Efficient Design of Controlled Offshore Systems
A Holistic Approach

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A Holistic Approach

MASTER OF SCIENCE THESIS

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Abstract

Designing a controlled offshore system is a challenging process, the system dynamics are often nonlinear and subjected to loads that are random in nature. Furthermore, the proper description of the system’s dynamical behaviour involves knowledge pertaining to several fields. Unfortunately, at the design level, the process is often segregated between the different fields which are using highly advanced technology from their own discipline on overly simplified models relating to the other disciplines.

This thesis focuses on creating a generic framework enabling appropriate modelling and analysis methods for controlled offshore systems considered as a whole. To create a virtual prototype of the system in a generic and modular way, the modelling is based on the formalism of multibody dynamics coupled to hydrodynamics. Generic methods for the linearisation of the system equations are introduced and implemented to enable analysis in both the time and frequency domains. The short term statistics are also provided in both domains, using spectral analysis in the frequency domain and time traces in the time domain. These different modules can be used to build the complete model, including control and study the system using classical techniques.

Lastly, an approach to efficiently compute the workability plots is proposed as they are an important performance indicator of offshore systems. The considerable increase in computational speed gained through this approach opens the door for new types of insights such as parameter optimisation and sensitivity analysis. This holistic view of the system should allow engineers to take the right decision during the early phase of the design process by allowing them to explore different integrated design solutions and understand the influence of design parameters on the system’s workability. The example of a motion compensated gangway is used throughout the thesis to illustrate how the developed framework and methods can be used in practice.
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Preface

My work on this thesis started in January 2017 at Royal IHC, when I came in contact with their research and development department (IHC MTI) in Delft. At first the topic of my research was focused on the analysis of a motion compensated gangway. This gradually shifted into creating a generic framework to model and analyse any controlled offshore systems. I am very proud to present you this thesis which, I hope, succeeded in contributing to better development of these systems in the future.

This would not have been possible without the support I received. I first of all would like to thank IHC MTI for giving me the opportunity to graduate in a very nice working environment. A special thanks to my supervisor from IHC MTI, Saumya Kadawathagedara, for helping me out at the critical moments and guiding me in all the right directions. I would like to thank my supervisor from the TU Delft, Jan-Willem van Wingerden, who despite having a very tight agenda always gave me accurate feedback. Furthermore, I wish to thank all the students at IHC MTI for the good times we had together.

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Chapter 1

Introduction

1-1 Problem Formulation

The offshore market is changing faster than ever before as complex technical challenges are emerging from new markets. On the other hand the growing competition is forcing contractors to decrease operational downtime. As a result offshore equipment is becoming increasingly complex and is bound to operate in harsher conditions. This trend is reflected by the growing demand for systems involving an increasing degree of control and automation. Companies like Royal IHC are compelled to develop innovative equipment designs that can tackle these technical challenges while keeping the downtime at a minimum. The research exposed in this thesis aims at supporting this process by developing a general and common framework in which controlled offshore systems can be modelled and analysed in a modular way. This proposition originates from the following observations:

- The maritime and offshore industry is relatively new to advanced controlled systems. The available modelling and analysis tools are still limited. They are in general specialised in a particular field and do not allow for proper representation of the system as a whole. As a result, mechanical structures and controllers are often designed separately which may hold potential optimal designs out of reach.

- Environmental conditions are among the most limiting factors of offshore operations. Equipment involved in these operations may have to cope with rough sea states composed of high waves and heavy wind loads. The so called workability provides an overview of the weather conditions at which the system can operate safely and effectively. It is usually computed at the end of the design process once most of the design choices are set. The computation time required to assess the system’s workability do not currently permit to provide insight on the influence of the design parameters. Thereby preventing opportunities to improve the workability and ultimately reducing the operational downtime.
1-2 Proposition

The block diagram of a prototypical controlled offshore system is depicted in Figure 1-1. The closed loop system consists of a controller acting on the system disturbed by exterior loading. The system’s states are estimated from sensor measurements through a kinematic chain.

![Figure 1-1: A prototypical closed loop configuration.](image)

In order to ensure that the developed framework will provide proper modelling and relevant analysis approaches, the following propositions are made.

**Proposition 1:** *Provide a proper method for modelling the integrated system.* This method should be able to model the closed loop system, as illustrated in Figure 1-1, in a generic and modular way.

**Proposition 2:** *Provide approaches allowing to evaluate the system using classic control analysis methods.* As depicted in Figure 1-1, the closed loop system is in general subjected to external disturbances and sensor noise. When designing a control system it is important to be able to evaluate the stability and robustness of the disturbed system. This analysis is often most effective in the frequency domain. Therefore the following modules should be developed:

- **Linearisation:** the integrated system is converted to a linearised state space model. The system can then be studied in the frequency domain.
- **Spectral Analysis:** the system’s response to stochastic disturbances can be studied efficiently by propagating their spectra to the output of the closed loop system.

**Proposition 3:** *Provide insight on the influence of design choices on the workability.* Computing the workability of a given system using the conventional methods is extremely time consuming. Generating these plots for various design parameters is currently not feasible. In order to reduce the computation times of such a plot, a different approach is proposed based on spectral analysis.
1-3 Methodology

Scope  Offshore systems come in a wide range of designs working in extremely diverse operational conditions. Methods and approaches developed in the sequel are restricted to the scope of equipment composed of articulated rigid bodies placed on board a vessel, performing seakeeping operations lasting for up to a few hours.

Implementation  The methods presented and developed in this thesis will be implemented in MATLAB as modules extending the existing multibody software package Dynamic Operations in Dredging and Offshore (DODO).

Validation and Verification  The verification of the implementation and the validity of the derived methods will be tested on variations of the four-bar linkage system represented below. Its properties will be discussed in more detail in Chapter 3.

Application  In order to illustrate how controlled offshore systems can be analysed using the developed framework, an application example is used throughout the thesis. The considered system consists of a motion compensated gangway operating in the North Sea.

Motion compensated gangways are systems used for the transfer of personnel and cargo between a vessel and an offshore platform. In rough sea conditions this task can be extremely risky. To allow for a safe and comfortable passage, the gangway is stabilised with respect to the platform using controlled actuators.
Gangway systems can be categorised in two main types based on two different strategies. The first strategy consist in stabilising the entire gangway by placing it on top of a motion compensated platform, as in Figure 1-3. The second strategy, focuses on stabilising the tip of the gangway, as the system presented on Figure 1-4. The design analysed in this thesis is of the second type. Its characteristics will be described in more detail in Chapter 2.
1-4 Thesis Outline

The models used to build a generic and modular model of the closed loop system are introduced in Chapter 2. Followed by Chapter 3, which introduces linearisation approaches in order to enable analysis in the frequency domain. The last chapter, Chapter 4, details the different approaches that can be adopted to analyse the system’s response before providing an example of their application to the motion compensated gangway. Finally, the conclusions and recommendations for future work are given in Chapter 5.
In this chapter, a design method is developed for controlled offshore systems. A connection is made with multibody dynamics, hydrodynamics and control in a generic way. An introduction to multibody dynamics is found in Section 2-1, followed by a method to couple hydrodynamics in Section 2-2. At the end of this chapter, in Section 2-3, a common procedure for designing a controller is described. At the end of each section the methods are applied to model a motion compensated gangway.

2-1 Multibody Dynamics

When modelling a dynamic system in three-dimensional space, one quickly runs into the problem of choosing reference frames and coordinate systems. There are however general methods that can be applied to model systems consisting of multiple linked bodies. One commonly used modelling framework is the one that is referred to in literature as multibody dynamics. The equations are written using a redundant coordinate set, meaning that all positions, velocities and accelerations of each individual body will be solved for. The bodies are linked to one each other using algebraic constraint equations.

In order to conveniently build up the multibody structure and apply forces, a suitable coordinate system has to be defined. Therefore, two types of coordinate systems are introduced: a body fixed- and a world-fixed frame. The inertia tensor, forces and constraints can then be applied on a specific body using its local coordinate system. This however, requires an extra step that translates these to the global world-fixed frame [1]. In the equations of motion all body coordinates are defined in this global coordinate system. This vector holds the translations and rotations of all bodies in a Cartesian coordinate system. The translations are expressed in the x, y and z coordinate of the Centre Of Gravity (COG) of the body. The rotations are described using Euler angles. Note that by using Euler rotations, the order of
rotation matters because it does not commute. Therefore all rotations have to be done in the same pre-defined order. First a rotation around the z-axis is made followed by a rotation around the rotated y-axis. Finally, a rotation around the twice rotated x-axis is done. This is shown in Figure 2-1.

Figure 2-1: The Euler rotations

The multibody equations consists of a combination between the Newton-Euler equations and the Lagrange multiplier method [2][3]. This results in a system of index-3 Differential-Algebraic Equations (DAEs), shown in eqs. (2-1) and (2-2). DAE systems are substantially more difficult to solve than regular ODE systems. Despite this fact, most multibody formulations are based on a Redundant Coordinate Set (RCS), as it is the only way to build a general framework in which any multibody system can be described.

\[
M(q)\ddot{q} + \Phi_q^T(q,t)\lambda = F(u,q,\dot{q},t) \tag{2-1}
\]
\[
\Phi(q,t) = 0 \tag{2-2}
\]

Here, \(q\) is the vector describing the body coordinates. These coordinates are used to calculate the inertia terms, \(M(q)\), which is a positive definite block diagonal matrix. Furthermore, the constraint forces are defined by \(\Phi_q^T(q,t)\lambda\). Where \(\Phi_q\) is the Jacobian of the constraint equations with respect to the coordinates \(q\) and describes the direction (in the generalised coordinate basis) in which the reaction forces act in order to keep the kinematic constraints satisfied. \(\lambda\) is the vector with the Lagrange multipliers and represent the magnitudes of the constraint forces in these direction. The external forces acting on the bodies, the Coriolis and centrifugal forces are collected in the force vector \(F\).

There are several ways to solve eqs. (2-1) and (2-2). One way of solving for the DAEs, is to introduce the second derivative of the constraint equations, which should also be equal to zero:

\[
\dot{\Phi} = \Phi_q\dot{q} + \Phi_t = 0 \tag{2-3}
\]
\[
\ddot{\Phi} = \Phi_q\ddot{q} + \Phi_{\dot{q}}\dot{q} + \Phi_{tt} = 0 \tag{2-4}
\]

Equations (2-1) and (2-4) form a system of Ordinary Differential Equations (ODEs), which are used to solve the original index-3 DAE. Since the constraints are not directly enforced through eq. (2-2), but through its second time derivative, the solution can drift away from the real solution. Several techniques can be used to prevent this numerical drift, see [4] and [5] for more detail on this topic. The equations are now transformed into the following form:

\[
\begin{bmatrix}
M(q) & \Phi_q^T(q,\dot{q}) \\
\Phi_q(q,\dot{q}) & 0
\end{bmatrix}
\begin{bmatrix}
\ddot{q} \\
\lambda
\end{bmatrix} =
\begin{bmatrix}
F(u,q,\dot{q},t) \\
c(q,\dot{q})
\end{bmatrix} \tag{2-5}
\]

where \(c = -\Phi_q\dot{q} - \Phi_t\) represents the so called convective acceleration terms.
2-1-1 Modelling of the Gangway

The motion compensated gangway has three degrees of freedom: slewing, luffing and telescoping. These are depicted in Figure 2-2. The slewing action, consists of a rotation around the vertical axis of pedestal, as shown by the red arrow. Luffing is a rotation around its horizontal axis, shown by the green arrow. Finally, the telescoping action represents the extension of the gangway, depicted as the blue arrow. These three actions are controlled in order to keep the tip stable with respect to the platform.

