Downhole interferometric illumination diagnosis and balancing

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SUMMARY

With seismic interferometry or the virtual source method, controlled sources can be redatumed from the Earth’s surface to generate so-called virtual sources at downhole receiver locations. Generally this is done by crosscorrelation of the recorded downhole data and stacking over source locations. By studying the retrieved data at zero time lag, downhole illumination conditions that determine the virtual source radiation pattern can be analyzed without a velocity model. This can be beneficial for survey planning in time-lapse experiments. Moreover, the virtual source radiation pattern can be corrected by multidimensional deconvolution or directional balancing. Such an approach can help to improve virtual source repeatability, posing major advantages for reservoir monitoring. An algorithm is proposed for so-called illumination balancing (being closely related to directional balancing). It can be applied to single-component receiver arrays with limited aperture below a strongly heterogeneous overburden. The algorithm is demonstrated on synthetic 3D elastic data to retrieve time-lapse amplitude attributes.

Key words: Seismic interferometry, virtual source, reservoir monitoring
1 INTRODUCTION

Conventional seismic imaging, characterization and monitoring relies heavily on the availability and quality of a subsurface velocity model. Especially if the shallow subsurface is strongly heterogeneous, accurate velocity models are hard to obtain, which can pose serious limitations in such environments. Bakulin & Calvert (2004) overcame this problem by placing receivers in horizontal or deviated wells below the major complexities and redatuming seismic sources from their actual locations at the earth’s surface to the well by a formalism based on crosscorrelation. The concept is generally referred to as the virtual source method (Bakulin & Calvert 2006) or seismic interferometry (Schuster et al. 2004; Schuster 2009). Since the required redatuming operators from the source level to the receiver level are actually measured, a velocity model is not required. Besides imaging below a complex overburden, the virtual source concept proved useful for a variety of applications, including reservoir monitoring (Bakulin et al. 2007), salt-flank imaging (Willis et al. 2006), sub-salt imaging (Vasconcelos et al. 2008) and various others (Curtis et al. 2006; Schuster 2009; Ramirez & Weglein 2009; Wapenaar et al. 2010a; Vasconcelos et al. 2011).

Over the past few years seismic interferometry has evolved significantly (Wapenaar et al. 2010b). Besides the common correlation-based approach, several authors use convolution (Slob & Wapenaar 2007; Poletto & Farina 2010), (single-trace) deconvolution (Vasconcelos & Snieder 2008; Poletto et al. 2011) and multidimensional deconvolution (Wapenaar et al. 2011). In the latter method, seismic redatuming is interpreted as an inverse problem, which can be solved in a least-squares sense. A major limitation of such an approach is that some sort of wavefield separation is required by definition, either by wavefield decomposition (Wapenaar et al. 2011) or time-gating (Van der Neut et al. 2011a). Moreover, in heterogeneous 3D media, the implementation of multidimensional deconvolution would ideally require 2D receiver arrays, which are not available in typical single-well acquisition.

It is well understood that interferometry by crosscorrelation and multidimensional deconvolution are closely related through the interferometric point-spread function (Wapenaar et al. 2011). The point-spread function quantifies the downhole illumination conditions, which will be imprinted on the retrieved virtual source data. As such, the point-spread function can be interpreted as the radiation pattern of the virtual source and deconvolution with the point-spread function (as done in multidimensional deconvolution) can be seen as optimization of this radiation pattern (Van der Neut & Bakulin 2009). In this paper, practical solutions are proposed to diagnose and optimize the radiation pattern of a virtual source in cases where acquisition conditions are limited. Implementation of the proposed algorithm will be demon-
strated with a single-component receiver array of limited aperture in a horizontal borehole below a complex overburden.

2 THEORETICAL DEVELOPMENTS

The main configuration of this study is shown in Figure 1. Sources are placed at surface locations \( \mathbf{x}_S^{(s)} \). Receivers are placed in a horizontal well at locations \( \mathbf{x}_R^{(r)} \). Subscripts \( S \) and \( R \) denote "source" and "receiver", whereas superscripts \( s \) and \( r \) are indices for the source and receiver numbers, respectively. For virtual sources, index \( v \) will be used. In the most common form of seismic interferometry (Wapenaar & Fokkema 2006), full wavefields are crosscorrelated and summed over source locations, yielding the following correlation function:

\[
\hat{C} \left( \mathbf{x}_R^{(r)}, \mathbf{x}_R^{(v)}, \omega \right) = \sum_s \hat{P} \left( \mathbf{x}_R^{(r)}, \mathbf{x}_S^{(s)}, \omega \right) \hat{P}^* \left( \mathbf{x}_R^{(v)}, \mathbf{x}_S^{(s)}, \omega \right) \Delta x_S.
\]

(1)

Here, \( \hat{P} \left( \mathbf{x}_R^{(r)}, \mathbf{x}_S^{(s)}, \omega \right) \) holds the recorded data from source \( \mathbf{x}_S^{(s)} \) to receiver \( \mathbf{x}_R^{(r)} \), \( \omega \) is the angular frequency, superscript \( * \) denotes complex conjugation and \( \Delta x_S \) is the source spacing. Equation 1 is written in the frequency-space domain, which is indicated by the caret. Various authors have shown that the causal part of the correlation function can be interpreted as an approximation of a Green’s function, as if there was a source at \( \mathbf{x}_R^{(v)} \) and a receiver at \( \mathbf{x}_R^{(r)} \), convolved with the auto-correlated source wavelet (Bakulin & Calvert 2006; Wapenaar & Fokkema 2006). For this to be true, it is required that the source array is sufficiently extended as to sample the Fresnel zones of the reflected wavefields (Snieder 2004; Schuster 2009; Costagliola 2010) and that the source array is tapered to avoid truncation artifacts (Mehta et al. 2008b). Note that equation 1 is subject to a far-field radiation approximation in which amplitude-versus-rayparameter characteristics are not preserved and free-surface reflections are not accounted for (Wapenaar & Fokkema 2006; Schuster 2009).

