Guidance Techniques for Near-Optimal
Lateral Escape Maneuvers in the
Presence of Windshear

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Title: Guidance Techniques for Near-Optimal Lateral Escape Maneuvers in the Presence of Windshear

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Abstract: This study treats three "missing" aspects of previous research on escape guidance strategies during microburst encounters. First, the performance of three longitudinal guidance laws in combination with one single lateral guidance law is evaluated when more than one microburst core is present. Altitude guidance proves to be the most robust and gives the most promising results, provided the reference altitude to which the aircraft is controlled, is selected properly. Therefore, in this study attempts are made to develop either a logic that adjusts the reference altitude during the escape, or a logic that predicts the "best" reference altitude at the initiation of the escape maneuver. Unfortunately, the results are not satisfactory. Third, the 3 degrees-of-freedom (DOF) model is extended to a full 6 DOF model in order to investigate the effects of inertia and windspeed variations over the aircraft's size on performance and controllability. Inertia decreases performance of all strategies. The windspeed variations cause additional body moments, but even for rather severe microbursts these moments are not expected to cause controllability problems. Finally some recommendations for future research are presented. For example, turbulence should be modeled and implemented to study controllability effects.

Keyword(s): Windshear, microburst escape strategies, guidance, dynamic inversion, 6 degrees-of-freedom model.
Preface

This report is a result of a thesis study at the Faculty of Aerospace Engineering of the Delft University of Technology. This study was started in September 1996. In this study, some aspects of guidance techniques for escape maneuvers in a windshear environment are treated. The purpose of this study was to fill in some "gaps" of the previous academic research on this subject.

I would like to thank the people of the disciplinary group of Aircraft Design and Flight Mechanics for their help, especially my mentor dr. ir. H. G. Visser for sharing his knowledge and experience.

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Michiel Selier
Summary

Windshear, in particular the weather phenomenon called “microburst”, presents a significant safety hazard for aircraft during take-off and landing. A microburst downdraft is a column of rapidly descending air resulting in radially diverging winds as it impacts the ground. Numerous accidents and incidents have occurred that are associated with windshear and microbursts.

In case a microburst cannot be avoided, the Federal Aviation Administration prescribes an escape strategy, named pitch guidance. In previous research, optimal escape trajectories in the presence of a microburst have been studied. To approach these optimal solutions, two more guidance strategies were developed, christened rate-of-climb guidance and altitude guidance.

In all three strategies maximum thrust is applied. Pitch guidance further commands a constant pitch angle of 15 degrees and rate-of-climb guidance prescribes a commanded rate-of-climb, depending on microburst intensity. Altitude guidance trades altitude for airspeed and penetrates the microburst at a low \textit{reference altitude} where the shear and downdraft are less hazardous.

In previous studies, simulations are performed with mostly 3 Degrees-of-Freedom (DOF) aircraft models, and mostly only one microburst core is present. Also, a logic for determining the “best” reference altitude, used in altitude guidance, is missing. Without such a logic the benefits of altitude guidance cannot be exploited to their full extend.

In this thesis, these three “gaps” of previous research are studied.

1. The presence of two microburst cores
2. A logic for determining a reference altitude
3. Extending the 3 DOF aircraft model to a 6 DOF aircraft model

The performance criterion for testing the three guidance strategies is minimizing the altitude drop, or maximizing the minimum altitude reached.

When two microbursts are present, altitude guidance proves to be the best guidance strategy in longitudinal and lateral maneuvering, provided the best reference altitude is selected. Pitch guidance has the largest altitude drop. The importance of a reference altitude logic is shown once more.

Several unsuccessful attempts are made to develop such a reference altitude logic. Three adaptive logics are studied, that alter the reference altitude during the maneuver. None of these logics approach the best possible trajectory. Also it is attempted to predict the microburst strength in advance, and estimate a reference altitude close to the best possible value.

In order to successfully apply altitude guidance in reality, the search for a reference altitude should be continued in future research. A few possible approaches are presented.
With two 6 DOF models, the effects of rotational inertia and windspeed variations over the finite aircraft’s size are studied. The presence of inertia has a significant effect on the aircraft’s trajectory, especially during lateral maneuvering. This results in a performance decrease for all strategies. The windspeed variations over the aircraft’s size result in wind induced yaw and roll moments that slightly alter the aircraft’s trajectory. The induced pitch moments are very small, and therefore, performance in the vertical plane is hardly affected. Also it is shown that for most microbursts no controllability problems occur. Only very severe microbursts can result in instability. The wind induced body moments increase the instability.

In future research, the presence of turbulence, and its effect on performance and controllability should be modeled and studied. Also, the control surface deflections should be modeled differently to prevent unrealistically and unacceptably high deflections and deflection rates. Perhaps also the ground effect and the effect of rain on aerodynamics and engine performance can be modeled.
Nomenclature

a  acceleration
b  wing span
b' span between point 1 and point 2 in Four Point Aircraft model
C_1, C_2 coefficients used in reference altitude logic
C_D  coefficient of drag
C_L  coefficient of lift
C_{\tau}  coefficient of roll moment
C_m  coefficient of pitch moment
C_n  coefficient of yaw moment
\bar{c}  mean aerodynamic chord
D  drag, diameter of contour of maximum outflow
E  specific energy
F  windshear hazard factor
F_{\text{thres}}  threshold F-factor
f  scaling factor for Soesman microburst model
g  acceleration of gravity
H  geopotential altitude
h  altitude
I_{xx}  moment of inertia about the x_b-axis
I_{xz}  product of inertia with respect to x_b and y_b-axis
I_{yy}  moment of inertia about the y_b-axis
I_{zz}  moment of Inertia about the z_b-axis
K  gain factor
L  lift, roll moment, transformation matrix
l_t  tail length
M  pitch moment
N  yaw moment
p  roll rate, air pressure
q  pitch rate
R  specific gas constant of air, radius of downdraft shaft
r  yaw rate, radial distance to microburst center
r_{\text{look}}  forward-look range
S  wing area
s  ground distance
T  thrust, air temperature
t  time
U_{\text{max}}  maximum horizontal outflow velocity
u  speed component in x-direction
u_c  pseudo control
V  speed, (no subscript: airspeed)
v  speed component in y-direction
W  aircraft weight, windspeed
w  speed component in z-direction
Y  side force
z_m altitude of maximum horizontal outflow velocity
\( \alpha \) angle of attack
\( \alpha_{\text{ref}} \) angle of attack where \( C_L \) stops to be linear
\( \beta \) sideslip angle
\( \gamma \) flight path angle
\( \delta \) control surface deflection
\( \delta_t \) thrust inclination angle
\( \eta \) throttle setting
\( \theta \) pitch angle
\( \lambda \) temperature gradient in the atmosphere
\( \mu \) aerodynamic bank angle
\( \rho \) air density
\( \tau \) time constant, time interval
\( \phi \) bank angle
\( \chi \) heading angle
\( \chi_w \) horizontal wind direction
\( \psi \) yaw angle

subscripts
0 sea level ISA, initial value, point 0 in FPA model
1 point 1 in FPA model
2 point 2 in FPA model
3 point 3 in FPA model
a aileron, ambient
av average
BE body axes to earth axes
BW body axes to wind axes
b body axes quantity
EB earth axes to body axes
EW earth axes to wind axes
e earth axes (inertial) quantity, elevator
ini value at initiation of escape maneuver
look forward-looking quantity
r radial, rudder
ref reference value
SL Sea Level
t tail
v vertical tail
w wind axes quantity, wind related quantity, wing
x component in x direction
y component in y direction
z component in z direction
\( \delta \) control surface deflections

superscripts
\( . \) time derivative
\( * \) quantity at the end of second phase of escape maneuver
acronyms and abbreviations

c.g. center of gravity
DOF Degrees Of Freedom
FAA Federal Aviation Administration
FPA Four Point Aircraft model
IAS Indicated Air Speed
ISA International Standard Atmosphere
LFA Linear Field Approximation
NDI Nonlinear Dynamics Inversion
OPA One Point Aircraft model
PD Proportional-Derivative
PID Proportional-Integral-Derivative
RC Rate of Climb
SEP Specific Excess Power
WDI Wind Difference index
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1. Introduction and Problem Discussion

Windshear and microbursts
Windshear, in particular the weather phenomenon called “microburst”, presents a significant safety hazard for aircraft during take-off and landing. A microburst downdraft is a column of relatively cold air, which descends rapidly. As it impacts the ground, winds diverge radially which leads to variation in wind speed and direction, i.e. windshear. This low frequency windshear is accompanied by high-frequency turbulence. A typical microburst lasts 5 to 10 minutes, and remains at full intensity for about 5 minutes (4).

Microbursts are frequently encountered in hot moist areas, like the South-East of the U.S., in a thunderstorm environment. The downdraft is then often accompanied by heavy rain with an intensity many times higher than ordinary showers (“wet” microbursts). In colder areas, like Colorado (U.S.), mostly “dry” microbursts are observed. The most hazardous microburst are those with a diameter between 1 and 2 km.

When an aircraft penetrates a microburst core, first a headwind is experienced that increases the aircraft’s performance. Then the headwind shifts rapidly into a tailwind, accompanied by a strong downdraft and aircraft loses airspeed and altitude. In fact, this energy drain may be sufficiently strong to prevent the aircraft from recovering, see figure 1-1.

Numerous accidents and incidents have occurred that are associated with windshear and microbursts. Notorious examples are the crashes of a PAN AM Boeing 727 at New Orleans Airport in 1982 and a Delta Airlines L-1011 at Dallas-Fort Worth International Airport in 1985.

It must be remarked that windshear encounters may also be critical from a controllability point of view. Controllability problems were found by Ávila de Melo et al in ref. [1]. The crash of a Martinair DC-10 at Faro Airport in December 1992 is associated with poor (auto) throttle management (3) in the presence of windshear.

Windshear Detection
When a potentially hazardous windshear is detected at a sufficiently early stage, the pilot may initiate an escape maneuver or avoid the microburst. Currently, reactive windshear warning systems are capable of detecting windshear situations based on in situ measured data. Nowadays, these reactive systems are installed in an increasing number of modern jet airliners.

In addition to the reactive systems, substantial research is conducted to develop so called “forward-look” systems, see refs. [5, 11, 12, 16]. Improving the alert time by a few seconds can already lead to substantial performance increase in an escape maneuver (11).

For forward-looking windshear detection, Doppler weather radar or lidar are used to measure windspeed and windspeed variations. Doppler radar measures the speed of precipitation and is therefore very useful for detecting wet microbursts. Lidar uses laser to measure the speed of aerosols relative to the aircraft and is used best for
detecting dry microbursts. A problem of both systems is the fact that only relative speed differences along the line-of-sight are measured, as will be discussed below.

Windshear Hazard Measurements
The severity of windshear is generally expressed in terms of the windshear hazard factor (F-factor), which depends on windspeed and windspeed gradients, as well as aircraft state variables like airspeed and flight direction.

In the case of forward-look sensor systems, the measured windshear hazard is based upon windspeed variations measured along the line-of-sight only. Vicroy \( ^{20} \) has developed methods to estimate the vertical windspeed from the measured horizontal windspeed along the line-of-sight, but generally, horizontal crosswinds are not taken into account.

This might lead to a hazardous situations when more than one microburst core is present or when the aircraft is involved in lateral maneuvering, for example during a curved (MLS) approach. A simple example is given in figure 1-2. The aircraft detects small windspeed variations along the line-of-sight. The pilot will experience a crosswind at the aircraft’s location. However, he cannot detect the difference between the constant windspeed and the microburst windspeed component perpendicular to the line-of-sight. In a lateral escape he might turn towards the microburst in stead of away.

Instrumentation
Another research area is the pilot’s “situational awareness” in microburst encounters. In ref. [16] a graphical (icon) presentation is preferred over “raw-data-display” which is difficult for a pilot to interpret in real time. A pilot should be informed of the threat, position and range of the windshear hazard.

Also it must be considered how an advised escape maneuver or go-around is presented to the pilot, once a hazardous windshear is detected. The pilot should be presented an advised pitch angle and, in the case of lateral escape, an advised bank angle.

Escape strategies and guidance
One may wonder about the necessity of developing open-loop and closed-loop escape maneuvers when so much effort is taken in developing forward look systems, that, in the future, can make microburst avoidance possible. However, the alert time for the presence of windshear might for some reason be too small for microburst avoidance. For example in the case of a missed alert of the forward-look sensor system. In such cases an escape maneuver must be initiated.

In the past, optimal open-loop trajectories have been studied and developed, see ref. [21]. Most of these studies only consider trajectories in the vertical plane. It was however shown by Visser \( ^{22} \) and Ávila de Melo and Hansman \( ^{1} \) that lateral maneuvering might prove beneficial and increase performance, when moderate bank angles (10°) are applied. In ref. [22], Visser studies 3 escape strategies which can be performed in the vertical plane and also in combination with a lateral guidance strategy. These strategies are named pitch guidance, rate-of-climb guidance and altitude guidance respectively.

All three strategies apply maximum thrust. Pitch guidance is the strategy presented in the FAA Windshear training aid \( ^{7} \). This strategy commands a constant
pitch angle of 15 degrees. Rate-of-climb guidance and altitude guidance are two candidate guidance laws, that were developed to approach the open-loop optimal solutions. Rate-of-climb guidance prescribes a commanded rate-of-climb depending on the energy drain. Altitude guidance trades altitude for airspeed and penetrates the microburst at a low reference altitude where shear and downdraft are less hazardous. In figure 1-3 examples of the vertical trajectory of these three strategies are plotted.

Problems studied in this report
This thesis study focuses on three aspects. These aspects are summarized and discussed below.

1. The presence of multiple cores
In most simulation studies only one microburst core is modeled. One may wonder about the effect of the presence of multiple cores on the performance of the various escape maneuvers, especially when lateral maneuvering is applied. Therefore the effects of multiple cores on pitch guidance, rate-of-climb guidance and altitude guidance will be investigated.

2. Logic for determining the “best” reference altitude for altitude guidance
As is mentioned above, the altitude guidance strategy requires a reference altitude at which the core is to be penetrated. The “best” reference altitude depends on the microburst strength and the advance warning time. A predictive logic for determining this “best” reference altitude is still missing. Though altitude guidance has proved to be fairly robust to uncertainty in microburst strength, such a logic is necessary before altitude guidance can be applied in reality. Therefore, an attempt will be made to develop such a logic and make altitude guidance more adaptive to microburst strength.

3. Extension of the 3 Degrees-of-Freedom model to 6 Degrees-of-Freedom
To bridge the gap between the (3 DOF) academic open-loop optimization studies and more realistic piloted simulations, a 6 DOF batch simulations will be performed. Points of attention will be the effects of rotational inertia and the effects of pitch, roll and yaw moments induced by windspeed variations over the aircraft’s size. The high-frequency turbulence that always accompanies a microburst is not modeled in this study. However, the implementation of turbulence should be one of the points of attention for future research. Since in this thesis only dry microbursts are simulated, the effect of heavy rain ingestion on engine performance should also be a point of interest in the future.

In chapter 2, the aircraft will be modeled and two microburst models are compared. Then, in the chapters 3 and 4 respectively, an aircraft controller will be designed to control the aircraft along a prescribed trajectory and the guidance strategies will be modeled and discussed in more detail. The effect of multiple cores on the escape strategies during longitudinal and lateral maneuvering will be studied in chapter 5. In chapter 6 an attempt will be made to find a logic for the best reference altitude in order to make altitude guidance more adaptive to microburst strength. Finally in the chapters 7 and 8 a 6 DOF model is presented, and lateral and longitudinal escape trajectories are simulated in order to study the influence of inertia and windspeed variations over the aircraft’s finite dimensions.
2. Modeling of Microburst Encounters

Before simulation can take place, the microburst encounters must be modeled. The aircraft type that is chosen and modeled is the Boeing 727-100. The initial values for all simulations are given. Two available microburst models are given: the Soesman model and the Bowles-Osegueda model. A standard atmosphere will be assumed. Note that for the small altitude variations in the simulations considered in this thesis constant values of air pressure, temperature and density would be sufficient as well.

2.1 The Aircraft Model: the Boeing 727-100

The aircraft model used to simulate microburst encounters is the Boeing 727-100. This aircraft has been chosen for 3 reasons:

1. This model is *used regularly* in literature \(^\text{4, 21, 22}\) for microburst research so results for one aircraft type can be compared. Also this makes it more easy to gather aircraft data.

2. The aircraft has *three engines*, and thus, in general, a lower thrust over weight ratio than aircraft with two engines. This makes the aircraft more vulnerable for windshear, as was found by de Bruin \(^\text{4}\).

3. The reason why this model is preferred over aircraft with four engines (even more vulnerable) is that the 727 has a shorter range and performs relatively more take-offs and landings than for example the 747. The 727 therefore has a higher chance on a microburst encounter.

Throughout the maneuver the configuration of the aircraft remains unchanged, so the model presented below is used throughout the entire maneuver. A remark should be made that no attempt is made to model ground effect.

**Lift, Drag and Thrust Modeling**

Lift, drag and maximum thrust are modeled the same way as in Visser \(^\text{21}\). Lift and drag are modeled as a 2\(^{\text{nd}}\) order function of angle of attack, thrust is modeled as a 2\(^{\text{nd}}\) order function of airspeed. In formula:

\[
T_{\text{max}} = a_0 + a_1 V + a_2 V^2
\]

(2-1)

\[
C_D = b_0 + b_1 \alpha + b_2 \alpha^2
\]

(2-2)

\[
C_L = c_0 + c_1 \alpha + c_2 (\alpha - \alpha_{\text{ref}})^2
\]

(2-3)

\(C_L\) varies linear with the angle of attack until \(\alpha_{\text{ref}}\) is reached (\(c_2 = 0\)). Beyond this angle of attack the \(C_L\)-\(\alpha\) curve is a quadratic polynomial in \(\alpha\) (\(c_2 \neq 0\)).
The thrust depends on the throttle setting, control variable \( \eta \):
\[
T = \eta T_{\text{max}} \quad \text{with} \quad 0 \leq \eta \leq 1
\]  
(2-4)

The values of the coefficients in equations (2-1) to (2-4) are given in Appendix A.

**Stability Derivatives**

For the 6 DOF model, besides forces, also moments act on the aircraft. It is assumed that longitudinal moments and coefficients do not influence lateral quantities, and vice versa. The stability derivatives that are used are also given in Appendix A. The values have been taken from ref. [19].

The pitch moment is assumed to be a linear function of angle-of-attack, angle-of-attack rate, pitch rate and elevator deflection. The roll and yaw moment are assumed to be linear functions of sideslip angle, roll rate, yaw rate, aileron deflection and rudder deflection. Now the nondimensional moments acting on the aircraft are:

\[
C_l = C_{l_\mu} \beta + C_{l_\alpha} \frac{pb}{2V} + C_{l_\alpha} \frac{rb}{2V} + C_{l_\delta_a} \delta_a + C_{l_\delta_r} \delta_r
\]  
(2-5)

\[
C_m = C_{m_\alpha} + C_{m_\alpha} \alpha + C_{m_\alpha} \frac{\bar{c}}{V} + C_{m_\alpha} \frac{qc}{V} + C_{m_\alpha} \delta_e
\]  
(2-6)

\[
C_n = C_{n_\alpha} \beta + C_{n_\alpha} \frac{pb}{2V} + C_{n_\alpha} \frac{rb}{2V} + C_{n_\delta_a} \delta_a + C_{n_\delta_r} \delta_r
\]  
(2-7)

Definitions of positive control surface deflections can be found in figure 2-1. The influence of windspeed variations on the aerodynamic moments is modeled in paragraph 7.2.

