mesh with 250 m mesh-widths.

Local bottom coordinates $x$, $y$ are used, with $x$ along AB and $y$ along AD and the point A is the origin $(x,y) = (0,0)$.

In the Netherlands, map references are often given in the local system, denoted as "Amersfoort" coordinates, denoted here as $(\tilde{x},\tilde{y})$. The origin $(\tilde{x},\tilde{y}) = (0,0)$ is located at a position of "Onze Lieve Vrouwe" Church at Amersfoort and the $\tilde{y}$-axis (for $\tilde{x} = 0$) points to the true North. The point A is given in terms of $(\tilde{x},\tilde{y})$ as

$$\tilde{x}_A = -113.8 \text{ km}$$
$$\tilde{y}_A = -27.4 \text{ km}.$$  

The orientation of ABCD is given by the angle $\beta$ between the positive $x$-axis and the North-direction. We have

$$\beta = -51.2^\circ.$$  

The values for the depths with respect to the level NAP in the 250 m mesh is obtained from the following maps:

- C7, nr. 80-71619 page 1002A
- C7, nr. 80-71618 page 1002
- C7, nr. 81-7792 page 1003A
- C7, nr. 81-7791 page 1003
- C7, nr. 81-7794 page 2004A
- C7, nr. 81-7793 page 2004
- C7, nr. 81-7796 page 2005A
- C7, nr. 81-7995 page 2005
The scale of these maps was for pages 1002 and 1002A 1:10000 and for the other pages it was 1:25000.

These maps were, in June 1982, the most recent available ones.

Initially some ray computations were performed with the programme STROBO of DHL. For this purpose the same region was taken, now with mesh-size of 500 m (and thus the extent was 21.5 by 29 km). Bottom contours with the 500 m mesh are shown in Figures 2 and 3 (scale 1:150000). Bottom contours of the 250 m mesh made with CREDIZ are shown in Figure 4.

It is clear from these Figures that the shoal, called the Hinderplaat, is an outstanding feature of this region. Moreover, seawards from the Hinderplaat a region exists in which occurs nearly parallel straight isobaths. Simplifying these by straight isobaths it is found from Figures 2-4 that the simplified isobaths make an angle of 82.66° with the positive x-axis. Therefore, the direction perpendicular to the simplified isobaths is 313.86° with respect to North. In the sequel we will consider therefore 315° w.r.t. N as the direction which is perpendicular to the simplified bottom contours.

It is noted that, at the Hinderplaat, the smallest depth is 0.1 m with respect to NAP. This means that the shoal falls partly dry during ebb (see also the photograph at the front page).

A serious problem is the accuracy of the depth measurements, especially in such shallow regions. This is already apparent from the maps with depth-soundings, because at the Hinderplaat less values are given than elsewhere. An overall estimate of the error in the values for the depths is a standard deviation of 15-20 cm. It seems reasonable to take a uniform distribution for this error with \( \sigma = 15 \text{ cm} \).

In June 1983 maps came available with depth-soundings taken during the summer of 1982. Of a small region in which the Hinderplaat is present the depths are digitized anew for the 250 m mesh. Care was taken that indeed the same region in space was taken and the digitizing was carried out by the same person (v.d.
Bosch), so that it can be expected that the smoothing was done in a similar way. This region is given by the points 48Δx to 64Δx and 30Δy to 56Δy, with Δx = Δy = 250 m. In Figures 5 and 6 bottom contours are given of respectively the 1981 and the 1982 situation. Comparing these Figures it is seen that the 2 m contour has moved somewhat in pos. y-direction, that for the 1982 situation part of the Hinderplaat became deeper (between 2 and 3 m) and that the deeper part (5 m) has become larger in extent. Again the shallowest part of the shoal is 0.1 m but these points remained in place. In Figure 7 the contours for the differences for the two situations are given (there is taken d(1982)−d(1981)).

It has to be noted that all computations with CREDIZ used the 1981 bottom, except T29a, see Section 7.6.

The location of the wave sensors in both the Amersfoort coordinates (\(\tilde{x},\tilde{y}\)) and the bottom coordinates (\(x,y\)) is given in the Table below. In this Table also the numerical designation of the wave sensors is given as they are used in order to store computed wave spectra and parameters in the Dutch wave information system DTBEST.

Note that we have two positions Wa-1 and Wa-2 for the Wavec buoy because the directional buoy has been moved to a new position on October 29. The same day the wave rider WR2 has been moved to a new position, denoted by WR7.

<table>
<thead>
<tr>
<th>Sensor</th>
<th>Code</th>
<th>(\tilde{x})</th>
<th>(\tilde{y})</th>
<th>x</th>
<th>y</th>
<th>d</th>
</tr>
</thead>
<tbody>
<tr>
<td>Wa-1</td>
<td>-101050</td>
<td>-22650</td>
<td>6960.2</td>
<td>11691.1</td>
<td>15.1</td>
<td></td>
</tr>
<tr>
<td>Wa-2</td>
<td>-99823</td>
<td>-25891</td>
<td>9947.3</td>
<td>9934.1</td>
<td>10.7</td>
<td></td>
</tr>
<tr>
<td>Ha-1</td>
<td>-100077</td>
<td>-27879</td>
<td>10884.9</td>
<td>8364.2</td>
<td>6.1</td>
<td></td>
</tr>
<tr>
<td>E-75</td>
<td>-92782</td>
<td>-33804</td>
<td>20392.9</td>
<td>8179.1</td>
<td>5.4</td>
<td></td>
</tr>
<tr>
<td>WR1</td>
<td>-99825</td>
<td>-25850</td>
<td>9920</td>
<td>9964.8</td>
<td>10.9</td>
<td></td>
</tr>
<tr>
<td>WR2</td>
<td>-99150</td>
<td>-27350</td>
<td>11386</td>
<td>9218.7</td>
<td>5.4</td>
<td></td>
</tr>
<tr>
<td>WR3</td>
<td>-98000</td>
<td>-26350</td>
<td>11659.6</td>
<td>10718.7</td>
<td>5.5</td>
<td></td>
</tr>
<tr>
<td>WR4</td>
<td>-96000</td>
<td>-28600</td>
<td>14624.2</td>
<td>10218.3</td>
<td>4.1</td>
<td></td>
</tr>
<tr>
<td>WR5</td>
<td>-97500</td>
<td>-29630</td>
<td>14100.5</td>
<td>8475.7</td>
<td>4.0</td>
<td></td>
</tr>
<tr>
<td>WR6</td>
<td>-98630</td>
<td>-30300</td>
<td>13639.7</td>
<td>7245.5</td>
<td>4.1</td>
<td></td>
</tr>
<tr>
<td>WR7</td>
<td>-96418</td>
<td>-29319</td>
<td>14748.9</td>
<td>9396.1</td>
<td>4.7</td>
<td></td>
</tr>
</tbody>
</table>

Table C.1 The position of the sensors (in m)
The transformation formulae between the bottom coordinates \((x,y)\), the computational coordinates \((x',y')\) and the Amersfoort coordinates \((\tilde{x},\tilde{y})\) are given below for ease of use.

Consider at first the computational region. It is possible in CREDIZ to define the computational region with respect to a centre of rotation, denoted here by \(E\), and the angle between the positive \(x\)-axis with the positive \(x'\) axis. Taking local coordinate frames \((\xi,\eta)\) and \((\xi',\eta')\) with \(E\) as origin, it is easy to find the transformation formulae between \((x,y)\) and \((x',y')\). It is noted that the angle \(\alpha\) between the \(x^+\) and \(x'^+\) axis is defined in the way as indicated in Figure C.1.

![Figure C.1](image1)

![Figure C.2](image2)

Consider the lay-out of the computational region as sketched in Figure C.1; note that here \(\alpha\) is negative. The transformation formulae between \((x,y)\) and \((x',y')\) then become:

\[
\begin{align*}
x' &= x'_E + (x-x_E) \cos \alpha + (y-y_E) \sin \alpha \\
y' &= y'_E - (x-x_E) \sin \alpha + (y-y_E) \cos \alpha
\end{align*}
\]  

(C.1)
\[ x = x_E + (x'-x'_E) \cos \alpha - (y'-y'_E) \sin \alpha \]
\[ y = y_E + (x'-x'_E) \sin \alpha + (y'-y'_E) \cos \alpha \]

We often used as centre of rotation E a point lying on the Hinderplaat (with depth 10 cm) which is given by the coordinates

\[ x_E = 13500 \text{ m} \quad , \quad y_E = 10500 \text{ m} \]
\[ x'_E = 8000 \text{ m} \quad , \quad y'_E = 6500 \text{ m} \]  \hspace{1cm} (C.2)

It is noted that for the incoming wave direction \( \theta = 315^\circ \) with respect to North (nautical convention), the angle \( \alpha \) between the x and x'-axis is \( \alpha = -6.2^\circ \).

The relation between the bottom coordinates \((x,y)\) and the Amersfoort coordinates \((\tilde{x},\tilde{y})\) is found as follows. The positive \( \tilde{y} \)-axis is directed to the North and therefore we find for \( \alpha \): \( \alpha = 90^\circ - \beta = 38.8^\circ \). The transformation formulae are then found to be

\[ \tilde{x} = \tilde{x}_A + x \cos \alpha + y \sin \alpha \]  \hspace{1cm} (C.3)
\[ \tilde{y} = \tilde{y}_A - x \sin \alpha + y \cos \alpha \]

and

\[ x = (\tilde{x} - \tilde{x}_A) \cos \alpha - (\tilde{y} - \tilde{y}_A) \sin \alpha \]  \hspace{1cm} (C.4)
\[ y = (\tilde{x} - \tilde{x}_A) \sin \alpha + (\tilde{y} - \tilde{y}_A) \cos \alpha \]

where

\[ \tilde{x}_A = -113800 \text{ m} \]
\[ \tilde{y}_A = -27400 \text{ m} \]
\[ \alpha = +38.8^\circ \]  \hspace{1cm} (C.5)

Instead of the Amersfoort coordinates \((\tilde{x},\tilde{y})\), also so-called R.D. coordinates are used sometimes as map references in the Netherlands. The use of the R.D. coordinates \((x_{RD},y_{RD})\) is sometimes preferred because only positive coordinates
are obtained then. Between \((\tilde{x}, \tilde{y})\) and \((x_{RD}, y_{RD})\) the simple transformation exist:

\[
\begin{align*}
  x_{RD} &= \tilde{x} + 155000 \ m \\
  y_{RD} &= \tilde{y} + 463000 \ m,
\end{align*}
\]  

(C.6)

so that the origin of the R.D.-coordinates lies in the North-West of France.

Another map-reference-system which is used sometimes (not for this project) is a local UTM (universal transverse Mercator) system. The Amersfoort coordinates are based on stereographic mapping. Denoting the coordinates of this UTM system by \((x_{UTM}, y_{UTM})\), where \(x_{UTM}\) is measured from 3° Eastern length and \(y_{UTM}\) is North of the equator, an approximate transformation between the UTM and Amersfoort coordinates (where the coordinates are given in m) consists of

\[
\begin{align*}
  \tilde{x} &= 15.093 \times 10^{-10} (y_{UTM}^2 - x_{UTM}^2) - 3.133 \times 10^{-10} (2x_{UTM}y_{UTM}) + \\
  &\quad + 0.015828557 y_{UTM} + 1.005095398 x_{UTM} - 805659.812 \\
  \tilde{y} &= -3.133 \times 10^{-10} (y_{UTM}^2 - x_{UTM}^2) - 15.093 \times 10^{-10} (2x_{UTM}y_{UTM}) + \\
  &\quad + 1.005095398 y_{UTM} - 0.015828557 x_{UTM} - 5778239.329.
\end{align*}
\]  

(C.7)

and

\[
\begin{align*}
  x_{UTM} &= 15.328 \times 10^{-10} (x^2 - y^2) + 1.632 \times 10^{-10} (2xy) + 0.99944931 \tilde{x} + \\
  &\quad - 0.03286329 \tilde{y} + 663394.88 \\
  y_{UTM} &= -1.632 \times 10^{-10} (x^2 - y^2) + 15.328 \times 10^{-10} (2xy) + 0.03286329 \tilde{x} + \\
  &\quad + 0.99944931 \tilde{y} + 5781192.70.
\end{align*}
\]  

(C.8)
APPENDIX D - Energy dissipation due to breaking waves

The dissipation of wave energy due to breaking waves is modelled in CREDIZ according to the method proposed by Battjes and Janssen (1978). Because this method assumes a stochastic wave field, some attention is given how to use this model in the regular wave model CREDIZ.

Because the Battjes-Janssen (BJ) model is a one-dimensional model we consider here the case of parallel bottom contours, expressed by $h = h(x)$, where $h$ denotes the local depth.

The BJ model

In one spatial dimension $x$ we have for the time averaged energy flux $P$:

$$\frac{dP}{dx} + D = 0 , \quad (D.1)$$

where $D$ is the time averaged dissipated power per unit area. For $P$ is taken $c_g E$ and the problem is to find an expression for $D$.

It is assumed that the non-broken waves obey a Rayleigh distribution with respect to the wave height $H$:

$$F(H) = P(H \leq H) = 1 - \exp[-\frac{1}{2}(H/H)^2] ; \quad 0 \leq H < H_m$$

$$= 1 , \quad H \geq H_m , \quad (D.2)$$

where $H$ is some reference wave height.