![Figure 2-2: The slewing action (red arrow), luffing action (green arrow) and telescoping action (blue arrow).](image)

The gangway is modelled using a multibody dynamic toolbox, which was already developed within Royal IHC. The toolbox, written in MATLAB, will be used as a basis for the modelling approach.

The gangway is composed of four bodies as can be seen in Figure 2-3. The Pedestal is the pillar that is fixed to the deck and elevates the gangway structure. In this model, the elevation is fixed. Attached to the Pedestal, is the Base. The Base can rotate around its vertical axis. This motion is referred to as the slewing action. The gangway is connected to the Base by a revolute joint, which allows Boom 1 to rotate around an axis which crosses through the revolute joint and is parallel to the local x-axis of the Base. This rotation represents the luffing action. The extendable body of the gangway, Boom 2, can slide along the local y-axis of Boom 1. This is the telescoping action. These three actions are controlled using hydraulic actuators to stabilise the tip of Boom 2 with respect to the platform.
The gangway model has been simplified in several ways. It is assumed that the gangway system can be described with only three rigid symmetric bodies each having a homogeneous mass distribution and no friction forces between them. For the purpose of this thesis, it is expected that these assumptions do not result to different conclusions.
2-2 Hydrodynamics

In this section a method to simulate accurate vessel dynamics is presented. The equations should be in a format that can be implemented in the multibody dynamics framework shown in eq. (2-5). The degrees of freedom of a vessel should be discussed first, before the equations of motion are explained. In literature, each motion is referred to as depicted in fig. 2-4. The three translations and three rotations are named surge, sway, heave, roll, pitch and yaw, respectively.

![Figure 2-4: All six degrees of freedom of the vessel.](image)

2-2-1 Equations of Motion

A commonly used method to model the vessel dynamics, is by the use of Cummins’ equation [6]. Cummins came up with the following time-domain model that describes the dynamics of a vessel with no or low forward speed subjected to an external force $F_v$:

$$(M + A)\ddot{q}_v(t) + \int_0^t B(t - \tau)\dot{q}_v(\tau)d\tau + Cq_v(t) = F_v(t)$$

(2-6)

where $q_v$ are the vessel coordinates expressed as three translations and rotations as shown in Figure 2-4. The matrices A, B and C contain the coefficients for the added mass, damping and restoration forces respectively. In order for the vessel to move, it has to displace the water particles. This additional inertia is referred to as the added mass. The friction that is involved in this interaction introduces dissipative forces. Cummins modelled these forces as a retardation function which describe certain fluid-memory effects an adds damping to the system. Furthermore, the gravitational and buoyancy forces are encompassed by the restoration terms, which can be seen as spring forces acting in the heave, roll and pitch direction. These forces keep the vessel floating and upright. To prevent the vessel from drifting in the other three remaining degrees of freedom, a dynamic...
positioning system can be added. This system tries to keep the vessel stationary by using its thrusters. In many offshore operations, keeping the vessel stationary is vital. For example, to be able to transfer cargo and personnel from a vessel to a platform at sea using a gangway, the vessel should keep the platform within its reach during the transfer. A dynamic positioning system can be fairly complex and comes in many forms. For the purpose of this thesis, it is modeled using virtual springs attached to the centre of gravity of the vessel and the desired reference coordinates in the surge, sway and yaw direction.

2-2-2 Integration with Multibody Dynamics

The equations of Cummins, eq. (2-6), can be easily implemented into the multibody framework as follows:

\[
\begin{bmatrix}
M_v(q_v) + A(q_v) & 0 & \phi_{q_v}^T(q, \dot{q}) \\
0 & M_s(q_s) & \phi_{q_s}^T(q, \dot{q}) \\
\phi_{qq}(q, \dot{q}) & \phi_{qq}(q, \dot{q}) & 0
\end{bmatrix}
\begin{bmatrix}
\ddot{q}_v \\
\ddot{q}_s \\
\lambda
\end{bmatrix}
= \begin{bmatrix}
F_v(t) - \int_0^t B(t - \tau) \ddot{q}_v d\tau - Cq_v \\
F_s(u, q_s, \dot{q}_s, t) \\
c(q, \dot{q})
\end{bmatrix}
\]  

(2-7)

Note that the vessel dynamics are coupled to the equipment on board through the constraint equations. The vessel dynamics are therefore influenced by the often heavy systems onboard.

2-2-3 Frequency Domain

The hydrodynamic coefficients in eq. (2-6), are obtained from either experimental data or through calculations. To obtain the coefficients through experiments, a scaled down version of the vessel is subjected to artificially created waves in a basin. The coefficients are then determined for a set of waves with different frequencies. This is however a costly and time consuming procedure. In practice the coefficients are often determined through potential theory of which the details are explained in [7].

It follows that Cummins’ equation has to be transformed to the frequency domain. This work was done by Ogilvie [8][9]. He related the coefficients in Cummins’ equation to the ones in the frequency domain:

\[-\omega^2[M + \tilde{A}(\omega)]\cos(\omega t) - \omega[B(\omega)\sin(\omega \tau)d\tau]\sin(\omega t) + C\cos(\omega t) = F(t) \]  

(2-8)

where the added mass and damping matrices, \( \tilde{A}(\omega) \) and \( \tilde{B}(\omega) \subset \mathbb{R}^{6 \times 6} \), are related to the time domain matrices as follows:

\[
\tilde{A}(\omega) = A - \frac{1}{\omega} \int_0^\infty B(\tau)\sin(\omega \tau) d\tau
\]

\[
\tilde{B}(\omega) = \int_0^\infty B(\tau)\cos(\omega \tau) d\tau
\]

(2-9)

The work of Ogilvie allowed for the formulation what is nowadays often referred to as the Response Amplitude Operator (RAO). The RAO, shown in eq. (2-11), describes the oscillatory motion of the vessel for a certain wave height \( \zeta_a \) and frequency \( \omega \), coming from a certain
relative direction $\theta$.

$$\text{RAO}(\omega, \theta) = \frac{a}{\zeta_a}(\omega, \theta) = \frac{F(\omega, \theta)}{C - (M + A(\omega))\omega^2 + iB(\omega)\omega}$$  \hspace{1cm} (2-11)

Here $F(\omega, \theta)$ are the wave loads per unit of wave height. Note that the RAO for a specific wave direction will give six transfer functions which relate the wave elevations to vessel excitation in all six degrees of freedom.

### 2-2-4 Modelling Irregular Waves

Depending on the location and the weather, the sea state changes constantly. To get an idea of how often a certain sea state occurs, scatter diagrams are constructed from measurements at sea, such as the Joint North Sea Wave Project (JONSWAP) [10]. From this data the so-called JONSWAP wave spectrum function has been formulated. This spectrum is a function of the significant wave height and the wave period. It is depicted for several sea-states in Figure 2-5.

![Wave spectrum (JONSWAP)](image)

**Figure 2-5:** The JONSWAP wave spectrum for different wave periods and significant wave heights.

From this graph it is clear that the power of the wave elevation is distributed over a range of frequencies. Therefore, it does not suffice to model the wave elevation signal with a regular wave, meaning a sine with a certain amplitude and frequency. Instead, a more accurate approximation can by achieved by creating irregular waves. This can be done by superimposing multiple regular waves with different frequencies and random phase shift, such that the spectrum of this generated signal matches that of JONSWAP, as shown in Figures 2-6 and 2-7.

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Figure 2-6: A visual representation on how a wave spectrum is formed and used to generate a wave elevation signal. Source: Journée [2015]

Figure 2-7: The generated wave elevation signal (top) and its spectrum compared with the analytical JONSWAP function (bottom).
2-3 Control

In order to control a multibody system, the actuators should be placed such that the controllability is optimised. Likewise, the sensors that are required for monitoring the necessary data for the control algorithm are to be placed such that the required level of observability is met. In turn, this data can be transformed into a useful input to the control algorithm. There are many ways to control a system. Finding the most suitable algorithm, that is capable of achieving the control objective, is up to the control engineer. In this section a controller is developed for the motion compensated gangway.

2-3-1 Controller Design for the Motion Compensated Gangway

There are several operational procedures involved during the transfer of personnel via a motion compensated gangway. First, the vessel is positioned near the platform, such that it is within reach of the gangway. At this point a dynamic positioning system is activated which tries to keep the vessel stationary. An operator can now manually steer the gangway towards the platform. Once the gangway tip is in front of the platform, the motion compensation can be activated. If the tip can be compensated within the safety criteria, the operator can attempt to make a secure connection with the platform.

The operational procedure that is studied in this thesis, is the moment that the gangway is motion compensated but not yet connected to the platform. This situation is shown in Figure 2-8.

Figure 2-8: The graphical model of the integrated motion compensated gangway in DODO.

The control objective for the motion compensated gangway is to stabilise the tip with respect to the world. In order to achieve this, the vessel motions have to be countered by the actuators.
With rougher weather conditions, this task becomes more difficult as the excitation of the vessel increase. To design such a controller, first the actuators and sensors are positioned. This is followed up by the kinematic equations that relate the sensor data to the input of the control algorithm. Finally, the open loop system is closed with a suitable controller.

**Actuator Placement**

To control the gangway system, hydraulic actuators are used. In this example, the hydraulics are not modelled, however it is assumed that this simplification will not influence the proof of concept of the proposed method.

Each actuator is responsible for either the slewing, luffing or telescoping action. The slewing actuator will be modelled as a controlled torque around the local z-axis of body 'Base' as shown in Figure 2-9. A reaction torque is applied on the Pedestal. The luffing motion is controlled by two hydraulic cylinders. The placement of these cylinders is not as obvious as the other actuators. Due to the large forces that are required for controlling this degree of freedom, the cylinders are placed such that the required torque can be reached without buckling the gangway. Therefore a bigger moment arm around the revolute joint, that connects the Base with Boom 1, is desired. In this example, the cylinders are attached at the far end of the Base and the most upper part of Boom 1, as shown in Figure 2-10. For the telescoping action, a hydraulic cylinder is placed between Boom 1 and Boom 2. This is depicted in Figure 2-11.

![Figure 2-9: The slewing torque acts around the local z-axis of body Base in the direction as shown by the rotating arrow in black.](image)
Figure 2-10: The luffing force is actuated by two hydraulic cylinders. They apply an action and reactions force on Boom 1 and the Base, respectively.

