Truthful Trading in Local Energy Markets

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by

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Abstract

With the increasing share of renewable and thus volatile distributed generation worldwide, small-scale energy producers, prosumers and consumers will become more and more involved in the overall energy system. These small-scale actors were formerly excluded from the energy market, as legislative restrictions about generation size and legal stipulations prohibited them from actively taking part in the bidding process. While DERs of intermittent nature (such as PV installations) already constitute a significant part of the generation mix, the wholesale electricity markets have not been designed taking their characteristics (production variability, low predictability, zero marginal cost of generation and strong site-specificity) into account, thus making their market integration harder.

Local electricity markets (LEMs) solve this issue by providing a local market platform to residential actors within a community. They empower small scale electricity producers, prosumers, and consumers and offer economic incentives for creating local electricity balances. Yet, definitions of LEMs, their concepts and market mechanisms are mostly case driven instead of comparative. Furthermore, mechanisms which induce truthful bidding from market participants have received little attention within the context of residential LEMs. This thesis attempts to address these gaps by comparing several truthful double auction mechanisms and proposing a market mechanism framework suitable for residential LEMs. A Monte Carlo simulation is conducted to compare mechanism performance indicators under various LEM scenarios. Main performance indicators include the quantity of energy traded locally, gains-from-trade between market participants and total revenue received by the market operator. Finally, recommendations on capturing the value of implementing truthful mechanisms are made for potential LEM stakeholders.
Acknowledgements

This thesis is written as part of the Master of Science degree in Sustainable Energy Technology at the Technical University of Delft.

I began this journey with the ambitious goal of searching for a market design framework that could up-end the current Dutch wholesale electricity market, allowing the common masses to truly enjoy freedom from their utility bills by managing their own energy generation and consumption. Like many amateurs entering unfamiliar territory, reality hit me fast and I quickly found out that a redesign of the wholesale electricity market is next to impossible. Thus began an exploration into smaller, and more specialized solutions that exists in the realm of “what could the future sustainable electrical system look like?”.

While this path involved many memories of confusion and frustration in terms of academic direction, I was fortunate to have met and be surrounded with a number of mentors that served as my compass. Throughout this entire journey, I totally enjoyed learning and gaining mastery of the subject matter, never once did I find myself not disengaged nor discouraged. I would therefore like to thank in general every professor, employee and student whom I had the pleasure meeting during this 9 month endeavor.

I would like to begin by expressing my enormous gratitude to my supervisor Dr.ir. Milos Cvetkovic for his continuous support and accommodation. While the scope of my study diverged somewhat from his field of expertise, he has supported and challenged me to constantly elevate myself throughout the entire course of my thesis in our weekly brainstorm sessions.

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I am very proud to conclude this important chapter of my life at this University and will always look back on these years with a special fondness.

L. Liu
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Executive Summary

The thesis is written in the context of a world in which distributed residential renewable energy sources, guided by economic and political goals, assumes an increasing share of the energy mix within the European energy sector. Given its potential economic, technological and social benefits, local energy markets (LEMs) could take on a significant role in the localizing power flow, increasing local renewable autocracy, empowering the residential agents to develop from passive price takers towards active market participants, and supporting the overall decentralization of the future electricity system. A local electricity market is a market platform for trading locally generated (renewable) electricity among residential agents within a geographically and socially close community. Security of supply is ensured through connections to a superimposed electricity system (e.g. national grid or adjacent local electricity markets. Thus, the electricity transition from the former fossil fuel-based generation structure towards a sustainable, renewable-based, multidimensional system becomes feasible.

LEMs are online virtual markets involving competitive behavior between rational and self-interested agents with a different objective than the platform provider that oversees the operation of the market. In a competitive environment with money, agents are incentivized to prioritize maximizing their profits and savings. In some cases when market rules allow, agents will make decisions to manipulate market outcomes in their favor. Thus the platform provider is tasked with designing and conducting a fair market platform for all participants. The study primarily focuses on the micro-economic theory of LEM mechanisms with the objective of inducing truthful behavior among market participants.

The competitive environment, the intrinsic susceptibility to market failure, and lack of attention in current research on the implementation of market mechanisms in local energy markets constituted as the main motivations behind the main question of this research:

Which market mechanism is suitable for a residential local energy market which defers non-truthful behavior of market participants?

These specific characteristics determined game theory and mechanism design theory as suitable tools to implement market mechanisms which aligns with the goals of this research. By capturing LEMs in a game theoretic framework, identifying suitable auction mechanisms and reflecting close-to-real-world electricity profiles of residential consumers and prosumers, this thesis sought to provide insights on macro perspectives of this market. The macro perspective takes on the standpoint of the LEM platform provider and examined the benefits and drawbacks in implementing the chosen truthful double auction mechanisms.

This resulted in a research approach which attempts to use a Monte Carlo method to identify influential parameters in the bid determination of the market participants such that the intended market outcomes are achieved. The research objective is therefore formulated as:

To compare and determine suitable truthful market mechanisms by incorporating realistic residential household electricity profiles as input for a LEM model.

In relation to the research objective, it is necessary to first give a clear description of this problem. This resulted in the formulation of the first sub-question:

• SQ 1: What are the main characteristics of LEMs and the stakeholders involved and What type of market design best describes residential local energy markets?

In Chapter 2, a literature review was conducted and the main characteristics and stakeholders of LEMs are identified. The general market design framework of LEM is proposed as a discrete-time sealed-bid double auction. Simplications made to the market framework allows us to align the market structure (the market rules and market mechanism) with the desired agent behavior to achieve the intended market outcome. In terms of market structure, the most important stakeholders of the LEM investigated in this study are: consumers who aim to minimize consumption costs, prosumers who aim
to either maximize profits or minimize consumption costs, and the auctioneer who aims to maximize total market value.

In order to motivate the choice for selecting mechanism design theory as the main approach for this study, the second sub-question is formulated:

- **SQ 2:** *What are the key concepts of game-theory and mechanism design and can these elements of economic theory be used to induce truthful behavior in the design of a residential local energy market?*

Chapter 3 served as a primer on the field of game theory and introduced the necessary conceptual building blocks in order to formulate the LEM structure as a game. In terms of the proposed LEM framework, the strategic form was determined to be the more suitable game representation. Furthermore, the LEM is formulated as a non-cooperative game played between residents within a community looking to either buy or sell up to a certain amount of energy. Every household is self-interested with a goal of gaining maximum profits for selling or maximum utility for purchasing. Each household holds some private information and thus have incomplete information about the game environment. Furthermore, households have imperfect information about the market because their own historical energy usage is considered private information and unknown to other households. The bids that households submit to the auctioneer in each round of the double auction is considered as pure strategies, and the best response strategy of each household should produce a competitive market equilibrium. Formulation of the LEM structure as a game requires us to choose a solution concept, or equilibrium concept. From a macro perspective, the dominant strategy equilibrium is chosen due to the powerful property in which each agent's best response strategy is to reveal its private information truthfully. Yet a game theoretical approach is not sufficient for us to ensure that a dominant strategy equilibrium exists, because the market rules have not been defined. We continue the discussion of inducing truthful behavior in market participants from a completely different approach in Chapter 4. While game theory is concerned with agent behavior in equilibrium (and thus the resulting strategy that it should play), mechanism design theory allows us to construct market rules to induce a game among the agents in a way that in an equilibrium state of the induced game, the desired system-wide solution is implemented. The mechanism design objectives that align with the goal of the research are identified as incentive compatibility and individual rationality. Incentive-compatible dominant-strategy mechanisms are chosen as applicable market mechanisms to be implemented in the LEM auction design. At this point, the second sub-question has thus been answered.

In Chapter 5, the usage of mechanism design theory results in the determination of two incentive-compatible double auction mechanisms, as well as an untruthful mechanism which serves as a point of reference. The Walrasian mechanism is not an incentive-compatible and serves as a benchmark for the comparison of truthful mechanisms. The impossibility theorem proposed by Myerson and Satterwaite creates a trade-off in achieving desired mechanism properties. The two incentive-compatible mechanisms explored in this study are the VCG mechanism and the Huang mechanism. The VCG mechanism is mathematically elegant in its ability to process offers with perfect efficiency and dominant truth-revealing strategies, yet it requires the market operator to subsidize trades. The Huang mechanism provides the market operator with a revenue for conducting trades, but its allocation rule requires a trade reduction which reduces the its efficiency.

Now that the theoretical aspects of the study has been addressed, Chapter 6 begins the formulation of the research design and methodology required to compare the mechanisms in a way which reflects the research objective. Thus the last sub-question of this study is formulated:

- **SQ 3:** *What are the main factors that influence market mechanism performance and how can we create a simulation environment that enables us to compare market mechanisms in the context of residential LEMs?*

The double auction market design proposed in Chapter 2 is further specified in able to conceptualize the model. Modeling the implementation of truthful double auction mechanisms requires careful determination of what constitutes as the “core” of the study and what is superfluous “noise”. Eliminating elements that have negligible influence on the objective of the study enables the modeler to enhance the effects of the core interactions. Developing critical assumptions and specifications of the model environment will simplify the model itself without distorting the results obtained. In line with the overall objective of the thesis project, the components attributed to the private information of market
participants are formulated and the auctioneer’s responsibilities within the LEM structure are clarified. Furthermore, we identified the following LEM model outputs which serve as important metrics on which the mechanism comparison is based: Market Clearing Price, Buyers’ Utility, Sellers’ Profit, Quantity Traded, and Budget Balance.

Chapter 7 serves as a description of the simulation methodology of the model, which takes on a Monte Carlo method in order to capture the various LEM scenarios. A focus is put on the macro-perspective of bid determination, where input parameters describe the mean $\mu$ and standard deviation $\sigma$ of the bid price ($s_v$ and $b_v$) and volume ($s_q$ and $b_q$). The simulation methodology first attempts to identify the most influential parameters in terms of bid determination, then seeks to validate the insights gained by modeling realistic generation and consumption data of residential households.

In Chapter 8, the results of the model simulation are presented. From the sensitivity analysis, it is observed that the $\mu_v$ and $\mu_{pv}$ parameters describing the mean valuation of the bids heavily influence the market outcome KPIs. This insight serves as a validation for the inference that truthful behavior of rational agents when valuing the energy supplied or demanded must be properly incentivized for the platform provider to reliably achieve intended market outcomes, which are the maximization of market value and localization of energy flows. The comparison of the three mechanisms was conducted under various residential microgrid LEM scenarios. Separate simulations were conducted under increasing residential PV penetration rates and under different population sizes. The results of the comparison indicates the VCG mechanism as a superior market mechanism for the LEM double auction structure.

At the end of Chapter 9, a number of areas for further research are appointed. These include methods to increase the functionality of the model and means to improve the accuracy of the predictions of the model. In this chapter, additional areas of further research with respect to the domain of public policy are also advised.
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In general, if any branch of trade, or any division of labour, be advantageous to the public, the freer and more general the competition, it will always be the more so.

Adam Smith

In the current landscape of electrical power production, transportation, and consumption, the energy sector is experiencing a paradigm shift. Specifically, within the last decade, this technological transition has been led by a significant increase in the viability and integration of renewable energy sources (RES) on the industrial, commercial, and residential scale. Additionally, innovative information and communication technology allow for efficient and complex coordination between control systems. The next-generation of power-grids are being designed with renewable energy in mind, which opens up new challenges and opportunities to integrate distributed generation into the energy supply system. Former top-down, centralized energy systems need to be adapted to take full advantage of the immense potential of decentralized energy generation and smart, interconnected end users. As the viability for the decentralization of electricity markets grows, this has led to ambitions to build solutions in which owners of solar panels can sell their production to other consumers on the local low-voltage distribution system. The paradigm shift in the socio-technological landscape of the energy sector is illustrated in Figure 1.1 below.

Figure 1.1: The electricity market is experiencing a shift to more distributed energy generation and an increasing share of prosumers supplying their energy [66]

1.1. Contextual Introduction
The increasing penetration of RES at the distribution grid level creates concerns about their successful integration in the existing electric grid. In November 2016, the European Commission (EC) presented the "Clean Energy for all Europeans" policy package [11]. Among the policies, priority is given to the empowerment of customers through more active involvement in the EU energy system, allowing them
better control over their energy consumption and an improved response to price signals by taking advantage of the local availability of renewable resources. According to the EC, in 2030, half of the EU's electricity will come from renewable energy sources (RES), and by 2050, its electricity should be 100% carbon-free. Most of this new intermittent capacity will continue to be deployed on the end-consumer premises, and must swiftly become fully market-integrated, to ensure RES cost-effectiveness. End-consumers of electricity are gaining the capability of self-generating their energy as well as optimizing their electricity consumption. The traditional consumer is slowly transforming into consuming producers, also known as prosumers. But the current electricity market provides little opportunity for the increasing number of residential producers to provide their added value to the overall energy network. Using conventional concepts to manage the bi-directional power flow is no longer valid, and more roles are being assigned to the distribution network operators and the energy consumers as well. Innovative market designs that take on a bottom-up approach have the potential to capture value otherwise lost by the current centralized electricity markets.

1. Introduction

1.1. Towards Decentralizing Energy Trade and Future Market Designs

These developments provide a framework for the establishment of local energy markets (LEMs), which can be broadly defined as marketplaces that enable prosumers and/or other local generating entities to trade energy volumes of their choosing within local communities. The introduction of LEMs in the EU's energy system will take shape through revisions of both the Electricity Directive and the Renewables Directive, which set legal and market participation principles for "local energy communities" and "renewable energy communities". Combined, the two documents advance that these communities

- can engage in energy generation, consumption, distribution, aggregation, storage, supply/sales, including through power purchase agreements, and/or energy efficiency services;
- are entitled to own, establish, lease, and autonomously manage community networks;
- should operate on the energy markets, directly or via aggregators or suppliers, on a level-playing field without distorting competition;
- must benefit from non-discriminatory treatment in their activities, rights, and obligations as final customer, generators, distribution system operators or aggregators;
- are subject to fair proportionate and transparent procedures and cost-reflective charges.

This last point highlights an integral issue in the design of modern liberalized electricity markets, which is that these markets are competitive, capitalistic and prone to gaming. Liberalized electricity markets refer to market structures where central management and coordination were replaced by decentralized decision-making about investments in and deployment of power plants. From a historical point of view, the decoupling of state ownership of electricity generation and consumption from transmission and distribution allowed for businesses to enter the energy sector. In [9], Cramton states that the main objective of the electricity market should fill is to provide reliable electricity at the lowest cost to consumers. This main objective can be broken down into two key components. Firstly, short-run efficiency (static efficiency), which means to optimize the use of the existing resources in such a way that it results in the lowest cost. Secondly, long-run efficiency (dynamic efficiency), which is ensuring that the market provides the right incentives for efficient long-term investments. In practice, this implies that there should be enough and efficiently installed capacity for the supply of electricity. Figure 1.2 below illustrates how different electricity markets operating on different timelines are coupled to ensure short-run and long-run efficiency.

The two components highlight a clash of incentives for market participants. On the one hand, competition is needed to supply electricity at low cost, but competition also significantly shrinks the profit margin for participating suppliers and prolongs the return of investment. On the other hand, there needs to be enough profit to be extracted from the market to fund future projects, where capital costs of utility scale power plants can amount to hundreds of millions of Euros. While long-term efficiency and security of supply are addressed through bilateral futures contracts, the short-term spot market is volatile and is primarily used by market participants to adjust their future contract positions up to real-time [41]. The nature of electricity as a commodity itself makes volatility of prices and volumes an intrinsic property for spot markets.
On these exchanges, the transmission and distribution of electrical power are prioritized from a business standpoint rather than with servicing end-customers in mind. With self-interested parties incentivized to maximize their profits, instances of malicious behavior by market participants to gain unfair advantage have been reported. Illustratively, the most infamous instance of a market participant caught gaming the market was the Enron scandal, which in turn caused the California energy crisis in 2000 and 2001 [6]. During that era where Californian electricity markets were gradually being deregulated, Enron exploited policy loop-holes to artificially create supply shortages, thus driving up spot market prices to gain huge profits. The historical legacy of electricity market liberalization and current institutional inertia prevent practical implementation of effective measures to induce truthful behavior from market participants.

The flaws in current liberalized electricity markets serve as potential aspects of improvement in designing electricity markets of the future. On a local scale, the characteristics of distribution grid level stakeholders and RES create opportunities for innovative market designs, which prioritize a social welfare-oriented market framework rather than a profit-maximizing one.

1.1.2. Customer-Oriented Energy Markets

Market design is concerned with rules that guide the market and the institutions that enable transactions. Traditionally, the economic theory took market institutions as static elements and only described the operational aspects. Two developments in economics changed this [51]. Firstly, game theory is the study of the "rules of the game" and the strategic interactions that are evoked. A game-theoretic approach involves taking the point of view of the market participant and analyzing how their interactions shape the outcomes of the market design. Secondly, mechanism design deals with the science behind creating "rules of the game" in such a way that certain goals are achieved. Mechanism design flips the design process by stating the desired outcomes and implementing the market rules, which induce participants' interactions to align with the objective of the market. These developments led to the introduction of the market design field, where an iterative approach is adopted to improve the function of markets by iterating between theory and practice. In [9], Cramton identified important aspects of electricity market design, such as simplicity, incentives, and fairness. Besides, he also argues that good market design begins with a good understanding of the market participants, their incentives, and the economic problem that the market is trying to solve. Within the context of LEMs, the main motivation for end-users to enter these markets is the ability to optimize their consumption and/or generation capacities to benefit from cost savings and profits. From the system operator's standpoint, the economic benefits consist of improvements to the overall efficiency of grid operations, minimization of system operation cost, and reduction of Green House Gas emissions [5].

While it is still uncertain how the mix of technologies and regulations will shape the future landscape of the energy sector, the study of market design for applications in local energy trading has gained increasing attention over the last decade. Several studies have been conducted utilizing a game-theoretical approach to model local energy markets for various residential scenarios [29][50][47][64]. These studies show performance improvements and additional value captured in smart-grid settings otherwise impossible in traditional macro-grid settings. In [38], Mengelkamp et al. conducted an extensive literature review on the state-of-the-art research on LEMs and acknowledged that the current work is shifting their focus from conceptual design and implementation towards increasingly realistic and practical applications of electricity trading. Furthermore, the studies display a trend towards service-oriented designs rather than profit-maximizing ones. The introduction of innovative technologies such as blockchain allows for residential local energy projects such as LO3's Brooklyn Microgrid
to be integrated on top of the existing low-voltage distribution grid [37]. The privacy and security offered by blockchain technology enable users to safely conduct financial transactions in tandem with the physical flow of power in the distribution network. As demonstrated in [28], LEMs can also serve as residential peer-to-peer trading platforms, allowing for non-conventional business models such as user-subscription fees to exist. While results from studies show multifaceted benefits, there lacks a focus on the comparison and analysis of market rules (from now on referred to as market mechanisms) as well as the prevention of potential market manipulation from bad actors within residential LEM designs.

1.2. Research Objective and Research Questions

In the previous sections, we expressed the motivation for this thesis project. As local energy markets increasingly prove to be viable solutions in addressing the decentralization of energy generation and consumption within residential communities, the implementation of market mechanisms should also be carefully considered to avoid the shortcomings of current liberalized electricity markets. The objective of this research is to analyze and establish a suitable market mechanism for a residential local energy market. The main research question is stated as follows:

Which market mechanism is suitable for a residential local energy market which defers non-truthful behavior of market participants?

The goal of this research is three-fold. The first part contextualizes the underlying characteristics of a residential local energy market and its participants. This conceptualization is necessary to clarify stakeholder incentives and the economic problem that LEMs are trying to solve. The second part deals with economic theory, utilizing game theory and mechanism design to determine suitable market mechanisms that induce truthful behavior from market participants. The Enron scandal has shown that poorly regulated markets can result in manipulation, gaming, and other inefficient conducts. The last part of the research question aims to make a comparison between market mechanisms through a modeling approach in order to recommend the market mechanism most suitable in the residential LEM context.

In dividing the main research question into manageable parts, the following sub-questions (SQ) are further postulated:

- **SQ 1:** What are the main characteristics of LEMs and the stakeholders involved, and What type of market design best describes residential local energy markets?
- **SQ 2:** What are the key concepts of game theory and mechanism design, and can these elements of economic theory be used to induce truthful behavior in the design of a residential local energy market?
- **SQ 3:** What are the main factors that influence market mechanism performance, and how can we create a simulation environment that enables us to compare market mechanisms in the context of residential LEMs?

1.3. Scope of the Research

The primary focus of this research is the adaptation of existing auction mechanisms within the framework of a residential LEM. Auction mechanisms and their properties have been extensively analyzed in applications such as art auctions, commodity exchanges, online ad auctions, and spectrum auctions. While the thesis project involves extensive discussions on the topics of market design and mechanism design, it should be made clear that the goal of the research is not to introduce a novel design but rather to utilize mechanism design theory to justify the efficacy of existing auction mechanisms within a novel environment - the residential local energy market. Where previous analysis of the local energy market involves the comparison of entirely different market designs, the contribution of this research entails the specific comparison of truthful auction mechanisms, which have not been conducted in the current field of study.

Therefore, the scope of this research is mainly based on microeconomic theory, game theory and mechanism design. The physical layer of the electricity system is not considered due to two reasons. Firstly, the aim of this research resides in the early phases of a developing technology (i.e., smart-grids), and a certain level of abstraction is required to provide a general solution that could then be
later adapted for specific use cases. Discussed in further detail in the following chapter, LEMs are applicable in various formats: as a virtual layer on top of the existing distribution grid; as an intermediary market for aggregators to provide flexibility services and demand response to the system operator; or as the central coordination entity in facilitating energy flow within microgrids. Secondly, to combine the LEM with the physical layer means that the grid typology and the physical constraints attributed to that typology needs to be accounted for. This limits the available scope to the typology under examination and offers little room for exploration in the dynamics of the market mechanism itself. While the technical exclusivity from few connections to the superimposed grid offered by a low voltage feeder or microgrid allows a clear physical boundary of the LEM, this work takes the virtual market boundaries of the participants as the decision criteria. Lastly, the goal of this study is to conduct a comparison of market mechanisms rather than the optimization of a single case. Thus simplifications to the overall model are made to accentuate mechanism insights otherwise buried within the dynamics of highly detailed and complex models. The research objective is, therefore formulated as:

To compare and determine suitable truthful market mechanisms by incorporating realistic residential household electricity profiles as input for a LEM model.

1.4. Thesis Structure
As Illustrated in Figure 1.3, this thesis project is divided into three main parts.

Figure 1.3: Overview of Thesis Structure

In Chapter 1, a background on the context and motivation of the thesis project is given, and the main research objective presented. Chapter 2 seeks to contextualize the research objective through a description of the LEM environment and stakeholders’ characteristics, thus enabling us to set the groundwork for economic analysis. This first part serves to introduce and engage the audience into the importance of the proposed research.

The second part of the thesis aims to introduce fundamentals of micro-economic theory, shedding light on the strategic aspects of stakeholder incentives that govern their behavior in the market. In Chapters 3, an overview of game theory is provided, and the resulting strategic behavior is investigated from the perspective of the market participants. From the perspective of the market operator, understanding the strategic implications of agent behavior allows us to identify market mechanisms that steer the participants towards making rational decisions that achieve predefined market objectives. Thus Chapter 4 introduces a sub-field of game theory called mechanism design, which allows us to do just that. With an understanding of market mechanisms and their desired properties, Chapter 5 introduces compatible mechanisms that are aligned with the goals of this study, and we formulate the economic problem to be solved within the context of LEMs.
1. Introduction

The last part of the thesis aims to bridge the gap between theoretical analysis with experimental analysis as well as present the results that allow us to answer the main research question. Chapters 6 adapts the economic analysis into a model framework that could then be used to conduct simulations. This chapter explicitly defines the boundaries and constraints of the simulation by adhering to the scope of the research. Furthermore, key performance indicators (KPIs) are identified, such that a comparison between mechanism performance could be made. Chapter 7 then seeks to detail the experimental setup (i.e., input variables and parameters) and LEM configurations (i.e., microgrid scenarios). Chapter 8 contains the results of the model simulation, which will be analyzed. The implications for the main research question will be drawn in terms of a recommendation for the suitable truth-inducing mechanism. Finally, Chapter 9 takes a critical reflection of the results obtained as well as provides recommendations for future work.
Residential Local Energy Markets

The Dutch energy transition aims to advance the political goal of a sustainable electricity system based on incorporating a greater representation of electricity generated by renewable means in the overall national electricity consumption [41]. For this reason, the Dutch government has been encouraging the transition to renewable energy with various instruments, among these are taxes on the use of fossil energy, a system of guarantees of origin for renewable power generation, and network operator commitments to prioritize the supply of renewable power (regardless of network congestion). The current regulation for incentivizing residential consumers to invest in more renewable generation capacity is a net-metering scheme, which allows homeowners feeding surplus solar generation into the distribution grid to offset their energy generation with regard to their power consumption on an annual basis. Yet this decoupling of decentralized generation from the overall electricity market will lead to new challenges for the electricity system. For example, peak solar irradiation hours will create massive feed-in from the surplus of household PV systems. This development will lead to either grid congestion or, in the worst case, direct curtailment of RES. Thus as the fraction of the residential population owning PV systems increase, the potential impact on the electricity system will become noticeable on a national scale. Yet these residential households are currently still passive price-takers [8]. These households are excluded from actively taking part in the electricity market due to their size, institutional, and regulatory restrictions. LEMs offer the opportunity to address these challenges. LEMs include the residential prosumers and consumers directly into the electricity market, provide the chance for small-scale, very efficient local markets, and increase the social acceptance by direct participation and involvement. This chapter first provides an in-depth overview of local energy markets, their characteristics, and the stakeholders involved. The focus is then directed to defining a LEM design and simplifying the market framework to accentuate the market mechanisms that facilitate economic interactions among market participants.

2.1. LEM Fundamentals

As LEMs are a relatively new concept, multiple definitions exist in academic literature. In [39], Menegelkamp consolidated existing definitions observed from literature reviews to a single holistic definition of LEMs:

A local electricity market is a market platform for trading locally generated (renewable) electricity among residential agents within a geographically and socially close community. Security of supply is ensured through connections to a superimposed electricity system (e. g. national grid or adjacent local electricity markets).

It is important to make the distinction between LEMs and the entities that LEMs are commonly integrated in. These separate entities reside within the future local distribution ecosystem; they consist of smart-grids, microgrids, peer-to-peer trading, energy sharing, and energy communities. LEMs should be considered as a virtual marketplace, independent from the actual physical implementation of power flows. While grid constraints ideally would be included in LEMs, a solely virtual LEM should also be possible. While LEMs induce local electrical balances, the autocracy of LEM implementation within
a closed system should never be the aim because it would require a tremendous amount of excess capacity at high costs to ensure security of supply. Rather, the security of supply within the LEM is ensured by the superimposition of the external macro-grid, which allows for unlimited purchase and feed-in of guaranteed electricity at retail rates.

2.1.1. LEM Structure and Stakeholders

In its most basic form, a LEM consists of sellers, buyers, a utility (or a DSO) and a platform provider. Sellers are typically small-scale producers who own generators or prosumers who are households with PV installations, while buyers are pure electricity consumers with no means of generating their electricity. The utility or DSO in charge of the superimposed grid is responsible for balancing power flow and managing congestion. In essence, the objective of LEMs is to clear the market based on predetermined rules, and this responsibility lies with the platform provider. Yet, this role could be taken over by other stakeholders (i.e., utility) depending on the regulatory framework in which the LEM resides. From a technical standpoint, a LEM could be built upon a microgrid, a low voltage feeder, or part of the distribution grid.