**Initial Values**

The aircraft is assumed to be initially situated on the glide slope in a stabilized approach. Therefore the initial values for the state variables are (unless indicated otherwise):

\[
\begin{align*}
x_e(0) &= -2500 \text{ m} & y_e(0) &= 0 \text{ m} & h(0) &= 131 \text{ m} \\
\alpha(0) &= 7.38^\circ & \beta(0) &= 0^\circ & \mu(0) &= 0^\circ \\
p(0) &= 0 \text{ rad/s} & q(0) &= 0 \text{ rad/s} & r(0) &= 0 \text{ rad/s} \\
\eta(0) &= 0.36 & V(0) &= 69.9 \text{ m/s}
\end{align*}
\]

Here \( \gamma \) and \( \chi \) are chosen in such way that the aircraft’s flight path is along the glide slope in the earth axis system, i.e. \( \gamma_e \) and \( \chi_e \) are equal to -3° and 0° respectively.

**2.2 Microburst Models**

Currently, various microburst models are available. De Bruin gives a summary and description of some models in his thesis \(^6\). For this thesis study, two microburst models, the Soesman model and the Bowles-Osegueda model, are considered. For both models the following three assumptions are made.
1. The absence of turbulence, that in reality is always present. With turbulence present, aircraft performance deteriorates due to the high-frequency disturbances, especially when the angle-of-attack limit is encountered. It is desirable that in future research the effects of turbulence on the performance during escape strategies are studied.

2. Only dry microbursts are considered. The effect of heavy rain on aerodynamics and engine performance is not taken into account. Descatoire et al.\(^{(6)}\) have studied rain effects on performance. The heavy rain decreases the maximum angle of attack and increases drag so that performance during wet microbursts is less than during dry microbursts. Also the effect of engine ingestion of very heavy rain may have performance decreasing effects on thrust, since the air quality is diminished. Rain effects can also be a point of future research.

3. The microbursts models represent full intensity microbursts that are time invariant.

2.2.1 The Soesman Microburst Model

The model that will be called Soesman model in this report is the model that is used by Visser in optimization studies for an open-loop guidance solution \(^{(21)}\). This is a 3 dimensional extension of the model presented in ref. [18]. Visser also used this model for closed loop guidance simulation \(^{(22)}\).

The model is time invariant and the flow is assumed to be incompressible. The model features two separate expressions for radial flow and downdraft. The model uses cylindrical coordinates since it is axisymmetric. The radial windspeed is independent of altitude; boundary layer effects are not taken into account.

The geometry of a microburst encounter is presented in figure 2-2.

The radial wind velocities can be calculated with:

\[
W_r = f_r \left[ \frac{100}{(r - D/2/200)^2 + 10} - \frac{100}{(r + D/2/200)^2 + 10} \right] \quad (2-8)
\]

and the downdraft by:

\[
W_h = -f_h \left[ \frac{0.4h}{(r/400)^4 + 10} \right] \quad (2-9)
\]

Where \(r\) is the radial distance to the microburst axis of symmetry. The origin of the earth axis coordinate system is in this study located at the runway threshold, with x
positive in the runway direction, and z downwards. If the microburst center is then located at \((x_c, y_c)\):
\[
r = \sqrt{(x - x_c)^2 + (y - y_c)^2}
\]
(2-10)

The windspeed components in x and y direction can easily be calculated from the radial windspeed \(W_r\) and the wind direction \(\chi_w\) with:
\[
W_x = W_r \cos \chi_w
\]
(2-11)
\[
W_y = W_r \sin \chi_w
\]
(2-12)

And the windspeed in z direction (downward) is of course:
\[
W_z = -W_h
\]
(2-13)

The microburst velocity profiles can be shaped by the two shaping factors \(f_r\) and \(f_h\), and the diameter of maximum horizontal outflow \(D\). In figure 2-3 radial and vertical windspeed components as functions of altitude and radial distance to the center of the microburst core are plotted for \(f_r = f_h = 2\) and \(D = 2000\) m.

2.2.2 The Bowles-Oseguera Microburst Model

A simple 3 dimensional analytic downburst model has been developed by R. L. Bowles and R. M. Oseguera \(^3\). This model is, like the Soesman model, axisymmetric (cylindrical coordinates), time invariant and flow is assumed to be incompressible. In this model however, boundary layer effects are taken in to account. This model is used in most simulations because of its simple analytic expressions and realistic character.

Different downbursts can be modeled by specifying four parameters. These are:
1. a characteristic horizontal dimension
2. the maximum horizontal or vertical wind velocity
3. the altitude where the maximum wind velocity is reached
4. the depth of the outflow

The expressions for radial and vertical velocities can be written as (see ref. [3]):
\[
W_r = \frac{U_{\text{max}}}{0.2357} \frac{R}{2r} \left(1 - e^{-(r/R)^2}\right) (e^{-n/z} - e^{-h/z})
\]
(2-14)
\[
W_h = -\frac{U_{\text{max}}}{0.2357R} e^{-(r/R)^2} \left[\varepsilon (e^{-h/z} - 1) - z^* (e^{-n/z} - 1)\right]
\]
(2-15)

Where \(U_{\text{max}}\) is the maximum horizontal outflow velocity, \(R\) is the radius of the downburst shaft, which according to TASS data, is a factor 1.1212 smaller than the radius of maximum horizontal outflow contour \(^3\). The distance to the microburst center is \(r\) and is defined the same way as for the Soesman model, see equation (2-10). The parameters \(z^*\) and \(\varepsilon\) are characteristic scale lengths associated with "out of boundary layer" and "in boundary layer" behavior respectively. These parameters are
2. Modeling of Microburst Encounters

related to \( z_m \), the altitude of where the maximum outflow velocity occurs. It is found (based on TASS data\(^9\)) that:

\[
\frac{z_m}{z^*} = 0.22
\]  
(2-16)

\[
\frac{z_m}{\varepsilon} = \frac{z_m}{z^*} \frac{z^*}{\varepsilon} = 0.22 \times 12.5 = 2.75
\]  
(2-17)

Radial and vertical windspeed components as functions of altitude and radial distance to the center of the microburst core, are plotted in figure 2-4 for \( U_{\text{max}} = 20 \text{ m/s} \), \( z_m = 100 \text{ m} \) and \( D = 2000 \text{ m} \).

2.2.3 Comparison between Soesman and Bowles-Oseguera

Differences:
When the two microburst models are compared, two differences are observed.

1. The boundary layer effects are taken into account in the Bowles-Oseguera model, but not in the Soesman model.

2. For the Soesman model, the windspeed gradients outside the maximum outflow contour are larger than for the (more realistic) Bowles-Oseguera model. This means that for the Soesman model, the performance increase due to favorable shear outside this contour is also larger. Outside the core, the aircraft's performance for the Soesman model is thus too optimistic. Inside the core, the two models show no significant differences.

Motivation of Choice:
In this thesis study, the Bowles-Oseguera model is preferred over the Soesman model. Since multiple cores will be simulated, there is a good chance that the aircraft will be flying outside at least one of the cores. Thus the behavior outside the core becomes important. Also the boundary layer effects modeled in the Bowles-Oseguera model supply a more realistic basis for the 6 DOF simulations.

2.3 Modeling of the Microburst Hazard Factor

As mentioned in chapter 1, the hazard of a microburst is often expressed in terms of the so-called windshear hazard factor (F-factor). The F-factor can be based on in situ or forward-look measured data. However, measuring and predicting the F-factor is extremely difficult, since besides the low frequency (mean) windspeed caused by windshear also high-frequency turbulence is present. Therefore the measured data should be filtered over a certain amount of time. The length of this time interval should be selected with great care. If the interval is too small, turbulence will blur the measurements which might cause nuisance alerts. If the interval is taken (too) large, the difference between the minimum and maximum values of F tends vanish. This can lead to missed alerts.
Another method to register windshear severity is the Wind Difference Index, studied by Mieles (12), which is defined as the difference between the longitudinal wind over a certain time interval. This index is less sensitive to the choice of the interval time than the filtered F-factor. When the time interval increases, the WDI converges to a constant value.

The WDI is used for ground-based radar. The WDI does not take into account the aircraft state, while the initial aircraft state highly influences the outcome of any maneuver in the presence of windshear. Therefore, for airborne measurements the F-factor is preferred over the WDI.

**F-factor model**

In this study the same definition for the F-factor is used as in refs. [21, 22]:

\[
F = \frac{(T - D)}{V} - \frac{\dot{E}}{V}
\]  

(2-18)

Where the specific energy rate is:

\[
\dot{E} = h + \frac{V}{g} \dot{V}
\]  

(2-19)

The F-factor presents the loss or gain in available specific excess power due to horizontal windshear and downdraft. *Note that positive values of F represent performance decreasing situations.*

Substitution of (2-19) and the force equations of motion, equations (C-15) to (C-17), into equation (2-18) results in the following expression for F:

\[
F = \frac{1}{g} \left( \dot{W}_x \cos \gamma \cos \chi + \dot{W}_y \cos \gamma \sin \chi + \dot{W}_h \sin \gamma \right) - \frac{W_h}{V}
\]  

(2-20)

From equation (2-20) it can be observed that F depends on the relative position to the microburst and the wind field itself (via windspeed gradients and \( W_h \)) *as well as aircraft state* (via \( V, \gamma \) and \( \chi \)). Since both microburst models presented in paragraph 2.2 are modeled by smooth analytical functions and no turbulence is present, filtering is not applied.

For forward-look evaluation of the F-factor, the windspeed components and gradients are calculated at a distance \( r_{look} \) along the flight path. When predicting the F-factor with a forward-looking sensor, besides wind, also the airspeed and flight direction should be predicted. In this study however, the forward-look F-factor \( F_{look} \) is based on the *current aircraft speed and flight direction*.

Once the measured F-factor at the aircraft’s position, or the forward-look F-factor exceeds a threshold, \( F_{thres} \), an escape maneuver is initiated.

**F-factor measurement in reality**

A procedure to calculate the F-factor on board, taken from ref. [22], is briefly described here.
The following quantities can be measured on board: ambient pressure $p_a$, ambient temperature $T_s$, indicated airspeed $V_{IAS}$, angle-of-attack $\alpha$, sideslip angle $\beta$, the acceleration components in the body axis system $a_{bx}$, $a_{by}$ and $a_{bz}$, yaw angle $\psi$, pitch angle $\theta$ and roll angle $\phi$. From these quantities the airspeed $V$, barometric altitude $h$ and ambient density $\rho$ can be easily derived. The equations of motion are written in terms of wind axis variables. The angles $\chi$, $\gamma$ and $\mu$ can be calculated from $\alpha$, $\beta$, $\psi$, $\theta$ and $\phi$ by transforming earth axis quantities to wind axis quantities, using the following relationship:

$$L_{EW} = L_{EB} \cdot L_{BW}$$

(2-21)

Where $L_{EW}$, $L_{EB}$ and $L_{BW}$ are wind-to-earth, body to earth and wind to body transformation matrices respectively (see Appendix C).

With the equations of motion (Appendix C) the wind components can be written as:

$$W_x = \dot{x} - V \cos \gamma \cos \chi$$

(2-22)

$$W_y = \dot{y} - V \cos \gamma \sin \chi$$

(2-23)

$$W_z = \dot{h} - V \sin \gamma$$

(2-24)

The total time derivatives of the wind components are:

$$\dot{W}_x = \ddot{x} - \frac{d}{dt} \left( V \cos \gamma \cos \chi \right)$$

(2-25)

$$\dot{W}_y = \ddot{y} - \frac{d}{dt} \left( V \cos \gamma \sin \chi \right)$$

(2-26)

$$\dot{W}_z = \ddot{h} - \frac{d}{dt} \left( V \sin \gamma \right)$$

(2-27)

When equations (2-25) to (2-27) are substituted in (2-20), the following expression for $F$ is found:

$$F = \frac{1}{g} \left( \dot{x} \cos \gamma \cos \chi + \dot{y} \cos \gamma \sin \chi + \dot{h} \sin \gamma - \dot{V} \right) - \frac{\ddot{h} - V \sin \gamma}{V}$$

(2-28)

The accelerations in the earth axis system can be obtained from the accelerations measured in the body axis system by transformation:

$$\begin{bmatrix} \ddot{x} \\ \ddot{y} \\ \ddot{z} \end{bmatrix} = L_{EB} \begin{bmatrix} a_{bx} \\ a_{by} \\ a_{bz} \end{bmatrix}$$

$$\ddot{h} = -\ddot{z}$$

(2-29)

The output signals of the accelerometers and gyro's need to filtered because they are blurred with high-frequency turbulence. A frequently applied way of filtering high-frequency turbulence is to average the $F$-factor over a time interval $\tau$. The average F-Factor then becomes:

$$F_{av} = \frac{1}{\tau} \int_{t_0}^{t} F \, dt$$

(2-30)

As is explained before, the value of $\tau$ should be selected with great care.
Comparison between Model and Measurements
In reality, the time averaged F-factor is still blurred by turbulence. A typical measured and averaged F-factor history consists of many small local peaks superimposed on the mean wind \(^{(13)}\). The F-factors calculated with the model presented above however, have a smooth history in time, due to the absence of atmospheric turbulence in the wind models.

The use of a smooth F-factor leads to some advantages compared to reality that must be kept in mind when interpreting simulation results. It is obviously much easier to base an alert or warning on the smooth function calculated from the model than on the measured F-factor. The local peaks in the measured F-factor can easily lead to nuisance alerts. Also, the utilization in reality of any guidance strategy that uses the F-factor in some part of the logic will be difficult due to the presence of turbulence.
3. Aircraft Controller Design

To control the aircraft along the desired trajectories, prescribed by the guidance strategies, a controller must be designed. For this purpose the high order problem will be split into several lower order problems using singular perturbations. Then the control variables are defined and discussed. With a second math technique, nonlinear inversion dynamics, the desired values of the control variables can be calculated from the desired trajectory.

3.1 Singular Perturbations

Singular perturbations can be used to split a high order problem into several lower order problems, if time scale separation is possible. This is the case when there are large order variations in the eigenvalues of variables. A short mathematical description is given in ref. [2].

For most trajectory problems, time scale separation is indeed possible. The aircraft state variables can be split into fast, slow and very slow state variables.

- fast system \( \dot{p}, \dot{q}, \dot{r} \)
- slow system \( \dot{\alpha}, \dot{\beta}, \dot{\mu} \)
- very slow system \( \dot{V}, \dot{y}, \dot{z}, \dot{x}, \dot{y}, \dot{z} \)

Now time separation can be performed, so that three lower order systems remain with different time scales. When a fast system is evaluated and integrated over a (small) time step, the slower time scale variables are assumed to remain constant at the values they have at the start of that time step. On the other hand when slower variables are to be evaluated over a time step, the faster variables are assumed to change instantaneous in that time interval.

Note that the engine dynamics equation only depends on the commanded throttle setting, independent of the other state variables and is therefore a separate system.

3.2 Control Variables

3 DOF

In 3 DOF the slow state variables \( \alpha, \beta \) and \( \mu \) are the control variables. They are assumed to be able of changing instantaneously since they are on a faster time scale than the very slow state variables.

6 DOF

In 6 DOF The control surface deflections \( \delta_w, \delta, \) and \( \delta_r \) are used to control the aircraft towards the desired trajectory prescribed by the escape guidance strategies. The
control surface deflections are assumed to change instantaneous, i.e. with step response. The control surface deflection limits are given in Appendix A.

It is also possible to limit the rate at which the control surface deflections can be deflected. Then three more differential equations should be added to the system, each having the form of:

\[ \dot{\delta} = \omega_0 (\delta_c - \delta) \quad \dot{\delta}_{\text{lim}} < \infty \]  

(3-1)

This would mean adding a very fast system to the other systems.

In this study however, this possibility will not be considered since the microburst induced windspeed has a low frequency, while the control surface deflections react a few orders faster. The effect of finite control surface deflection rates will thus be negligible. However, when in future research high-frequency turbulence is to be modeled, adding the very fast system might be a point of consideration.

### 3.3 Nonlinear Inversion Dynamics

Nonlinear inversion dynamics (NDI) is a mathematical technique that can be used to control the aircraft to a desired trajectory. A short mathematical description and example is given by Mulgund \(^{(4)}\) and also by Boone \(^{(5)}\).

Normally in flight simulation, control surface deflections are inputs. With these values the fast state variables (p, q and r) are evaluated. Then slow state variables (\(\alpha, \beta, \text{ and } \mu\)) are integrated using fast state variables \(p, q\) and \(r\). With these slow state variables, the very slow state variables \((V, \gamma, \chi, x, y, h)\) can be evaluated.

The time separation makes it possible to divide the problem into three loops. One for the very slow state variables, one for the slow state variables and one for the fast state variables. In this thesis however, **two** loops will be mentioned.

1. The outer loop: the loop concerning the control of the very slow state variables which is used in *all simulations*.

2. The inner loop, concerning the slow and fast state variables. This loop is *only used in 6 DOF simulations*.

#### Outer-Loop Inversion Dynamics

The guidance strategies however, prescribe very slow state variables. In order to control the aircraft toward this trajectory, the normal process is inverted. From prescribed values for the very slow state variables, desired slow state variables can be calculated. The detailed process is given in chapter 4.

#### Inner Loop Inversion Dynamics

For 6 DOF simulation, another step of inversion dynamics is taken in order to calculate desired body rates. From these desired body rates, the desired control surface deflections can be calculated. When these desired control surface deflections are in
return used as input control surface deflections for the 6 DOF model, the aircraft is controlled towards the prescribed trajectory. The inner loop inversion dynamics will be further explained in paragraph 7.1.2.

Note: For 3 DOF simulation the slow state variables are the (instantaneous) control variables, therefore 3 DOF simulation only has the outer loop.
4. Modeling of Guidance Strategies

From the guidance strategies, commanded aerodynamic angles ($\alpha_c$, $\beta_c$, $\mu_c$) and a commanded throttle setting ($\eta_c$) will be calculated to guide the airplane using NDI. The aerodynamic angles can be used directly as control variables (3 DOF) or used as input for a NDI routine for the inner loop in order to calculate the desired control surface deflections (6 DOF, see paragraph 7.1.2).

During any escape maneuver it is desired to use maximum thrust, so $\eta_c$ is set equal to 1 during the entire escape. Also it is desired to avoid sideslip. The commanded sideslip angle is therefore set equal to zero at all times ($\beta_c = 0^\circ$). The commanded angle of attack is calculated using the vertical strategies, and the commanded bank angle is calculated using the lateral guidance strategy.

The maneuver is divided into three phases:

1. Initially, the glide slope is followed by the aircraft, until an escape maneuver is initiated (switch to phase 2 when $F > F_{\text{thres}}$ or $F_{\text{look}} > F_{\text{thres}}$).
2. An escape maneuver is performed and maximum thrust is applied (switch to phase 3 when the Energy drain has stopped, i.e. $\dot{E} > 0$).
3. A climb-out is performed.

A short description of the three vertical guidance strategies studied in this thesis is given, as well as a mathematical model. These vertical guidance strategies can be combined with a lateral guidance strategy, which is also described and modeled. Finally a control logic for thrust is given and discussed.

4.1 Vertical Guidance strategies

Three vertical guidance strategies are modeled and studied in this thesis, pitch guidance, rate-of-climb guidance and altitude guidance.

4.1.1 Pitch Guidance

The escape maneuver currently advised by the FAA (\textsuperscript{7}) consists of applying maximum thrust and holding the sum of angle of attack and flight path angle (pitch angle) constant to a target of 15° throughout the entire escape.

The commanded angle-of-attack value $\alpha_c$ can be calculated from the demand to keep the sum of angle-of-attack and flight path angle equal to a target pitch:

$$\alpha_c = \theta_{\text{ref}} - \gamma , \quad 0 \leq \alpha_c \leq \alpha_{\text{max}} \quad (4-1)$$

With $\theta_{\text{ref}}$ selected as 15° according to FAA procedures (\textsuperscript{7}). This guidance law holds throughout phase 2 and 3.
Error Due to Lateral Maneuvering
Although during lateral maneuvering the pitch angle is not equal to the sum of angle of attack and flight path angle, this strategy will be named pitch guidance throughout this report. The error between the real pitch angle and the sum of angle-of-attack and flight path angle is small as will be shown below.