The maximum wave height is given by

$$H_m = \frac{0.88}{k} \tanh\left(\frac{\gamma}{0.88} kh\right) , \quad (D.3)$$

which expression is a variation of the one proposed by Miche. It is noted that in shallow water, $kh \ll 1$, one has $H_m + \gamma h$ and in deep water, $kh \gg 1$, $H_m + 0.88/k$, or $H_m/\lambda + 0.14$ where $\lambda$ is the wave length, $\lambda = 2\pi/k$. 
We consider in fact a whole spectrum of waves wherein the highest and steepest waves (according to \( H_m \)) are considered as being broken or breaking. The probability \( Q_b \) that for some \( x \) a waveheight is associated with a breaking or broken wave \( (H \geq H_m) \) is

\[
Q_b = P(H \geq H_m) = \exp(-\frac{1}{2}(H_m/H)^2).
\]  

(D.4)

As a characteristic wave for the wave field is taken \( H_{rms} \). One has, by definition,

\[
H_{rms}^2 = \int_0^\infty H^2 dF(H) = \int_0^H H^2 dF(H) + H_m^2 \Delta F(H_m) = \int_0^H H^2 dF(H) + Q_b H_m^2.
\]

One then obtains

\[
H_{rms}^2 = 2H^2(1-Q_b).
\]  

(D.5)

The not yet specified reference wave height \( \hat{H} \) can be eliminated from (D.4) and (D.5). One obtains

\[
Q_b = \exp[-(1-Q_b)/b^2], \quad b = H_{rms}/H_m.
\]  

(D.6)

For a bore of height \( H \) the dissipated power per unit of breadth is approximately:

\[
D' \sim \frac{1}{2} g H^3 (g/h)^{1/2},
\]  

(D.7)

where \( c = (gh)^{1/2} \) has been substituted.

For periodic waves with frequency \( f \) one would then obtain

\[
D = D'/\lambda = D'f/c \sim \frac{fD'}{(gh)^{1/2}} \sim \frac{1}{2} g f H^3 / h,
\]

(D.8)

where \( D \) is the dissipated power per unit area.

Expression (D.8) is applied for waves with \( H \geq H_m \) in a random wave field. Because the probability of \( H \geq H_m \) is \( Q_b \), we obtain from (D.8), with \( f \) some representative frequency of the spectrum,
\[ D \sim \frac{iQ_b}{4} \rho g f H_m^2/h. \]  

(D.9)

Because there will be mostly \( H_m/h = O(1) \) during breaking, and the estimation of \( D \) is based on order-relations, Battjes and Janssen took for \( D \) the expression

\[ D = \frac{\alpha}{4} Q_b \rho g f H_m^2, \]  

(D.10)

where \( \alpha \) is a proportionality constant which is \( O(1) \) when the estimate of \( D \) is any good.

The change in mean water level, \( \eta \), due to radiation stress effect is also accounted for in the BJ model. The momentum equation reads

\[ \frac{dS}{dx} + \rho g h \cdot \frac{dn}{dx} = 0, \quad h = d + \eta \]  

(D.11)

with

\[ S_{xx} = \frac{c}{c - \frac{R}{4}} E. \]  

(D.12)

Here \( d \) is the still-water depth and \( c_g \) is obtained from the linear dispersion relation \( \omega_r^2 = gk \tanh kh. \)

Summarizing, the BJ model for dissipation of wave energy due to breaking consists of the energy equation (D.1) and the momentum equation (D.11) where \( P = c_g E, \quad E = (1/8) \rho g H_{rms}^2, \quad Q_b \) is solved iteratively from (D6) and \( D \) is given by (D.10) and the maximum wave height \( H_m \) is defined by (D.3). The free parameters \( \alpha \) and \( \gamma \) have standard values \( \alpha = 1, \gamma = 0.80, \) in which case the BJ model corresponds quite well with laboratory measurements.

Energy dissipation for breaking waves in CREDIS

In CREDIS essentially the BJ model is taken in order to obtain a dissipation function due to breaking waves. No changes in mean water level due to radiation stress effects are taken into account in CREDIS. However, current-refraction and amplitude effects on the wave celerity are taken into account in CREDIS.
In CREDIZ is used a dissipation function \( W \) which is related to \( D \) by

\[
D = WE ,
\]

where \( E \) is the local wave energy. Because the governing equations of CREDIZ describe the propagation of regular waves, the expression to be taken for \( E \) in CREDIZ is the expression which is valid for regular waves, viz.,

\[
E = \frac{1}{6} \rho g H^2 \quad (D.14)
\]

From (D.10) and (D.14) then is obtained for the dissipation function \( W_b \) in CREDIZ:

\[
W_b = 2\alpha Q_b f(H_m/H)^2 ,
\]

or, with \( \omega_r = 2\pi f \),

\[
W_b = \frac{\alpha}{\pi} \omega_r Q_b (H_m/H)^2 , \quad (D.15)
\]

which is the expression given in (A.17).

The Battjes and Janssen model is formulated in terms of \( H_{\text{rms}} \) as a convenient measure for the total energy \( E \) (and thus the variance \( m_o \)). So is the fraction of broken or breaking waves, \( Q_b \), a function of the maximum wave height \( H_m \) and of \( H_{\text{rms}} \). The BJ model could equally well be formulated in terms of the significant wave height \( H = \frac{H_m}{m_o} \). It is easily seen that for \( Q_b \) then is obtained, because \( H_{\text{rms}}^2 = \frac{2}{2} \frac{m_o}{m_o} \)

\[
Q_b = \exp\left[-2(1-Q_b)/(H_m/H_s)^2\right] . \quad (D.16)
\]

When interpreting the resulting wave height \( H = 2a \) from the regular wave propagation model CREDIZ in terms of prototype situations, we take \( H \) from CREDIZ to be directly comparable with the significant wave height \( H_{m_o} \) as occurs in nature. Thus, also the boundary condition \( H \) is taken, if possible, as \( H_{m_o} \) from measurements at a sensor nearby.

In CREDIZ therefore, \( Q_b \) is evaluated from expression (D.6) where is simply taken (defined)
\[ H_{\text{rms}} = \frac{H}{\sqrt{2}} \]  

(D.17)

This is of course the same as taking (D.16) with \( H_s = H \).

The relevant equations as used in CREDIZ to obtain the decrease in wave height due to breaking waves are thus given by

\[ H_m = \frac{2\pi \gamma_d}{k} \tanh \left( \frac{\gamma_s}{2\pi \gamma_d} kh \right) \]

\[ Q_b = \exp[-2(1-Q_b)/(H/H_m)^2] \]  

(D.18)

\[ W_b = \frac{\alpha}{\pi} \omega Q_b (H_m/H)^2, \]

where in practice is always taken \( 2\pi \gamma_d = 0.88 \) and \( \gamma_s \) is the same as the parameter \( \gamma \) of BJ. With expressions (D.18) the parameters \( \gamma_s \) and \( \alpha \) are directly comparable to the parameters \( \gamma, \alpha \) of BJ.

The evaluation of \( Q_b \) in CREDIZ

It is noted that \( Q_b \) in (D.18) is implicitly given. Some iteration has thus to be carried out in order to obtain the numerical value. The following procedure is chosen. Writing the expression for \( Q_b \) as

\[ q = \exp[-(1-q)/b^2], \quad b = \frac{H}{\sqrt{2}} / H_m \equiv \frac{H_{\text{rms}}}{H_m}, \]

as initial value \( q_0 \) is taken:

\[ q_0 = (2b-1)^2 \quad ; \quad 0.5 < b < 1 \]  

(D.19)

\[ = 0 \quad ; \quad b \leq 0.5 \]

Applying once the Newton-Raphson procedure yields the next approximation \( q_1 \) as:

\[ q_1 = q_0 - b^2 \cdot \frac{q_0 - \exp[(q_0-1)/b^2]}{b^2 - \exp[(q_0-1)/b^2]}. \]  

(D.20)
It has been found by carrying out further Newton-Raphson iterations that only one iteration with the initial value \( q_o \) from (D.19) gives \( q \) accurate in at least three significant figures.

In CREDIZ the value for \( Q_b \) is given by

\[
\begin{align*}
Q_b &= 0 \quad ; \quad b \leq 0.30 \\
Q_b &= q_1 \quad ; \quad 0.30 < b < 0.90 \\
Q_b &= q_o \quad ; \quad 0.90 \leq b \leq 1.0
\end{align*}
\]  \hspace{1cm} (D.21)

It is noted that for \( b = 0.3 \) one has \( q_1 = 1.49 \times 10^{-5} \) and therefore the energy dissipation due to breaking can be neglected for \( b \leq 0.3 \). The value \( b = 0.3 \) can be regarded as the incipience of breaking. For \( b = 0.9 \) one has \( q_1 = 0.646 \) and \( q_o = 0.640 \) so that \( q_o \) gives a value which is accurate enough (\( q + 1 \) for \( b + 1 \)).

As an example of \( Q_b \) values, where \( q_1 \) is used everywhere, the following Table is presented.

\[
\begin{array}{ccccccc}
b & 0.2 & 0.3 & 0.4 & 0.5 & 0.6 & 0.7 \\ 
Q_b & 1.39 \times 10^{-11} & 1.49 \times 10^{-5} & 1.95 \times 10^{-3} & 1.98 \times 10^{-2} & 7.65 \times 10^{-2} & 19.18 \times 10^{-2} \\
\end{array}
\]

\[
\begin{array}{ccccccc}
b & 0.8 & 0.9 & 0.95 & 1 \\ 
Q_b & 37.85 \times 10^{-2} & 64.57 \times 10^{-2} & 81.15 \times 10^{-2} & 1 \\
\end{array}
\]

Table D.1

It is noted that the initial guess \( q_o \) was in fact the one that was used in previous CREDIZ-versions as the ultimate value for \( Q_b \); this was found to given large errors, especially for wave propagation over the Hinderplaat. A discussion of these earlier CREDIZ-versions is given in Section 4.2.
APPENDIX E - Energy dissipation due to bottom friction

E.1 Introduction

In this Appendix we consider wave-induced bottom friction. As a starting point is taken the formulation of Putnam and Johnson (1949) who give a dissipation function $D$ which is valid for regular waves. This formulation is extended for random waves in a very simple way. We consider both the case of the full and the clipped Rayleigh-distribution for the wave heights. A serious shortcoming is that no wave period is considered. Because in the dissipation formulation for breaking waves and in the CREDIZ model also only one wave period is present, the present investigations are at least consistent with these.

We consider again the one-dimensional model

$$\frac{d}{dx} \left( \frac{c_E}{g} \right) + D = 0 ,$$  \hspace{1cm} (E.1)

where now $D = D_f$ is the mean power per unit area which is dissipated due to bottom friction.

Putnam and Johnson (1949) give the expression

$$D_f = \frac{4}{3} \pi \rho f_w \left( \frac{H}{T \sinh kh} \right)^3 .$$  \hspace{1cm} (E.2)

With $T = 2\pi/\omega$ and

$$s_o = \omega / \sinh kh ,$$  \hspace{1cm} (E.3)

this can be written as

$$D_f = \frac{1}{6\pi} \rho f_w s_o^3 H^3 .$$  \hspace{1cm} (E.4)

Consider now a uni-directional random wave field, with a distribution function $F(H,T)$. From the Putnam and Johnson formulation for $D_f$ for regular waves the random waves counterpart for $D_f$ could be obtained from the following expression:
\[ D_f = \frac{4}{3} \pi^2 \rho \int_0^\infty \int_0^\infty \left( \frac{H}{\text{T} \cdot \sinh kH} \right)^3 \text{d}F(H, T) \quad (E.5) \]

Because a realistic simultaneous distribution function \( F(H, T) \) has a quite complicated form and because the validity of (E.5) is by far not certain, we restrict ourselves in the following to one significant period \( T \) and use for \( F(H) \) either the full or the clipped Rayleigh distribution:

\[ F(H) \equiv P(\hat{H} \leq H) = 1 - \exp\left[-\frac{1}{2}(H/\hat{H})^2\right] \quad ; \quad 0 \leq H < \infty \quad (E.6) \]

or

\[ F(H) \equiv P(\hat{H} = H) = 1 - \exp\left[-\frac{1}{2}(H/\hat{H})^2\right] \quad ; \quad 0 \leq H < H_m \quad (E.7) \]

\[ = 1 \quad ; \quad H \geq H_m , \]

where the probability at breaking or broken waves \( Q_b \) is given by

\[ Q_b = \exp\left[-\frac{1}{2}(H_m/\hat{H})^2\right] . \]

We will evaluate in the following the function \( D_f \) given by

\[ D_f = \frac{1}{6\pi} \rho \int_0^\infty \int_0^\infty H^3 \text{d}F(H) , \quad (E.8) \]

where \( F(H) \) is given either by (E.6) or by (E.7).

The integral in (E.8), given as

\[ I = \int_0^\infty H^3 \text{d}F(H) \quad (E.9) \]

is to be evaluated.

**E.2 The full Rayleigh distribution**

With \( F(H) \) given by (E.6) one obtains

\[ I = - \int_0^\infty H^3 \exp\left[-\frac{1}{2}(H/\hat{H})^2\right] \text{d}H \]

and a simple calculation, using \( \int_0^\infty \exp(-z^2/2) \text{d}z = \sqrt{\pi/2} \), yields
I = 3H^3(\pi/2)^{1/2}.

Because \[ H_{\text{rms}}^2 = \int_0^\infty H^2 dF(H) = 2H^2 \]
we have then

\[ I = \frac{3}{4} \sqrt{\pi} H_{\text{rms}}^3, \quad (E.10) \]

and thus, from (E.8),

\[ D_f = \frac{1}{8\sqrt{\pi}} \rho f_w s_o H_{\text{rms}}^3. \quad (E.11) \]

This formulation (E.11) is used for the dissipation function in the one-dimensional wave model (see Appendix F).