Figure 2-11: The telescoping force induces a sliding motion of Boom 2 along Boom 1.
Sensor Placement

Now that the hydraulic actuators have been placed, the equations have to be formed that compute the setpoints of the cylinder extensions. Since the goal of the system is to compensate the vessel motions, the setpoints are defined by the position and orientation of the vessel, which are measured by a Motion Reference Unit (MRU). This device, obtains its reference from fusing multiple sensors, such as gyroscopes and GPS. From this measurement, the actuator setpoints are computed using forward kinematics. These setpoints are then compared to the extension of the cylinders at that time. The resulting offset can be used as an input to the controller which in turn tries to minimise the error.

Kinematics

From the moment that the operator enables the motion compensation, the sensor outputs at that time are used to determine the position of the end-effector in global coordinates. This location is stored as a fixed tracking point and used to determine the actuator setpoints using kinematic relations.

For each body, the relative translation and rotation of the body fixed frames can be expressed using homogeneous transformation matrices [11]. However, not all system states are measured and can therefore not be used directly by the control algorithm. For the motion compensated gangway, the setpoints have to be determined using only the position and orientation of the vessel. This is done by first introducing several auxiliary frames. The multibody system state is expressed in a world-fixed coordinate Frame 0. All coordinates expressed in locally defined body frames are transformed to this frame in order to perform computations with them. The first body fixed frame is placed at the centre of gravity of the vessel and is labelled as Frame 1. In this frame, the position of the gangway on deck of the vessel is defined. At this position a new auxiliary frame is introduced. This frame is shown as Frame 2 in Figure 2-12. It is fixed to the vessel and its z-axis is always parallel to the z-axis of the Base. The y-axis of this Frame 2 is lined out with the gangway when the slewing angle is equal to zero. To determine the slewing angle setpoint, the fixed tracking point is defined in Frame 2. In the figure this vector is drawn in black. From this vector, the reference slewing angle can be determined by computing the angle between this vector and the y-axis of Frame 2. This angle is depicted in Figure 2-13, where a top view of the system is shown.

In turn, a similar approach is done for computing the setpoints of the luffing and telescoping action. For these setpoints a new frame is introduced. This frame is depicted as Frame 3 in Figure 2-12. It is placed on the position where the revolute joint would be, that connects the Base to Boom 1, when the slewing angle is at its setpoint. The axes of this frame are aligned with the local body frame of the Base at its reference attitude. It can therefore only be placed once the slewing angle setpoint is known. By defining the position of the reference point in this Frame 3, shown in magenta in Figure 2-12, the luffing angle can be determined. This is done by computing the angle this vector makes with the y-axis of Frame 3, as shown in Figure 2-14. The telescoping setpoint is obtained by taking the length of this vector.

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Figure 2-12: By expressing the tracking point as a vector in two auxiliary frames, the reference for the slewing, luffing and telescoping action can be determined.

Figure 2-13: By expressing the reference point in Frame 2, the slewing angle setpoint can be determined.
When computing the actuator setpoints, only the vessel states can be used. In order to have enough information, it is assumed that all body dimensions are known and that the kinematic relations are stationary, i.e. the system configuration does not change. Then by composing the homogeneous matrices, every point in frame $a$ can be expressed in another frame $b$, [11]. These transformation matrices have the following form:

$$H_{a}^{b} = \begin{bmatrix} R_{a}^{b} & T_{a}^{b} \\ 0 & 1 \end{bmatrix}$$

Where $R_{a}^{b}$ and $T_{a}^{b}$ denote the relative rotation matrix and translation vector, respectively.

This homogeneous transformation matrix can be used to express the reference point $p$ of the gangway tip, defined in Frame 0, in one of the auxiliary frames $n$ using the following transformation:

$$p_{n} = H_{n}^{0} \cdot r_{0}$$

The homogeneous transformation matrix is a useful tool in linked multibody systems due to its convenient properties. They can be multiplied to form new transformation matrices:

$$H_{0}^{2} = H_{0}^{1} H_{1}^{2}$$

Furthermore, the inverse of these transformation matrices can be computed fairly easy:

$$(H_{0}^{2})^{-1} = H_{2}^{0} = \begin{bmatrix} (R_{0}^{2})^{T} & -(R_{0}^{2})^{T}T_{0}^{2} \\ 0 & 1 \end{bmatrix}$$

Once the setpoints for the slewing angle, luffing angle and the telescoping distance have been computed, they are translated to the setpoints for the hydraulic cylinder extensions using trigonometric rules. These values are in turn compared to the actual extensions to obtain an error which the controller has to minimise.
The Controller

After the actuator setpoints have been calculated using the kinematic equations, it is the task of the controller to track them. Because the actuators are not modelled, the controller will directly apply forces on the bodies. This means that based on the gangway state and actuator setpoints, the controller is able to provide the desired forces.

The controller has two objectives. The first goal is to always compensate for the gravitational forces on the bodies. However, it has been shown in practice that it is more efficient to passively compensate for the gravity forces\(^1\). For this example, the assumption is made that the gravity forces are perfectly compensated by a passive system. Therefore the focus is on achieving the second controller objective, which is the stabilisation of the gangway end-effector, by compensating for the vessel motion.

The slewing, luffing and telescoping actuators are controlled with three parallel PD-controllers which will steer the actuators to their computed setpoints. The "textbook" version of a PD-controller is written as:

\[
C(s) = \frac{U(s)}{Y(s)} = K \left(1 + sT_d \right)
\]

(2-12)

where \(U(s)\) and \(Y(s)\) are the Laplace transforms of the controller output \(u(t)\) and the error defined by the difference of the reference and the process output \(y(t)\), respectively. \(K\) is the proportional gain and \(T_d\) the derivative time. However, a pure derivative is unrealistic as it results in very high gains for high frequency signals. This gain can be limited by adding a filter, which adds an additional pole to the compensator at high frequencies:

\[
sT_d \approx \frac{sT_d}{1 + sT_d/N}
\]

(2-13)

where \(N\) is the filter divisor which is typically in the range of 3 to 20. The more realistic controller can now be written as follows:

\[
C(s) = K \left(1 + \frac{T_d s}{1 + T_d s/N} \right)
\]

(2-14)

Before the controller can be implemented into the multibody system an additional step is required. Because eq. (2-5) is solved numerically, the controller eq. (2-14) should be discretised. The proportional term is untouched because this is a purely static part. The discretisation of the derivative term \(D\) on the other hand can be approximated as [12]:

\[
D(kh) = \frac{T_d}{T_d + Nh} D(kh - h) - \frac{K T_d N}{T_d + Nh} \left(y(kh) - y(kh - h)\right)
\]

(2-15)

where \(h\) is the time step for which the dynamics are solved numerically.

The proportional and derivative coefficients of these controllers are typically selected and tuned by the control engineer. This can be done iteratively or through analysis in the frequency domain. In Section 4-6-3, the control parameters are tuned based on the analysis in the frequency domain. This does require a linearised model of the multibody system which is developed in Chapter 3.

\(^1\)The official website of Seaqualize
The Generalised Plant

All the subsystems have now been modelled and are ready to be linked to each other. Figure 2-15 provides a schematic view of the closed loop system.

Here the multibody model is encapsulated in box $G$. $E_1$ and $E_2$ represent the kinematic equations which give the actual actuator states and their setpoint, respectively. Furthermore, the actuator states and the actuator references are obtained through kinematic equations which are represented by blocks $E_1$ and $E_2$. The multibody state vector $x$ is translated to the current actuator extensions through the kinematic relation described by block $E_1$. These actuator extensions are measured directly in practice. Therefore, the full body state $x$ can be used for their computation. For $E_2$ however, which computes the actuator references, only the vessel states can be used, as these are the only states that are measured using the MRU. The symbol $C$, are the kinematic equations that compute the distance between the tip of the gangway and its reference point using the system state vector. These four subsystem result in a Multiple Input Multiple Output (MIMO) generalised plant $P$. The inputs consist of the actuator sensor noise $n_1$, the MRU noise $n_2$ and disturbance forces $d$ induced by the waves. These noise and disturbance signals are stochastic by nature. Furthermore, the controlled forces $u$ are computed by $K \cdot v$.

In the next chapter, a generic approach is developed to linearise the closed loop system, such that it can be analysed in the frequency domain.
Chapter 3

Linearisation of the Multibody System

Numerically solving the multibody dynamic eqs. (2-1) and (2-2) is computational heavy. When studying or designing an offshore system, one is often interested in assessing indicators such as workability. This requires multiple time simulations over a long period of time. This process is time consuming and impractical. A solution to this problem is to convert the original index-3 Differential-Algebraic Equation (DAE) to a Linear Time-Invariant (LTI) system, which is much easier to analyse and solve. In Chapter 4, this LTI system is used for an analysis in the frequency domain.

In this chapter a general introduction on the linearisation is given in Section 3-1. Two methods are presented that can be used to linearise the multibody equations in Section 3-2. One of the methods is then applied to the motion compensated gangway example in Section 3-3.

3-1 Introduction to the Linearisation Problem

Linearisation of a continuous nonlinear function \( f(x, u) \) can be achieved by taking the gradient with respect to all variable around a certain system state and input. The linear function can then be written as a first order Taylor series expansions as shown in eq. (3-1).

\[
f(x, u) - f(\bar{x}, \bar{u}) \approx \frac{\partial f(x, u)}{\partial x} \bigg|_{\bar{x}, \bar{u}} \cdot (x - \bar{x}) + \frac{\partial f(x, u)}{\partial u} \bigg|_{\bar{x}, \bar{u}} \cdot (u - \bar{u}) \quad (3-1)
\]

Here, \( f(x, u) \) is a continuous nonlinear function of the system state \( x \) and input \( u \). The linearisation is done around a certain workpoint \((\bar{x}, \bar{u})\).

For high order systems, a state-space representation is often used as it can compactly describe the dynamics. The states are the system variables and are usually denoted by \( x \). These states are influenced by their dynamic relations and external inputs \( u \). The Taylor series expansion as shown in eq. (3-1) can be written in such a state-space representation as follows:

\[
\begin{align*}
\dot{x}_\Delta &= Ax_\Delta + Bu_\Delta \\
y_\Delta &= Cx_\Delta + Du_\Delta
\end{align*}
\]
Where, \( x_\Delta = x - \bar{x} \) and \( u_\Delta = u - \bar{u} \)

Here the matrices \( A, B, C \) and \( D \) are defined as:

\[
A = \left. \frac{\partial f(x,u)}{\partial x} \right|_{\bar{x},\bar{u}} \\
B = \left. \frac{\partial f(x,u)}{\partial u} \right|_{\bar{x},\bar{u}} \\
C = \left. \frac{\partial g(x,u)}{\partial x} \right|_{\bar{x},\bar{u}} \\
D = \left. \frac{\partial g(x,u)}{\partial u} \right|_{\bar{x},\bar{u}}
\]

The linearisation of \( f(x,u) \) is valid around \( f(\bar{x},\bar{u}) \). As the system states start to deviate from this point, the accuracy of the linear representation of the system decreases.