When the sideslip angle is assumed to be small (since the commanded sideslip angle is always zero), the real pitch angle can be approximated with (see fig. 4-1):

$$\theta_{\text{real}} \approx \gamma + \frac{\alpha}{\cos \phi} \quad (4-2)$$

The error between the real pitch angle and sum of $\alpha$ and $\gamma$ is then:

$$\left| \theta_{\text{real}} - (\alpha + \gamma) \right| = \left| \alpha \left( 1 - \frac{1}{\cos \phi} \right) \right| \quad (4-3)$$

This error increases as $\alpha$ and $\phi$ increase. Thus when maximum angle-of-attack and maximum banking occur, this error has its maximum value. In normal flight conditions the angle-of-attack and aerodynamic bank angle do not exceed the prescribed limits. The bank angle $\phi$ is then assumed to be approximately equal to the aerodynamic bank angle $\mu$. With $\alpha = \alpha_{\text{max}} = 17.2^\circ$ and $\phi \approx \mu_{\text{max}} = 10^\circ$ the maximum absolute value of the error in pitch angle is $0.0046$ rad or $0.27^\circ$.

4.1.2 Rate-of-Climb Guidance

Rate-of-climb guidance (RC-guidance) is presented by Visser in ref. [22]. The aim of RC-guidance is to approximate the open-loop optimal trajectories. Visser found that the optimal strategy is to:

1. initially descend to the minimum altitude
2. stay in the vicinity of the minimum altitude while in the high shear/downdraft region
3. initiate a climb once this region is passed.

The basic idea behind RC-guidance is to design a guidance law that couples the time derivative of specific energy of the aircraft to a commanded rate of climb. This is thus an energy managing method, that trades airspeed for altitude and vice versa.

In the initial phase of the escape, altitude is traded for airspeed. Flying on the lower altitude, where the shear and downdraft and thus the energy drain is less, minimizes the energy loss, while keeping a safe distance above ground level. Then when the high shear region is passed, airspeed is traded back to altitude.

The commanded angle of attack is obtained with the use of dynamics inversion in the same way as Visser. Differentiation of the kinematic equation for rate of climb (see equation (C-3)) gives:

$$\dot{h} = V \dot{\gamma} \cos \gamma + \dot{V} \sin \gamma + \dot{W}_h \quad (4-4)$$
With the approximations \( \cos(\alpha+\delta) \approx 1-1/2(\alpha+\delta)^2 \) and \( \sin(\alpha+\delta) \approx (\alpha+\delta) \), substitution of equations (C-15) and (C-16) into (4-4) gives:

\[
\hat{h} = g \left[ \frac{T \left( 1 - \frac{1}{2}(\alpha+\delta)^2 \right) - D}{\sin \gamma} + \frac{L}{W} \cos \gamma \cos \mu \right] \cos \gamma \cos \mu \left( \frac{L + T(\alpha + \delta)}{W} - 1 \right) = u_c \quad (4-5)
\]

Where \( u_c \) is a pseudo control, that is now related to \( \alpha \) with a quadratic polynomial (for given \( T \) and \( \mu \)). For RC-guidance, the pseudo control is:

\[
u_c = -K_h (\hat{h} - \hat{h}_c) \quad (4-6)
\]

Similar to Mulgund \((14)\), the commanded rate-of-climb is scheduled to the energy rate by a gain \( K_e \):

\[
\dot{h}_c = K_e \dot{E} \quad (4-7)
\]

with:

\[
\dot{E} = \dot{h} + \frac{VV}{g} \quad (4-8)
\]

The commanded angle of attack can now be calculated from solving equation (4-5) for \( \alpha \).

The gain \( K_h \) is set to 0.25, a value that is selected by Visser \((22)\) because with this gain the closed-loop and open-loop solutions matched best. The choice of the gain \( K_e \) is made by the following logic:

In phase 2:

\[
\begin{align*}
K_e &= 0.25 & \text{in the performance increasing shear, i.e. dE/dt}>0. \\
K_e &= 0.75 & \text{once the energy drain starts (trade altitude for airspeed).} \\
K_e &= 0 & \text{when F has reached a (temporarily) peak value (altitude hold).}
\end{align*}
\]

In phase 3:

\[
K_e = 0.5 \quad \text{Use 50\% of energy rate for climbing.}
\]

Note that forward-look sensing is necessary if the escape maneuver is to be initiated while the aircraft is still the performance increasing shear.

### 4.1.3 Altitude Guidance

Altitude guidance has the same background as rate-of-climb guidance, namely to approximate the open-loop optimal solutions. In the initial phase of the escape maneuver, altitude is traded for airspeed by descending to reference altitude. Then an attempt is made to keep a constant altitude until the high shear region is passed. In the third phase a climb-out is performed.
For altitude guidance, the same method is followed as for RC-guidance, only here the
pseudo control \( u_e \) is defined as:
\[
 u_e = -(2\zeta \omega_0 \dot{h} + \omega_0^2 (h - h_c)) 
\]  
(4-9)

This law directs the aircraft towards the commanded altitude. Here, only PD-control is
applied but in future research, like in Mulgund (19), PID control can be used.

In phase 1, the aircraft is controlled along the glide slope.

In phase 2, the commanded altitude is set equal to a reference altitude, chosen by the
pilot. The choice of this altitude is the most difficult and critical part of this guidance
strategy, as will be discussed in chapter 6.

In phase 3, a switch is made to RC-guidance, with the gain \( K_e = 0.5 \).

4.2 Lateral Guidance

For lateral guidance, two possibilities are available. A lateral escape maneuver, where
the aircraft turns away (or towards) the microburst core, and a longitudinal escape
maneuver where the bank angle is used to correct deviations from a selected longitudinal
course.

As is mentioned before, the commanded sideslip angle is always set to zero, since
sideslip is to be avoided. Sideslip causes a considerable drag increase, which degrades
performance. So in any escape maneuver: \( \beta_e = 0 \).

4.2.1 Longitudinal Escape

Besides lateral escape maneuvers, also longitudinal maneuvers are studied in this
thesis. For these maneuvers, the commanded bank angle is calculated with the
following guidance law:
\[
 \mu_e = K_1 (y_c - y) - K_2 \dot{y}, \quad |\mu_e| \leq \mu_{max} 
\]  
(4-10)

This law controls the aircraft to a commanded \( y \) coordinate in the Earth axis system.
For the gain factors the following values are selected:
\[
 K_1 = 0.02 \\
 K_2 = 0.1 
\]

This law is used in all three phases for longitudinal escape maneuvers.

4.2.2 Lateral Escape

Phase 1
In phase 1, the same control law is used as in the longitudinal case. The aircraft is
controlled to fly a stabilized approach along the glide path.
Phase 2 and Phase 3
From the open-loop optimal trajectories found by Visser (21), it can be observed, that in
the after-shear region the aircraft "lines up" with a wind radial, i.e. \( \chi - \chi_w \) tends to go to
zero. Therefore in ref. [22], Visser presents the following, simple guidance law for
lateral escape that tries to control the "wind heading difference" (= \( \chi_w - \chi \)) to zero:

\[
\mu = K \mu \left( \chi_w - \chi \right), \quad |\mu| \leq \mu_{\text{max}} \quad (4-11)
\]

Where \(-180^\circ \leq \chi \leq 180^\circ\) and \(-180^\circ \leq \chi_w \leq 180^\circ\).

The question can be raised how this strategy reacts to the presence of constant horizontal
wind. A pilot is able to evaluate the direction of a crosswind. However, he is not able to
observe the difference between constant, "normal" wind and winds induced by a
microburst. When in figure 1-2 for example, at the aircraft’s position the constant wind
speed is larger than the lateral microburst windspeed component, the pilot might, in an
attempt to turn away from the microburst, turn right into the core.

To avoid such hazardous situations, variations in crosswind speed and direction
should be processed. The most ideal situation would be combining the airborne
measurements (crosswinds and winds along line-of-sight) with ground based windspeed
measurements.

\[4.3 \text{ Thrust Laws}\]

Phase 1
In phase 1, the aircraft tries to fly along the glide slope. The one remaining control
variable, the throttle setting \( \eta \), is used to control the aircraft’s speed. In ref. [15] Psiaki
and Park have developed and analyzed thrust laws. They found that a law that controls
the \textit{airspeed or the inertial speed, whichever is smaller}, to the nominal value gives
best results in the presence of windshear.

When just the airspeed is controlled to it’s nominal value, thrust is reduced in
the headwind phase of a microburst encounter since the headwind is experienced as a
gain in airspeed. The core is entered with reduced thrust, and thus reduced specific
excess power. This may cause hazardous situations since, due to engine dynamics,
valuable seconds are lost before maximum thrust is available.

If the inertial speed is controlled, no prominent thrust reduction will occur
since the inertial speed doesn’t change as quick as the airspeed. So \textit{in case of a
microburst}, thrust is not significantly reduced. This gives the aircraft better
performance in an escape maneuver since maximum thrust is available in a shorter
period. However if the airspeed drops below the inertial speed, the airspeed is to be
controlled and (increased) to prevent stalling.

Thus in ref. [15], it is stated that it is important to use thrust to keep the \textit{airspeed or
inertial speed, whichever is smaller}, at or above the nominal value. In this study
however, since it is certain that a microburst is present, the airspeed will rise and the
inertial speed will be the smallest. Thus, for simplicity, the thrust law presented in
equation (4-12) will be used in the initial phase, when the aircraft is flying along the glide scope.

\[ \eta_c = \eta_0 + \left(1 - \frac{V_e}{V_{eo}}\right), \quad 0 \leq \eta_c \leq 1 \] (4-12)

Where \( \eta_0 \) and \( V_{eo} \) are the nominal values for throttle setting and inertial speed respectively. In simulation, these nominal values are set equal to the initial conditions.

Note that only \textit{the inertial speed} is controlled.

**Phase 2 and Phase 3**

When the escape maneuver is initiated, maximum thrust is set throughout the entire maneuver, so \( \eta_c = 1 \).
5. Three DOF Simulation with Two Microburst Cores

In most previous research, the guidance strategies have been tested in simulations where just one microburst core is present. It is however quite interesting to know how the performance of these strategies is affected by the presence of a second or even more cores, especially during lateral maneuvering. An example of the presence of multiple cores is the crash at Faro Airport, 21 December 1996, where three downbursts were present (refs. [8, 10]).

When multiple cores are present, the benefits of lateral maneuvering are no longer guaranteed. In figure 5-1-a for example, turning left most likely results in the best performance. In figure 5-1-b however, it’s not possible to point out the best escape maneuver without knowledge of the location and strength of both microbursts. In this case longitudinal maneuvering can be more beneficial than lateral maneuvering, depending on microburst parameters. The problem of multiple cores research is the unlimited number of scenarios, since microburst location as well as strength can be varied. Also it is difficult to define a worst case scenario.

Another interesting case is presented in figure 5-2. One can imagine that the lateral guidance law, in an attempt to control the aircraft along a wind radial, might change direction and “slalom” past the two cores, see figure 5-2-c. This once again depends on microburst location and strength. When core 2 is located far along the glide slope (figure 5-2-a), the first core will have the largest contribution to the resulting wind vector, and the aircraft will turn right. When the (stronger) core 2 is rather close, it will dominate the wind direction, resulting in a left turn (figure 5-2-b). An unsuccessful attempt was made to simulate the scenario of case C by varying microburst location and strength. All attempted scenarios resulted in either case A or case B. Apparently very few (coincidental) combinations of microburst parameters result in such a “slalom” maneuver, maybe none at all. However, the slalom maneuver might give better performance in some cases, but in order to perform this maneuver the lateral guidance logic must be altered. Perhaps an optimization study of such cases might give better insight into the “best” maneuver.

So it is clear that in the presence of multiple cores, lateral maneuvering does not always result in better performance than longitudinal maneuvering, or perhaps a slalom maneuver. However, there is an unlimited number of scenarios possible and there is no obvious worst case scenario. Therefore, in this study the following two interesting scenarios are investigated to get insight into the effects of the presence of multiple cores on the escape strategies.

1. **Symmetric case.** Two microburst cores aligned on the glide path, longitudinal escape only. The distance between the cores is varied.

2. **Asymmetric case.** One core is placed on the glide path and a second core 950 m offset of the glide path. The difference in performance for turning away and turning towards the second core is studied (lateral escape).
Both cases will be simulated in a reactive, as well as a forward-look scenario. The reactive simulations base the F-factor evaluation on the aircraft state and windspeed in the center of gravity. The forward-look simulations evaluate the forward-look F-factor \( F_{\text{look}} \) on the current aircraft state and the windspeed components at a distance of 250 m along the line-of-sight. \( F_{\text{look}} \) is also based on crosswind speeds and vertical windspeed components. In reality only the windspeed along the line-of-sight can be measured.

The *performance criterion* used to judge the effectiveness of the three escape strategies will be the minimum altitude that is reached. It's also possible to use a combination of minimum altitude reached and specific energy at simulation termination \(^{(22)}\), since a good energy reserve is very useful in such a highly turbulent environment. The difficulty with such a criterion is the choice of the weight factors for minimum altitude and final energy. There is an unlimited amount of possibilities for placing emphasis on either altitude or energy. Thus, for reasons of simplicity the minimum altitude reached will be the criterion.

For both cores the Bowles-Osegueru model has been used. This choice is based on the fact that a Bowles-Osegueru model gives more realistic wind gradients outside the core, than the Soesman model. When multiple cores are simulated the windspeed components outside the core are important since depending on the core locations, the aircraft might be flying outside at least one of the cores. *Further, it is assumed that the resulting wind velocities and wind gradients can be calculated with linear superposition.*

The axis system that is used to describe the location of both cores is the Fixed Earth (inertial) axis system see Appendix B.

*Note:*
Initial values and aircraft data differ from those discussed in paragraph 2.1. The reason for this is that these simulations are performed with previously developed FORTRAN programs by Visser. The initial values and aircraft data are given in table 5-1 (see also ref. [21]).

<table>
<thead>
<tr>
<th>aircraft weight:</th>
<th>( W = 667233 \text{ N} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>wing area:</td>
<td>( S = 144.9 \text{ m}^2 )</td>
</tr>
<tr>
<td>( x(0) = -2500 \text{ m} )</td>
<td>( y(0) = 0 \text{ m} )</td>
</tr>
<tr>
<td>( h(0) = 131 \text{ m} )</td>
<td>( E(0) = 384.326 \text{ m} )</td>
</tr>
<tr>
<td>( \gamma(0) = -3^\circ )</td>
<td>( \chi(0) = 0^\circ )</td>
</tr>
<tr>
<td>( \eta(0) = 0.333 )</td>
<td></td>
</tr>
</tbody>
</table>

*Table 5-1: Initial values and aircraft data used in two core studies*

These conditions represent a stabilized glide slope approach for the given weight and wing area.
Also a \textit{F-factor threshold value of 0.04} is used, just as in Visser’s optimization studies \cite{21}. This means that the aircraft will initiate the escape immediately. This will result in better performance in lateral maneuvering than with a normally used threshold value, between 0.1 to 0.15. The low threshold value can be viewed as an extra forward-look range.

The various simulations will be performed with a 3 DOF aircraft model.

\subsection*{5.1 Three Degree-of-Freedom Aircraft Model}

The 3 equations of motion, describing the dynamics of a 3 DOF (point mass) aircraft model are derived in Appendix C. The force equations are derived in the wind axis system. The force equations of motion are:

\begin{equation}
\dot{V} = \frac{1}{m} \left[ T \cos(\alpha + \delta) - D \cos \beta + Y \sin \beta \right] - \dot{W}_x \cos \gamma \cos \chi + \nonumber \\
- \dot{W}_y \cos \gamma \sin \chi - \dot{W}_h \sin \gamma - g \sin \gamma \tag{5-1}
\end{equation}

\begin{equation}
\dot{\gamma} = \frac{1}{V} \left[ \frac{L + T \sin(\alpha + \delta)}{m} \cos \mu + \right. \nonumber \\
+ \frac{T \cos(\alpha + \delta) \sin \beta - D \sin \beta - Y \cos \beta}{m} \sin \mu + \nonumber \\
- g \cos \gamma + \dot{W}_x \cos \gamma \cos \chi + \dot{W}_y \sin \gamma \sin \chi - \dot{W}_h \cos \gamma \left. \right] \tag{5-2}
\end{equation}

\begin{equation}
\dot{\chi} = \frac{1}{V \cos \gamma} \left[ \frac{L + T \sin(\alpha + \delta)}{m} \sin \mu + \right. \nonumber \\
- \frac{T \cos(\alpha + \delta) \sin \beta + D \sin \beta + Y \cos \beta}{m} \cos \mu + \nonumber \\
+ \dot{W}_x \sin \chi - \dot{W}_y \cos \chi \left. \right] \tag{5-3}
\end{equation}

The three kinematic relations are:

\begin{equation}
\dot{x} = V \cos \chi \cos \gamma \sin \chi + \dot{W}_x \tag{5-4}
\end{equation}

\begin{equation}
\dot{y} = V \cos \gamma \sin \chi + \dot{W}_y \tag{5-5}
\end{equation}

\begin{equation}
\dot{h} = V \sin \gamma + \dot{W}_h \tag{5-6}
\end{equation}

And finally an equation for engine dynamics is added:

\begin{equation}
\dot{\eta} = \frac{1}{\tau_n} (\eta - \eta) \tag{5-7}
\end{equation}

Control variables are \(\alpha (= \alpha_c)\), \(\beta (= \beta_c)\), \(\mu (= \mu_c)\) and \(\eta_c\). Note that \(\beta\) remains zero throughout the simulation. Therefore the side force \(Y\) is also zero.
5.2 Longitudinal Escape

A study is performed to the effect of the presence of a second microburst on the performance and behavior of pitch-, RC- and altitude guidance during a longitudinal escape. For altitude guidance a reference altitude of 60 m will be used for all longitudinal cases. For this purpose, the following scenario will be used.

Two cores are placed on one line, namely the glide slope path (y_c=0). Both cores are equal in intensity and dimension and the following parameters are chosen.

\[ U_{\text{max}} = 15 \text{ m/s} \quad z_{\text{m}} = 100 \text{ m} \quad D = 2000 \text{ m} \]

Forward-looking Scenario (r_{look} = 250 m)

The first core is placed at \( x_{c1} = -1250 \text{ m} \) (1250 m before the runway threshold), see figure 5-3. Because the effect of the distance between the two cores is to be studied, the second core is placed in three different locations: \( x_{c2} = 750 \text{ m}, 250 \text{ m}, \) and \( 0 \text{ m} \) respectively.

Simulation Results

When the cores are far apart (\( x_{c2} = 750 \text{ m} \)) the plane behaves like flying through two separate microbursts. The second one is entered at lower altitude. This can easily be seen in the plots of the altitude, energy and F-factor, figure 5-4, where two peak values can be found.

When the second core is placed closer to the first, both cores tend to be experienced as one (long) microburst core. This behavior is best observed from the plots of the F-factor, where for smaller distances the two separate peaks tend to join in one peak. This is caused by the superposition of the windspeed components and windspeed gradients, that leads to one peak value between the two cores. This effect is the strongest for the altitude guidance technique, where the second peak has disappeared already at \( x_{c2} = 0 \text{ m} \), see figure 5-5. For pitch guidance the second peak in the F-factor value is present for smaller distances than for the other two strategies.

When the performance of the three strategies in the presence of the second core is compared, it seems that altitude guidance is the strategy that is the most robust to microburst strength and location. The plot of the altitude versus time, figures 5-4-a and 5-5-a, is quite similar to the case with one core. The minimum altitude is in some cases lower than for the other strategies, but that is due to the selected value of the reference altitude. At this lower altitude the energy drain is less, resulting in a higher specific energy. Also aircraft has no problems in holding this altitude.

