Impact of the space and satellite environment on the optical path differences of Darwin

V.J. Sterken

19 May 2005
TNO report

050006
Impact of the space and satellite environment on the optical path differences of Darwin

Date May 19, 2005
Author(s) V.J.Sterken
Copy no No. of copies 50
Number of pages
Number of appendices
Sponsor TNO - Science and Industry
Project name
Project number

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Supervisors:
Ir. A.Kamp TUDelft - Faculty of Aerospace Engineering
Drs. T.C.van den Dool TNO - Science and Industry
Every student who enters upon a scientific pursuit, especially if at a somewhat advanced period of life, will find not only that he has much to learn, but much also to unlearn …

As a first preparation, therefore, for the course he is about to commence, he must loosen his hold on all crude and hastily adopted notions, and must strengthen himself, by something of an effort and a resolve, for the unprejudiced admission of any conclusion which shall appear to be supported by careful observation and logical argument, even should it prove of a nature adverse to notions he may have previously formed for himself, or taken up, without examination, on the credit of others. Such an effort is, in fact a commencement of that intellectual discipline which forms one of the most important ends of all science.

J.F.W. Herschel, Collingwood, April 12, 1840.
Preamble

Framework of the study
This study is performed in order to acquire a M.sc. degree in Aerospace Engineering at the Delft University of Technology, department of Astrodynamics and Satellite Systems. The work is sponsored and performed under the authority of TNO - Science and Industry, in Delft, the Netherlands.

The aim of the study is to investigate the level of disturbances and their effects on the position of Darwin and its precursor mission in Low Earth Orbit, to the nanometer level. The assignment for this study has been changed along the way. Originally, it was set-up for two students and consisted of a literature study followed by an analytical part. One part of the assignment (assigned to Remco de Kok, [2]) focused on the equations of motion, necessary to describe the relative motion of the Darwin satellites. The other part (assigned to the author of this work) focused on the investigation of the disturbance forces that could potentially cause relative displacements larger than one nanometer. In February 2004, the assignment was redefined and split-up in a study of low-frequency disturbances (up to about 1 Hz), and high-frequency disturbances (above about 1 Hz). This redefinition was motivated by TNO’s need to determine the necessary control bandwidth of the Optical Delay Lines (ODL) being developed at TNO - Science and Industry. The breadboard ODL for Darwin, developed at TNO, is shown on the front page of this study\(^1\) as well as an impression of the Darwin satellites\(^2\) in L\(_2\).

However, until now very few to nothing is known in the literature about high-frequency disturbance forces, especially in L\(_2\). Even if it was known, building up a model for every existing force that alters Darwin and its precursor mission, including its variability, would have been a life-long task, not fitting in the timeframe of this study. Both at first seemed to be a troublesome trap, not permitting the author to solve the question TNO - Science and Industry had posed. But soon it was decided to follow another approach: the level of disturbance forces in LEO is determined by a top-down method instead of from the bottom up, by using accelerometer data from the already existing LEO-mission GRACE, and Fourier transforming the data to the frequency domain. A description of the low-frequency and, for as far as it is known, the high-frequency environment is made, with an emphasis on its temporal and spatial variability. Using both the description of the L\(_2\)-environment and the LEO results, a qualitative consideration is given of the expected disturbance forces in L\(_2\). In addition, it was decided to address the so-called “internal disturbances”, originating from mechanisms in the spacecrafts themselves.

Reading instructions
The reader who is interested in the general content of this study is recommended to read the Executive summary. The introduction of this summary already gives a good overview of what this study comprises. The reader who is more interested in a particular part of the study, after having read the summary, is referred to the relevant Section of the report. The

\(^1\)Courtesy of TNO - Science and Industry
\(^2\)Courtesy of ESA
introduction and the conclusions are written in the Executive summary only, while the basis and method of working are also explained in the report itself.

Acknowledgements
In the very first place, I would like to express my special gratitude to Ernst Kouwe, Edward Jansen and Sytze Kampen for their non-suspended support during the period of the research. Ernst, Edward and Sytze have showed their exceptional ability to manage with people, which I admire and from which I have tried to learn as much as possible. While listening with full comprehension and patience, without judging and while giving many good advices, they have helped me going through the study and through the more difficult days. I would like to thank them for the good talks, the many encouraging words, the good understanding and for the fun that was sometimes associated with it.

Furthermore I would like to thank my supervisors Aldert Kamp and Teun van den Dool for their solid supervision. From Aldert I have learned to prepare meetings very well, which clearly payed off during this graduation study. Aldert as well as Teun have stressed many times the importance of being pragmatic when necessary, which was a very welcome advice.

I am very thankful to Arno Wielders for the support during the first phase of the study, and for providing me with this extremely interesting subject that perfectly fits to my personal interests in the space environment.

The author also wishes to thank Eamonn Daly, Jeroen van den Ameede, Pieter Visser, Akke Suiker, Johannes Burger, Marcel ter Brake, Mikael Kilter, Anders Karlsson, Martijn Termeer, Nacho Andres and Jaap Wijker, for their goodwill and the valuable contributions and discussions on the subject.

Furthermore I have really enjoyed Erik Boslooper and Jos Groote-Schaarsberg being my roommates. We had lots of fun while exercising synchronous-thee-and-coffee-drinking. I would also like to thank Fred Kamphues for always being ready to take pictures whenever fun was made.

Finally I express my gratitude to TNO - Science and Industry for providing me with excellent computer facilities and for sponsoring this study. I have enjoyed watching movies, ice skating, performing a trustworthy IQ-test and table-bouldering, together with the people of the space division of TNO - Science and Industry.
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## Constants

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<th>Value</th>
<th>Unit</th>
<th>Source</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gravitation constant</td>
<td>[ G = 6.67259 \cdot 10^{-11} ] m³/(kgs)²</td>
<td>[22]</td>
<td></td>
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<tr>
<td>Earth equatorial radius</td>
<td>[ R_E = 6378136.3 ]</td>
<td>m</td>
<td>[22]</td>
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<tr>
<td>Geocentric mass</td>
<td>[ M_E = 5.97370 \cdot 10^{24} ] kg</td>
<td>[22]</td>
<td></td>
</tr>
<tr>
<td>Earth gravitational parameter</td>
<td>[ GM_E = 398600.4415 ]</td>
<td>kg³/s²</td>
<td>[22]</td>
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<tr>
<td>Mean Sun-Earth distance</td>
<td>[ 1AU = 1.4959787 \cdot 10^8 ] km</td>
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<tr>
<td>L₂-Sun distance</td>
<td>[ 1.51105517 \cdot 10^8 ] km</td>
<td>[24]</td>
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<tr>
<td>Earth heliocentric eccentricity</td>
<td>[ 0.016708158 ]</td>
<td>–</td>
<td>[24]</td>
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<td>Variation L₂ due to eccentric heliocentric EM-orbit</td>
<td>[ ±0.000168 ] AU</td>
<td></td>
<td></td>
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<tr>
<td>Solar constant at 1 AU</td>
<td>[ 1367 \pm 10 ] W/m²</td>
<td>[24]</td>
<td></td>
</tr>
<tr>
<td>Solar constant at L₂</td>
<td>[ 1340 \pm 10 ] W/m²</td>
<td>[24]</td>
<td></td>
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<tr>
<td>Mean synodic solar rotation period</td>
<td>[ 27.2753 ] days</td>
<td>[24]</td>
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<td>Solar disk diameter seen from L₂</td>
<td>[ 0.525 ] deg</td>
<td>[24]</td>
<td></td>
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<tr>
<td>Earth disk diameter seen from L₂</td>
<td>[ 0.487 ] deg</td>
<td>[24]</td>
<td></td>
</tr>
<tr>
<td>Moon disk diameter seen from L₂</td>
<td>[ 0.133 ] deg</td>
<td>[24]</td>
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<tr>
<td>Speed of light in a vacuum</td>
<td>[ c = 299792 ] km/s</td>
<td>[76]</td>
<td></td>
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<tr>
<td>Planck's constant</td>
<td>[ h = 6.625 \cdot 10^{-34} ] Js</td>
<td>[36]</td>
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<tr>
<td>Bolzmann constant</td>
<td>[ k = 1.38066 \cdot 10^{-23} ] J/K</td>
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<tr>
<td>Number of Avogadro (mole)</td>
<td>[ N_A = 6.022 \cdot 10^{23} ] 1/mol</td>
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<td>Universal gas constant</td>
<td>[ k = 8.314 ] J/molK</td>
<td>[26]</td>
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<td>Permeability of free space</td>
<td>[ \mu_0 = 4\pi \cdot 10^{-7} ] N/A</td>
<td>[48]</td>
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<tr>
<td>Permittivity of free space</td>
<td>[ \epsilon_0 = 8.854 \cdot 10^{-12} ] F/m</td>
<td>[48]</td>
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<tr>
<td>Coulomb's constant</td>
<td>[ k_c = \frac{1}{4\pi \epsilon_0} ]</td>
<td>[10]</td>
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<tr>
<td>Electron mass</td>
<td>[ m_e = 9.1094 \cdot 10^{-31} ] kg</td>
<td>[48]</td>
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<td>Electron charge</td>
<td>[ e = 1.6022 \cdot 10^{-19} ] C</td>
<td>[48]</td>
<td></td>
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<tr>
<td>Hydrogen atom mass</td>
<td>[ m_H = 1.673534 \cdot 10^{-27} ] kg</td>
<td>[48]</td>
<td></td>
</tr>
<tr>
<td>Proton mass</td>
<td>[ m_p = 1.672623 \cdot 10^{-27} ] kg</td>
<td>[48]</td>
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## Units

<table>
<thead>
<tr>
<th>Unit</th>
<th>Full name</th>
<th>Explanation</th>
</tr>
</thead>
<tbody>
<tr>
<td>arcsec</td>
<td>arc second</td>
<td>(\frac{1}{3600}) of a degree or (\frac{2\pi}{3600 \cdot 60 \cdot 60}) rad</td>
</tr>
<tr>
<td>mas</td>
<td>mili-arcsec</td>
<td>(\frac{1}{1000}) of an arcsec</td>
</tr>
<tr>
<td>pc</td>
<td>parsec</td>
<td>distance from the Earth to a star of which the parallax angle would be 1 arcsec (\sim 3.26) lightyears (\sim 3.08568025 \cdot 10^{16}) m</td>
</tr>
<tr>
<td>Jy</td>
<td>Jansky</td>
<td>(10^{-26}) W/m²Hz, flux (density)</td>
</tr>
<tr>
<td>J</td>
<td>Joule</td>
<td>Ns, energy</td>
</tr>
<tr>
<td>T</td>
<td>Tesla</td>
<td>(10^4) Gauss, magnetic flux density</td>
</tr>
<tr>
<td>nT</td>
<td>nano Tesla</td>
<td>(10^{-5}) Gauss</td>
</tr>
<tr>
<td>W</td>
<td>Watt</td>
<td>1 J/s, power</td>
</tr>
<tr>
<td>A</td>
<td>Ångström</td>
<td>(10^{-10}) m</td>
</tr>
<tr>
<td>eV</td>
<td>electron Volt</td>
<td>(1.602 \cdot 10^{-19}) J, energy</td>
</tr>
<tr>
<td>erg</td>
<td>erg</td>
<td>(10^{-7}) J</td>
</tr>
<tr>
<td>° C</td>
<td>degree Celsius</td>
<td>° C = K - 273.15, degrees Celsius, temperature</td>
</tr>
<tr>
<td>mole</td>
<td>mole</td>
<td>6.022 \cdot 10^{23}</td>
</tr>
</tbody>
</table>
Acronyms

ACE Advanced Composition Explorer
ASD Acceleration Spectral Density
AU Astronomical Unit
CFHX Counterflow Heat Exchanger
CHAMP Challenging Minisatellite Payload
CME Coronal Mass Ejection
DC Direct Current
DSD Displacement Spectral Density
ESA European Space Agency
FEEP Field Emission Electric Propulsion
FFDEM Formation Flying Demonstration Mission
FSD Force Spectral Density
GRACE Gravity Recovery and Climate Experiment
HEOS-2 Highly Eccentric Orbit Satellite 2
L2 The second Lagrange point (in this study: of the Sun-Earth system)
LEO Low Earth Orbit
LISA Laser Interferometer Space Antenna
NASA National Aeronautics and Space Administration
ODL Optical Delay Line
OPD Optical Path Difference
OSU91 Ohio State University gravity model 1991
PSD Power Spectral Density
RMS Root Mean Square
SMART-2 Small Missions for Advanced Research in Technology

(second SMART mission)
SOHO Solar and Heliospheric Observatory
TTN Three Telescopes Nuller
WMAP Wilkinson Microwave Anysotropy Probe
Executive summary

1 Introduction

Since ancient times\(^1\), people have been wondering whether there is extraterrestrial life in the universe. Since then, this question remained an issue of high social and religious importance. One of the steps undertaken to solve this question is the search for Earth-like planets in other solar systems than ours. Until now, astronomers have discovered over 100 extra-solar planets. However, these planets were detected by indirect methods, like planetary transits and doppler shift measurements of the wobble of the accompanying star. As a consequence, most of these planets are big Jupiter-like planets. For detecting Earth-like extra-solar planets in a direct way, determining whether or not it has an atmosphere and what its composition is, telescopes with a much higher spatial resolution than commonly used, are needed. This could be achieved by using bigger telescopes but these are limited due to mechanical constraints. Here, interferometry shows a possible solution. By correctly combining the light received by several telescopes, it is possible to “see” further with a better resolution.

The primary objective of ESAs Darwin mission is detecting and characterizing Earth-like extra-solar planets, in a direct way. Six telescopes are to be positioned at the second Sun-Earth Lagrange point L\(_2\), while a central “hub” satellite combines the light received by these telescopes (Figure 1). In order to be able to perform interferometry, the optical paths between the telescopes and the hub need to be equal and kept stable to a nanometer-level precision. The satellites thus need to be positioned and controlled very accurately with respect to each other. This control is done by means of very precise thrusters for the

\(^1\)The Greek philosopher Epicurus (about 300 BC) and the Roman Lucretius (about 75 BC) already mentioned that other “worlds” like ours are likely to exist in the universe. Epicurus was convinced that we have to believe in life on those worlds.
centimeter-level disturbances. Optical delay lines (ODL) will be used to reduce the optical path differences (OPD) to the nanometer level. The ODL is an accurately moving mirror unit that adjusts the OPDs for the purpose of interferometry. A breadboard ODL for Darwin is being developed at TNO - Science and Industry.

In order to properly design the ODL control and to determine the required control bandwidth, it is necessary to know the spectrum of the internal and external disturbance forces acting on the spacecrafts. The internal disturbance forces may arise due to vibrations induced by on-board equipment, while the external disturbance forces are a consequence of the space environment.

In order to help determining the ODL control bandwidth, the order of magnitude of the internal and external disturbance spectra for Darwin and the Formation Flying Demonstration Mission\(^2\) (FFDEM) are examined in this study. This is done in three major parts:

- The internal disturbance forces for Darwin
- The external disturbance forces for FFDEM
- The external disturbance forces for Darwin

The internal disturbance forces for FFDEM are not treated in this study because the satellite subsystems are not designed so far. The study is also limited to the translational disturbances of Darwin, except for the micrometeoroids and space debris effects, where also rotations are looked at.

At first, during the study of the internal disturbance forces for Darwin, the three most important sources of disturbance are dealt with. These are the microthrusters, the sorption coolers and the optical delay lines themselves. Secondly, for determining the order of magnitude of the external disturbance forces on FFDEM, accelerometer data of two current missions in similar orbits, GRACE and CHAMP, are analyzed. Finally, a thorough literature study is made about the space environment in L\(_2\), where the Sun seems to be the driving mechanism of its variability. The micrometeoroids and space debris are treated apart because there is no direct connection with the solar variability. From the LEO study results and the literature study of the space environment in L\(_2\), conclusions are drawn for the external disturbance forces on Darwin in L\(_2\).

The requirement of the OPD of Darwin depends on the rejection rate that Darwin should achieve. The rejection rate is the ratio of constructive and destructive interference intensities when performing nulling interferometry\(^3\). This rate is defined to be 10\(^5\) which leads to a maximum allowed OPD of 5 nm\(_{\text{RMS}}\).

The internal disturbance forces turn out to have a flat acceleration spectrum (quasi-white-noise) that leads to a displacement spectrum of -40 dB/dec. The corresponding RMS-value\(^4\) of the OPD, in a frequency band of 10\(^0\) – 10\(^3\) Hz is smaller than the OPD-stability requirement. However, the margins are not large as it still has the same order of magnitude as the Darwin OPD requirement. The displacements derived from the GRACE accelerometer data are decaying slightly faster than the -40 dB/dec slope (about -48 dB/dec). From an extrapolation of these data to the higher frequencies it is concluded that the RMS-value of the OPD in LEO due to external disturbance forces is also smaller than the OPD requirement, but also still has the same order of magnitude. The external environment in L\(_2\) is not calculated quantitatively, but the literature study suggests that the L\(_2\)-environment

\(^2\)The FFDEM is a mission scenario for a precursor Darwin technology demonstration mission in a Low Earth Orbit.

\(^3\)The concept of nulling interferometry is explained in Section 2.1.

\(^4\)Root Mean Square value
is definitely more “quiet” than in LEO, and therefore it is plausible to conclude that the external disturbance forces in L₂ are smaller than in LEO.

From these results it is concluded and recommended that the Darwin satellites are equipped with an optical delay line, having a control bandwidth of at least 10 Hz. In this manner, the residual OPDs will remain certainly below the nanometer level.

2 Darwin and FFDEM

2.1 Interferometry

When attempting to detect a planet in another solar system in a direct way, two major problems arise:

1. The planet appears very close to the star.
   For distinguishing the planet from the star, a resolving power of 40 mas⁵ is necessary, which corresponds to a telescope diameter of several tens of metres. Because building such large telescopes is not possible from a mechanical point of view, interferometry is an appropriate solution. By constructively combining the light received by several telescopes, such a resolution can be achieved. The resolution then mainly depends on the maximum distance between the telescopes, instead of on the diameter of a single telescope aperture. The distance between these telescopes is called the interferometer baseline.

2. The star is far more brighter than the planet itself.
   Detection of the planet seems impossible because the star outshines the planet. Two solutions are used as remedy. In the first place, the satellite observations are made in the infrared. This is because the star radiates a factor of 10⁶ more energy than the planet in the infrared, while in the visible part of the spectrum this number is 10⁹. However, for observing in the infrared, space based telescopes are needed. Secondly, a technique called “nulling interferometry” is used to eliminate the signal from the star and to increase the signal from the planet by applying destructive and constructive interferometry respectively. The principle of nulling interferometry is illustrated in Figure 2, from [12, 3].

In order to combine the light coming from the star and planet, optical delay lines are necessary to correct for the optical path differences that arise due to e.g. mechanical disturbances of the satellites, or due to atmospheric effects in the case of Earth-based interferometry. The optical delay lines acquire information on the OPD by the output of the fringe sensor. This is a sensor that measures the fringes (patterns of constructively and destructively combined light) such that corrections for the OPD are fed back to the ODL itself. The fringe sensor frequency always has to be at least twice as much as the ODL control bandwidth to avoid aliasing. In practice however, this is often a factor 10. This means that if all OPDs above 1 Hz have to be counteracted by the ODL, a fringe sensor of 10 Hz is needed. If the OPDs above 1 Hz still remain too large (i.e. larger than 5 nmRMS), then an ODL control frequency of 10 Hz or more becomes a necessity. An ODL control bandwidth of 10 Hz implies a sample rate of the fringe sensor of about 100 Hz.

Apart from detecting and characterizing extra-solar planets, Darwin will also perform observations on other astrophysical objects. Therefore Darwin has two modes to operate in: the imaging mode and the nulling mode.

⁵A resolving power of 40 milli-arcsec corresponds with resolving a feature on the Moon with a length of 80 m as seen from the Earth. For comparison, the human eye has a resolving power of one arcminute which corresponds with resolving a lunar feature of approximately 100 km long.
2.2 The Darwin mission

The Darwin satellites are to fly in a Lissajous orbit around the second Sun-Earth Lagrange point L₂. This Lagrange point lies at 0.01 AU “behind” the Earth as seen from the Sun. It was selected for Darwin for its stable natural space environment, and because from this place in space, the telescopes have an undisturbed view on the heavens with both the Sun and the Earth in their “back”. Throughout the year, as this Lagrange point moves along with the revolution of the Earth about the Sun, Darwin gets a view of 360° on the sky. The Lagrange point L₂, and a possible Lissajous orbit for Darwin in L₂ are shown in Figure 3, from [12, 3].

The design of Darwin on which this study is based, consists of 6 telescope satellites, or free flyers that fly in a single plane. The central satellite or the hub satellite receives the light coming from the 6 free flyers and combines it for the purpose of interferometry. The configuration is shown in Section 1, Figure 1, from [12]. By altering the distance between the free flyers and the hub, the baseline of the interferometer changes and thus also the resolution of the observations.

The free flyers weigh about 500 kg each, and have the shape of a telescope tube with a length of about 2 m, attached to a sunshield with diameter of 3.5 m. The free flyer layout
is shown in Figure 4 (left), from [3]. Apart from the primary telescope, they also have a relay telescope to relay the starlight to the hub. The sunshield protects the satellite structure from solar heating effects and from straylight into the optics. The satellites are cooled down passively to 40 K by the sunshield and a double telescope tube. The free flyers are equipped with Field Emission Electric Propulsion (FEEP) thrusters to correct their attitude and relative positions for the centimeter-level disturbances. The FEEP s can be turned on continuously during the observations.

Figure 4: A free-flying telescope (left) and the hub satellite (right), from [3]. The sunshield of the hub is not shown here.

The Darwin hub has a mass of about 400 kg, and is equipped with six receiver-telescopes with a diameter of 0.2 m. The hub is cooled down passively to 40 K, while the detector is cooled down to 6–8 K [3] actively, by a sorption cooler. The hub also has a sunshield (diameter at least 5 m) and FEEP s. In addition, the hub also carries the optical delay lines for correcting the optical path differences and allowing nulling interferometry. Both the telescope and hub satellites have a metrology system to measure and help control the satellite positions with respect to each other. The hub satellite without sunshield is shown in Figure 4 (right), from [3].

The master satellite is a relay satellite for the communication of the data between the hub and the Earth. It has no sunshield and does not fly in the same plane as the other seven Darwin satellites. It weighs about 180 kg and has the shape of a 1 m$^3$ cube. The master satellite has two solar panels and is equipped mainly with a communication system, metrology systems and its own propulsion system.

2.3 Darwin configurations

During this study the configuration of Darwin was not fully defined yet. The point of departure was the hexagonal configuration as described in Section 2.2. In June 2004 the Darwin Z-array came up, while in December 2004 the Darwin TTN$^+$-configuration came up.

The Darwin Z-array consists of four spacecrafts flying in a Z-shaped configuration, consisting of two relay satellites and two detector satellites. The design of the telescopes is based on the original design of the free flyers. The relay satellites consist of a telescope, the FEEP propulsion system and Optical Delay Lines. In addition, the detector satellites have a detector that needs to be cooled down to 6–8 K by active cooling. Therefore the detector satellites also possess a sorption cooler and an accompanying radiator.

The Darwin TTN$^+$ configuration also consists of four satellites, but three of them are telescope satellites and one of them is the beam combiner. The four satellites are flying in two
possible configurations during the same mission. The first one is the triangular configuration, comparable with the original hexagonal configuration, but with only three telescope satellites instead of six. The second is a linear configuration, where the three telescope satellites fly in a linear array in front of the beam combiner. The satellite designs of this configuration are based on the earlier discussed hexagonal configuration. The main difference are the larger dimensions of the Darwin TTN+ satellites.

Because for the Darwin Z-array each telescope is based on the free flyer design and because the detector telescopes all possess an ODL, the FEEP and a sorption cooler, the calculations made in this study are based on a detector telescope of the Z-array. This was considered to be a worst-case for the free flyers. Another realistic possibility would have been to consider the hub satellite of the hexagonal configuration, which contains FEEP, a sorption cooler, as well as an amount of 6 delay lines.

2.4 The Formation Flying Demonstration Mission

ESA is planning a preparatory interferometry mission in a Low Earth Orbit in order to prepare and test new technologies for the Darwin mission. One scenario of such a preparatory technology demonstration mission is described in [6]. The Formation Flying Demonstration Mission consists of two satellites that are capable of flying in formation and performing interferometry, at an altitude of 561 km in a polar orbit. They weigh about 120 kg and have a dimension of 0.6 × 0.6 × 0.7 m. The target duration of the mission is 17 months. The OPD requirement for FFDEM is assumed to be the same as for Darwin, in this study.

3 The internal disturbance forces for Darwin

3.1 Introduction

For determining the order of magnitude of the OPD of Darwin due to the internal disturbance forces, the three most important sources of disturbance were examined. These were:

1. The Field Emission Electric Propulsion (FEEP) microthrusters
2. The sorption coolers
3. The Optical Delay Lines (ODL)

At first, a description of these systems is given and their vibrational behaviour is examined. Secondly, a satellite model is built based on a mass-spring model that simulates the behaviour of a free flyer satellite, excited by a stochastic load. Finally, the vibration model outputs of the disturbing equipment are used as input for the satellite model, in order to calculate the displacements of the free flyers. A conclusion is then drawn on the level of disturbance due to internal disturbance forces, especially in the ranges of $10^0 - 10^3$ Hz (1 Hz ODL control bandwidth) and $10^1 - 10^3$ Hz (10 Hz ODL control bandwidth), for different cases of resonance.

3.2 Sources of vibration

The vibration levels from the FEEP, ODL and sorption cooler are examined. There are also other potential sources of disturbance such as the mirror switch, amplitude modulator, beam angle actuator, according to [3] also the DC/DC-converter, etc., but these were not included in this study because no information is available on them so far. A Force Spectral Density\(^7\)

\(^6\)Based on a detector satellite of the Darwin Z-array for what concerns the on-board systems.

\(^7\)The Force Spectral Density is the square root of the Power Spectral Density of the appropriate disturbance force. The Power Spectral Density describes how much “power” a signal or force contains in function of its frequency.
(FSD) is constructed for each one of the three main sources of disturbance. These FSDs are summarized in Figure 5. The sum of the internal disturbance forces turn out to remain within the level of $10^{-3} - 10^{-5} \text{ N/\sqrt{Hz}}$. They therefore can be regarded as quasi-white noise. The three subsystems and their vibration levels are explained in the following:

1. **The Field Emission Electric Propulsion thrusters**
   The FEEPs are microNewton thrusters whose thrust force is based on the emission of an ionized propellant, that is accelerated by an electric field. This propellant is usually Caesium or Indium. The FSDs of several FEEPs are known from on-ground testing and they are inspected in this study. These all seem to be smaller than the FEEP requirement-FSD that is posed on Darwin. This requirement-FSD is finally used as a worst-case input for the satellite vibration model.

2. **The sorption coolers**
   The sorption coolers are known to be the only “vibration-free” coolers that are suitable for Darwin. They use adsorption as the driving mechanism to fill the cooler cells with gas. In this manner, no vibrations are induced due to the lack of a mechanical pumping mechanism. Still, some microvibrations might remain from other parts of the cooler. The three main sources of disturbance in the cooler are examined. These are the vibrations due to the movement of the gas flow in the compressor cells, the movement of the valves, and the counterflow heat exchanger.

3. **The Optical Delay Lines**
   The Optical Delay Line is the mirror unit that moves on a carriage in order to correct for the OPDs. The carriage is supported by magnetic bearings and is actuated by a voice coil actuator. Two sources of disturbance are taken into account. These are the disturbances due to the movement of the carriage of the ODL, and the magnetic bearing sensor noise.

![Figure 5: Overview of the FSD of the internal disturbance forces. The sum of all disturbance sources is roughly constant. They can almost be regarded to be white noise.](image-url)
3.3 Satellite model

After having examined the vibration levels induced by each disturbance source, a satellite model is built. This is done using a mass-spring model. A Finite Elements Model (FEM) was considered, but too many assumptions had to be made such that the accuracy of the FEM model would hardly be better then the mass-spring model presented in this study.

In the first instant, the satellite is considered to be a rigid body, where the stochastic forces directly work on the whole satellite mass. Secondly, the satellite is split-up into two masses connected to each other with a spring and a damper. This model simulates the resonance behaviour of Darwin. The first mass is the satellite itself where the optical bench is placed, while the second mass represents a subsystem in resonance. The effects of the resonance on the first mass (i.e. the spacecraft with optical bench, but without the resonating subsystem) are examined. The disturbance forces are applied to the first mass of the model, where the three main sources of disturbance are assumed to be located with respect to the resonating mass (e.g. the sunshield).

For the “two degrees of freedom”-model (2-DOF), two methods are used: a direct method and a method called “modal analysis”. The two models (rigid body model and 2-DOF model) yield the same results if no resonance is present, while the two methods (direct and modal analysis) also yield the same results in the case of resonance. This is used to cross-check the models and the methods. Because the modal analysis method brings along more insight in the problem and because this method is easily expanded in future studies to more degrees of freedom, emphasis is laid on modal analysis.

3.4 Results of the rigid body model

The satellite models are fed by the vibration model outputs of the disturbance sources. The output for the rigid body model is a Displacement Spectral Density (DSD) that has a slope of about -40 dB/dec. This nearly constant slope of the DSD is a logical consequence of the model input approaching a white noise. The displacement RMS-value is calculated for the rigid body motion. For a fringe sensor of 10 Hz (1 Hz ODL control bandwidth), the remaining OPD-value is 1.5 nm_{RMS}. This OPD result has the same order of magnitude as the Darwin OPD requirement of 5 nm_{RMS}. On the one hand, the RMS-value is dominated by the vibrations induced by the CFHX of the sorption cooler, and this part of the cooler was modeled very pessimistically. The real values of the vibrations will improve in the final design of the cooler and in addition, they will be internally compensated. On the other hand, disturbance sources not included in this study could still increase the OPD. Also resonance effects are not yet taken into account. The latter is discussed in Section 3.5. The second largest disturbance force is due to the ODL movement.

The RMS-values for the rigid body motion are summarized in Table 1 for different frequency bandwidths. Notice that for a fringe sensor of 100 Hz (ODL control bandwidth of 10 Hz), the remaining OPD is 0.024 nm_{RMS}. Even with the pessimistically modeled CFHX forces, a fringe sensor of 10 Hz fulfills the Darwin requirements, though its margin is not large.
Table 1: The results of the rigid body study, the RMS-values for the OPD between 2 Free Flyers of Darwin (m = 493 kg).

<table>
<thead>
<tr>
<th></th>
<th>$10^{-4} - 10^4$ Hz</th>
<th>$10^4 - 10^5$ Hz</th>
<th>$10^5 - 10^6$ Hz</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>FEEPs requirement</strong></td>
<td>3.187 nm</td>
<td>0.101 nm</td>
<td>0.003 nm</td>
</tr>
<tr>
<td><strong>Sorption Cooler (RMS)</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Gas movement</td>
<td>10.240 nm</td>
<td>0.005 nm</td>
<td>0.000 nm</td>
</tr>
<tr>
<td>Valve</td>
<td>1.433 nm</td>
<td>0.004 nm</td>
<td>0.000 nm</td>
</tr>
<tr>
<td>CFHX</td>
<td>64.250 nm</td>
<td>1.293 nm</td>
<td>0.000 nm</td>
</tr>
<tr>
<td><strong>ODL (RMS)</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>ODL magnetic bearings</td>
<td>0.019 nm</td>
<td>0.019 nm</td>
<td>0.019 nm</td>
</tr>
<tr>
<td>ODL movement</td>
<td>0.128 nm</td>
<td>0.067 nm</td>
<td>0.002 nm</td>
</tr>
<tr>
<td><strong>Total (RMS)</strong></td>
<td>79.257 nm</td>
<td>1.489 nm</td>
<td>0.024 nm</td>
</tr>
</tbody>
</table>

3.5 Results of the 2-DOF model

For the 2-DOF model several parameters like the damping ratio, the resonance frequency and the mass of the resonating subsystem are altered. The effect of resonance is visible in the DSD and cumulative RMS-figure (Figures 6 and 7, respectively). The case illustrated is a simulation of the resonance behaviour of the sunshield, assuming that it has a resonance frequency of 2 Hz and a mass of 46 kg. The sunshield is taken as an example because it has the lowest resonance frequency of all subsystems, and is thus worst-case for what concerns the total RMS-value of the OPD above 1 Hz. Apart from the resonance peak at 2 Hz, the DSD also shows the -40 dB/dec slope originating from the rigid body motion. The cumulative RMS-figure shows the cumulative RMS-value of the relative displacements of two free flyer satellites, in a bandwidth of $10^{-4} - 10^3$ Hz, where the lower frequency $x$ is free to be determined by the reader.

The closer the sunshield resonance frequency is to $10^0$ Hz, the bigger the total RMS-value will be for the bandwidth of $10^0 - 10^3$ Hz. If a fringe sensor of 10 Hz is used (ODL control bandwidth of 1 Hz), then it is recommended to construct the sunshield in such a manner, that no resonance occurs between 0.5 and 2 Hz, because in that case the 5 nm$_{RMS}$ OPD requirement is easily exceeded. The study shows that a resonance frequency of 1 Hz yields 14 nm$_{RMS}$, while a resonance frequency of 1.5 Hz results in a displacement-RMS of 3.5 nm$_{RMS}$ and 2 Hz yields 2 nm$_{RMS}$. Except for this, resonance at higher frequencies has not a major influence on the total OPD. Of course, only the resonance of one subsystem is looked at, at the time. The sum of all resonances occurring could still increase the RMS-values.

The damping ratio that was assumed turns out to have a big influence on the resonance behaviour and final RMS-values of the relative satellite displacements. For the illustration case, a damping ratio of $\zeta = 0.1\%$ is chosen, which is a realistic and commonly used number in space calculations. When taking a damping ratio that is five times as small, an OPD value of 5 nm$_{RMS}$ instead of 1.5 results. However, it is not likely that the damping ratio will be much smaller than assumed in the illustration case, the opposite is more eligible.

Also the mass of the subsystem in resonance has an influence on the final OPD. As long as the resonating mass of the spacecraft is lower than 50 kg, the OPD value remains below 2 nm$_{RMS}$. A mass of 100 kg yields an OPD value of about 3 nm$_{RMS}$, at a resonating frequency of 2 Hz.

Using a 100 Hz fringe sensor (assuming that it can control the OPDs until about 10 Hz
Figure 6: Overview of the Displacement Spectral Densities for the 2-DOF example case with resonance at 2 Hz.

Figure 7: The cumulative RMS values ($10^5 - 10^3$ Hz) for the 2-DOF example case with resonance at 2 Hz.
in reality) yields a residual displacement in the order of 0.02 nm\textsubscript{RMS}. A 100 Hz fringe sensor is advised because when using a 10 Hz fringe sensor, the margins of the OPD results with respect to the Darwin requirements are small. The accuracy of the model does not allow to rely on such small margins.

4 The external space environment in L\textsubscript{2} and in LEO

4.1 Introduction

The space environment in the Sun-Earth L\textsubscript{2}-region still remains largely unexplored. Until now, only one satellite has been placed in L\textsubscript{2}\textsuperscript{8}. However, from the measurements of missions like SOHO, ACE and Cluster, a lot of insight is gained in the complex space environment surrounding the Earth.

The driving mechanism behind the variability of the space environment is the activity of the Sun. Therefore, first an overview is given of the variability and activity of the Sun. The geomagnetic field is described and its interaction with the Interplanetary Magnetic Field (IMF), leading to the description of the magnetosphere in L\textsubscript{2}. Then, the plasma environment is treated which is related to the characteristics of the magnetosphere. In addition, for LEO, the neutral environment, that also depends on the solar variability, and the gravitational environment are described. This space environment description, together with the results of the external disturbance forces in LEO (Section 5), will finally lead to a conclusion about the disturbance forces in L\textsubscript{2}, caused by the variable space environment (Section 7 and 8).

4.2 The Sun and its variability

On the contrary to what it appears like, the Sun is not a steady “quiet” celestial body at all. It has many peculiarities which are mentioned in this Section.

On the long-term, the Sun is living through cycles of 11 years in which its activity fluctuates once a cycle. This cycle is called the solar cycle, with the Solar Maximum and Solar Minimum indicating the period of maximum and minimum solar activity, respectively. On the short-term, the Sun’s variability is referred to as the active Sun. The most important aspects of the variability are summed as follows:

1. **The solar wind**
   The Sun continuously expells an amount of charged, but as a whole neutral, particles into space. This stream has a velocity varying between 400 km/s and 800 km/s. It carries along a magnetic field with it and therefore has a big influence on the magnetic environment surrounding the Earth.

2. **Sunspots**
   The Sun has dark regions on its surface, that change during the solar cycle. These regions are called sunspots and are linked to the magnetic fields on the surface of the Sun. They have a correlation with the solar cycle and the solar radiation in the Ultra Violet part of the spectrum, which has on its turn an influence on the expansion of the Earth atmosphere.

3. **Solar flares**
   A solar flare is a sudden outburst of energy coming from an active region on the solar surface. It is characterized by a major increase of particularly UV and X-ray radiation. A solar flare can lead to a sudden expansion of the Earth’s atmosphere, and hereby suddenly increase the drag on satellites in LEO temporarily.

\textsuperscript{8}This mission is NASA’s astronomical mission WMAP (Wilkinson Microwave Anisotropy Probe). This satellite maps the sky at microwave frequencies. It arrived in L\textsubscript{2} in October 2001.
4. CMEs
Sometimes the Sun expels huge amounts of energetic plasma from the corona, at a very high speed. These events are called Coronal Mass Ejections (CMEs). Their speed varies between 200 and 2000 km/s. Along their path, they change plasma properties and the local magnetic field. They are a major cause of the geomagnetic storms: significant disturbances of the magnetic field surrounding the Earth, due to solar activity. Typical storm durations are in the range of an hour to a few hours.

5. Solar radiation
The radiation flux of the Sun at 1 AU is about 1400 W/m$^2$ and fluctuates according to solar activity with an amount of 0.2% throughout the solar cycle. The total amount of radiation flux received at the Earth also varies throughout the year for 3.3%, due to the eccentricity of the Earth’s orbit. During solar activity, the total flux changes only slightly for less than a percent (e.g. 0.027% for solar flares). In the UV-part of the spectrum this amount is larger (from an average of 118 W/m$^2$ to 177 W/m$^2$) but the changes occur on a longer timescale than for flares.

4.3 The magnetic environment in L$_2$

4.3.1 The magnetic field of the Sun, the Earth and the Interplanetary Magnetic Field
The Sun has a weak dipolar magnetic field that switches polarity each solar cycle. In addition, it has a maze of strong local magnetic fields on its surface, that continuously change in magnitude and space. These strong local magnetic fields are correlated to the sunspots. The magnetic field of the Sun is shown in Figure 8 (from [11]) for the solar maximum and the solar minimum. The Sun’s magnetic field has a “current sheet” that separates the regions of opposite polarity. Because the Sun’s magnetic axis is inclined with respect to its rotation axis, this current sheet is warped (and wrinkled) with respect to the equator. It is therefore sometimes referred to as the Sun’s ballerina skirt.

![Figure 8: The magnetic field of the Sun, from [11].](image)

The solar magnetic field is “taken along” by the solar wind plasma into the solar system. This magnetic field is called the Interplanetary Magnetic Field (IMF). The strength of the IMF at 1 AU is about 5 nT. Also the IMF is warped and wrinkled in the same manner. It has a heliospheric current sheet comparable to the current sheet of the Sun. Sometimes the Earth and L$_2$ remain in a positive magnetic field sector one time, and in a negative one the other time.

The Earth has a dipolar magnetic field with a strength of 0.3 Gauss at the equator and 0.6 Gauss at the poles. The Earth’s magnetic axis is offset and tilted from the Earth rotation axis with an angle of about 11.5°. Its magnetic north pole corresponds to the geographic
south pole and vice versa. The Earth’s magnetic field strength depends on the latitude and longitude of the observer, as well as the height above the surface and the time. It is described by a dipole model, with a spherical harmonics model added up. The influence of the space environment on the Earth magnetic field is not always implemented in Earth magnetic field models.

4.3.2 The structure of the magnetosphere

The Earth’s magnetic field looses its strength with increasing altitude. At 10 to 14 Earth radii (R_E) towards the Sun, it equals the IMF strength. The interface between the Earth magnetic field and the IMF is called the *magnetopause* (see Figure 9, from [13]). Inside the magnetopause, the Earth magnetic field dominates. Outside, the IMF reigns. About 3 R_E “upstream” from the magnetopause, a shock is formed. This *bow shock* is the result of the “supersonic solar wind” being “decelerated” by the Earth’s magnetosphere. The solar wind plasma then flows around the magnetosphere, between the magnetopause and the bow shock. This region is called the *magnetosheath*. Sunward from the bow shock, there is the undisturbed free solar wind. The solar wind conditions have a large effect on the position of the bow shock, the size and the shape of the Earth’s magnetosphere. On the night-side of the Earth, the magnetosphere is stretched out to variable distances of up to 500 R_E. This part of the magnetosphere is called the *magnetotail*. Also the length of the magnetotail depends a lot on the solar wind conditions.

**Structure of Earth’s magnetosphere:**

**Lateral View**

![Diagram of the structure of the Magnetosphere](image)

Near L_2, at distances of about 236 R_E on the night-side of the Earth, the cross-section of the magnetotail is subdivided in divisions as shown in Figure 10, from [5]. It consists of a *boundary layer* (also called *plasma mantle*), a *plasma lobe* and a *plasma sheet*. The plasma sheet is on its turn divided into a *central plasma sheet*, enclosed by a *plasma sheet boundary layer*. Inside the central plasma sheet, is situated the *neutral sheet*: a “surface” where the magnetic field is equal to zero, comparable with the current sheet of the Sun.

---

9 During geomagnetic storms, the magnetopause can shift down to 6.6 R_E from the Earth. This means that geostationary satellites at about 5.6 R_E are not far from dwelling in the solar wind plasma of the magnetosheath at that moment.
At the northern side of the neutral sheet, the magnetic field lines point towards the Earth, while on the southern side of the neutral sheet they are pointed away, in accordance with the Earth’s magnetic polarity.

4.3.3 The magnetosphere in L$_2$

A possible orbit for Darwin is a Lissajous orbit with an amplitude of 39 R$_E$ in the $x$-direction, 125 R$_E$ in the $y$-direction and 47 R$_E$ in the $z$-direction in the Lagrange-point coordinate system\textsuperscript{10}. The bow shock is known to be at a distance of about 75 to 100 R$_E$ from the neutral sheet (see Figure 10). The magnetopause is situated at 25 to 33 R$_E$ from the neutral sheet. The plasma sheet occupies only a few Earth radii in the $z$-direction. It can be concluded that if the magnetosphere is in a static situation, Darwin will encounter all parts of the magnetosphere during its orbit and will even fly through the undisturbed solar wind. The differences between the magnetic and plasma properties of the different regions inside the plasma sheet are expected not to be significant. However, the differences between the solar wind, the magnetosheath and magnetotail are larger. An estimation of the magnetic field strength in these regions is shown in Table 2. The magnetic field strengths associated with the different regions of the magnetosphere in L$_2$ are used in Section 6 for calculating the Lorentz forces on the Darwin satellites.

Figure 10: A cross section of the Magnetosphere, from [5].

<table>
<thead>
<tr>
<th></th>
<th>Solar wind</th>
<th>Magnetosheath</th>
<th>Lobe</th>
<th>Plasma Sheet</th>
</tr>
</thead>
<tbody>
<tr>
<td>$B$</td>
<td>5 nT</td>
<td>higher than solar wind</td>
<td>30 nT</td>
<td>10 nT</td>
</tr>
</tbody>
</table>

Table 2: Typical values for the magnetic field strength in the different plasma regimes, from [1].

4.3.4 The variability of the magnetosphere in L$_2$

The length and the position of the magnetosphere depends strongly on the dynamic pressure and conditions of the free solar wind. Darwin’s orbit in L$_2$ has a period of about 180 days,

\textsuperscript{10}The origin of this coordinate system is in L$_2$. The $x$-axis points in the same direction as from the Sun to the Earth, while the $y$-axis points in the direction of the revolution of the Earth around the Sun. The $z$-axis fits to a right-handed coordinate system.
which is significantly longer than the variability of the magnetosphere itself. Therefore Darwin can be regarded as “hanging still” in the magnetosphere at different positions, while the magnetosphere moves and twists about the Lagrange point. The neutral sheet is often offset with an amount of 40 R$_E$ from its nominal position. In addition, it is also often twisted to a certain amount, depending on the solar wind conditions.

As the solar wind has a big influence on the geomagnetic environment, it will be of no surprise that a sudden burst of, for instance, a CME, will change the magnetic environment abruptly for a few hours. When a CME occurs, directed towards the Earth and turning the IMF southward (opposite to the direction of the Earth's magnetic field), then a magnetic storm can occur. When this happens, the magnetic field lines of the Earth magnetic field, and the southward turned IMF, reconnect to each other and CME-plasma is “allowed” to enter the Earth’s magnetosphere. A similar mechanism occurs at the night-side of the Earth, called geomagnetic substorms. These are a result of the temporal building up of energy in the magnetotail. In the case of the IMF turned southward, the magnetic fields reconnect at typically 6 to 20 R$_E$ downstream of the Earth. During the reconnection, a stream of solar wind plasma is injected towards the Earth and away from the Earth towards L$_2$. These substorms are responsible for the aurorae that are often seen at higher latitudes on Earth, when the injected plasma enters the higher regions of the atmosphere.

Apart from the irregular storms and substorms, also often shock waves and oscillations are formed in the solar wind, due to the differences in solar wind speed.

4.4 The plasma environment in L$_2$

On the contrary to what it appears like, interplanetary space is not empty. It is filled with a mixture of electrons and ions which form together an electrically neutral substance. This substance is called a plasma. The plasma in L$_2$ is fully ionized and originates from the solar wind. It has a low density, but a high energy and it is therefore called a hot plasma.

A space plasma has the property that a magnetic field can move along with the plasma. It is said that the magnetic field is “frozen-in” in the plasma. The solar wind plasma thus carries along the magnetic field of the Sun, which is then called the IMF. In the opposite way, the plasma also flows along magnetic field lines very easily, but only rarely and slowly across them. If reconnection occurs, suddenly an opening for the plasma is formed across the former magnetic field line “obstruction”, and plasma is injected. The injection of plasma e.g. at L$_2$ involves not only a disturbance of the magnetic field, but often also a change in plasma properties.

As long as a plasma moves in the same direction as is the case for e.g. the solar wind, no current originates. However, if for some reason, the electrons or ions drift to another direction, such currents arise. In L$_2$ the currents that could possibly be encountered by Darwin are the tail current, the neutral sheet current and the field aligned currents. These could contribute to spacecraft electromagnetic charging.

Two spacecrafts loaded with a net charge $q$, and formation flying in space will repel each other due to inter-satellite Coulomb forces. However, when this spacecraft is flying through a space plasma, this Coulomb forces are dimmed by the “shielding effect” of the plasma. The amount of shielding that the plasma provides depends on a plasma property called plasma Debye length. If a charged satellite (regarded as a point load) is placed in such a plasma, it will attract the negatively charged electrons, and keep away from it the positive ions. The Debye length is the distance from the charged satellite that a plasma needs in order to gain back its quasi-neutrality. Beyond that distance, no influence of the charge is noticeable. Within that distance, the electric field is reduced. The Debye length is calculated...
by [4, 9]:

\[ \lambda_D = \sqrt{\frac{\varepsilon_0 k T_e}{n_0 e^2}} \approx 69 \cdot \sqrt{\frac{T_e}{n_0}} \]  

(1)

where \( T_e \) is the temperature of the electrons, \( n_0 \) is the plasma density far from the satellite, \( k \) is Boltzmann’s constant, \( \varepsilon_0 \) is the permittivity of free space and \( e \) is the elementary charge. At a distance of 300 km in LEO, the Debye length is about 0.2 cm. Therefore, the Coulomb forces of two formation flying satellites in LEO will be nihil. However, in a Geostationary Earth Orbit (GEO), the plasma Debye length varies between 140 m and 1400 m. The Debye length in the free solar wind is in the order of 10 m. In the plasmasheet it is about 300 m while in the lobes it is about 700 m. It must be stressed that very few is known up to now about the properties of the plasma in L2 and that the plasma parameters in L2 are estimations, deduced from magnetosphere models. It can be concluded that Darwin will regularly fly through regions with a large Debye length, and that therefore the inter-satellite Coulomb forces could be an issue. The plasma properties are used in Section 7 for the disturbance force estimations in L2.

4.5 The gravitational environment in LEO

The Earth is not a pure sphere, but it has many mountains and valleys and internal mass anomalies, that cause a non-uniform mass distribution. When a satellite is flying in a Low Earth Orbit, it experiences a “disturbance force” when compared to the nominal situation (Newton’s 3rd law). The spatial variations are then felt by the LEO-satellite as temporal variations. These spatial variations of the Earth’s gravity field are described in a spherical harmonics expansion model where the gravity field is described as the sum of a central force, and a series describing the deviations for this gravity field. In this expansion, Legendre polynomials are used of a certain order and degree. The order and degree of the model determines the spatial resolution of the model. In the study case in Section 6.2, the order and degree of the “OSU91”-model was 360. This corresponds with a resolution of 55 km. If a satellite is flying at 300 km altitude, the maximum disturbance felt by the satellite due to the non-uniform Earth gravity field has a frequency of 0.14 Hz.

4.6 The neutral environment in LEO

FFDEM is flying in a Low Earth Orbit at 561 km, which is the lower part of the exosphere, the upper layer of the atmosphere. The main constituents at this height are atomic Oxygen, Helium and Nitrogen. The temperatures are in the order of 700 K to 1600 K, depending on solar activity. The atmospheric densities vary between \(10^{-14}\) and \(10^{-11}\) kg/m\(^3\), also depending on the solar activity. The atmospheric densities and temperatures depend on the height above the Earth’s surface. However, from the previous numbers it is already clear that atmospheric properties also depend a lot on the solar activity. The Earth’s atmosphere in particular responds quickly to the solar UV-radiation\(^{11}\). During the solar cycle of 11 years, the atmospheric density can alter with a factor of 100 to 300. On the short-term events like solar flares may alter the atmospheric density in a timeframe of only minutes to hours. The higher the orbit is, the more pronounced the influence of the Sun on the atmospheric density is. The effect of the solar radiation on the satellite drag (due to expansion of the atmosphere) is is demonstrated from accelerometer data in Section 6.