### 3-2 Linearisation of the Multibody Equations

For the multibody system, described by eqs. (2-1) and (2-2), the Taylor series expansion cannot be applied directly. This is due to the fact that they are written as index-3 DAEs. In order to linearise these equations, they have to be transformed. There are two ways to formulate the equations of motion for a multibody system. One can either choose a set of generalised coordinates that are compatible with the kinematic constraints, such that the constraints are implicitly satisfied. This approach is discussed in Section 3-2-1. The other option is to keep the original set of redundant coordinates. Linearisation from this coordinate set is discussed in Section 3-2-2.

#### 3-2-1 Minimal Coordinates Set

Starting from the Redundant Coordinate Set (RCS) formulation this approach reformulates the equations in terms of a Minimal Coordinate Set (MCS) \( z \), using a so called velocity transformation. This transformation recasts the DAEs into Ordinary Differential Equations (ODEs) that are then linearised.

The chore of the method lies in expressing the tangent terms of the transformed system as a function of the known matrices of the original system. Since the transformation from RCS to MCS is nonlinear, these expressions can be quite complex.

Once the linearised MCS system is formed, the transfer functions for the open and closed loop systems can be easily computed as shown at the end of this chapter.

**Velocity Transformation**

The equations of motion formed using a general multibody package are given in terms of a RCS \( q \) as was shown in eqs. (2-1) and (2-2) which are repeated here for convenience:

\[
\begin{align*}
M(q)\ddot{q} + \Phi^T_q(q,t)\lambda &= F(u,q,\dot{q},t) \\
\Phi(q,t) &= 0
\end{align*}
\] (3-2)
Deriving the kinematic constraint equations twice consecutively with respect to time, leads to the following relations on the velocities and accelerations.

\[ \Phi q \dot{q} + \Phi_t = 0 \]
\[ \Phi q \ddot{q} + \dot{\Phi} q \dot{q} + \dot{\Phi}_t = 0 \]  \hspace{1cm} (3-3)

The vectors \( b \) and \( c \) are introduced:

\[ b = -\Phi_t \]
\[ c = -\dot{\Phi} q \dot{q} - \dot{\Phi}_t \]  \hspace{1cm} (3-4)

When \( \Phi_q \) has full rank, i.e. the set of kinematic constraints are non redundant, eq. (3-4) can be used to define a transformation from the non compatible generalised velocities, accelerations and virtual displacements \( \dot{q}, \ddot{q}, \delta q^* \) to the set of generalised coordinates \( \dot{z}, \ddot{z}, \delta z^* \) which are compatible with the constraints [13][14][15]:

\[ \dot{q} = R \dot{z} + Sb \]
\[ \ddot{q} = R \ddot{z} + Sc \]  \hspace{1cm} (3-5)

where \( R \) is a matrix containing the vectors spanning the null space of the Jacobian of the constraints \( \Phi_q \) and matrix \( S \) is the right side inverse of \( \Phi_q \).

\[ \Phi_q R = 0 \]
\[ \Phi_q S = I_{Ncons} \]  \hspace{1cm} (3-6)

Using the transformation (3-5) and projecting the equations of motion onto the null space of \( \Phi_q \), the following reduced equations on the minimum set of coordinates \( z \) are obtained:

\[ R^T M R \ddot{z} = R^T F - R^T M Sc \]  \hspace{1cm} (3-7)

Also noted as:

\[ \overline{M} \ddot{z} = \overline{F} \]  \hspace{1cm} (3-8)

The matrix is now written as an ODE, where the transformed mass matrix \( \overline{M} \) and force vector \( \overline{F} \) have the following expressions:

\[ \overline{M} = R^T M R \]
\[ \overline{F} = R^T F - R^T M Sc \]  \hspace{1cm} (3-9)

**Linearisation**

The system of ODEs (3-8) can now be linearised around the equilibrium point \( z_0, \dot{z}_0, \ddot{z}_0 \) leading to:

\[ \overline{M} \delta \ddot{z} + \overline{C} \delta \dot{z} + \overline{K} \delta z = \overline{F}_l \delta u \]  \hspace{1cm} (3-10)
where the tangent terms are:

\[
\begin{align*}
\overline{M}_t & = \overline{M} = R^T M R \\
\overline{C}_t & = -\frac{\partial F}{\partial \dot{z}} \\
\overline{K}_t & = -\frac{\partial F}{\partial z} = -\left( \frac{\partial F}{\partial q} \frac{\partial q}{\partial z} + \frac{\partial F}{\partial \dot{q}} \frac{\partial \dot{q}}{\partial z} \right) \\
\overline{F}_t & = \frac{\partial F}{\partial \dot{u}} 
\end{align*}
\] (3-11)

### Computing the Tangent Terms

The expressions in (3-11) look deceptively simple. The transformation (3-8) is indeed non-linear as \( R \) and \( S \) are dependent on \( q \). This introduces terms such as \( R_q \) that need to be expressed in terms of the known matrices of the original system. Similarly, the partial derivatives of \( \dot{q} \) with respect to \( z \) and \( \dot{z} \) also need to be expressed in terms of known matrices to be able to exploit the derivation chain rule. The derivation of these expressions are found in [14]. The required matrices and vectors to construct the linear state-space are now given:

\[
\begin{align*}
\frac{\partial q}{\partial z} & = \frac{\partial \dot{q}}{\partial \dot{z}} = R \\
\frac{\partial \dot{q}}{\partial z} & = -S \dot{\Phi}_q R 
\end{align*}
\] (3-12)

which gives

\[
\begin{align*}
\overline{K}_t & = -\overline{F}_q R + \overline{F}_{\dot{q}} \dot{\Phi}_q R \\
\overline{C}_t & = -\overline{F}_q R \\
\overline{F}_q & = R_q^T (F - M S c) + R^T (F_q - M q S c - M (S c)_q) \\
\overline{F}_{\dot{q}} & = R^T (F_q - M S c_q)
\end{align*}
\] (3-14)

where

\[
\begin{align*}
R_q & = -R^T \Phi_{qq}^T S^T \\
(S c)_q & = S (\Phi_{qq}^T S c + c_q)
\end{align*}
\] (3-15)

The linearised equations can be written in a state-space representation as follows:

\[
\begin{bmatrix}
\delta \dot{z} \\
\delta \ddot{z}
\end{bmatrix} = 
\begin{bmatrix}
0 & I \\
-M_t^{-1} \overline{K}_t & -M_t^{-1} \overline{C}_t
\end{bmatrix}
\begin{bmatrix}
\delta z \\
\delta \dot{z}
\end{bmatrix} +
\begin{bmatrix}
0 \\
-M_t^{-1} \overline{F}_t
\end{bmatrix} \delta \dot{u}
\] (3-16)

The equations in the closed loop often require the redundant system states as an input. Therefore the minimal coordinates set has to be transformed back to the original redundant states using eq. (3-12):
\[
\begin{bmatrix}
\delta q \\
\delta \dot{q}
\end{bmatrix} = \begin{bmatrix}
R & 0 \\
-S\Phi_q R & R
\end{bmatrix}
\begin{bmatrix}
\delta z \\
\delta \dot{z}
\end{bmatrix}
\]

(3-17)

### 3-2-2 Redundant Coordinates Set

The dynamic equations can also be linearised while preserving the RCS. The set of DAEs is then linearised as follows:

\[
\begin{align*}
M_t \delta \ddot{q} + C_t \delta \dot{q} + K_t \delta q + \Phi^T \delta \lambda &= F_t \delta u \\
\Phi_q \delta q &= 0
\end{align*}
\]

(3-18)

where:

\[
\begin{align*}
M_t &= M \\
K_t &= \frac{\partial M}{\partial q} + \frac{\partial \phi_q^T \lambda}{\partial q} - \frac{\partial F}{\partial q} \\
C_t &= -\frac{\partial F}{\partial \dot{q}} \\
F_t &= \frac{\partial F}{\partial u}
\end{align*}
\]

(3-19)

The system (3-18) can be cast into a descriptor form:

\[
E \delta \dot{x} = A \delta x + B \delta u
\]

(3-20)

Here, \( x \) represents the state of the system: \( x = \begin{bmatrix} q & \dot{q} & \lambda \end{bmatrix}^T \)

The matrices \( E, A, B \) have the following form:

\[
\begin{align*}
E &= \begin{bmatrix} I & 0 & 0 \\
0 & M_t & 0 \\
0 & 0 & 0 \end{bmatrix} \\
A &= \begin{bmatrix} 0 & I & 0 \\
-K_t & -C_t & -\phi_q^T \\
-\phi_q & 0 & 0 \end{bmatrix} \\
B &= \begin{bmatrix} 0 \\
\frac{\partial F}{\partial u} \\ 0 \end{bmatrix}
\end{align*}
\]

(3-21)

Note that the matrix \( E \) is singular due to the algebraic equations enforcing the kinematic constraints in the last block line of the system. The DAE nature of the system is unaltered by the linearisation step and transformation to a descriptor form.

Retrieving the state space form from (3-20) is not straight forward and requires the use of special techniques. In [16], the authors present a few of these approaches. An approach based on the generalised Schur decomposition (or QZ-decomposition) was selected and is briefly outlined hereafter.

The method exploits the generalised Schur decomposition of the pencil matrix \((E, A)\), which factorises both matrices as \( E_{qz} = QEZ \) and \( A_{qz} = QAZ \). Where \( Q \) and \( Z \) are unitary matrices, and \( E_{qz} \) and \( A_{qz} \) are upper quasi-triangular real matrices.

Using this factorisation the systems can be written as:

\[
E_{qz} \delta \dot{x} = A_{qz} \delta x + B_{qz} \delta u
\]
where the transformed states $\delta \dot{x}$ and $B_{qz}$ are:

$$Z\delta \dot{x} = \delta x, \quad B_{qz} = QB \quad (3-22)$$

The factorisation is reordered such that a selected set of $r < n$ leading eigenvalues appear in the leading (upper left corner) diagonal blocks of the triangular pencil $(E_{qz}, A_{qz})$, which are associated to the dynamics of the system.

$$\begin{bmatrix} E_{qz11} & E_{qz12} \\ 0 & E_{qz22} \end{bmatrix} \begin{bmatrix} \delta \dot{x}_{qz1} \\ \delta \dot{x}_{qz2} \end{bmatrix} = \begin{bmatrix} A_{qz11} & A_{qz12} \\ 0 & A_{qz22} \end{bmatrix} \begin{bmatrix} \delta x_{qz1} \\ \delta x_{qz2} \end{bmatrix} + \begin{bmatrix} B_{qz1} \\ B_{qz2} \end{bmatrix} \delta u \quad (3-23)$$

The following change of variable is introduced to decouple the derivatives $\delta \dot{x}_{qz1}$ and $\delta \dot{x}_{qz2}$:

$$x_{qz1} = \tilde{x}_{qz1} + Tx_{qz2}, \quad T = -E_{qz11}^{-1}E_{qz12}. \quad (3-23)$$

Leading to the system:

$$\begin{bmatrix} E_{qz11} & 0 \\ 0 & E_{qz22} \end{bmatrix} \begin{bmatrix} \delta \dot{x}_{qz1} \\ \delta \dot{x}_{qz2} \end{bmatrix} = \begin{bmatrix} A_{qz11} & \tilde{A}_{qz12} \\ 0 & A_{qz22} \end{bmatrix} \begin{bmatrix} \delta x_{qz1} \\ \delta x_{qz2} \end{bmatrix} + \begin{bmatrix} B_{qz1} \\ B_{qz2} \end{bmatrix} \delta u \quad (3-24)$$

The algebraic part of the equations, can be solved in a staggered way exploiting the structure of $E_{qz22}$. When $E_{qz22}$ is null $\delta x_{qz2}$ can directly be substituted into the dynamic part. The dynamic part of the equations can then be cast in a state-space form by inverting $E_{qz11}$:

$$\begin{align*}
\delta \dot{x}_{qz1} &= \tilde{A}\delta \dot{x}_{qz1} + \tilde{B}u \\
z &= \tilde{C}\delta \dot{x}_{qz1} + \tilde{D}u
\end{align*} \quad (3-25, 3-26)$$

where,

$$\begin{align*}
\tilde{A} &= E_{qz11}^{-1}A_{qz11} \\
\tilde{B} &= E_{qz11}^{-1}(B_{qz1} - \tilde{A}_{qz12}A_{qz22}^{-1}B_{qz2}) \\
\tilde{C} &= C_{qz1} \\
\tilde{D} &= -\left(C_{qz2} + C_{qz1}T\right)A_{qz22}^{-1}B_{qz2}
\end{align*} \quad (3-27, 3-28, 3-29, 3-30)$$
To verify that the linearisation was properly implemented and to assess the applicability of the methods, both are tested on a multiple loop four-bar linkage depicted in Figure 3-1.