Note that for \( x_{c2} = 750 \text{ m} \), the minimum altitude drops below the reference altitude. This is due to the fact that the switch to rate-of-climb guidance is made before the second core is entered. To avoid such behavior, a switch back to altitude guidance (phase 2) should be implemented.

RC guidance is also fairly robust. Though the history of the altitude with respect to time shows some differences, the minimum altitude for all cases doesn’t differ much. Better performance is perhaps possible if other gain factors in the guidance law are chosen. The currently used gain factors are the same values Visser used in ref. [22] and are tuned for the performance through only one (Soesman model) core.
It is shown that the *pitch guidance* strategy is the most sensitive for the presence of the second core. The performance of the pitch guidance strategy decreases considerably with decreasing distance between the two cores, as can be seen in figures 5-4-a and 5-5-a.

### 5.3 Lateral Escape

In the previous paragraph the effect of a the presence of a second core during longitudinal maneuvering is studied. The presence of a second core during lateral maneuvering is even more critical since due to banking, performance in the vertical plane is reduced. Therefore is it useful to study the effects of the presence of a second downburst that is placed right on the aircraft’s trajectory in the one core case. The pilot can now turn towards or away from the second core. The robustness of pitch-, RC- and altitude guidance during lateral maneuvering can then be compared.

To create this scenario one microburst core is placed straight in front of the aircraft (on the $y = 0$ line). Then a lateral escape is simulated with all three strategies. The second microburst is now placed in such way that it is situated approximately on the escape paths flown with all three strategies. This core has an maximum outflow velocity of 50% of the first microburst core.

Now lateral escape maneuvers are again performed for two cases. In one case the plane turns left, away from the second core and in the second case the plane turns right, into the second core.

For altitude guidance, the reference altitude is chosen near the best value in the case of turning away from the second core. This value is kept the same for the turn towards the second core in order to study the effect of a “wrong turn”.

**Reactive and Forward-looking Scenario**

For the reactive scenario, the microburst parameters chosen for the reactive scenario are given in table 5-2. To simulate forward-look sensing with a forward-look range of 250 m, both microburst cores are placed 250 m in positive $x_e$-direction.

<table>
<thead>
<tr>
<th></th>
<th>Core 1</th>
<th>Core 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>$U_{max}$ [m/s]</td>
<td>20</td>
<td>10</td>
</tr>
<tr>
<td>$z_m$ [m]</td>
<td>100</td>
<td>100</td>
</tr>
<tr>
<td>$D$ [m]</td>
<td>2000</td>
<td>2000</td>
</tr>
<tr>
<td>$x_e$ [m] (Reactive case)</td>
<td>-1500</td>
<td>-1000</td>
</tr>
<tr>
<td>$y_e$ [m]</td>
<td>0</td>
<td>950</td>
</tr>
</tbody>
</table>

*Table 5-2: Parameters for Simulation of a "Wrong Turn"*

This scenario is plotted in figure 5-6.
Simulation Results
Now both turns are simulated. The minimum altitudes reached in the simulations for reactive and forward-look sensing are displayed in table 5-3.

<table>
<thead>
<tr>
<th></th>
<th>( r_{\text{look}} = 0 \text{ m} )</th>
<th></th>
<th>( r_{\text{look}} = 250 \text{ m} )</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>turn away</td>
<td>turn to</td>
<td>turn away</td>
</tr>
<tr>
<td>alt. ( (h_c = 60) )</td>
<td>59.0</td>
<td>41.8</td>
<td>93.1</td>
</tr>
<tr>
<td>climb-rate</td>
<td>44.7</td>
<td>35.0</td>
<td>64.1</td>
</tr>
<tr>
<td>pitch</td>
<td>36.0</td>
<td>21.8</td>
<td>75.1</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>( h_c = 95 )</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>climb-rate</td>
<td></td>
<td>64.1</td>
<td>53.9</td>
</tr>
<tr>
<td>pitch</td>
<td></td>
<td>75.1</td>
<td>40.7</td>
</tr>
</tbody>
</table>

Table 5-3: Comparison of minimum altitude in the presence of a second core

The results show an obvious decrease in performance (minimum altitude) if the aircraft turns towards the second core in stead of turning away, see also figures 5-7 and 5-8.

The results also show that when the wrong turn is made, rate-of-climb guidance adapts quite well to the extra microburst core, with an additional altitude drop of 22% for reactive and 16% for the forward-look scenario. Even better results are perhaps possible when the gain factor \( K_h \) in the outer-loop guidance law are altered, since the results for this strategy depend significantly on the value of \( K_h \).

Surprisingly, altitude guidance gives a better performance for the reactive scenario than for the forward-looking scenario. This is due to the fact that for the reactive situation the reference altitude is closer to the best possible altitude. The best reference altitudes for turning towards the second core are 54 m and 61 m for reactive and forward-looking respectively. Then the minimum altitude reached is 52.5 m for \( r_{\text{look}} = 0 \text{ m} \) and \( h_{\text{min}} = 60.0 \text{ m} \) for \( r_{\text{look}} = 250 \text{ m} \). Again it is shown that altitude guidance depends highly on the selected reference altitude. When the selected value is too high, like in the forward-look scenario, a large altitude drop due to the encountering of the angle-of-attack limit results. If this altitude is selected near the best possible altitude, this strategy produces the best results. A prediction logic for \( h_{\text{ref}} \) is therefore indispensable.

Pitch guidance again proves to be not very robust with respect to microburst location and strength. For the reactive scenario, the minimum altitude is 50% lower than in the forward-looking scenario for both cases. When the aircraft turns towards the second core instead of away, \( h_{\text{min}} \) decreases about 40% for reactive and 45% for the forward-look scenario.
6. Search for a Reference Altitude Logic

The altitude guidance strategy consists on the choice of a reference altitude. When the escape maneuver is initiated, the aircraft descends to this altitude, and when possible, this altitude is kept constant until the high shear/downdraft region is passed. The results of this strategy are the most promising \(^{(23)}\) of the three strategies, provided the best reference altitude is selected.

If the selected altitude is too high, the energy drain will cause the aircraft to encounter the angle-of-attack limit. As long as the desired angle-of-attack is equal to the limit value, the aircraft will perform a phugoid motion, resulting in a larger maximum altitude drop. An example is given in table 5-2 and the figures 5-7-b and 5-8-b.

However, if the reference altitude is selected lower than necessary, the aircraft will build up a high speed at a low altitude. The headwind increases energy at the initiation (higher altitude), but at the low selected altitude the energy drain is less than expected, which leads to a high airspeed. Though this is not as much a problem from a performance point of view, flying at low altitude with high speed in a highly turbulent environment is for a (human) pilot an undesired situation as well.

The above illustrates the importance of a proper selection for the reference altitude. Finding a solution for this problem can be approached in two ways:

1. Develop a logic that makes the reference altitude adaptive to microburst strength by adjusting \( h_{ref} \) during the maneuver.
2. Predict the reference altitude in advance by estimating the energy drain.

Without such a reliable prediction or adaptive logic, altitude guidance cannot be used effectively in random microburst encounters. First a brief study is performed to discover relations between reference altitude and microburst strength.

The Optimal Reference Altitude

Just like in the previous chapter, the performance criterion for the altitude guidance strategy is minimizing the altitude drop, i.e. maximizing the minimum altitude reached. When the altitude obtains values below 10 meters, the aircraft is assumed to be crashed and simulation is terminated.

In order to get insight in the relation between microburst strength and size, various simulations with the 3 DOF model and one downburst core are performed. For each microburst size and strength the best reference altitude is obtained by trial and error. It shows that, independent of microburst strength or size, the best value of \( h_{ref} \) is a value where during the maneuver the angle-of-attack limit is reached just before the energy drain stops. Since the energy drain is low, almost zero, at that moment, the resulting altitude drop due to phugoid motion is small, almost negligible, until the moment that the climb-out is initiated. A remark must be made that only integer values are used in evaluating the best reference altitude.
6.1 An Adaptive Logic for the Reference Altitude

First it is attempted to develop a logic that adjusts the reference altitude during the escape maneuver. When the escape is initiated an initial reference altitude $h_{ref0}$ is selected. This reference altitude is at first relatively high (for example 70% of the altitude at initiation). Then when the aircraft is within 1 m of the reference altitude and the angle of attack approaches the limit while the energy drain is still large, the reference altitude is lowered. Three concepts of such a logic are studied.

If $\alpha \geq C_1 \alpha_{max}$ and $|h-h_{ref}| < 1$ m, then:

A. Step Logic:
$$h_{ref,new} = h_{ref,old} - \Delta h$$
$$h_{ref} \geq h_{ref,min}$$

B. Scale Factor Logic:
$$h_{ref,new} = C_2 h_{ref,old}$$
$$h_{ref} \geq h_{ref,min}$$

C. F-Factor Logic:
$$h_{ref,new} = h_{ref,old}(1-F)$$
$$h_{ref} \geq h_{ref,min}$$

Where $C_1$ is the fraction of $\alpha_{max}$ where a switch to a lower reference altitude is performed, and $C_2$ is the ratio between the new and old reference altitude. $F$ is the current F-factor at the aircraft’s location. Also a minimum reference altitude is specified to prevent the reference altitude from obtaining unacceptably low values.

Test Results:
The three logics are tested for different values of $\Delta h$, $C_1$, and $C_2$. It proves that the flown trajectory is about equal for all combinations of $\Delta h$, $C_1$, and $C_2$. The following examples will illustrate the behavior of the three guidance logics.

The three logics are tested in a reactive scenario with a F-factor threshold value of 0.04. This relatively low threshold value can be viewed as a forward-look range. The following microburst data are selected:

$$U_{max} = 22.7 \text{ m/s} \quad D = 2000 \text{ m} \quad z_m = 100 \text{ m}$$

$$x_{mb} = -1500 \text{ m} \quad y_{mb} = 0 \text{ m}$$

First, the best reference altitude is evaluated. The best reference altitude proves to be $h_{ref} = 59$ m; in that case the minimum altitude reached is: $h_{min} = 57.8$ m. The best trajectory is plotted in figure 6-1. Now for the three logics, two simulations are performed with a different value of $h_{ref0}$:

1. $h_{ref0} = 90$ m
2. $h_{ref0} = 65$ m

Logic A ($\Delta h = 5$ m, $C_1 = 0.8$):
In figure 6-2 the altitude is plotted for simulations with and without the use of logic A. The initial reference altitudes are $h_{ref0} = 90$ m and $h_{ref0} = 65$ m. The minimum altitudes reached are shown in table 6-1.

As can be observed, the performance of the logic is better for the value of $h_{ref0} = 65$ m, that is closest to the best value of 59 m.
Table 6-1: Minimum altitudes reached with and without use of logic A

For \( h_{ref} = 90 \text{ m} \), the aircraft initially approaches this reference altitude. Then, when the angle of attack reaches 80% of the limit value, the reference altitude is decreased. However, as can be seen in figure 6-3, the reference altitude decreases several times until the value has reached 25 m while for \( h_{ref} = 65 \text{ m} \), the reference altitude is 50 m in the end.

Apparently, when the initial reference altitude is very high above the best reference altitude, the angle-of-attack reaches the 80% limit while the aircraft is trying to level off after descending to the new reference altitude.

Also the comparison is made between the minimum altitude reached with logic A and the minimum altitude reached without any logic, when for both the initial reference altitudes 90 and 65 m are used, see fig. 6-2 and table 6-1. It can be observed immediately that the trajectories with and without logic A practically coincide. Logic A the minimum altitude with about 1 to 1.5 meter, so using the logic is beneficial but the positive effect is very small.

It can be concluded that this logic has no satisfactory behavior for these values of \( \Delta h \) and \( C_1 \), especially not when the initial reference altitude isn’t near the best reference altitude. Varying the values of \( \Delta h \) and \( C_1 \), however, doesn’t increase the benefits of this logic, as will be explained below.

In this example, in the first few steps, \( \Delta h \) is too small so the angle-of-attack threshold \( 0.8 \alpha_{max} \) is reached after a short period again, resulting in another decrease of the commanded altitude. For larger values of \( \Delta h \), the performance in the first few steps is better. However, at low reference altitudes, the altitude step is then too large to perform “fine tuning” of the reference altitude. This will lead larger to altitude decreases than necessary.

Varying the value of \( C_1 \) has hardly any effect on performance. Once the limit \( C_1 \alpha_{max} \)-limit has been reached for the first time, the reference altitude is decreased too often because the angle-of-attack remains high, in the proximity of this limit. This occurs for every value of \( C_1 \), especially when the value of \( \Delta h \) is low.

It can be concluded that this particular logic seems to have no significant improvements compared to flying without an altitude reference logic.

Logic B (\( C_1 = 0.8, C_2 = 0.8 \)):
As can be seen in fig. 6-4 and table 6-2, the second logic has a slightly better performance for \( h_{ref} = 90 \text{ m} \), but still no significant improvement compared to flying without an altitude reference logic.
Table 6-2: Minimum altitudes reached with or without use of logic B

The reason for this slight improvement is the fact that this logic has no constant altitude step, but one that depends on the current reference altitude. For higher values of $h_{ref}$, $\Delta h$ will be higher than for low reference altitudes. This solves the problem of constant altitude step applied in logic A.

However, the reference altitude is still decreased too often as a result of the constant value of $C_1$. This still causes the reference altitude to decrease when the high shear/downdraft area is almost passed, resulting in unnecessary low reference altitudes, see figure 6-5. Therefore, the performance of logic B does not approach the performance of the best trajectory.

Logic C: ($C_1 = 0.8$)

This third logic relates the altitude step to the local F-factor, a measure for the local microburst strength. When the 80% of the angle-of-attack limit is reached in a high shear/downdraft zone, the drop of the commanded altitude will be larger than when the 80% limit is reached in a lower shear/downdraft zone. This logic thus depends on microburst strength. The results of this logic are given in figures 6-6 and 6-7 and table 6-3.

Table 6-3: Minimum altitudes reached with or without use of logic C

As can be seen in table 6-3, the results of logic C are slightly better than logic B for $h_{ref} = 90$ m. However, for $h_{ref} = 65$ m, the minimum altitude reached is slightly less than for logic B.

Logic C is also beneficial compared to the simulations without logic, but when the initial reference altitude is selected much to high, the minimum altitude reached with this logic is 27.1 m lower than the best trajectory.

With this logic the reference altitude does not drop as much as with the logics A and B. For $h_{ref} = 90$ m the final reference altitude is 54.8 m. The reference altitude (figure 6-7) actually increases at the end. In that phase, the F-factor has a negative value. However, this does not influence the trajectory since in that phase RC guidance is used. For $h_{ref} = 65$ m the final reference altitude is 47.7 m.
The performance of logic C also does not approach the performance of the best trajectory, but especially when the initial reference altitude is selected high, this logic has the best results of the three.

An important aspect of this logic is the measurement of the F-factor. The implementation and appliance of this logic in reality shall not be easy, due to the difficulties in windshear hazard measurement. The question may be raised if the evaluation of such an important parameter as the reference altitude should depend on such an uncertain environmental parameter.

Conclusions and recommendations
In this paragraph, three logics are presented and tested which adjust the reference altitude during the escape maneuver. With none of these logics the best trajectory is approached.

All logics start the escape maneuver with a fairly high reference altitude. The reference altitude is lowered when the angle-of-attack approaches the limit value. These logics prove to be slightly beneficial, but the results do not approach the performance of the “best” reference altitude and with these logics, the advantage of altitude guidance over pitch guidance or RC guidance is lost.

Selecting a moderate value, say between 60 m and 70 m, for the initial reference altitude will give better results than choosing a high initial reference altitude for most microburst encounters. Two major scenario’s are then possible.

1. In the case of a moderate or weak microburst, the moderate reference altitude will be lower than the “best” reference altitude, this indeed results in a high speed, but the aircraft still has a moderate buffer and the angle-of-attack is not approached.

2. In the case of a more severe microburst, the moderate initial reference altitude might be near to, or somewhat higher than, the best reference altitude. In this case, the aircraft will approach the angle-of-attack limit, but the resulting altitude drop is not large enough to result in a situation the aircraft cannot recover from.

In the rare case of a very severe microburst, when the best reference altitude is much lower than the initial moderate altitude, the altitude drop will be large and a crash may result. There will always be severe cases that will always result in a crash, no matter which escape strategy is used. Such cases must however, be treated as exceptions.

Based on this observation that selecting a moderate initial reference altitude is a “safe” approach for an escape maneuver, two different approaches are suggested for future research.

1. Instead of starting from a high initial reference altitude, and decreasing the reference altitude when necessary, the maneuver can also be started with a moderate initial reference altitude. Then energy is stored as airspeed. When the energy drain is less than expected, the reference altitude can be
increased stepwise, tradingairspeed for altitude. If the energy drain is larger than expected the reference altitude should be lowered. The step in reference altitude should depend on the current energy rate, a quantity that can be measured with total energy-rate sensors.

2. A second approach worth trying in future research is choosing a moderate reference altitude initially. When the angle-of-attack limit is approached, a switch from altitude guidance to (the energy managing) RC guidance can be made.

### 6.2 A Prediction of the Reference Altitude in Advance

A different approach might be attempting to estimate the best reference altitude at initiation of the escape maneuver, using an energy balance. When the Escape maneuver is initiated at time \( t = t_{\text{ini}} \), the specific energy is:

\[
E_{\text{ini}} = h_{\text{ini}} + \frac{V_{\text{ini}}^2}{2g}, \tag{6-1}
\]

As was observed in the optimal reference altitude study, the angle-of-attack limit is reached just before the energy drain stops for the best reference altitude. When this limit is reached (at time \( t = t' \)), the aircraft is still in a level flight on an altitude near the reference altitude, so the airspeed equals the 1-g stall speed at that moment. The aircraft's specific energy is then:

\[
E' = h_{\text{ref}} + \frac{V_s^2}{2g} \tag{6-2}
\]

From the law of energy conservation, the energy balance can be derived:

\[
E' = E_{\text{ini}} + \int_{t_{\text{ini}}}^{t'} \dot{E} \, dt \approx E_{\text{ini}} + \int_{s_{\text{ini}}}^{s'} \frac{\dot{E}}{V + V_w} \, ds \tag{6-3}
\]

Where \( \dot{E} \) is the specific energy flow to the aircraft, and \( s \) is the ground distance covered by the aircraft. Substitution of (6-1) and (6-2) in (6-3) gives a formula for \( h_{\text{ref}} \):

\[
h_{\text{ref}} = h_{\text{ini}} + \frac{V_{\text{ini}}^2}{2g} - \frac{V_s^2}{2g} + \int_{s_{\text{ini}}}^{s'} \frac{\dot{E}}{V + V_w} \, ds \tag{6-4}
\]

In between \( t = t_{\text{ini}} \) and \( t = t' \), there has been an energy flow due to the microburst's energy drain and Specific Excess Power (SEP). The SEP, most likely, has a positive contribution to the energy balance due to the appliance of maximum thrust. The energy flow is (see also equation (2-18)):

\[
\dot{E}(t) = \frac{(T - D)V}{W} - VF \tag{6-5}
\]
Substitution into equation (6-4) gives:

\[ h_{ref} = h_{ini} + \frac{V_{ini}^2}{2g} - \frac{V_s^2}{2g} + \int_{s_i}^{s_f} \frac{1}{V + V_s} \left( \frac{T - D}{W} - V_F \right) ds \]  

(6-6)

Initial speed and altitude are known, as well as the stall speed. The only unknown quantity is the integral that represents the energy flow. If this energy flow can be predicted accurately at initiation of the escape maneuver, a good estimation of the best reference altitude can be made at the moment the escape maneuver is initiated.