In practice, the retrieved response by equation 1 is not always optimal and several alternative formulations have been proposed (Schuster & Zhou 2006; Snieder et al. 2009). In the virtual source method, for example, the full field at \( \mathbf{x}_R^{(r)} \) is correlated with the time-gated direct field at \( \mathbf{x}_R^{(v)} \) (Bakulin & Calvert 2006). A similar approach is taken in so-called perturbation-based interferometry (Vasconcelos et al. 2009) by crosscorrelating the scattered field from the target reflectors at \( \mathbf{x}_R^{(r)} \) with the incident field from the overburden at \( \mathbf{x}_R^{(v)} \); that is

\[
\hat{C}_{sc,inc} \left( \mathbf{x}_R^{(r)}, \mathbf{x}_R^{(v)}, \omega \right) = \sum_s \hat{P}_{sc} \left( \mathbf{x}_R^{(r)}, \mathbf{x}_S^{(s)}, \omega \right) \hat{P}_{inc}^* \left( \mathbf{x}_R^{(v)}, \mathbf{x}_S^{(s)}, \omega \right) \Delta x_S.
\]

(2)
Here subscripts \( \text{sc} \) and \( \text{inc} \) stand for "scattered" and "incident" constituents, respectively, which can often be separated by time-gates (Van der Neut et al. 2011a). The retrieved response \( \hat{C}_{\text{sc,inc}}(x_r^{(r)}, x_s^{(s)}, \omega) \) can be interpreted as an approximation of the scattered wavefield as if there was a virtual source at \( x_r^{(v)} \) and a receiver at \( x_r^{(r)} \). The effectiveness of time-gating can be understood in several ways. Vasconcelos et al. (2009) and Van der Neut et al. (2011a) show that with equation 2 less spurious cross-terms are retrieved than with equation 1. Alternatively, it can be shown that time-gating improves the radiation pattern of a virtual source (Van der Neut & Bakulin 2009). Applying wavefield decomposition prior to crosscorrelation proved even more successful (Mehta et al. 2007, 2010a). However, such decomposition requires multi-component receivers, which are often not available.

Representations for interferometry by multidimensional deconvolution of decomposed wavefields have been derived by Wapenaar et al. (2011). Applications have been demonstrated for redatuming elastic wavefields below a complex overburden (Van der Neut et al. 2011b), virtual crosswell imaging (Minato et al. 2011), controlled-source electromagnetic exploration (Hunziker et al. 2012) and ground penetrating radar (Slob 2009). Slight modifications have been introduced for time-gated incident and scattered wavefields (Van der Neut et al. 2011a). In any case, one starts with the formulation of a convolutional forward equation, which reads for time-gated wavefields:

\[
\hat{P}_{\text{sc}}(x_r^{(r)}, x_s^{(s)}, \omega) = \sum_v \hat{X}_{\text{sc,inc}}(x_r^{(r)}, x_r^{(v)}, \omega) \hat{P}_{\text{inc}}(x_r^{(v)}, x_s^{(s)}, \omega) \Delta x_R, \tag{3}
\]

where \( \Delta x_R \) is the receiver spacing and \( \hat{X}_{\text{sc,inc}}(x_r^{(r)}, x_r^{(v)}, \omega) \) is a dipole impulse response as if there was an incident field emitted at \( x_r^{(v)} \) and a scattered field received at \( x_r^{(r)} \). If sufficient sources are available, equation 3 can be inverted in the least-squares sense for \( \hat{X}_{\text{sc,inc}}(x_r^{(r)}, x_r^{(v)}, \omega) \). It can be shown that this solution obeys the following normal equation (Van der Neut et al. 2011a):

\[
\hat{C}_{\text{sc,inc}}(x_r^{(r)}, x_s^{(s)}, \omega) = \sum_v \hat{X}_{\text{sc,inc}}(x_r^{(r)}, x_r^{(v)}, \omega) \hat{C}_{\text{inc,inc}}(x_r^{(v)}, x_s^{(w)}, \omega) \Delta x_R, \tag{4}
\]

where \( w \) is another receiver index and the point-spread function is introduced as

\[
\hat{C}_{\text{inc,inc}}(x_r^{(r)}, x_s^{(w)}, \omega) = \sum_s \hat{P}_{\text{inc}}(x_r^{(v)}, x_s^{(s)}, \omega) \hat{P}^*_{\text{inc}}(x_r^{(w)}, x_s^{(e)}, \omega) \Delta x_S. \tag{5}
\]

The left-hand-side of equation 4 is exactly the correlation function as defined in equation 2 (with index \( v \) replaced by \( w \)). The results as obtained with interferometry by crosscorrelation
can thus be interpreted as the desired dipole impulse response $\tilde{X}_{\text{sc,inc}}(x^{(r)}_R,x^{(v)}_R,\omega)$ blurred in time and space with the point-spread function. Given this, the point-spread function can be used to study the radiation pattern of a virtual source. Computing the point-spread function is closely related to beamforming (Lacoss 1971). It should be noted, however, that the point-spread function characterizes the illumination at a single virtual source location, whereas with beamforming one analyzes the average illumination along the receiver array.