![Figure 3-1: A schematic representation of the N-loop four-bar linkage system.](image)

This system consist of \( N \) loops made up of identical rigid rods of length \( l = 1m \) with uniformly distributed mass \( m = 1kg \). Each loop has a spring-damper force connecting the point \( B_k \) to point \( A_{k-1} \). With a damping of \( c = 1Ns/m \) and a spring stiffness of \( k = 25N/m \) with natural length \( l_0 = \sqrt{2}m \). The only external forces acting on the system are the gravity forces with \( g = 9.81m/s^2 \).

The multiple loop four-bar linkage is representative of system with geometric nonlinearity arising from multibody dynamics. It has one degree of freedom parameterised by the angle \( \theta \). The exact eigenvalues can be derived analytically. They consist of a pair of conjugate complex eigenvalues \( \sigma \) which are summarised in Tables 3-1 and 3-2 for several values of \( N \), with and without damping. These eigenvalues are compared to the ones obtained from the linearised systems using the methods discussed in Sections 3-2-1 and 3-2-2.

**Table 3-1: Eigenvalues of the \( N \)-loop four-bar linkage without damping.**

<table>
<thead>
<tr>
<th>( N )</th>
<th>( \phi )</th>
<th>( \sigma )</th>
<th>( \sigma_{MCS} )</th>
<th>( \sigma_{RCS} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2.2343</td>
<td>( \pm 2.1477i )</td>
<td>( \pm 2.1476i )</td>
<td>( \pm 2.1476i )</td>
</tr>
<tr>
<td>5</td>
<td>1.8922</td>
<td>( \pm 1.5455i )</td>
<td>( \pm 1.5455i )</td>
<td>( \pm 1.5455i )</td>
</tr>
<tr>
<td>10</td>
<td>1.8454</td>
<td>( \pm 1.4352i )</td>
<td>( \pm 1.4351i )</td>
<td>( \pm 1.4351i )</td>
</tr>
</tbody>
</table>

**Table 3-2: Eigenvalues of the damped \( N \)-loop four-bar linkage.**

<table>
<thead>
<tr>
<th>( N )</th>
<th>( \sigma )</th>
<th>( \sigma_{MCS} )</th>
<th>( \sigma_{RCS} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>(-0.2424 \pm 2.1340i)</td>
<td>(-0.2424 \pm 2.1339i)</td>
<td>(-0.2424 \pm 2.1339i)</td>
</tr>
<tr>
<td>5</td>
<td>(-0.2350 \pm 1.5276i)</td>
<td>(-0.2350 \pm 1.5275i)</td>
<td>(-0.2350 \pm 1.5275i)</td>
</tr>
<tr>
<td>10</td>
<td>(-0.2325 \pm 1.4162i)</td>
<td>(-0.2325 \pm 1.4162i)</td>
<td>(-0.2325 \pm 1.4162i)</td>
</tr>
</tbody>
</table>

As can be seen from Tables 3-1 and 3-2, both linearisation methods preserve the eigenvalues of the system. The matrix \( A \), formed using these two approaches, is therefore able to appro-
appropriately represent dynamic behaviour of these types of systems.

The four-bar linkage system is modelled in DODO to be able to simulate the system response. The implemented linearisation method is now further validated through a spectral analysis. For this, a single four-bar linkage is perturbed by a disturbance force which is generated from a JONSWAP spectrum. This force is applied in the direction of the y-axis on the top bar. An estimate of the transfer function that relates this input to the angle deviation $\delta \theta$ is obtained through a spectral analysis. The estimate is then compared to the transfer function obtained from the linearised system. The result is shown in Figure 3-2. As is seen, the transfer function is very similar to the spectral analysis of the nonlinear time simulation.

![Bode Diagram](image)

**Figure 3-2:** The closed loop transfer function compared to a spectral analysis of the output signal from the disturbed system.
3-3 Linearisation of the Integrated Motion Compensated Gangway

Now that two methods are found that can effectively linearise the multibody equations, they can be applied on the motion compensated gangway system. In this case, the velocity transformation is used. In Figure 3-3, the closed loop of the gangway example is shown again. The blocks $G$, $E_1$, $E_2$ and $C$ need to be linearised in order to form the transfer functions from each input to each output. In Section 3-3-1 blocks $C$, $E_1$ and $E_2$ are linearised which represent the kinematic equations. In Section 3-3-2 block $G$ is linearised, which represents the multibody dynamic equations.

![Figure 3-3: The schematic representation of the linearised closed loop system.](image)

3-3-1 Linearisation of the Kinematics

The kinematic equations translate the body states $x$ to the tip position error, the actuator states and the actuator references. To determine the end-effector position, the states of body $Boom$ 2 are used. The local position of the end-effector, expressed in the body-fixed frame of $Boom$ 2, is a static vector. This vector has to be transformed to global coordinates. In Figure 3-3, block $C$ represents this kinematic relation. In turn, the difference between the end-effector reference point and its actual position results in the performance signal $z$.

The kinematic equations in block $C$, $E_1$ and $E_2$ are all functions of the body states $x$. For the linearisation of these equations, the partial derivatives to the body states are taken:

$$
\begin{align*}
    z &= C(x) \\
    s &= E_1(x) \\
    r &= E_2(x) \\
    \delta z &= \frac{\partial C}{\partial x} \bigg|_x \delta x \\
    \delta s &= \frac{\partial E_1}{\partial x} \bigg|_x \delta x \\
    \delta r &= \frac{\partial E_2}{\partial x} \bigg|_x \delta x
\end{align*}
$$
3-3-2 Linearisation of the Multibody Gangway System

For the linearisation of closed loop gangway system, first a workpoint, $\vec{x}$ and $\vec{u}$, has to be chosen. The system state $\vec{x}$ should be a member of the reachable set, which spans the workspace of the gangway end-effector as shown in Figure 3-4.

The reference point is always placed at the end-effector, when the system is in its initial state. This way, after the linearisation, any tip position variation is equal to the tip deviation from the reference point. Furthermore, the vessel is in equilibrium. This means that the position and orientation of the vessel is the same as it would be in a sea with no wave or drift forces. It is important that in this state, all the constraint equations are satisfied.

Once the system states are defined, the corresponding input signal can be calculated, which brings the system into an equilibrium. This is the case when the total force for each individual body is equal to zero. Because there are no gravitational forces on the gangway bodies, these do not have to be compensated by the actuators to create an equilibrium. The slewing, luffing and telescoping force are thus equal to zero in the workpoint as long as the end-effector is directly positioned in the reference position. Furthermore, the hydrodynamic forces are also equal to zero as the vessel state is in equilibrium. Once a workpoint is defined, the linearisation of the multibody equations can be done using the equations derived in Section 3-2-1.

As is shown in Figure 3-3, there are three external input channels. The actuator sensor noise $n_1$, the Motion Reference Unit (MRU) noise $n_2$ and the wave disturbance $d$ are all stochastic signals. The output $z$ is a vector in $\mathbb{R}^3$ containing the tip deviation from the reference point.
in x, y and z direction. The vector \( v \) are the extension errors of the hydraulic actuators. These are used as the input to the controller \( K \). The resulting generalised plant is described as follows:

\[
\begin{bmatrix}
\delta z \\
\delta v
\end{bmatrix} =
\begin{bmatrix}
0 & 0 & CG_2 & CG_1 \\
I & -E_2 & (E_1 - E_2)G_2 & (E_1 - E_2)G_1
\end{bmatrix}
\begin{bmatrix}
\delta n_1 \\
\delta n_2 \\
\delta d \\
\delta u
\end{bmatrix}
\] (3-31)

The linearised multibody system \( G \) has been split into \( G_1 \) and \( G_2 \), which represent the transfer functions from \( \delta u \) and \( \delta d \) to \( \delta x \) respectively. To see how the stochastic disturbances influence the output \( z \), the transfer function matrices are created of the closed loop system:

\[
\begin{align*}
\delta d \rightarrow \delta z : & \quad C \left( \left( I - G_1 K (E_2 - E_1) \right)^{-1} G_2 \right) \\
\delta n_1 \rightarrow \delta z : & \quad C \left( \left( I - G_1 K (E_2 - E_1) \right)^{-1} G_1 K \right) \\
\delta n_2 \rightarrow \delta z : & \quad C \left( \left( I - G_1 K (E_2 - E_1) \right)^{-1} G_1 KE_2 \right)
\end{align*}
\]
Chapter 4

System Analysis

The previous chapters introduced the formalisms used to model controlled systems in an offshore environment (Chapter 2) and approaches enabling the study of their nonlinear behaviour by linear approximation (Chapter 3). This chapter focuses on the analysis of the system’s responses which are generally performed either in the time domain or the frequency domain. Both methods are first discussed before discussing a new approach aiming at providing the required speed and accuracy needed to perform control tuning and parameter sensitivity analysis on the system’s workability.

4-1 Workability

Vessel and equipment are constantly subjected to unpredictable environmental loading that vary greatly in characteristics. When designing control systems for offshore equipment, it is important to evaluate their performance and robustness in these environments.

It is therefore interesting to analyse the performance of such systems using workability plots, they indeed provide an overview of the system’s performance over a range of sea states for a given criteria. Since ocean waves are stochastic processes in nature, the workability criteria is expressed in terms of the probability of matching a given performance goal. The range of sea states are characterised by the corresponding range of spectrum parameters. For instance, to describe sea states occurring in the North Sea during the months of June to August using the JONSWAP spectrum, the grid is defined as: $H_s \in [0, 8]m$, $T_p \in [3, 10]s$ and $\Theta \in [0, 360]deg$.