### E.3 The clipped Rayleigh distribution

With \[ F(H) \] given by (E.7) one has

\[ I = \int_0^\infty H^3 dF(H) = H_0 \int_m^H H^3 dF(H) + Q_b H_m^3 = \]

\[ = \hat{H}^3 \int_m^H (H/H)^3 dF(H) + Q_b H_m^3 = \hat{H}^3 J + Q_b H_m^3. \]

The integral \( J \) is to be evaluated. With the substitution

\[ z = H/H, \quad z_m = H_m/H, \]

one obtains

\[ J = -z_m^3 Q_b + \int_{z_m}^z 3z^2 \exp(-z^2/2)dz, \]

and thus,

\[ I = \hat{H}^3 \int_{z_m}^z 3z^2 \exp(-z^2/2)dz = \hat{H}^3 K \quad (E.12) \]

The integral \( K \) is now to be evaluated. It is possible to express \( K \) in terms of the incomplete Gamma function \( \Gamma(a, x) \) or in terms of the error function \( \text{erf}(x) \). One has
\[ \gamma(a,x) = \int_0^x t^{a-1} e^{-t} \, dt ; \]  
\[ \text{erf}(x) = \frac{2}{\sqrt{\pi}} \int_0^x e^{-t^2} \, dt . \] 

(E.13)  
(E.14)

At first the expression in terms of the incomplete Gamma function is given. Using the substitution \( t = z^2/2 \) one finds for \( K \):

\[ K = 3\sqrt{2} \gamma\left(\frac{3}{2}, z_m^2/2\right), \]

and thus \( I \) is given as

\[ I = 3\sqrt{2} H^3 \gamma\left[\frac{3}{2}, \frac{H_m^2}{H^2}\right] . \]

Using

\[ H_{\text{rms}}^2 = 2(1-Q_b)H^2 \]  
(E.15)

we obtain

\[ I H_{\text{rms}}^{-3} = \frac{3}{2} (1-Q_b)^{-3/2} \gamma\left[\frac{3}{2}, \frac{H_m^2}{H_{\text{rms}}^2} (1-Q_b)\right] \]  
(E.16)

Using the identities

\[ \gamma(a+1,x) = a \gamma(a,x) - x^a e^{-x} \]
\[ \gamma\left(\frac{1}{2}, x^2\right) = \sqrt{\pi} \text{erf}(x) , \]  
(E.17)

(E.16) can be expressed in terms of the Error function. One obtains, noting that \( \exp[-(H_m^2/H_{\text{rms}}^2)(1-Q_b)] = Q_b \),

\[ I H_{\text{rms}}^{-3} = -\frac{30_b}{2(1-Q_b)} \cdot \frac{H_m}{H_{\text{rms}}} + \frac{3(\pi/2)^{1/2}}{[2(1-Q_b)]^{3/2}} \cdot \text{erf}\left[\frac{H_m}{H_{\text{rms}}} (1-Q_b)^{1/2}\right] . \]  
(E.18)

It is of course possible to derive (E.18) directly from (E.12). This is achieved by integrating partially once and subsequently using the substitution \( t^2 = z^2/2 \).
We now consider the limiting behaviour of (E.18) or (E.16) for the cases $H_{\text{rms}}/H \to 0$ (thus $Q_b \to 0$) and $H_{\text{rms}}/H \to 1$ ($Q_b \to 1$).

Consider at first the case $H_{\text{rms}}/H \to 0$; the expression (E.18) is used now.

One has $Q_b (H_{\text{rms}}/H)^{-1} \to 0$ for $H_{\text{rms}}/H \to 0$; this is clear from the numerical example as shown in next Table.

<table>
<thead>
<tr>
<th>$H_{\text{rms}}/H$</th>
<th>0.3</th>
<th>0.2</th>
<th>0.1</th>
<th>0.07</th>
<th>0.06</th>
</tr>
</thead>
<tbody>
<tr>
<td>$Q_b$</td>
<td>1.49x10^{-5}</td>
<td>1.39x10^{-11}</td>
<td>3.72x10^{-44}</td>
<td>2.34x10^{-89}</td>
<td>&lt; 10^{-99}</td>
</tr>
</tbody>
</table>

Because $(H_{\text{rms}}/H)(1-Q_b) \to \infty$ for $H_{\text{rms}}/H \to 0$ one obtains $\text{erf}(.) \to 1$. Therefore:

$$\lim_{H_{\text{rms}}/H \to 0} I.H_{\text{rms}}^{-3} = \frac{3}{4} \sqrt{\pi} = 1.3293 ,$$

(E.19)

which result is indeed the same as (E.10), which was derived using the full Rayleigh distribution (and thus, $Q_b = 0$).

In the case $H_{\text{rms}}/H \to 1$, and thus $Q_b \to 1$ it is easier to use expression (E.16). One has the convergent series expansion

$$\gamma(a,x) = x^a \sum_{n=0}^{\infty} \frac{(-x)^n}{(a+n)n!} ; \quad a > 0 .$$

(E.20)

From (E.16) is then obtained for $1 - Q_b \to 0$, putting $x = (1-Q_b) = H_{\text{rms}}/H_{\text{rms}}^2$,

$$I.H_{\text{rms}}^{-3} = \frac{3}{2} \left( \frac{H_{\text{rms}}}{H_{\text{rms}}^2} \right)^3 \left[ \frac{1}{3/2} + \sum_{n=1}^{\infty} \frac{(-x)^n}{(a+n)n!} \right] ; \quad a = 3/2$$

and thus,

$$\lim_{H_{\text{rms}}/H \to 1} I.H_{\text{rms}}^{-3} = 1 .$$

(E.21)

For a few values $H_{\text{rms}}/H$ the expression (E.18) is evaluated numerically. For the function $\text{erf}(x)$ we use the approximation:
\[ \text{erf}(x) = 1 - (a_1 t + a_2 t^2 + a_3 t^3) e^{-x^2} + \varepsilon(x), \]

with \( t = (1 + px)^{-1} \), \( \varepsilon(x) \leq 2 \times 10^{-5} \)

\[ p = .47047, \ a_1 = .3480242 \]

\[ a_2 = -.0958798, \ a_3 = .7478556 \]

(E.22)

There is obtained

<table>
<thead>
<tr>
<th>( \frac{H_{\text{rms}}}{H_m} )</th>
<th>( Q_b )</th>
<th>( I.H_{\text{rms}}^{-3} )</th>
<th>( I.H_{\text{rms}}^{-3}/(\frac{3}{4} \sqrt{\pi}) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>.3</td>
<td>1.495 \times 10^{-5}</td>
<td>1.3293</td>
<td>1.0000</td>
</tr>
<tr>
<td>.4</td>
<td>.0020</td>
<td>1.3254</td>
<td>.9970</td>
</tr>
<tr>
<td>.5</td>
<td>.0198</td>
<td>1.3022</td>
<td>.9796</td>
</tr>
<tr>
<td>.6</td>
<td>.0765</td>
<td>1.2555</td>
<td>.9445</td>
</tr>
<tr>
<td>.7</td>
<td>.1918</td>
<td>1.1942</td>
<td>.8984</td>
</tr>
<tr>
<td>.8</td>
<td>.3785</td>
<td>1.1279</td>
<td>.8485</td>
</tr>
<tr>
<td>.9</td>
<td>.6457</td>
<td>1.0622</td>
<td>.7991</td>
</tr>
<tr>
<td>.99</td>
<td>.9605</td>
<td>1.0060</td>
<td>.7568</td>
</tr>
</tbody>
</table>

Table E.2

It is now concluded that the dissipation function \( D_b \) for bottom friction is not very sensitive for the accounting of the occurrence of wave breaking as reflected in the use of the clipped Rayleigh distribution for the wave height. This is especially true in the range \( .4 \leq \frac{H_{\text{rms}}}{H_m} \leq .6 \) for which most of the dissipation due to wave breaking takes place. For \( \frac{H_{\text{rms}}}{H_m} = 0.6 \) one has with the clipped distribution a \( D_b \) which is 94% in value of the \( D_b \) for the unclipped Rayleigh distribution. In view of the fact that the choice of the friction factor \( f_w \) is a highly uncertain one, there is not enough reason to choose expression (E.18) or (E.16) in favour of (E.10), although the use of (E.16) or (E.18) is more consistent when dissipation due to breaking is included than (E.10) is.
APPENDIX F - Description of the 1-D model

F.1 Introduction

In this Appendix we give some details about the existing 1-D models which are used in the present investigations.

Because these models have been extended during the present research programme, the model which includes most physical processes is described here (we started with the model described in Appendix D). The other programmes are easily obtained from this one by dropping various terms.

At first the refraction part is described (Section F.2), together with the effect on the mean free surface level due to radiation stress.

F.2 Refraction

As a starting point the wave action conservation law is taken where a term $D/\omega_r$ is added to account for dissipation:

$$\frac{3}{\rho t} \left( \frac{E}{\omega_r} \right) + \nabla \left[ \left( \frac{c_g + \hat{U}}{\omega_r} \right) \frac{E}{\omega_r} \right] + \frac{D}{\omega_r} = 0$$  \hspace{1cm} (F.1)

where $E$ is the mean wave energy per unit area and $D$ is the average dissipated power per unit area. We consider stationary waves. With a current field $\hat{U}(x,y)$ present one has

$$\nabla \left[ \left( \frac{c_g + \hat{U}}{\omega_r} \right) \frac{E}{\omega_r} \right] + \frac{D}{\omega_r} = 0$$  \hspace{1cm} (F.2)

with

$$\omega = \omega_r + \hat{k} \cdot \hat{U}$$  \hspace{1cm} (F.3)

$$\omega_r^2 = gk \tanh kh \quad , \quad k = |\hat{k}|.$$  \hspace{1cm} (F.4)

At this moment the effect of current-refraction is not yet included. Therefore the case $\hat{U} = 0$ is considered, and one has $\omega = \omega_r = \text{constant}$. Equation (F.2) can then be written as
\[ \nabla \cdot \left( \frac{c}{g} \right) + D = 0 \]  

We note that in (F.5) the vector \( \frac{+}{g} \) is directed along a wave ray, with coordinate \( s \) along the ray. Let the angle between the ray and the positive \( x \)-axis be, locally, \( \theta \). Then (F.5) can be written as

\[ \frac{\partial}{\partial x} \left( \frac{c}{g} \cos \theta \cdot E \right) + \frac{\partial}{\partial y} \left( \frac{c}{g} \sin \theta \cdot E \right) + D = 0 , \]  

where \( \frac{+}{g} = \frac{c}{g} \).

We consider now a bathymetry with parallel bottom contours; only in such a case a 1-D model may be of some use. We have then \( h = h(x) \), and thus, the \( x \)-axis is perpendicular to the iso-baths. In this case we have

\[ k_2 = k \sin \theta = \text{constant} \quad \text{or} \quad \frac{\sin \theta}{c} = \text{constant} \quad \text{(Snell)} \]  

and from (F.6) it follows that

\[ \frac{d}{dx} \left( \frac{c}{g} \cos \theta \cdot E \right) + D = 0 . \]  

We seek information concerning the wave height along a straight line, along which the coordinate \( \xi \) is running.

Moreover, the incoming waves at \( \xi = 0 \) need not be directed in \( \xi \)-direction. The positive \( \xi \)-axis makes an acute angle \( \psi \) with the positive \( x \)-axis. Along the \( \xi \) axis the still-water depth \( d \) is given at distances \( \Delta \xi \). Note that we have to consider the wave height changes along the \( \xi \)-axis because in practise the bathymetry is only partly one-dimensional.

We have the situation as sketched in Figure F.1.
Because $d\xi = dx/cos\psi$ one obtains from (F.8):

$$\frac{d}{d\xi}(c \cdot g \cdot cos\theta \cdot E) + D \cdot cos\psi = 0$$  \hspace{1cm} (F.9)

Equation (F.9) is to be solved together with Eq. (F.7).

The effect of radiation stress on the mean surface level is expressed by

$$\frac{3S_{ij}}{3x_{ij}} + \rho g h \frac{\partial n}{\partial x_j} = 0 , \quad h = d + n ,$$  \hspace{1cm} (F.10)

where the radiation stress tensor is given as

$$S_{ij} = \frac{k}{2} \left( k_{i-j} \cdot \frac{2c}{k} + \delta_{ij} \frac{2c}{c} - 1 \right)$$  \hspace{1cm} (F.11)

In our case, with $h = h(x_1)$ and the wave direction being locally $\theta$ with respect to the positive $x$-axis, we have with $k_1 = k \cdot cos\theta$, $k_2 = k \cdot sin\theta$:

$$\frac{dS_{xx}}{dx} + \rho g \frac{dn}{dx} = 0 ,$$  \hspace{1cm} (F.12)

where

$$S_{xx} = \left( \frac{c}{c} + \frac{2}{(1+cos^2\theta) - \frac{1}{2}} \right) \cdot E \right.$$  \hspace{1cm} (F.13)

Along the $\xi$-axis we have, from (F.12),

$$\frac{dS_{xx}}{d\xi} + \rho g \frac{dn}{d\xi} = 0 .$$  \hspace{1cm} (F.14)

**F.3 Dissipation**

The dissipated power per unit area, $D$, is given as

$$D = D_b + D_f ,$$

where $D_b$ is the power dissipated due to wave breaking and $D_f$ is the dissipated power due to bottom friction. The expressions for $D_b$ and $D_f$ are given in (D.10) and (E.11) as
\[ D_b = \frac{1}{4} \alpha \rho g Q_b f H_m^2 \]  \hspace{1cm} (F.15)

and

\[ D_f = \frac{1}{8\sqrt{\pi}} \rho f_w \frac{\omega_r H_{\text{rms}}}{\sinh kh}^3 . \]  \hspace{1cm} (F.16)

It is noted that when current refraction is to be taken into account, the following expression for \( D_b \) should be taken (see (D.8)):

\[ D_b = \frac{3}{4} Q_b \rho g \frac{\omega_r}{2\pi} H_m^2 , \]  \hspace{1cm} (F.17)

with \( \omega_r \) the relative wave frequency.