**Estimation of the SEP**

The aircraft weight can assumed to be constant during the escape. For \( T, D \) and \( V \) it might be possible to assume mean values for the entire escape. After initiation, the shape of the history of these three variables with time is approximately the same from the moment of initiation.

A *very limited* study of the mean values is performed in this thesis. Three reactive and three forward-look scenarios are simulated. For all cases \( U_{max} = 22.7 \text{ m/s}; x_{mb} = -1500 \text{ m} \) (-1250 m for forward-look); \( y_{mb} = 0 \). Lateral maneuvering is applied in all cases.

The remaining microburst parameters are:

A. \( z_m = 100 \text{ m} \)  
B. \( z_m = 75 \text{ m} \)  
C. \( z_m = 125 \text{ m} \)  
D. \( z_m = 100 \text{ m} \)  
E. \( z_m = 75 \text{ m} \)  
F. \( z_m = 125 \text{ m} \)

\[ \begin{array}{ccc}
A & 2.08 & 0.938 & 1.27 \\
B & 2.18 & 0.930 & 1.38 \\
C & 2.19 & 0.884 & 1.37 \\
D & 2.25 & 0.969 & 1.20 \\
E & 2.70 & 0.962 & 1.26 \\
F & 3.03 & 0.936 & 1.27 \\
\end{array} \]

*Table 6-4: Thrust, Drag and airspeed ratios*

The values of mean thrust, airspeed and drag are quite similar for the reactive scenarios A, B and C. The values of airspeed and drag are similar in the forward-look cases D, E and F also. The mean thrust ratio is not so constant in forward-look scenarios, but for simplicity, a mean value is evaluated anyway. With the values in
table 6-4 a first approximation can be made for the mean values of thrust, drag and airspeed of the form:

\[ T_m = k_T T_{ini} \]
\[ V_m = k_V V_{ini} \]
\[ D_m = k_D D_{ini} \]  \hspace{1cm} (6-7)

Substitution of (6-7) in equation (6-6) gives:

\[ h_{ref} = h_{ini} + \frac{V_{ini}^2}{2g} - \frac{V_2^2}{2g} + \frac{(k_T T_{ini} - k_D D_{ini}) k_V V_{ini} (t' - t_{ini}) - \int_{t_{ini}}^{t'} VF \, ds}{W} \]  \hspace{1cm} (6-8)

Where for the time being, for the Boeing 727, the mean values from table 6-5 are selected:

<table>
<thead>
<tr>
<th></th>
<th>reactive (A, B, C)</th>
<th>forward-look (D, E, F)</th>
</tr>
</thead>
<tbody>
<tr>
<td>( k_T ) =</td>
<td>2.15</td>
<td>2.66</td>
</tr>
<tr>
<td>( k_V ) =</td>
<td>1.34</td>
<td>0.96</td>
</tr>
<tr>
<td>( k_D ) =</td>
<td>0.92</td>
<td>1.25</td>
</tr>
</tbody>
</table>

**Table 6-5: Mean thrust, drag and airspeed ratio**

Due to the limited time available for this thesis study, these values were not tested extensively. Therefore these values should be validated by future research, especially the mean thrust in a forward-looking scenario.

**Estimation of the F-Factor**

For forward-look sensing, the F-factor is known in advance for the area within the forward-look range. Predicting the maximum F-factor is however very difficult, unless of course, a maximum is already measured within the forward-look range. A possible solution would be finding a characteristic shape for the F-factor history versus time. An example is shown in figure 6-8.

When an escape is initiated, an attempt can be made to fit the shape of the history that is measured until then with the characteristic shape. This way it might be possible to make a prediction of the maximum F-factor and the diameter of the microburst.

For reactive measurement, a similar approach can be followed. The amount of collected data is equal, but in the reactive case, the escape is initiated at a stage where the aircraft is located further towards the microburst center. The time to evaluate a reference altitude and control the aircraft towards this altitude is very limited.

A substantial amount of time is put in to an attempt to define such a characteristic shaping function for the F-factor history (Bowles-Oseguera model). However, no “general” shaping function or accurate prediction method for the energy drain is found that fits most of the possible combinations of maximum outflow velocity, altitude of maximum outflow velocity and core diameter.
Predicting the energy drain in advance from airborne measurements only will always remain difficult, because of the presence of turbulence, and since only the windspeed component along the line-of-sight are measured. Vicroy (20) has studied the possibility to estimate the vertical windspeed from the horizontal windspeed. His work has not been used in the attempt defining a characteristic shaping function in this study. Perhaps his work can provide a useful basis for future attempts.
7. Six Degree-of-Freedom Aircraft Model

To bridge the gap between 3 DOF batch simulation and real time piloted simulations, 6 DOF batch simulations need to be performed. With a 6 DOF model the influence of inertia on the escape strategy performance can be studied as well as the influence of moments induced by windspeed variations over the finite aircraft size. As stated in the introduction, the effect of turbulence is not studied in this thesis, but this should be the next point of focus for future research.

Two 6 DOF aircraft models will be used to study these effects.

1. The One Point Aircraft model differs from the 3 DOF point-mass model by allowing for the influence of inertia. Now body rates are limited and aerodynamic angles do not change instantly (as control variables) but are time and system dependent state variables. The control variables are now the control surface deflections.

2. The Four Point Aircraft model is the similar to the One Point Aircraft model, but now the effect of windspeed variations over the aircraft’s dimensions is taken into account.

7.1 One Point Aircraft Model

The one point aircraft model evaluates the equations of motion and windspeed components in only one point, the aircraft’s center of gravity. The aircraft is controlled by the control surfaces. The deflections are calculated from the commanded aerodynamic angles using NDI. In the NDI subroutine, gain factors for the slow and fast state dynamics should be selected to get satisfactory behavior of the very slow state variables.

7.1.1 Equations of Motion

For 6 DOF the force equations of motion and kinematic relations for x, y and h (derived in Appendix C) are the same as those for 3 DOF simulations.

\[
\dot{V} = \frac{1}{m} \left[ T \cos(\alpha + \delta) - D \cos \beta + Y \sin \beta \right] - \bar{W}_x \cos \gamma \cos \chi +
\]
\[
- \bar{W}_y \cos \gamma \sin \chi - \bar{W}_h \sin \gamma - g \sin \gamma
\]

(5-1)
\[
\dot{y} = \frac{1}{V} \left[ \frac{L + T \sin(\alpha + \delta)}{m} \cos \mu + \frac{T \cos(\alpha + \delta) \sin \beta - D \sin \beta - Y \cos \beta}{m} \sin \mu + g \cos \gamma + \dot{W}_s \sin \gamma \cos \chi + \dot{W}_r \sin \gamma \sin \chi - \dot{W}_h \cos \gamma \right] \\
\dot{x} = \frac{1}{V \cos \gamma} \left[ \frac{L + T \sin(\alpha + \delta)}{m} \sin \mu + \frac{-T \cos(\alpha + \delta) \sin \beta + D \sin \beta + Y \cos \beta}{m} \cos \mu + \dot{W}_s \sin \chi - \dot{W}_r \cos \chi \right] \\
\dot{t} = V \cos \gamma \cos \chi + \dot{W}_x \\
\dot{y} = V \cos \gamma \sin \chi + \dot{W}_y \\
\dot{h} = V \sin \gamma + \dot{W}_h \\
\]

Now the moment equations and the kinematic relations for the aerodynamic angles are added. These equations are derived in Appendix C. In the moment equations the aerodynamic moments are divided into moments due to control surface deflections (subscript d) and moments due to other aerodynamic effects (subscript a). This approach is necessary to calculate the control surface deflections in the inner loop NDI routine, see paragraph 7.1.2. The moment equations are:

\[
\dot{p} = \frac{I_{xx}L_a + I_{xx}N_a}{I_{xx}I_{xx} - I_{xx}^2} + \frac{I_{xx}L_d + I_{xx}N_d}{I_{xx}I_{xx} - I_{xx}^2} + \frac{I_{xx}(I_{xx} - I_{yy} + I_{xx})pq + [I_{xx}(I_{yy} - I_{xx}) - I_{xx}^2]qr}{I_{xx}I_{xx} - I_{xx}^2} \\
(7-1)
\]

\[
\dot{q} = \frac{1}{I_{yy}} \left[ M_a + M_d + (I_{xx} - I_{xx})(r^2 - p^2) \right] \\
(7-2)
\]

\[
\dot{r} = \frac{I_{xx}L_a + I_{xx}N_a}{I_{xx}I_{xx} - I_{xx}^2} + \frac{I_{xx}L_d + I_{xx}N_d}{I_{xx}I_{xx} - I_{xx}^2} + \frac{I_{xx}(I_{xx} - I_{yy} + I_{xx})pq - I_{xx}(I_{yy} - I_{xx}) + I_{xx}^2}{I_{xx}I_{xx} - I_{xx}^2} \] \\
(7-3)
The kinematic relations for the aerodynamic angles are (Appendix C):
\[ \dot{\alpha} = -p \cos \alpha \tan \beta + q - r \sin \alpha \tan \beta - \frac{1}{\cos \beta} \left( \dot{y} \cos \mu + \dot{\chi} \cos \gamma \sin \mu \right) \]  
(7-4)
\[ \dot{\beta} = p \sin \alpha - r \cos \alpha - \dot{y} \sin \mu + \dot{\chi} \cos \gamma \cos \mu \]  
(7-5)
\[ \dot{\mu} = p \frac{\cos \alpha}{\cos \beta} + r \frac{\sin \alpha}{\cos \beta} - \dot{y} \tan \beta \cos \mu + \dot{\chi} \left( \cos \gamma \tan \beta \sin \mu + \sin \gamma \right) \]  
(7-6)

These relations are functions of the time derivatives of \( \gamma \) and \( \chi \). Since those are functions of the aerodynamic forces, it's also possible to write the kinematic relations for the aerodynamic angles as functions of the aerodynamic forces. This is however not necessary, as long as in the program the force equations are evaluated first.

Finally the equation for engine dynamics, the same as for the 3 DOF model, is added.
\[ \dot{\eta} = \frac{1}{\tau_e} (\eta_c - \eta) \]  
(5-7)

7.1.2 Inner Loop Inversion Dynamics

With NDI, the desired control surface deflections can be calculated from the commanded aerodynamic angles supplied by the guidance strategies.

The procedure (see ref. [2]) is as follows:

From the commanded aerodynamic angles, desired aerodynamic angle rates are calculated using proportionate feedback control:
\[ \dot{\alpha}_d = \omega_\alpha (\alpha_c - \alpha) \]
\[ \dot{\beta}_d = \omega_\beta (\beta_c - \beta) \]  
(7-7)
\[ \dot{\mu}_d = \omega_\mu (\mu_c - \mu) \]

Here \( \omega_\alpha, \omega_\beta \) and \( \omega_\mu \) are gain factors for the proportionate feedback. Now commanded body rates can be calculated when equations (7-4) to (7-6) are written as:
\[
\begin{bmatrix}
\dot{p}_c \\
\dot{q}_c \\
\dot{r}_c
\end{bmatrix}
= G_i^{-1}
\begin{bmatrix}
\dot{\alpha}_d \\
\dot{\beta}_d \\
\dot{\mu}_d
\end{bmatrix}
- \begin{bmatrix}
- \frac{1}{\cos \beta} (\dot{y} \cos \mu + \dot{\chi} \cos \gamma \sin \mu) \\
\dot{y} \sin \mu + \dot{\chi} \cos \gamma \cos \mu \\
\dot{\gamma} \tan \beta \cos \mu + \dot{\chi} (\tan \beta \cos \gamma \sin \mu + \sin \gamma)
\end{bmatrix}
\]  
(7-8)

With:
\[
G_i = \begin{bmatrix}
-\cos \alpha \tan \beta & 1 & -\sin \alpha \tan \beta \\
\sin \alpha & 0 & -\cos \alpha \\
\cos \alpha \sec \beta & 0 & \sin \alpha \sec \beta
\end{bmatrix}
\]  
(7-9)
With the commanded body rates, desired values can be calculated for the time derivatives of the body rates, again using proportionate control.

\[
\begin{align*}
\dot{p}_d &= \omega_p (p_c - p) \\
\dot{q}_d &= \omega_q (q_c - q) \\
\dot{r}_d &= \omega_r (r_c - r)
\end{align*}
\] (7-10)

Where \(\omega_p\), \(\omega_q\), and \(\omega_r\) are also gain factors. Finally, using equations (7-1) to (7-3), the desired control surface moments can be calculated.

\[
\begin{bmatrix}
L_{\delta} \\
M_{\delta} \\
N_{\delta}
\end{bmatrix}
= G_2^{-1}
\begin{bmatrix}
\dot{p}_d \\
\dot{q}_d \\
\dot{r}_d
\end{bmatrix}
- \begin{bmatrix}
f_p \\
f_q \\
f_r
\end{bmatrix}
\] (7-11)

Where:

\[
G_2 = \begin{bmatrix}
I_{xx} & 0 & I_{xx} \\
0 & I_{yy} & 0 \\
I_{xx} & 0 & I_{xx}
\end{bmatrix}
\] (7-12)

and

\[
\begin{align*}
f_p &= \frac{I_{xx}L_a + I_{yy}N_a}{I_{xx}I_{zz} - I_{zz}^2} + \frac{I_{zz}(I_{xx} - I_{yy} + I_{zz})pq + [I_{zz}(I_{yy} - I_{zz}) - I_{zz}^2]qr}{I_{xx}I_{zz} - I_{zz}^2} \\
f_q &= \frac{1}{I_{yy}} \left[ M_a + (I_{xx} - I_{zz})pr + I_{xx} (r^2 - p^2) \right] \\
f_r &= \frac{I_{xx}L_a + I_{yy}N_a}{I_{xx}I_{zz} - I_{zz}^2} + \frac{[I_{xx}(I_{xx} - I_{yy}) - I_{xx}^2]pq + I_{xx}(I_{xx} - I_{yy} + I_{zz})qr}{I_{xx}I_{zz} - I_{zz}^2}
\end{align*}
\] (7-13)

The commanded control surface deflections become:

\[
\begin{bmatrix}
\delta_a \\
\delta_r
\end{bmatrix}
= G_3^{-1}
\begin{bmatrix}
\frac{L_{\delta}}{\dot{q}Sb} \\
\frac{M_{\delta}}{\dot{q}Sc} \\
\frac{N_{\delta}}{\dot{q}Sb}
\end{bmatrix}
\] (7-14)

With:

\[
G_3 = \begin{bmatrix}
C_{n\delta} & 0 & C_{n\delta} \\
0 & C_{m\delta} & 0 \\
C_{n\delta} & 0 & C_{n\delta}
\end{bmatrix}
\] (7-15)
Now the control surface deflection that are necessary for flying the commanded trajectory are known. These deflections are now put into the 6 DOF aircraft model as control variables.

A schedule of the NDI aircraft controller is presented in figure 7-1.

gain scheduling
Also some attention is paid to gain factor scheduling of the inner loop inversion dynamics routine. The inner loop time constants should be at least 3 to 5 times faster than the outer loop time constants \(^4\). The maximum outer loop gain factors (very slow variables) are \(K_v = 0.75\) and \(K_h = 0.25\) respectively. The gain factors for the slow variables should therefore have a value of at least 2.25 and the gain factors for the very fast variables should at least be 6.75. The gain factors should not be selected too high either. If the gains are selected high, the aircraft will also react on gust winds. The maximum limit is restricted by the rate at which the control surfaces can be deflected.

When the response of the 6 DOF model is investigated for several combinations of gain factors, it shows that the gain factors for \(p\), \(q\) and \(r\) hardly affect the behavior of the outer loop variables like \(V\) and \(h\). However, the values of \(\omega_p\), \(\omega_q\) and \(\omega_r\) should indeed be at least 3 times the values of \(\omega_a\), \(\omega_p\) and \(\omega_r\) to assure good damping for the aerodynamic angles. The outer loop behavior is primarily controlled by the slow state gain factors.

To get a satisfactory outer loop behavior, without selecting unreasonably high gains, the following gain factor values are chosen:

\[
\omega_a = \omega_b = \omega_c = 2.5
\]
\[
\omega_p = \omega_q = \omega_r = 7.5
\]

7.2 Four Point Aircraft Model

To study the effect of the windspeed variations over the finite aircraft size a Four Point Aircraft model (FPA) will be used. This FPA model, originally developed by Etkin is discussed in ref. [9]. The equations of motion and the NDI routine for the FPA model remain the same as for the OPA model but the FPA model calculates the windspeed components at 4 points, see fig 7-2. Between these points the windspeed components can be linearly interpolated. The points are:

- point 0: center of gravity \((x_0 = 0, y_0 = 0, z_0 = 0)\)
- point 1: right wing tip at 85% of the semi span
- point 2: left wing tip at 85% of the semi span
- point 3: at the aerodynamic center of the tail planes

Points 0, 1, and 2 are chosen on a straight line, i.e. the effect of sweepback is neglected. The distance between Points 1 and 2 is 85% of the wingspan. This value is generally used and is based on the fact that, for a general (elliptic) spanwise lift
distribution, the wing tips have little contribution to total lift. Note however that the value of the span between point 1 and point 2 (b') may change considering the spanwise lift distribution, for example for wings equipped with winglets.

Linear Field Approximation
A Linear Field Approximation (LFA) calculates the wind gradients at the aircraft's center of gravity. For the purpose of modeling (large wavelength) windshear that is treated in this thesis, LFA would also be sufficient. This can for example be seen from the following case where the windspeed gradient $\partial w_\alpha / \partial y$ is calculated using LFA and the FPA model.

The aircraft's location is $(x_e, y_e, h) = (-1500, 500, 100)$ and the heading and flight path angle are $0^\circ$. Microburst data is $(x_e, y_e) = (-1500, 0)$; $U_{max} = 22.7$ m/s; $z_a = 100$ m; $D = 2000$ m.

The calculated vertical windspeed gradient in lateral direction $\partial w/\partial y$ is FPA model
in point 1 ($y = 514$ m) $w_1 = 4.317$ m/s
in point 2 ($y = 486$ m) $w_2 = 4.471$ m/s
Thus $\Delta w / \Delta y = -5.50 \times 10^{-3}$ 1/s

LFA in c.g. $\partial w / \partial y = -5.52 \times 10^{-3}$ 1/s

The difference between these two models is thus 0.35% on a location where this gradient is relatively high.

Motivation of Choice for the FPA model
The choice for a FPA model is made with the eye on possible future research on the effects of high-frequency turbulence during a windshear encounter. For small wave lengths (smaller than the aircraft's wingspan) the LFA tends to exaggerate the simulated aircraft response. The reason for this can easily be seen in figure 7-3.

The influence of low frequency windshear modeled with the PFA model is assumed to only have effect on the aerodynamic moments. The difference in windspeed over the aircraft's size are experienced as additional body rates, which cause additional moments through the stability derivatives.

The low-frequency windshear induced windspeed components have no effect on the aerodynamic forces, contrary to high-frequency gust windspeeds. The influence of the mean windshear windspeed is already modeled in the equations of motion.
The body rates experienced by the aircraft due to windspeed variations are defined as:

\[ p_w = \frac{w_{w1} - w_{w2}}{b^l} \]  \hspace{1cm} (7-16)

\[ q_w = \frac{w_{w3} - w_{w0}}{l_i} \]  \hspace{1cm} (7-17)

\[ r_{1w} = \frac{u_{w1} - u_{w2}}{b^l} \]  \hspace{1cm} (7-18)

\[ r_{2w} = \frac{v_{w0} - v_{w3}}{l_i} \]  \hspace{1cm} (7-19)

Where \( u_w, v_w \) and \( w_w \) are the windspeed components in the body axis system, \( p_w \) is the experienced roll rate, \( q_w \) is the experienced pitch rate, \( r_{1w} \) is the experienced yaw rate of the wing and \( r_{2w} \) is the experienced yaw rate of the fuselage and vertical tail.