The workability is in general evaluated for a discrete number of sea states computed at the nodes of a workability grid and represented as scatter plots for a given incidence angle $\Theta$ or polar plots for a given peak period $T_p$.

Figure 4-1 provides an example of a polar plot representing the workability of the motion compensated gangway at a given wave period $T_p$. The wave height $H_s$ is represented on the radial axis and wave incidence angle $\Theta$ on the angular axis. The bold black line describes
a workability limit enclosing the workable sea states, the dashed lines indicate the accuracy boundaries for this estimation.

![Diagram](image)

**Figure 4-1:** The orientation of the motion compensated gangway and vessel with respect to the coordinate axis are specified on the left figure. On the right, the workability plot of this system for $T_p = 8s$ is depicted. The dotted lines mark the uncertainty boundaries, note that they increase with wave height.

Generating the workability plots discussed above requires the computation of the system’s response for each node of the grid followed by the evaluation of the probability of reaching the performance goal.

In the next section, the properties of the input and output processes will be discussed as they justify the validity of the approaches used in time and frequency domains.

### 4-2 Short Term Statistics

**Some notations and definitions**

Let’s introduce a stochastic process representing some irregular ocean waves, denoted by $U$. The random event $U_i$ is associated to the signal $u_i(t)$ representing the wave elevation. The set of all the available functions $u_i(t)$ is called the ensemble of the random process and will be denoted $\Omega$. Two kinds of probabilities can be calculated on $U$, those on the ensemble set and those over time.

The expected value of an event $U_i$ is for instance an ensemble statistic. Note that ensemble statistics are in general dependent on time:

$$E[U(t)] = \int_{\Omega} U(\omega, t) p(\omega) d\omega$$  \hspace{1cm} (4-1)
The average (or mean) of a given realisation $u_i(t)$ is a time statistic. Note that time statistics are in general dependent on the event:

$$m(u_i) = \lim_{T \to \infty} \frac{1}{T} \int_0^T u_i(t) dt$$

(4-2)

**Input Process**

Ocean waves are stochastic processes generated by wind. These processes retain their features over a period of up to a few days, they are therefore generally considered as stationary over the course of an operation. As mentioned in Chapter 2, irregular waves can be seen as a superposition of wave components with different frequencies and amplitudes. It is not difficult to verify that such models generate a Gaussian, ergodic and stationary process.

**Output Process**

When the system can be appropriately represented by a LTI system, the short term statistics of the output process are relatively easy to obtain. As the input process is stationary, ergodic and Gaussian, the output process of the LTI will share these same statistical properties. This is illustrated in Figure 4-2 for the motion compensated gangway. The histograms on the left display the distribution of components $\Delta x$, $\Delta y$ and $\Delta z$ of the tip variation, computed using time simulations and fitted with a normal distribution (in red).

**Figure 4-2**: Statistics of the time response of a simulated motion compensated gangway. On the left, a normal distribution has been fitted on the data. On the right, the evolution of the standard deviation with simulation time is shown.
These properties come in handy when computing the short term statistics of the output process, since multivariate Gaussian processes are fully characterised by the mean and standard deviation of their components.

By exploiting the ergodic property of the process, any statistics on the ensemble can be estimated by computing the statistics over time instead. For instance, the mean or variance of the output process can be computed by the variance of the time data:

\[
E[Y] = m(y_i) = \lim_{T \to \infty} \frac{1}{T} \int_0^T y_i(t) dt
\]

(4-3)

\[
E[Y^2] = V(y_i) = \lim_{T \to \infty} \frac{1}{T} \int_0^T y_i^2(t) dt
\]

(4-4)

Furthermore, the output processes are zero mean valued, as they represent the output of the linearised equations of the system which oscillates around a static equilibrium.

These important properties will be exploited to retrieve the statistics of the system’s response on the course of an operation. The following section shortly describes and discusses the approaches in the time and frequency domain.

Remark: the properties of the input process are in general not preserved and transmitted to the output process when the system is not LTI.

### 4-3 Time Domain Analysis

#### 4-3-1 Linear or Nearly Linear Systems

As pointed out in the previous section, for LTI systems, the output process of the controlled offshore system is:

- Gaussian, the probability distributions are thus fully characterised by the mean and variance of the process.

- Ergodic, meaning that the ensemble mean and variance can be computed using the mean and variance of a single realisation over time.

Hence, the process to evaluate the workability for each sea state in the time domain consists of the following steps.

1. Generate a wave-induced force on the vessel by using a wave spectrum \( S(\omega) \) to describe the wave elevation components \( A_k \). By using the RAO of the considered vessel, this wave elevation is translated to a load acting in all six degrees of freedom.

2. Compute the time response of the system to the wave loads by numerical integration of the coupled equations eqs. (2-1) and (2-2).
3. Estimate the standard deviation of the output process on the ensemble $\Omega$ by computing the standard deviation of the output on the generated time data. Which defines the probability density function.

4. Form the probability distributions functions and evaluate the workability.

For slow dynamics, such as for the vessel, the numerical time integration of step 2 can be performed with relatively large time steps without running into stability problems. However, the equipment installed on board have faster dynamics that require smaller time steps. As discussed in the previous section, evaluating time statistics require a simulation window that spans a long period of time to capture the characteristics of the process. This is illustrated in Figure 4-2, right column, which depicts the evolution of the standard deviation as a function of the length of the simulated time window $T_{\text{sim}}$. Observe that the curve stabilises after a few thousand seconds. These lengthy simulations with small time steps, result in large computation time. The simulation for a single workability point can easily take several hours to compute, meaning that it can take weeks to obtain the workability plots of a given system. The computation of the motion compensated gangway response over a period of 6000 seconds for instance requires about 5 to 6 hours, for a single workability point. The complete workability for a grid resolution of $1s \times 0.5m \times 30^{\circ}$ would take about 3 months to compute. In practice, a much rougher resolution is taken to be able to compute the workability plots within about 3 to 4 weeks.

4-3-2 Nonlinear Systems

In the nonlinear case, without prior knowledge on the properties of the output process, a proper treatment of these cases would involve a study of the distribution of $Y$. This can be done by using Monte Carlo simulations or by solving the Fokker Plank equations if the process can be considered as Markovian, see [17, 18] for more details on these approaches. Although the procedure described for studying linear systems in the time domain are time consuming, their implementation is more straightforward and the computation time considerably faster than the available procedures for the nonlinear case. As a result, the output process is in practice often treated as if it was stationary and ergodic. The statistics of the process are simply evaluated on a single realisation and the distributions assumed to be Gaussian or Rayleigh without much justifications.

4-4 Frequency Domain Analysis

Stochastic processes are usually characterised by their probability distribution functions. If well behaved, they also admit a spectral representation. The spectral density representation is particularly useful when considering control systems. Given a wide sense stationary process $Y$, with zero mean and autocorrelation $R_y(\tau)$, the spectrum $S_{yy}(\omega)$ of $Y$, also called Power
Spectral Density (PSD), is the Fourier transform of the autocorrelation:

\[
S_{yy}(\omega) = \int_{-\infty}^{\infty} R_y(\tau) e^{j\omega \tau} d\tau
\]

\[
R_y(\tau) = \frac{1}{2\pi} \int_{-\infty}^{\infty} S_{yy}(\omega) e^{j\omega \tau} d\omega
\]

Since the autocorrelation is a real and even function, the spectrum is a real and even function of frequency \( \omega \).

The spectra of the input and output process are related through the Wiener-Khinchin relation as:

\[
S_{yy}(\omega) = H(\omega)S_{uu}(\omega)H^*(\omega)
\]  \hspace{1cm} (4-5)

where \( H(\omega) \) is the transfer function of the system [19]. Since \( \sigma_y^2 = R_y(0) \), the variance of the output process is represented by the area under power spectrum curve:

\[
\sigma_y^2 = R_y(0) = \int_{0}^{\infty} S_{yy}(\omega) d\omega
\]  \hspace{1cm} (4-6)

The probability on the performance criteria can now be computed by forming the probability density functions of the output process using the estimated standard deviations and deducing the probability from it, as exposed in Appendix A.

If the output process \( Y \) represents an error, the spectral analysis can also be used to perform Dynamic Error Budgeting (DEB). By observing the output spectrum \( S_{yy}(\omega) \) the frequencies at which power was added to the error become apparent. This technique is discussed in more detail in Appendix B.

As the spectrum \( S_{yy} \) is often graphed on a logarithmic scale, the comparison between areas underneath broad peaks at lower frequencies and narrow peaks at higher frequencies becomes difficult. The so called Cumulative Power Spectrum (CPS) is commonly represented instead. It is computed by integrating the power spectrum \( S_{yy}(\omega) \) as follows:

\[
CPS_y(\omega) = \int_{0}^{\omega} S_{yy}(\alpha) d\alpha
\]  \hspace{1cm} (4-7)

The CPS thus results in a monotonically increasing function, ranging from \( CPS_y(0) = 0 \) to \( CPS_y(\infty) = \sigma_y^2 \). If instead of the variance, the standard deviation \( \sigma_y \) is the parameter of interest, the so called Cumulative Amplitude Spectrum (CAS) can be computed. It is simply the root of the CPS function:

\[
CAS_y(\omega) = \sqrt{CPS_y(\omega)}
\]  \hspace{1cm} (4-8)

The frequency domain approach is very convenient and extremely efficient. The computation of the complete workability can be obtain within minutes for the grid resolution of \( 1s \times 0.5m \times 30^\circ \) instead of several weeks using the time domain approach. It is however limited to linear or weakly nonlinear systems. This will be briefly highlighted and discussed at the end of the following section.
4-4 Frequency Domain Analysis

4-4-1 Validation of the Frequency Analysis for Controlled Multibody Systems

In order to validate the implementation of the spectral analysis module, several tests were performed. In this section, the tests performed on the four-bar linkage example are presented. This system was previously used to validate the linearisation methods in Chapter 3. This time only a single loop is considered. Additionally, a disturbance $F_d$ and control force $F_c$ are applied on the system as depicted in Figure 4-3.

![Figure 4-3: The controlled four-bar linkage system under external disturbance.](image)

The stochastic disturbance force $F_d$ is applied to the top bar along the y-axis. It is a stationary and ergodic Gaussian process generated with the JONSWAP spectrum, using the same decomposition as described in Section 2-2-4 for the wave elevation of irregular waves. The force $F_c$, applied on the top bar along the x-axis is governed by a Proportional Integral Derivative (PID) controller, to control the angle $\theta$ and track the reference value $\theta_{ref}$.

The complete system is modelled in DODO and linearised around the reference state using the implemented linearisation module introduced in Section 3-3-2. The closed loop transfer function relating the disturbance force $F_d$ to the angle tracking error $\Delta \theta$ is then formed. In turn, the spectrum $S_Y(\omega)$ of the system response is computed with the spectral analysis module, using the Wiener-Khinchin relation eq. (4-5). The input spectrum, closed-loop transfer function and output spectrum for this example are shown in Figure 4-4, from left to right respectively.

