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Summary of the research on bolted beam-to-column connections

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Preface

This report embodies the result of much research and many detailed and intensive discussions within the Committee "Connections" of the Staalbouwkundig Genootschap (Structural Steelwork Association).
In those discussions the emphasis was not only on economic aspects, but more particularly also on aspects of structural safety. For example, in the case of calculations based on elastic theory it will, in order to obtain the necessary safety, often be necessary to ensure that particular plastic deformations can occur.
Besides making their valuable contributions to the Committee's activities, the members often ensured that manpower and material were made available to enable the extensive testing of specimens to be carried out. The tests were performed in the Stevin Laboratory of the Delft University of Technology. At the time of completion of this report the Committee "Connections" comprised the following members:

Prof.ir. J. Berenbak	: Delft University of Technology Hollandia-Kloos, Krimpen a/d IJssel
Ir. L.P. Bouwman	: Delft University of Technology
Ir. F.S.K. Bijlaard	: TNO-IBBC, Rijswijk
H.G.A. Evers	: Evers Staalconstructies, Hillegom
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Notation

A_s	(m^2)	stressed cross-sectional area of bolt
$\hat{B}_t$	(N)	tensile design strength of a bolt in the ultimate limit state
D	(N)	shear force
E	(N/m^2)	modulus of elasticity (Young's modulus)
F	(N)	actual force
F_c		strength (loadbearing capacity) of a frame
F_E		Euler buckling load of a frame
$\hat{F}$	(N)	design strength in the ultimate limit state
F_o	(N)	force at time "zero"
F_v	(N)	prestressing force
$\hat{F}_p$	(N)	design strength of a plate in the ultimate limit state
G		point on moment-rotation diagram where the calculated uniformly distributed load causes just the permissible deflection of a beam
I	(m^4)	moment of inertia of beam
M_e	(Nm)	elastic moment of beam in the ultimate limit state
M_p	(Nm)	plastic moment of beam
M_v	(Nm)	moment in the connection
$\hat{M}_v$	(Nm)	design strength (moment capacity) of the connection in the ultimate limit state
R		reduction factor
a	(m)	distance
b	(m)	width
b_m	(m)	effective length of plate or flange
b_{mo}	(m)	effective length of packing plate
c	(Nm/rad)	spring constant
c_k	(Nm/rad)	spring constant of column
f	(m)	deflection at mid-span
f_y	(N/m^2)	guaranteed yield point
h	(m)	depth of beam or connection, or height of column
i		subscript, or number of bolt rows
k		ratio of stiffness of connection to stiffness of beam
l	(m)	span of beam

m	(m)	distance from centre of bolt to point of fixity of the plate
m_e	(m)	distance from centre of bolt to point of fixity of the end plate
m_k	(m)	distance from centre of bolt to point of fixity of the column flange
m_p	(N)	plastic moment per unit length of plate or flange
m_{po}	(N)	plastic moment per unit length of packing plate
n	(m)	distance from centre of bolt to point of action of prying force
n		number
n'	(m)	distance from centre of bolt to free edge of plate
p	(m)	bolt spacing
q	(N/m)	uniformly distributed load per unit length of beam
$\hat{q}$	(N/m)	uniformly distributed load in the ultimate limit state of the beam
t	(m)	plate thickness or flange thickness
t_f	(m)	flange thickness
t_w	(m)	web thickness
t_e	(m)	end plate thickness
t_d	(s)	duration of time
α		factor which, multiplied by the plastic moment per unit length, gives the plate strength (see Fig. 2.5)
β		ratio of plate strength to bolt strength
γ		$n/m < 1.25$
δ	(m)	deflection or displacement
ϕ	(rad)	angular rotation
λ_1 and λ_2		ratios
σ	(N/m^2)	actual stress.

Introduction

The object of this report is to improve the understanding of the behaviour of bolted beam-to-column connections.
This has been made possible by supplementary research which has been carried out since the publication of the "Code of practice for the calculation and detailing of bolted beam-to-column connections" in 1978.
The research is concerned chiefly with beam-to-column connections which can resist bending moment and comprise two or more horizontal rows of bolts in end plates which, on the tension side of the connection, either

- do not project beyond the flange of the beam, so-called flush end plate, or
- project beyond the flange of the beam, so-called extended end plate,

connected either to

- column flanges with stiffeners, or to
- column flanges without stiffeners.

The connections dealt with in this report are chiefly:

- semi-rigid connections;
- partial-strength connections.

These designations are used also in Eurocode 3/3/.

A connection is semi-rigid if the design stiffness of the connection (moment-rotation ratio) is less than the stiffness of the connected beam over a length equal to half the depth of the column section.
In a partial-strength connection the design strength of the connection is less than the strength of the connected beam.
A connection which is completely rigid is not necessarily a full-strength connection, and vice versa.
In analysing the structure (calculating the distribution of forces) in accordance with elastic theory the stiffness (or rigidity) of the connections is important. In a calculation in accordance with elementary plastic theory their strength is important.
In Chapter 1 it is explained why a distinction is drawn between structures which are analysed in accordance with plastic theory and with elastic theory respectively.

Then follows a discussion of the effects that the reduced stiffness and strength of the structural connections have upon the results of the calculations.
It is also considered how the calculations can be simplified by imposing requirements as to stiffness and deformation capacity of the connections. In this context a distinction is drawn between calculations for braced frames (without sidesway) and unbraced frames (with sidesway).
Chapter 2 gives the formulae with which the stiffness and strength of a connection formed with an end plate and one or more rows of bolts can be calculated.
The design rules for the tensile strength of connections formed with T-stubs, extended end plate and flush end plate are found to be identical. The rules are summarized in a chart in which the strength can be read if the plate strength/bolt strength ratio of a plate portion with a group of bolts is known. This is illustrated with the aid of design examples. The plate strength/bolt strength ratio is found also to be a criterion of deformation capacity.
The formula for the calculation of stiffness is based on the summation of the flexibilities of the various components of the connection. It can be taken into account that components which have not yet reached failure are less flexible than those which determine the design strength of the connection. Design examples elucidate the method.
The available possibilities for increasing the strength and stiffness of the connection are also indicated in Chapter 2. With reference to some test results it is shown that the strength of connections is influenced only by very high normal stresses in the columns. This must be taken into account with the aid of the reduction formulae given.
Interaction of shear stresses and normal stresses in the column web need not be considered. Test results which provide confirmation for the theory set fort in Chapter 2 are discussed in Chapters 3 and 4. The tests reported in Chapter 3 were carried out for the purpose of standardization of structural connections formed with angle cleats and extended end plates. It appears from these tests that in the design of structural connections it is necessary to take account of the differences in the load-deformation relationships of the components of the connection, unless these components possess sufficient deformation capacity.

To be certain of sufficient deformation capacity, the upper limit of the yield point would have to meet particular requirements. Since this is impracticable, standardization of angle cleats for structural design calculations is based on elastic theory.
In standardizing the end plates it is not known in advance whether the required deformation could be provided also by other parts of the connection, e.g., the flange of the column. In the case of extended end plates the requirement of sufficient deformation capacity results in such small thicknesses for these plates that their strength is too greatly reduced if it were desired to employ these end plates in a structure designed on the basis of elastic theory.
This has led to the conclusion that it is the design rules, not the structural components, that should be standardized. The tests described in Chapter 4 were performed in order to show that structural connections, as specified in Chapter 2, can resist large loads due to explosions if it is ensured that the connections possess deformation capacity.
Each Chapter is followed by a summary of the main points. The results of the research have been incorporated in EC 3 /3/.

A substantial proportion of the design rules presented here has already been incorporated in the draft entitled "Concept N.P.R.-verbindingen" /2/. The present report is intended as explanatory comment on the above mentioned proposal and draft.

Chapter 1:

Influences of connection behaviour and design methods on the behaviour of frames.

1.1 Introduction

The draft design rules /2/ state the requirements to be satisfied by the structural connections. They are summarized in Fig. 1.1. The present report is concerned chiefly with semi-rigid or partial-strength connections. The moment-rotation behaviour of semi-rigid connections must be known in order to be able to calculate the interaction of the parts to be connected. The actual stiffness and strength of the connections can be calculated only with a limited degree of accuracy. Therefore it is necessary to have a conception of the influence of these properties upon the behaviour of the frame structure.

1.1.1. Data relating to frames

Fig. 1.2 presents an overview of the possibilities that play a part in designing a structural frame and its connections.
In the first place, the distribution of forces in the frame and the connections can be analysed in accordance with elastic theory or with plastic theory. Alternatively, different theories may be used for the frame and for the connections, subject only to the limiting condition that a connection which remains elastic up to failure cannot be employed in a structure designed on the basis of plastic theory unless the connection is stronger than the connected beam. In the latter case the requisite deformation can be provided by the plastic hinge that develops adjacent to the connection.
In the analysis of the frame it may be necessary to take account of second-order effects. This means that the product of the lateral displacement of the columns and the normal forces in the columns is so great that it gives rise to additional moments and displacements.

REQUIREMENTS APPLICABLE TO CONNECTIONS.
The behaviour of the connection must be in conformity with the assumptions on which the analysis of the structure, i.e., the calculation of the distribution of forces in it, is based. The structure may be analysed in accordance with elastic theory or in accordance with plastic theory.
For an analysis in accordance with elastic theory the connections may be assumed to be: - either hinged, - or fully rigid, - or semi-rigid. In all cases the connections must be able to resist the calculated forces. Hinged connections must be able to undergo the required rotation without causing moments of any appreciable magnitude to develop in the structural members in consequence of the action of internal forces in the connections. Fully rigid connections are moment-resisting connections in which the deformations have no effect on the calculated distribution of forces and on the deformations of the structure. Semi-rigid connections are moment-resisting connections whose flexibility influences the distribution of forces in the structure. The moment-rotation behaviour must be known if the structure is to be accurately analysed.
For an analysis in accordance with plastic-theory the connections may be assumed to be: - either full-strength (moment-resisting) - or partial-strength (moment-resisting) connections. In the case of full-strength connections the design moment of the connection must not be less than the plastic moment of the connected structural member. In the case of partial-strength connections the connection may be weaker than the connected member, provided that the rotational capacity is so great that all the plastic hinges required for the collapse mechanism can develop.

Fig. 1.1: Requirements applicable to connections.

These additional moments and displacements may in turn affect the behaviour of the frame and must therefore be taken into account.
In relation to this it makes a great difference whether or not the frame is laterally supported, i.e., whether or not it is braced or restrained against "sidesway". The terms "braced frame" (without sidesway and "unbraced frame" (with sidesway) are employed in this context.

1.1.2. Data relating to connections

The overview presented in Fig. 1.2 applies only to partial-strength connections. The exceptions for full-strength connections will be mentioned in due course. Partial-strength connections may be either rigid or semi-rigid. If the distribution of forces in the connection is determined on the basis of elastic theory, the connection will in general be more rigid than if the distribution of forces is determined on the basis of plastic theory. The validity of this statement is conditional upon so detailing the connection that the principles of the elastic or of the plastic theory are valid. This will be illustrated with the aid of some examples.
Application of elastic theory presupposes that plane sections remain plane in a member that undergoes bending (Bernoulli's assumption). This is utilized for calculating the distribution of forces in the bolts connecting an end plate to a column. For this it must be ensured that the end plate and the column flange remain plane when subjected to load. If this requirement is satisfied, a rigid connection will as a rule be obtained because the deformation is caused only by strain in the bolts.
In applying plastic theory it is assumed that the connecting elements can undergo so much deformation that a component in which early failure occurs will deform to such an extent that other parts of the connection can also fail. This has certain consequences for the design of a beam-to-column connection which does not allow of elastic analysis because, for example, the column flange is too thin. In that case the end plate or the column flange must be able to deform so much that not only the row of bolts in the extended part of the end plate attains its design strength, but also the rows located in the less deformed parts of the end plate or column flange between the beam flanges (see Chapter 3).

Data of connection		Partial-strength connection			
		Distribution of forces in connection calculated by:			
		Elastic theory		Plastic theory	
Rigidity		Fully rigid	Semi-rigid	Fully rigid	Semi-rigid
Likelihood of occurrence of stated rigidity		Very likely	Possible	Less likely	Very likely
Data of frame		Applicabillity of connections with the above qualifications in braced and unbraced frames calculated according to the mentioned theories.			
Load distribution according to:	Lateral rigidity				
First order of elasticity	braced	no problem normally applied	joint behaviour should be known	no problem	no problem
	unbraced			[4]	joint' should be known
Theory of elasticity including geometrical non linearity	braced	[4]	Sugimoto 34 Jones 35	[4]	no problem
	unbraced		joint behaviour should be known		may behaviour dangerous [7]
First order theory of plasticity	braced	not permitted		[4]	no problem
	unbraced				joint behaviour should be known
Theory of plasticity including geometrical non linearity	braced			[4]	[1]
	unbraced				joint behaviour should be known

Fig. 1.2: Overview of the possibilities associated with the design of a structural frame and connections.

The consequence is that such a connection is in general less rigid than a connection designed on the basis of elastic theory. From the foregoing it appears that it is sometimes necessary to apply plastic theory unless stiffeners for the column flanges are installed. The principle adopted in this report is that the designer will try to avoid as much as possible the use of stiffeners because fitting and welding them are expensive. In some cases, however, their use cannot be avoided, and for this reason the design procedure for connections with stiffeners is also indicated.

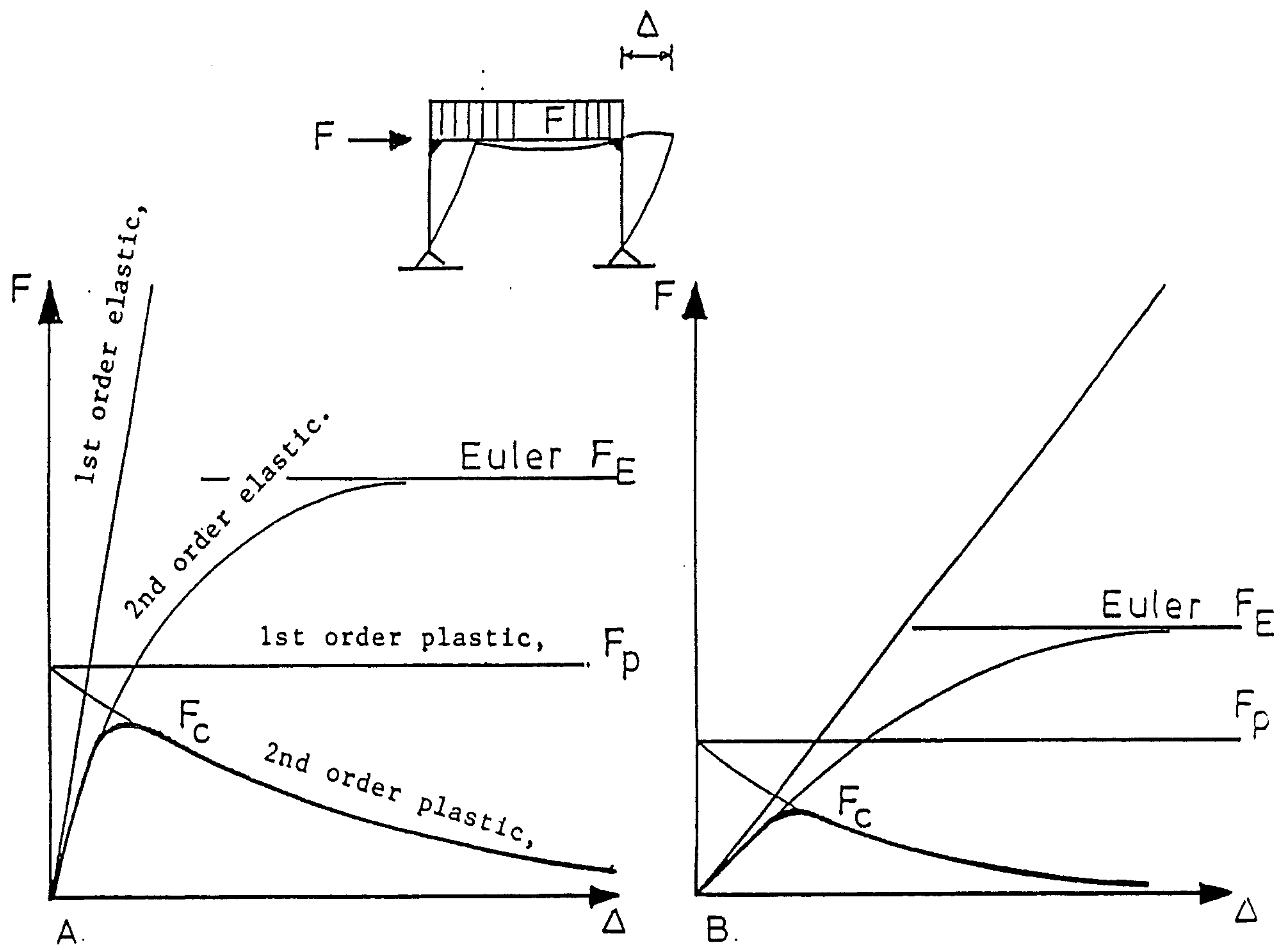
1.1.3. Relations between the frame and the behaviour of the connections

Which relations between the design methods for frames and for the structural connections require further elucidation is apparent from Fig. 1.2. The behaviour of frames designed by the elastic method, with fullrigid connections likewise of elastic design, is known. This used to be the design procedure in all cases.
Connections designed by the elastic method which behave elastic ally up to failure are not permissible in frames of plastic design unless such connections are stronger than the connected beams.
On the other hand, connections designed by the plastic method are allowed to be used in frames designed by the elastic method. In such cases the stiffness of the connection need not be known if the frame is not susceptible to second-order effects. But in the frame is susceptible to second-order effects, there are misgivings as to employing plastically designed connections which are semi-rigid.
The above mentioned points will be examined in this Chapter. In Fig. 1.2 the names of Sugimoto /34/ and Jones /35/ are mentioned with reference to braced frames designed by the elastic method and taking account of second-order effects. Those investigators use the stiffness of the connection with attached beam to reduce the effective (buckling) length of the braced column.
Snijder et al. /33/ have shown that this entails consequences for the strength of the beam, because the resistance that the connection offers to column buckling has to be provided by the beam.

In this Chapter only those situations will be considered where the rotational resistance of the connection is utilized for increasing the overall stability of the frame or the strength and stiffness of the connected beam.
For those situations where the rotational resistance stiffness of the connection with the attached beam is utilized for reducing the effective length of the column the reader is referred to the above-mentioned publications.
First, the most general case of the unbraced frame is discussed.
Next, ways and means of simplifying the analysis are dealt with which are possible on increasing the stiffness of the connection and/or the overall rigidity of the frame. Particularly for braced frames it proves possible to simplify the analysis of the frame (calculation of the distribution of forces) and of the deformation behaviour.

1.2 Elastic analysis or plastic analysis

According to the draft design code /2/ the structure can be analysed, i.e., the distribution of forces in it be calculated, on the basis of elastic theory or of plastic theory. This is true of all structures in which the distribution of forces is not susceptible to the effect of lateral displacement (sidesway such as braced frames or very rigid unbraced frames). In ref. /4/, which gives a classification into several categories, these structures are assigned to categories I and II, comprising frames in which the lateral displacements have no influence on the distribution of forces. In category II, however, the risk of instability of an individual column is not negligible, and additional rules for checking are available for such frames.
In the case of unbraced frames in categories III and IV a first-order plastic analysis will not always suffice. This will be explained with reference to Fig. 1.3. In an unbraced frame the column ends can undergo displacement in relation to the system lines of the frame in its undeformed condition. Because of these displacements the axial forces in the columns cause additional bending moments (second-order moments) and additional displacements (second-order displacements).



A. Situation with fully rigid and full-strength connections in an unbraced frame compared with the situation in case B.

B. Situations with reduced strength and stiffness of the connections in an unbraced frame compared with the situation in case A.

In consequence of reduced stiffness the Euler buckling load decreases and the deformations increase. The Euler buckling load is determined from an elastic calculation.

In consequence of reduced strength the load at which the frame collapses, as determined by a first-order plastic analysis, becomes less.

Fig. 1.3: Schematic representation of the influence of reduced stiffness (or rigidity) of the connections on the design strength of an unbraced frame.

As a result of the additional moments it may occur that the ultimate (or failure) strengths of the connections and other structural components are exceeded at a lower magnitude of the load on the frame than follows from a first- order plastic analysis. The maximum load that can be supported is therefore partly dependent on the product of lateral displacement and vertical load on the columns.
Fig. 1.3 schematically indicates the load-deformation characteristics that can be determined with the respective methods of analysis. The characteristic which approximates most closely to reality is represented by the heavy curve. This characteristic is obtained when the additional moments (second-order effects) due to lateral displacement of the frame and also the development of plasticity in parts of the structure are taken into account.
For complex structures the actual force-displacement curve can be satisfactorily approximated only by making use of a computer program which duly takes account of second-order effects and the development of plasticity in parts of the structure.
The draft code /4/ is so arranged that the maximum load represented by the curve can be approximated by performing a first-order elastic and a first-order plastic analysis and applying the relevant checking rules.
The Euler buckling load /5/ of the structure is determined from the first-order elastic analysis. The maximum load F_c is calculated from the formula:

$$1/F_c = 1/F_E + 1/F_p \qquad (1.1)$$

where:

F_c = loadbearing capacity of the frame

F_p = loadbearing capacity of the frame determined on the basis of first-order plastic theory involving the formation of a collapse mechanism

F_E = theoretical elastic buckling load (Euler) of the frame.

The maximum load F_c is in this case the design strength of the frame.
The force-displacement diagrams in Fig. 1.3b show what happens if, instead of fully-rigid and full-strength connections, semi-rigid and partial-strength connections are employed.

Because of the lower stiffness of the connections the frame is less rigid, so that the Euler buckling load is reduced. And because of the lower strength of the connections the loadbearing capacity of the frame determined by first-order plastic theory is likewise reduced. The combination of reduced rigidity and/or strength of the frame results in lower design strength.
The Euler buckling load of the frame is calculated for a particular stiffness (or rigidity) of the connections. This stiffness can be calculated with the formulae given in section 2.9. These formulae are so contrived as to enable the actual moment-rotation characteristic to be approximated as closely as possible (see the dotted curve in Fig. 1.4a).
Another possibility is to adopt a bilinear diagram as the approximation to the moment-rotation characteristic. The ascending branch of the bilinear diagram connects the origin to the point obtained from the stiffness analysis at the design strength of the connection (see the dotted straight line in Fig. 1.4a). It is a safe assumption to adopt this last-mentioned stiffness in calculating the Euler buckling load of the frame. This will be illustrated with the aid of a simple example (see Fig. 1.4b).
Consider an infinitely stiff column supported by a spring. This column and spring assembly is a model often used in studying the stability of a frame. The spring corresponds to the lateral stiffness of the frame. More accurate results are obtained by conceiving the frame as composed of several infinitely stiff columns with springs, because the actual buckled shape of the frame can thus be better approximated. For the theoretical background see ref. /5/.

The stiffness of the frame is in this case represented by a spring with a characteristic as shown in the right-hand diagram in Fig. 1.4b. The value of the vertical force F for which the infinitely stiff member just ceases to be in equilibrium follow from determinations of the equilibrium of moments about the hinge at the base of that member. It is called the critical load. Its value is found rapidly to decrease when the force acting in the spring reaches the second branch of the spring characteristic (see Fig. 1.4c). The critical load is comparable to the Euler buckling load of the frame, the stiffness of the frame being represented by the spring.

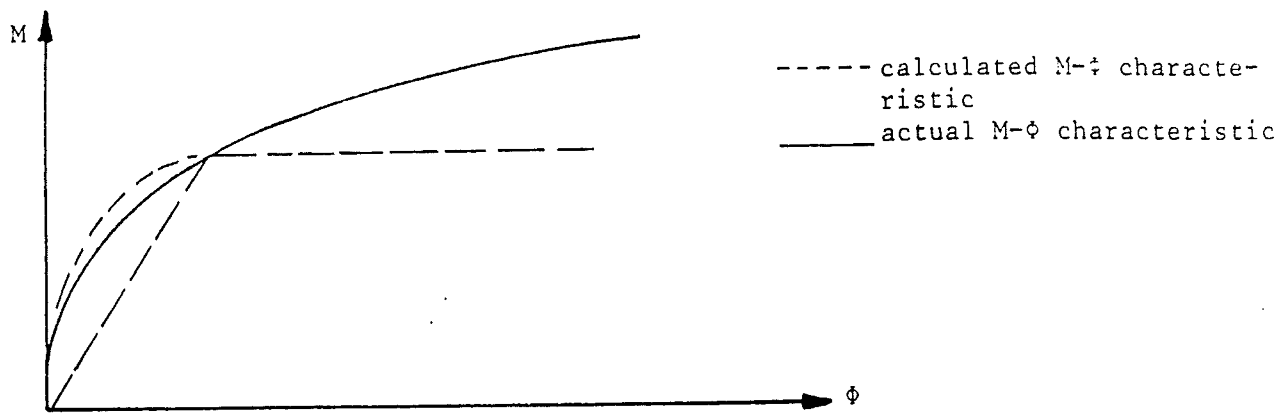


Fig. 1.4a: Moment-rotation characteristic of the structural connections.

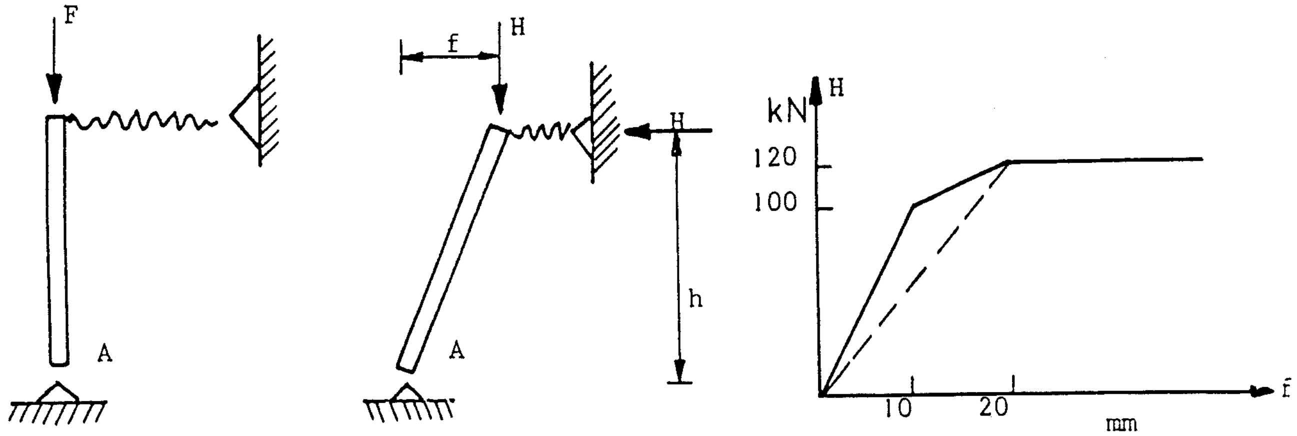
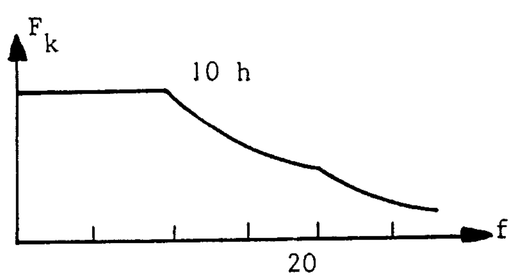


Fig. 1.4b: Schematization of the structure with spring characteristic for lateral displacement.

From the equilibrium of moments about point a.follows:			
f 10 mm	F * f = f * 100/10	* h	F_k = 10 h kN
f = 11 mm	F * 11 =(100 + 1 * 20/10)	* h	F_k = 9,3 h kN
f = 12 mm	F * 12 =(100 + 2 * 20/10)	* h	F_k = 8,7 h kN
f = 13 mm	F * 13 =(100 + 3 * 20/10)	* h	F_k = 8,2 h kN
f = 14 mm	F * 14 =(100 + 4 * 20/10)	* h	F_k = 7,7 h kN
f = 20 mm	F * 20 =(100 + 20)	* h	F_k = 6 h kN
f = 30 mm	F * 30 = 120	* h	F_k = 4 h kN



Conclusion: The critical load rapidly decreases if the spring stiffness is no longer linear,but equilibrium continues to be possible if the vertical load decreases.

Fig. 1.4C: Approximation of the critical load of the structure.

Fig. 1.4: Influence of the stiffness of the connections on structural stability.

If we continued to work with the initial spring stiffness in determining the Euler buckling load while increasing the load. no equilibrium would be possible.

But if we adopt the critical load calculated with the spring stiffness represented by the dotted line in Fig. 1.4b, equilibrium is possible until the load attains the design strength of the spring.

The conclusion is that a safe value for the Euler buckling load of the frame is calculated when the spring stiffnesses of the connections are adopted which exist when the connections attain their design strength.

In the above example it has been assumed that the force-deformation characteristic of the frame can be represented as a diagram comprising three branches. In reality the force-deformation characteristic of the frame is a curve which is influenced by the loads in the connections. By taking account of the stiffnesses of the connections as described above we assume all the connections to be loaded to the design strength.

In reality the forces in many of the connections will be smaller than corresponding to the design strength when the frame is subjected to its design load; the stiffnesses of those connections are therefore greater than the values adopted in the calculations. This means that many reserves of safety are incorporated in the proposed method of analysis, which has been devised in this way because the actual stiffnesses of the connections can be determined only with a limited degree of accuracy, as is explained in section 2.9.

In the foregoing it has chiefly been explained how the flexibility of semi-rigid connections affects the distribution of forces in the structure.

It will be shown below that this influence is negligible in many structures, more particularly those structures which according to ref. /4/ should be checked in accordance with criteria of the categories I and II. These will, in the further treatment of the subject, be referred to as "braced" structures. The structures assignable to the categories III and IV will be referred to as "unbraced".

First, we shall take a closer look at the assertion that elastically designed connections must not be used in frames analysed on the basis of plastic theory if the connections are not full-strength connections, whereas there are no restrictions upon using plastically designed connections if the frame is analysed on the basis of elastic theory.

1.3 Plastically designed connections in elastically designed frames

In the analysis of a frame (calculation of the distribution of forces in the frame) on the basis of elastic theory it is not the limit state of the frame that is calculated, but the bending moments that occur in it in consequence of the design load.
If the ratio of the Euler buckling load of the structure to the design load is low (< 10), second-order effects must be taken into account.
This is done by applying an amplification factor n/(n-1) to all the moments that cause lateral displacement /6/, where n denotes the ratio of the Euler buckling load to the design load. If the moments obtained in this way are less than the design strengths of the connections, the structure is satisfactory.
The design strengths of the connections are calculated with the formulae given in Chapter 2. Those formulae have been derived on the basis of plastic theory, which means that plastic deformations may have occurred when the connections have attained their design strengths.
In section 1.2 it has already been explained that this is taken into account in calculating the Euler buckling load if we adopt for the stiffnesses of the connections those values which correspond to the design strength and can be calculated with the formulae given in section 2.9. These are lower stiffness values than actually occur in those connections which are less several loaded, i.e., not up to their design strength.
On the other hand, the moments which are produced in the connections by the vertical loads on the beams are found to decrease when the connections are reduced in stiffness. This will be considered in section 1.7.
Therefore, in order to ensure that the actual moment will not be larger than the calculated value, the stiffness of the connection should be adopted at its highest possible value in the calculation. This is at variance with the earlier advice that relatively low stiffness associated with the design strength of the connection should be adopted.
However, in section 1.7 it is also shown that the adoption of a lower stiffness for the connection in the calculation will not adversely affect the connection if it has sufficient capacity for deformation. This can be obtained by allowing plastic deformation in the connection.

This means that unbraced frames will be safely analysed if low values of the stiffnesses of the connections are introduced, provided that the connections can deform sufficiently. If they cannot deform sufficiently (elastically designed connections), it would in principle be necessary to carry out two analyses - one with low stiffnesses, the other with high stiffnesses of the connections. The second of these analyses is not necessary if the connections are stronger than the connected beams or have sufficient deformation capacity.
In the case of braced frames a safe analysis is obtained by neglecting the flexibility of the connections (see section 1.7). However, a more economical design will be obtained by adopting a low stiffness of the connection in the calculation, provided that sufficient deformation of the connection is possible.
From the foregoing it follows that in frames which are to be designed on the basis of elastic theory it is advisable to use connections which can undergo plastic deformation. These connections should be designed on the basis of plastic theory. In calculating the Euler buckling load and the deformations of the structure it will be necessary to allow for the lower stiffness that plastically designed connections may have.

1.4 Elastically designed connections in plastically designed frames; required deformation capacity and associated stiffnesses of connections

In a structure in which more than one plastic hinge must develop before the assumed collapse mechanism is attained the first plastic hinge and the then following ones will have to deform so much that the last hinge can also develop. This applies both to braced and to unbraced frames.
Partial-strength connections which remain elastic up to failure must not be used in such structures because these connections have insufficient deformation capacity.
A method of determining the required rotational or deformation capacity of connections in braced frames is given in section 1. The required rotational capacity in unbraced frames is higher. From simple "manual" calculations it can be inferred that this required rotational capacity need be no more than 0.04 radian for most frames.

Basing oneself on this value it may be asked whether the calculation of the Euler buckling load as laid down in section 1.3 is indeed appropriate. This will be examined with reference to the moment-rotation characteristics and force-deformation characteristics as represented in Fig. 1.5. Suppose that the distribution of forces in the structure shown in Fig. 1.5 is such as to give rise to the successive formation of the plastic hinges 1 to 4, as found in a first-order plastic analysis (see Figs. 1.5b and 1.5c). Now if the first hinge develops a rotation of 0.04 radian by the time the last hinge is formed, the first hinge will have the notional stiffness as indicated by the dotted line in Fig. 1.5a. The stiffness is less than the notional stiffness with which the Euler buckling load has been calculated.

Application of formula (1.1) with which the vertex of the actual force-deformation characteristic is approximated is based on a combination of the results of the first-order plastic analysis and the calculation of the Euler buckling load without taking account of the reduction in stiffness due to the formation of plastic hinges. In the procedure proposed here, the Euler buckling load is calculated for a stiffness of the structure which is less than envisaged by formula (1.1).

In reality not all the connections will have attained their design strength when the vertex of the force-deformation characteristic has been reached. The proposed procedure is therefore on the safe side.

1.5 Design example of an unbraced frame

The effects of semi-rigid and partial-strength connections on the strength are well illustrated in an example which Tautschnig /7/ gives in his dissertation (see Fig. 1.2). He has written a computer program which takes account of second-order effects and can also take account of actual moment-rotation characteristics determined experimentally (see Fig. 1.2). He compares the results of the calculations for this example with those for a frame assumed to have full-strength fully rigid connections and those for a frame in which the connections have the bilinear moment-rotation characteristic with the design stiffness and strength obtained from the formulae of Bakker and Voorn /8/.

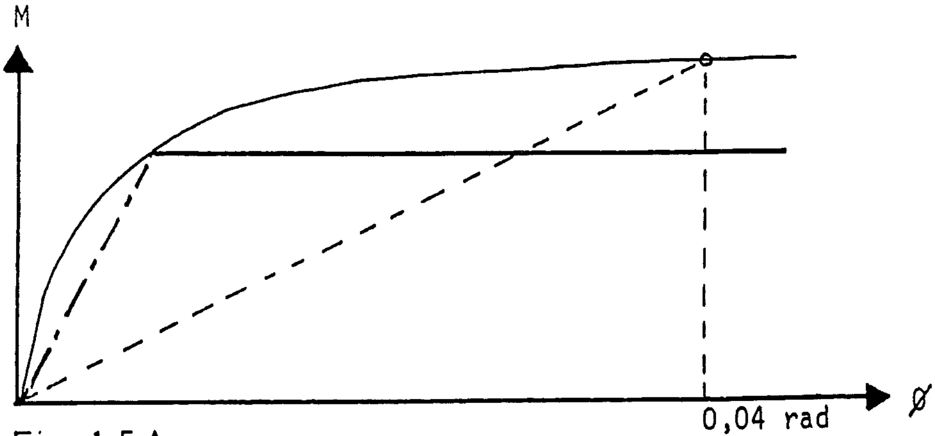


Fig. 1.5A

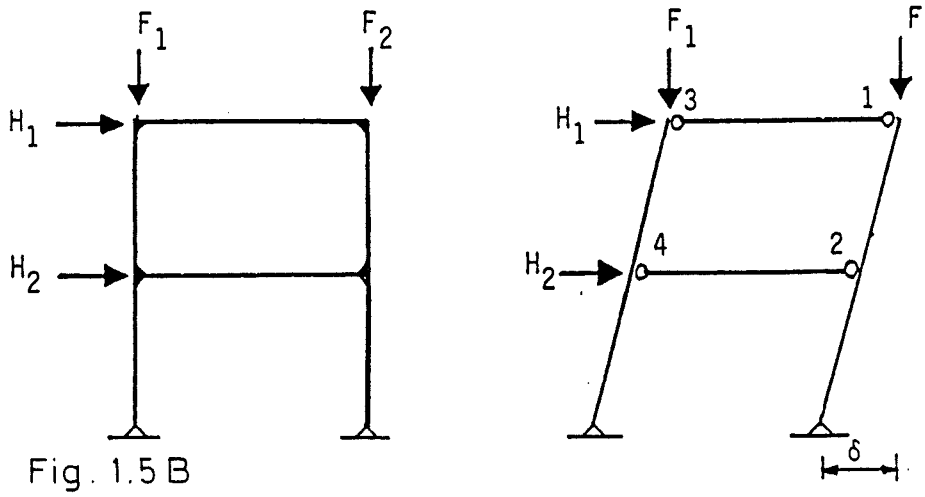


Fig. 1.5B

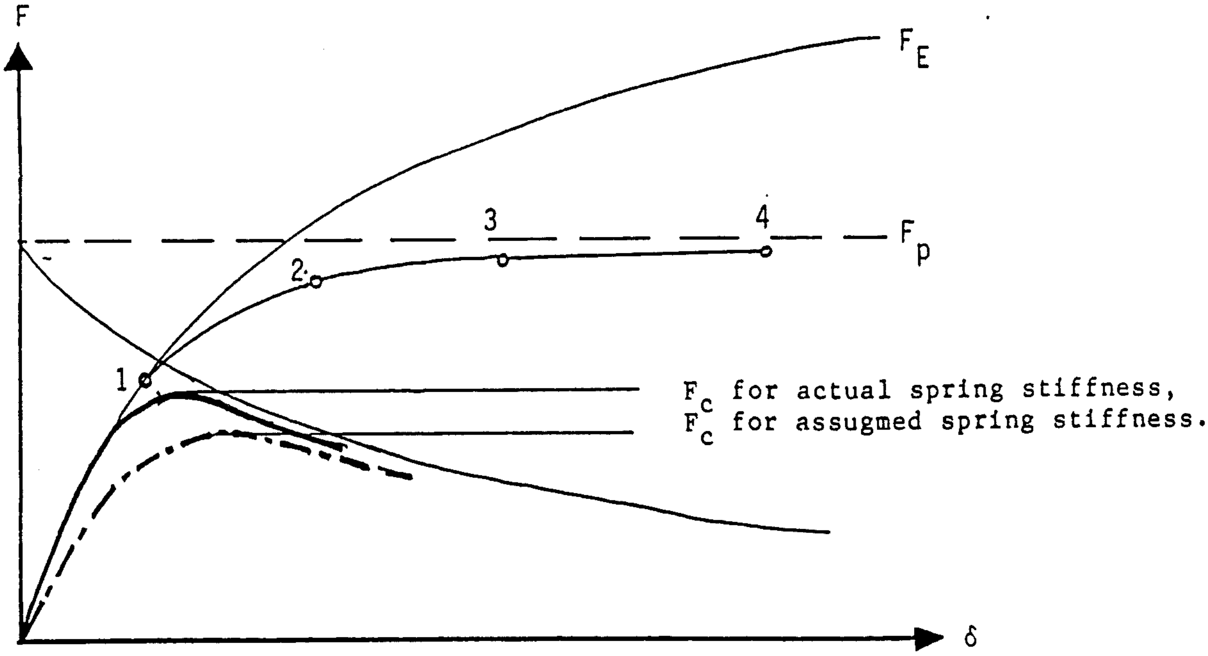


Fig. 1.5C

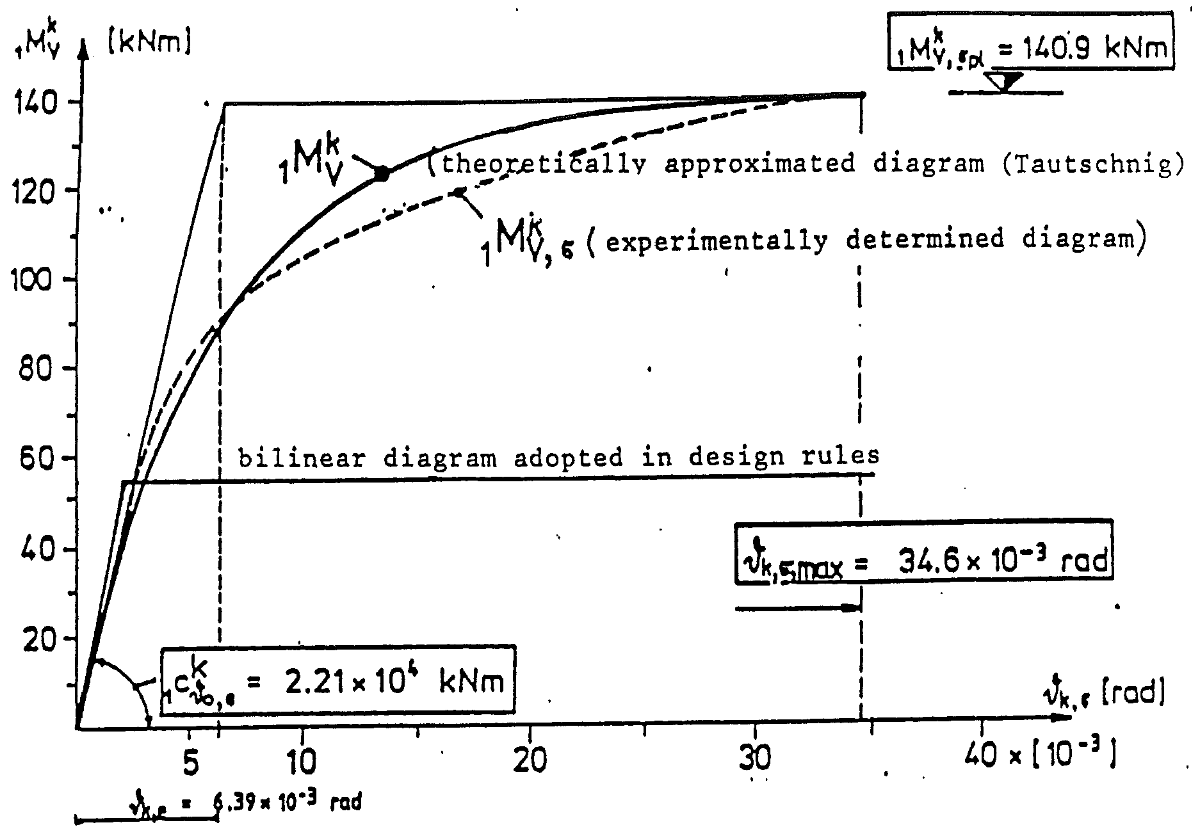
Fig. 1.5: Spring stiffness of the connection to be taken into account is not determined by the required rotational capacity;

On the basis of these calculations Tautschnig arrives at the, in our opinion, incorrect conclusion that connections without stiffeners should, for the present, not be employed in unbraced frames because of their sensitivity to lateral load.