In the case that current-refraction is considered, it is also necessary to account for the current-induced dissipation. Care should be taken that then also in absence of waves Eq. (F.2) is fulfilled. This is not investigated further here.

\textbf{F.4 Wave growth due to wind}

The effect of wave growth due to wind is also included in the 1-D model in an approximate way. Use is made of growth curves as given in the wave prediction programme GONO, see Sanders et al (1981) or Janssen et al (1983). Because this method is essentially discrete, it is not possible to define the resulting dissipation function as \( D = D_b + D_f - D_{wg} \), with \( D_{wg} > 0 \) expressing the wave growth due to wind.

The growth-curve has the general appearance

\[ \frac{gH_s}{W^2} = \beta \tanh[F(gt/W)] , \]

where \( W \) is the wind velocity component in the \( \xi \)-direction, measured at 10 m above mean sea level and \( F(.) \) is some function.

Introducing the non-dimensional wave height \( z \) and the non-dimensional time \( \tau \) by

\[ z = \frac{gH_s}{W^2} , \quad \tau = \frac{gt}{W} , \]  \hspace{1cm} (F.18)
the chosen growth-curve has the form

\[ z/\beta = \tanh[p \tau^q] \quad \text{ } (F.19) \]

The numerical coefficients \( p \) and \( q \) as used in GONO are given as:

\[ \begin{align*}
  p &= c_2 \quad , \quad q = c_3 \quad \text{for} \quad \tau \leq 13.10^3 \\
  p &= c_4 \quad , \quad q = c_5 \quad \text{for} \quad \tau > 13.10^3
\end{align*} \quad (F.20) \]

and

\[ \begin{align*}
  \beta &= 0.22 \\
  c_2 &= 4.62 \times 10^{-4} \\
  c_3 &= 0.7786 \\
  c_4 &= 1.91 \times 10^{-3} \\
  c_5 &= 0.6286 \\
  c_6 &= 0.62763 ,
\end{align*} \quad (F.21) \]

where \( c_6 \) is the value obtained for \( z/\beta \) from \( z/\beta = \tanh[c_2(13.10^3)^c_3] \).

Consider now the effect of wave growth due to wind over the distance \( \Delta \xi \) from \( \xi_i \) to \( \xi_{i+1} = \xi_i + \Delta \xi \). At the point \( \xi_i \) the energy \( E_i = (1/8) \rho g H_{rms_i}^2 \) is known. Therefore, at \( \xi = \xi_i \), one has \( H_{rms_i} = H_{rms_i} \sqrt{2} \). For the case of unsaturated waves (i.e., \( z/\beta < 1 \)) expression (F.19) is inverted in order to give the (non-dimensional) time \( \tau \) which would be needed to give waves of non-dimensional wave height \( z_i = gH_{rms_i}/W^2 \). One obtains

\[ \tau_i = \left[ \frac{1}{p} \arctanh(z_i/\beta) \right]^{1/q} . \quad (F.22) \]

For \( z_i/\beta \leq c_6 \) the coefficients \( c_2, c_3 \) are to be used for \( p, q \); for \( z_i/\beta > c_6 \) coefficients \( c_4, c_5 \) are used in (F.22).

The (dimensional) time-span \( \Delta t \) which is needed for the wave field to travel the distance \( \Delta \xi \) is given by
\[ \Delta t = \frac{\Delta \xi}{c \cos(\theta - \psi)} \]  

\( (F.23) \)

With \( \Delta t = g \Delta t/W \), relations (F.19) and (F.20) are applied for \( \tau_{i+1} = \tau_i + \Delta \tau \) yielding \( z_{i+1} \) and thus \( H_{s_{i+1}} \). The wave growth due to wind over the distance \( \Delta \xi \) is then given by \( H_{s_{i+1}} - H_{s_i} \) and the amount \( (H_{s_{i+1}} - H_{s_i})/\sqrt{2} \) is simply added to the previously calculated value \( H_{rms_{i+1}} \).

The solution proceeds now as follows.
At first Eqs. (F.7) and (F.9) are solved over the space-step \( \Delta \xi \). A Runge-Kutta procedure is used.
Subsequently the wave growth due to wind is determined.
Thereafter the mean surface level \( d+\eta \) is obtained from (F.14). Because \( Q_b \) depends on the water level this procedure is repeated a few times; only the wave growth part needs to be calculated only once in this iteration. A more detailed description of the numerical procedure will appear elsewhere.
APPENDIX G - A relation between $\alpha$ and $\gamma$

The purpose of this Appendix is given in Section 4.4.3. It concerns the relation between the free parameters $\alpha$ and $\gamma$. This relation is derived such that for combinations of $(\alpha, \gamma)$ the dissipation function $D_b$ remains of the same value in a certain application.

We have

$$H_m = \frac{0.88}{k} \tanh\left[\frac{\gamma}{0.88} k h\right]$$  \hspace{1cm} (G.1)

$$Q_b = \exp[-(1-Q_b)/b^2] \quad , \quad b = H_{rms}/H_m$$  \hspace{1cm} (G.2)

$$D_b = \frac{a}{4} \rho g \int_0^L H_m^2 .$$  \hspace{1cm} (G.3)

The standard values are $(\alpha, \gamma) = (1, 0.80)$. These standard values are denoted by $\alpha_n, \gamma_n$ in this Appendix.

With

$$\mu = (\gamma - \gamma_n)/\gamma_n ,$$  \hspace{1cm} (G.4)

expression (G.1) for $H_m$ can be written as

$$H_m = \frac{0.88}{k} \tanh\left[(1+\mu) \frac{\gamma_n}{0.88} k h\right]$$  \hspace{1cm} (G.5)

The maximum wave height for $\gamma = \gamma_n$ (thus, $\mu = 0$) is denoted by $H_n$. Using the expansion

$$\tanh((1+u)z) = \tanh z + uz[1-\tanh^2 z] +$$

$$-\frac{1}{4} (uz)^2 \tanh z [1-\tanh^2 z] + \ldots,$$

$H_m$ can be expressed in terms of $H_n$ as

$$\tilde{H}_m = H_n + \mu \gamma_n \left[ 1 - \left( \frac{k n}{0.88} \right)^2 \right] + \frac{\mu k n}{0.88} \left[ 1 - \left( \frac{k n}{0.88} \right)^2 \right] H_n .$$  \hspace{1cm} (G.6)
Taking $\gamma = 0.6$ ($\mu = -0.25$) a relative error of 1.67% in $\tilde{H}_m$ corresponding to the true $H_m$ is obtained for $kh = 1$. When only first-order terms in $\mu$ are retained, the relative error is 3.17% for $kh = 1$. In the case $kh = 0.5$ these errors are 0.62% and 1.31% respectively.

It is clear that the first approximation in (G.6) (only linear in $\mu$) suffices for our purposes here:

$$\tilde{H}_m \approx H_n + \mu Y_n h \left[ 1 - \left( \frac{b}{0.88} \right)^2 \right].$$  \hspace{1cm} (G.7)

Expression (G.7) is now used in the expression for $D_b$. Because also $Q_b$ depends on $H_m$, it is desirable that an expression for $Q_b$ is used which is explicit in terms of $H_{rms}/H_m$. We take

$$\tilde{Q}_b = 2.4 \left( \frac{H_{rms}}{H_m} \right)^7.$$ \hspace{1cm} (G.8)

This approximation is compared with the exact expression for $Q_b$ (Eq. (D.6)) for some values in the Table below.

<table>
<thead>
<tr>
<th>$H_{rms}/H_m$</th>
<th>$Q_b$</th>
<th>$\tilde{Q}_b$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.3</td>
<td>$1.49 \times 10^{-5}$</td>
<td>$0.52 \times 10^{-5}$</td>
</tr>
<tr>
<td>0.4</td>
<td>$0.195 \times 10^{-2}$</td>
<td>$0.393 \times 10^{-2}$</td>
</tr>
<tr>
<td>0.5</td>
<td>$1.98 \times 10^{-2}$</td>
<td>$1.88 \times 10^{-2}$</td>
</tr>
<tr>
<td>0.6</td>
<td>$7.65 \times 10^{-2}$</td>
<td>$6.72 \times 10^{-2}$</td>
</tr>
<tr>
<td>0.7</td>
<td>$19.2 \times 10^{-2}$</td>
<td>$19.8 \times 10^{-2}$</td>
</tr>
</tbody>
</table>

Table G.1

It is seen from Table G.1 that especially for the lower values of $H_{rms}/H_m$ $\tilde{Q}_b$ is not a very good approximation to $Q_b$. Inserting expression (G.8) in (G.3), one obtains

$$\tilde{D}_b = 0.6 \rho g f \frac{H_{rms}}{H_m}^7 \frac{H_m^5}{H_{rms}}.$$ \hspace{1cm} (G.9)

Inserting approximation (G.7) for $H_m$ into this expression then yields
\[
\tilde{D}_b = 0.6 \alpha \rho g f H_{rms}^7 \left[ H_n + \nu \gamma_n h \left[ 1 - \left( \frac{H}{.88} \right)^2 \right] \right]^{-5}.
\]

From the expression (G.10) for the dissipation function \( \tilde{D}_b \) a relation between \( \alpha \) and \( \gamma \) can be obtained under the condition of a constant value of \( \tilde{D}_b \). We take now also a constant value for \( H_{rms} \) although in practise \( H_{rms} \) will be different for a certain location along the ray, which location may be characterized by \( kh \). That \( H_{rms} \) is different may be seen as follows. For the incipience of breaking to be resulting in measurable loss of energy over a short distance, one should have at least \( H_{rms}/H > 0.30 \); besides, for \( H_{rms}/H < 0.30 \) there is simply defined \( Q_b = 0 \) in CREIZ version C-3. When \( \gamma \) is decreased from its standard value \( \gamma_n = 0.80 \) there will be obtained a value \( Q_b \neq 0 \) at a location further off-shore than is the case for \( \gamma = \gamma_n \); say that \( H_{rms}/H > 0.30 \) at location \( (kh)_1 \) for the first time, when \( \gamma = \gamma_n \) is taken. Off-shore from this location an increase in \( \alpha \) has no effect because \( D_b \) remains zero. However, when decreasing \( \gamma \) the value 0.30 is reached at a point off-shore from \( (kh)_1 \) so that the breaking region begins earlier.

An increase in \( \alpha \) becomes effective at a point further in-shore than a decrease in \( \gamma \).

To simplify matters we take \( H_{rms} \) to be constant at some location. Then one obtains from (G.10)

\[
\alpha \left[ kH_n + \nu \gamma_n kh \left[ 1 - \left( \frac{H}{.88} \right)^2 \right] \right]^{-5} = \text{constant}
\]

This functional relation depends still on \( kh \). Therefore the relation (G.11) is worked out for a few \( kh \) values. At first it is noted that it follows that the relation (G.11) can be rewritten as

\[
\alpha (\gamma + \varepsilon)^{-5} = \text{const},
\]

where

\[
\varepsilon = \frac{kH_n}{kh} \left[ 1 - \left( \frac{H}{.88} \right)^2 \right] - \gamma_n.
\]

Substituting the expression for \( H_n \) in (G.13), one obtains

\[
\varepsilon = (kh)^{-1} 0.88 \tanh(\gamma_n kh/.88)[1-\tanh^2(\gamma_n kh/.88)]^{-1} - \gamma_n.
\]
The value of $\varepsilon$ is now computed for several values $kh$, where $\gamma_n = 0.80$ is used. We obtain the values shown in next Table:

$$
\begin{array}{cccccccccc}
kh & 0.10 & 0.20 & 0.30 & 0.40 & 0.50 & 0.60 & 0.70 & 0.80 & 0.90 & 1.00 \\
\varepsilon & 0.0044 & 0.0177 & 0.0403 & 0.0724 & 0.1148 & 0.1648 & 0.2342 & 0.3135 & 0.4080 & 0.5196 \\
\end{array}
$$

Table G.2

It is clear from Table G.2 that for shallow water, $kh \ll 1$, the simple relationship

$$
 \alpha \gamma^{-5} = \text{constant} \quad \text{kh} \ll 1 \quad (G.15)
$$

is obtained. This relation could easily be derived directly by noting that $H_m + \gamma h$ for $kh + 0$; substitution of $H_m = \gamma h$ in the dissipation function $\tilde{D}_b$ given in (G.9) leads then to (G.15).

It is noted that in the HV-ray substantial breaking occurs at $\xi = 5800$ m, for which typical values $kh = .45 - .55$ are obtained.

A few examples of corresponding pairs $(\alpha_n, \gamma)$, $(\alpha, \gamma_n)$ are given below; relation (G.12) is used with $\varepsilon$-values from (G.14) for both $kh = 0$ ($\varepsilon = 0$) and $kh = 0.4$ ($\varepsilon = 0.0724$). These pairs are obtained from

$$
\alpha(n + \varepsilon)^{-5} = \alpha_n(n + \varepsilon)^{-5}.
$$
<table>
<thead>
<tr>
<th>$\alpha_n$</th>
<th>$\gamma_n = .80$</th>
<th>$\alpha_n$</th>
<th>$\gamma_n = .80$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\gamma$</td>
<td>$\alpha$</td>
<td>$\gamma$</td>
<td>$\alpha$</td>
</tr>
<tr>
<td>.80</td>
<td>1</td>
<td>.80</td>
<td>1</td>
</tr>
<tr>
<td>.75</td>
<td>1.38</td>
<td>.75</td>
<td>1.34</td>
</tr>
<tr>
<td>.70</td>
<td>1.95</td>
<td>.70</td>
<td>1.84</td>
</tr>
<tr>
<td>.696</td>
<td>2</td>
<td>.687</td>
<td>2</td>
</tr>
<tr>
<td>.65</td>
<td>2.82</td>
<td>.65</td>
<td>2.57</td>
</tr>
<tr>
<td>.606</td>
<td>4</td>
<td>.606</td>
<td>3.509</td>
</tr>
<tr>
<td>.60</td>
<td>4.214</td>
<td>.60</td>
<td>3.677</td>
</tr>
<tr>
<td>.589</td>
<td>4.632</td>
<td>.589</td>
<td>4</td>
</tr>
</tbody>
</table>

Table G.3 Pairs of $(\alpha_n, \gamma)$ and $(\alpha, \gamma_n)$ satisfying (G.12) for the cases $kh = 0$ and $kh = 0.40$
APPENDIX H - Measurements of the ideal condition

In this Appendix some details are given concerning parameters at the wave sensors found for the measurements belonging to the ideal condition. The selected measurements at the Wavec and at Ha-l are given in Table 5.1 in Section 5.2. Application of the time window, as mentioned in Section 3.5 (Eq. (3.5)) yields at the five selected times the following available measurements at the various wave sensors.