Additional stakeholders may include Energy Service Companies (ESCOs), aggregators, and balance responsible parties (BRPs) to provide a wider range of electricity products and services residential households are accustomed to [39]. An overview of stakeholder definitions and their objectives is provided in Table 2.1 below.

<table>
<thead>
<tr>
<th>Stakeholder(s)</th>
<th>Description</th>
<th>Objectives</th>
<th>References(s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Consumers</td>
<td>Electricity end consumers, often residential households</td>
<td>Reduction of electricity costs, increase of share of local/green RES</td>
<td>[30]</td>
</tr>
<tr>
<td>Producers</td>
<td>(Small-scale) electricity producers, e.g., a Combined Heat-and-Power (CHP) plant</td>
<td>Increase of profits, supply local community</td>
<td>[32]</td>
</tr>
<tr>
<td>Prosumer</td>
<td>(Small-scale) residential consumer, who also produces electricity</td>
<td>Reduction of costs, increase of profits, increase of autonomy, increase of share of local/green RES</td>
<td>[46]</td>
</tr>
<tr>
<td>Platform Provider</td>
<td>Provision of a virtual platform for trading on a LEM</td>
<td>Provide efficient market (e.g. high liquidity), maximization of profits</td>
<td>[7]</td>
</tr>
<tr>
<td>Utilities</td>
<td>Supply electricity customers, control feed-in of prosumers and producers, can access the LEM and the wholesale market</td>
<td>Maximization of profits, optimization of portfolio, identification of new business models, increase of customer satisfaction</td>
<td>[30]</td>
</tr>
<tr>
<td>DSO</td>
<td>Operates, maintains and controls the distribution network</td>
<td>Reduction of overall cost and congestion, maintenance of supply security, increase of network efficiency</td>
<td>[45][32]</td>
</tr>
<tr>
<td>ESCOs</td>
<td>Offer various services to local actors, e.g. electricity source portfolio optimization</td>
<td>Maximization of profits, increase of customer satisfaction</td>
<td>[30]</td>
</tr>
<tr>
<td>Aggregator</td>
<td>Represents several local actors, may provide additional services (e.g. portfolio optimization)</td>
<td>Maximization of aggregated value of representatives, optimization of own portfolio, maximization of profits</td>
<td>[30][32]</td>
</tr>
<tr>
<td>BRP</td>
<td>Responsible for balancing the forecasted and actual schedule of a balance group (e.g. a LEM)</td>
<td>Provide electricity balance at minimal cost, optimize balance group portfolio</td>
<td>[30][61]</td>
</tr>
</tbody>
</table>

Table 2.1: LEM stakeholders and their objectives
2.1.2. Potential Value Offered by LEMs
We highlight the most prominent benefits that LEMs contribute to the future electricity system. These can be divided between benefits to the end-consumer and potential value-added for the network operator.

Customer Level:
LEMs enable prosumers to make optimal use of local distributed energy resources such as energy storage and particularly RES. This provides them with a higher level of energy independence and control when compared to the traditional electricity “price-taker consumers”, which are essential motivators for their engagement in LEMs[59]. More importantly, LEMs enable prosumers to trade their energy generation surplus within the boundaries of their local communities, thus strengthening the customer’s position in the energy market/system from a passive to an active role. Expanding the broader energy market to the local level is a significant sectoral change that will drive disruption and innovation, but consequently also competition. Competition motivates companies to develop further their services and products in the interest of customers, in terms of both variety and price. Furthermore, the continued development of LEMs will increase pressure over traditional power industry players to adapt their operations towards more customer-oriented approaches. LEMs will connect communities and drive their participant customers into achieving common goals, such as reducing costs of energy, emitting less greenhouse gas emissions, or becoming more energy self-sufficient[30]. This involvement and group-like behavior, alongside a sense of mission, contribute to a growing perception of transparency and trust in the energy system as a whole, which results in more engagement and commitment.

Network Operators Level:
Customer-owned DER generation present in the distribution level will impact the everyday operations of distribution and transmission networks in various ways. It can reduce the network operators’ need to make new investments and reinforcements in the distribution grid, not only because of DER capacity additions but also due to increased flexibility and more efficient overall network operations. These capacity additions at the grid edge also decrease stress in the distribution, due to a decrease in power demand. Network losses reduce as well since there is less load at both the transmission and distribution levels. In general, the development of LEMs opens up new market opportunities driven by new business models geared towards value creation from the network operators’ point of view. Different forms of value-added and real-time services can be provided, for example, real-time energy monitoring and billing, local balancing of supply and demand, and microgrid real-time energy management.

2.1.3. State of the Art
Due to the nature of electricity being a highly customizable product exposed to various technical and commitment constraints, there are numerous factors and possibilities to design, run, and implement LEMs. The developments in the information and communication technologies and the introduction of the blockchain increased the number of research discussing the applications of energy markets at the microgrid and distribution level [54][56][21][10][27]. Using the blockchain technology, Mengelkamp et al. [37] discussed the required components for designing an energy market for the microgrids. Kang et al. [26] presented an electricity trading market for electric vehicles (EVs) using consortium blockchain. Noor et al. [44] applied game-theoretic approaches and blockchain to enable transactions between individuals in the microgrid. Guerrero et al. [15] compared the centralized against the distributed trading approaches in the low voltage network under different optimization goals to demonstrate the importance of local energy markets. Ampatzis et al. [2] derived design choices for a residential LEM consisted of residential customers with PV generation, energy storage, and inelastic demand and derived design choices to realize market-based control for the coordination of residential RES. Bahrami and Amini [4] developed a decentralized energy trading algorithm, where uncertainties of generation were considered. Khorasany et al. [28] presented an hour-ahead energy market, where a market subscription charge was used as a price signal. In Zhou et al. [68], multiple energy sharing mechanisms based on a multi-agent framework were evaluated. The authors discussed the economic and technical benefits of the presented models for residential prosumers. Hwang et al. [23] proposed a transaction model as a service for the prosumers. The authors focused on increasing the energy efficiency of the system while maintaining and secure and transparent transactions.
The literature mentioned only highlights a small portion of the on-going research on LEM applications and implications it has for the future electricity system. For a comprehensive overview of current developments in LEM research, the interested reader is referred to Mengelkamp’s work in [38].

### 2.1.4. LEM Market Design Considerations

In the previous subsection, the scenarios presented in the literature were limited, given the various possible configurations of market design, microgrid typology, and prosumer makeup. Yet the behavioral economics that governs how market participants make decisions is not confined by physical constraints of the LEM, but rather based on the market structure. Agent behavior represents the objectives and preferences of individual market participants in a LEM. The scope of the research presumes agents can actively take part in the LEM by placing bids, which include their desired price and quantity, and the objective of this research is to induce agents to behave in such a way that they reveal their true preferences. The market structure determines the actual matching of bids and financial transactions [39] and consists of market rules. The combination of agent behavior and their interactions within the market structure determines the market outcome of a LEM. Some examples of objectives for market outcomes may be the increase of supply security or efficient integration of RES. To align the market objective with the main research question, the LEM outcome should be focused on efficient allocation of local energy generation and consumption as well as maximization of profits for market participants.

In defining agent behavior and market outcome objectives, the design of the LEM structure should be carefully considered such that the aforementioned objectives are met. The market structure consists of the market rules and market mechanisms that enable economic interactions among market participants.

### 2.2. Contextualizing LEM Design for Research Objective

In this section, we specify the attributes necessary for a complete market design based on the taxonomy of market models from [62] and market framework suggested by [48]. It is important to begin by clarifying the fact that LEMs are, in essence, double auction markets [38], which is in line with the auction market format of wholesale electricity markets. This is attributed to the characteristics of agent behavior - their ability to conduct trade of a commodity freely both as a seller or as a buyer on an online platform. In terms of stakeholders involved, the scope of this thesis investigates the interactions between residential consumers and prosumers who own PV systems, while the platform provider - from now on referred to as the auctioneer - aims to achieve the predefined market outcome and induce agent behavior in line with the objectives mentioned in the previous section.

DERs which provide flexibility and loads that offer demand response is considered to be beyond the scope of this study. While electrical energy storage serves as the most effective flexibility solution for redistributing the intermittency of solar generation, this adds an unnecessary layer to the optimization problem. From the auctioneer’s point of view, it still categorizes market participants as either buyers or sellers. The optimization of energy storage capacity during the bid formation process remains as the agent’s responsibility and is not required in the determination of market mechanisms. On the other hand, the demand of consumers is inelastic. While research on smart grids usually assumes elastic demand to realize demand response schemes, currently flexible appliances have a negligible presence in households. With these stakeholder attributes clarified, their individual preferences are stated in terms of optimization objectives:

- **Consumer**: Minimize its cost of consumed energy with the upper-bound constraint of satisfying its expected demand.
- **Prosumer**: Minimize its cost of consumed energy if PV generation is not enough for self-consumption. If there is a surplus in generation even after self consumption, maximize its profit of the energy traded at the LEM.
- **LEM Operator/Auctioneer**: Maximize total market value at each round of the auction instance.

### 2.2.1. LEM Double Auction Attributes

The contextualization of LEM attributes is the first step towards local efficient electricity market design. In [62], a taxonomy of electricity markets is presented according to such attributes. The degree of
2.2. Contextualizing LEM Design for Research Objective

competition, trading horizon, and dispatch intervals should be taken into account in the market design since they greatly affect the optimization problem for the LEM stakeholders. The next step is to describe the double auction market structure by determining the bidding restrictions, bid acceptance, and price determination. The bidding restrictions describe the double auction format, which includes the trading horizon, trading time intervals, and composition of the submitted bid regarding energy quantity and price. In combination, bid acceptance and price determination describe the market mechanism implemented for the LEM. The determination of suitable market mechanisms within the context of the residential LEM is the focal point of this study and will be elaborated in detail in Chapters 4 and 5.

Degree of Competition and Market Participant Behavior

Economics dictate that market efficiency can be reached through competition when the market reaches a competitive equilibrium \[57\]. The exercise of market power by participants in the market results in the market outcome moving away from the competitive equilibrium, thus creating inefficiencies within the LEM. Buyers and sellers exercise market power in a double-auction market by misreporting their marginal costs, their true willingness to pay, or their true volumes. Non-truthful behavior occurs when market signals provide enough incentive for market agents to increase their profit by manipulation of the competitive market equilibrium. In the context of LEMs, the auctioneer has no control of what participants report or if they report truthfully. In a competitive market environment, it is in every participant’s self-interest to maximize profits and thus providing them a strong incentive to lie. Market mechanisms which do not induce truthful behavior will be susceptible to gaming by rational agents with market power. When this happens, the LEM ceases to operate due to over/underbidding, or in worse cases, the market collapses due to mistrust of exploitation between participants. While, at first glance, no individual residential consumer or prosumer has significantly more capacity to enable it to exercise market power. The potential introduction of a stakeholder with higher capacity than the others, i.e., school or supermarket, is a threat to the degree of competition in the local community. Thus, inducing truthful behavior is an essential attribute of the LEM double auction.

Bidding Restrictions and the Double Auction Format

A discrete sealed-bid double-sided auction is chosen to provide a fair trading environment and maintain the prosumer’s privacy. The market is chosen to be discrete-timely to synchronize all traders’ communication with the market trading platform and provide a fair environment to all traders where communication speed does not play a role. In a discrete-timely auction, the market is cleared at predefined time intervals. However, in a continuous-timely auction, the market is cleared as soon as a matching bid is available. Thus, faster traders can have an advantage in a continuous-timely auction, which can lead to an inherently flawed auction. The trader’s speed is not only a function of the decision making speed or even the available computational power but also the communication infrastructure. Given the real-life situation in microgrids, it is practically hard to guarantee a synchronized reaction a using continuous-timely auction. Hence, discrete-timely trading is favored in this situation to maintain a fair environment for all market participants. Additionally, continuous auction formats have the disadvantage of being potentially less efficient than discrete auctions. The reason for this loss in efficiency is easy to spot. The continuous clearing rule results in myopic matching; when the clearing operation is performed the auctioneer has only a partial view of the aggregate supply and demand in the LEM. This results in the auctioneer impatiently clearing the market before every participant has the opportunity to place their bid.

One may notice that so far, there has been no mention in the LEM context regarding the settlement rules, which refers to the way deviations from the contracts real-time are handled. While volatility and unpredictability of household demand and PV generation serve as the primary motivation for implementing the LEM with a small trading horizon, the scope of this thesis considers market participants’ bids as deterministic due to the absence of physical constraints. This means that their bid volumes describing their expected generation or consumption is identical to their actual generation or consumption, without any uncertainty. The same argument is valid for the duration of the dispatch intervals regarding PV generation.

Even under the pretense of imbalances caused by the volatility of generation and consumption, the sealed-bid format still makes sense because the households have the ability to constantly update their bids as the uncertainty towards energy generation and consumption becomes smaller the closer it gets to dispatch. Thus if the accuracy of the bids reflects the true generation and consumption in
the future, then the question of truthfully reporting one’s price and quantities should be independent of participants’ gains-from-trade.

2.2.2. Proposed LEM Design
To summarize, the market design proposed for this study is based on a sealed-bid double-sided auction with private information and discrete auction instances. The format of the bids only includes a quantity and price for one dispatch interval, and individual market participants are allowed only one bid per auction instance. The market participants have the ability to improve their private positions of their reported bids up until the end of the auction interval. Once the auction closes, the auctioneer collects all reported bids and execute trades wherever sellers bid prices are lower than buyers bid prices, with the aim to maximize total market value. The quantities of the unmatched bids are served via the superimposed macro-grid.

2.3. Summary
In this chapter, a holistic definition of LEM was first provided, followed by an overview of the potential stakeholders that could extract value from LEM implementation. The current research in LEMs is briefly described to highlight the diversity of applications LEMs can potentially reside in. Finally, a double-auction market design is proposed to serve as a framework for the determination of suitable market mechanisms that induce truthful behavior of market participants. The implementation of a truthful market mechanism begins with taking a game-theoretic approach to the economic interactions between the market participants and aligning it with the auctioneer’s optimization objective. This will be further discussed in the following chapter.
3

Game Theory

Game theory is a branch in mathematics that deals with the analysis of games, where the outcome of one player does not depend on his or her own strategy but also on the strategies of other participants. The first half of the chapter will introduce the topic of game theory, providing definitions and terminologies. The remaining sections will classify residential local energy markets as a game.

The applications of game theory wide and far-reaching. Whether you are considering your next move when playing chess, driving through traffic to reach work on time, or negotiating on a deal to earn a profit, the interactions and decisions made by other participants operating in the same environment will have an effect on your outcome. Thus, finding the optimal action, or strategy, to take in these situations becomes less straightforward, because the optimality of your action depends on the optimality of others’ actions. As this holds for all players involved, a circularity threatens. What game theory is trying to accomplish is to introduce solution concepts that describe the optimal strategy each player should take. Its development shows that mathematical reasoning could be applied to studying complex human interactions. In our modernized world, digitization and the internet of things create a growing need for optimizing, not just human-to-human interactions, but also human-to-machine and machine-to-machine interactions. Thus, the emergence of interfacing game theory with engineering sciences such as network and computer science catapulted game theory to the center-stage of problem solving in modern times. In regards to the scope of this thesis project, the notable advancements in developing game theory as a science are the following: equilibrium analysis of games, auction theory, and mechanism design theory. But before expanding on the different branches of game theory, it is essential to introduce the main concepts and terminologies that will be discussed within this thesis.

3.1. Concepts and Terminologies
The term game in game theory corresponds to an interaction involving decision-makers or players who are rational and intelligent. The rationality of players implies that the players choose strategies to maximize a well defined individualistic payoff. Intelligent simply means that players are capable of calculating their best strategies. In this sense, game theory is a tool for logical analysis that models conflict as well as cooperation between the players and provides a principled way of predicting the result of the interactions. All games consist of the following elements:

- **Players**
  Collection of strategic decision-makers within the context of the game.

- **Actions**
  The moves available to each agent throughout the entirety of the game.

- **Information**
  The information available to each agent at a given point in the game.

- **Payoff**
  The payoff which a player receives from arriving at a particular outcome by choosing a certain set of actions. Often the payoff is measured by a utility function.
The rules of the game determine how the elements of the game will be defined.

### 3.1.1. Representational forms of games

There are two main representations of games, namely *normal form games* (also called *strategic form games*) and *extensive form games*. The main difference between these two representations is how the strategies play out in time.

*Extensive form games* describe games in the form of a game tree, which is a diagram that shows which strategies players choose at different points in time. Illustrated in Figure 3.1a, the extensive form is mostly used to represent sequential games where a series of decisions are made, such as chess.

This thesis will mainly focus on *normal form games*, which are often used to describe simultaneous move games. Here the players make simultaneous decisions in a static setting where the outcome is not dependent on the timing of an action. All possible outcomes from all combinations of players’ strategies can be described in matrices. One example of a normal form game is rock-paper-scissors, as illustrated in Figure 3.1.

Here it is important to differentiate between the terms *action* and *strategy*. An *action* is a move available to the player when called upon to make a decision at any point within the game. A *strategy* consists of the complete set of actions the player makes in every stage of the game in response to other players’ actions. In looser terms, a strategy could be viewed as a complete contingent plan. The difference becomes clear when attempting to describe a sequential game in normal form. The normal form is used to represent strategies (not action) in a game. To appreciate the difference between strategies and action, it’s probably best to first consider a game in extensive form.

In Figure 3.1a, suppose two players move sequentially, and that Player 2 observes Player 1’s choice before making his decision. Here we can call \{A,B\} Player 2’s actions (actually the correct jargon here is behavior strategies, but this level of nuance is unnecessary at this point). However, they are not his strategies, for strategies in this case is a "full contingent plan" that specifies an action at each information set the player moves. Since Player 2 moves at two information sets (one after L and the other after R), so one of his strategies would be “choose A if Player 1 chooses L and choose A if Player 1 chooses R”, or simply denoted as AA.

To represent this game using the normal/strategic form, we would arrive at the matrix presented in Figure 3.1b. It’s not a 2-by-2 game, but 2-by-4, precisely because strategies and actions are not the same.

Normal form is a complete representation of every possible combination of actions available to the player, which is described by a strategy set. Each combination in a player’s set of possible strategies corresponds to an outcome of the game. The set of all possible outcomes for a player is referred to as the player’s *payoff function* or *utility function*. From [43], the definition of a normal form game follows:

**Definition 3.1.1. (Normal Form Game)** A normal form game Γ is a tuple \( \langle N, (S_i)_{i \in N}, (u_i)_{i \in N} \rangle \), where
• \( N = \{1, 2, ..., n\} \) is a set of players;
• \( S_1, S_2, ..., S_n \) are sets called the strategy sets of the players 1, ..., n, respectively; and
• \( u_i : S_1 \times S_2 \times \cdots \times S_n \to \mathbb{R} \) for \( i = 1, 2, ..., n \) are mappings called the utility functions or payoff functions.

### 3.1.2. Classification of Games

A classification of well-known games and their counterparts are introduced to define additional concepts. Depending on the game’s environment and rules, multiple classifications mentioned below can be applied to the same game.

**Non-cooperative Games and Cooperative Games:**
Non-cooperative games are also called competitive games. In these games, the agent is the basic modeling unit while in cooperative games, the group of agents is the basic modeling unit. Commitments made among agents are enforceable in cooperative games, while in non-cooperative games it is not the case.

**Pure Strategy Games and Mixed Strategy Games:**
A pure strategy game provides a complete description of what actions each player will take during a game. In specific, it defines the action a player will take for any situation he could face. A mixed strategy game includes the assignment of a probability to each pure strategy. This allows for a player to randomly select a pure strategy. As probabilities are continuous, infinitely many mixed strategies are available to players. Mixed strategies are often used to model stochastic behavior while pure strategies describe a set of predetermined behavior.

**Games with Perfect Information and Games with Imperfect Information:**
A perfect information game is where all agents are fully informed about the entire past history of the game before taking an action. Thus in a perfect information game, the information set known to each agent is the complete set of moves all agents made in all previous rounds played in the game. Any game that does not have this property is treated as an imperfect information game. These types of games typically apply to sequential move games and are described in extensive form. Simultaneous move games are all treated as games with imperfect information.

**Games with Complete Information and Games with Incomplete Information:**
In complete information games, every aspect of the game is common knowledge. This includes all agents’ utility functions, strategy sets, payoff functions ...etc. Incomplete information games are ones which at the start of the game, some agents hold private information about the game that other players do not know.

### 3.1.3. Equilibria in Game Theory

With the concepts of strategy sets and utility functions in place, it is clear that a player’s utility of an outcome depends not only on his or her own strategy, but also on the strategies of the other players. Some strategies yield an outcome with higher utility than others and thus each player has a preference for certain strategies over others. The private information held by an individual relating to preferences of that individual is commonly referred to as type. The key assumption in game theory is that players are rational in that they will consistently pursue decisions that maximize the expected value of their utility. Thus, a player’s preferred outcome could coincide with other players’ preferred outcomes and when rational decision making interacts, the decision problems have to be analyzed together like a system of simultaneous equations. Game theory provides a useful framework in applying solution concepts, or equilibria, within these systems of equations. Equilibria is a central notion in game theory because this is effectively the goal for every game theorist: to reliably predict a stable outcome based on rational thinking.

Equilibria in game theory is closely associated with Pareto optimality. Pareto optimality for a group of individuals describes a situation where one cannot make an individual’s outcome better off without making another worse off. When talking about equilibria in game theory, it is referring to the equilibrium
strategy of each player rather than the outcome. A best response strategy is the preferred strategy a player chooses to most effectively counter strategies adopted by others. Given a normal form game, the best response strategy of a player is defined as follows [43]:

**Definition 3.1.2. (Best Response Strategy)** Given a normal form game $\Gamma = (N, (S_i), (u_i))$ and a strategy profile $s_{-i} \in S_{-i}$, we say $s_i \in S_i$ is a best response strategy of player $i$ with respect to $s_{-i}$ if $u_i(s_i, s_{-i}) \geq u_i(s_i', s_{-i}) \forall s_i' \in S_i$.

Based on the assumption of rationality, players will always prefer and not unilaterally deviate from their best response strategy. Interestingly, there could exist multiple best response strategies within a player’s strategy set if the strategies yield identical payoffs. In a game where all players result in choosing their best response strategy, an equilibrium solution for the game has been reached. Let us illustrate this with the well-known Prisoners’ dilemma game.

**Prisoners’ Dilemma:**
Two prisoners go on trial for a crime and each one is given the choice of confessing to the crime or to remain silent. Their payoffs are rewarded by taking into account both their choices. If both prisoners remain silent, then they will both serve a short term of 2 years. If only one of them confesses, his term is reduced to 1 year and he will be used as a witness against the other prisoner, who in turn gets a sentence of 5 years. Finally if they both confess, both will each get sentenced to 4 years due to cooperating with the authorities. We can summarize the outcomes of the game in a cost matrix as illustrated in Figure 3.2.

**Figure 3.2: Prisoner’s Dilemma Game Matrix**

<table>
<thead>
<tr>
<th></th>
<th>P2</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>P1</strong></td>
<td>Confess</td>
</tr>
<tr>
<td>Confess</td>
<td>4</td>
</tr>
<tr>
<td></td>
<td>4</td>
</tr>
<tr>
<td>Silent</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>5</td>
</tr>
</tbody>
</table>

Each prisoner has two strategies, to confess or not to confess. The strategies of Prisoner 1 correspond to the rows, and the strategies of Prisoner 2 correspond to the columns of the matrix. The entries of the matrix are outcomes for the prisoners if they followed the outcome’s corresponding strategies. In analyzing what Prisoner 2’s best response strategy is, we first look at Prisoner 1’s available strategies and compare Prisoner 2’s options when Prisoner 1 picks a certain strategy. If Prisoner 1 decides to confess, Prisoner 2’s best response strategy should be to confess, because that will only result in him getting sentenced 4 years instead of 5. In the case where Prisoner 1 chooses not to confess, Prisoner 2’s best response strategy is also to confess. Intuitively in this game, confessing is also the best response strategy for Prisoner 1 no matter what Prisoner 2 does. Thus the game has only one stable solution, which is where both prisoners confess. In all other three cases, at least one prisoner can switch from “No Confess” to “Confess” to improve his own payoff.

The stable solution highlighted in the Prisoners’ Dilemma is referred to as the infamous Nash equilibrium, named after mathematician and economic Nobel Laureate John Nash. The location of the Nash
equilibrium is at a strategy coordinate (a strategy, or set of strategies for each player) in which any unilateral change in strategy of each of the participants, will not result in an incremental improvement to either of the players. This solution concept only insists that each agent’s strategy offers a best response against the Nash equilibrium strategies of the other players. It is important to note that the notion of stability is dependent on the fact that each prisoner is knowledgeable of his counterpart’s available strategies and payoffs. If the prisoners did not have complete information of either his counterpart’s available strategies or respective payoffs, then they could not determine a best response strategy, or any contingency plan for that matter. Thus, the Nash equilibrium is simple yet strict in defining a solution concept, demanding that all information is common knowledge.

While complete information games provide a convenient and useful abstraction for strategic situations, it is often inapplicable in real-world situations because agents are more likely to possess private information unknown to their competitors. In a residential local energy market (LEM), enforcing complete information is intuitively impossible to achieve. Among the aspects of the energy market that might not be public information are:

- Cost and utility functions of individual agents
- Possible actions of participating agents
- Identities of other agents

Fortunately in game theory, various types of solution concepts exist that do not necessarily require an agent to have complete information to determine its best response strategy. Carefully selecting the type of equilibrium model is therefore essential as these differ greatly in their underlying assumptions of market behavior and therefore on the eventual equilibrium solution. We first characterize residential local energy markets as a game in order to determine a suitable equilibrium concept.

### 3.2. Characterization of Local Energy Markets as a Game

A residential LEM can be seen as a non-cooperative game played between residents within a community looking to either buy or sell up to a certain amount of energy. Every household is self-interested with a goal of gaining maximum profits for selling or maximum utility for purchasing. As mentioned in the previous section, each household holds some private information (such as its identity and renewable generation costs) and thus have incomplete information about the game environment. Furthermore, households have imperfect information about the market because their own historical energy usage is considered private information and unknown to other households. During each trading period, each household’s action is to submit a single sealed-bid \( X(P, Q) \) to the market operator. The bid consists of a reported valuation \( P \) and quantity \( Q \). Thus households’ strategies cover every possible bid combination of a price and quantity pair. The strategy set is constrained by bid combinations which ensure that the outcome will result in non negative profits (if its a seller) or utilities (if its a buyer). The bids themselves are considered pure strategies due to the fact that they are deterministic upon being chosen. To summarize, a game-theoretical description of residential LEMs is presented below.

#### 3.2.1. LEM Game Theoretical Formulation

The residential LEM game consists of a set of \( N \) sellers \( i = 1, 2, \ldots, n \) and a set of \( M \) buyers \( k = 1, 2, \ldots, m \).

For sellers, the \( i \)'th seller has a set of actions \( X_i(P, Q) = \{ P_i, Q_i \} \), s.t. \( P_i \geq 0, 0 \leq Q_i \leq E_{gen} \), where \( E_{gen} \) is the expected energy generation of seller \( i \). Here \( j_i = 1, 2, \ldots, J \) describes the individual actions available to seller \( i \). \( T_i \) is a set of seller types, where \( \forall t_i \in T_i \) is the private information of the \( i \)'th seller. A utility function \( u_i : T_i \times X_1 \times X_2 \times \ldots, X_n \rightarrow \mathbb{R} \) where \( u_i(t_i, x_1, x_2, \ldots, x_n) \) is the utility achieved by seller \( i \) if his private value type is \( t_i \) while actions taken by all sellers is \( x_1, \ldots, x_n \). The strategy profile of the \( i \)'th seller can thus be described as \( s_i : T_i \rightarrow X_i \) and seller \( i \)'s goal is to max \( s_i : T_i \rightarrow X_i(P_i, Q_i) \).

