Capacity Estimation Methods

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Definition of Capacity

Maximum number of trains $N$ that may be operated using a defined part of the infrastructure at the same time during a defined time period $[1/7]$; $T$: Time period $[24\ h = 1440\ min; 1\ h = 60\ min]$

• Theoretical capacity $C = \frac{T}{\sum(t_{\text{h min}} + \Delta t)}$
  Maximum number of trains $N_f$ at scheduled order and speed without timetable margins; $t_{\text{h min}}$: minimum headway time
  $\Delta t$: running time difference between successive trains

• Practical capacity $C_p = \frac{T}{\sum(t_{\text{h min}} + \Delta t + t_b)}$
  Maximum number of trains $N_p$ at scheduled order and speed including running time supplements $t_r$, buffer times $t_b$ and track possession time for infrastructure inspection and maintenance
Assignment of headway times

A. To stations

B. To track sections

- $t_{dd,A}$: "depart-depart" headway at station A
- $t_{da,A}$: "depart-arrive" headway at station A
- $t_{aa,A}$: "arrive-arrive" headway at station A
- $t_{ad,A}$: "arrive-depart" headway at station A
- $t_{dd,B}$: "depart-depart" headway at station B
- $t_{da,B}$: "depart-arrive" headway at station B
- $t_{aa,B}$: "arrive-arrive" headway at station B
- $t_{ad,B}$: "arrive-depart" headway at station B
Impact of speed and number of tracks on capacity

\[ T = 60 \text{ min} \]

12/h

8/h

1 fast line

0

double track

12

1 slow line

8

double track single track, sidings

2/h

2+3
double track single track

2 lines

5/h

1 line

1 line

Capacity estimation
Impact of station density, service pattern and overtakings on capacity

- **Capacity estimation**

**T = 60 min**

- 4/h
  - 1 overtaking
  - 2+2 stations

- 6/h
  - 3 overtakings
  - 2 lines

- 8/h
  - 1 overtaking
  - 4+4 stations

- 6/h
  - 2 overtakings
  - 4+2 stations

**5 stations**

- 4/h
  - 2+2 stations

- 6/h
  - 3+4

- 8/h
  - 4+4 stations

**3 stations**

- 6/h
  - 2 lines

**4 stations**

- 6/h
  - 2 lines
Capacity depends on

• Timetable
  – Train speed and homogeneity
  – Train order
• Infrastructure
  – Alignment
  – Number and length of tracks
  – Number of stations
  – Number of lines
  – Signalling & safety system
  – Travel time differences
  – Minimum headways
  – Timetable margins
  – At-grade crossings, flyovers, speed reductions, steep gradients
  – Single (bidirectional), passing loop
  – Double, merging/diverging/crossing, terminal, stabling
  – Fixed block {one-section/multiple track sections}
  – Automatic Train Protection (ATP)
  – Automatic Train Control (ATC)
  – Automatic Train Operation (ATO)
  – Moving block
• Rolling stock
• Weather
• Human behavior
Capacity balance

Number of trains

Average speed

Stability

Heterogeneity

Mixed-train exploitation
Metro-train exploitation

Source: UIC, 2004
Trade-off between waiting time and capacity

![Diagram showing the trade-off between waiting time and capacity.](image)

**Trade-off between waiting time and capacity**

- **a)** High variation of minimum line headways
- **b)** Low variation of minimum line headways

- Scheduled waiting time
- Waiting time in scheduling
- Waiting time in current operation

- **C_u**: Timetable capacity
- **C_{max}**: Maximum capacity
- **T**: Traffic flow

- **t_w**: Time waited

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**Capacity estimation**
## Classification of capacity estimation models

### A. Graphical (rule based)
- Train diagramming

### B. Analytical
- Track occupation
- Waiting time, stability margin
- Queueing

### C. Simulation
- Macroscopic
- Microscopic

### D. Combinatorial Optimisation
- (Mixed) Integer Linear Programming
- Heuristics (local search, genetic, tabu search)
- Stochastic programming, Light robustness, Recoverable robustness, …