The results of the calculations are considered in Appendix B. The analysis of the frame with the bilinear moment-rotation characteristic of the connections has been done over again because the values which Tautschnig used are not in agreement with our present rules. The design strength of the connections in the frame without stiffeners is determined by shearing of the column web. In that case the design rules given by Bakker and Voorn are no longer in agreement with our present rules either. Experimental research /9/ has shown the interaction of compression and shear according to the Huber-Hencky-von Mises yielding criterion to be negligible.

The results of the calculations are presented in Fig. 1.6. It emerges that the design strength of the frame for which Tautschnig takes into account the actual moment-rotation characteristics of the connections is considerably higher than the value calculated with the bilinear moment-rotation characteristics. This difference is due to the large difference between the actual failure moment of the connection and the design strength (factor of 2.6). This large difference had already been observed earlier in tests in which the failure moment is determined by shear of the column flange /10/. The reality may be much more favourable than the design rules suggest. Tautschnig rightly considers that this favourable effect should be utilized, since the design strength of many structural frames is indeed governed by column web shearing. The solution that he proposes is not yet practicable, however, because it is based on the experimental determination of the moment-rotation characteristics of the connections to be used (see design example in Appendix A).

That the design strength of the frame calculated with the actual moment-rotation characteristic is so much higher than that calculated with the bilinear diagram is of little importance in the present context if the requirement is imposed that the lateral displacement under service load is not allowed to exceed 1/200 of the height. If the frame attains this permissible displacement, the load according to the calculation is less than the design strength of the frame divided by a factor of 1.5. Therefore the deformation criterion governs.



moment-rotation characteristics used in the calculations of appendix B

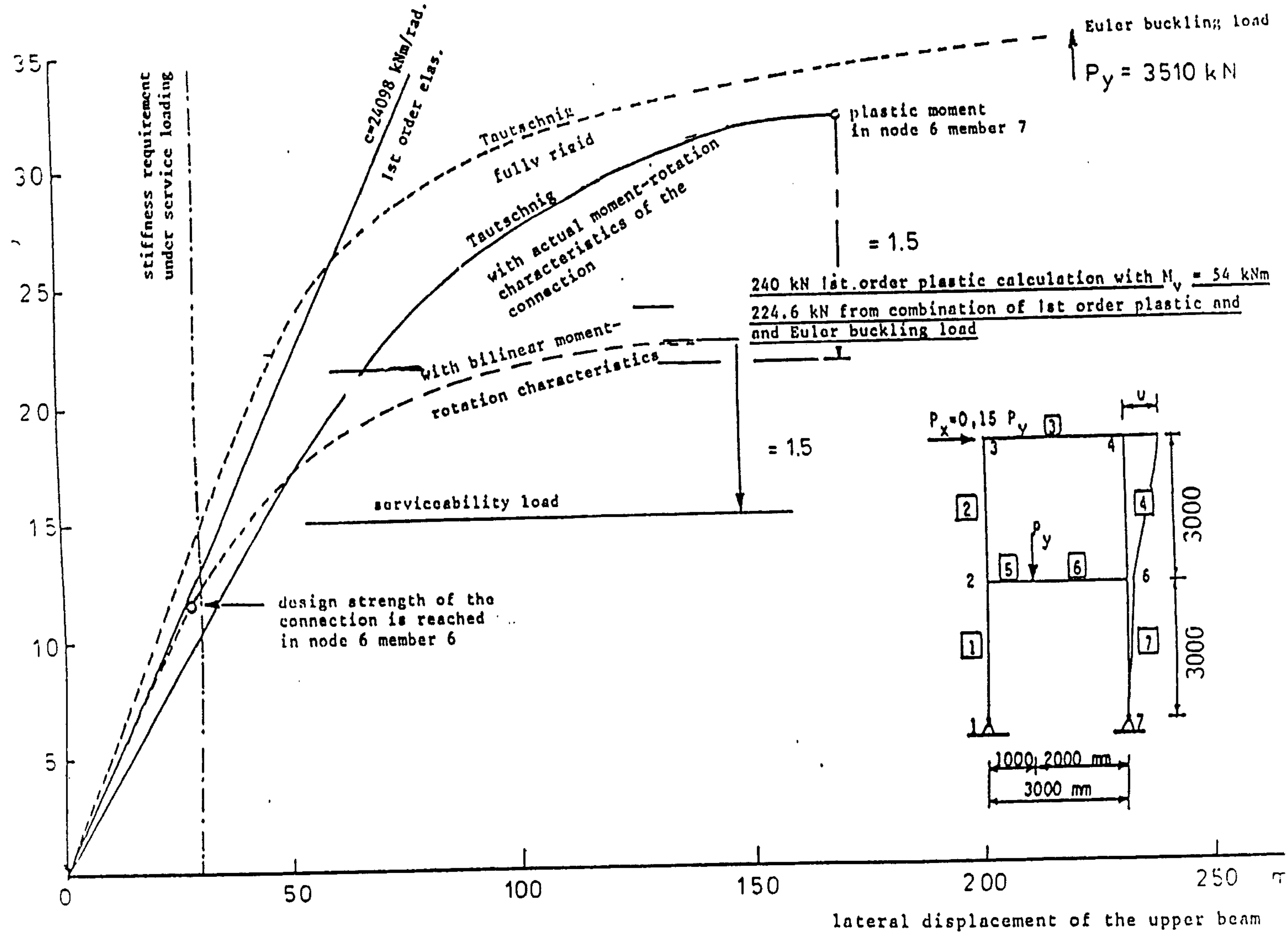
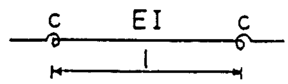
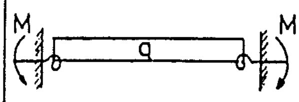
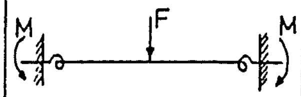
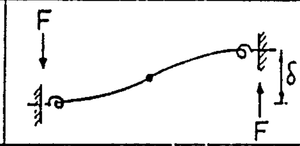
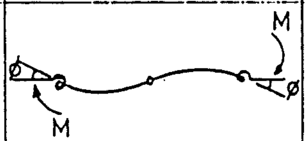
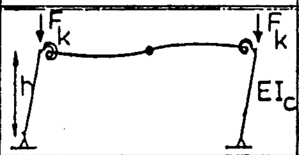
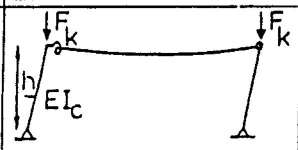


Fig. 1.6: Summary of force-displacement curves for an unbraced frame calculated by various methods (see Appendix A);

If the stiffness requirement is reduced to 1/100 of the height, the design strength becomes the governing criterion if the calculation is based on the bilinear diagram of the connection. In that case the design strength of the frame (224.6 kN), approximated with formula (1.1) by combining the Euler buckling load (3510 kN) and the loadbearing capacity based on first-order plastic theory (240 kN), comes closest to the last-mentioned value, because the ratio of the two values in question is high ($F_E/F_p = 14.6$).
If the calculation were based only on elastic theory and the bilinear diagram, the design strength of the frame would turn out very low (116 kN). Yet this is preferable to the conclusion that Tautschnig draws from his calculations, namely, that connections without stiffeners should, for the present, not be used in unbraced frames because of their sensitivity to load causing lateral displacement (sidesway). It is not clear why this should not be permitted, provided that the lower stiffness of those connections without stiffeners is taken into account.
Furthermore, the influence of the lower stiffness of the connections is strongly dependent on the stiffness ratios of the frame. This will be shown in the next section.

1.6 Can the flexibility of the connection be neglected?

This will depend on the stiffness ratios, as will be explained with reference to Fig. 1.7, which represents the relations between loads, moments and deformations for some commonly encountered situations for unbraced frames. Each of these relations is expressed in the corresponding relation for infinitely rigid connection multiplied by a reduction factor R which is a function of the factor k.
The latter is the ratio of the spring stiffness of the connection to the flexural stiffness of the beam, i.e., $k = c\ell/EI$, where c denotes the design stiffness of the connection. The reduction factor R in all cases increases from 0 to 1 with increasing value of k.
The last two columns in Fig. 1.7 give the reduction factor R with respect to the situation for $k = \infty$. Under the table of factors these relations have been plotted in graphic form for the examples 1, 3 and 4.

		review of formulas R is a function of $k = \frac{cl}{EI}$		$\frac{I_c l}{I h}$	$\frac{R(k=\dots)}{R(k=\infty)}$ k = 0,5	k = 25
1		$\frac{M}{q} = R\frac{l^2}{12}$	$R = \frac{k}{k+2}$		0,20	0,93
2		$\frac{M}{F} = R\frac{l}{8}$	$R = \frac{k}{k+2}$		0,20	0,93
3		$\frac{F}{\delta} = R\frac{12EI}{l}$	$R = \frac{k}{k+6}$		0,08	0,81
4		$\frac{M}{\phi} = R\frac{4EI}{l}$	$R = \frac{k^2+3k}{k^2+8k+12}$		0,10	0,84
5		$\frac{M}{\phi} = R\frac{3EI}{l}$	$R = \frac{k}{k+3}$		0,14	0,89
6		$\frac{M}{\phi} = R\frac{6EI}{l}$	$R = \frac{k}{k+6}$		0,08	0,81
7		$F_k = R\frac{\pi^2 EI_c}{4h^2}$	$R = \frac{1}{1+\frac{k+6}{k}\frac{I_c l}{I h}\frac{\pi^2}{24}}$	10 2 1 0,5 0,1	0,09 0,16 0,22 0,33 0,68	0,84 0,90 0,93 0,96 0,99
8		$F_k = R\frac{\pi^2 EI_c}{8h^2}$	$R = \frac{1}{1+\frac{k+3}{k}\frac{I_c l}{I h}\frac{\pi^2}{12}}$	10 2 1 0,5 0,1	0,16 0,21 0,27 0,36 0,69	0,90 0,93 0,95 0,97 0,99

B

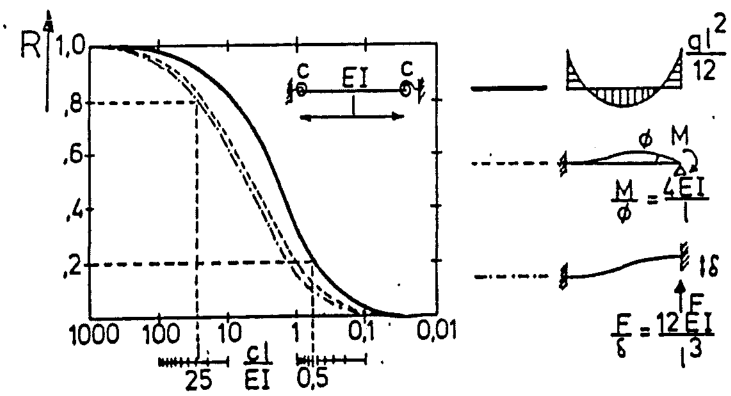


Fig. 1.7 : Relations between loads and deformations as a function of the ratio joint/stiffness/member stiffness and the belonging reduction factor

It appears that for k = 1000 the connection is to be rated as fully rigid, because R = 1. For cases where k > 25 the reduction factor has diminished by less than 20%.
From Fig. 1.3 it is apparent that the design strength of a frame is determined by a combination of the Euler buckling load and the loadbearing capacity determined on the basis of first-order plastic theory. From the examples 7 and 8 in Fig. 1.7 it follows that for k > 25 the Euler buckling load has diminished by less than 10% in comparison with its value for k = ∞ in the case of normal column and beam proportions.
From the information contained in Fig. 1.7 it is inferred that the stiffness of the connections need not be considered in the calculations when k > 25. In all other cases it is possible also to employ semi-rigid connections, provided that the spring stiffness of the connection is taken into account.

1.7 Influence of the stiffness of connections in a braced frame

In analysing a braced frame it is meaningful to take account of the stiffness of the connections only if:

a. it is desired to determine the deformation under service load;
b. the design strength of a beam is defined as the load at which the design strength of the connection is attained.

In both cases an elastic analysis is envisaged. In the second case a higher design strength is obtained if a lower design stiffness of the connection is adopted. This contrasts with the situation for unbraced frames, where a reduction in the stiffness of the connection may cause greater second-order effects so that larger moments occur.
The favourable effect of low stiffness of the connections in a braced frame is apparent from Fig. 1.8. In this diagram the values on the horizontal axis represent the design strength of the connection as a dimensionless quantity obtained by dividing it by the plastic moment of the beam. Those on the vertical axis represent the design strength of the beam as the product of uniformly distributed load and the square of the span divided by the plastic moment of the beam.

The relation between the design strength of the beam on the vertical axis and the design strength of the connection on the horizontal axis is given by the sloping lines through the origin. The slope of each of these lines depends on the ratio k of the stiffness of the connection to the stiffness of the beam. The derivation of this relation is given in Fig. 1.8.
From the diagram it appears that the strength of the beam decreases if the stiffness of the connection increases. But the diagram does not allow for the case where the stiffness of the connection may be so low that the plastic moment of the beam at mid-span is attained before the design strength of the connection. The effect of this situation is shown in Fig. 1.9. The intersections of the horizontal lines in this diagram and the sloping lines in Fig. 1.8 are all located on one straight line. This line represents the situation where the mid-span plastic moment and the design strength of the connection are attained simultaneously (see Fig. 1.10). Theoretically a beam mechanism then develops. This is the starting point of a first-order plastic analysis. Despite the stiffness of the connection, this situation can always be attained provided that the connection possesses sufficient deformation capacity. An example will serve to illustrate this.
Consider a connection with a moment-rotation characteristic as shown in Fig. 1.11. With the formulae of Chapter 2 the design strength (moment capacity) of the connection is calculated as 0.4 M_p, and a spring stiffness value c is obtained which, in combination with the data of the beam, gives k = 6. The actual spring stiffness is found to be higher. It will be shown here that this does not matter. For this purpose the so-called beam line concept will be employed, which will now first be explained:
For a simply-supported beam carrying uniformly distributed load q, with equal end moments M acting at the supports, a formula for the rotation at the ends can be derived (see Fig. 1.1).
Next, a linear relationship between the counteracting support moment and the rotation of the end of the beam can be plotted in a moment-rotation diagram. The so-called beam line intersects the vertical axis of the diagram at a point corresponding to the moment in a beam with fully fixed ends ($q\ell^2/12$) and intersects the horizontal axis at a point corresponding to the end rotation of a simply-supported beam ($q\ell^3/24EI$).

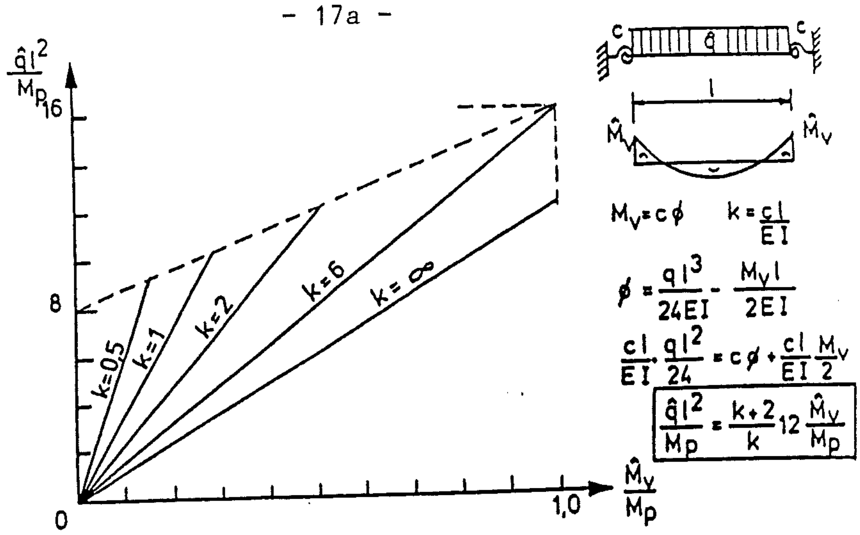


Fig. 1.8 : Design strength of a beam in a braced frame as a function of the joint strength and the ratio between joint stiffness and beam stiffness, $k = \frac{cl}{EI}$, in the case if comparatively stiff joints.

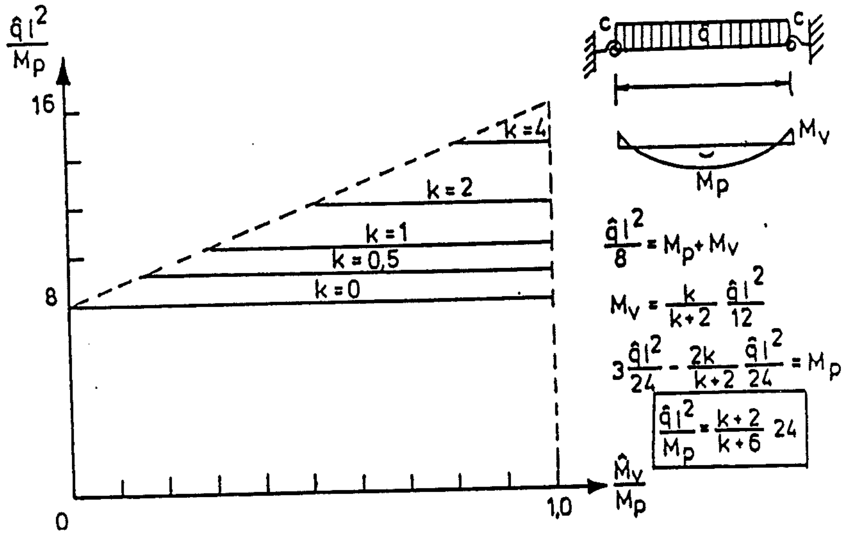


Fig. 1.9 : Design strength of a beam in a braced frame as a function of the joint strength and the ratio between joint stiffness and beam stiffness, $k = \frac{cl}{EI}$ in the case of joints with comparatively low stiffness.

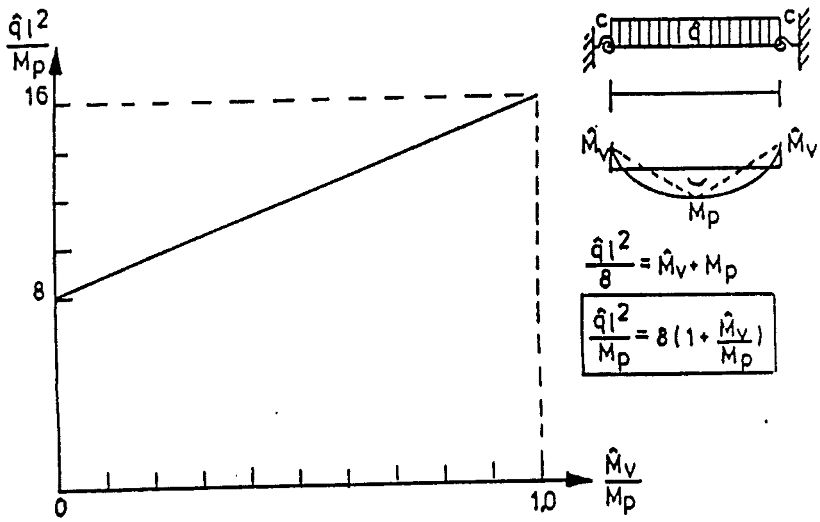


Fig. 1.10 : Design strength of a beam in a braced frame as a function of the joint strength, calculated according to the theory of plasticity.

The intersection of the beam line with the actual moment-rotation characteristic represents the situation of the connection (moment and rotation) or a particular load.

If the actual moment-rotation characteristic of a connection is known, it can be determined what moment and what rotation occur in the connection under various loading conditions. For the moment-rotation characteristic in Fig. 11 this has been done in Fig. 1.13a. The uniformly distributed loads have been so chosen that the design strength $\hat{M}_v = M_p$ is just attained in the connections. These loads have been read, for various values of the stiffness of the connections, in Fig. 1.13b, which is a summary of the diagrams in Figs. 1.8 to 1.11.

The vertical line at $\hat{M}_v = M_p$ first intersects the line $k = \infty$ at a point corresponding to $\hat{q}\ell^2/M_p = 4.8$ and then intersects the lines representing other stiffnesses at points corresponding to the values listed in the following table:

$\hat{M}_v = 0,4\ M_p$ k	$\frac{\hat{q}l^2}{M_p}$
∞	4,8
6	6,4
3	8
2	9,6
1,5	11,2
1	10,3
0,5	9,2
0	8

The values thus found have been plotted in Fig. 1.13a. It emerges that the highest load can be attained when k = 1.5. For that stiffness ratio there exists in the beam an elastic moment distribution which corresponds to a distribution as adopted in an analysis based on plastic theory, namely, 0.4 M_p at the connection and M_p at mid-span.

A designer wishing to base himself only on elastic theory but adopting too low a stiffness for the connection, so that k = 1 is in fact applying plastic theory. Now this need not be dangerous, provided that the connection possesses so much deformation capacity that it can adjust itself to the assumed stiffness at k = 1.5 (shown dotted in Fig. 1.13a).

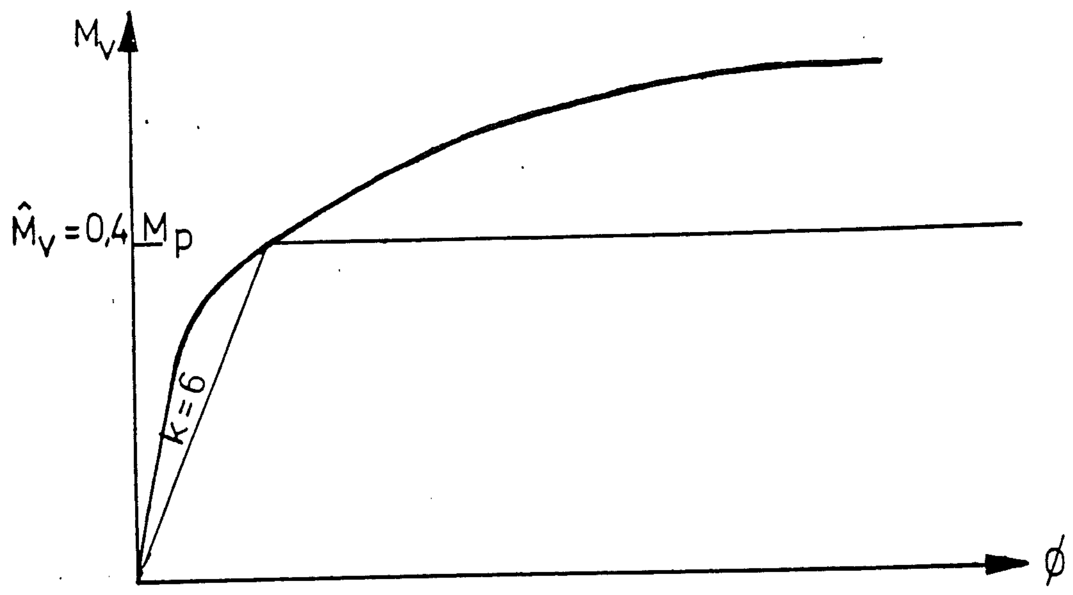


Fig. 1.11: Example of a moment-rotation characteristic and the calculated biliniar moment-rotation characteristic.

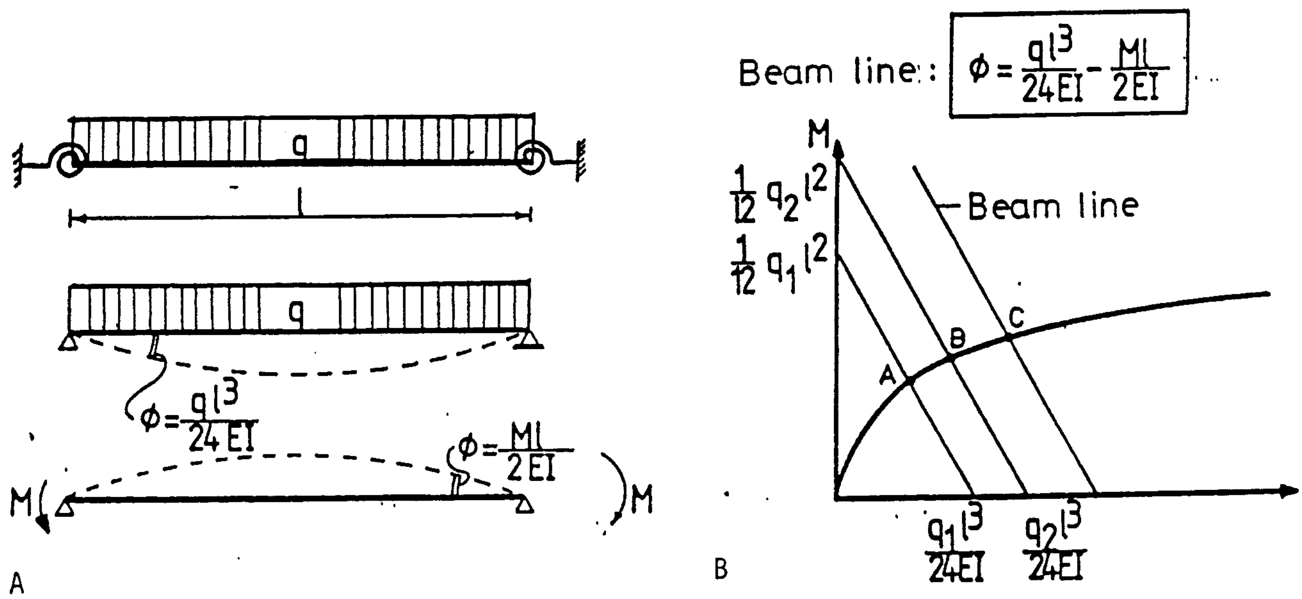


Fig. 1.12: The actual situation in the connection is determined by the intersection of the beam line with the experimentally determined moment-rotation characteristic of the connection

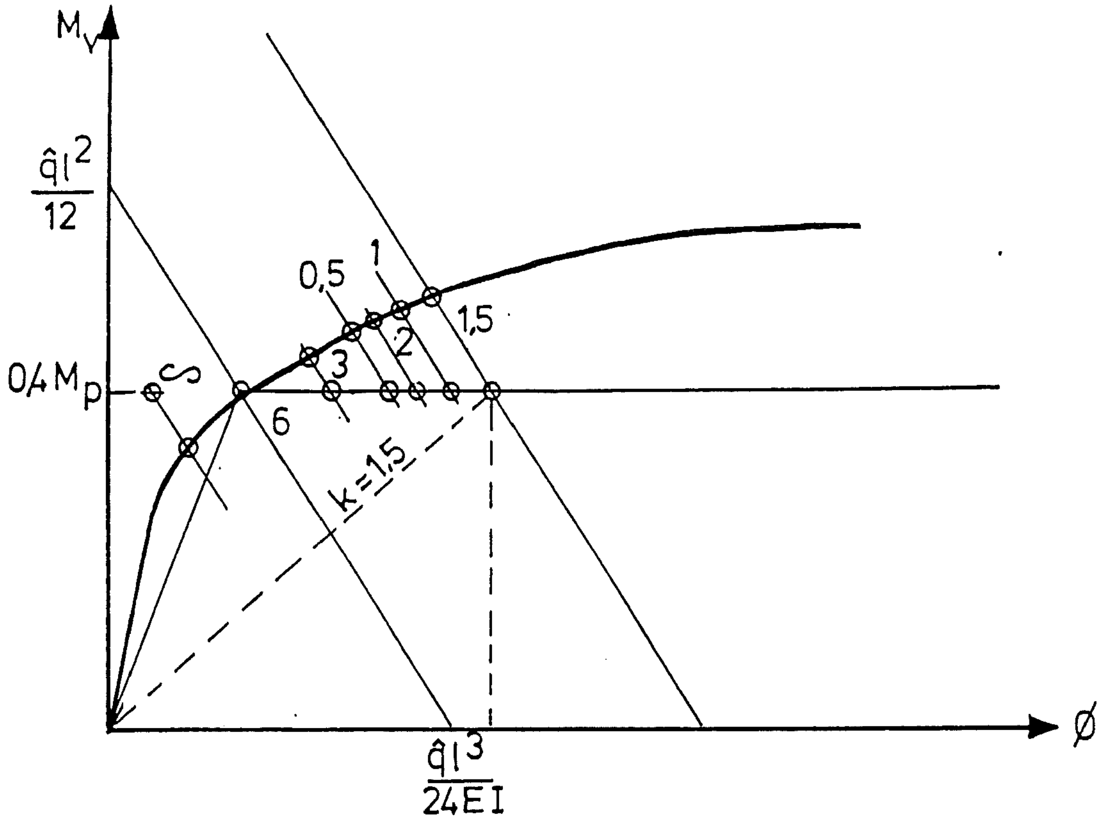


Fig. 1.13a: Intersections of the beam lines with the experimentally determined moment-rotation characteristic and the calculated moment-rotation characteristic.
$\hat{q}$ has been calculated on the basis of elastic theory with the aid of the diagram given below, for $\hat{M}_v$ = 0.4 M_p and k=6.

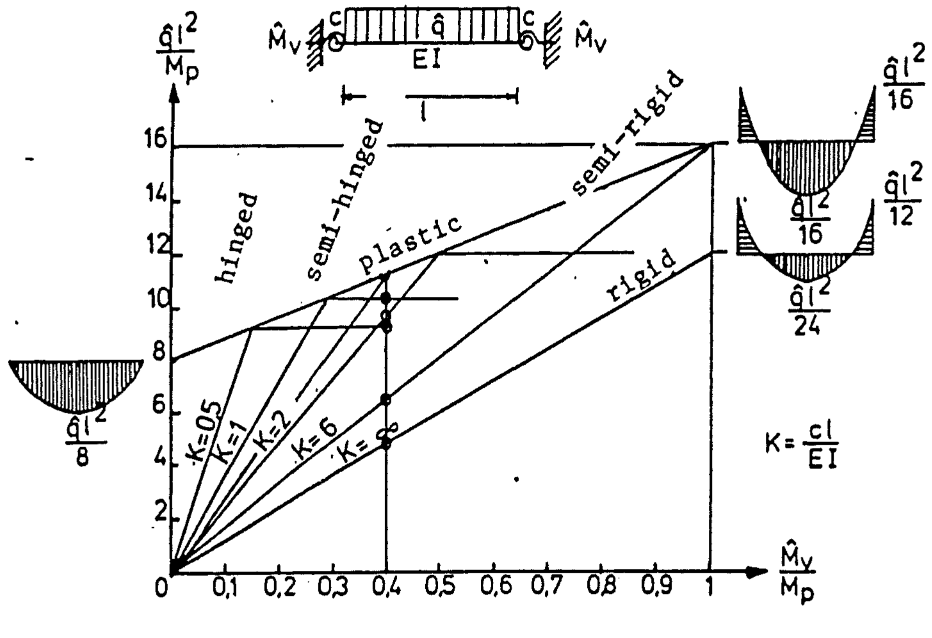


Fig. 1.13b: Relations between loading and the strength and stiffness of the connections and the beam in a braced frame;

Fig. 1.13: Relations between the actual situations and the calculated strength and stiffness of the connections.

In other words: in calculating the design strength of a braced frame it is permissible to neglect the stiffness of the connection if the latter has enough deformation capacity to enable the mechanism assumed in the analysis to be duly attained.

In establishing the formulae for calculating the design strength (moment capacity) of the connection, as presented in Chapter 2 it has been ensured that the connection does indeed possess adequate deformation capacity. This has been achieved by checking all the test results against the beam line concept and formulating further requirements on the basis of this, as will be explained in the next section.
The stiffness of the connection under service load is likewise of importance in calculating the deflection of the beam. In establishing the design rules one of the aims has been to avoid having to calculate deflection. This has been achieved by imposing limits upon the span of the beam, which will also be explained in the next section.

1.8 Considerations with a view to simplifying the analysis of a braced frame

The conclusion from the foregoing is that the stiffness of the connection can permissibly be ignored in calculating the strength of the beam of the connection in question possesses sufficient deformation capacity. This capacity should be so great that the mechanism assumed in the analysis of the frame can indeed be attained.
Another condition to be fulfilled is that under service load the connection is of such stiffness that the deflection at mid-span of the beam does not become too large. Both these conditions have received due attention in verifying the correctness of the design formula with experimental results. This will be explained here.
In the experimentally determined moment-rotation characteristics of a connection (see Fig. 1.14) the beam line has been drawn which corresponds to a load calculated on the basis of a beam mechanism and a particular span/depth ratio (ℓ/h) of the beam. The load is $q = 8\ (\hat{M}_v + M_p)/\ell^2$.

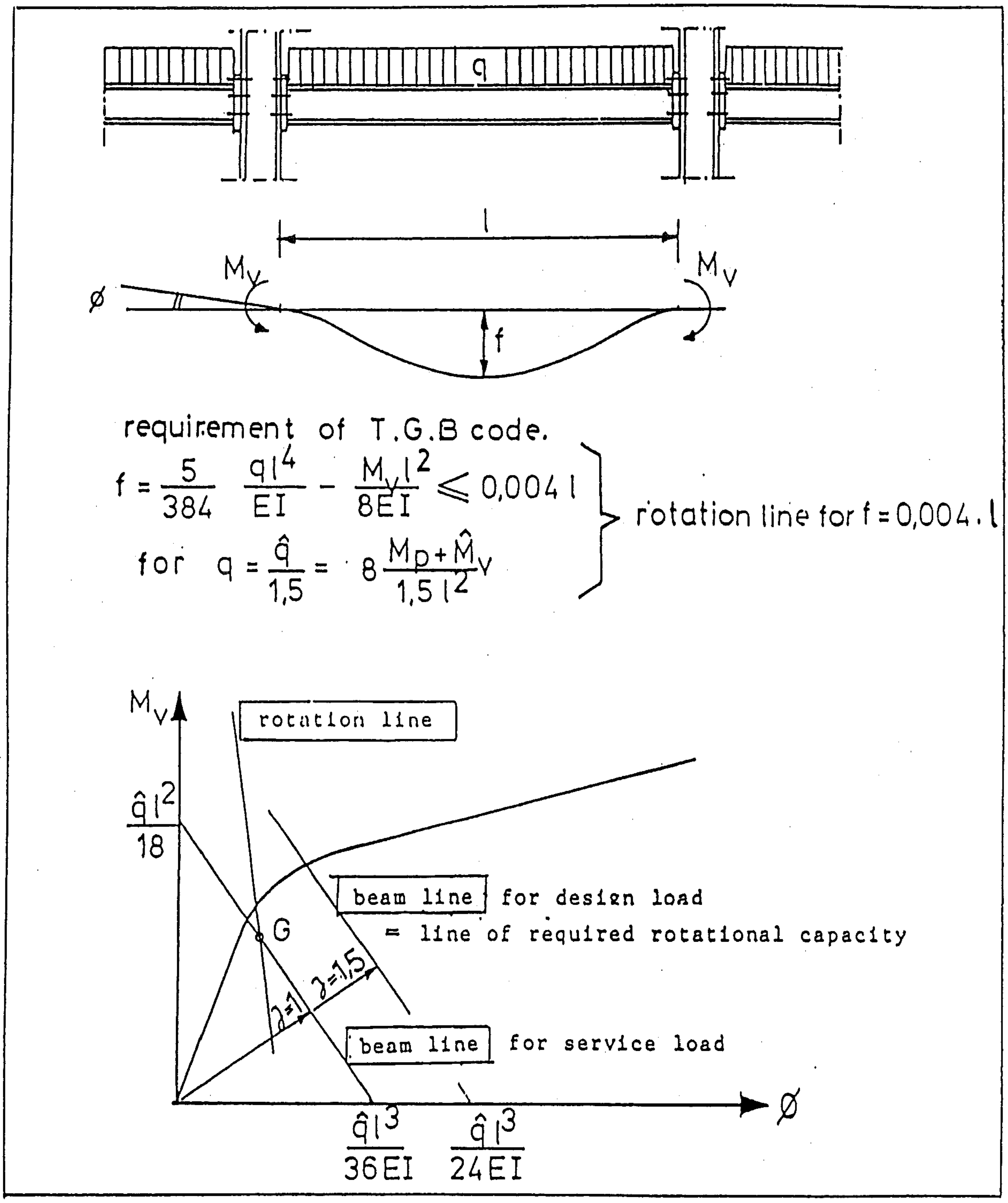


Fig. 1.14: The requirement of sufficient stiffness is represented by point G in the moment-rotation diagram;

The moment-rotation characteristic must intersect the beam line, otherwise there will not be sufficient rotational capacity. Next, the beam line associated with a load which is lower by a factor of 1.5 (= the load factor), i.e., $q = 8\ (\hat{M}_v + M_p)/1.5\ \ell^2$, has been drawn.
The moment-rotation characteristic must intersect this last-mentioned line at a rotation which is less than, or equal to, the rotation corresponding to the permissible deflection of the beam under service load. According to the T.G.B. code this deflection is 0.004ℓ.

This deflection requirement can also be represented by a line in the diagram for the moment-rotation characteristic. The formula for this line is given in Fig. 1.14, where it is called the rotation line. The moment-rotation characteristic must therefore pass to the left of the intersection of the beam line and rotation line (point G in Fig. 1.14). If it fails to do this, it means that the beam exceeds the permissible deflection under a load which is less than that calculated on the basis of the strength formulae and the beam mechanism.
The structure then does not satisfy the stiffness requirement. In that case the designer may adopted a lower value for the load or for the length of the beam. In either event the intersection point G, but also the beam line for the rotational capacity, will move nearer the origin of the diagram. This is apparent, for example, from a comparison of the diagrams presented in Fig. 1.15.
These diagrams give the minimum requirements for the moment-rotation characteristics for spans equal to 20, 25, 30 and 40 times the depth of the beam. They are based on the theoretical considerations described above. Hence they correspond to load determined on the basis of beam mechanisms with end restraint moments which are equal to the said strengths (moment capacities) of the connections, which have been divided by the plastic moment of the beam to obtain dimensionless quantities.
These diagrams are determined by the location of the point G, the design strength $\hat{M}_v$ and the beam line which has to be passed in order to have sufficient rotation capacity for the beam mechanism (see Fig. 1.15b).
There is moreover a line applicable to cases where the beam span/depth ratios are small. Connections possessing low stiffness are then permissible.