![Figure 4-4: From left to right, the power spectrum of the disturbance $S_{uu}(\omega)$, the amplitude of the controlled system transfer function $|H(\omega)|$ and the output spectrum $S_{yy}(\omega)$.](image)
The estimation of the standard deviation using spectral analysis is compared to the estimation obtained from time simulations. The results are depicted in Figure 4-5. The standard deviation represents the area under the power spectrum curve, but can be read more conveniently as the end value of the CAS. As can be observed from Figure 4-5, the results from the time domain and frequency domain are in good agreement: the end-value of this spectrum matches the standard deviation of the time response shown on the right.

![Figure 4-5: From left to right, the Cumulative Power Spectrum (CPS), the Cumulative Amplitude Spectrum (CAS), the standard deviation obtained using the time domain approach](image)

**Limitations**

In combination with the linearisation module, the spectral analysis module offers a much more efficient approach to study the output process. Its use is however limited to the case where the system’s behaviour is only weakly nonlinear and can be appropriately represented by its linear counterpart. If its state deviates too much from the linearisation point, the accuracy of the approach starts to decrease. To get a better grasp on how the disturbance affects the accuracy, a range of cases in which the disturbance force applied on the four-bar linkage is gradually increased have been studied. The comparison with the standard deviations computed by each approach is presented in Figure 4-6.
As can be observed, in this case the standard deviation of the linearised system can differ by up to 15% for the largest disturbances.

This brief study is by far not exhaustive as it is presented for a single wave period. The standard deviations $\sigma_{y,\omega}$ computed in the frequency domain are compared to those computed in the time domain $\sigma_{y,t}$, but as stressed in Section 4-3-2, both of these approaches are only valid for ergodic and stationary processes. A proper quantification of the error would require a full nonlinear time domain using Monte Carlo simulations or Fokker-Plank equations which are not yet implemented in the tool.

This study however gives an indication of how and when the linear approximation stops representing the system appropriately. For amplification factors above 8 the system’s behaviour is clearly nonlinear, whereas below this amplification factor the system seem to behave rather linearly. This suggest that a linearisation around a different state might appropriately represent the system over a broader range of disturbances.

**Figure 4-6:** Trend in the standard deviation as disturbance increases.
### 4-5 A Mixed Frequency-Time Domain Approach

One important goal of this thesis is to provide insight on the influence of design choices on the workability (Proposition 3). This involves the computation of several workability plots during the design phase. As emphasised in the previous sections, both the frequency and time domain approaches feature clear benefits and drawbacks. The power of the time domain approach resides in its ability to handle nonlinear behaviours, while the strength of the frequency domain approach is the remarkable speed at which it can compute the response of linear systems. Considering the computation of workability plots, the two approaches complement one another. Based on these observations a mixed approach to generate workability plots more efficiently was developed.

The workability is defined by the sea states where the probability that the system performance criteria are met is judged acceptable. In order to find the conditions where the system is performing on its limit, the entire grid defining the sea states is explored. The mixed approach relies on using the frequency domain when speed is required and the time domain approach when accuracy and validity are crucial. Furthermore, in order to maximise both speed and accuracy, the workability space is explored strategically by finding a path that minimises the number of required time simulations. The workability space is therefore explored by sections taken at given wave periods. The considerations above led to the following proposed algorithm:

1. A set of computations is run in the frequency domain over a given section grid. The boundary points $M_b$ are positioned on the grid at all the workable points adjacent to a non-workable point. This step is used to quickly scan the workability space and get an indication of where the workability boundary is located.

2. The workability labels are evaluated and updated at these boundary points $M_b$ in the time domain.

3. The position of the boundary points are then adjusted depending on the label they got assigned at the previous step. All boundary points that got labelled workable are moved forward one grid step $\Delta H_s$ along the $H_s$ axis. If it is labelled non-workable they are moved backwards one grid step along the $H_s$ axis.

4. A new set of simulations is computed at the relocated boundary points $M_b$, to evaluate their workability labels. Steps 3 and 4 are repeated, continuing in the same direction until meeting the first non-workable state when moving forward along the $H_s$ axis, or workable state when moving in the other direction. A given boundary point is then considered to be converged. Once converged, the $M_b$ moving forward along the $H_s$ axis are moved back to their previous (workable) position.

The estimation obtained using this algorithm can only achieve the accuracy of the resolution of the grid space. The exact solution to the workability condition is thus not obtained.

Using this approach the number of computations performed in the time domain is significantly reduced for weakly nonlinear systems, thereby drastically reducing the computation time of the workability plots.
4-6 Workability Study of the Integrated Motion Compensated Gangway System

The statistics of a system response to stochastic disturbances can be determined through spectral analysis under certain circumstances as was presented in the previous section. In order to apply this to study a motion compensated gangway, a few assumptions need to be made. The disturbances, which in this case are the wave loads, are stationary. This means that the characteristics of the studied sea state does not change over the considered time window. The second assumption is that the system can be approximated by a linear time-invariant system.

For the spectral analysis of the motion compensated gangway, the linearised model is used as was developed in Section 3-3. In this example, the performance parameter will be the tip deviation of the motion compensated gangway from its fixed reference point. This tracking error will be used to determine the workability of the system and the criteria is defined as follows:

Workability definition: The gangway is considered to be out of its safe working area if it is incapable of tracking the reference point within a radius of 10 cm with a probability of 99%.

![Figure 4-7: The controller tries to keep the tip of the gangway within the green sphere.](image)

The tip deviation, denoted by $\delta e$, is a variable with a multivariate normal distribution. The theory on constructing the probability function for this variable is explained in Appendix A.
4-6-1 Computing the Workability Graph

The workability graph marks the sea states in which the system can safely and efficiently operate. Therefore the performance of the system is analysed for a set of sea states. The points of interest are the sea states where the system performs on the limit. This is the case when the following equality holds:

\[ P(\delta c < 0.10) = 0.99 \]  \hspace{1cm} (4-9)

As was explained in Section 2-2-4, the wave spectrum is parameterised by the peak period and the significant wave height. Furthermore, the wave loads acting upon the vessel depend on the angle at which they hit the hull. The combination of these three parameters for which eq. (4-9) holds, form a three dimensional boundary structure, as depicted in Figure 4-8. It would take years to generate this structure using time simulations, whereas a spectral analysis can compute this within several minutes.

Figure 4-8: This figure shows the full workability structure. A horizontal slice of this structure results in a commonly seen polar plot (highlighted in yellow). A vertical slice would result in the scatter diagram (highlighted in green).

A horizontal slice of this structure is depicted in Figure 4-9, for a peak period of \( T_p = 8 \). The boundary line shown on the polar plot divides the workable from the unsafe sea states.

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The workability graph in Figure 4-9 is constructed through spectral analysis using a linearised version of the multibody equations. To estimate the accuracy of workability, the probabilities obtained using spectral methods are compared to those generated from time simulations. Because the time simulations are computationally expensive, only a selection of the found limiting sea states are used for simulations in the time domain. In Section 4-6-1 the results of this comparison show that the linear probability is underestimated and is therefore on the conservative side. This indicates that the performance of the system is better than estimated by the linearised system.

Table 4-1: Comparison of the probability estimation.

<table>
<thead>
<tr>
<th>Wave Direction [deg]</th>
<th>Hs [m]</th>
<th>Linear Estimation</th>
<th>Nonlinear Estimation</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>6.43</td>
<td>0.9901</td>
<td>0.9997</td>
</tr>
<tr>
<td>90</td>
<td>1.45</td>
<td>0.9921</td>
<td>0.9999</td>
</tr>
<tr>
<td>180</td>
<td>4.05</td>
<td>0.9900</td>
<td>0.9994</td>
</tr>
<tr>
<td>270</td>
<td>1.45</td>
<td>0.9908</td>
<td>0.9998</td>
</tr>
</tbody>
</table>
4-6-2 Parameter Sensitivity Analysis

The workability graph such as in Figure 4-1 are usually produced for one conceptual design of the gangway system. If the concept design is properly parameterised, one can test several parameter sets. This shows how sensitive the total performance of the system is for certain parameter changes. Using this knowledge the system parameters can be tuned. To illustrate this, several cases have been studied of which two examples are presented now.

Mechanism Parameters

It is interesting to know how much the total length of the gangway affects the performance. A longer gangway provides a bigger reach and could therefore handle a larger vessel excitation. The downside of having a longer gangway is that the increase in mass and inertia can make it more difficult to control due to actuator limitations. In Figure 4-10 three cases are depicted where the total length of the gangway is varied while keeping the ratio of the boom lengths constant. These plots clearly reveal that a longer gangway, although providing more reach, eventually reduces the workability.

![Figure 4-10: The workability plot for different gangway lengths.](image)
Layout Parameters

Another interesting parameter to study is the position of the gangway on deck. Placing it further away from the rotation axis of the vessel results in amplified disturbances that deteriorate the performance.

The vessel being asymmetric, placing the gangway near the stern will give a workability that has a very different distribution than when placing it near the bow. This is clearly visible from Figure 4-11. The fact that placing the gangway near the centre is a better design solution may seem obvious, but it is nevertheless interesting to see to which extent the positioning of the gangway influences the performance. As can be seen in the figure, the influence can be quite drastic for a relatively change in position on deck.

![Figure 4-11: The workability plot for different gangway positions.](image)

Remarks

Workability analysis presented above are computed based on the system’s response to wave loads. In practice the disturbances arise from multiple sources, such as current and wind. The presented framework is able to handle these types of disturbances as long as they can be considered as stationary ergodic Gaussian processes. Furthermore, the performance of a system is usually assessed by considering multiple criteria. For the motion compensated gangway for instance, additional criteria representing limitations on the actuators or the dynamic positioning systems would typically be considered. Again, this can be handled in the current framework by defining and tracking the different criteria. The system is labelled non-workable as soon as one of the criteria fails.
4-6-3 Controller Tuning

The three PD-controllers actuating the hydraulic cylinders of the motion compensated gangway have been modelled in Section 2-3. Each controller, has the following the form:

\[ C(s) = K \left( 1 + \frac{T_d s}{1 + T_d s/N} \right) \]  \hspace{1cm} (4-10)

Using the linearised multibody equations, the proportional gain \( K \) and the derivative time \( T_d \) can now be tuned in the frequency domain to achieve a stable and fast response. When tuning these parameters for a real system, restrictions will arise from the physical limits such as the bandwidth of the actuators or the maximum power it can output. To reflect this, the phase margin will be limited to a minimum of 30 degrees and the bandwidth to a minimum of 3 rad/s. The gain is increased to a point where the system response results in a realistic behaviour. The open loop transfer function of luffing action is depicted in Figure 4-12. The controller for this degree of freedom has the following parameters: \( K = 1E6, T_d = 1/3, N = 20 \).

![Bode Diagram](image.png)

**Figure 4-12:** The controller

The effect of the controller performance is directly visible in the workability plots. For example, the effect of increasing the proportional gain by 10% is shown in Figure 4-13. From this graph it is clear that this improves the performance of the system.
Figure 4-13: The workability plots for the system with the original control parameters (blue) and with the altered proportional gain (orange).