<table>
<thead>
<tr>
<th></th>
<th>Open track</th>
<th>Station</th>
<th>Network</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>A</strong> Graphical</td>
<td>✔</td>
<td>✔</td>
<td>✔</td>
</tr>
<tr>
<td><strong>B</strong> Analytical</td>
<td>✔</td>
<td>✔</td>
<td></td>
</tr>
<tr>
<td><strong>C</strong> Simulation</td>
<td>✔</td>
<td>✔</td>
<td>(✓)</td>
</tr>
<tr>
<td><strong>D</strong> Combinatorial Optimisation</td>
<td>✔</td>
<td>✔</td>
<td>(✓)</td>
</tr>
</tbody>
</table>

- ✔: Included
- (✓): Included with some modifications
- (✓): Included with strict conditions
Infrastructure model

A. Macroscopic model

B. Microscopic model
Drawbacks of macroscopic capacity estimation model

• Inaccuracy of scheduled travel (running and dwell) times
  – Linear train graphs: time loss due to acceleration, coasting and deceleration unknown/not disaggregated
  – Scale: rounded-up to full minutes
  – Discrete point modelling of trains: variation of train length neglected

• Validation of scheduled minimum time headways missing
  – Use of given standard minimum headway values (safety constraints)
  – Variation of train speed and minimum headway times neglected
  – Impact of ATP, ATC neglected

• Timetable margins unknown
  – Standard running time supplements not verified nor differentiated
  – Amount of buffer times unknown

⇒ Insufficient precision and reliability of capacity estimation!
Microscopic model of station track yard and route nodes
Analytical capacity estimation

Diagram:

- Infrastructure data
  - Train data
    - Dynamics of train movement
    - Timetable
    - Calculation of train paths
    - Calculation of blocking times
    - Calculation of minimum line headway for all train combinations
    - Determining the relative frequency of train combinations
    - Calculation of average minimum line headway
    - Calculation of line exploitation rate
Microscopic infrastructure and rolling stock model

**Input**
- Graph modelling of track infrastructure
  - Track section and platform lengths, radii, gradients and max. speed
  - Location and distance between signals, switches, crossings, insulation joints, overhead contact line separators,
- Specification of signalling and safety systems
  - Blocking and clearance, signal aspects, overlaps
  - Train detection, location of track circuits/axle counters/fouling points
  - Interlocking, set-up and (partial) release of routes
  - Train protection, train control
  - Train regulation
- Train length, weight, resistance, tractive effort-speed diagram
Calculation of (scheduled) blocking time overlap

Path of 2nd train must be postponed by this amount to eliminate scheduling conflict in the regarded block section.

- \( t_b \): blocking time
- \( t_{be} \): begin of blocking time
- \( t_{be} \): end of blocking time
Capacity consumption: ‘compression’ of blocking time diagrams

Drawbacks: timetable dependency, transferability
Microscopic capacity estimation

1. Estimation of blocking times $t_{bl\,i}$ of trains per line
2. Determination of minimal headway time $t_{h\,ij}$ between trains
   - at departure (according to different train sequences)
   - at arrival (stations)
   - at conflict points (merging/crossing of lines, long block, speed limit)
3. Determination of prevailing minimal headway times;
   mean minimal headway $t_{hm} = \sum (t_{h\,ij} \cdot p_{ij})$; $p_{ij} = n_i \cdot n_j / n^2$
4. Estimation of (scheduled/feasible) number of train path $n/n_{max}$
5. Estimation of total track occupation time of compressed
   (scheduled) train graph $T_{toc} = n \cdot t_{hm}$
6. Estimation of scheduled track occupation $\rho_s = T_{toc}/T$ [%]
7. Estimation of maximal track occupancy $\rho_{max} = T_{toc\,max}/T$ [%]
Impact of block length and speed on blocking time

\[ T_{\text{block}} = t_{\text{sw}} + t_{\text{sight}} + \left( l_{\text{br}} + l_{\text{block}} + l_{\text{safety}} + l_{\text{v}} \right) / v + t_{\text{cl}} \]