Similarly, the \( k \)'th buyer has a set of actions \( Y_k(P, Q) = \{ P_k, Q_k \} \), s.t. \( P_k \geq 0, 0 \leq Q_k \leq E_{con} \), where \( E_{con} \) is the expected energy consumption of buyer \( k \). \( L_k \) is a set of buyer types with individual buyer types denoted as \( \forall t_k \in T_k \). The utility function \( u_k : T_k \times Y_1 \times Y_2 \times \ldots, Y_m \rightarrow \mathbb{R} \) where \( u_k(t_k, y_1, y_2, \ldots, y_m) \) is the utility achieved by buyer \( k \) if his private value type is \( t_k \) while actions taken by all buyers is \( y_1, y_2, \ldots, y_m \). We define the utility function as the savings buyers experience in relation to their own willingness to buy and thus maximizing savings
is the desirable strategy. The strategy profile of the $k$’th seller can thus be described as $s_k : T_k \rightarrow Y_j$ and buyer $k$’s goal is to max $s_k : T_k \rightarrow Y_j(P^j_k, Q^j_k)$.

Seller $i$ must choose an action $x_i$ when knowing its private information $t_i$. The term $t_{-i}$ indicates the private information of all agents other than $i$. Note that $t_{-i}$ does not affect the seller’s utility directly but rather how the other agents behave, which can be denoted as $x_{-i}$. This in turn ultimately affects seller $i$’s utility $u_i(t_i, x_i, x_{-i})$. The total behavior of seller $i$ in such a setting is captured by the seller’s strategy $s_i(t_i) = x_i$, which specifies which action $x_i$ is taken for every possible type $t_i$. It is these strategies $s_1, ..., s_n$ that we want to be in equilibrium.

The Nash equilibrium solution to the LEM game requires that each agent’s strategy $s_i(t_i)$ is the best response to $s_{-i}(t_{-i})$ for every possible value of $t_{-i}$. Specifically, we have for all $i$, all $t_1, ..., t_n$ and all alternative actions $s_i(t_{-i})$ (or denoted as $x'_i$) available to $i$, we have that $u_i(t_i, s_i(t_i), s_{-i}(t_{-i})) \leq u_i(t_i, x'_i, s_{-i}(t_{-i}))$. The notion of complete information is reflected by the term $s_{-i}(t_{-i})$, which means that agent $i$ is knowledgeable of the other agents’ strategies $s_{-i}$ as functions. This requirement does not hold for residential local energy markets: notice how in the utility function $(u_i(t_i, x_i, x_{-i})$ agent $i$ considers all possible actions $x_{-i}$ of others’ but does not know anything about $s_{-i}$. How can we model strategic behavior of the agents and reach an equilibrium when the agents may not have enough information to determine their best response strategies?

### 3.2.2. Dominant Strategy Equilibria

Sometimes, an agent’s best response strategy remains the same regardless of the strategies or actions chosen by the other agents. This is known as the **dominant strategy** for that agent and is defined as the following [43]:

**Definition 3.2.1. (Dominant Strategy)** A strategy $s'_i \in S_i$ is said to be a dominant strategy for player $i$ if it dominates every other strategy $s_i \in S_i$. That is, $\forall s_i \neq s'_i$, $u_i(s'_i, s_{-i}) > u_i(s_i, s_{-i})s_i \in S_i$

**Definition 3.2.2. (Dominant Strategy Equilibrium)** A strategy profile $(s'_1, ..., s'_n)$ is called a dominant strategy equilibrium of the game $\Gamma = (N, (S_i), (u_i))\forall i = 1, 2, ..., n$, the strategy $s'_i$ is a strongly dominant strategy for player $i$.

The independence of agent $i$’s best response strategy from strategies of others’ makes the notion of dominant strategies applicable in incomplete information environments as well. The notion of dominant strategy in incomplete information games requires that $s^* i$ be true for all possible actions $x$ for each agent, without it knowing anything about $t_{-i}$ or $s_{-i}$. From a game theoretical stand-point, this solution concept may seem too good to be true: how likely is it that an agent has a single best response to all $s_{-i}$? Indeed in usual cases one does not expect games with incomplete information to dominant strategy equilibria. However, in the context of mechanism design, where we get to design the rules of the game, we can sometimes ensure that they do exist. The notion of dominant strategy equilibria is central to the scope of this thesis project and is a highly desirable property sought in mechanism design. These concepts will be discussed in detail in the following chapter.

### 3.3. Summary

This chapter served as a primer on the field of game theory. Agent decisions are governed by their rational behavior in striving to maximize their own utilities. On this basis, game theory is the study of player interactions and the subsequent payoffs within the framework of a game. In a commodity exchange market such the LEM, the decisions and actions that market participants make naturally have a profound effect on their own as well as other participants’ utilities. Thus creating a contingency plan for all possible combinations of actions taken within the game space allows the participant to determine the decision it should make to receive the highest payoff. In game theory, the contingency plan of a market participant is referred to as its strategy set, and the decision that provides the participant with the most utility when compared to every other strategy within its strategy set is called the best response strategy. When each agent ends up playing their own best response strategy, an equilibrium solution of the game emerges. But most real-world problems are not so transparent in the information available to all participants, and each individual will hold some private information which makes determination of one’s best response strategy very difficult. In Section 3.2, we have characterized LEMs incomplete
and imperfect information games. Even after we have identified that the dominant strategy equilibrium solution concept induces participants to always reveal their private information no matter their strategy set, a game theoretical approach is not sufficient enough to ensure that an equilibrium solution will be formed. In Chapter 8, the LEM will be taken on from the perspective of mechanism design, which is the micro-economic theory behind engineering the rules of an auction game.
Mechanism Design

Mechanism design is a sub-field of game theory that can be loosely viewed as the reverse engineering of games. A mechanism is a game, in which each agent is required to choose an action among a set of possible actions. A social designer then determines an outcome based on the chosen actions. The main goal of mechanism design is to create a set of protocols that satisfy socially desired objectives. It is best to view the goals of the designed mechanisms in terms of social choice. A social choice is simply an aggregation of the agents’ preferences (list of agent strategies). While game theory is concerned with the individual strategies of the agents, mechanism design attempts to implement desired social choices in a strategic setting - assuming that the agents each act rationally and intelligently in a game theoretic sense. In other words, a mechanism defines the strategies available to agents and the final outcome based on the agents’ chosen strategies.

The essential technique that mechanism design uses is to induce a game among the agents in a way that in an equilibrium state of the induced game, the desired system-wide solution is implemented. However, an important aspect of mechanism design is that the mechanism designer only knows the agents’ announced preferences (chosen strategy) but not their actual preferences. In auction markets, an agent’s actual preferences is referred to as its private valuation (or truthful valuation), and announced preference as its reported valuation. When an agent’s private valuation is identical to its reported valuation, the agent is described as being truthful. In the design of energy markets, this alignment of the agent’s self-interest and the system operator’s objective for creating the market has profound effects on market performance. From the market designer’s perspective, only by knowing the agents’ actual preferences can it solve for an optimal social choice. Because markets are essentially zero-sum games, anything other than the truth will reduce market efficiency - not every agent is trading at his/her ideal state and thus certain agents will inevitably bear this externality. On the other hand, rational agents will always deviate from their actual preference if being untruthful yields a higher utility for themselves.

Take a simple example where an energy generator has 2 kWh available for sale but anticipates a higher profit by submitting a bid of 1 kWh. According to game theory, the generator will always choose to bid 1 kWh even though this result withholds 1 kWh from potential consumers that may otherwise benefit from it. To the market operator whose social objective is to maximize welfare for all agents, this loss of trade negatively impacts the efficiency of the market as a whole. Thus implementing rules that induce truthfulness becomes a highly desirable property in mechanism design. In Chapter 3, we have characterized LEM’s as multi-unit double auction markets and introduced the concept of dominant strategy equilibria. The remainder of this chapter will be dedicated to the introduction of important concepts and implementation of dominant strategies in mechanism design.

4.1. Key Concepts in Mechanism Design

As you may have noticed, functions defined so far have been intentionally abstract. We have already introduced the notion of dominant strategy equilibrium as the desired solution concept of auction mechanisms. Yet it is still unclear how the mechanism implements a protocol, and what outcome function and price functions are required to induce truthful behavior in agents. To bridge this gap, an example illustrating the implementability of a single-sided auction mechanism is shown in Appendix A. This
section aims to provide the audience with a brief introduction on the important concepts in mechanism design theory.

4.1.1. Direct Mechanisms
There are two approaches to elicit the private valuation from the agents in a truthful way, which is called direct-revelation mechanisms and indirect-revelation mechanisms. We first cover direct mechanisms.

We say that a mechanism is a direct-revelation mechanism if the only action available to the agent is to submit their reported valuations. From a game theoretical point of view, it does not involve player types or strategy sets. The social choice function \( f : V \rightarrow A \) and payment function \( p : V \rightarrow \mathbb{R} \) directly maps agent valuations to an outcome. The mechanism protocol directly seeks the private valuation from the agents by asking them to reveal their true types. This does not mean the agents are obligated to reveal their private valuations \( v_i \). Instead agents submit their reported valuations \( \hat{v}_i \). It is the auctioneer’s objective to ensure that the allocation and payment protocol incentivizes agents to directly report their private valuation \( \hat{v}_i = v_i \). When this is the case, the direct mechanism is referred to as an incentive compatible mechanism.

4.1.2. Indirect Mechanisms
An indirect-revelation mechanism is a generalized notion of a mechanism where the auctioneer provides a choice of actions to each agent and specifies an outcome for each action profile. The main idea is that each agent now has some private information \( x_i \in T_i \) which determines the agent’s valuation function over a set of outcomes \( A \), denoted as \( v_i : T_i \times A \rightarrow \mathbb{R} \). An incomplete information game is induced by the mechanism, where the agent has a set of available actions \( X_i \), while being informed of the outcome function \( a : X_1 \times X_2 \times ... \times X_m \rightarrow A \) and payment function \( p : X_1 \times X_2 \times ... \times X_m \rightarrow \mathbb{R} \). The agent’s goal is to maximize its utility given by \( u_i(t_i, x_1, ..., x_n) = v_i(t_i, a(x_1, ..., x_n)) - p_i(x_1, ..., x_n) \). Instead of submitting their reported valuation, agents play certain strategies that indirectly capture the private information, or \( x_1 , ..., x_n = s_1(t_1), ..., s_n(t_n) \). In this case, the auctioneer’s objective is to implement a social choice function \( F : T \rightarrow A \) such that \( f(t_1, ..., t_n) = a(x_1, ..., x_n) \). This last expression equates the auctioneer’s intended social outcome when knowing private valuations to the social outcome resulting from agent strategies induced by the incomplete information game.

In the context of energy auction markets, the best way to emphasize the difference between direct and indirect mechanisms is to look at what exactly are the agents submitting to the auctioneer. In a direct mechanism, each agent is asked to submit its valuation for its energy demand directly to the auctioneer. Whereas in an indirect mechanism, the agents are asked to submit a bid. The intent is that the bid submitted will be dependent on the agent’s private information, and based on this the agent has a strategy for bidding.

Indirect mechanisms are appealing and useful because they provide us with practical ways of implementing social choice functions. They generally describe dynamic mechanisms such as continuous auctions or combinatorial auctions. Unfortunately, indirect mechanisms are generally extremely hard to implement due to the complexity of its rules and available action space of agents. Finding a profile of equilibrium strategies may sometimes be impossible under such circumstances. On the other hand, direct mechanisms generally describe static games and are sometimes too expensive for agents because they place high demands on information revelation. Additionally, they are useful in the development of theory but rarely practical in the real world.

4.1.3. Revelation Principle
Fortunately, mechanism design theorists developed a fundamental relationship between an indirect mechanism \( \mathcal{M} \) and direct mechanism \( \mathcal{D} \) with respect to a given social choice function \( f \). The revelation principle states that under weak conditions any mechanism can be transformed into an equivalent direct-revelation mechanism that implements the same social choice function. This proves to be a powerful tool, as it enables us to restrict our inquiry about truthful implementation of a social choice function to the class of direct revelation mechanisms only.

4.1.4. Mechanism Design Objectives
The following design objectives are most commonly considered in mechanism design for auction markets:
4.2. Double Auction Mechanism Design Environment

Incentive Compatibility (IC): A mechanism is incentive compatible if all of the agents maximize their utilities when truthfully revealing any private information asked for by the mechanism. This property is also known as truthfulness or strategy-proofness.

Efficiency: Also known as social welfare maximization, this objective reiterates the importance of the social choice function implemented. In the case of auction settings, the objective is to match a buyer who values the commodity the most to a seller asking for the lowest price to achieve maximal gains-from-trade (GFT).

Individual Rationality (IR): A mechanism is individually rational if it gives its traders non-negative utility $u_i$. This implies that for any possible $(v_i, v_{-i})$, $v_i(f(v_i, v_{-i})) - p_i(v_i, v_{-i}) \geq 0$ for all $i$ [1]. In other words, the mechanism’s allocations do not make any trader worse off than had the trader not participated.

Budget-Balance (BB): A mechanism is exactly budget balanced if the total payment that buyers and sellers make equals zero, so no money is injected into or removed from the mechanism. A weakly budget-balanced describes the scenario where the total payment is non-negative, which infers that the mechanism does not run at a loss and generates some form of revenue for the auctioneer.

4.2. Double Auction Mechanism Design Environment

This study investigates two-sided markets, where several sellers who hold items for sale and several buyers who consider buying these items. A double auction is a mechanism for organizing a two-sided market. In a double auction setting, there are $n$ agents who are rational and intelligent, interacting strategically among themselves towards making a collective decision. A denotes the set of auction outcomes or alternatives for a social choice function $f$ the auctioneer chooses. Note that the $A$ describes all possible resulting allocations agents can receive. The preference of an agent $i$ is now captured by a valuation function $v_i : A \rightarrow \mathbb{R}$ that describes the monetary value that the agent will obtain from each chosen allocation. In this case, $v_i(a)$ denotes the monetary value that $i$ assigns to it receiving allocation $a$. Note that $v_i(a)$ is the private valuation of an agent and does not depend on the other agents’ values.

The double auction mechanism $\mathcal{M}$ includes an outcome function $f : V \rightarrow A$ and price functions $p_i : V \rightarrow \mathbb{R}$ for each agent $i$. In energy trading (or in any commodity exchange), the social choice function $f$ implemented is naturally welfare-maximizing. In addition to implementing $f$, the mechanism designer charges the agent an additional amount of money $p_i$ for the chosen outcome $a$. Thus agent $i$’s utility is defined as $u_i = v_i(a) - p_i$. Utilities of this form are quasi-linear due to its separable and linear dependence on money. Compared to the game-theoretical description of LEM’s in Section 3.2.1, this quasi-linear utility is the translation of buyers’ utility $u_i(t_1, x_1, ..., x_n)$ and sellers’ utility $u_k(t_k, y_1, ..., y_m)$ for a direct mechanism. Due to the revelation principle, it is possible to disregard an agent’s types $T$ and actions $(X, Y)$, restricting attention to the truth-revealing direct revelation mechanism. The implicit assumption is that an agent aims to maximize its resulting utility $u_i = v_i(f(v_i, v_{-i})) - p_i(v_i, v_{-i})$.

The feasibility of the mechanism is determined by using game theory to analyze the equilibrium strategies of all agents. The mechanism $\mathcal{M}(f, p_1, ..., p_n)$ successfully implements social choice function $f(v_1, ..., v_n)$ if and only if the outcome computed with equilibrium strategies $v_1, ..., v_n$ is a solution to the social choice function for all possible agent preferences.

4.2.1. Implementation of Dominant Strategies

Recall from game theory the concept of dominant strategy equilibrium where strategy $s^*(t)$ maximizes the agent’s utility regardless of other agents’ strategies. For mechanism design, we can translate an agent’s strategy $s(t)$ to its valuation $v$.

Definition 4.2.1. A direct mechanism is truthful (or incentive compatible, or strategy-proof) if the dominant strategy of each agent is to reveal its true type (or private valuation). That is for every $v_{-i} \in V_{-i}$ and $v_i, v_i' \in V_i$, $v_i(f(v_i, v_{-i})) - p_i(v_i, v_{-i}) \geq v_i(f(v_i', v_{-i})) - p_i(v_i', v_{-i})$ [1].

Incentive compatibility and dominant strategy equilibrium are inexorably linked; these two concepts constitute important pillars in mechanism design. As mentioned before, strategy-proofness is a highly
useful property. Dominant strategy implementation is very robust to assumptions about agents, such as the information and rationality of agents, and has been proved to be the strongest solution concept in game theory. There are other equilibrium concepts that could be implemented by an incentive compatible mechanism, such as the Bayesian-Nash equilibrium, but we restrict the scope of this thesis to focusing only on dominant strategy equilibrium because it makes the least assumptions about agents. In the context of residential local energy markets, the agents conducting trades are most likely the households’ building management systems. These interactions are conducted online, and often automated. Computationally, dominant strategy implementation allows an agent to compute its optimal strategy without modeling the preferences and strategies of other agents. Thus implementation of dominant strategy incentive compatible mechanisms.

Notice that the social choice and payment functions are embedded in the agent’s utility. The mechanism designer has full control over the construction of functions $f$ and $p$. We are now ready to state the required properties from an incentive compatible direct mechanism for the framework of the residential local energy market. A mechanism is incentive compatible if and only if it satisfies the following conditions for every agent $i$ and every $v_{-i}$:

- The payment $p_i$ is not dependent on $v_i$, but on the alternative outcome chosen $a = f(v_i, v_{-i})$. In reference to direction mechanisms, the agent’s only available action could not be manipulated to increase its payoff.

- The mechanism optimizes for each player, meaning for every $v_i$, $f(v_i, v_{-i}) \in \arg\max_a (v_i(a) - p_a)$. This is of mutual understanding when the auctioneer announces the specifications of the mechanism to agents.

4.3. Summary

In this chapter, an overview of mechanism design theory has been given. In Chapter 4, the formulation of the LEM as a game and its solution concept allows us to identify and align desired properties in mechanism design to be able to achieve the research objective. Thus, a framework on implementations of double auction mechanisms which produces dominant strategy equilibria is given. In the next chapter, a non-truthful mechanism will first be introduced, followed by two dominant strategy incentive-compatible mechanisms to be explored in this thesis.
Truthful Double Auction Mechanisms for Local Energy Markets

In the previous chapter, an overview of mechanism design is given. In this chapter, the double auction mechanisms to be investigated in this study will be presented. We first introduce an important theorem that limits the desired objectives one can achieve in mechanism design, followed by the introduction of the truthful mechanisms which will be explored in this study.

5.1. The Impossibility Theorem

In an important and influencing result, Myerson and Satterthwaite have shown an impossibility in double auction mechanisms: [42]

**Theorem 5.1.1. Myerson and Satterthwaite Theorem:** There does not exist a double auction that is truthful, efficient, individually rational and (weakly) budget-balanced.

Myerson and Satterthwaite demonstrate this impossibility in a two-agent one-good example, for the case that trade is possible but not certain. The consequence of this result is that we achieve at most three of the four objectives/properties stated in Section 4.1.5. Thus there will inevitably be a trade-off when choosing which social choice function and equilibrium concept to implement.

The most commonly used double-auction mechanism is the Walrasian mechanism [3]. While this mechanism attains maximum gains-from-trade (GFT), this mechanism is unfortunately not incentive-compatible – agents have an incentive to misreport their valuations in order to manipulate the price. In this study, this popular mechanism will be implemented as a benchmark to gauge the cost of implementing incentive-compatible mechanisms.

The impossibility result initiated a search for double-auction mechanisms that are IC, IR, and BB and attain an approximately maximal GFT. McAfee [34] proposed a truthful, individually rational and budget-balanced double auction that is not efficient. McAfee’s key idea is trade reduction, i.e. reducing the match that gives the least social welfare increase if necessary. McAfee also showed that the proposed auction approaches efficiency if the number of traders approaches infinity, this is referred to as asymptotic efficiency. This trade reduction idea has inspired some further work dealing with similar problems in different static exchange environments. For example, Gonen et al. [14] proposed a general trade reduction framework for different exchange environments including multi-unit and combinatorial cases.

Instead of efficiency, other properties have also been extensively considered for sacrifice. The well-known VCG mechanism chooses to forego BB. Wurman et. al [67] proposed single-unit double auctions that are efficient, IR, BB but only partially truthful, i.e. truthful only for either buyers or sellers, and they also showed that there is no multi-unit double auction that has the same properties given that a trader’s valuation for each unit is independent of how many units he trades if partial satisfaction is possible or traders do not allow partial satisfaction (take-it-or-leave-it). Under a similar setting to the one studied by [67], Huang et al. [22] proposed multi-unit double auctions that are IR, weakly BB, and IC, but not efficient. The most recent advancement is aptly named MUDA, a multi-unit double auction.
proposed by Segal-Halevi et al. [55]. Their mechanism is IR, DSIC, strongly BB, but also not efficient. Furthermore, the MUDA procedure introduces complexity by splitting the market into two sub-markets, left and right, by sending each trader to each side with a probability 1/2, independently of the others.

Ultimately, the VCG and Huang mechanisms are chosen to be investigated for this study. The VCG mechanism represents an intriguing case in mechanism design as it is the only mechanism that achieves both maximum efficiency and dominant-strategy incentive compatibility. On the other hand, the Huang mechanism is less efficient but owns the interesting property of being weakly budget-balanced, meaning that the auctioneer enjoys revenue from operating the LEM. Figure 5.1 provides an overview of the mechanisms investigated and their properties. The chosen mechanisms are first presented and their properties analyzed.

![Mechanism Properties](image)

**Figure 5.1: Overview of chosen mechanisms and their properties**

### 5.2. Walrasian Mechanism

Regularly used in financial markets, the Walrasian market model acts as the fundamental framework for a free market economy and became a standard part of our conception of capitalistic markets. Léon Walras originally developed this model to capture the underlying order in the production and exchange of goods in a competitive system [25]. Drawing from the concept of utility maximization as the driving force behind all economic behavior, sellers and buyers supply and demand commodities as a function of their preferences, actual holdings of commodities, and all prices. In their most basic forms, the truthful double auction mechanisms explored in this thesis project are extensions of the Walrasian mechanism, and thus the Walrasian mechanism is used as the benchmark when analyzing mechanism performance.

The Walrasian auction mechanism describes an exchange process called tâtonnement. Characteristics of a tâtonnement exchange process include the following: 1) there is only one price at one time, 2) there is an information system notifying all agents of the price, 3) there is a mechanism for determining the quantities and prices and 4) transactions at non-equilibrium prices are not allowed. Most importantly, the pricing rule dictates that the adjustments made by the auctioneer to the trading price \( \frac{dp}{dt} \) must follow the same sign as excess demand \( E = D - S \). When the Walrasian auction is done in practice, buyers and sellers calculate their own demand for the commodity at every possible price and submit these preferences to the auctioneer. Thus, each agent reveals its full utility function to the auctioneer. By taking into account all agents’ preferences, the auctioneer determines the market equilibrium by adjusting the uniform trading price of the commodity until aggregate supply meets aggregate demand. This process is iterative. If excess demand \( E \) is positive, this is a signal from the market that there is potential to satisfy unfulfilled demand and thus the auctioneer raises the system price. In contrast, the auctioneer lowers the system price if the excess demand \( E \) is negative. From a mathematical optimization point of view, the tâtonnement process boils down to essentially being a Linear Programming (LP) problem, where incremental adjustments made to the trading price converge to a final market clearing price based on constraints in quantity. In other words, the auctioneer’s goal is to find the optimal allocated quantities such that social welfare is maximized. The tâtonnement technique used for price adjustments closely mimics the Simplex algorithm used to solve LP problems and will be explained further in detail later on.
5.2.1. Walrasian Mechanism Problem Formulation

The Walrasian double auction is a type of simultaneous auction, or sealed-bid auction. In game theory, simultaneous auctions are treated as static games. Agents have incomplete information about the game; other agents’ identities and payoff functions are unknown to individual agents. Therefore, any interaction between agents leading up to the market outcome should not affect their strategy set.

Assume that a market is made up of \( m \) buyers and \( n \) sellers, where each buyer \( i \) wants to purchase \( x_i \) amount of a good and each seller \( j \) has \( y_j \) amount to sell. For our residential LEM, each agent is allowed only a single bid composed of a discrete quantity and price. Let \( b_i = (b_{vi}, b_{qi}) \) be the bid placed by buyer \( i \), where \( b_{vi} \) is the reported per unit valuation of the ordered quantity \( b_{qi} = x_i \). Similarly, let \( s_j = (s_{vj}, s_{qj}) \) be the bid placed by seller \( j \), where \( s_{vj} \) is the reported per unit valuation of the ordered quantity \( s_{qj} = y_j \). The true valuation for each buyer \( i \) and seller \( j \) are denoted as \( b_{vi} \) and \( s_{vj} \), this information is private and only known to the agents themselves. The reported and true valuations for each agent are assumed to be static throughout the market period and this is the case for all mechanisms explored.

First, we take a look at individual gains-from-trade of the market outcome from the agents’ point of view. Let the quantity buyer \( i \) receives from the market outcome be denoted as \( x_i^* \). We use the * symbol to denote the outcome of agent allocations obtained by solving Eq. 5.3. The market clears with a uniform trading price, denoted as \( p_t \), at which all allocated buyers and sellers commit their trades. Buyer \( i \)’s utility is the additional value it receives when purchasing a good at \( p_t \) and can be described as:

\[
u_{bi} = (b_{vi} - p_t)x_i^* \tag{5.1}\]

Seller \( j \)'s profit is the amount of revenue it makes from each trade subtracted by its costs. Profit can be described as:

\[
u_{sj} = (p_t - s_{vj})y_j^* \tag{5.2}\]

The Walrasian mechanism seeks to maximize total market value described by the sum of utilities and profits. As the auctioneer received utility functions from all agents, finding the market equilibrium consists of solving a linear programming optimization problem described as follows:

\[
\begin{align*}
\max_{x,y} & \quad \sum_{i=1}^{m} (b_{vi} - p_t)x_i + \sum_{j=1}^{n} (p_t - s_{vj})y_j \\
\text{s.t.} & \quad \sum_{i=1}^{m} x_i - \sum_{j=1}^{n} y_j = 0 \quad \forall i \in m, \forall j \in n \\
& \quad x_i \leq b_{qi} \quad \forall i \in m \\
& \quad y_j \leq s_{qj} \quad \forall j \in n \\
& \quad x_i \geq 0, y_j \geq 0 \quad \forall i \in m, \forall j \in n
\end{align*}
\]

The inequality constraints describe the range of quantities at which each agent is willing to trade. The equality constraint, which is also the market clearing constraint, ensures that the aggregate generation and consumption is balanced at all times.

The allocation rule is determined by the LP solution vectors \( \hat{x}^* \) and \( \hat{y}^* \). Each buyer \( i \) receives a quantity equal to \( x_i^* \). Each seller \( j \) supplies a quantity equal to \( y_j^* \). As mentioned before, tâtonnement is the pricing rule used to reach system equilibrium by finding the market clearing price (MCP). From the LP problem described in Eq. 5.3, the MCP is not an explicit solution for the objective function. Nevertheless, solving a linear program usually provides more information about an optimal solution than merely the values of the decision variables. For every linear optimization problem, known as the primal, there is an associated problem known as its dual. Shadow prices (also referred to as dual variables or marginal values) are the Lagrange multipliers to the primal problem and, simultaneously, the solutions to the dual problem.