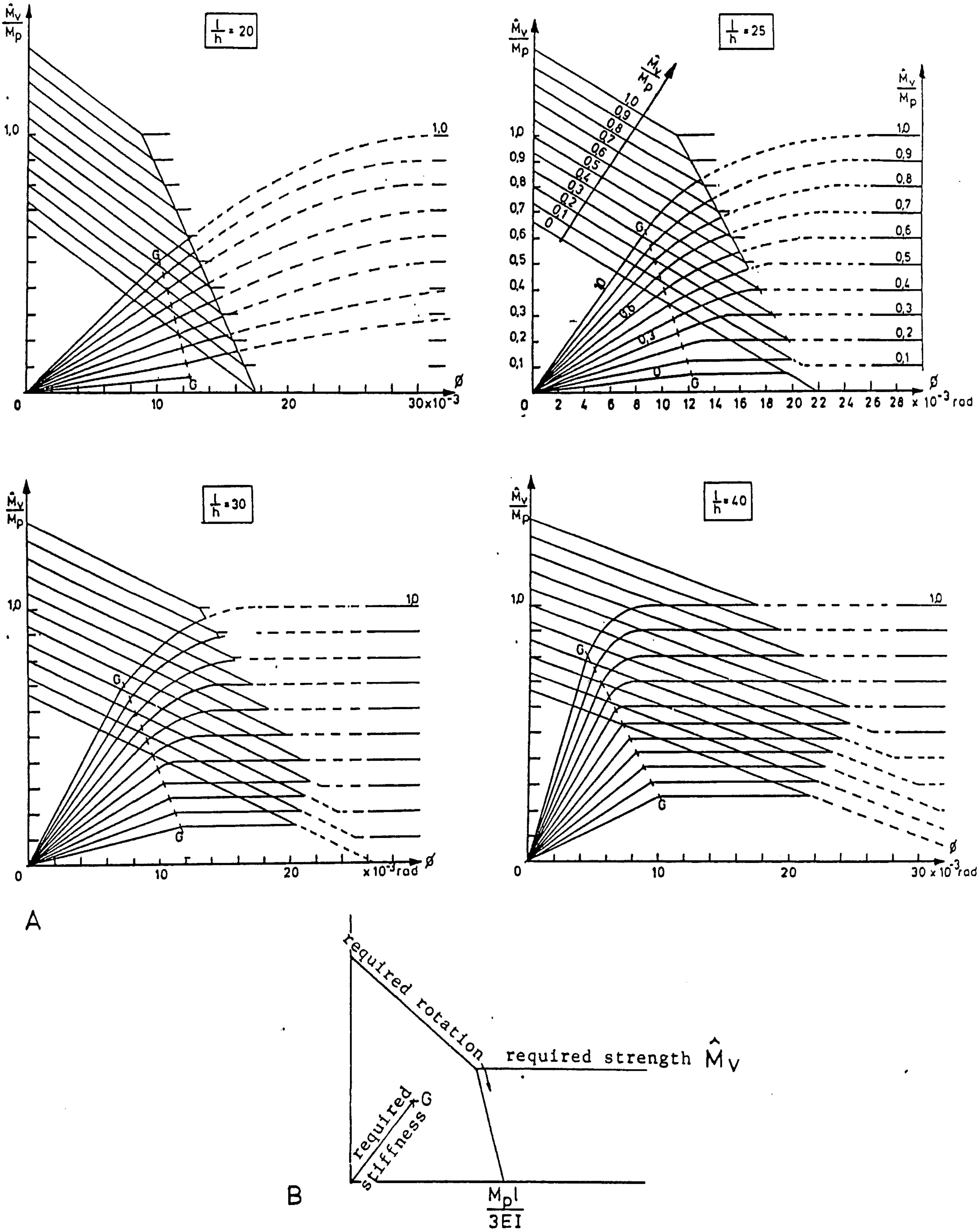


Fig. 1.15: Diagrams with minimum requirements for moment-rotation characteristics;

In such cases it may occur that the plastic moment at mid-span is attained before the design strength of the connection. The line representing this situation extends from the intersection of the beam line with the design strength $\hat{M}_v$ to the point $\phi = M_p.\ell/3EI$ on the horizontal axis, where $\hat{M}_v = 0$.

Figs. 1.16a and b give typical examples of the requirements for short beam spans, as envisaged in Fig. 1.15. Figs. 1.16c and d give typical examples of the requirements for long beam spans. In Fig. 1.16c it is notable that in order to pass to the left of point G the moment-rotation characteristic has to attain a higher value of the moment than the design strength of the connection. This means that the design strength of the connection is exceeded already under service load. In order to avoid this, the span/depth ratio of the beam should be so chose as not to exceed the values listed in the following table. The values have been obtained from Fig. 1.15 and are also stated in the draft design rules for structural connections:

if $\frac{\hat{M}_v}{M_p}$ is:	$\frac{\ell}{h}$ must satisfy:
< 0,1	< 20
< 0,2	< 25
< 0,3	< 30
< 0,4	< 35
< 0,5	< 40

Four moment-rotation characteristics I to IV are also indicated in Fig. 1.16 and will be discussed here.

Connection I with $\hat{M}_v = 0.8\ M_p$ is typically a connection designed for an elastic analysis. If the calculation adopts $\hat{M}_v = 0.8\ M_p$ and a beam mechanism is envisaged, this connection has insufficient deformation capacity for short and for long spans, as is apparent from Figs. 1.16b and d.

The connection does, however, possess sufficient deformation capacity if the load is calculated on the basis of $\hat{M}_v = 0.2\ M_p$ (see Fig. 1.16a). For that purpose it is assumed that $0.2\ M_p$ occurs at the connection and that M_p occurs at mid-span.

In reality, however, 0.8 M_p occurs at the connection and 0.4 M_p at mid-span, this being a moment distribution conforming to elastic theory.
This shows that, although $\hat{M}_v = 0.2\ M_p$ is adopted in the calculation, the parts of which the connection is composed, such as welds, must be designed for a higher value of the moment, namely, 0.8 M_p in this case.
However, this is an extreme example.
From checks on the test results it has been established that the welds in connections which possess sufficient deformation capacity to enable a beam mechanism to develop should be designed to resist a moment which is 1.4 times the design strength of the connection (see Fig. 2.16). If these connections are employed in an unbraced frame, their parts should be designed for a moment equal to 1.7 times the design strength (see explanatory notes on Fig. 2.16).
The parts of the connection, however, need never be made stronger than the connected beam, although it appears from Fig. 1.15 that a connection possessing very high stiffness with a design strength equal to M_p may intersect the line of sufficient rotational capacity at a point corresponding to 1.33 M_p. This situation could arise of the yield point of the parent material of the beam is 33% higher than the guaranteed yield point, which is quite possible because the relevant codes do not lay down an upper limit for the yield point. The moment distribution is then in agreement with the distribution calculated from elastic theory for $\hat{q} = 16\ M_p/\ell^2$, with 1.33 M_p at the connection and 0.67 at mid-span. In that case, however, it is no longer a connection that satisfies the condition that sufficient rotational capacity is to be provided. But if this last-mentioned condition is satisfied, it will suffice to design parts of the connection, such as welds, for M_p at most. How connections may be designed for achieving the required deformation capacity is explained in section 2.7.
Connection II in Fig. 1.16d is an example of a connection that satisfies all requirements.
Connection III does indeed satisfy the requirement of sufficient deformation capacity, but not the stiffness requirement in the case of a high span/length ratio (40) of the beam. Such a connection, with a design strength of 0.8 M_p, which does not satisfy the stiffness requirement under service load for $\ell/h = 40$ has not been encountered in checking the test results.

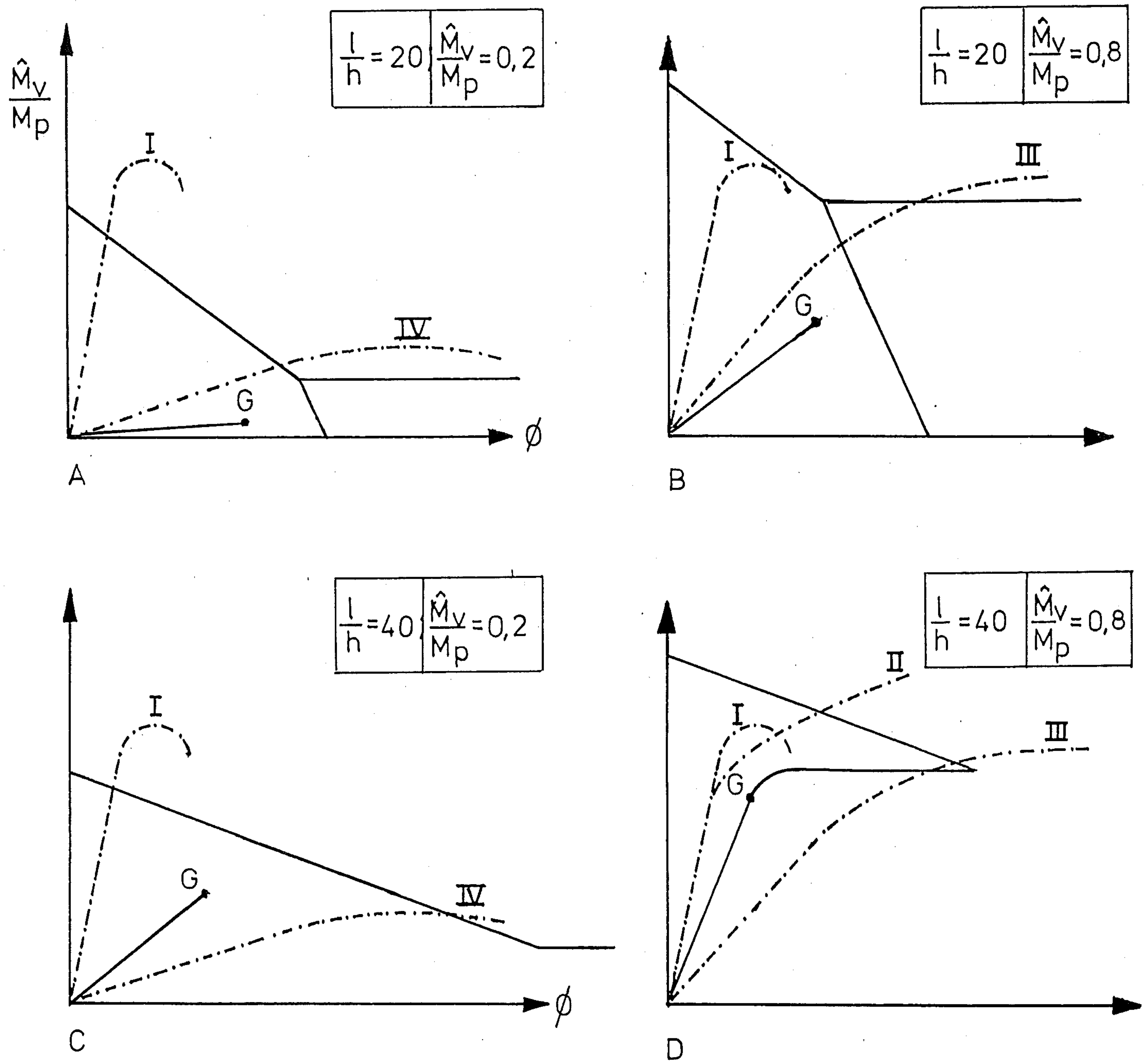


Fig. 1.16: Examples of minimum requirements and moment-rotation characteristics which are and are not satisfactory.

On the other had, there have been cases as represented by connection IV, with a design strength of 0.2 M_p, which satisfies the stiffness requirement for $\ell/h = 20$ and also (only just) for $\ell/h = 25$, but not for $\ell/h = 40$. An example of such a connection is one having a flush end plate and designed to provide deformation capacity.

On the basis of all the checks of the test results against the minimum requirements represented in Fig. 1.15 the rules indicated in Fig. 1.17 have been established. They are based on the assumption that the columns are infinitely stiff. This is the case if, among other possibilities, the two sides of the column are subjected to loading by a beam moment of equal magnitude.

If the column is not infinitely stiff, the diagrams in Fig. 1.15 are not valid; in that case the stiffness of the connection and that of the column must together be taken into account as indicated in Fig. 1.17. The stiffness of the connection can be calculated with the formulae presented in section 2.9.

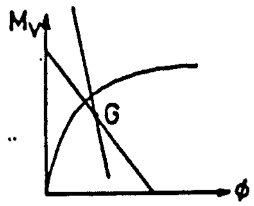
The diagrams in Fig. 1.15 are not valid for loading conditions other than uniformly distributed load either. If the diagrams are used for other loadings, however, the result will be on the safe side because the area of the positive bending moment diagram in such cases is always less than in the case of a beam carrying uniformly distributed load, except for a beam supported on a hinge-type bearing at one end. In those exceptional cases the designer will have to prepare his own diagrams or charts for checking whether a moment-rotation characteristic of any particular connection satisfies the basic assumtions. For this purpose ref. /11/ can prove useful; it deals comprehensively with the derivation of the formulae relating to Fig. 1.15.

The diagrams as represented in Fig. 1.15 provide a better conception of the possibilities. An example of this is the use of Fig. 1.15a, from which it appears that for a low span/depth ratio of the beam the connection need possess no stiffness in order to satisfy the stiffness requirement under service load, while on the other hand the design strength of the connection (e.g., 0.1 M_p) can be utilized for increasing the load. This knowledge can occasionally prove useful in the design of beams with shear connections which are just inadequate in strength, whereas they can be shown to be adequate when the low strength of the connections is taken into account.

Simplified analysis for stiffness.

Service loading:

$$q \leq 8 \frac{M_p + \hat{M}_v}{1,5 l^2}$$

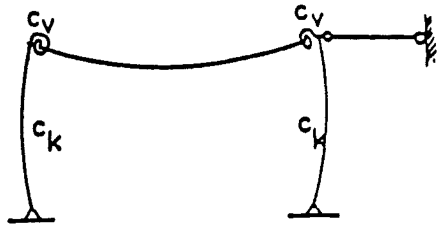


Requirement:
deflection < 0.004 ℓ

For braced frames this is satisfied provided that the length of the beam:

ℓ/h	if $\hat{M}_v/M_g$
<40	< 1,0
<30	< 0,5
<25	< 0,3

This does not apply if the columns undergo deformation



In that case: $\frac{1}{c} = \frac{1}{c_k} + \frac{1}{c_v}$

The above rules are valid only if the loading has been determined on the basis of a beam mechanism. In the case of elastically designed beams the loading is in general less and a longer span can be chosen.

Fig. 1.17: Rules derived from test results for simplifying the stiffness analysis of beams in braced frames;

1.9 When should the flexibility of semi-rigid connections be taken into account?

It follows from the foregoing that the flexibility of structural connections can be neglected in analysing:

- braced frames;
- unbraced frames with connections for which the ratio of their stiffness to the stiffness of the beam is $k = c\ell/EI > 25$.

In all other cases of unbraced frames the actually occurring moments may exceed the calculated moments if the flexibility of the connections is neglected. The reason why this possibility exists is that the actual stiffness of the frame is less than the calculated stiffness, so that the lateral displacements and the moments which cause them are increased.
On the other hand, the moments in the connections due to the vertical load on the beam are decreased if the flexibility of the connections is taken into account.
Which of the two above mentioned effects predominates will depend on the ratio of the horizontal to the vertical load and on the sensitivity of the structure to second-order effects. This is something that will have to be investigated for each case individually. The data given in Fig. 1.7 can be used for obtaining a quick indication of the sensitivity of the structure to the consequences of neglecting the flexibility of the connections.
In braced frames there is no need to check the deformations under service load if the beam span does not exceed certain values indicated in Fig. 1.17.
In the case of unbraced frames the checking of the lateral displacement is subject to the same considerations as those already mentioned with regard to analysing the frames. If the ratio of the stiffnesses of connection and beam is less than 25 ($k = c\ell/EI < 2$) it is permissible to neglect the flexibility of the connections in the analysis.

1.10 Main points of Chapter 1

Depending on the stiffness (or rigidity) in the lateral direction, a distinction is drawn between braced frames (without sidesway) and unbraced frames (with sidesway).
A braced frame can be analysed, i.e., the distribution of forces that occurs in it can be calculated, in accordance with elastic theory or plastic theory. An unbraced frame can be analysed in accordance with elastic theory or can be approximated by a combination of the results of a calculation based on first-order plastic theory and a calculation of the Euler buckling load based on elastic theory. The behaviour of semi-rigid and partial-strength connections influences the results of the calculations.
A connection is said to be semi-rigid if its design stiffness is less than that of the connected beam over a length equal to half the depth of the column section.
A connection is said to be a partial-strength connection if its design strength is less than that of the connected beam.
These two distinctive properties may occur in combination with each other, but not necessarily. The design strength and stiffness of the connection can be determined with formulae given in Chapter 2. These formulae have been checked against test results. The design strength is found to corespond to the bending moment at which large plastic deformations commence or the moment which is lower by at least a factor of 1.4 than the moment at which bolt failure occurs. The ascending branch of the moment-rotation characteristic is appoximated with the design stiffness.
In determining the distribution of forces in accordance withelastic theory the design stiffness is important only if the stiffness ratio $k = c\ell/EI$ is less than 25.
In the case of a braced frame a higher design strength is calculated for the frame if lower values are adopted for the design stiffnesses of the structural connections.
In reality the stiffness of a connection is allowed to be greater, provided that the connection possesses so much deformation capacity that the more favourable distribution of forces can be attained.

If the actual stiffness of the connection is adopted in the analysis and all possible loading conditions are considered, the connection need not possess deformation capacity.
Nor need the connection possess deformation capacity if its design strength is greater than that of the connected beam. In determining the Euler buckling load of an unbraced frame it is necessary to take account of the stiffness of the connections which exists when the design strength of the connections is attained.
The maximum design strength of the structure is attained with a first-order plastic analysis. In that case it is not necessary to take account of the stiffness of the connections, but maximum deformation capacity of the partial-strength connections is essential. To procide this deformation capacity the welds must be made stronger than the design strength indicates ($1.4\ \hat{M}_v < M_p$ for braced, $1.7\ \hat{M}_v < M_p$ for unbraced frames).

Partial-strength connections which remain elastic up to failure are unsuitable for structures whose design strength is determined on the basis of plastic theory.
Connections whose design strength is determined on the basis of plastic theory may be employed in elastically designed structures.
In unbraced frames with connections for which $k = c\ell/EI < 25$ it will have to be investigated how sensitive the frame is to the consequence of neglecting the flexibility of the connections. For this purpose the information given in Fig. 1.7 may be used.
Beams in braced frames satisfy the stiffness requirement under service load if the beam span/depth ratios do not exceed certain values as indicated in Fig. 1.7.

Chapter 2:

Design rules for stiffness, strength and rotational capacity.

2.1 Introduction

This chapter will first deal with the design method for the tension side of flexurally stiff (bending moment resisting) structural connections. The treatment of the subject will more or less follow the development sequence. First, a design method for T-stub connections /12/ was developed next, a method for column flanges without stiffeners and with the projecting end plate /13/; then a method for column flanges with stiffeners and the flush end plate /14, 15/ with one or more rows of bolts on the tension side of the connection. For this purpose the plastic and the elastic theory were applied, and the results of the calculations were checked against test results.

It proves possible to assemble the design rules for all the above mentioned connection components in one diagram, the only difference being the effective (structurally co-operating) length of the plates.
A table is given for determining this effective length (see Fig. 2.12).
The use of the rules for each component is illustrated in design examples.
It will be apparent from the above mentioned diagram whether the design strength of the connection is calculated on the basis of elastic theory or plastic theory or a combination of the two. Also, the diagram provides an insight into the deformation of the connection. This will be explained in due course.

In Chapter 1 it has been shown that the welds joining the end plate to the beam should be stronger than the connection itself if the latter has to provide deformation capacity. The required extra strength of the bolts already follows from the results of the formulae.

It has also been explained in Chapter 1 in which cases the stiffness of the connection should be taken into account. In the present chapter the stiffness formulae are discussed which have been established on the basis of test results and elastic theory. Next, some special subjects are dealt with, such as increasing the strength and the stiffness of the connection, and necessary reductions of the design strength of the connection because of the interaction of stresses in the web or the flanges of the column.

2.2 Strength of T-stub connections

In tests performed on T-stubs as shown in Fig. 2.1a there were found to be three failure mechanisms. These are represented schematically in Fig. 2.1b; below them are indicated the shear force diagrams and bending moment diagrams in the flange plates. In the case of mechanism I the flanges of the T-stubs undergo yielding at the bolt row and at the transitions from the flanges to the web. Since the difference in bending moments is equal to the area of the shear force diagram, it follows that the external load is:

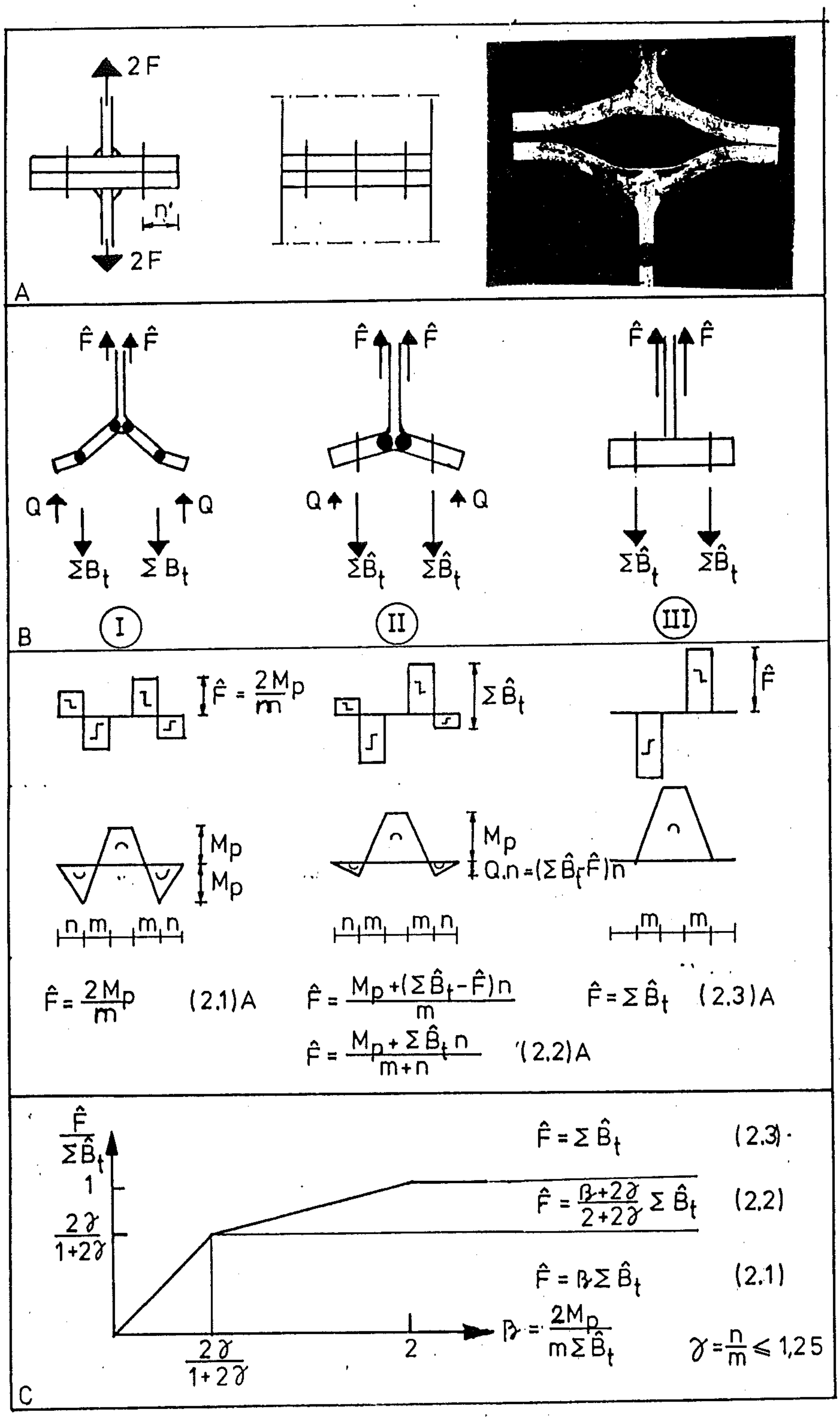
$$\hat{F} = 2\ M_p/m \qquad (2.1a)$$

where:

M_p = plastic moment of the flange over the width of the T-stub.

m = distance from the bolts to a line at one-fifth of the radius of the transition or the width of the weld between web and flange of the T-stub.

If the bolts are not strong enough, mechanism II develops.
Tests have shown that in this case first the plastic moment at the flange-to-web transition is attained, in consequence of which the plates bend and a so-called prying force acts at their edges. Failure of the connection occurs if the prying force together with the externally applied force exceeds the strength of the bolts. From the bending moment diagram it follows that:



Explanation: n' = edge distance

n = distance from the bolts to the assumed point of application of the prying force.

assumption: n = n' as long as n' ≤ 1,25 m
otherwise n = 1,25 m

Fig. 2.1: Derivation of the formulae for the analysis of T-stubs and the diagram based on these, with formulae forming the basis for the analysis of all connection components involving a plate loaded perpendicularly to its plane.

$$M_p + (\sum \hat{B}_t - \hat{F})\, n = \hat{F}.m$$

which can be rewritten as:

$$\hat{F} = \frac{M_p + \sum \hat{B}_t.n}{m + n} \qquad (2.2a)$$

where:

$\sum \hat{B}_t$ = design strength of all the bolts on one side of the T-stub.

n = distance from the bolts to the assumed point of application of the prying force.

If the edge distance n' is not too large, the prying force will act at the edge of the plate. Checks of formula (2.2a) against test results have shown that the distance n should not be made more than 1.25 m. According to the Netherlands code of practice (T.G.B. Staal 1972 the design strength of the bolts should be taken as equal to 0.7 of their ultimate strength. If a load factor of 1.5 is applied, as is done in the Netherlands code, this definition of the design strength of a bolt corresponds to a factor of safety of more than 2 against failure.

If, in accordance with the Eurocode, the design strength of the bolts is taken as equal to 0.8 of the ultimate strength and if a load factor of 1.5 is likewise applied, the factor of safety against failure will be 1.88. From formula (2.2) it is apparent that this reduction in safety is to some extent reflected in the design strength of the connection, but this effect diminished according as $\sum \hat{B}_t.n$ approaches M_p more closely. For $\sum \hat{B}_t.n = M_p$ the failure mechanism I is attained.

If the flanges are very thick, the strength of the bolts may be insufficient to enable yielding of the flange plates to take place. If the bolts yield, whereas the plates merely deform elastically, the plates will become detached from each other; the externally acting tensile force is then equal to the design strength of the bolts. In that case mechanism III develops, for which:

$$\hat{F} = \sum \hat{B}_t \qquad (2.3a)$$

The design strength of a connection can be determined by substitution of the formulae (2.1a), (2.2a) and (2.3a). The lowes result determines which mechanism is the governing failure mechanism.
The three formulae can be rewritten and be represented as in Fig. 2.1c, namely, the formulae (2.1), (2.2) and (2.3). The values on the vertical axis of the diagram represent the ratio of the design strength of the connection (combination of plate and bolts) to the total design strength of the bolts. This ratio has its maximum value of 1 if formula (2.3) corresponds to the governing condition. The values on the horziontal axis represent the ratio β of the design strength of the plate to that of the bolts. The design strength of the plate is determined with formula (2.1a).
The relation between the vertical and the horizontal axis is given by the three straight lines of which the diagram is composed. The slope of the second line (mechanism II) depends on the ratio $\gamma = n/m$, which can have a maximum value of 1.25. The slope of the first line, starting at the origin (mechanism I), is always 45^{o}.
The design procedure described above is valid only for connections subject to static loading. In the case of dynamic loading it is necessary to ensure that the bolts will not be cyclically stressed by load alternations, for under such conditions they can, as tests have shown /16/, resist only 7% of the static strength. Load alternations in the bolts can be prevented by giving the bolts a prestress which is greater than the external tensile force acting on them and by ensuring that the resultant of the contact pressure due to the prestress coincides with the line of action of the external load /17/. In this way the external load will be resisted by reduction of the contact pressure instead of by tension in the bolts.
Fig. 2.2 presents a design example which illustrates the use of the formulae and diagram.

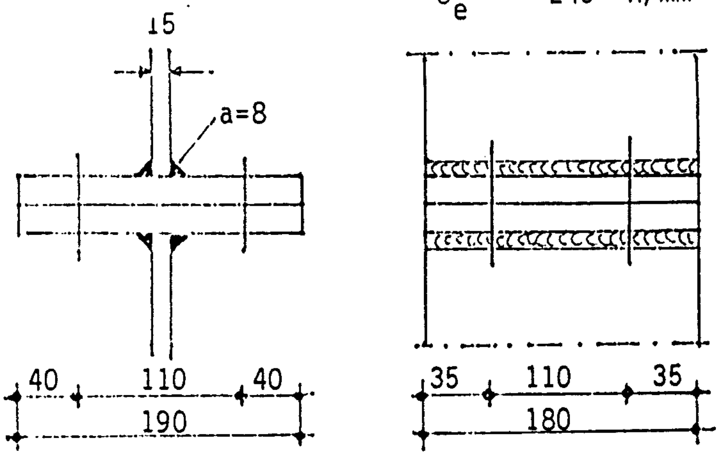
2.3 Strength of column flanges without stiffeners

In the testing of column specimens as illustrated in Fig. 2.3 relating to example 2 the failure mechanisms found to occur were similar to those of T-stubs.

Design example 1: T-stubs, Bolts M24 8.8.

$\hat{B}_t$ = 197,75 kN ΣB_t=395,5 kN

σ_e = 240 N/mm²



$$m = \frac{110 - 15 - 0{,}8 * 16\sqrt{2}}{2} = 38{,}44 \text{ mm}$$

n' = 40 mm

n = 40 mm < 1,25 * 38,44

With formulae:

Plate thickness 15 mm		25 mm	30 mm
(2.1) $\hat{F} = \frac{2M_p}{m} = \frac{2 * 180 * \frac{1}{4} * 15^2 * 240}{38{,}44} =$	126,4 kN	351,1 kN	505,7 kN
(2.2) $\hat{F} = \frac{M_p + \Sigma \hat{B}_t \cdot n}{m+n} = \frac{180 * \frac{1}{4} * 15^2 * 240 + 395500 \times 40}{38{,}44 + 40} =$	232,6 kN	287,7 kN	325,6 kN
(2.3) $\hat{F} = \Sigma \hat{B}_t$	395,5 kN	395,5 kN	395,5 kN
Relevant formula.	(2.1)	(2.2)	(2.2)

With diagram:

$$\gamma = \frac{n}{m} = \frac{40}{38{,}44} = 1{,}04$$

$$\frac{2\gamma}{1+2\gamma} = 0{,}676$$

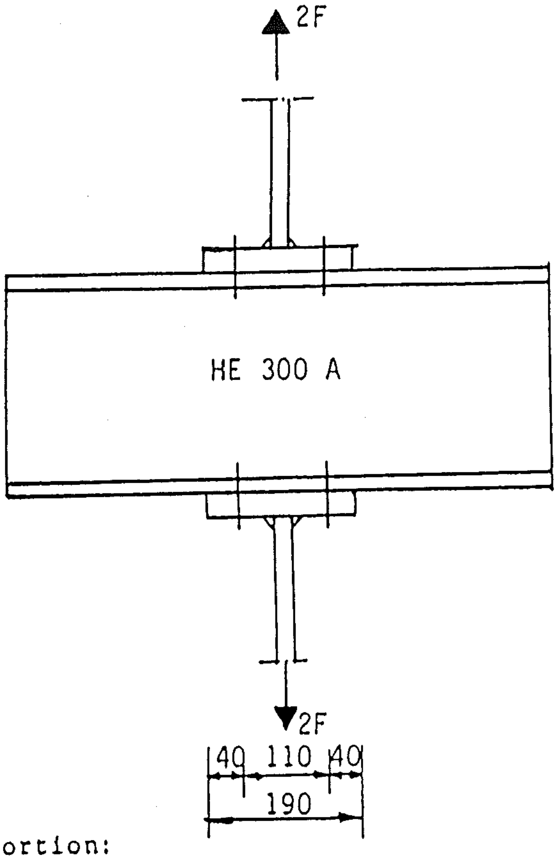
Plate thickness 15 mm.	25 mm	30 mm
$\beta = \frac{2M_p}{m \hat{B}_t} = \frac{126{,}4 \text{ kN}}{395{,}5 \text{ kN}}$ (zie boven)	$\frac{351{,}1 \text{ kN}}{395{,}5 \text{ kN}} =$	$\frac{505{,}7 \text{ kN}}{395{,}5 \text{ kN}} =$
$\beta = 0{,}319 < \frac{2\gamma}{1+2\gamma} = 0{,}676 <$	0,888	1,28
$\beta < \frac{2\gamma}{1+2\gamma} \rightarrow \hat{F} = \beta \Sigma \hat{B}_t = 0{,}319 * 395{,}5 =$	$\frac{\beta+2\gamma}{2+2\gamma} \Sigma \hat{B}_t =$	$\frac{\beta+2\gamma}{2+2\gamma} \Sigma \hat{B}_t =$
= 126,4 kN	287,7 kN	325,7 kN

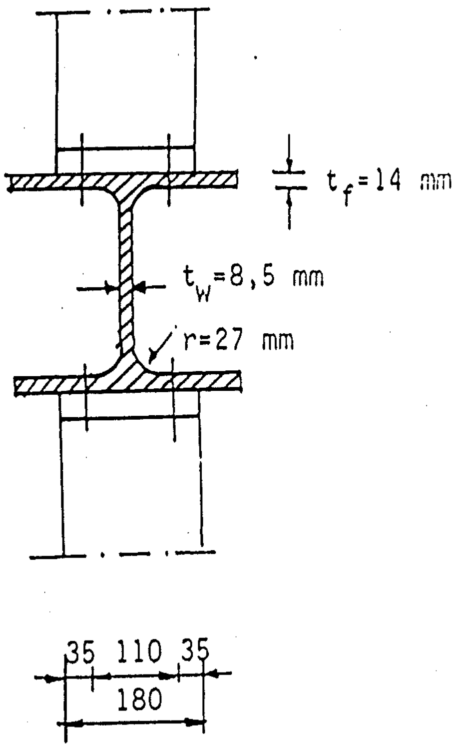
Fig. 2.2: Example of the application of formulae (2.1) and (2.2) and the diagram in Fig. 2.1C to T-stubs;

Design example 2: Column flanges with T-stubs HE 300A

$h = 290$ mm
$b = 300$ mm

Material Fe 360 with $\sigma_e = 240\ N/mm^2$
Bolts M24 8.8 with $\hat{B}_t = 197{,}75$ kN





Column portion:

$$m = \frac{110 - 8{,}5 - 0{,}8 * 2 * 27}{2} = 29{,}15 \text{ mm}$$

$$n' = \frac{300 - 110}{2} = 95 \text{ mm}$$

$n = 1{,}25 * 29{,}15 = 36{,}43$ mm
$n = 35$ mm (determined by T-stub)

(2.4) effective length $b' = a + 4m + 1{,}25n' = 110 + 4 * 29{,}15 + 1{,}95 * 95 = 345$ mm

With formulae: (2.1 t/m (2.3)

$$(2.1)\quad \hat{F} = \frac{2\ M_p}{m} = \frac{2 * 345 * \frac{1}{4} * 14^2 * 240}{} = 278{,}4 \text{ kN}$$

$$(2.2)\quad \hat{F} = \frac{M_{p+}\ \Sigma\hat{B}_t . n}{} = \frac{345 * \frac{1}{4} * 14^2 * 240 + 395500 * 35}{2{,}25 + 29{,}15} = 277{,}9 \text{ kN}$$

$$(2.3)\quad \hat{F} = \Sigma\hat{B}_t = 395{,}5 \text{ kN}$$

With diagram :

$$\gamma = \frac{n}{m} = \frac{35}{29{,}15} = 1{,}20 \qquad \beta = \frac{278{,}4}{395{,}5} = 0{,}704$$

$$\frac{2\gamma}{1+2\gamma} = 0{,}706$$

$$\hat{F} = 0{,}704 * 395{,}5 = 278{,}4 \text{ kN}$$

Combination with T-stubs of design example 1.

Plate thickness $t = 15$ mm $\hat{F} = 126{,}4$ kN (T-stub is governing component)
$t = 25$ mm $\hat{F} = 278{,}4$ KN (column flange is governing component).

Fig. 2.3: Example of the application of formulae (2.1) to (2.2) and formula (2.4);

With the aid of the yield line theory /13/ it has been shown that the design strength of the connections to column flanges can be calculated with the formulae (2.1), (2.2) and (2.3) if the portion of column in question is conceived as having a notional effective length equel to:

$$b = a + 4 + 1.25\ n' \qquad (2.4)$$

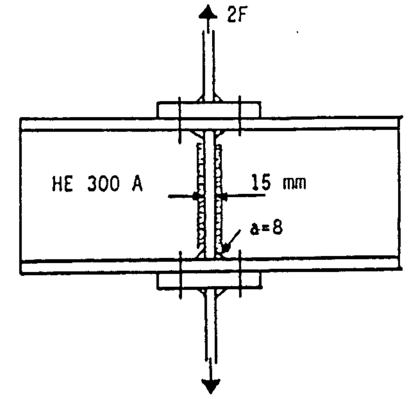
where a denotes the distance between the outermost bolts.
It has furthermore been established that the design strengths of T-stubs and column flanges can be determined independently of each other /13/.

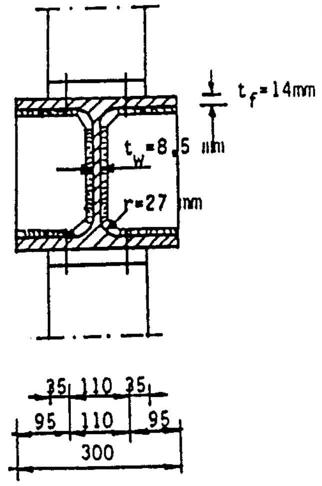
2.4 Strength of column flanges with stiffeners

In tests on column specimens with stiffeners as shown in Fig. 2.4 relating to example 3 the same three failure mechanisms as those of T-stubs were found to occur. It emerges, too, that the same formulae and diagram can be employed if the design strength of the plate as determined from the diagram in Fig. 2.5 is introduced.
This last-mentioned diagram has been compiled by analysis of the yield line mechanisms observed in the tests and by making use of theoretical approximations. On the horizontal axis is indicated the location of the bolts in relation to the web of the column. The bolt location is represented by the ratio of the distance m_1 (from the bolt to the radiused transition) to the width of the column flange, i.e., $\lambda_1 = m_1/(m_1 + n')$.
On the vertical axis of the diagram is indicated the ratio of the distance m_2 (from the bolt to the weld of the stiffner) to the width of the column flange, i.e., $\lambda_2 = m_2/(m_1 + n')$.
The design strength is the product of the plastic moment per unit length of the plate and the value α read from the coordinates of the bolt from the diagram. For this purpose it is permissible to interpolate between the curves for which the values of α are indicated.
The design strength of the combination of the bolt and plate is determined with the aid of the same formulae and diagram as employed for the T-stubs, except that for $\hat{F}_p = 2M_p/m$ we must now substitute the value $\hat{F}_p = \alpha m_p$,

Design example 3: Column flanges with stiffeners and T-stubs,

Bolts : M24 8.8 ... $\hat{B}_t$ = 197,75 kN
Material: Fe 360 ... σ_e = 240 N/mm^2





$m_1 = \frac{110 - 8,5 - 54}{2} = 23,75$ mm

$m_2 = \frac{110 - 15 - 2 * 8 \sqrt{2}}{2} = 36,18$ mm

$\lambda_1 = \frac{23,75}{23,75+95} = 0,20$ $\lambda_2 = \frac{36,18}{23,75+95} = 0,304 \rightarrow \alpha = \pi$ (see diagram in Fig. 2.5)

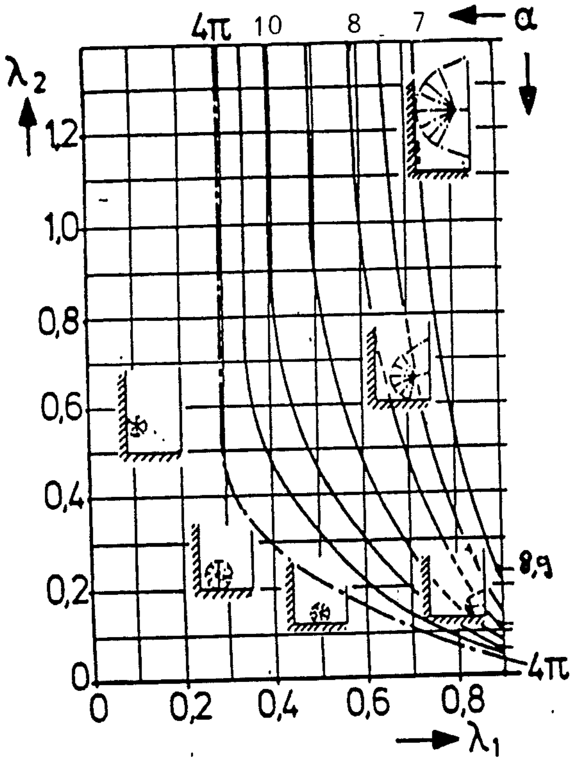
$\hat{F}_p = 4 * \frac{1}{4} * 14^2 * 240 = 147,8$ kN $\gamma = \frac{n}{m} = \frac{35}{23,75} = 1,47$

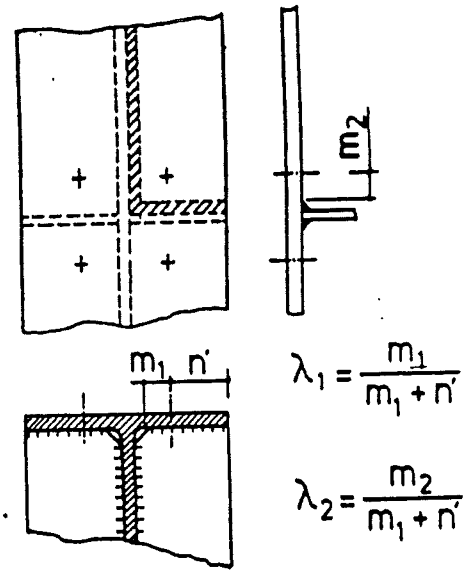
$\beta = \frac{147,8}{197,75} = 0,747$ $\frac{\gamma}{1+2\gamma} = 0,747$

$\hat{F} = 0,747 * 395,5 = 295,3$ kN

Combination with T-stubs of example 1;
Plate thickness t = 15 mm ... (T-stub is governing component)
t = 25 mm ... (T-stub is governing component)
t = 30 mm ... (column flange is governing component).

Fig. 2.4: Design example for column with stiffeners.





In EC 3, January 1988, this graph is made consistant with the formula for effective length per bolt in a flange without stiffener $b_m = 4\ m + 1,25\ n'$.

Fig. 2.5: Diagram for determining the design strength of an end plate or column flange with stiffener loaded by a bolt.

where:

$m_p = \frac{1}{4} t^2 . f_y$

t = thickness of plate or flange

f_y = guaranteed yield point of plate or flange material.

In this text the terms "plate" and "flange" are both used because it has been found that the part of the end plate between the beam flanges can be designed in the same way as the column flange with stiffeners. A design example is presented in Fig. 2.4.