It is noted that $\langle T \rangle = \frac{T}{z}$.

<table>
<thead>
<tr>
<th>time = 2209;06:00</th>
<th>sensor</th>
<th>time</th>
<th>WST</th>
<th>$H_m$</th>
<th>TH1/3</th>
<th>$\langle T \rangle$</th>
<th>$m_o$</th>
<th>$H_{ife}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>WR1</td>
<td>05:50</td>
<td>160</td>
<td>138</td>
<td>5.7</td>
<td>4.0</td>
<td>1196</td>
<td></td>
<td></td>
</tr>
<tr>
<td>WR3</td>
<td>06:20</td>
<td>130</td>
<td>118</td>
<td>5.8</td>
<td>5.0</td>
<td>878</td>
<td></td>
<td></td>
</tr>
<tr>
<td>WR4</td>
<td>06:10</td>
<td>130</td>
<td>53</td>
<td>3.5</td>
<td>2.3</td>
<td>182</td>
<td></td>
<td></td>
</tr>
<tr>
<td>WR5</td>
<td>05:40</td>
<td>160</td>
<td>110</td>
<td>5.7</td>
<td>5.0</td>
<td>766</td>
<td></td>
<td></td>
</tr>
<tr>
<td>WR5</td>
<td>06:00</td>
<td>140</td>
<td>102</td>
<td>5.5</td>
<td>4.4</td>
<td>654</td>
<td></td>
<td></td>
</tr>
<tr>
<td>WR6</td>
<td>06:30</td>
<td>120</td>
<td>128</td>
<td>6.1</td>
<td>4.5</td>
<td>1030</td>
<td></td>
<td></td>
</tr>
<tr>
<td>HA-1</td>
<td>06:00</td>
<td>140</td>
<td>150</td>
<td>5.8</td>
<td>4.2</td>
<td>1412</td>
<td></td>
<td></td>
</tr>
<tr>
<td>E-75</td>
<td>06:00</td>
<td>140</td>
<td>54</td>
<td>4.2</td>
<td>2.9</td>
<td>212</td>
<td></td>
<td></td>
</tr>
<tr>
<td>LEG</td>
<td>06:00</td>
<td>140</td>
<td>151</td>
<td>5.8</td>
<td>12</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>EURO-3</td>
<td>06:00</td>
<td>140</td>
<td>154</td>
<td>5.7</td>
<td>16</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>time = 2209;17:00</th>
<th>sensor</th>
<th>time</th>
<th>WST</th>
<th>$H_m$</th>
<th>TH1/3</th>
<th>$\langle T \rangle$</th>
<th>$m_o$</th>
<th>$H_{ife}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>HA-1</td>
<td>17:00</td>
<td>140</td>
<td>125</td>
<td>6.4</td>
<td>4.4</td>
<td>984</td>
<td></td>
<td></td>
</tr>
<tr>
<td>LEG</td>
<td>17:00</td>
<td>140</td>
<td>137</td>
<td>6.8</td>
<td></td>
<td>32</td>
<td></td>
<td></td>
</tr>
<tr>
<td>EURO-3</td>
<td>17:00</td>
<td>140</td>
<td>139</td>
<td>6.9</td>
<td></td>
<td>42</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Sensor</td>
<td>Time</td>
<td>WST</td>
<td>$H_m$</td>
<td>TH1/3</td>
<td>$\langle T \rangle$</td>
<td>$m_o$</td>
<td>$H_{life}$</td>
<td></td>
</tr>
<tr>
<td>--------</td>
<td>-------</td>
<td>-----</td>
<td>-------</td>
<td>-------</td>
<td>----------------</td>
<td>-------</td>
<td>------------</td>
<td></td>
</tr>
<tr>
<td>WR1</td>
<td>00:00</td>
<td>150</td>
<td>139</td>
<td>6.5</td>
<td>5.1</td>
<td>1214</td>
<td></td>
<td></td>
</tr>
<tr>
<td>WR2</td>
<td>00:00</td>
<td>150</td>
<td>142</td>
<td>6.2</td>
<td>4.5</td>
<td>1262</td>
<td></td>
<td></td>
</tr>
<tr>
<td>WR3</td>
<td>00:20</td>
<td>170</td>
<td>148</td>
<td>6.0</td>
<td>3.8</td>
<td>1370</td>
<td></td>
<td></td>
</tr>
<tr>
<td>WR4</td>
<td>00:20</td>
<td>170</td>
<td>59</td>
<td>4.8</td>
<td>3.7</td>
<td>222</td>
<td></td>
<td></td>
</tr>
<tr>
<td>WR5</td>
<td>00:00</td>
<td>150</td>
<td>128</td>
<td>5.2</td>
<td>4.8</td>
<td>1032</td>
<td></td>
<td></td>
</tr>
<tr>
<td>WR5</td>
<td>00:20</td>
<td>170</td>
<td>111</td>
<td>6.3</td>
<td>4.9</td>
<td>780</td>
<td></td>
<td></td>
</tr>
<tr>
<td>E-75</td>
<td>00:00</td>
<td>150</td>
<td>59</td>
<td>4.4</td>
<td>2.9</td>
<td>222</td>
<td></td>
<td></td>
</tr>
<tr>
<td>LEG</td>
<td>00:00</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>EURO-3</td>
<td>00:00</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Sensor</th>
<th>Time</th>
<th>WST</th>
<th>$H_m$</th>
<th>TH1/3</th>
<th>$\langle T \rangle$</th>
<th>$m_o$</th>
<th>$H_{life}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>WR1</td>
<td>01:30</td>
<td>140</td>
<td>124</td>
<td>6.7</td>
<td>5.1</td>
<td>966</td>
<td></td>
</tr>
<tr>
<td>WR2</td>
<td>01:30</td>
<td>140</td>
<td>113</td>
<td>6.6</td>
<td>4.6</td>
<td>1002</td>
<td></td>
</tr>
<tr>
<td>WR4</td>
<td>00:40</td>
<td>180</td>
<td>55</td>
<td>5.0</td>
<td>3.3</td>
<td>190</td>
<td></td>
</tr>
<tr>
<td>WR5</td>
<td>00:40</td>
<td>180</td>
<td>109</td>
<td>6.4</td>
<td>5.1</td>
<td>748</td>
<td></td>
</tr>
<tr>
<td>WR6</td>
<td>00:40</td>
<td>180</td>
<td>126</td>
<td>7.0</td>
<td>5.8</td>
<td>996</td>
<td></td>
</tr>
<tr>
<td>LEG</td>
<td>01:00</td>
<td></td>
<td>161</td>
<td>6.4</td>
<td></td>
<td>28</td>
<td></td>
</tr>
<tr>
<td>EURO-3</td>
<td>01:00</td>
<td></td>
<td>169</td>
<td>6.8</td>
<td></td>
<td>36</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Sensor</th>
<th>Time</th>
<th>WST</th>
<th>$H_m$</th>
<th>TH1/3</th>
<th>$\langle T \rangle$</th>
<th>$m_o$</th>
<th>$H_{life}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>WR4</td>
<td>01:50</td>
<td>130</td>
<td>41</td>
<td>4.9</td>
<td>3.6</td>
<td>108</td>
<td></td>
</tr>
<tr>
<td>WR5</td>
<td>01:40</td>
<td>130</td>
<td>101</td>
<td>6.4</td>
<td>5.1</td>
<td>748</td>
<td></td>
</tr>
<tr>
<td>WR5</td>
<td>02:10</td>
<td>130</td>
<td>83</td>
<td>5.7</td>
<td>4.0</td>
<td>534</td>
<td></td>
</tr>
<tr>
<td>WR6</td>
<td>02:10</td>
<td>130</td>
<td>119</td>
<td>7.1</td>
<td>5.7</td>
<td>888</td>
<td></td>
</tr>
<tr>
<td>LEG</td>
<td>02:00</td>
<td></td>
<td>147</td>
<td>7.2</td>
<td></td>
<td>27</td>
<td></td>
</tr>
<tr>
<td>EURO-3</td>
<td>02:00</td>
<td></td>
<td>133</td>
<td>6.5</td>
<td></td>
<td>31</td>
<td></td>
</tr>
</tbody>
</table>
It is thus seen that the water level (WST) attains a rather large range of values for those wave registrations which are defined to occur simultaneously. This variation has effect on the observed wave heights.

Next we give for each sensor the parameters for the obtained measurements. Both the mean and the standard variation (s.d.) of $H_{m0}$ and TH1/3 are given. The values of $<T> = T_z$ are also given although this parameter has a far larger statistical variability than TH1/3, see Goda (1979).

### Buoy WR1

<table>
<thead>
<tr>
<th>WST</th>
<th>$H_{m0}$</th>
<th>TH1/3</th>
<th>$&lt;T&gt;$</th>
<th>$m_0$</th>
</tr>
</thead>
<tbody>
<tr>
<td>160</td>
<td>138</td>
<td>5.7</td>
<td>4.0</td>
<td>1196</td>
</tr>
<tr>
<td>150</td>
<td>139</td>
<td>6.5</td>
<td>5.1</td>
<td>1214</td>
</tr>
<tr>
<td>140</td>
<td>124</td>
<td>6.7</td>
<td>5.1</td>
<td>966</td>
</tr>
</tbody>
</table>

**mean** 150 133.7 6.3
**s.d.** 10 8.4 0.5

### Buoy WR2

<table>
<thead>
<tr>
<th>WST</th>
<th>$H_{m0}$</th>
<th>TH1/3</th>
<th>$&lt;T&gt;$</th>
<th>$m_0$</th>
</tr>
</thead>
<tbody>
<tr>
<td>140</td>
<td>113</td>
<td>6.6</td>
<td>4.6</td>
<td>1002</td>
</tr>
<tr>
<td>150</td>
<td>142</td>
<td>6.2</td>
<td>4.5</td>
<td>1262</td>
</tr>
</tbody>
</table>

**mean** 145 127.5 6.4
**s.d.** 7.1 20.5 0.3

### Buoy WR3

<table>
<thead>
<tr>
<th>WST</th>
<th>$H_{m0}$</th>
<th>TH1/3</th>
<th>$&lt;T&gt;$</th>
<th>$m_0$</th>
</tr>
</thead>
<tbody>
<tr>
<td>130</td>
<td>118</td>
<td>5.8</td>
<td>5.0</td>
<td>878</td>
</tr>
<tr>
<td>170</td>
<td>148</td>
<td>6.0</td>
<td>3.8</td>
<td>1370</td>
</tr>
</tbody>
</table>

**mean** 150 133.3 5.9
**s.d.** 28.3 21.2 0.1
<table>
<thead>
<tr>
<th>Buoy WR4</th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>WST</td>
<td>$H_m$</td>
<td>TH1/3</td>
<td>$&lt;T&gt;$</td>
<td>$m_o$</td>
</tr>
<tr>
<td>130</td>
<td>53</td>
<td>3.5</td>
<td>2.3</td>
<td>182</td>
</tr>
<tr>
<td>170</td>
<td>59</td>
<td>4.8</td>
<td>3.7</td>
<td>222</td>
</tr>
<tr>
<td>180</td>
<td>55</td>
<td>5.0</td>
<td>3.3</td>
<td>190</td>
</tr>
<tr>
<td>130</td>
<td>41</td>
<td>4.9</td>
<td>3.6</td>
<td>108</td>
</tr>
</tbody>
</table>

Mean 152.5  52  4.6
SD  26.3  7.7  0.7

<table>
<thead>
<tr>
<th>Buoy WR5</th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>WST</td>
<td>$H_m$</td>
<td>TH1/3</td>
<td>$&lt;T&gt;$</td>
<td>$m_o$</td>
</tr>
<tr>
<td>160</td>
<td>110</td>
<td>5.7</td>
<td>5.0</td>
<td>766</td>
</tr>
<tr>
<td>140</td>
<td>102</td>
<td>5.5</td>
<td>4.4</td>
<td>654</td>
</tr>
<tr>
<td>150</td>
<td>128</td>
<td>5.2</td>
<td>4.8</td>
<td>1032</td>
</tr>
<tr>
<td>170</td>
<td>111</td>
<td>6.3</td>
<td>4.9</td>
<td>780</td>
</tr>
<tr>
<td>180</td>
<td>109</td>
<td>6.4</td>
<td>5.1</td>
<td>748</td>
</tr>
<tr>
<td>130</td>
<td>101</td>
<td>6.4</td>
<td>5.1</td>
<td>748</td>
</tr>
<tr>
<td>120</td>
<td>101</td>
<td>6.4</td>
<td>5.1</td>
<td>748</td>
</tr>
</tbody>
</table>