The shadow price on a particular constraint represents the change in the value of the objective function per unit increase in the right hand-side value of that constraint. In the context of our double
auction LP problem, the right hand-side values of the inequality constraints \( x_i \leq b_{q_i} \) and \( y_j \leq s_{q_j} \) constitute as the maximum amounts of energy the agents are willing to trade for. Since the agents’ reservation prices \( (b_{q_i}, s_{q_j}) \) are static, each agent values energy at its reservation price regardless of the amount it was allocated. Thus the shadow price of each constraint is equivalent to the reservation price each agent reports to the auctioneer respectively.

When looking at the market-clearing constraint \( (\sum_{i=1}^{m} x_i - \sum_{j=1}^{n} y_j = 0) \), this equality constraint puts a limit on the total market value. Thus the shadow price of the equality constraint represents the MCP, which can be viewed as the marginal per unit valuation of total quantity traded. The MCP could thus be found by solving the dual problem and computing the dual variable associated with the equality constraint from the primal problem.

### 5.2.2. Walrasian Mechanism: An Example

The Walrasian mechanism is intuitively simple when illustrated by a graphical example. Suppose there are 6 buyers and 6 sellers each looking to trade 1 kWh of energy for the current trading window. Each buyer has a private reservation price and reports \( b_{q_i} \), and each seller has a private reservation price \( s_{q_j} \) and reports \( s_{q_j} \). For now let us assume the reported prices of all agents are equal to their private reservation prices. Without loss of generality, we assume the following:

\[
b_{q_1} > b_{q_2} > ... > b_{q_m}
\]  
\[
s_{q_1} > s_{q_2} > ... > s_{q_m}
\]

(5.4)

(5.5)

The auctioneer arranges the demand volumes according to the price order shown in Eq. 5.4 and supply volumes according to the price order shown in Eq. 5.5.

Prices reported by buyers are set and ordered as follows:

<table>
<thead>
<tr>
<th>Buyer</th>
<th>Reservation Price [c/kWh]</th>
<th>60</th>
<th>50</th>
<th>40</th>
<th>30</th>
<th>20</th>
<th>10</th>
</tr>
</thead>
</table>

Table 5.1: Buyers’ reservation prices for single unit bids

Similarly, for the sellers:

<table>
<thead>
<tr>
<th>Seller</th>
<th>Reservation Price [c/kWh]</th>
<th>10</th>
<th>20</th>
<th>30</th>
<th>40</th>
<th>50</th>
<th>60</th>
</tr>
</thead>
</table>

Table 5.2: Sellers’ reservation prices for single unit-bids

By placing the buyers and sellers in separate merit-orders, the aggregate supply and demand functions can be plotted in discrete steps as shown in Figure 5.2.

The maximum social welfare of this market period is 90 cents, which is found by subtracting the area underneath the demand function with the area underneath the supply function up to the equilibrium quantity \( q_e \). The Walrasian MCP can also be intuitively determined by finding the average valuation between the last pair of buyer and seller, which is 35 cents/kWh. These results are validated by solving the LP problem with the example’s bid parameters. Using equations 5.1 and 5.2, the utility for buyers and sellers are tabulated below.

All double auction mechanisms with market value maximizing social choice functions follow this approach in determining the agents’ merit-order. The commodity is always allocated to the buyer with the highest marginal utility from the seller with the lowest marginal cost. However in competitive markets, agents are not required to submit bids that reflect their true marginal utility and costs. The VCG and Huang mechanisms define allocation and pricing rules that induce agents in revealing their true reservation prices.
5.2. Walrasian Mechanism

5.2.3. Walrasian Mechanism Discussion

The Walrasian mechanism has some desirable properties, but ultimately does not induce truthful behavior from the agents. It is individually rational because the agents in the merit order will commencing trading at a uniform market clearing price that does not produce negative utility for either sellers or buyers; it is efficient because all the items are held by the agents who value them the most; it is budget balanced because all monetary transfers are kept between buyers and sellers. The mechanism’s pricing rule causes it to be prone to manipulation in terms of price. We use the example shown in Figure 5.2 to illustrate how this is the case.

We first assume that all the agents in the example from the previous section are all reporting their truthful valuations ($\hat{s}_i = s_i \forall i; \hat{b}_j = b_j \forall j$). Consider seller $s_3$, who is part of the pair of agents that determines the market clearing price. By reporting its true valuation $s_{3\ell} = 30$ (cents/kWh), seller $s_3$
receives a payoff of \(35 - 30 = 5\) cents (see Table 5.4). Yet for him, this is not the maximum possible payoff he can receive. For example if \(s_3\) reports a price \(\hat{s}_{3}\) = 35 (cents/kWh), the market clearing price of the auction will in fact be increased to 37.5 (cents/kWh). This manipulation of price is illustrated in Figure 5.3. By reporting a higher bid price, seller \(s_3\) is able to increase his own profits to 37.5 – 30 = 7.5 cents, as well as increasing the profit of all in-merit sellers in the process. Assuming that the other sellers stay truthful, seller can actually gain a maximum profit of 9.9 cents if it happens to bid just below \(s_4\)’s reported valuation at \(\hat{s}_{4}\) = 39.9 (cents/kWh).

![Double-Sided Auction: price manipulation](image)

**Figure 5.3: Price manipulation of a Walrasian double auction**

### 5.3. VCG Mechanism

The VCG mechanism is named after its inventors William Vickrey, Edward Clarke, and Theodore Groves. It defines a broad class of dominant strategy incentive compatible mechanisms that efficiently allocates a public good based on a social choice function. The VCG mechanism is proved to be the unique direct mechanism that enjoys strategy-proofness, perfect efficiency, and individually rationality, but this comes at a cost. Unfortunately, the mechanism is not budget balanced.

William Vickrey’s inquiry into auctions marked the first serious attempt by an economist to use game theory to explain the dynamics of auctions and to design new rules to achieve superior performance. In his seminal paper[63] which originated auction theory, he demonstrated that a particular pricing rule makes it a dominant strategy for bidders to report their values truthfully, even when they know that their reported values will be used to allocate goods efficiently. Due to the influence of Vickrey’s original work on truth inducing pricing rules, a brief summary is provided on how the VCG mechanism is extended from a single-sided auction implementation to the multi-unit double auction setting.

The first analysis of Vickrey’s mechanism was carried out on single item auctions, and is often referred to as the second-price sealed-bid auction, or simply the Vickrey auction. The Vickrey auction awards a single item to the highest bidder, but the winner pays the amount of the second-highest bid. For example, if the winning bidder bids 11 and the highest losing bid is 10, the winner of the bid pays 10. With these rules, a winning bidder can never affect the price it pays, so there is no incentive for any bidder to misrepresent its value. This very simple and elegant idea achieves something that is quite remarkable: it reliably computes a maximization function of private valuations that are each held secretly by a different self-interested player. From another point of view, this may be seen as the mechanics for the implementation of Adam Smith’s invisible hand, because despite private information and pure
5.3. VCG Mechanism

The generalization of VCG mechanisms is defined as follows [1]:

**Definition 5.3.1.** A mechanism $M(f, p_1, ..., p_n)$ is called a Vickrey-Clarke-Groves (VCG) mechanism if:

- $f(v_1, ..., v_n) = \text{argmax}_a \sum_{i=1}^n v_i(a)$; that is, $f$ maximizes social welfare
- For some functions $h_1, ..., h_n$, where $h_i : V_i \rightarrow \mathbb{R}$ (i.e. $h_i$ does not depend on $v_i$), we have that for all $v_1 \in V_1$, ..., $v_n \in Vn$ the payment rule is defined as:

$$p_i(v_1, ..., v_n) = h_i(v_{-i}) - \sum_{j \neq i} v_j(f(v_1, ..., v_n)).$$

The adaptability of the VCG mechanism to meet the requirements of various auction formats can be inferred from first point of Definition 5.3.1. Any social choice function $f(v_1, ..., v_n)$ that seeks to maximize social welfare is implementable by the VCG mechanism; this is an extremely desirable aspect as it allows the market designer to make less assumptions about properties relating to the market framework. The summation term $v_i(a)$ allows agents to declare their entire utility functions to the auctioneer (this private information is still kept from other agents). Under the same residential LEM framework, it is clear that the VCG mechanism optimizes for the same social choice function as the Walrasian mechanism described in Eq. 5.3. From the first point of Definition 5.3.1 and objective function in Eq. 5.3, the VCG mechanism allocation rule is determined by the LP solution vectors $\hat{x}^*$ and $\hat{y}^*$. Each buyer $i$ receives a quantity equal to $x_i^*$. Each seller $j$ supplies a quantity equal to $y_j^*$. The pair of seller and buyer bids that define the market equilibrium are allowed to be partially accepted in respect to quantity.

From the second point of Definition 5.3.1, the first term $h_i(v_{-i})$ is an arbitrary function on the the reported valuations of every agent except for $i$. The specific form of the function $h_i(v_{-i})$ used in this study is the Pivotal, or Clarke, mechanism: $h_i(v_{-i}) = \sum_{j \neq i} v_j(b)$, where $b = \text{argmax}_a \sum_{j \neq i} v_j(a)$. This term has no strategic implications for agent $i$ since it does not depend, in any way, on the action it chooses to take.

The main idea lies in the second term $- \sum_{j \neq i} v_j(f(v_1, ..., v_n))$, which describes the social welfare of all other agents with agent $i$’s participation. Thus when this term is added to the agent’s own value $v_i(f(v_1, ..., v_n))$, the sum becomes exactly the total social welfare of $f(v_1, ..., v_n)$. Together, these two terms form the pricing rule of the VCG mechanism. Intuitively, agent $i$ pays an amount equal to the total damage that it causes the other players, which is the difference between the social welfare of the others with and without $i$’s participation. The nature of the pricing rule allows for price discrimination.
rather than using a single price to clear the market, each household is thus charged a personalized payment for the quantity of energy traded.

The VCG payment rule for buyers can be formulated as follows:

\[ p_i = (\arg \max_{k=1}^{m} \sum_{j=1}^{n} (b_{vk}x_k - s_{vj}y_j)) - \sum_{k=1}^{m} \sum_{j=1}^{n} (b_{vk}x_k - s_{vj}y_j') \] (5.6)

Similarly for sellers:

\[ p_j = (\arg \max_{k=1}^{m} \sum_{j=1}^{n} (b_{vi}x_i - s_{vj}y_j)) - \sum_{k=1}^{m} \sum_{j=1}^{n} (b_{vi}x_i' - s_{vj}y_j') \] (5.7)

### 5.3.2. VCG Mechanism: An Example

To highlight the mechanics of a VCG double auction, a simple market consisting of agents looking to trade unit quantities is illustrated in Figure 5.4. Following Eq. 5.4 and Eq. 5.5, the 6 sellers and 6 buyers are placed in their respective merit orders. Similar to the Walrasian double auction example, social welfare of agents for this auction instance can be hand calculated by subtracting the area underneath the demand curve by the area underneath the supply curve up to the market equilibrium quantity. The total social welfare is found to be \((90 + 80 + 70 + 60) - (10 + 20 + 30 + 40) = 200\) cents.

Prices reported by buyers and sellers are tabulated in Tables 5.5 and 5.6.

<table>
<thead>
<tr>
<th>Buyer</th>
<th>Reservation Price [c/kWh]</th>
</tr>
</thead>
<tbody>
<tr>
<td>b₁</td>
<td>90</td>
</tr>
<tr>
<td>b₂</td>
<td>80</td>
</tr>
<tr>
<td>b₃</td>
<td>70</td>
</tr>
<tr>
<td>b₄</td>
<td>60</td>
</tr>
<tr>
<td>b₅</td>
<td>30</td>
</tr>
<tr>
<td>b₆</td>
<td>20</td>
</tr>
</tbody>
</table>

Table 5.5: Buyers’ reservation prices for single unit bids

<table>
<thead>
<tr>
<th>Seller</th>
<th>Reservation Price [c/kWh]</th>
</tr>
</thead>
<tbody>
<tr>
<td>s₁</td>
<td>10</td>
</tr>
<tr>
<td>s₂</td>
<td>20</td>
</tr>
<tr>
<td>s₃</td>
<td>30</td>
</tr>
<tr>
<td>s₄</td>
<td>40</td>
</tr>
<tr>
<td>s₅</td>
<td>50</td>
</tr>
<tr>
<td>s₆</td>
<td>60</td>
</tr>
</tbody>
</table>

Table 5.6: Sellers’ reservation prices for single unit-bids
Consider seller $s_1$ who will receive a final allocation of 1 unit of energy from solving the LP problem (or from observing aggregate supply and demand functions of the auction instance). To find the per-unit price at which seller $s_1$ receives, we apply Eq. 5.7. The first term solves for the social choice function as if seller $s_1$ had not entered the market, which shifts all sellers’ indices up one spot in the merit order. In this case, seller $s_1$’s bid is considered the externality incurred by seller $s_1$. The total social welfare from the first term $\arg\max \sum_{k=1}^{n} \sum_{i=1}^{m} (p_{yi}x_i - s_{yk}y_k)$ can be hand calculated as $(90 + 80 + 70 + 60) - (20 + 30 + 40 + 50) = 160$ cents. The second term equals the total social welfare of the original merit order excluding seller $s_1$’s welfare, which can be calculated as $(90 + 80 + 70 + 60) - (20 + 30 + 40) = 210$ cents. Thus seller $s_1$ pays a price of $p_1 = 160 - 210 = -50$ cents, where the negative sign translates to the payment $s_1$ receives for generating one unit of energy. Note that this is the entire payment made to seller $s_1$, and the per-unit price $p_1$ trades as is equal to the total payment divided by the total units allocated (in this case the rate is -50 cents/unit). By performing similar procedures to all agents within the merit-order, the resulting trading prices and utilities of the auction instance can be found in Tables 5.7 and 5.8.

<table>
<thead>
<tr>
<th>Buyer</th>
<th>$b_1$</th>
<th>$b_2$</th>
<th>$b_3$</th>
<th>$b_4$</th>
<th>$b_5$</th>
<th>$b_6$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Reservation Price [c/kWh]</td>
<td>90</td>
<td>80</td>
<td>70</td>
<td>60</td>
<td>30</td>
<td>20</td>
</tr>
<tr>
<td>MCP [c/kWh]</td>
<td>40</td>
<td>40</td>
<td>40</td>
<td>40</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Allocation [kWh]</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Utility [c]</td>
<td>50</td>
<td>40</td>
<td>30</td>
<td>20</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Total Utility [c]</td>
<td>140</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 5.7: Buyer outcomes from VCG double-auction mechanism
5.3.3. VCG mechanism discussion

In this sub-section, we examine some aspects that make VCG mechanisms the only class of mechanisms that are allocatively-efficient and strategy-proof among direct-revelation mechanisms. While the revelation principle extends this uniqueness to general mechanisms (indirect mechanisms), the sealed-bid format implemented for the residential LEM double auction suggests a focus on incentive-compatible direct mechanisms. While dominance of a strategy does not guarantee that an agent will choose it, it ensures that deviating from it is not profitable. So dominance of each valuation \( v_i \) can be viewed as a statement that in the considered auction lying does not pay off. Analysis of the VCG mechanism could be deconstructed into three parts respective of its creators: the Vickrey auction, Grove’s mechanism and Clarke’s mechanism.

As mentioned previously, truth-revelation is a dominant strategy in the Vickrey second-price sealed-bid auction because an agent’s bid determines the range of prices that it will accept, but not the actual price it pays. Thus the price an agent pays is completely independent of its bid price. This fundamental virtue can be inferred from the premise of Definition 5.3.1, which describes the Vickrey-Clarke-Groves family of mechanisms.

The VCG mechanism is often simply called the Groves mechanism for problems in which agents have quasi-linear preferences. Specifically, the Groves mechanism selects the allocation \( a \) that maximizes the total reported value \( v(a) \) over all agents, and enforces a payment rule where the agent’s reported valuation is excluded from the payment calculation. Consider the buyer’s utility function as defined in Equation 5.1 in combination with the payment rule from Definition 5.3.1. The utility to buyer \( i \) from reporting bid strategy \((\hat{b}_{vi}, \hat{b}_{qi})\) while its private valuation is \( b_{vi} \) can be formulated as the following:

\[
u_{bi}(b_{vi}) = b_{vi}x^*_i - h_i(v^*_i) + \sum_{k=1}^{m} \sum_{j=1}^{n} (b_{vk}x^*_k - s_{uj}y^*_j)
\]

(5.8)

The only effect of the buyer \( i \)'s reported valuation \( \hat{b}_{vi} \) is on its final quantity allocated \( x^*_i \), and it can maximize its utility in Equation 5.8 by announcing \( b_{vi} = \hat{b}_{vi} \). This is possible due to direct revelation, because the Groves mechanism explicitly maximizes the terms \( x^*_i \) and \( y^*_j \) describing the resulting selection that maximizes social welfare, which is formed by the utility functions of the buyers and sellers. Intuitively, the utility function that the agent attempts to maximize is aligned with the objective function of the LP problem the auctioneer tries to maximize. We can ignore the term \( h_i(v^*_i) \) because it is independent of an agent’s reported bid. Buyer \( i \) is not incentivized to misrepresent its reported quantity since there is no way to reliably manipulate its reported volume to increase its utility. Since the mechanism is efficient, buyer \( i \) will be allocated the entirety of its reported quantity if it resides within the merit order and if its bid is not partially accepted at the market equilibrium. Under-reporting the desired consumption quantity will only diminish buyer \( i \)'s desired allocation and over-reporting will not bring it any additional benefit. Thus, truth-revelation is the dominant strategy of buyer \( i \), whatever the reported bids of other agents.

Strictly speaking, the Clarke mechanism is a special form of the Groves mechanism in which the payment rule, \( h_i(v^*_i) \), is carefully set to achieve individual-rationality while also maximizing the payments made by the agents to the mechanism. The first terms in Equations 5.6 and 5.7 describes the Clarke pivot rule, which leaves the strategy-proofness and efficiency of the Groves mechanism unchanged since the market allocation is still maximized independent of the report from buyer \( i \).

From the results in Tables 5.7 and 5.8, one can observe a budget deficit incurred on the market operator for executing the trades. The four buyers who receive allocations make a total payment of 160 cents to the auctioneer, while the four sellers who won the trades are expecting to receive a total payment of 200 cents from the auctioneer. This deficit must be subsidized by the auctioneer, because

<table>
<thead>
<tr>
<th>Seller</th>
<th>( s_1 )</th>
<th>( s_2 )</th>
<th>( s_3 )</th>
<th>( s_4 )</th>
<th>( s_5 )</th>
<th>( s_6 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Reservation Price [c/kWh]</td>
<td>10</td>
<td>20</td>
<td>30</td>
<td>40</td>
<td>50</td>
<td>60</td>
</tr>
<tr>
<td>MCP [c/kWh]</td>
<td>50</td>
<td>50</td>
<td>50</td>
<td>50</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Allocation [kWh]</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Profit [c]</td>
<td>40</td>
<td>30</td>
<td>20</td>
<td>10</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Total Profit [c]</td>
<td>100</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 5.8: Seller outcomes from VCG double-auction mechanism
to transfer these costs back to the participants would result in the mechanism failing to uphold its *individual-rationality* property.

![Double-Sided Auction: VCG Mechanism](image)

The effect of discriminatory pricing can be illustrated by altering the example from the previous subsection. Consider the original auction instance shown in Figure 5.4 with identical assumptions, which will now be rearranged by setting the bid quantity of seller $s_1$ from 1 unit to 1.5 units. The aggregate supply and demand functions for this alternate auction instance is shown in Figure 5.5. Our analysis considers both the original auction instance (Figure 5.4) and the alternate version (Figure 5.5). From Eq. 5.7, the first term of the payment rule becomes $(90 + 80 + 70 + 60) - (20 + 30 + 40 + 50) = 160$ cents. This term is expected to be identical to the original example illustrated in Figure 5.4 as the bids from all other agents remain unchanged. The second term from Eq. 5.7 gives $(90 + 80 + 70 + 60) - (20 + 30 + 20) = 230$ cents. The auctioneer pays seller $s_1$ a sum of $\text{abs}[160 - 230] = 70$ cents for 1.5 units, resulting in a per-unit price of 46.67 cents/unit of energy. The remaining market clearing prices for each seller are calculated in Table 5.9 below.

<table>
<thead>
<tr>
<th>Seller</th>
<th>$s_1$</th>
<th>$s_2$</th>
<th>$s_3$</th>
<th>$s_4$</th>
<th>$s_5$</th>
<th>$s_6$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Reservation Price [c/kWh]</td>
<td>10</td>
<td>20</td>
<td>30</td>
<td>40</td>
<td>50</td>
<td>60</td>
</tr>
<tr>
<td>MCP [c/kWh]</td>
<td>46.67</td>
<td>45</td>
<td>45</td>
<td>50</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Allocation [kWh]</td>
<td>1.5</td>
<td>1</td>
<td>1</td>
<td>0.5</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Profit [c]</td>
<td>55</td>
<td>25</td>
<td>15</td>
<td>5</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Total Profit [c]</td>
<td>100</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 5.9: Seller outcomes from VCG double-auction mechanism

The alternate auction instance also highlights the mechanism’s resistance to cheating amongst participants. On one hand, this simple illustration serves as an indication that sellers who act rationally are not incentivized to over-report their bid volumes due to the potential decrease in marginal revenue (MCP). From a practical standpoint assuming that seller $s_1$ had been untruthful about its volume and wins the bid, there is also the question regarding how it can physically supply the remaining 0.5 units of energy not in its possession. On the other hand, the increased externality caused by $s_1$ over-bidding on volume can be observed in the resulting payments made to the other sellers. The comparison between auction results in Tables 5.7 and 5.9 show sellers $s_2$ and $s_3$ receiving less profit even though their
bids and final allocations remain identical. Loosely speaking, this potential loss in profit is stems from the increase in “welfare externality” caused by seller $s_i$’s bid being partially pushed out by seller $s_j$’s irrational decision to increase its reported volume. To put this in context of residential LEMs, artificially increasing the bid quantities does not serve any purpose for individual sellers if the seller could not physically deliver on its reported volume. Doing so would be considered individually irrational while simultaneously decreasing the overall welfare of the other sellers. In special cases, the VCG mechanism is susceptible to this form of cheating when a buyer and seller within the merit-order conspire by artificially increasing their reported volumes by the same exact amount - supply in one case and demand in the other to increase their respective Vickrey payments. Hoobs et al. \[20\] examined the potential use of the VCG process in electricity markets and pointed out this possibility, but posited that this form of cheating is easily detectable by monitoring the net flow at the point of connection for each participating agent. We round up the discussion by highlighting the virtues of the VCG mechanisms which have been discussed in this section.

Virtues:

- The dominant strategy property adds reliability to the efficiency prediction, because it means that the conclusion is not sensitive to assumptions about what bidders may know about what each others’ values and strategies.
- VCG does not impose any restrictions on the bidders’ possible rankings of different outcomes, meaning that the basic rules of the VCG auction can be further adapted if the auctioneer wishes to impose some extra constraints.
- The final and most important virtue of the VCG mechanism is that its average revenues are not less than that from any other efficient mechanism.

### 5.4. Huang Mechanism

The Huang mechanism was first proposed by Huang et al.[22] for the design of a multi-unit double auction e-market. It has since been extended and implemented in several research projects within the field of smart-grid energy trading [ref][ref][ref]. The Huang mechanism is strategy-proof with respect to reservation price, weakly budget-balanced and individually rational. Unfortunately, the mechanism implements a social choice function which produces inefficient outcomes.

#### 5.4.1. Huang Mechanism Problem Formulation

The problem formulation follows a similar framework Eq. 5.3, but makes adjustments to its final allocation. We focus on the allocation rule that governs the Huang mechanism. The Huang mechanism arranges the agents’ merit orders by following the same approach in Eq. 5.4 and Eq. 5.5. At the equilibrium where aggregate supply and aggregate demand meet, lets us denote the last pair of buyer and seller that still adds value to social welfare as buyer $K$ and seller $L$. Figure 5.6 below illustrates this intersection point.

At the market equilibrium, the supply and demand functions can intersect at points satisfying either one of two possible cases:  

**Case 1** (as shown in Figure 5.6):

$$b_{vK} \geq s_{vL} \geq b_{vK+1} \text{ and } \sum_{j=1}^{L-1} s_{qj} \leq \sum_{i=1}^{K} b_{qi} \leq \sum_{j=1}^{L} s_{qj}$$  \hspace{1cm} (5.9)

Or **Case 2**:

$$s_{vL+1} \geq b_{vK} \geq s_{vL} \text{ and } \sum_{i=1}^{K-1} b_{qi} \leq \sum_{j=1}^{L} s_{qj} \leq \sum_{i=1}^{K} b_{qi}$$  \hspace{1cm} (5.10)

These cases describe the two possible scenarios that arise in multi-unit auctions where, due to the equality constraint, an agent’s bid is only partially accepted within the merit order. In the Walrasian
Figure 5.6: A multi-unit double auction market implementing the Huang mechanism

example shown in the previous section, all agents are bidding single unit volumes and thus aggregate trading volumes on the supply and demand sides match perfectly. This is often not the case in real world markets with a divisible commodity such as energy. We refer back to Figure 5.6 to continue our discussion.

The mechanism checks whether inequality: \( \sum_{i=1}^{K-1} b_{qi} \geq \sum_{j=1}^{L-1} s_{qj} \) or \( \sum_{i=1}^{K-1} b_{qi} \leq \sum_{j=1}^{L-1} s_{qj} \) holds. The mechanism is determining if there is an over-demand or an over-supply in the resulting market when the last matching pair of trades are removed. The first inequality describes an over-demand while the second inequality infers that the market is in over-supply. Note that the mechanism omits seller K and buyer L in commencing trade; this is in fact a necessity. To induce truthful bidding, Huang et al.[22] applied a Vickery-like auction on each side of the market, removing the pair of matching offers that determines the MCP. In mechanism design, this technique is called a trade reduction, as illustrated by the absence of quantity traded \( B \). In doing so, the Huang implementation of the social choice function in Eq. 5.3 fails to reach its theoretical maximum, and thus provides an explanation as to why the efficiency property does not hold for the Huang mechanism.

When implementing the allocation rule, the auctioneer ensures maximum social welfare while bringing quantity supplied and quantity demanded to a balanced state. In Figure 5.6, the remaining K-1 buyers are demanding a total of 8 units while the L-1 sellers are offering up to 5 units, creating the aforementioned ‘over-demand’ state. To balance supply and demand, the mechanism’s allocation rule performs a ‘balancing’ reduction on the side of the market where there is a surplus. From the auctioneer’s point of view, this surplus is a “burden” that must be fairly resolved. The rules of this second reduction can be stated as follows:

**Rule 1:**
If inequality \( \sum_{i=1}^{K-1} b_{qi} \geq \sum_{j=1}^{L-1} s_{qj} \) holds, the market is in over-demand. All sellers with indices \( j < L \) sell their entire volumes and each buyer with indices \( i < K \) buy a volume equal to:

\[
x_i^* = b_{qi} - \left( \sum_{i=1}^{K-1} b_{qi} - \sum_{j=1}^{L-1} s_{qj} \right)/(K - 1)
\]  
(5.11)

**Rule 2:**
If inequality \( \sum_{i=1}^{K-1} b_{qi} \leq \sum_{j=1}^{L-1} s_{qj} \) holds, the market is in over-supply. All buyers with indices \( i < K \) buy
their entire volumes and each seller with indices \( j < L \) buy a volume equal to:

\[
y_j^* = s_{q_j} - \left( \sum_{j=1}^{L-1} s_{q_j} - \sum_{i=1}^{K-1} b_{q_i} \right) / (L - 1)
\]  

(5.12)

The second term in equations 5.11 and 5.12 introduce a fair way of redistributing the “burden” among agents within the merit order. However, there will be cases (such as the situation presented in Figure 5.6, where there is at least one agent on the surplus side whose bid volume is less than the “burden” itself. As an example, consider the ‘over-demand’ case and Rule 1 is applied to all buyers. Buyer \( a \) is within the merit order \( a < K \) whose bid quantity \( b_{qa} < (\sum_{i=1}^{K-1} b_{qi} - \sum_{j=1}^{L-1} s_{q_j}) / (K - 1) \). In the event where this happens, buyer \( a \) will not purchase any energy and the resulting mismatch \( \sum_{i=1}^{K-1} b_{qi} - \sum_{j=1}^{L-1} s_{q_j} \) is then treated as a “reimbursement” and averaged over the remaining \( K - 2 \) buyers. This procedure is continued until each buyer trades a positive volume. In the case of over-supply, a similar procedure is applied on the seller’s side of the market. Hence, this final part of the allocation rule is an iterative process. Further discussion on the resulting allocations for each agent within the market presented in Figure 5.6 is provided in the next section.