2.5 Strength of a connection with projecting end plate and four bolts on the tension side

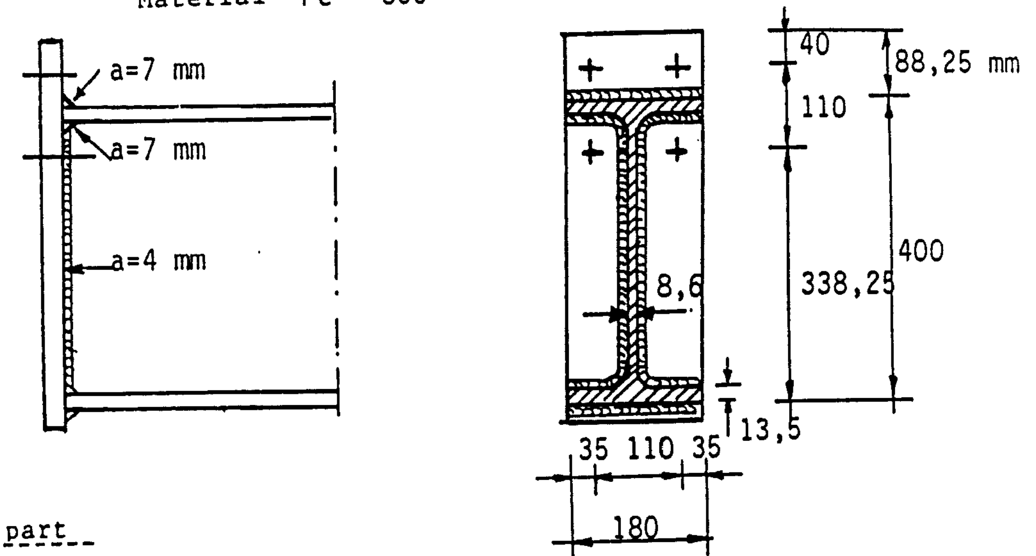
In the case of a beam connection with projecting end plate, as shown in Fig. 2.6, the strength of three component parts of the connection should first be calculated:

- the side of the column, with or without stiffener;
- the projecting part of the end plate;
- the part of the end plate between the beam flanges.

The method applicable to T-stubs can be applied also to the projecting part of the end plate. For the part of this plate situated between the flanges of the beam the method applicable to the column flange with stiffener can be employed (see design example 4, Fig. 2.6).
The force transmitted by a bolt is equal to the lower of the values calculated for the strength of the column flange with bolt or of the end plate with bolt. Also, it is necessary to satisfy the condition that there is equilibrium between the forces on the tension side and those on the compression side of the connection. The force on the compression side is limited by the design strength of the column web without stiffeners on the compression side of the connection (see Section 2.11) and the design strength of the column web in shear (see Section 2.11). Shearing of the column web may occur in consequence of unequal load acting on the two sides of the column, as may occur with knee or T-stub connections (see design example 5, Fig. 2.7).

Design example 4: Connection with projecting end plate,

Bolts M24. 8.8
Material Fe 360



Projecting part

$$m = \frac{110 - 13,5 - 0,8 * 2 * 7\,2}{2} = 40,33 \text{ mm} \qquad n = 40 \text{ mm}$$

with plate thickness t = 15 mm	t = 25 mm	t = 30 mm
$\hat{F} = \frac{2M_p}{m} = \frac{2 * \frac{1}{4} * 180 * 15,0^2 * 240}{40,33} = 120,5$ kN	334,7 kN	482 kN
$\hat{F} = \frac{M_p + \Sigma\hat{B}_t \cdot n}{m+n} = \frac{\frac{1}{4}*180*15^2*240+395500*40}{40,33+40} = 227,2$ kN	281,0 kN	317,9 kN
$\hat{F} = \Sigma\hat{B}_t = 395,5$ kN	395,5 kN	395,5 kN

Part between the flanges,

$$m_1 = \frac{110 - 8,6 - 8\sqrt{2}}{2} = 45 \text{ mm} \qquad n = 35 \text{ mm} \qquad \lambda_1 = \frac{45}{35+45} = 0,56$$

$$m_2 = \frac{110 - 13,5 - 2 * 7\sqrt{2}}{2} = 38,5 \text{ mm} \qquad \lambda_2 = \frac{38,5}{35+45} = 0,481$$

$\alpha = 9,3$ from diagram in Fig. 2.5

Plate thickness t = 15 mm

$$\hat{F}_p = 9,3 * \tfrac{1}{4} * 15^2 * 240 = 125,6 \text{ kN} \qquad \beta = \frac{125,6}{197,75} = 0,632$$

$$\gamma = \frac{n}{m} = \frac{35}{45} = 0,777 \rightarrow \frac{2\gamma}{1+2\gamma} = 0,608$$

$$\hat{F} = \frac{\beta+2\gamma}{2+2\gamma}\,\Sigma\hat{B}_t$$

$$\hat{F} = \frac{\beta+2\gamma}{2+2\gamma}\,\Sigma\hat{B}_t = \frac{0,632 + 2 * 0,777}{2 + 2* 0,777} * 395,5 = 243,3 \text{ kN}$$

Plate thickness t = 25 mm.

$$\hat{F}_p = 9,3 * \tfrac{1}{4} * 25^2 * 240 = 348,88 \text{ kN} \qquad \beta = \frac{348,8}{197,75} = 1,764$$

$$\gamma = 0,777 \rightarrow \frac{2\gamma}{1+2\gamma} = 0,608$$

$$\hat{F} = \frac{\beta+2}{2+2\gamma}\,\Sigma\hat{B}_t$$

$$\hat{F} = \frac{\beta+2\gamma}{2+2}\,\Sigma\hat{B}_t = \frac{1,764 + 2 * 0,777}{2+2 * 0,777} * 395,5 = 369,4 \text{ kN}$$

Plate thickness t = 30 mm.

$$\hat{F}_p = 9,3 * \tfrac{1}{4} * 30^2 * 240 = 502,2 \text{ kN} \qquad \beta = \frac{502,2}{197,75} = 2,54 > 2 \rightarrow \hat{F} = \Sigma\hat{B}_t = 395,5 \text{ kN}$$

Fig. 2.6: Design example for connections with projecting endplate.

Design example 5: End plate in combination with HE 300A (shearing);
(for design strengths of the tension side see examples 2 and

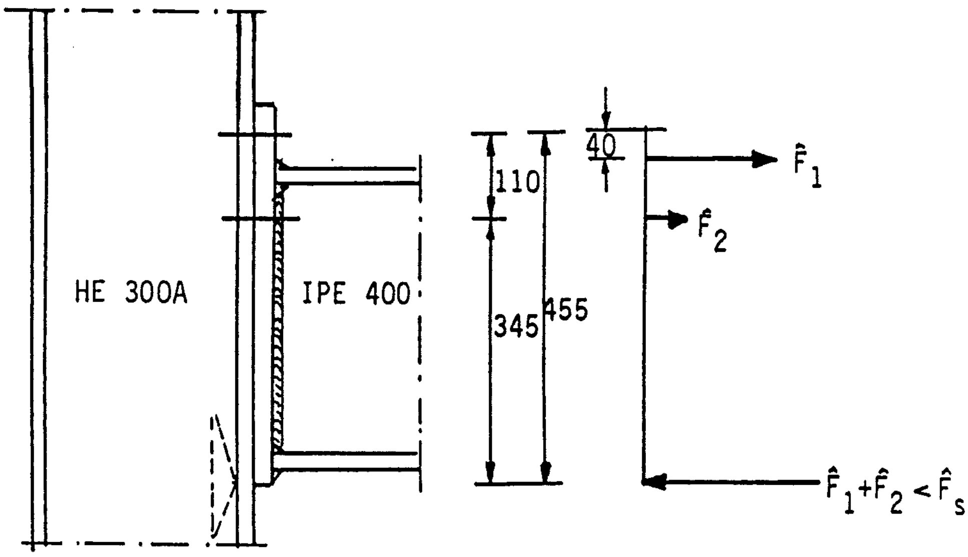
Design strength for shearing of column web:

$\hat{F}_s = 0,58\ (h-2t_f)\ t_w \cdot \sigma_e = 0,58\ (290-2*14)\ *8,5 * 240 = 310$ kN (see Section 2.11) (2.

Design strength for compression side of column web:

$\hat{F}_c = 5 * (t_f+r)\ .t_w \cdot \sigma_e = 5(14+27) * 8,5 * 240 = 418$ kN (see Section 2.11) (2.8)

End plate 15 mm



Shear 310 kN

Top row of bolts flange, see example 2 278,4 kN; end plate, projecting part 120,5 kN } 120,5 kN

Bottom row of bolts flange, 278,4 kN; end plate between flanges 243,3 kN } 243,3 kN

The design strength of the shearing zone is 310 kN < 363.8 kN.

Bottom row of bolts 310-120.5 = 189.5 kN

$\hat{M}_v = 120,5 * (0,455-0,040) + 189,5 * 0,345 = 50 + 65 = 115$ kNm

End plate as T-stub $2 * 120,5 * 0,4 = 96$ kNm

End plate 25 mm

end plate, projecting part 281 kN; flange 278,4 kN } 278,4 kN

Bottom row of bolts 310 - 278,4 = 31,6 kN

$\hat{M}_v = 278,4 * 0,415 + 31,6 * 0,345 = 115,5 + 10,9 = 126,4$ kNm

End plate as T-stub $310 * 0,4 = 124$ kNm

Fig. 2.7: Design example for connection with end plate if shearing is the governing mode.

If the sum of the design strengths of the connecting components on the tension side is greater than that on the compression side or greater than the design strength of the column web in shear, the strength on the tension side should be reduced to such an extent that equilibrium is achieved (see design example 6, Fig. 2.8). This reduction may begin with the bolts located closest to the centre of compression, i.e., the centre of reaction on the compression side.

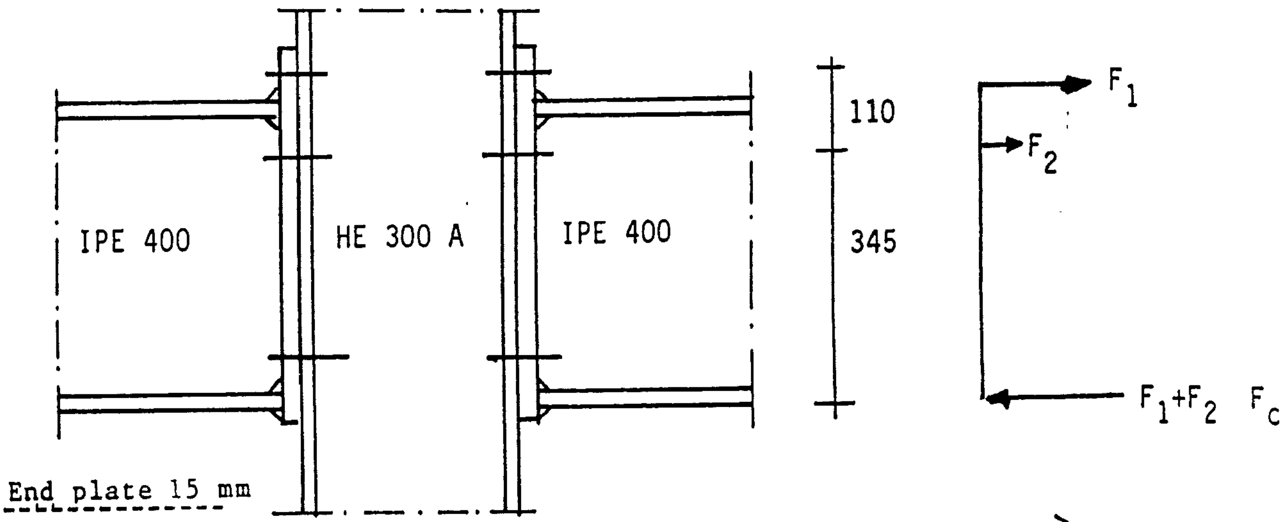
The design strength of the connection is equal to the sum of the products of the forces determined as indicated above and their lever arms with respect to the centre of compression. In determining the lever arms it must be taken into account that in the case of the connecting components similar to T-stubs the calculated tensile force is located at the transition from web to flange and that in all other components it is located at the row of bolts.

To simplify the calculation, the end plate may likewise be conceived as a T-stub and the supporting effect of the beam web be neglected.
The resultant of the forces is aligned in the continuation of the tension flange of the beam. In that case there is only one lever arm to consider.
The calculation procedures described above are incorporated in the design examples 4 to 7. Example 7 is contained in Appendix B, Fig. B.1.

2.6 Strength of a connection with more than one row of bolts on tension side between the beam flanges

The following applies to connections formed with a flush end plate as well as those formed with a projecting end plate, provided that the projecting part of the end plate can undergo sufficient deformation to enable the yield line pattern between the beam flanges to develop (see note under Fig. 2.10). The projecting part of the end plate should be taken into account in the same way as has been described for the projecting end plate with four bolts (see Section 2.5).

Design example 6: End plate in combination with HE 300A (column loaded symmetrically), Bolts M24 8.8,



End plate 15 mm

Top row of bolts	flange, see example 2,	: 278,4 kN	} 120,5 kN
	end plate, projecting part	: 120,5 kN	
Bottom row of bolts	flange.	278,4 kN	} 243,3 kN
	end plate between flange.	243,3 kN	

The design strength of the compression side, see example 5 418 kN > 363,8 kN

$\bar{M}_V$ = 120,5 * (0,455-0,040) + 243,3 * 0,345 = 50 + 84 = 134 kNm

End plate as T-stub: $\bar{M}_V$ = 2 * 120,5 * 0,4 = 96 kNm

End plate 25 mm

Top row of bolts	flange	: 278,4 kN	} 278,4 kN
	end plate, projecting part	: 281 kN	
Bottom row of bolts	flange	278,4 kN	} 278,4 kN
	end plate between flanges	369,4 kN	

The design strength of the compression side, see example 5 418 kN < 556,8 kN

Bottom row of bolts 418-278.4 = 139.6 kN

$\bar{M}_V$ = 278,4 * (0,455-0,04) + 139,6 * 0,345 = 115,5 + 48,2 = 163,7 kNm

End plate as T-stub: $\bar{M}_V$ = 418 * 0,4 = 167 kNm

A stiffener between the flanges on the tension side is of little use here, because then:

Top row of bolts	flange with stiffener, see example 3	295,5 kN	} 281 kN
	end plate, projecting part.	281 kN	

$\bar{M}_V$ = 281 * (0,455-0,04) + (418 - 281) * 345 = 116,6 + 47,3 = 163,8 kNm

Fig. 2.8: Design example for connection with projecting end plate if behaviour is governed by the compression side;

Here, too, it is necessary to do a separate calculation for the column flange (with or without stiffeners) and the part of the end plate between the beam flanges.
It will first be explained how the bolt forces in a group of bolts and then how the design strength of the connection should be determined.

2.6.1. Bolt forces in a group of bolts

Tests have shown that in the part between the beam flanges then first develops a yield line pattern around the bolt which is located nearest to the tension flange of the beam and the beam web (see Fig. 2.9).
If another bolt is added, the yield lines extend to it, but the force in the first bolt remains the same as that corresponding to the original yield line pattern. This effect has been ascertained from bolt force measurements in tests /14, 15/. What consequences this has for the design method are indicated schematically in Fig. 2.10 and 2.11.
To start with, the design strength of the plate with one bolt is determined, as indicated in Figs. 2.1A, B, C. Next, the effect of adding a bolt is considered. It is possible that the second bolt, e.g., as shown in Fig. 2.10D, does nothing, because it has been placed within the yield mechanism formed by the first bolt.

The difference between the design strengths of two groups of bolts is the maximum force that a row of bolts can transmit. The force must be determined from the difference between the two bolt groups because the addition of a bolt may change the type of failure mechanism, e.g., from a mechanism with bolt failure (II) to a mechanism with complete yielding of the end plate or column flange (I). That is why it is necessary first to determine the design strengths of a number of bolt groups in the connection.
This number is equal to the number of bolt rows on the tension side of the connection which it is desired to include in the design calculation.
The design strength of a bolt group is determined with the same formulae as those used for the column flange without stiffeners or with the diagram in Fig. 2.1C.

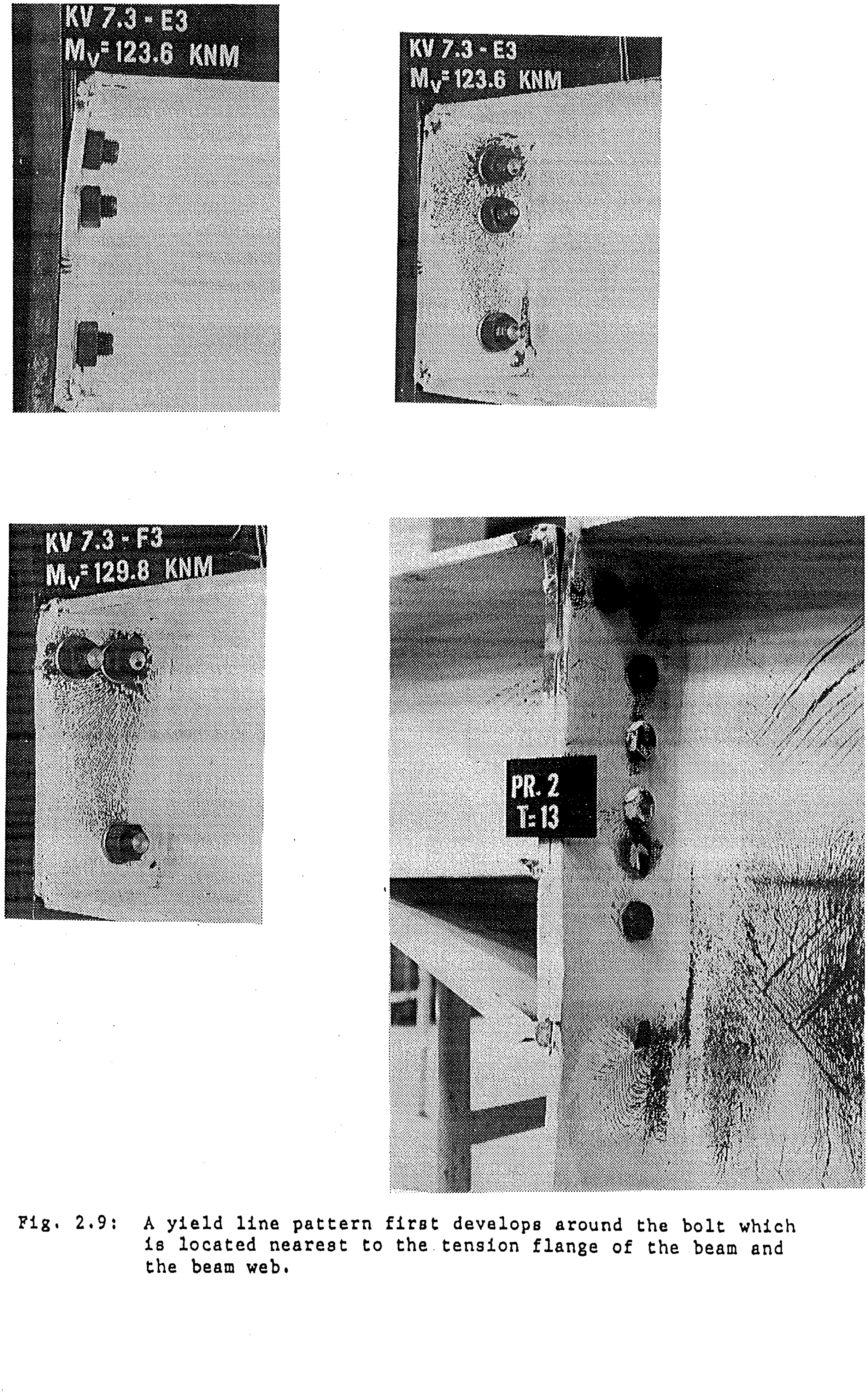


Fig. 2.9: A yield line pattern first develops around the bolt which is located nearest to the tension flange of the beam and the beam web.

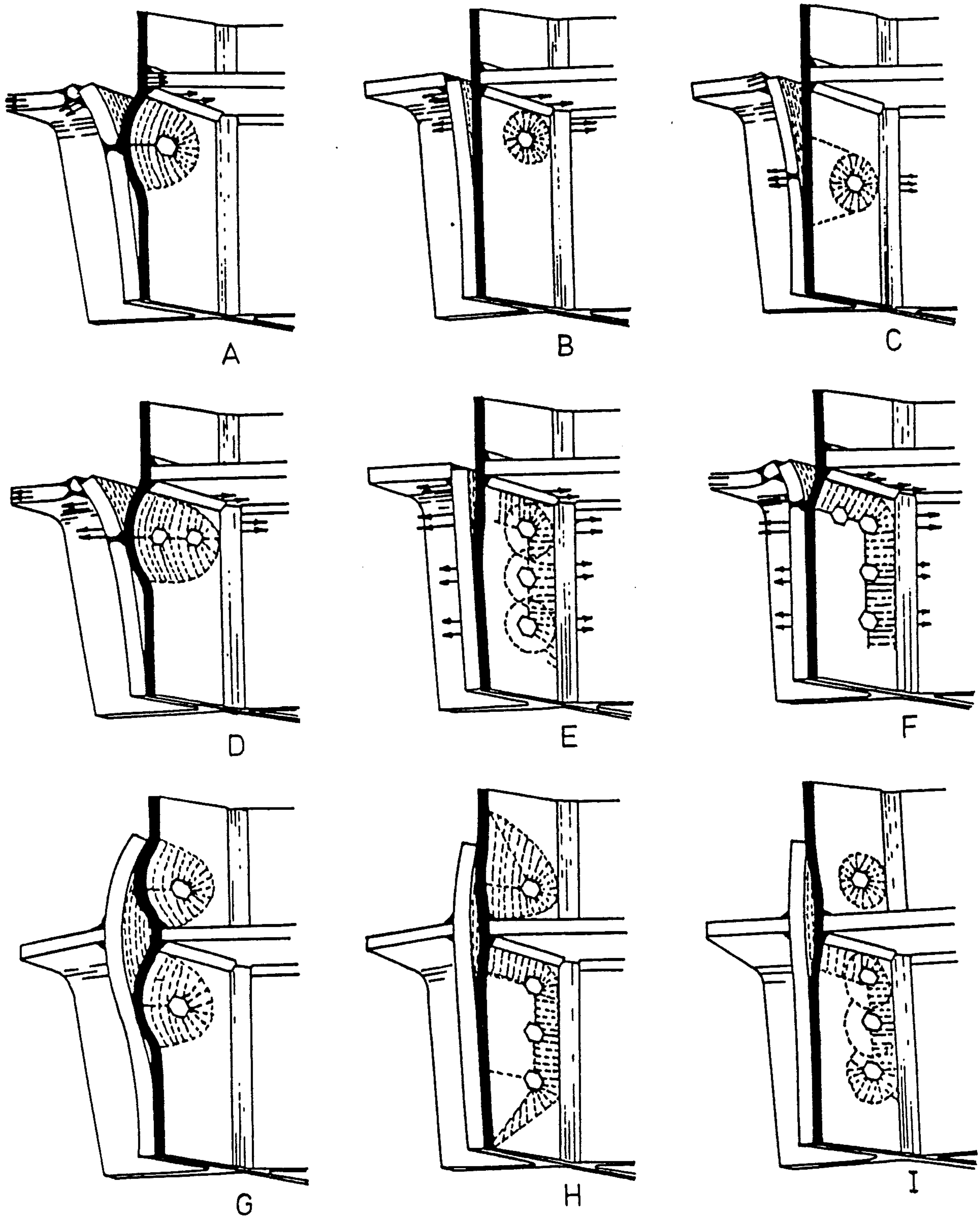


Fig. 2.10: The effectiveness of added bolts depends on the yield line pattern formed by the first bolt.

Note: In the connections shown in Figs. 2.10 G, H and I the yield line pattern in the part between the beam flanges can develop only if the part outside the flanges has sufficient deformation capacity. In Chapter 3 this is explained for a comparable situation with the aid of test results. Sufficient deformation capacity is obtained by ensuring that yielding of the plate or the flange occurs.

This same diagram is reproduced in Fig. 2.13, but now with the formulae relating to T-stubs, in order to highlight the similarity between the two methods. This similarity is apparent when the plastic moment of the T-stub is replaced by the product of the effective length b_m and the plastic moment per unit length m_p of the end plate or column flange. The strength of the bolt is then equal to the sum of the design strengths of the bolts in the group. The design strength of the plate or column flange is:

$$\hat{F}_p = 2\ b_m \cdot m_p / m \qquad (2.5)$$

where:

b_m = effective length of a bolt group.

The effective length of a bolt group is equal to the sum of the effective lengths as given in the table in Fig. 2.12. As the table shows, only the effective lengths associated with the first bolt vary, depending on the mechanism that may be formed. The effective length for the added bolts is always equal to the distance between the bolt rows because the yield line pattern of the first row is increased by an amount equal to the bolt spacing in consequence of the added row. This is shown schematically in Fig. 2.11.

The total effective length for a group of bolts can never exceed the total length of the welds along the tension part of the beam flange and beam web. In the vertical direction there is no advantage in installing additional bolts if the effective length already extends to the compression side, unless the plate is still fully elastic ($\beta_1 > 2$).
In the last mentioned case the addition of a bolt will have the effect of reducing the ratio β of plate strength to bolt strength, so that the failure mechanism involving complete yielding of the plate is more closely attained $(\beta_1 < 2\gamma/(1+2\gamma))$.
It has been shown in /13/ that the design strengths of the end plate and column flange can be determined independently of each other and that the lower strength is the governing strength with regard to the bolts.
An example will serve to illustrate this.

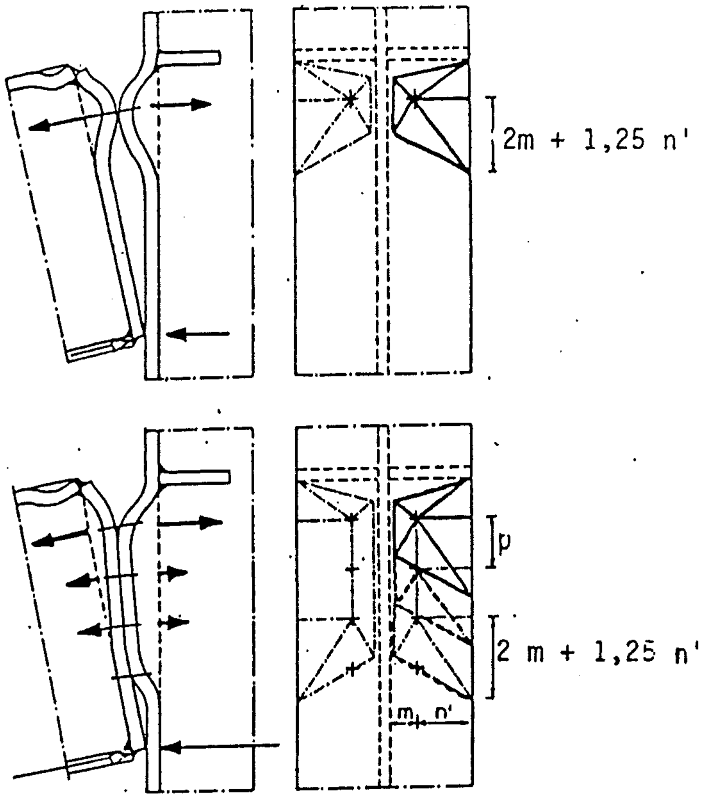
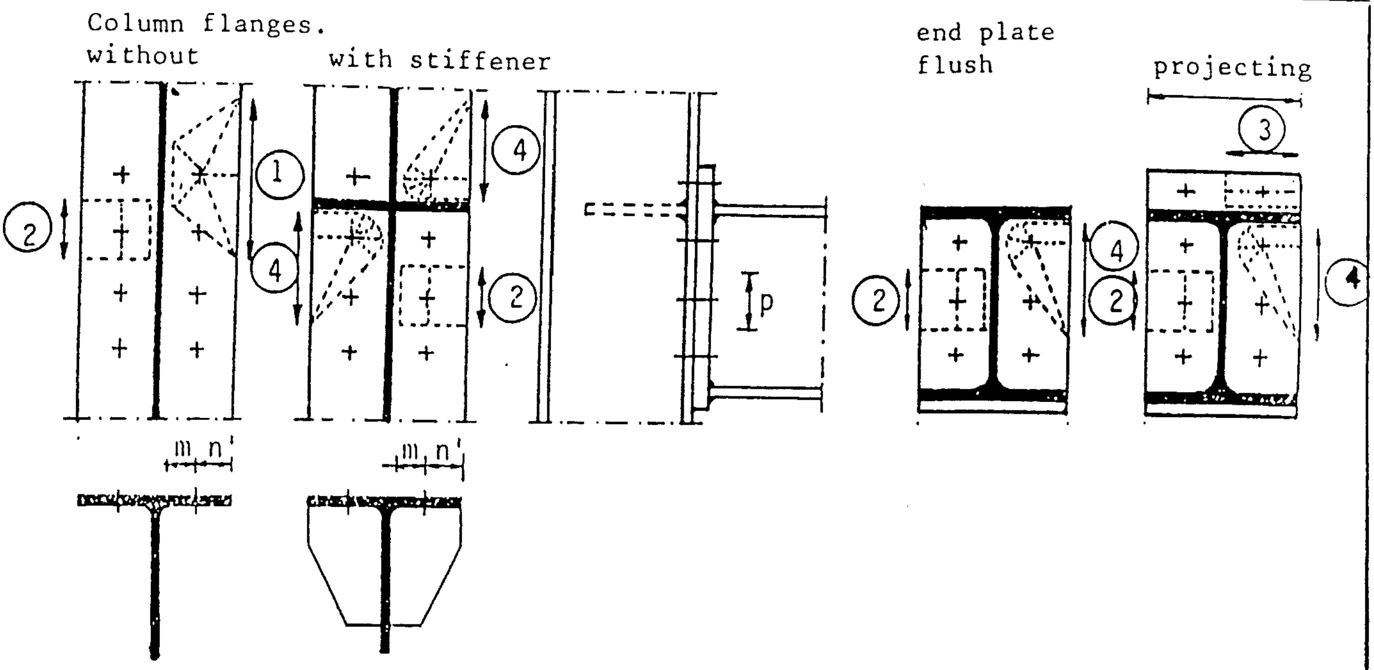
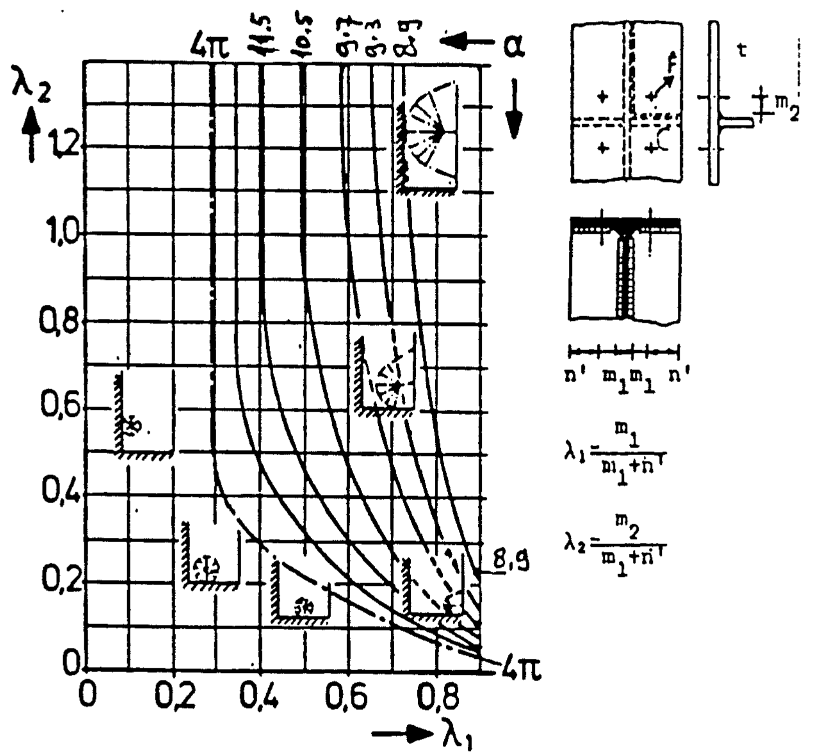


Fig. 2.11: Extension of the yield line pattern proceeds from the first row of bolts. The effective length is limited by the dimensions of the end plate or column flange.



1	column flange without stiffener	first bolt row	$b_m = 4\ m + 1{,}25\ n'$
2	column flange end plate	second bolt row	$b_m = p$
3	extended part of end plate	per side per bolt	$b_m = 4\ m + 1{,}25\ n' < \frac{b}{2}$
4	column flange with stiffener end plate	first bolt row (see diagram for between flanges)	$b_m = \frac{\alpha m}{2}$ (see graph for α)
5	bolt group	between flanges	$b_m = \Sigma\ b_m$



Note: This is the same diagram as in fig. 2.5.

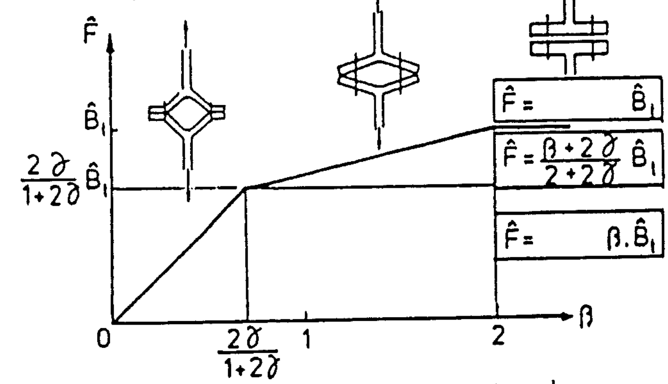
Fig. 2.12. Table for determining the effective length of the end plate or column flange.

Formulae for T-stubs — for two sides

$\hat{F} = \frac{4 . b_m . m_p}{m}$	complete yielding of plate
$\hat{F} = \frac{2 . b_m . m_p + \Sigma \hat{B}_t . n}{m + n}$ $\quad n \leq 1{,}25\, m$	yielding of plate + bolts
$\hat{F} = \Sigma \hat{B}_t$	bolt failure.

Now for part of plate or flange with one bolt with:

$$\beta = \frac{2 . b_m m_p}{m . \hat{B}_t} \quad \text{and} \quad \gamma = \frac{n}{m}$$



complete yielding of part of plate or flange	yielding of part of plate or flange for restraint and yielding of bolt.	bolt failure

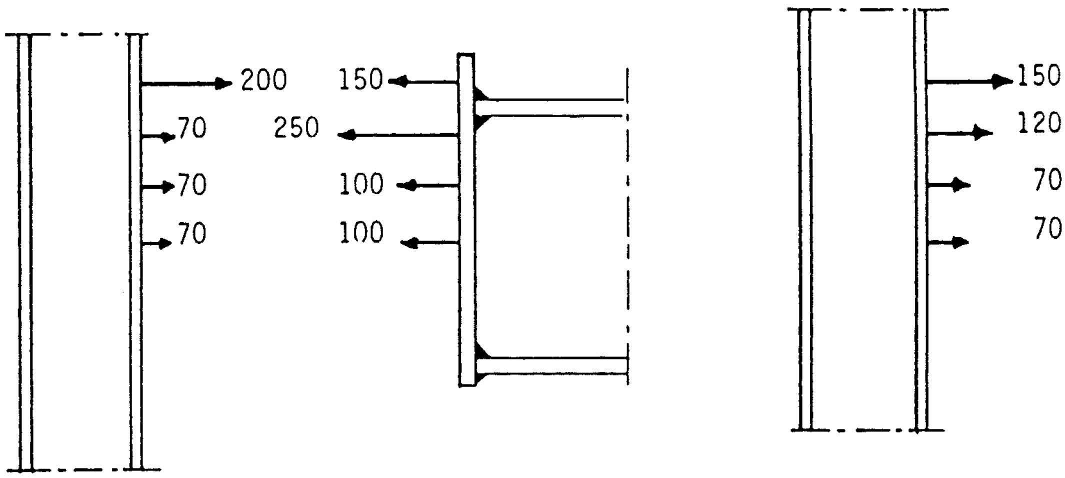
Fig. 2.13: Similarity between the design methods for the T-stubs and the end plate or column flange with bolt row is evident on rearranging the formulas for the T-stubs;

Suppose that the bolt force distributions as represented in Fig. 2.14a have been determined. The large bolt force in the top bolt row in the column flange is due to the large effective length assigned to the first bolt row. The yield line pattern is extended in consequence of the bolt rows located below that top row. For these rows only the bolt spacing is available as the effective length. Hence the bolt forces in these rows are smaller. The bolt force distribution obtained for the end plate is characterized by the fact that the bolt row in projecting part of this plate can transmit less force than the first bolt row in the column flange.
In this case the strength of the projecting part of the end plate must be adopted in calculating the design strength of the connection. The second bolt row in the column flange can now transmit more force.
This redistribution of forces cannot be assumed to occur if the column flange is supported by a stiffener located between the first and the second bolt row. In that case the bolt groups above and below the column stiffener function independently of each other. In the foregoing example it has also been assumed that the connection can deform to such an extent that the yield line pattern can develop. There are conceivable situations where this is not necessarily so, as will be considered in the next section.

2.6.2. Design strength of the connection

If the design strength of the bolt rows is known, the design strength of the connection ($\hat{M}_v$) can be determined. Various possibilities have to be taken into account. All of them (108) have been assembled in Fig. 2.15 and numbered from 1 to 108. The associated formulae for calculating the design strength of the connection as given in Fig. 2.16.

The failure mechanisms which may occur in the beam flange and the end plate are indicated schematically in Fig. 2.15. This table is divided into four sections in which are accommodated the combinations of the column flange with or without stiffener with the flush end plate or the projecting end plate. On the left in each combination is shown the mechanism of the column flange, and on the right the mechanism of the end plate.



A. Calculated bolt force distribution.

B. Bolt force distribution to be adopted.

Fig. 2.14 Calculated bolt force distributions and derivation of a bolt force distribution to be adopted, taking account of redistribution to be adopted,taking account of redistribution.

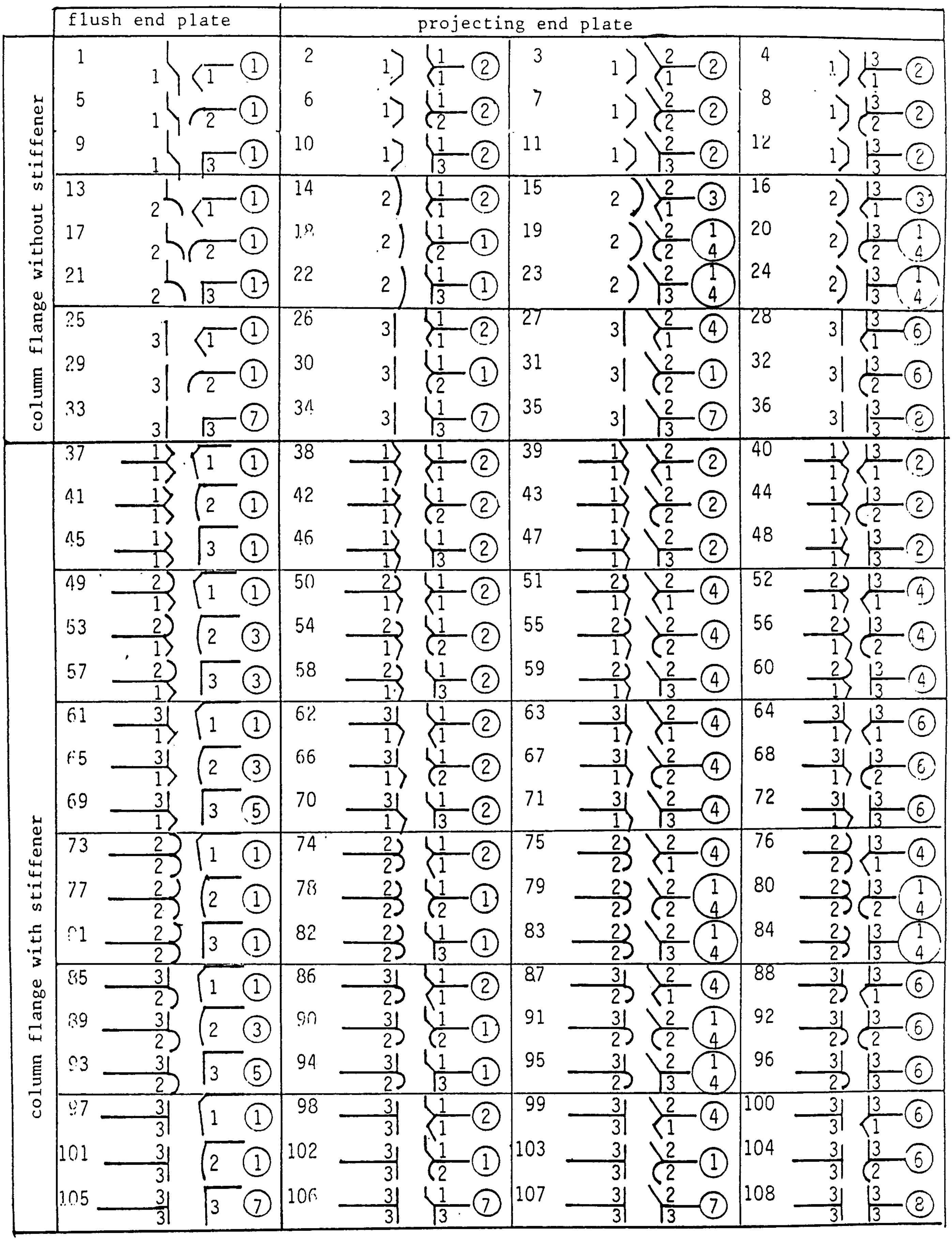


Fig. 2.15 Possible failure mechanisms with indication of the formulae to be used as given in Fig. 2.16

Is the following condition satified? $\Sigma \hat{F}_i \leq \hat{F}_c$ or $\leq \hat{F}_s$			
yes			no
	Flush end plate	Extended end plate	restrict the number of bolt rows to be taken into account in such a way that $\sum_{i=1}^{n} \hat{F}_i \leq \hat{F}_c$ or $\hat{F}_s$ where n is the number of bolt rows or parts thereof to be taken into account. Next, apply one of the formulae (1) to (8) given in this table
plastic theory	(1) $\hat{M}_v = \sum_{i=1}^{n} \hat{F}_i . h_i$	(2) $\hat{M}_v = (\sum_{i=1}^{n} \hat{F}_i . h_i + \hat{F}_e . h)$	
plastic theory	(3) $\hat{M}_v = (\hat{F}_1 . h_1 + \hat{F}_2 . h_2)$	(4) $\hat{M}_v = 2\ \hat{F}_e . h$ (T-stub)	
elastic theory	(5) $\hat{M}_v = 2\ \hat{B}_t \dfrac{(h_1^2 + h_2^2)}{h_1}$	(6) $\hat{M}_v = 4\ \hat{B}_t . h$ (T-stub)	
elastic theory	(7) $\hat{M}_v = 2\ \hat{B}_t \dfrac{\Sigma h_i^2}{h_1}$	(8) $\hat{M}_v = 2\ \hat{B}_t \dfrac{\Sigma h_i^2 + h_e^2}{h_e}$	

$\hat{F}_e$ = strength of the bolt row in the extended part of the end plate

$\hat{F}_i$ = strength of the ith bolt row between the beam flanges

= $(\hat{F}_{n_i} - F_{n_{i-1}})$ or $\hat{F}_i = \dfrac{h_i}{h_1} 2\ \hat{B}_t$ if $\beta_i \geq 2$.