Mean 150  108.9  6.0
SD  21.6  9.5  0.5

<table>
<thead>
<tr>
<th>Buoy WR6</th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>WST</td>
<td>$H_m$</td>
<td>TH1/3</td>
<td>$&lt;T&gt;$</td>
<td>$m_o$</td>
</tr>
<tr>
<td>120</td>
<td>128</td>
<td>6.1</td>
<td>4.5</td>
<td>1030</td>
</tr>
<tr>
<td>180</td>
<td>126</td>
<td>7.0</td>
<td>5.8</td>
<td>996</td>
</tr>
<tr>
<td>130</td>
<td>119</td>
<td>7.1</td>
<td>5.7</td>
<td>888</td>
</tr>
</tbody>
</table>

Mean 143.3  124.3  6.7
SD  32.1  4.7  0.6
staff E-75

<table>
<thead>
<tr>
<th>WST</th>
<th>$H_{m_0}$</th>
<th>TH1/3</th>
<th>$&lt;T&gt;$</th>
<th>$m_o$</th>
</tr>
</thead>
<tbody>
<tr>
<td>140</td>
<td>58</td>
<td>4.2</td>
<td>2.9</td>
<td>212</td>
</tr>
<tr>
<td>150</td>
<td>60</td>
<td>4.4</td>
<td>2.9</td>
<td>222</td>
</tr>
</tbody>
</table>

---

mean 145 59.0 4.3
s.d.   7.1  1.4  0.14

LEG

<table>
<thead>
<tr>
<th>WST</th>
<th>$H_{m_0}$</th>
<th>TH1/3</th>
<th>$H_{1fe}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>140</td>
<td>151</td>
<td>5.8</td>
<td>12</td>
</tr>
<tr>
<td>140</td>
<td>137</td>
<td>6.8</td>
<td>32</td>
</tr>
<tr>
<td>161</td>
<td>6.4</td>
<td>28</td>
<td></td>
</tr>
<tr>
<td>147</td>
<td>7.2</td>
<td>27</td>
<td></td>
</tr>
</tbody>
</table>

---

mean 149.0 6.6
s.d.   9.9  0.6

buoy EURO-3

<table>
<thead>
<tr>
<th>WST</th>
<th>$H_{m_0}$</th>
<th>TH1/3</th>
<th>$H_{1fe}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>140</td>
<td>154</td>
<td>5.7</td>
<td>16</td>
</tr>
<tr>
<td>140</td>
<td>139</td>
<td>6.9</td>
<td>46</td>
</tr>
<tr>
<td>169</td>
<td>6.8</td>
<td>36</td>
<td></td>
</tr>
<tr>
<td>133</td>
<td>6.5</td>
<td>31</td>
<td></td>
</tr>
</tbody>
</table>

---

mean 148.8 6.5
s.d.   10  16.3 0.5
It is thus seen from these Tables that the $<H_{m_o}>$ values for the wave sensors are obtained for different values of $<WST>$. Because the $<H_{m_o}>$ values are used for the measured values in comparison with the with CREDIZ computed values $<H>$, some allowance should be made for this fact in interpreting the results of the comparisons.
APPENDIX J - Variability of the bottom topography

In this Appendix we give some details about the variability observed for values obtained in the 500 by 500 m regions around the locations of the wave sensors. With a mesh-size of 125 m one obtains 25 values in one such block. The point in the middle is located at the position of the wave sensor.

At first we give the depth-values in the blocks as they are obtained from the CREDIZ computations T23-T26 for the ideal condition. These depth-values $h$ are inclusive the water level WST = 1.43 m; the values of $h$ in the blocks below is expressed in dm, the same as given in the output of CREDIZ. In all blocks we have from left to right the $y$-direction and from top to bottom the $x$-direction.

At the various sensors we have for $h = d + \text{WST}$ in dm:

<table>
<thead>
<tr>
<th></th>
<th>Wa $h = d + 14.3$ dm</th>
<th></th>
<th>WR1 $h = d + 14.3$ dm</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>166 167 167 167 167</td>
<td></td>
<td>136 137 138 140 143</td>
</tr>
<tr>
<td></td>
<td>165 165 166 166 166</td>
<td></td>
<td>134 134 135 137 139</td>
</tr>
<tr>
<td></td>
<td>164 164 165 166 166</td>
<td></td>
<td>130 130 131 132 134</td>
</tr>
<tr>
<td></td>
<td>163 163 164 165 165</td>
<td></td>
<td>124 124 125 126 127</td>
</tr>
<tr>
<td></td>
<td>162 163 163 164 166</td>
<td></td>
<td>115 115 115 117 119</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>WR2 $h = d + 14.3$ dm</th>
<th></th>
<th>WR3 $h = d + 14.3$ dm</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>61 60 61 66 69</td>
<td></td>
<td>65 67 69 67 65</td>
</tr>
<tr>
<td></td>
<td>61 60 61 67 69</td>
<td></td>
<td>65 68 69 66 66</td>
</tr>
<tr>
<td></td>
<td>62 60 61 68 69</td>
<td></td>
<td>65 69 68 66 66</td>
</tr>
<tr>
<td></td>
<td>62 60 63 68 68</td>
<td></td>
<td>66 69 68 66 66</td>
</tr>
<tr>
<td></td>
<td>62 60 64 69 68</td>
<td></td>
<td>66 68 67 66 66</td>
</tr>
<tr>
<td>WR4</td>
<td>h = d + 14.3 dm</td>
<td>WR5</td>
<td>h = d + 14.3 dm</td>
</tr>
<tr>
<td>------</td>
<td>----------------</td>
<td>------</td>
<td>----------------</td>
</tr>
<tr>
<td>42</td>
<td>38</td>
<td>35</td>
<td>33</td>
</tr>
<tr>
<td>46</td>
<td>41</td>
<td>35</td>
<td>32</td>
</tr>
<tr>
<td>54</td>
<td>50</td>
<td>46</td>
<td>44</td>
</tr>
<tr>
<td>61</td>
<td>61</td>
<td>60</td>
<td>58</td>
</tr>
<tr>
<td>62</td>
<td>63</td>
<td>64</td>
<td>63</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>WR6</th>
<th>h = d + 14.3 dm</th>
<th>WR7</th>
<th>h = d + 14.3 dm</th>
</tr>
</thead>
<tbody>
<tr>
<td>54</td>
<td>53</td>
<td>51</td>
<td>48</td>
</tr>
<tr>
<td>55</td>
<td>56</td>
<td>56</td>
<td>52</td>
</tr>
<tr>
<td>55</td>
<td>56</td>
<td>56</td>
<td>54</td>
</tr>
<tr>
<td>55</td>
<td>55</td>
<td>55</td>
<td>55</td>
</tr>
<tr>
<td>55</td>
<td>55</td>
<td>55</td>
<td>55</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Ha-1</th>
<th>h = d + 14.3 dm</th>
<th>E-75</th>
<th>h = d + 14.3 dm</th>
</tr>
</thead>
<tbody>
<tr>
<td>79</td>
<td>81</td>
<td>82</td>
<td>83</td>
</tr>
<tr>
<td>76</td>
<td>78</td>
<td>80</td>
<td>80</td>
</tr>
<tr>
<td>72</td>
<td>74</td>
<td>77</td>
<td>77</td>
</tr>
<tr>
<td>70</td>
<td>72</td>
<td>74</td>
<td>74</td>
</tr>
<tr>
<td>68</td>
<td>69</td>
<td>70</td>
<td>71</td>
</tr>
</tbody>
</table>

Instead of using the central value in a block as the depth for that wave sensor, we use the mean $\langle h \rangle$ taken over the central 3x3 values and give also the standard deviation $s(h)$ computed from these 9 values. In order to see the difference with using all 5x5 values for obtaining $\langle . \rangle$ and $s(.)$, both for 3x3 and for 5x5 values $\langle h \rangle$ and $s(h)$ are given in the next Table.
<table>
<thead>
<tr>
<th>Sensor</th>
<th>3x3</th>
<th>5x5</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>&lt;h&gt;</td>
<td>s(h)</td>
</tr>
<tr>
<td>Wa</td>
<td>16.49</td>
<td>0.11</td>
</tr>
<tr>
<td>WR1</td>
<td>13.04</td>
<td>0.46</td>
</tr>
<tr>
<td>WR2</td>
<td>7.17</td>
<td>0.28</td>
</tr>
<tr>
<td>WR3</td>
<td>7.10</td>
<td>0.17</td>
</tr>
<tr>
<td>WR4</td>
<td>5.30</td>
<td>0.27</td>
</tr>
<tr>
<td>WR5</td>
<td>4.74</td>
<td>1.07</td>
</tr>
<tr>
<td>WR6</td>
<td>5.50</td>
<td>0.13</td>
</tr>
<tr>
<td>WR7</td>
<td>6.08</td>
<td>0.10</td>
</tr>
<tr>
<td>Ha-1</td>
<td>7.62</td>
<td>0.29</td>
</tr>
<tr>
<td>E-75</td>
<td>6.91</td>
<td>0.22</td>
</tr>
</tbody>
</table>

Table J.1 Mean depth <h> in m, inclusive WST = 1.43 m

It is seen from this Table that s(h) is significantly larger in the 5x5 case than in the 3x3 case. It was therefore decided to take only averages over the 250 m square around the location of the sensor.

The 25 values of h are given for each sensor so that one can get any idea of the variability in wave height which is obtained in the same regions.
APPENDIX K - Details of the hindcast

In this appendix some details concerning the computations for the hindcast as described in Chapter 6 are given. The values $\langle H \rangle$ and $s(H)$ obtained at the locations of the wave sensors with CREDIZ computations T27-T31 and T32a are given in Tables K.1 and K.2. In Table K.3 the values $s(H)/\langle H \rangle$ are given as percentages. The wave heights $H$ obtained from CREDIZ at the locations of the wave sensors (where no averaging is applied over the block of 250 by 250 m) are given in Table K.4; these values are not used further, but are given for completeness so that the reader can compare $\langle H \rangle$ with $H$, using also the given values $s(H)$.

The "measured" values $\hat{H}_s$ which are lying on the smooth curves through the measurements, as given in Figures 68-71, are given in Table K.5. These values are simply read from the figures (in fact, figures on a large scale were used for that purpose). The $\hat{H}_s$ -values serve now to compare the computed $\langle H \rangle$-values with. In Table K.6 the relative error $\delta = (\langle H \rangle - \hat{H}_s)/\hat{H}_s$ is given as a percentage.
<table>
<thead>
<tr>
<th></th>
<th>T27</th>
<th>T32a</th>
<th>T29</th>
<th>T28</th>
<th>T30</th>
<th>T31</th>
</tr>
</thead>
<tbody>
<tr>
<td>test time</td>
<td>17:00</td>
<td>20:00</td>
<td>22:00</td>
<td>23:00</td>
<td>02:00</td>
<td>04:00</td>
</tr>
<tr>
<td>WST</td>
<td>-10</td>
<td>20</td>
<td>85</td>
<td>170</td>
<td>150</td>
<td>45</td>
</tr>
</tbody>
</table>

sensor

<table>
<thead>
<tr>
<th></th>
<th>Wa</th>
<th>256.2</th>
<th>303.6</th>
<th>320.2</th>
<th>350.4</th>
<th>288.0</th>
<th>270.0</th>
</tr>
</thead>
<tbody>
<tr>
<td>WR1</td>
<td>251.8</td>
<td>297.8</td>
<td>312.0</td>
<td>340.4</td>
<td>282.4</td>
<td>262.9</td>
<td></td>
</tr>
<tr>
<td>WR2</td>
<td>225.1</td>
<td>253.6</td>
<td>280.0</td>
<td>306.2</td>
<td>270.0</td>
<td>238.2</td>
<td></td>
</tr>
<tr>
<td>WR3</td>
<td>217.6</td>
<td>244.2</td>
<td>269.8</td>
<td>301.1</td>
<td>275.6</td>
<td>240.0</td>
<td></td>
</tr>
<tr>
<td>WR4</td>
<td>0</td>
<td>5.8</td>
<td>38.0</td>
<td>68.4</td>
<td>69.2</td>
<td>13.3</td>
<td></td>
</tr>
<tr>
<td>WR5</td>
<td>7.4</td>
<td>86.2</td>
<td>112.9</td>
<td>146.2</td>
<td>138.0</td>
<td>94.2</td>
<td></td>
</tr>
<tr>
<td>WR6</td>
<td>124.7</td>
<td>136.7</td>
<td>154.4</td>
<td>183.6</td>
<td>163.8</td>
<td>126.7</td>
<td></td>
</tr>
<tr>
<td>WR7</td>
<td>11.6</td>
<td>18.4</td>
<td>67.3</td>
<td>97.6</td>
<td>64.0</td>
<td>29.8</td>
<td></td>
</tr>
<tr>
<td>Ha-1</td>
<td>230.4</td>
<td>261.1</td>
<td>281.1</td>
<td>304.0</td>
<td>276.7</td>
<td>240.7</td>
<td></td>
</tr>
<tr>
<td>E-75</td>
<td>8.2</td>
<td>10.4</td>
<td>48.4</td>
<td>92.2</td>
<td>42.2</td>
<td>52.4</td>
<td></td>
</tr>
</tbody>
</table>

Table K.1 Wave heights $<H>$ in cm.