An important advantage of multidimensional deconvolution is that free-surface reflections are correctly handled. A deeper look in the fundamental representations of correlation-based methods shows that a homogeneous halfspace above the source array should be assumed to derive equations 1 and 2 (a direct consequence of the so-called far-field radiation approximation) (Wapenaar & Fokkema 2006). For this reason, free-surface related multiples are not taken care of and can cause spurious arrivals in the retrieved responses (Van der Neut et al. 2011a). The representations for multidimensional deconvolution do not require any assumption on the medium above the sources. As multidimensional deconvolution changes the boundary conditions, free-surface interactions are eliminated from the retrieved responses (Wapenaar et al. 2011). To remove all free-surface interactions, the incident field should contain all surface-related multiples, which is often hard to achieve by time-gating (but not by wavefield decomposition).

Multidimensional deconvolution should not be confused with (single-)trace deconvolution as proposed by Vasconcelos & Snieder (2008). In the latter method, a reflection response $\hat{X}_{TD}(x^{(r)}_R,x^{(v)}_R,\omega)$ is obtained by summing deconvolution pairs, according to

$$\hat{X}_{TD}(x^{(r)}_R,x^{(v)}_R,\omega) = \frac{\hat{P}_{\text{sc}}(x^{(r)}_R,x^{(s)}_S,\omega) \hat{P}_{\text{inc}}^*(x^{(v)}_R,x^{(s)}_S,\omega)}{\hat{P}_{\text{inc}}(x^{(v)}_R,x^{(s)}_S,\omega) \hat{P}_{\text{inc}}^*(x^{(v)}_R,x^{(s)}_S,\omega) + \epsilon^2}, \quad (6)$$

where a small constant $\epsilon^2$ is added for numerical stability and subscript $TD$ stands for trace deconvolution. Another approach is to neglect the off-diagonal contributions in the inversion process of multidimensional deconvolution. This is equivalent to choosing $v = w$ in equation 4, ignoring $\Delta x_R$ and removing the summation sign, yielding

$$\hat{X}_{DD}(x^{(r)}_R,x^{(v)}_R,\omega) = \frac{\hat{C}_{\text{sc,inc}}(x^{(r)}_R,x^{(v)}_R,\omega)}{\hat{C}_{\text{inc,inc}}(x^{(v)}_R,x^{(v)}_R,\omega) + \epsilon^2}. \quad (7)$$

Equation 7 is closely related to illumination compensation as sometimes applied in seismic migration (Nemeth et al. 1999), which balances the illumination strength along the receiver array. Since essentially the off-diagonal contributions in the point-spread function are neglected,
this method is referred to as diagonal deconvolution, indicated by the subscript \( DD \). Note that equation 6 describes a summation over deconvolved pairs, whereas equation 7 describes a deconvolution of summed correlated pairs, which is essentially different.

3 PRACTICAL ISSUES

In practice it is not always easy to isolate wavefields with time-gates prior to crosscorrelation. However, the correlation function of full fields (equation 1) can be interpreted as a sum of four constituents, reading in the time-space domain:

\[
C\left(\mathbf{x}^{(r)}_{R}, \mathbf{x}^{(v)}_{R}, t\right) = C_{\text{inc,inc}}\left(\mathbf{x}^{(r)}_{R}, \mathbf{x}^{(v)}_{R}, t\right) + C_{\text{sc,inc}}\left(\mathbf{x}^{(r)}_{R}, \mathbf{x}^{(v)}_{R}, t\right) + C_{\text{inc,sc}}\left(\mathbf{x}^{(r)}_{R}, \mathbf{x}^{(v)}_{R}, t\right) + C_{\text{sc,sc}}\left(\mathbf{x}^{(r)}_{R}, \mathbf{x}^{(v)}_{R}, t\right).
\]

Note that \( C_{\text{inc,inc}} \) is the point-spread function (equation 5), having its main contribution at \( t = 0 \) (and \( v = r \)). \( C_{\text{sc,inc}} \) and \( C_{\text{inc,sc}} \) host the scattered contributions (reflections) being located at positive and negative times, respectively. \( C_{\text{sc,sc}} \) is generally weak and mostly concentrated at \( t = 0 \) (for \( v = r \)).

The constituents of \( C \) around \( t = 0 \) are referred to as the virtual source function, which can loosely be interpreted as the source function of the data as retrieved with interferometry by crosscorrelation. The virtual source function is dominated by the point-spread function, but also contains contributions from \( C_{\text{sc,sc}} \). The virtual source function does not contain later arrivals from the point-spread function relating to multiples in the incident wavefield. It is assumed that the virtual source function can be isolated from the correlated data by placing a time-gate around \( t = 0 \). Theoretically, multidimensional deconvolution can be applied by deconvolving the causal contributions in \( C \) (which are dominated by \( C_{\text{sc,inc}} \)) by the virtual source function (approximating \( C_{\text{inc,inc}} \)). A similar procedure has been used before to apply multidimensional deconvolution on ambient noise correlations (Wapenaar et al. 2011). In practice, however, multidimensional deconvolution with time-gated fields can be hard, especially when events are interfering and conditions for wavefield separation are not optimal. For such reasons, an alternative strategy is proposed in the following.