$\hat{F}_{n_i}$ = strength of a bolt group up to and including the ith bolt row

= $\dfrac{\beta_i + 2\gamma}{2 + 2\gamma}\ i.2\ \hat{B}_t$ if $\beta_i > \dfrac{2\gamma}{1 + 2\gamma}$.

= $\beta_i . i . 2\hat{B}_t$ if $\beta_i \leq \dfrac{2\gamma}{1 + 2\gamma}$.

β_i = β for the plate with bolt group up to and including the ith bolt row

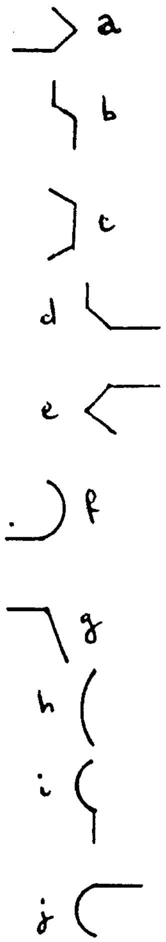
h_e = distance from the bolt row in the projecting part of the end plate to the centre of compression

h_i = distance from the ith bolt row to the centre of compression

h = distance from the tension flange of the beam to the centre of compression

Fig. 2.16 : Formulae for calculating the design strength of the connection (see Fig. 2.15)

Note: $\hat{F}_c$ = ultimate strength of compression zone
$\hat{F}_s$ = ultimate strength of shear zone



1. = complete yielding of plate or flange $\beta < \frac{2\gamma}{1 + 2\gamma}$ schematized to:
   a ... column flange with stiffener
   b ... column flange without stiffener, flush end plate
   c ... column flange without stiffener, extended end plate
   d ... extended part of the end plate
   e ... part of the end plate between the beam flanges.
2. = partial yielding of plate or flange with bolt failure $\frac{2\gamma}{1 + 2\gamma} < \beta < 2$ schematized to:
   f ... column flange with stiffener
   g ... column flange without stiffener, flush end plate
   h ... column flange without stiffener, extended end plate
   i ... extended part of the end plate
   j ... part of the end plate between the beam flanges.
3. = bolt failure without yielding of plate or flange $\beta > 2$; in this case the plate or flange remains virtually undeformed, as is indicated by straight line segments.

The tubular presentation in Fig. 2.15 is so arranged that on a horizontal line the mechanism of the column flange and the mechanism of the end plate between the flanges remain the same, while the projecting part of the end plate becomes progressively stiffer.

In the vertical direction as presented in the table the three possibilities for the part of the end plate between the beam flanges are repeated, while the column becomes progressively stiffer.

This means that the situations relating to the column flange with stiffener, with equal mechanisms on each side of the stiffener, are repetitions of the three groups of situations relating to the column flange without stiffener.

Thus the situations 37-48 are equivalent to the situations 1-12; the situations 73-84 are equivalent to 13-24; and the situations 97-108 are equivalent to 25-36. The number of possibilities has accordingly been reduced to 72.

Each combination is marked by a number enclosed in a circle. This number refers to the formula in Fig. 2.16 with which the design strength of the connection should be calculated. The odd numbers refer to the formulae for the flush end plate, the even ones to those for the projecting end plate. The higher the number, the stiffer is the connection. For numbers above the connection behaves elastically.

In Fig. 2.15, with reference to the possibilities with projecting end plate, situations are indicated in which formulae should be used, although this formula is actually applicable to the flush end plate.
Such situations correspond to the case where the part of the end plate between the beam flanges is so strong in relation to the projecting part that the part between the flanges cannot be expected to deform to such an extent that the calculated force will indeed be attained in the projecting part. Such situations were encountered in the tests, as will be reported in Chapter 3. The co-operation of the projecting part of the end plate can then best be neglected.
On the other hand, it may occur that the projecting part of the end plate is much stiffer than the part between the beam flanges, e.g., in the case where the column flange has a stiffener.
Above the stiffener there is, for example, only on bolt, which is not sufficiently strong to bring about yielding of the column flange. Below the stiffener there are several bolts, and these do cause complete or partial yielding of the column flange. In that case the part above the stiffener is so stiff that the mechanism between the beam flanges cannot develop before the bolt in the projecting part fails. Then only the bolts near the stiffener can be taken into account. Therefore formula (4) or (6) should be used for a projecting end plate, and formula (3) or (5) for a flush end plate if in the latter case the stiffener is installed between the first and the second row of bolts.
With the projecting end plate there then exists the situation where this end plate can be calculated as a T-stub, subject to certain conditions being satisfied, as appears from Fig. 2.15. Fig. 2.15 also indicates situations for which it cannot be decided in advance whether the connection should be calculated as a flush end plate or as a T-stub. For example, see the cases 19, 20, 23 and 24, or 79, 80, 83 and 84. In those cases the formulae (1) and (4) will have to be applied in order to see which of them yields the lower value.
Obviously, the formulae (7) and (8) in Fig. 2.16 give the highest strength of the connection. For applying these formulae the designer should be sure that the plates concerned remain straight up to failure of the bolts.
The analysis can then be based on elastic theory.

The deformation capacity in that case is very low. Most deformation capacity is obtained if the formulae (1) and (2) in Fig. 2.16 can be applied. Probably the lowest strength is obtained if the formulae (3) and (4) have to be applied.

It is also indicated in Fig. 2.16 that the condition $2 \sum \hat{F}_i < \hat{F}_c$ or $< \hat{F}_s$ should be verified, i.e., whether the sum of the forces on the tension side can equilibrate the design strength of the compression side $\hat{F}_c$ or equilibrate the shearing panel of the column web $\hat{F}_s$. The formulae for calculating $\hat{F}_c$ and $\hat{F}_s$ are given in Section 2.11.
If the sum of the tensile forces exceeds $\hat{F}_c$ or $\hat{F}_s$, the total tensile force should be reduced. This can be achieved by succesively neglecting the forces, proceeding from the centre of compression, until equilibrium is reached. In that case the design strength of the connection is equal to the sum of the products of the remaining forces and their lever arms with respect to the centre of compression.
Fig. 2.17a schematically shows the situation where the formulae (1) to (4) should be applied because the end plate or the column flange is not sufficiently stiff and strong to justify the assumption of elastic behaviour. Should it nevertheless be desired to apply elastic analysis, then the ratio of plate strength to bolt strength will have to be increased.
For column flanges it then becomes necessary to employ stiffeners, as shown in Fig. 2.17b, where a simple rule for calculating the end plate thickness is also indicated. This rule has been adopted from the American literature /18/ and is based on the assumption that the tensile force in the beam flange is transmitted entirely through bending from the end plate - i.e., without supporting action from the beam web - the top row of bolts.

2.6.3. Distribution of forces in the web behind end plate or column flange

If an elastic analysis is dispensed with and if the distribution of forces as imposed by the plate strength to bolt strength ratio is accepted, the end plate thickness can be adapted to suit this.

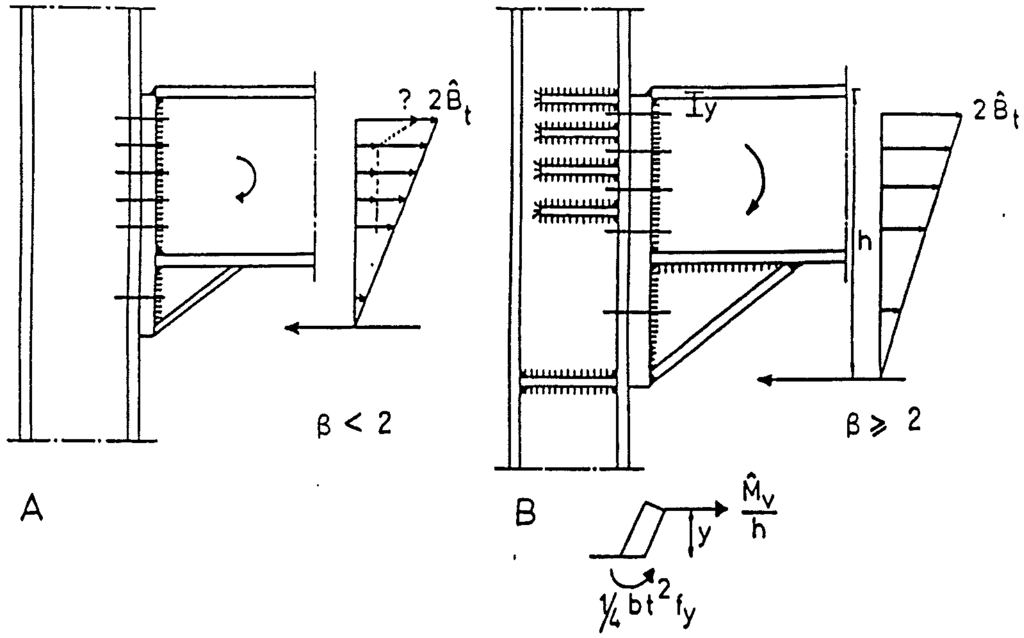


Fig. 2.17: Possible distributions of forces in the connections depend on the plate strength/bolt strength ratio β.

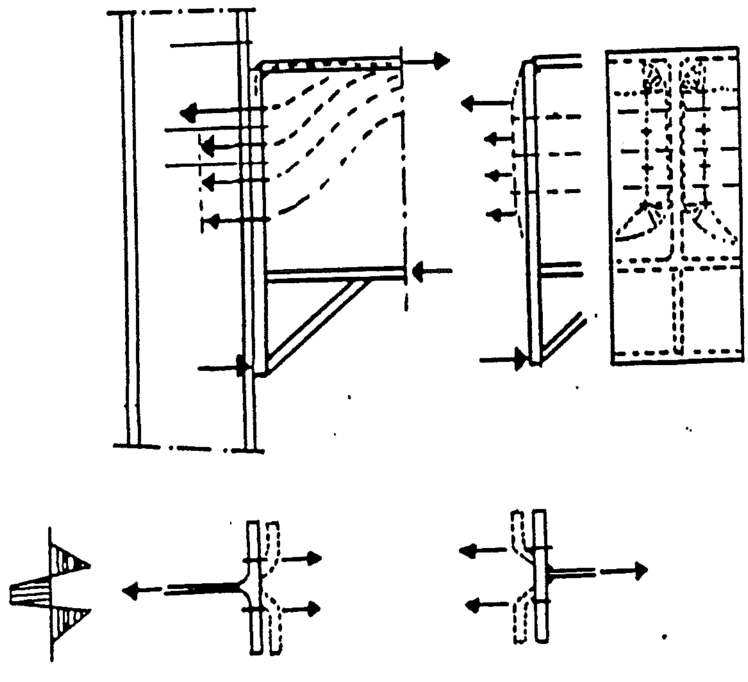


Fig. 2.18: If the thickness of the end plate is adapted to the forces that the column flange can transmit, the forces must pass via the beam web to the beam flange.

It should then be realized that the design method is based on the assumption that the forces are transmitted via the beam web to the beam flange and that the web must be capable of so functioning /19/ (see Fig. 2.18). If the end plate need not develop any deformation capacity, the simple (elastic) method of calculating the thickness of the end plate, as described above, is the least time-consuming. It will always be necessary to check that the bolt forces can be transmitted by the webs behind the column flange and end plate. For this purpose it is permissible to take account of redistribution of forces in the web. The effective length of the web can be taken as equal to the effective length in the flanges or the end plate.

2.7 Deformation capacity

The deformation capacity of a connection may be due to:

a. yielding of the column web in consequence of shearing;
b. yielding of the column web on the compression side of the connection;
c. yielding of the column flange or end plate on the tension side of the connection.

In general, the phenomenon mentioned in point a. will provide the greatest deformation capacity. It cannot occur in symmetrically loaded connections, however. In that case the phenomenon mentioned in point c. must be presumed. From tests /14, 15, 27/ it has emerged that considerable deformation capacity is obtained from the tension side of the connection if:

$$\beta < \frac{2\gamma}{1 + 2\gamma}$$

either a. the whole bolt group in the case of column flanges without stiffeners (situations 1-12 in Fig. 2.15),

or b. the bolt groups in the parts of the column flange above and below the stiffener (situations 37-48 in Fig. 2.15),

or c. the bolt group in the end plate, provided that the part there of projecting outside the flanges also deforms sufficiently (situations 13, 14, 25, 26, 49, 50, 61, 62, 73, 74, 85, 86, 97 and 98 in Fig. 2.15).

In these cases there occurs complete yielding of one of the plate components mentioned. In the intermediate range $\frac{2\gamma}{1 + 2\gamma} < \beta < 2$ the bolt strength is sufficient to cause yielding at the at the transition form plate to web, but the bolts fail before the prying force at the edge of the plate becomes so large that yielding of the plate at the bolts also occurs. The deformation then remains limited. From the test results an approximate formula for the deformation capacity in that situation has been deduced, namely if:

$$2\gamma/(2\gamma + 1) < \beta < 1{,}75$$

then the rotational capacity is:

$$\phi = \frac{10.6 - 4\beta}{1.3h}$$

where:

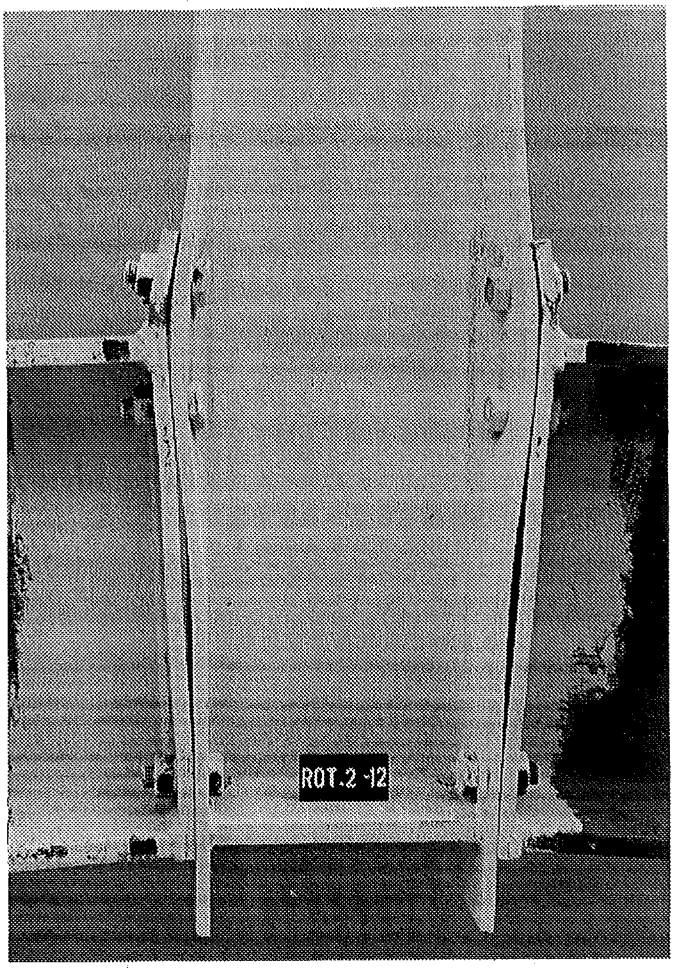
h = the distance in mm between the first bolt row from the tension flange and the centre of compression.

If $\beta > 2$, then no deformation of the plate occurs. The connection behaves elastically up to failure of the bolts. The deformation will in that case have to be supplied by the strain of the bolts. Safe values for the plastic strain are 2 mm for 8.8 bolts and 1 mm for 10.9 bolts.
It is however, important to avoid having the deformation capacity provided by failure of the bolts. To ensure that this condition is satisfied, it is seesntial that $\beta < 1.75$.
This latter value has been chosen because in checking the test results with respect to deformation capacity the design strength adopted for the bolts was 0.7 of the guaranteed ultimate strength. It is likely, however, that this requirement for the design strength will be raised to 0.8 in the foreseeable future. In anticipation of this the value $\beta = 2$ has been reduced to $\frac{7}{8} \times 2 = 1.75$.

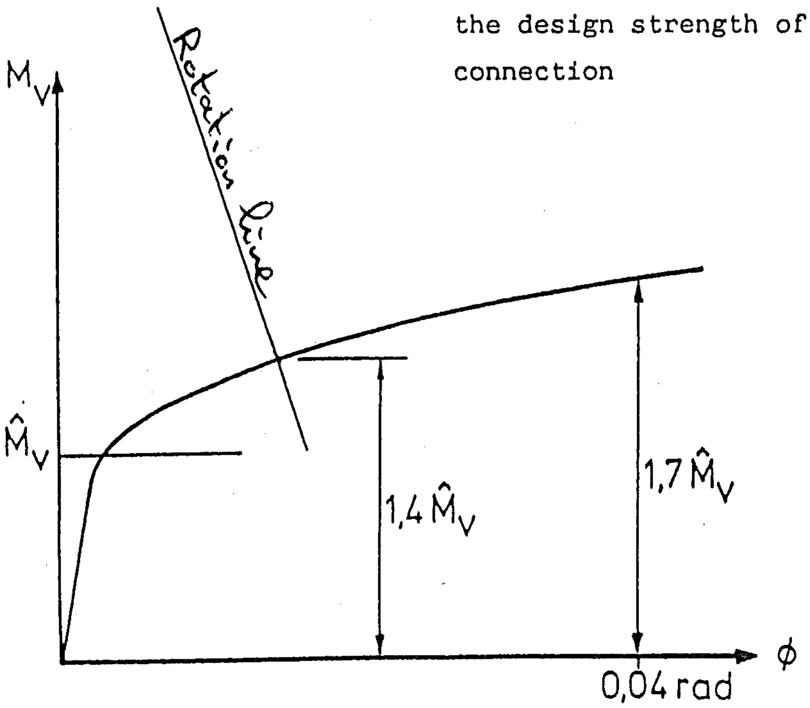
2.8 Deformation capacity and bolt force capacity

In Chapter 1 it has been explained that the components of the connection must be stronger than the design strength of the connection if the latter is to provide rotational capacity. The welds between the end plate and beam must be designed to a bending moment which is higher than the strength of the connection by a factor 1.4 or 1.7 in a braced or an unbraced frame respectively (see Fig. 2.19).

It may be asked whether this applies also to the bolts. That this is not so is apparent from Fig. 2.20. The connection show on the right in Fig. 2.20a has an end plate and column flange which are so thick in relation to the bolt strength that plate and flange remain elastic up to failure of the bolts. The plate strength to bolt strength ratio is higher than 2, as is indicated in Fig. 2.20b. The moment-rotation characteristic is almost linear (see Fig. 2.20c).
Suppose that a thinner end plate is used, so that the plate strength to bolt strength ratio becomes lower than $2\gamma/(1 + 2\gamma)$. As a result, deformation capacity is obtained because more than $1/(1 + 2\gamma)$ of the design strength of the bolt is used for causing the plate to yield at the bolt row by the action of the prying force.
In the example: $\gamma = 1.25$ and therefore $1/(1 + 2\gamma) = 28.5\%$, say 30%.
The design strength of the connection becomes about 30% lower than that of the connection, which behaves elastically up to failure of the bolts (see Fig. 2.20c). In consequence of the large deformations of the end plate, membrane forces are developed in it, which ensure that the resistance of the connection continues to increase with increasing rotation. With this procedure 30% of the bolt force capacity is therefore sacrificed in order to obtain deformation capacity. As a result the moment-rotation characteristic continues to rise, and it is not correct to demand extra bolt force capacity to maintain safety against failure at this higher moment.
In the situation where the rotational capcity is due to shearing of the column web or failure of the compression side it is, however, necessary to provide extra bolt force capacity if the end plate or column flange remains elastic up to failure of the bolts, i.e., if $\beta > 2$.



A



B

With increasing deformation the force on the tension side of the connection continues to increase because embrane forces develop inthe deformade plate components. The welds must be able to transmit this increase inforce. It appears from the test results that the moment-rotation characteristic intersects the line of required rotational capacity for a value opf the moment which is 1.4 times the design strength inthe case of braced frames. Analyses of unbraced frames show that the required rotational capacity is no more than 0.04 radian. As the test results indicate, the moment is then higher by a factor of 1.7 than the design strength of the connection

Fig. 2.19 : In the case of connections which have to provide rotational capacity the welds should be made stronger than the end plate with bolts.

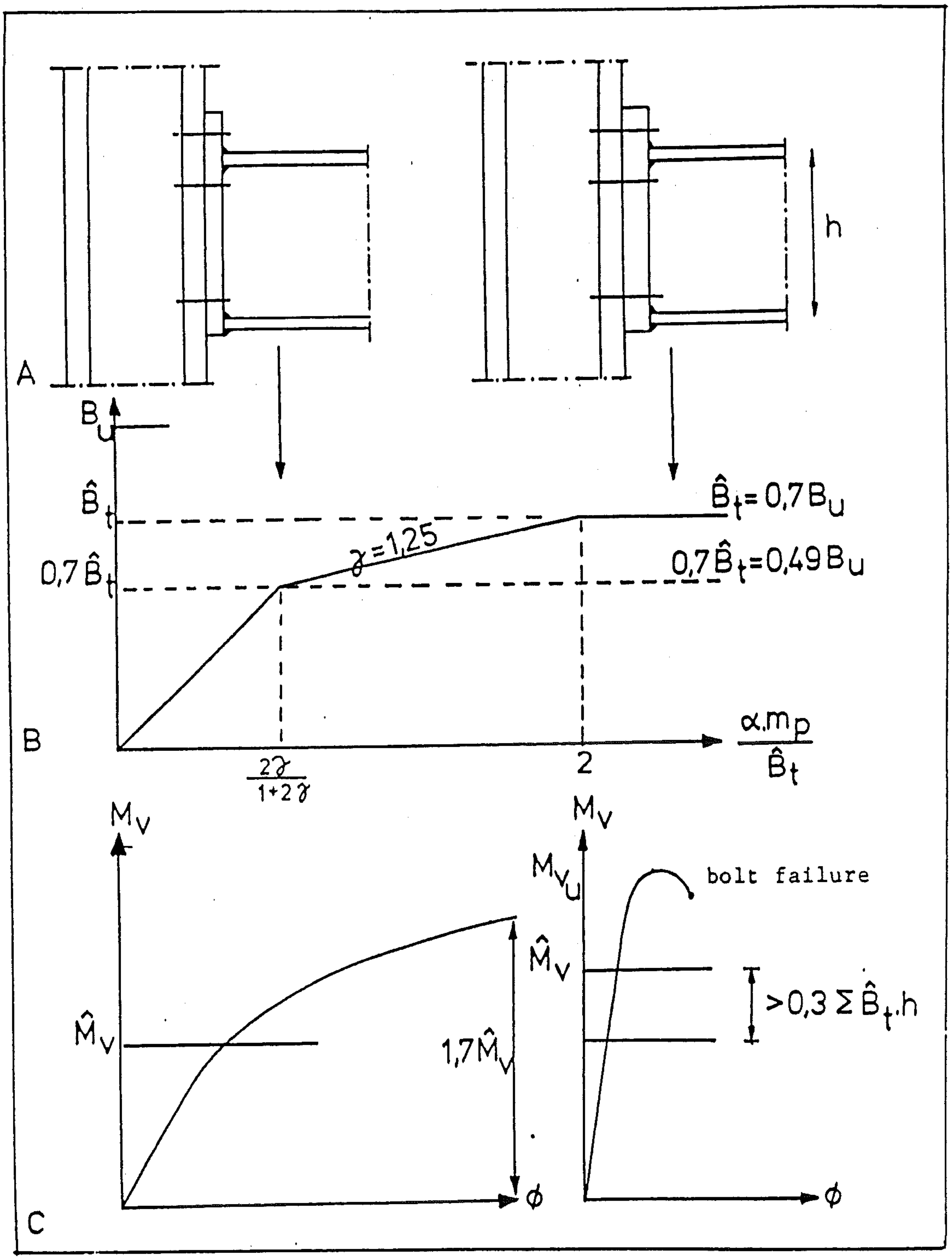


Fig. 2.20: Comparison of the behaviour of connections with and without deformation capacity, from which it appears that no extra bolt force capacity is necessary.

2.9 Stiffness

The method of analysis for stiffness is based on observations made from tests and on theoretical approximations. It provides insight into the contribution that each component of the connection makes to the deformation. From Chapter 1 it appears that the introduction of too low a stiffness of the connection into the calculation presents no problems if the connection possesses sufficient rotational capacity. By undergoing plastic deformation the connection adjusts itself to the under-estimated stiffness.
On the other hand, if too high a value is adopted for the stiffness of the connection, this is liable to have adverse consequences for the actual stiffness and stability of the structure.
That is why the formula in Fig. 2.21 gives an approximation of the stiffness of the connection in a situation where plastic deformations may permissibly have already occurred, namely, the design strength of the connection. However, the formulation has been so contrived that the initial stiffness of the connection can also be approximately determined.
It remains an approximation because its actual stiffness depends on the force with which the bolts are tightened and on the location of the contact pressure /17/. This latter aspect will be considered in the next section.

The with aid of formula (2.6) in Fig. 2.21 the spring stiffness of the connection corresponding to the design strength $\hat{M}_v$ of the connection is determined. The basic principle is that the flexibility is a summation of the flexibilities due to (see Fig. 2.22):

a. shear deformation of the column web,
b. compressive deformation of the column web,
c. tensile deformation of the column web) at the bolt row
   column flanges) outside the beam
   bolts end plate) flanges

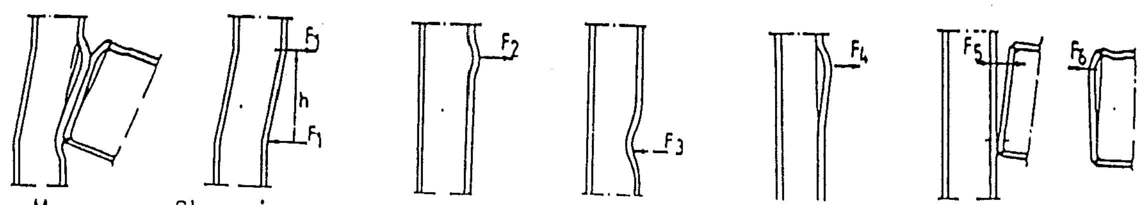
if the projecting end plate has sufficient stiffness and strength, and otherwise at the first bolt row between the flange. The correct choice of the bolt row must be determined from an analysis of the two possibilities. The lower of the two values obtained is the governing value. This will be further considered in the context of the design examples (see example 17, Fig. C10).

The stiffness factor $c = \frac{M}{\phi}$ is $c = \left(\sum_{i=1}^{6} f_i\right)^{-1} E.h^2$ (2.6)

where $f_i = \frac{1}{c_i} \times \left(\frac{F_i}{\hat{F}_i}\right)^2$

f_i = flexibility factor of component i
c_i = stiffness factor of component i
F_i = actual force in component i
$\hat{F}_i$ = design strength of component i

The total deformation is composed of deformations due to:



$\frac{M}{\phi}$	Shearing of column web	Tension in column web	Compression in column web	Tension in column web	Tension in bolts	Tension on end plate
c_i for F_i :	$c_1 = 0{,}24 t_w$	$c_2 = 0{,}8 t_w$	$c_3 = 0{,}8 tw$	$c_4 = \frac{t_f^3}{4m^2}$	$c_5 = \frac{A_s}{l_b}$	$c_6 = \frac{t_e^3}{12\lambda_2 m^2}$

The deformation on the tension side is determined by one bolt row. The effect of the other bolt row is taken into aacount by multiplying by the quotient of the design strengths of the actual connection and the notational connection with one bolt row.

Fig. 2.21 : Schematic representation of the method of analysis of the stiffness of the connection.

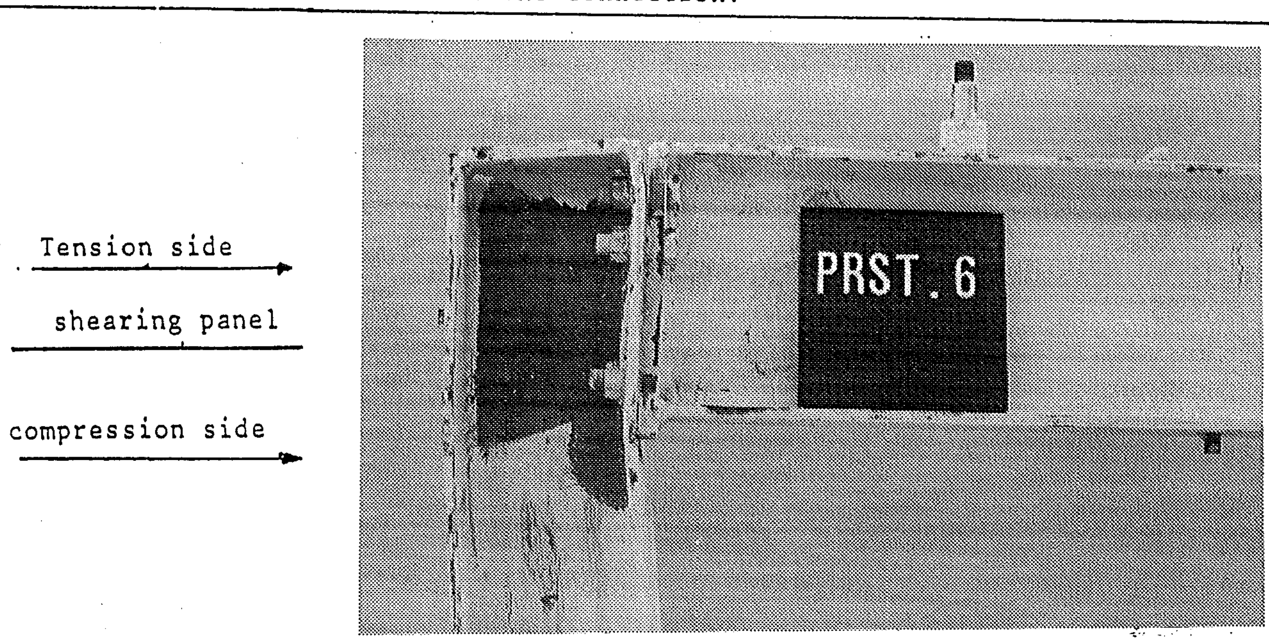


Fig. 2.22 : Possible deformations in the connection in the ultimate limit state.

Design example 12: Stiffnes analysis

Same connection as in example 5: behaviour at failure governed by shearing zone and projecting part of end plate(end plate thicjness t_e = 15mm)

First bolt row between flanges determines rotation.

Component or mechanism	Formula of C_i for $\hat{F}_i$	$(\frac{\hat{F}_i}{F_i})^2$	C_i for $\hat{M}_v$	C_i for $\hat{M}_v/1,5$
Shearing of column web	$C_1 = 0,24\ t_w = 0,24 * 8,5$	1	2	4,6
TEnsion on column web	$C_2 = 0,8\ t_w = 0,8 * 8,5$	$(\frac{704}{310})^2$ *	15	15
Compression on column web	$C_3 = 0,8\ t_w = 0,8 * 8,5$	$(\frac{418}{310})^2$	12	15
Tension-ebending of column web	$C_4 = \frac{t_f^3}{4m^2} = \frac{14^3}{4 * 29,15^2}$	$(\frac{278}{189})^2$	1,7	1,8
Tension in bolt	$C_5 = \frac{2A_s}{l_b} = \frac{2 * 353}{60}$	$(\frac{243}{189})^2$	19,5	26,5
Tension-bending of end plate	$C_6 = \frac{t^3}{12\lambda_2 m^2_1} = \frac{15^3}{12 * 0,481 * 45^2}$	$(\frac{243}{189})^2$	0,48	0,65

* $\hat{F}_i$ for tension= $b_m \cdot t_w \cdot \sigma_e = 345 * 8,5 * 240 = 704$ kN

$$(\frac{\hat{F}_i}{F_i})^2 \leq 2,25$$

Spring stiffness for the notional connection with one bolt row.

$$c = (\frac{1}{2} + \frac{1}{15} + \frac{1}{12} + \frac{1}{1,7} + \frac{1}{19,5} + \frac{1}{0,48}) * 2,1 * 10^5 * 345^2 =$$

$$c = \frac{0,296 * 2,1 * 10^5 * 345^2}{10^6} = 7410 \text{ kNm/rad} \quad \text{for notional connection}$$

For the actual connection

$$c = \frac{115}{189 * 0,345} * 7281 = 1,76 * 7410 = 13042 \text{ kNm/rad for } \hat{M}_v = 115 \text{ kNm}$$

$$\emptyset = \frac{115}{13042} = 8,82 * 10^{-3} \text{ rad for 115 kNm}$$

For 1.5-fold lower loading, i.e. for 77 kNm

$$c = (\frac{1}{4,6} + \frac{1}{15} + \frac{1}{15} + \frac{1}{1,8} + \frac{1}{26,5} + \frac{1}{0,65}) * 2,1 * 10^5 * 345^2$$

$$c = \frac{0,402 * 2,1 * 10^5 * 345^2}{10^6} * 1,76 = 10.0686 * 1,76 = 17721 \text{ kNm/rad}$$

$$\emptyset = \frac{77}{17721} = 4,35 \times 10^{-3} \text{ rad for 77 kNm}$$

Fig. 2.23 : Analysis for the stiffness of the connection considered in example 5.

The deformation on the tension side is assumed to be independent of the number of bolt employed. This means that the stiffness of a connection can first be calculated with one bolt row. The actual stiffness of the connection can then be found by multiplying this result by the quotient of the strengths of the actual and of the notional connection.
The flexibility of each component of the connection is expressed in the reciprocal value of a stiffness factor C_1. This factor can be multiplied by a quadratic term if the load on the component is lower than the design strength thereof. The deformation of a component is assumed to increase linearly with the load up to 67% of the design strength of the component and then increases quadratically. Because the stiffness factor C_1 is given in conjunction with the design strength of the component, this means that at 67% of the strength the stiffness is 2.25 times as great as that which is calculated for the design strength of the component.

In determining the stiffness of the notional connection with on bolt row it is, for calculating the load on the compression side of the shearing zone, necessary to take account of the force which equilibrates the sum of the forces of all the bolt rows, however. Fig. 2.23 presents a design example for the connection already considered in example 5.
Calculating the stiffness comprises the following operations:

a. Determine the load of the compression zone,
the shearing zone,
the first bolt row.
b. Determine the quotients of the design strengths and loads of the above mentioned components and square these values; the squares must not exceed 2.25.
c. Determine for each component the stiffness factor C_1 and multiply these factors by the quadratic terms already calculated.
d. Find the sum of the reciprocal values of the stiffness factors thus determined.
e. Take the reciprocal value of the sum obtained in this way and multiply this value by the modulus of elasticity and the square of the distance between the centre of compression and the bolt row assumed.
f. Multiply the value thus obtained by the quotient of the design strengths of the actual connection and of the notional connection with one bolt row.

With the calculation procedure outlined above it is also possible to calculate the stiffness of a connection at a lower value of the load than its design strength. Up to 67% of the design strength this spring stiffness is constant and approximately twice as high as it is at the design strength. Fig. 2.24 summarizes the moment-rotation characteristics which have, in Appendix C, been calculated by the above method for the connections of examples 5 to 8.

The dash lines represent approximations at loads lower than the design strengths. For these lines only the point corresponding to 67% of the design strength has been calculated (see design examples).

In example 17 it is investigated whether, in determining the spring stiffness of the projecting end plate, the calculation should be based on a notional connection with one bolt row in the projecting part of the end plate or with one bolt row between the beam flanges. In borderline cases, both calculations will have to be performed in order to determine which of them yields the lower governing value.

In the other examples the choice is found to be more or less obvious if the projecting part of the end plate is of such low stiffness that it fails before the connection does. This is the case in the connections with an end plate thickness of 15 mm in the examples 5 and 6. The behaviour of these connections is comparable to that of a flush end plate connection, as appears also from Fig. 2.24. A small increase in design strength of the connection, so that it no longer fails due to shearing of the column web, results in large deformations of the end plate: in consequence, the design stiffness rapidly diminishes with increasing design strength.

2.10 Increasing the stiffness of the connection

It is sometimes necessary to increase the stiffness of the connection. For that purpose, as the formula for the spring stiffness indicates, the most effective measure consists in increasing the distance between tension and compression side, because the distance h_s occurs squared in the formula (see Fig. 2.25a).

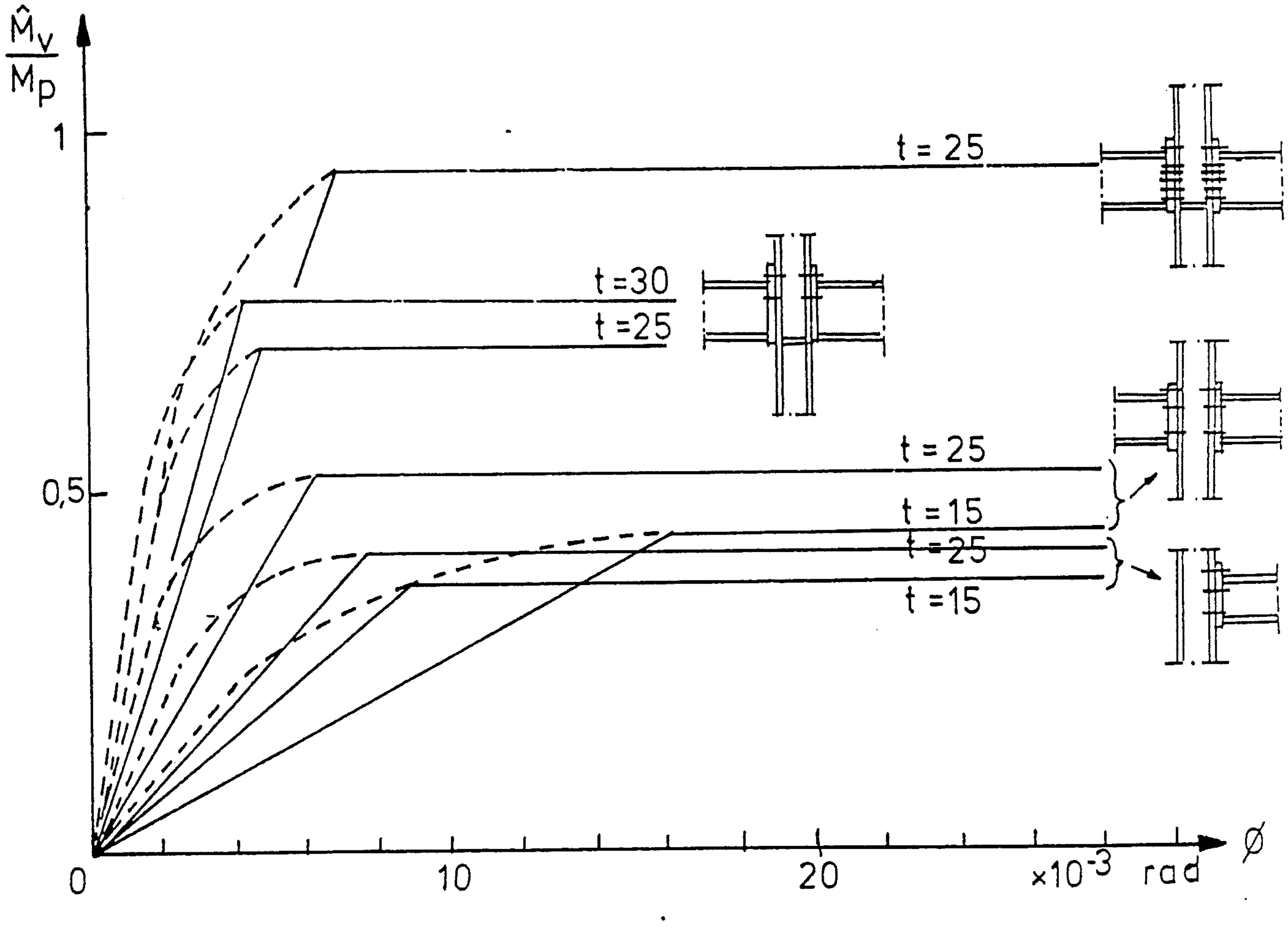
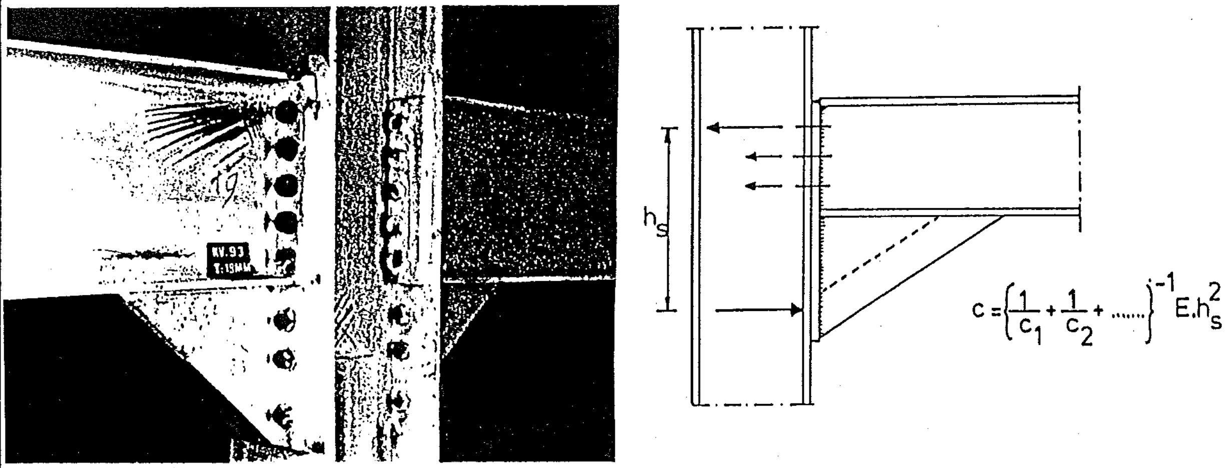
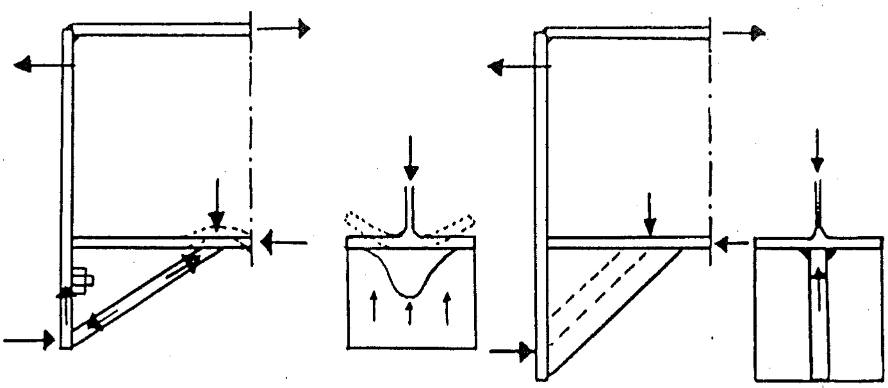


Fig. 2.24: Summary of moment-rotation characteristics of examples 12 to 18 (see Appendix B).