\(^{11}\)The solar UV-flux can unfortunately not be measured directly on the Earth. Fortunately there is a link between the radio flux of the Sun at 10.7 cm and the UV-flux. Therefore, for prediction purposes of the atmospheric density, the 10.7-cm-flux is usually measured. From the 10.7-cm flux, the F10.7 index is derived and used as an indicator of the UV-flux of the Sun.
5 Micrometeoroid impacts on Darwin

5.1 Introduction

In this Section the micrometeoroid environment and its effects on Darwin is assessed. First, the micrometeoroid environment in L\(_2\) is described after which calculations are made to estimate the order of magnitude of the micrometeoroid flux that could disturb the observations of Darwin. All particulates that could displace Darwin for more than 5 nm are included. In order to be able to calculate this flux, a model of Darwin is assumed. Translations as well as rotations are looked at.

5.2 The micrometeoroid environment in L\(_2\)

5.2.1 Micrometeoroid populations in L\(_2\)

In the vicinity of L\(_2\) the particulate environment consists only of micrometeoroids. These are divided in two categories: the isotropic micrometeoroid environment, and the peak meteoroid streams. The first poses the largest risk for spacecrafts. The latter is a result of the Earth (and L\(_2\)) encountering the remnants of a comet. There is a possibility that a third population of dust might exist in L\(_2\) due to its quasi-stable characteristics. The solar radiation pressure pushes these particles away from L\(_2\) in the same manner as it does with Darwin\(^{12}\). A quick calculation by hand showed that the \(\frac{A_m}{m}\)-ratio of at least a part of the local dust population has the same order of magnitude as the \(\frac{A_m}{m}\)-ratio of Darwin. The size and the flux of these particulates are assumed to be similar as for the isotropic micrometeoroids. However, their velocity is only about 10 km/s. This population has not been proven to exist yet, but it could be interesting to consider it in a worst-case situation.

The impact velocities of the interplanetary isotropic particulates vary between about 10 and 70 km/s. A mean value of 20 km/s is recommended and used. For a worst-case impact 72 km/s is used. For the dust population in L\(_2\) an impact velocity of 10 km/s is assumed.

The model used assumes an isotropic flux while in reality there are certain directions in which the flux is higher. The directional distribution of the particulates is neglected in this study.

5.2.2 The micrometeoroid model

For calculating the amount of micrometeoroids hitting Darwin, that could result in a displacement of more than 5 nm, the Grün model is used. This model gives the flux of interplanetary micrometeoroids that have a mass, larger than a certain threshold value, set by the author. This threshold value is derived from the OPD requirement, assuming that the FEEP’s counteract the disturbance immediately and not the ODL. The Grün model comprises particulates with masses between \(10^{-18}\) and 1 g. It holds for distances of 0.3 to 20 AU, though at large distances from the Earth it becomes less reliable. The flux is given by:

\[
F_{\text{net,0}}(m) = c_0 \cdot (F_1(m) + F_2(m) + F_3(m))
\]  

(2)

where:

\[
c_0 = 3.15576 \cdot 10^7
\]

\[
F_1(m) = (2.2 \cdot 10^3m^{0.306} + 15)^{-4.38}
\]

\[
F_2(m) = 1.3 \cdot 10^{-9}(m + 10^{11}m^2 + 10^{27}m^4)^{-0.36}
\]

\[
F_3(m) = (1.3 \cdot 10^{-16}(m + 10^6m^2)^{-0.85}
\]

The flux is given in number of particulates with mass \(m\) or bigger, per unit area per year.

\(^{12}\)Due to the solar radiation pressure it is said that a “new” artificial Lagrange point arises, where satellites remain.
at a distance of 1 AU from the Sun. A correction is made for other distances than 1 AU from the Sun:
\[ F(m, r) = F_{\text{met,0}}(m) \cdot \left( \frac{r}{r_0} \right)^{-1.8} \]  
(3)

The assumptions made for the calculations are summarized in Table 3.

<table>
<thead>
<tr>
<th>Case</th>
<th>Model</th>
<th>Velocity</th>
<th>Flux ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>Average</td>
<td>Grün: isotropic; average of streams</td>
<td>20 km/s</td>
<td>1</td>
</tr>
<tr>
<td>Worst-case</td>
<td>Grün; stream (Quadrantids); local population</td>
<td>20 km/s, 72 km/s, 10 km/s</td>
<td>1, 8, 1</td>
</tr>
</tbody>
</table>

Table 3: The assumptions made in this study for calculating the average and worst-case micrometeoroid flux in L2.

5.2.3 The satellite model

A satellite model (masses, moments of inertia and surface areas) is assumed, based on the lay-out of the Darwin free flyers. It is also assumed that the FEEPs react immediately after a particulate impact and no ODL is used to correct for these disturbances. The assumptions of the maximum allowed disturbances are shown in Figure 11. The 3D reality is approached with a 2D-impact model and directional effects are not taken into account.

5.2.4 Translation

Three cases are looked at for the translations, which are shown in Figure 12, while the corresponding requirements (5 nm, 5 nm and 70 µm respectively), are shown in Figure 11. The threshold values of micrometeoroid masses that cause an OPD of 5 nm or more, are calculated for each case using [?]:
\[ m_{\text{thresh}} = \frac{1}{\epsilon \cdot v_{\text{imp}}} \sqrt{2m_{\text{sc}}x_{\text{max}}F_T} \]  
(4)
The fluxes corresponding to the threshold values of the mass are then multiplied by the impact surface of the Darwin free flyer in order to get an estimate of the amount of micrometeoroids per year or per day, that cause Darwin to move for 5 nm or more. A parametric study is made for a spacecraft with mass of 500 kg, using $\mu N$ and $mN$ FEEPs, for the average case and a worst-case, as defined in Table 3. The amount of particulate impacts per year or per day are given in Tables 4 and 5.

For the average micrometeoroid conditions, when using only $\mu N$ FEEPs as is done during the observations, one Darwin free flyer will be displaced temporarily for 5 nm or more, about 5 times a day (1850 times a year). For the 6 free flyers and one hub, this would mean that Darwin will be disturbed about 35 times a day, using only the $\mu N$ FEEPs for corrections. Even if the $mN$ FEEPs should be used for correcting for the impacts with immediate effect, about 3 times a day, the Darwin interferometer would be subject to a micrometeoroid impact that displaces one of the satellites more than 5 nm. These numbers are high, if remembering that the observation time needed to detect extra-solar planets is 6.5 hours, and the observation time for spectrometry is 125 hours [3]. In worst-case conditions, during a micrometeoroid stream and if the local dust population is present in L$_2$, the amount of impacts on one free flyer increases to about 10 per day when using $mN$ FEEPs.

On the one hand, the 5 nm limit is not a “hard” limit above which interferometer operations would fail, but a $1\sigma$-value of the OPD requirement. This means that in theory, in 68.3% of the time, the OPD should remain below the 5 nm limit and not in 100% of the time as assumed in our calculations. Therefore the estimation of the amount of particulates that could possibly disturb the interferometer observations is rather “pessimistic” because the 5 nm limit does not directly determine the failure or success of the observations. On the other hand, the assumption that the FEEPs are activated immediately after the impact is an “optimistic” estimation of the impact count. The question whether the impacts can disturb the interferometer observations will depend on the total amount of time that one of the satellites is displaced significantly. This integrated time should be less than a fraction of the nulling depth ($\frac{1}{\rho}$) times the observation time$^{14}$. A more detailed study is necessary to calculate the real effect of the impacts on the observations. The order of magnitude of the amount of impacts (based on the $1\sigma$-limit) indicates that the effect of micrometeoroid impacts should certainly be taken into account in the design of the control system of Darwin.

Possibly, it will be necessary to deploy the $mN$ FEEPs for correcting the disturbances instead of the $\mu N$ FEEPs. These can introduce extra disturbances. Their force spectral densities

---

$^{14}$According to this rule of thumb, for planet detection, this maximal integrated OPD-error-time would be in the order of 0.02 sec, while during spectroscopy it would be about 0.5 sec.
Table 4: The amount of impacts on Darwin, per year and per free flyer satellite, for average micrometeoroid environment conditions - translation.

<table>
<thead>
<tr>
<th>$F_T$ [N]</th>
<th>$m_{\text{thresh}}$ [g]</th>
<th>$v_{sc}$ [m/s]</th>
<th>Flux $\phi$ in $L_2$ [$#m^{-2}yr^{-1}$]</th>
<th>Amount of impacts with OPD $\geq$ 5 nm [$#$ per year]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Case 1,2</td>
<td>$x_{\text{max}} = 5 \cdot 10^{-9}$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$10^{-3}$</td>
<td>$7 \cdot 10^{-1}$</td>
<td>$1.4 \cdot 10^{-7}$</td>
<td>2</td>
<td>150</td>
</tr>
<tr>
<td>$10^{-4}$</td>
<td>$2 \cdot 10^{-1}$</td>
<td>$4.5 \cdot 10^{-8}$</td>
<td>5</td>
<td>400</td>
</tr>
<tr>
<td>$10^{-5}$</td>
<td>$7 \cdot 10^{-8}$</td>
<td>$1.4 \cdot 10^{-8}$</td>
<td>12</td>
<td>900</td>
</tr>
<tr>
<td>$10^{-6}$</td>
<td>$2 \cdot 10^{-8}$</td>
<td>$4.5 \cdot 10^{-9}$</td>
<td>24</td>
<td>1850</td>
</tr>
<tr>
<td>Case 3</td>
<td>$x_{\text{max}} = 70 \cdot 10^{-6}$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$10^{-3}$</td>
<td>$8 \cdot 10^{-9}$</td>
<td>$1.7 \cdot 10^{-5}$</td>
<td>0.01</td>
<td>0.1</td>
</tr>
<tr>
<td>$10^{-6}$</td>
<td>$2 \cdot 10^{-6}$</td>
<td>$5.3 \cdot 10^{-7}$</td>
<td>0.6</td>
<td>5</td>
</tr>
</tbody>
</table>

Table 5: The total amount of impacts on Darwin, per day and per free flyer satellite, in a worst-case situation (meteoroid stream) - translation.

<table>
<thead>
<tr>
<th>$F_T$ [N]</th>
<th>Grün [# per year]</th>
<th>Quadrantids [# per year]</th>
<th>Local dust population [# per year]</th>
<th>Total amount of impacts [# per day]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Case 1,2</td>
<td>$x_{\text{max}} = 5 \cdot 10^{-9}$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$10^{-3}$</td>
<td>150</td>
<td>3500</td>
<td>80</td>
<td>10</td>
</tr>
<tr>
<td>$10^{-4}$</td>
<td>400</td>
<td>8000</td>
<td>230</td>
<td>24</td>
</tr>
<tr>
<td>$10^{-5}$</td>
<td>900</td>
<td>16000</td>
<td>560</td>
<td>48</td>
</tr>
<tr>
<td>$10^{-6}$</td>
<td>1850</td>
<td>28300</td>
<td>1220</td>
<td>86</td>
</tr>
<tr>
<td>Case 3</td>
<td>$x_{\text{max}} = 70 \cdot 10^{-6}$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$10^{-3}$</td>
<td>0.1</td>
<td>35</td>
<td>0.05</td>
<td>0.1</td>
</tr>
<tr>
<td>$10^{-6}$</td>
<td>5</td>
<td>1200</td>
<td>3</td>
<td>3</td>
</tr>
</tbody>
</table>

should be investigated as was done in Section 3 for the $\mu$N FEEPs, in order to evaluate the disturbances coming from on-board systems\textsuperscript{15}.

In the calculations of the impact count on the free flyers, it is assumed that only the FEEPs are turned on to correct for the disturbances of the satellite (and thus the OPD). In Table 4 the velocity changes of the free flyer after impact are shown for each threshold mass. In average micrometeoroid conditions and an impact frequency of 3 times a day on the whole constellation, this velocity change is about 140 nm/s. In other words, without correction of the FEEPs, the satellite would be displaced 5 nm within 0.03 sec. Therefore the time between impact of the micrometeoroid and the response of the FEEPs should be very small, leading to a high sample frequency for the control loop. Also the bandwidth of the control needs to be high (about 30 Hz) in order to correct the OPD in time. This can be achieved with a combination of FEEP and ODL control, using a high sample frequency (and corresponding fringe sensor) of at least 100 Hz and an appropriate design of the ODL control leading to a high control bandwidth. From these observations it is recommended to simulate the response of Darwin to micrometeoroid impacts in more detail, and to determine its effect on the performance of the interferometer in more detail.

\textsuperscript{15}In this report only $\mu$N FEEPs were considered, because only these are planned to be used during the observations themselves.
5.2.5 Rotation

For the rotations the same method is used where the maximum angle after micrometeoroid impact is defined by:

\[ m_{\text{thresh}} = \frac{1}{e \cdot d \cdot v_{\text{imp}}} \sqrt{2I_{G,sc} \theta_{\text{max}} M_T} \]  

(5)

The maximum angle after impact is assumed 8.4 mas (Darwin requirement). The moments of inertia are estimated to be 268 kg m\(^2\) in the x- and y-direction, and 415 kg m\(^2\) in the z-direction. Calculations are made in a slightly different way as for the translation, because the threshold value of the micrometeoroid mass depends on the position of impact with respect to the center of mass of the satellite. The Grün equation is approximated in the range of the relevant masses between \(10^{-6}\) and \(10^{-8}\) g by a linear curve on a logarithmic scale. This expression is used to calculate the flux, depending on the distance from the position of impact to the center of mass of the satellite. These fluxes are then multiplied by their accompanying ring surface, in the case of the sunshield:

\[ \Phi_{\text{tot}} = 2 \cdot \int_0^R \phi(r) \cdot 2\pi r \, dr \]  

(6)

where \(\Phi_{\text{tot}}\) is the total amount of particulate impacts per year on the sunshield of Darwin. The factor 2 stands for the two sides of the sunshield. In the case of an impact on the telescope tube a similar equation is used:

\[ \Phi_{\text{tot}} = \int_0^R \phi(r) \cdot 2a \, dr \]  

(7)

The amount of impacts that cause a satellite rotation of 8.4 mas or more for the rotations are of the same order of magnitude as for the translation, at least in the case of the sunshield impacts (for the rotations the amount of impacts is approximately a factor 2 smaller than in the translation case). The impacts on the tube are a factor 50 smaller than on the sunshield.

6 The external disturbance forces for FFDEM

6.1 Introduction

In order to assess the feasibility of an FFDEM interferometry mission in LEO, the order of magnitude of the disturbance forces in such an orbit are assessed as a function of their frequency. This assessment shows whether an ODL is needed to compensate these disturbances and if so, for which bandwidth. In addition, the results of this Section are used in combination with the information on the space environment (Section 4) for determining the level of external high-frequency disturbances on Darwin (Section 7). The method used, is based on the accelerometer measurements of the gravity missions GRACE and CHAMP. Because the accelerometer measures only non-gravitational disturbance forces, first a consideration about the gravitational forces is given, based on a gravity field model. Then, the GRACE and CHAMP missions are introduced as well as the characteristics of their accelerometers and a description of the accelerometer data. The analysis is based on Fourier transforming the time-domain data into the frequency domain. Several goals are set and parameters are changed in order to reveal their influence on the results. Finally, conclusions are drawn about the level of disturbances in a bandwidth of \(10^0 - 10^3\) Hz by using extrapolation of the disturbance spectrum, and recommendations are given.

6.2 The gravitational disturbance forces in LEO

The gravitational disturbance forces are considered to be very low-frequent in nature. However, when a satellite flies in an orbit around the Earth which has a non-uniform mass...
distribution, it experiences gravitational forces that change with the time relatively fast. The question solved here is how fast these forces change and what effect they have on a satellite in LEO. The displacements of a satellite at an altitude of 300 km due to the gravitational forces were derived in [10], from a gravitational model, called OSU91. The results in [10] have been used in this study to determine the RMS-values of the gravitational disturbances in a bandwidth of $10^0 - 10^3$ Hz, by means of extrapolation. The case of 300 km is a worst-case for the gravitational forces in comparison with the real altitude of FFDEM (561 km) because the effect of the higher order gravitational forces diminishes as the orbit altitude increases. The displacements are examined and extrapolated, such that a realistic estimate of the RMS-values at higher frequencies can be made, assuming that extrapolation is allowed. The OPD due to gravitational forces in LEO is in the order of $10^{-4} \text{ nm}_{\text{RMS}}$.

6.3 The accelerometer data analysis

The non-gravitational disturbances in LEO are being measured by the current gravity missions GRACE and CHAMP. Both satellites have very accurate accelerometers on-board. The analysis was based on the GRACE accelerometer data, while CHAMP data was used for a cross-verification. Both GRACE and CHAMP have polar orbits at initial altitudes of approximately 450 to 500 km. These missions are comparable to FFDEM, except for their mission objectives, their masses and dimensions of the spacecraft.

The data analyzed are the Level 1B data of GRACE. These data contain the 1 Hz accelerations that are derived from the original 10 Hz data. These data are suitable for solving the order of magnitude and the trend of the accelerations with increasing frequency, but no “real” information about the accelerations above 0.035 Hz is available, due to low-pass filtering, applied to the Level 1B data. For determining what exactly happens between 0.035 Hz and 0.1 Hz, the Level 1A data should be assessed. Due to data distribution restrictions, these are not available to the public unless going through a request procedure. This procedure was started during the project, but no time was left for getting the data and analyzing them. It is known that certain events happened for GRACE at a frequency of 3.5 Hz. These oscillations in the accelerations are called “twangs” and are probably a result of the heating of a Teflon foil on the bottom of GRACE [7].

The calculated Acceleration Spectral Density (ASD) shows a descending trend in accelerations with respect to frequency. On the low-frequent side of the ASD, several peaks are visible that match with the orbital frequency of the satellite and twice the orbital frequency. These are a result of the eccentricity of the orbit, the pear-shape of the Earth, the solar-radiation pressure, seasonal effects, etc. The 15-days averaged ASD (left) and DSD (right) for GRACE are shown in Figure 13.

The analysis of the GRACE data has the following goals:

- **Establish a representative Acceleration and Displacement Spectral Density for the given dataset**
  The mean was taken of the ASDs of 15 days and compared to a one-day DSD. Taking the mean of 15 days seemed to reduce the noise and make the signal clear. A mean of 15 days is taken as a good representation of the ASD average signal.

- **Separate AOCS and orbit maintenance spectrum from external disturbances**
  The time-domain data contained a lot of spikes, which probably result from AOCS and orbit maintenance manoeuvres. These spikes are very “sharp” and cause the general level of the noise to increase. The result is an ASD that is not the real one, but an

\[ \text{Ohio State University model 1991} \]
upper limit of the true ASD. In order to reduce this increase in noise-level, the spikes are filtered from the data, using a magnitude criterium. This results in a decrease of the noise level at the higher frequencies. The general trend of the accelerations seem to become a straight descending line on a double-logarithmic scale. However, using the magnitude criterium means that—using different magnitudes for filtering—results in too many and/or too few spikes being removed. A more sophisticated filter should be programmed using the spacecraft orbit maintenance data and the AOCS data. When the spikes are removed in this manner, it is also interesting to see whether there are still any spikes left or not, and what their source is. However, for determining the influence of the spikes on the data, the magnitude criterium is satisfactory.

- **Investigate the effect of solar variability**
  The solar variability is looked at by comparing a solar active period to a quiet period just before. Also, data from two eras in the solar cycle are compared to each other. Three parameters seem to have a big influence on the magnitude of the disturbance forces. These are the solar activity (events like flares and CMEs), the period in the solar cycle, and the orbit altitude of the spacecraft. It turns out that the solar activity has more influence on the order of magnitude of the disturbances than the orbit altitude. In addition, it seems like this difference becomes smaller for higher frequencies. However, when the noise is further reduced at high frequencies, then this difference could possibly remain larger than assumed here.

- **Cross-validate data**
  The accelerometer data of CHAMP are processed in the same way as GRACE. These seem to have several consistencies like the positions of the signal peaks and the influence of the solar variability. Their level of disturbances in the same epochs are different due to the orbit altitude difference between CHAMP and GRACE (GRACE was launched two years later). Unfortunately, the decay level of the ASDs and DSDs seemed to be higher in the case of CHAMP than for GRACE. It is probable that this is an effect of the filtering applied to the data (e.g. in the case of GRACE the spikes were still present). Furthermore the Fourier transform was checked using Parseval’s theorem, which states that the RMS-value of the time-domain data is equal to the RMS-value of the frequency domain data.

- **Determine a (realistic) worst-case scenario for the RMS calculation**
  For a worst-case scenario the orbit data of GRACE have been taken, because the decay
of the ASD is less steep than for CHAMP. The data of the 2002-epoch are analyzed. The data are not filtered so that in any case not too many spikes are removed and the slope remains less steep than the real one without noise would be. The result is a worst-case slope of the disturbance spectrum instead of the real one. The plotted DSD is linearly extrapolated after which the RMS-value of the displacements in the bandwidth of $10^0 - 10^3$ Hz is determined. The total OPD between two satellites is in the order of one nanometer for every direction. Reducing the noise in the ASD could lead to a steeper slope and a smaller RMS-value. On the other hand, the effect of solar variability on the high frequencies is not known and could possibly also have an influence and enlarge the RMS-value of the relative displacements of the satellites.

The relative displacements of two LEO satellites, in a bandwidth of $10^0 - 10^3$ Hz turn out to be in the order of 0.1 up to 1 nm, based on the extrapolated data. The disturbance forces in LEO are modeled very roughly using an extrapolation. This holds in the risk of having a larger RMS-value in reality. On the other hand, the extrapolation was made on a worst-case slope that is less steep than in reality, because of the noise at higher frequencies not being filtered away. Therefore the RMS-values are upper limits of the disturbances in LEO. Although the estimated RMS-value of the OPD due to external disturbances in LEO is small and worst-case, its order of magnitude is still approximately equal to the order of magnitude of the OPD requirement. Therefore is is recommended to use an OPD control mechanism with a control bandwidth of 10 Hz (and an associated sensor frequency of 100 Hz).

7 The external disturbance forces for Darwin

7.1 Introduction

The variable space environment discussed in Section 4 can lead to variable disturbance forces in $L_2$. The different disturbance forces resulting from the environments discussed are shown in Figure 14, together with the associated environments from which their variability is derived. In this Section, the Lorentz force, inter-satellite Coulomb forces, solar radiation pressure force and charged particle drag are examined. Also a conclusion is drawn on the general level of high-frequency disturbances in $L_2$, based on the information on its environment (Section 4) and the OPD-results for LEO (Section 6).

![Figure 14: An overview of the environments discussed in Section 3 and the disturbance forces associated with them.](image-url)
7.2 The Lorentz force

A satellite with a nett charge, flying through a magnetic field with a certain velocity, will be subject to the Lorentz force:

$$\vec{F}_L = q \cdot (\vec{v} \times \vec{B})$$

where $q$ is the nett charge of the satellite, $\vec{v}$ is the velocity of the satellite and $\vec{B}$ is the magnetic field vector. From Section 3 we know that fluctuations in the magnetic field in the Lagrange point will be not larger than about 25 nT. Using in this value for the variation in magnetic field strength, assuming a spacecraft potential of 500 V and estimating the spacecraft velocity at 200 m/s in L$_2$ yields values for the Lorentz forces of about $10^{-13}$ N, and accelerations between $10^6$ and $10^7$ nm/s$^2$. Even if assuming an ICME is passing by with a velocity of 550 km/s, the order of magnitude of the Lorentz forces for a satellite charged to 500 V is about $10^{-3}$ nm/s$^2$. This force, and its variations is negligible in L$_2$.

7.3 The Coulomb force

Two charged satellites flying in each others neighbourhood will attract or repel each other according to their nett charge. The force exerted between these satellites is called the Coulomb force. It is calculated using:

$$F_c = \frac{1}{4\pi\varepsilon_0} \cdot \frac{Q_{sc1} \cdot Q_{sc2}}{d^2} \cdot e^{-\frac{d}{\lambda_D}}$$

where $Q_{sc1}$ and $Q_{sc2}$ are the nett charges of the satellites, $d$ is the distance in between them and $\varepsilon_0$ is the permittivity of free space, equal to $8.854 \cdot 10^{-12}$ C$^2$/N.m$^2$. $d$ is chosen to be 50 m in accordance with the the smallest baseline used in the hexagonal Darwin concept. The Coulomb force is restricted by the plasma Debye length $\lambda_D$ that is explained in Section 4. This “shielding” effect is visible in Equation 9 in the term $e^{-d/\lambda_D}$. From Section 4 we also know that the largest Debye lengths in L$_2$ are about 700 m. Assuming a spacecraft potential of 500 V yields inter-spacecraft Coulomb accelerations in the order of $10^{-3}$ nm/s$^2$, which is negligible.

7.4 The solar radiation pressure force

The solar radiation pressure force is the biggest non-gravitational force in L$_2$. However, its variability is only limited. The Sun emits a flux of 1340 W/m$^2$, in L$_2$, leading to a solar radiation pressure acceleration of 344 nm/s$^2$ for a spacecraft of 500 kg. Differences in total solar irradiance are known due to events like solar flares. These seemed to increase the solar irradiance by 0.027%, which leads to an increase of solar radiation pressure acceleration of 0.1 nm/s$^2$. Variations in the near-UV occur during the solar cycle with a magnitude of about 60 W/m$^2$. These variations would lead to a solar radiation pressure acceleration of 15 nm/s$^2$, which is large, but the timescale on which these variations take place is not known. It is hereby plausible that the solar radiation pressure only causes low-frequency disturbance forces in normal conditions.

7.5 Charged particle drag

A charged spacecraft flying through a plasma experiences a drag force that is larger than if it were not charged. This drag force consists of three components: the direct collisions of particles, the particle scattering and the wake effects. The drag due to the direct collision of the particles is comparable to the neutral drag in the atmosphere. It is estimated that the drag force variations due to direct collisions will not be larger than 1.5 nm/s$^2$. The second component, the particle scattering, mainly depends on the charge of the satellite and could increase the drag significantly. However, most satellites are designed with active systems in order to avoid such charging and therefore this component of the drag will depend
largely on the specific design of Darwin. The third component also largely depends on the charge of the satellite as well as on the difference between the satellite velocity (with respect to the plasma) and the thermal velocity of the plasma particles. This thermal velocity is estimated to be in between about 50 and 75 km/s while the spacecraft velocity with respect to the plasma is about 400 to 800 km/s, if assuming it is flying in the free solar wind. This difference indicates that the wake effects of the charged particle drag could be of importance.

7.6 The high-frequency disturbance forces in L₂ due to the space environment

The literature study in L₂ suggested that L₂ is a region with a highly variable space environment, but the considerations in this Section show that in despite of these variations, most of the disturbance forces are negligible. However, for charged particle drag, the spacecraft charging is a sources of large uncertainty for the final order of magnitude of this force and its variability. It is necessary to perform a charging analysis on Darwin and to re-assess the charged particle drag if these charges are significant.

The other forces in L₂ that were first believed to be a potential source of highly variable disturbances seem to be negligible. Only the variations in the near-UV cause a significant change in solar radiation pressure force on Darwin, although their timescales are not clear. Because there is no atmosphere in L₂ and furthermore the environment is more “quiet” than in LEO, it is plausible to assume that the OPD due to the high-frequency disturbance forces in L₂ will not exceed the value of the LEO case. Therefore it is concluded that also in L₂ the high-frequency disturbances will not be larger than 1 nm_{RMS}.

8 Conclusions

The order of magnitude of the optical path disturbances of Darwin and its precessor mission (FFDEM) are determined. This is done in order to define a minimum frequency at which the OPD control loop should operate such that it successfully compensates for the high-frequency disturbances (> 1 Hz) on Darwin. These disturbances are twofold of nature. At first, there are disturbances that arise from on-board systems. Secondely, the turbulent space environment causes a spacecraft to be disturbed too. The question is what the order of magnitude of these disturbances is with respect to their frequency.

The internal disturbances arising from on-board systems are derived for the FEEPs, the sorption cooler and the ODL, and put into a mass-spring spacecraft model to simulate the resulting OPDs. The general slope of the OPD decays almost linearly with -40 dB/dec. This is logically, because the sum of these disturbance forces is nearly white noise. The frequencies at which resonances are present cause additional peaks on that slope. For a fringe sensor frequency of 10 Hz and an ODL control bandwidth of 1 Hz, the resulting OPD due to internal disturbance forces is smaller than the OPD requirement of 5 nm_{RMS}. However, its order of magnitude still equals the Darwin OPD requirement. The model used is rough in comparison with reality, and some internal disturbances may be present that were not modeled yet. The margins are thus very small and cannot be relied on. Therefore, it is recommended to use a fringe sensor of 100 Hz, such that all disturbances up to 10 Hz are compensated in all cases. In this manner, the order of magnitude of the OPD remains safely below the nanometer-level. It is recommended to perform a follow-up study of the internal disturbance forces, using updated information on the subsystems and possible disturbance sources not included in this study. It is advised to perform the follow-up study with a Finite Elements Model of the Darwin spacecrafts, such that the cumulative effects of all resonance phenomena can be incorporated at once, as soon as more detailed material and design parameters are known for Darwin.
The disturbance forces in a Low Earth Orbit are estimated using existing data from the current gravity missions GRACE and CHAMP. These fly in a Low Earth Orbit, similar to that of FFDEM. The data of the non-gravitational accelerations of these satellites are transformed to the frequency domain using a Fourier transformation. The displacements of the satellites also show a nearly linear decay with increasing frequency. The slope is about -48 dB/dec, which implies that the accelerations themselves decay very slightly with increasing frequency. The RMS-value of the OPD for FFDEM is calculated from an extrapolation of the Fourier transformed data. The OPD for FFDEM in LEO, based on the extrapolation, is also smaller than, but still has the same order of magnitude as the Darwin OPD requirement itself. Also here the method based on extrapolation is rough and no more information is known in reality above 0.035 Hz, except that features like “twangs” might occur. The margins between the resulting OPD and the Darwin OPD requirement are small, but on the other hand a worst-case slope of the GRACE data was used to calculate them. Therefore, the OPD value for LEO is an upper limit for the external disturbance forces. For a mission in LEO, also an ODL control bandwidth and fringe sensor frequency of 10 respectively 100 Hz is recommended. It is advised to perform a follow-up study, using the level-IA data of GRACE in order to reveal the true disturbances above 0.035 Hz, included the “twangs”.

A literature study is made on the space environment in L$_2$ for Darwin. This environment is very stable in comparison with Low Earth Orbit, but yet it is not “quiet” at all. The Sun has a major influence on the variability of the magnetospheric and plasma environment. Darwin will fly through all kinds of plasma and magnetic field regions, though their effect turned out to be negligible (e.g. the magnetic field is variable but not strong). From the literature study of the space environment and from the low OPD results in LEO, it is concluded that the L$_2$ space environment probably does not cause any significant OPD at high frequencies. In L$_2$ there is no atmosphere, which is one of the two main causes of disturbance in LEO next to the solar radiation pressure force. The biggest (non-gravitational) disturbance force in L$_2$ is the solar radiation pressure force but no evidence is found that there are fluctuations on the high-frequency level in this study. Care must be taken for spacecraft charging issues. A spacecraft charging analysis is strongly recommended for Darwin. The space environment in L$_2$ remains very unknown until today, and especially the high-frequency environment needs further investigation in order to confirm the outcome of this study. It is advised to apply the method used for the LEO case on L$_2$ too, using data of other satellites, like ACE and SOHO. The data of e.g. the solar variability or plasma densities could be transformed in a PSD and then converted into Forces Spectral Densities and Displacement Spectral Densities.

In addition, the micrometeoroid impacts on Darwin certainly need a deeper research. In average conditions Darwin is hit about 35 times a day if only, and immediately the $\mu$N FEEP$\mathrm{s}$ are used to counteract the OPD. Because the spacecraft of 500 kg reacts relatively slowly to the FEEP thrust, it is recommended to not only use the FEEP$\mathrm{s}$ immediately after impact, but also the ODL, such that the OPD is compensated for optically before it reaches the value of 5 nm$\mathrm{RMS}$, in spite of the FEEP activation. In addition, due to the change of velocity Darwin acquires after an impact, it is advised that the FEEP$\mathrm{s}$ and the ODL have a control bandwidth of at least 30 Hz in order to be able to compensate for the impact in time. A thorough study of the effect of the impacts on the performance of the interferometer operations is strongly recommended, as well as a design study for an appropriate ODL and FEEP control.

During this study very few was found in the literature about the high-frequency disturbances on satellites in space. Although it becomes of increasing importance for missions like LISA and Darwin. It is of scientific and engineering interest to have accelerometers in space in the future, that measure such accelerations in the frequency range between about $10^{-1}$ and $10^2$ Hz.
The three main issues of concern for Darwin turned out to be the internal disturbance forces, the particulate impacts and the spacecraft charging effects. Because the OPD due to the external forces in LEO was in between 0.1 and 1 nm\text{RMS} and a worst-case dataset was taken, it is expected that the ambient space environment causes no larger OPDs than the disturbances from on-board equipment do. According to this study, it is advised to have Darwin equipped with an optical delay line with a control bandwidth of at least 10 Hz (fringe sensor of 100 Hz) in order to cope with the OPD disturbances arising from mainly the on-board instruments. Further research is strongly recommended for the spacecraft charging and micrometeoroid impact issues, as well as calculations for the internal disturbance forces, using updated information and a complete Finite Elements Model.
Bibliography


Chapter 1

The Darwin Mission

1.1 Background information on Darwin

1.1.1 Introduction

In 1995 the very first exoplanet was discovered using an indirect detection method with a ground-based telescope [55]. In the past 8 years a number of more than 100 other exoplanets have been detected. However, these were all Jupiter-like planets, not suitable for developing life. For the direct detection of Earth-like planets more advanced technologies are required. One of the ways is to search for Earth-like planets in other solar systems through space interferometry.

Darwin is the infrared space interferometry mission of the European Space agency that will be launched in the next decade, probably in 2015. Its main purpose is detecting habitable planets in other solar systems, next to mapping other astrophysical objects with an unprecedented precision. Not only its goals are ambitious, also advanced techniques need to be developed. These are supposed to be tested in-flight before during a Darwin precursor mission: the Formation Flying Demonstration Mission (FFDEM). Although Darwin will fly at the second Sun-Earth Lagrange point L₂, FFDEM will fly in a polar Low Earth Orbit [30]. Darwin’s basic design consists of 6 space telescopes flying in formation—that form together with the hub satellite and the communications satellite—one single space interferometer (see Fig. 1.1, from [86]).

When using interferometry, the optical paths between the satellites need to be controlled in an extremely accurate way, to a fraction of the wavelength of the light measured. For this purpose, control systems are used that operate with a specific frequency. This frequency depends amongst others on the frequency and level of the disturbance forces acting on the satellites. This study intends to estimate the level of disturbances on Darwin and FFDEM and their effects on the Optical Path Differences (OPD).

In order to be able to reveal the level of disturbances and their frequency dependence for Darwin and FFDEM, first, the concept and lay-out of both missions need to be well-understood.

This Section highlights the background information on Darwin and FFDEM. In Section 1.1, the goals of Darwin are discussed, and how these are achieved while having to defeat some particular problems. Also the lay-out of Darwin is described. Section 1.2 is meant to give the reader more insight in what interferometry actually is after which the requirements for Darwin are explained in Section 1.3. During this study, the design of Darwin was not fixed yet and three different mission scenarios showed up. This study is mainly based on the first

---

1 However, these Jupiter-like planets may harbor moons capable of sustaining life.
and basic mission scenario of Darwin. The other two scenarios are described in Section 1.4, as well as the implication of the changes for this study. Finally, an introduction to Darwin’s precursor mission, FFDEM, is given in Section 1.5.

![Figure 1.1: The Darwin mission, from [86].](image)

### 1.1.2 Goals of Darwin

The main purpose of Darwin is the direct detection of Earth-like planets in other solar systems within our Galaxy. Once Darwin has detected such a planet, it will investigate whether or not it has an atmosphere, and scan its composition for tracers of life that are based on life on Earth, as we know it. If signs of possible life are detected, it will have an immense impact on several areas in society such as religion, science and philosophy. These findings will not only contribute to the search for extra-terrestrial life but the studies will also contribute to theories about the origin of our own solar system [7].

Apart from the search for extra-solar Earth-like planets, Darwin will also sweep the sky with an unprecedented spatial resolution\(^2\), that is at least 100 times more detailed than possible with ground-based telescopes. This detailed imaging will possibly lead to important advances in long-lasting astrophysical quests. Darwin will look at the planets in the clouds surrounding their parent star, which will help us to reconstruct the formation of our solar system, and trace the origin of the planets.

With its high resolution, Darwin will also be able to look at the very distant galaxies, some of them still in the process of formation. This makes it very interesting for improving current theories and models on the formation of galaxies, and their evolution. Not only will Darwin look at their global structures, it will also make big steps toward the solution of the deep secrets in the centers of galaxies, including our own. Black holes are believed to appear at these centers, but it is not possible to see them as even light cannot escape from these objects. Moreover, their expected radius is not greater than our own solar system and thus hardly resolvable with a normal telescope. Due to Darwin’s high spatial resolution it is possible to study the movement of the clusters in their neighbourhood. From these movements, information can be attained on the mass and location of the supposed black holes.

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\(^2\)Darwin’s spatial resolution is 0.65 micro-arcseconds, which is comparable to distinguishing a 1-meter statue standing on the Moon as seen from the Earth.
1.1.3 Methods used for detecting extra-solar planets

Until now only Jupiter-sized planets have been detected, and only with indirect detection methods. Most of the indirect detection methods rely on the observation of the wobble of the accompanying star, while it moves about the barycenter of the star and the planet. The planets that are discovered in that way until now have masses of 0.5 to 10 times the mass of Jupiter. They have an orbital radius of 0.05 (hot Jupiters) to 2 AU and are situated near stars at a distance of 10 to 20 pc from the Sun [3]. The indirect methods however, are not suitable for detecting Earth-like planets. Earth-like planets have a mass of approximately \( \frac{1}{300} \) Jupiter’s mass and they induce an almost undetectable wobble of the star. For the detection of these planets direct methods like nulling interferometry in the infrared are needed. An overview of the most important detection methods used is given in Table 1.2 and Table 1.1. This Table is adapted from [21] and [6]. Fig. 1.2 (from [81] and [21]) illustrates the principle of the radial velocity method, and the transit method.

![Figure 1.2: The radial velocity method (doppler) and the transit method, from [81] and [21].](image)

<table>
<thead>
<tr>
<th>method</th>
<th>principle</th>
<th>limits</th>
<th>mission</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>transits</strong></td>
<td>When the planet passes in front of the star a small fraction of the star is occulted. This is observable by a decrease of intensity of the light received from the star (Fig. 1.2).</td>
<td>Only very few planet-orbits pass between the star and the observer on Earth thus a lot of observations are needed before detecting one.</td>
<td>Will be used during the COROT mission. On June 8th 2004 a similar thing will happen in our solar system, namely the Venus transit. It occurs only very occasionally.</td>
</tr>
<tr>
<td>indirect</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>nulling interferometry</strong></td>
<td>Direct information is available from the planet, the starlight is nulled out.</td>
<td>On Earth the atmosphere is a limiting factor.</td>
<td>Will be used on the DARWIN mission.</td>
</tr>
<tr>
<td><strong>direct</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 1.1: The most important methods of planetfinding–1.
### Table 1.2: The most important methods of planet finding

<table>
<thead>
<tr>
<th>method</th>
<th>principle</th>
<th>limits</th>
<th>mission</th>
</tr>
</thead>
<tbody>
<tr>
<td>radial velocity</td>
<td>Due to gravity of planet the star wobbles. This wobble is seen along the line of sight as if the star moves forth and back. This can be measured with a spectrograph because the light shifts in blue or in red as the star moves toward or away from observer.</td>
<td>The wobbles of Earth-like planets are too small, thus only Jupiter-sized and Neptunus-like planets can be detected.</td>
<td>Will be used during the GAIA mission and is used with ground-based spectrographs.</td>
</tr>
<tr>
<td>astrometric</td>
<td>The star wobble is detected by precise measurements of the position of the star, perpendicular to the line of sight.</td>
<td>From Earth, such precise measurements are nearly not possible. Interferometry, adaptive optics or space telescopes are needed. The wobbles of Earth-like planets are too small, only Jupiter-sized planets can be detected.</td>
<td>Will be used during the GAIA and SIM missions.</td>
</tr>
<tr>
<td>polarimetry</td>
<td>Light propagates in waves which have a wavelength and a direction. The light from the star is unpolarized, its electromagnetic field has an arbitrary direction. However, the light reflected by the planet has a specific direction and is thus polarized. Finding polarized light means that a planet could circle around a star.</td>
<td>-</td>
<td>Is used on the ground-based 2nd generation VLT instruments (UVES).</td>
</tr>
</tbody>
</table>

**1.1.4 What to detect**

What is Darwin exactly looking for? Darwin is searching for extra-solar planets that carry signs of possible life as we know it on Earth. These habitable planets have a mass between 0.5 and 10 times the Earth’s mass. Planets with a mass smaller than $\frac{1}{2} M_{\text{Earth}}$ are thought to be too small to keep their atmosphere, and planets with a mass greater than $10 M_{\text{Earth}}$ will mostly be gas giants. These planets also have to be orbiting in the habitable zone which is the orbit radius where water can be found in its liquid form. The presence of an atmosphere and liquid water will be considered as requirements for possible life.

When Darwin has detected a habitable planet, it will perform spectroscopy to determine the composition of its atmosphere. Darwin will look at certain components that could be an indicator of life, as we know it on Earth. Some of these components, referred to as biomarker gases, are given in Table 1.3, adapted from [21] and [50].
1.1.5 Problems

However, when detecting a planet in another solar system, two major problems arise:

1. The planet appears very close to the star and both bodies need to be distinguished from each other. This means that the telescopes will need to have a much higher resolving power than the ones used presently on Earth. Resolving power depends rather on the aperture than on the surface area of the primary mirror, as given by Rayleigh’s formula [36]:

\[
\sin \phi = 1.22 \cdot 10^{(-10)} \cdot \frac{\lambda}{D}
\]  

In this formula the wavelength \( \lambda \) is given in Ångström \((10^{-10} \text{ m})\) and \( D \) is the aperture in \( m \) [36]. For detecting a habitable planet in another solar system a resolving power of at least 40 mas is needed. This corresponds to resolving a planet at 1 AU from a central star, at a distance of 10 parsec from the Earth, or to distinguishing a feature with a length of approximately 80 meters on the lunar surface as seen from Earth. For a comparison, the human eye has a resolving power of one arcminute which means we can only resolve features on the Moon that are larger than 100 km.

For reaching the required resolving power of 40 mas a telescope with a diameter of a few tens of meters is needed. However, building a telescope of 10 m or more and putting it into space is not a trivial matter. Here, the technique of interferometry offers an excellent solution. A stellar interferometer is a telescope that consists of two or more monolithic telescopes at a certain distance from each other, the baseline. By combining the light beams coming from the several telescopes, one obtains information about the object of interest with the same angular resolution as a monolithic telescope with a diameter equal to the interferometer baseline. Making use of free-flying tele-

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Table 1.3: Spectral lines of the *biomarker* gases.

<table>
<thead>
<tr>
<th>Formula</th>
<th>Name</th>
<th>Part of the spectrum</th>
</tr>
</thead>
<tbody>
<tr>
<td>CO₂</td>
<td>carbon dioxide</td>
<td>15 micrometer</td>
</tr>
<tr>
<td>H₂O</td>
<td>water</td>
<td>6-8 micrometer</td>
</tr>
<tr>
<td>O₃</td>
<td>ozone</td>
<td>9.6 micrometer</td>
</tr>
<tr>
<td>CH₄</td>
<td>methane</td>
<td>7.7 micrometer</td>
</tr>
</tbody>
</table>

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Figure 1.3: Spectral lines of some planetary gases, from [21].
scopes in space, a variable baseline can be achieved up to 1000 m and more. More about interferometry is written in Section 1.2.

2. The star is far more brighter than the planet, which makes it very difficult to detect the planet. To tackle this issue, three contributions to the solution show up:

- **Observations in the infra-red:** the contrast in the signal from the star and the planet is much more in the infrared than in the visible. In the visible the star radiates $10^9$ times more energy than the planet does, while in the infrared this ratio is only $10^6$. This is a result of the fact that planet’s peak of radiation is situated in the IR-region, while stars peak in the visible. Another advantage of observing in the infrared is that the most gases of interest absorb in the infrared (Table 1.3).

- **Space based telescopes:** apart from the resolution requirement, there are two other reasons for observing from space, related to the infrared observations. Space based telescopes are necessary because the Earth’s atmosphere blocks most of the mid-infrared waves. A second reason is that everything on Earth radiates in the IR-region, with an average peak at about 10 µm. This means that also the telescope radiates thermal energy itself. Therefore the telescope and detectors need to be cooled down to a few Kelvin. This is almost impossible on Earth, but feasible in space. The Darwin telescopes for instance, will be cooled down to 40 K and the detectors themselves to 6 or 8 K [21].

- **Nulling interferometry:** nulling interferometry is a technique for interferometers, comparable to the coronograph for monolithic telescopes. Both result in the light of the main star being blocked or nulled out, while the light from the accompanying object (the planet in this case) still comes through. Interferometry is a matter of combining the light beams in a coherent way, which implies keeping both light paths equal to within a fraction of the observing wavelength (≈nm). Nulling interferometry is based on the same principle, but one of the light paths is delayed by exactly half a wavelength.

### 1.1.6 Nulling-imaging

Darwin will detect and observe exo-planets in the so-called *nulling-mode*. Apart from using nulling interferometry, there is a possibility that Darwin will also do imaging of extended astrophysical objects with a very high resolution. For these operations Darwin would operate in *imaging mode*.

In the nulling mode Darwin will detect exoplanets at a distance up to 20 or 25 parsec. After having detected a planet, Darwin will look at its spectrum. Integration times can take up to 6.4 hours for detection, and 125 hours for measuring the spectra. The nulling mode puts the most strict constraints on the optical path differences and is therefore considered to be important for this study.
1.1.7 Lay-out

1. The constellation/formation

The Darwin constellation consists of three main elements: 6 free-flying telescopes, one hub satellite and one master satellite. The free-flying telescopes each collect the incoming light and forward it to the hub satellite that serves as a beam combiner. The free-flying telescopes, as well as the hub satellite, fly in one single plane. The master satellite, which is not in the same plane as the other satellites, mainly takes care of the communications. The master satellite will probably also be used to measure the out-of-plane movements of the telescope satellites. All the satellites are probably equipped with metrology systems to check the inter-satellite distances.

The free-flying telescopes are flying in a Robin-Laurence configuration. This means that they are positioned in a hexagonal shape. Spacecraft separations between the hub and the free flyers will vary from 50 to 500 m. The total mass of the constellation will be less than 4240 kg, which is restricted by the maximum payload capability of the Ariane 5 launcher.

The information about the lay-out and satellites is based on [21]. The constellation design is not yet fixed and can still change in the future. This is discussed in Section 1.4.

2. The satellites

The free-flying telescopes

The free-flying telescopes each collect the light coming from the star and planet and forward it to the hub. The six free-flying telescopes are identical. They have a primary mirror with a diameter of 1.5 meter and they operate in the infrared (wavelength of 6–18 µm). In order to avoid interference of the radiation emitted by the telescope itself, the spacecrafts are being cooled passively to 40 K. This is done in the first place by mounting a sunshield on the spacecraft, that prevents it from being directly heated by the Sun. The sunshield has a diameter of 7 meter and is totally flat. Darwin can be tilted with respect to the Sun to a maximum of 45 degrees, which is related to the dimensions of the sunshield in such a manner that the telescope tube always remains in the shadow. At the bottom side of the sunshield, there is a platform with the necessary equipment for metrology, control and power.

The thermal stability is enhanced by two so-called V-groove-shields that are not flat but are inclined by a slight angle of 5 degrees. These shields must help to prevent the radiation coming through the sunshield from heating the telescope. Also the double telescope tube has the function of providing thermal stability. The free-flyers each have a mass of 493 kg [21]. The telescope itself is a Cassegrain telescope and is shown in Figure 1.4.

The hub satellite

The hub satellite is meant to collect and combine the beams coming from the six free-flying telescopes. After the detection of the resulting signals, they are sent to the communication satellite that redirects them to Earth. As it flies in the middle of the hexagonal-shaped formation of free-flyers, it also has a hexagonal shape, with on each face a beam-receiver-telescope with a diameter of 0.2 m (equal to the diameter of the outgoing-beam-telescopes on the free-flyers). Also the hub flyer has a large sunshield,
with a diameter of at least 5 m. It has a mass of 396 kg. The hub spacecraft has a so-called optical bench, which consists of a platform with 6 delay lines and other elements. These delay lines are designed to delay the light beams so that they arrive at the same time at the detector when the beams are combined. More about delay lines will be written in Section 1.2 about interferometry.

The hub spacecraft and the instruments on the hub also need to be cooled down to 40 K. The detector, however, has to be actively cooled to approximately 6 K. Also the hub is shown in Figure 1.4.

The master satellite

The master satellite’s main function is to send and receive information between the hub satellite and the Earth. Besides, the master satellite will keep an eye on the out-of-plane drifts of the seven other satellites. This is possible with simple metrology because the master is the only satellite not flying in the same plane as the other satellites, but at a distance of hundreds of meters behind them.

The master satellite is a small cube of approximately 1 m$^3$, that weighs only 179 kg. It has no sunshield like the other satellites, but it has 2 rectangular solar panels. It is equipped with mainly communication systems, metrology systems, and its own propulsion system.

3. The orbit and trajectory

Darwin will fly in a Lissajous orbit about the second Lagrange point of the Sun-Earth system. This point is located at 1.5 million km (0.01 AU) behind the Earth as seen from the Sun (Fig. 1.5, from [86, 3]). The main advantage of an orbit at L$_2$ is the uninterrupted view on the stars, because the Sun, as well as the Earth and the Moon are all in the same direction relative to Darwin. As Darwin moves along with the L$_2$ point in an orbit around the Sun, it can cover the whole celestial sphere in exactly one year. Another advantage is the relatively quiet environment at L$_2$. There are no radiation belts, the Earth albedo has a low value, the microgravity environment is low-frequent and limited, and the biggest disturbance force is considered to be the solar radiation pressure force. Still, there is one disadvantage at L$_2$ and that is its instability. Darwin will have to keep itself in the right orbit by regular stationkeeping. However, this costs on average only 2 ms$^{-1}$y$^{-1}$ and can be managed quite well by a correction manoeuvre about every month. A possible trajectory for Darwin is shown...
in Fig. 1.5. This trajectory will be assumed during this study.
Darwin is scheduled to be launched by Ariane 5 in 2015. The orbit to $L_2$ will be a
direct transfer orbit, which will take about 100 to 200 days. Its nominal lifetime will
be 5 years.