Consider the following \( \tau - p \) transform (or slant stack) at fixed \( \mathbf{x}^{(v)}_{R} \):

\[
\mathcal{C}\left(p, \mathbf{x}^{(v)}_{R}, \tau\right) = \sum_{r} C\left(\mathbf{x}^{(r)}_{R}, \mathbf{x}^{(v)}_{R}, \tau + p (r - v) \Delta x_{R}\right) \Delta x_{R}.
\]

Here, \( p \) is the rayparameter and \( \tau \) is the intercept time. \( r \) and \( v \) are integers taking the
values of indices $r$ and $v$. In the Appendix, it is shown that if the overburden is homogeneous and the receiver array is illuminated with a maximum propagation angle of $\alpha_{\text{max}}$, the $\tau - p$ transform of the correlation function converges to

$$
\hat{C} \left( p, x_R^{(v)} , \tau \right) = \begin{cases} 
\hat{S}(\tau) & \text{for } p^2 \leq c_a^{-2} \\
0 & \text{for } p^2 > c_a^{-2} 
\end{cases}
$$

(10)

Here, $\hat{S}(\tau)$ is the auto-spectrum of the source wavelet and $c_a = c / \sin(\alpha_{\text{max}})$ is the apparent velocity, with $c$ being the propagation velocity. In Figure 2a the time-space domain representation of such ideal virtual source function is shown for a case with $c_a = 2000 \text{m/s}$. As expected from equation 10, the $\tau - p$ transform reveals an imprint of the source auto-spectrum $\hat{S}(\tau)$ for $-c_a^{-1} \leq p \leq c_a^{-1}$, see Figure 2b.

If the overburden is complex and/or sources are not uniformly distributed, illumination conditions can be less ideal and the virtual source function can be rayparameter-dependent within the illuminated band ($-c_a^{-1} \leq p \leq c_a^{-1}$). To compensate for this, a filter is introduced that balances the virtual source function at each rayparameter and virtual source location within this band, according to

$$
\hat{C}' \left( p, x_R^{(v)} , \tau \right) = \hat{f} \left( p, x_R^{(v)} , \tau \right) * \hat{C} \left( p, x_R^{(v)} , \tau \right),
$$

(11)

where $*$ denotes temporal convolution. Here $\hat{C}'$ is the updated correlation function and $\hat{f}$ is a matching filter, as commonly used in adaptive subtraction (Verschuur & Berkhout 1997; Abma et al. 2005). To find $\hat{f}$, the following minimization problem is solved in a local window around $\tau = 0$:

$$
\text{minimize } \| \hat{D}(\tau) - \hat{f} \left( p, x_R^{(v)} , \tau \right) * \hat{C} \left( p, x_R^{(v)} , \tau \right) \|_2,
$$

(12)

where subscript 2 denotes the $\ell_2$-norm and $\hat{D}(\tau)$ is the desired virtual source function. An obvious choice would be $\hat{D}(\tau) = \hat{S}(\tau)$ for $-c_a^{-1} \leq p \leq c_a^{-1}$, such that the observed virtual source function is forced to align with the ideal virtual source function (equation 10). An alternative route is to choose for $\hat{D}(\tau)$ a delta function $\delta(\tau)$, which is numerically computed as a vector with zeros and one non-zero entry at $\tau = 0$. In this way the data are also temporally deconvolved by the filter and knowledge of the average source auto-spectrum is not required. The proposed approach is referred to as illumination balancing, being closely related to the directional balancing method of Curtis & Halliday (2010). In both cases the observed correlated data $(C)$ is matched with an ideal virtual source response $(D)$. Curtis
& Halliday (2010) apply their method to full gathers in the frequency-wavenumber domain for surface-wave retrieval. Illumination balancing is applied in a local window around zero intercept time in the $\tau - p$ domain for interferometric redatuming. It has been assumed that the virtual source function is flat in the $\tau - p$ domain at each receiver location and $\tau = 0$. If the medium is strongly heterogeneous at the receiver level, this assumption will no longer be met. If knowledge of the local medium properties is available, this might be used for a more sophisticated estimate of $\tilde{D}(\tau)$.

One important practical issue that has not been addressed is dimensionality. Ideally, multidimensional deconvolution (or illumination balancing) requires the downhole wavefields to be sampled in 2 spatial directions in a 3D medium. However, the borehole samples these fields only in 1 spatial direction. To allow multidimensional deconvolution (or illumination balancing) for this configuration, a synthesized-2D solution is proposed. Directions $x_1$ and $x_2$, respectively parallel and perpendicular to the well, are distinguished. By inspection of the correlation gather in the $x_2$-direction, the interferometric Fresnel zones (Costagliola 2010) of the major reflections can be identified, as will be shown later. Based on this, a sub-array of sources is selected being broad in the $x_1$-direction and narrow in the $x_2$-direction, but large enough to contain all relevant stationary-phase points. Correlations of all sources in the 2D sub-array are stacked to generate the correlation functions (equation 1) to be used for multidimensional deconvolution or illumination balancing in the $x_1$-direction. As the source array is narrow in the $x_2$-direction, illumination variations outside the stationary-phase region, which are not accounted for by the synthesized-2D assumption, are avoided to some extent.

4 RESERVOIR MONITORING EXAMPLE

Various time-lapse applications of seismic interferometry and the virtual source method have been presented in the past (Sens-Schönfelder & Wegler 2006; Bakulin et al. 2007; Wapenaar et al. 2010b). Changes in surface statics and overburden velocities are naturally compensated by crosscorrelation or deconvolution (Schuster et al. 2004) and, as a consequence, virtual sources tend to be more repeatable than physical sources (Mehta et al. 2010b). Most of these studies have focused attention on the kinematics of the retrieved signals. It has been synthetically demonstrated, however, that relative amplitude-versus-rayparameter information can be retrieved with interferometry by multidimensional deconvolution (Van der Neut 2011). Since the scattered wavefield is deconvolved by the incident wavefield (including variations in the overburden), multidimensional deconvolution offers the potential to differentiate changes in the overburden from changes in the target reservoir. This is highly desirable in time-lapse
processing, since changes in the shallow subsurface tend to cumulate and often mask the responses of deeper target horizons (Spetzler & Kvam 2006).