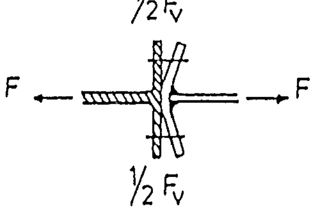


A. Increasing the lever arm by means of a haunch under the beam is very effective because the lever arm occurs squared in the stiffness formulea. Tests have shown that such a haunch need not have a flange.



B. In the case of connections without stiffeners the haunch without flange is to be recommended because the forces must be transmitted into the webs of the column and beam.

Als: $F_v > F$



$$\frac{m_k^2}{t_{f_k}^3} + \frac{l_b}{A_s} + \frac{m_e^2}{t_e^3} = 0$$

C. Prestressing the bolts with a force larger than the tensile force has the effect that the bending of the end plate and column flange can be neglected, provided that the contact pressure acts in alignment with the column web and beam web.

Fig. 2.25: Possibilities for increasing the stiffness of the connection.

Increasing this distance, i.e., the depth of the connection, can be achieved quite simply by installing a thick haunch plate under the beam, as shown in the left-hand diagram of Fig. 2.25b.
With the aid of the appropriate formulae given in the code of practice it must be shown that this inclined plate will not buckle prematurely. If no stiffeners are needed for transmitting the compressive force into the web of the beam or of the column, this method of increasing the depth is in better agreement with the pattern of forces than is the solution with a solid haunch as shown on the right in Fig. 2.25b, where the transmissio of force is concentrated at the end plate and beam flange and then spreads into the haunch. With the inclined thick haunch plate the force is distributed over the height (or depth) of the connection. A design method for this last-mentioned type of haunch and the connecting welds is proposed in /19/.

Another possibility for increasing the stiffness of the connection is by prestressing the bolts. This is, however, meaningful only if the contact pressure due to the prestress is aligned with the column web and beam web. Such alignment is indeed quite likely, because the end plate will, as a result of shrinkage of the welds, acquire a shape as shown in Fig. 2.25c. If it is ensured that the contact pressure is greater than the maximum force occurring on the tension side of the connection, then the transmission of force will be accomplished by reduction of the contact pressure, in which case the deformations due to bending of the end plate, bending of the column flange and tension in the bolts can be neglected in calculating the stiffness of the connection (see Fig. 2.25c).

2.11 Increasing the strength of the connection

The usual methods of strengthening the connection, more particularly by providing stiffeners between the column flanges, will not be considered here. These are adequately dealt with in the existing literature.

2.11.1. Tension side (packing plates)

The column flanges can be strengthened with packing plates as shown in Fig. 2.26, as a result of which the plate strength $\hat{F}_p = 2b_m \cdot m_p/m$ is increased by a factor $(1 + b_o m_{po}/2b_m \cdot m_p)$,

where: b_o = length of the packing plate $< b_m$,
m_{po} = plastic moment per unit length of packing plate,
b_m = effective length of column flange of T-stub flange,
m_p = plastic moment per unit length of column flange.

In this factor and in the formulae given in Fig. 2.26 it is clearly apparent that the plastic moment at the web-to-flange transition is not increased by the packing plates. Hence it follows that there is no point in using these plates if $\beta > 2\gamma/(1 + 2\gamma)$, because then yielding occurs only at the web-to-flange transition and failure of the bolts is the governing condition. In the case where $\beta < 2\gamma/(1 + 2\gamma)$ the strength of the connection can be substantially increased, e.g., by a factor 1.5, by giving the packing plate a length equal to the effective length and a thickness equal to that of the column flange.

2.11.2. Compression side

For the compression side of the connection a thickening (strengthening) plate applied to the column web as in Fig. 2.27c does not provide an adequate solution. It introduces eccentricity which cancels the advantage of increasing the cross-sectional area.

The solution illustrated in Fig. 2.27d does indeed achieve an increase in the strength of the compression side of the connection, but the number of experimental results is insufficient for establishing a formula for calculating the increase. Tests have shown that strengthening of plates as illustrated in Fig. 2.27a can be calculated with the formula:

$$\hat{F}_c = [t_{f_1} + 2t_e + 2a\sqrt{2} + 5(t_{f_k} + r_k)]\,(t_{w_k} + \tfrac{1}{2}t_o)\,f_y \qquad (2.8)$$

where:

a = thickness of fillet weld between end plate and beam flange (throat thickness),

t_{f_1} = thickness of beam flange,

t_e = thickness of end plate,

t_{f_k} = thickness of column flange,

r_k = radius of column web-to-flange transition,

t_{w_k} = thickness of column web,

t_o = thickness of strengthening plate.

It appears from this formula that only half the thickness of the strengthening plate is taken into account. If the width/thickness ratio of this plate is above 40, plug welds must be employed. The throat thickness of the weld along the column flange should be equal to the thickness of the strengthening plate. The horizontal welds should have a throat thickness equal to $\frac{1}{2}t_o \sqrt{2}$.

2.11.3. Shearing zone

If the column web is too weak in shear, it can be strengthened with a plate as shown in Fig. 2.27b. The distance a. should be made as small as possible, but sufficient to enable the fillet weld joining the plate to the web to be properly executed.
Local yielding will occur over the distance a., but the resulting deformations are small. The strength of the column web can be calculated with the formula:

$$\hat{F}_s = [(h_k - 2t_{f_k})\, t_{w_k} + b_o \cdot t_o]\ 0.58\ f_y \qquad (2.9)$$

where:

h_k = depth of column section,

b_o = width of strengthening plate,

t_o = thickness of strengthening plate.

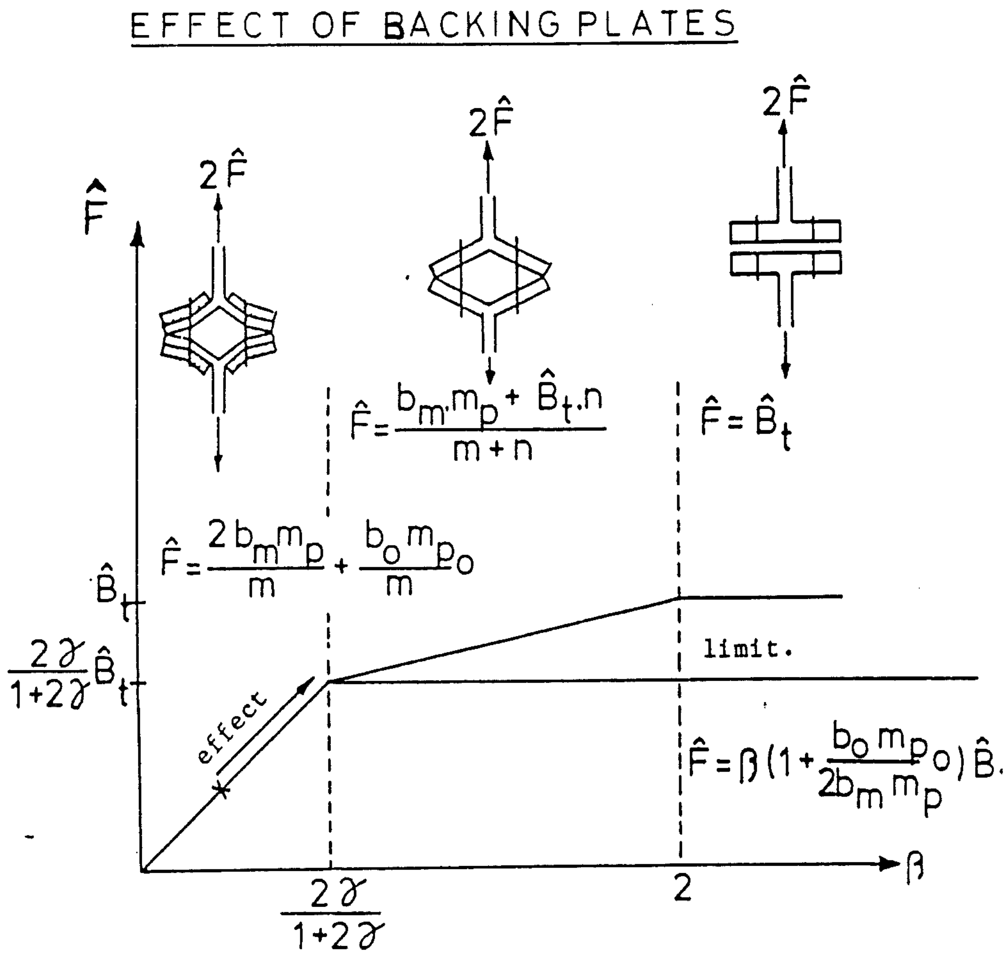


Fig. 2.26: Effect of packing plates.

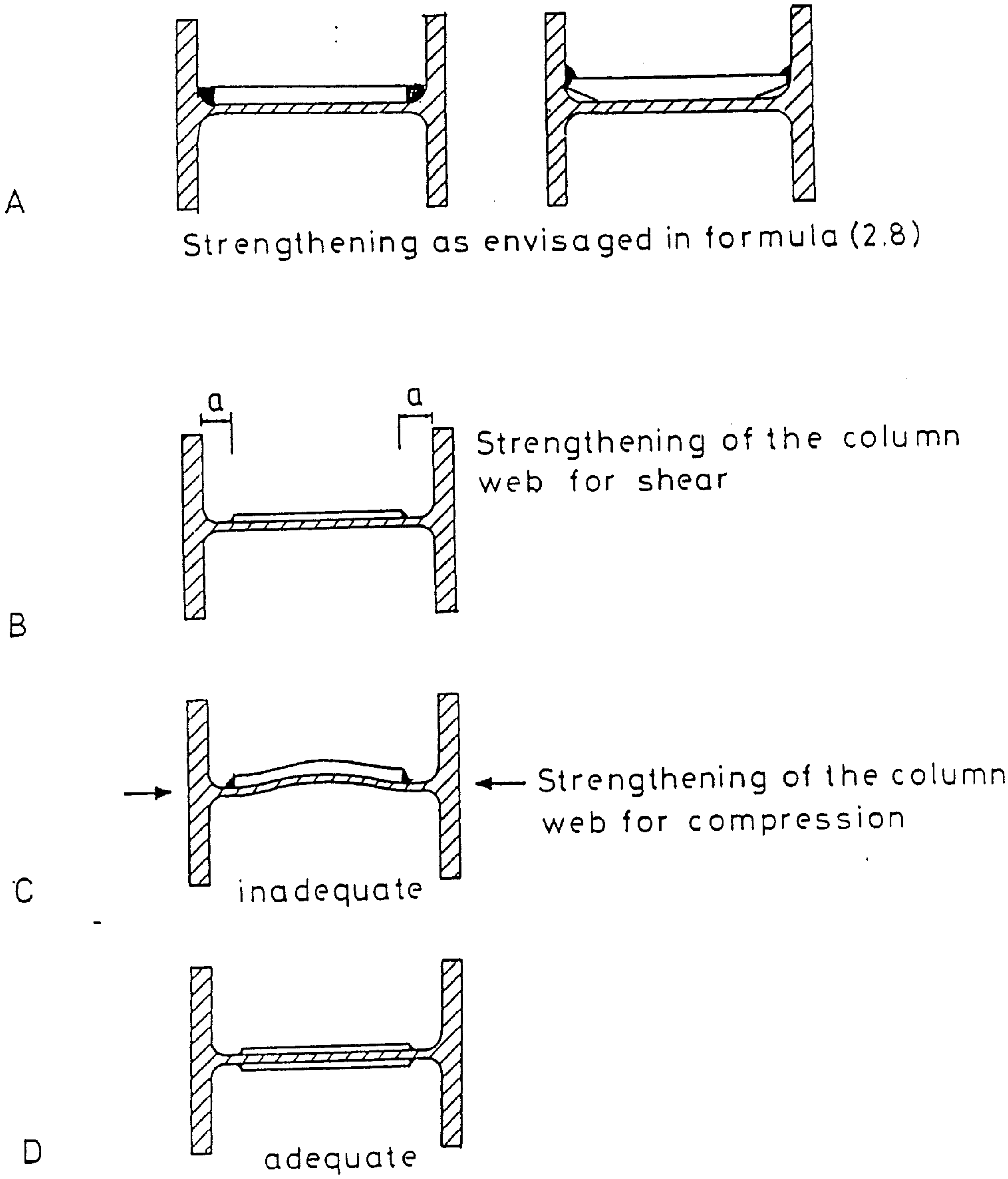


Fig. 2.27: Strengthening of column web with thickening plates.

2.12 Interaction of mechanisms

2.12.1. Compression and shearing of column web

Tests /9/ have shown that interaction of normal and shear stresses, as given by the Huber-Hencky-von Misses criterion, need not be taken into account in the determination of the design strengths of the shearing zone and compression zone of the column web. This means that these strengths can be determined independently of each other with the formulae (2.8) and (2.9).

2.12.2. Compression and compression in column web

Tests /9/ have shown that an axial compressive stress σ_n in the column web exceeding one-half of the yield stress will adversely affect the strength on the compression side of the connection. This is taken into account by applying a reduction factor $(1.25 - 0.5\sigma_n/f_y)$ to the value obtained with formula (2.8).

2.12.3. Compression and bending in column web

Because of the large deformations of the column flanges, as shown in Fig. 2.19, a compressive force in the column flange can be expected to have an adverse effect on the strength in the tension zone of the connection. The problem has been investigated in some tests using a set-up as shown in Fig. 2.28, where a specimen after undergoing testing is also illustrated /20/. Fig. 2.29 presents some characteristic test results. First, a reference test without axial compressive force in the flanges was performed (test A). The moment-rotation curve obtained in this test is shown in the two bottom diagrams in Fig. 2.29.

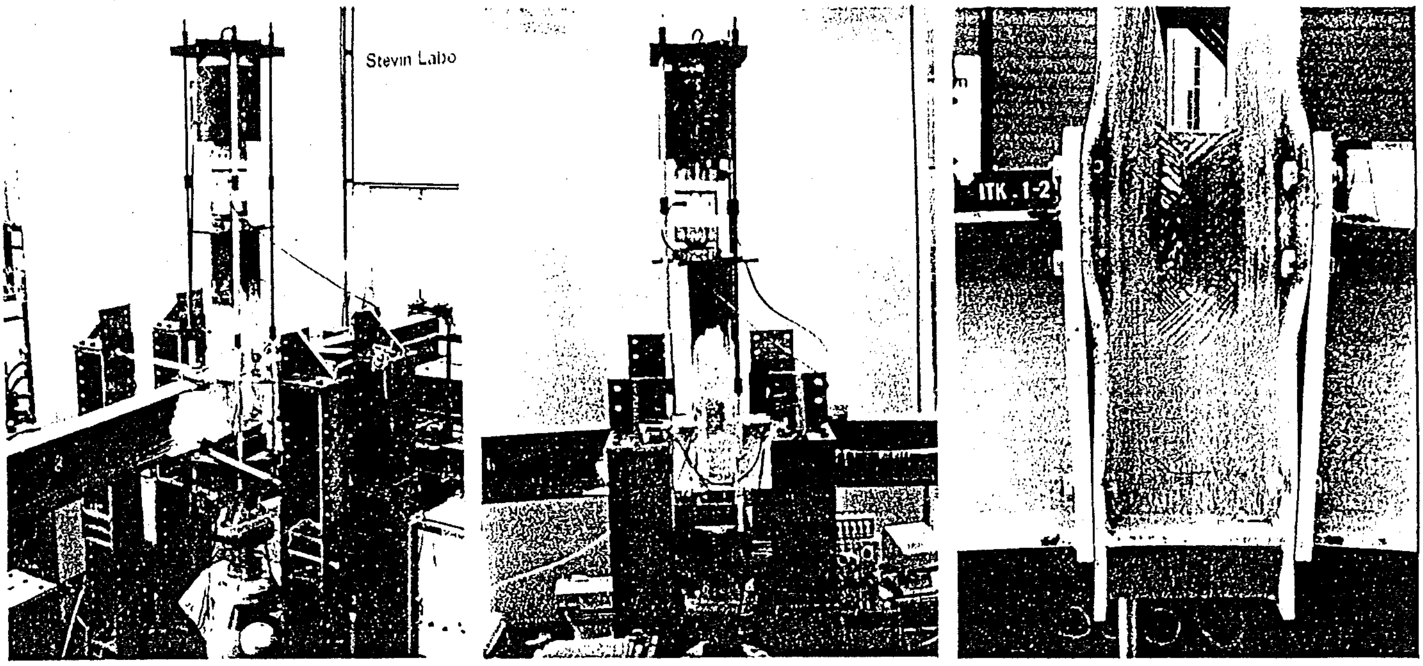


Fig. 2.28: Test set-up and specimen with which the effect of an axial compressive force was investigated.

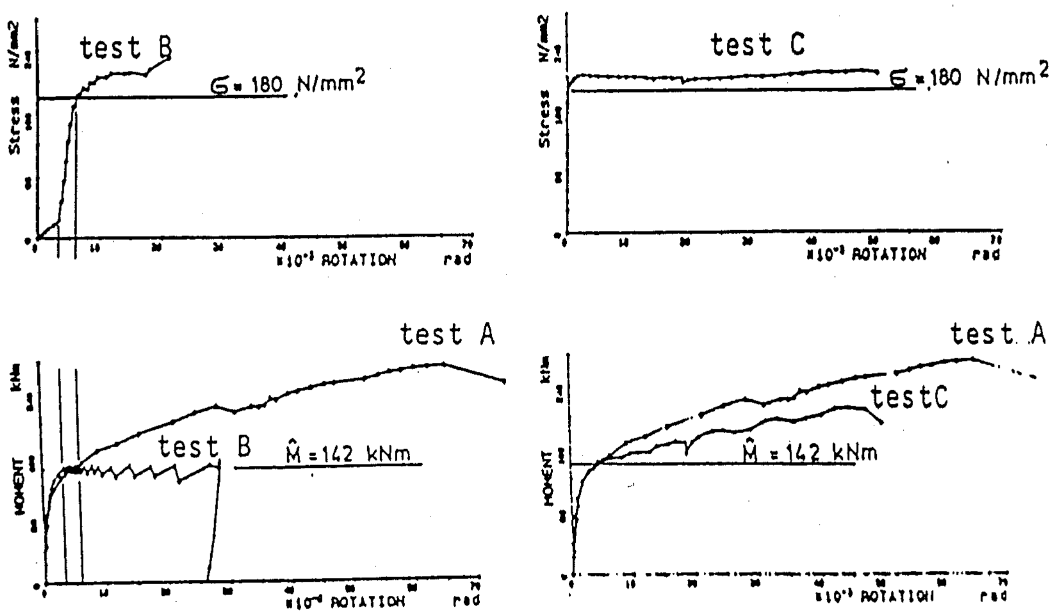


Fig. 2.29: Comparison of moment-rotation curves for various loading situations.

Next, a test was performed in which a moment corresponding to the design strength in the ultimate limit state ($\hat{M}_v$ = 142 kNm) was applied.
The compressive force was then increased and it was investigated at what value of the compressive stress the rotation, for constant moment, began to increase (test B).
As appears from the top left-hand diagram in Fig. 2.29, this occurred at a stress of 180 N/mm^2. The actual yield point of the material was 252 N/mm^2, so that this test can be regarded as representative of steel grade Fe 360. A test was then performed in which an initial stress of 180 N/mm^2 was applied in the flanges, after which the moment was increased (test C). The results are represented in the top right-hand diagram of Fig. 2.29, from which it can be inferred that the strength decreases when the stress in the column flange increases above 180 N/mm^2. For this reason the draft code for N.P.R. connections and draft Eurocode 3 include a reduction factor for the plate strength. This factor has to be applied in the compressive stress in the column flange exceeds 180 N/mm^2. It is expressed by:

$$R = 2.5 - 2\sigma_n/f_y$$

where σ_n denotes the normal stress in the longitudinal direction of the column flange. Therefore:

$$\hat{F}_p = \frac{2b_m \cdot m_p}{m} \cdot R \qquad (2.10)$$

If it is desired to obviate the adverse consequences, the column flanges will have to be strengthened with stiffeners or a column with larger cross-sectional dimensions be chosen.

2.13 Main points of Chapter 2

The design strength of a connection is: $\hat{M}_v = \hat{F}_i \cdot h_i$.
where: $\hat{F}_i$ = the force that a bolt row can transmit,
h_i = the distance from the bolt row to the centre of compression.

The force, that a bolt row can transmit is determined as the difference between the design strengths of the bolt groups with and without the bolt row in question. The design strength of a bolt group is calculated from the plate strength/bolt strength ratio:

$$\beta = \hat{F}_p/\sum \hat{B}_t = 2b_m \cdot m_p/m \cdot \sum \hat{B}_t$$

as represented in the formula (2.1), (2.2) and (2.3) in Fig. 2.10.
For connections with column flanges not provided with stiffeners the plate strength $\hat{F}_p$ must be reduced by a factor:

$$R = 2.5 - 2\ \sigma_n/f_y \text{ is } \sigma_n > 180 \text{ N/mm}^2$$

The effective length for a bolt group is equal to the sum of the effective lengths of the bolt rows, as shown in Fig. 2.21. The force that a bolt row can transmit will, for reasons of equilibrium, never exceed:

a. the design strength of the compression side of the connection $\hat{F}_c$ determined with formula (2.8);
b. the design strength of the shearing zone of the column web $\hat{F}_s$ determined with formula (2.9);
c. the design strength of the column web or beam web behind the bolt group.

A connection has sufficient deformation capacity if the design strength is governed by:

a. shearing of the column web;
b. failure of the column web on the compression side, or if the plate strength/bolt strength ratio $\beta = \hat{F}_p/\sum \hat{B}_t < 2\gamma/(1 + 2\gamma)$.

The design stiffness of the connection can be determined with formula (2.6), as represented in Fig. 2.21. The stiffness of the connection can be increased by prestressing of the bolts only if it is certain that the contact pressure will duly be aligned with the column web and beam web. Alternatively, the stiffness of the connection can be increased by increasing the lever arm between the tension and the compression side or by providing more bolt rows.

The strength of a connection can be increased by thickening the column flange (see Section 2.11) or by installing packing plate behind the column flanges. This latter solution is meaningful only if bolt failure is not the governing condition, i.e., if $\beta < 2\gamma/(1 + 2\gamma)$ (see Section 2.11.3.).

Compression in the longitudinal direction of the column affects the strength of the connection for the following values of the stress:

- $> 120\ N/mm^2$ for the compression side of the connection (see Section 2.12.2.)
- $> 180\ N/mm^2$ for the tension side of the connection (see Section 2.12.3.).

The design strengths of all the components of the connections can be calculated independently of one another.

Chapter 3:

Standardization

3.1 General

There is an increasing need to standardize structural connections.
Various manuals giving guidance on the subject, as exemplified in Fig. 3.1, are already in existence in other countries /21, 22/.
It may well be asked why these standardizations are not simply adopted in the Netherlands. One reason is that steelwork constructors in this country make use of them anyway, if required.
The principal argument against adopting them wholesale is, however, that the strengths stated for these connections are based on elastic analysis without sufficient attention being paid to deformation capacity.
From Chapter 1 of the present report it emerges that the calculation of the design strength of a beam in a braced frame can be simplified if connections possessing deformation capacity are employed. Also plastic theory can be expected to be used more and more in calculating the design strengths of unbraced frames because more and better facilities (computers and software are becoming available. In such cases, too, it should be possible to employ connections which possess deformation capacity.
It would not be justified to introduce standardized solutions which are liable to become obsolete within a few years.
Research has been carried out on the standardization of angle cleats (as web cleats on beams), which is now nearing completion /23/, and on the standardization of projecting end plates. This research has yielded rules which have been incorporated in Chapter 2. One of the most important rules is that in the design of connections it is necessary to take account of the differences in force-deformation relations of the various components of the connections, unless the components possess deformation capacity.
Another important conclusion from the research is that the design rules, not the connections, should be standardized.
These points will be considered here with reference to a discussion of the research.

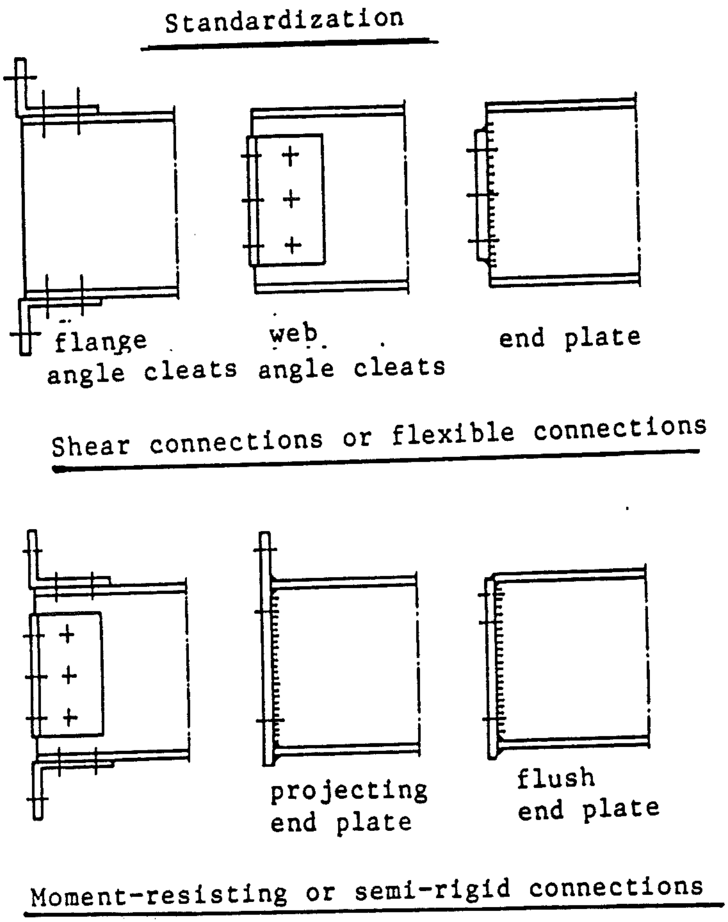
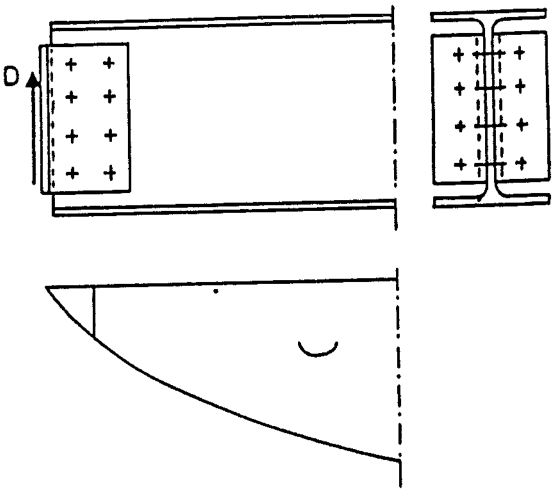


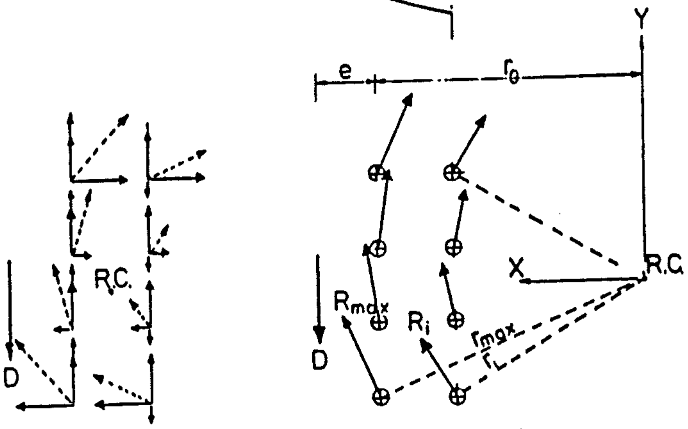
Fig. 3.1: Types of connection to be standardized or already standardized.

3.2 Angle cleats

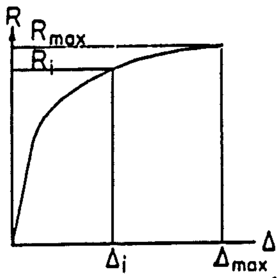
The reaction force of the beam is assumed to act at the end of the beam. The bolt group with which the angle cleats are connected to the web of the beam must transmit the reaction force plus a bending moment (see Fig. 3.2a). If the analysis is performed in accordance with clause 6.2.3.2.1. of the Eurocode, i.e., based on elastic theory, the distribution of forces as shown in the left-hand diagram of Fig. 3.2b is obtained.
The reaction force is conceived as equally distributed over the bolts. In principle this means that plastic deformation is taken into account, because it is not possible to drill the holes in the cleats and in the beam with equal dimensional tolerances. In determining the distribution of forces due to the bending moment it is assumed that the forces are distributed proportionally to the distances from the centre of rotation. It is also assumed that the centre of rotation coincides with the centroid of the bolt group. The dotted arrows indicate the resultants of the forces thus determined. It appears that more particularly the bolts which are located far from the reaction force, but close to the assumed centre of rotation, are very inefficiently utilized.
If an elasto-plastic analysis is performed in accordance with clause 6.2.3.1.1 of the Eurocode, a distribution of forces as shown in the right-hand diagram of Fig. 3.2b can be expected. In some cases the reaction force may turn out to be as much as 20% larger than that found by elastic analysis. The precondition is, however, that the outermost bolts can undergo so much deformation that those closer to the centre of rotation will deform to such an extent that they can transmit the force assumed in the calculation.
It appeared questionable whether this would indeed still be possible in the case of bolts with 2 mm clearance in their holes. To reduce this clearance did not appear to be justified because the gain due to larger loads would be offset by higher cost of erection. It was decided to carry out tests /25/ to verify the design method described by Fisher and Struik /24/, the formulae for which are given in Fig. 3.2c. For that purpose a test set-up was constructed for applying loading in the manner shown schematically in Fig. 3.3a.



A.



B. Elastic 6.2.3.2.1. — elasto-plastic 6.2.3.2.2.



$r_i^2 = x_i^2 + y_i^2$

$\Delta_i = \frac{r_i}{r_{max}} \Delta_{max}$

$\downarrow$ from experimentally determined

R_i R-Δ diagram.

$$\Sigma F_x = 0 \longrightarrow \sum_{i=1}^{n} R_i \frac{y_i}{r_i} = 0$$

$$\Sigma F_y = 0 \longrightarrow \sum_{i=1}^{n} R_i \frac{x_i}{r_i} - D = 0$$

$$\Sigma M = 0 \longrightarrow \sum_{i=1}^{n} r_i R_i - D(e + r_0) = 0$$

$\Big\} \; r_0 \rightarrow r_i \rightarrow R_i \downarrow D$

C.

Fig. 3.2: Elasto-plastic method of analysis for angle cleats on web of beam.

In this arrangement two bolt groups are simultaneously loaded in bending and shear. Some results are shown in Fig. 3.3b. Clearly, the deformation at the outer rows was more than sufficient to enable the bolts in the rows farther inwards as to transmit forces which had been taken into account. All the test results were verified with a computer program in which the actual force-deformation relation of plate and bolt had been incorporated. This relation had been determined by means of some detail tests with and without clearance of the bolt in the hole. The computer program was based on a design procedure as described by Fisher and Struik. The test results were in good agreement with the calculated values, justifying the conclusion that an elasto-plastic approach gives satisfactory results, provided that the actual load-deformation relation is known.

The last-mentioned condition was decisive in not basing the standardization of angle cleats on elasto-plastic analysis. More particularly, the reason is that with standardization the force-deformation relations that occur are not known because an upper limit to the yield point of the material is imposed. In the actual yield point of the plate material is much higher than the guaranteed value and if the design value of the bolt shearing force is not far from the upsetting force, shearing of the bolts may actually occur. In that case it becomes questionable whether thye assumed distribution of forces is indeed established.
On the basis of these tests it was decided, however, that the upsetting factors given in the Netherlands code (T.G.B.-Staal '72) were too low. For the purpose of standardization an upsetting factor of 3 has been adopted if the edge distance is more than 3d. This is in agreement with the Eurocode.

3.3 Extended end plates

Connections formed with extended end plates belong to the category of moment-resisting connections. From the diagram in Fig. 1.9b it was apparent that beams connected in this way can be utilized most effectively by basing their design on plastic theory.

This is true also if the connections are to be rated as partial-strength connections; in that case they should possess deformation capacity, however. For the projecting end plate with four bolts at the tension flange, as shown in Fig. 3.4a, the available bolt force capacity is insufficient to form a full-strength connection for European wide-flanged beams greater than about 300 mm in dept if M27-10.9 bolts are the maximum permissible size. On the other hand, IPE beams can be formed with full-strength connections in this way.

If a smaller bolt diameter and lower grade of bolt are chosen, the limits will of course be lower. The above considerations lead to the conclusion that most standardized connections formed with a projecting end plate and four bolts are not full-strength connections and must provided deformation capacity if the load on the beam is determined on the basis of a beam mechanism.

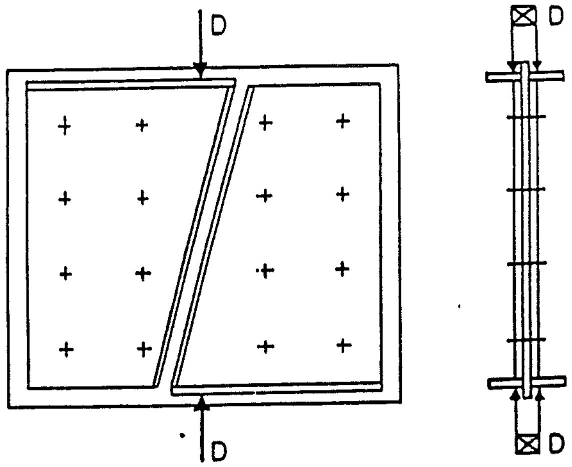
It has so far been assumed that connections derive their deformation capacity from:

a. deformation of the column flanges on the tension side;
b. deformation of the column web on the compression side and, in the case of knee connections, shearing of the column web.

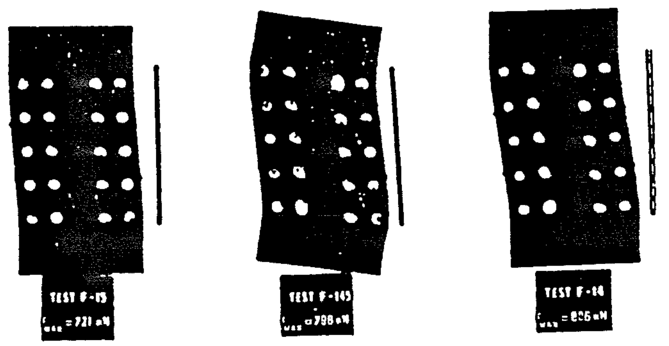
If connections with end plates are employed in structures as shown in Fig. 3.4c (secondary beams connected to main girders) the deformation capacity will have to be provided by the end plates. In Section 2.7 it has been noted that this is possible if complete yielding of the end plate occurs. To achieve this the plate strength/bolt strength ratio should be made less than 0.7 if the ratio $\gamma = n/m$ is taken as larger than 1.25.

It was decided, as a first approach, to base the standardization of projecting end plates on this knowledge. There was doubt as to the validity of the generally accepted rule that a projecting end plate can be analysed as a T-stub.

The question was whether this basic approach could be retained even if the end plate must provide deformation capacity. In addition, the yield point of the end plate material is of importance. If the yield point is much above the guanranteed value, the plate strength/bolt strength ratio β may in reality turn out to be higher than the desired value of 0.7.



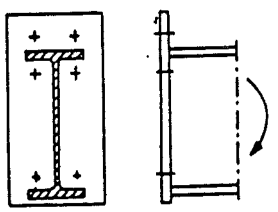
A



B

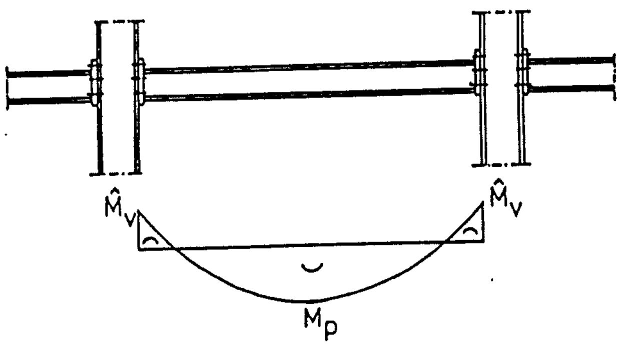
Fig. 3.3: Test set-up (schematic) and test results with which the deformation capacity under direct compression (upsetting) was verified.

<u>Standardized end plates</u>

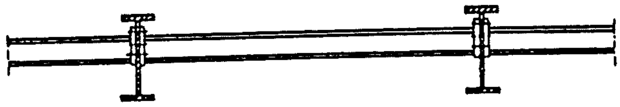


Heavy-section beams h > 300 mm
$4\ \hat{B}_t \cdot h < M_p$
Hence: rotational capacity required

A



B



C

Fig. 3.4: Standardized end plates should provide deformation capacity if they are not designed for full-strength connection.

This could result in bolt failure before the plate undergoes sufficient deformation. This is represented schematically in Fig. 3.5c with the moment-rotation characteristic which does not cross the line of required rotation (see Section 1.8). In view of these considerations some check tests were carried out in which the yield point of the end plate was deliberately varied, whereas the thickness of the end plate was calculated with respect to the guaranteed yield point. The data of the test specimens are given in Fig. 3.6 /26/.

In this case the distance m is the distance to the toes of the angle cleats connecting the end plate to the beam web, because the end plate is conceived as a T-stub. Fig. 3.8a shows a typical moment-rotation diagram. The bolts between the flanges ("inner" bolts in Fig. 3.8b) fracture before those ("outer") in the projecting part of the end plate did (see Fig. 3.7). This behaviour was attributable to the supporting effect of the beam web, so that the plate strength/bolt strength ratio for the "inner" bolts was not 0.7, but approximately 2, as appears from the results assembled in Fig. 3.8c. The theory that bolt failure occurs when $\beta > 2$, as explained in Section 2.2, is thus confirmed.

A cleaner conception of this test result is obtained on making a comparison of the force-displacement diagrams of the "inner" and "outer" parts with their bolts (see Fig. 3.9). Because of the greater stiffness of the inner part the bolt located there is already on the point of failure when the force in the outer part is just beginning to increase. In other words, the inner part must undergo more deformation (as shown dotted) in order to allow the plate in the outer part to deform sufficiently to enable the force adopted in the calculation to develop.

Another method to enable the outer part to cooperate would be to choose a very thick end plate, so that the stiffness-increasing effect of the beam web would be eliminated. However, this solution cannot be adopted here, since the connection must provide rotational capacity, but it does explain why it was formerly found that the projecting end plate could be analysed as a T-stub, whereas this is now found not to be correct.

From the summarized test results plotted in Fig. 3.10 it is inferred that a distance $m > 30$ mm, the use of 10.9 bolts and material with a high yield point have an adverse effect on deformation capacity if the end plates are designed as T-stubs for $\beta = 0.7$.