In contrast to Huang’s allocation rule, the pricing rule is rather straightforward. Instead of a single uniform market clearing price for both sides of the market, sellers with indices \( j < L \) trade at a uniform price \( s_{vl} \) and buyers with indices \( i < K \) trade at a uniform price \( b_{vk} \). Thus the Huang mechanism produces two market clearing prices, a buyer’s MCP and a seller’s MCP for each trading period:

- **Buyers’ MCP**: \( p_b = b_{vk} \)
- **Sellers’ MCP**: \( p_s = s_{vl} \)

The Huang mechanism is weakly budget balanced due to this pricing rule. By being the last pair of agents within the merit order, the \( K^{th} \) buyer and \( L^{th} \) seller induce a price margin between marginal utility and marginal cost (\( p_b - p_s \)) for the auctioneer across all allocated units. As visualized by the area \( A \) in Figure 5.6, this margin is the amount of revenue per unit quantity traded the auctioneer can expect to receive in Huang’s double auction.

### 5.4.2. Huang Mechanism: An Example

We continue the discussion of the Huang mechanism with the market example illustrated in Figure 5.6. Unlike the example used to illustrate the Walrasian mechanism, it is immediately clear that there are several agents bidding at greater volumes. Similar to the VCG example, the aim of providing an example is to showcase the mechanism in operation while revealing its weaker properties. In this particular example, the market is comprised of five buyers and 4 sellers, where the \( K^{th} \) buyer \( b_5 \) and the \( L^{th} \) seller \( s_3 \) form the market equilibrium. Tables 5.10 and 5.11 provides an overview of the private valuation and desired quantity of each agent entering the double auction market.

<table>
<thead>
<tr>
<th>Buyer</th>
<th>( b_1 )</th>
<th>( b_2 )</th>
<th>( b_3 )</th>
<th>( b_4 )</th>
<th>( b_5 )</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Reservation Price [c/kWh]</strong></td>
<td>70</td>
<td>60</td>
<td>50</td>
<td>40</td>
<td>20</td>
</tr>
<tr>
<td><strong>Quantity Demanded [kWh]</strong></td>
<td>1</td>
<td>1</td>
<td>6</td>
<td>1</td>
<td>2</td>
</tr>
</tbody>
</table>

Table 5.10: Reservation prices and quantities of buyers

<table>
<thead>
<tr>
<th>Seller</th>
<th>( s_1 )</th>
<th>( s_2 )</th>
<th>( s_3 )</th>
<th>( s_4 )</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Reservation Price [c/kWh]</strong></td>
<td>10</td>
<td>20</td>
<td>30</td>
<td>40</td>
</tr>
<tr>
<td><strong>Quantity Demanded [kWh]</strong></td>
<td>4</td>
<td>1</td>
<td>6</td>
<td>1</td>
</tr>
</tbody>
</table>

Table 5.11: Reservation prices and quantities of buyers

After removing \( b_5 \) and \( s_3 \) from trade, the aggregate quantity demanded for the remaining 3 buyers is found to be at 8 kWh, while the aggregate quantity supplied by the 2 sellers is observed at 5 kWh. By satisfying Equation 5.9, Equation 5.11 is applied to the buyers side of the market. Each buyer is...
5.4. Huang Mechanism

required to share an equal portion of the 3 kWh ‘burden’ caused by over-demand, which in this example amounts to 1 kWh per agent. In this example, the allocation rule effectively pushes buyers $b_1$ and $b_2$ out of the market even though both buyers are technically still considered to be within the merit order (they are considered to be trading 0 kWh each rather than being removed from trade). This particular example exposes the inefficiency of the Huang mechanism at implementing a welfare maximizing social choice function, as compared to the Walrasian and VCG mechanism which efficiently allocates quantity to agents who value it the most. In this particular scenario, buyer $b_3$ benefits the most from being a large consumer. The resulting market outcomes of the Huang mechanism is tabulated in Table 5.12 for buyers and in Table 5.13 for sellers.

### Table 5.12: Buyer outcomes from Huang double-auction mechanism

<table>
<thead>
<tr>
<th>Buyer</th>
<th>$b_1$</th>
<th>$b_2$</th>
<th>$b_3$</th>
<th>$b_4$</th>
<th>$b_5$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Reservation Price [c/kWh]</td>
<td>70</td>
<td>60</td>
<td>50</td>
<td>40</td>
<td>20</td>
</tr>
<tr>
<td>MCP [c/kWh]</td>
<td>40</td>
<td>40</td>
<td>40</td>
<td>40</td>
<td>40</td>
</tr>
<tr>
<td>Allocation [kWh]</td>
<td>0</td>
<td>0</td>
<td>5</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Utility [c]</td>
<td>0</td>
<td>0</td>
<td>50</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Total Utility [c]</td>
<td>50</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

### Table 5.13: Seller outcomes from Huang double-auction mechanism

<table>
<thead>
<tr>
<th>Seller</th>
<th>$s_1$</th>
<th>$s_2$</th>
<th>$s_3$</th>
<th>$s_4$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Reservation Price [c/kWh]</td>
<td>10</td>
<td>20</td>
<td>30</td>
<td>40</td>
</tr>
<tr>
<td>MCP [c/kWh]</td>
<td>30</td>
<td>30</td>
<td>30</td>
<td>30</td>
</tr>
<tr>
<td>Allocation [kWh]</td>
<td>4</td>
<td>1</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Profit [c]</td>
<td>80</td>
<td>10</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Total Profit [c]</td>
<td>90</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

The total utility on the buyer’s side of the market is 50 cents, enjoyed solely by buyer $b_3$, while on the sellers side a total profit of 90 cents is shared between sellers $s_1$ and $s_2$. The auctioneer’s revenue is induced by a price margin of 10 cents per unit traded, and by allocating 5 units the auctioneer receives 50 cents in total.

5.4.3. Huang mechanism discussion

There are a number of aspects regarding the Huang’s mechanism properties which will be discussed in further detail in this subsection. Under certain assumptions, the mechanism is mathematically proven in [22] to be incentive-compatible, weakly budget-balanced, and individually rational. The aim of this discussion is to clarify the market assumptions made by the creators of the Huang’s mechanism in achieving these theoretical properties and examine the rules required for the residential LEM implementation for these properties to be retained.

Talk about the special case presented in the figure that incentivizes the sellers to over-report their volumes to receive allocation.

The primary reason for Huang’s mechanism being strategy-proof is that everyone who trades always pays the price proposed by someone else. As seen in the previous section covering the VCG mechanism, this is a general principle to follow when designing a strategy-proof mechanism. Additionally, strategy-proofness of a mechanism usually requires some specific utility function form; in the VCG mechanism, this is achieved when each agent’s utility function is *quasi-linear*. For the Huang mechanism, the requirement is that the trading volume and price of each agent are separable in its utility function. This is an assumption made about the agents as well as the nature of the market. By virtue of *quasi-linear* utility functions described in Equations 5.1 and 5.2, a combination of the Huang mechanism’s uniform pricing rule and the trade reduction technique allows the separation of price and quantity values. For example, price and quantity are related if the auction follows a discriminatory pricing format, where a seller charges buyers at different rates depending on the amount being demanded. Another example of instances where agent preferences become interdependent is where the commodity being traded possesses inherent interdependent valuation and quantity preferences, such as a fruit...
market. Fruits are a type of commodity which have expiration dates. Thus when pricing fruits, traders take into account the quantity being sold and often offers a discount price if a buyer wants to buy at higher volumes. Fortunately, energy does not expire. Instead, it has a unique set of physical properties that sets it apart from other commodities and poses an entirely different set of challenges in able to properly conduct trade and physical delivery. For the scope of this thesis project, implementing rules within the residential LEM framework can align the requirements of Huang’s mechanism with the auctioneer’s intended goal, which is to retain strategy-proofness. In the residential LEM framework, each agent is constrained to submitting a single sealed bid to the auctioneer, thus eliminating the possibility of multiple utility or cost curves for the same commodity being submitted to the market. Additionally, the sealed-bid itself is a tuple consisting of a reported valuation and quantity pair as described in Subsection 5.2.1. These values are discrete and are thus separable, ensuring that the Huang mechanism’s utility function requirements are fulfilled.

We now take a look at how the Huang mechanism incentivizes agents to truthfully report their private valuations. From Figure 5.6, suppose buyer $b_2$ with reservation price $60 \text{ c/kWh}$ intentionally misrepresent their reported valuation, denoted as $r_{b_2}$. This buyer is within the merit order; and if $r_{b_2} \geq 60$, over-bidding will give buyer $b_2$ the same utility as if he bids at $60 \text{ c/kWh}$. On the other hand if buyer $b_2$ under-bids at an amount $r_{b_2} \leq 60$, it may risk losing the trade it otherwise would win while still receiving the identical utility even if it does win the trade. Now consider buyer $b_{K+1}$ who is out of the merit order. If it over-reports for the purpose of entering the merit order, i.e. $r_{b_{K+1}} \geq 50$, the result will incur negative utility if the buyer is included in the final trade. Even if the buyer is not included in the final trade, it still receives zero utility; under-bidding also results in zero utility. Thus there is no individual incentive for buyers anywhere on the aggregate demand curve to deviate from submitting its reservation price. The same reasons can be applied to the seller’s side of the market.

Another important assumption made by the Huang et al. is that the volume of each agent is public information [22]. This requirement infers that the volumes reported by all agents are inherently truthful quantities. Their justification for making this assumption is by first positing the opposite assumption (that agents’ volumes are private information) to prove this result inconclusive, and then associating this lack of analytical feasibility to justify the original assumption. In a double-sided auction market, sellers have the incentive to under-report their volumes while buyers have the incentive to over-report. While this is the case, strategies of under/over reporting are difficult for the sellers to successfully implement due to several reasons. Firstly, only buyers and sellers within the final merit order can use this strategy, but an individual agent does not know whether it will be included unless it submits a bid. Secondly, agents cannot decide on how much to under/over report without full information of the entire market, which in the residential LEM context the mechanism does not provide complete information. This aspect is illustrated in Figure 5.6 where buyer $b_2$’s bid volume effectively pushes buyers $b_1$ and $b_2$ out of the final allocation. Had $b_1$ and $b_2$ been knowledgeable of the bid of all participants, they would be incentivized to increase their bid to capture positive gains-from-trade. Yet to achieve this would mean to backtrack on the LEM market design and allow for a perfect information game to be played among the agents, which does not provide any practical analytical value nor does it align with the scope of this thesis. Lastly, even if all the agents have complete information about the market, they would still have a difficult time optimizing for the correct amount to under/over report, because all agents now have an incentive to misrepresent their volume and an individual agent must take other’s strategies into account when formalizing its own. This last aspect turns the determination of an equilibrium strategy for all agents into an iterative induction game that becomes infeasible without further information. Thus the authors of [22] cite this indeterminacy as sufficient proof to assume public information of agent volumes.

Traditionally, economist view efficiency as a measure of how much the maximum market value a mechanism can induce. While the inefficiency of Huang’s mechanism at allocating trades becomes pronounced in markets with small population sizes, the mechanism still achieves asymptotic efficiency. Asymptotic efficiency is a theoretical attribute to scenarios where the sample size, which in this case is the population of agents in the market, grows indefinitely. In other words, the mechanism is expected to induce maximum market value when the number of agents reaches infinity. Reaching asymptotic efficiency is the first and foremost objective in mechanism design, while the measured efficiency of the mechanism at sample sizes less than infinity is referred to as the allocative efficiency. Allocative efficiency is defined as the ratio of the sum of utility and profits of agents included in the final trade divided by the theoretical maximum social welfare the market could potentially achieve. For the example in
Figure 5.6, the allocative efficiency is measured at $\frac{140}{290} = 48.3\%$. To help visualize the concept of allocative efficiency, Figure 5.7 shows the relationship between population size and efficiency of Huang’s mechanism between different buyer compositions. Sellers and buyers bids are drawn from independent probability distribution functions (PDFs). Any arbitrary probability distribution function is implementable for the purpose of analysis, the only requirement is that the distribution functions are constrained within the first quadrant to ensure positive sampled values. Firstly, all bidding quantities are both drawn from a standard uniform distribution function supported on the interval $[0,1]$. The sellers’ valuations are drawn from a normal distribution function with mean of $8 \frac{c}{kWh}$. In three separate buyer compositions, buyer valuations are drawn from PDFs with the following mean valuations: 8, 9, an 10 $\frac{c}{kWh}$. Thus the three scenarios provides an indication to the relative rate at which the allocative efficiency increases under increasing average price margins (0, 1, and 2 cents); this margin represents the difference in average valuations between the buyers population and the sellers population. The results show a clear convergence towards asymptotic efficiency, where a larger price margin is observed to converge at a faster rate. Nonetheless, the Huang mechanism displays poor allocative efficiency when the number of agents are low (0-100 agents). This particular property should be carefully considered by the residential LEM operator if the project should expect participation from a relatively low number of residential households.

![Figure 5.7: Allocative efficiency of Huang mechanism with increasing buyer and seller price margins](image)

5.5. Summary

This chapter introduces the main double auction mechanisms which will be explored in the third part of this thesis project. In mechanism design, Theorem 5.1.1 states the impossibility of retaining all desired mechanism objectives mentioned in Section 4.1.4. In terms of prioritizing mechanism objectives, IC and IR are necessary properties that serves the scope of this thesis: achieving truthfulness is the goal of the study, and retaining net positive transfers allow the voluntary participation of rational agents. Among the three mechanisms introduced, the Walrasian mechanism does not satisfy IC, the VCG mechanism is not BB, and the Huang mechanism is not efficient. The properties of each mechanism have been extensively discussed with provided examples. At this point, part two of the thesis has been concluded and we move into part three where the next chapter discusses the experimental setup used to compare mechanism performance.
Research Design

The goal of modeling and simulation is to define and implement a general framework for the multi-unit double auction market design. The model framework needs to capture the mechanism protocols and their underlying assumptions to perform an unbiased comparison, yet remain flexible to be able to explore various scenarios within the context of residential LEMs. Once the model framework is defined, determining an appropriate simulation approach allows the comparison of truthful mechanisms and evaluation of stakeholder benefits.

While models are helpful tools for providing solutions to real-world problems, they remain but an abstraction of reality. The complexities, intricacies and unpredictability of reality will always diminish the validity of the model itself. Thus, modeling the implementation of truthful double auction mechanisms requires careful determination of what constitutes as the “core” of the study and what is superfluous “noise”. Eliminating elements that have negligible influence on the objective of the study enables the modeler to enhance the effects of the core interactions. Developing critical assumptions and specifications of the model environment will simplify the model itself without distorting the results obtained.

The goal of this study is to determine a suitable market mechanism within the context of residential LEMs which defers bad actors from market manipulation. In Chapter 2, a centralized market design for residential local energy markets was proposed. In Chapter 5, we have introduced suitable mechanisms that have the desired incentive-compatible property and their implementation protocols. Having elaborated the specific characteristics of the research problem it is now possible to conceptualize the problem in a model. This chapter serves as an overview of the experimental set-up created for the comparison of different double auction mechanisms. While LEMs could be applied to various grid compositions and residential communities, a microgrid energy market is the chosen composition for the experimental framework of this study. A microgrid is a subsystem within the distribution grid architecture that exists on the neighborhood/district level; the size of these microgrids ranges from the scale of one street block, to a low-voltage feeder, or a low voltage distribution grid under one substation [33]. The main characteristic of a microgrid is a single point of common coupling with a higher level grid, which allows for both grid-connected and islanded operation modes. This type of grid architecture requires LEMs to be implemented. More importantly, the functionality of microgrids aligns with scope of this study, reinforcing the concept of localizing decentralized energy generation and consumption.

6.1. Conceptualization and Model Specification

This section aims to introduce the underlying assumptions made in the context of the residential LEM model and proceeds to clarify important model properties. The model assumptions follow closely the market structure proposed in Chapter 2, which is that of a discrete-time sealed-bid double auction. A residential microgrid is the chosen environment on which the LEM is superimposed, where the physical and temporal dynamics of the real-world system and its participants are removed layer-by-layer through assumptions. Finally, the general procedure of the LEM model is outlined.
6.1.1. Context of the Model
In this section, the main assumptions made within the model are explicitly stated in able to contextualize the simulation. Furthermore, the roles of the auctioneer and market participants within the LEM model are defined.

LEM Model Context:

- Model time frame consists of independent hourly multi-unit double auctions. Bidding, market clearing, and settlement procedures are conducted independently for each time frame. Each round of the auction determines the amount of energy to be physically allocated for the upcoming hour.

- In terms of the microgrid on which the LEM is implemented, we make an assumption that the microgrid is coupled to a national, low-voltage distribution grid. Motivation for local trade within the microgrid is two-fold; firstly, transmission costs within the microgrid will be more efficient due to smaller transmission costs and second, dependence and autonomy of the microgrid is increased, reducing the demand on the national grid. Only in case of depletion or overflow the macro-grid will either provide or absorb energy.

- The physical layer of system balance and delivery of energy is not considered within the model. This model assumes a copper-plate, meaning the LEM is implemented on a microgrid coupled to the macro-grid with instantaneous energy delivery, zero transport losses and no physical constraints. Balance of system frequency, voltage, reactive power and 3-phase power are out of the scope of this study.

- Finally, the most significant assumption is the deterministic nature of renewable generation and residential consumption. Uncertainty and stochastic behavior of supply and demand are removed from the model context. In other words, quantities sellers submit to the auctioneer are exactly the quantities they will generate during energy delivery.

Stakeholder Roles:

- **Auctioneer**: oversees the function and operation of the LEM, enforcing the allocation and payment rules defined by the implemented auction mechanism. Furthermore, the auctioneer treats the auction rounds independently and the solves the scheduling problem one auction round at a time. Combinatorial auctions, where products spanning different time horizons may be offered, are not considered in this study.

- **Traders**: consist of household prosumers and consumers with a demand to either sell or procure energy from the LEM. Traders only interact with the auctioneer and do not conduct bilateral trades among themselves.

General comments on model assumptions:

Notice how these assumptions depart from the observations of the real world problem but instead create an environment solely focused on the microeconomics of truthful mechanisms. This separation from physical constraints and removal of stochastics in renewable generation allow us to merit the property of truthfulness purely on the efficacy of direct mechanisms - agents directly submit their reported valuations based on their marginal costs and utility. This idealized version of the market is useful in providing an upper bound benchmark for comparing market performance among market mechanisms. When integrating physical constraints into the model, distribution and conversion losses contribute to the reduction of overall market liquidity. Additionally, market performance is partially diminished ex-post if the microgrid operator needs to manage congestion during peak hours, or if market participants incorporate the uncertainties of solar generation and demand consumption into their bid determination process.

Looking ahead to possible future work stemming this study, the revelation principle allows the auctioneer to implement indirect mechanisms that incorporate constraints between the physical and virtual layers of energy trading within a smart-grid, while keeping the implementability of the social choice function intact from the original direct mechanisms. In more sophisticated models, the agents’ strategies become dynamic as temporal and stochastic effects of their actions carry over between market periods. But while these are necessary milestones to achieve real-world implementation of truthful double auction mechanisms for residential LEMs, this study lays down the groundwork for these future advancements.
6.1.2. Properties of the Model
The market participants are modeled and their properties explicitly stated. We consider the components that form the private information of market participants essential to their bid determination process, and the market rules which is the public information known to all market participants beforehand.

**Auctioneer Properties:**
- The auctioneer conducts an hourly multi-unit double auction
- Its objective is to maximize social welfare of market participants based on the social choice function:

**Household Properties:**
- Households have discrete demand profiles for each hourly auction period
  - \( q_d = \) consumption scalar value in kWh
  - \( v_d = \) household’s private valuation of consumption in \( c/kWh \) (euro cents per kilowatt hour)
- A percentage of the household population own PV systems
  - Capacity = 5\( kW_p \) for all households
  - \( q_{pv} = \) production scalar value in kWh
  - \( v_{pv} = \) LCOE of PV system in \( c/kWh \)
- All households act rationally and seek to maximize a quasi-linear utility function
  - Utility functions described by Eq. 5.1 and Eq. 5.2
- Note that the quantities are deterministic- representing the absence of stochastic real-time consumption and renewable generation characteristics of real-world scenarios.

**Market Properties:**
- Each hourly auction period consists of 3 main phases: 1) bidding, 2) market clearing, 3) delivery & settlement.
- Auction procedure is performed sequentially (at no point in time does the auctioneer or any household optimize for two or more periods simultaneously). The auctioneer must conclude all phases for the current hour before beginning operations for the next period.
- Each household is only allowed a single sealed bid per market period.
- No entry costs or transaction costs are required to participate in the double auction market.
- Demand response and other flexibility services are not considered.

6.1.3. LEM Procedure
Defining the core assumptions of research design will in turn govern agent behavior and the auction procedures. Thus by incorporating the assumptions made from the previous subsections, we outline the three phases of LEM operation below.

**I Bid Determination Phase:**
1) Households formulate bid quantities by the expected net energy flow at the point of grid connection for the upcoming hour.
   - Prosumers with PV systems will always consume any available solar generation to meet demand: \( q_d - q_{pv} \). Any extra generation will be submitted to the LEM \( q_{pv} - q_d \), while any demand unsatisfied by the PV system will be procured on the LEM \( q_d - q_{pv} \).
   - Consumers without PV systems will always rely on the LEM to satisfy demand \( q_d \).
2) Households formulate their reported valuations to be submitted to the auctioneer, either as a buyer or a seller.
   • For buyers, the goal is to procure energy at a cheaper rate than retail electricity rates.
   • For sellers, the goal is to sell energy at a higher rate than their generator marginal costs.
3) Households submit a single bid to the auctioneer in the form of tuples consisting of a valuation and quantity pair.
   • Seller’s bid: \((s_v, s_q)\), where \(s_q = (q_{pv} - q_d)\), and \(s_v = v_{pv}\)
   • Buyer’s bid: \((b_v, b_q)\), where \(b_q = (q_d)\) or \((q_d - q_{pv})\) and \(b_v = v_d\)

II Market Clearing Phase:
1) Auctioneer opens the market period and accepts bids from agents (households) for the current hour slot.
2) Households announce their private valuation to the auctioneer in the form of the bid tuple \((s_v, s_q)\) or \((b_v, b_q)\).
3) Auctioneer initiates gate closure and implements social welfare maximization according to the auction mechanism’s social choice function \(f\) and payment function \(p\).
4) Bids fall under one of three cases:
   • Within merit order: bid quantity fully allocated
     \(y^* = s_q\), or \(x^* = b_q\)
   • Partially within merit order: bid quantity partially allocated
     \(y^* \leq s_q\), or \(x^* \leq b_q\)
   • Out of merit order: bid quantity fully rejected
     \(y^* = 0\), or \(x^* = 0\)
   • Operator announces the resulting allocation vector \(\hat{a} = [a_1, ..., a_n]\) and payment vector \(\hat{p} = [p_1, ..., p_n]\) for all agents.
   • Note that allocation vector \(\hat{a}\) represents all values from buyers’ allocation vector \(\hat{x}^*\) and sellers’ allocation vector \(\hat{y}^*\).

III Delivery and Settlement Phase:
1) Physical delivery of successful trades within the LEM is assumed to be instantaneous with the announcement of outcome vector \(\hat{a}\).
2) For the households with reported bids unsatisfied (partially or not at all):
   • Unallocated generation will be supplied to the macro-grid at a grid feed-in price.
     \(p_{\text{Feed-in}} \leq p_{\text{LEM}}\)
   • Unallocated consumption will be supplied to the macro-grid at retail rate of electricity (RRoE).
     \(p_{\text{RRoE}} \geq p_{\text{LEM}}\)

It is important to note that the third phase does not constitute as part of the core of this study and is not included in the Monte Carlo simulation. We are concerned with the outcomes of the market clearing phase, which will allow us to compare mechanism performance. The purpose of the third phase is to provide a framework for agents to act rationally within the LEM microgrid context.

6.2. Simulation Approach for Modeling Market Performance
Now that the model protocol has been outlined, the determination of model inputs and outputs is required. The model outputs allow for direct comparison of market mechanism performance while model inputs shape the various scenarios which the LEM could experience. From the auctioneer’s perspective, it is required to consider the content of the bids only after they are reported, thus the bid formulation process is not considered as part of the model and market performance can analyzed from a macro standpoint. From the market participant’s perspective, bid formulation is the essential part of the model.
as it directly determines how the participant performs within the market. The aggregated outcome of each individual's performance thus forms the overall market performance.

To help determine which perspective the model should take, we take this opportunity to recall the goal of the research objective here:

**To compare and determine suitable truthful market mechanisms by incorporating realistic residential household electricity profiles as input for a LEM model.**

The objective of this study is to analyze and compare the market outcomes of multiple mechanisms under different residential LEM scenarios. The simulation approach should include the following aspects:

- The LEM model environment consisting of the market mechanisms introduced in Chapter 5 and the LEM operation procedure outlined in Section 6.1.3.

- Market environment parameters that contribute to the bid determination process, such as LCOE and the retail electricity rate.

- LEM scenario parameters that describe different market configurations, such as the population makeup of participants (i.e. population size) and installed RES capacities participating in the market (i.e. population demographic of RES participation within LEM).

The analysis of market mechanisms has engaged academics to propose several approaches to generate meaningful results in order to validate their designs. Two simulation approaches that are commonly used are Agent-based simulations and Monte Carlo simulations.

**Agent-based:**
Agent-based simulations are particularly useful when the market designer has full understanding of the factors driving rational behavior in market participants. They are capable of including more complex influencing factors and are especially effective by mimicking human behaviors or machine control logic to arrive at optimal bidding strategies\[12\]. In these models, each agent arrives at an optimal bidding strategy by learning from past experiences obtained from the direct interaction with the environment. However, although interesting results on the behavior of the market may be obtained from the approach, these models are less effective in generalizing market performance in various environments as influencing factors differ case-by-case. This simulation approach is therefore not very effective in a general comparison between double auction mechanisms.

**Monte Carlo Methods:**
Monte Carlo (MC) methods are a subset of computational algorithms that use the process of repeated random sampling to make numerical estimations of unknown parameters. They allow for the modeling of complex situations where stochastic variables are involved, and the model outcome is measurable. The uses of MC are incredibly wide-ranging, and have led to a number of groundbreaking discoveries in the fields of physics, game theory, and finance. There are a broad spectrum of Monte Carlo methods, but they all share the commonality that they rely on random number generation to solve stochastic problems. Given a range of values for each variable, a Monte Carlo simulation will randomly select a number within each range, and see how they combine — and repeat the process tens of thousands or even millions of times. Sample variables are randomly drawn from independent probability density functions (PDF). No two iterations of the simulation might be identical, but collectively they build up a realistic picture of the outcome. This differs from deterministic simulations, where the results of the simulation are 100% predictable due the non-existence of random properties within the model, thus rendering Monte Carlo simulations pointless.