In the following, interferometric illumination diagnosis and balancing will be applied for reservoir monitoring below a complex 3D overburden. Elastic wavefields are modeled synthetically using a 3D base survey model with free surface, based on an existing oilfield in Oman (Korneev et al. 2008). In Figure 3a, a 2D slice (in the \((x_2, x_3)\)-plane) of the shear-wave velocity model is shown. Horizontal vibrators are located on a 2D grid (having 141 x 39 source locations with 8m source spacing in both directions) at the earth’s surface. All sources are polarized in the \(x_2\)-direction. At 340m depth, a geophone array is deployed with 10m receiver spacing, measuring particle velocity in the \(x_2\)-direction. This array is placed in a horizontal well that is oriented in the \(x_1\)-direction (perpendicular to the slice of Figure 3a). With seismic interferometry, virtual source data will be generated as if both (virtual) sources and receivers are located in the well. Since the polarization of the vibrators is perpendicular to the receiver line in which virtual source data are retrieved, Sh-waves dominate the gathers, approximately obeying a scalar wave equation that can be decoupled from the system of P- and Sv-waves.

In Figure 3b, a slice of the medium below the receivers is shown in the \((x_1, x_3)\)-plane. After modeling the responses of the base survey, changes between 0% and 15% are introduced in the medium parameters (density, P- and S-wave velocity) in various parts of the reservoir, including six dashed rectangles (anomalies) that are indicated in Figure 3b. Medium parameters in the overburden remain unchanged. Monitor survey data are forward modeled in the evolved medium with the same source and receiver coordinates as used for modeling the base survey.

In Figures 4a and 4b, common receiver gathers are shown for the base survey and the time-lapse response (monitor survey minus base survey), with source locations varying in the \(x_2\)-direction (at \(x_1 = 0m\)). In Figures 5a and 5b common receiver gathers are shown with source locations varying in the \(x_1\)-direction (at \(x_2 = 0m\)). It should be noticed that the illuminating wavefield is strongly distorted by the complex overburden, especially in the \(x_1\)-direction (being the direction of the well). To find the stationary-phase region in the \(x_2\)-direction, auto-correlation gathers are created by auto-correlating the wavefields at the central receiver, see Figures 6a and 6b. Based on visual inspection, a subset of sources is selected, indicated by the dashed blue lines. This subset is assumed to cover the interferometric Fresnel zone of the dominant reflections (Costagliola 2010). Source tapers are applied in both \(x_1\)- and \(x_2\)-directions to mitigate finite source aperture effects (Mehta et al. 2008b).

Virtual sources are generated at all 41 receiver locations using crosscorrelation of full
wavefields (equation 1). In Figure 7a the retrieved response is shown at zero subsurface offset (meaning that \( r = v \) at each trace). The virtual source function is removed from the retrieved gather by muting all information at \( t < 0.08s \), as indicated by the dashed red line in the figure. In Figure 7b the time-lapse response is shown for all 41 virtual-source-receiver pairs at zero offset. The anomalies that are marked by dashed black rectangles in Figure 3b can clearly be observed in Figure 7b and are indicated by similar markers. Note that additional time-lapse responses can be found between 0.30s and 0.40s. These spurious events are probably caused by travel-time mismatch in the deeper section originating from the velocity changes in the shallower section (Spetzler & Kvam 2006).

5 ILLUMINATION DIAGNOSIS

As mentioned before, the retrieved virtual source data do not only contain the scattered field, but also the virtual source function around \( t = 0 \), mainly stemming from crosscorrelations of the incident field with itself. In the following, the virtual source function at a receiver location in the center of the array \( (v = 21) \) will be studied carefully. First, this is done before crosscorrelated traces are stacked over surface sources. In Figure 8a the contribution of one particular surface source to the correlation function (including the virtual source function) is shown. This figure is obtained by crosscorrelating the field observed at the virtual source location with the fields observed at all receivers for the chosen surface source. The following \( \tau - pq \) transform is defined as an extension to the \( \tau - p \) transform (equation 9):

\[
\tilde{C}(p, q, x_R^{(v)}, \tau) = \sum_r C(x_R^{(v)}, x_R^{(v)}, \tau + p((r - v) \Delta x_R) + q((r - v) \Delta x_R)^2) \Delta x_R.
\]  

(13)

Here \( q \) is referred to as the curvature. Taking the \( \tau - pq \) transform of the correlation function at \( v = 21 \) and \( \tau = 0 \), the virtual source function can be unraveled in \( p \) and \( q \), see Figure 8b. A dominant peak is found at \((p_{dom}, q_{dom}) \approx (-1.8 \cdot 10^{-4}s/m, -1.1 \cdot 10^{-6}s/m^2)\) where the subscript \( dom \) stands for dominant. To illustrate that the estimated dominant ray parameter and curvature provide a good parameterization of the event in Figure 8a, the function \( t = p_{dom}(r \Delta x_R) + q_{dom}(r \Delta x_R)^2 \) is drawn in this figure. Note that it matches the dominant event well.

Virtual source data are created by stacking crosscorrelated traces over the available surface sources (equation 1). Such stacking process will have an effect on the retrieved virtual source function. In Figure 9a the correlation function (including the virtual source function) is shown after the sources of one chosen \( x_2 \)-coordinate are stacked along the \( x_1 \)-direction.
In Figure 9b, the corresponding scan over $p$ and $q$ is shown. A dominant orientation can no longer be observed. Note that the illuminated zone is limited in rayparameters to a band of approximately $-4 \cdot 10^{-4} s/m \leq p \leq 4 \cdot 10^{-4} s/m$. Outside this band, the virtual source has not been illuminated well and, as a consequence, virtual source radiation vanishes. This behavior is consistent with equation 10. It can be concluded that the maximum propagation angle is approximately $\alpha_{\text{max}} = \arcsin \left( \frac{c - 1}{c_{\text{max}} c} \right) \approx 17.5^\circ$, with $c_{\text{max}} = p_{\text{max}} = 4 \cdot 10^{-4} s/m$ and $c \approx 750 m/s$ at the receiver level.