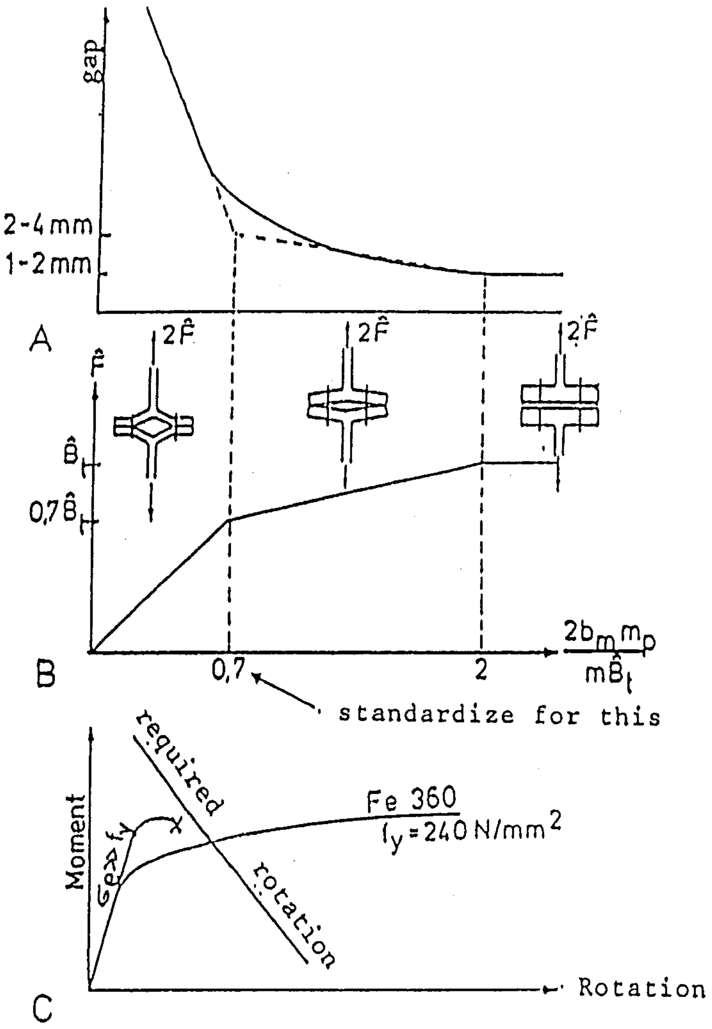
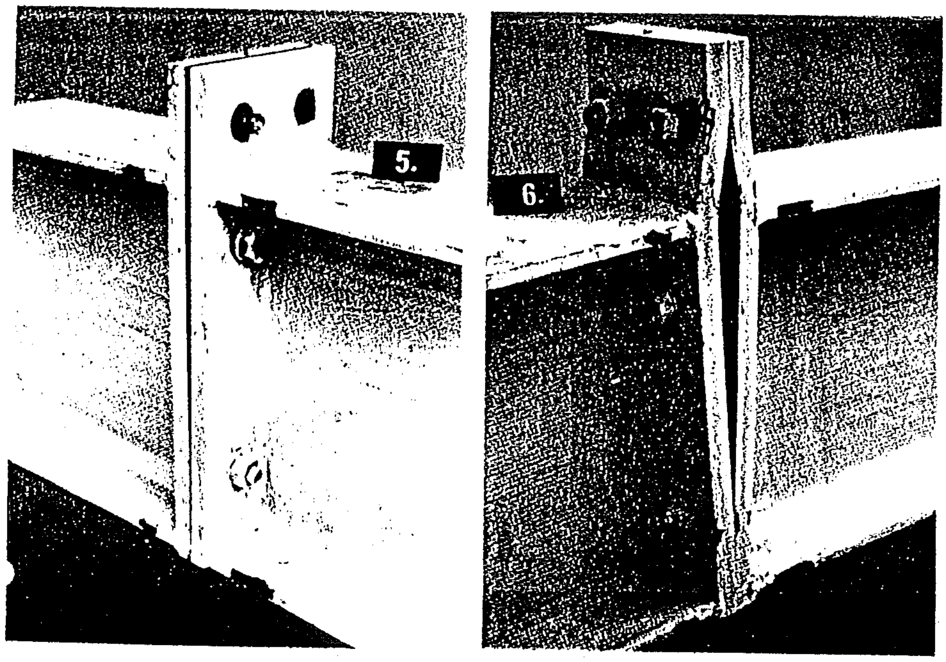


Fig. 3.5: Basis for standardization of end plates β = 0.7

Test	Bolt spacing m mm	Bolt grade	Plate thikness mm	Actual yield-poinț N/mm²	$\frac{M_v}{M_p}$
1	30	8.8	16	250	0.48
2	40	8.8	20	237	0.52
3	60	8.8	22	268	0.45
4	30	8.8	16	341	0.48
5	40	8.8	20	366	0.52
6	60	10.9	22	260	0.44
7	30	10.9	19	263	0.63
8	40	8.8	22	270	0.54
9	60	10.9	25	240	0.59
10	30	10.9	19	378	0.62
11	40	8.8	22	260	0.54
12	60	10.9	25	356	0.60
13	40	8.8	20	345	0.52

Fig. 3.6: Data relating to test-specimens for which the rotational capacity was verified.



Afb.3.7: Proefstukken na belasten.Bouten tussen liggerflenzen gebroken.

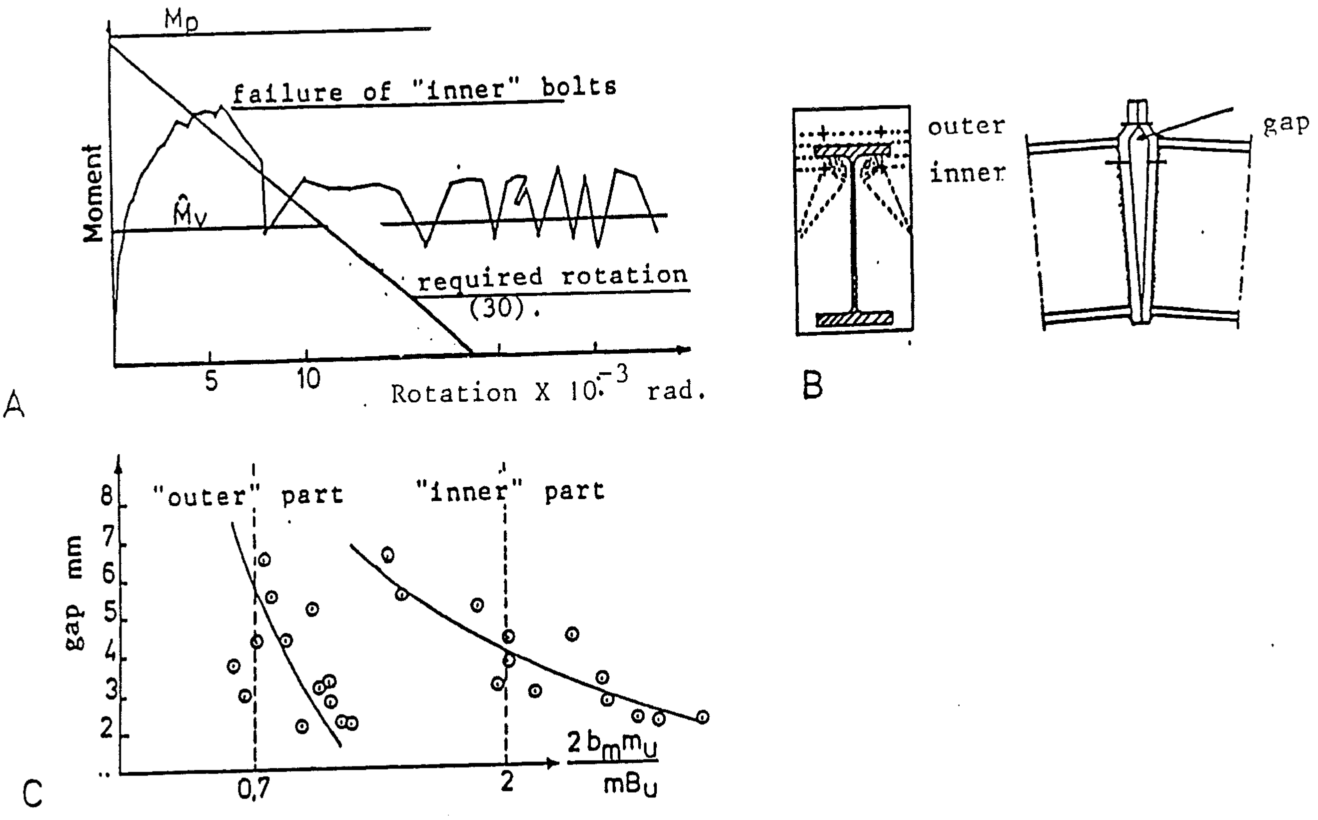


Fig. 3.8: Explanation for premature bolt failure. For the part between the beam flanges: $\beta > 2$.

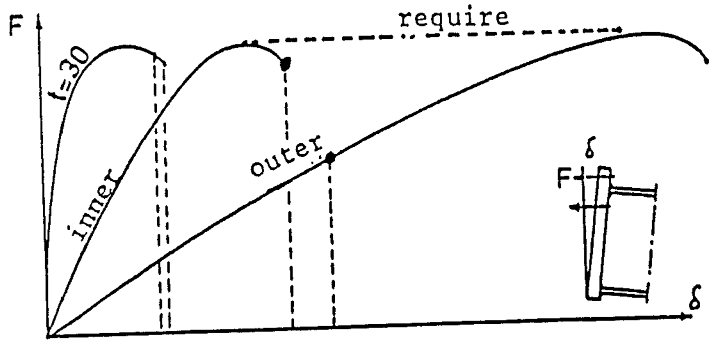


Fig. 3.9: Comparison of the force-displacement diagrams of "inner" and "outer" parts.

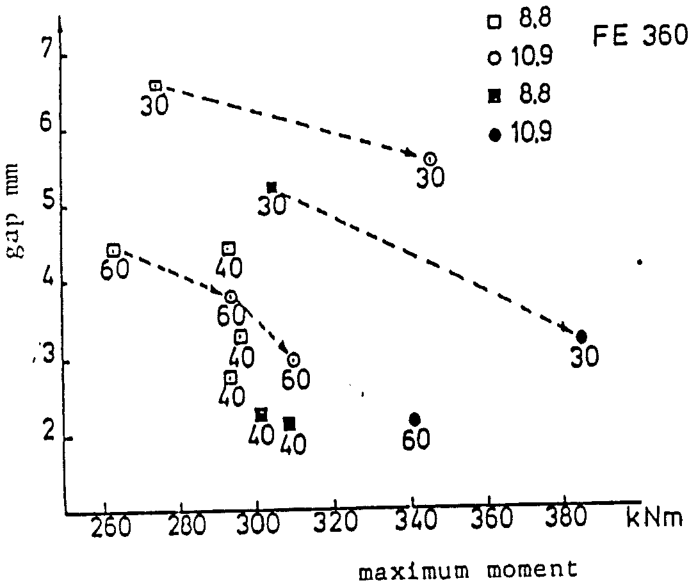


Fig. 3.10 Test results.

test	girder	column	bolt	plate thickness	actual yield point N/mm^2	$\frac{A_v}{H_p}$
1	IPE 400	-	8.8	12	214	0.34
2		-	10.9	14	312	0.46
3		-	8.8	12	385	0.34
4		-	10.9	14	345	0.47
5	HE 500A	-	8.8	14	312	0.26
6		-	10.9	16	195	0.34
7		-	8.8	14	345	0.26
8		-	10.9	16	360	0.34
9	IPE 400	HE 240A	8.8	12	214	0.34
10	IPE 400	HE 200B	10.9	14	312	0.47
11	HE 500A	HE 300A	8.8	14	312	0.19
12			10.9	16	195	0.19

Fig. 3.11: Data relating to second test series.

For this reason fresh tests were proposed /26/, and subsequently carried out, in which m = 30 mm was adopted and the end plate thickness was so chosen that β = 0.7 for the inner part of the end plate.
The data relating to these tests are given in Fig. 3.11, and some results are shown in Fig. 3.12. It was decided not only to vary the yield point of the end plate material and the grade of bolts employed, but also to employ portions of column in some of the tests. This was done for the following reason. If the plate strength/bolt stength ratio is less than 0.7, the end plates are fairly thin. The question is then whether, with thin column flanges and thin end plates, the stiffness requirements under service load, as discussed in Section 1.8, are still satisfied.
That this is indeed the case was confirmed by the test results /27/.
Fig. 3.13 summarizes the results of the first and the last series of tests. The plate strength/bolt strength ratios in the ultimate limit state are indicated on the horizontal axis of the diagram; the maximum widths of the gap formed are indicated on the vertical axis.
The last series of tests is found to give considerably more favourable results than the first. It confirms the basic conception that rotational capacity is obtained by choosing a value of less than 0.7 for the plate strength/bolt strength rather.
The main conclusion to be drawn from the whole research relating to the standardization of projecting end plates is summarized in Fig. 3.14.
The investigations aimed at obtaining rotational capcity. It emerges that this is achieved at the expense of the strength of the connection because the bolt force capacity is limited and only 70% of it can be utilized.
If infinitely stiff connections are assumed on the basis of elastic theory, the end plates should in fact be able to resist large forces, whereas the rotational capacity need not fulfil particularly exacting requirements.
This means that two types of "projecting end plates" would have to be standardized: one suited to elastic theory, the other to plastic theory. However, in addition to this there are so many other boundary conditions governed by the steelwork fabricating workshop and design office that it turns out to be more sensible to standardize the design methods than the structural details.

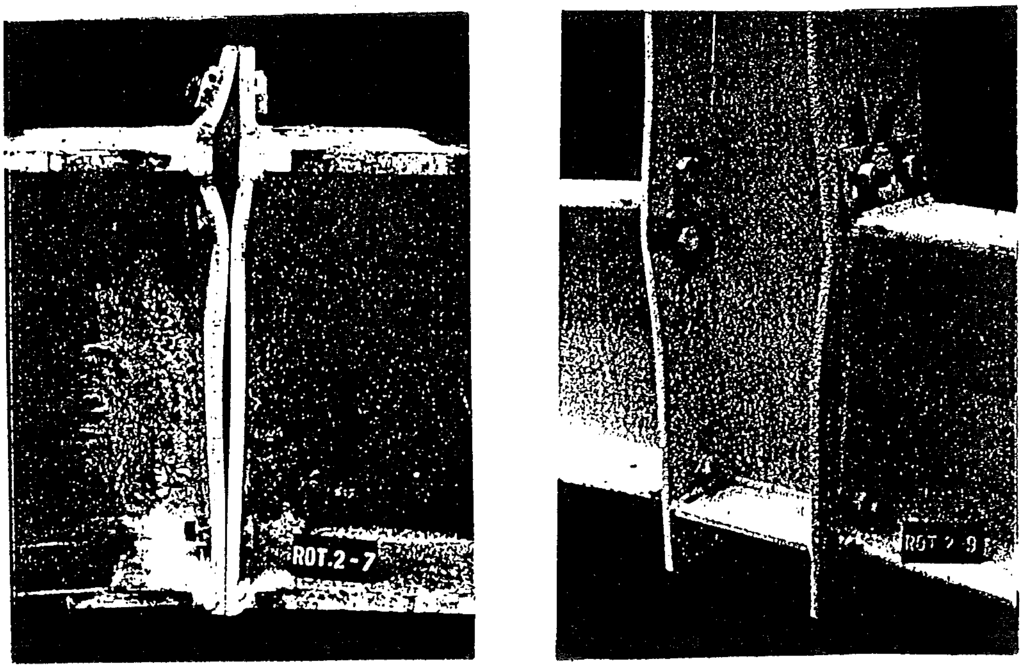
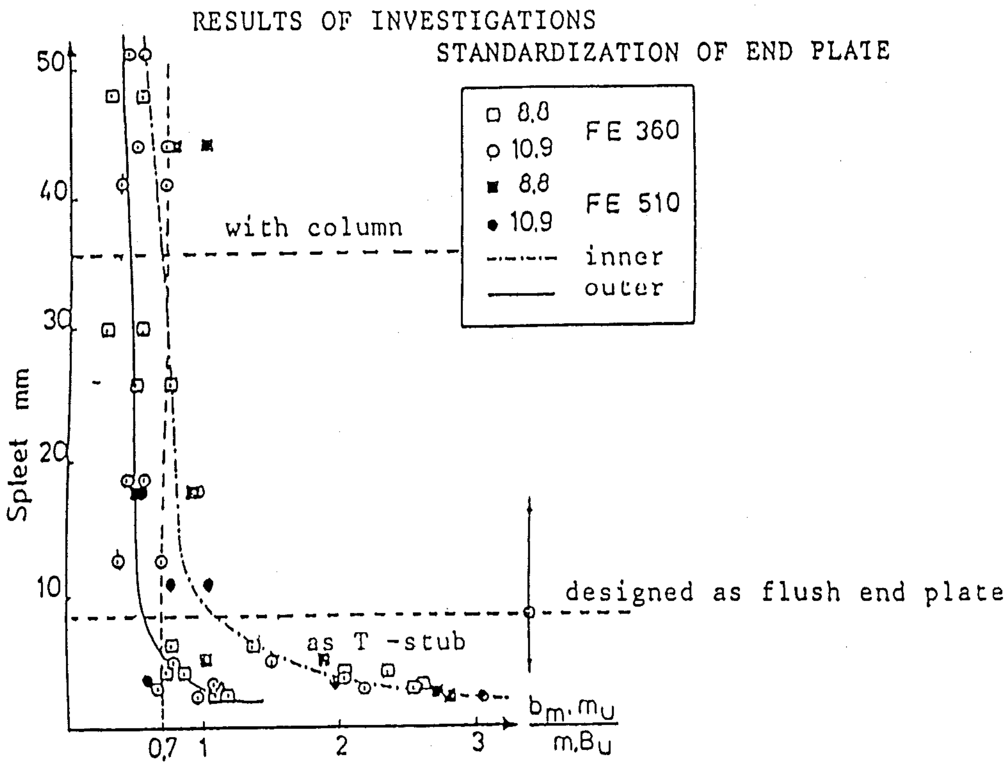


Fig. 3.12: Specimens of the second test series with β = 0.7, without bolt failure.



conclusion: Designed as flush end plate gives:
more rotation, less strength,
some influence of yield point and
grade of bolt.

Fig. 3.13: Summary of all the test results.

In future, design rules should be presented in such a form that they are directly suitable for incorporation in computer programs which can be run on microcomputers, so that even any small design office will have access to, and be able to make use of, the available information.

Main points of Chapter 3

It appears from the test that the distribution of forces in the connection of beam webs by means of angle cleats can be analysed on the basis of plastic theory if upsetting of the bolts is the governing criterion. The uncertainty as to the upper limit of the yield point of the beam material is the reason why this theory has not been applied to the standardization of angle cleats.
The tests have shown that a projecting end plate cannot be analysed as a T-stub if this plate is to be so designed that it will provide deformaion capacity. On account of the supporting effect of the web of the beam the deformation capacity of the part between the beam flanges may be limited by premature bolt failure. To obtain deformation capacity from the end plate it is necessary to make the plate strength/bolt strength ratio for the part between the beam flanges less than 0.7.
Since elastically designed structures require strength rather than deformation capacity, it is not meanigful to standardize end plates for plastically designed structures. End plates which are so standardized as to possess maximum strewngth are in general not suitable for structures in which a considerable amount of rotational capacity is required.
The above dilemma has led to the conclusion that what should be standardized are not the end plates, but the design rules. In view of the increasingly widespread use of microcomputers these rules will have to be made suitable for direct incorporation in computer programs.

CONCLUSION

1. Standardization is necessary
2. No structural details
3. Design methods suitable for microcomputers are required

Fig. 3.14 : Main conclusion from research relating to standardization.

Chapter 4:

Connections subjected to impact loading

4.1 Introduction

The behaviour of structural connections which are subjected to impact loading will be considered here, because the test result provide confirmation for the theory presented in Chapter 2.
The tests were carried out in order to establish design rules for connections in explosion-resistant buyildings associated with chemical plants. These buildings contain monitoring and controle equipment which must continue to function after an explosion. They are allowed to suffer damage to such an extent that rebuilding may be necessary, but they must not collapse.
To design the loadbearing structural system to resist the extremely severe loads due to an explosion, which may be between 20 and 70 times the wind loading, would be uneconomical. That is why the designer utilizes kinetic energy to absorb the difference between the external loading and the static resistance of such a building. When the external loading has diminished to less than the static resistance, this kinetic energy must still be adsorbed by plastic deformations of the structure. The kinetic energy and the plastic deformations it causes will be less according as the strength, stiffness and mass of the building are greater. For the theoretical background the reader is referred to /28, 29, 30/. Here follows a description of the test results confirming the theory of Chapter 2.

4.2 Tests

The loading pattern was as shown in Fig. 4.1b. The duration of the loading was 20 milliseconds. Figs. 4.2 and 4..3 schematically represent the connections that were tested. Those in Fig. 4.2 were formed with end plates on HE-100A beams.

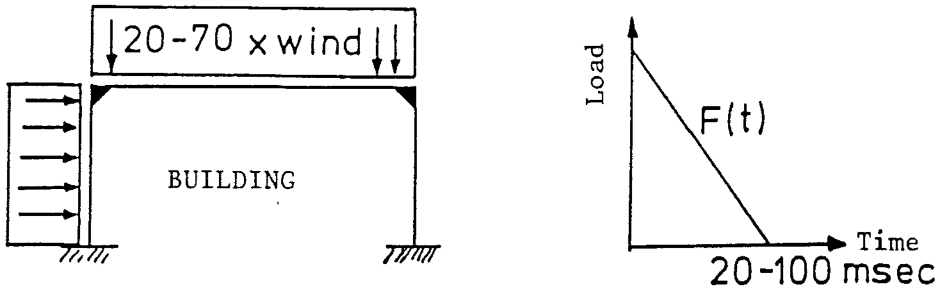


Fig. 4.1 Loading on explosion-resistant buildings for monitoring equipment.

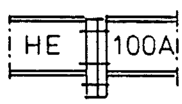
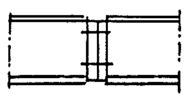
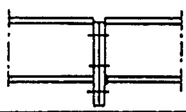
Prf.	Configuration	Bolts	end plate thickness	$\frac{2b_m \cdot m_p}{m \cdot B_t}$	$\frac{M_v}{M_p}$
2 3		8.8 10.9	20 20	> 2. > 2.	0.83 1.0
4 5		8.8 10.9	20 20	> 2. > 2.	0.3 0.37
6		8.8	10	0.9	0.76

Fig. 4.2 Data relating to end plate connections that were tested.

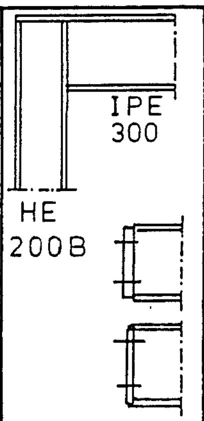
prf		Type	$\frac{M_v}{M_p}$	$\frac{2b_m \cdot m_p}{m \cdot Bt}$	governing
1		welded	0.42		shearing
2		end plate t = 20	0.36	flange 1.2	shearing
3		end plate t = 12	0.25	end plate 0.6	end plate

Fig. 4.3 Data relating to knee connections that were tested.

All the test specimens except No. 6 failed by bolt fracture. This confirms the theory that bolt fracture occurs when the plate strength/bolt strenght ratio is greater than 2.
According to /31/ there exists a risk of brittle failure if the increase in strain per unit time dε/dt exceeds 1000% per second.
This did not occur here, as is apparent from the moment-rotation diagrams in Fig. 4.4. The elastic deformations of the structural members associated with the connections were so large that the strain increase per unit time remained below that value, despite the suddenness of loading. The moment-rotation diagrams in Fig. 4.5 relate to the connections between IPE 300 beams and HE 200 B columns as schematically represented in Fig. 4.3.
The failure modes are cleary revealed in Fig. 4.6.
In none of the connections was failure found to occur. All the specimens were rapidly loaded three times in order to find out whether they would possess sufficient deformation capacity also if the duration of the loading were greater than 20 milliseconds. The right-hand diagram in Fig. 5 shows the summation of the moment-rotation diagrams of the welded connection together with those of a specimen of similar construction, but loaded slowly.
From a comparison of the last-mentioned diagrams it emerges that the conclusion arrived at by other investigators /32/, namely, that rapid loading raises the yield point, is correct. This was found to be the case in all the specimens.
This fact could be utilized to adopt a higher value for the "kink" in the bilinear force-deformation diagram in the analysis of a structure under explosion loading than in the analysis of a structure under static loading. In his tests Van Beek found this "kink" to be higher by a factor of between 1.5 and 1.9 than the design moment calculated by the method given in Chapter 2.

4.3 Main points of Chapter 4

The conclusion to be drawn from this research is that connections subjected to impact loading can be so designed that failure will not occur.

Prf.	Configuration	Bolts	end plate thickness	$\frac{2b_m \cdot m_p}{m \cdot \hat{B}_t}$	$\frac{M_v}{M_p}$
2	HE 100A	8.8	20	> 2.	0.83
3		10.9	20	> 2.	1.0
4		8.8	20	> 2.	0.3
5		10.9	20	> 2.	0.37
6		8.8	10	0.9	0.76

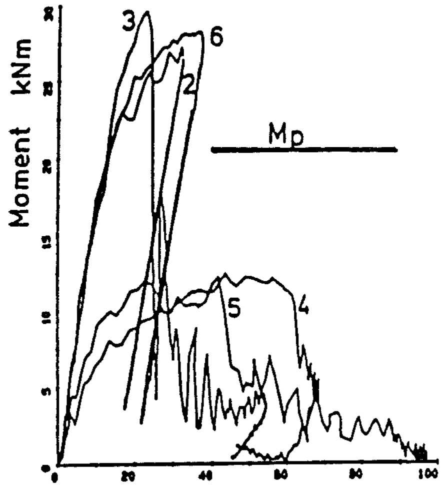


Fig. 4.4 Moment-rotation characteristics of the end plate connections that were tested.

prf		Type	$\frac{M_v}{M_p}$	$\frac{2b_m \cdot m_p}{m \cdot \hat{B}_t}$	governing
1	IPE 300	welded	0.42		shearing
2	HE 200B	end plate t = 20	0.36	flange 1.2	shearing
3		end plate t = 12	0.25	end plate 0.6	end plate

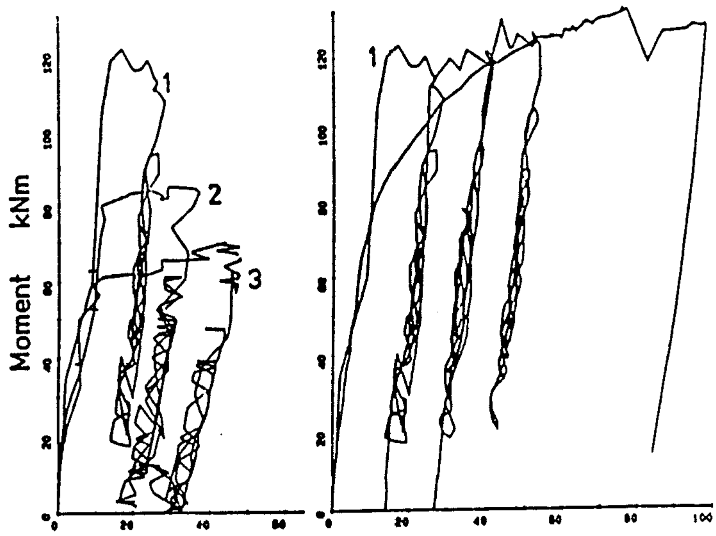
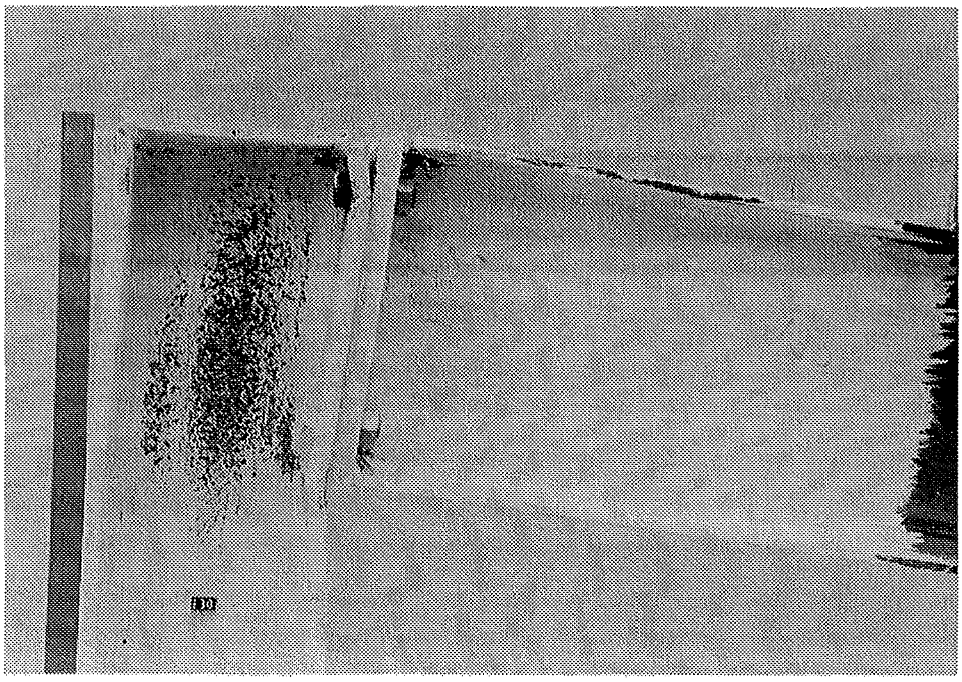
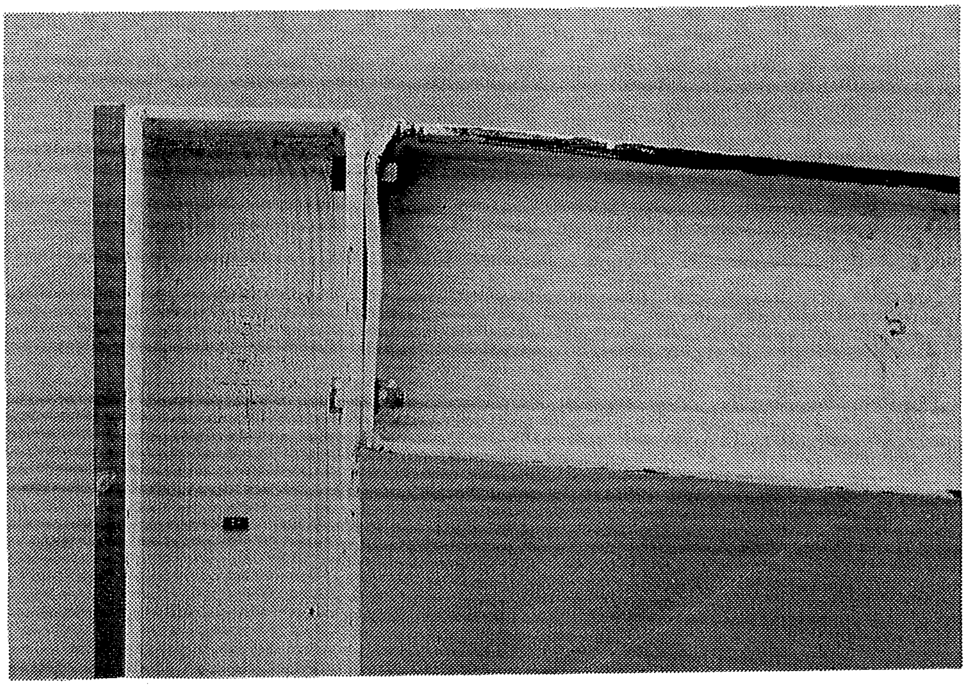


Fig 4.5. Moment-rotation characteristics of knee connections that were tested.



Rapidly loaded knee connection with end plate thickness t = 20 mm. Failure governed by shearing of column web.



Rapidly loaded knee connection with end plate thickness t = 12 mm. Failure governed by end plate.

Fig. 4.6. Knee connections that were tested.

For this purpose the rules given in Chapter 2 for statically loaded connections may be used. In the case of knee connections there is no need to impose any special deformation requirements on the end plate connections if shearing of the column web is the governing criterion.
This latter failure mode provides sufficient deformation to ensure that the increase in the strains will not be of such magnitude as to give rise to brittle failure and provides sufficient deformation capacity to absorb the energy. In end plate connections in beams the deformation must be provided by the plates in order to avoid bolt fracture.

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Appendix A

Example of a calculation of the design strength of a frame with semi-rigid and partial-strength connections.

Introduction

The frame shown in Fig. A.1 was chosen because Tautschnig /7/ had analysed it with the aid of a computer program taking account of the actual moment-rotation characteristic of the connections. The actual experimentally determined moment-rotation characteristic is shown in Fig. A .2, together with the curve that Tautschnig adopted in his analysis and also the bilinear diagram corresponding to the design rules as given here.

The great difference between the actual diagram and the bilinear approximation is notable. This difference is due to the rule that the column web is deemed to have failed in shear if the theoretical shear stress $\sigma_e/\sqrt{3}$ occurs over the area of the web. From the test results it always emerges that the load can be further increased quite considerably after this theoretical failure stress has been exceeded /10/. Tautschnig approximates the actual curve by assuming that the column web does not fail until the design strength of the compression side of the connection has been attained. He proposes that the actual curve of each connection in the structure be approximated by a combination of three curves to be obtained by tests performed for three basic cases:

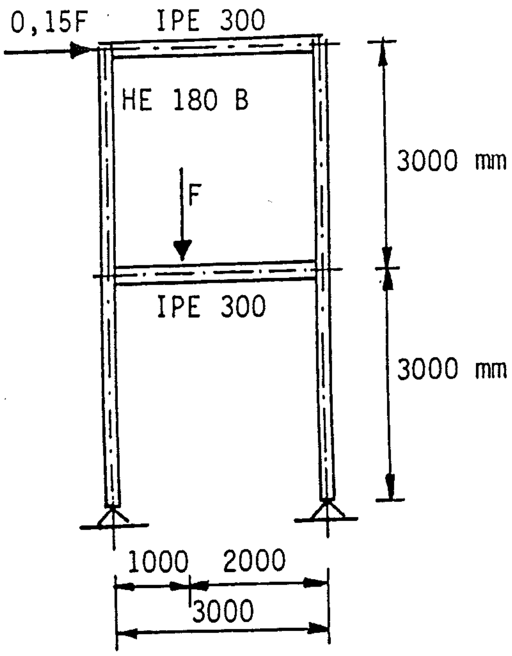
- column web loaded in direct (pure) compression;
- connection loaded chiefly in bending;
- connection loaded chiefly in shear.

He illustrates this proposal with a design example in which he employs design rules applied in the Netherlands to show that these rules are too conservative. This conclusion may well be correct, but more test results will be needed to enable the behaviour of every structure to be analysed with greater accuracy, and not all designers have computer programs at their disposal whereby the actual moment-rotation curve can be taken into account.

Nevertheless, Tautschnig's example is of considerable interest in demonstrating how conservative our design rules are.
Fig. A.3 shows the force-displacement curves which Tautschnig calculated for the frame, as well the force-displacement curve calculated with the bilinear approximation of Fig. A.2.
Fig. A.4 shows the moment distribution and deformations calculated by Tautschnig of a load of 195 kN acting vertically and 29.25 kN acting horizontally on the frame with infinitely stiff connections. The results calculated for a similar frame taking account of the actual moment-rotation curve of the connections, as well as those calculated with the bilinear approximation of the moment-rotation curve, are also shown.
In this last-mentioned case a fresh calculation was performed for the frame provided with a hinge substituted for the connection after the latter had attained its design strength.
For comparison Fig. A.5 gives the moment distribution which would have been obtained at 195 kN if the design strength of the connection had not been exceeded. Also shown are the moment distribution on attainment of the design strength and the moment distribution which is added to it after the design strength is exceeded. The moment distribution found here turns out to be different from Tautschnig's.
There are two reasons for this. The first is that Tautschnig adopts a spring constant of 22100 kNm/rad for the connection, whereas we obtain a spring constant of 24098 kNm/rad with the design rules. This difference in stiffness is, however, more than compensated by the difference in the lengths adopted for the beams. Tautschnig takes the length of the beam between the lateral faces of the columns, whereas here the system length has been adopted, with a spring at the intersection of the system lines.

The second reason that can be adduced to account for the difference in moment distribution is that Tautschnig uses a computer program that takes account of deformations due to axial forces and to second-order effects. On the other hand, in het present case the distribution of the moments has been calculated with the Hardy Cross method of analysis, not taking account of deformations due to axial forces. Second-order effects have, however, been taken into account by multiplying all the moments causing sidesway by a factor m/(n-1).

For the frame without plastic hinge the Euler buckling load is 3510 kN, so that n = 18 and n/(n-1) = 1.06 for a vertical load F = 915 kN, while n = 29.7 and n/(n-1) = 1.035 on attainment of the design strength of the connection for F = 116 kN.
For the frame with plastic hinge the Euler buckling load is 2300 kN, so that n = 11.8 and n/(n-1) = 1.09 for a vertical load F = 195 kN.



IPE 300: $EI = 17556$ kNm^2

$M_p = 151$ kNm

HE 180B : $EI = 8043$ kNm^2

$M_p = 116$ kNm

connection. $c = 24098$ kNm/rad.

$k = \frac{cl}{EI} = 4{,}12$

$M_v = 54$ kNm (35,7% M_p)

Fig. A.1: Frame analysed with fully rigid and semi-rigid and with full-strength and partial strength connections.

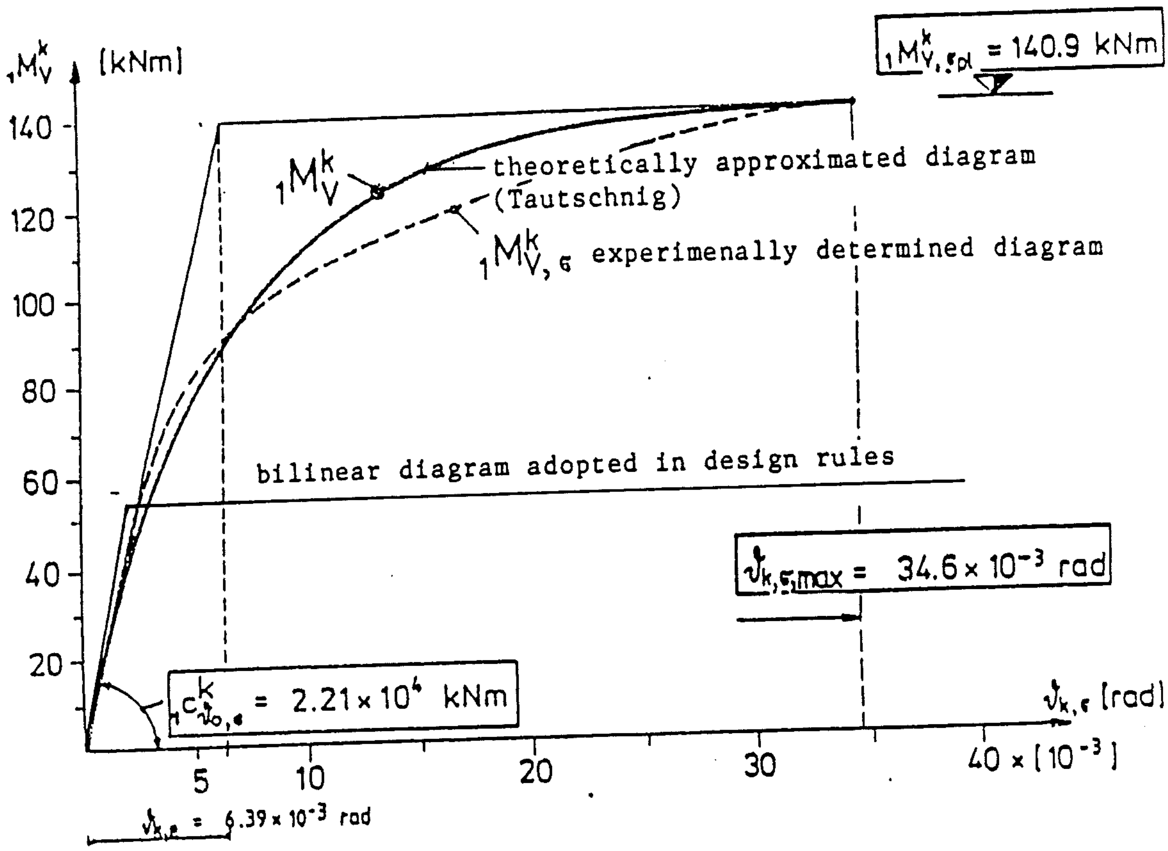


Fig. A.2: Moment-rotation diagrams of the connections to which the calculations relate.