- Minimum blocking time at very short block length and low speed \( \approx 80 \text{ km/h} \)!
Capacity consumption levels recommended by UIC leaflet 406

<table>
<thead>
<tr>
<th>Type of line</th>
<th>Peak hour</th>
<th>Daily period</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dedicated suburban passenger traffic</td>
<td>85 %</td>
<td>70 %</td>
</tr>
<tr>
<td>Dedicated high speed lines</td>
<td>75 %</td>
<td>60 %</td>
</tr>
<tr>
<td>Mixed traffic lines</td>
<td>75 %</td>
<td>60 %</td>
</tr>
</tbody>
</table>
Recommended area of traffic flow

Traffic energy \( E = N \cdot v^2 \left[ \frac{\text{km}^2}{\text{h}^2} \right] \)

Relative sensitivity of waiting time

\( \Rightarrow \) Problems: validation of waiting time and speed distributions; combination of different dimensions; transferability

‘Optimal’ analytical route capacity and cost-benefit model

⇒ Shortcomings:
- complicated parameter calibration;
- validation?
- optimality gap unknown

Source: Schwanhäußer, 2009
Analytical junction occupation model

Source: Mussone & Wolfler Calvo, 2013

⇒ Shortcoming: Switch, sight and approach time time neglected
Conditional probability junction
capacity estimation model

Consecutive delay survival probability for
northbound departing trains as function of
number of trains passing at level crossing

Consecutive delay survival probability for a
northbound departing train as function of
train frequency passing at level crossing

Source: Yuan, 2006

⇒ Shortcomings:…
Queuing models to estimate train delays and capacity

Queueing system
- A: Arrival process with interarrival time distribution $A$
  - Arrival time $t_k$, interarrival time $a_k = t_{k+1} - t_k$ of customer $k$
- B: Service process with service distribution $B$
  - Service time $b_k$
- $n$: Number of service channels
- Queue discipline: First Come First Served (FCFS), Priority (PR)

Notation:
- $A/B/n$ are systems with queue capacity $m = \infty$ (general $A/B/n/m$)
# Classification of queuing models

Variation coefficients

\[
k = 1/V_A^2 = (\sigma^2 A / E^2 A)^{-1}, \quad V_A: \text{variance of arrival headway times}
\]

\[
l = 1/V_B^2 = (\sigma^2 B / E^2 B)^{-1}, \quad V_B: \text{variance of service times (minimal headway)}
\]

<table>
<thead>
<tr>
<th>Arrival distribution</th>
<th>Service distribution</th>
<th>big variance (0 &lt; l &lt; 1)</th>
<th>Markov (l = 1)</th>
<th>small variance (1 &lt; l &lt; \infty)</th>
<th>deterministic (l = \infty)</th>
</tr>
</thead>
<tbody>
<tr>
<td>big variance</td>
<td></td>
<td></td>
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</tr>
<tr>
<td>Markov</td>
<td>(k = 1)</td>
<td>M/M/1</td>
<td>M/GI/1</td>
<td>M/D/1</td>
<td></td>
</tr>
<tr>
<td>small variance</td>
<td>(1 &lt; k &lt; \infty)</td>
<td>GI/M/1</td>
<td>GI/GI/1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>deterministic</td>
<td>(k = \infty)</td>
<td>D/M/1</td>
<td></td>
<td>D/D/1</td>
<td></td>
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</tbody>
</table>

M: Markovian process (exponential)
GI: General independent process (iid)
D: Dirac process (deterministic, variance = 0)

⇒ **Drawbacks:** iid assumption invalid for periodic timetable and heavy traffic; service distributions unknown
Comparison with estimated waiting time of clockface timetable

Relative deviation between $GI/GI/1$ queuing system and 2-train clockface timetable

⇒ Modelling of clockface process by estimated waiting time and variation coefficient of interarrival time must be rejected!