By estimating the dominant rayparameter $p_{\text{dom}}$ of each surface source, a map can be constructed of $p_{\text{dom}}$ as a function of the surface source location, see Figures 10a and 10b. With dashed colored lines the different rayparameters that are provided in Table 1 are indicated. The band of illumination ($-4 \cdot 10^{-4} s/m \leq p \leq 4 \cdot 10^{-4} s/m$) can be clearly observed. The constructed map provides useful information on which source contributes to the retrieval of which rayparameter. Note that the dominant rayparameter is estimated in the $x_1$-direction only. For this reason, it is sensitive for the $x_1$-coordinate of the source, but relatively insensitive for its $x_2$-coordinate.

Mateeva et al. (2007) demonstrated the advantage of steering a virtual source by selecting only those sources at the surface that contribute to the desired illumination. To achieve this, ray tracing was applied for the direct arrival in a velocity model. However, by creating a map of source-dependent illumination (Figures 10a and 10b), appropriate sources can be selected without a velocity model. This is demonstrated by repeating the experiment with a smaller source array. With the help of Figure 10b, only the sources that radiate in the range $2 \cdot 10^{-4} s/m \leq p \leq 4 \cdot 10^{-4} s/m$ are selected. These sources are indicated by the green zone in Figure 10b. The selected source array is referred to as selected $\partial S_{\text{selected}}$. In Figure 11a the virtual source function is shown in case sources are stacked over selected $\partial S_{\text{selected}}$ only. Note that a preferred direction can be observed, which is even more apparent after a scan over rayparameter and curvature, see Figure 11b. The imposed radiation limitation $2 \cdot 10^{-4} s/m \leq p \leq 4 \cdot 10^{-4} s/m$ can well be observed. The dominant rayparameter and curvature are picked and the function $t = p_{\text{dom}}(r \Delta x_R) + q_{\text{dom}}(r \Delta x_R)^2$ is drawn in Figure 11a. Note that this function follows the main trend in the virtual source function well.

In Figures 12a and 12b the retrieved time-lapse responses are shown using all sources and only sources in $\partial S_{\text{selected}}$, respectively. Rays are traced in the gathers with rayparameters as indicated in Table 1. Note that in Figure 12b only anomalies in the green zone (corresponding to the selected rayparameters $2 \cdot 10^{-4} s/m \leq p \leq 4 \cdot 10^{-4} s/m$) are detected.
This is a consequence of limiting the illumination from the surface and, consequently, limiting the virtual source radiation direction.

Through a survey with a sparse but wide source grid, the general dominant rayparameter might be mapped as a function of source location (as in Figure 10a). With the help of such an illumination map, denser source arrays can be designed to steer the virtual source in the direction of interest. Alternatively, in time-lapse experiments, a base survey can be conducted with a wide grid of sources and, depending on the target of interest, a smaller sub-grid can be selected for monitoring. When the imaging target is more complex, as for instance in salt-flank and sub-salt imaging (Willis et al. 2006; Mateeva et al. 2007; Vasconcelos et al. 2008), steered virtual-source radiation may also simplify the interpretation of the retrieved records considerably.

6 ILLUMINATION BALANCING

Mehta et al. (2010b) discuss two problems that can hamper reservoir monitoring with downhole receivers: 1) non-repeatable source coupling and 2) obstructions of the source array (i.e. missing sources). Concerning non-repeatable source coupling, a distinction can be made between the emitted phase and amplitude spectra. When creating virtual source data with equation 1 or 2, crosscorrelation naturally compensates for non-repeatability of the phase spectra. However, non-repeatability of the amplitude spectra and missing sources are not accounted for by correlation-based theory. Illumination balancing is one possible way to improve the retrieved signals, as will be demonstrated with an example.

In Figure 13a, a map is shown of the surface source array. The source coupling is assumed to be similar in the base and monitor survey, except for zone X that is indicated in grey. In this zone, high frequencies have been relatively more attenuated, as shown in Figure 13b. The sources have been scaled such that the frequency-integrated autospectrum \( \int \hat{S}(\omega)d\omega \) of the base survey is equal to that of the monitor survey at each source location in the survey. Apart from the source coupling issues in zone X, there is an obstruction in the source array (i.e. missing sources) of the monitor survey, indicated as zone Y in Figure 13a.

After crosscorrelation and stacking (equation 1), the retrieved data are resorted with respect to downhole common midpoint locations. At each common midpoint location, the correlation function is transformed to the \( \tau - p \) domain and evaluated at \( \tau = 0 \) to analyze the virtual source function. In the upper panel of Figure 14a the virtual source function of the base and monitor surveys are shown at common midpoint location \( x_2 = -50m \), being situated right above the center of anomaly B (see Figure 3b). Note that the virtual source
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function of the base and monitor survey at this common midpoint location is different in terms of frequency content, especially near \( p = 0 \text{s/m} \). This is caused by the non-repeatable sources in zone X (Figure 13b), crossing this common midpoint location at the surface (Figure 13a). Using equation 12, the virtual source functions of both surveys are matched to a desired function \( D(\tau) \) (in this case a delta function) at each common midpoint location and rayparameter. The virtual source functions after matching are shown in the lower panel of Figure 14a. Note that the mismatch in frequency content has largely been suppressed. Illumination balancing is implemented by applying the obtained matching filters to the reflected fields at each common midpoint location and rayparameter. Results of this operation are shown in Figure 14b.