It is assumed that asymmetric winds only have influence on asymmetric moments, and that the pitch moment is only affected by the experienced pitch rate, i.e. the symmetrical and asymmetrical effects are not coupled. The aerodynamic moments, induced by these experienced body rates are:

\[ C_{n_r} = C_{n_r} \frac{p_w b}{2V} + C_{n_s} \frac{r_{1w} b}{2V} + C_{n_e} \frac{r_{2w} b}{2V} \]  \hspace{1cm} (7-20)

\[ C_{m_s} = C_{m_s} \frac{q_w c}{V} \]  \hspace{1cm} (7-21)

\[ C_{n_r} = C_{n_r} \frac{p_w b}{2V} + C_{n_s} \frac{r_{1w} b}{2V} + C_{n_e} \frac{r_{2w} b}{2V} \]  \hspace{1cm} (7-22)

Finally these moment contributions are calculated and added to the “normal” aerodynamic moments. The effects of windspeed variations over the aircraft's dimensions are now taken into account.
8. Six Degree-of-Freedom Simulation Results

The goal of the 6 DOF simulations in this study is explained in chapter 7. With the One Point Aircraft model and the Four Point Aircraft model described in that chapter, several lateral and longitudinal escape maneuvers will be performed. Comparing the results of the OPA model with the 3 DOF model results gives insight into the effect of inertia, while comparing the OPA model and FPA model will show the effect of variation of windspeed components over the aircraft’s finite size.

8.1 6 DOF Lateral Escape Results

A lateral escape is simulated with reactive windshear hazard measurement only. With the 3 DOF model, and both 6 DOF models (OPA model and FPA model) all three guidance strategies are simulated. The used microburst parameters are:

\[ x_c = -1500 \text{ m} \]
\[ y_c = 0 \text{ m} \]
\[ U_{\text{max}} = 22.7 \text{ m/s} \]
\[ D = 2000 \text{ m} \]
\[ z_m = 100 \text{ m} \]

For altitude guidance, the reference altitude selected is the best reference altitude for the FPA model. This reference altitude is 41 m.

8.1.1 Model Behavior and Performance

Effects of Inertia and Finite Size

In the figures 8-1 to 8-3 the influence of the inertia can be observed. For the 3 DOF model the aerodynamic angles change instantaneous, while for the OPA and FPA models these angles have a delay in following the desired aerodynamic angles. This has a direct effect on the lateral trajectory. The lateral displacement reached by the 6 DOF models is clearly less than for the 3 DOF model. Figure 8-4, the lateral trajectory for pitch guidance, illustrates this. Note that the final part of the trajectory in fig. 8-4 for all models is aligned with the microburst center at (-1500,0), i.e. the final part of the trajectory is flown along a wind radial.

The difference in behavior between the OPA model and FPA model can also be observed in figure 8-4. The FPA model has slightly less lateral displacement than the OPA model. Due to the weather cocking effect the FPA model has a tendency to yaw towards the microburst core. The roll rate and pitch rate are approximately equal throughout the entire maneuver but the yaw rate shows a small difference between t = 10 sec. and t = 25 sec, see figure 8-5. The OPA model has a slightly higher yaw rate, and therefore a slightly larger lateral displacement.
Performance
In table 8-1 the final specific energy is presented.

<table>
<thead>
<tr>
<th></th>
<th>Pitch guidance</th>
<th>RC-guidance</th>
<th>altitude guidance</th>
</tr>
</thead>
<tbody>
<tr>
<td>3 DOF</td>
<td>298.5</td>
<td>315.8</td>
<td>317.8</td>
</tr>
<tr>
<td>OPA</td>
<td>284.5</td>
<td>299.5</td>
<td>305.0</td>
</tr>
<tr>
<td>FPM</td>
<td>284.1</td>
<td>297.1</td>
<td>304.6</td>
</tr>
</tbody>
</table>

Table 8-1: Final specific energy in meters

The 6 DOF models, due to finite body rates, climb out slower than the 3 DOF model and obtain a final specific energy that is 4 to 5 percent lower than the 3 DOF model. Inertia effects are thus responsible for a small energy loss.

The difference between the OPA model and FPA model is small. The FPA model finishes with slightly less energy but the differences are less than 1%. Thus for this particular case the wind variation over the aircraft’s finite size has little effect on the specific energy.

In table 8-2 the minimum altitude reached, is given.

<table>
<thead>
<tr>
<th></th>
<th>Pitch guidance</th>
<th>RC-guidance</th>
<th>altitude guidance</th>
</tr>
</thead>
<tbody>
<tr>
<td>3 DOF</td>
<td>13.1</td>
<td>29.9</td>
<td>39.5</td>
</tr>
<tr>
<td>OPA</td>
<td>11.9</td>
<td>32.6</td>
<td>38.0</td>
</tr>
<tr>
<td>FPA</td>
<td>11.8</td>
<td>32.8</td>
<td>37.8</td>
</tr>
</tbody>
</table>

Table 8-2: Minimum altitude reached, in meters

For *pitch guidance* the angle of attack shows little difference in the area where the limit is reached. The behavior of the three models is thus quite similar. The minimum altitude reached (table 8-2 and figure 8-6-a), is for the 3 DOF model 1.2 m higher than for the 6 DOF models. This small difference can be explained from the difference in lateral displacement, see figure 8-4. The 3 DOF model turns away faster from the high shear/downdraft area.

For *RC-guidance* a surprising result is observed. The minimum altitude reached for the 3 DOF model is actually lower than the for the 6 DOF models, see figure 8-6-b and table 8-2. The reason for this unexpected behavior must be the limited rotation rates inertia of the 6 DOF models, since this is the only difference between the OPA model and the 3 DOF model. When the energy drain decreases, commanded rate of climb increases to a less negative value, which will result in a higher commanded angle of attack.

As can be seen in figure 8-2, the 3 DOF model reaches the angle-of-attack limit for \( t = 22 \) sec. The 6 DOF model however, due to the inertia, has not reached that value. The consequence of this is, that the 3 DOF model will ascend earlier due to higher lift (fig. 8-6-b). It enters a region where the shear and downdraft are stronger. When the angle of attack limit is reached the aircraft is forced down again, below the minimum altitude reached by the 6 DOF models.
Apparently, the gain factors $K_e$ and $K_h$ used in the RC guidance law do not have the best possible values for 3 DOF simulation of this particular maneuver. For the 6 DOF models these gains are more appropriate than for 3 DOF, though it is possible that other gains may result in even better performance for the 6 DOF models. Selecting the proper gain factors for RC guidance is an important issue. The best values depend on the aircraft model as well as the maneuver that is to be performed.

For altitude guidance the altitude plot (figure 8-6-c) shows little difference between the three models. The reason for this is that for all models, the logic controls the aircraft towards the same reference value: 41 m. All models have a small "overshoot" when descending towards this altitude, so the minimum altitude reached is somewhat less. The 3 DOF model has less overshoot due to the unlimited body rotation rates, and therefore the minimum altitude reached is 1.5 m higher than for both 6 DOF models.

All models reach the angle-of-attack limit about 2 seconds before the switch to RC guidance is made, see figure 8-3-a. Since the energy drain has almost stopped at that moment, the altitude drop due to the resulting phugoid is very small, as can be seen in figure 8-6-2. The differences between the models in minimum altitude are quite small because as long as the angle-of-attack limit isn't reached by one of the models, the aircraft is controlled towards the same reference altitude.

The reference altitude that is used here is the best value for the FPA model. When however, for the 3 DOF model and OPA model the best reference altitude is evaluated, the best reference altitude proves to be also 41 m for both models. So again a somewhat surprising result is found. For altitude guidance, the 3 DOF model does not perform better than the 6 DOF models in terms of best reference altitude for this particular scenario. The minimum altitude reached however, is higher for the 3 DOF models, due to the absence of rotational inertia, as is remarked before.

When simulations are performed with a different microburst strength, $U_{\text{max}} = 15 \text{ m/s}$, the best reference altitudes for the three models are again equal, namely 66 m. In table 8-3 the best reference altitudes and the minimum altitudes reached are displayed.

<table>
<thead>
<tr>
<th>Model</th>
<th>$U_{\text{max}} = 15 \text{ m/s}$</th>
<th>$U_{\text{max}} = 22.7 \text{ m/s}$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$h_{\text{ref, best}}$ [m]</td>
<td>$h_{\text{min}}$ [m]</td>
</tr>
<tr>
<td>3 DOF</td>
<td>66</td>
<td>64.92</td>
</tr>
<tr>
<td>OPA</td>
<td>66</td>
<td>64.12</td>
</tr>
<tr>
<td>FPA</td>
<td>66</td>
<td>64.16</td>
</tr>
</tbody>
</table>

Table 8-3: Minimum altitude and best reference altitude for two microbursts

It can be concluded that the different models have no effect on the best reference altitude. The 3 DOF model however, reaches a higher minimum altitude due to the infinite body rates. The difference in minimum altitude between the OPA model and FPA model is very small in both cases. Two conclusions may be drawn with respect to the influence of finite aircraft size:
1. Apparently the induced pitching moments do not cause a significant performance decrease for these particular cases.

2. The difference in lateral displacement does not affect the vertical trajectory significantly.

Also it can be concluded that for this particular scenario, altitude guidance is the best strategy, provided that the best reference altitude is selected. The minimum altitude as well as the specific energy are higher than for RC guidance. Pitch guidance has the largest altitude drop.

8.1.2 Aircraft Controllability for Lateral Maneuvering

Sign and Value of the Control Surface Deflections
An important aspect of the usefulness of the lateral escape maneuver is the control surface deflections necessary to perform the maneuver. The control surface deflections versus time are plotted for the lateral escape maneuver in figures 8-7 to 8-9. As can be easily seen, large control surface deflections are required to fly the maneuver.

For example, the aileron deflections show very sudden sign changes, with amplitudes equal to the deflections limits. The high amplitudes are caused by the choice of feedback gains in the inner control loop, see paragraph 7.1.2. With these feedback gains, a large difference between the actual bank angle and commanded bank angle results in large aileron deflections. The large amplitudes can be avoided by lowering the gains, but this would result in very slow responses for small course corrections. Another option can be simply restricting the values of the desired control surface deflections to values, lower than the operational control surface deflection limits.

The sign changes are caused by the infinite deflection rates, which follow sign changes in the commanded bank angle, see figure 8-10 for example. When the escape maneuver is initiated, the desired bank angle changes instantly from $0^\circ$ to $10^\circ$. The positive roll moment results in a negative (left) yaw moment since $C_{np}$ is negative. The aileron deflection also results in a negative yawing moment since no aileron differential is applied. The causes a (small) displacement of the aircraft to the left of the glide slope. On that side of the glide slope, the wind radial has a component in negative $\gamma_r$-direction, which results in a negative commanded bank angle and thus causing positive aileron deflection. However due to the positive bank angle the aircraft has obtained at that time, the aircraft turns to the right side of the glideslope in a few seconds, resulting in a second sign change of the commanded bank angle and thus aileron deflection. Finally a third sign change in aileron deflection occurs to minimize overshoot when the bank angle approaches the commanded bank angle.

These sign changes can be minimized (or even avoided) by applying control surface deflection dynamics with finite deflection rates. Similar to the selection of gains in paragraph 7.1.2, the time constants for the deflections should be chosen at least 3 times faster than the time constants for the fast state variables. However they should be sufficiently slow to prevent the sudden deflection rates discussed above.
Effect of Finite Size
The influence of wind induced moments are visible in the plots for the control surface deflections (figs. 8-7 to 8-9). It can be easily observed that the windspeed variations over the aircraft’s size have no significant effect on the elevator deflections. In the previous paragraph, it is already observed that the induced pitching moments are very small and do not influence the aircraft’s behavior in the vertical plane.

The effects of windspeed variation over the aircraft’s finite size during lateral maneuvering, discussed in paragraph 8.1.1, can be traced back to the aileron and rudder deflections. In the initiation phase of the escape maneuver the OPA model and FPA model have similar deflections, but between 10 and 25 seconds, the deflections show a constant difference between the models. This difference is caused by the induced roll and yaw moments occurring in the FPA model.

For example, when the downburst core is at the left side of the aircraft, the vertical windspeed variations over the wing are experienced by the aircraft as a positive roll speed. This induces an additional negative roll moment, i.e. to the left. To compensate, a positive roll moment (to the right) should be initiated by aileron deflection, i.e. a negative contribution. In general the aircraft has a tendency to roll towards the microburst core. Due to the weather cocking effect there is also a tendency to yaw towards the core axis of symmetry (see also ref. [1]).

The constant difference in aileron and rudder deflection between OPA model and FPA model, discussed above, is in the order of 3°to 5°. This difference in this particular case it is *not sufficiently large to cause any controllability problems during lateral maneuvering*. Since the microburst used in this simulation can be classified as severe, it can be concluded that in general the windspeed variations over the aircraft’s finite size are not sufficiently large to cause controllability problems during lateral maneuvering. However, in this study turbulence is not modeled. The controllability aspects of the presence of high-frequency turbulence should therefore be a next point of attention.

### 8.2 Longitudinal Core Penetration Offset of Microburst Axis

In ref. [1] Ávila de Melo and Hansman report controllability problems during an attempt to simulate a flight along a straight line offset the axis of symmetry of a very severe microburst. It is shown in the previous paragraph that during lateral maneuvering the yaw and roll effects of windspeed variation over the aircraft’s finite size are not sufficiently large to cause significant controllability problems. Therefore the next objective is to investigate the controllability during a longitudinal escape maneuver similar to those performed in ref. [1].

To simulate a worst case scenario, the core will be penetrated at a distance from the microburst axis where both the radial and the vertical wind gradient are relatively high. For a Bowles-Oseguera microburst such a condition exists at approximately one-third of the maximum outflow diameter.

Two different microbursts will be simulated:

A. Has the same strength and size as in the previous paragraph
B. Has the same parameters as the severe case used in ref. [1] (case B), which corresponds to an event at Andrews Air Force Base on August 1, 1983.
The size, strength and location of both cases are given in table 8-4. The altitude of maximum outflow is not specified in ref. [1], since in that study the microburst model is invariant with altitude. In this study this altitude will be selected 100 m.

<table>
<thead>
<tr>
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<tr>
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<tr>
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<td>1219</td>
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*Table 8-4: Microburst data for two longitudinal cases*

The initial values are chosen as discussed in paragraph 2.1, with one exception. Due to the asymmetric location of the microburst, it is not easy to choose the initial values of $\gamma$ and $\chi$ in such way that the inertial quantities $\gamma_e$ and $\chi_e$ are both along the glide slope path, -3° and 0° respectively. Since the vertical trajectory is the most critical and important in this study, it is decided to set only $\gamma_e$ equal to -3°. The value of $\chi$ is chosen such that the deviation from the glide slope is small.

For altitude guidance, once again the best reference altitude is evaluated and used in both case A and B. These best reference altitudes are $h_{\text{ref}} = 59$ m for case A and $h_{\text{ref}} = 20$ m (!) for case B.

*Note*: The effect of inertia is already shown in the lateral maneuvers (paragraph 8.1), and these effects are expected to be similar for longitudinal maneuvers. Therefore only simulations with the OPA model and FPA model are performed.

### 8.2.1 Case A: Severe Microburst

**Performance and Behavior**

In the figures 8-11 and 8-12 the vertical and horizontal trajectory is plotted respectively. Note that the direction of the y-axis in figure 8-12 is reversed with regard to the earth-axis system.

The behavior of both models is quite similar to the lateral case. For each strategy, the vertical trajectory is approximately equal for both models, as can be observed in figure 8-11. The elevator deflections also show no significant difference (figures 8-13-b, 8-14-b and 8-15-b).

In figure 8-12 it can be observed that some very small lateral deviations from the glide slope ($\gamma_e = 0$) do occur for both the OPA model as the FPA model. These deviations occur directly at the start of the simulation since the inertial heading angle is not equal to zero, but about 0.06°.

In table 8-5 the minimum altitude and minimum energy are presented, as well as the final specific energy.
Table 8-5: Minimum altitudes reached for case A

When the three strategies are compared, altitude guidance proves to have the least altitude drop. Pitch guidance has the largest altitude drop. RC-guidance clearly has a higher specific energy. Pitch guidance and altitude guidance have a similar minimum energy, but altitude guidance finishes with slightly more energy than pitch guidance.

In contrary to the lateral case, no strategy can be pointed out to be the "best", since altitude guidance has a smaller altitude drop, provided that the best reference altitude is selected, but RC-guidance has the best energy performance.

Controllability and Effect of Finite Size

As is stated above, the vertical trajectories are similar for both strategies but the lateral trajectories differ slightly. This difference is explained with the weather cocking and roll effects induced by the windspeed variations. Figure 8-12 shows that these effects are most prominent when the aircraft is located at approximately $x_s = -1500$, i.e. when it passes the microburst core. This is quite easily explained, since here the vertical windspeed gradients are largest, resulting in the largest roll moment. As can be seen in the plots of the control surface deflections, figures 8-13 to 8-15, small rudder and aileron deflections are necessary to correct these deviations.

As in the lateral case, the control surface deflections show a difference due to the weather cocking and roll effects, see paragraph 8.1.2. In the first part of the simulation the aileron and rudder deflections of the FPA model are therefore lower (more negative) compared to the OPA model control surface deflections, similar as in the lateral case. Note that at the end of the simulation for the FPA model control surface deflections are higher (more positive) than for the OPA model. This is due to the switch in sign of the yaw rate experienced by the wing ($r_{1w}$) when the aircraft passes the microburst core.

The values of aileron and rudder deflections are in the order of 3 to 4 degrees, since the maneuver is longitudinal. However, the elevator deflection does obtain very high values, as in the lateral case. As in the lateral case, sudden changes in the control surface deflections do occur, especially when the escape is initiated. The reasons for this behavior are presented in paragraph 8.1.2.

For this particular case, in the absence of high-frequency turbulence, it is most certainly possible to perform an approximately straight (longitudinal) maneuver. However, the pilot's workload will be high considering the history of the control surface deflections with time, see figures 8-13 to 8-15. The controllability problems described by Avila de Melo and Hansman [1] do not occur in that extent, even though the microburst used in this study can be classified as severe, and the microburst is penetrated at a location where windspeed gradients are high. Therefore in paragraph 8.2.2, similar simulations will be performed using the severe microburst described in ref. [1].
As in the previous paragraph, it is recommended to implement limited control surface deflection rates, and perhaps lower deflection limits in order to prevent the extreme sign changes and maximum values. Also the effect of turbulence on controllability should be a point of attention in future research.

8.2.2 Case B: Very Severe Microburst

Performance and Behavior
The hazard of this particular microburst is the small distance (1219 m) in which the airspeed shifts from a maximum headwind of 30.8 m/s (60 kts) to a tailwind of 30.8 m/s. From the altitude plots, figure 8-16, the severity of microburst B can be immediately observed. The maximum F-factor reached is 0.39 (!) for all strategies. For a longitudinal escape straight through the center of the microburst core the F-factors would be even higher.

*Pitch guidance and RC guidance* are not sufficient to perform a successful escape maneuver and both result in a crash, i.e. the altitude drops below 10 m. Pitch guidance and RC guidance enter the core at high altitude, where the shear and downdraft are extremely high. RC guidance relates the commanded climb rate to the energy rate. In the core, the energy drain is extensive which causes a high rate of descent from which the aircraft is unable to recover.

Only with *altitude guidance* the aircraft recovers from the strong energy drain and stays above the crash limit of 10 m, though the minimum altitude reached is only 16.8 m. The reference altitude used is the best reference altitude, \( h_{ref} = 20 \text{ m} \). When the reference altitude had been selected a few meters higher or lower, the aircraft would have crashed (\( h < 10 \text{ m} \)) with altitude guidance as well.

This extreme example once again states the potential of altitude guidance, provided a good reference altitude can be selected.