![Figure 1.5: The orbit and trajectory of Darwin, from [86, 3].](image)

**1.1.8 Precision formation flying**

In order to be able to fly the satellites on relative distances controlled to the nanometer, a
sophisticated control and metrology system is inevitable. The metrology system of Darwin
takes care of this by measuring the relative distances to a nanometer level after which the
data is processed and the delay lines or eventually propulsion system is activated. All this
is done at a frequency of probably 10 Hz or 100 Hz, depending on the level of disturbance
forces at higher frequencies (i.e. above 1 Hz). The control system acts in three different
operational modes designed for a different goal: the baseline control mode, the fringe ac-
quision mode and the normal operation mode. All three modes have different precision
requirements. Only the most stringent requirements are considered in this work (normal
operation mode).

The metrology system provides Darwin with range measurements as well as angle measure-
ments and fringes of a reference star. The control system takes the metrology measurements
to control FEEP thrusters. When the satellites are disturbed with a frequency higher than
the control loop or metrology frequency, nanometer stability may not be provided anymore.
The propulsion system will probably consist of Field Emission Electric Propulsion units,
capable of thrusting in a range of $1 \mu N$ to a magnitude of $mN$.

**1.1.9 Development**

A few technological innovations need to be made before Darwin can be launched into space.
Studies have already been made by Alcatel for Darwin, and Astrium for its precursor mission
as it was defined in earlier stages (SMART-2). Also a study is started for the Formation
Flying Demonstration Mission [30].

Improvements need to be made in formation flying technology and strategies as well as
for certain subsystems like coolers and FEEPs. These subsystems will need to be developed
further, and checked especially in order to avoid disturbances on the satellites. Also metrology and laser monitoring of the relative positions will be developed so that the satellites can be controlled to a centimeter level, and the delay lines to a nanometer level. Also nulling interferometry is a new technology that needs further development and testing.

1.2 Interferometry with Darwin

1.2.1 Interference

Interference is the phenomenon of the interaction of waves. Darwin uses infrared waves. When two similar light waves (same wavelength, phase and polarity) are combined, their throughs and peaks will amplify each other and form a wave with stronger intensity. However, if those waves are combined with a time delay between them that corresponds to exactly half a wavelength, then the throughs of one wave compensate the peaks of the other one and the resulting wave dies out. When the waves are put together in phase, and thus reinforce each other, one speaks of constructive interference. The other case, in which two waves cancel each other, is called destructive interference. When two light beams are put together, the resulting brightness is thus a function of the difference in arrival times of the two waves.

The difference between constructive and destructive interferometry is only a matter of half a wavelength. The wavelengths in the visible have magnitudes of $10^{-7}$ to $10^{-6}$ m, thus a very high accuracy is needed when combining the two light beams. This also holds for Darwin (wavelength range of 6–18 µm). Radio waves have wavelengths of 1 mm and larger, thus the accuracy for combining the beams is less stringent. Therefore, it is no wonder that optical interferometry has been well developed only half a century after radio interferometry.

1.2.2 The stellar interferometer

About two centuries ago, Thomas Young demonstrated the concept of interference of light waves with his so-called two-pinhole experiment. He put a light source behind a plate with two holes in it, and projected the light on a screen. The result was a pattern of light and dark bands corresponding to constructive and destructive interference, also called interference fringes. Later on, the contrast of the fringes seemed to be related to the distance between the two separated beams, and to the geometry of the source (e.g. the pinholes, or a star). In 1868 Fizeau suggested to calculate the diameters of stars from measurements of the optical interference patterns. Although the interferometers of today make use of advanced technologies, the basic principle of interferometry has not changed since then.

Fizeau’s proposal was to put a mask with two slits on the aperture of a telescope. The light beams through the slits would interfere and form fringes. The fringes would then disappear at a certain separation of the two slits, which is related to the size of the star. It was his successor, E. Stephan who applied the idea and put a mask with two slits in front of a telescope at Marseille observatory. The angular resolution of the observations seemed to depend only on the distance between the slits, as is clear from Formula 1.1 from Section 1.1.5. However, Stephan’s 80 cm telescope was not large enough to have the fringe contrast to become zero. He concluded that the diameter of the stars must be smaller than 0.158 arcsec.

The next great step in the development of interferometers was the research done by A. Michelson. Apart from his research on the speed of light, he also mounted a set of mirrors on top of a telescope for constructing his interferometer (Fig. 1.6, from [82]). He was able to deduce the diameters of Jupiter’s moons by means of interferometry.

In the case of Stephan, the distance the light has traveled from the star to the detec-
tor was exactly the same, as the telescope has a parabolic mirror on which the mask was placed. Nowadays, the two apertures are not located on one single parabolic mirror, but the apertures are actually two separate telescopes at a distance from each other. This distance is called the baseline. The principle is the same, but now the light paths have to be equalized by means of modern technology. For combining the light beams with the same wavelength, phase and polarization, accuracies are needed of about a fraction of the observing wavelength. This could be compared with the manufacturing accuracy of the primary mirror of a telescope. Modern interferometry requires very sensitive optical detectors, lasers, computers, extremely accurate mechanical systems, vibration control, closed control loops and data processing techniques.

1.2.3 Principles of interferometry and nulling interferometry

Interferometry is not about getting a direct image of the sky. Interferometry is about gaining an image of the skies by using only a limited number of pieces from the telescope mirror as performed by Stephan, or apertures as it is now. The advantage is that high-resolution astronomy can be done without building enormous telescopes. Several apertures with a certain baseline is enough. Mixing the pieces of information collected, and using a Fourier transform, one reconstructs the original image, with a resolution as good as with the entire original telescope mirror.

These pieces of information are not just images, but quantities that are a result of the interference of the combined light beams.

**Fringes** have two main properties out of which we can get the desired information: the contrast between the light and dark bands, and their phase, or position of the central fringe with respect to the Optical Path Difference (OPD) equal to zero.

The contrast between light and dark bands is called **fringe visibility**. It is a number between 0 and 1 defined as

\[ V = \frac{(I_{\text{max}} - I_{\text{min}})}{(I_{\text{max}} + I_{\text{min}})} \]  

(1.2)

\( I_{\text{max}} \) is the intensity of the light regions and \( I_{\text{min}} \) is the intensity of the dark regions. If the visibility is equal to one, there is perfect contrast. The observed object is said to be unresolved. In reality instrumental and atmospheric effects keeps the visibility away from 1. If the visibility is zero, the object is resolved and no fringes are present anymore.
The visibility is a function of four parameters: the baseline $B$, the intensity distribution of the star, the spectral bandwidth and the geometry of the light source.

The **baseline** is the line connecting two apertures. On a monolithic telescope this baseline would be equal to the diameter of the primary mirror. The baseline defines the angular resolution of the interferometer: the bigger the baseline, the better the angular resolution.

The visibility plotted against the baseline is called the **visibility curve** (Fig. 1.7). The visibility has a maximum when the baseline is zero and decreases with larger baselines. At a certain point, the resolving point, the visibility drops to zero and no fringes are present anymore. At this point the star is resolved and its diameter can be deduced from the data. It is this point that Stephan tried to find with his 80 cm telescope in order to deduce the diameter of the stars. At longer baselines than the resolving point the visibility increases again, but not more than 10% of its initial value, after which it drops again.

![Figure 1.7: The visibility curve](image)

The **visibility function** is a complex function that consists of those two variables. Its amplitude is determined by the fringe visibility, while its phase is given by the position of the central fringe.

Observing using interferometry means taking measurements for many baselines with different length and orientation, of those two variables. Taking the Fourier transform of the visibility function at all those different lengths and orientations, one can reconstruct the original image again. The more measurements are taken with different baselines, the more accurate the image will be. The free-flying formation concept of Darwin eases taking measurements with many different baselines.

If two telescopes receive the same light beams from the same star, and corrections have been made for the OPD so that fringes are formed, the light beam of one of the apertures needs only to be shifted in phase for half a wavelength to create destructive interferometry. On this principle **nulling interferometry** is based (Fig. 1.8, from [86, 21]). The central star is nulled out in this way, to detect the light of the accompanying planet. This is because the planet is situated at a slight angle from the star. This angle causes an optical path difference. At a certain baseline this optical path difference will allow fringes to be formed of the planet, while the star is still nulled out.
1.2.4 The Darwin interferometer

1. The array

Darwin consists of 6 free-flying telescopes. A number of 6 apertures results in 15 possible baselines that can be used at the same single moment\(^3\). These telescopes will fly at distances from the hub, varying from 50 to 500 m. Apart from varying baseline lengths, the total formation of telescopes will rotate continuously about its line of sight in order to make measurements at more and different baselines.

The light coming from the star is thus collected by the 6 telescopes and forwarded to the hub satellite where it will need some corrections for the OPD before being recombined. This is done by the Optical Delay Lines.

2. Delay lines

In order to reach the nanometer stability of the baselines, 6 delay lines will probably be implemented in the hub satellite of Darwin. A delay line can be best presented as a little sled on a rail with a set of mirrors on it. By moving the sled with a high frequency and to a nanometer level, the OPD is kept stable. Systems like FEEP-thrusters and metrology will keep the baselines stable during the observing time to an accuracy of 1 cm. This is why the delay lines of the original Darwin design would only have a length of 2 cm. An example of Delay Lines as used in Chili for the Very Large Telescope Interferometer are shown in Fig. 1.9, from [83].

The frequency at which the sled is moved is not yet determined and will depend on the level of the disturbance spectrum. The delay lines are controlled by the fringe tracker. This device tracks the fringes of a reference star using a fringe sensor, and sends signals to the delay lines and control subsystems to correct the OPD if it deviates too much to form proper fringes.

Once the OPD is zero, the light beams coming from the different telescopes are combined by the beam combiners after which a detector detects the incoming photons.

\(^3\)The number of baselines defined for an array of \(n\) telescopes is equal to \(\frac{n(n-1)}{2}\).
1.2.5 Rejection factor $\rho$

The rejection rate $\rho$ is defined as the ratio of constructive and destructive interference intensities

$$\rho = \frac{I_{\text{max}}}{I_{\text{min}}}$$ (1.3)

The rejection factor is related to the visibility by

$$\rho = V = \frac{(I_{\text{max}} - I_{\text{min}})}{(I_{\text{max}} + I_{\text{min}})} = \frac{(\rho - 1)}{(\rho + 1)}$$ (1.4)

For a perfect monochromatic and coherent plane wave and an ideal interferometer, the rejection rate is equal to infinity. For Darwin it is calculated that $\rho$ should be $10^5$ [21], although a value of $10^6$ is advised and used during this study [77]. The rejection factor depends on different influences that are listed in Table 1.4.

<table>
<thead>
<tr>
<th>Influences</th>
<th>examples</th>
</tr>
</thead>
<tbody>
<tr>
<td>Optical defects</td>
<td>figuring, polishing, coating defects, dust</td>
</tr>
<tr>
<td>OPD residuals</td>
<td>tilt differences, intensity differences, OPD differences, wavefront errors, difference in polarization</td>
</tr>
<tr>
<td>Pointing errors</td>
<td>pointing errors of individual telescopes, or of the constellation as a whole</td>
</tr>
</tbody>
</table>

Table 1.4: Influences on the rejection rate $\rho$.

1.3 Requirements for Darwin

The Darwin mission needs to satisfy a few demands that are derived from the scientific needs. The relevant requirements for this study are recapitulated in this Section.

During the Darwin mission different modes will be used, the “imaging” and “nulling”, and three different control and metrology modes. They all have different requirements, of which only the most strict ones are discussed and used in this study. The strictest requirements are linked to the nulling mode and the normal operations mode. The most important requirements for the nulling mode are listed in Table 1.5.
Baseline accuracy
OPD
transverse offset between optical axes of transmitter and receiver
FF pointing error
pointing error hub
array attitude accuracy
observing time nulling - planet detection
observing time nulling - spectrometry
stellar rejection ratio (rho)
temperature inner telescope tube and optical subsystems
temperature service modules (isolated from optical platform)
 asymmetry
metrology frequency

<table>
<thead>
<tr>
<th>Requirement/Property</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Baseline accuracy</td>
<td>&lt; 1 cm</td>
</tr>
<tr>
<td>OPD</td>
<td>&lt; 5 nm</td>
</tr>
<tr>
<td>transverse offset between optical axes of transmitter and receiver</td>
<td>70 µm</td>
</tr>
<tr>
<td>FF pointing error</td>
<td>8.4 mas</td>
</tr>
<tr>
<td>pointing error hub</td>
<td>1 arcsec</td>
</tr>
<tr>
<td>array attitude accuracy</td>
<td>0.1 deg</td>
</tr>
<tr>
<td>observing time nulling - planet detection</td>
<td>6.4 hours</td>
</tr>
<tr>
<td>observing time nulling - spectrometry</td>
<td>125 hours</td>
</tr>
<tr>
<td>stellar rejection ratio (rho)</td>
<td>$10^5(-10^6)$</td>
</tr>
<tr>
<td>temperature inner telescope tube and optical subsystems</td>
<td>&lt; 40 K</td>
</tr>
<tr>
<td>temperature service modules (isolated from optical platform)</td>
<td>&lt; 300 K</td>
</tr>
<tr>
<td>temperature hub detector</td>
<td>&lt; 6–8 K</td>
</tr>
<tr>
<td>asymmetric gradients in hub optical bench</td>
<td>&lt; 30 mK</td>
</tr>
<tr>
<td>metrology frequency</td>
<td>10 Hz (not fixed yet)</td>
</tr>
<tr>
<td>design lifetime</td>
<td>5–10 yr</td>
</tr>
</tbody>
</table>

Table 1.5: Requirements and properties of the Darwin mission, from [21].

The frequency of the metrology is assumed to be 10 Hz. This means that in theory, only perturbations up to 5 Hz (Nyquist frequency) can be controlled. In reality this is about 1 Hz. When perturbations exist with a magnitude of a nanometer or more and a frequency of at least 1 Hz, then these cause the rejection factor $\rho$ to decrease and thus fringes to be less clear. The relationship between the rejection factor $\rho$ and the OPD $\delta$, is [21]:

$$\frac{1}{\rho} = 2\pi^2 \frac{(OPD)^2}{\lambda^2} \quad (1.5)$$

In [21] it is assumed that 50% of the degradation in $\rho$ is caused by stellar leakages, and another 50% is due to optical path differences. If this holds, then for an observation at 10 micrometer and $\rho$ equal to $10^5$, the OPD due to vibrations must remain smaller than 5 nmRMS. An additional requirement is that the spectrum of the OPD must remain lower than $[P_{OPD}]^{(\frac{1}{2})} = 7.5 \text{ nm Hz}^{(-\frac{1}{2}4)}$ [21]. However, according to [77], a rejection factor of $\rho = 10^6$ might be necessary. In that case, the RMS-value of allowed optical path differences due to vibrations becomes in the order of 1 nmRMS. This study assumes $\rho = 10^5$ as given in [21, 39, 40] and a maximum OPD of 5 nmRMS.

The main sources of vibrations will be of internal cause. Subsystems like FEEP’s and DC-DC converters can cause vibrations on all the spacecrafts [21]. For the hub, extra care must be taken for the active coolers of the detector. External sources of vibrations may also exist. The cumulative effect of all the disturbance forces (their quadratic sum, assuming that there is no correlation between the sources of vibration) must finally remain smaller than 1 nmRMS. In [21] ESA states that the FEEP noise must remain smaller than 1.7 $\mu$N at 1 Hz.

1.4 The Darwin Z-array/TTN+

1.4.1 Introduction

The hexagonal configuration of Darwin as it is described in Chapter 1 has been changed during the duration of this study, and still might be changed in the future before the final

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4In [21] a value of 7.5 nm^2 Hz^{(-\frac{1}{2}4)} was stated. The author assumes in this study that the right units are nm Hz^{(-\frac{1}{2}4)}, corresponding to the unit of the square root of a Power spectrum $P_{OPD}$.\[4\]
concept is defined. Two other concepts were reviewed until now: the *Darwin Z-array* and *Darwin TTN+*. These changes will obviously have consequences on the results of the disturbance force study, although the final order of magnitude of the disturbance levels does not depend largely on the different concepts. For this reason they are all based on the basic hexagonal Darwin configuration. The calculation of the internal disturbance forces is based on a mix of the original Darwin concept and the Darwin Z-array. The particulate environment is based on the old concept. This Section discusses the new configurations and compares it with the old one.

### 1.4.2 The Darwin Z-array configuration

The Darwin Z-array consists of only 4 satellites, in a Z-shaped configuration: two *Detector satellites* and two *Relay satellites*. All of them have a telescope unit on-board. The scientific observations are made in two steps: the light coming from the star or planet is received by two telescopes (one Relay and one Detector) and “forwarded” to the two Relay satellites [39]). In the Relay satellites the light is nulled, and sent on its turn to the two Detector satellites. There, the nulled light is modulated and detected. There is no fifth satellite for the communications, but the detector satellites are equipped with an antenna. The shortest two inter-satellite distances are 11 to 34 m, while the long arms of the parallelogram are at least three times longer. The interferometer array is rotated continuously around its line of sight.

The satellite designs are all based on the former free-flyer telescopes of the hexagonal configuration. The relay satellites are equipped with a telescope unit, 5 transfer telescopes, FEEP thrusters, a nulling mechanism (containing the ODL), and the necessary metrology equipment. The detector satellite also has a telescope and FEEPs, but it only has 3 transfer telescopes. In addition it has a modulation and detection mechanism for detecting the fringes. The latter yields an active cooling system for the detector. The detector satellite also contains an antenna for communications with the Earth. The diameters of the mirrors are 2.5 m instead of the former 1.5 m.

### 1.4.3 The Darwin TTN+ configuration

The so-called Darwin TTN+ configuration also consists of 4 satellites: 3 telescope satellites and one beam combiner. The design of the satellites is also based on the design of the free flyers and hub of the hexagonal configuration. The diameter of the main mirror is 3.15 m instead of the former 1.5 m. The Darwin TTN+ array can fly in two possible configurations. The first one is a triangular configuration, where the beam combiner flies in the middle of the three telescope satellites. The second is a linear configuration, where the three telescope satellites fly next to each other, in front of the beam combiner. The difference between the two is mainly that in the linear configuration more light can reach the detector, while in the triangular configuration the localization of the planets is better [40]. The Darwin TTN+ would be launched by a Soyuz/Fregat launcher instead of Ariane 5.

### 1.4.4 Configuration used in this study

The model used for the internal disturbance forces is based on the detector spacecraft of the Darwin Z-array, because this spacecraft contains the FEEPs, ODL as well as the sorption cooler. In such a manner a “worst-case” is made of the internal disturbance forces. Another possibility would have been to model the disturbance forces of the hub of the original hexagonal configuration. This hub also contains the sorption cooler, as well as 6 delay lines and FEEPs. Because this assignment only reveals an order of magnitude of the disturbance forces, the assumed configuration finally does not matter a lot. For the micrometeoroid impacts, assumptions are made for a satellite, based on the basic design of the free flyers of the hexagonal configuration.
The outcome of this study for a certain configuration depends mainly on the amount and kind of subsystems aboard, the mass of the satellite and the radius of the sunshield.

1.5 Formation Flying Demonstration Mission

ESA is considering a Formation Flying Demonstration Mission (FFDEM) in order to prepare for future missions like Darwin. During this mission, the concept of formation flying will be demonstrated, as well as some new technologies like the FEEP-s, that will be used for Darwin. The mission scenario on which this study is based, is taken from [30]. In this article, a demonstration mission is proposed with two microsatellites having a mass of approximately 120 kg and measuring only $0.6 \times 0.6 \times 0.7$ m. These satellites are based on AMP, the Advanced Microsatellite Platform. The satellites have a baseline of 50 m. They are flying in a Low Earth Orbit, at 561 km initial altitude and $90^\circ$ inclination, with a velocity of 7.6 km/s. These are the most difficult conditions for formation flying purposes [30]. This is probably in order to test the formation flying technology thoroughly for all possible orbits, though the reason of the orbit choice is not mentioned in [30]. The target duration of the FFDEM is 17 months. The requirements posed on FFDEM were assumed in this study to be the same as for Darwin, concerning the maximum allowed OPDs.
Chapter 2

The internal disturbance forces for Darwin

2.1 Introduction

Apart from external disturbance sources like the solar radiation pressure or aerodynamic drag, the Darwin satellites also suffer from disturbances that arise from on-board equipment. Most of these disturbances have a high-frequency component, higher than 1 Hz. Care must be taken when such a frequency reaches a value near the eigenfrequencies of the satellite subsystems, because then the disturbances could reach higher values than normally.

In this Chapter the internal disturbance sources are described from the following subsystems that are believed to be the major potential disturbance sources:

1. FEEPs

2. Sorption coolers

3. Optical delay lines

There are many other systems on Darwin that can possibly cause high-frequency disturbances too. These are listed in Section 2.4.4 and are not looked at furthermore in this study, because for some of them no information is found yet on their characteristics (often it is technology in development), while for others it is not sure if they are going to be used on Darwin, or if they do cause disturbances at all. They need to be examined later, during the Darwin study.

A model is built to examine the effects of these disturbance forces on the satellites, and finally the results of the internal disturbance forces are discussed. In this Chapter the satellite refers to a Darwin free flyer satellite. Only the Darwin free flyers have been modeled. It is assumed that they all have one sorption cooler, one optical delay line and 16 FEEPs, based on the design of the detector satellite of the Darwin Z-array.

Section 2.2 describes the satellite properties that are input for modelling the satellite. Section 2.3 summarizes the vibration requirements of the Darwin OPDs. In Section 2.4 the different disturbance sources are discussed in detail, and the way in which they are modeled. Section 2.5 discusses the model used for calculating the effect of the disturbances on a satellite. Section 2.6 describes the input parameters used for the model. In Section 2.7 the results of the study are shown, after which a conclusion is drawn in Section 2.8.
2.2 Darwin satellite properties

The basic layout of the Darwin satellites has been described in Chapter 1. In this Section attention will be paid to properties of the satellites that are needed for the construction of the model.

The Darwin satellites have a deployable sunshield, with an eigenvalue that will be assumed to be in the order of 0.5 to 5 Hz [38]. The sunshield has a mass of about 46 kg, solar arrays included.

The spacecraft without sunshield has a mass of about 447 kg. The Soyuz-Fregat launcher requires the spacecraft to have an eigenfrequency of higher than 15 Hz in the lateral direction and 35 Hz in the longitudinal direction. This means that all components must have eigenfrequencies above this number\(^1\). Typical values are about 60 Hz [38], or more (e.g. 785 Hz for the ODL stator [72]).

2.3 Darwin satellite and subsystem requirements

The Optical Path Difference (OPD) due to vibrations, after OPD regulation must remain smaller or equal to 5 nm\(_{\text{RMS}}\) [21]. The cumulative effect of all vibration sources apart (their quadratic sum), after OPD regulation, must not exceed this value. If the OPD regulation mechanism operates at 10 Hz, then in theory the maximum disturbance frequency that can be controlled is the Nyquist frequency\(^2\) of 5 Hz. In practice, however, the OPD disturbances can only be controlled up to 1 or probably 2 Hz [17]. In this study 1 Hz is assumed to be the maximum disturbance force frequency that can be controlled. This means that the quadratic sum of the RMS-values of all the disturbance forces with a frequency component above 1 Hz must be lower than 5 nm RMS, if a fringe sensor of 10 Hz is used.

The ODL has a mechanical length of only 1 cm. In order to prevent the ODL from saturating (i.e. reaching the end of its stroke), the free flyer satellites are equipped with a sensor with a resolution of 100 \(\mu m\) that measures the position of the ODL [72]. This information is fed back to the FEEPs. In this manner the FEEPs are activated in time to correct for the satellite displacement as soon as the ODL threatens to exceed its 1 cm-stroke, and no ODL saturation takes place. Especially in the case of large (sudden) disturbance forces the ODL could otherwise be saturated quickly. Therefore also the disturbances with frequencies just below 1 Hz are important to be looked at. In this study most of the disturbance forces are looked at in a frequency band of 0.01 Hz up to 1000 Hz, but the main focus of this study are the forces with frequencies above 1 Hz. The low-frequent, deterministic forces have been studied in [15].

For certain specific subsystems, the following requirements are also given:

- The Force Spectral Density (FSD) of the FEEPs noise must not exceed the FSD as shown in Figure 2.1 (from [41]).
- The sorption cooler noise should be lower than 1 \(\mu N/\sqrt{\text{Hz}}\) [12].
- The total optical delay line noise should be lower than 1 nm RMS in closed loop [74].
- The ODL should be able to counteract disturbance forces that are specified in the following Displacement Spectral Density (Table 2.1 and Fig. 2.2, from [72]):

\(^1\)The eigenfrequency of the Sunshield is different (larger) when it is folded up than if unfolded.
\(^2\)Appendix B.1 gives an overview of some basic understandings in random vibration analysis and signal processing.
Figure 2.1: The Force Spectral Density of the FEEP requirements, from [41].

<p>| | |</p>
<table>
<thead>
<tr>
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<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Up to 1 Hz</td>
<td>constant</td>
</tr>
<tr>
<td>Above 1 Hz</td>
<td>-40 dB/dec</td>
</tr>
<tr>
<td>Total RMS</td>
<td>100 nm RMS</td>
</tr>
</tbody>
</table>

Table 2.1: The microvibration Displacement Spectral Density that the ODL should be able to counteract, from [72].

Figure 2.2: The microvibration Displacement Spectral Density that the ODL should be able to counteract, from [72].
These requirements are met for frequencies above 1 Hz of all subsystems. One part of the cooler (the “CFHX”, see Section 2.4.2), exceeds the requirement below 10 Hz, but this part of the cooler was modeled very pessimistically and in addition, the forces will be internally compensated. In reality, also this part of the cooler will meet the requirements above 1 Hz. These requirements are not used furthermore in this study, except for the FEEP’s Force Spectral Density requirement, which is used as a worst-case for the FEEPs.

2.4 Internal disturbance sources

2.4.1 The FEEPs

FEEP description

Darwin needs thrusters that control the satellites to a centimeter-level precision. One of the thrusters that meets the requirements for Darwin are the FEEPs thrusters (Field Emission Electric Propulsion). Two kinds of FEEPs are used on Darwin: the mN-FEEPs and the μN-FEEPs. In this Section, only the μN-FEEPs are discussed because only these are used during the scientific operations that have the most demanding requirements. The FEEPs are assumed to be turned on all the time during these operations.

Two kinds of FEEPs are currently in development, that could be used for Darwin [41, 42]. These are the Caesium FEEPs developed by ALTA\textsuperscript{3}, and the Indium FEEPs, developed by ARC\textsuperscript{4}. The Caesium FEEPs are slit-based, while the Indium FEEPs are needle-based, but still for both the main principle remains the same: ions are accelerated along an electric field, that results in a certain thrust level. Both kinds of FEEPs will be shortly discussed now.

- Caesium FEEPs
  The main parts of the slit-based FEEP thruster are the emitter, the extractor plate or accelerator and a neutralizer as shown in Figure 2.3, from [85]. The emitter contains a reservoir with a propellant. This propellant, usually a metal like Caesium, is stored in a solid phase. The emitter has a narrow slit of only about 1 or 2 μm, and has a length of basically 1 mm to a few cm [64]. On the accelerator a voltage is applied (usually 11 kV) such that an electric field is formed between the emitter and extractor, typically about $10^9$ V m. This field causes the atoms of the propellant to be ionized and accelerated. The neutralizer is necessary to provide a stream of electrons, in order to prevent spacecraft charging. The propellant has a low ionization potential, a low

\textsuperscript{3}Centrospazio, Pisa, Italy
\textsuperscript{4}Austrian Research Centers Seibersdorf in Austria

![Figure 2.3: The Caesium FEEP thruster, from [85].](image-url)
melting point and a high atomic weight. Still, because it is stored in a solid phase, a heater is necessary in the FEEP system. Not much fuel is needed. For a mission like Darwin, 1.1 kg of fuel is enough for the entire 5-year lifetime [41] in the case of the Caesium FEEPs of ALTA. The total FEEP system mass for 16 FEEP thrusters will be almost 20 kg [21, 41]. This number includes all subsystems belonging to the FEEPs system, like a heater and a power unit.

• **Indium FEEPs**

The Indium FEEPs are based on the same principle as the Caesium FEEPs, but they have a needle point instead of a slit, where the ions are accelerated (Figure 2.4, from [29]). The propellant is stored in a liquid form at the bottom of the needle. As soon as a voltage is applied, the propellant forms a thin film on the outside of the needle and is accelerated towards the accelerator (or **extractor electrode** in Figure 2.4). The total system mass of the Indium FEEPs is almost twice as much as the Caesium FEEPs (nearly 40 kg for 12 thrusters). The total amount of fuel for the Darwin mission is 1.4 kg [41]. On the other hand, one of the advantages of the Indium thrusters is that contamination is less severe than in case of Caesium thrusters.

![Figure 2.4: The Indium FEEP thruster, from [29].](image)

**Accommodation on the satellite**

A Darwin free-flyer satellite is assumed to have 4 clusters, each containing 2 FEEP thrusters each, on the service module (a small part of the spacecraft “below” the sunshield), and another 4 clusters on the payload module (the part of the free flyer “above” the sunshield containing the optical instruments) [41]. This adds up to a total of 16 FEEP thrusters for microneutron corrections. It is assumed that all of them are activated at the same time, and that all of them point to the same direction. This is not the case in reality, but it is a worst-case scenario. The 16 mN FEEP thrusters are not turned on during scientific observations and are therefore not taken into account in this study. The FEEP noises are also assumed to have no correlation.

**Noise model**

The FEEP noise has been modeled using an approximation of the FEEP noise measurements from [41, 29, 71]. This approximation is shown in Figure 2.5 for different kinds of FEEPs. In Figure 2.5, the noise FSD per single FEEP thruster is given. The FEEP noise for one thruster stays below the given requirements for the FEEPs, for most frequencies. However,
there is a maximum of 16 FEEP thrusters firing at the same time, and in the same direction, in a worst-case assessment. Therefore, the FEEP noise RMS-value is summed quadratically 16 times. The thruster noises as shown in Figure 2.5 are deduced from the thrust noise Power Spectral Densities as given in [41], [29] and [71]. The FEEP requirement is selected as a worst case in this study. The LISA A thruster has a maximum thrust force of 50 $\mu$N. The LISA C FEEP has a maximum thrust force of 25 $\mu$N and the Caesium FEEP has a maximum thrust force of 100 $\mu$N. For the other FEEPs the thrust force is unknown to the author. In the very high and very low frequency range ($10^{-1}$ – $10^{4}$ Hz and $10^{-4}$ – $10^{-3}$ Hz), some of the PSDs are extrapolated (Figure 2.5). In the range from $10^{-3}$ – $10^{3}$ all the data are real and not extrapolated.

![Figure 2.5: An overview of the approximated FEEP noise per thruster, based on the FSDs from [41, 29, 71].](image)

### 2.4.2 The sorption coolers

**Sorption cooler description**

The Darwin free flying telescopes are primarily cooled down in a passive way, to a temperature of only 40 K (50 K for the hub). This is done by mounting a large sunshield at the bottom of the satellites, that blocks out all direct sunlight on the instrument part of the satellite. The detector however, needs to be cooled down to 4 K, which is only possible by

---

5 Summing them before or after the computations does not matter, because the model is assumed to be linear.
means of an active cooler.

The *Joule-Thompson sorption cooler* has been selected currently as a suitable cooler for the Darwin mission. This cooler is a 2-stage, H/He carbon-based cooler, that makes use of passive precooling\(^6\).

The sorption cooler for Darwin actually consists of two sub-sorption coolers: one for Hydrogen and one for Helium. The Hydrogen stage pre-cools the Helium for the Helium stage. In this study only the Helium stage is looked at, because for the Hydrogen stage no information on the vibration levels is known yet.

Each JT-sorption cooler consists of two main parts: the *compressor unit* and the *cold stage*, as shown in Figure 2.6, from [12]. Figure 2.6 shows the basic design of a sorption cooler, in general. The compressor unit, in turn consists of at least four *sorption compressor cells*. This number is necessary in order to provide continuous cooling. Every compressor cell is accompanied by two *check valve units* to control the gas flow. Also buffers are added to the design, that are not shown in Figure 2.6. Figure 2.8 shows the last design of the cooler, where the buffers are uncluded. These buffers are extra volumes added to the compressor unit. One of the main reasons is to prevent a too high pressure when the cooler is not working (e.g. before launch). The *high-pressure line*\(^7\) consists of a high-pressure buffer, and the *Counterflow Heat Exchanger* (CFHX), which is part of the cold stage. The cold stage also has a cryostat (Figure 2.6).

The properties of the cooler for Darwin as it was defined in [8] are summarized in Table 2.2.

<table>
<thead>
<tr>
<th></th>
<th>1st stage</th>
<th>2nd stage</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cooling substance</td>
<td>H</td>
<td>He</td>
</tr>
<tr>
<td>Cycle period</td>
<td>420 sec</td>
<td>720 sec</td>
</tr>
<tr>
<td>Number of compressor cells</td>
<td>4 cells (2 times 2)</td>
<td>8 cells (2 times 4)</td>
</tr>
<tr>
<td>Number of valves</td>
<td>8 valves</td>
<td>16 valves</td>
</tr>
</tbody>
</table>

Table 2.2: Subparts and properties of the JT-sorption cooler, from [8].

The main principle of the JT-sorption cooler is based on the fact that a large amount of gas can be adsorbed on solids like highly porous Carbon [11]. The process is described in the following five steps:

1. The sorption compressor cell is filled with gas at a low temperature and low pressure, by means of adsorption. One of the valves opens as soon as a small pressure difference is present in the pipeline. This corresponds with \(D\) in the adsorption diagram of Figure 2.7, from [11].

2. The temperature of the Carbon is raised which also causes the gas pressure to increase. The valves are closed. This is illustrated with \(A\) in the adsorption diagram.

3. One of the valves opens again, and the gas flows out of the sorption compressor cell. This corresponds with \(B\) in the adsorption diagram. This is done in a controlled way while the pressure is kept high and the temperature increases.

---

\(^6\)Passive precooling to such low temperatures as 40 K is possible thanks to the stable thermal environment in \(L_2\). In an Earth orbit the Earth radiation would not permit passive precooling.

\(^7\)Also not shown in Figure 2.6
4. In the cold stage, the gas flows to the cold side through the *Counterflow Heat Exchanger* that acts as a pre-cooler. The gas is then cooled down further in the JT-unit to a low temperature at a low pressure.

5. Finally the gas flows back through the CFHX to the compressor unit and the cycle is repeated. This corresponds with C in the adsorption diagram.

Figure 2.6: The different parts of the sorption cooler, from [12].

Figure 2.7: The adsorption diagram, from [11].
Figure 2.8 shows the real design of the cooler. On the left is shown the hydrogen stage and on the right the helium stage. The helium stage consists of two compressor stages. Each one of them has four compressor cells, accompanied by two check valves per cell. It has a medium-pressure buffer and a low-pressure buffer. The function of the high-pressure buffer is taken over by the pipelines themselves because their volume is small enough to do so. The first stage of the compressor also has an extra valve.
Vibration levels

This Section summarizes the vibration levels that are expected, based on the assessments of [12] and [8]. The vibration levels are sorted by source of vibration. Special attention must be paid to the assumptions that have been made for the calculations of these vibration levels: the subject is still in development and thus vibration levels can still alter. The assumptions have been defined in direct consultation with the developer of the cooler [8].

1. Sorption compressor cell
   There are three main causes for vibrations of the sorption compressor cells:
   
   (a) The compressor cells can change in length due to changes in temperature.
       According to [12], for the first stage of the compressor this results in a maximum force of 26 nN. The second stage has a resulting maximum force of 22 nN. These forces are internally compensated and extremely small. Therefore this force is neglected.
   
   (b) The compressor cells can change in length due to changes in pressure.
       This can result in a strain change of \( \Delta \varepsilon = 1.6 \cdot 10^{-4} \) [12], that results in forces that are even smaller than the forces induced by a change in temperature. Also these forces are neglected for the same reasons as the previous one.
   
   (c) Forces may arise due to the gas flow in and out of the compressor cells.
       The gas is accelerated and decelerated while flowing in and out of the compressor cells, due to the adsorption and desorption process [12]. This results in a force on the particular compressor cell. Also in this case the forces are internally compensated (the total momentum within the cooler is conserved\(^8\)). This force is larger than the previous ones and has been taken into account for this study, assuming that for a worst-case, there is no internal compensation. It is assumed that the compressor cells are freely moving and exert forces on its surroundings [8]. The Force Spectral Density of the force exported by the gas movement from the cells, is shown in Figure 2.9. The FSD of the force from the gas flow is equal to about 6 \( \mu N/\sqrt{Hz} \) at 0.1 Hz and 0.2 \( \mu N/\sqrt{Hz} \) at 1 Hz. This FSD counts for the second compressor stage only. The first stage is modeled using the same FSD as the second stage because they are very similar concerning the exported forces, and the second stage is the worst case. The computations are given in Appendix B.2.

2. Check valves
   The valves may introduce disturbances due to the movement of the valve-bosses. This will lead to disturbances of smaller than 0.1 \( \mu N/\sqrt{Hz} \) [8]. The forces from the valves are not internally compensated, but on the other hand they are very small. The exported force from one valve is shown in Figure 2.10. The computations for this FSD are given in Appendix B.3. Only one valve is looked at in this study, and it is assumed that no other valves open at the same time. The Force Spectral Density of the valve force is about 1.5\( \mu N/\sqrt{Hz} \) at 0.1 Hz and about 0.14\( \mu N/\sqrt{Hz} \) at 1 Hz.

3. The CFHX and the temporary zero outflow of the cells
   The Counterflow Heat Exchanger (CFHX) can also cause disturbance forces. It is a spiral wound double tube, and therefore it acts like a spring when the pressure inside is altered [8]. When the cells are heated one by one, it may occur that one cell is stopped while in the other one the pressure is not high enough yet for the valve to be opened. At that moment, the flow from the compressor cells into the high-pressure line is stopped temporarily [8]. This can induce forces that depend on two variables:

---

\(^8\)Yet, the center of mass of the gas is changed when the gas is moving. This effect has to be looked into in a future study.
Figure 2.9: The Force Spectral Density of the force exerted by the cells due to the movement of the gas.

Figure 2.10: The Force Spectral Density of the force exerted by a valve.
$t_0$ and $\tau_C$. $t_0$ is the zero-inflow period. $\tau_C$ is a time constant, that represents the time-dependence of how fast a change in pressure in the high-pressure buffer translates into a change in mass flow in the cryostat.

It is expected that the zero-inflow period can be reduced to less than 1 s. In this study 0.5 s is assumed, while the time constant is 686 s at this moment. This leads to a constant force spectral density of $135 \mu N/\sqrt{Hz}$ between 0.1 and 1 Hz. However, it is expected that $\tau_C$ can be improved up to 2700 s, that leads to a force spectral density of $34 \mu N/\sqrt{Hz}$ between 0.1 and 1 Hz. Above 1 Hz, the FSD decreases. The FSD for the CFHX force is shown in Figure 2.11. The computations are given in Appendix B.4.

These forces will result only in internal stresses on the supporting structure and will have almost no effect on the satellite [8]. Apart from that, these forces will be reduced, because it is expected that the zero-inflow period probably can be reduced to zero, and furthermore the problem can also be diminished by adapting the volume of the high-pressure buffer [8]. Still, as a worst-case, these forces are included in the model.

2.4.3 The optical delay line

ODL description

The optical delay line is basically a carriage containing a set of mirrors, that moves back and forwards in order to compensate for the OPDs induced by disturbance forces. The stroke of the carriage is 1 cm (OPD stroke 2 cm) and its movement precision is 1 nm. The ODL gets its information on the OPD from the fringe sensor at a rate, currently specified as 10 Hz. The ODL has a dimension of $30 \times 10 \times 10$ cm and a weight of maximum 10 kg. The target weight of the final Darwin ODL is 6 kg.
The ODL consists of two main parts:

1. **The rotor**
   The rotor is the moving part of the ODL. It has a mass of 0.65 kg. It consists of the cats eye, voice coil magnet, the magnetic bearing targets and the construction that connects these parts. The moving part of the ODL (the rotor) for Darwin is shown in Figure 2.12, from TNO/TPD.

   (a) **The cats eye**
   The cats eye consists of two mirrors on the carriage, along which the light from the star or planet is reflected. One of the mirrors is parabolic, the other one has a flat surface.

   (b) **The voice coil actuator**
   The voice coil actuator\(^9\) is the driving mechanism of the carriage. It is placed behind the cats eye. The voice coil actuator is basically the same system that is used in radio loudspeakers [43]. There, the electric signal is sent through a coil, inside a magnet. Due to the current of the coil and the magnetic field of the magnet, a Lorentz force is created that makes the magnet move together with the paper that is used in front of the loudspeaker. This makes us hear the sound waves. The case of the ODL is not much different; the movement of the magnet (and thus also the ODL carriage) is controlled very accurately, in proportion to the current applied to the coil. These coils are generally used whenever an actuation is necessary with a high frequency and high precision.

   (c) **The magnetic bearings**
   The magnetic bearings\(^10\) are in fact a guiding mechanism that keeps the carriage

---

\(^9\)In development by SRON, Utrecht, the Netherlands
\(^10\)In development by Micromega-Dynamics, Liège, Belgium
on its track, without suffering from friction. It consists of a sensor, a permanent magnet and an extra set of active coils that produce a variable magnetic field in addition to the permanent magnet. There are 5 magnetic bearings and one reserve bearing for redundancy.

2. The stator
The stator consists of all the other parts of the ODL, without the rotor. This comprises sensors, the coil for the voice coil actuator, etc.

Vibration levels
The vibrations of the optical delay lines can be divided in three subcategories: the vibrations induced by the movement of the carriage, by the magnetic bearing sensor noise, and the other self-induced vibrations due to OPD control [72]. The latter can be divided in another three sources of vibrations.

1. Vibrations induced by the movement of the carriage
The movement of the carriage, that has a mass of 0.65 kg, causes disturbances on the satellite. The forces exerted on the satellite by this movement, are derived from the requirements that apply to the ODL (Section 2.3). The satellite is subject to external and internal disturbance forces. These forces have an effect on the satellite that has been assumed to be the same as given in the Displacement Spectral Density of Figure 2.2 (Section 2.3), which is an assumption of the disturbance forces that are expected. The ODL has to counteract the change in OPD at the same rate as the satellite position is being disturbed (worst-case11). This means that an approximation of the forces on the satellite from the movement of the ODL can be calculated, starting from the assumed Displacement Spectral Density of the satellite. Multiplying the DSD by \( m(2\pi f)^2 \) yields the Force Spectral Density of the force that the ODL exerts on the satellite:

\[
F = ma = mx\omega^2 = mx(2\pi f)^2.
\]

\( m \) is the mass of the ODL rotor. The result is shown in Figure 2.13. However, before this force was calculated, Figure 2.2 (Section 2.3) was put through a filter which explains why the constant Force Spectral Density is not constant above 1 Hz, but it decays above \( 2 \cdot 10^1 \) Hz. The resulting force that the ODL exerts on the satellite is 8.8 \( \mu \)N RMS.

2. The magnetic bearing sensor noise
The noise in the magnetic bearing sensor causes small disturbance forces, because this noise is also compensated by the active bearings [72]. The PSD of the force of one bearing (resulting from this sensor noise) on the satellite, is shown in Figure 2.14, from [18]. The straight-line approximation for this PSD that was used in the calculations is shown in Figure 2.14. There are 5 magnetic bearings, thus the PSD is summed, assuming that there is no correlation between the noises. It is also assumed that the bearings do not have an influence on each other nor on the voice coil, and vice versa [72].

3. Other self-induced vibrations due to OPD control
These vibrations are described shortly in this Section, but they will be left out of this study. This is mainly because these vibrations are fully controllable by optimizing and finetuning the design of the delay lines [17]. They are assumed to be zero in this study.

(a) The voice coil current source noise
The noise in the current source for the voice coil leads to a position noise of 8 nm RMS in open loop, for frequencies higher than 0.1 Hz [72]. At a fringe sensor sample rate of 10 Hz, 3 nm RMS is left in closed loop, after OPD regulation [72].

\(^{11}\)The OPD stroke of the ODL is equal to twice the ODL mechanical stroke. Depending on the direction of the disturbance force, the ODL movement is equal to (worst-case) or half as much as the displacement the satellite experiences due to disturbance forces.
This number can be adapted to below 1 nm, if the fringe sensor operates at a higher frequency.

(b) **DAC noise**

The Digital to Analog Converter (DAC) converts digital signals to analog voltages or currents. This generates voltage noise that leads to a current noise in the voice coil. This noise resembles the voice coil current source noise and will lead to a residual of 3 nm RMS in closed loop [16], if no corrective measures are taken and/or if a 10 Hz fringe sensor is used.

(c) **DAC Quantization noise**

The DAC quantization noise is the noise that occurs as a result of the errors in the quantization of the signal during the conversion from digital to analog. The quantization noise also depends on the fringe sensor frequency. The quantization noise can be reduced by special measures like oversampling: sending force commands at a higher frequency, than position measurements are received [74]. Without special measures, the residual OPD in closed loop, using a fringe sensor of 10 Hz, is 287 nm [74]. Using a 100 Hz fringe sensor and an oversampling factor of 3, the OPD is reduced to 0.749 nm [74], and at 1000 Hz no oversampling or other techniques are necessary. It is clear that the quantization noise is dominant if no measures are taken for a fringe sensor of 10 Hz.

However, it is stressed that these problems will be eliminated in the ODL design process.

![Force Spectral Density of the ODL movement force](image)

Figure 2.13: The force resulting from the movement of the ODL.
2.4.4 Other sources of vibration

As mentioned in the introduction, there are many other potential sources of vibration beside the three that were discussed in this Chapter. According to [21], the DC/DC-converters may also introduce high-frequency disturbances. For the optics, potential sources of disturbance (no matter how small they could be) are the amplitude modulator (modulates the amplitude of two beams of light before being combined), the mirror switch (used to switch the phase of the incoming light, in demand) and beam angle actuators (corrects the angle of the beam of light before coming into the optical fibres). These are all modulating optical systems, that will probably be used in the future on Darwin. If so, they should be treated in the same way as done in this study for checking the disturbances that they may induce.

Furthermore, differential warming of the satellite or of parts of the satellite can lead to thermal problems in two ways. First, thermal cracking may occur: small sudden thermal deformations that cause shocks. Secondly, the thermal balance of the satellite could be disturbed that leads to a displacement of the satellite by emission. This is however not a high-frequency disturbance and it is therefore not looked at in this study. However, it is a major concern because Darwin is equipped with radiators for the cooler, on the hub. The thermal cracking needs to be avoided by carefully designing the satellite. These thermal problems are fortunately less worse in L2 than in LEO, because the thermal environment is much more stable in L2.

For these items hardly any information is found by the author within the timeframe of this study. It is necessary to take these into account in a future Darwin study, though they are neglected in this study.
2.5 Satellite model

2.5.1 Introduction

This Section describes the satellite model used for determining the effects of the disturbance forces on the movement of the satellite. The approach is as follows: first, a rigid body model is used for determining the order of magnitude of the satellite displacements due to the disturbance forces. Secondly, the satellite is split-up into two components connected to each other with a spring and a damper (Section 2.5.4, Figure 2.17). One of the two masses represents a certain subsystem (e.g. the sunshield) with a corresponding eigenfrequency, while the other mass represents the rest of the satellite, that is excited by the disturbance spectra that were derived for the FEEPs, ODL and sorption cooler. Using this mass-spring model, the peaks that might occur during resonance, are examined. The mass-spring model is actually an extension of the rigid body model.

2.5.2 Validation

The models are verified by using two different approaches for the vibration model. On the one hand, modal analysis has been used, while on the other hand the direct method has been applied. The two methods give the same results. A third check is made for the modal analysis by putting the transfer function of the second mode equal to zero. Doing this, the results are the same as for the rigid body motion. While modal analysis is more complicated than the direct method, it also gives more insight in the problem, it permits you to have a closer look at one of the modes separately, and it permits to expand the model to more degrees of freedom easily, if necessary.

2.5.3 Rigid body model

Before examining the resonance properties of the satellite, its rigid body motion as a result of the disturbance forces is looked at in this Section. The basic equation of motion for a rigid body is:

\[ m\ddot{x} = f \] (2.1)

where \( f \) is the disturbance force acting on a satellite, \( m \) is the satellite mass and \( x \) is the displacement of the satellite.

When taking the Fourier transform of both sides of the equation, using:

\[
\begin{align*}
   f(t) &= \frac{1}{2\pi} \int_{-\infty}^{\infty} F(\omega) e^{i\omega t} d\omega \\
   x(t) &= \frac{1}{2\pi} \int_{-\infty}^{\infty} X(\omega) e^{i\omega t} d\omega \\
   \ddot{x}(t) &= -\frac{\omega^2}{2\pi} \int_{-\infty}^{\infty} X(\omega) e^{i\omega t} d\omega
\end{align*}
\] (2.2)

then Equation 2.1 becomes:

\[-\omega^2 m X(\omega) = F(\omega)\] (2.3)

with

\[
\begin{align*}
   F(\omega) &= \frac{1}{2\pi} \int_{-\infty}^{\infty} f(t) e^{-i\omega t} dt \\
   X(\omega) &= \frac{1}{2\pi} \int_{-\infty}^{\infty} x(t) e^{-i\omega t} dt
\end{align*}
\] (2.4)

The transfer function is:

\[ H(\omega) = \frac{X(\omega)}{F(\omega)} = -\frac{1}{m\omega^2} \] (2.5)

For Power Spectral Densities, the following equation holds [54]:

\[ S_{\text{out}} = H^*(\omega)H(\omega)S_{\text{in}} = |H(\omega)|^2 S_{\text{in}} \] (2.6)
Figure 2.15: The Displacement Spectral Density for the rigid body motion as result of a white noise input of $1 \mu N/\sqrt{Hz}$.