Before analyzing the reflected signals in more detail, the change in the virtual source function’s peak amplitude is studied at each common midpoint location and rayparameter. This is done by subtracting virtual source function peak amplitudes of the monitor and base surveys and normalizing with the peak amplitude of the base survey. The resulting map is shown in Figure 15a. The imprint from zone Y can clearly be observed at high \( x_1 \) and high \( p \). Recall that in zone X the frequency-integrated auto-spectra \( \int \hat{S}(\omega) d\omega \) of the base and monitor survey are similar at each source location. Therefore the contributions from this zone to the virtual source function’s peak amplitude of the base and monitor survey are almost similar, such that no imprint can be observed in Figure 15a.

The relative change in reflected amplitudes from the upper anomalies (indicated by A, B and C in Figure 3b) is estimated. The reflections in the upper zone \( (0.10 \text{s} \leq \tau \leq 0.17 \text{s}) \) that contain these anomalies are time-gated in the common midpoint gathers. The time-gate is indicated in Figure 14b. The amplitude spectrum of each time-gated trace is summed over all frequencies to estimate the total reflected amplitude in that trace. The change in reflected amplitudes is computed by taking the difference of the monitor and base survey and normalizing with the base survey. In Figure 15b, the estimated change of the reflected amplitude is shown before illumination balancing was applied. A reference response is computed by generating amplitude-versus-rayparameter curves using Shuey’s approximation (Shuey 1985) at each common midpoint location, using contrasts as estimated from the velocity and density models; see Figure 16a. By comparing Figures 15b and 16a, it is observed that the retrieved amplitude change without illumination balancing is masked by 1) an imprint from zone X masking the target anomalies and 2) an imprint from zone Y giving the impression of a second anomaly at high \( x_1 \) and high \( p \). Note the correlation of the fake second anomaly caused by zone Y with the change of the virtual source function’s peak amplitude observed in Fig-
ure 15a. The change of reflectivity after illumination balancing is shown in Figure 16b. As intended, the imprints from zones X and Y have been attenuated by illumination balancing and the underlying anomalies are better exposed.

In theory, exact amplitude-versus-rayparameter curves can be retrieved with interferometry, which can be inverted for the amplitude-versus-rayparameter intercept and gradient, as demonstrated for a simple synthetic model by Van der Neut (2011). However, with few single-component downhole receivers in complex 3D media, the retrieval of true-amplitude signals is hard in practice. In Figure 17a, the amplitude-versus-rayparameter curve of anomaly B (see Figure 3b) is shown after averaging over common midpoint locations $-75m \leq x_1 \leq -25m$. Note that even after illumination balancing and averaging, the retrieved amplitude-versus-rayparameter response is still noisy. Although retrieving the amplitude-versus-rayparameter gradient might be hard, the amplitude-versus-rayparameter intercept can be estimated relatively well. To demonstrate this, the retrieved change of reflectivity is averaged over rayparameters (neglecting rayparameter dependence) at each common midpoint location. Results are shown in Figure 17b. Note that without illumination balancing the target anomaly is obstructed by the imprints from zone X, whereas the fake second anomaly caused by the missing sources in zone Y can be misleading for interpretation. Implementation of illumination balancing has resulted in improved localization and characterization of the anomaly, although the reflectivity change in anomaly A is slightly overestimated. By combining the observed changes in reflectivity with time shift information, for instance from virtual refraction data (Tatanova et al. 2011), an inversion for the change of medium parameters might be possible.

7 DISCUSSION

In the previous example, non-repeatability at the source side has been addressed. However, multidimensional deconvolution or illumination balancing might also be used to eliminate the effects of changes in the overburden. It is noticed that illumination balancing is closely related to the self-decon method of Mehta et al. (2008a), where the source spectra are deconvolved in the correlation gathers prior to source summation. Note that Mehta et al. (2008a) apply deconvolution at each source location, similar to equation 6, whereas illumination balancing is derived from equation 4 and involves deconvolution at each rayparameter after source summation. Illumination balancing can also be used to remove the imprints of missing sources, which is not possible with the self-decon method. An alternative way to deal with the missing sources is interpolation in the correlation gather, as suggested by Poliannikov & Willis (2011).

In this study a Ricker wavelet was used for the source function. In realistic scenarios, the
source function will generally be more complex and an initial deconvolution step might have to be applied prior to interferometry to be able to differentiate the virtual source function from the reflected signals (Poletto et al. 2010). A relatively tight source spacing of 8m has been used in the synthetic example to prevent aliasing of S-waves. For P-waves generated by vertical vibrators, larger source intervals can probably be tolerated. 3D effects remain challenging in the application of multidimensional deconvolution or illumination balancing. Under the current implementation, synthesized-2D data are created by stacking over a number of source lines in the x_2-direction (perpendicular to the receiver array). Constructive interference is assumed at the stationary points and destructive interference elsewhere in the gather. To further eliminate contributions outside the interferometric Fresnel zones, velocity filtering or SVD-based filtering (as proposed by Melo et al. (2010)) might be applied to the correlation gathers in the x_2-direction prior to source-line summation.

Illumination balancing is applied after τ − p transformation of the correlation function. Instead, interferometric redatuming itself can be applied in the τ − p domain (Tao 2011) and illumination balancing can follow. Similar concepts may also be applied in passive seismic interferometry. Draganov et al. (2009) showed how a reflection response of the earth’s subsurface can be retrieved by crosscorrelation of ambient seismic noise records. The success of this method strongly depends on the distribution of subsurface noise sources. Instead of crosscorrelating long records of seismic noise, virtual source data can also be retrieved by selecting events in passive records and crosscorrelating these separately. This is the approach taken in event-driven passive seismic interferometry (Draganov et al. 2010). Each passive source gives a contribution to the virtual source function. By studying these individual contributions, a virtual source can be manufactured that radiates uniformly within a certain range of rayparameters. A comparable route has been taken for lithospheric scale imaging by Ruigrok et al. (2010). Almagro Vidal et al. (2011) show that analysis of the virtual source function can also be used to distinguish surface waves from body waves in correlated panels of passive data. If passive source distributions are limited, virtual source radiation will be limited accordingly. Studying the virtual source function can reveal such limitations. Illumination balancing in the τ − p domain might be a robust way to remove the directivity imprint of passive sources from retrieved reflection data.