This conclusion may seem trivial but the information provided by these type of workability plots are a lot more insightful than it may look. The gain in global performance can be quantified as increase or decrease of downtime by defining an appropriate measure. As opposed to the high-tech industry, performance in the offshore industry is not focused on precision but rather more on operability. Analysis approaches such as those presented above are hence much more valuable for decision making.
Conventional modelling and analysis tools typically offer a narrow discipline-specific view of controlled offshore systems, thereby depriving engineers from valuable insights on its global performances. The main goal of this thesis was to support and improve the design of controlled offshore systems from a holistic standpoint by developing a generic and modular tool providing a suitable modelling and analysis approach.

To do so, the in-house multibody software package of Royal IHC, DODO, was first extended to encompass the modelling of control systems and to enable analysis in the frequency domain. Two additional modules were implemented to perform spectral analysis and to compute probabilities. By combining the different components of this modular framework a wide range of systems can be studied. These are however bound to the scope of systems consisting of rigid articulated mechanism installed on a floating vessel, performing seakeeping operations in stationary weather conditions. It might therefore be interesting to broaden this scope by expanding the developed framework to include:

### Spectral Analysis of Non-stationary Processes

In order to enable the study of long-term statistics it would be interesting to investigate how non-stationary spectral analysis approaches [20] can be integrated to the current framework. They are most commonly based on the notion of time-dependent spectra. Unfortunately, the time-dependent spectra do not the carry the same kind of physical interpretation as the Power Spectral Density (PSD). Consequently, analysis approaches relying on the physical interpretation of the PSD, such as dynamic error budgeting, are difficult to extended using these theories. Typical applications would therefore be the study of operations taking place over longer time scales or in unstable weather conditions.

### Linear and Nonlinear Approximations

- In the examples presented in Section 4-6 the systems were linearised around the static equilibrium. Although convenient, this is obviously not the best solution. As suggested by the results of Section 4-6-1, a better linear approximation might improve the accuracy
of the frequency domain approach thereby drastically reducing computational time. One idea is to linearise the system around an averaged state considering a small sample of system responses computed in the time domain.

- Another commonly used approach to study nonlinear systems is based on the Volterra series expansion. Although this approach also relies on a local estimation, it usually gives a better approximation of the system’s behaviour as it is able to represent some of its nonlinear features. It also has the advantage of providing a nonlinear model in the frequency domain.

**Non-ergodic Processes**

If the output process is known to be ergodic and stationary, any nonlinear system, regardless of its degree of nonlinearity, can be appropriately analysed within the current framework. However, without prior knowledge on the output process of a nonlinear system, the proper study of its statistical properties require further development.

- An easy extension would be to develop a module performing Monte Carlo simulations of the system’s response by numerically integrating the nonlinear coupled equations eq. (2-7). This brute force approach is however computationally inefficient.
- Another method that might be interesting to consider is based on deriving the Fokker-Planck equations of the process and numerically solving these differential equations to obtain the probability distributions of the output process. This approach however requires the study and development of approaches to integrate it into the current generic framework.

**Optimisation**

The analysis on the gangway in Chapter 4 illustrated how the gained insight over the influence of design parameter on the system’s workability could be used to tune and improve its performance. However, it is in general not possible to reach an optimum solution by tuning parameters separately. It would therefore be interesting to integrate an optimisation algorithm to guide the design towards solutions that minimises a certain average expected downtime.
Appendix A

Probability Estimation

In section 4-6, the following workability criteria was introduced:

A given sea state is workable, if \( P(\delta \epsilon < r_{lim}) \geq P_{lim} \)

This appendix shortly describes how the probability \( P \) is estimated.

In the sequel, the random process representing the variation of position of the gangway tip with respect to the tracking point will be denoted \( \delta X \). Its components along the \( x \), \( y \) and \( z \) axis are denoted as \( \delta X_i, \ i=1,2,3 \) respectively. The variation of the distance from tip position to tracking point \( \delta \epsilon \) is also a random process defined as \( \delta \epsilon = (\delta X^T \delta X)^{1/2} \).

**Assumptions**

Without additional knowledge on the process it is in general not possible to compute probabilities on it. The computations exposed below assume that the multivariate process \( \delta X \) has a normal distribution.

This assumption is valid whenever the system is well represented by a Linear Time Invariant system. Since the output of a LTI system to a Gaussian input process (with continuous spectrum) is a Gaussian process itself.\(^1\)

This is of course not necessarily true for workability points lying outside of the validity zone of the linearisation. The output processes are nevertheless often assumed to have a Gaussian distribution.

**Chi distribution**

Chi distributions represent the Euclidean distance to the origin of a multivariate Gaussian process \( Y \) with independent components \( Y_i \). In other words it represents the distribution of the process \( D = \sqrt{\sum_{i=1}^{k} Y_i} \). The parameter \( k \) is called the degrees of freedom of the distribution.

\(^1\)The input sea states represented by the Joint North Sea Wave Project (JONSWAP) spectrum are Gaussian processes with continuous spectra.
Its cumulative distribution function \( F(d, k) \) is given by:

\[
F(d, k) = \frac{\gamma(k/2, d^2/2)}{\Gamma(k/2)}
\]

where \( \Gamma \) denotes the ordinary gamma function and \( \gamma \) the lower incomplete gamma function.

The probability that the distance \( D \) is less then a given \( d_{\text{lim}} \) therefore writes

\[
P(d < d_{\text{lim}}) = F(d_{\text{lim}}, k)
\]

**Decorrelation**

Although \( \delta X \) is a multivariate Gaussian process, its components \( \delta X_i \) are not independent. They are obviously correlated through the kinematic constraints. It is however not difficult to decorrelate these components by defining a transformation to an eigenbasis of the covariance matrix \( \Sigma_X \).

Consider the transformed process \( \delta Y = \Lambda_k^{-\frac{1}{2}} V_k^T \delta X \), with the notations introduced below. It is a multivariate Gaussian process with covariance matrix \( \Sigma_Y = I_{k \times k} \), and its components are independent standard normal processes.

\[
\Sigma_X = E(\delta X \delta X^T) = VA V^T
\]

\( \Lambda \): diagonal matrix containing the eigenvalues of \( \Sigma_X \)

\( V \): unitary matrix containing the eigenvectors of \( \Sigma_X \)

If \( \Sigma_X \) is singular, \( V \) and \( \Lambda \) are further partitioned as:

\[
\Lambda = \begin{bmatrix}
\Lambda_k & 0 \\
0 & 0
\end{bmatrix}, \quad V = \begin{bmatrix}
V_k & V_n
\end{bmatrix}
\]

\( \Lambda_k \) and \( V_k \) being the partitions of \( \Lambda \) and \( V \) associated to the \( k \) non-zero eigenvalues, and \( V_n \) the partition of \( V \) associated to the zero eigenvalues.

**Relating limit radii in the original and transformed basis**

The Chi distribution can now be used to estimate the probability that the tip is confined within a given domain of space. Note that since it is associated to the Euclidean distance of the process \( \delta Y \), \( P_{\text{lim}D} = P(D < d_{\text{lim}}) \) gives the probability that the tip deviation is contained within a sphere of radius \( d_{\text{lim}} \) in the transformed basis. This defines an ellipsoid in the original basis.

To relate the limit radius in the new basis, \( d_{\text{lim}} \), to the specified limit radius in the original basis, \( r_{\text{lim}} \), one can observe that

\[
D^2 = \delta Y^T \delta Y \leq \frac{\delta X^T \delta X}{\max(\Lambda_k)}
\]

Since we are interested in the probability that \( \delta X^T \delta X < r_{\text{lim}}^2 \), by computing the probability:

\[
P_{\text{lim}=\epsilon} = P(D < \frac{r_{\text{lim}}}{\sqrt{\max(\Lambda_k)}})
\]

We get an estimate of the probability \( P_{\text{lim}=\epsilon} \). As illustrated in Figure A-1, the confidence interval containing the tip point with probability \( P_{\text{lim}} \) is an ellipsoid included in the limit sphere of the workability criteria. It is therefore clear that this probability overestimates the chances of the tip to be outside the limit sphere, meaning that the workability is underestimated.
**Figure A-1:** Representation of the limit sphere in which the tip deviation should be contained and the limit ellipse for which we can compute the probability of containing the tip.
Dynamic Error Budgeting (DEB) is a technique which is used to analyse how stochastic disturbances propagate through a system. This knowledge assists the engineer to make decisions during the design phase. It has been effectively applied on high performance mechatronic systems that are sensitive to sensor noise and vibrations, for example in [21] [22].

In any controlled closed loop system, disturbances enter at various points. This is illustrated in Figure B-1. Each of these disturbance signals, $d_i$, are translated to the output in a different manner, depending on where they enter the closed loop.

Dynamic Error Budgeting helps to visualise the contribution of each disturbance source to the total performance error $e$. This is typically done by propagating the input spectra of the disturbances through the closed loop system to obtain the spectrum of the performance variable. This spectral analysis allows to get this insight into the disturbance rejection of the system as a function of frequency. By analysis of this function, the system’s mechanical design and controller $C$ can be adjusted such that performance improves.

![Figure B-1: The contribution of each disturbance channel $d_i$ to the total performance $e$ [23].](image-url)
The DEB approach cannot always be used. There are several conditions that have to be met in order to get an accurate result. The assumptions on which this method is built upon are the following:

- All dynamics can be accurately modelled by a linear time-invariant system.
- The disturbances must be stationary. This means that the statistical properties do not change over time.
- Only stochastic disturbances are allowed at the inputs. If there are deterministic disturbances present, they should be implemented in the model.

The contribution of a disturbance to the system’s performance can be visualised using a Cumulative Power Spectrum (CPS) as was explained in section 4-4. When multiple disturbances enter at various points in the closed loop this method can subdivide the total performance error into their accountable disturbance sources. An example of this is presented in [21], where DEB is illustrated on a high-tech mechatronic positioning system. The CPS functions of this example are depicted in Figure B-2.

![Figure B-2: The subdivision of the CPS of the total performance error into the disturbance sources as a function of frequency.](image)

A similar approach can be applied on a controlled offshore system when several disturbances perturb the system. For the motion compensated gangway, this method would allow to see the significance of the sensor accuracy to the system performance.
Bibliography


Glossary

List of Acronyms

CAS       Cumulative Amplitude Spectrum
CPS       Cumulative Power Spectrum
COG       Centre Of Gravity
DAE       Differential-Algebraic Equation
DEB       Dynamic Error Budgeting
DODO      Dynamic Operations in Dredging and Offshore
JONSWAP   Joint North Sea Wave Project
LTI       Linear Time-Invariant
MCS       Minimal Coordinate Set
MIMO      Multiple Input Multiple Output
MRU       Motion Reference Unit
ODE       Ordinary Differential Equation
PID       Proportional Integral Derivative
PSD       Power Spectral Density
RAO       Response Amplitude Operator
RCS       Redundant Coordinate Set