Source: Bär, 2009
Combinatorial optimisation models

1. Integer Linear Programming (ILP)  
   (Review see Cacciani & Toth, 2012)

2. Genetic Algorithms (GA)
   } 

3. Simulated Annealing (SA)
   } Disregarded here

4. Constraint Programming (CP)
   } 

5. Job-Shop Scheduling
   (Mascis & Pacciarelli, 2002; D’Ariano et al. 2007, 2008, 2009…
Integer Linear Programming (ILP)

- Objective functions
  - Maximize customer satisfaction (minimize travel times, waiting times, number of transfers)
  - Maximize infrastructure use (train throughput)
  - Minimize changes w.r.t. requested arrival & departure times
  - Minimize costs of train operation
  - Increase robustness (avoid/minimize delay propagation)

- Constraints
  - Periodicity (regularity, running, dwell and transfer times)
  - Safety (minimal headway times)
  - Infrastructure discontinuities (number of tracks, terminals, switches, crossings, platform length)
  - Rolling stock composition, performance and assignment
  - etc.
Capacity estimation

Integer Linear Programming (ILP)

Cyclic train timetabling

• Periodic Event Scheduling Problem (PESP)
  • Constraint Graph (directed graph with node set N and arc set A)
  • Constraint propagation algorithm
• Given timetable of train operator assumed to be feasible, i.e. route conflicts within stations (interlocking areas) neglected
• Timetable consists of fixed running times between stations that may be altered within certain bounds (time windows)
• Solving PESP of larger networks and complex nodes is \textit{NP hard}
• Relaxation of (some) constraints in order to find a near-optimal solution and/or speed-up computation time
• Mixed Integer Linear Programming (MILP) tolerates mix of non- and integer constraints
Integer Linear Programming (ILP)

Non-cyclic train timetabling

• More appropriate in competitive market with generally conflicting train path requests
• Arrival and departure times represented by continuous variables (MILP)
• Binary variables expressing the train order at departures
• Discretising the time (each node corresponding a time instant, arcs representing train travel or stop at station)
• Costs associated e.g. with the deviation from preferred departure time, travel times and dwell times
• Set of arcs representing assumed feasible travel times between and dwell times per train at each station (time-space graph equals train path)
• Trains may be cancelled
• Easier to solve than PESP of periodic timetable (more flexible)
Resource-Tree Conflict Graph

Source: Caimi et al, 2011
Drawbacks of (M)ILP

Exception: ILP relaxation of Resource-Tree Conflict Graph by Caimi et al, 2011

- Possible headway conflicts on open and station tracks due to speed limitations and/or longer approach and block signal distances neglected
- Route conflicts in interlocking areas neglected (feasibility assumption)
- Shunting movements generally neglected
- Availability of sufficient parking/stabling tracks not guaranteed

⇒ Validity of given timetable and used safety constraints not proven!
⇒ Lack of accuracy compared to analytical, queuing and microscopic simulation models that may endanger timetable feasibility!
⇒ Optimality claim may lead to misunderstanding
⇒ Stochastic Programming and Robust MILP train timetabling models cannot provide optimal allocation of timetable margins due to inherent restrictions of macroscopic models!
Conclusion

- Track capacity is influenced by the timetable, infrastructure, signalling and safety systems, rolling stock, weather and human behaviour
- Macroscopic capacity estimation models simplify infrastructure, route and signalling constraints but can support strategic network and timetable planning
- Microscopic capacity models can accurately estimate minimum headways, capacity consumption and timetable margins for different signalling and safety systems
- (M)ILP cannot substitute more accurate analytical capacity analysis and microscopic timetable simulation
- Micro-macroscopic (meso-scopic) models can bridge the gap between accurate analytical and efficient combinatorial optimisation
Literature


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