Figure 2.16: The cumulative RMS for the rigid body motion, as a result of a white noise input of $1 \mu N/\sqrt{Hz}$. 
This yields:

\[
S_{\text{out}} = \frac{1}{m^2\omega^4} S_{\text{in}}
\]  

(2.7)

where \( S_{\text{in}} \) is the PSD of the disturbance force in N^2/Hz and \( S_{\text{out}} \) is the PSD of the satellite displacement in m^2/Hz. The RMS-value of the displacement in a certain frequency band, for a one-sided PSD\(^{12}\) is then equal to:

\[
\text{RMS} = \sqrt{\int_{f_{\text{low}}}^{f_{\text{up}}} S_{\text{out}} df}
\]  

(2.8)

For illustration, if a white noise input of 1 \( \mu \text{N}/\sqrt{\text{Hz}} \) is applied to the model, the total RMS-value of the satellite displacement is 0.0297 nm in a frequency band of \( 10^0 - 10^1 \) Hz. The corresponding Displacement Spectral Density and cumulative RMS graphs are shown in Figures 2.15 and 2.16 respectively. These Figures may be compared to the Figures of the 2-DOF-model in Section 2.5.5.

2.5.4 Modal analysis

This Section explains the method used for solving the vibration problem with modal analysis. It is illustrated using the 2 degrees of freedom (2-DOF) mass-spring model, applied to the sunshield. The 2-DOF mass-spring model consists of 2 masses, a spring and a damper,

![Diagram of 2-DOF mass-spring model](image)

Figure 2.17: The 2-DOF mass-spring model for viscous damping (A) and for structural damping (B).

as shown in Figure 2.17. However, in this case the damping is modeled as structural damping instead of viscous damping because the damping is only light and originates from the structure itself. Therefore the damping coefficient \( c \) is written as a complex function of the spring constant \( k \), and is equal to \( i \cdot k \cdot \eta \). The problem can be solved now as a mass-spring system with only a spring, that has a complex spring constant equal to \( k(1 + i\eta) \) where \( i = \sqrt{-1} \) (Figure 2.17).

The input parameters for this example, the case of the sunshield, are given in Table 2.3.

Mass \( m_1 \) represents the mass of the satellite and all the subsystems, without the sunshield.

\(^{12}\)See Appendix B.1
Table 2.3: The parameters, used in the model for the sunshield (solar panels included).

<table>
<thead>
<tr>
<th>$f_n$, estimated [Hz]</th>
<th>$m_1$ [kg]</th>
<th>$m_2$ [kg]</th>
<th>$k$</th>
<th>$\eta$</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>447</td>
<td>46</td>
<td>6586.25</td>
<td>0.002</td>
</tr>
</tbody>
</table>

Mass $m_2$ represents the sunshield mass (solar panels included). For a 2-DOF system as shown in Figure 2.17, the eigenvalue with damping does not differ much from the eigenvalue in the case without damping. In the undamped case, $k$ and $\omega_n$ are related as shown in Equation 2.9:

$$\omega_n = \sqrt{\frac{k}{m_1} + \frac{k}{m_2}}$$

(2.9)

Because the eigenvalues are almost similar in the case with and without damping, the value of $k$ will also not differ much and we can make a good approximation for $k$ using Equation 2.9. For linear systems the value of $\eta$ is estimated from the relationship $\eta = 2 \zeta$ where $\eta$ is the structural damping coefficient and $\zeta$ is the viscous damping ratio.

1. Equations of motion
   The first step to take is deriving the equations of motion from the model in Figure 2.17:
   $$M\ddot{x}(t) + Kx(t) = F(t)$$
   (2.10)
   or:
   $$\begin{bmatrix} m_1 & 0 \\ 0 & m_2 \end{bmatrix} \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} + \begin{bmatrix} k(1 + i\eta) & -k(1 + i\eta) \\ -k(1 + i\eta) & k(1 + i\eta) \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} F_1(t) \\ F_2(t) \end{bmatrix}$$
   (2.11)

2. The eigenfrequencies
   The next step is to find the eigenfrequencies of the system. These eigenfrequencies are:
   $$\omega_1 = 0$$
   (2.12)
   and
   $$\omega_2 = \sqrt{\frac{k(1 + i\eta)(m_1 + m_2)}{m_1 m_2}}$$
   (2.13)

   The eigenfrequency $\omega_1$ corresponds to the rigid body motion of the satellite. This means that all masses move at the same speed and in the same direction. Resonances are out of question in this case, because there is no relative motion between the masses. This motion was discussed in Section 2.5.3. The eigenfrequency $\omega_2$ corresponds to the relative vibrational motion of the two masses.

3. Modal matrix
   Once the eigenfrequencies are found, the corresponding eigenvectors or eigenmodes are calculated. Putting the two eigenvectors in one matrix, yields the modal matrix:
   $$z = \begin{bmatrix} 1 & \frac{m_1}{m_2} \\ 1 & \frac{m_1}{m_2} \end{bmatrix}$$
   (2.14)
4. Normalized modal matrix

The modal matrix is now normalized with respect to the mass matrix of the system and it becomes:

$$z_N = \frac{1}{\sqrt{m_1 + m_2}} \begin{bmatrix} 1 & \sqrt{\frac{m_2}{m_1}} \\ 1 & -\sqrt{\frac{m_1}{m_2}} \end{bmatrix} \quad (2.15)$$

5. Uncoupling the equations of motion

At this moment the equations of motion are still coupled and will be uncoupled by performing a transformation from the physical coordinate system, to the principal coordinate system [33]. Also the initial conditions (if applicable) and the applied forces or their PSDs are converted to the principal coordinate system. The new equations of motion in the principal coordinate system are:

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} \ddot{x}_{p1} \\ \ddot{x}_{p2} \end{bmatrix} + \begin{bmatrix} 0 \\ k(m_1 + m_2) \end{bmatrix} \begin{bmatrix} x_{p1} \\ x_{p2} \end{bmatrix} = \begin{bmatrix} F_{p1} \\ F_{p2} \end{bmatrix} \quad (2.16)$$

Notice that $\left(k(1+i\eta)(m_1 + m_2)\right)_{m_1m_2}$ is equal to $\omega_2^2$.

6. Solving the equations using the transfer functions

The two uncoupled equations of motion can now be solved independently. The method used for solving the equations is the transfer function method. The transfer function corresponding to the first equation and the first mode ($\omega_1 = 0$) is:

$$H(\omega)_{p11} = -\frac{1}{\omega^2} \quad (2.17)$$

This transfer function corresponds to the rigid body motion. For the second mode ($\omega_2 = \sqrt{\frac{k(1+i\eta)(m_1 + m_2)}{m_1m_2}}$) the transfer function is:

$$H(\omega)_{p22} = \frac{m_1m_2}{k(1+i\eta)(m_1 + m_2) - \omega^2m_1m_2} = \frac{1}{\omega_2^2 - \omega^2} \quad (2.19)$$

The absolute value of the transfer functions is plotted in Figure 2.18 for the case of the sunshield. The parameters used are:

- $m_1 = 447$ kg, corresponding to the spacecraft mass without sunshield
- $m_2 = 46$ kg, corresponding to the sunshield mass
- $\eta = 0.002$, $\eta = 2\zeta$ in which $\zeta$ is assumed to be 0.1%
- $f_{n,\text{estimated}} = 2$ Hz ($f_{n,\text{estimated}}$ is used for estimating a spring stiffness $k$)

7. Backtransforming

The last step is backtransforming the solution from the principal coordinate system to the physical coordinate system. This is done using:

$$S_X = z_N S_{XP} z_N^T \quad (2.20)$$

While backtransforming the displacement PSDs, actually the contributions of the two modes to the total displacement of the mass are summed, per mass. The output is
Transfer functions $|H_{p11}|$ and $|H_{p22}|$ for each mode

Frequency [Hz]

![Graph showing transfer functions](image)

Figure 2.18: The absolute values of the transfer functions, for each mode.

...a matrix that consists of the Power Spectral Densities of the displacement of the two masses ($S_{X_{11}}$ and $S_{X_{22}}$) and their cross spectral densities ($S_{X_{12}}$ and $S_{X_{21}}$). These are shown in Figure 2.20 and are the same as if using the direct method as described in the next Section (Section 2.5.5). Their cumulative RMS-values are shown in Figure 2.21 in a frequency range between 1 and 10 Hz.

Then the RMS-value of the deviation is calculated for a certain frequency band of interest using Equation 2.21 (for a one-sided PSD).

$$\text{RMS} = \sqrt{\int_{f_{lw}}^{f_{up}} S_{out} \, df} \quad (2.21)$$

Resuming, during the modal analysis a sequence of transformations has been made as follows:

$$S_F \rightarrow S_{FP} \rightarrow S_{XP} \rightarrow S_X \quad (2.22)$$

### 2.5.5 The direct method

The equations we start from are the basic equations of motion, deduced from Figure 2.17:

$$M\ddot{x}(t) + Kx(t) = F(t) \quad (2.23)$$

or:

$$\begin{bmatrix} m_1 & 0 \\ 0 & m_2 \end{bmatrix} \begin{bmatrix} \ddot{x}_1 \\ \ddot{x}_2 \end{bmatrix} + \begin{bmatrix} k(1 + i\eta) & -k(1 + i\eta) \\ -k(1 + i\eta) & k(1 + i\eta) \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} F_1(t) \\ F_2(t) \end{bmatrix} \quad (2.24)$$

Taking the Fourier transform of these equations yields:

$$(k(1 + i\eta) - \omega^2 m_1)X_1 - k(1 + i\eta)X_2 = F_1$$

$$-k(1 + i\eta)X_1 + (k(1 + i\eta) - \omega^2 m_2)X_2 = F_2 \quad (2.25)$$
or:

\[
\begin{bmatrix}
  (k(1 + i\eta) - \omega^2 m_1) & -k(1 + i\eta) \\
  -k(1 + i\eta) & (k(1 + i\eta) - \omega^2 m_2)
\end{bmatrix}
\begin{bmatrix}
  X_1 \\
  X_2
\end{bmatrix}
= \begin{bmatrix}
  F_1 \\
  F_2
\end{bmatrix}
\]

(2.26)

\[
Z \cdot X = F
\]

\[
\frac{X}{F} = Z^{-1} = H
\]

(2.27)

\[
H = \frac{1}{|\det(Z)|}
\begin{bmatrix}
  (k(1 + i\eta) - \omega^2 m_2) & k(1 + i\eta) \\
  k(1 + i\eta) & (k(1 + i\eta) - \omega^2 m_1)
\end{bmatrix}
\]

(2.28)

where:

\[|\det(Z)| = \{(k(1 + i\eta) - \omega^2 m_1) \cdot (k(1 + i\eta) - \omega^2 m_2)\} - \{(-k(1 + i\eta)) \cdot (-k(1 + i\eta))\}\]

Figure 2.19 shows the transfer functions $H_{11}$ and $H_{22}$ for the direct method.

\[Z \cdot X = F\]

\[
\frac{X}{F} = Z^{-1} = H
\]

(2.27)

\[
H = \frac{1}{|\det(Z)|}
\begin{bmatrix}
  (k(1 + i\eta) - \omega^2 m_2) & k(1 + i\eta) \\
  k(1 + i\eta) & (k(1 + i\eta) - \omega^2 m_1)
\end{bmatrix}
\]

(2.28)

where:

\[|\det(Z)| = \{(k(1 + i\eta) - \omega^2 m_1) \cdot (k(1 + i\eta) - \omega^2 m_2)\} - \{(-k(1 + i\eta)) \cdot (-k(1 + i\eta))\}\]

Figure 2.19: The absolute values of the transfer functions, for the direct method.

Now the Power Spectral Densities of the mass displacements can be calculated, using Equation 2.29 [58, 54]:

\[
S_X = H S_F H^* 
\]

(2.29)

$S_X$ is the matrix of the displacement PSDs ($S_{X1}$ and $S_{X2}$), and their cross-spectral densities ($S_{X12}$ and $S_{X21}$). The results are the same as for the normal analysis method, and are shown in Figures 2.20 and 2.21. Then the RMS-value of the displacements is again calculated
for a certain frequency band of interest using Equation 2.30 (for a one-sided PSD).

\[
\text{RMS} = \sqrt{\int_{f_{\text{low}}}^{f_{\text{up}}} S_{\text{out}} \, df}
\]  

(2.30)

For illustration purposes, a white noise input of \(1 \mu N/\sqrt{\text{Hz}}\) has been applied to the model. In Figure 2.20 the displacement Spectral Density is shown, as a result of the input. Figure 2.21 shows the cumulative RMS-value in the frequency band of \(10^0 - 10^1\) Hz. The RMS-value for the displacement of mass 1 is 0.0589 nm, while for mass 2 it is 0.5045 nm. For the rigid body motion, the RMS-value for the displacement of the total satellite was 0.0297 nm in the same frequency band. The influence of resonance is thus clearly present in the RMS-value.

It is clear from Figure 2.20 that resonance occurs at a frequency of 2 Hz, as was taken in the assumptions (Table 2.3). It is also clear from the Figure that the lowest mass (\(m_2\) in this case) suffers from the biggest displacements. While \(m_2\) gets in resonance, \(m_1\) first vibrates in anti-resonance, and then joins the second mass, in resonance. The relative behaviour of the masses can be seen from the cross-spectral densities of the two displacements. Both \(S_{X12}\) and \(S_{X21}\) are the same. Figure 2.21 shows the parts of the spectrum that add a lot to the RMS-value. The steeper the curve is, the more this part of the spectrum adds to the final RMS-value. It is clear that at lower frequencies, the RMS-value of the displacements is bigger, and also at the resonance frequency.

Figure 2.20: The Displacement Spectral Density for mass 1, mass 2 and the cross-spectral densities of the two movements, as result of a white noise input of \(1 \mu N/\sqrt{\text{Hz}}\).
2.6 Input and parameters for the model

2.6.1 Disturbance forces

The disturbance forces from each subsystem are described in Section 2.4. They are compared to each other in Figure 2.22. In this Figure the Force Spectral Densities are summed (quadratically) for the subsystems that consist of more than one part. 16 FEEP thrusters are assumed to be working at the same time (worst-case). There are 5 ODL magnetic bearings that export forces and only one valve is assumed to open at the same time in the cooler. It is also assumed that the force from the movement of the gas in the cooler compressor cells is experienced three times, once for each cluster of four cells. There are also two Counterflow Heat Exchangers that export forces, one for each stage. All the forces of the systems are doubled (quadratically) to take into account the fact that the OPD is a difference in the distance between two satellites, and not just the displacement of one satellite. All disturbance forces are assumed to be working on \( m_1 \), the spacecraft itself. Also the Figures in this Section and in Section 2.7 give the displacements of \( m_1 \), because this mass contains the optical bench.

In Figure 2.22 the cooler and the FEEP are the most pronounced at lower frequencies \((10^{-2} - 10^{0} \text{ Hz})\). At frequencies in the range of \(10^{0} - 10^{3} \text{ Hz}\), the most important frequencies in this study, all values of the internal disturbance forces are in the same range. This range has a span of roughly \(10^{-4} - 10^{-6} \text{ N}\). The only increasing disturbance force at higher frequencies is the ODL magnetic bearings force. However, the higher the frequency of the disturbance force is, the lesser its contribution is to the total displacement of the satellite, so this is not necessarily a problem.

The study is limited to the frequency of \(10^{-2} \text{ Hz}\) on the low frequent side. The upper
frequency of $10^{3}$ Hz is chosen because at higher frequencies their contribution to the total OPD is negligible.

### 2.6.2 Parameters of the models

For the rigid body model, a free flyer with mass of 493 kg has been assumed. For the 2-DOF model, a parametric study is made in which the satellite model parameters are altered. In reality, a satellite has a certain amount of sub-parts, or subsystems, that will resonate if the satellite is excited with a disturbance force that has a frequency component equal to the eigenfrequency of the subsystem. With the 2-DOF model, the resonance behaviour of one such component apart can be simulated.

An example case is set-up for the resonance case where mass $m_1$ is equal to the satellite without sunshield, and $m_2$ is the sunshield mass (447 resp. 46 kg). The sunshield resonance frequency is assumed to be 2 Hz and the damping coefficient $\eta$ is equal to 0.002. Also a parametric study is performed for the high frequencies from $10^0 - 10^3$ Hz. The parameters that are altered are the mass of the resonating part, its eigenfrequency and the damping coefficient. The results are given in Section 2.7.
2.7 Results

2.7.1 Results for the rigid body motion

The Displacement Spectral Densities due to internal disturbance forces are presented in Figure 2.23, for the rigid body motion. Figure 2.24 shows the cumulative RMS-value for each disturbance force. It gives the RMS-values for every frequency band of $10^x - 10^{x+1}$ Hz where $x$ can be chosen arbitrarily by the reader between $10^{-2}$ and $10^3$ Hz.

The RMS-values for the satellite displacements are calculated. The RMS-values of the

![Displacement Spectral Density of the satellite](image)

Figure 2.23: Overview of the Displacement Spectral Density for every internal disturbance source for the rigid body motion.

OPD (nm RMS) are given in Table 2.4 for the rigid body motion. Table 2.4 summarizes the results for 6 different bandwidths. These can be roughly divided into the low-frequency bands and the high-frequency bands, with 1 Hz as a partition value. The most important bandwidth for this study is $10^0 - 10^3$ Hz because the disturbance forces in this bandwidth determine what kind of fringe sensor frequency is needed. If a 10 Hz fringe sensor is used, then the total RMS-value of the disturbance forces above 1 Hz must remain below the given requirement of 5 nm RMS (Section 2.3). If this is not the case, then a fringe sensor with higher frequency is necessary. The frequency band between 10 Hz and 1000 Hz tells something about the remaining disturbance forces, when a fringe sensor with higher frequency (100 Hz) is used instead of 10 Hz.

The results for the bandwidths containing the frequencies $10^{-2}$ and $10^{-1}$ Hz give information about the bigger forces at the lower frequencies. These forces are mainly counteracted by the FEEPs. However, if these forces are so big that they cause the ODL to saturate before the FEEPs can counteract them, then a problem arises. This is also the case when
the satellite disturbances exceed the ODL speed. Therefore it is interesting to have a look at the low-frequent forces. A thorough study of these forces in relation with the FEEP and ODL capabilities is necessary during the Darwin study. However, this is beyond the scope of this study which is confined mainly to the high-frequent disturbance forces. Only an indication of the magnitude of the satellite displacements due to these forces is given for the rigid body motion, and for the example case of the 2-DOF model.

From Table 2.4 it is clear that in the frequency band from $10^{-2} - 10^3$ Hz, the sorption cooler and the FEEPs are the cause of the highest disturbance vibrations. In the frequency band of $10^0 - 10^3$ Hz especially the CFHX calls for attention. The RMS-value for the OPD due to the CFHX is about 1 nmRMS. This number seems to be high, but on the other side, it must be stressed that the assumptions were very worst-case, and that the design is still in development (Section 2.4.2). The other values cause no worry because they remain well below the OPD requirement of 5 nmRMS (Section 2.3), even with the worst-case assumptions that were used. If the CFHX of the sorption cooler is further developed and the vibration levels are as low as predicted (Section 2.4.2), then the internal disturbance forces that were discussed in this study will cause no problem for the use of a 10 Hz fringe sensor, if neglecting resonance phenomena.

In Table 2.4 it is also shown that the difference between the bands $10^{-2} - 10^3$ Hz and $10^{-2} - 10^0$ Hz is not substantial. This is because the disturbances of higher frequencies do not contribute as much to the OPD as the disturbances at lower frequencies.
### 2.7.2 Results for the 2-DOF system

#### Example case

The results of the example case of the 2-DOF model are summarized in Table 2.5. It is assumed that the sunshield of the free flyer resonates at 2 Hz [38], the sunshield has a mass of 46 kg, and the damping coefficient $\eta$ is equal to 0.002.

A comparison of Table 2.5 and Table 2.4 shows us that the influence of resonance is very restricted in this case. Due to resonance, the RMS-value of the OPD is increased by only about $\frac{1}{2}$ nmRMS in the frequency range between 1 Hz and 1000 Hz. This does not change the order of magnitude of the previous results. The corresponding Displacement Spectral Densities and cumulative RMS are shown in Figures 2.25 and 2.26.

#### Parametric study - change of resonance frequency

For the rigid body motion and for the example case, the RMS-values of the OPD remain below 5 nmRMS in the range of $10^0 - 10^3$ Hz. Therefore it is interesting to know whether or not resonance can cause them to exceed this requirement. For this reason, only the frequency range of $10^0 - 10^3$ Hz is looked at during the parametric study.

Table 2.6 shows the effect on the OPD when the resonance frequency is altered. The example case is already a bad case in comparison with higher resonance frequencies. The results get worse very easily as soon as the resonance frequency is smaller than 2 Hz. If the resonance frequency is below 1 Hz, then the results for the band of $10^{-2} - 10^3$ Hz will be worse, but for the high-frequency part alone, the influence of the resonance becomes lower. This means that the worst results are caught at a resonance frequency of 1 Hz (Table 2.6) for the high-

<table>
<thead>
<tr>
<th></th>
<th>$10^{-2} - 10^1$ Hz</th>
<th>$10^{-1} - 10^3$ Hz</th>
<th>$10^0 - 10^4$ Hz</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>FEEPs requirement</strong> (RMS)</td>
<td>0.421 $\mu$m</td>
<td>3.187 nm</td>
<td>0.101 nm</td>
</tr>
<tr>
<td><strong>Sorption Cooler</strong> (RMS)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Gas movement</td>
<td>1.246 $\mu$m</td>
<td>10.240 nm</td>
<td>0.005 nm</td>
</tr>
<tr>
<td>Valve</td>
<td>0.521 $\mu$m</td>
<td>1.433 nm</td>
<td>0.004 nm</td>
</tr>
<tr>
<td>CFHX</td>
<td>2.752 $\mu$m</td>
<td>64.250 nm</td>
<td>1.293 nm</td>
</tr>
<tr>
<td><strong>ODL</strong> (RMS)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>ODL magnetic bearings</td>
<td>0.019 nm</td>
<td>0.019 nm</td>
<td>0.019 nm</td>
</tr>
<tr>
<td>ODL movement</td>
<td>0.132 nm</td>
<td>0.128 nm</td>
<td>0.067 nm</td>
</tr>
<tr>
<td><strong>Total</strong> (RMS)</td>
<td>5.091 $\mu$m</td>
<td>79.257 nm</td>
<td>1.489 nm</td>
</tr>
</tbody>
</table>
Table 2.5: The results of the example case with resonance: the RMS-values for the OPD between 2 free flyers of Darwin ($m_1 = 447$ kg, $m_2 = 46$ kg, $f_r = 2$ Hz, $\eta = 0.002$).

<table>
<thead>
<tr>
<th></th>
<th>$10^{-2} - 10^3$ Hz</th>
<th>$10^{-1} - 10^4$ Hz</th>
<th>$10^0 - 10^4$ Hz</th>
<th>$10^1 - 10^4$ Hz</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>FEEPs requirement</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(RMS)</td>
<td>0.421 $\mu$m</td>
<td>3.189 nm</td>
<td>0.202 nm</td>
<td>0.004 nm</td>
</tr>
<tr>
<td><strong>Sorption Cooler</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(RMS)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Gas movement</td>
<td>1.246 $\mu$m</td>
<td>10.236 nm</td>
<td>0.006 nm</td>
<td>0.000 nm</td>
</tr>
<tr>
<td>Valve</td>
<td>0.521 $\mu$m</td>
<td>1.432 nm</td>
<td>0.006 nm</td>
<td>0.000 nm</td>
</tr>
<tr>
<td>CFHX</td>
<td>2.752 $\mu$m</td>
<td>64.212 nm</td>
<td>1.536 nm</td>
<td>0.000 nm</td>
</tr>
<tr>
<td><strong>ODL</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(RMS)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>ODL magnetic bearings</td>
<td>0.007 nm</td>
<td>0.022 nm</td>
<td>0.022 nm</td>
<td>0.021 nm</td>
</tr>
<tr>
<td>ODL movement</td>
<td>0.174 nm</td>
<td>0.171 nm</td>
<td>0.133 nm</td>
<td>0.002 nm</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td>5.136 $\mu$m</td>
<td>79.262 nm</td>
<td>1.905 nm</td>
<td>0.027 nm</td>
</tr>
</tbody>
</table>

Figure 2.25: Overview of the Displacement Spectral Densities for the 2-DOF example case with resonance at 2 Hz.

frequency part ($10^0 - 10^3$ Hz). Recall that the CFHX forces are modeled as very worst-case and therefore the OPD results in case of resonance at 1 Hz are very high. Assuming that the CFHX force will become about 10% of the value in this study, then the RMS-value of the OPD in case of 1 Hz resonance still remains within the margin of 5 nm RMS. However, even then it is very close to the limit. Also other forces like the external disturbance forces and sloshing are not added yet. It is therefore recommended that the satellite is designed without any resonance frequencies in the range between 0.5 Hz and 2 Hz, if a fringe sensor of only 10 Hz is used.
The higher the resonance frequency is, the more the total OPD converges to the case of the rigid body. Notice that in the case of $f_r = 100$ Hz there is a very small local maximum in the OPD. This is caused by the fact that the magnetic bearings of the ODL export bigger forces at higher frequencies (Figure 2.22). This resonance phenomenon is only very small because the forces at higher frequencies contribute the least to the OPD.

In Table 2.6 the masses $m_2$ are changed from 46 kg for low resonance frequencies, to 20 kg for the higher resonance frequencies. This is because the bigger parts on the satellite, e.g. the sunshield, usually have lower eigenfrequencies than the smaller parts like the scientific instruments.

The satellite has in reality a lot of eigenfrequencies from the different subsystems and instruments aboard. In this study only one eigenfrequency at a time is looked at. The lowest eigenfrequency is expected from the sunshield at about 2 Hz. Then, no eigenfrequencies are expected anymore in the range between roughly 2 Hz and a few tens of Hz. The influence of these higher eigenfrequencies on the total OPD is very small. In order to check whether their sum has a significant influence on the total RMS-value of the OPD, a FEM analysis is recommended as soon as more details (materials, subsystems) are known for Darwin.

**Parametric study - change of damping coefficient**

Another parameter that influences the results is the damping coefficient that is assumed for this model. Usually a damping ratio of $\zeta = 0.1\%$ is representative for a spacecraft. However, Table 2.7 shows the influence of other damping coefficients. On the one hand this coefficient
<table>
<thead>
<tr>
<th>Influence of the resonance frequency</th>
<th>( f_r = 0.5 \text{ Hz}, )</th>
<th>( f_r = 1 \text{ Hz}, )</th>
<th>( f_r = 1.5 \text{ Hz}, )</th>
<th>( f_r = 2 \text{ Hz}, )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( m_2 = 46 \text{ kg} )</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( f_r = 30 \text{ Hz}, )</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( m_2 = 20 \text{ kg} )</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>FEEPs requirement ([\text{nm}_{\text{RMS}}])</strong></td>
<td>0.113</td>
<td>0.426</td>
<td>0.289</td>
<td>0.202</td>
</tr>
<tr>
<td><strong>Sorption Cooler ([\text{nm}_{\text{RMS}}])</strong></td>
<td>0.006</td>
<td>0.037</td>
<td>0.009</td>
<td>0.006</td>
</tr>
<tr>
<td>Gas movement</td>
<td>0.005</td>
<td>0.023</td>
<td>0.011</td>
<td>0.006</td>
</tr>
<tr>
<td>Valve</td>
<td>1.459</td>
<td>12.926</td>
<td>2.970</td>
<td>1.536</td>
</tr>
<tr>
<td>CFHX</td>
<td>0.021</td>
<td>0.021</td>
<td>0.022</td>
<td>0.022</td>
</tr>
<tr>
<td>ODL magnetic bearings</td>
<td>0.075</td>
<td>0.281</td>
<td>0.191</td>
<td>0.133</td>
</tr>
<tr>
<td>ODL movement</td>
<td>1.679</td>
<td>13.714</td>
<td>3.492</td>
<td>1.905</td>
</tr>
<tr>
<td><strong>Total ([\text{nm}_{\text{RMS}}])</strong></td>
<td>1.010</td>
<td>1.010</td>
<td>1.010</td>
<td>1.010</td>
</tr>
</tbody>
</table>

Table 2.6: The results for the OPD RMS-values between 2 free flyers of Darwin, using different resonance frequencies (\( \eta = 0.002 \)).

<table>
<thead>
<tr>
<th>Influence of the damping ratio</th>
<th>( \eta = 2 )</th>
<th>( \eta = 0.002 )</th>
<th>( \eta \div 2 )</th>
<th>( \eta \div 5 )</th>
<th>( \eta \div 10 )</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>FEEPs requirement ([\text{nm}_{\text{RMS}}])</strong></td>
<td>0.158</td>
<td>0.202</td>
<td>0.267</td>
<td>0.372</td>
<td>0.422</td>
</tr>
<tr>
<td><strong>Sorption Cooler ([\text{nm}_{\text{RMS}}])</strong></td>
<td>0.005</td>
<td>0.006</td>
<td>0.006</td>
<td>0.008</td>
<td>0.011</td>
</tr>
<tr>
<td>Gas movement</td>
<td>0.005</td>
<td>0.006</td>
<td>0.006</td>
<td>0.012</td>
<td>0.020</td>
</tr>
<tr>
<td>Valve</td>
<td>1.363</td>
<td>1.536</td>
<td>2.041</td>
<td>4.081</td>
<td>7.827</td>
</tr>
<tr>
<td>CFHX</td>
<td>0.022</td>
<td>0.022</td>
<td>0.022</td>
<td>0.023</td>
<td>0.024</td>
</tr>
<tr>
<td>ODL magnetic bearings</td>
<td>0.104</td>
<td>0.133</td>
<td>0.176</td>
<td>0.245</td>
<td>0.278</td>
</tr>
<tr>
<td>ODL movement</td>
<td>1.657</td>
<td>1.905</td>
<td>2.318</td>
<td>4.741</td>
<td>8.582</td>
</tr>
</tbody>
</table>

Table 2.7: The results for the OPD RMS-values between 2 free flyers of Darwin, using different damping coefficients (\( m_1 = 447 \text{ kg}, m_2 = 46 \text{ kg}, f_r = 2 \text{ Hz} \)).

has a significant influence on the OPD, but on the other hand, the probability that \( \eta \) is smaller than 0.002 is very little: in reality the satellite will tend to have bigger damping coefficients instead of smaller ones.

**Parametric study - change of mass of the resonating part**

The last parameter to alter is the mass of the resonating body. The smaller this mass is, the lesser the influence is on the OPD of mass \( m_1 \), which is the satellite itself, including optical bench. The case of \( m_2 = 20 \text{ kg} \) is representative for the parts of the satellites like scientific instruments or specific subsystems. As long as \( m_2 \) remains within reasonable limits (e.g.
50 kg), it does not cause a big difference with the case of the rigid body motion. The case of $m_2 = 46$ kg represents the resonance of the sunshield (at 2 Hz). It can also be assumed as a very worst-case for resonance of the spacecraft structure. This structure usually has a mass of about 10% of the spacecraft mass [49], thus about 40 kg, but it has usually a larger resonance frequency than 2 Hz (about 10 to 50 Hz), which leads in reality to a smaller RMS-value of the OPD than assumed here. Two bigger masses are shown too in Table 2.8, namely 100 kg and 150 kg, although it is not expected that such big masses will resonate as a whole and also not at 2 Hz.

### Table 2.8: The results for the OPD RMS-values between 2 free flyers of Darwin, using different masses in resonance ($\eta = 0.002$, $f_r = 2$ Hz).

<table>
<thead>
<tr>
<th>Influence of the resonating mass</th>
<th>$m_2 = 20$ kg</th>
<th>$m_2 = 46$ kg</th>
<th>$m_2 = 100$ kg</th>
<th>$m_2 = 150$ kg</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>FEEP's requirement</strong> ($\mu m_{RMS}$)</td>
<td>0.123</td>
<td>0.202</td>
<td>0.448</td>
<td>0.759</td>
</tr>
<tr>
<td><strong>Sorption Cooler</strong> ($\mu m_{RMS}$)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Gas movement</td>
<td>0.005</td>
<td>0.005</td>
<td>0.008</td>
<td>0.012</td>
</tr>
<tr>
<td>Valve</td>
<td>0.005</td>
<td>0.006</td>
<td>0.012</td>
<td>0.021</td>
</tr>
<tr>
<td>CFHX</td>
<td>1.319</td>
<td>1.536</td>
<td>2.567</td>
<td>4.098</td>
</tr>
<tr>
<td><strong>ODL</strong> ($\mu m_{RMS}$)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>ODL magnetic bearings</td>
<td>0.020</td>
<td>0.022</td>
<td>0.027</td>
<td>0.034</td>
</tr>
<tr>
<td>ODL movement</td>
<td>0.081</td>
<td>0.133</td>
<td>0.295</td>
<td>0.501</td>
</tr>
<tr>
<td><strong>Total</strong> ($\mu m_{RMS}$)</td>
<td>1.553</td>
<td>1.904</td>
<td>3.357</td>
<td>5.425</td>
</tr>
</tbody>
</table>

Table 2.8: The results for the OPD RMS-values between 2 free flyers of Darwin, using different masses in resonance ($\eta = 0.002$, $f_r = 2$ Hz).

### The frequency range $10^1 − 10^3$ Hz for the 100 Hz fringe sensor

If a 100 Hz fringe sensor is used, then the frequency range in which the remaining OPD may not exceed 5 $\mu m_{RMS}$ is $10^1 − 10^3$ Hz. Table 2.9 summarizes the results for this frequency range for the rigid body case, the example case and for resonance frequencies of 1 and 10 Hz. 10 Hz is the worst-case for a fringe sensor of 100 Hz. Table 2.9 shows that the use of a 100 Hz fringe sensor is sufficient in all cases.

### Table 2.9: The results for the OPD RMS-values between 2 free flyers of Darwin in the range from 10 to 1000 Hz ($m_1 = 447$ kg, $m_2 = 46$ kg, $\eta = 0.002$).

<table>
<thead>
<tr>
<th>Influence of the resonating mass</th>
<th>Rigid body</th>
<th>$f_r = 2$ Hz</th>
<th>$f_r = 1$ Hz</th>
<th>$f_r = 10$ Hz</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>FEEP's requirement</strong> ($\mu m_{RMS}$)</td>
<td>0.003</td>
<td>0.004</td>
<td>0.004</td>
<td>0.013</td>
</tr>
<tr>
<td><strong>Sorption Cooler</strong> ($\mu m_{RMS}$)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Gas movement</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
</tr>
<tr>
<td>Valve</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
</tr>
<tr>
<td>CFHX</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.002</td>
</tr>
<tr>
<td><strong>ODL</strong> ($\mu m_{RMS}$)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>ODL magnetic bearings</td>
<td>0.019</td>
<td>0.021</td>
<td>0.021</td>
<td>0.023</td>
</tr>
<tr>
<td>ODL movement</td>
<td>0.002</td>
<td>0.002</td>
<td>0.002</td>
<td>0.009</td>
</tr>
<tr>
<td><strong>Total</strong> ($\mu m_{RMS}$)</td>
<td>0.024</td>
<td>0.027</td>
<td>0.027</td>
<td>0.047</td>
</tr>
</tbody>
</table>
2.8 Conclusions

The study of the rigid body motion indicates that the requirement of 5 nm\(\text{RMS}\) is met, if a fringe sensor of 10 Hz is used. In other words, for the rigid body motion, the residual displacements due to internal forces after OPD regulation are smaller than the requirement. However, the margin is not very large. In reality the disturbance forces of the CFHX will become less during the design, and internally compensated. On the other hand, the possible effects from disturbing equipment listed in Section 2.4.4 are not yet taken into account, mainly because of a lack of data on their disturbing behaviour. It is recommended to do so during the Darwin study. This study demonstrates that the numbers of the remaining OPD above 1 Hz, have about the same order of magnitude as the requirement. If using a 100 Hz fringe sensor, this uncertainty is removed. Using the 100 Hz fringe sensor gives some margin to the fact that also e.g. external forces may increase the OPD and the method is rough in comparison with reality. The requirement of the DSD remaining below 7.5 nm/\(\sqrt{\text{Hz}}\) is met for all cases, for frequencies above 2 Hz. Furthermore the DSD has a slope of roughly -40 dB/dec. This is a result of the sum of the disturbance forces being nearly white noise. An exception on this slope is when resonance occurs. There a peak in the local DSD arises. These peaks do not contribute significantly to the OPDs as long as they are single peaks at high frequencies.

The model of the satellite was cross-checked using two methods for solving the problem (modal analysis and the direct method), as well as two models where the one was a part of the other one (rigid body model and resonance model).

Depending on the parameters of the 2-DOF model, the OPD requirement is met using a 10 Hz fringe sensor. As long as the sunshield resonance frequency is higher than 2 Hz or lower than 0.5 Hz, and the damping ratio \(\zeta\) is equal to 0.1% or more, then the 10 Hz fringe sensor will satisfy. If a fringe sensor of 10 Hz is used, then it is advised to design the sunshield such that its resonance frequency is not in between 0.5 and 2 Hz. Also here, the margins are not large and thus still a sensor of 100 Hz is advised.

This study focused on the high-frequent disturbance forces, above 1 Hz. The main purpose was evaluating the feasibility of using a 10 Hz fringe sensor. On the other hand, the lower-frequency disturbance forces, between \(10^{-2}\) and \(10^{0}\) Hz are also very important to look at. This is because if these forces are too big, the satellite may be displaced faster than the FEEP's and ODL can handle (e.g. saturation of the ODL). It is advised to perform a study about these lower-frequent forces and to calculate the velocities the satellite gets due to the disturbances.

This study is a first attempt to find out the order of magnitude of the disturbances on the OPD. A more profound study is necessary using a complete Finite Elements Model, when more details about the satellite design are known. One resonance peak does not alter the OPD a lot, but a collection of resonance frequencies might still do so. The FEM model should also make use of updated information on the disturbing subsystems and the Darwin material properties. Subsystems not taken into account should be looked at in the follow-up study. Examples of such systems are DC/DC-converters, mirror switches, thermal cracks, amplitude modulators, pupil derotators and beam angle actuators.
Chapter 3

The external space environment for Darwin and FFDEM

3.1 Introduction

Darwin will fly in a Lissajous orbit about the second Sun-Earth Lagrange point L\textsubscript{2}. This point is located at 0.01 AU “behind” the Earth as seen from the Sun. FFDEM, as defined in [30], will fly in a polar Low Earth Orbit at 561 km, where the space environment is much more variable than in L\textsubscript{2}. However, the space environment in L\textsubscript{2} is not “quiet” at all. There are many aspects of the variable L\textsubscript{2} environment that may contribute to the disturbance level of Darwin. In order to find out how relevant these disturbances are, in this Chapter, the local space environment in L\textsubscript{2} is examined from the literature available. Chapter 6 then follows with a deduction of the disturbances on Darwin, starting from this space environment investigation. Not much is published yet about the space environment in L\textsubscript{2}, whereas it becomes of increasing importance due to the growing number of missions towards L\textsubscript{2}. This Chapter also aims for the L\textsubscript{2} space environment to be more well-known, for the benefit of future missions in L\textsubscript{2}.

The starting point of this Chapter is the Sun, because the solar variability is the driving mechanism behind the variability of the space environment. A glimpse of the solar variability is given in Section 3.2. The elements of the space environment that are vulnerable to the Solar variability, are the magnetic environment (Section 3.3), the plasma environment (Section 3.4) and the Earth atmospheric or “neutral environment” (Section 3.5). For each one of them, the spatial and temporal variations are described. In addition the gravitational environment in LEO is described in Section 3.6, which is a base for the consideration given in Section 5.2 about the gravitational disturbance forces in LEO.

3.2 The solar-terrestrial environment for Darwin and FFDEM

3.2.1 The Sun and the space environment

The variability of the Sun is of large importance for the low- and high-frequency variability of the total local space environment. In this Section the Sun is discussed and its connection to the space environment in general.

Going out from the center of the Sun, or the core, one comes upon a radiative zone, a convective zone and a photosphere (Fig. 3.1 from [91], [53]). The latter is the outer surface and at the same time the visible part of the Sun. Upon the photosphere, the Sun has its
own atmosphere that consists of a chromosphere and a corona. The chromosphere can be observed as a tiny red ring during solar eclipses. The corona is the huge outer part of the Sun’s atmosphere that is also visible during solar eclipses. It is heated up to one million K by a mechanism that is still not well understood. It is generally believed that solar magnetic fields are the drivers for coronal heating. The corona is the part of the Sun that has a large influence on the space weather conditions in interplanetary space, as it is the source of the solar wind.

The Sun rotates about its axis in the same direction as the planets do, but its rotation rate is not constant, like it would be if the Sun were a rigid body. Instead, its rotation rate is higher in the low equatorial latitudes, and lower at polar regions. A mean value of 27 days is mostly accepted for the rotation period. The Sun has a weak dipolar magnetic field, but in addition it has a maze of strong local magnetic fields on its surface, that continuously change in magnitude and space. When the Solar magnetic field is carried away together with the solar wind, it is called the \textit{Interplanetary Magnetic Field}. Table 3.1 lists some of the Sun’s characteristics, from [27].

![Figure 3.1: The different layers of the Sun, and the strong local magnetic fields on its surface, from [91], [53].](image)

The solar wind

The Sun does not only emit electromagnetic radiation, but it also expells charged particles into space. This stream of charged but as a whole neutral particles is called the solar wind. It has a velocity range of approximately 300 to 800 km/s depending on its origin. Solar wind originating at low latitudes has a relatively low velocity and comes from the corona.

<table>
<thead>
<tr>
<th>Black Body temperature</th>
<th>$T = 5762$ K</th>
</tr>
</thead>
<tbody>
<tr>
<td>Radius photosphere</td>
<td>$R_{s, ph} = 695.910^6$ m</td>
</tr>
<tr>
<td>Mean distance from Earth</td>
<td>1 AU $= 149.59810^6$ m</td>
</tr>
<tr>
<td>Solar flux (at 1 AU)</td>
<td>$\phi = 1360.45$ W/m$^2$</td>
</tr>
<tr>
<td>Solar pressure (at 1 AU)</td>
<td>$P_0 = 4.5610^{-6}$ N/m$^2$</td>
</tr>
<tr>
<td>View angle of the solar diameter at 1 AU</td>
<td>31°59.3° = 0.5331°</td>
</tr>
</tbody>
</table>

Table 3.1: Some of the Sun’s most important properties, from [27].
They are called *equatorial streamers*. This wind is variable in speed, composition, and temperature. At high latitudes, the solar wind originates at the so-called *coronal holes*, regions of the corona with a relatively low density. It has a high speed, but it is very stable in comparison to the slow solar wind. Some solar wind parameters are given in Table C.1 in Appendix C.1. Note that the proton density for the slow wind is much higher than for the fast wind. However, due to the differences in wind speed, the proton flux will be almost the same.

The solar wind consists of electrons, protons and to a lesser extent ions. It has a density of 5 electrons and 5 protons per cm$^3$ at a distance of 1 AU. The real plasma density in L$_2$ or in LEO will be lower, due to the Earth magnetosphere [48]. Apart from these particles the solar wind also carries along a magnetic field that has a strength of about 5 to 6 nT at a distance of 1 AU from the Sun [?, 48]. This wind sometimes has a positive magnetic polarity, and sometimes a negative one. The solar wind has also high-frequency fluctuations in the order of seconds to tens of seconds at a distance up to about 0.1 AU from the Sun [69]. Whether these fluctuations are conserved until in L$_2$ (1.01 AU) is not known.

**Sunspots**

Sunspots are dark regions on the photosphere, whose quantity is associated with the *solar cycle* or *sunspot cycle*. The spots were already noticed by Theophrastus in 325 BC, but were rediscovered by Galileo Galilei (1610–1613, see Fig. 3.2, from [80]). He deduced the mean solar rotation period of the Sun by observing the sunspots during its rotation. In 1848, Wolf devised a method for counting the numbers of sunspots and sunspot groups, and related these numbers to the solar activity. This counting method is still in use today, so that the numbers can be compared with very old but valuable data. This sunspot number can be correlated to the solar flux at a wavelength of 10.7 cm (see Appendix C.1). The amount of sunspots depends on the phase of the solar cycle in which the Sun finds itself. Each 11 years a maximum of sunspot activity and associated sunspot number occurs, as well as a minimum. Hence, the solar cycle is therefore often called the sunspot cycle. The sunspots have very strong magnetic fields associated with it, in a range of a few thousand Gauss (4000 G = 0.4 T).

![Figure 3.2: Close-up of a sunspot, and a drawing of the sunspots made by Galileo in the 17th century, from [80].](image)

**Coronal mass ejections**

When the Sun expells a huge amount of energetic plasma particles from the corona at a very high speed, a *coronal mass ejection* or CME occurs. Such CMEs can move at 200 to 2000 km/s. A division is made between slow CMEs and fast CMEs according to their velocity.

---

1The protons are the heaviest and most abundant elements of the solar wind, so the energy that is transported by that solar wind will mainly depend on the proton flux.
(slower or faster than 400 km/s respectively). Fast CMEs produce shock waves in front of them, slow CMEs don’t. CMEs change plasma properties like temperature, number density, velocity and magnetic fields. They are the major cause of geomagnetic storms: significant disturbances of the geomagnetic field due to solar activity. The amount of CMEs is the largest just before and just after solar maximum (a mean of 3 to 10 a day), which is ten times as much as during solar minimum (a mean of 0.2 to 1 a day). It might be of importance for Darwin how many CMEs are intercepted in the L2-point if they would happen to disturb the satellite position. The spatial distribution of the CMEs is therefore of importance. Halo CMEs are CMEs that are pointed toward the Sun-Earth axis. A halo CME is most geo-effective (causing a large geomagnetic storm), at least if it is pointed towards the Earth and not away from it. The number of occurrence of halo CMEs pointed towards the Earth could not be found in the literature by the author at this time.

Flares
A flare is a sudden outburst of energy coming from an active region on the solar surface. It is characterized by a major increase of particularly UV and X-ray emission from the Sun. Also gamma rays and additional radio bursts occur during flares. A flare usually lasts for a few hours and several flares per hour can occur during solar maximum. At solar minimum, microflares still occur, at a rate of about 10 per hour.

Some characteristic properties of CMEs and Flares are shown in Table C.2 in Appendix C.1. Note that CMEs mostly become geo-effective, while flares not always do.

These radio bursts were already noticed on Earth in the mid-19th century by the early telegraph operators. A link between the radio disturbances and the aurora was quickly made.
Solar particle event

Sometimes a flare is accompanied by an repel of very energetic particles with energies of up to 100 MeV. These are called solar particle events but they are very uncommon. If they happen, they occur mostly during solar max.

Very often during big storms, these mechanisms act at the same time.

3.2.3 Solar radiation

The Sun emits not only particles (solar wind) but also electromagnetic radiation. This electromagnetic radiation is the source of the largest non-gravitational disturbance force in $L_2$: the radiation pressure force. The total solar electromagnetic flux in $L_2$ is equal to 1340 W/m² [24]. This flux is the sum of the total energy the Sun emits, integrated over the entire electromagnetic spectrum and per surface unit. It is called the solar constant or solar irradiance, although this constant varies throughout the solar cycle.

The solar flux has a maximum in the visible part of the spectrum. However, in the visible, its variation is only limited, while the variations in other parts of the spectrum are larger but do not contribute as much to the total solar irradiance. An overview of the average and worst-case flux from the Sun is shown in Table 3.2, from [22]. It illustrates quantitatively that only the near-UV variability will have a significant effect on the total solar irradiance and thus the solar radiation force.

On the long-term variation of the solar radiance a lot is found in the literature, although the short-term fluctuations are much more unknown. The following statements are found in the literature:

- A solar flare can cause an increase of the solar irradiance by 270 ppm (0.027%) [78], as was the case in October 2003.
- Variations of 0.1% are observed in the total solar irradiance in timescales between 1 second and 11 years [48].
- The solar irradiance changes through the solar cycle with an amount of about 0.2% [67].
- The solar irradiance can decrease by an amount up to 0.34% by the presence of sunspots [78].
The annual change in total solar irradiance changes by 3.3% due to the eccentricity of the Earth’s orbit (and thus also in L₂).

These factors will be used for the consideration on the solar radiation pressure variability in Chapter 6. It is not sure on which timescales the near-UV variability exactly takes place, but it is plausible to accept that it will not be on a timescale of only seconds.

<table>
<thead>
<tr>
<th>Type</th>
<th>Wavelength [nm]</th>
<th>Average flux [W/m²]</th>
<th>Worst-case flux [W/m²]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Near-UV</td>
<td>180 – 140</td>
<td>118</td>
<td>177</td>
</tr>
<tr>
<td>UV</td>
<td>&lt; 180</td>
<td>2.3 \cdot 10^{-4}</td>
<td>4.6 \cdot 10^{-4}</td>
</tr>
<tr>
<td>UV</td>
<td>100 – 150</td>
<td>7.5 \cdot 10^{-4}</td>
<td>1.5 \cdot 10^{-4}</td>
</tr>
<tr>
<td>EUV</td>
<td>10 – 100</td>
<td>2 \cdot 10^{-3}</td>
<td>4 \cdot 10^{-3}</td>
</tr>
<tr>
<td>X-rays</td>
<td>1 – 10</td>
<td>5 \cdot 10^{-3}</td>
<td>1 \cdot 10^{-4}</td>
</tr>
<tr>
<td>Flare X-rays</td>
<td>0.1 – 1</td>
<td>1 \cdot 10^{-4}</td>
<td>1 \cdot 10^{-3}</td>
</tr>
</tbody>
</table>

Table 3.2: The variability of the solar electromagnetic flux in different parts of the spectrum, from [22].

### 3.3 The magnetic space environment for Darwin and EFFDM

#### 3.3.1 Introduction

In this Section the magnetic environment surrounding the Earth is described, and its variability. The magnetic environment in L₂ is focused on, because only these will lead to calculations of the disturbance forces for Darwin. The consequences of the magnetic environment for Darwin will be deduced in Chapter 6.