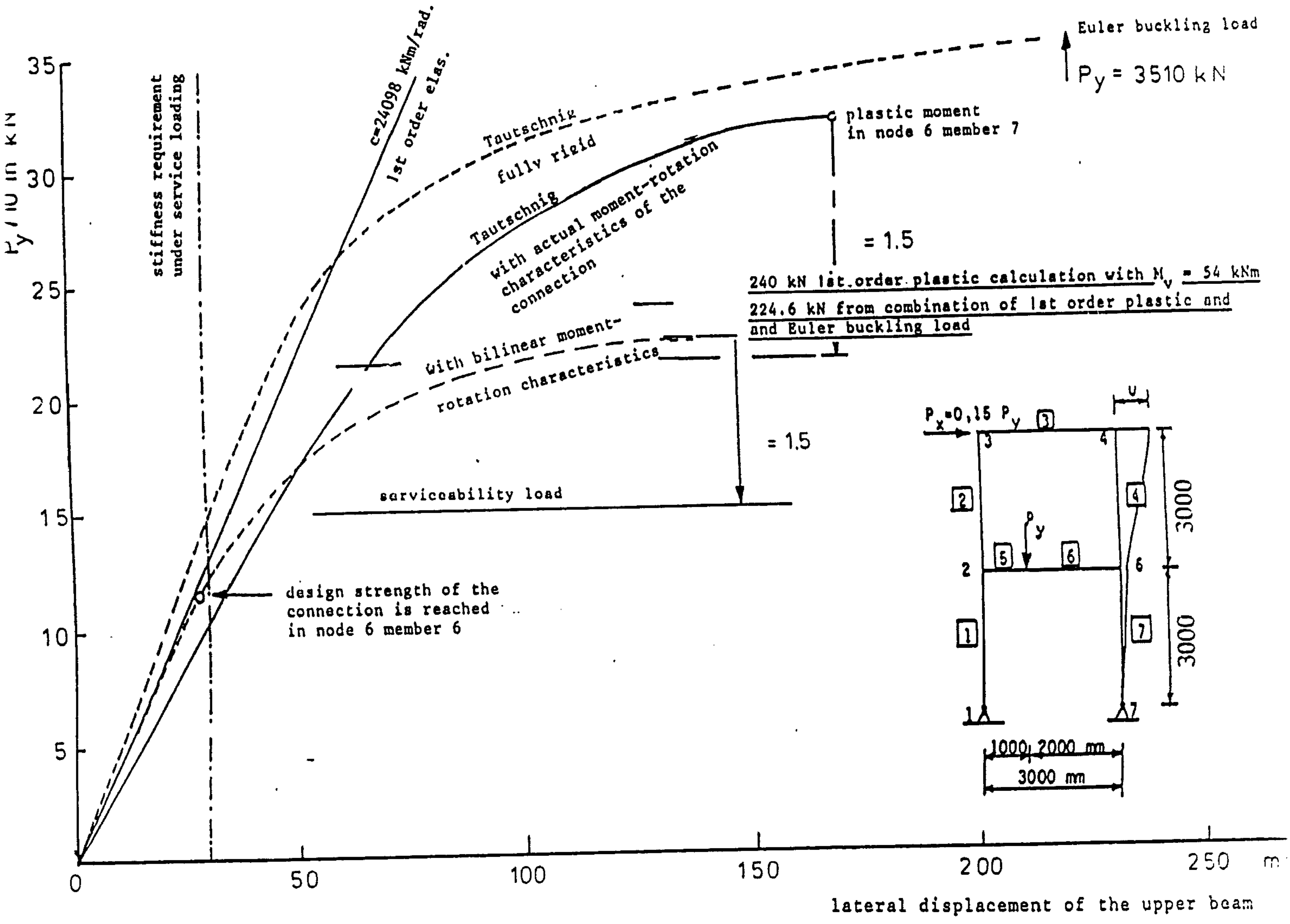
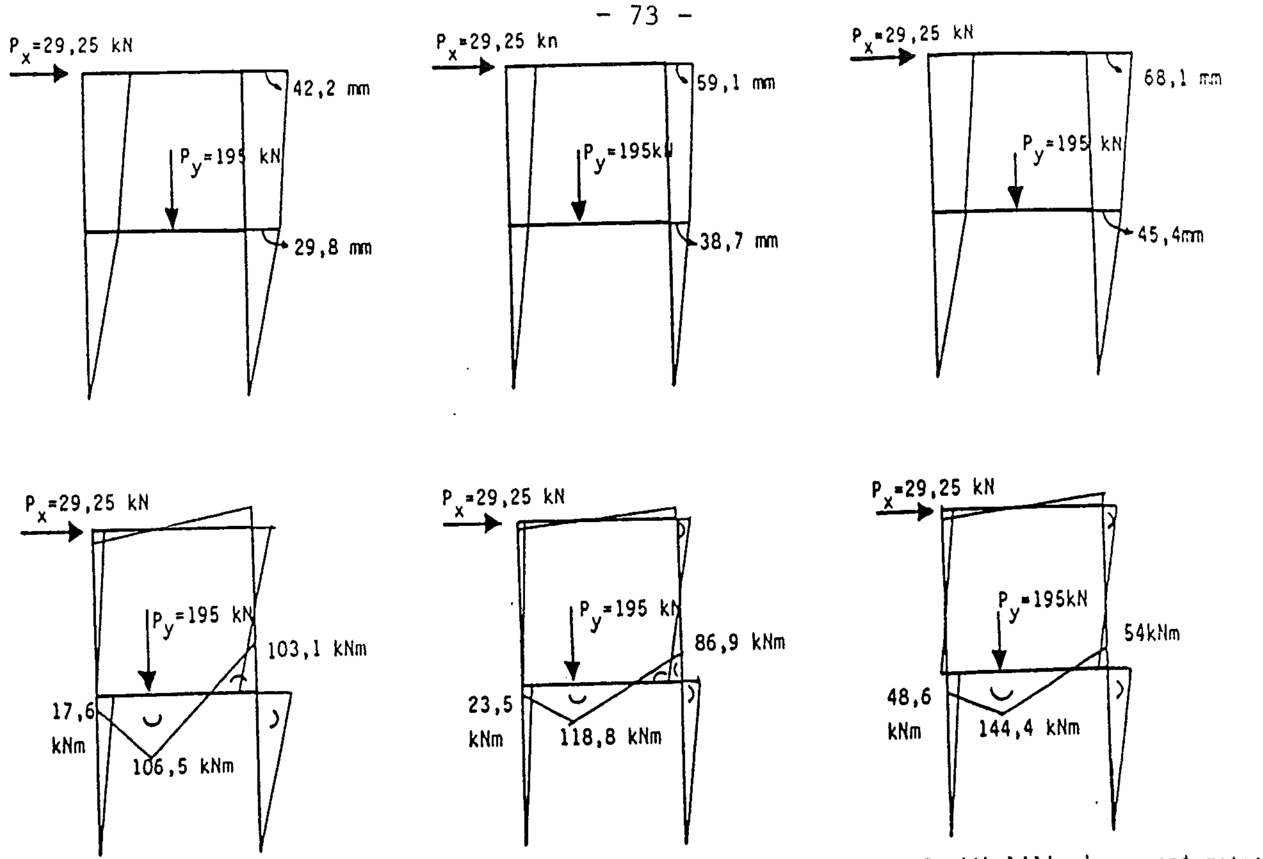


Fig. A.3: Influence of the stiffness and strength of the connections on the force-displacement behaviour of an unbraced frame. (zie Fig. 1.6).

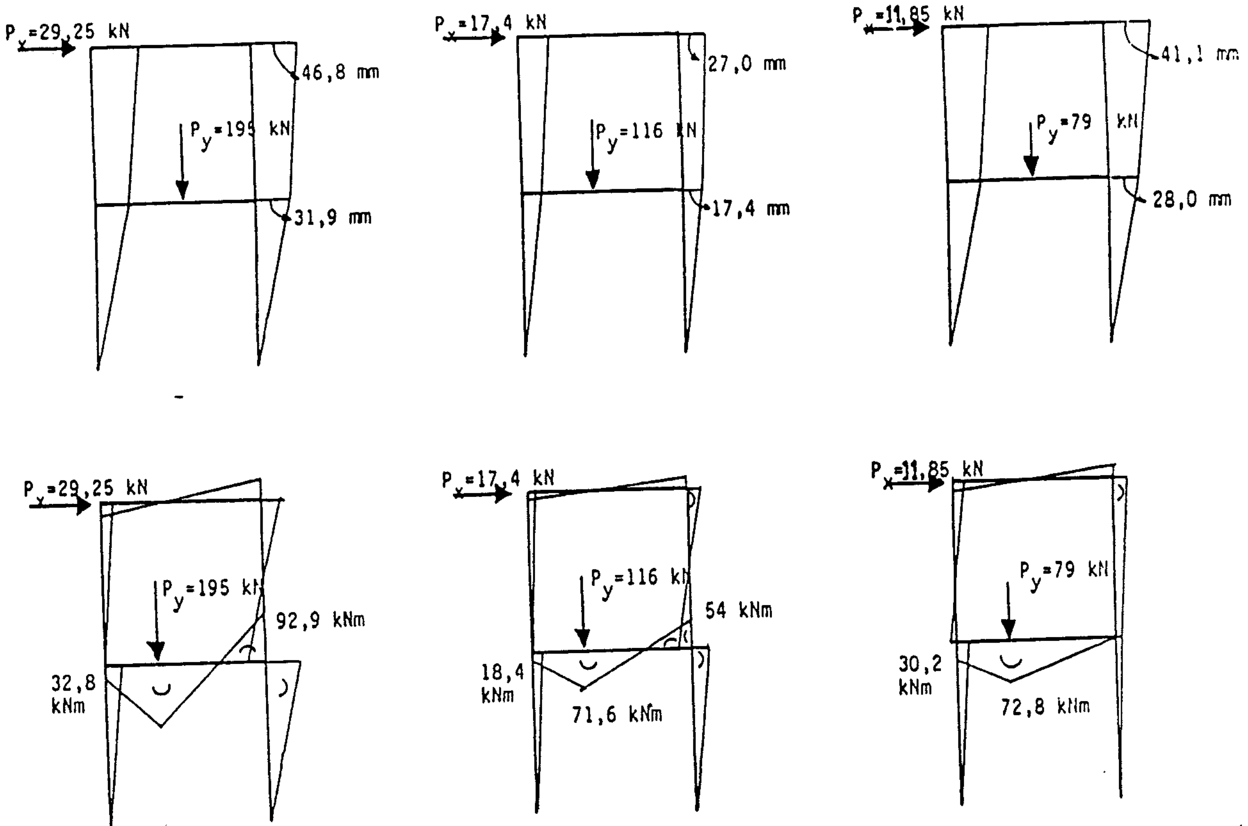


A with fully rigid connections according to Tautschnig.

B with actual moment-rotation diagrams, according to Tautschnig.

C with bilinair moment-rotation diagrams c = 24098 kNm/rad.

Fig. A.4: Moment distributions and deformations for the various assumptions.



A linear moment-rotation diagram, c = 24098 kNm/rad. no reduction due to design-strength of connection.

B linear moment-rotation diagram, c = 24098 kNm/rad. F_E = 3510 kNm restricted by design strength of connection to $\hat{M}_v$ = 54 kNm

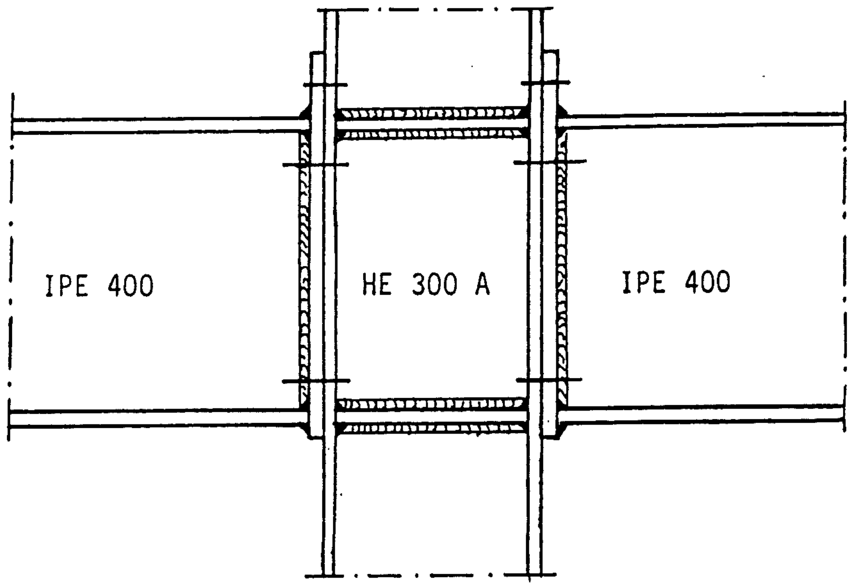
C same as A and B, but now with hinge at where strength is exceeded, F_E = 2300kNm.

Fig. A.5: Moment distributions and deformations if partial-strength connections must be taken into account.

Appendix B

Design examples

Design example 7 : Column
Symmetricallly loaded with stifeners on tension side
and compression side. Bolts M24 8.8



End plate 25 mm

Top row of bolts	flange with stiffener (example 3)	295.5 kN	281 kN
	end plate, projecting part (example 4)	281 kN	
bottom row of bolts	flange with stiffener (example 3)	295.5 kN	295.5 kN
	end plate between flanges (example 4)	369.4 kN	

M_v = 281 * (0.455 - 0.04 + 295.5 * 0.345 + 116.6 + 101.9 = 218.5 kNm

End plate as T-stub: M_v = 2 * 281 * 0.4 = 225 kNm

End plate 30 mm

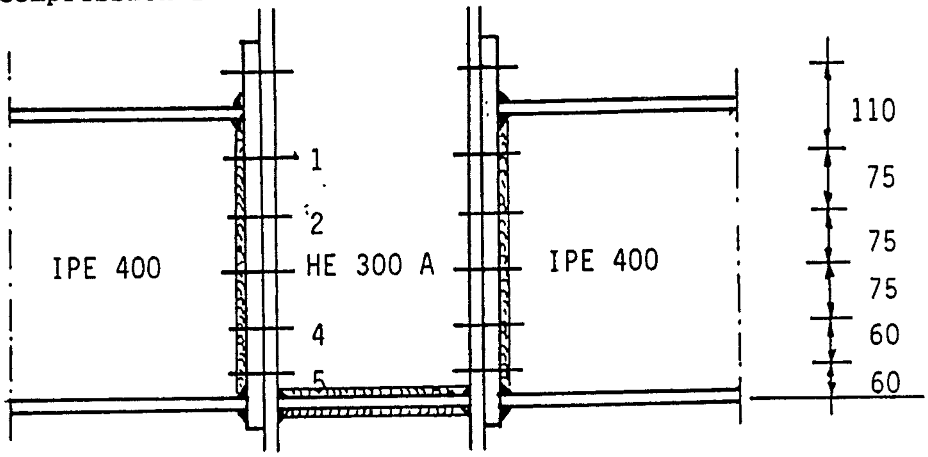
Top row of bolts	flange with stiffener (example 3)	295.5 kN	295.5 kN
	end plate, projecting part ((example 4)	317.9 kN	
bottom row of bolts	flange with stiffener (example 3)	395.5 kN	295.5 kN
	end plate between flanges (example 4)	395.5 kN	

M_v = 295.5 * 0.455 + 295.5 * 0.345 = 236.4 kNm < M_p = 312 kNm

End plate as T-stub: M_v = 2 * 295.5 * 0.4 = 236.4 kNm

Figure B.1 : Design example for symmetrically loaded column with stiffeners.

Design example 8 : Column with stiffeners and more bolt row.
From example 6 it appears that tere is nothing to be gaine from constructing this connection without stiffeners on the compression side.



End plate 25 mm

From example 6 it is also apparent that a 25 mm thick end plate is adequate , but for the sake of completeness this will be checked here.

Projecting part 281 kN
Between flanges first bolt row 369.4 kN } (see example 4)

$b_m = \frac{9.3 * 45}{2} = 209.3$ mm

Second bolt row

Effective length, two bolt rows $b_m = 209.3 + 75 = 284.3$ mm

Strength of plate $F_p = \frac{2.b_m.m_p}{m} = \frac{2 * 284.3 * 1/4 * 25^2 * 240}{45} = 473.8$ kN

$\beta = \frac{\text{plate strength}}{\text{bolt strength}} = \frac{473.8}{395.5} = 1.198$

$\gamma = 0.777$ (see example 4)

$\frac{\beta + 2\gamma}{2 + 2\gamma} = 0.774$

F for second bolt row = 612.6 - 369.4 = 243.2 kN

Third bolt row $b_m = 284.3 + 75 = 359.3$ mm

Plate strength = $\frac{359}{284} * 474 = 599$ kN $\rightarrow \beta = \frac{599}{3*197.75} = 1.00$

$\frac{\beta + 2\gamma}{2 + 2\gamma} = 0.72 \rightarrow F = 0.718 * 3 * 2 * 197.75 = 853$ kN

F for third bolt row = 853 - 613 = 240 kN.

Before calculating the moment it should first be investigated whether the column flanges can transmit the forces calculated above. This will be done in example 9. The calculationof the moment is given in example 11.

Fig. B.2 : Design example for end plate with more bolt rows.

Design example 9 : <u>Column with stiffeners and more bolt rows</u>
<u>Column flange with no stiffeners on tension side</u>

Connection similar in construction to that in example 8.
From example 2 it follows that the two topmost of bolts can each transmit 278.4 kN. Here follows a fresh calculation, because in example 2 it was assumed that the bolt rows are symmetrically loaded, whereas it is here supposed that the top bolt row develops te yield line pattern in which the second bolt row is located. In itself there is little point in this, because the projecting part o the end plate can resist only 281 kN (see example 8).

<u>top bolt row:</u> effective length 4 m + 1.25 n';
in column flange $m = 29.15$ mm
$n' = 95$ mm see example 2
$b_m = 4 * 29.15 + 1.25 * 95 = 235$ mm

plate strength $\frac{2b_m.m_p}{m} = \frac{2 * 235 * 1/4 * 14^2 * 240}{29.15} = 189.6$ kN

$\beta = \frac{189.6}{197.95} = 0.96 = \frac{\text{plate strength}}{\text{bolt strength}}$ $\quad \frac{\beta + 2\gamma}{2 + 2\gamma} = 0.764$

$\gamma = \frac{n}{m} = \frac{35}{29.15} = 1.2 \quad \frac{2\gamma}{1+2\gamma} = \frac{2.4}{34.} = 0.706$

$F = 0.764 * 395.5 = 302$ kN

<u>F for first bolt row: between the beam flanges</u> then 2 * 278.4 - 302 = 254.8 kN
This is meaninglful only if the end plate thickness is chosen as > 25 mm, otherwise F for the top bolt ros will remain 281 kN instead of 302 kN as calculated here.

<u>Second bolt row</u>
Effective length of three bolt rows is 345 + 75 = 420 mm

plate strength $\frac{2b_m.m_p}{m} = \frac{2 * 240 * 1/4 * 14^2 * 240}{29.15} = 338.9$ kN

$\beta = \frac{\text{plate strength}}{\text{bolt strength}} = \frac{338.9}{3 * 197.75} = 0.571$ complete yielding of plate therefore F for three bolt rows = 2 x 338.9 = 677.8 kN

$\frac{\gamma}{1 + 2\gamma} = 0.706$

F for second bolt row = 677.8 - 556.8 = 121 kN

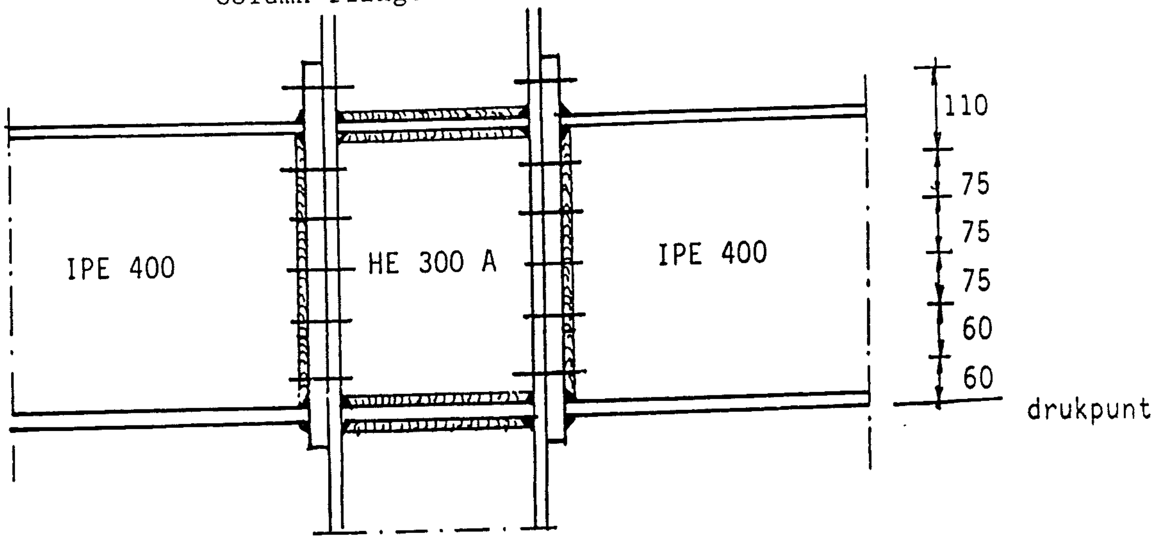
F for third bolt row = $\frac{2b_m.m_p}{m} = \frac{4 * 75 * 1/4 * 14^2 * 240}{29.15} = 121$ kN

It can now be decided whether the column flange or the end plate is the governing component. From a comparison it appears that the forces are determined by the column flange.

Fig. B.3 : Design example for column flanges with more bolt rows.

Design example 10 : Column with stiffeners and more bolt row.

Column flange with stiffeners. Bolt M24 8.8



Top bolt row and bolt row 1: see example 3

<u>Second bolt row</u>

Effective length first + second bolt rows = $\frac{4\pi * 23,75}{2}$ + 75 = 224 mm

Plate strength $\frac{2b_m . m_p}{m} = \frac{2 * 224 * \frac{1}{4} * 14^2 * 240}{23,75} = 222$ kN

$\beta = \frac{\text{plate strength}}{\text{bolt strength}} = \frac{222}{395,5} = 0,5614$

$\frac{2\gamma}{1+2\gamma} = 0,747$

} complete yielding of plate

$\hat{F}$ for two bolt rows = 2 * 222 = 444 kN

$\hat{F}$ for 1st bolt row = 295,5 kN

$\hat{F}$ for 2nd bolt row = 148,5 kN

Each following bolt row is : $2 * \frac{2 * 75 * \frac{1}{4} * 14^2 * 240}{23,75} = 148,5$ kN

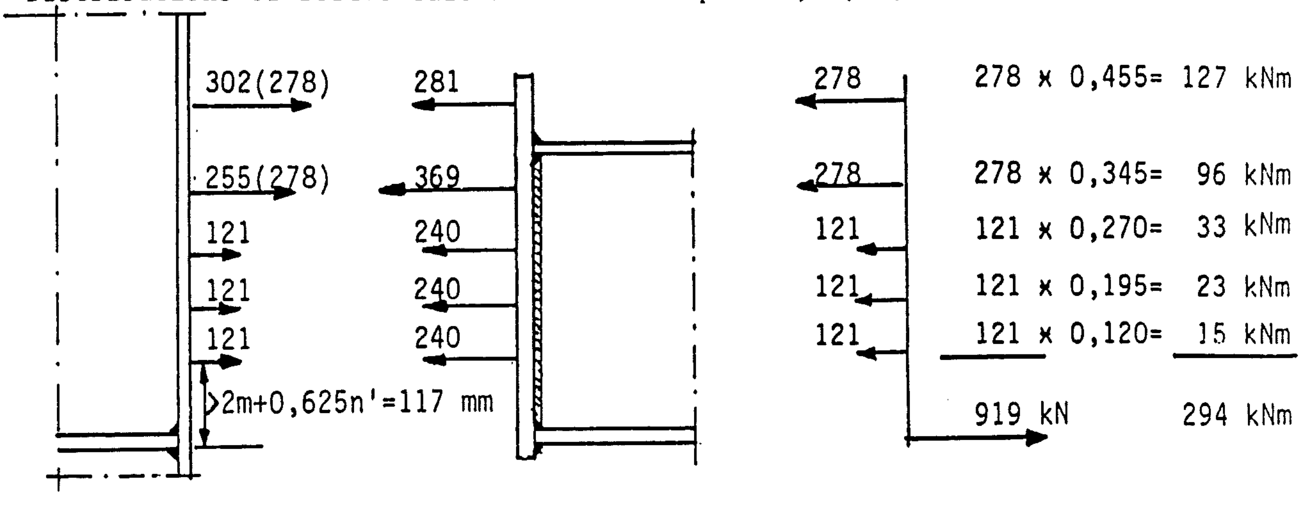
because complete yielding of the plate has already been attained at the second bolt row.

The difference between the design strengths for the column flange <u>with</u> or <u>without</u> stiffeners for the second and following bolt rows is due to the difference in m.
In the case of the column without stiffeners 1/5 of the transition radius is added to this. This must be done also for the part where the supporting effect of the stiffener is zero, so that $\hat{F}$ for the second and following bolt rows is 121 kN (see example 9).

Fig. B.4 : Design example for column flanges with stiffeners and more bolt rows.

Design example 11 : Calculations and comparisons of design strengths of column flanges with and without stiffeners and with several bolt rows.

Distributions of forces calculated in examples 3, 4, 6, 7, 8, 9 and 10.

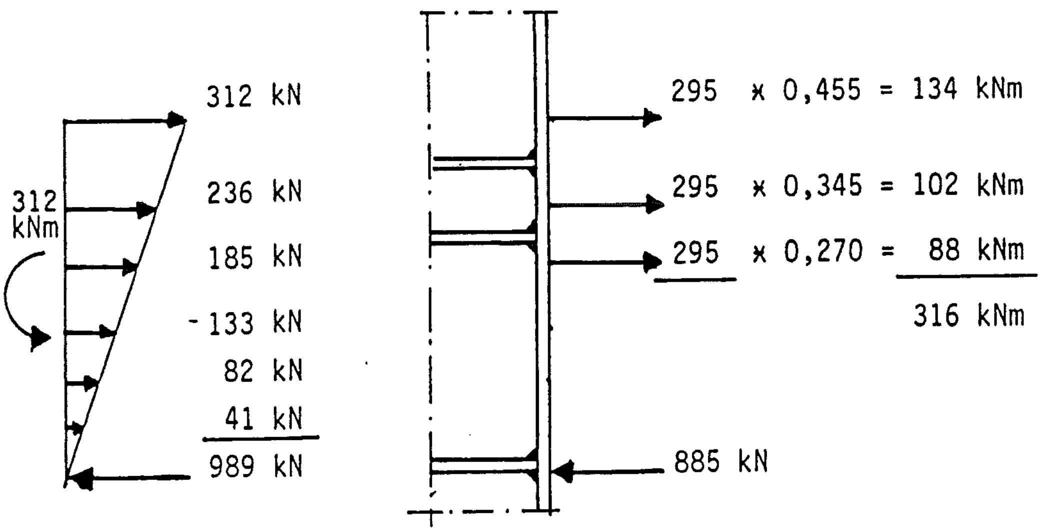


Column flanges without stiffeners on tension side (see ex. 3 and 9)

End plate 25 mm (see examples 4 and 8)

Combination on column flange and end plate.

With the basic assumption that the connection must be able to attain the plastic moment of the beam: M_p = 312 kNm.



With end plate and column flange assumed to be infinitely stiff.

With results of example 3 and 9

Conclusions:
The distribution of forces for end plate and column flange assumed to be infinitely stiff can be resisted, with redistribution of forces, by a column flange with two stiffeners on the tension side. The end plate should then be made 27 mm thick. With the method of analysis proposed here the column flange without stiffeners can develop with five bolt rows a moment of 294 kNm (0.94 M_p).

Fig. B.5 : Design example with design strength of connections.

Design example 13 : Stiffness analysis

Connection of example 5 : shearing zone governs behaviour; end plate thickness t_e = 25 mm

Topmost bolt row determines rotation.

Component or mechanism	Formula for C_i for $\hat{F}_i$.	$(\frac{\hat{F}_i}{F_i})^2$	C_i for M_v	
Shearing of column web	$C_1 = 0{,}24\ t_W = 0{,}24 * 8{,}5$	1	2	4,5
Tension on column web	$C_2 = 0{,}8\ t_W = 0{,}8 * 8{,}5$	$(\frac{704}{310})^2$	15	15 see example 12
	$C_3 = 0{,}8\ t_W = 0{,}8 * 8{,}5$	$(\frac{418}{310})^2$	15	
Compression on column web	$C_4 = \frac{t_f^3}{4m^2} = \frac{14^3}{4*29{,}15^2} =$	$(\frac{278}{278})^2$	0,8	1,8
Tension in bolt	$C_5 = \frac{2A_s}{l_b} = \frac{2*35^3}{60}$	1	11,8	26
Tension-bending of end plate	$C_6 = \frac{t_e^3}{4m^2} = \frac{25^3}{4*40^2}$	$(\frac{281}{278})^2$	2,5	5,5

$$C = \left(\frac{1}{2} + \frac{1}{15} + \frac{1}{12} + \frac{1}{0{,}8} + \frac{1}{11{,}8} + \frac{1}{2{,}5} \right)^{-1} * 2{,}1 * 10^5 * (455-40)^2$$

$$C = \frac{0{,}420 * 2{,}1 * 10 \quad * 415^2}{10^6} = 15166 \text{ kNm/rad for notational connection}$$

$$C = 15166 * \frac{126{,}4}{278{,}4 * 0{,}415} = 15166 * 1{,}09 = 16531 \text{ kNm/rad}$$

$$\phi = \frac{126{,}4}{16531} = 7{,}65 * 10^{-3} \text{ rad for } 126{,}4 \text{ kNm}$$

For 1.5-fold lower loading, i.e. for 84 kNm.

$$C = \left(\frac{1}{4{,}5} + \frac{1}{15} + \frac{1}{15} + \frac{1}{1{,}8} + \frac{1}{26} + \frac{1}{5{,}5}\right) * 2{,}1 * 10^5 * (455-40)^2$$

$$C = \frac{0{,}884 * 2{,}1 * 10^5 * 415^2}{10^6} * 1{,}09 = 34844 \text{ kNm/rad}$$

$$\emptyset = \frac{84}{34844} = 2{,}41 * 10^{-3} \text{ for 84 kNm.}$$

Fig. B.6 : Stiffness analysis for a connection in which the projecting part of the end plate is so strong that the lever arm for the stiffness is reckoned as extending up to the topmost bolt. The strength of the connection is governed by shearing on the column web.

Design example 14 : Stiffness analysis

Connection of example 6 : behaviour at failure governed by compression zone and projecting part of end plate (end plate thickness t_e = 15 mm).

First bolt row between flanges determines rotation.

Component or mechanism	Formula for C_i for $\hat{F}_i$	$\frac{\hat{F}_i}{F_i}$	C_i for $\bar{M}_v$	C_i for $\frac{\bar{M}_v}{1,5}$
Shearing of column web	$C_1 = \infty$			
Tension on column web	$C_2 = 0,8\ t_w = 0,8 * 8,5$	$(\frac{704}{364})^2$	15	15
	$C_3 = 0,8\ t_w = 0,8 * 8,5$	$(\frac{418}{364})^2$	9	15
Compression on column web	$C_4 = \frac{t_f^3}{4m^2} = \frac{14^3}{4 * 29,15^2}$	$(\frac{278}{243})^2$	1	1,8
Tension in bolt	$C_5 = \frac{2A_s}{l_b} = \frac{2 * 353}{60}$	$(\frac{243}{243})^2$	11,8	26,5
Tension-bending of end plate	$C_6 = \frac{t_e^3}{12\lambda_2 m_1^2} = \frac{15^3}{12*0,48*45^2}$	$(\frac{243}{243})^2$	0,29	0,65

$$C = \left(\frac{1}{15} + \frac{1}{9} + \frac{1}{1} + \frac{1}{11,8} + \frac{1}{0,29}\right)^{-1} * 2,1 * 10^5 * 345^2$$

$$C = \frac{0,2122 * 2,1 * 10^5 * 345^2}{10^6} = 5306 \text{ kNm/rad for notational connection}$$

$$C = \frac{134}{243 * 0,345} * 5306 = 1,6 * 5306 = 8490 \text{ kNm/rad}$$

$$\emptyset = 15,8 * 10^{-3} \text{ rad for } 134 \text{ kNm}$$

For 1.5-fold lower loading, i.e. for 89 kNm

$$C = \left(\frac{1}{15} + \frac{1}{15} + \frac{1}{1,8} + \frac{1}{26,5} + \frac{1}{0,65}\right)^{-} * 2,1 * 10^5 * 345^2$$

$$C = \frac{0,4415 * 2,1 * 10^5 * 345^2}{106} * 1,6 = 17656 \text{ kNm/rad}$$

$$\emptyset = 5,04 * 10^{-3} \text{ rad for } 89 \text{ kNm}$$

Fig. B.7 : Stiffness analysis for connection of example 6 with 15 mm end plate thickness. The lever arm extends to the first bolt row between the beam flanges.

Design example 15 : Stiffness analysis

Connection of example 6 : compression zone governs behaviour; end plate thickness t_e = 25 mm.

Topmost bolt row determines rotation.

Component or mechanism	Formula for C_i for $\hat{F}_i$	$(\frac{\hat{F}_i}{F_i})^2$	C_i for $\hat{M}_v$	C_i for $\frac{\hat{M}_v}{1,5}$
Shearing of column web	$C_1 = \infty$		∞	∞
Tension on column web	$C_2 = 0,8\ t_w = 0,8 * 8,5 =$	$(\frac{704}{418})^2$	15	15
	$C_3 = 0,8\ t_w = 0,8 * 8,5$	$(\frac{418}{418})^2$	6,8	15
Compression on column web	$C_4 = \frac{t_f^3}{4m^2} = \frac{14^3}{4 * 29,15^2}$	$(\frac{278}{278})^2$	0,8	1,8
Tension in bolt	$C_5 = \frac{2A_s}{l_b} = \frac{2 * 353}{60}$	$(\frac{278}{278})^2$	11,8	26,6
Tension-bending of end plate	$C_6 = \frac{t_e^3}{4m^2} = \frac{25^3}{4 * 40^2}$	$(\frac{281}{278})^2$	2,5	5,5

$$C = \left(\frac{1}{15} + \frac{1}{6,8} + \frac{1}{0,8} + \frac{1}{11,8} + \frac{1}{2,5}\right)^{-1} * 2,1 * 10^5 * (455-40)^2$$

$$C = \left(\frac{0,51 * 2,1 * 10^5 * 415^2}{10^6}\right) = 18561 \text{ kNm/rad} \quad \text{for notational connection}$$

$$C = \frac{164}{278 * 0,415} * 18561 = 1,42 * 18561 = 26385 \quad \text{kNm/rad}$$

$$\emptyset = \frac{164}{26385} \doteq 6,2 * 10^{-3} \text{ rad for } 164 \text{ kNm}$$

For 1.5-fold lower loading, i.e. for 109 kNm

$$C = \left(\frac{1}{15} + \frac{1}{15} + \frac{1}{1,8} + \frac{1}{26,6} + \frac{1}{5,5}\right)^{-1} * 2,1 * 10 * (455 - 40)^2$$

$$C = \left(\frac{1,1 * 2,1 * 10 * 415^2}{10^6}\right. * 1,42 = 56493 \text{ kNm/rad}$$

$$\emptyset = \frac{109}{56493} = 1,93 * 10^{-3} \text{ rad. for } 109 \text{ kNm}$$

Fig. B.8 : Stiffness analysis for connection of example 6 with 25 mm end plate thickness

Design example 16 : Stiffness analysis

Connection of example 7 : Symmetrically loaded, with stiffeners on tension side and compression side; projecting part of end plate governs behaviour (end plate thickness 25 mm).

First bolt row between flanges determined rotation.

Component or mechanism	Formula for C_i for $\hat{F}_i$	$(\frac{\hat{F}_i}{F_i})^2$	C_i for $\hat{M}_v$	C_i for $\frac{\hat{M}_v}{1,5}$
Shearing of column web	$C_1 = \infty$		∞	∞
Tension on column web	$C_2 = \infty$		∞	∞
	$C_3 = \infty$		∞	∞
Compression on column web	$C_4 = \frac{t_f^3}{12\lambda_2 m_1^2} = \frac{14^3}{12*0,30*23,7^2}$	$(\frac{296}{296})^2$	1,33	3
Tension in bolt	$C_5 = \frac{2A_s}{l_b} = \frac{2*353}{60}$	$(\frac{296}{296})^2$	11,8	26,5
Tension-bending of end plate	$C_6 = \frac{t_e^3}{12\lambda_2 m_1^2} = \frac{25^3}{12*0,41*45^2}$	$(\frac{369}{296})^2$	2,4	3,5

$$C = \left(\frac{1}{1,33} + \frac{1}{11,8} + \frac{1}{2,4}\right)^{-1} * 2,1*10^5 * 345^2$$

$$C = \frac{0,8087 * 2,1*10^5 * 345^2}{10^6} = 21401 \text{ kNm/rad}$$

$$C = 21401 * \frac{219}{296 * 0,345} = 45896 \text{ kNm/rad}$$

$$\emptyset = \frac{219}{45896} = 4,77 * 10^{-3} \text{ rad for } 219 \text{ kNm}$$

For 1.5-fold lower loading, i.e. for 146 kNm

$$C = \left(\frac{1}{3} + \frac{1}{26,5} + \frac{1}{3,5}\right)^{-1} * 2,1*10^5 * 345^2$$

$$C = \frac{1,52257 * 2,1*10^5 * 345^2}{10^6} = 38057 \text{ kNm/rad}$$

$$C = \frac{219}{296 * 0,345} * 38057 = 2,14 * 38057 = 81615 \text{ kNm/rad}$$

$$\emptyset = \frac{146}{81615} = 1,79 * 10^{-3} \text{ rad for } 146 \text{ kNm}$$

Fig. B.9 : Stiffness analysis or connection of example 7 with stiffeners on tension side and compression side.

Design example 17 : Stiffness analysis

Connection of example 7 : symmetrically loaded, with stiffeners on tension side and compression side; column flanges govern behaviour (end plate thickness 30 mm).

It will be shown that the first bolt row between the flanges continues to determine rotation.

Tension-bending of column flange	$C_4 = \frac{t_f^3}{12\lambda_2 m_1^2} = \frac{14^3}{12*0,3*23,7^2}$	1,33	3	3
Tension in bolt	$C_5 = \frac{2A_s}{l_b} = \frac{2*353}{60}$	11,8	26,5	26,5
Tension-bending of end plate of topmost bolt row	$C_6 = \frac{t_e^3}{4m^2} = \frac{30^3}{4*40^2}$	$(\frac{398}{296})^2$	4,86	9,5
First bolt row	$C_6 = \frac{t_e^3}{12\lambda_2 m_1^2} = \frac{30^3}{12*0,41*45^2}$	$(\frac{396}{296})^2$	4,85	6,1

Topmost bolt row

$$C = \left(\frac{1}{1,33} + \frac{1}{11,8} + \frac{1}{4,86}\right)^{-1} * 2,1 * 10^5 * (455-40)^2 = 0,959 * 2,1 * 10^5 * 415^2$$

Bolt row between flanges (first bolt row)

$$C = \left(\frac{1}{1,53} + \frac{1}{11,8} + \frac{1}{4,85}\right)^{1} * 2,1 * 10^5 * 345^2 = 0,959 * 2,1 * 10^5 * 345^2$$

Topmost bolt row

$$C = 34696 * \frac{236}{295 * 0,455} = 61003 \text{ kNm/rad}$$

Bolt row between flanges (forst bolt row)

$$C = 23970 * \frac{236}{295 * 0,345} = 55582 \text{ kNm/rad (governing value)}$$

$$\emptyset = \frac{236}{55582} = 4,25 * 10^{-3} \text{ rad for 236 kNm}$$

For 1.5-fold lower loading, i.e. for 157 kNm

$$C = \left(\frac{1}{3} + \frac{1}{26,5} + \frac{1}{6,1}\right)^{-1} * 2,1 * 10^5 * 345^2 = 1,87 * 2,1 * 10^5 * 345^2$$

$$C = 46.720 \text{ kNm/rad}$$

$$C = 46720 * \frac{236}{295 * 0,345} = 108335 \text{ kNm/rad}$$

$$\emptyset = \frac{157}{108335} = 1,45 * 10^{-3} \text{ rad for 157 kNm}$$

Fig. B.10 : Stiffness analysis for connection with stiffeners on tension side and compression side of the connection. Checking of the lever arm indicates that the bolt row between the beam flanges determines the stiffness.

Design example 18 : Stiffness analysis

Connection of example 8 : symmetrically loaded, with stiffeners on compression side and more bolt rows. Column flange governs behaviour (end plate thickness 25 mm).

Topmost bolt row determines rotation.

Component or mechanism	Formula for C_i for $\hat{F}_i$	$\left(\frac{\hat{F}_i}{F_i}\right)$	C_i for M_v	C_i for $\frac{M_v}{1,5}$
Shearing of column web	$C_1 = \infty$	∞	∞	∞
Tension on column web	$C_2 = 0,8\ t_w = 0,8 * 0,85$	$\left(\frac{704}{576}\right)^2$	9,6	15
	$C_3 = \infty$	∞	∞	∞
Compression on column web	$C_4 = \frac{t_f^3}{4m^2} = \frac{3}{4 * 29,15^2}$	$\left(\frac{278}{278}\right)^2$	0,8	1,8
Tension in bolt	$C_5 = \frac{2A_s}{l_b} = \frac{2 * 353}{60}$	$\left(\frac{278}{278}\right)^2$	11,8	26,6
Tension-bending of end plate	$C_6 = \frac{t_e^3}{12\lambda_2 m_1^2} = \frac{25^3}{12 * 0,41 * 45^2}$	$\left(\frac{369}{278}\right)^2$	2,8	3,5

$$C = \left(\frac{1}{9,6} + \frac{1}{0,8} + \frac{1}{11,8} + \frac{1}{2,8}\right)^{-1} * 2,1 * 10^5 * 345^2 = 0,556 * 2,1 * 10^5 * 345^2$$

C = 13916 kNm/rad for notational connection

$$C = \frac{294}{278 * 0,345} * 13916 = 3,06 * 13916 = 42660 \text{ kNm/rad bij } \hat{M}_v = 294 \text{ kNm(see example 1)}$$

$$\phi = \frac{294}{42660} = 6,89 * 10^{-3} \text{ rad for } \hat{M}_v = 294 \text{ kNm}$$

For 1.5-fold lower loading, i.e. for M_v = 196 kNm

$$C = \left(\frac{1}{15} + \frac{1}{1,8} + \frac{1}{26,6} + \frac{1}{3,5}\right)^{-1} * 2,1 * 10^5 * 345^2 - 1,057 * 2,1 * 10^5 * 345^2$$

$$C = \frac{294}{278 * 0,345} * 1,057 * 2,1 * 345^2 * 10^{-1} = 81034 \text{ kNm/rad}$$

$$\phi = 2,42 * 10^{-3} \text{ rad for } M_v = 196 \text{ kNm}$$

Fig. B.11 : Stiffness analysis for connection with stiffeners and more bolt rows.