For this study, the mechanisms are the primary focus. In a mechanism design environment, the private valuations of agents are variables and often treated as stochastic components describable by random sampling. Accurately modeling the household’s demand profile over a wide range of household demographics poses a huge challenge and often impractical. Additionally, there is a lack of research in quantifying the valuations assigned to residential energy consumption patterns, thus making the bid formulation in agent-based models difficult to validate.
Thus by making reasonable assumptions to the model context, random sampling of the load profiles in terms of valuation and quantity can be constrained to ranges that reflect real-world properties. MC simulations thus prove as a useful tool for the designer to measure KPIs of auction markets without requiring stringent assumptions of agent behavior. For the purpose of this study, a Monte Carlo simulation approach be used to evaluate and compare multiple scenarios within the context of residential LEMs. The model outcomes quantify the utility of market participants and market liquidity under different economic and technical constraints. Since the MC approach focuses on the macro-perspective of the market and clusters individual agent behavior by describing the population via probability distributions, the influence that these parameters associated with the probability distributions have on the market outcome must also be investigated.

6.2.1. Defining Key Performance Indicators of LEM Model for Mechanism Comparison

The focus of this study is on the local energy market. The experimental setup for these auctions involves comparing different mechanisms. The model KPI’s allowing for comparison between double auction mechanisms contain the following metrics:

- **Market Clearing Price** ($MCP_1$, $MCP_2$): to compare the prices of transactions in each mechanism, the average transaction price for buyers and sellers are calculated (note that only the VCG mechanism implements discriminatory pricing).

- **Buyer’s Utility** ($U_B$): measures the savings consumers capture by participating in the local energy market.

- **Seller’s Profit** ($P_S$): measures the profits prosumers capture by participating in the local energy market.

- **Quantity Traded** ($Q_T$): measures the total energy “localized” within the LEM community.

- **Budget Balance** ($BB$): measures the net payment made from the agents to the mechanism. This is the surplus left over from the transactions after all trades between buyers and sellers have been finalized.

Figure 6.1 below illustrates the conceptual framework the experimental set-up.

The following chapter will cover the configuration of the input parameters, LEM scenario outcomes and discussion of results obtained from simulations utilizing the Monte Carlo approach.

6.2.2. Research Tools and Resources

The nature and complexity of the problem requires a great deal of computational resources and solving can therefore not be done by hand. A computer program that can easily process vast amounts of data is therefore required. The proposed mechanisms and the auction model have therefore been programmed in Python v3.7.4. Pyomo [17][18] is used to formulate the LP optimization problem and Gurobi [16] was used as an all-purpose optimization solver.

6.3. Summary

This chapter aims to bridge the theoretical part of the study with the analytical part by first conceptualizing the LEM framework in able to set model boundaries. In line with the overall objective of the thesis project, the components attributed to the private information of market participants are formulated and the auctioneer’s responsibilities within the LEM structure is clarified. Two simulation approaches which allows for different analytical purposes are considered and discussed. The Monte Carlo method is determined as the more suitable simulation approach for comparing the performance of different market mechanisms. We identified LEM model outputs which serve as important metrics on which the mechanism comparison is based. In Chapter 7, an overview of the Monte Carlo simulation is provided and the LEM model inputs will be defined in detail.
Figure 6.1: Monte Carlo Experiment Framework
Research Methods and Input Data

The research methodology consists of two parts: a sensitivity analysis which identifies the influence of model input parameters on model output metrics, and simulations of residential LEM configurations where the influence of the number of prosumers and overall population size on mechanism performance are studied.

7.1. Description of LEM Model Inputs

We first provide a description how the input data is generated and the parameters attributed to the input variables. Referring back to the model procedure in Section 6.1.3, the input variables are defined as the bid valuations and quantities that describe the market participant’s private information, namely: \((s_v, s_q)\) and \((b_v, b_q)\). Because the LEM model takes on a macro perspective on market performance, the input variables describe the characteristics of the market participants as a group rather than as individuals. This is in line with the Monte Carlo simulation, where the input data of each market participant is treated as a sample drawn from a range of possible values describing the group's characteristics. The range of possible values can thus be captured by parameterized probability density functions (PDFs), where the parameters consist of the mean \(\mu\), standard deviation \(\sigma\). Additionally, the PDF exists on an interval \([min, max]\) which act as the boundaries for the range of possible values. An overview of the input variables and their parameters is provided in Table 7.1 below:

<table>
<thead>
<tr>
<th>Input Variable</th>
<th>Input Parameter</th>
</tr>
</thead>
<tbody>
<tr>
<td>(s_v)</td>
<td>(\mu_{sv}, \sigma_{sv})</td>
</tr>
<tr>
<td>(s_q)</td>
<td>(\mu_{sq}, \sigma_{sq})</td>
</tr>
<tr>
<td>(b_v)</td>
<td>(\mu_{bv}, \sigma_{bv})</td>
</tr>
<tr>
<td>(b_q)</td>
<td>(\mu_{bq}, \sigma_{bq})</td>
</tr>
</tbody>
</table>

Table 7.1: LEM model input variables and input parameters

7.2. Monte Carlo Simulation Overview

The first part of the simulation takes advantage of a sensitivity analysis which utilizes a Monte Carlo-based design. The goal of the sensitivity analysis aims to capture the composition of the agents composition and relate its significance to the various mechanism KPIs. A sensitivity analysis will in turn instruct the modellers as to the relative importance of the inputs in determining the output. The analysis focuses on ‘factor prioritization’ of the bid determination process, specifically the mean \(\mu\) and standard deviation \(\sigma\) describing each PDF characterizing the input variables. \(\mu\) and \(\sigma\) are essential parameters which capture buyer and seller characteristics. Take for example the \(\mu\) and \(\sigma\) a distribution from which buyer quantity, \(b_q\), is drawn from. A high \(\mu\) value may represent a residential community where households’ have high energy demand, while a high \(\sigma\) value may represent a diverse LEM community consisting of households of different sizes and consumption patterns. Will a group of buyers with highly diverse bid quantities negatively affect the amount of energy traded within the LEM? Will this have a greater effect on the Huang mechanism or the VCG mechanism? These are just some of
the questions a sensitivity analysis can help address. Because these parameters indirectly influence the output KPIs, this relationship must be investigated. A total of 8 dimensional factors, or parameters, are considered for this part of the Monte Carlo simulation.

In the second part, PDFs contextualized for the residential LEM environment are constructed using parameters reflecting real-world values. Different scenarios involving the microgrid participants and population size are studied. This is important because it allows us to gain insight on how the auction mechanisms differ under various configurations and how trade-offs between mechanism properties (IC, IR, BB, and Efficiency) influences the market outcomes for the market participant and the auctioneer alike. Because the model inputs for part one and two have identical formats (both draw samples from PDFs), model outputs in part two are analyzed and validated with trends found from the sensitivity analysis in part one.

7.3. Part 1: Influence of Input Parameters on Model Output

Assuming \( M, i = 1, \ldots, m \), the demand valuations \( b_{qi} \) and demand volumes \( b_{qi} \) of the \( m \) buyers are drawn from two independent probability density functions \( F(b_{qi}) \) and \( G(b_{qi}) \), both with continuous densities \( f \) and \( g \). We denote \( \mu_{bq} \) and \( \sigma_{bq} \) as parameters for \( F(b_{qi}) \), \( \mu_{bq} \) and \( \sigma_{bq} \) as parameters for \( G(b_{qi}) \).

Assuming \( N, j = 1, \ldots, n \), the supply valuations \( s_{qj} \) and supply volumes \( s_{qj} \) of the \( n \) sellers are drawn from two independent probability density functions \( H(s_{qj}) \) and \( K(s_{qj}) \), both with continuous densities \( h \) and \( k \). We denote \( \mu_{sq} \) and \( \sigma_{sq} \) as parameters for \( H(s_{qj}) \), \( \mu_{sq} \) and \( \sigma_{sq} \) as parameters for \( K(s_{qj}) \).

For the purpose of this simulation, all density functions \( f, g, h, \) and \( k \) are supported on the compact interval \([0,1]\). The sensitivity analysis focuses on adjusting the \( \mu \) and \( \sigma \) parameters of the PDFs to investigate their correlation with the KPIs. Figure 7.1 illustrates an example of a truncated normal PDF and CDF for parameter values \( \mu = 0.5 \), and \( \sigma = 1 \).

![Truncated Normal Distribution: default](image)

Figure 7.1: Truncated normal distribution PDF and CDF

7.3.1. Sensitivity Analysis of Input Parameters

We start with a broader analysis of KPI’s in a general setting. When comparing mechanism performance, detailed models may not capture the full range of insights. Thus, the sensitivity analysis aims to address the following question:

- What are parameters which have the highest impact on KPI’s for each truthful double auction mechanism?
Variance-based Sensitivity Analysis of Input Parameters:
Variance-based sensitivity methods have been a powerful tool for modellers to deal with quantitative uncertainty of non-deterministic input values. They study how the variance of the output depends on the uncertain input factors and can be decomposed accordingly[53]. We follow Sobol’s Monte Carlo-based implementation due to the following advantages:

- Sensitivity measure is model-free, the same sequence of input values can be used for any model.
- Capacity to capture the influence of the full range of variation of each input parameter
- Ability to capture interaction effects among input parameters

The Sobol Method utilizes a quasi-random sequence of values for the Input PDFs

For the complete description of the Sobol Variance-based Method, please refer to Chapter 4 of [53]. The results obtained from performing the variance-based approach are Sobol indices $S_i$. A Sobol index is an arbitrary number that represents the decomposition of the effect each input parameter has on the variance of the model KPI.

Here a brief explanation on the correlation between the variance of a model output, denoted by $V(Y)$, and its Sobol indices is provided. The main idea of variance is based on finding the derivative $dY/dX$ of a model output $Y$ versus an input $X_i$, which can be thought of as a mathematical definition of the sensitivity of $Y$ versus $X_i$. When studying how the mean, denoted as $E(Y|X_1)$, of the output $Y$ changes when input $X_i$ is fixed at a given value over a range of uncertainty, we would then take the variance of this measure over all possible values of $X_i$. This variance is denoted as $V(E(Y/X_1))$. The first-order Sobol index of a model input $X_i$ describes the parameter’s independent influence on output $Y$, formulated as:

$$S_i = V[E(Y|X_i)]/V(Y)$$

(7.1)

Just as important, interactions effects between two parameters $X_i$ and $X_j$ on the variance of output $Y$ can be investigated through the second-order Sobol index described by the following equation:

$$S_{ij} = V[E(Y|X_i,X_j)]/V(Y)$$

(7.2)

Lastly, total effect measure provides the educated answer to the question: ‘Which parameter can be fixed anywhere over its range of variability without affecting the output?’. The total Sobol index for input parameter $X_i$ is formulated as:

$$S_{T_i} = 1 - V[E(Y|X_{-i})]/V(Y)$$

(7.3)

A Sobol index with a high value for an input parameter $X$ indicates it as an influential parameter on output $Y$, while close-to-zero values represent non-influential parameters. Parameter prioritization is thus identified by comparing Sobol indices of each parameter $X$ for the same output $Y$. For this analysis, the Sobol sequence of quasi-random numbers is used as input values, the sequence is generated using Saltelli’s extension of the Sobol sequence[19]. Table 7.2 provides an overview of the input parameters $X$. Table 7.3 reiterates the KPI’s as model outcomes $Y$ for the Sobol sensitivity analysis.

<table>
<thead>
<tr>
<th>$X_1$</th>
<th>$X_2$</th>
<th>$X_3$</th>
<th>$X_4$</th>
<th>$X_5$</th>
<th>$X_6$</th>
<th>$X_7$</th>
<th>$X_8$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\mu_{bV}$</td>
<td>$\mu_{bQ}$</td>
<td>$\mu_{bV}$</td>
<td>$\mu_{bQ}$</td>
<td>$\sigma_{bV}$</td>
<td>$\sigma_{bQ}$</td>
<td>$\sigma_{bV}$</td>
<td>$\sigma_{bQ}$</td>
</tr>
</tbody>
</table>

Table 7.2: Sobol Indices: Input parameters $X$

<table>
<thead>
<tr>
<th>$Y_1$</th>
<th>$Y_2$</th>
<th>$Y_3$</th>
<th>$Y_4$</th>
<th>$Y_5$</th>
<th>$Y_6$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$MC\bar{P}_b$</td>
<td>$MC\bar{P}_2$</td>
<td>$U_b$</td>
<td>$P_3$</td>
<td>$Q_T$</td>
<td>$BB$</td>
</tr>
</tbody>
</table>

Table 7.3: Sobol Indices: Output KPIs $Y$
7.3.2. Investigation of Trends Between Input Parameters and KPI’s
Influence of input parameters are further investigated using a different approach. While the Sobol indices were created by drawing parameters from a uniform distribution, this section describes an ad-hoc approach to observe trends in LEM mechanism KPI’s in response to input parameters when sampled from a truncated normal distribution such as one shown in Figure 7.1.

- What trends can be observed on KPI’s of each truthful mechanism when adjusting parameters one-at-a-time?

The approach consists of fixing all input parameters but one as constants, the parameter under investigation is then adjusted over a range of discrete values. We perform this for all 8 dimensional input parameters of the model across all three double auction mechanisms. This part of the analysis is meant to compliment the results obtained from the variance-based Sobol sensitivity analysis. For each parameter, 500 auction iterations are simulated to ensure integrity of the results obtained. The discrete parameter values for the mean and standard deviation of PDF’s are:

- \( \mu : [0, 0.5, 1] \)
- \( \sigma : [0.01, 0.5, 1, 1.5, 2] \)

A total of \((4 \mu \text{ dimensions x } 3 \mu \text{ values}) + (4 \sigma \text{ dimensions x } 5 \sigma \text{ values})\times 500 \text{ iterations} = 16,000\) simulations were run for the investigation of trends between input parameters and mechanism KPIs.

7.4. Part 2: The Residential LEM Model
This final part of the Monte Carlo simulation attempts to answer the last sub-question:

*How can we create a simulation environment that enables us to compare market mechanisms in the context of residential LEMs?*

Consequently, answering the sub-question will allow us to address the objective of this research:

To compare and determine suitable truthful market mechanisms by incorporating realistic residential household electricity profiles as input for a LEM model.

From Chapter 5, the first part of this objective has been satisfied by introducing truthful mechanisms, but each have their own strengths and weaknesses. This part of the Monte Carlo simulation puts a focus on constructing realistic input data such that model outputs could provide insights on the feasibility and efficacy of implementing truthful mechanisms in a residential LEM context, allowing us to quantify benefits for a market operator to determine the more suitable mechanism to implement. In line with this goal, simulating market operation for an entire year is impractical for the scope of this thesis as it does not provide added value in analyzing the dynamics of mechanism behavior. The mechanism comparison analysis is performed based on a model simulation over a complete day. May 1st was chosen arbitrarily as the simulation date, with the only preference being summertime where the magnitude of solar generation is more pronounced.

7.4.1. Modeling a Residential Microgrid Community
We assume the microgrid coupled to a low-voltage distribution grid implements a Feed-in-Tariff cost support scheme. Since we assume no storage capacity is installed within the microgrid, there are two situations that could occur.

- When the residential community’s aggregate production outstrips aggregate consumption, the excess energy is exported to the LV distribution grid for a rate equal to the Feed-in-Tariff (FiT).
- Contrarily, the microgrid will import energy to satisfy consumers in the case where aggregate consumption outstrips generation. Households will thus be billed the retail energy price for consuming imported energy. We will refer to this as the retail rate of electricity (RRoE).
These conditions incentivize the households to conduct trading within the LEM as the price difference between the FiT and retail rate of electricity provide a margin for profit and utility. These definitions are consistent in the sense that no buyer would pay more than the retail electricity rate it would otherwise purchase from the LV distribution grid. Similarly, no sellers would sell their units cheaper than the incentive (FiT) that they would expect to receive. In summary, the process of agents’ bid formulation is:

- Households enter the market as either Sellers or Buyers, depending on their state of net energy balance.
- Sellers value their energy at prices greater than the FiT.
- Buyers value their energy at prices lower than the RRoE.
- Bids are compiled and fed into the model as input data.

### 7.4.2. Modeling Input Data

As mentioned in previous chapters, the revelation principle allows us to simply model agent preferences by assuming they report their true valuations in a direct mechanism. By drawing random samples from a known probability distribution, we implicitly assign them to individual agents as their true valuations: \((b_v, b_q)\) and \((s_v, s_q)\). Determination of realistic values for the input variables can be broken down into three parts:

- Agent valuations \(b_v\) and \(s_v\) are sampled from predetermined PDFs.
- Household PV generation, \(q_{pv}\), is chosen to be deterministic using meteorological data.
- Household consumption, \(q_d\), is sampled from PDFs created via a kernel density estimation of consumption profiles taken from a load profile generator.

Variables \(b_q\) and \(s_q\) are determined by \(q_d\) and \(q_{pv}\) via the procedure outlined in the Bid Determination Phase in Section 6.1.3. Figure 7.2 illustrates the bid determination process for the sellers, while Figure 7.3 illustrates the process for the buyers.

---

**Figure 7.2: Diagram describing sellers’ bid determination process**
Agent Valuations

The strategic bidding problem, which aims to optimize agent bid valuations $b_v$ and $s_v$, has been extensively researched and modelled using various approaches. As such, the modeling literature can be classified into four different groups of models: 1) optimization models, 2) game-theoretically based models, 3) agent-based models and 4) hybrid models [31]. These approaches would be suitable for the analysis of a single market structure, not for a general comparison between market mechanisms.

Instead, the strategic aspect of bid formulation is removed and this study adopts an approach similar to Gode and Sunder’s work [13] on zero-intelligence (ZI) agents. This approach was used to model continuous double auction markets, where agents do not actively strategize their bid valuations to maximize utility but rather submit valuations randomly by drawing from a known distribution. In this study, we adapt this bidding approach to our static double auction, taking one step further by refining the distributions to reflect how rational agents would submit truthful prices within a residential LEM context. The assumptions are explained and resulting PDFs are presented below:

- **Buyers Valuation PDF $F(b_v)$**: We assume that all participants within the community have similar demographics and thus consumption involves mostly household activities (e.g. appliances). If households were to value their consumption truthfully, then the variation between pricings of submitted bids is assumed to be small. During each auction period, the main factor contributing to price differentiation among households can be viewed as different consumption habits (e.g. households run their critical loads at different auction periods). Taking into account the buyers’ incentives, the probability density function $F(b_v)$ is continuous and supported on the compact interval $[0, RRoE]$, with a mean $\mu_{b_v} = RRoE$ and a standard deviation $\sigma_{b_v} = 0.1$.

- **Sellers Valuation PDF $H(s_v)$**: Pricing of sellers’ bids follows the truthful reporting of marginal costs of PV generation, which in this case it is the LCOE of rooftop PV systems. We take the LCOE as the bid price due to the fact that most FiT schemes offered by the utility companies are lower than the LCOE of rooftop PV. Thus rational sellers would not bid at prices that will not provide a return in investment for their systems. They also will not submit bids priced higher than the RRoE, the reasoning being that rational buyers, in this case, will simply purchase electricity from the macrogrid. Taking into account the sellers’ incentives, the probability density function $H(s_v)$ is continuous and supported on the compact interval $[LCOE, RRoE]$, with a mean $\mu_{s_v} = RRoE$ and a standard deviation $\sigma_{s_v} = 0.01$. Note that the standard deviation $\sigma_{s_v}$ is one magnitude smaller...
than that of the buyers $\sigma_{pv}$ because we assume that the LCOE between different brands of PV systems are much more price competitive and thus have similar LCOE’s.

Figure 7.4 is a visualization of $F(b_v)$ and $H(s_v)$.

![Figure 7.4: Valuation PDF and CDF](image)

Household PV Generation

In the LEM model, the amount of solar generation, $q_{pv}$, at each hour is assumed to be identical for all households. The model framework assumes households owning PV systems have an identical system configuration of 5 kWp. Solar radiation data taken from the KNMI meteorological database is used to estimate the hourly PV production following the simplified equation:

$$q_{pv} = \eta x GH x S$$ (7.4)

The PV system efficiency is denoted by $\eta$. GH is the hourly global radiation in W/m². The system size of 5 kWp is denoted by S. The resulting discrete hourly generation profile for May 1st is tabulated in Figure 7.5.

![Figure 7.5: Hourly energy generation of a 5 kWp PV system](image)
Household Consumption: Hourly Load Profiles and Kernel Density Estimation

Since measured profiles are rarely available, capturing probability density functions that reflect household consumption \( \langle q_t \rangle \) patterns can be constructed using accredited load profile synthesizers. For this study, household load profiles are constructed from a load profile generator called the LPG\[49\]. It takes on a different load modeling approach than device-oriented models such as the CREST demand model\[35\]. The LPG model considers the person as the central element of the model, which associates the person to a number of different desires\[49\]. The basic idea behind the model is that people will do whatever gives them the greatest satisfaction at any given time. The LPG program has been validated with real-world energy consumption data from Germany. An illustration of the model’s propagation of desires to device activation is provided below in Figure 7.6.

![Figure 7.6: LPG model framework](image)

For the LEM model, we utilize LPG’s 60 predefined, validated households based on German statistical data and measurements to build our probability density function. The predefined households contain a wide range of demographics which provides a sufficient action space for the formulation of agents’ bid quantities. More information on the LPG program, household demographics and program outputs are provided in Appendix. While LPG provides high-resolution annual load profiles (1 minute per step), we compile hourly consumption data and limit the time frame to May 1st.

Before transforming the set of consumption data into PDFs, an additional step is taken to further randomize and expand the load profile database from 60 profiles to 5000 using Python’s `random.choice()` function. This procedure dampens the significance of individual household demographics from a system-wide point of view. The database is stored in a pandas.DataFrame as illustrated in Figure 7.7 below.

![Figure 7.7: Python DataFrame of randomly sampled household load profiles](image)
Our goal is to draw samples from a PDF which captures the magnitude (kWh) and likelihood (density) of energy consumption, \( q_d \), for the designated hour. It is evident that custom-made PDFs for each hour are required to sample meaningful input variables for the auction model.

For this purpose, we apply the Kernel Density Estimation (KDE) technique for each hourly consumption data. KDE is a non-parametric way of estimating the PDF of a random variable via data smoothing where inferences about the population are made, based on a finite data set [65]. Without going into much detail, the process involves building a histogram of the finite data-set and applying a kernel function.

The smoothing parameter, also called bandwidth, defines the overall fitness of the kernel function to the plotted histogram and is an important piece of KDE. For the same input data, different bandwidths can produce very different results. For this study, the bandwidth is selected by following a cross-validation approach, where the model is fit to part of the data, and then a quantitative metric is computed to determine how well this model fits the remaining data. Such an empirical approach to model bandwidth selection is very flexible, and can be used regardless of the underlying data distribution. The Gaussian kernel function is chosen for this study and the Python module used for this application is `scipy.stats.gaussian_kde`. The bandwidth calculated after a 20-fold cross-validation (meaning 20 parts of the data-set are fitted and compared) is found to be 0.002. In Figure 7.8, the histogram of consumption at 12AM on May 1st is plotted against the PDF using the KDE approach. Figure 7.9 shows an overview of the PDF’s over an entire day. The PDFs describe the action space in relation to the demand of all agents within the residential LEM.

![KDE and Histogram of Residential Consumption for HR 0: bandwidth = 0.002](image.png)

Figure 7.8: Kernel density estimation of sampled residential household hourly consumption

### 7.4.3. LEM Scenario Configurations

From the point of view of an LEM platform provider, exploring various microgrid configurations allows the problem owner to account for a wide range of grid typologies. Current real-world LEM projects are modest in size, ranging from 10-50 households [66]. Thus it is worth investigating into future scenarios where LEM participants are numbered in the hundreds. Additionally, the share of renewable generation mix within local communities is widely expected to increase as LCOE of rooftop solar decreases annually [24]. Modeling grid configurations with high penetration of RES should The impact of population size and percentage of residential PV penetration will be investigated through the scenario configurations presented below:

- **Base-case:**

---

1Kernel density estimation is a way to estimate the probability density function (PDF) of a random variable in a non-parametric way. `gaussian_kde` works for both uni-variate and multi-variate data. It includes automatic bandwidth determination. The estimation works best for a unimodal distribution; bimodal or multi-modal distributions tend to be oversmoothed.
– Population: 50 households  
– Renewables penetration: 25%, 50%, 75%, 100%

• **Scenario 1:**
  – Population: 100 households  
  – Renewables penetration: 25%, 50%, 75%, 100%

• **Scenario 2:**
  – Population: 500 households  
  – Renewables penetration: 50%, 75%

![Figure 7.9: Kernel density estimation of hourly consumption for an single day](image)

### 7.5. Summary

This chapter served to elaborate and provide clarity on the Monte Carlo simulation methodology and construction of input data. The first part of the simulation consists of a sensitivity analysis of input parameters $\mu$ and $\sigma$ that shape the probability density functions from which the agents’ bid variables $(b_v, b_q)$ and $(s_v, s_q)$ are drawn from. The second part of the simulation description constructs the model environment which reflects a residential LEM microgrid. The scenarios investigating microgrid configurations are defined. From this understanding the conceptual description and logic of the two types of simulations are presented and the methodology, input data generation, and research steps are explained. In the following chapter, the results of this study are presented.
Simulation Results and Discussion

From Chapter 7, we have provided the details of the Monte Carlo simulation methodology. Additionally, the determination of input data for each Monte Carlo simulation has been discussed. This chapter presents the findings obtained from the simulations described in the previous chapter. The results and their implications are discussed to provide an answer to the main research question.

8.1. Evaluation of Variance-based Sensitivity Analysis

In this section, first-order, second-order and total Sobol Indices for each double auction mechanism will be investigated. First-order ($S_I$) and total ($S_T$) indices are presented in the form of heat-maps while second-order interaction effects ($S_{IJ}$) are presented in a tabulated form. The number of combinations consisting of 2 out of the 8 input factors is large and a large portion of interactions have no observed effect on the model metrics, thus only the first 4 interaction effects that have the greatest influence on output variance will be presented. The rest of the interaction effects will be fully recorded in Appendix.

8.1.1. First-Order Sobol Indices

Figures 8.1, 8.2, 8.3, 8.4 and 8.5 show the $S_I$ indices of the KPI's, or metrics, for each mechanism. The first-order index represents the main effect contribution of each input parameter to the variance of the output. A direct comparison between Sobol indices of the same parameter is possible due to the mechanisms using identical parameterized inputs (i.e. Variance of seller's profit $V(\text{Profit})$ due to input parameter $m_{uy}$ can be compared across all three mechanisms). Each row of the heat-map corresponds to the metric being evaluated while the columns represent the Sobol indices of input parameters. The sum of $S_I$ indices across each metric is always less than 1, meaning that the LEM model is non-additive and the input parameters are not independent. Thus there are interaction effects between parameters at play here and we will eventually continue the discussion on interaction effects in the next section with Total-Effect Indices.
8. Simulation Results and Discussion

Figure 8.1: First-Order Sobol Indices of Buyers MCP KPI

Figure 8.2: First-Order Sobol Indices of Sellers MCP KPI
8.1. Evaluation of Variance-based Sensitivity Analysis

**Figure 8.3:** First-Order Sobol Indices of Buyers Utility KPI

**Figure 8.4:** First-Order Sobol Indices of Sellers Profit KPI
Simulation Results and Discussion

Figure 8.5: First-Order Sobol Indices of Quantity Traded KPI

Contribution of $\sigma$ Parameters to Metric Variance:
Generally, the results show clear trends in standard deviation ($\sigma$) parameters across all three mechanisms. The ($\sigma$) parameters, representing the diversity of agents’ desired price and quantity, have a negligible contribution to the variance of model outputs. This infers that market outcomes for the chosen double auction mechanisms are not sensitive to a wide range of bid compositions. Interestingly, this is observed to be true all except for the main effect contribution of parameter $\sigma_{px}$ to the variance of buyers’ utility ($U_B$) and $\sigma_{px}$ to the variance of sellers’ profits ($P_S$). Loosely speaking, the $\sigma_{px}$ and $\sigma_{px}$ parameters dictate the slopes of the demand and supply curves respectively. For example in a single-sided auction market with perfect competition, the slope of supply curve is flat all sellers have identical marginal costs, thus the market clearing price becomes exactly the marginal cost of all agents and no seller gains a profit from the trade. If sellers enter the market with lower marginal costs, the total profit of the sellers becomes greater than zero. As the metrics Utility and Profit measure the market outcomes from a macro perspective, the steepness of the demand and supply curve contributes directly to the amount of welfare extracted by the market participants. We compare the three mechanisms’ $S_1$ indices of the $\sigma_{px}$ and $\sigma_{px}$ parameters for $V(U_B)$ and $V(P_S)$ in Table 8.1.