#### 3.3.2 The magnetic field of the Sun

The magnetic activity of the Sun is the driver for all its active events (CMEs etc.) through which the Sun has an influence on the environment near the Earth. The Sun has a dipole magnetic field that switches polarity every 11 years, during solar maximum. After 22 years the original polarity has come back. This 22-year cycle is called the magnetic cycle of the Sun and it is composed of two sunspot cycles. In addition, the Sun has a maze of very strong local magnetic fields peering through the solar surface. These are situated mainly near sunspots and active regions. They fluctuate and change very rapidly, depending on the phase of the solar cycle. Also the polarity of sunspot pairs switches in the 22-year magnetic cycle, like the dipolar magnetic field.

In general and specifically during solar minimum, the magnetic field lines near the solar poles are open and they extend into space (see Fig. 3.5, from [84]). These field lines correspond to the fast solar wind at the coronal holes. At equatorial latitudes the field lines are in general “closed” or “looped” as seen in Fig. 3.5. At higher altitudes these loops change into “coronal streamers” of which the magnetic field lines also extend into space, but with opposite polarity at the equator. Between the two fields of opposite polarity, a neutral layer is situated, or a so-called current sheet. The Sun’s magnetic axis is inclined with respect to its rotation axis. Hereby this current sheet is warped and also wrinkled with respect to the equator\(^4\). It is often referred to as the Sun’s ballerina skirt.

\(^4\)Apart from the warp of the current sheet, the magnetic field lines are also not shaped radially outward, but in a spiral pattern. This is referred to as the “Archimedean spiral pattern” but is of no concern for the work presented here.
3.3.3 The interplanetary magnetic field

The interplanetary magnetic field (IMF) can be considered as an extension of the solar magnetic field into interplanetary space. It has a magnitude of approximately 5 nT at one AU. Just like the Earth’s magnetic field decreases with distance from the Earth’s surface, also the IMF becomes weaker with increasing distance from the Sun. Furthermore, the strength of the IMF depends on the level of solar activity. During geomagnetic storms the IMF strength could be as large as 18 nT. The IMF has the same ballerina shape as the solar magnetic field, and hence, the Earth and L2-satellites are located in a positive magnetic field sector at one time, and in a negative one the other time. The neutral sheet at the solar magnetic equator corresponds to the heliospheric current sheet that divides the regions of positive and negative magnetic polarity through the heliosphere.

3.3.4 The magnetic field of the Earth

The Earth has a dipolar magnetic field with a strength of approximately 0.3 Gauss at the equator, and 0.6 Gauss at the poles. The Earth’s magnetic axis is offset from the Earth’s center and tilted by a varying value of approximately 11.5° with respect to the Earth’s rotation axis. The magnetic field fluctuates both in strength, and in position of the axis. At the moment, its magnetic north pole corresponds to the geographic south pole and vice versa. In the age-old history of the Earth, it has switched polarity regularly.

The Earth’s magnetic field not only allows us to navigate, but it also protects us from the dangerous charged particles coming from e.g. cosmic rays, solar particle events and from the solar wind. The Earth’s magnetic field strength depends on the latitude and longitude of the observer, as well as the height above the surface and the time. At a certain height, the Earth’s magnetic field has lost so much strength that it becomes affected by the solar wind. At low altitudes the Earth magnetic field is thus quasi-static, but at high altitudes there can be big fluctuations in magnetic field strength and plasma properties due to the external variations of the magnetic field. These fluctuations are measured and described by means of indices that are discussed in Appendix C.2.

The Earth’s magnetic field is modeled in a dipole model, added with higher order terms described by a spherical harmonics expansion. Usually the influences from the IMF are not included in this model. The magnetic field model of the Earth is described in detail in Appendix C.2.

---

4The heliosphere is the region surrounding the solar system in which the solar wind dominates on the interstellar gas.
3.3.5 The structure of the magnetosphere

The Earth’s magnetic field looses its strength with increasing height above the surface. At a certain height, usually 10 to 14 \( R_E \) [24], it equals the interplanetary magnetic field (IMF). The interface between the Earth’s magnetic field and the IMF is called the magnetopause (Fig. 3.6, from [89]). The magnetopause encloses the magnetosphere, which is defined as the region where the Earth’s magnetic field dominates. Outside, the interplanetary magnetic field dominates.

About 3 \( R_E \) sunward of the magnetopause a bow shock is formed. This shock is the result of the “supersonic” solar wind that is decelerated at the magnetosphere. On the sunward side of the bow shock, the free solar wind reigns. The solar wind has a strong effect on the position of the bow shock and magnetopause, and thus also on the size and shape of the Earth’s magnetosphere. During severe geomagnetic storms, the magnetopause can even shift down to 6.6 \( R_E \)\(^{5}\) [73].

Figure 3.6: The structure of the Magnetosphere, from [89].

The region in between the bow shock and the magnetopause is called the magnetosheath. It is at its thinnest on the sunlit side of the Earth, and it dilates with increasing distance from the Sun. In the magnetosheath, the plasma flows “around” the magnetosphere in just the same manner as the free solar wind does “around” the magnetosheath and bow shock.

Inside the magnetosphere, at a distance of approximately 1–3 \( R_E \) and 3–9 \( R_E \) from the Earth are situated the Earth’s radiation belts or Van Allen belts. The plasmasphere is a region in between the night-side of the Earth and the inner radiation belt (see Fig. 3.6, from [89]).

The magnetosphere does not really have the shape of a sphere, but is stretched out by the interaction of the solar wind and the Earth’s magnetic field. On the night side of the Earth, it can stretch out up to about 500 \( R_E \). This stretched-out part of the magnetosphere

\(^{5}\)Notice that geostationary satellites flying at 5.6 \( R_E \) from the Earth are then almost encountering the free solar wind in such cases.
at the night-side of the Earth is called the magnetotail and depends strongly on the solar
wind conditions. Hence, $L_2$ at some times is in the magnetotail and at other times outside.

The magnetotail is subdivided in a boundary layer or plasma mantle, a plasma lobe and
a plasma sheet (see Fig. 3.7, from [24]). The plasma sheet in turn is divided in a central
plasma sheet enclosed by the plasma sheet boundary layer. Inside the central plasma sheet is
situated the neutral sheet, a “surface” where the magnetic field is equal to zero, comparable
with the neutral sheet of the Sun. At the northern side of the neutral sheet, the magnetic
field lines of the Earth point towards the Earth while on the southern side of the neutral
sheet they are pointed away, in accordance with the Earth’s polarity.

![Figure 3.7: A cross section of the Magnetosphere, from [24].](image-url)

### 3.3.6 Variability of the magnetosphere

As the solar wind has a big influence on the geomagnetic environment, it will be of no sur-
prise that a sudden burst of, for instance, a CME will change the geomagnetic environment
abruptly for a certain timespan. It is therefore useful, in case of a geomagnetic storm, to
look for what will change, in what timespan and to which extent. These situations can be
considered as worst-case scenarios for this study.

Already in 1724 George Graham and Anders Celsius independently discovered at differ-
ent locations that their magnetic compass needles showed some irregular variations on a
temporal scale of smaller than a day. At that moment they witnessed for the first time a
geomagnetic storm. A geomagnetic storm typically occurs when a CME hits the Earth and
turns the IMF southward\(^6\), opposite to the direction of the Earth’s magnetic field. When
this happens, the magnetic field lines of the IMF and the Earth will reconnect at the sun-
ward side of the magnetosphere, and a boost of solar wind plasma will be injected into the
magnetosphere.

It is clear that as the plasma brings along its own magnetic field, that the magnetic field\(^7\)

---

\(^6\)This condition is called $B_z$-south or $B_z < 0$.

\(^7\)See Section 3.4.
near the Earth will be disturbed when the plasma enters the magnetosphere. These geomagnetic storms occur irregularly, but they occur most frequently just before and just after solar maximum. Their effects are observable on a global scale. The most severe geomagnetic storms are caused by halo CMEs.

Apart from the irregular and severe storms, also regular and more moderate storms occur. They are initiated by the fast solar wind that is coming from the coronal holes. Especially during solar minimum the fast solar wind originates more and more in the equatorial regions. When this fast wind interacts with the slower solar wind, shock waves and oscillations are formed with notably strong magnetic fluctuations. These cause less severe storms that are mostly restricted to the polar regions of the Earth.

Apart from geomagnetic storms, also geomagnetic substorms occur. These are the result of a temporal building of energy in the magnetotail. If in this case the IMF is turned southward, reconnection occurs, typically at a distance of about 6 to 20 $R_E$ downstream of the Earth [48]. Plasma material is then accelerated away from the reconnection point towards the Earth, and also towards $L_2$. This could have implications for the magnetic and plasma environment near $L_2$, but surely too in LEO. These substorms are more moderate and regular than the geomagnetic storms and are restricted to the auroral regions.

<table>
<thead>
<tr>
<th>Time-span</th>
<th>Storm 1</th>
<th>Storm 2</th>
<th>Substorm</th>
</tr>
</thead>
<tbody>
<tr>
<td>Frequency</td>
<td>irregular</td>
<td>less irregular</td>
<td>regular</td>
</tr>
<tr>
<td></td>
<td>most just before and after solar max</td>
<td>most just before and after solar max</td>
<td></td>
</tr>
<tr>
<td>B (magnetic field strength)</td>
<td>$&gt; -10$</td>
<td>$&gt; -5$</td>
<td>$&gt; -3 nT$</td>
</tr>
<tr>
<td>Location</td>
<td>global</td>
<td>polar regions, auroral zones</td>
<td>polar regions, auroral zones</td>
</tr>
</tbody>
</table>

Table 3.3: The three types of geomagnetic storms, including substorms.

The storms change the direction and strength of the magnetic field, and they change the size and orientation of the magnetosphere. According to [13], the magnetic field strength during ICMEs can become up to -20 nT. The minus-sign denotes the IMF to be turned southward. The magnetic effects are summarized in Table 3.3. The storms also have consequences for the plasma environment.

### 3.3.7 The geomagnetic environment in $L_2$

$L_2$ is situated at 236 $R_E$ downstream of the Earth, what means that Darwin will fly in the outer part of the Earth’s magnetotail. Under average solar wind conditions, the magnetotail can be assumed to be cylindrical [24]. However, its size and its orientation depend on the solar wind dynamic pressure and its direction. Typical values for the distance from the neutral sheet to the bow shock (see Figure 3.7) in $L_2$ are 75 to 100 $R_E$. The magnetopause in $L_2$ is typically restricted to a radius of 25 to 33 $R_E$ from the neutral sheet. The plasma sheet occupies only a few Earth radii in the $z$-direction, but up to 10 $R_E$ in the $y$-direction. Neverthless, the plasma sheet is often known to be tilted. The centerline of the neutral sheet varies according to the direction of the solar wind. At a distance of $L_2$ it can be offset by

\[^8\] Coordinates are given in the Lagrange-point coordinate system. The origin of this coordinate system is in $L_2$. The $x$-axis points in the same direction as from the Sun to the Earth, while the $y$-axis points in the direction of the revolution of the Earth around the Sun. The $z$-axis fits to a right-handed coordinate system.
up to 40 \(R_E\) [24].

In [21], a possible orbit in \(L_2\) is shown with an amplitude of 39 \(R_E\) in the \(x\)-direction, 125 \(R_E\) in the \(y\)-direction and 47 \(R_E\) in the \(z\)-direction. This means that if the plasma regimes remained stable in its position, the spacecraft would encounter the plasma sheet, as well as the magnetosphere and the magnetosheath, and Darwin would even find itself in the free solar wind. A typical orbital period in \(L_2\) is approximately half a year or 180 days. This is much longer than the variations that occur in the magnetotail. As a result, the spacecraft can be regarded as hanging “still” in various positions, while the magnetic and plasma environment continuously changes [24].

The difference in plasma regimes inside the plasmasphere are hard to notice for a spacecraft [24], but still a major distinction can be made between magnetotail, magnetosheath and free solar wind. In those plasma regimes the numbers for magnetic field strength and particle fluxes are necessary for the calculations of the disturbance forces on Darwin. The typical values of the magnetic field strength for each plasma regime are given in Table 3.4, from [5].

<table>
<thead>
<tr>
<th>Solar wind</th>
<th>Magnetosheath</th>
<th>Lobe</th>
<th>Plasma Sheet</th>
</tr>
</thead>
<tbody>
<tr>
<td>(B)</td>
<td>5 nT</td>
<td>higher than solar wind</td>
<td>30 nT</td>
</tr>
</tbody>
</table>

Table 3.4: Typical values for the magnetic field strength in the different plasma regimes, from [5].

### 3.4 The plasma environment for Darwin and EFFDM

#### 3.4.1 Introduction

Space is not empty. This idea already raised in the ancient Greek days, about 350 BC, when Aristotle mentioned the existence of ether through which light was supposed to propagate. This idea was revised in the seventeenth century by different scientists. Later on, in 1887 Michelson and Morley conducted an experiment to prove that the ether as it was defined, did not exist.

Today it is known, thanks to the space era, that space is yet not empty at all. It is filled with a mixture of electrons and ions, which form together an electrically neutral substance. This substance is called a plasma. Some plasmas are fully ionized (as is the case in \(L_2\)), others are only partially ionized (e.g. in the ionosphere). The main difference between the plasmas in \(L_2\) and in LEO is that in \(L_2\) it has a very low density but a high energy (hot plasma), while the plasma in LEO has a high density and low energies (cold plasma)\(^9\).

A spacecraft flying through a plasma can become charged partially or wholly, and discharged again e.g. when the plasma environment changes abruptly or when the spacecraft enters or leaves the Earth’s shadow. Spacecraft charging leads on its turn to effects like inter-satellite Coulomb forces, changes in the surface properties, or increased contamination. Also torques can appear due to induced potentials and spacecraft wake effects in LEO can lead to a different charge in a different part of the spacecraft (differential charging). Therefore the typical plasma environment characteristics are looked at in this Section. Some aspects of the plasma environment in LEO are described in Appendix C.3.

\(^9\)An exception are the polar regions, where also low-density and high-energy plasma can be present.
3.4.2 Plasma properties

Magnetic field

A space plasma has the property that a magnetic field can move along with the plasma, if the plasma is conductive enough. It is said that the magnetic field is frozen into the plasma. For this reason the solar wind carries along with it the magnetic field of the corona, where the solar wind originates, into interplanetary space where it becomes the interplanetary magnetic field (IMF). Similarly, the plasma within the Earth’s magnetosphere flows easily along the magnetic field lines, but only very slowly and few across them. This means that if two spacecrafts are flying at the same magnetic local time and if they are flying along the same magnetic field line (hence, having the same L-coordinate, see Appendix C.2), they will encounter the same plasmas [22]. The motion of the single plasma particles along the Earth’s magnetic field lines is described in Appendix C.3 for background knowledge.

Plasma currents

A plasma is quasi-neutral because the amount of electrons is nearly the same as the amount of protons. As long as those two move into the same direction, no electric currents arise, as is the case in the free solar wind. However, in the Earth’s radiation belts for instance, electrons tend to drift to the East while protons drift to the West due to their opposite charge sign and the drift motion (see Appendix C.3). This causes the so-called ring-current around the Earth, above $2 \text{R}_E$. These currents can transport mass, charge, momentum, energy and they can create magnetic fields [5].

A list of electric currents is given in Table 3.5 from [5]. The electric currents in the ionosphere are mostly located between 100 and 150 km and will not affect the satellites of the Formation Flying Demonstration Mission. In $L_2$ the Darwin satellites will possibly encounter the tail current, neutral sheet current and field aligned currents. These could possibly play a role in spacecraft charging effects.

<table>
<thead>
<tr>
<th>Magnetospheric currents</th>
<th>Location</th>
<th>Ionospheric currents</th>
<th>Location between 100 and 150 km</th>
</tr>
</thead>
<tbody>
<tr>
<td>Magnetopause current</td>
<td>Magnetopause</td>
<td>Auroral electrojets</td>
<td>auroral oval</td>
</tr>
<tr>
<td>Tail current</td>
<td>Magnetotail</td>
<td>Sq currents</td>
<td>dayside mid-latitude ionosphere</td>
</tr>
<tr>
<td>Neutral sheet current</td>
<td>Neutral sheet</td>
<td>Equatorial electrojet</td>
<td>Magnetospheric equator</td>
</tr>
<tr>
<td>Ring current</td>
<td>Radiation belts</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Field aligned currents</td>
<td>Along magnetic field lines</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 3.5: The electric current systems in the magnetosphere and ionosphere (from [5]).

Plasma injections

In case of a geomagnetic storm, magnetic fields reconnect and they cause a boost of plasma to be injected in the magnetosphere on the sunlit side of the Earth. Also during substorms, plasma is shot away from the reconnection point ($X$-line) in the magnetotail, towards and away from the Earth. The kind of plasma that is injected is solar wind plasma.
These storms and substorms thus not only cause the magnetic field to change, but they also disturb the local plasma properties at a certain location and during a certain timespan. These short plasma variations can possibly lead to very small disturbances in satellite orbit, or could cause charging and discharging effects.

**Debye shielding**

For a unit charge $q$ in an empty space, the potential at a distance $r$ from $q$ is equal to [73, 10]:

$$V = \frac{1}{(4\pi\epsilon_0)} \cdot \left(\frac{q}{r}\right)$$  \hspace{1cm} (3.1)

in which $\epsilon_0$ is the permittivity of free space ($8.854 \cdot 10^{-12}$ C$^2$N$^{-1}$m$^{-2}$).

If a unit charge that is positively loaded, is placed in a plasma, it will attract the negatively charged electrons and keep away from it the positive ions. The result is that the effect of the charge will be limited in space. This limitation is characterised by the *plasma Debye length*:\footnote{Peter Debye, Dutch scientist born in Maastricht [1884-1966]} This length is the distance from the point load that the plasma needs in order to gain back its quasi-neutrality or *load homogeneity*. Beyond that distance, no influence of the charge is noticeable. Within that distance, the electric field is reduced. The Debye length is calculated by [22]:

$$\lambda_D = \sqrt{\frac{\epsilon_0 k T_e}{n_0 e^2}}$$  \hspace{1cm} (3.2)

A rule of thumb for the Debye length is [73]:

$$\lambda_D \approx 69 \cdot \sqrt{\frac{T_e}{n_0}}$$  \hspace{1cm} (3.3)

In these Equations, $n_0$ is the plasma density far from the point load, $T_e$ is the electron temperature, $e$ is the elementary charge ($1.6022 \cdot 10^{-19}$ C), $\epsilon_0$ is the permittivity of free space ($8.854 \cdot 10^{-12}$ C$^2$N$^{-1}$m$^{-2}$) and $k$ is the Bolzmann constant ($1.38066 \cdot 10^{-23}$ J/K).

For a unit load $q$ in a plasma, the potential at a distance $r$, depends on the Debye length as follows [73, 44]:

$$V(r) = \frac{1}{(4\pi\epsilon_0)} \cdot \left(\frac{q}{r}\right) \cdot \exp\left(\frac{-r}{\lambda_D}\right)$$  \hspace{1cm} (3.4)

Comparing Equation 3.4 with Equation 3.1, we notice that the plasma has a shielding effect on the potential from the point load.

At an altitude of 300 km in LEO the Debye length is approximately 0.2 cm. The mutual influence of two charged spacecrafts in Low Earth Orbit on each other, will therefore be nihil. However, in GEO the plasma Debye length varies between 140 m to 1400 m [45]. There, two charged spacecrafts in formation flight, might have a significant influence on each other. The Debye lengths in the magnetosphere are given in Table 3.6.

### 3.4.3 The plasma environment in L$2$

Darwin will encounter many different plasma regimes during its orbit about L$2$. Not only because its orbit is rather large, but also due to the variability of the magnetotail in that region [24]. During its orbit, Darwin will encounter the free solar wind, the magnetosheath and the magnetotail. Inside the magnetotail the satellites will encounter some more different
plasma regimes when they pass through the plasmasphere, lobes and boundary layers. An overview of the main plasma characteristics in these plasma environments that were found in the literature is given in Table 3.6. The data taken from [24] are calculated by hand from the tables with cumulative probabilities ([24]p.27–28). The plasma Debye lengths from [5] are roughly taken out of a graph ([5]p.4). Effects like storms and substorms are not included yet.

<table>
<thead>
<tr>
<th>Source</th>
<th>parameter</th>
<th>Solar wind</th>
<th>Magnetosheath</th>
<th>Boundary layer</th>
<th>Lobe</th>
<th>Plasmasheet</th>
</tr>
</thead>
<tbody>
<tr>
<td>[5]</td>
<td>$T_e$(eV)</td>
<td>9</td>
<td></td>
<td>43</td>
<td>430</td>
<td></td>
</tr>
<tr>
<td>[24]</td>
<td></td>
<td>35</td>
<td>75</td>
<td>336</td>
<td>231</td>
<td></td>
</tr>
<tr>
<td>[22]</td>
<td></td>
<td>8.6</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>[48]</td>
<td></td>
<td>12</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>[24]</td>
<td>$T_i$(eV)</td>
<td>23</td>
<td>125</td>
<td>299</td>
<td>1352</td>
<td>1192</td>
</tr>
<tr>
<td>[22]</td>
<td></td>
<td>10</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>[48]</td>
<td></td>
<td>10</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>[5]</td>
<td>$n_e$(cm$^{-3}$)</td>
<td>5</td>
<td>higher than solar wind</td>
<td>10$^{-2}$</td>
<td>0.5</td>
<td></td>
</tr>
<tr>
<td>[48]</td>
<td></td>
<td>5</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>[24]</td>
<td>$n_i$(cm$^{-3}$)</td>
<td>12</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>[60]</td>
<td></td>
<td>2.8</td>
<td>0.6</td>
<td>0.2</td>
<td>0.3</td>
<td></td>
</tr>
<tr>
<td>[48]</td>
<td></td>
<td>5</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>[5]</td>
<td>$V$(km/s)</td>
<td>400-800</td>
<td></td>
<td>100 (Earthward)</td>
<td>[5]</td>
<td></td>
</tr>
<tr>
<td>[5]</td>
<td>$\lambda_D$(m)</td>
<td>10</td>
<td></td>
<td>700</td>
<td>300</td>
<td></td>
</tr>
</tbody>
</table>

Table 3.6: Characteristics in the different plasma environments in the magnetosphere, found in the literature.

- **Solar wind**
  The solar wind is in general dense and relatively slow when coming from the Sun, but it becomes cooler and less dense as it approaches Earth. Its velocity increases mostly in the beginning and is rather constant at a distance of 1 AU. Some general correlations between the solar wind and the distance to the Sun are described in Table 3.7 (from [60]). The solar wind plasma is thus slightly different in L$_2$ compared to 1 AU, though this difference is very small compared to the temporal variations in the solar wind [60].

| Solar wind plasma density | $n \propto R_{\odot}^{-2.1 \pm 0.3}$ |
| Solar wind electron temperature | $T_e \propto R_{\odot}^{-0.33}$ |
| Solar wind proton temperature | $T_p \propto R_{\odot}^{-0.37}$ |

Table 3.7: The solar wind dependencies on the distance from the Sun, from [60].

- **Magnetosheath**
  The bow shock causes the solar wind to be compressed, heated, slowed down and deflected (the plasma becomes *subsonic*). The plasma always flows in antisolar direction and at a higher speed than in the magnetotail, but still slower than the solar wind. The temperatures here are in general colder than in other parts of the magnetosphere.
• Boundary layer
The boundary layer is a typical transition region from the magnetosheath to the lobe. It is less dense than the magnetosheath, but still denser than the lobes.

• Magnetotail (general)
In the magnetotail the plasma environment is very variable on a temporal scale of down to 10 minutes [60]. The composition of the magnetotail plasma is nearly the same as the one of the solar wind, because the solar wind is the origin of the magnetotail’s plasma. According to [60], the electron and ion temperatures in the plasmasheet are higher than in the lobe and boundary layer, which are on its turn higher than the magnetosheath temperatures. However, this is in contrast with the data from [24].

• Lobe (magnetotail)
The magnetotail lobe’s main characteristic is its very low density. Its temperatures are higher than the magnetosheath and the flow velocity is relatively slow.

• Plasmasheet (magnetotail)
The plasmasheet is the region inside the magnetotail where most of the plasma is located. It is characterised by a high plasma temperature and a higher density than the lobes, but it is still not as dense as the magnetosheath and the solar wind. In the plasmasheet, plasma flows mostly away from the Earth, though very often it also travels towards the Earth. The sunshield of the Darwin telescopes can thus be exposed to the plasma from both sides. It typically has a speed of a few 100 km/s when flowing towards the Earth [60]. The plasma then moves along the magnetic field lines that end in the auroral regions of the ionosphere.

• Neutral sheet (magnetotail)
According to [24] the neutral sheet plasma density is about 1 cm$^{-3}$ and the electron temperature $T_e$ is 6894 eV. For the plasma sheet boundary layer the plasma density is 0.1 cm$^{-3}$ while $T_e$ is 103 eV. However, the difference between neutral sheet, plasma sheet boundary layer and the plasma sheet itself is only found in one article [24].

3.5 The neutral environment for Darwin and FFDEM

3.5.1 Introduction
The activity of the Sun has been discussed in Section 3.2. In this Section the influence of the solar activity on the neutral atmosphere at a height of 561 km is focused, and thus also its influence on the drag of the FFDEM satellites. The main types of flow at this height, are described in Appendix C.4.

The neutral atmosphere is known to be very difficult to model properly, due to its high variability. The atmospheric density and temperatures depend a lot on the solar flux, geomagnetic indices, tides, winds, etc. Modeling the atmosphere to a time variation of 1 Hz is therefore a difficult task. The spatial resolution of such a model would then be 7 km or less. Building such a model during this study is not feasible. Therefore the accelerometer-data-approach is used to investigate the high-frequency disturbances in LEO (Chapter 5). This Section therefore does not describe the atmosphere models in great detail, but it still gives the necessary background information or a brief overview of the atmospheric phenomena. First, a general description of the atmosphere is given in Section 3.5.2, after which the variability of the atmosphere is discussed in Section 3.5.2.

3.5.2 Description of the neutral atmosphere
The neutral atmosphere is the small film of gas that is sticked to the Earth. It stretches out to about 1000 km or 1500 km in height, depending on the solar flux. It is divided into
several layers, depending on their temperature. These layers are called spheres while the transition zones between them are called pauses. Their transition heights are not absolute but are defined differently in the literature, and they also depend on the solar flux. In this study we will assume the heights as given in [22].

The two major parts of the atmosphere are the homosphere and the heterosphere with a transition at about 90 km. In the homosphere the composition of the atmosphere is uniform due to turbulent mixing, vertical winds etc. and the gas is treated as a perfect gas. The heterosphere has no uniform composition anymore. There, different constituents dominate at different heights. The main cause of this is diffuse equilibrium: heavier constituents tend to stick around at lower altitudes, while lighter ones remain in higher regions. In the heterosphere there is nearly no mixing.

The homosphere consists of the troposphere, stratosphere and the mesosphere. The heterosphere consists of the thermosphere and the exosphere. The orbit of the European Formation Flying Demonstration Mission is located at the bottom of the exosphere.

Temperature
The variation of the temperature with the height differs from layer to layer. The temperature in the exosphere is almost constant and converges to an asymptotic limit that strongly depends on the level of solar activity\textsuperscript{11}. However, the atmospheric temperature still has a strong correlation with solar activity as is illustrated in Table 3.8.

Density
The density is one of the most important factors for determining the aerodynamic forces on a satellite. In contrast to the temperature, the density does vary a lot with a change in altitude. Apart from the height, also other influences on the density are present like solar flux and geomagnetic activity. The density variations with respect to time and space are discussed in Section 3.5.2, and are also shown in Table 3.8.

The neutral atmosphere characteristics at 561 km height
The orbit of EFFDM is a circular polar orbit at 561 km height. At this height, the satellite will fly through the lower part of the exosphere. The exosphere begins at a height of about 250 to 400 km depending on the solar activity. Typical values of the temperature, density, pressure, mean molecular weight and dominant constituents are given in Table 3.8 for a height of 560 km. This Table is taken from [22] where the data were produced by the MSISE-90 atmospheric model. The characteristics are given for a low solar activity level ($F_{10.7} = 70, A_p = 0$), for a mean solar activity level ($F_{10.7} = 140, A_p = 15$) and for an extremely high solar activity level ($F_{10.7} = 380, A_p = 300$). The values are averaged over the diurnal, seasonal and latitudinal variations [22].

The variability of the atmosphere
Apart from the height, the atmospheric density depends the most on the solar variability. On the long term—in a timespan of 11 years—the atmospheric density varies along with the solar UV flux, with a factor of 100 up to 300. On the shorter terms also variations of 27 days (rotation period of the Sun) and one day (rotation period of the Earth) or even less, are observed. Also seasonal effects are present. However, all these effects have a low frequency and can be easily compensated by an accurate thrust force.

On the short term, a sudden change in solar output will have a remarkable effect on the atmospheric density and temperatures. Events like a solar flare or a geomagnetic storm may

\textsuperscript{11}According to the MSISE-90 model the difference in temperature between the two heights is less than 1 K for low solar activity, as well as medium and high solar activity [70]. The MSISE model is explained in [22, 70]. It is available on internet via http://nssdc.gsfc.nasa.gov/.
<table>
<thead>
<tr>
<th>Parameter</th>
<th>Unit</th>
<th>Low solar activity</th>
<th>Mean solar activity</th>
<th>Extremely high solar activity</th>
</tr>
</thead>
<tbody>
<tr>
<td>Temperature</td>
<td>[K]</td>
<td>699.1631</td>
<td>1011.5245</td>
<td>1621.9803</td>
</tr>
<tr>
<td>Density</td>
<td>[kg/m³]</td>
<td>1.93 \times 10^{-14}</td>
<td>2.85 \times 10^{-13}</td>
<td>9 \times 10^{-12}</td>
</tr>
<tr>
<td>Pressure</td>
<td>[N/m³]</td>
<td>1.55 \times 10^{-8}</td>
<td>1.71 \times 10^{-7}</td>
<td>7.81 \times 10^{-6}</td>
</tr>
<tr>
<td>Mean molecular weight</td>
<td>[kg/mol]</td>
<td>7.2141</td>
<td>14.0125</td>
<td>15.9067</td>
</tr>
<tr>
<td>Dominating constituents</td>
<td>[in order of abundance]</td>
<td>He, O, H, N, N₂, O₂, Ar</td>
<td>O, He, N, N₂-H, O₂, Ar</td>
<td>O, N, N₂-He, O₂, H, Ar</td>
</tr>
</tbody>
</table>

Table 3.8: Characteristics of the neutral atmosphere at a height of 560 km, from the MSISE-90 model, [22].

alter the atmospheric characteristics drastically in a timespan of only minutes to hours [57]. The higher the orbit, the more the solar influence on the atmospheric density is pronounced. The influence of solar variability on the temperature, density, pressure, mean molecular weight and dominating constituents is clear from Table 3.8 for a height of 560 km. Another aspect of the variability in the atmosphere are the vertical and horizontal winds. The vertical winds in general have a slow speed, below 1 cm/s [22]. Most of the horizontal winds are due to a pressure gradient under the influence of the Coriolis acceleration. They can have velocities of up to 1 km/s at a height of 300 km [22]. They are known to be influenced more by the geomagnetic activity (Ap-index) rather than by the solar electromagnetic radiation (F₁₀.₇-index).

3.6 The gravitational environment for Darwin and FF-DEM

3.6.1 Introduction

Gravity forces are among the main sources of disturbance for spacecrafts travelling both in LEO and in L₂. They affect the trajectory as well as the attitude of the spacecraft (gravity gradient torque). The gravitational environments in L₂ and in LEO are described in this Section for as far as necessary to understand the considerations of the gravity forces in Section 5.2. The low-frequency component of the gravitational disturbance forces has been studied in [15].

Section 3.6.2 describes the gravitational environment in L₂. In Section 3.6.3 the gravitational environment for a satellite in LEO is explained and more specifically the gravity distribution of the Earth. Satellites also attract each other by gravitational force. Though it is a non-stochastic force (i.e. with no high-frequency behaviour), it is looked at in Appendix F.1 for illustrating that even forces that seem to be negligible to us, are still not so small on the nanometer-level.

3.6.2 The gravitational disturbance forces in L₂

The two-body problem

The 3\textsuperscript{rd} law of Newton expresses that two masses \( m_1 \) and \( m_2 \) attract each other with a force \( F \) that is proportional to the product of their masses, and inversely proportional to the square of the distance \( r \) between them [22], [26].

\[
F = \frac{Gm_1m_2}{r^2}
\]  (3.5)
G is the universal constant of gravitation \(G = 6.67259 \times 10^{-11} \text{ m}^3/(\text{kg s})^2\). This force is the gravitational force, to which this Section is dedicated.

When only the gravitational force of two masses is taken into account, the two-body problem is considered. A satellite in an orbit around the Earth is considered in the first place as a two-body problem, where the satellite is one of the masses and the Earth is the other one. All other forces are disturbance forces. If no such forces would be present, the orbit of the satellite would be a conic section.

The three-body problem

The Lagrange point L\(_2\) is the second of five locations in the Sun-Earth/Moon system, where there is an equilibrium of the gravitational forces from the Earth/Moon-system, the Sun, the Coriolis force and the centrifugal force. This is seen in a rotating frame of reference that has an origin in the mass centre of the system, and co-rotates with the two main bodies. As a consequence, in the Lagrange point L\(_2\) the gravitational forces from the Earth and the Sun have about the same order of magnitude. Therefore it is not realistic to only take into account the gravitational force from the Earth. The motion of a satellite in the Lagrange points is thus considered as a three-body problem. A good analytical approximation for the three-body problem is the Circular Restricted Three Body Problem (CR3BP) where the third mass is infinitesimally small in comparison with the other two masses (“restricted”) and the motion of the two main bodies is assumed to be circular. The motion of the third mass, in the rotating frame of reference, is described by Equation 3.6 in which the gravity, Coriolis and centrifugal forces are clearly visible [76].

\[
\frac{\delta^2 \vec{r}}{\delta t^2} = -G \left( \frac{m_1}{r_1^3} \vec{r}_1 + \frac{m_2}{r_2^3} \vec{r}_2 \right) - 2\vec{\omega} \times \frac{\delta \vec{r}}{\delta t} - \vec{\omega} \times (\vec{\omega} \times \vec{r})
\] (3.6)

The explanation and equations of motion in the CR3BP are described in detail in [76]. In the CR3BP of the second Lagrange point L\(_2\), the two primary bodies are the Sun and the Earth/Moon-system. If the Moon would be treated apart we would have to deal with the four body problem, which would become too complicated. Instead, the Earth and Moon are treated as one single mass placed at their barycentre. The influence of the motion of the Moon and the Earth about their barycenter are regarded as perturbations, as well as the ellipticity of the Earth/Moon orbit about the Sun-Earth barycentre. The third body is the satellite itself that has a negligible mass in comparison with the two main bodies.

The gravitational disturbances as a result of the eccentricity of the Earth/Moon orbit about the Sun, the orbit of the Moon and Earth about their barycentre, the inclination of the Moon with respect to the Earth and the gravity of other planets are all low-frequent phenomena. Therefore they are not important in this study.

3.6.3 The gravitational disturbance forces in LEO

In a Low Earth Orbit, the satellites also suffer from the gravitational perturbations of other planets and the Sun. For giving an indication, at an altitude of 400 km, an acceleration of 0.56 \(\mu\text{m/s}^2\) is a common size, while in a geostationary orbit this is 3.5 \(\mu\text{m/s}^2\) [76]. These perturbations are the same low-frequent type of forces as in L\(_2\) and will therefore not be discussed further in this Section.

The Earth is not a pure sphere with a uniform gravity field, but it has many lumps and bumps that cause spatial variations in the gravitational field. Also temporal variations exist due to movements of masses (ice, water, local air pressure, Earth, ocean and atmospheric tides, etc.). When a satellite is flying through the variable gravity field of the Earth, it experiences “disturbance forces” with respect to the nominal situation. Due to the velocity of
the spacecraft, spatial variations in the gravity field are experienced as temporal variations in the orbit of the satellite. The question in this study is how big these disturbances are, and what frequency they have.

The spatial variations of the Earth’s gravity field are described by means of a spherical harmonics expansion of the gravity field. In this expansion, the gravity field is described as the sum of a central force, and a series describing the deviations from the central gravity field. The gravity field can be described by means of a potential $U$ for which holds:

$$\mathbf{F}_g = -\nabla U$$

(3.7)

where $\mathbf{F}_g$ is the gravitational force. The geopotential $U$ satisfies the Laplace equation:

$$\nabla^2 U = \frac{\partial^2 U}{\partial x^2} + \frac{\partial^2 U}{\partial y^2} + \frac{\partial^2 U}{\partial z^2} = 0$$

(3.8)

The general solution of the Laplace equation for the geopotential is equal to [26]:

$$U = -\frac{GM_\oplus}{r} \left( 1 + \sum_{n=1}^{\infty} \sum_{m=0}^{n} \left( \frac{R_\oplus}{r} \right)^n P_{nm}(\sin\phi) \{ C_{nm} \cos m \lambda + S_{nm} \sin m \lambda \} \right)$$

(3.9)

where $M_\oplus$ is the mass of the Earth and $P_{nm}$ is a Legendre polynomial of order $m$ and degree $n$. It is defined by [26]:

$$P_n(x) = \frac{1}{2^n n!} \frac{d^n}{dx^n} (x^2 - 1)^n$$

(3.10)

In Equation 3.9, $\lambda$ is the longitude, $\phi$ is the latitude and $r$ is the distance from the satellite to the center of the Earth. These coordinates are given in the geocentric reference frame, that has an origin in the center of Mass of the Earth, and co-rotates with it. $C_{nm}$ and $S_{nm}$ are spherical harmonic coefficients. The Legendre polynomials divide the Earth in several parts, while a magnitude is added to it by means of the harmonic coefficients. This combination describes the severity of the Earth potential for the parts of the Earth which it is divided into. If $m$ is equal to zero, the Earth is divided into strips as illustrated in Figure 3.8. These harmonics are called the zonal harmonics. If $m = n$, the Earth is divided into sections (see Figure 3.8). These are called the sectoral harmonics. Finally if $m \neq n$ the result is a combination of both. These are called the tesseral harmonics. For these coefficients also holds [26]:

$$J_{nm} \cos m(\lambda - \lambda_{nm}) = C_{nm} \cos m \lambda + S_{nm} \sin m \lambda$$

(3.11)

For $m = 0$ and $n = 2$ the coefficients $C_{2,0}$ and $S_{2,0}$ are equal to $-J_2$ and 0 respectively. The $J_2$ term describes the flattening of the Earth, while the $J_3$-term describes the pear-form of the Earth.

Through these coefficients, Equation 3.9 is in fact a description of the Earth’s gravity potential. The model is best explained by the following: imagine the Earth being divided into several parts corresponding to the spherical harmonics coefficients. Each part or each such coefficient has a magnitude that describes the magnitude of the Earth potential. The higher the coefficient, the bigger the number of parts which the Earth gravity potential is divided into. The sum of all these coefficients is finally a good approximation for the real gravity potential. If only the lower coefficients are considered, the model is less detailed. If also higher coefficients are considered, then the model becomes very reliable. This model is used in the consideration in Section 5.2.

As concluded in Section 3.6.2, only the gravity forces in LEO are considered to possibly have a high-frequent component in the nanometer-level. In L2 the gravitational forces are low-frequent and will not be considered further in this study anymore.
3.6.4 Other gravitational effects

Gravity gradient torque

A satellite flying in a gravity field will experience a torque as a result of the differential gravitational force acting on it. There are two cases in which this can occur: when the satellite is not a uniform bar, the gravity pull will differ according to its mass distribution and tend to change the satellite attitude. A second case can be illustrated or explained well from the following [49]: When a bubble of fluid would float in space in an orbit around e.g. the Earth, the gravity on the lower side of the bubble would be bigger than the gravity pull on the upper side of the bubble. The result is that the fluid would be deformed from a sphere into a more pear-like form. When this bubble is replaced by a satellite (e.g. an elongated object inclined with respect to the gravity field), the gravity pull would induce a torque. This principle is often used as a control mechanism, from which it can be concluded that it is not negligible. The gravity gradient torque is well-known for satellites in an Earth orbit, but also in L₂ the gravity of the Sun and the Earth cause such a torque [15]. This force is regarded as a low-frequent force and will not be elaborated in this study any further.

Relativity

In the 17th century Newton stated his law of gravitation (Equation 3.5). This law is still used today to perform calculations, even in space engineering and in the two and three-body problems. However, Einstein’s theory of relativity is in fact a correction of this law for objects that have a certain velocity. This effect was observed for the first time for the planet Mercurius but also for man-made satellites this effect is small, but not negligible [76]. This force is also a low-frequent phenomenon and therefore it is not worked-out in detail.
Chapter 4
Micrometeoroid impacts on Darwin

4.1 Introduction

The importance of micrometeoroid impacts on the OPD of Darwin may be considerable. Each time a particulate with given mass and velocity strikes one of Darwin’s satellites, it can disturb the interferometer operations [37]. Impacting particulates give the satellite an excitation (translation and rotation), or even cause vibrations, damage or degradation. Not only the possibility of the loss of scientific data plays a role, but also the question of “how much propellant would be needed to counteract these impacts during Darwin’s lifetime if no delay lines are used”, needs to be investigated in future mission analysis.

The “particulate environment” in LEO consists of micrometeoroids and man-made space debris and is described in Appendix D.5. In L_2, only the micrometeoroids are present. The effect of an impact does not depend on whether it is a micrometeoroid or a space debris impact, but rather on the specific characteristics of the particulate itself, like its mass and mass density, its size, its impact velocity and the direction of the impact. A brief overview is given in Table 4.1, adapted from [20]. Indirect effects are also possible: circa 1% of the vaporized ejecta is plasma. This could interfere with the spacecraft potential, causing contamination or triggering arc discharges. Also within each impact, light flashes are generated that could possibly interfere with measurements. Meteoroid impacts on telescope mirrors also lead to a degradation of optical properties. An impact typically lasts for less than a few microseconds and leads to a shockwave in the material [20].

This Chapter focuses only on the influence of the impacts on the OPD of Darwin, and in particular on the amount of impacts that alter the OPD. The indirect effects are neglected. Of most interest for this study is the particulate flux. This particulate flux is defined as the amount of particulates that hit a spacecraft surface in a unit of time. The total amount of particulates that hit the spacecraft is thus [22]:

\[ N = F \times A \times T \]  \hspace{1cm} (4.1)

where \( F \) is the particulate flux, \( A \) is the total surface area that is hit by the particulates, and \( T \) is the exposure time (e.g. the lifetime of the satellite). In general the flux increases when the particulate size decreases [68]. The cumulative flux is the accumulated flux of all particulates that exceed a certain threshold for a certain quantity, like their mass.

In this Chapter, the cumulative flux is meant whenever discussing the “particulate flux”. First, the micrometeoroid environment in L_2 is discussed. The minimum value of the particulate impact momentum \((mv)_{imp}\) that affects the interferometer operations (i.e. OPD
\[ \geq 5 \text{ nm} \), is determined, which yields a certain threshold value for the particulate mass. It is assumed that the FEEP\( s \) are activated immediately after an impact (and not the ODL). Subsequently, the cumulative flux of all particulates with mass \( m \geq m_{\text{thresh}} \) is calculated using the micrometeoroid model that is first described. Using the calculated accumulated flux and the dimensions of the Darwin free flyers, the order of magnitude of the amount of OPD disturbances bigger than 5 nm due to particulate impacts, is calculated. If Darwin is disturbed only once a year, then the particulate environment can be considered as unimportant because such sporadic cases only cause very few disturbed scientific measurements. If it happens more frequently (e.g. a few times per minute), the design of the satellite would have to be adapted to cope with the sudden disturbances due to a particulate impact. This could be done by making sure that the feedback control loop acts quick enough to counteract for the disturbance, before the 5 nm level is reached. FEEP\( s \) with a certain thrust value that is high enough can be used, in combination with the ODL being put into action immediately.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Effect</th>
</tr>
</thead>
<tbody>
<tr>
<td>Size</td>
<td></td>
</tr>
<tr>
<td>(&lt; 1 \mu m)</td>
<td>Degradation</td>
</tr>
<tr>
<td>(10 - 100\mu m)</td>
<td>Penetration of coatings, solar cells</td>
</tr>
<tr>
<td>(&gt; 1 \mu m)</td>
<td>Damage</td>
</tr>
<tr>
<td>Velocity</td>
<td></td>
</tr>
<tr>
<td>(&lt; 5 \text{km/s})</td>
<td>Ejecta are solid fragments</td>
</tr>
<tr>
<td>(5 \text{km/s} &lt; v &lt; 20 \text{km/s})</td>
<td>Ejecta are a mix of vaporized fragments, solid fragments and molten droplets</td>
</tr>
<tr>
<td>(&gt; 20 - 25 \text{km/s})</td>
<td>Ejecta are vaporized</td>
</tr>
</tbody>
</table>

Table 4.1: Possible effects after impact according to mass and velocity, adapted from [20].

### 4.2 The micrometeoroid environment in L\(_2\)

The particulate environment in L\(_2\) only consists of meteoroids. These natural particulates are present in basically two populations: a constant presence of particulates (the isotropic particulate environment), and peak meteoroid streams that are a result of the Earth (and thus also L\(_2\) flying regularly through the remnants of e.g. a comet. However, according to [20], the isotropic particulate environment dominates the risk of spacecraft failure or disturbance and the streams of particulates that occur a few times a year do not have a significant influence on that risk. Therefore in the first computation of the micrometeoroid flux, no extra micrometeoroid stream is taken into account\(^1\). For a worst-case scenario, an extra micrometeoroid stream is added to this flux.

Apart from the intersplanetary micrometeoroids (isotropic and streams), also a third population, consisting of dust could be present in the vicinity of L\(_2\) [68]. This dust is supposed to have a velocity of a few km/s up to 10 km/s. Its size and flux are comparable to the size and flux of the interplanetary particulates [68]. However, this has never been proven yet, and probably the solar radiation pressure would “push” these particulates in anti-sunward direction into a shifted artificial L\(_2\)-point. If these particulates have a different area-to-mass ratio than Darwin does, then this population certainly does not need to be taken into account. However, a few single calculations (see Appendix D.1) show that there is a possibility that a certain part of this particulate population is shifted as much as Darwin will be from the

---

\(^1\)Except for the yearly average of these streams that is included in the model that is used.
real Lagrange point, as a result of the solar radiation pressure. This population of L₂-dust is not taken into account in an average flux estimation, but it is for the worst-case assessment, together with the micrometeoroid streams.

4.2.1 Modeling the micrometeoroid environment in L₂

A commonly used model for the interplanetary particulate flux is the Grün model, as described in [31]. The Grün model is known for its simplicity and ease of use due to the analytical formulation of the particulate flux. The model gives the interplanetary flux for particulates with mass \( m \) or bigger, on a randomly oriented surface at 1 AU, viewed from an angle of \( 2\pi \) steradian [22, 31]. This flux is equal to:

\[
F_{\text{met},0}(m) = c_0 \cdot (F_1(m) + F_2(m) + F_3(m))
\]

where
\[
c_0 = 3.15576 \cdot 10^7
\]
\[
F_1(m) = (2.2 \cdot 10^3 m^{0.306} + 15)^{-4.38}
\]
\[
F_2(m) = 1.3 \cdot 10^{-9} (m + 10^{11} m^2 + 10^{27} m^4)^{-0.36}
\]
\[
F_3(m) = 1.3 \cdot 10^{-16} (m + 10^6 m^2)^{-0.85}
\]

The flux is given in number of particulates with mass \( m \) or bigger, per unit area per year. Equation 4.2 is called Grün’s equation. Particulates with masses between \( 10^{-18} \text{g} \) and 1g are included in the model. The factor \( c_0 \) converts the flux given per second into a flux given per year. In this model, \( F_1(m) \) represents the large particulates with a mass larger than \( 10^{-9} \text{g} \), \( F_2(m) \) represents medium particulates with \( 10^{-14} < m < 10^{-9} \text{g} \) and \( F_3(m) \) represents the small particulates with a mass smaller than \( 10^{-14} \text{g} \). Grün’s equation is plotted in Figure 4.1.

Figure 4.1: The Grün equation.

---

\(^2\)The Grün model is derived from HEOS-2 and PIONEER-8 and 9 measurements, [70].

\(^3\)The isotropic flux summed by the yearly average of the particulate streams.
The flux predicted by Grün’s model for distances other than 1 AU is calculated by [37, 31]:

\[ F(m, r) = F_{\text{met},0}(m) \cdot \left(\frac{r}{r_0}\right)^{-1.8} \]  

(4.3)

It applies for distances from the Sun between 0.3 and 20 AU, but it still deviates significantly from other more recent and sophisticated models like the Divine model [37]. Because recent models are much more complicated—requiring a lot of CPU time—for the order of magnitude calculation the simple analytical model of Grün is preferred. In [37] an analytical approximation of the Divine model is given, in order to simplify calculations. However, for distances near 1 AU, Grün also seems to be very accurate compared to the other models [37, 20]. At a distance of 1.01 AU, the Grün model still satisfies. In this study Grün is used, but the non-circular movement of \( L_2 \) about the Sun due to the elliptical orbit of the Earth is neglected.

### 4.2.2 Micrometeoroid characteristics

Apart from the flux, in this Section the other particulate characteristics are discussed briefly.

- **Mass density** \( \rho \)

The mass density of meteoroids is of importance to transfer particulate masses into particulate sizes and vice versa. However, the mass density of the micrometeoroid particulates is very uncertain. The mass density of the meteoroids generally varies between 0.15 g/cm\(^3\) and 8 g/cm\(^3\) [20]. On average and in calculations 2 g/cm\(^3\) is recommended by [22, 20] while [24, 51] recommend to use 1 g/cm\(^3\). If the mass density is not chosen as a constant, the mass density rules can be used that are given in Table 4.2, adapted from [22, 24, 51]. In [31] a value of 2.5 g/cm\(^3\) is assumed and used for the Grün model.

<table>
<thead>
<tr>
<th>Mass</th>
<th>Mass density</th>
</tr>
</thead>
<tbody>
<tr>
<td>&lt; 10(^{-9})g</td>
<td>2 g/cm(^3)</td>
</tr>
<tr>
<td>10(^{-6}) – 0.01g</td>
<td>1 g/cm(^3)</td>
</tr>
<tr>
<td>&gt; 0.01g</td>
<td>0.5 g/cm(^3)</td>
</tr>
</tbody>
</table>

Table 4.2: The relation between mass density of a micrometeoroid and its size according to [22, 24, 51].