<table>
<thead>
<tr>
<th></th>
<th>T27</th>
<th>T32a</th>
<th>T29</th>
<th>T28</th>
<th>T30</th>
<th>T31</th>
</tr>
</thead>
<tbody>
<tr>
<td>test time</td>
<td>17:00</td>
<td>20:00</td>
<td>22:00</td>
<td>23:00</td>
<td>02:00</td>
<td>04:00</td>
</tr>
<tr>
<td>WST</td>
<td>-10</td>
<td>20</td>
<td>85</td>
<td>170</td>
<td>150</td>
<td>45</td>
</tr>
</tbody>
</table>

sensor

<table>
<thead>
<tr>
<th></th>
<th>Wa</th>
<th>0.7</th>
<th>0.9</th>
<th>0.7</th>
<th>0.9</th>
<th>0</th>
</tr>
</thead>
<tbody>
<tr>
<td>WR1</td>
<td>3.4</td>
<td>5.6</td>
<td>3.6</td>
<td>2.4</td>
<td>12.0</td>
<td>6.9</td>
</tr>
<tr>
<td>WR2</td>
<td>5.0</td>
<td>7.1</td>
<td>6.3</td>
<td>6.1</td>
<td>4.5</td>
<td>4.8</td>
</tr>
<tr>
<td>WR3</td>
<td>5.0</td>
<td>7.2</td>
<td>7.4</td>
<td>8.9</td>
<td>3.6</td>
<td>4.4</td>
</tr>
<tr>
<td>WR4</td>
<td>0</td>
<td>1.9</td>
<td>21.9</td>
<td>26.2</td>
<td>20.3</td>
<td>5.3</td>
</tr>
<tr>
<td>WR5</td>
<td>7.1</td>
<td>7.0</td>
<td>7.6</td>
<td>8.5</td>
<td>9.0</td>
<td>7.5</td>
</tr>
<tr>
<td>WR6</td>
<td>11.2</td>
<td>12.5</td>
<td>16.1</td>
<td>16.5</td>
<td>10.4</td>
<td>16.8</td>
</tr>
<tr>
<td>WR7</td>
<td>5.3</td>
<td>7.3</td>
<td>30.3</td>
<td>50.5</td>
<td>36.6</td>
<td>15.1</td>
</tr>
<tr>
<td>Ha-1</td>
<td>9.8</td>
<td>6.6</td>
<td>10.0</td>
<td>8.7</td>
<td>14.5</td>
<td>6.4</td>
</tr>
<tr>
<td>E-75</td>
<td>3.4</td>
<td>3.4</td>
<td>3.8</td>
<td>5.5</td>
<td>6.7</td>
<td>3.4</td>
</tr>
</tbody>
</table>

Table K.2 Standard deviations $s(H)$ in cm.
<table>
<thead>
<tr>
<th>time</th>
<th>17:00</th>
<th>20:00</th>
<th>22:00</th>
<th>23:00</th>
<th>02:00</th>
<th>04:00</th>
</tr>
</thead>
<tbody>
<tr>
<td>sensor</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Wa</td>
<td>0.3</td>
<td>0.3</td>
<td>0.2</td>
<td>0.3</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>WR1</td>
<td>1.4</td>
<td>1.9</td>
<td>1.2</td>
<td>0.7</td>
<td>4.2</td>
<td>2.6</td>
</tr>
<tr>
<td>WR2</td>
<td>2.2</td>
<td>2.8</td>
<td>2.3</td>
<td>2.0</td>
<td>1.7</td>
<td>2.0</td>
</tr>
<tr>
<td>WR3</td>
<td>2.3</td>
<td>2.9</td>
<td>2.7</td>
<td>3.0</td>
<td>1.3</td>
<td>1.8</td>
</tr>
<tr>
<td>WR4</td>
<td>-</td>
<td>33</td>
<td>58</td>
<td>38</td>
<td>29</td>
<td>40</td>
</tr>
<tr>
<td>WR5</td>
<td>9.6</td>
<td>8.1</td>
<td>6.7</td>
<td>5.8</td>
<td>6.5</td>
<td>8.0</td>
</tr>
<tr>
<td>WR6</td>
<td>9.0</td>
<td>9.1</td>
<td>10.4</td>
<td>9.0</td>
<td>6.3</td>
<td>13.3</td>
</tr>
<tr>
<td>WR7</td>
<td>12</td>
<td>63</td>
<td>45</td>
<td>52</td>
<td>57</td>
<td>51</td>
</tr>
<tr>
<td>Ha-1</td>
<td>4.3</td>
<td>2.5</td>
<td>3.6</td>
<td>2.9</td>
<td>5.2</td>
<td>2.7</td>
</tr>
<tr>
<td>E-75</td>
<td>41</td>
<td>33</td>
<td>7.9</td>
<td>6.0</td>
<td>16</td>
<td>6.5</td>
</tr>
</tbody>
</table>

Table K.3 Values $s(H)/\langle H \rangle$ in %.

<table>
<thead>
<tr>
<th>test</th>
<th>T27</th>
<th>T32a</th>
<th>T29</th>
<th>T28</th>
<th>T30</th>
<th>T31</th>
</tr>
</thead>
<tbody>
<tr>
<td>time</td>
<td>17:00</td>
<td>20:00</td>
<td>22:00</td>
<td>23:00</td>
<td>02:00</td>
<td>04:00</td>
</tr>
<tr>
<td>WST</td>
<td>-10</td>
<td>20</td>
<td>85</td>
<td>170</td>
<td>150</td>
<td>45</td>
</tr>
<tr>
<td>sensor</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Wa</td>
<td>256</td>
<td>304</td>
<td>320</td>
<td>350</td>
<td>288</td>
<td>270</td>
</tr>
<tr>
<td>WR1</td>
<td>250</td>
<td>296</td>
<td>316</td>
<td>342</td>
<td>290</td>
<td>270</td>
</tr>
<tr>
<td>WR2</td>
<td>226</td>
<td>254</td>
<td>280</td>
<td>304</td>
<td>270</td>
<td>242</td>
</tr>
<tr>
<td>WR3</td>
<td>218</td>
<td>242</td>
<td>270</td>
<td>304</td>
<td>274</td>
<td>244</td>
</tr>
<tr>
<td>WR4</td>
<td>0</td>
<td>8</td>
<td>22</td>
<td>46</td>
<td>76</td>
<td>14</td>
</tr>
<tr>
<td>WR5</td>
<td>72</td>
<td>86</td>
<td>116</td>
<td>150</td>
<td>142</td>
<td>96</td>
</tr>
<tr>
<td>WR6</td>
<td>138</td>
<td>148</td>
<td>148</td>
<td>176</td>
<td>160</td>
<td>122</td>
</tr>
<tr>
<td>WR7</td>
<td>10</td>
<td>20</td>
<td>94</td>
<td>90</td>
<td>46</td>
<td>28</td>
</tr>
<tr>
<td>Ha-1</td>
<td>230</td>
<td>260</td>
<td>276</td>
<td>296</td>
<td>272</td>
<td>234</td>
</tr>
<tr>
<td>E-75</td>
<td>10</td>
<td>10</td>
<td>52</td>
<td>96</td>
<td>40</td>
<td>52</td>
</tr>
</tbody>
</table>

Table K.4 Computed values $H$ (not averaged) in cm.
### Table K.5 "Measured" $\tilde{H}_s$-values lying on the smooth curves.

<table>
<thead>
<tr>
<th>test</th>
<th>T27</th>
<th>T32a</th>
<th>T29</th>
<th>T28</th>
<th>T30</th>
<th>T31</th>
</tr>
</thead>
<tbody>
<tr>
<td>time</td>
<td>17:00</td>
<td>20:00</td>
<td>22:00</td>
<td>23:00</td>
<td>02:00</td>
<td>04:00</td>
</tr>
<tr>
<td>WST</td>
<td>-10</td>
<td>20</td>
<td>85</td>
<td>170</td>
<td>150</td>
<td>45</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>sensor</th>
<th>Wa</th>
<th>WR1</th>
<th>WR2</th>
<th>WR3</th>
<th>WR4</th>
<th>WR5</th>
<th>WR6</th>
<th>Ha-1</th>
<th>E-75</th>
<th>LEG</th>
<th>Euro-3</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>260</td>
<td>305</td>
<td>338</td>
<td>356</td>
<td>300</td>
<td>285</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>260</td>
<td>325</td>
<td>360</td>
<td>370</td>
<td>330</td>
<td>305</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>245</td>
<td>275</td>
<td>325</td>
<td>356</td>
<td>350</td>
<td>280</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>216</td>
<td>-</td>
<td>270</td>
<td>285</td>
<td>-</td>
<td>275</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>40</td>
<td>48</td>
<td>62</td>
<td>72</td>
<td>70</td>
<td>44</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>66</td>
<td>75</td>
<td>105</td>
<td>145</td>
<td>115</td>
<td>66</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>114</td>
<td>120</td>
<td>160</td>
<td>186</td>
<td>206</td>
<td>135</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>205</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>65</td>
<td>70</td>
<td>95</td>
<td>106</td>
<td>95</td>
<td>65</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>320</td>
<td>374</td>
<td>388</td>
<td>378</td>
<td>295</td>
<td>292</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>306</td>
<td>364</td>
<td>350</td>
<td>338</td>
<td>325</td>
<td>308</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

### Table K.6 Error $\delta = (\langle H \rangle - \tilde{H}_s) / \tilde{H}_s$ in %

<table>
<thead>
<tr>
<th>sensor</th>
<th>Wa</th>
<th>WR1</th>
<th>WR2</th>
<th>WR3</th>
<th>WR4</th>
<th>WR5</th>
<th>WR6</th>
<th>Ha-1</th>
<th>E-75</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>-1.5</td>
<td>-3.2</td>
<td>-8.1</td>
<td>+0.7</td>
<td>-100.0</td>
<td>+12.1</td>
<td>+9.4</td>
<td>+12.4</td>
<td>-87.4</td>
</tr>
<tr>
<td></td>
<td>-0.5</td>
<td>-8.4</td>
<td>-7.8</td>
<td>-0.1</td>
<td>-87.9</td>
<td>+14.9</td>
<td>+13.9</td>
<td>-</td>
<td>-85.1</td>
</tr>
<tr>
<td></td>
<td>-5.3</td>
<td>-13.3</td>
<td>-13.8</td>
<td>-0.1</td>
<td>-38.7</td>
<td>+7.5</td>
<td>-3.5</td>
<td>-</td>
<td>-49.1</td>
</tr>
<tr>
<td></td>
<td>-1.6</td>
<td>-8.0</td>
<td>-14.0</td>
<td>+5.6</td>
<td>-5.0</td>
<td>+0.8</td>
<td>-1.3</td>
<td>-</td>
<td>-13.0</td>
</tr>
<tr>
<td></td>
<td>-4.0</td>
<td>-14.4</td>
<td>-22.9</td>
<td>-</td>
<td>-1.1</td>
<td>+20.0</td>
<td>-20.5</td>
<td>-</td>
<td>-55.6</td>
</tr>
<tr>
<td></td>
<td>-5.3</td>
<td>-13.8</td>
<td>-14.9</td>
<td>-12.7</td>
<td>-</td>
<td>-68.9</td>
<td>-6.1</td>
<td>-</td>
<td>-19.4</td>
</tr>
</tbody>
</table>
It was noted in Section 6.4 that simply the measured value $H_s$ at the Wavec was used as boundary condition for the computations, except in one case. It turns out that these values lie below the values $\tilde{H}_s$ on the smooth curves through the measurements, see Figure 68. Because the boundary of the computational mesh lies off-shore from the position of the Wavec, slightly lower values $\langle H \rangle$ at Wa are to be expected. This is, e.g., the case for T32a (20:00) where $\delta = -0.5\%$; these effects are thus insignificant. As can be seen in Table K.6, the errors at Wa are sometimes $-5.3\%$ which is due to the fact that $H_s < \tilde{H}_s$. Because the computed values $\langle H \rangle$ are compared with $\tilde{H}_s$ some correction is now necessary, at least for the off-shore locations. As is seen in Section 7.7 where a comparison between T32 and T32a (wave height at boundary of 3.48 vs. 3.06 m), at the in-shore locations WR4, WR5, WR6, WR7 and E-75 the same waveheight $\langle H \rangle$ is found for both T32 and T32a. Because the wave period used in T32 and T32a is $T = 7.7$ s and also $T = 7.1$ and $T = 8.3$ in the other computations is used, a correction for the off-shore locations seems justified. Whether in some cases some correction at WR6 and WR5 is justified is not known to us now, so we do not correct the computed values at WR5 and WR6.

We now correct the $\langle H \rangle$-values at the off-shore locations Wa, WR1, WR2, WR3, Ha-1 simply by adding $\delta \langle H \rangle$ to $\langle H \rangle$, where $\delta$ is the error at Wa, as given in Table K.6. The corrected values are denoted by $\langle \hat{H} \rangle$ in the sequel. Introducing the relative error $\hat{\delta}$ by $\hat{\delta} = (\langle \hat{H} \rangle - \tilde{H}_s)/\tilde{H}_s$ we obtain from Table K.6:

<table>
<thead>
<tr>
<th>test</th>
<th>T27</th>
<th>T32a</th>
<th>T29</th>
<th>T28</th>
<th>T30</th>
<th>T31</th>
</tr>
</thead>
<tbody>
<tr>
<td>time</td>
<td>17:00</td>
<td>20:00</td>
<td>22:00</td>
<td>23:00</td>
<td>02:00</td>
<td>04:00</td>
</tr>
<tr>
<td>WST</td>
<td>-10</td>
<td>20</td>
<td>85</td>
<td>170</td>
<td>150</td>
<td>45</td>
</tr>
</tbody>
</table>

sensor

<table>
<thead>
<tr>
<th></th>
<th>Wa</th>
<th>WR1</th>
<th>WR2</th>
<th>WR3</th>
<th>WR4</th>
<th>WR5</th>
<th>WR6</th>
<th>Ha-1</th>
<th>E-75</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>-1.7</td>
<td>-7.9</td>
<td>-8.0</td>
<td>-6.4</td>
<td>-10.4</td>
<td>-8.5</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>-6.6</td>
<td>-7.3</td>
<td>-8.5</td>
<td>-12.4</td>
<td>-18.9</td>
<td>-9.6</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>+2.2</td>
<td>-5.2</td>
<td>+7.2</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td></td>
</tr>
<tr>
<td>-100.0</td>
<td>-87.9</td>
<td>-38.7</td>
<td>-5.0</td>
<td>-1.1</td>
<td>-68.9</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>+12.1</td>
<td>+14.9</td>
<td>+7.5</td>
<td>+0.8</td>
<td>+20.0</td>
<td>+42.7</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>+9.4</td>
<td>+13.9</td>
<td>-3.5</td>
<td>-1.3</td>
<td>-20.5</td>
<td>-6.1</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>+13.9</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>-87.4</td>
<td>-85.1</td>
<td>-49.1</td>
<td>-13.0</td>
<td>-55.6</td>
<td>-19.4</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table K.7 Error $\hat{\delta} = (\langle \hat{H} \rangle - \tilde{H}_s)/\tilde{H}_s$ in %.
The corrected values $\hat{H}$ follow from Table K.1 as given in Table K.8 below.