<table>
<thead>
<tr>
<th>Metric</th>
<th>Buyers Utility: $U_B$</th>
<th>Sellers Profit: $P_S$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Parameter</td>
<td>$\sigma_{px}$</td>
<td>$\sigma_{px}$</td>
</tr>
<tr>
<td>Walrasian: $S_1$</td>
<td>0.102</td>
<td>0.096</td>
</tr>
<tr>
<td>VCG: $S_1$</td>
<td>0.079</td>
<td>0.067</td>
</tr>
<tr>
<td>Huang: $S_1$</td>
<td>0.111</td>
<td>0.111</td>
</tr>
</tbody>
</table>

Table 8.1: Comparing contribution of private valuation parameters $\sigma_{px}$ and $\sigma_{px}$ to variance of buyers’ Utility and sellers’ Profit

The influence of price variation on profits and utility is observed to be weakest for the VCG mechanism. This is attributed to the VCG payment rule, where the gains-of-trade for individual agents are rely discriminatory pricing instead of a uniform pricing. Thus the welfare enjoyed by all agents are not determined by the last pair of buyer and seller at the market equilibrium. In the same line of argument, $S_1(\sigma_{px})$ and $S_1(\sigma_{px})$ is more prominent in the Huang mechanism than in the Walrasian mechanism. This is attributed the Huang pricing rule requiring the sellers and buyers each trade at a different market clearing prices, as compared to the Walrasian pricing rule requiring all trades be conducted at a single uniform price. When taking the average price, the Walrasian mechanism dampens the contribution of the individual valuations of the last matched pair of bids. Thus, the effect of the parameters $\sigma_{px}$
and $\sigmaqv$, which indicates the diversity of bid valuations, indirectly becomes dampened as well.

### Contribution of $\mu$ Parameters to Metric Variance:

When we turn our focus to the mean ($\mu$) parameters, it is clear that these input parameters make the largest contributions to the overall variance of LEM model output metrics for all three mechanisms. The trends in factor prioritization of the metric measuring market clearing price $V(MCP)$ is observed to have the same order in terms of parameter importance, with $\mu_{qv}$ ranked as the most influential on the variance of MCP and followed by $\mu_{bv}$. Tables 8.2 and 8.3 provides an overview of the contributions the 8 parameters have on the variance of profits, utility and quantity traded.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Buyers Utility: $U_B$</th>
<th>Sellers Profit: $P_S$</th>
<th>Quantity Traded: $Q_T$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Walrasian: $S_1$</td>
<td>$\mu_{bv}$ 0.081</td>
<td>$\mu_{qv}$ 0.132</td>
<td>$\mu_{bv}$ 0.272</td>
</tr>
<tr>
<td>VCG: $S_1$</td>
<td>$\mu_{bv}$ 0.064</td>
<td>$\mu_{qv}$ 0.120</td>
<td>$\mu_{bv}$ 0.221</td>
</tr>
<tr>
<td>Huang: $S_1$</td>
<td>$\mu_{bv}$ 0.058</td>
<td>$\mu_{qv}$ 0.131</td>
<td>$\mu_{bv}$ 0.226</td>
</tr>
</tbody>
</table>

Table 8.2: Comparing influence of average bid valuation parameters $\mu_{bv}$ and $\mu_{qv}$ on agent utility and quantity traded

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Buyers Utility: $U_B$</th>
<th>Sellers Profit: $P_S$</th>
<th>Quantity Traded: $Q_T$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Walrasian: $S_1$</td>
<td>$\mu_{bq}$ 0.007</td>
<td>$\mu_{sq}$ 0.140</td>
<td>$\mu_{bq}$ 0.100</td>
</tr>
<tr>
<td>VCG: $S_1$</td>
<td>$\mu_{bq}$ -0.005</td>
<td>$\mu_{sq}$ 0.153</td>
<td>$\mu_{bq}$ 0.092</td>
</tr>
<tr>
<td>Huang: $S_1$</td>
<td>$\mu_{bq}$ 0.022</td>
<td>$\mu_{sq}$ 0.146</td>
<td>$\mu_{bq}$ 0.100</td>
</tr>
</tbody>
</table>

Table 8.3: Comparing influence of average bid quantity parameters $\mu_{bq}$ and $\mu_{sq}$ on agent benefits and quantity traded

On closer inspection, $\mu$ parameters associated with bid valuations ($s_v$ and $b_v$) are more influential than $\mu$ parameters associated with bid quantities ($s_q$ and $b_q$), with the exception of the contribution of parameters $\mu_{qv}$ and $\mu_{sq}$ on $V(U_B)$. While this is anticipated for gains-of-trade in terms of utility and profits, the contribution of $\mu_{bv}$ and $\mu_{qv}$ parameters on the variance of quantity traded $V(Q_T)$ is observed to be greater than the contribution of $\mu_{bq}$ and $\mu_{sq}$ parameters, which are parameters directly associated with the amount of available volume at any time within the market. This infers that truthfulness of valuation has a greater effect on market outcomes than truthfulness in quantity.

The parameter $\mu_{bv}$ contributes most to the variance in sellers’ profits $V(P_S)$, while having less influence on the variance of buyers’ utilities $V(U_B)$. The interesting observation stemming from this is the relative influence $\mu_{bv}$ and $\mu_{qv}$ have on $V(U_B)$ and $V(P_S)$. While it is expected that buyers’ valuations have a more pronounced effect on the seller’s side of the market and vice versa in a buyer’s market, the influence of the $\mu_{bv}$ over $\mu_{qv}$ is much greater in $V(P_S)$ than the influence of the $\mu_{sv}$ over $\mu_{bv}$ in $V(U_B)$. Furthermore, the parameter $\mu_{bv}$ ranks higher than $\mu_{qv}$ in terms of its contribution to the variance of quantity traded $V(Q_T)$. An inference made from these above results could be that the buyers have greater market power in double auction markets.

From Table 8.2, it is also worth noting that the Sobol indices $V(Q_T)$ from the Walrasian and VCG mechanisms are identical, which is attributed to both mechanisms having maximum allocative efficiency. The Huang mechanism’s social choice function is not efficient and thus a greater effect on variance is observed.

### 8.1.2. Total-Effect Sobol Indices

Figures 8.6, 8.7, 8.8, 8.9 and 8.10 show the $S_T$ indices of the model metrics for each mechanism. Generally, $S_{XI} = 0$ implies that an input factor $X_i$ is non-influential and can be fixed anywhere in its distribution without affecting the variance of the output. It can be observed that all parameters have some level of influence on the variance of the LEM model metrics.
8. Simulation Results and Discussion

Figure 8.6: Total-Effect Sobol Indices of Buyers MCP KPI

Figure 8.7: Total-Effect Sobol Indices of Sellers MCP KPI
8.1. Evaluation of Variance-based Sensitivity Analysis

Figure 8.8: Total-Effect Sobol Indices of Buyers Utility KPI

Figure 8.9: Total-Effect Sobol Indices of Sellers Profit KPI
Simulation Results and Discussion

Figure 8.10: Total-Effect Sobol Indices of Quantity Traded KPI

Total-Effect Indices of $V(U_B)$ and $V(P_S)$

When comparing general $S_T$ trends across all three mechanisms, ranking the parameters based on each metric can reveal the total influence each parameter has on each mechanism. Table 8.4 ranks each input parameter in terms of its influence on the variance of buyers’ utility $U_B$, and sellers’ profit $P_S$. The top two most influential parameters for buyers are $\mu_{\nu v}$ and $\mu_{\nu q}$, in that order. The top two most influential parameters for sellers are $\mu_{\nu v}$ and $\mu_{\nu q}$, in that order. This observation is intuitive for each side of the market, as the individual welfare of agents is most reliant on bids from the opposite side of the market to successfully conduct a trade. However, it can be noticed for the case of the VCG mechanism that the standard deviation of buyers valuation, $\sigma_{\nu v}$, has a greater effect on sellers profits than in the Huang mechanism.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>$\mu_{\nu v}$</th>
<th>$\mu_{\nu q}$</th>
<th>$\mu_{\nu v}$</th>
<th>$\mu_{\nu q}$</th>
<th>$\mu_{\nu v}$</th>
<th>$\mu_{\nu q}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\nu$</td>
<td>0.525</td>
<td>0.402</td>
<td>0.540</td>
<td>0.220</td>
<td>0.534</td>
<td>0.220</td>
</tr>
<tr>
<td>$\mu$</td>
<td>0.320</td>
<td>0.320</td>
<td>0.279</td>
<td>0.320</td>
<td>0.279</td>
<td>0.320</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>0.296</td>
<td>0.296</td>
<td>0.318</td>
<td>0.280</td>
<td>0.341</td>
<td>0.318</td>
</tr>
</tbody>
</table>

Table 8.4: Comparing influence of bid price average $\mu_{\nu v}$ and $\mu_{\nu q}$ on agent utility and quantity traded

Total-Effect Indices of $V(Q_T)$

In a similar manner, total effect indices for the metric quantity traded for each truthful mechanism are ranked in Table 8.5 below. Factor prioritization shows the same behavior in both mechanisms, validating the the $\mu$ parameters as the most influential for the LEM model metrics.
### 8.1. Evaluation of Variance-based Sensitivity Analysis

#### 8.1.3. Conclusion for Sobol’s Sensitivity Analysis on LEM Model Metrics

The results from this section suggest that the bid price submitted to the double auction mechanisms have profound effects on market performance in terms of agent revenue and residential LEM energy localization. This reinforces a major inference of this study, which may also seem redundant: *truthful behavior of rational agents when valuing the energy supplied or demanded must be properly incentivized for the market operator to reliably achieve stable market outcomes*. The argument is that if truthful behavior is not incentivized, market outcomes may be extremely difficult to predict unless the platform provider resort to means such as projection of historical data or stochastic forecasting to gauge long term market performance. This inference is thus achieved by the implementation of truthful mechanisms.

#### 8.1.4. Investigation of Trends Between Input Parameters and KPIs

The aim of this trend analysis of PDF parameters was to provide a second set of results which corroborates with the results obtained from the sensitivity analysis of parameters. While the Sobol SA analysis emphasizes on the variance of the output KPIs through a sequence of quasi-random input values, the trend analysis compares normalized output KPIs by relying on a default set of parameters, each time only adjusting one single parameter. Illustrated in Figure 7.1, the default value for all $\mu$ parameters is 0.5, while for $\sigma$ parameters it is 1. The set of KPIs obtained from performing the Walrasian mechanism simulations (with default parameters) are utilized to normalize Huang and VCG mechanism KPIs.

Unfortunately, the results from the trend analysis did not reveal additional insights, but rather confirmed the significance of the $\mu$ parameters. In Figure 8.11, the normalized buyer’s MCP values from the Huang double auction are plotted on the vertical axis respective to the adjusted parameter values plotted on the horizontal axis. The $\sigma$ parameter adjustments are grouped on the left sub-plot while the $\mu$ parameter adjustments are grouped on the right. Figure 8.12 follows the same principle but is conducted for the VCG double auction. In both figures, there is a clear distinction between the magnitudes at which the MCP changes between adjusting the $\sigma$ and the $\mu$ parameters, with the latter having a greater effect. Trends such as the adjustment of $\mu_{eq}$, attributed to the average quantity of sellers’ bids, having an inverse effect on buyers’ MCP for both mechanisms are self-evident (greater quantities offered by the sellers tilt the market towards the buyer’s side, resulting in a lower trading price). The remaining KPI trends for both mechanisms and their brief explanations can be found in Appendix B.
8.2. Residential LEM Scenario Simulations

This section presents the results for a single day, May 1st, of the model simulations. Simulations are conducted separately for each of the LEM configurations described in Section 7.4.3. The Walrasian, VCG and Huang mechanisms are simultaneously run with identical input data in the same simulation for each LEM configuration. Each LEM configuration has been iterated 100 times and the results averaged. This study analyzes the model results mainly from a macro perspective on the market.

A comment that should be made regarding the simulation process is the simulation time. Processing a LEM configuration of 50 households takes approximately thirty minutes, whilst processing 500 households amounts up to over 30 hours of computational time. The huge increase in simulation time is due to the unavoidable computation required to solve the payment function of the VCG mechanism. A LP problem needs to be solved to determine the final payment for each individual agent. Thus for a 500 household model with 100 iterations, the LP problem is solved 50,000 times for a single round of the hourly auction. Therefore, a choice was made to conduct a limited number of simulation configurations with carefully selected model parameters.

8.2.1. Base-Case Scenario with 50 Households

We first illustrate the market outcomes for an entire day for the configuration of 50 households with 50% of the population owning PV penetration. Figure 8.13 shows the quantity traded for each hour as well as the overall aggregate demand and supply offered in the local energy market, providing an indication of the amount of expected energy being imported/exported at the micro-grid interconnection.
8.2. Residential LEM Scenario Simulations

Figure 8.14 shows the development of market clearing price over the entire day. Figure 8.15 shows the results obtained from buyers’ utility while Figure 8.16 shows the results obtained from sellers’ profits. Figure 8.17 provides insights on the expected revenue for the micro-grid operator.

From Figure 8.13, the results from overall quantity traded show a trend of LEM sufficiency primarily dictated by solar generation profile. Peak consumption hours are not aligned with the intermittency of PV generation. It is important to note that the aggregate supplies are the combined bid quantities of the sellers, not the generation of the PV systems themselves. The households first satisfy their own consumption before offering any additional energy in the form of supply bids. This insight strengthens the case for other decentralized energy resources installed within the microgrid. The trends in aggregate supply and demand also show a sharp increase of disproportion between buyers and sellers near the peak hours at 6 AM and 6 PM, due to household activities during those hours. This disproportionate market is then rapidly skewed in the opposite direction due to the fact that half of the population begins to utilize their PV yields, thus the market rapidly transitions from a buyers market to a more balanced state around 10 AM. If household consumption behavior is assumed to follow this predictable pattern, this may create a risk and expose opportunities for households without PV systems to implement de-
mand response measures, such as load shifting, to purchase energy at hours where more supply bids are being offered. This could be seen from Figure 8.14 below where there is a clear dip in price at 10 AM when the market reaches a more balanced state between aggregate supply and demand. At the peak hours of 6 AM and 6 PM, the market clearing price of the Huang mechanism is observed to be exceptionally lower than both Walrasian and VCG mechanisms. But this effect should not be accounted for as true mechanism performance since the market is disproportionately skewed towards the buyers side (refer to Figure 8.13 such that there are essentially no trades being conducted within the LEM.

Figure 8.15: Base-case Scenario: Buyers’ Utility

Figure 8.15 shows the utility of the buyers from successful trades during the market periods. The amount of collective utility is directly influenced by the margin of the LEM lower bound (LCOE) and upper bound (RRoE) of the microgrid environment. From a macro perspective, the VCG mechanism provides buyers the highest total gains-from-trade, this effect is more prominent especially during hours with larger quantities traded.

Figure 8.16: Base-case Scenario: Sellers’ Profit
From Figure 8.16, the same similarities between the mechanisms can be seen for sellers’ profits, where revenue trends closely with the solar generation profile. The slight decrease in buyers’ profit at the 10 AM auction round coincides with the greater amount of surplus generation being offered on the market than volume demanded. Observed in Figure 8.14, this skews the market towards a buyer’s market and thus, in response, market clearing prices become lower.

In Figure 8.17, the amount of net market revenue provides an estimation of the expected return for the microgrid operator. This amount seems almost insignificant in comparison to the investment costs required to set up the microgrid and LEM itself. This insight should be an indication for the LEM operator that implementing a Huang mechanism within the LEM business model should not be targeted towards creating profit from market operations. In the same vein, implementing a VCG mechanism does not require a substantial subsidy as one would think.

Table 8.6 shows the cumulative KPI’s for the entire day over various amounts of renewable penetration levels for the VCG and Huang mechanisms. The VCG mechanism shows advantages over Huang not only in daily cumulative profit and utility but also in the amount of quantity traded as well. Energy localization is calculated by finding the ratio between the amount of energy retained, or ‘localized’, in the microgrid over the total aggregate supply and demand offered in the LEM. This value increases as the percentage of households owning PV systems increase, which is expected. Self-sustainability hits a threshold near 75% renewables penetration, signifying that the liquidity of the market is constrained by the inflexibility of consumption and generation patterns without support from other forms of DERs. The overall difference between the two mechanism’s ability to keep energy localized stays relatively constant at 4%, even as the renewable penetration rate increases, signifying that the degree of renewable penetration does not influence the performance of either truthful mechanisms.

<table>
<thead>
<tr>
<th>PV Penetration</th>
<th>25%</th>
<th>50%</th>
<th>75%</th>
<th>100%</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mechanism</td>
<td>VCG</td>
<td>Huang</td>
<td>VCG</td>
<td>Huang</td>
</tr>
<tr>
<td>$P_S$</td>
<td>€ 0.98</td>
<td>0.60</td>
<td>€ 1.25</td>
<td>0.92</td>
</tr>
<tr>
<td>$U_B$</td>
<td>€ 0.30</td>
<td>0.21</td>
<td>€ 0.72</td>
<td>0.54</td>
</tr>
<tr>
<td>$BB$</td>
<td>€ -0.06</td>
<td>0.15</td>
<td>€ -0.09</td>
<td>0.16</td>
</tr>
<tr>
<td>$Q_T$</td>
<td>kWh 20.41</td>
<td>14.87</td>
<td>kWh 31.81</td>
<td>26.09</td>
</tr>
<tr>
<td>Energy localization</td>
<td>% 15.77</td>
<td>11.49</td>
<td>% 24.88</td>
<td>20.41</td>
</tr>
</tbody>
</table>

Table 8.6: Results of cumulative mechanism KPIs for Base-Case Scenario over increasing levels of renewable penetration
Visualization of Trends in Base-case Scenario KPI's

The trends in LEM model KPIs for each scenario are visualized to highlight the relationship between market performance and renewable penetration rates. Only the trends for truthful mechanisms are shown in this comparison. Figure 8.18 shows a comparison between trends in utilities and profits while Figure 8.19 plots the quantity traded in the market against RES penetration rate.

While in both figures it is clear that the VCG mechanism has superior performance in terms of KPI's, one additional feature the mechanism shows is it's efficiency in allocating maximum social welfare. An intersect point at approximately 90% renewable penetration can be seen where buyers' utility outstrips the sellers' profits. This is attributed to the fact as more consumers become prosumers, the aggregate supply volumes increase during the day and results in more auction rounds becoming a buyers market. Because the LEM model simulation runs both mechanisms with identical sets of input data, a similar intersection point is seen to be forming for the Huang mechanism, yet the inefficiency brought by the trade reduction procedures within its social choice function prevents this intersection point from forming at a lower renewable penetration rate.

![Base Case Scenario: Daily Utility and Profit of VCG and Huang mechanisms](image1)

**Figure 8.18: Base-case Scenario: Utility and Profit**

![Base Case Scenario: Quantity Traded for VCG and Huang mechanisms](image2)

**Figure 8.19: Base-case Scenario: Quantity Traded**
Reflection on Base-Case Scenario Results
A particular point of interest for the microgrid LEM platform provider is Budget Balance. While this revenue and deficit respectively for the Huang and VCG mechanisms are relatively small, it poses the interesting question of whether increasing the population size to a certain degree could bring in enough profit for the microgrid operator to make a justifiable business case around monetizing the Huang mechanism. The same goes for the VCG mechanism: will the deficit grow large enough that subsidizing the trades become financially infeasible for the microgrid operator? This aspect will be investigated in the next section as we increase the population size of the LEM.

8.2.2. Scenarios A and B: 100 and 500 Households
In a similar format as the last section, we discuss the results obtained from performing simulations for Scenario A and Scenario B. Table 8.9 shows the results obtained for the microgrid Scenario A with 100 households participating, and with RES penetration rates of 25%, 50%, 75% and 100%. Table 8.8 shows the results obtained for the microgrid Scenario B with 500 households participating, and simulated only for 50% and 75% RES penetration rates. The main reason behind this is due to the fact that the best results observed in terms of market liquidity occur when at least half the population own rooftop PV systems, the best set of results is observed at 75% renewables penetration. Thus simulating for 25% and 100% penetration rates for Scenario B was deemed impractical due to the computational intensity mentioned at the start of this section.

Since the input parameters of probability density functions are kept constant, the results obtained for daily cumulative values of buyers’ utility, sellers’ profit and quantity traded develop a similar linear relationship across penetration rates for LEMs with larger populations. From a macro perspective, it is expected that more participants means more trade conducted, which leads to larger cumulative utility and profit. The main insight gathered from both Scenario A and B allows us to answer the question raised at the end of the previous section:

*Is the Budget-Balance property of truthful mechanisms a significant factor to be considered for the microgrid operator as population size increases?*

The results show that the revenue received from implementing the Huang mechanism and budget deficit induced by the VCG mechanism does not share relationship with the population size. Even as the quantity traded within the LEM scenarios increases, which is the case for both scenarios, the budget balance property for both mechanisms do not experience much alteration. For Scenario A, the VCG mechanism daily budget deficit caps at 0.11 cents, while the maximum Huang mechanism daily revenue is observed at 28 cents. Comparing the Budget Balance results for 50% and 75% RES penetration across Scenario A and Scenario B shows that the difference is negligible. The reasoning for this development can be found by taking a look at the results obtained from conducting the Sensitivity Analysis at the beginning of this chapter (Section 8.1). Budget balance is linked to buyers’ utility and sellers’ profit, these KPIs are influenced primarily by the pricing of agents’ bids as seen from the factor prioritization of Sobol indices. Thus because the margin of the upper and lower bound of the range of valuations rational agents would report are kept constant over the interval [LCOE, RRoE], the budget-balance property of both mechanisms are thus constrained as well. This insight provides an interesting point of discussion concerning the feasibility of the LEM business model and separately the regulatory policies for potential pricing schemes that could be implemented in the LV-distribution grid.

<table>
<thead>
<tr>
<th>PV Penetration</th>
<th>25%</th>
<th>50%</th>
<th>75%</th>
<th>100%</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mechanism</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>VCG</td>
<td>Huang</td>
<td>VCG</td>
<td>Huang</td>
</tr>
<tr>
<td>$R_B$</td>
<td>ε</td>
<td></td>
<td>€</td>
<td></td>
</tr>
<tr>
<td>$U_B$</td>
<td>0.49</td>
<td>0.40</td>
<td>1.38</td>
<td>1.19</td>
</tr>
<tr>
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<td>0.211</td>
<td>-0.09</td>
<td>0.19</td>
</tr>
<tr>
<td>$Q_T$</td>
<td>kWh</td>
<td></td>
<td></td>
<td></td>
</tr>
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<td>34.23</td>
<td>65.46</td>
<td>59.16</td>
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<tr>
<td>Energy Localization</td>
<td>%</td>
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<td></td>
<td></td>
</tr>
<tr>
<td>15.6</td>
<td>13.2</td>
<td>25.4</td>
<td>22.9</td>
<td>29.8</td>
</tr>
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</table>

Table 8.7: Results of cumulative mechanism KPIs for Scenario A over increasing levels of renewable penetration
8. Simulation Results and Discussion

### Table 8.8: Results of cumulative mechanism KPIs for Scenario B over increasing levels of renewable penetration

<table>
<thead>
<tr>
<th>PV Penetration</th>
<th>Mechanism</th>
<th>VCG</th>
<th>Huang</th>
<th>VCG</th>
<th>Huang</th>
</tr>
</thead>
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<td>$P_T$</td>
<td>€</td>
<td></td>
<td>€</td>
<td></td>
</tr>
<tr>
<td>50%</td>
<td>13.06</td>
<td>12.65</td>
<td>12.47</td>
<td>12.12</td>
<td></td>
</tr>
<tr>
<td>75%</td>
<td>10.34</td>
<td>10.07</td>
<td>10.10</td>
<td>0.20</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$U_R$</td>
<td>€</td>
<td></td>
<td>€</td>
<td></td>
</tr>
<tr>
<td>50%</td>
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<td>6.40</td>
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<tr>
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<td>10.07</td>
<td>10.07</td>
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</tr>
<tr>
<td></td>
<td>$B_B$</td>
<td>€</td>
<td></td>
<td>€</td>
<td></td>
</tr>
<tr>
<td>50%</td>
<td>-0.08</td>
<td>0.20</td>
<td>-0.10</td>
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<td></td>
</tr>
<tr>
<td>75%</td>
<td>1.20</td>
<td>1.20</td>
<td>1.20</td>
<td>1.20</td>
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</tr>
<tr>
<td></td>
<td>$Q_T$ kWh</td>
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</tr>
<tr>
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</tr>
<tr>
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<tr>
<td></td>
<td>Energy Localization %</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>50%</td>
<td>12.93</td>
<td>12.67</td>
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</tr>
<tr>
<td>75%</td>
<td>15.12</td>
<td>14.8</td>
<td>14.8</td>
<td>14.8</td>
<td></td>
</tr>
</tbody>
</table>

**Visualization of Trends in Scenario A KPIs**

Finally, the developments in KPI's for Scenario A are visualized in this section to show the relationship between market performance and renewable penetration rates. Scenario B is purposefully excluded due to the predictable relationship developed between the three scenarios. Figure 8.20 shows a comparison between trends in utilities and profits while Figure 8.21 plots the quantity traded in the market against penetration rate. Parallels can be drawn from the figures below and Figures 8.18 and 8.19 from the previous section.
The main insight gained from increasing the population are the intersection points in Figure 8.20 where cumulative utility of the buyers is equal to the cumulative profits of the sellers. Compared to an LEM with fewer agents, the intersection point is reached at approximately 90% renewables penetration. The effects from the trade reduction of the Huang mechanism can be observed to be reduced. This observation confirms that the Huang mechanism will indeed reach asymptotic efficiency as the number of agents within the market reaches infinity.

8.2.3. Implication of Results from LEM Scenario Simulations

We reiterate the crucial results obtained from running simulations with different residential microgrid LEM configurations across the three mechanisms. This will allow us explicitly answer the main research question:

Which market mechanism is suitable for a residential local energy market which defers non-truthful behavior of market participants?

Evaluation of Mechanism Budget Balance Between Population Sizes

Table 8.9 compiles the expected net revenue the microgrid LEM platform provider can expect to gain from various LEM configurations. While the results are idealized due to the many simplifications made to the LEM model, it reveals important attributes regarding the Budget Balance property of the truthful mechanisms. Under the pretense that all agents are rationally revealing their truthful valuations directly to the auctioneer, the resulting budget revenue from the Huang mechanism and deficit incurred by the VCG mechanism is constrained by the available margin at which the market participants can conduct trade. This margin is formed by the policy and regulatory framework of the LV-distribution grid with which the microgrid is connected to. A similar effect applies even if, instead of existing within the microgrid, the LEM is superimposed on the distribution grid itself. To solve the budget deficit issue for the VCG mechanism, the platform provider could consider implementing a membership scheme for households wishing to participating in the LEM.