8 CONCLUSION

Traditionally, seismic interferometry has been applied by crosscorrelation. From a theoretical standpoint it is better to replace crosscorrelation by an inversion process, referred to as multi-
dimensional deconvolution. This process can be interpreted as optimizing the radiation pattern of a virtual source at a receiver location. Although the advantages of such an approach have been demonstrated before, implementation can be hard, since some type of wavefield separation (either by decomposition or time-gating) has to be applied prior to inversion. If wavefield separation is not an option, the virtual source radiation pattern can still be analyzed by studying the crosscorrelations of full wavefields at zero time lag. By illumination balancing, the directivity in the radiation pattern can be reduced. This is especially interesting for reservoir monitoring, since the process allows us to remove non-repeatability in the illuminating source fields from the desired time-lapse response. In theory, correct amplitude-versus-ray-parameter information can be retrieved by accurate interferometric redatuming. In practice, achieving this is hard in cases with limited 1D downhole receiver arrays below a complex 3D overburden.

APPENDIX A: THE CORRELATION FUNCTION IN A HOMOGENEOUS MEDIUM

Consider a horizontal source array of finite aperture at a distance $Z$ above a horizontal receiver array of infinite aperture. Source and receiver spacings are equal. A reference virtual source is generated in the center of the receiver array which is illuminated with a maximum angle $\alpha_{\text{max}}$. Since a homogeneous medium is shift-invariant, the correlation function at the reference virtual source can be expressed as

$$\hat{C}\left(x^{(r)}, \omega\right) = \sum_s \hat{P}\left(x^{(r)} - x^{(s)}, \omega\right) \hat{P}^*\left(-x^{(s)}, \omega\right) \Delta x_S.$$ (A1)

Here, $\hat{P}\left(x^{(r)}, \omega\right)$ is a shot record from a source that is located directly above the virtual source location. Equation A1 can be transformed to the frequency-wavenumber domain, for which the following discrete $f-k$ transform is introduced:

$$\tilde{C}(k, \omega) = \sum_r \hat{C}\left(x^{(r)}, \omega\right) \exp\left(jkx^{(r)}\right) \Delta x_R,$$ (A2)

where $j$ is the imaginary unit and $k$ is the horizontal wavenumber. Applying $f-k$ transformation to equation A1 yields the following (for $\Delta x_R = \Delta x_S$):

$$\tilde{C}(k, \omega) = \tilde{P}(k, \omega) \tilde{P}^*(k, \omega).$$ (A3)
When dipole sources are deployed (for instance vertically oriented vibrators at the free surface), the incident field can be expressed as (Wapenaar et al. 2011):

$$\tilde{P}(k, \omega) = \tilde{s}(\omega) \times \begin{cases} \exp \left(-j \text{sgn}(\omega) \sqrt{\omega^2c_a^{-2} - k^2Z} \right) & \text{for } k^2 \leq \omega^2c_a^{-2} \\ \exp \left(-\sqrt{k^2 - \omega^2c_a^{-2}}Z \right) & \text{for } k^2 > \omega^2c_a^{-2} \end{cases}$$  \hspace{1cm} (A4)

where $c_a = c / \sin \alpha_{max}$, $c$ is the propagation velocity and $\tilde{s}(\omega)$ is the spectrum of the source wavelet. Substitution of equation A4 into A3 yields

$$\tilde{C}(k, \omega) = \tilde{S}(\omega) \times \begin{cases} 1 & \text{for } k^2 \leq \omega^2c_a^{-2} \\ \exp \left(-2\sqrt{k^2 - \omega^2c_a^{-2}}Z \right) & \text{for } k^2 > \omega^2c_a^{-2} \end{cases}$$  \hspace{1cm} (A5)

Here $\tilde{S}(\omega) = |\tilde{s}(\omega)|^2$ is the source auto-spectrum. The $\tau - p$ transform as introduced in equation 9 can be expressed for the shift-invariant medium as

$$\hat{C}(p, \tau) = \sum_r C \left( x^{(r)}, \tau + px^{(r)} \right) \Delta x_R$$  \hspace{1cm} (A6)

By applying forward temporal Fourier transformation to equation A6 and using the shift property, it follows that

$$\int_{-\infty}^{+\infty} \hat{C}(p, \tau) \exp(j\omega \tau) d\tau = \sum_r \hat{C} \left( x^{(r)}, \omega \right) \exp \left( j\omega px^{(r)} \right) \Delta x_R.$$  \hspace{1cm} (A7)

By taking $k = \omega p$, the right-hand side of equations A7 and A2 are identical. It follows that the left-hand side of equation A7 is equal to $\hat{C}(k, \omega)$, which is given by equation A5 and consequently

$$\int_{-\infty}^{+\infty} \hat{C}(p, \tau) \exp(j\omega \tau) d\tau = \tilde{S}(\omega) \times \begin{cases} 1 & \text{for } p^2 \leq c_a^{-2} \\ \exp \left(-2\sqrt{p^2 - c_a^{-2}}Z \right) & \text{for } p^2 > c_a^{-2} \end{cases}$$  \hspace{1cm} (A8)

Applying inverse temporal Fourier transformation to equation A8 yields equation 10 in the main text. Here the evanescent field (at $p^2 > c_a^{-2}$) has been neglected.

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