- **Impact velocity**

The impact velocities of the interplanetary micrometeoroids vary a lot in a range from 11–72 km/s [22]. A mean value of 20 km/s is recommended to use [22]. For a worst-case impact event, 72 km/s is used. For the third population of dust in \( L_2 \), an average impact velocity of 10 km/s is assumed in this study, according to [68]. Several velocity distribution functions are defined in the different models. However, here one constant average or worst-case velocity is accepted.

- **Directional distribution**

In Grün’s model, the direction from which the impacting particulates come is isotropic. Nowadays it is known that the particulates come from specific directions [24] but for the estimation of the order of magnitude of the micrometeoroid flux, the isotropic flux model (Grün) is used. Other directional effects are also induced when flying in a Low Earth Orbit (see Appendix D.5).

### 4.2.3 Worst-case scenario for the particulate flux

For a worst-case scenario, two extra fluxes are summed to the flux of the Grün model:
1. A severe micrometeoroid stream
Summing this flux to the Grün model flux, would represent a worst-case that is even more worse than in reality because the Grün model already includes a yearly average of these particulate streams. The Quadrantids are taken as a worst-case stream, that has a maximum flux ratio\textsuperscript{4} of eight [22]. These streams can last for a few hours up to a few days [22].

2. An estimated L\textsubscript{2} dust particulate flux
An estimated flux for the dust population in L\textsubscript{2} is added, assuming that there is such a particulate population in the vicinity of L\textsubscript{2} as described in [68]:
- velocities of a few km/s up to 10 km/s
- the flux and size of the particulates is comparable to the interplanetary flux

Finally, an overview of assumptions of the interplanetary micrometeoroid environment is summarized in Table 4.3.

<table>
<thead>
<tr>
<th>Case</th>
<th>Model</th>
<th>Velocity</th>
<th>Flux ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>Average</td>
<td>Grün:</td>
<td>20 km/s</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>· isotropic</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>· average of streams</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Worst-case</td>
<td>· Grün</td>
<td>20 km/s</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>· stream (Quadrantids)</td>
<td>72 km/s</td>
<td>8</td>
</tr>
<tr>
<td></td>
<td>· local population</td>
<td>10 km/s</td>
<td>1</td>
</tr>
</tbody>
</table>

Table 4.3: The assumptions made in this study for calculating the average and worst-case micrometeoroid flux in L\textsubscript{2}.

4.3 Estimation of the micrometeoroid flux for Darwin
In this Section, the micrometeoroid flux in L\textsubscript{2} is estimated. This flux tells us how frequently Darwin will be hit by a micrometeoroid that alters the OPD more than is allowed (5 nm). The flux is calculated using Grün’s model as described in Section 4.2.1. The assumptions made are summarized as follows:

- The satellite is assumed to be a rigid body. Vibrations of the satellite due to the impact are not taken into account.
- The FEEP\textsubscript{s} are represented as a force, respectively a torque that acts immediately after the micrometeoroid impact took place. The ODL is assumed not to be activated for corrections of the OPD due to an impact.
- The line of action of the FEEP\textsubscript{s} force/torque acts through the center of mass of the spacecraft.
- The satellite is assumed to be a single tube with a flat circular sunshield attached to the bottom.
- The center of mass (c.o.m.) of the satellite is assumed to be on the axis of symmetry of the tube and at 0.75 m from the sunshield (see Appendix D.2).
- The coordinate axes are shown in Figure 4.2. Assumed is that $I_{xx} = I_{yy} \neq I_{zz}$.

\textsuperscript{4}The flux ratio is the ratio of the cumulative flux of the stream versus the average cumulative sporadic flux.
The 3D-problem is approached by 2D-representations of the satellite. This approach is good enough for a general investigation of the order of magnitude of the micrometeoroid flux.

Impacts are assumed to happen perpendicular to the surface of Darwin (no oblique impacts).

Directional effects are not taken into account. The flux is isotropic and on a randomly oriented flat plate, in L₂.

In this study the FEEP have a power of respectively 1 µN and 1 mN. The FEEP torque is assumed 3 µN and 3 mN respectively (arbitrary values). The threshold values of the maximum micrometeoroid mass are based on the pointing and stability requirements for Darwin. The maximum OPD that may occur is 5 nm for the lateral motion between hub and free flyer and the transverse motion parallel to the z-axis of the satellites, and 70 µm for the transverse motion perpendicular to the free flyer-hub axis and in the plane of the sunshield. The rotations are restricted to 8.4 mas (see Figure 4.2). In these calculations it is supposed that one satellite (e.g. the hub) is hanging still with respect to the other one that is being struck by a particulate (the free flyer). The method used is adapted from the one described in [37].

![Figure 4.2: The requirements for Darwin.](image)

### 4.3.1 Translation

Three possibilities exist for translation, as drawn in Figure 4.3. The first case is when the micrometeoroid strikes the satellite sunshield, perpendicular to the surface. In this case the satellite will get a transverse motion with respect to the hub (and a rotational motion too, discussed in Section 4.3.2). The maximum allowable deviation of the satellite along the z-axis is 5 nm with respect to its nominal position. In the second case the satellite is struck on the tube, where the line of action of the impact crosses the z-axis of the telescope tube and acts parallel to the free flyer-hub direction. For this case the maximum allowable deviation is also 5 nm. In the third case, the micrometeoroid strikes the tube in the same way as before, but perpendicular to the free flyer-hub direction. For this motion a maximum deviation of 70 µm exists.
Figure 4.3: The three cases for translation after a micrometeoroid impact on the Darwin free flyers.

The deviation of the spacecraft from its nominal position in case of translation is [37]:

\[ x(t) = \frac{1}{m_{sc}} \int_0^t (e \cdot m v)_{imp} - F_T s \, ds \]  

(4.4)

where \( x \) is the deviation of the spacecraft, \( F_T \) is the FEEP thrust force immediately after impact, \( t \) is the time and \( (mv)_{imp} \) is the momentum of the impacting particulate. \( e \) is the momentum enhancement factor. This factor ranges from 2 to 10, and has an average of 5 [57]. The derivation of Equation 4.4 is given in Appendix D.3.

\[ x(t) \] is a parabolic function that has a maximum at [37]:

\[ x_{\text{max}} = \frac{1}{m_{sc}} \frac{(e \cdot m v)_{imp}^2}{2 F_T} \]  

(4.5)

The threshold value for the meteoroid impact mass that will give rise to a maximum displacement of the satellite \( x_{\text{max}} \) is then [37]:

\[ m_{\text{thresh}} = \frac{1}{e \cdot v_{imp}} \sqrt{2 m_{sc} x_{\text{max}} F_T} \]  

(4.6)

The flux is calculated for the three translation cases for different values of the FEEP thrusters. The results for the average micrometeoroid conditions are shown in Table 4.4.

The mass of the spacecraft is assumed to be 500 kg. First, the threshold value for the mass \( (m_{\text{thresh}}) \) is calculated, based on the OPD-requirements for cases 1, 2 and 3. The first two cases have the same requirement of 5 nm and are treated together. The third case is treated briefly, because the strictest requirements are most important, and these are valid for the first and second case, which are hereby considered as the most important ones. Also the satellite velocity after impact is calculated. This is done using:

\[ v_{sc} = \frac{(mv)_{imp} e}{m_{sc}} \]  

(4.7)

Using Grün, the micrometeoroid flux is calculated for masses equal to or bigger than the threshold value. In order to calculate the amount of times per year that one free flyer satellite is hit by a micrometeoroid large enough to disturb the interferometer observations, the fluxes of the first and second case (Z- and X-direction) are multiplied with twice the surface area of the sunshield (totally 77 m²). In this manner the micrometeoroid impacts on the satellite from an angle of \( 4\pi \) steradian are covered. The flux of the third case (Y-direction) is multiplied with the surface of the telescope tube (9.4 m²). Table 4.5 shows the impacts per year, for the three micrometeoroid populations apart, that form together the worst-case.
They are calculated in the same way as the average case, taking into account the assumptions of Table 4.3. Also the total amount of impacts per day is given, for the worst-case.

<table>
<thead>
<tr>
<th>( F_T \ [N] )</th>
<th>( m_{\text{thresh}} \ [g] )</th>
<th>( v_{sc} \ [m/s] )</th>
<th>Flux ( \phi ) in ( L_2 ) ([# \text{m}^{-2} \text{yr}^{-1}])</th>
<th>Amount of impacts per year with ( \text{OPD} \geq 5 \text{ nm} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Case 1,2 ( x_{\text{max}} = 5 \cdot 10^{-9} )</td>
<td>( 10^{-3} )</td>
<td>( 7 \cdot 10^{-7} )</td>
<td>( 1.4 \cdot 10^{-7} )</td>
<td>( 2 )</td>
</tr>
<tr>
<td></td>
<td>( 10^{-4} )</td>
<td>( 2 \cdot 10^{-7} )</td>
<td>( 4.5 \cdot 10^{-8} )</td>
<td>( 5 )</td>
</tr>
<tr>
<td></td>
<td>( 10^{-5} )</td>
<td>( 7 \cdot 10^{-8} )</td>
<td>( 1.4 \cdot 10^{-8} )</td>
<td>( 12 )</td>
</tr>
<tr>
<td></td>
<td>( 10^{-6} )</td>
<td>( 2 \cdot 10^{-8} )</td>
<td>( 4.5 \cdot 10^{-9} )</td>
<td>( 24 )</td>
</tr>
<tr>
<td>Case 3 ( x_{\text{max}} = 70 \cdot 10^{-6} )</td>
<td>( 10^{-3} )</td>
<td>( 8 \cdot 10^{-8} )</td>
<td>( 1.7 \cdot 10^{-5} )</td>
<td>( 0.01 )</td>
</tr>
<tr>
<td></td>
<td>( 10^{-4} )</td>
<td>( 2 \cdot 10^{-8} )</td>
<td>( 5.3 \cdot 10^{-7} )</td>
<td>( 0.6 )</td>
</tr>
</tbody>
</table>

Table 4.4: The amount of impacts on Darwin, per year and per free flyer satellite, for average micrometeoroid environment conditions - translation.

| \( F_T \ [N] \) | Grü"un \[# per year\] | Quadrantids \[# per year\] | Local dust population \[# per year\] | Total amount of impacts per day |
|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|
| Case 1,2 \( x_{\text{max}} = 5 \cdot 10^{-9} \) | \( 10^{-3} \) | \( 150 \) | \( 3500 \) | \( 80 \) | \( 10 \) |
| | \( 10^{-4} \) | \( 400 \) | \( 8000 \) | \( 230 \) | \( 24 \) |
| | \( 10^{-5} \) | \( 900 \) | \( 16000 \) | \( 560 \) | \( 48 \) |
| | \( 10^{-6} \) | \( 1850 \) | \( 28300 \) | \( 1220 \) | \( 86 \) |
| Case 3 \( x_{\text{max}} = 70 \cdot 10^{-6} \) | \( 10^{-3} \) | \( 0.1 \) | \( 35 \) | \( 0.05 \) | \( 0.1 \) |
| | \( 10^{-4} \) | \( 5 \) | \( 1200 \) | \( 3 \) | \( 3 \) |

Table 4.5: The total amount of impacts on Darwin, per day and per free flyer satellite, in a worst-case situation (meteoroid stream) - translation.

From Tables 4.4 and 4.5 it can be concluded that if mN FEEPs were used to counteract the impacts, an average of one impact in two days is achieved for one satellite that would disturb the OPD by more than 5 nm. This number depends on assumptions like the momentum enhancement factor and the Grü"un model assumptions. However, the FEEP's thrust force after impact is the largest influence. For smaller thrust forces, the amount of impacts becomes larger, up to 5 per day for \( \mu \)N FEEP's (for one satellite). In a worst-case, during a micrometeoroid stream, the amount of impacts can rise up to 10 per day for the mN FEEPs and above 80 per day for the \( \mu \)N FEEP's for one satellite. The amount of impacts from the local dust population in \( L_2 \) is only about half as much as for the average case. For the total amount of micrometeoroid impacts, these numbers should be multiplied by 7 (the sum of the surfaces of 6 free flyers and an approached surface of the hub, times the flux).
4.3.2 Rotation

Analogue to the previous case, the angular deviation of the spacecraft is given by

$$\theta(t) = \frac{1}{I_{G,sc}} \int_0^t (e \cdot d \cdot mv)_{imp} - M_T s \, ds$$  \hspace{1cm} (4.8)

where $\theta$ is the angular deviation of the spacecraft, $M_T$ is the torque provided by the FEEP thrusters, $t$ is the time, $d$ is the distance from the line of action of the impact to the center of mass of the satellite, $(mv)_{imp}$ is the micrometeoroid impact momentum and $e$ is the momentum enhancement factor. $\theta(t)$ is also a parabolic function with a maximum at:

$$\theta_{max} = \frac{m_{sc}}{2I_{G,sc}M_T} (e \cdot d \cdot mv)_{imp}^2$$  \hspace{1cm} (4.9)

The minimum particulate mass that will give rise to a rotation of the satellite that is bigger than a certain maximum angle $\theta_{max}$ is:

$$m_{thresh} = \frac{1}{e \cdot d \cdot v_{imp}} \sqrt{2I_{G,sc}\theta_{max}M_T}$$  \hspace{1cm} (4.10)

A study is made for a maximum angular deviation of $\theta_{max} = 8.4$ mas (OPD requirement for rotation). The moments of inertia are estimated in Appendix D.2. These are:

$I_{xx} = I_{yy} = 268$ kgm$^2$
$I_{zz} = 415$ kgm$^2$

Two cases are presented: the first one with an impact on and perpendicular to the sunshield, but with its line of action not going through the center of mass of the satellite. In the second case the impact happens on the tube, where the line of action of the impact does not cross the center of mass of the satellite, but still crosses the axis of symmetry so that no rotation effects about the z-axis are induced (2D-case).

In the case of rotation, we cannot just multiply the spacecraft surface with the accumulated flux as is done for the translations. This is because the deviation of the satellite for a particular micrometeoroid mass, not only depends on its impact momentum, but also on the perpendicular distance $d$ from the place of impact to the axis of rotation.

**Impact on the sunshield**

In case of a micrometeoroid impact on the sunshield, the method to apply is to split the sunshield in infinitesimally small rings. For each ring with a certain radius $r$, the corresponding threshold value for the micrometeoroid mass should be calculated as well as the associated micrometeoroid flux. These fluxes should be multiplied by their accompanying ring surface $(2\pi r \, dr)$ and summed over the total sunshield. In a mathematical formulation this is:

$$\Phi_{tot} = \int_0^R \phi(r) \cdot 2\pi r \, dr$$  \hspace{1cm} (4.11)

where $\Phi_{tot}$ is the total amount of particulate impacts per year on the sunshield that cause a deviation of the satellite of more than $\theta_{max}$ (8.4 mas). $R$ is the maximum sunshield radius (3.5 m), $\phi(r)$ is the particulate flux for micrometeoroids that alter the spacecraft more than 8.4 mas, at a certain radius from the center of the sunshield, and $r$ is the (variable) radius on that sunshield.

The next step is to derive an analytical expression for $\Phi_{tot}$ and to calculate it. In the following derivation of $\Phi_{tot}$, the perpendicular distance $d$ between the place of impact and the center of mass of the satellite is equal to the parameter $r$. The threshold mass $m_{thresh}$ is inversely proportional to the perpendicular distance $r$ from the center of mass of the satellite:

$$m_{thresh} = \frac{c}{r}$$  \hspace{1cm} (4.12)
where \( c \) depends on the momentum enhancement factor, the mass moment of inertia, the FEEPs thrust torque used for correction after impact, and the maximum allowed angle \( \theta_{\text{max}} \) (see Equation 4.10). These threshold masses are calculated and all lie in a range between \( 10^{-6} \) and \( 10^{-8} \) g. The Grün equation (see Figure 4.1) is approximated in the range of \( m = 10^{-6} - 10^{-8} \) by a linear (upper-limit) curve on a double-logarithmic scale. The equation of the Grün approximation is:

\[
\log(\phi) = -\frac{14}{18} \log(m) - 4.3 \tag{4.13}
\]

or:

\[
\phi = 10^{-4.3} \cdot e^{-\frac{7}{9} \log(c/r)} - 4.3 
\]

or:

\[
\phi = 10^{-4.3} \cdot e^{-\frac{7}{9} \log(c/r)} \cdot \frac{9}{25} \cdot R^{\frac{16}{9}}(4.15)
\]

Filling in Equation 4.15 in Equation 4.11 yields for a sunshield with diameter of \( R \):

\[
\Phi = 2 \pi c^{-\frac{7}{9}} 10^{-4.3} \cdot \frac{9}{25} \cdot R^{\frac{16}{9}}(4.16)
\]

This equation yields the amount of impacts on one side of the sunshield. For the total amount of disturbances from impacts on the sunshield, this amount must be doubled. For the total amount of disturbances on 6 free flyers and the hub, it needs to be multiplied by 7.

**Impact on the telescope tube**

For impacts on the telescope tube, the same method is used except that Equation 4.11 is replaced for the telescope tube by:

\[
\Phi_{\text{tot}} = \int_0^R \phi(r) \cdot 2a \, dr \tag{4.17}
\]

where \( R \) is the distance from the center of mass to the side of the tube. \( r \) is the distance from the place of impact to the center of mass of the satellite, and \( a \) is the diameter of the telescope tube (0.75 m). Calculating this integral yields:

\[
\Phi_{\text{tot}} = 2a c^{-\frac{7}{9}} 10^{-4.3} \cdot \frac{9}{16} \cdot R^{\frac{16}{9}}(4.18)
\]

Equation 4.18 needs to be doubled for a flux on the tube from both sides. Because the center of mass of the free flyer is offset with respect to the tube (See Appendix D.2), Equation 4.18 needs to be calculated twice and then summed together (once for the integral of 0 towards the sunshield (0.75 m) and once for towards the other side (1.25 m)). The results of the calculations are listed in Table 4.6 for the average micrometeoroid conditions and for the worst-case. The total fluxes are given for one free-flyer satellite. These numbers should be multiplied according to the amount of satellites considered. The amount of impacts for the rotations have the same order of magnitude as for the translation cases. The difference is approximately a factor 2. Also the effects of the rotations from particulate impacts, on the performance of Darwin, should be looked at during the Darwin study.
<table>
<thead>
<tr>
<th>$T_T$ [Nm]</th>
<th>Grün [# per year]</th>
<th>Quadrantids [# per year]</th>
<th>Local dust population [# per year]</th>
<th>Total amount of impacts per day</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sunshield</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$3 \cdot 10^{-5}$</td>
<td>80</td>
<td>1750</td>
<td>45</td>
<td>5</td>
</tr>
<tr>
<td>$3 \cdot 10^{-4}$</td>
<td>200</td>
<td>4250</td>
<td>115</td>
<td>13</td>
</tr>
<tr>
<td>$3 \cdot 10^{-3}$</td>
<td>500</td>
<td>10500</td>
<td>280</td>
<td>30</td>
</tr>
<tr>
<td>$3 \cdot 10^{-2}$</td>
<td>1200</td>
<td>25000</td>
<td>700</td>
<td>74</td>
</tr>
<tr>
<td>Tube</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$3 \cdot 10^{-5}$</td>
<td>2</td>
<td>35</td>
<td>1</td>
<td>0.1</td>
</tr>
<tr>
<td>$3 \cdot 10^{-4}$</td>
<td>4</td>
<td>85</td>
<td>3</td>
<td>0.3</td>
</tr>
<tr>
<td>$3 \cdot 10^{-3}$</td>
<td>10</td>
<td>210</td>
<td>5</td>
<td>0.6</td>
</tr>
<tr>
<td>$3 \cdot 10^{-2}$</td>
<td>25</td>
<td>520</td>
<td>15</td>
<td>1.5</td>
</tr>
</tbody>
</table>

Table 4.6: The total amount of impacts, per day and per satellite, in a worst-case situation (meteoroid stream) - rotation.

4.4 Conclusions

The micrometeoroid environment for L2 is assessed in this Section. The amount of micrometeoroids impacting on Darwin is predicted using an interplanetary micrometeoroid model (Grün, [31]). It is assumed that the FEEPs are turned on immediately after impact in order to counteract the effect of the impact on the OPD. Depending on the thrust force of the FEEPs, the minimum value of the mass of the impacting micrometeoroid is determined that causes disturbance of more than 5 nm. Consequently the flux associated with these particulates and particulates with a bigger mass is determined.

The following conclusions are drawn:

- **Impact count**
  In average micrometeoroid conditions, the Darwin constellation is expected to be displaced temporarily by a micrometeoroid impact for about 35 times a day if assumed that the impact is immediately counteracted by the $\mu$N FEEPs. Even if the mN FEEPs would have been used, this impact count is still about 3 a day. In a worst-case condition, taking into account micrometeoroid streams and the local dust population in L2, the number of impacts causing a displacement of 5 nm using mN FEEPs to counteract them, increases to an amount of 10 impacts a day. These numbers are high, especially when recalling the observation times being 6.4 and 125 hours for the planet detection and spectrometry respectively. This means that if no measures are taken, the observations could be disturbed regularly, depending on how large the effect is of one such impact on the observations.

- **Rotations**
  The same calculations are done for the rotational motion of the satellites, with 8.4 mas as the requirement. The results of the rotation have the same order of magnitude as for the translation.

- **mN FEEPs**
  Depending on the effect of the micrometeoroid impacts on the optical performance of Darwin (which is considerable), the possibility exists that it will be necessary to deploy the mN FEEPs for correcting the disturbances instead of the $\mu$N FEEPs. These were
intentionally not meant to be used during the observations, and can introduce extra
disturbances on Darwin in the classification of “internal disturbance forces”. Their
force spectral densities should be investigated as was done in Section 2 for the $\mu N$
FEEPs. Also the design of the OPD control system, using mN FEEPs as well as $\mu N$
FEEPs and the ODL should not be underestimated if the mN FEEPs turn out to be
a necessity.

- Model accuracy

On the one hand, the 5 nm limit is not a “hard” limit above which interferometer
operations would fail, but a $1\sigma$-value of the OPD requirement. This means that in
time, 68.3% of the time, the OPD should remain below the 5 nm limit and not
in 100% of the time as assumed in our calculations. Therefore the estimation of the
amount of particulates that could possibly disturb the interferometer observations is
rather “pessimistic” because the 5 nm limit does not directly determine the failure or
success of the observations. On the other hand, the assumption that the FEEPs are
activated immediately after the impact, is an “optimistic” estimation of the impact
count. The question whether the impacts can disturb the interferometer observations
will depend on the total amount of time that one of the satellites is displaced signif-
icantly. This integrated time should be less than a fraction of the nulling depth ($\frac{1}{2}$
times the observation time. A more detailed study is necessary to calculate the real
effect of the impacts on the observations. The order of magnitude of the amount of
impacts (based on the $1\sigma$-limit) indicates that the effect of micrometeoroid impacts
should certainly be taken into account in the design of the control system of Darwin.

- Velocity and control frequency

When a micrometeoroid impacts on a Darwin satellite, the satellite will move with a
certain speed. This speed can reach values of 140 nm/s at impact frequencies on Dar-
win of three times a day (the mN FEEPs case in average micrometeoroid conditions).
This means that if no correcting measures are taken, that Darwin would be displaced
by 5 nm in only 0.03 sec. Therefore, the time between impact of the micrometeoroid
and the response of the on the FEEPs should be very small, leading to a high sample
frequency of the control loop. The response of the satellite (and thus the OPD) on
the FEEPs force is much “slower” than in case of the ODL. In other words, also the
control bandwidth needs to be high enough (at least 30 Hz), in order to correct for the
OPD in time. This can be achieved with a combination of FEEP and ODL control,
using a high sample frequency of at least 100 Hz and an appropriate design of the
ODL control, leading to a high control bandwidth. The design of such a control is not
straightforward and needs to be simulated in case the micrometeoroids do disturb the
observations.

From this study it is concluded that the order of magnitude of the micrometeoroid impacts
possibly implies disturbances in the observations. It is recommended to perform a thorough
study in which the effect of these micrometeoroids on the observations is revealed. If the
observations are disturbed, measures have to be taken, and a OPD control using both the
FEEPs and ODL simultaneously should be simulated and studied in detail. Also the use of
mN FEEPs can possibly become a necessity.
Chapter 5

The external disturbance forces for FFDEM

5.1 Introduction

In this Chapter the order of magnitude of the high-frequent disturbance forces\(^1\) in a Low Earth Orbit is determined. This is done in order to know whether or not a preparatory interferometry demonstration mission in a Low Earth Orbit is feasible and what frequency of the ODL would be reasonable.

The question of how big these disturbance forces are, is answered by analyzing the data from the CHAMP and GRACE accelerometers. These accelerometers measure the non-gravitational accelerations of the satellites. The approach used is top-down: instead of building a model for each disturbance force and calculating the effect of them on the satellite, the total accelerations are measured and analyzed. They are not split-up into different sources of disturbance forces.

The main characteristics of the Formation Flying Demonstration Mission (FFDEM) have been summarized in Section 1.5. Because the CHAMP and GRACE accelerometers do not yield information on the magnitude of the high-frequent gravitational forces, a consideration about these is given in Section 5.2, based on a theoretical Earth gravity model, called “OSU91”. Section 5.3 describes the CHAMP and GRACE mission characteristics, as well as the data used for the analysis of the non-gravitational forces. The data analysis objectives are explained in Section 5.4 while the results of the analysis are given in Section 5.5.

5.2 The gravitational forces for FFDEM

This Section gives a consideration of the high-frequency part of the gravitational forces for the Formation Flying Demonstration Mission (FFDEM). It is based on the model explained in Section 3.6. The low-frequent gravitational forces like planetary perturbations and relativity corrections are omitted because this study focuses on the high-frequency content of the disturbances. The only gravitational force that might have a high-frequency content, is the gravitational force due to non-uniformity of the Earth’s mass distribution. The consideration given in this Section is thus only focused on this kind of gravitational forces.

The gravity model used is called “OSU91” [63]. It is a model of the gravity potential up to order and degree 360 [75], which corresponds to a resolution of 55 km [22]. With a satellite velocity of 7.725 km/s at a height of 300 km, this transforms into a maximum

\(^1\)The bandwidth of interest is between \(10^0\) and \(10^4\) Hz.
disturbance frequency of 0.14 Hz, equivalent to a time period of 7 sec). It seems impossible
to extract information about the high-frequency component of the gravitational force using
this model. Still a notion of the magnitude of these high-frequent gravitational forces can
be acquired by extrapolation.

A frequency spectrum has been calculated in [75] using OSU91, for a satellite flying in
a polar Low Earth Orbit at 300 km height. The orbit disturbances are given in centimetres
(cm) and the frequency in cycles per revolution\(^2\) (cpr), for each direction. This spectrum
is shown in Figure 5.1, for the radial direction (top figure), the along-track (middle figure)
and cross-track direction (lower figure).

Figure 5.2 shows an approximation of the radial orbit perturbations spectrum on a semi-
logarithmic scale, in analogy with Figure 5.1. Plotting this graph on a double logarithmic
scale yields Figure 5.3. From the amplitude spectrum, a PSD of the displacements and a
DSD have been constructed. The latter is shown in Figure 5.4. From the DSD, the cumu-
lative RMS is calculated. Figure 5.5 shows that the RMS-value of the displacements due to
gravitational forces has a magnitude of about \(2 \cdot 10^{-4}\) nm in the bandwidth of \(10^0 - 10^3\) Hz,
if accepting that the data are extrapolated to higher frequencies.

Figure 5.1 is a worst-case, because the satellite in the model is flying at 300 km, while
FFDEM is assumed to fly at 561 km (initial height). The higher the orbit, the less the
higher gravitational harmonics are “felt” by the satellite. Hereby it is concluded that the
gravitational disturbance forces do not cause a significant high-frequent disturbance in the
orbit of satellites in LEO. Remark that the absolute movement of the satellite is looked
at here. For the relative movement, the quadratic sum of the RMS-value is taken for two
satellites, which yields a total OPD RMS-value of \(2.8 \cdot 10^{-4}\) nm.

\(^2\)The cycles per revolution can be transformed into Herz by dividing it by the orbital period in seconds
(5431 sec in this case).
Figure 5.1: Spectrum of the gravitational orbit perturbations for a satellite in LEO (polar orbit, 300 km), from [75].
Figure 5.2: Approximation of the disturbance spectrum of Figure 5.1 (radial direction), plotted on a semi-logarithmic scale.

Figure 5.3: The disturbance spectrum of Figure 5.2, plotted on a log-log-scale.
Figure 5.4: The Displacement Spectral Density of the disturbance spectrum of Figure 5.1 (radial direction).

Figure 5.5: The cumulative RMS, deduced from Figure 5.4. At $10^0$ Hz (RMS-value between $10^0 - 10^3$ Hz) the graph indicates an RMS-value of $2 \cdot 10^{-4}$ nm for one satellite.
5.3 The GRACE and CHAMP data

5.3.1 The CHAMP mission

CHAMP (Challenging Mini-satellite Payload) is a German\textsuperscript{3} geoscience mission with as goal to model the gravity field of the Earth, and the Earth’s magnetic field. The satellite was launched on July 15\textsuperscript{th}, 2000, and was designed to fly for a period of at least five years. Its orbit is near-circular with an eccentricity of 0.003 [9]), and near-polar with an inclination of 87.3\textdegree [65]. The altitude of the orbit after launch is 454 km, but the satellite will fly at approximately 200 km at the end of its lifetime due to the atmospheric drag force [9]. The satellite has a weight of 522 kg and a length of about 8 m. Half of this length is taken by the boom (see Figure 5.6). CHAMP’s orbit is tracked using the Global Positioning System (GPS) and Satellite Laser Ranging from Earth. In order to determine the Earth’s gravity field accurately, an accelerometer measures the non-gravitational accelerations of the satellite. This is done by measuring the accelerations of the satellite with respect to a proof-mass inside of it. The accelerometer adds information to the models of e.g. the atmospheric density and radiation pressure that have large uncertainties. CHAMP’s accelerometer is the first one brought into orbit with such a high resolution (better than $10^{-8}$ m/s\textsuperscript{2} [52]). Its sample frequency is 1 Hz.

![Image of CHAMP satellite](image.png)

Figure 5.6: An illustration of the CHAMP satellite, from [88].

5.3.2 The GRACE mission

The Gravity Recovery and Climate Experiment (GRACE) Mission is launched in order to determine an accurate model of the Earth’s gravity field and its changes in time. The mission was launched on March 17\textsuperscript{th} 2002, has a lifetime of five years and is operated by NASA and DLR\textsuperscript{4} [14]. It consists of two satellites, sometimes ironically called “Tom & Jerry”, chasing each other with a distance of 220 km between them. They are identical satellites (with exception of the radio frequencies), flying in the same plane (“co-planar orbits”). The inter-satellite distance can change by up to 50 km, after which stationkeeping manoeuvres are needed, every 30 to 60 days [14]. The satellites are flying in a Low Earth Orbit, at 500 km height at the beginning of the mission, and 300 km at the end of the mission. The

\textsuperscript{3}CHAMP is on watch by the GeoForschungsZentrum Potsdam (GFZ), Germany

\textsuperscript{4}Deutsches Zentrum für Luft- und Raumfahrt, Germany
orbit is a near-polar orbit with an inclination of 89° and an eccentricity of 0.005 [14]. The determination of the gravity field is done by means of inter-satellite range and range-rate measurements. Use is made of a microwave tracking system, on-board GPS-equipment and an accelerometer. The accelerometer is an updated version of the CHAMP accelerometer. It has a resolution of better than 10^{-9} m/s^2 and a sample frequency of 10 Hz. The data from this accelerometer is used for building a frequency spectrum of the orbit disturbances in LEO, in this study. The CHAMP data are used as a cross-check for these. Figure 5.7 shows an illustration of the twin satellites, from [90]. Table 5.1 gives an overview of the main characteristics of the CHAMP and GRACE satellites.

![GRACE twin satellites](image)

Figure 5.7: An illustration of the GRACE twin satellites, from [90]. The real distance between the two satellites is about 220 km.

<table>
<thead>
<tr>
<th>Orbit</th>
<th>CHAMP</th>
<th>GRACE</th>
</tr>
</thead>
<tbody>
<tr>
<td>height</td>
<td>454 km (BOL), 200 km (EOL)</td>
<td>500 km (BOL), 300 km (EOL)</td>
</tr>
<tr>
<td>inclination</td>
<td>87.3°</td>
<td>89°</td>
</tr>
<tr>
<td>eccentricity</td>
<td>0.003</td>
<td>0.005</td>
</tr>
<tr>
<td>Mass per satellite</td>
<td>522 kg</td>
<td>487 kg</td>
</tr>
</tbody>
</table>

Table 5.1: The main characteristics of the CHAMP and GRACE satellites.

**5.3.3 The STAR accelerometers**

The accelerometers used for CHAMP and GRACE were developed by ONERA\(^5\). The STAR (“Space Three-axis Accelerometer for Research”) accelerometer used for CHAMP consists of a proof mass that floats in a cage. It is suspended by electrostatic forces. The non-gravitational accelerations of the satellite are measured by determining the proof mass position and rotation with respect to its caging. The GRACE mission is equipped with its follow-up accelerometer SuperSTAR that surpasses STAR in resolution and sample frequency. The characteristics of both accelerometers are listed in Table 5.2.

It must be stressed that for the CHAMP accelerometer, the measurements in the radial direction are not reliable due to a hardware problem.

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\(^5\)Office National d’Etudes et de Recherches Aerospatiales, France
The coordinate axes for CHAMP are defined as depicted in Figure 5.8, from [9]. It shows the accelerometer reference frame \( (acc) \) and spacecraft reference frame \( (s/c) \).

The definition of the coordinate axes for GRACE are shown in Figure 5.9. Shown is the accelerometer reference frame \( (acc) \), the spacecraft reference frame \( (s/c) \) and the science reference frame \( (srf) \). In reality the spacecraft reference frame is shifted by 0.1 mm of the accelerometer reference frame, but for this study the difference is of no importance. The \( X_{s/c} \)-axis is in the flight direction for the trailing satellite, while it is in the opposite direction for the leading satellite. The two X-axes thus point to each other. This is because the axes are defined with respect to the GRACE geometry and because the two identical satellites are turned towards and “looking” at each other.
Table 5.2: Specifications of the accelerometers of CHAMP and GRACE, in a bandwidth of $10^{-4} - 10^{-1}$ Hz for CHAMP and $5 \cdot 10^{-5} - 10^{-1}$ Hz for GRACE [9, 65, 28, 34].

<table>
<thead>
<tr>
<th></th>
<th>STEP CHAMP</th>
<th>STEP GRACE</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Sample frequency</strong></td>
<td>1 Hz (raw data)</td>
<td>10 Hz (raw data)</td>
</tr>
<tr>
<td><strong>Measurement range</strong></td>
<td>0.1 Hz (Level 1B data)</td>
<td>1 Hz (Level 1B data)</td>
</tr>
<tr>
<td>Radial</td>
<td>$\pm 10^{-4}$ m/s$^2$</td>
<td>$\pm 5 \cdot 10^{-5}$ m/s$^2$</td>
</tr>
<tr>
<td>Along-track</td>
<td>$\pm 10^{-4}$ m/s$^2$</td>
<td>$\pm 5 \cdot 10^{-5}$ m/s$^2$</td>
</tr>
<tr>
<td>Cross-track</td>
<td>$\pm 10^{-3}$ m/s$^2$</td>
<td>$\pm 5 \cdot 10^{-4}$ m/s$^2$</td>
</tr>
<tr>
<td>Yaw</td>
<td>$\pm 10^{-3}$ rad/s$^2$</td>
<td>$\pm 1 \cdot 10^{-2}$ rad/s$^2$</td>
</tr>
<tr>
<td>Roll</td>
<td>$\pm 10^{-3}$ rad/s$^2$</td>
<td>$\pm 1 \cdot 10^{-2}$ rad/s$^2$</td>
</tr>
<tr>
<td>Pitch</td>
<td>$\pm 10^{-3}$ rad/s$^2$</td>
<td>$\pm 1 \cdot 10^{-3}$ rad/s$^2$</td>
</tr>
<tr>
<td><strong>Resolution</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Radial</td>
<td>$3 \cdot 10^{-8}$ m/s$^2$/√Hz</td>
<td>$10^{-10}$ m/s$^2$/√Hz</td>
</tr>
<tr>
<td>Along-track</td>
<td>$3 \cdot 10^{-9}$ m/s$^2$/√Hz</td>
<td>$10^{-10}$ m/s$^2$/√Hz</td>
</tr>
<tr>
<td>Cross-track</td>
<td>$3 \cdot 10^{-9}$ m/s$^2$/√Hz</td>
<td>$10^{-9}$ m/s$^2$/√Hz</td>
</tr>
<tr>
<td>Yaw</td>
<td>$1 \cdot 10^{-7}$ rad/s$^2$/√Hz</td>
<td>$5 \cdot 10^{-6}$ rad/s$^2$/√Hz</td>
</tr>
<tr>
<td>Roll</td>
<td>$5 \cdot 10^{-7}$ rad/s$^2$/√Hz</td>
<td>$5 \cdot 10^{-6}$ rad/s$^2$/√Hz</td>
</tr>
<tr>
<td>Pitch</td>
<td>$5 \cdot 10^{-7}$ rad/s$^2$/√Hz</td>
<td>$2 \cdot 10^{-7}$ rad/s$^2$/√Hz</td>
</tr>
</tbody>
</table>

5.3.4 Level 1B data

The accelerometer data of GRACE are obtained via podaac.jpl.nasa.gov where the GRACE Level 1B and Level 2 data are publicly available. The most important differences between the data levels are listed in Table 5.3. For the purpose of this study the best analysis would be to perform the analysis for the Level 1A and Level 1B data simultaneously, where the Level 1A data would manually have to be processed further towards Level 1B, except for the downsampling to 1 Hz and for the filtering at 0.035 Hz by a so-called “CRN-filter” [34], such that the high-frequent features above 0.035 Hz are also visible. During this study a procedure of obtaining the Level 1A data was started, but no time was left in this study to finish the procedure and to perform a thorough analysis on this data. Therefore this study is based on the Level 1B data and a cross-check through the CHAMP Level 1B data.

The data used from CHAMP are the Level 2 data, available from the GeoForschungsZentrum Potsdam (GFZ)\(^6\). These data have a sample rate of 0.1 Hz (the original raw data sample rate is 1 Hz) and are not filtered at 0.035 Hz. The CHAMP data are used to verify the results of the GRACE data. In this way two independent datasets from two independent missions are examined for cross-checking. The reason why the GRACE data were chosen as the data on which this analysis is based on, is the higher sample rate and a better accuracy. The filtering of the data was discovered after seeing the results of the Fourier transform.

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\(^6\)http://isdc.gfz-potsdam.de/champ/
| Level 0 | - Raw data  
| Sample rate: 10 Hz |
| Level 1A | - Non-destructive processing of Level 0 data  
| - Sensor calibration factors applied  
| - Binary measurements to engineering units  
| - Time corrections where necessary  
| - Quality flags added  
| Sample rate: 10 Hz |
| Level 1B | - Irreversible processing of Level 1A data  
| - Sample rate reduced  
| - Publicly available to the scientific community  
| - Filtering at 0.035 Hz(!)  
| Sample rate: 1 Hz |
| Level 2 | - Include orbits of GRACE  
| - Harmonic coefficients for the Earth gravitational potential  
| - Publicly available to the scientific community  
| Sample rate: 1 Hz |

Table 5.3: Description of the GRACE data sets, from [14, 79].

5.4 Analysis objectives

- **Establish a representative ASD/DSD for the given dataset**
  A Fourier transform is made for one day of accelerometer data, from which a Power Spectral Density (PSD), Acceleration Spectral Density (ASD) and Displacement Spectral Density (DSD) are derived. When taking the mean of several such Fourier transforms, the noise level reduces and the signal becomes clearer. An analysis is made for one day and the average of 15 days in order to demonstrate the decay in noise with the amount of days analyzed. The dates used are 01-08-2002 and the period of 01-08-2002 until 15-08-2002.

- **Separate AOCS and orbit maintenance spectrum from external disturbances**
  In the time-series of the GRACE data there are many spurious spikes, especially in the cross-track direction. These pulses are among other things the result of the thruster activation for AOCS and orbit maintenance manoeuvres\(^7\). They have an influence on the result of the Fourier analysis. The data were filtered for these spikes and the difference between the filtered and non-filtered data is shown. This is done for the data from 01-08-2002 until 15-08-2002 (an average of 15 days).

- **Investigate the effect of the solar variability**
  It is known that through sudden solar events, satellites can suffer drastically from an increase of atmospheric drag (Section 3.5.2). The question is if and how the solar variability influences the acquired spectra. Therefore a period with high solar activity is compared to a “quiet” period just before. The selected period is the period between 18-10-2003 and 5-11-2003, where an amount of 140 solar flares occurred in only 18 days [78].

Also a 15-days run is made of the 2002-data (01-08-2002 until 15-08-2002) as well as the 2004 data (15-06-2004 until 30-06-2004) in order to reveal the influence of the 11-year Solar cycle. The data from 2002 are the closest data to the Solar Maximum\(^8\)

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\(^7\)See Appendix E.2  
\(^8\)The Solar Maximum occurred near 2001.
available. The data from 2004 are the most recent data available and thus furthest away from the Solar Maximum.

Apart from the difference in Solar activity, also the influence of the altitude of the orbit is visible. In 2004 the satellite is flying lower than in 2002 due to atmospheric drag. This results in a different drag force and thus a different Fourier spectrum. From this case and the case in which an “active” period of the Sun was compared to the “quiet” period just before, a conclusion is drawn on both the influence of solar variability and the orbit altitude, on the spectrum.

- **Cross-validate data**
  In order to check the GRACE results, the same kind of analysis has been made from accelerometer data acquired by the CHAMP satellite. The case done is the average of 15 days from 01-08-2002 until 15-08-2002 without thruster filtering.

- **Determine a worst-case scenario for the RMS-calculation**
  From the previous cases, a worst-case scenario is chosen to determine the maximum Displacement Spectral Density that can occur. From this spectrum the RMS-value is calculated for a bandwidth of $10^0 - 10^3$ Hz.

### 5.5 Data analysis results

This Section presents the results of the data analysis that is performed. The method used for analysis is described in Appendix E.1. The results are scaled down to a satellite with a mass of 120 kg in accordance with the FFDEM satellite masses. The results are not corrected for the cross-section surface of FFDEM which implies that the surface $A$ is assumed to be the same as for GRACE and CHAMP. In all cases the GRACE A data were used (satellite A of the GRACE mission). The result of the Fourier transformation was checked using Parseval’s theorem (see Appendix E.1), for GRACE as well as CHAMP.

#### 5.5.1 General result

Figure 5.10 (left) shows the mean ASD for 15 days of data from 01-08-2002 until 15-08-2002, for the cross-track direction. The thruster spikes were removed in order to reduce the noise level. The result shows us that the accelerations generally decay slightly and almost linearly on a logarithmic scale with the frequency. The ASD’s clearly show a few peaks with well-known frequencies:

The first peak occurs at $1.8 \cdot 10^{-4}$ Hz which corresponds to the orbital frequency of approximately 5550 sec and an orbit altitude of about 450 km. One of the components of this peak is the difference in atmospheric drag per orbit, as a result of the eccentricity of GRACE’s orbit. This eccentricity is small (0.005), but still not zero. It results in a maximum difference between the perigee and the apogee of about 5 km. At a height of 450 km, 5 km difference in height yields a difference in atmospheric density of about $1 \cdot 10^{-13}$ kg/m$^3$ [22] and a drag force difference of a few microNewtons with a period of once per orbit. In the same manner, the pear-shape of the Earth, the solar radiation pressure, the day and night difference in atmospheric density and the density differences due to seasons on Earth contribute to the magnitude of the first peak in the ASD. Most of these contributions have different phases with respect to each other, but the phase information is not visible in the ASD.

The second peak is located at $3.6 \cdot 10^{-4}$ Hz or twice the orbit frequency. Contributions to this peak are the ellipsoid-shape of the Earth, and differences in air density between the Earth poles and equator (a satellite flies twice per orbit above the equator and a pole).
Figure 5.10: The 15-days average of the ASD (left) and DSD (right) for the period of 01-08-2002 until 15-08-2002, in the cross-track direction.

The peaks beyond $3.6 \cdot 10^{-4}$ Hz are also clear signals until a frequency of about $2 \cdot 10^{-3}$ Hz.

From the ASD a DSD is calculated and shown in Figure 5.10 on the right. From an extrapolation of this DSD, a careful conclusion can be drawn about the general trend of the disturbance forces in the frequency domain and thus about the high-frequent displacements of the satellites. An exception to this are the high-frequent disturbances with an exceptional and unknown character like the “twangs” that are observed on GRACE. These are sudden oscillations in the accelerometer data at 3.5 Hz that remain unexplained until today [34]. An example of such a twang is shown in Figure 5.11, from [34]. Typical for the twangs of GRACE is that it occurs systematically at 3.5 Hz while their magnitude can varies from twang to twang. One of the possible explanations is that these are a result of unequal heating of the Teflon radiator foil, that is attached on the bottom of the GRACE satellites at certain intervals. A thorough inspection of the 10 Hz unfiltered data (Level 1A) would be very useful for the purpose of this study, if the data is available.

Figure 5.11: An example of a “twang” in the time domain and the frequency domain, from [34].
5.5.2 Establish a representative ASD for the given dataset

The acceleration spectral density (ASD) for one day of data is shown on the left of Figure 5.12, while the mean of the ASD’s of 15 days is shown on the right. Only the cross-track direction is shown for illustration purposes. The other two directions are very much alike. Taking the mean of the Fourier transforms of the data from different days reduces the noise significantly, and makes the peaks in the signal more clear. At a frequency of 0.035 Hz the signal is filtered by the CRN-filter during the processing of the Level 1A data to Level 1B. This is visible in both figures as a fast decay and narrowing of the signal at 0.035 Hz, that changes into a thin smooth line.

Figure 5.12: The one-day-ASD for the linear accelerations of GRACE (left) and the mean of the ASD’s of 15 days (right).

5.5.3 Separate AOCS and orbit maintenance spectrum from external disturbances

The method used for removing the spikes in the time-domain data is described in Appendix E.2. The spikes are a result of AOCS and orbit maintenance manoeuvres, but possibly other sources could contribute too. The time-series of data for the cross-track direction with spikes is shown in Figure 5.13 for 01-08-2002. The spikes have been removed by a spike selection criterium depending on their magnitude. The criterium is described in Appendix E.2. In Figure 5.14 (top) the ASD for one day and 15 days is shown for the cross-track direction using a “soft” filter. The middle of Figure 5.14 shows the same for a harder filter and the bottom figure shows the results using a filter that was overdone. When a soft filter is used, some redundant spikes remain, which is visible in the ASD by a slightly lower level of the noise and the signal, at about $10^3$ and $3 \cdot 10^{-2}$ Hz. When a hard filter is used, too many spikes are removed, and some parts of the signal disappear. Some unusual features then appear in the spectrum, as seen in the one-day spectrum of the hard filter, and the one-day and 15-days spectra of the overdone filter. Though the hard filter filters away too many peaks, it is visible that the effects of the peaks that do not belong to the data (i.e. spikes that are a result of non-natural disturbance forces), is an increase

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If spikes occur due to another source of disturbance than the thruster pulses, these are of importance for this study too. Therefore it is recommended in the conclusions that the spikes are filtered using the AOCS data during a follow-up study, instead of a magnitude criterium for filtering them.
of magnitude of the noise. It seems like if the peaks are removed, that the spectrum approaches a descending straight line. The unfiltered data are thus an upper limit of the real non-gravitational, natural accelerations. This could be verified by performing the analysis with a filter as described in Appendix E.2, in a future study. This filter would remove spikes using what is known about their source (AOCS & orbit maintenance) instead of filtering through a criterium. The filter as used in this study may also filter spikes not coming from AOCS and orbit maintenance. Also for this reason it is desirable to filter the data using the recommended method.

Figure 5.13: The timeseries of the GRACE A data on 01-08-2002 for the cross-track direction. The orbit maintenance spikes at the maxima and minima are clearly visible.

5.5.4 Investigate the effect of the environment variability

The ASD for the “active” Sun between 18-10-2003 and 5-11-2003 is shown in figure 5.15 (left) for the cross-track measurements, while the right side of Figure 5.15 shows the ASD for the more quiet period just before (01-09-2003 until 18-09-2003)\(^\text{10}\). Not much differences are found between the active and quiet ASD’s in the outline of the spectrum. However, the level of the magnitude of the first few peaks is considerably higher in the case of an active Sun in comparison with the quiet Sun. This difference seems to decay as the frequency of the disturbance increases: the signal crosses the \(10^9 \text{ nm/s}^2/\sqrt{\text{Hz}}\)-level in both cases at about \(3 \cdot 10^{-3} \text{ Hz}\).

In Figure 5.16 the cross-track ASD’s for the periods of 01-08-2002 until 15-08-2002 (left) and 15-06-1004 until 30-06-2004 (right) are shown. Both periods are relatively “quiet” periods, though 2002 is closer to the solar maximum. Though the orbit is lower in 2004 than in 2002, still the magnitude of the first peaks of the signal is higher in 2002 than in 2004, which implies that the solar activity is much more important than the orbital altitude. This is consistent with the results of the comparison between the case of an active and a quiet Sun. This conclusion is also verified with the MSISE-90 atmospheric model\(^\text{11}\), from which the values of the atmospheric densities are given in [22] for different heights and solar activities. Table 5.4 shows the densities for a low, medium and high solar activity at 440 and 460 km. The differences between the orbit heights of 20 km seem to be less than the differences between the solar activity cases which are different by one order of magnitude. Therefore, in Table 5.5 the orbit heights counted from 460 km are shown, assuming that the

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\(^{10}\) The solar activity was checked from [78] and from: http://www.bbso.njit.edu/Research/ActivityReport/brep200309.html

\(^{11}\) http://nssdc.gsfc.nasa.gov/space/model/atmos/msise.html
Figure 5.14: The one-day-ASD for the linear accelerations of GRACE A (left) and the mean of the ASD’s of 15 days (right), using a soft (top), hard (middle) and overdone (bottom) spike filter, for the cross-track direction.
solar activity remains the same but the air density increases by a factor 10. From Table 5.4 and 5.5 it can be concluded that the order of magnitude of the disturbance forces, which are dominated in LEO by atmospheric drag and solar radiation pressure, is dominated by the solar activity instead of orbital altitude, at altitudes comparable with GRACE and FFDEM (about 350 km up to about 550 km). Not much can be concluded about the high-frequent forces unless the data are filtered using the thruster and AOCS data, and unless the noise level is lowered. It can only be concluded that the differences in the ASD between solar active and quiet periods, decreases with increasing frequency. The same conclusions are drawn for the other two directions (along-track and radial).