<table>
<thead>
<tr>
<th>PV Penetration</th>
<th>25%</th>
<th>50%</th>
<th>75%</th>
<th>100%</th>
</tr>
</thead>
<tbody>
<tr>
<td>50 HH</td>
<td>€</td>
<td>-0.06</td>
<td>0.15</td>
<td>-0.09</td>
</tr>
<tr>
<td>100 HH</td>
<td>€</td>
<td>-0.05</td>
<td>0.211</td>
<td>-0.09</td>
</tr>
<tr>
<td>500 HH</td>
<td>€</td>
<td>-0.08</td>
<td>0.20</td>
<td>-0.10</td>
</tr>
</tbody>
</table>

Table 8.9: Budget Balance: Cumulative results from all scenarios

8.2.4. Evaluation of LEM Energy Localization Between Population Sizes

Here we compare the truthful mechanisms by measuring their effectiveness in localizing energy flows within the community. Trends are observed as the population size increases. We have seen in the previous sections in the Base Scenario and Scenario A that energy localization of the microgrid plateaus after $\frac{3}{4}$ of the population consists of prosumers. Figure 8.22 plots the trend in energy localization by fixing the renewable penetration rate at 50% constant while adjusting the population size itself. Figure 8.23 shows a similar comparison but with 75% of the population as prosumers.

It is important to note that the observed drop in percentage of energy localized as population size increases is mainly attributed to the imbalance between supply and demand at peak hours. This is a direct effect of the state of the market make-up and reinforces the need for DER’s and ancillary services, but does not diminish the effectiveness of the mechanisms themselves. That being said, the VCG mechanism again shows superior performance at all LEM configurations in energy localization within the microgrid community.
Comparison of VCG and Walrasian Mechanism

In the previous sections, we have found that the VCG mechanism produces superior market outcomes in terms of the following metrics: Buyers’ Utility, Sellers’ Profit, and Quantity Traded. Thus it can be concluded that the VCG mechanism is a more suitable incentive compatible mechanism to be implemented for an LEM. We take one further step in the evaluation by comparing the simulation results from the non-truthful Walrasian mechanism with the VCG mechanism. Since both Walrasian and VCG mechanisms are efficient, the quantity traded at all times will be identical, and the only difference in the market outcomes between the two mechanisms are the prices at which the households trade at. Thus, we investigate the trends in buyers’ utility and sellers’ profits from the resulting simulations conducted. Figures 8.24 and 8.25 illustrates the trends in gains-of-trade over increasing RES penetration rates for the Base-case Scenario and Scenario A, respectively. It can be observed that the VCG mechanism is “at least as good as” the Walrasian mechanism in terms of utility experienced by the residential community. Thus the argument could be made that the VCG mechanism should be implemented over an
untruthful mechanism such as the Walrasian mechanism. The LEM platform provider simply needs to implement a business model that accommodates for the minor budget deficit incurred from the VCG implementation.

Figure 8.24: Comparison of Walrasian and VCG mechanism performance, Base Case: Utility and Profit

Figure 8.25: Comparison of Walrasian and VCG mechanism performance, Scenario A: Utility and Profit

8.3. Summary

In this chapter, the results of the Monte Carlo simulations were presented and their implications discussed in-depth. The results obtained from the Sobol Sensitivity Analysis show a high influence the parameters $\mu_{sv}$ and $\mu_{bv}$ have on the variance of LEM model output KPIs. This reinforces the importance of implementing truthful mechanisms for the LEM platform provider to be able to project long-term performance of the market. The results from the simulations conducted for various residential LEM scenarios indicate that the VCG mechanism is the more suitable incentive-compatible mechanism to be implemented due to superior performance in most mode output KPIs. When comparing the VCG mech-
anism to the non-truthful Walrasian mechanism, it is observed that the VCG mechanism is "at least as good" in terms of providing utility for the microgrid LEM participants.
Conclusions and Recommendations

9.1. Research Objective
The thesis is written in the context of a world in which distributed residential renewable energy sources, guided by economic and political goals, assumes an increasing share of the energy mix within the European energy sector. Given its potential economic, technological and social benefits, LEMs could take on a significant role in the localizing power flow, increasing local renewable autocracy, empowering the residential agents to develop from passive price takers towards active market participants, and supporting the overall decentralization of the future electricity system. Thus, the electricity transition from the former fossil fuel-based generation structure towards a sustainable, renewable-based, multidimensional system becomes feasible. The study primarily focuses on the micro-economic theory of LEM mechanisms with the objective of inducing truthful behavior among market participants.

LEMs are online virtual markets involving competitive behavior between rational and self-interested agents with a different objective than the platform provider that oversees the operation of the market. These specific characteristics determined game theory and mechanism design theory as suitable tools to implement market mechanisms which aligns with the goals of this research. By capturing LEMs in a game theoretic framework, identifying suitable auction mechanisms and reflecting close-to-real-world electricity profiles of residential consumers and prosumers, this thesis sought to provide insights on macro perspectives of this market. The macro perspective takes on the standpoint of the LEM platform provider and examined the benefits and drawbacks in implementing the chosen truthful double auction mechanisms.

This resulted in a research approach which attempts to use a Monte Carlo method to identify influential parameters in the bid determination of the market participants such that the intended market outcomes are achieved. The research objective is therefore formulated as:

To compare and determine suitable truthful market mechanisms by incorporating realistic residential household electricity profiles as input for a LEM model.

9.2. Answers to the Research Questions
In this section, a summary of the main findings which addresses the research sub-questions is provided. The sub-questions serve to compartmentalize the main research objective into manageable milestones. By addressing these sub-questions, the theoretical part of the study, which clarifies on the scope of the research, is merged with the analytical part such that the research objective could be achieved. In relation to the research objective, it is necessary to first give a clear description of this problem. This resulted in the formulation of the first sub-question:

• SQ 1: What are the main characteristics of LEMs and the stakeholders involved and What type of market design best describes residential local energy markets?

In Chapter 2, a literature review was conducted and the main characteristics and stakeholders of LEMs are identified. The general market design framework of LEM is proposed as a discrete-time
Conclusions and Recommendations

sealed-bid double auction. Simplifications made to the market framework allows us to align the market structure (the market rules and market mechanism) with the desired agent behavior to achieve the intended market outcome. In terms of market structure, the most important stakeholders of the LEM investigated in this study are: consumers who aim to minimize consumption costs, prosumers who aim to either maximize profits or minimize consumption costs, and the auctioneer who aims to maximize total market value.

In order to motivate the choice for selecting mechanism design theory as the main approach for this study, the second sub-question is formulated:

- **SQ 2:** What are the key concepts of game-theory and mechanism design and can these elements of economic theory be used to induce truthful behavior in the design of a residential local energy market?

Chapter 3 served as a primer on the field of game theory and introduced the necessary conceptual building blocks in order to formulate the LEM structure as a game. In terms of the proposed LEM framework, the strategic form was determined to be the more suitable game representation. Furthermore, the LEM is formulated as a non-cooperative game played between residents within a community looking to either buy or sell up to a certain amount of energy. Every household is self-interested with a goal of gaining maximum profits for selling or maximum utility for purchasing. Each household holds some private information and thus have incomplete information about the game environment. Furthermore, households have imperfect information about the market because their own historical energy usage is considered private information and unknown to other households. The bids that households submit to the auctioneer in each round of the double auction is considered as pure strategies, and the best response strategy of each household should produce a competitive market equilibrium. Formulation of the LEM structure as a game requires us to choose a solution concept, or equilibrium concept. From a macro perspective, the dominant strategy equilibrium is chosen due to the powerful property in which each agent’s best response strategy is to reveal its private information truthfully. Yet a game theoretical approach is not sufficient for us to ensure that a dominant strategy equilibrium exists, because the market rules have not been defined. We continue the discussion of inducing truthful behavior in market participants from a completely different approach in Chapter 4. While game theory is concerned with agent behavior in equilibrium (and thus the resulting strategy that it should play), mechanism design theory allows us to construct market rules to induce a game among the agents in a way that in an equilibrium state of the induced game, the desired system-wide solution is implemented. The mechanism design objectives that align with the goal of the research are identified as incentive compatibility and individual rationality. Incentive-compatible dominant-strategy mechanisms are chosen as applicable market mechanisms to be implemented in the LEM auction design. At this point, the second sub-question has thus been answered.

In Chapter 5, the usage of mechanism design theory results in the determination of two incentive-compatible double auction mechanisms, as well as an untruthful mechanism which serves as a point of reference. The Walrasian mechanism is not an incentive-compatible and serves as a benchmark for the comparison of truthful mechanisms. The impossibility theorem proposed by Myerson and Satterwaite creates a trade-off in achieving desired mechanism properties. The two incentive-compatible mechanisms explored in this study are the VCG mechanism and the Huang mechanism. The VCG mechanism is mathematically elegant in its ability to process offers with perfect efficiency and dominant truth-revealing strategies, yet it requires the market operator to subsidize trades. The Huang mechanism provides the market operator with a revenue for conducting trades, but its allocation rule requires a trade reduction which reduces its efficiency.

Now that the theoretical aspects of the study has been addressed, Chapter 6 begins the formulation of the research design and methodology required to compare the mechanisms in a way which reflects the research objective. Thus the last sub-question of this study is formulated:

- **SQ 3:** What are the main factors that influence market mechanism performance and how can we create a simulation environment that enables us to compare market mechanisms in the context of residential LEMs?

The double auction market design proposed in Chapter 2 is further specified in able to conceptualize the model. Modeling the implementation of truthful double auction mechanisms requires careful
9.3. Conclusion to the Main Research Question

The competitive environment, the intrinsic susceptibility to market failure, and lack of attention in current research on the implementation of market mechanisms in local energy markets constituted as the main motivations behind the main question of this research:

*Which market mechanism is suitable for a residential local energy market which defers non-truthful behavior of market participants?*

From the results of simulations of a LEM superimposed on a grid-connected microgrid, the VCG mechanism outperforms the Huang mechanism in all KPI’s other than Budget Balance. Thus the analysis has shown that the VCG mechanism is the more suitable choice for LEM implementation. The main drawback of VCG experienced in this study is the computational intensity required to solve LP problems, and is especially so when dealing with combinatorial double auctions (auctions where different commodities are being exchanged such as heat and power markets). This needs to be addressed for practical implementation of double auctions using this mechanism. Studies have addressed this issue by introducing approximation techniques [40] that allow for constructing more computational efficient VCG mechanisms while retaining the truthful property.

In relation to the implementation of an untruthful mechanism, the VCG mechanism is “at least as good as” the Walrasian mechanism in terms of monetary utility brought for the residential community. Buyer utilities and seller profits are observed to be greater for the case of the VCG mechanism, this is attributed to the VCG payment rule described in Equations 5.6 and 5.7. While both the Walrasian and VCG mechanisms achieve maximum efficiency when implementing the social choice function, the VCG mechanism creates a budget deficit for the auctioneer, and this amount is directly translated into the additional utility and profit enjoyed by the agents included in the final trade. While this deficit implies that a subsidy is necessary for the operation of the market implementing a VCG mechanism, we can
view this as the “cost of truthfulness” that the auctioneer has to pay for reducing the risk of manipulation and market failure.

The implication that the platform provider runs a deficit raises the question of why a market operator would ever want to implement the VCG mechanism. From a business model standpoint, membership schemes to for participants to join the LEM in the form of a subscription fee could offset this deficit while retaining the strategy-proof property. The fact that the VCG deficit is constrained by the available profit margin of the upper bound macro-grid price and and lower bound of LCOE means that the auctioneer can accurately estimate the deficit incurred and redistribute it within the payment scheme of the subscription model. From a policy standpoint, the subsidy could also be provided directly by the government or the public sector as part of the realization of the electricity transition towards a more sustainable future. The potential economic, technological, social and environmental value that LEMs potentially offers far outweighs the observed “cost of truthfulness”.

### 9.4. Recommendations for Future Work

The results of this study has provided many insights on the viability and performance of implementing truthful mechanisms. However, the implications drawn from these results can be greatly advanced by further research as more attention is required to expand the accuracy and realism of the simulation model.

With regard to the current LEM simulation model and the design framework, the introduction of other DER technologies within the residential household will add a level of complexity within the model that brings it closer to realistic settings. It is becoming increasingly commonplace for residential households to invest in energy storage systems (ESS) as a stand alone product or in conjunction with PV systems. Market trends show that a combination of declining costs and governmental subsidies are responsible for the boost in residential ESS in recent years [60]. Thus ESS is projected to play a large role in future electricity systems in terms of providing much needed flexibility and real-time balancing services. It also provides interesting interactions among market participants during trading hours where there is a large amount of surplus consumption or generation. As mentioned in Section 9.2, the intermittency of solar generation creates periods during the day where the majority of the household demand could not be met locally or that surplus in solar energy are forced to be redistributed elsewhere. At competitive prices for ESS, it market participants may be incentivized to leverage their ESS in the LEM in able to increase their total utility. The inclusion of ESS introduces another layer of complexity to the simulation model, such as consideration of trading time horizon in the agents’ optimization problem. This will drastically alter agent behavior, where optimization is not just constrained the current static auction but includes the dynamic programming over a time horizon. Regarding how this addition will effect the methodology of the research design, a separate battery optimization model will be required as part of the Bid Determination Phase of the LEM model. Additionally, agent identities are required to be indexed and stored within the model such that individual market outcomes of each auction iteration will be mapped to the respective agent IDs. Individual market data is imperative for the optimization of the battery model respective to households with ESS installed.

In terms of the market participants, we recommended to incorporate a wider range of possible residential level actors within the LEM. While this study focused on residential households of different consumption requirements, other potential players include schools, store-fronts, hospitals and public spaces, and they serve as integral parts of the residential built environment. These market participants can impact the market outcomes due to each having unique consumption profiles and private valuation of electricity. More importantly, these actors are larger in scale compared to the residential households have capability of exercising market power. Thus the implications of the importance of strategic behavior become ever more apparent. Exploring how truthful mechanisms perform in these diversified local settings can provide valuable insights on mechanism feasibility. This addition will not drastically alter the current methodology of the experimental set-up. Rather a separate set of generation data and KDE function would be required for each type of LEM actor. Including these in the Bid Determination Phase of the market model and defining the necessary parameters in the LEM Composition Parameters is sufficient.

As electricity is a physical good that cannot be stored without (high) costs and requires a physical grid for delivery, market models either directly include representations of the physical grid system, or assume a congestion-free (copper-plate) grid system [62]. In able to keep the focus on the household
behavior as well as market mechanism performance, this study considered the copper-plate model and removed the stochastic properties of electricity generation and consumption. A suggestion in future work regarding incorporating the physical layer with the LEM virtual layer within a model simulation is the interfacing of analytical software such as PowerFactory alongside Python. As to modeling the stochastic properties of household consumption and generation, this may be more challenging. While generation forecasting is an extensively studied field, there has been little research done on residential consumption forecasting. Additionally, obtaining real-world smart-meter data in a residential distribution systems is challenging as this data is usually owned by utilities and treated as intellectual property. Yet development in these two areas will result in a more accurate depiction of residential LEM operation within the future electricity system.

Another area of further research involves the development of residential bidding strategies. The liberalization introduced competition into electricity markets before the turn of the millennium. With the new paradigm shift towards the decentralization of energy generation, LEMs will in turn deal with the same challenges as established electricity markets. As households are typically non-experts in electricity trading, the development of residential bidding strategies becomes more pressing. As electricity is considered a low involvement good, households require automated bidding strategies that take over the LEM bidding process. Incorporating agent-based models that strategically formulate agent bids and combining it with the existing LEM model simulations is the logical next step in achieving a more comprehensive understanding of market dynamics. Substituting such a sophisticated model in place of the PDFs that determine buyers and sellers reported valuations during the Bid Determination Phase would be indeed an interesting game-theoretical approach in exploring residential energy markets.

It is important to highlight the fact that mechanism design is still a relatively new field of study, especially in the energy sector partly due to the numerous regulatory and physical constraints that are necessary for market operation. There are constantly new auction mechanisms being proposed, and this study has covered only two. A suggestion would be to adopt other dominant-strategy incentive-compatible mechanisms into the current LEM framework such as the MUDA [55] or McAfee [34] mechanisms. The current research methodology can be used to implement these mechanisms without any alterations.

A final area of further research involves regulatory policies which enable LEMs to fit in with the current energy sector. Take the Dutch electricity market for example. Even with the liberalization of the Dutch electricity market in 1998, the retail electricity sector is still heavily regulated and populated by only a few incumbent utilities[58]. The ownership structure revolving around Balance Responsible Parties allows some room for LEMs to enter the market, yet the role that LEMs have in the current Dutch electricity market structure is still vague. As mentioned in [36], a compatible legal environment is most important for real-life implementation of LEMs. Additionally, regulations differ from country to country, and finding a one-size-fit-all LEM solution is very unlikely. Thus studying, identifying and developing key regulatory policies which facilitate the diffusion of LEM into the energy sector is just as important as the development of the LEM itself.
Implementability of Mechanisms

In this section, the concept of social choice function and implementability of a direct mechanism will be discussed through the use of a simple example: a single sided-auction market dealing with the procurement of an indivisible object. The example closely follows the one provided in Chapter 15 in [43].

A.1. An Example: Procurement of an Indivisible Object

We consider a buying agent (from now on referred to as agent 0) and two selling agents (referred to as agent 1 and agent 2), such that we have 3 participants in the market. So we have \( N = \{0, 1, 2\} \). An indivisible object is to be procured from one of the sellers in return for a monetary payment. We can represent the entire outcome space of this situation by \( x = (a_0, a_1, a_2, p_0, p_1, p_2) \). The numbers \( a_0, a_1, a_2 \) indicate the allocations of the trade and \( p_0, p_1, p_2 \) represent the payments made by each participant. For the sellers \( i = 1, 2 \), \( a_i = 1 \) if the object is procured from seller \( i \) and 0 otherwise. For the buyer \( i = 0 \), \( a_0 = 0 \) if buyer receives the object and consequently \( a_0 = 1 \) if the buyer does not receive the object; the logic behind setting the values for the buyer in this way will become apparent.

The set \( X \) of all feasible outcomes can be described by the following:

\[
X = \{(a_0, a_1, a_2, p_0, p_1, p_2) : a_i \in \{0, 1\}, \sum_{i=0}^{2} a_i = 1, p_i \in \mathbb{R}, i = 0, 1, 2\}
\]

Notice how the term \( \sum_{i=0}^{2} a_i = 1 \) must always equal 1, attributed to the fact that the total quantity exchanged in this situation should be 1. The three scenarios in terms of \( \{a_0, a_1, a_2\} \) that can happen are thus:

- \( \{1, 0, 0\} \): No procurement has been made by the buyer (Buyer 0)
- \( \{0, 1, 0\} \): Agent 1 (Seller 1) wins the trade and sells its object to Buyer 0
- \( \{0, 0, 1\} \): Agent 2 (Seller 2) wins the trade and sells its object to Buyer 0

For \( x = (a_0, a_1, a_2, p_0, p_1, p_2) \), the utilities of the selling agents 1 and 2 can be formulated as:

\[
u_i(x, v_i) = u_i((a_0, a_1, a_2, p_0, p_1, p_2), v_i) = -a_i v_i + p_i; \quad i = 1, 2
\]

where \( p_i \in \mathbb{R} \) can be viewed as seller \( i \)'s valuation of the object, or its willingness to sell. In line with rational behavior, this value must always be positive. We make a few further assumptions regarding valuations:

- The buyer has a known value \( v_0 \) for the object. This valuation does not depend on the choice of the seller from whom the item is purchased.
- In game theory, valuations \( v_1 \) and \( v_2 \) are referred to as the types of Sellers 1 and 2, respectively.
We now provide two social choice functions that can be applied to this simple auctions and then show how one of the example is implementable while the other isn’t the case. Let us consider the following social choice function (SCF1):

- Buyer 0 buys the object from the seller with the lowest willingness to sell. If both the sellers have the same type, the buyer will buy the object from Seller 1.
- Buyer 0 pays to the allocated seller, say seller $i$, amount equal to $v_i$.

The above social choice function $f(v) = (a_0(v), a_1(v), a_2(v), p_0(v), p_1(v), p_2(v))$ can be written as a list of the following relationships:

- $a_0(v) = 0$
- $a_1(v) = 1$ if $v_1 \geq v_2$
- $a_1(v) = 0$ if $v_1 < v_2$
- $a_2(v) = 1$ if $v_1 > v_2$
- $a_2(v) = 0$ if $v_1 \leq v_2$
- $p_1(v) = a_1(v)v_1$
- $p_2(v) = a_2(v)v_2$
- $p_0(v) = -(p_1(v) + p_2(v))$

Now we consider another social choice function, which has the same allocation rule as the one we have just stated, but has a different payment rule. The buyer now pays the winning seller a payment equal to the second lowest willingness to sell. The new social choice function (SCF2) will be the following:

- $a_0(v) = 0$
- $a_1(v) = 1$ if $v_1 \leq v_2$
- $a_1(v) = 0$ if $v_1 > v_2$
- $a_2(v) = 1$ if $v_1 > v_2$
- $a_2(v) = 0$ if $v_1 \leq v_2$
- $p_1(v) = a_1(v)v_2$
- $p_2(v) = a_2(v)v_1$
- $p_0(v) = -(p_1(v) + p_2(v))$

Notice how the only difference between the two social choice functions is just the change in the payment rule (highlighted in bold). While this may seem like a small adjustment, it greatly affects how the agents choose their strategies. We now investigate the implementability, through direct mechanisms, of SCF1 and SCF2.

A.2. Implementability of SCF1
Let us assume that the agent types $v_1$ and $v_2$ are drawn independently from a uniform distribution over interval $[0,1]$. These samples will be treated as true valuations, or the private information, of Seller 1 and Seller 2. The following analysis show that SCF1 is not implementable.

Suppose Seller 2 announces his true value $v_2$. Let us say the valuation of Seller 1 is $v_1$ and he announces $\hat{v}_1$. If $v_1 \geq \hat{v}_1$, then Seller 1 is the winner and his profit will be $v_1 - \hat{v}_1$. If $v_2 < \hat{v}_1$, then Seller 2 is the winner and Seller 1’s profit is zero. Since Seller 1 wishes to maximize his expected utility, he solves the problem
\[ \max_{v_1} (v_1 - \hat{v}_1) P \{ v_2 \geq \hat{v}_1 \} \]

Since \( v_2 \) is uniformly distributed on \([0,1]\),
\[ P \{ v_2 \geq \hat{v}_1 \} = \hat{v}_1 \]

Thus Seller 1 tries to solve the problem
\[ \max_{v_1} (v_1 - \hat{v}_1) \hat{v}_1 \]

The solution to this maximization problem is
\[ \hat{v}_1 = \frac{v_1}{2} \]

Thus if Seller 2 is truthful, the best response for Seller 1 is to announce \( \frac{v_1}{2} \).

Similarly if Seller 1 always announce his true valuation \( v_1 \), then the best response for Seller 2 is to announce \( \frac{v_2}{2} \) when his true valuation is \( v_2 \). As one can see, there is no incentive for the Sellers to announce their true valuations. An auctioneer who wishes to realize the above social choice function finds that rational players will not reveal their true private values. Thus SCF1 is not implemented.

A.3. Implementability of SCF2

Now let us consider the other case and show that SCF2 can be implemented. Suppose \( v_i \) is the true valuation of Seller \( i \) (\( i = 1, 2 \)). Let us say Seller 2 announces his valuation as \( v_2 \). In this social choice function, there are two cases that will arise: (1) \( v_1 \leq \hat{v}_2 \) and (2) \( v_1 > \hat{v}_2 \).

Case 1: \( v_1 \leq \hat{v}_2 \)

Let \( \hat{v}_1 \) be the announcement of Seller 1. Here we consider the two cases again:
- If \( \hat{v}_1 \leq \hat{v}_2 \), then Seller 1 wins and his payoff is \( \hat{v}_2 - v_1 \geq 0 \).
- If \( \hat{v}_1 > \hat{v}_2 \), then Seller 1 loses the auction and his payoff is 0.
- Thus in this case, the maximum payoff possible is \( \hat{v}_2 - v_1 \geq 0 \).

If Seller 1 announces his true valuation \( \hat{v}_1 = v_1 \), then Seller 1’s payoff equals \( \hat{v}_2 - v_1 \), which happens to be the maximum possible payoff shown. Thus announcing \( v_1 \) is a best response for Seller 1 whenever \( v_1 \leq \hat{v}_2 \).

Case 2: \( v_1 > \hat{v}_2 \)

Once more we consider the two cases:
- If \( \hat{v}_1 \leq \hat{v}_2 \), then Seller 1 wins and his payoff is \( \hat{v}_2 - v_1 < 0 \). This will result in Seller 1 receiving negative profit (he will trade at a loss).
- If \( \hat{v}_1 > \hat{v}_2 \), then Seller 1 loses the auction and his payoff is 0.
- Thus in this case, the maximum payoff possible is 0.

If he announces his true valuation \( \hat{v}_1 = v_1 \), the payoff for Seller 1 is 0, which in this case is his best response.

Thus from this section we can see that no matter what Seller 2 reports, the optimal response for Seller 1 is always to report his true valuation. In the same line of argument, Seller 2 will rationally report his true valuation no matter what Seller 1 reports. Formally speaking, a weakly dominant strategy for both sellers is to report their true valuations. We say ‘weakly’ due to the fact that the dominant strategy is ‘at least as good as’ the second best strategy. Our analysis has brought us to the conclusion that SCF2 can be implemented even though the valuations of the agents are private information.

A.4. Summary

In this Appendix we have first introduced a simple auction for the procurement of a single object. The auction procedure and relationships are described in detail. A direct mechanism for a social choice function works by trying to extract private information (true valuation, or type) from the agents truthfully. We have shown that for the case of SCF1, a direct mechanism could not be implemented. On the other hand, SCF2 could be implemented by a direct mechanism in dominant strategies.
Investigation of Trends Between Input Parameters and KPI’s

The relationship between input distribution function parameters and the resulting market outcomes is plotted and categorized by KPIs in respect to the truthful mechanisms. The first section covers trends for the VCG mechanism, while the second section focuses on the Huang mechanism.

B.1. VCG Mechanism Trends

![Figure B.1: Adjustment of parameters for VCG Buyer MCP](image-url)
B. Investigation of Trends Between Input Parameters and KPI's

Figure B.2: Adjustment of parameters for VCG Seller MCP

Figure B.3: Adjustment of parameters for VCG Seller Profit

Figure B.4: Adjustment of parameters for VCG Buyer Utility
Figure B.5: Adjustment of parameters for VCG Quantity Traded

Figure B.6: Adjustment of parameters for VCG Budget Balance
B.2. Huang Mechanism Trends

Figure B.7: Adjustment of parameters for Huang Buyer MCP

Figure B.8: Adjustment of parameters for Huang Seller MCP
B.2. Huang Mechanism Trends

Figure B.9: Adjustment of parameters for Huang Seller Profit

Figure B.10: Adjustment of parameters for Huang Buyer Utility

Figure B.11: Adjustment of parameters for Huang Quantity Traded
Figure B.12: Adjustment of parameters for Huang Budget Balance


[48] Simon Parsons and Marek Marcinkiewicz. Everything you wanted to know about double auctions but were afraid to (bid or) ask. 2006.