RC guidance can perhaps give better results if in the initial phase of the escape the preservation of and investment in airspeed is emphasized. The aircraft will then descend earlier, gain more airspeed and enter the microburst at a lower altitude. Another improvement for such extreme microburst encounters can be specifying a maximum rate of descent. This prevents the aircraft of descending too fast at low altitude. Also the switch to altitude hold can be made as soon as a specified minimum altitude is reached. Finally, specifying other gain factors in the inner loop can improve performance, but then the behavior in mild microbursts can be less satisfactory.

Controllability and Effects of Finite Size
As in the case A and the lateral case, the vertical trajectory practically coincides for the OPA model and the FPA model. Once again the vertical performance is not influenced by the windspeed variations over the aircraft’s finite dimensions. The controllability for *altitude guidance* shows no such problems and is similar to case A. However, for *pitch guidance and RC guidance*, all aircraft state variables have become unstable for both the OPA model and the FPA model. This can readily be observed in the plots of the lateral displacement, figure 8-17, the control surface
deflection plots, figures 8-18 to 8-20, and the sideslip angle, figure 8-21. Where the rudder and aileron deflections are in the order of 3 to 4 degrees in case A, here they obtain values near the deflection limits. Also, the sideslip angle is no longer approximately zero for pitch- and RC guidance.

The instability is not caused by numerical instability since the instability remains when the time interval for integration is decreased. Moreover, altitude guidance did not show any instability with the same time interval. The reason for the instability is the rapid descent and low altitude, from which the aircraft is unable to recover. The longitudinal state variables become unstable due to the rapid descent. Then, due to coupling effects in the equations of motion, the lateral variables become unstable as well.

In the first part of the simulation, the differences between the OPA model and FPA model are similar to those in case A. The differences in control surface deflections are somewhat larger due to the higher intensity of microburst B. When the aircraft becomes unstable the difference between both models increases. The lateral instability of the FPA model is larger, as can be seen in the plots for the roll rate \( p \).

The instabilities are not caused by the finite body rates. When a 3 DOF simulation is carried out, instability is also present, though the amplitudes increase not as rapid as for the 6 DOF simulations. Thus, inertia increases the instability.

**Conclusions**

For this (very severe) microburst it can be concluded that only altitude guidance recovers from the high energy drain and doesn’t show any instability or controllability problems. The OPA model and FPA model behave similar to case A.

Controllability problems do occur for pitch and RC guidance. Both strategies make the aircraft enter the core at high altitude and cause high descent rates. The aircraft is unable to recover and crashes. Longitudinal instability as well as lateral instability occurs.

The instability does also occur in 3 DOF simulations. The instability is worse for the OPA model due to the finite body rates. The windspeed variations over the aircraft’s finite size increase the instability even more.

In ref. [1], pitch guidance is used as is recommended by the FAA. The controllability problems for this guidance strategy found by Ávila de Melo and Hansman are confirmed by this study. Also the potential of altitude guidance in performance and controllability is emphasized.

As is stated earlier, this microburst can be classified as very severe. In case A, classifiable as a severe microburst, no controllability problems occurred. It must be remarked that it is always possible to define a microburst strong enough to cause performance and stability problems. Microbursts of the intensity as in case B do not occur frequently and should certainly not be taken as standard in the development of escape maneuvers.
9. Conclusions and Recommendations

In this thesis, three aspects of guidance during near-optimal escape maneuvers in the presence of microburst are studied.

First, since in most previous research only one core is treated, the effects of the presence of a second microburst core on aircraft performance are investigated for pitch guidance, rate-of-climb guidance and altitude guidance.

Second, an attempt is made to find a logic for the "best" reference altitude. Altitude guidance is the most promising strategy of the three mentioned earlier. However, without such a logic, altitude cannot be applied effectively in reality and the advantage over pitch guidance and rate-of-climb guidance is lost.

Third, to bridge the gap between the academic 3 Degrees-of-Freedom (DOF) model open-loop optimal solutions and more realistic piloted simulations, the 3 DOF is extended to two full 6 DOF models, the One Point Aircraft model (OPA) and the Four Point Aircraft model (FPA). The OPA model differs from the 3 DOF model by the presence of rotational inertia. The FPA model also accounts for windspeed variations over the aircraft's dimensions, that induce pitch, roll and yaw moments.

9.1 Conclusions

Simulations with Two Microburst Cores
The microburst model used in the simulations is the Bowles-Oseguera downburst model. First, two cores are placed on one line and the distance is varied. For large distances the aircraft experiences two cores and there are two F-factor peak values. When the distance is decreased, the two cores are experienced as one core, due to superposition of the windspeed components and windspeed gradients.

Altitude guidance is the most robust strategy. Due to the constant reference altitude, the trajectory is approximately the same for all cases. Due to the relatively lower altitude at which both cores are penetrated, less energy is lost than for the other two strategies. The energy managing rate-of-climb guidance strategy is also fairly robust. The performance of pitch guidance reduces significantly when the distance between the two cores is decreased, and they are experienced as one stronger core.

Also the robustness of the strategies during a lateral escape is studied. For this purpose, one core is placed on the glide slope, and a second on the right of the glide slope. Now the results of a left turn (away from the second core) and a right turn (towards the second core) are compared.

Again rate-of-climb guidance and altitude guidance prove to be the most robust to microburst strength and location. Altitude guidance is potentially the best strategy, provided that the best possible reference altitude is selected. Pitch guidance has again the largest decrease in performance.
Reference Altitude Logic
A logic for the reference altitude is indispensable. Two approaches are used to
develop such a logic, adaptive logics and predictive logics. Adaptive logics start at a
high initial reference altitude, and lower the reference altitude when the angle-of-
attack limit is approached. Predictive logics make an estimation of the energy drain at
the initiation of the escape maneuver, on which a reference altitude is based.

Three adaptive logics are tested. The first method uses a constant altitude decrease,
the second method multiplies the current reference altitude with a constant factor,
while the third method multiplies the reference altitude with a factor that depends on
the F-factor, and thus on the energy drain.

All of the trajectories with use of logics are better than those without logic, but
with the same initial reference altitude. However, none of these adaptive logics resulted
in a performance that approaches the performance of the “best” possible trajectory.

In order to develop a predictive logic, a formula is derived for the “best” reference
altitude, based on the observation that for the “best” trajectory the angle-of-attack
reaches its maximum when the energy drain stops. In this formula, the energy rate
needs to be integrated in time. The energy rate depends on the specific excess power
(SEP) and the F-factor.

Since the thrust, airspeed and drag have a characteristic shape for the best
trajectory, a constant “mean” value is estimated for the SEP. This approach should be
validated by future research.

In order to estimate the F-factor history, a search was initiated for a
characteristic shaping function that depends on microburst strength and size.
However, the attempts to develop a general shaping function for the F-factor history
failed due to the unlimited amount of combinations of microburst parameters and the
limited amount of time available for this thesis study.

6 DOF simulations
For the 6 DOF models, like in the 3 DOF case, altitude guidance proves to be the most
promising strategy, provided the best reference altitude is selected. When this
reference altitude cannot be adequately estimated, rate-of-climb guidance is the best
option. Both strategies easily outperform pitch guidance.

The inertia of the aircraft has a substantial influence on the aircraft’s trajectory,
especially during lateral maneuvering. The lateral displacement reached by the 6 DOF
models is significantly less due to the finite body rates. In general, the altitude is
slightly higher for the 3 DOF model than for the 6 DOF models. Only for rate-of-
climb guidance the performance is less. For 3 DOF the gain factors in the outer loop
are not optimal, but due to inertia effects, they are more appropriate for the 6 DOF
models.

The wind speed variations over the aircraft’s finite size result in additive body
moments. The induced pitch moments prove to be extremely small and do not
influence the aircraft’s trajectory. The induced roll and yaw moments however are
large enough to influence the trajectory. The aircraft has a tendency to roll and yaw
(weather cocking effect) towards microburst core. This influences the rudder and aileron deflections necessary to control the aircraft along the prescribed trajectory. During a longitudinal escape the vertical trajectory is not affected by the induced moments. During lateral maneuvering, the FPA model performs slightly less due to the slightly smaller lateral displacement.

When the very severe microburst described in ref. [1] is penetrated offset of the axis of symmetry in a longitudinal maneuver, the aircraft becomes unstable. However, for most microbursts, the induced moments are expected to be too small to cause any controllability problems.

The time histories of the control surface deflections that are calculated with Nonlinear Dynamics Inversion (NDI) are not very realistic. The values maximum deflections are reached frequently, especially for the elevator. The rate at which the deflections change and the sudden sign changes that occur would increase the pilots workload enormously, if a (human) pilot is able to realize these deflections at all.

**General Remarks**
A remark must be made that the used rate-of-climb guidance gain factors are the ones selected by Visser [22], and are tuned to give good results in 3 DOF cases with only one (Soesman) microburst present. For each different scenario the best value for the gain factors will be different. Tuning these gain factors for a specific scenario will most probably result in better performance for rate-of-climb guidance.

### 9.2 Recommendations for Future Research

**Reference altitude**
In order to exploit the benefits of altitude guidance, the search for a reference altitude logic should be continued. For an *adaptive logic*, that corrects the reference altitude during the maneuver two approaches are suggested.

1. Entering the microburst at a moderate altitude, and increase the altitude stepwise if the energy drain is less than expected. The value of the step should depend on the value of the energy rate, which in reality can be measured with total energy-rate sensors.

2. Initially choose a moderate reference altitude. If the energy drain proves to be too large and the angle-of-attack is approached, a switch to rate-of-climb guidance can be made.

Another option is creating a *predictive logic*. Such a logic estimates the total energy drain before the escape maneuver is initiated and uses this prediction to evaluate a reference altitude that is close to the "best" value. In the case of forward-look sensing the windspeed along the line-of-sight can be measured. Using this (almost) horizontal windspeed Vicroy predicts the vertical windspeed component [20]. In a similar way the energy drain can perhaps be estimated. A possible approach is to define a standard characteristic shaping function for the F-factor history. Fitting the F-factor history that is registered at the initiation of the escape to the first part of the standard shaping
function might then give an estimation of the microburst's strength and size. When this data is combined with measured or calculated crosswinds an the microbursts location can perhaps be estimated as well.

**Modeling of Microburst Encounters and Aircraft**

In addition to the study of controllability problems due to (mean) windspeed variations over the aircraft's size, the next step for future research is the *modeling of high-frequency turbulence*. Turbulence may have a significant effect on the controllability and performance of the aircraft for the various guidance strategies.

Especially when turbulence is modeled, attention should be paid to the modeling and evaluation of the *control surface deflections*. The rate of deflection should be given a finite value by modeling the control surface deflections with, for example, a first order PD control law. This creates a more realistic behavior in the presence of turbulence and will avoid the high deflection rates and sudden sign changes that are found in this study. Also, proper gain scheduling should help avoiding the large values of the control surface deflections that are observed in this thesis.

Microbursts are often accompanied by *heavy rain*. The effects of heavy rain on aerodynamics and engine performance are not modeled in this thesis. Especially the engine performance might decrease in the presence of this heavy rain, resulting in a decreasing aircraft performance in escape maneuvers.

Also, *ground effect* is not modeled in this thesis. Ground effect may have a positive influence on the performance during an escape maneuver.
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(b)
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Appendix A: Boeing 727-100 Data

General data (source: van der Veen \(^{(19)}\))

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<td>(I_{yy})</td>
<td>6.92*10(^6)</td>
<td>kg m(^2)</td>
</tr>
<tr>
<td>(I_{zz})</td>
<td>4.80*10(^6)</td>
<td>kg m(^2)</td>
</tr>
<tr>
<td>(I_{xz})</td>
<td>2.50*10(^3)</td>
<td>kg m(^2)</td>
</tr>
</tbody>
</table>

Stability derivatives (source: van der Veen \(^{(19)}\))

**longitudinal derivatives:**

\[ C_{n_s} = -1.21 \text{ rad}^{-1} \]
\[ C_{n_a} = -1.77 \]
\[ C_{m_s} = -1.5 \text{ rad}^{-1} \]
\[ C_{n_r} = -29.5 \]

**lateral derivatives:**

\[ C_{r_p} = -0.974 \]
\[ C_{r_s} = 0.520 \]
\[ C_{r_p} = 0.100 \]
\[ C_{r_a} = 0.240 \]
\[ C_{r_s} = -0.139 \]
\[ C_{r_p} = -0.380 \]
\[ C_{r_s} = 0.380 \]
\[ C_{r_p} = -0.046 \]
\[ C_{r_s} = 0.003 \]
\[ C_{r_p} = 0.143 \]
\[ C_{n_p} = -0.400 \]
\[ C_{n_s} = -0.205 \]
\[ C_{n_p} = 0.022 \]
\[ C_{n_s} = -0.097 \]
Aerodynamic and thrust data (source: Visser (21))

**Thrust coefficients** \((T = a_0 + a_1 V + a_2 V^2)\):

- \(a_0 = 198280\) N
- \(a_1 = -350.08\) N \((\text{m/s})^1\)
- \(a_2 = 0.69063\) N \((\text{m/s})^2\)

- \(\tau_1 = 3\) s

**Drag coefficients** \((D = b_0 + b_1 \alpha + b_2 \alpha^2)\):

- \(b_0 = 0.15751\)
- \(b_1 = 0.0768\) rad\(^{-1}\)
- \(b_2 = 2.524\) rad\(^2\)

**Lift coefficients** \((L = c_0 + c_1 \alpha + c_2 (\alpha - \alpha_{\text{ref}})^2)\):

- \(c_0 = 0.7076\)
- \(c_1 = 5.97\) rad\(^{-1}\)
- \(c_2 = 0\) rad\(^2\) \quad \text{if} \ 0 \leq \alpha \leq \alpha_{\text{ref}}\)
- \(-5.95\) rad\(^2\) \quad \text{if} \ \alpha_{\text{ref}} \leq \alpha \leq \alpha_{\max}\)

**Angle of attack limits:**

- \(\alpha_{\text{ref}} = 0.2269\) rad
- \(\alpha_{\max} = 0.3002\) rad
Appendix B: Axis Systems, Forces and Moments

In refs. [6, 17] various axis systems are presented, that will be used in this thesis.

Fixed Earth Axis System \((Ox_\text{e}y_\text{e}z_\text{e})\)

The earth axis system is a Cartesian axis system, fixed to earth. For the purpose of this thesis, it can be assumed that the earth is non rotating and flat, and moves with a constant speed with respect to an inertial axis system fixed somewhere in space, and is therefore an inertial axis system itself.

Unless indicated otherwise, the earth axis system is defined as follows:
The origin is situated on the runway threshold. The \(x_\text{e}\)-axis is positive in runway direction. The \(z_\text{e}\)-axis is positive downward, and the \(y_\text{e}\)-axis stands perpendicular on the \(Ox_\text{e}z_\text{e}\) plane. For an aircraft approaching the runway, the \(y_\text{e}\)-axis thus points to the right.

Body Axis System \((Ox_by_bz_b)\)

The body axis system is fixed to the aircraft. The origin of the body axis system is in the aircraft's center of gravity. The \(x_b\)-axis is pointing forward, through the nose. The \(z_b\)-axis lies in the aircraft's plane of symmetry, perpendicular to the \(x_b\)-axis, and is positive downward. The \(y_b\)-axis is perpendicular to the \(Ox_bz_b\) plane, positive in starboard direction.

The orientation of the body axis system can by obtained from the Earth axis system by rotation about the Euler angles, \(\psi, \theta\) and \(\phi\) respectively. Where \(\psi\) is the yaw angle, \(\theta\) is the pitch angle and \(\phi\) is the bank angle.

Wind Axis System or Air Path Axis System \((Ox_wy_wz_w)\)

Like the body axis system, the origin of the wind axis system is also in the aircraft's center of gravity. The \(x_w\)-axis lies along the airspeed vector. The \(z_w\)-axis lies again in the plane of symmetry, perpendicular to the \(x_w\)-axis and is positive downward. The \(y_w\)-axis stands perpendicular to the \(Ox_wz_w\) plane and is pointing out to the starboard side of the aircraft.

The orientation of the wind axis system can be obtained from the earth axis system by rotation about the Euler angles, \(\chi, \gamma\) and \(\mu\) respectively. Where \(\chi\) is the heading angle, \(\gamma\) is the flight path inclination angle and \(\mu\) is the aerodynamic bank angle.

The angle of attack \(\alpha\) is the angle between the projection of the \(x_w\)-axis on the \(Ox_bz_b\) plane, and the \(x_b\)-axis. The sideslip angle \(\beta\) is the angle between the projection of the \(x_w\)-axis on the \(Ox_bz_b\) plane, and the \(x_w\)-axis.

Forces and Moments

The forces \(X, Y\) and \(Z\) are components of the total force that acts on the aircraft (the sum of aerodynamic force and thrust) along the positive \(x_b, y_b\) and \(z_b\) axes respectively.

The thrust vector \(T\) lies in the aircraft's plane of symmetry for most planes (unless, for example, thrust vectoring is applied). The angle between the thrust vector and the \(x_w\)-axis
is the thrust inclination angle, $\delta_t$. This angle is positive when the thrust vector has a component in upward direction, along the negative $z_s$-axis.

The lift force $L_s$ is defined as the component of the aerodynamic force along the $z_s$-axis. $L$ is positive in the negative $z_s$ direction. The drag $D$ and side force $S$ are usually defined as the components of the aerodynamic force along the negative $x_s$-axis and $y_s$-axis respectively. In this thesis however, the drag $D$ is defined as the component of the aerodynamic force that lies in the plane of symmetry, perpendicular to $L$ and positive in the opposite direction of the projection of the airspeed vector on the plane of symmetry. This is done to simplify the evaluation of the value of the drag. Instead of the side force $S$, the force $Y$ (positive along positive $y_s$-axis) is used. Since the thrust $T$ lies in the plane of symmetry for most aircraft, the force $Y$ consists only of an aerodynamic contribution.

The moments of the aircraft $L_s$, $M$ and $N$ are the moments around the positive $x_s$, $y_s$ and $z_s$ axes respectively.
Appendix C: General Equations of Unsteady Motion

Before a 6 DOF simulation can be carried out, the equations of motion and kinematic equations need to be derived. In this chapter a derivation of the general equations of unsteady motion in the presence of windshear will be derived.

The used axis systems and the definition of forces and moments can be found in Appendix B.

*Force equations in wind axis system*

Newton’s second law is only valid in an inertial reference frame or a frame that moves with a constant speed relative to this frame. Here a flat, non-rotating earth will be considered, and the axis system fixed to earth will be taken as inertial reference frame.