Figure 5.15: The ASD for the period with a high solar activity (left) and a preceding period with low solar activity (right), for the cross-track direction.

Figure 5.16: The ASD for the period in 2002 (close to the solar maximum, on the left) and 2004 (right), in the cross-track direction.
Table 5.4: The atmospheric densities at 440 km and 460 km for different levels of solar activity, from the MSISE-90 model [22].

<table>
<thead>
<tr>
<th>Activity Level</th>
<th>Density ρ_{440} [kg/m^3]</th>
<th>Density ρ_{460} [kg/m^3]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Low solar activity</td>
<td>2.23 \cdot 10^{-13}</td>
<td>1.42 \cdot 10^{-13}</td>
</tr>
<tr>
<td>Medium solar activity</td>
<td>1.96 \cdot 10^{-12}</td>
<td>1.40 \cdot 10^{-12}</td>
</tr>
<tr>
<td>High solar activity</td>
<td>3.23 \cdot 10^{-11}</td>
<td>2.60 \cdot 10^{-11}</td>
</tr>
</tbody>
</table>

Table 5.5: The height for which a density difference of a factor 10 originates, according to the MSISE-90 model [22].

<table>
<thead>
<tr>
<th>Activity Level</th>
<th>10 times less dense</th>
<th>Reference height (Reference density)</th>
<th>10 times as much dense</th>
</tr>
</thead>
<tbody>
<tr>
<td>Low solar activity</td>
<td>360 km</td>
<td>460 km</td>
<td>580 km</td>
</tr>
<tr>
<td>Medium solar activity</td>
<td>330 km</td>
<td>460 km</td>
<td>600 km</td>
</tr>
<tr>
<td>High solar activity</td>
<td>260 km</td>
<td>460 km</td>
<td>690 km</td>
</tr>
</tbody>
</table>

5.5.5 Cross-validate data

The data from CHAMP are processed as described in Appendix E.3. Two cases were run: one for the near-solar-maximum-period (01-08-2002 until 15-08-2002) and one for the active solar period (18-10-2003 until 05-11-2003). The data were not filtered for thruster spikes, but these were also less pronounced and sharp than in the case of GRACE. The ASD of the cross-track measurements of the 2002-period is shown in Figure 5.17 (left) while the ASD from the active period in 2003 is shown on the right side of Figure 5.17. The data show consistency with the previous results for 2002 and 2003 (see Figures 5.16 and Figure 5.15) except for the general level of the disturbance forces which is considerably larger in the case of CHAMP.

Also the decay level is higher for CHAMP than for GRACE, at least at frequencies above about 4 \cdot 10^{-3} Hz. The reason why the CHAMP level of disturbance is about one order of magnitude higher, is because the satellite was launched two years earlier than GRACE. Thereby it flies at a height of about 150 to 200 km lower than GRACE at the same time in 2002, where it experiences a much higher atmospheric drag. This difference is in agreement with the height differences for a factor 10 of difference in atmospheric drag as shown in Table 5.5. A possible explanation for the difference in decay level could be that the data were filtered in a different way. A second possibility is that the level of noise is higher in the case of GRACE because of the nature of the spikes in the time-domain data (in the case of GRACE the filtered data have also a steeper slope than the non-filtered data, see Figure 5.10 (left) and 5.12 (right)). For the worst-case scenario the data of GRACE are used. The X-axis has the same scale in all figures in order to be able to compare them more easily.

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12The only days not included in the mean of the ASD’s are 30-10-2003 and 03-08-2002 because these datafiles had a different length. A mean of 14 days was taken instead of 15 days.

13And except for the radial direction as a result of the disfunctioning radial accelerometer measurements. This is illustrated in Appendix E.3.
5.5.6 Determine a worst-case scenario for the RMS-calculation

For selecting a reasonable worst-case scenario, several parameters can be taken into account, like orbit altitude, period in the solar cycle, solar events and the way of filtering of the data. As a worst-case scenario for an FFDEM satellite in LEO at 561 km, the GRACE case has been taken in 2002. The data have been chosen for this case to not to be filtered in order to consider the ASD with a slope as slight as possible. Furthermore in all the ASDs the mass of the satellite was changed to match for the mass of FFDEM (120 kg). The acceleration of the FFDEM in comparison with the ones of GRACE and CHAMP depends on the mutual $A_m$-ratios of the satellites. The mass of FFDEM is about 4 times as small than the masses of GRACE and CHAMP while its cross-sectional area varies from almost equal to the CHAMP cross-section in the along-flight direction, up to three times the cross-section of GRACE in the same direction. In the radial and cross-track direction the areas of GRACE and CHAMP are bigger, but in order to consider the worst-case, the accelerations are only corrected for the mass of the satellites while the areas are kept the same. However, the influence of the $A_m$-ratio plays a smaller role in the order of magnitude of the accelerations than the solar activity and the orbital altitude do. The ASD and DSD for the 2002-case without filtering, are shown for the three directions in Figure 5.18.

The DSD of Figure 5.18 is approached by a piecewise linear curve that delimits the upper bound of the signal. This approximation is used to extrapolate the DSD and to calculate the worst-case RMS-value as a function of the bandwidth, with an emphasis on the frequencies between $10^0$ and $10^3$ Hz. The RMS-value for the displacements in this bandwidth is 0.61 nm for one satellite, in the cross-track and radial direction and 0.47 nm in the along-track direction. This yields for the relative OPD an average RMS value of about 0.7 nm, which is not too far below the requirements of Darwin (5 nmRMS). If the satellite would be flying in an extreme case during the solar maximum, at a low altitudes (e.g. at the end of its lifetime) and if there are also exceptional solar events, then the RMS-value could be easily ten times more. The approximated DSD and cumulative RMS-values are shown in Figures 5.19 and 5.20 respectively.
Figure 5.18: The ASD (left) and DSD for the selected realistic worst-case scenario.
Figure 5.19: The approximated DSD for the selected worst-case scenario. The cross-track DSD disappears behind the radial DSD because they have the same values.

Figure 5.20: The cumulative RMS for the selected worst-case scenario. The RMS-value of the displacement of one satellite is indicated in the Figure (0.61 nm). Also here the cross-track cumulative RMS disappears behind the radial cumulative RMS.
5.6 Conclusions

The RMS-value of the OPD between two satellites of FFDEM is in the order of 0.1 to 1 nm in a frequency range of $10^0 - 10^3$, based on the extrapolated data. This comes close to the magnitude of the Darwin and FFDEM requirements. For the bandwidth between $10^1$ and $10^3$ Hz the RMS-value of the OPD is about 0.01 nm. Apart from the general trend of the disturbances, exceptional events like “twangs” are not included in the RMS-value. This general trend is a nearly linear decay of the DSD at a rate of $-48$ dB/dec. The requirement of the DSD remaining below $7.5 \text{ nm/}\sqrt{\text{Hz}}$ is met for frequencies above 1 Hz.

**Gravitational forces**

The contribution of the gravitational forces is negligible in comparison with the non-gravitational forces in the bandwidth of $10^0 - 10^3$ Hz. The results of the gravitational forces could be made more accurate by using the “OSU91” model itself and extrapolating from 0.14 Hz (the resolution of the model) instead of starting from the figures in the literature [75] and extrapolating from $2.7 \cdot 10^{-3}$ Hz.

**Solar activity and altitude**

The order of magnitude of the disturbance spectra, especially at the lower frequencies, depends a lot on the altitude of the satellite, but even more on the solar activity. This is because the Earth’s atmosphere responds to the solar activity. The solar cycle as well as occasional solar events have a big influence. The worst-case was done for the solar maximum, but if at such a solar maximum also solar active events take place or if the satellite is flying lower, the order of magnitude of the disturbances could be ten times bigger than calculated before.

**Filtering of the data**

It is recommended to filter the data from the thruster spikes more accurately using housekeeping information. Also the data spikes could be Fourier transformed as well to reveal more information about these. In addition, the source of the eventually remaining spikes should be revealed. Only then the true slope of the spectrum might become more visible instead of using the upper limit where the signal is drowned in the noise.

**Mass and filter**

The mass and area assumptions, and the filter used are of lesser importance for the order of magnitude of the displacements than the orbit and the solar activity itself.

**Checks**

The calculations were checked by a cross-check between CHAMP and GRACE which showed some consistency on the one hand, but a different decay on the other hand, probably due to filtering differences of the data. In addition, the Fourier transform was checked using Parseval’s theorem (Appendix E.1).

**Level 1A data**

It is recommended to analyze the Level 1A data and process it to Level 1B, without smoothing the original 1 Hz-data to 0.1 Hz, and without the CRN-filter at 0.035 Hz. In this manner, exceptional events like “twangs” and other unknown events that might appear could be studied. These events are not visible in the Level 1B data, but are of big importance for determining the ODL frequency. If events like “twangs” appear, then the ODL should work with a frequency that is higher than the highest frequency of these events. The “twangs” however, probably depend on the design of the GRACE spacecraft (Teflon foils). For Darwin, care must be taken in the design to avoid such effects.

**Method used**
The method used in this LEO study permits to evaluate the order of magnitude and the course of the disturbance forces in LEO for different frequencies (except for the events like “twangs” at high frequencies). The method, using only Level 1B data and filtering the thruster spikes by a magnitude criterium, does not permit to draw reliable conclusions about the exact signal at high frequencies, and to yield the difference in the signal pattern between solar active periods and quiet periods.

**Orbit count**

The amount of orbits per day is not an integer count, which introduces a certain noise level comparable to that of the thruster spikes. However, the data are averaged over 15 days which reduces it again. Taking the mean of e.g. 100 days would not be an improvement for the noise, because due to the “jump” from the first to the last data point always introduces a certain noise level. A more complicated algorithm could be written to eliminate this effect and to lower the noise.

**Orbit choice**

The orbit choice of FFDEM was chosen as a polar orbit. Even for polar orbits, interferometry is possible, though a non-polar orbit is recommended from a disturbance point of view. At the poles the solar events are most noticeable, and also atmospheric upwelling events exist.

**Velocity and rotations**

In this study only the linear accelerations and displacements are looked at. It is necessary to have a look at the rate of the displacements and at the rotational behaviour of the satellite too, though the results of the linear displacements already suggest that these are also very small.

**GRACE A and GRACE B**

In this study the displacement of one satellite is looked at at the time, and the total RMS-value of the satellite is summed quadratically to get the RMS-value of the OPD. It is interesting to also have a look at the relative measurements made between GRACE A and GRACE B, or to compare the accelerometer data from GRACE A and B at the same epoch and to make a PSD of the difference of both.
Chapter 6

The external disturbance forces for Darwin

6.1 Introduction

In Chapter 3 the space environment in L\textsubscript{2} is described (and some aspects of the LEO environment), in order to reveal the disturbance forces on Darwin and in particular their variability. Unfortunately very little is known about the environment in L\textsubscript{2}, especially about its high-frequency variability. Therefore, the variable disturbance forces in L\textsubscript{2} are not described quantitatively but only qualitatively for as much as is known about them. The better-known low-frequency aspect of the disturbance forces in L\textsubscript{2} are discussed in [15]. These are mainly the gravitational forces and the solar radiation pressure force.

Figure 6.1: An overview of the environments discussed in Section 3 and the disturbance forces associated with them.

An overview of the variable environments and the forces associated with them, is shown in Figure 6.1. An inventory of all forces in L\textsubscript{2} and in LEO is given in Appendix A. The forces of which was believed that they could possibly cause a variable disturbance on the OPD of Darwin are discussed in this Section for as far as information was found about them.

As discussed in Section 3.4, the Darwin satellites in L\textsubscript{2} will be flying through different types of plasma, in which they can become charged. The resulting forces, Lorentz and Coulomb,
are discussed in Section 6.2 and 6.3. The solar radiation pressure force and the charged particle drag force from the solar wind are addressed in Sections 6.4 and 6.5 respectively. Finally, conclusions are drawn on the level of high-frequency disturbances in \( L_2 \), from the considerations made in this Chapter and the OPD-results of the LEO study (Chapter 5).

### 6.2 Lorentz force

Whenever a charged particle moves with a velocity through a magnetic field, it is subject to the so-called Lorentz force. This force is equal to:

\[
\vec{F}_L = q \cdot (\vec{v} \times \vec{B})
\]

where \( q \) is the charge of the particle, \( \vec{v} \) is its velocity vector through the magnetic field and \( \vec{B} \) is the magnetic field vector. When a satellite is charged with a nett charge and flying through a magnetic field, the same happens and the satellite is subject to a Lorentz force.

From Section 3.3, we know that the magnetic field of the Earth has values of about 30000 (equator) up to 60000 nT (poles). However, the IMF has a magnetic field strength of only 5 nT. Even during magnetic storms the IMF acquires a strength of only about 20 nT [13]. The maximum value of the magnetic field strength in the region of \( L_2 \) has been found in the literature to be 30 nT, in the lobes of the magnetotail [5]. A fluctuation of magnetic field strength of 25 nT is used as a worst-case assumption for calculating the Lorentz force. If the potential of the satellite is known, an approximation of the charge can be calculated by:

\[
Q_{sc} = V \cdot 4\pi\epsilon_0 \cdot r_0
\]

where the satellite is assumed to be a sphere with radius \( r_0 \), \( V \) is its nett acquired potential and \( \epsilon_0 \) is the permittivity of free space, equal to \( 8.854 \cdot 10^{-12} \) C\(^2\)/Nm\(^2\). The orbital velocity of Darwin is estimated to be about 200 m/s in \( L_2 \), \( r_0 \) is assumed to be 1 m, and a spacecraft potential of 500 V is assumed. Filling in these numbers in Equation 6.1 yields a Lorentz force with an order of magnitude of \( 10^{-13} \) N. This yields accelerations in the order of \( 10^{-6} \)–\( 10^{-7} \) nm/s\(^2\) for a 500 kg satellite. Even in a variable magnetic environment, and even if acquiring variable nett charges as a result of the variable plasma environment, the Lorentz forces, in normal conditions, are negligible. When a solar event like an ICME is considered, having a change in magnetic field strength of 15 nT and travelling at 550 km/s, the Lorentz accelerations become of an order of magnitude of \( 10^{-4} \)–\( 10^{-3} \) nm/s\(^2\), for satellites charged with 50 or 500 V. Such bad conditions are exceptional and hereby do not necessarily need to be taken into account for Darwin.

### 6.3 Coulomb force

The Coulomb forces between two charged spacecrafts are given by:

\[
F_c = \frac{1}{4\pi\epsilon_0} \cdot \frac{Q_{sc1} \cdot Q_{sc2}}{d^2}
\]

where \( Q_{sc1} \) and \( Q_{sc2} \) are the nett charges of the satellites and \( d \) is the distance in between them. When taking into account the plasma Debye length (see Section 3.4), and again assuming spherical satellites with radius \( r_{sc} \), the Coulomb force is equal to:

\[
F_c = 4\pi\epsilon_0 \cdot \frac{V_{sc1} V_{sc2} r_{sc1} r_{sc2}}{d^2} \cdot e^{-\frac{d}{\lambda_D}}
\]

From Section 3.4 we know that the plasma Debye length in \( L_2 \) varies between about 10 and 700 m depending on the region in the magnetotail that Darwin is positioned. This yields for two satellites in \( L_2 \) at a distance of 50 m (smallest baseline of Darwin) and a potential
of 50 V, a Coulomb force of $10^{-10}$ and $10^{-12}$ N for a plasma Debye length of 700 and 10 m respectively. This corresponds to accelerations of about $10^{-4}$ nm/s$^2$ and $10^{-6}$ nm/s$^2$ for a 500 kg satellite. A satellite potential of 500 V leads to accelerations of about $10^{-3}$ nm/s$^2$ for $\lambda_D = 700$ m. This is only the force between two satellites. Darwin consists of 7 satellites and thus the Coulomb force will be larger, but not more than one order of magnitude. Even if variations take place in the plasma environment, the Coulomb forces remain negligible. A satellite in L$_2$ acquiring a charge of much more than 500 V in L$_2$ is not expected, though this should be verified using a spacecraft charging model. A more complete consideration on the Coulomb forces is given in Appendix F.2.

### 6.4 Solar radiation pressure force

The solar radiation pressure force is the largest non-gravitational disturbance force in L$_2$. Even in LEO at 561 km, its order of magnitude is as large as the drag force of the atmosphere. However, its variations are rather small, as discussed in Section 3.2.3. The solar radiation pressure force is given by [46, 32]:

$$dF_{\text{tot}} = -P \cdot dA \cdot \cos(\theta) \cdot \left[(1 - \rho_S) \cdot \vec{e} + \rho_S \cdot 2\cos(\theta) \cdot \vec{n} + \rho_D \cdot \frac{2}{3} \vec{n}\right]$$  \hspace{1cm} (6.5)

where:

- $P$ is the pressure exerted on the satellite ($P = \frac{\phi}{c}$ with $\phi$ the total solar irradiance (energy flux) and $c$ the speed of light).
- $A$ is the cross-sectional surface of the satellite, lit by the sun.
- $\theta$ is the angle of incidence of the solar rays (angle between the surface edge of Darwin’s sunshield and the solar rays).
- $\rho_S$ is the specular reflection coefficient. This describes the amount of photons that are specularly reflected.
- $\rho_D$ is the diffuse reflection coefficient. This coefficient describes the amount of photons that are diffusely reflected ($\rho_S + \rho_D = \rho$).
- $\vec{e}$ is the vector that points in the same direction as the incident rays of photons, directed towards the Sun.
- $\vec{n}$ is the vector that is normal to the surface, and pointing towards the Sun-side of the surface.

From Equation 6.5 it is clear that the solar radiation pressure acceleration depends on several parameters. These are the total solar irradiance, the angle at which the surface is positioned with respect to the Sun, the surface that is sunlit with respect to the mass of the satellite, and the optical properties of the materials of the satellite.

**Variations in total solar irradiance**

The variations in the total solar irradiance are very small. The solar flux varies by about 0.1% in timescales from 1 sec to 11 years [48]. From [78], we know that during optical flares, the total solar irradiance can change up to 270 ppm (0.027%). On the other hand, sunspots can cause the total solar irradiance to decrease by 0.34%. Also in the near-UV (wavelength between 180 and 400 nm), the solar flux can vary from an average of 118 W/m$^2$ to a worst-case of 177 W/m$^2$ [4]. Neglecting the material properties of the sunshield and assuming an incidence angle $\theta$ of 0° yields a solar radiation pressure acceleration of a Darwin free flyer of 344 nm/s$^2$ (mass assumed is 500 kg). A difference of 0.34% makes a difference of about 1 nm/s$^2$ for the solar radiation pressure force. However, sunspots do not originate very
suddenly and are therefore not a high-frequency cause of change in disturbance force. For what concerns a solar flare, its influence is about 0.1 nm/s\(^2\) on the solar radiation pressure, if it occurs. It must be stressed that this solar flare was a worst-case flare, and that an acceleration of 0.1 nm/s\(^2\) causes no worry for the OPD control mechanism of Darwin. For variations in the near-UV, the difference in solar radiation pressure is about 15 nm/s\(^2\). This seems to be a large number, but the author does not know on what timescales this variation takes place and how worst-case this assumption is, i.e. how often this happens. In normal conditions, variations of maximum 0.1% have to be taken into account, leading to a change of solar radiation pressure acceleration of about 0.3 nm/s\(^2\). A comment must be made on the use of the Darwin TTN+ array. These free flyers would have larger sunshields that lead to a larger solar radiation pressure force and larger differences in its variability. However, an assumed sunshield diameter of 10 m would lead to a difference due to a solar flare of only 0.7 nm/s\(^2\), and also the mass of these satellites will be larger which reduces the accelerations.

The \(\frac{A_m}{m}\)-ratio

A difference in \(\frac{A_m}{m}\)-ratio between the Darwin satellites could lead to different solar radiation pressure accelerations. If one satellite is subject to more disturbance forces (e.g. particulate impacts) than the other one, it could occur that their mass is different, due to a different fuel consumption history. A change of mass of 1 kg (which is very large) would lead to a difference in solar radiation pressure acceleration of 0.7 nm/s\(^2\). However, this is considered an effect of low-frequency nature, and is discussed in [15]. A second effect of a different mass is a torque that could arise, depending on the center of pressure of the solar radiation pressure force, and the center of mass. If the center of mass is shifted, an extra disturbance torque due to solar radiation pressure will originate. This is also low-frequent of nature.

Material properties

Equation 6.5 indicates that the properties of the sunshield play a role in the total solar radiation pressure force. Contamination from e.g. FEEPs or outgassing could change these.

Attitude angle

Finally, variations in the solar radiation pressure can also occur when the sunshield is tilted with respect to the Sun. However, as Darwin is allowed to have only deviations of smaller than 8.4 mas, it is not expected to be significant.

It is plausible to accept that the solar radiation pressure mainly causes a low-frequent disturbance accelerations and that the variability that was found causes only sub-nanometer-level disturbances, that are easily be corrected by the \(\mu\)N FEEP\'s during the observations.

6.5 Charged particle drag

Darwin will be flying in different kinds of plasmas, each with their own velocity and particle densities. Comparable to the drag in LEO, the solar wind causes a “drag force” on the satellites. This perturbation is expected to be orders of magnitude smaller than the solar radiation pressure force. Table 6.1 shows the energy fluxes and momentum fluxes of the solar wind and the solar radiation pressure, from [48].

<table>
<thead>
<tr>
<th></th>
<th>Solar Wind</th>
<th>Solar Radiation Pressure</th>
</tr>
</thead>
<tbody>
<tr>
<td>Energy flux at 1 AU</td>
<td>0.00016 J/m(^2)s</td>
<td>1400 J/m(^2)s</td>
</tr>
<tr>
<td>Momentum flux at 1 AU</td>
<td>8 \times 10^{-10} kg/ms</td>
<td>5 \times 10^{-4} kg/ms</td>
</tr>
</tbody>
</table>

Table 6.1: Comparison of the solar wind and the solar radiation, from [48].
If the satellite is charged, the (also charged) plasma particles will follow a different path and cause different drag forces depending on (amongst others) the charge of the satellite (see Figure 6.2, from [1]). There are three contributions to the charged particle drag: drag due to the direct collisions of particles with the spacecraft, drag due to particle scattering and drag due to wake effects.

1. **Direct collisions**

The drag due to direct collisions also exists when the satellite is not charged. It is comparable with the aerodynamic drag from neutral particles, in the atmosphere. The displacement of the satellite due to the direct collisions of the charged particles, is calculated using:

\[ F = \frac{1}{2} \rho V^2 C_D A \]  

(6.6)

where \( \rho \) is the particle density, \( C_D \) is a drag coefficient (assumed to be equal to 3), \( A \) is the cross-sectional area of the satellite (the surface area of the sunshield is assumed) and \( V \) is the velocity of the satellite with respect to the plasma. From Table 3.6 in Section 3.4, we use the following parameters to calculate the drag force from the free solar wind:

- \( V = 800 \text{ km/s for a worst-case, } V = 400 \text{ km/s for a “quiet” case} \)
- \( C_D \approx 3 \)
- \( A = \pi \bar{b}^2 \) where \( \bar{b} \) is a function in between 0 and 1, depending on the satellite charge. An area equal to the sunshield area is assumed here with \( \bar{b} \) assumed to be equal to zero (\( A = 38.4 \text{ m}^2 \) corresponding to a sunshield diameter of 3.5 m).
- The solar wind consists mainly of Hydrogen atoms. The mass of a Hydrogen atom is equal to \( 1.673534 \cdot 10^{-27} \text{ kg} \) [48].
- The particle density is assumed to be 12 particles per cm\(^3\) [24]. Also values of 5 and 8 are found in the literature.

For the fast solar wind and a solar wind density of 12 particles per cm\(^3\), the acceleration of a Darwin free flyer would become about 1.5 nm/s\(^2\). For the slow solar wind this is only 0.4 nm/s\(^2\). Using the differences in ion density between the different regions
of the magnetosphere in $L_2$ (Table 3.6), we conclude that the variations in the accelerations of Darwin due to the direct collisions of the charged particle drag will not be larger than $1.5 \text{ nm/s}^2$. As is the case with the solar radiation pressure acceleration, also the charged particle drag depends on the $\frac{1}{m}$ ratio of the satellites.

2. **Particle scattering**

The particle scattering mainly depends on the spacecraft potential that it has acquired. Usually satellites are equipped with active systems to reduce or avoid electromagnetic charging and therefore it depends strongly on the Darwin satellites design. This is not only done to avoid nett charges on the satellite, but also to avoid different charges on different parts of the satellite. This could possibly damage some subsystems when the charges are “triggered” (e.g. when flying through a different kind of plasma), and discharged through a mechanism called “arc discharges”.

3. **Wake effects**

The spacecraft wake effects mainly depend on the difference between the spacecraft velocity and the thermal velocity of the charged particles. The velocity of Darwin with respect to the solar wind is in between 400 and 800 km/s. Its thermal velocity is calculated by [48]:

$$V_{th} = \sqrt{\frac{3kT}{m}}$$  \hspace{1cm} (6.7)

where $k$ is the Bolzmann constant ($1.38066 \cdot 10^{-23} \text{ J/K}$), $T$ is the particle temperature (about 10 to 20 eV for the solar wind) and $m$ is the particle mass ($1.67 \cdot 10^{-27} \text{ kg}$ for hydrogen). Filling in these values yields a thermal velocity of about $50 - 75 \text{ km/s}$. Hence, the charged particle drag due to wake effects would require more attention.

The drag due to particle scattering and wake effects are two sources of large uncertainty for the total magnitude of the charged particle drag. It is necessary to perform a charging analysis of the Darwin satellites during the Darwin study. Depending on the results of the charging analysis, the drag from solar wind might be re-assessed because a higher level of charging may increase the general level of drag force significantly and also its accompanying variations. These variations will probably not be of a very high frequency, though the possibility exists that if they occur, it happens fast. Furthermore the influence of a CME must be looked at, taking into account the particle densities that are associated with these.

6.6 **Conclusions**

From Section 3 we have learned that the biggest disturbances in LEO are due to the atmospheric drag force and the solar radiation pressure force. From the consideration in 6.4 it is concluded that on the short-term the solar radiation pressure force is rather constant and will in no case lead to variations larger than $1 \text{ nm/s}^2$. The Lorentz force and the Coulomb forces also do not lead to significant differences in accelerations on the short-term. The variations in charged particle drag from the spatial differences in the plasma density in $L_2$ do not lead to variations in the satellite acceleration of more than $1.5 \text{ nm/s}^2$ at the first sight. The impact of CMEs is not looked at yet, and spacecraft charging should be simulated in order to calculate the increase in charged particle drag due to wake effects and particle scattering. This could increase the accelerations significantly, and their variations.

The accelerations from “sudden” variations in the space environment (plasma density fluctuations, solar flares, etc.) at a first look seem to cause no worry for Darwin. The RMS-values of the OPD in Low Earth Orbit is found to be between 0.1 and $1 \text{ nmRMS}$ in Chapter 5. Because in $L_2$ there is no atmosphere, and only the charged particle drag causes drag forces that are probably smaller than in LEO, it is plausible to conclude that also in $L_2$ the real
high-frequency disturbance forces will remain below 1 nm\textsubscript{RMS}.

The literature survey in L\textsubscript{2} suggested that L\textsubscript{2} is a highly variable region, but the considerations in this Section showed that despite the variability, the influence of the associated disturbance forces is negligible for this study. However, it is based on the limited information that was available on the high-frequency variability of the environment, and could still increase significantly due to spacecraft charging. Therefore it is advised to perform a follow-up study to verify the considerations of this Section. Such a study could make use of the data collected by SOHO, ACE and Cluster, on the magnetic field variations, solar wind density variations and solar irradiance variations. These data should be Fourier transformed as was done in Chapter 5 after which calculations have to be made for the disturbance forces themselves. Probably also no high-frequency information will be available in these data, but an extrapolation could be considered. Also a simulation of the Darwin satellites charging is strongly recommended.
Bibliography


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[84] Courtesy of www.sp.ph.ic.ac.uk/~forsyth/reversal/.


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Appendix A

Inventory of Disturbance forces

A.1 Gravitation

The gravitational force is the biggest of the forces acting on the satellite formations of Darwin and FFDEM. The issues to be taken into account concerning gravitational forces are:

A.1.1 Earth’s gravity field

The non-spherical Earth causes spatial variations in its gravity field. As a consequence of these spatial variations, satellites flying in LEO suffer from temporal variations as they encounter the spatial differences in gravitational force. As shown in Section 5.2, this gravitational force has no significant component (i.e. above $5 \text{ nm}_{\text{RMS}}$) at frequencies higher than about $5 \cdot 10^{-2} \text{ Hz}$. In addition, the Earth’s gravity field changes due to i.e. tidal effects of the atmosphere, the oceans and the Earth itself. However, these changes are low-frequent.

A.1.2 Gravitational force from the Sun and other planets

The Sun and planets like Jupiter, Saturn and the Moon, also exert a significant gravitational disturbance forces on the satellites. Because the satellites are not orbiting the Sun nor these planets, the spatial variations in their gravity field do not cause such a temporal variation in the satellite’s orbit, as is in the case of satellites in LEO. Therefore the gravitational force from the Sun and other planets is regarded as a disturbance force with only a low frequency content.

A.1.3 Gravitational forces between the formation flying satellites

Two masses in each other’s sphere of influence exert a gravitational force on each other. This is also the case with formation flying satellites, although this force is so small that it is commonly accepted to disregard it. However, when doing research in the nanometer-level-world, precaution should be taken when disregarding small forces. One cannot rely on experience or common sense anymore. For illustration, gravitational force between the satellites are calculated in Appendix F.1 although they are further of no importance in this study, because they are not high-frequent of nature.

A.1.4 Relativistic effects

Newton’s law of gravity is commonly used in space calculations. However, Einstein’s law of relativity is nowadays a better representation for gravitational force. This correction is commonly regarded as a disturbance force when Newton’s laws were used in orbit calculations.
This “disturbance force” is also low-frequent in conformity with the other gravitational forces, and thus not considered in this study.

A.2 Drag force
Whenever a satellite flies in a medium of particles, a drag force results. The particle densities are extremely low in space, and thus the drag forces are limited. Still, at 561 km, the drag force is one of the two biggest non-gravitational disturbance forces, next to the solar radiation pressure force. It is a “surface force” and therefore it depends on the satellite geometry and its attitude.

A.2.1 Atmospheric drag (LEO)
A satellite flying in LEO at 561 km flies through an atmosphere with a mean density of $2.85 \times 10^{-13} \text{ kg/m}^3$ [22]. This number seems to be very small, but essentially the satellites in such an orbit do suffer from aerodynamic drag forces. As is the case with gravitational forces, the spatial variations in density of the atmosphere lead to temporal variations of the drag force on the satellite. These spatial variations are consequences of e.g. the Earth’s shadow or temperature differences between the poles and the equator. The atmosphere also shows temporal variations: it reacts strongly on the variations in solar radiation and geomagnetic activity. Also local vertical and horizontal winds contribute to the temporal variations in the atmosphere. Because the drag forces are the most important fluctuating forces for the perturbations in LEO, the LEO neutral environment and its variability is studied in Section 3.5 while the effect of the sum of all disturbance forces in LEO, where the drag is part of, is examined in Chapter 5.

A.2.2 Solar wind
The solar wind is a stream of particles coming from the Sun. It has a low density but on the other hand it has many density fluctuations. It is worth to have a closer look at the influence of the Solar wind on the relative distances of the Darwin satellites. A consideration about this is given in Section 6.5.

A.3 Radiation pressure forces
The radiation pressure force has a magnitude comparable to the atmospheric drag force, at a height of 561 km. The (solar) radiation pressure is used in the propulsion concept of “solar sailing” and thereby proves to be a force of vital importance for this study. Like atmospheric drag, also the radiation pressure forces are surface forces.

A.3.1 Solar radiation pressure
The Sun radiates with an energy of $\pm 1367 \text{ W/m}^2$ [?] at 1 AU distance from the Sun. Although this number is called the “solar constant”, it is not constant at all. On the long term, the solar cycle (11 years) and the variable distance of the Earth from the Sun affects this number by 0.2% and 3.3% respectively. On the short term, space weather effects of the “active Sun” could affect the amount of energy radiated. In the visible wavelengths these variations are small, but in the near-UV, they are significant. These solar variability is discussed in Section 3.2, while the influence of the solar radiation variability on Darwin is discussed in Section 6.4.

\footnote{The mean density at the surface of the Earth is about 1.17 kg/m$^3$ [22].}
A.3.2 Earth and Moon Albedo

The Earth and the Moon reflect the light coming from the Sun. The ratio of the light reflected from a planet to the light incident on it is called the “albedo”. The albedo of the Earth has both spatial and temporal variations (e.g., clouds that move). The albedo of the Moon is not negligible. They are in the order of 30% for the Earth and 6% for the Moon [22, 2], although for short periods the albedo of the Earth can vary between 5% and 60% [22]. In L_2 the Earth and Moon albedos are considered low-frequent.

A.3.3 Earth Infrared Radiation

The Earth is a thermal system that remains in equilibrium by receiving and absorbing energy from the Sun, and re-radiating it in the infrared part of the energy spectrum. The Earth IR is comparable with the albedo, but the main difference is that the Earth IR radiation also happens on the night-side of the Earth, while the albedo and solar radiation pressure do not occur there, because of the Earth shade. The Earth IR radiation is on average 230 W/m², and it can vary from 150 W/m² to 350 W/m² [22].

A.3.4 Satellite thermal emission

Just like the Earth, also the satellite is a thermal system that has to remain in equilibrium. Also the satellite receives and radiates energy. This energy has two main sources: the external (Sun, Earth and Moon albedo and IR) and internal (heat dissipation from subsystems on-board). The contribution of the internal sources can be large, and therefore the thermal system of Darwin has to be designed cleverly. One of the potential problems could be the radiators, needed for the sorption coolers. These can cause accelerations caused by thermal emission. Concerning the external forces, especially differential warming of the satellite could cause displacements or deformation of the satellite, or “thermal cracks” as mentioned in Section 2.4.4.

A.4 Due to charging

Satellites can acquire a variable charge when flying through different kinds of plasma. This charging leads to effects like Coulomb forces between the formation flying satellites, Lorentz forces and it could increase contamination problems. Important to know is how fast a satellite’s charge changes in time, when flying through the magnetosphere or at L_2. The Earth’s magnetosphere and the plasma environment are described in detail in Section 3.3 and 3.4 in order to be able reveal the time-scale and the magnitude of the charges and these effects. However, for calculating charging effects, a dedicated model should be used. Therefore, a qualitative (semi-quantitative) description of the charging effects on Darwin is described in Section 6.3 and 6.2.

A.4.1 Inter-satellite Coulomb forces

Satellites flying in space in general acquire a nett charge. Two formation flying satellites that are equally charged will repel each other. The inter-satellite Coulomb forces are studied in [44] and [45] with as goal to use them for satellite orbit control, while charging them in a controlled manner. Therefore it was first believed to be possibly a significant disturbance force in L_2. The calculations in Section 6.3 proved the opposite.

A.4.2 Lorentz forces of charged satellite through IMF

A satellite with a nett charge q, and flying through a magnetic field $\vec{B}$ with a velocity $\vec{V}$, will experience Lorentz forces. This force is looked at in Section 6.2 and also proved negligible.
A.4.3 Charged particle drag

A charged satellite flying through a plasma (e.g. the solar wind) will experience an increase of drag, which is a consequence of its charge. This drag is discussed in Section 6.5 and could be of importance for Darwin, depending on the outcome of the spacecraft charging study that is advised in the same Section.

A.5 Exceptional events

There are exceptional events that may disturb the Darwin mission only temporarily, but on a regular base. It is necessary to know to what extent these events will affect the Darwin mission.

A.5.1 Micrometeoroid impacts

On a regular basis, micrometeroids impacting on the Darwin satellites will alter their OPDs and possibly disturb interferometry observations. An estimate of the order of magnitude of these impacts is made in Section 4 and turns out to be significant for Darwin. A follow-up study on the effect of this amount of impacts on the interferometer observations is recommended as well as a study on the control system needed to compensate for them, if necessary.

A.5.2 Geomagnetic storms and solar flares

At irregular times the Sun expells a large amount of particles and radiation. These can affect the Earth magnetic field, the atmosphere, the plasma around the Earth, etc. In this study, these solar active events are often used as worst-case scenarios.

A.6 Internal environment

Apart from the external disturbance forces also disturbances could occur that were generated by Darwin itself, as a result of vibrations of some of the on-board equipment. The effects of the internal environment are described in Chapter 2. The internal environment of FFDEM is not studied because the instruments and subsystems are not yet specifically defined for this mission.

A.6.1 FEEP

The Field Emission Electric Propulsion or FEEP thrusters will control the Darwin satellites to the centimeter-level. These FEEP's could possibly induce vibrations to the satellite. The magnitude of the vibrations are investigated in Section 2.4.1.

A.6.2 ODL

The Optical Delay Line is the mechanism on-board Darwin, that will correct the Optical Path Differences (OPD's) to the nanometer-level. This delay line is a small carriage with two mirrors that is moved according to the OPD, in order to cancel it out. The ODL-induced vibrations are looked at in Section 2.4.3.

A.6.3 Sorption cooler

The detector used on Darwin, needs to be cooled down to a level of 8 K [21]. Though the satellites are cooled down in a passive way to 40 K [21], still an active cooler is needed. The Sorption Coolers are suitable for this task, because they are basically vibration-free. The residual vibrations still coming from the Sorption coolers are examined in Section 2.4.2.
A.6.4 Other internal disturbance sources

The three main sources of disturbance are examined in Chapter 2, though also others could possibly disturb which no information was available during this study. These sources are the amplitude modulator (modulates the amplitude of two beams of light before being combined), the mirror switch (used to switch the phase of the incoming light, in demand) and beam angle actuators (corrects the angle of the beam of light before coming into the optical fibres). These are all modulating optical systems, that will probably be used in the future on Darwin. Furthermore, according to [21] also the DC/DC-converters are a potential source of disturbance.

A.7 Effects of the space environment on Darwin and FFDEM, besides the OPD

The subject of this study was restricted to the disturbance forces that can alter the OPD of the satellites. However, a large span of other space environment effects could affect the interferometer observations. Apart from the OPD fluctuations, also these should be taken into account in a future study. Some of them are listed here:

1. **Contamination & Outgassing**
   The satellite mirrors and optical instruments can be polluted by outgassing of materials, propellant of the FEEP, or dust and plasma from space.

2. **Corrosion, erosion and degradation**
   In a Low Earth Orbit, flying through the atmosphere that contains atomic oxygen can lead to corrosion effects. This is called “Atomic Oxygen Attack” in the space environment studies. Also surface erosion and degradation occurs when flying through the atmosphere.

3. **Charging problems**
   Surface charging and differential surface charging leads to discharging when a trigger is met. Such a trigger could be a difference in space environment, or contamination. The discharging can damage the satellite locally. Also charging enhances contamination problems.

4. **Thermal expansion**
   Differential thermal expansion can lead to “thermal cracks”. These are well-known phenomena, e.g. on Hubble, but they are still not yet well-understood and modeled. Care has to be taken in the design of Darwin to avoid such effects.

5. **Interference with observation**
   Spacecrafts in LEO can suffer from spacecraft glow. Their sensors could be disturbed by high-energetic particles. Darwin is flying outside the magnetosphere, thus radiation can reach Darwin easily.
Appendix B

The internal disturbance forces for Darwin

B.1 A summary of the most important concepts in the study of random vibrations

In this Appendix the most important quantities for random vibrations are resumed.

- **Average**
  \[
  \bar{x} = \lim_{T \to \infty} \frac{1}{T} \int_{0}^{T} x(t) \, dt
  \] (B.1)

- **Mean square**
  \[
  \bar{x}^2 = \lim_{T \to \infty} \frac{1}{T} \int_{0}^{T} x^2(t) \, dt
  \] (B.2)

  The mean square is a measure of the magnitude of the fluctuations in a signal [35].

- **Root mean square**
  \[
  x_{\text{RMS}} = \sqrt{\bar{x}^2}
  \] (B.3)

- **Variance**
  \[
  \sigma^2 = \langle x^2 \rangle - \langle x \rangle^2
  \] (B.4)

- **Standard deviation**
  \[
  \sigma = \sqrt{\langle x^2 \rangle - \langle x \rangle^2}
  \] (B.5)

  The standard deviation is also called the 1σ-value. An interpretation of the 1σ-value of a signal, is that during 68.3% of the time, the signal remains below this 1σ-value. For 2σ, this percentage is 95% and for 3σ it is 99.7%.

- **Autocorrelation function**
  \[
  R_{xx}(\tau) = \lim_{T \to \infty} \frac{1}{T} \int_{0}^{T} x(t)x(t+\tau) \, dt
  \] (B.6)
The autocorrelation function tells us how quickly the value of a variable changes, and thus how long it would take us to collect enough random signals before the collection of samples is representative [35]. The autocorrelation function is often used to detect a small signal in a large amount of noise, because the autocorrelation of random white noise is zero.

- **Cross-correlation function**

\[
R_{xy}(\tau) = \lim_{T \to \infty} \frac{1}{T} \int_{0}^{T} x(t)y(t+\tau) \, dt
\]  

(B.7)

The cross-correlation function gives information about the interdependence of two signals.

- **Power Spectral Density**

\[
S_{xx}(\omega) = \frac{1}{2\pi} \int_{-\infty}^{\infty} R_{xx}(\tau)e^{-i\omega\tau} \, d\tau
\]  

(B.8)

The Power Spectral Density (PSD) is the Fourier transform of the autocorrelation function of the signal.

- **Conversion from PSD to RMS**

Usually, a PSD is symmetric about the frequency \( f = 0 \). Therefore the RMS-value of the PSD for a frequency band between \( f_{\text{low}} \) and \( f_{\text{up}} \) is equal to:

\[
\text{RMS} = \sqrt{\int_{f_{\text{low}}}^{f_{\text{up}}} S \, df + \int_{-f_{\text{up}}}^{-f_{\text{low}}} S \, df}
\]  

(B.9)

Since the PSD is symmetric about zero, the concept of the one-sided PSD is introduced, which is in fact the positive part of the PSD, that has an amplitude of twice the original two-sided PSD. For the one-sided PSD the RMS-value in a certain frequency band is:

\[
\text{RMS} = \sqrt{\int_{f_{\text{low}}}^{f_{\text{up}}} S_{\text{one-sided}} \, df}
\]  

(B.10)

One-sided PSDs are commonly used the most.

- **The summation of PSDs**

If a signal \( z \) is equal to \( x + y \), then the PSD of \( z \) is equal to:

\[
S_{zz} = S_{xx} + S_{yy} + S_{xy} + S_{yx}
\]  

(B.11)

If an amount of \( n \) equal PSDs is summed, that are uncorrelated, then the sum of the PSDs is equal to:

\[
S_{\text{total}} = nS_{xx}
\]  

(B.12)

If the PSD’s are 100% correlated, and if the signals are in phase with respect to each other, then the sum of the PSDs is equal to:

\[
S_{\text{total}} = n^2S_{xx}
\]  

(B.13)

- **The summation of RMS-values**

If two signals are uncorrelated, then the two of them are summed quadratically:

\[
x_{Z,RMS} = \sqrt{x_{X,RMS}^2 + x_{Y,RMS}^2}
\]  

(B.14)

If the signals are 100% correlated, and if the signals are in phase with respect to each other, then the sum of them is equal to:

\[
x_{Z,RMS} = x_{X,RMS} + x_{Y,RMS}
\]  

(B.15)
• Nyquist frequency
  The Nyquist frequency is equal to \( \frac{1}{2} \) sampling frequency.

• White noise
  White noise is a random signal that has a constant PSD and an autocorrelation function of zero.

B.2 Calculation of the disturbance force from the gas movement in the sorption compressor cells

The force resulting from the movement of the gas in a compressor cell, is given by [12]:

\[
F = \frac{\dot{m}^2}{\rho A_{\text{out}} \ [10^{-6} \text{ kg m/s}^2]} \tag{B.16}
\]

A tube diameter of 0.61 mm\(^2\) has been used in [12]. From [8] it is known that this diameter will be three times as big in reality, thus it is assumed to be 1.83 mm\(^2\) in this study. For the density, the lowest density of Helium that is present in the cooler has been taken (1.25 kg/m\(^3\)), which is worst-case.

An approximation for the mass-flow has been made from [12]. The mass flow curve has been divided into four intervals, in which it is approached by linear interpolation. This approximated mass flow is then used to calculate the forces at each frequency, using Equation B.16. The values used for the approximation of the mass flow are given in Table B.1.

<table>
<thead>
<tr>
<th>Frequency</th>
<th>(10^{-2})</th>
<th>(10^{-1})</th>
<th>(5 \times 10^{-1})</th>
<th>(5.3)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\dot{m}^2)</td>
<td>1.2</td>
<td>0.3965</td>
<td>5 \times 10^{-2}</td>
<td>10^{-4}</td>
</tr>
</tbody>
</table>

Table B.1: The values of \(\dot{m}^2\) that were used for approximating the mass flow

For a cycle with cycle time \(t_c\), the force from the gas flow has a peak on every frequency that is a multiple of the cycle frequency. Each peak has a power of \(\frac{1}{2} F^2\), where \(F\) is the peak force at a certain frequency. A small frequency band \(B\) contains \(\frac{B}{t_c}\) or \(B t_c\) peaks. Hence, the total power in this frequency band is equal to \(\frac{1}{2} F^2 t_c\). The Force Spectral Density is therefore equal to \(\sqrt{\frac{1}{2} F^2 t_c} \ ([\text{N/\sqrt{Hz}}])\).

In this case the power of one peak is equal to \(F^2\) because the forces that were deduced from the mass flow, are not the peak forces, but the RMS-value of the forces.

For the second stage the worst-case exported force due to the movement of the gas in the cells is equal to about \(0.2 \mu \text{N/\sqrt{Hz}}\) at 1 Hz. At 0.1 Hz, this is about \(6 \mu \text{N/\sqrt{Hz}}\).

B.3 The calculation of the input disturbance force from a valve in the sorption cooler

The forces coming from steel valves are equal to 0.11 \(\mu \text{N RMS}\) at 0.1 Hz, and 0.01 \(\mu \text{N RMS}\) at 1 Hz, for both stages [12]. These forces hold for steel valves of 15 mg. The valves will have a mass of only 6 mg in reality [8]. This reduces the forces to about 0.044 \(\mu \text{N RMS}\) at 0.1 Hz, and 0.004 \(\mu \text{N RMS}\) at 1 Hz. The Force Spectral Density is calculated in the same way as in Appendix B.2. This results in an FSD of about \(1.5 \mu \text{N/\sqrt{Hz}}\) at 0.1 Hz and \(0.14 \mu \text{N/\sqrt{Hz}}\) at 1 Hz.
B.4 The calculation of the input disturbance force from the CFHX

For frequencies between 0.1 and 1 Hz, the force from the zero-inflow into the high-pressure line for a small $t_0$, is equal to [8]:

$$F_{CFHX} = \frac{2t_0^2}{R\tau_C}F_{DC}$$  (B.17)

where:
· $F_{DC} = 0.927$ N is the force that results when the mass flow is zero [12, 8].
· $t_R$ is the desorption period of each single compressor cell, equal to 200 s.
· $t_0$ is the zero-inflow period, equal to 0.5 s.
· $\tau_C$ is a time constant, equal to 686 s.

This leads to an exported maximum force of about 3.3 $\mu$N between 0.1 and 1 Hz. The Force Spectral Density is calculated in the same manner as in Appendix B.2 and B.3, only here, we have to use the factor $\frac{1}{2}$ for calculating the power of one peak. The forces that are given are maximum or peak forces (not RMS). This leads to a constant Force Spectral Density of about $34\mu$N/$\sqrt{Hz}$ between 0.1 and 1 Hz.

From [8], the force at 10 Hz is equal to about $10^{-8}$ N. The corresponding FSD is about $0.1\mu$N/$\sqrt{Hz}$. The other forces are linearly interpolated and extrapolated from the two values at 10 Hz and 1 Hz, which is a good approximation for the Forces given in [8].

B.5 Calculation of the value of the PSD, for a PSD that increases or decreases with a constant slope of $n$ dB/oct

It is given that the PSD has a certain slope of $n$ dB/oct.

- An amount of **decibels** is defined by:
  $$\# dB = 10 \cdot 10^{\log\left(\frac{S_2}{S_1}\right)}$$  (B.18)

  where $S_1$ is a reference value and $S_2$ is the output power.

- An **octave** is the spacing between two frequencies for which holds:
  $$\frac{f_2}{f_1} = 2^1$$  (B.19)

  or for $x$ octaves:
  $$\frac{f_2}{f_1} = 2^x$$  (B.20)

  The amount of octaves in a certain interval is:
  $$\frac{f_2}{f_1} = 2^x \Rightarrow x = \frac{10\log(f_2)}{10\log(2)}$$  (B.21)

- The reduction in $S$ expressed in $n$ dB, if $f$ increases one octave is equal to:
  $$10 \cdot 10^{\log\left(\frac{S(2f)}{S(f)}\right)} = n dB$$  (B.22)
where $S(2f)$ and $S(f)$ are the power spectral densities at a frequency of $2f$ and $f$ respectively, and $n$ is the amount of decibels in one octave.

\[
\Rightarrow \frac{S(2f)}{S(f)} = 10^n \text{(B.23)}
\]

- Because the amount of dB in one octave is $n$, and the amount of octaves per frequency band is $x$, then the amount of dB in an arbitrary frequency band $f_1 - f_2$ is equal to $x \cdot n$ and. Using the definition of the decibel (Equation B.18) it follows:

\[
x \cdot n = 10 \cdot 10^\log\left(\frac{S(f_2)}{S(f_1)}\right) \text{(B.24)}
\]

or finally:

\[
S(f_2) = S(f_1) 10^{\left(\frac{10\log(f_2)}{10\log(2)}\cdot n\right)} \text{(B.25)}
\]

where $n$ is the given slope in $\#$ dB/oct [25].