<table>
<thead>
<tr>
<th>test</th>
<th>T27</th>
<th>T32a</th>
<th>T29</th>
<th>T28</th>
<th>T30</th>
<th>T31</th>
</tr>
</thead>
<tbody>
<tr>
<td>time</td>
<td>17:00</td>
<td>20:00</td>
<td>22:00</td>
<td>23:00</td>
<td>02:00</td>
<td>04:00</td>
</tr>
<tr>
<td>WST</td>
<td>-10</td>
<td>20</td>
<td>85</td>
<td>170</td>
<td>150</td>
<td>45</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>sensor</th>
<th>Wa</th>
<th>WR1</th>
<th>WR2</th>
<th>WR3</th>
<th>WR4</th>
<th>WR5</th>
<th>WR6</th>
<th>WR7</th>
<th>Ha-1</th>
<th>E-75</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>260.0</td>
<td>305.0</td>
<td>338.0</td>
<td>356.0</td>
<td>300.0</td>
<td>285.0</td>
<td>255.6</td>
<td>299.3</td>
<td>328.5</td>
<td>345.8</td>
</tr>
<tr>
<td></td>
<td>228.5</td>
<td>254.9</td>
<td>294.8</td>
<td>311.1</td>
<td>280.8</td>
<td>250.8</td>
<td>220.9</td>
<td>254.4</td>
<td>284.1</td>
<td>305.9</td>
</tr>
<tr>
<td></td>
<td>0</td>
<td>5.8</td>
<td>38.0</td>
<td>68.4</td>
<td>69.2</td>
<td>13.3</td>
<td>74.0</td>
<td>86.2</td>
<td>112.9</td>
<td>146.2</td>
</tr>
<tr>
<td></td>
<td>124.7</td>
<td>136.7</td>
<td>154.4</td>
<td>183.6</td>
<td>163.8</td>
<td>126.7</td>
<td>11.6</td>
<td>18.4</td>
<td>67.3</td>
<td>97.6</td>
</tr>
<tr>
<td></td>
<td>233.4</td>
<td>262.4</td>
<td>296.0</td>
<td>308.9</td>
<td>287.8</td>
<td>253.5</td>
<td>8.2</td>
<td>10.4</td>
<td>48.4</td>
<td>92.2</td>
</tr>
<tr>
<td></td>
<td>52.4</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table K.8 Corrected values $\hat{H}$ in cm.

Some kind of measure for the overall error for each computation is desirable. Because of the large errors which occur at E-75 and also at WR4, we present three measures for the overall errors:

1) all sensors are included
2) only E-75 is not included
3) both E-75 and WR4 are not included.

A simple measure for the overall error (based on $\hat{\delta}$) is $<|\hat{\delta}|>$ with $s(|\hat{\delta}|)$ as a measure for the spread.

A different measure is provided by the root-mean-square error, $\varepsilon_{\text{rms}}$, and the bias, $b$, which are defined as

\[
\begin{align*}
\hat{b} & = \frac{1}{n} \sum_{s} (\hat{H} - \bar{H}_s) / \frac{1}{n} \sum_{s} \bar{H}_s \\
\hat{\varepsilon}_{\text{rms}} & = \left( \frac{1}{n} \sum_{s} (\hat{H} - \bar{H}_s)^2 \right)^{\frac{1}{2}} / \frac{1}{n} \sum_{s} \bar{H}_s
\end{align*}
\]

In the sequel $<|\hat{\delta}|>$ and $s(|\hat{\delta}|)$ will be given, where $\hat{\delta}$ is taken from Table K.7; values are given for the three cases mentioned above. See Tables K.9 and K.10.
Thereafter the bias $\hat{b}$ and the rms-error, $\hat{\varepsilon}_{\text{rms}}$, are given, using $\hat{H}$ and $\tilde{H}$ from Tables K.8 and K.5; see Tables K.11 and K.12.

<table>
<thead>
<tr>
<th>test</th>
<th>T27</th>
<th>T32a</th>
<th>T29</th>
<th>T28</th>
<th>T30</th>
<th>T31</th>
</tr>
</thead>
<tbody>
<tr>
<td>time</td>
<td>17:00</td>
<td>20:00</td>
<td>22:00</td>
<td>23:00</td>
<td>02:00</td>
<td>04:00</td>
</tr>
<tr>
<td>WST</td>
<td>-10</td>
<td>20</td>
<td>85</td>
<td>170</td>
<td>150</td>
<td>45</td>
</tr>
<tr>
<td>all sensors</td>
<td>29.2</td>
<td>36.2</td>
<td>17.2</td>
<td>6.6</td>
<td>21.1</td>
<td>23.2</td>
</tr>
<tr>
<td>not E-75</td>
<td>20.8</td>
<td>26.4</td>
<td>11.9</td>
<td>5.5</td>
<td>14.2</td>
<td>23.9</td>
</tr>
<tr>
<td>not E-75, WR4</td>
<td>7.7</td>
<td>11.0</td>
<td>6.5</td>
<td>5.6</td>
<td>17.5</td>
<td>14.9</td>
</tr>
</tbody>
</table>

Table K.9 Overall error $\langle|\hat{\delta}|\rangle$ in %.

<table>
<thead>
<tr>
<th>test</th>
<th>T27</th>
<th>T32a</th>
<th>T29</th>
<th>T28</th>
<th>T30</th>
<th>T31</th>
</tr>
</thead>
<tbody>
<tr>
<td>time</td>
<td>17:00</td>
<td>20:00</td>
<td>22:00</td>
<td>23:00</td>
<td>02:00</td>
<td>04:00</td>
</tr>
<tr>
<td>WST</td>
<td>-10</td>
<td>20</td>
<td>85</td>
<td>170</td>
<td>150</td>
<td>45</td>
</tr>
<tr>
<td>all sensors</td>
<td>40.2</td>
<td>39.1</td>
<td>18.6</td>
<td>4.8</td>
<td>18.5</td>
<td>23.9</td>
</tr>
<tr>
<td>not E-75</td>
<td>35.2</td>
<td>34.6</td>
<td>13.3</td>
<td>4.3</td>
<td>8.4</td>
<td>26.1</td>
</tr>
<tr>
<td>not E-75, WR4</td>
<td>5.1</td>
<td>4.0</td>
<td>2.1</td>
<td>4.8</td>
<td>4.7</td>
<td>15.6</td>
</tr>
</tbody>
</table>

Table K.10 $\sigma(|\hat{\delta}|)$ in %.

<table>
<thead>
<tr>
<th>test</th>
<th>T27</th>
<th>T32a</th>
<th>T29</th>
<th>T28</th>
<th>T30</th>
<th>T31</th>
</tr>
</thead>
<tbody>
<tr>
<td>time</td>
<td>17:00</td>
<td>20:00</td>
<td>22:00</td>
<td>23:00</td>
<td>02:00</td>
<td>04:00</td>
</tr>
<tr>
<td>WST</td>
<td>-10</td>
<td>20</td>
<td>85</td>
<td>170</td>
<td>150</td>
<td>45</td>
</tr>
<tr>
<td>all sensors</td>
<td>-5.4</td>
<td>-13.1</td>
<td>-8.4</td>
<td>-4.4</td>
<td>-15.3</td>
<td>-8.8</td>
</tr>
<tr>
<td>not E-75</td>
<td>-0.8</td>
<td>-7.1</td>
<td>-5.4</td>
<td>-3.7</td>
<td>-11.7</td>
<td>-8.2</td>
</tr>
<tr>
<td>not E-75, WR4</td>
<td>-2.8</td>
<td>-2.3</td>
<td>-3.7</td>
<td>-3.7</td>
<td>-12.5</td>
<td>-5.6</td>
</tr>
</tbody>
</table>

Table K.11 Bias $\hat{b}$ in %.
<table>
<thead>
<tr>
<th>Test</th>
<th>T27</th>
<th>T32a</th>
<th>T29</th>
<th>T28</th>
<th>T30</th>
<th>T31</th>
</tr>
</thead>
<tbody>
<tr>
<td>Time</td>
<td>17:00</td>
<td>20:00</td>
<td>22:00</td>
<td>23:00</td>
<td>02:00</td>
<td>04:00</td>
</tr>
<tr>
<td>WST</td>
<td>-10</td>
<td>20</td>
<td>85</td>
<td>170</td>
<td>150</td>
<td>45</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>sensor type</th>
<th>18.3</th>
<th>22.1</th>
<th>13.5</th>
<th>9.9</th>
<th>22.2</th>
<th>14.5</th>
</tr>
</thead>
<tbody>
<tr>
<td>all sensors</td>
<td>12.4</td>
<td>15.1</td>
<td>10.1</td>
<td>9.6</td>
<td>19.2</td>
<td>13.9</td>
</tr>
<tr>
<td>not E-75</td>
<td>8.0</td>
<td>9.6</td>
<td>8.6</td>
<td>9.2</td>
<td>18.3</td>
<td>11.6</td>
</tr>
</tbody>
</table>

Table K.12 Error $\varepsilon_{\text{rms}}$ in %. 
REFERENCES

Voorbereiding op golfmetingen o.m. met behulp van golfmeetboeien t.b.v. de
toetsing van het numeriek-wiskundig model CREDIZ
Rijkswaterstaat, notitie WWKZ-82.S215, October 1982

ABRAMOWITZ, M. and I.A. STEGUN (1968)
Handbook of Mathematical functions
Dover, 1968

 AHLBERG, J.H., E.N. NILSON and J.L. WALSH (1967)
The theory of splines and their applications

BARNETT, T.P. (1969)
On the generation, dissipation, and prediction of ocean wind waves
J. Geophys. Res. 73(2), Jan. 1968 pp. 513-529

Computation of set-up, longshore currents, run-up and overtopping due to wind-
generated waves

BATTJES, J.A. and J.P.F.M. JANSSEN (1978)
Energy loss and set-up due to breaking of random waves

Refraction and diffraction of water waves; wave deformation by a shoal; com-
parison between computations and measurements
Delft Hydraulics Lab., Report WI54-VIII, March 1982

BERKHOFF, J.C.W., N. BOOIJ and A.C. R ADDER (1982)
Verification of numerical wave propagation models for simple harmonic linear
water waves
Coastal Engineering 6, 1982, pp. 255-279
Gravity waves on water with non-uniform depth and current
Thesis Delft Technical University, May 1981

Surface propagation over an uneven bottom; spectral calculations of refraction
over a shoal
Delft Hydraulics Laboratory, Report W301 part III, November 1981

GODA, Y. (1979)
A review on statistical interpretation of wave data
Report of the Port and Harbour Institute 18 (1), March 1979, pp. 5-32

GCP-II (1972)
A general purpose contouring program, user's manual
April 1972

HAAN, R.H.E. de (1980)
Wave direction research in the entrance of the Eastern Scheldt for the design
of the storm surge barrier
Rijkswaterstaat, Deltadienst, Report DDWT 80.015, 1980 (in Dutch)

GONO, a coupled hybrid wave prediction model
preprint May 1983, submitted for publication

KRASITSKIY, V.P. (1974)
Toward a theory of transformation of the spectrum on refraction of wind waves
(translated from the Russian)
Izv., Atmospheric and Oceanic Physics 10(1), 1974, pp. 39-44

LAWSON, J.D. and J.H. MORRIS (1978)
The extrapolation of first-order methods for parabolic partial differential
equations. I.
MARDIA, K.V. (1972)
Statistics of directional data

PATANKAR, S.V. and B.R. BALIGA (1978)
A new finite-difference scheme for parabolic differential equations
Numerical Heat Transfer 1, 1978, pp. 27-37

PERT, G.J. (1981)
Physical constraints in numerical calculation of diffusion

RADDER, A.C. (1979)
On the parabolic equation method for water-wave propagation
J. Fluid Mech. 95 (1), Nov. 1979, pp. 159-176

ROSKAM, A.P. (1983)
Inwinning en verwerking van meetgegevens voor het project "Toetsing CREDIZ"

Golfklimaat tijdens de meetcampagne voor het project Toetsing CREDIZ
Rijkswaterstaat, WWKZ in preparation

SANDERS, J.W., W.J.P. de VOOGT and J. BRUINSMA (1981)
Fysisch golfonderzoek Noordzee
MLTP-2 Scientific Report

A nonreflecting plane boundary for wave propagation problems

STELLING, G.S. (1983)
On the construction of computational methods for shallow water flow problems
Thesis Delft Technical University, December 1983
Some notes (manuscript) and personnel communications

STIVE, M.J.F. (1983a)
Energy dissipation in waves breaking on gentle slopes

STIVE, M.J.F and M.W. DINGEMANS (1983)
Verification of the refraction-diffraction wave propagation model CREDIZ in a realistic laboratory situation
Delft Hydraulics Lab., Report S581, December 1983

Verification of a one-dimensional wave energy decay model

Transformation of wave height distribution

A practical method to obtain wave and storm surge conditions for prediction and probabilistic calculations

WU, J. (1980)
Wind-stress coefficients over sea surface near neutral conditions - a revisit

WU, J. (1982)
Wind-stress coefficients over sea surface from broeze to hurricane
J. Geophys. Res. 87 (C12), Nov. 1982, pp. 9704-9706