The groundspeed in the earth axis system is:

\[
\begin{bmatrix}
\dot{x} \\
\dot{y} \\
\dot{z}
\end{bmatrix}^E = L_{EW} \begin{bmatrix}
\dot{V} \\
0 \\
0
\end{bmatrix}^W + \begin{bmatrix}
W_x^E \\
W_y^E \\
W_z^E
\end{bmatrix}^E
\]

(C-1)

Where \( L_{EW} \) is the transformation matrix from the wind axis system to the earth axis system. This matrix is:

\[
L_{EW} = \begin{bmatrix}
\cos \gamma \cos \chi & \sin \mu \sin \gamma \cos \chi & \cos \mu \sin \gamma \cos \chi \\
-\cos \mu \sin \chi & + \sin \mu \sin \chi \\
\cos \gamma \sin \chi & \sin \mu \sin \gamma \sin \chi & \cos \mu \sin \gamma \sin \chi \\
+ \cos \mu \cos \chi & - \sin \mu \cos \chi & \\
- \sin \gamma & \sin \mu \cos \gamma & \cos \mu \cos \gamma
\end{bmatrix}
\]

(C-2)

So when written in scalar notation, the inertial speeds are:

\[
\begin{align*}
\dot{x}_E &= V \cos \gamma \cos \chi + W_x \\
\dot{y}_E &= V \cos \gamma \sin \chi + W_y \\
\dot{z}_E &= -\dot{h} = -V \sin \gamma + W_z
\end{align*}
\]

(C-3)

When the expressions in (C-3) are differentiated with respect to time, the inertial acceleration is obtained.

\[
\begin{align*}
\ddot{x}_E &= \dot{V} \cos \gamma \cos \chi - V \dot{\gamma} \sin \gamma \cos \chi - V \dot{\chi} \cos \gamma \sin \chi + \dot{W}_x \\
\ddot{y}_E &= \dot{V} \cos \gamma \sin \chi - V \dot{\gamma} \sin \gamma \sin \chi + V \dot{\chi} \cos \gamma \cos \chi + \dot{W}_y \\
\ddot{z}_E &= -\dot{V} \sin \gamma - V \dot{\gamma} \cos \gamma + \dot{W}_z
\end{align*}
\]

(C-4)
These accelerations can be transformed to the wind axis system with:

\[
\begin{bmatrix}
a_x \\
a_y \\
a_z
\end{bmatrix}^w = L_{WE}^T
\begin{bmatrix}
\dot{x} \\
\dot{y} \\
\dot{z}
\end{bmatrix}^E
\]  
\hspace{1cm} (C-5)

Where the matrix \(L_{WE}\) is the transposed of \(L_{EW}\).

\[
L_{WE} = L_{EW}^T
\]  
\hspace{1cm} (C-6)

When this transformation is performed the following expressions are found:

\[
a_x = \dot{V} + \dot{W}_z \cos \gamma \cos \chi + \dot{W}_y \cos \gamma \sin \chi - \dot{W}_x \sin \gamma
\]

\[
a_y = -V \dot{\gamma} \sin \mu + V \dot{\chi} \sin \mu \cos \gamma + \dot{W}_z (\sin \mu \sin \gamma \cos \chi - \cos \mu \sin \chi) + \dot{W}_y (\sin \mu \sin \gamma \sin \chi + \cos \mu \cos \chi) + \dot{W}_x \sin \mu \cos \gamma
\]  
\hspace{1cm} (C-7)

\[
a_z = -V \dot{\gamma} \cos \mu - V \dot{\chi} \sin \mu \cos \gamma + \dot{W}_z (\cos \mu \sin \gamma \cos \chi + \sin \mu \sin \chi) + \dot{W}_y (\cos \mu \sin \gamma \sin \chi + \sin \mu \cos \chi) + \dot{W}_x \cos \mu \cos \gamma
\]

Or, in vector notation:

\[
\begin{bmatrix}
a_x \\
a_y \\
a_z
\end{bmatrix}^w = \begin{bmatrix}
\dot{V} \\
-V \dot{\gamma} \sin \mu + V \dot{\chi} \cos \mu \cos \gamma \\
-V \dot{\gamma} \cos \mu - V \dot{\chi} \sin \mu \cos \gamma
\end{bmatrix}^E + L_{WE}^T
\begin{bmatrix}
\dot{W}_x \\
\dot{W}_y \\
\dot{W}_z
\end{bmatrix}^E
\]  
\hspace{1cm} (C-8)

Now Newton’s second law is applied on the aircraft in the earth axis system. The mass of the aircraft can be assumed constant during the escape maneuver, and the aircraft is considered to be a rigid body:

\[
\vec{F} = m \ddot{\vec{a}}_E
\]  
\hspace{1cm} (C-9)

Or:

\[
\begin{bmatrix}
F_x \\
F_y \\
F_z
\end{bmatrix}^E = m \begin{bmatrix}
\ddot{x} \\
\ddot{y} \\
\ddot{z}
\end{bmatrix}^E \Leftrightarrow L_{WE}^T
\begin{bmatrix}
F_x \\
F_y \\
F_z
\end{bmatrix}^E = m L_{WE}
\begin{bmatrix}
\ddot{x} \\
\ddot{y} \\
\ddot{z}
\end{bmatrix}^E \Leftrightarrow \begin{bmatrix}
F_x \\
F_y \\
F_z
\end{bmatrix}^w = \begin{bmatrix}
a_x \\
a_y \\
a_z
\end{bmatrix}^w
\]  
\hspace{1cm} (C-10)

The forces acting on the aircraft can be expressed in the wind axis system:

\[
\begin{bmatrix}
F_x \\
F_y \\
F_z
\end{bmatrix}^w = L_{WB}^T
\begin{bmatrix}
T_x \\
T_y \\
T_z
\end{bmatrix}^B - \begin{bmatrix}
D \cos \beta - Y \sin \beta \\
D \sin \beta + Y \cos \beta
\end{bmatrix} L_{WE}
\begin{bmatrix}
0 \\
0 \\
mg
\end{bmatrix}
\]  
\hspace{1cm} (C-11)

Where \(L_{WB}\) is the transformation matrix from the body axis system to the wind axis system.
\[ L_{wb} = \begin{pmatrix} \cos \alpha \cos \beta & \sin \beta & \sin \alpha \cos \beta \\ -\cos \alpha \sin \beta & \cos \beta & -\sin \alpha \sin \beta \\ -\sin \alpha & 0 & \cos \alpha \end{pmatrix} \]  \tag{C-12}

The thrust vector is easier calculated in the body axis system since for most aircraft the direction of the thrust is constant relative to the body axis system (no thrust vectoring is applied).

So the force equations in the wind axis system are:

\[
L_{wb} \begin{bmatrix} T_x \\ T_y \\ T_z \end{bmatrix} + D \begin{bmatrix} \cos \beta - Y \sin \beta \\ D \sin \beta + Y \cos \beta \\ L \end{bmatrix} + L_{we} \begin{bmatrix} 0 \\ 0 \end{bmatrix} + \begin{bmatrix} mg \\ -V \gamma \sin \mu + V \dot{\gamma} \cos \mu \cos \gamma \\ -V \gamma \cos \mu - V \dot{\gamma} \sin \mu \cos \gamma \end{bmatrix} = \begin{bmatrix} W_x \\ W_y \\ W_z \end{bmatrix} \tag{C-13}
\]

so that:

\[
\begin{bmatrix} 
V \\
-V \gamma \sin \mu + V \dot{\gamma} \cos \mu \cos \gamma \\
-V \gamma \cos \mu - V \dot{\gamma} \sin \mu \cos \gamma 
\end{bmatrix} = L_{wb} \frac{1}{m} \begin{bmatrix} T_x \\ T_y \\ T_z \end{bmatrix} \begin{bmatrix} D \cos \beta - Y \sin \beta \\ D \sin \beta + Y \cos \beta \\ -L \end{bmatrix} + \begin{bmatrix} W_x \\ W_y \\ g - W_z \end{bmatrix} \tag{C-14}
\]

The force equations of motion become (scalar notation):

\[
\dot{V} = \frac{1}{m} \left[ T \cos(\alpha + \delta) - D \cos \beta + Y \sin \beta \right] - W_z \cos \gamma \cos \chi + \nonumber
\]

\[
-W_y \cos \gamma \sin \chi - \dot{W}_n \sin \gamma - g \sin \gamma \tag{C-15}
\]

\[
\dot{\gamma} = \frac{1}{V} \left[ \frac{L + T \sin(\alpha + \delta)}{m} \cos \mu + \nonumber\right]
\]

\[
\frac{T \cos(\alpha + \delta) \sin \beta - D \sin \beta - Y \cos \beta}{m} \sin \mu + \nonumber
\]

\[
-g \cos \gamma + \dot{W}_z \sin \gamma \cos \chi + \dot{W}_y \sin \gamma \sin \chi - \dot{W}_n \cos \gamma \tag{C-16}
\]
\[ \dot{\chi} = \frac{1}{V \cos \gamma} \left[ \frac{L + T \sin(\alpha + \delta)}{m} \sin \mu + \right. \\
\left. + \frac{T \cos(\alpha + \delta) \sin \beta + D \sin \beta + Y \cos \beta}{m} \cos \mu + \right. \\
\left. + \dot{\theta} \sin \chi - \dot{\psi} \cos \chi \right] \]  

(C-17)

**Moment equations in body axis system**

In the inertial earth axis system the moment equation is (see ref. [6]):

\[ \vec{M}_E = \dot{\vec{h}}_E \]  

(C-18)

The moment equation is transformed to the body axis system, since in this axis system the aircraft’s moments of inertia are constant, which simplifies the equations considerably.

In body axis:

\[ \vec{M}_B = \vec{L}_{BE} \vec{M}_E = \dot{\vec{h}}_B + \vec{\omega}_B \times \vec{h}_B \]  

(C-19)

The resultant moment on the aircraft in body axis is:

\[ \vec{M}_B = \begin{bmatrix} L \\ M \\ N \end{bmatrix} \]  

(C-20)

And the angular momentum for an aircraft is (when the contributions of elastic vibrations and moving hinges, like ailerons, elevators etc. are neglected):

\[ \vec{h}_B = \begin{bmatrix} h_x \\ h_y \\ h_z \end{bmatrix} = \begin{bmatrix} I_{xx} & -I_{xy} & -I_{xz} \\ -I_{yx} & I_{yy} & -I_{yz} \\ -I_{zx} & -I_{zy} & I_{zz} \end{bmatrix} \begin{bmatrix} p \\ q \\ r \end{bmatrix} \]  

(C-21)

The moment equations for a rigid aircraft (constant moments of inertia with respect to time) are:

\[ \begin{bmatrix} L \\ M \\ N \end{bmatrix} = \begin{bmatrix} I_{xx} & -I_{xy} & -I_{xz} & -I_{yx} & I_{yy} & -I_{yz} & -I_{zx} & -I_{zy} & I_{zz} \end{bmatrix} \begin{bmatrix} p \\ q \\ r \end{bmatrix} \]  

(C-22)

Since the aircraft is symmetric in the Oxz plane, \( I_{yx} = I_{zx} = I_{yz} = I_{zy} = 0 \). Also \( I_{xx} = I_{xx} \).

When this is substituted in (C-22) the angular body rates can be calculated from:

\[ \begin{bmatrix} I_{xx} & 0 & -I_{xx} \\ 0 & I_{yy} & 0 \\ -I_{xx} & 0 & I_{zz} \end{bmatrix} \begin{bmatrix} p \\ q \\ r \end{bmatrix} = \begin{bmatrix} L \\ M \\ N \end{bmatrix} \begin{bmatrix} I_{xx} & 0 & -I_{xx} \\ 0 & I_{yy} & 0 \\ -I_{xx} & 0 & I_{zz} \end{bmatrix} \begin{bmatrix} p \\ q \\ r \end{bmatrix} \]  

(C-23)
so that
\[
\begin{bmatrix}
\dot{p} \\
\dot{q} \\
\dot{r}
\end{bmatrix} = \begin{bmatrix}
I_{xx} & 0 & -I_{zt} \\
0 & I_{yy} & 0 \\
-I_{zt} & 0 & I_{zz}
\end{bmatrix}^{-1} \begin{bmatrix}
L \\
M \\
N
\end{bmatrix} + \begin{bmatrix}
I_{zt} pq + (I_{yy} - I_{zz})qr \\
I_{zt} (r^2 - p^2) + (I_{zz} - I_{xx})pr \\
-I_{zt} pq + (I_{xx} - I_{yy})pq
\end{bmatrix}
\]  
(C-24)

when the first matrix is inverted:
\[
\begin{bmatrix}
\dot{p} \\
\dot{q} \\
\dot{r}
\end{bmatrix} = \begin{bmatrix}
\frac{I_{zt}}{I_{xx} I_{zz} - I_{zt}^2} & 0 & \frac{I_{zt}}{I_{xx} I_{zz} - I_{zt}^2} \\
0 & \frac{I_{zt}}{I_{yy} I_{zz} - I_{zt}^2} & 0 \\
\frac{I_{zt}}{I_{xx} I_{zz} - I_{zt}^2} & 0 & \frac{I_{zt}}{I_{xx} I_{yy} - I_{zt}^2}
\end{bmatrix} \begin{bmatrix}
L + I_{zt} pq + (I_{yy} - I_{zz})qr \\
M + I_{zt} (r^2 - p^2) + (I_{zz} - I_{xx})pr \\
N - I_{zt} qr + (I_{xx} - I_{yy})pq
\end{bmatrix}
\]  
(C-25)

In the moment equations the aerodynamic moments are divided into moments due to control surface deflections (subscript δ) and moments due to other aerodynamic effects (subscript a). This approach is necessary to calculate the control surface deflections in the inner loop NDI routine, see paragraph 7.1.2.
\[
\dot{p} = \frac{I_{zt} L + I_{zt} N_a + I_{zt} L_d + I_{zt} N_d}{I_{xx} I_{zz} - I_{zt}^2} + \frac{I_{zt} (I_{yy} - I_{xx}) pq + [I_{zt} (I_{yy} - I_{zz}) - I_{zt}^2]qr}{I_{xx} I_{zz} - I_{zt}^2}
\]  
(C-26)

\[
\dot{q} = \frac{1}{I_{yy}} [M_a + M_d + (I_{zz} - I_{xx}) pr + I_{zt} (r^2 - p^2)]
\]  
(C-27)

\[
\dot{r} = \frac{I_{zt} L + I_{zt} N_a + I_{zt} L_d + I_{zt} N_d}{I_{xx} I_{zz} - I_{zt}^2} + \frac{[I_{zt} (I_{yy} - I_{xx}) + I_{zt}^2]pq - I_{zt} (I_{xx} - I_{yy} + I_{zt})qr}{I_{xx} I_{zz} - I_{zt}^2}
\]  
(C-28)

Note that no wind terms are explicitly present in the moment equations. However, wind influences enter the moment equations through the aerodynamic moments, which depend on the flow around the aircraft and thus depend on the windspeed components.

**Kinematic Equations**
Besides the 6 equations of motion, 6 kinematic relations will be derived. Three equations for translational motion are already given in equation C-4. Those are the speeds in the earth axis system.
\[
\begin{align*}
\dot{x}_E &= V \cos \psi \cos \chi + W_x \\
\dot{y}_E &= V \cos \psi \sin \chi + W_y \\
\dot{z}_E &= -\dot{h} = -V \sin \psi + W_z
\end{align*}
\]  
(C-3)
The remaining three can obtained from a relation for rotational motion. When the rotation of the aircraft (body axis system) with respect to the earth axis system is written in the wind axis system, the following relation holds:

\[
(\omega_{B/E})^w = (\omega_{B/A})^w + (\omega_{A/E})^w
\]  

(C-29)

This can be written as:

\[
\begin{pmatrix}
\cos \alpha \cos \beta & \sin \beta & \sin \alpha \cos \beta \\
-\cos \alpha \sin \beta & \cos \beta & -\sin \alpha \sin \beta \\
-\sin \alpha & 0 & \cos \alpha
\end{pmatrix}
\begin{bmatrix}
p \\ q \\ r
\end{bmatrix}
= \begin{bmatrix}
\dot{\alpha} \cos \beta \\
\dot{\gamma} \cos \mu + \dot{\chi} \cos \gamma \sin \mu \\
-\dot{\chi} \sin \mu + \dot{\chi} \cos \gamma \cos \mu
\end{bmatrix}
\]

\[= \begin{bmatrix}
\frac{\mu - \dot{\chi} \sin \gamma}{\cos \beta} \\
\dot{\gamma} \cos \mu + \dot{\chi} \cos \gamma \sin \mu \\
-\dot{\chi} \sin \mu + \dot{\chi} \cos \gamma \cos \mu
\end{bmatrix}
\]

(C-30)

From this the following relations are easily obtained.

\[
\dot{\alpha} = -\frac{1}{\cos \beta} (-p \cos \alpha \sin \beta + q \cos \beta - r \sin \alpha \sin \beta - \dot{\gamma} \cos \mu - \dot{\chi} \cos \gamma \sin \mu)
\]

\[
\dot{\beta} = p \sin \alpha - r \cos \alpha - \dot{\gamma} \sin \mu + \dot{\chi} \cos \gamma \cos \mu
\]  

(C-31)

\[
\dot{\mu} = p \cos \alpha \cos \beta + q \sin \beta + r \sin \alpha \cos \beta - \dot{\chi} \sin \gamma - \dot{\alpha} \sin \beta
\]

Finally, when \( \alpha \) is substituted in the \( \mu \) equation:

\[
\dot{\alpha} = -p \cos \alpha \tan \beta + q - r \sin \alpha \tan \beta - \frac{1}{\cos \beta} (\dot{\gamma} \cos \mu + \dot{\chi} \cos \gamma \sin \mu)
\]

\[
\dot{\beta} = p \sin \alpha - r \cos \alpha - \dot{\gamma} \sin \mu + \dot{\chi} \cos \gamma \cos \mu
\]  

(C-32)

\[
\dot{\mu} = p \frac{\cos \alpha}{\cos \beta} + r \frac{\sin \alpha}{\cos \beta} - \dot{\gamma} \tan \beta \cos \mu + \dot{\chi} (\cos \gamma \tan \beta \sin \mu + \sin \gamma)
\]

Note that the windspeed gradients enter these equations through \( \dot{\gamma} \) and \( \dot{\chi} \), which are calculated in the force equations.
Appendix D: program description

To perform 3 DOF and 6 DOF simulations, two FORTRAN 77 programs can be used. The program GUIDOPA.F performs either a 3 DOF simulation or an OPA model (6 DOF) simulation. The program GUIDFPA.F performs also a 3 DOF simulation or uses the (6 DOF) FPA model.

Both programs use several input and output files. A summary and short description of these files is given below.

**Input Files**
- **727.IN** aircraft data: weight, dimensions and moments of inertia
- **INITIAL.IN** initial values of all state variables (V, γ, χ, x, y, h, η, α, β, μ, p, q, r)
- **STRAT.IN** strategy related quantities: h<sub>eff</sub>, θ<sub>α</sub>, η<sub>α</sub>, α<sub>max</sub>, μ<sub>max</sub>, F<sub>thres</sub>
- **WIND.IN** microburst data: U<sub>max</sub>, Z<sub>mb</sub>, D<sub>mb</sub>, J<sub>mb</sub>, r<sub>look</sub>

**Output Files**
- **CONTROL.OUT** t, phase, η, α<sub>c</sub>, μ<sub>c</sub>, C<sub>L</sub>, p, q, r, δ<sub>α</sub>, δ<sub>δ</sub>, δ<sub>ε</sub>
- **DATA.OUT** t, phase and extra data like F, F<sub>look</sub>, E, p<sub>sw</sub>, q<sub>sw</sub>, r<sub>lw</sub>, r<sub>sw</sub>
- **STATE.OUT** t, phase, V, χ, γ, η, x, y, h, α, β, μ

**Program Usage**
When the programs are run, the user can first choose between 3 DOF or 6 DOF simulation. Further, the user can select the escape strategy that is to be simulated:
- pitch guidance (constant α + γ)
- RC guidance
- altitude guidance

**Main Program Structure**
The program consists of one main routine and 11 subroutines. These subroutines are summarized and described below.

- **guidan** calculates α<sub>c</sub>, β<sub>c</sub>, μ<sub>c</sub> and η<sub>c</sub> from prescribed trajectory and strategy
- **atmos** calculates air temperature, pressure and density
- **wind3d** calculates windspeed components on given location
- **trumax** calculates maximum thrust
- **dyninv** evaluates required δ<sub>α</sub>, δ<sub>δ</sub> and δ<sub>ε</sub> from α<sub>c</sub>, β<sub>c</sub> and μ<sub>c</sub>
- **aero** calculates aerodynamic forces, moments and experienced body rates
- **deriv1** evaluates very slow state equations of motion
- **deriv2** evaluates slow state equations of motion
- **deriv3** evaluates fast state equations of motion
- **print** prints the output variables to screen and output files
- **rk4** 4<sup>th</sup> order Runge-Kutta integration routine

In the main program the time interval and the termination time can be specified.