### B.6 Normal mode analysis

#### B.6.1 Eigenvalues

The basic equations we start from are the same as Equation 2.23 (Section 2.5), except that the forcing function $F$ is set to zero:

\[
M\ddot{x}(t) + Kx(t) = 0 \text{ (B.26)}
\]

or:

\[
\begin{bmatrix}
m_1 & 0 \\
0 & m_2
\end{bmatrix}
\begin{bmatrix}
x_1 \\
x_2
\end{bmatrix} +
\begin{bmatrix}
k(1+i\eta) & -k(1+i\eta) \\
-k(1+i\eta) & k(1+i\eta)
\end{bmatrix}
\begin{bmatrix}
x_1 \\
x_2
\end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \text{ (B.27)}
\]

Substitution of $x = z e^{i\omega t}$ yields:

\[
-\omega^2 Mz + Kz = 0 \text{ (B.28)}
\]

or:

\[
(K - \omega^2 M)z = 0 \text{ (B.29)}
\]

The trivial solution of Equation B.29 corresponds to $z = 0$. The nontrivial solution is found when the determinant of $(K - \omega^2 M)$ is equal to zero:

\[
\begin{vmatrix}
k(1+i\eta) - \omega^2 m_1 & -k(1+i\eta) \\
-k(1+i\eta) & k(1+i\eta) - \omega^2 m_2
\end{vmatrix} = 0 \text{ (B.30)}
\]

or:

\[
\begin{cases}
(k(1+i\eta) - \omega^2 m_1)(k(1+i\eta) - \omega^2 m_2) - k^2(1+i\eta)^2 = 0 \\
\omega^2 (m_1 m_2) \omega^2 - k(1+i\eta)(m_1 + m_2) = 0
\end{cases} \text{ (B.31)}
\]

The solution yields two eigenfrequencies:

\[
\omega_1 = 0 \text{ (B.32)}
\]

and

\[
\omega_2 = \sqrt{\frac{k(1+i\eta)(m_1 + m_2)}{m_1 m_2}} \text{ (B.33)}
\]
### B.6.2 Eigenvectors

For finding the eigenvectors, we start from Equation B.29:

\[
\begin{align*}
(k(1 + i\eta) - \omega^2 m_1) \cdot z_1 - k(1 + i\eta) \cdot z_2 &= 0 \\
-k(1 + i\eta) \cdot z_1 + (k(1 + i\eta) - \omega^2 m_2) \cdot z_2 &= 0
\end{align*}
\]

(B.34)

\(z_1\) will be taken as a reference and \(z_2\) is written as function of \(z_1\) from Equation B.34\(^1\).

\[
z_2 = \frac{k(1 + i\eta) - \omega^2 m_1}{k(1 + i\eta)} \cdot z_1
\]

(B.35)

For finding the eigenmodes or eigenvectors, the eigenfrequencies corresponding to the particular mode are filled in, in the previous equation.

For \(\omega_1\) the rigid body mode is obtained:

\[
\begin{align*}
\omega_1^2 &= 0 \\
\downarrow \\
z_2 &= z_1 \\
\downarrow \\
z_1 &= \begin{bmatrix} 1 \\ 1 \end{bmatrix}
\end{align*}
\]

(B.36)

\(\omega_2\) yields:

\[
\begin{align*}
\omega_2^2 &= \frac{k(1+i\eta)(m_1+m_2)}{m_1m_2} \\
\downarrow \\
z_2 &= -\frac{m_1}{m_2}z_1 \\
\downarrow \\
z_2 &= \begin{bmatrix} -1 \\ m_2 \\ m_1 \end{bmatrix}
\end{align*}
\]

(B.37)

The Modal matrix is a matrix of which the columns consist of the eigenvectors:

\[
z = \begin{bmatrix} 1 & 1 & \end{bmatrix}
\]

(B.38)

### B.6.3 Uncoupling the equations of motion

Equation B.27 is coupled and will be uncoupled by a transformation from the Physical coordinate system, to the Principal coordinate system [33]. This uncoupling is done by performing the following two operations:

\[
\begin{align*}
M_N &= z_N^T M z_N \\
K_N &= z_N^T K z_N
\end{align*}
\]

(B.39)

Before this operation is done, the modal matrix \(z\) needs to be normalized to the mass, \(z_N\). This is done according to Equation B.40 [33].

\[
z_N^T M z_N = 1.0
\]

(B.40)

For each eigenvector \(z_i\), the normalized eigenvector is equal to [33]:

\[
z_{Ni} = \frac{z_i}{\sqrt{z_i^T M z_i}} = \frac{z_i}{q_i}
\]

(B.41)

\(^1\)Which one of the two equations is not important, both give the same results.
Thus:

\[ q_1 = \sqrt{(z_{11})^2 m_1 + (z_{21})^2 m_2} = \sqrt{m_1 + m_2} \quad \text{(B.42)} \]

and

\[ q_2 = \sqrt{(z_{12})^2 m_1 + (z_{22})^2 m_2} = \sqrt{\frac{m_1}{m_2} (m_1 + m_2)} \quad \text{(B.43)} \]

The normalized modal matrix becomes:

\[ z_N = \begin{pmatrix} \frac{1}{q_1} & 1 - \frac{\sqrt{m_2}}{m_1} \\ \frac{1}{q_2} & 1 - \frac{\sqrt{m_1}}{m_2} \end{pmatrix} \quad \text{(B.44)} \]

or:

\[ z_N = \frac{1}{\sqrt{m_1 + m_2}} \begin{pmatrix} 1 & \sqrt{\frac{m_2}{m_1}} \\ 0 & 1 - \sqrt{\frac{m_1}{m_2}} \end{pmatrix} \quad \text{(B.45)} \]

If

\[ z_N = \begin{pmatrix} z_{N11} & z_{N12} \\ z_{N21} & z_{N22} \end{pmatrix} \quad \text{(B.46)} \]

then for each component of \( z_N \), the first index indicates the mass that is moved, while the second index indicates the particular mode that contributes to the movement.

Substituting Equation B.45 in Equations B.39 yields:

\[ M_N = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \]

\[ K_N = \begin{pmatrix} 0 & 0 \\ 0 & k (1 + i\eta) (m_1 + m_2) \end{pmatrix} \]

(B.47)

Remark that the elements on the diagonal of the \( K_N \) matrix are equal to the square of the eigenfrequencies, which is also an indication of the correctness of Equation B.45.

The equations of motion in the principal coordinate system are now:

\[ \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} \ddot{x}_{p1} \\ \ddot{x}_{p2} \end{pmatrix} + \begin{pmatrix} 0 & 0 \\ 0 & \frac{k (1 + i\eta) (m_1 + m_2)}{m_1 m_2} \end{pmatrix} \begin{pmatrix} x_{p1} \\ x_{p2} \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \quad \text{(B.48)} \]

These two uncoupled equations can be solved independently in the principal coordinate system, and each one of them represents one mode.

In the case where the force on the system is not equal to zero, this force, and (if applicable) the initial conditions must be transformed in the new coordinate system too [33]. We start from Equation 2.23.

\[ M \ddot{x}(t) + Kx(t) = F(t) \quad \text{(B.49)} \]

Premultiplication by \( z_N^T \):

\[ z_N^T M \ddot{x}(t) + z_N^T Kx(t) = z_N^T F(t) \quad \text{(B.50)} \]

Insertion of the unit matrix \( I = z_N z_N^{-1} \):
\[ z_N^T M (z_N z_N^{-1}) \ddot{x}(t) + z_N^T K (z_N z_N^{-1}) x(t) = z_N^T F(t) \] (B.51)

Substitution of \( M_N = z_N^T M z_N \) and \( K_N = z_N^T K z_N \):

\[ M_N z_N^{-1} \ddot{x}(t) + K_N z_N^{-1} x(t) = z_N^T F(t) \] (B.52)

Thus for the principal coordinate system holds:

\[
\begin{align*}
\ddot{x}_p &= z_N^{-1} \ddot{x}(t) \\
x_p &= z_N^{-1} x(t)
\end{align*}
\] (B.53)

The initial conditions are transformed using:

\[
\begin{align*}
x_{p,0} &= z_N^{-1} \dot{x}_0(t) \\
x_{p,0} &= z_N^{-1} x_0(t)
\end{align*}
\] (B.54)

The input forces are transformed using:

\[ F_p = z_N^T F \] (B.55)

or

\[
\begin{align*}
F_{p1} &= z_{NT_{11}} F_1 + z_{NT_{12}} F_2 \\
F_{p2} &= z_{NT_{21}} F_1 + z_{NT_{22}} F_2
\end{align*}
\] (B.56)

The Power Spectral Densities are transformed in a similar manner as the forces, taking into account the cross-correlation [54]:

\[ S_{F_p} = z_N^T S_F z_N \] (B.57)

where:

\[
S_{F_p} = \begin{bmatrix} S_{F_{p11}} & S_{F_{p12}} \\ S_{F_{p21}} & S_{F_{p22}} \end{bmatrix} \quad S_F = \begin{bmatrix} S_{F11} & S_{F12} \\ S_{F21} & S_{F22} \end{bmatrix}
\] (B.58)

\( S_{F12} \) and \( S_{F21} \) are the cross-correlations of \( S_{F11} \) and \( S_{F22} \). However, in most of the cases the two forces \( F_1 \) and \( F_2 \) are not correlated and thus \( S_{F12} = S_{F21} = 0 \). Notice that \( S_{Fp12} \) and \( S_{Fp21} \) are different from zero.
B.6.4 Computation of the transfer functions in the principal coordinate system

The uncoupled equations of motion in the principal coordinate system are:

\[ M_N \ddot{x}_p(t) + K_N x_p(t) = F_p(t) \]

\[
\begin{bmatrix}
1 & 0 \\
0 & 1 \\
\end{bmatrix}
\begin{bmatrix}
\ddot{x}_{p1} \\
\ddot{x}_{p2} \\
\end{bmatrix}
+ 
\begin{bmatrix}
0 & k(1+i\eta)(m_1+m_2) \\
\frac{k(1+i\eta)(m_1+m_2)}{m_1m_2} & 0 \\
\end{bmatrix}
\begin{bmatrix}
x_{p1} \\
x_{p2} \\
\end{bmatrix}
= 
\begin{bmatrix}
F_{p1} \\
F_{p2} \\
\end{bmatrix}
\]

(B.59)

The equations can be solved using the same method as described in Section 2.5.3 for the rigid body motion (the transfer function method). After taking the Fourier transform the equations B.59 become:

\[
\begin{cases}
-\omega^2 X_{p1} = F(\omega)_{p1} \\
(-\omega^2 + \frac{k(1+i\eta)(m_1+m_2)}{m_1m_2})X_{p2} = F(\omega)_{p2}
\end{cases}
\]

For the first mode (\(\omega_1 = 0\)) the transfer function in the principal coordinate system is:

\[ H(\omega)_{p11} = \frac{X_{p1}}{F_{p1}} = -\frac{1}{\omega^2} \]

(B.61)

For the second mode (\(\omega_2 = \sqrt{\frac{k(1+i\eta)(m_1+m_2)}{m_1m_2}}\)) the transfer function in the principal coordinate system is:

\[ H(\omega)_{p22} = \frac{X_{p2}}{F_{p2}} = \frac{m_1m_2}{k(1+i\eta)(m_1+m_2) - \omega^2m_1m_2} \]

(B.62)

or in the complex notation \((a + bi)\):

\[ H(\omega)_{p22} = \frac{m_1m_2(km_1 + km_2 - \omega^2m_1m_2)}{(km_1 + km_2 - \omega^2m_1m_2)^2 + (k(\eta m_1 + m_2))^2} + \frac{k\eta(m_1 + m_2)m_1m_2}{(km_1 + km_2 - \omega^2m_1m_2)^2 + (k(\eta m_1 + m_2))^2} \cdot i \]

(B.63)

and the matrix \(H_p\) is:

\[ H_p = \begin{pmatrix}
\frac{X_{p1}}{F_{p1}} & 0 \\
0 & \frac{X_{p2}}{F_{p2}}
\end{pmatrix} \]

(B.64)

For the uncoupled equations in the principal coordinate system, \(H_{p12}\) and \(H_{p21}\) are zero.
B.6.5 Computation of the output PSD in the principal coordinate system

For the Power Spectral Densities holds [58, 54]:

\[ S_{X_{p}} = H_{p}S_{F_{p}}H_{p}^{*} \]  

(B.65)

where \( H_{p}^{*} \) is the complex conjugate of \( H_{p} \), and

\[
S_{X_{p}} = \begin{bmatrix}
S_{X_{p}11} & S_{X_{p}12} \\
S_{X_{p}21} & S_{X_{p}22}
\end{bmatrix}
\quad S_{F_{p}} = \begin{bmatrix}
S_{F_{p}11} & S_{F_{p}12} \\
S_{F_{p}21} & S_{F_{p}22}
\end{bmatrix}
\]  

(B.66)

In the principal coordinate system, where \( H_{p12} \) and \( H_{p21} \) are zero, Equation B.65 becomes:

\[
\begin{cases}
S_{X_{p}11} = H_{p11}H_{p11}^{*}S_{F_{p}11} \\
S_{X_{p}12} = H_{p11}H_{p22}^{*}S_{F_{p}12} \\
S_{X_{p}21} = H_{p22}H_{p11}^{*}S_{F_{p}21} \\
S_{X_{p}22} = H_{p22}H_{p22}^{*}S_{F_{p}22}
\end{cases}
\]  

(B.67)

B.6.6 Back-transforming

In Section B.6.3 it was shown that the relationship between the physical coordinates and the principal coordinates is:

\[ x_{p} = z_{N}^{-1}x(t) \]  

(B.68)

Premultiplication by \( z_{N} \) yields:

\[ z_{N}x_{p} = z_{N}(z_{N}^{-1}x) \]  

(B.69)

Substitution of \( I = z_{N}z_{N}^{-1} \) yields the equation used for backtransforming the principal coordinates into the physical coordinates:

\[ x = z_{N}x_{p} \]  

(B.70)

Backtransforming the displacement Power Spectral Densities is done in a similar manner as the displacements (also compare this to Equation B.57):

\[ S_{X} = z_{N}S_{X_{p}}z_{N}^{T} \]  

(B.71)
Appendix C

The space environment

C.1 The solar-terrestrial environment

Sunspot numbers and UV flux

Wolf’s system of deducing the solar activity according to the amount of sunspots works as described in the following. For the sunspot activity number holds: \( R = k(10g + s) \). In this formula, \( s \) is the number of individual spots, \( g \) is the number of sunspot groups and \( k \) is the “observatory factor”. As Wolf was observing from Zurich, the observatory factor is equal to one for the Zurich observatory. Nowadays the sunspot numbers from a worldwide network of observatories are combined into one daily international sunspot number \( R_i \). Also monthly sunspot numbers \( R_k \) are calculated and a twelve-month mean sunspot number \( R_{12} \) exists that is related to the solar flux at a wavelength of 10.7 cm (\( F_{10,7} \) index) [4]. The \( F_{10,7} \) index has a strong correlation with the UV flux from the Sun, which determines the amount of expansion of the Earth’s atmosphere during solar active periods. The latter is important for the amount of drag experienced by satellites in LEO. The radio emission from the Sun at 10.7 cm can be measured on the Earth as well as the sunspot number, while the amount of UV-radiation cannot. Therefore especially \( F_{10,7} \) is commonly used for determining the solar activity.

\[
R_{12} = \frac{1}{12} \sum_{n=5}^{n+5} (R_k) + \frac{1}{2} \cdot (R_{(n+6)} + R_{(n-6)}) \quad (C.1)
\]

\[
F_{12} = 63.7 + 0.728 R_{12} + 8.9 \times 10^{-4} R_{12}^2 \quad (C.2)
\]
Properties of the solar wind, CMEs and flares

Table C.1 lists some solar wind parameters, adapted from [48] while the characteristics of CMEs and flares are given in Table C.2.

<table>
<thead>
<tr>
<th>Quantity</th>
<th>Unit</th>
<th>General Solar wind</th>
<th>Fast SW</th>
<th>Slow SW</th>
</tr>
</thead>
<tbody>
<tr>
<td>$N_p$ (number of protons)</td>
<td>cm$^{-3}$</td>
<td>$5e^-/5p^+$</td>
<td>3$p^+$</td>
<td>10.7$p^+$</td>
</tr>
<tr>
<td>$V_p$ (proton speed)</td>
<td>km/s</td>
<td>400 - 800</td>
<td>750</td>
<td>348</td>
</tr>
<tr>
<td>$F_p^+ = N_p \cdot V_p$ (proton flux)</td>
<td>m$^{-2}$s$^{-1}$</td>
<td>$10^{12} - 10^{13}$</td>
<td>1.99 $10^{12}$</td>
<td>3.66 $10^{12}$</td>
</tr>
<tr>
<td>$T_p^+$ (proton temperature)</td>
<td>K</td>
<td>120 000</td>
<td>280 000</td>
<td>55 000</td>
</tr>
<tr>
<td>$T_e^-$ (electron temperature)</td>
<td>K</td>
<td>140 000</td>
<td>130 000</td>
<td>190 000</td>
</tr>
<tr>
<td>$N_{He^+}/N_p^+$ (ratio of Helium and protons)</td>
<td></td>
<td>0.036 (constant)</td>
<td>0.025 (variable)</td>
<td></td>
</tr>
</tbody>
</table>

Table C.1: Some solar wind properties, taken from [48]. The proton and electron temperatures however, differ from those in [24] and therefore need to be reconsidered.
## Flare

**What is it**

sudden big release of energy from active region on the Sun, often radio bursts associated after flare; snow effect: particles hit the spacecraft (s/c)

**Constituents**

$10^{15}$ gram, 5–50 billion tons (halo CME) (E)UV, X-ray, radio noise is higher

**speed**

(200–) 2000 km/s speed of light

**duration**

several days 1–2 hours

**energy**

10 MeV [48] up to 100 MeV (10 MeV [48], $10^{25}$ J)

**frequency**

at solar$_{\text{MAX}}$: amount of CMEs is larger, largest amount just before and just after solar$_{\text{MAX}}$

solar$_{\text{MAX}}$: several per hour

solar$_{\text{MIN}}$: still more than 10 X-ray microflares per hour

**Interplanetary**

ICME

**Consequences**

disturbance of Earth magnetic field causes geomagnetic storm, s/c desorientation, false sensor readings, s/c damage (proton events in detector and electronics), s/c charge and drag aurora

disturbance of atmosphere

### Table C.2: Comparison of CMEs and Flares.

<table>
<thead>
<tr>
<th>What is it</th>
<th>CME</th>
<th>Flare</th>
</tr>
</thead>
<tbody>
<tr>
<td>low-medium energetic flux of particles/plasma erupted from the corona protons (p+)</td>
<td>sudden big release of energy from active region on the Sun, often radio bursts associated after flare; snow effect: particles hit the spacecraft (s/c)</td>
<td></td>
</tr>
<tr>
<td>Constituents</td>
<td>$10^{15}$ gram, 5–50 billion tons (halo CME)</td>
<td>(E)UV, X-ray, radio noise is higher</td>
</tr>
<tr>
<td>speed</td>
<td>(200–) 2000 km/s</td>
<td>speed of light</td>
</tr>
<tr>
<td>duration</td>
<td>several days</td>
<td>1–2 hours</td>
</tr>
<tr>
<td>energy</td>
<td>10 MeV [48]</td>
<td>up to 100 MeV (10 MeV [48], $10^{25}$ J)</td>
</tr>
<tr>
<td>frequency</td>
<td>at solar$<em>{\text{MAX}}$: amount of CMEs is larger, largest amount just before and just after solar$</em>{\text{MAX}}$</td>
<td>solar$<em>{\text{MAX}}$: several per hour, solar$</em>{\text{MIN}}$: still more than 10 X-ray microflares per hour</td>
</tr>
<tr>
<td>Interplanetary</td>
<td>ICME</td>
<td></td>
</tr>
<tr>
<td>Consequences</td>
<td>disturbance of Earth magnetic field causes geomagnetic storm, s/c desorientation, false sensor readings, s/c damage (proton events in detector and electronics), s/c charge and drag aurora</td>
<td>disturbance of atmosphere</td>
</tr>
<tr>
<td>geo-effectiveness</td>
<td>halo CME most effective</td>
<td>flares not always geo-effective</td>
</tr>
</tbody>
</table>
C.2 The magnetic environment

Geomagnetic indices

This Appendix is a glossary of the most common geomagnetic indices, from [22, 70].

- **The K index**: This index gives an indication of the general magnetic activity relative to a “quiet day”. The magnetic activity is measured during three hours at a certain local observatory. The K index has a quasi-logarithmic scale, going up from 0 to 9 in steps of \((\frac{1}{3})^d\) (Table C.3).

- **The Kp index**: This is the mean of the K indices of a network of 13 observatories. This index is thus also taken over a 3-hour interval. It is sometimes called the planetary 3-hour range index.

- **The Ap index**: The Ap index or planetary 3-hour range equivalent index is derived from Kp. Its relation to Kp is given in table C.3. The Ap-index has a scale going up from 0 to 400. The Ap index also indicates the disturbance of the geomagnetic field directly by multiplying it by 2. For example, if \(K_p = 4^+\) then \(a_p = 32\) and the variation in the magnetic field is 64 nT.

- **The Ap index**: The Ap index is the average of all ap indices in one day (thus an average of 8 ap values). It is also called the geomagnetic index or equivalent daily amplitude. Its units are the same as for the ap index (2 nT).

- **The Ak index**: The Ak index is the daily station index, which is essentially Ap for only one specific station.

- **TheDst index**: The Dst index, or disturbance storm-time index is a measure of the Terrestrial ring current or “equatorial electrojet”. It is measured by observatories around the equator. It has a correlation with the daily planetary index Ap, except that after a storm Dst recovers a little bit slower. This indicates that the ring current disappears slower than the disturbance in the polar regions due to a storm.

- **The F10.7 index**: The “10.7 cm solar radiation flux” (adjusted to 1 AU) is a measure of the solar output at a wavelength of 10.7 cm (2800 MHz). It has a strong correlation with the UV-radiation of the Sun and thus also with the changes in the upper atmosphere (expansion). It is also a good measure for the activity of the Sun, and is correlated with the solar sunspot number \(R\) in the following manner:

\[
F_{10.7} = 63.7 + 0.728R + 8.9 \times 10^{-4}R^2 \quad (C.3)
\]

The \(F_{10.7}\) index is given in units of \(10^4\) Jansky or \(10^{-22}\) J/sm²Hz. The index is often corrected for the difference in Sun-Earth distance throughout the year. This is sometimes referred to as the S index, and is most interesting for solar physicists. For the purpose of this study the real observed flux is more important (the S index). The change in solar flux throughout the solar cycle clearly indicates two components: a slowly-varying one of the “quiet Sun” and the more impulsive one coming from the “active Sun”, on shorter terms.

- **The AE index**: The Auroral electrojet index is a measure of the magnetic activity in the auroral zones.

In [70] some reference index values are given for a long-term interval (between 1 day and one month) and for a short-term interval (less than a day). In our case we could hang on to the long-term index for average magnetic field disturbance conditions, while the short-term case could be used to look at the more extreme conditions. The case of high-frequency disturbances (>1Hz) in magnetic field still needs to be examined later on in this graduation study.
\[
\begin{array}{cccccccccc}
K_p & 0 & 0_+ & 1_- & 1_0 & 1_+ & 2_- & 2_0 & 2_+ & 3_- & 3_0 & 3_+ & 4_- & 4_0 & 4_+ \\
K_p & 5_- & 5_0 & 5_+ & 6_- & 6_0 & 6_+ & 7_- & 7_0 & 7_+ & 8_- & 8_0 & 8_+ & 9_- & 9_0 \\
a_p & 0 & 2 & 3 & 4 & 5 & 6 & 7 & 9 & 12 & 15 & 18 & 22 & 27 & 32 \\
a_p & 39 & 48 & 56 & 67 & 80 & 94 & 111 & 132 & 154 & 179 & 207 & 236 & 300 & 400 \\
\end{array}
\]

Table C.3: Conversion of the \(K_p\) index into the \(a_p\) index [22, 70].

<table>
<thead>
<tr>
<th>(K_p)</th>
<th>(a_p)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>0_+</td>
<td>2</td>
</tr>
<tr>
<td>1_-</td>
<td>3</td>
</tr>
<tr>
<td>1_0</td>
<td>4</td>
</tr>
<tr>
<td>1_+</td>
<td>5</td>
</tr>
<tr>
<td>2_-</td>
<td>6</td>
</tr>
<tr>
<td>2_0</td>
<td>7</td>
</tr>
<tr>
<td>2_+</td>
<td>9</td>
</tr>
<tr>
<td>3_-</td>
<td>12</td>
</tr>
<tr>
<td>3_0</td>
<td>15</td>
</tr>
<tr>
<td>3_+</td>
<td>18</td>
</tr>
<tr>
<td>4_-</td>
<td>22</td>
</tr>
<tr>
<td>4_0</td>
<td>27</td>
</tr>
<tr>
<td>4_+</td>
<td>32</td>
</tr>
<tr>
<td>5_-</td>
<td>39</td>
</tr>
<tr>
<td>5_0</td>
<td>48</td>
</tr>
<tr>
<td>5_+</td>
<td>56</td>
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<td>6_-</td>
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<td>6_+</td>
<td>94</td>
</tr>
<tr>
<td>7_-</td>
<td>111</td>
</tr>
<tr>
<td>7_0</td>
<td>132</td>
</tr>
<tr>
<td>7_+</td>
<td>154</td>
</tr>
<tr>
<td>8_-</td>
<td>179</td>
</tr>
<tr>
<td>8_0</td>
<td>207</td>
</tr>
<tr>
<td>8_+</td>
<td>236</td>
</tr>
<tr>
<td>9_-</td>
<td>300</td>
</tr>
<tr>
<td>9_0</td>
<td>400</td>
</tr>
</tbody>
</table>

Table C.4: The reference values for \(A_p\) [22].

**Coordinate transformation to geomagnetic coordinates**

This appendix gives the equations needed to transform geographical coordinates into geomagnetic coordinates [61].

\[
\sin(\phi) = \sin(\Phi) \cdot \sin(\Phi_0) + \cos(\Phi) \cdot \cos(\Phi_0) \cdot \cos(\Lambda - \Lambda_0) \tag{C.4}
\]

\[
\sin(\lambda) = \frac{\cos(\Phi) \cdot \sin(\Lambda - \Lambda_0)}{\cos(\phi)} \tag{C.5}
\]

\[
\cos(\lambda) = \frac{\sin(\Phi_0) \cdot \cos(\Phi) \cdot \cos(\Lambda - \Lambda_0) - \sin(\Phi) \cdot \cos(\Phi_0)}{\cos(\phi)} \tag{C.6}
\]

\[
\cos(\lambda) = \frac{\sin(\Phi_0) \cdot \cos(\Phi) \cdot \cos(\Lambda - \Lambda_0) - \sin(\Phi) \cdot \cos(\Phi_0)}{\cos(\phi)} \tag{C.7}
\]

In these formulas \(\Phi\) and \(\Lambda\) are the geographical coordinates and \(\phi\) and \(\lambda\) are the geomagnetic coordinates. \(\Phi_0\) and \(\Lambda_0\) refer to the position of the geomagnetic axis with respect to the geographical axis.

**The magnetic field model of the Earth**

A basic approximation of the magnetic field of the Earth especially holds for middle-low altitudes and it is called the “ideal dipole model”. In this model, the magnetic field strength is assumed as a simple dipole: the field strength does not depend on the longitude, and influences from outside the Earth are not included. Apart from the dipole component, higher order terms are not taken in account. According to this model the magnetic potential of the Earth is equal to [61]:

\[
V = \frac{M \cdot \hat{r}}{r^3} = -\frac{M}{r^2} \cdot \sin(\phi) \tag{C.8}
\]

in which \(M\) is the Earth’s magnetic moment \((M = 7.9 \times 10^{30} \text{ nTcm}^3 [22])\), \(r\) is the radial distance and \(\phi\) is the geomagnetic latitude\(^1\). In the radial and tangential direction, the strength of the Earths magnetic potenital is equal to [61]:

\[
Z = \frac{\delta V}{\delta r} = \frac{2 \cdot M \cdot \sin(\phi)}{r^3} \tag{C.9}
\]

\[
H = \frac{1}{r} \cdot \frac{\delta V}{\delta \phi} = -\frac{M \cos(\phi)}{r^3} \tag{C.10}
\]

\(^1\)In all previous formulas geomagnetic coordinates are used. These can be deduced from the geographic coordinates and the position of the magnetic dipole axis by means of the relations as given in Appendix C.2. 
For the magnetic field strength of the dipolar magnetic field holds [61]–[73]:

\[ B = \nabla V \]  
(C.11)

or

\[ B^2 = Z^2 + H^2 \]  
(C.12)

and

\[ B = \frac{M}{r^3} \cdot \sqrt{1 + 3 \cdot \sin^2(\phi)} \]  
(C.13)

From this Equation it follows that at the geomagnetic equator, the magnetic field strength will be:

\[ B_e = H_e = \frac{M}{R_e^3} \]  
(C.14)

\( (R_e \) is the mean Earth radius) while at the poles the magnetic field strength is:

\[ B_p = Z_p = 2 \cdot \frac{M}{R_e^3} \]  
(C.15)

The slope of the magnetic field line is defined by the quotient of \( H \) and \( Z \):

\[ \frac{H}{Z} = \frac{r \cdot \delta \phi}{\delta r} = -\frac{1}{2} \cdot \frac{1}{\tan(\phi)} \]  
(C.16)

or:

\[ \frac{1}{r} \delta r = -2 \cdot \tan(\phi) \delta \phi \]  
(C.17)

Integrating this equation yields [73]:

\[ r = L \cdot R_e \cdot \cos^2(\phi) \]  
(C.18)

in which \( L \) is a constant of integration. The different magnetic field lines are thus described by the constant \((L \cdot R_e)\). At the equator, \( L \) is a measure of the distance of the field line to the Earth’s magnetic field axis. The shape of the magnetic field lines is determined by \( \cos^2(\phi) \). However, the magnetic field along a field line is not constant. For its strength along a field line one can substitute equation (C.18) into equation (C.13):

\[ B = \frac{M}{R_e^3} \cdot \frac{1}{L^3} \cdot \sqrt{1 + 3 \cdot \sin^2(\phi) \cdot \cos^6(\phi)} \]  
(C.19)

The Earth’s geomagnetic field can thus be described with only two parameters: the field strength \( B \), and the field line \( L \). These geomagnetic coordinates are very often used when considering the geomagnetic environment or defining the position of e.g. the Earth’s radiation belts.

A more complicated and precise model is often used too, in which one makes use of the spherical harmonics expansion of the magnetic potential \( V^2 \). This model will mostly be used for high-precision calculations at lower and higher altitudes. In contrast with the simple dipolar model, this model does take into account the higher order components of the intrinsic magnetic field, its change with longitude, and influences from outside the Earth.

\(^2\)Compare this with the spherical harmonics expansion of the Earth’s gravity field.
However, at altitudes higher than 3 $R_e$, a separate external model should be used \[70\]. In the spherical harmonics expansion model $V$ is defined as:

$$V = R_e \sum_{n=1}^{\infty} \sum_{m=1}^{\infty} P_n^m \left( \sin(\phi) \right) \cdot \left( \frac{R_e}{r} \right)^{(n+1)} \cdot \left( g_n^m \cos(m\lambda) + h_n^m \sin(m\lambda) \right) + \left( \frac{2}{r} \right)^{-n} \left( A_n^m \cos(m\lambda) + B_n^m \sin(m\lambda) \right)$$

where:

- $r$ = geomagnetic radial distance
- $\phi$ = geomagnetic latitude
- $\lambda$ = geomagnetic longitude
- $R_e$ = radius of the Earth (for the International Geomagnetic Reference field $R_e=6371.2$ km)
- $P_n^m (\cos(\theta))$ = Legendre polynomials (sometimes called “Schmidt functions”)
- $g_n^m, h_n^m, A_n^m, B_n^m$: Legendre/Schmidt coefficients, of which $g_n^m$ and $h_n^m$ refer to the magnetic field sources inside the Earth, and $A_n^m$ and $B_n^m$ refer to external magnetic field sources or disturbances.

The International Geomagnetic Reference Field uses this representation of the magnetic potential of the Earth \[22\].

Variations in the Earth’s magnetic field strength

The Earth’s magnetic field contains several internal and external sources of variation and disturbances, of which the external ones are the most variable on the short term. The Earth rotation must be taken into account in the first place: while a person standing on the Earth’s surface enters the night-side of the Earth, the magnetotail part of the magnetosphere is experienced at the same time. Seasonal variations in magnetic field strength are a result of the position of the Earth’s magnetic dipole axis that is not perpendicular to the ecliptica. The fact that the dipole axis does not cross the geographic center of the Earth but is shifted towards one side, results in a stronger magnetic field at the surface on one side of the Earth than on the other side. This major longitudinal variation in magnetic field strength is called the South Atlantic Anomaly. It will has to be taken into account when working on LEO-satellites. Also smaller longitudinal and latitudinal variations are included in the spherical harmonics model of the internal magnetic field.

Furthermore the magnetic field strength will depend on the distance above the Earth’s surface. The further away from the Earth, the weaker the magnetic field becomes. These internal anomalies are all included in the spherical harmonics expansion model of the magnetic field. In some cases also the external sources of variation are included \[70\]. These variations are a result of the interaction of the (variable) solar wind with the magnetosphere, and the plasma currents that occur in the magnetosphere.

### C.3 The plasma environment

#### Single particle motion

The motion of a single plasma particle is discussed when it is subject to an external magnetic and/or electric field. This is the case for the plasma surrounding the Earth. Not only a charged particle will be influenced by these fields, but also a charged spacecraft’s motion can possibly be affected. The particle motion also has its consequences for how satellites become charged.

Three basic motions of the charged particle are distinguished. We start from the summation of Coulomb’s law and Lorentz law. A charged particle moving through an electromagnetic
field experiences a force equal to [73, 61]:

$$\vec{F} = m \cdot \vec{a} = q(\vec{E} + \vec{v} \times \vec{B})$$  \hspace{1cm} (C.21)

where $q$ is the particle charge, $\vec{E}$ is the electric field, $\vec{v}$ is the particle velocity and $\vec{B}$ is the magnetic field.

**Cyclotron movement**

Imagine there is no electric field, but only a magnetic field through which a particle with charge $q$ moves. This charged particle will undergo a force that only affects its trajectory, not the magnitude of its velocity. Hence, the particle will gyrate about a magnetic field line. This motion is called the cyclotron movement. The radius of gyration ($Larmor$ radius) is equal to:

$$r_{\text{Larmor}} = \frac{mv_{\perp}}{qB}$$  \hspace{1cm} (C.22)

This radius corresponds to a cyclotron gyration frequency [73, 61]:

$$f_c = \frac{1}{2\pi} \left( \frac{qB}{m} \right)$$  \hspace{1cm} (C.23)

**Drift movement**

If there is also an electric field, or another arbitrary force perpendicular to the magnetic field, the particle will accelerate or slow down when it moves into the direction of the electric field line. The particle slows down as it moves upwards, and it speeds up as it moves downwards. This, together with the cyclotron movement, results in a drift to the right or left according to the sign of the charge of the loaded particles. The drift velocity is equal to [73, 61]:

$$\vec{v}_d = \frac{\vec{E} \times \vec{B}}{B^2}$$  \hspace{1cm} (C.24)

and for an arbitrary perpendicular force [73, 61]:

$$\vec{v}_d = \vec{F}_\perp \times \frac{\vec{B}}{qB^2}$$  \hspace{1cm} (C.25)

These drifts also occur in special cases like if there is a gradient in the magnetic field or when the field lines are curved [5]. In case of the Earth, electrons and ions will drift in opposite directions due to their opposite charge signs.

**Magnetic mirroring or bouncing**

The magnetic field lines of the Earth are not parallel at all places, but they converge near the poles. These field lines can be analyzed by considering a parallel component of a field line with respect to the other one ($B_p$) and a component perpendicular to them ($B_n$). Due to the Lorentz force, $B_p$ will lead to the cyclotron movement of charged particles while $B_n$ will cause a Lorentz force that is parallel with the direction of $B_p$. If the magnetic field lines are converging, this Lorentz force will tend to pull the particles back (slow down), if the magnetic field lines are diverging, the particles are accelerated. At a certain point above the poles, the field lines are converging such that a particle is completely stopped and its motion is reversed. This point is called the mirroring point. We say that a particle is trapped between two mirroring points. In theory a particle is trapped in the magnetic field forever, but in reality particles can collide, and escape the bouncing movement.
These three mechanisms are the cause of the so-called Van Allen radiation belts that surround the Earth. These belts were discovered in 1958 with the Explorer mission, lead by James van Allen. The radiation belts are also called trapped radiation belts due to the magnetic mirroring. Not too much attention is paid to these belts, as none of the Darwin or EFFDM spacecrafts will encounter them. Still they might be useful for understanding the complete picture.

The plasma environment in Low Earth Orbit: the ionosphere

The ionosphere is the part of the atmosphere, located roughly between an altitude of 60 to 1000 km, depending on the solar activity. Many textbooks define it in a slightly different altitude range, but the most important fact is that the satellites flying in a Low Earth Orbit fly through the ionosphere. In the ionosphere, the Extreme Ultraviolet radiation from the Sun causes the atoms to split up into ions and electrons (photo-ionization). Hence, the ionosphere is a mix of atmospheric neutrals and the UV-induced plasma. Actually it is a transition region between the neutral atmosphere and the fully ionized space plasma of the plasmasphere. The plasma in the ionosphere is colder and much denser than in the outer parts of the magnetosphere. Its temperature is nearly the same as the neutral temperature at a certain corresponding height. The ionosphere corotates with the Earth. It consists of different layers:

- The D-layer is this part of the ionosphere between 60 and 100 km [32].
- The E-layer is the region between 100 and 150 km [32].
- The F-layer is the region between 150 and 600 or even 1000 km [32]. The F-layer is composed of two separate layers. The F1-layer is the layer at about 180 km, the F2-layer is the layer at about 300 km.

The D, E and F1-layer all disappear during the night and are built up during the day. The F2-layer is permanent and is the most important one for us because the EFFDM will fly near this layer, at 561 km altitude.

The particle density in the ionosphere depends on the height, the solar cycle, time of the day, the season and on active solar events (flares, CME’s etc.). It has a peak in the F-layer at about 300 km and then decays exponentially with increasing height. The plasma at 561 km is dominated by H\(^+\) and O\(^{+3}\).

Equatorial regions—the plasmasphere

At heights above the density peak (300 km) the plasma density drops quickly. However, at equatorial regions and mid-latitude regions this density drop becomes less severe. At these latitudes and heights, the ionosphere passes over to a plasmasphere. This plasmasphere contains plasma of ionospheric origin. It stretches out to about four Earth radii and it also corotates with the Earth magnetic field. The plasmasphere is less dense than the ionosphere, but it is still denser and cooler than the other parts of the magnetosphere. This plasmasphere is relatively variable. The outer boundary of the plasmasphere is called the plasmapause.

Auroral regions

At higher latitudes, at about 60 to 70 degrees, the ionospheric density is very irregular and sometimes varies vertically and horizontally in a spatial range of a few meters. The origin of the irregularities is the plasma stream that flows from the plasmasheet along the magnetic field lines, into the auroral oval. This gives rise to the well-known phenomenon of the aurora at heights of about 100 km. Especially during substorms the plasma in these regions is highly variable. Sometimes the plasma densities can raise with a factor of 100 during those
storms. For a satellite travelling in LEO, passing the auroral regions equals to a disturbance in a time-span of approximately only one minute [73].

**Polar caps**
Inside the auroral oval, at the poles, the ionospheric density is less variable and can be relatively low. Just like the magnetic field lines of the plasmasphere end in the auroral oval, the lobe magnetic field lines end up in the polar caps.

### C.4 The neutral environment

**Types of flow**
The type of the flow in which a spacecraft is flying has an influence on the aerodynamic forces acting on it, and on how to model them. Therefore in this Appendix the type of flow is determined for an orbit at 561 km altitude.

**Mean free path length $L$**
The mean free path length of a molecule in a certain flow region is the average distance a molecule can travel before colliding with another one. This path length in general increases with increasing height above the Earth surface. For giving an indication, at a height of 120 km, $\lambda$ is approximately 3 m, at 200 km it is 200 m and at 250 km it is approximately 700 m [26]. Especially for altitudes below 200 km the mean free path length can be calculated by [22]:

$$\lambda = \frac{kT}{\sqrt{2\pi d_{avg}^2 p}}$$

where $\lambda$ is the mean free path length of a molecule in m, $d_{avg}^2$ is the mean collision diameter of the molecule (e.g. $3.62 \times 10^{-10}$ m for $\text{N}_2$), $p$ is the air pressure in N/m$^2$, $T$ is the temperature in K and $k$ is the Bolzmann constant ($1.38066 \times 10^{-23}$ J/K).

Filling in these values\(^4\) in equation C.26 yields an approximation for the mean free path length of 107 km for low solar activity, 140 km for mean solar activity and 5 km for high solar activity. These values are very large.

**Knudsen number**
The Knudsen number is the ratio of the mean free path length and a characteristic (linear) dimension of the spacecraft [19]:

$$K_n = \frac{\lambda}{D}$$

If $K_n \ll 0.1$, the flow is assumed to be a continuum flow that obeys the Navier-Stokes equations of aerodynamics. When $K_n \gg 1$, the flow is called a collisionless flow or free molecule flow. Usually a limit of $K_n \geq 10$ is used. Whenever $K_n$ has a magnitude near unity, one has a transitional flow or Knudsen flow. For satellites in orbit, the flow is nearly always a free molecule flow or a transitional flow. In case of a free molecule flow a collision between two molecules only seldomly happens, and happens on average when the molecule has traveled a distance equal to the mean free path length. The mean free path length of the molecules at 560 km height is far beyond the typical characteristic dimension of the satellite.

\(^4\)Assume in all cases that the mean collision diameter is equal to the mean collision diameter for $\text{N}_2$. Therefore keep in mind that these values for $\lambda$ are just approximations, good enough for determining an order of magnitude.
which is in the order of 1 m or less. Therefore we clearly have to deal with a free molecule flow in the case of EFFDM.

**Speed ratio**

The speed ratio is the ratio of the spacecraft velocity with respect to the atmosphere, and the average thermal molecular velocity, assuming a gas with a Maxwellian distribution function for the molecule velocities:\(^5\)

\[
S = \frac{v_{sc}}{v_{th}} = \frac{v_{sc}}{\sqrt{\frac{2kT}{m_p}}}
\]

where \(v_{sc}\) is the velocity of the spacecraft with respect to the atmosphere, \(v_{th}\) is the thermal velocity of the molecules and \(m_p\) is the molecular weight of the dominating molecules in mg. If \(S \gg 1\), then the spacecraft velocity with respect to the atmosphere is the most important contributor for the gas interaction with the spacecraft. This gives rise to a higher flux on the ram side of the spacecraft while a lower flux is maintained on the wake side. This flow is called a *supersonic flow* in analogy with the supersonic flows in a continuum. If \(S \ll 1\) then the average thermal velocity of the molecules will cause nearly the same fluxes at both sides of the spacecraft [32]. In this case the flow is called *subsonic*. The speed ratio has a slight influence on the drag coefficient of the spacecraft.

In our case the mass of the most dominant constituent in mean conditions (Oxygen) is \(2.657 \times 10^{-20}\) mg[62]. This yields a thermal energy of about 1.03 km/s and a speed ratio of about 7. A spacecraft velocity with respect to the atmosphere of more or less 7.583 km/s is assumed according to its orbital height. In reality it will be slightly larger or smaller depending on the speed of the atmosphere with respect to the Earth. For low solar activity the dominant constituent is Helium with a mass of \(6.6466 \times 10^{-21}\) mg[62]. This yields a thermal speed of 1.7 km/s and a speed ratio of about 4.45. For high solar activity a thermal speed of 1.3 km/s is reached that leads to a speed ratio of 5.8. In general the speed ratio will thus vary between about 4 and 7 and we have to deal with a supersonic flow. This means that the EFFDM spacecrafts are probably subject to wake-effects. There exists a possibility that the formation flying satellites affect each other’s orbit if one spacecraft enters the wake of the other one. However, the speed ratio is not extremely high, thus these effects are expected not to be extreme.

**Ramside compression**

As already mentioned in Section C.4, the speed ratio has an influence on the drag coefficient of the satellite. In case of \(\frac{v_{sc}}{v_{th}} \gg 1\) it is assumed that no *ram side compression* occurs. This means that the dimensions of the spacecraft are much smaller than the mean free path length and that the spacecraft velocity is not too big in comparison with the molecule thermal velocity:

\[
\lambda \cdot v_{th} = D \cdot v_{sc}
\]

Without ram compression the incoming particles only collide with the spacecraft (ram) surface, but the reflected particles do not interact with the other incoming ones. The consequence is that the transfers of the impulse of the incoming and outgoing molecules can be treated separately and summed afterwards [19]. In this case \(\frac{v_{sc}}{v_{th}}\) lies between 860 for high solar activity and 24,000 for low solar activity. These numbers are high, and thus no ram side compression will occur.

---

\(^{5}\)The Maxwellian distribution function for the molecule velocities is explained in [32]. It has a minor importance in this study.
Appendix D

The micrometeoroid and space debris environment

D.1 The shift of particulates in $L_2$ due to solar radiation pressure

Assume that the sunshield of Darwin is positioned perpendicular to the incoming light rays from the Sun, that it has a diameter of 7 m, and the spacecraft has a mass of 500 kg. Darwin’s area-to-mass ratio is then:

$$\frac{A}{m} = \frac{154}{500} = 3.25 \quad (D.1)$$

Assume now three particulates for each particulate range in the mass-density rules (Table 4.2 in Section 4.2.2). For each particulate it is assumed that it has a spherical geometry so that its volume can be calculated from the mass and mass density. Its profile (a disk) is taken as an approximation for the surface exposed to the sunlight at a right angle. From Table D.1 it is clear that some of these particulates in $L_2$ will have the same area-to-mass ratio as Darwin.

<table>
<thead>
<tr>
<th>Mass</th>
<th>Mass density</th>
<th>particulate profile</th>
<th>$A/m$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$10^{-12}g$</td>
<td>2 g/cm$^3$</td>
<td>$7.6 \cdot 10^{-13}m^2$</td>
<td>760</td>
</tr>
<tr>
<td>$10^{-5}g$</td>
<td>1 g/cm$^3$</td>
<td>$2.6 \cdot 10^{-7}m^2$</td>
<td>2.6</td>
</tr>
<tr>
<td>1g</td>
<td>0.5 g/cm$^3$</td>
<td>$1.9 \cdot 10^{-4}m^2$</td>
<td>0.19</td>
</tr>
</tbody>
</table>

Table D.1: Properties of the three particulates for calculating the area-to-mass ratio.

D.2 Mass moments of inertia of the Darwin free flyers

The mass moments of inertia of the free flyers are estimated in this Appendix. The spacecraft is represented by a summation of four main parts that are shown in Figure D.1: the telescope tube, the telescope mirror, the instruments on-board and the sunshield (solar arrays included).

First, the mass moments of inertia of each part are calculated with respect to their own center of mass.

- **Telescope tube**
  
  Assumed is a tube thickness of 1 mm and that the tube is made of Aluminum with a
mass density of 2770 kgm$^{-3}$ [66]. Its outer and inner radii are assumed to be 0.75 and 0.749 m, while its height is assumed to be 2 m.

Mass estimation:

$$m = \rho \pi h (r_2^2 - r_1^2) = 26 \text{ kg}$$

The mass moments of inertia are:

$$I_{xx} = I_{yy} = \frac{1}{12} m (3(r_1^2 + r_2^2) + L^2) = 16 \text{ kgm}^2$$

$$I_{zz} = \frac{1}{2} m (r_1^2 + r_2^2) = 14.6 \text{ kgm}^2$$

- **Primary mirror**

  The primary mirror is approached as if it were a thin flat disk. The mass of the primary mirror has been estimated with the following rule of thumb [23]:

  $$m = 22D^{2.7} = 66 \text{ kg}$$

  where $D$ is the mirror diameter.

  The mass moments of inertia are:

  $$I_{xx} = I_{yy} = \frac{1}{4} mr^2 = 9.3 \text{ kgm}^2$$

  $$I_{zz} = \frac{1}{2} mr^2 = 18.6 \text{ kgm}^2$$

- **On-board instruments**

  The instruments on board are represented as a solid cylinder, with a radius of 0.75 m and a height of 0.5 m. Its mass is assumed to be the total spacecraft mass minus the mass of the other three parts of the spacecraft:

  $$m = 493 \text{ kg} - 66 \text{ kg} - 46 \text{ kg} - 26 \text{ kg} = 355 \text{ kg}$$

  The mass moments of inertia are:

  $$I_{xx} = I_{yy} = \frac{1}{4} mr^2 + \frac{1}{12} ml^2 = 57.3 \text{ kgm}^2$$

  $$I_{zz} = \frac{1}{2} mr^2 = 99.8 \text{ kgm}^2$$
• Sunshield

The sunshield has a radius of 3.5 m and a mass of 40 kg. The mass of the solar panels—that is only 6 kg—is added to the mass of the sunshield. The difference in radius has been neglected because the mass of the solar panels is already small. The sunshield and solar panels are assumed to be one circular flat disk with a mass of 46 kg.

\[
I_{xx} = I_{yy} = \frac{1}{4}mr^2 = 141 \text{ kgm}^2 \\
I_{zz} = \frac{1}{2}mr^2 = 282 \text{ kgm}^2
\] (D.11) (D.12)

The center of mass of the spacecraft is assumed to be on the z-axis of the telescope, and in the middle of the on-board-instruments-cylinder. This is a good approximation, because if multiplying each single center of mass by the distance to the middle of this cylinder, we almost get an equilibrium.

\[
46 \text{ kg} \cdot 0.75 \text{ m} \approx (66 \text{ kg} \cdot 0.25 \text{ m}) + (26 \text{ kg} \cdot 0.75 \text{ m})
\] (D.13)

The mass moment of inertia can now be calculated with respect to the total center of mass of the satellite with the Steiner equation:

\[
I = \bar{I} + md^2
\] (D.14)

where \(I\) is the mass moment of inertia of one spacecraft part with respect to the total center of mass of the satellite, \(\bar{I}\) is the mass moment of inertia of one of the spacecraft parts with respect to its own center of mass, \(m\) is the mass of this spacecraft part and \(d\) is the distance between the center of mass of the spacecraft part and the center of mass of the whole spacecraft. The total mass moment of inertia of the spacecraft is the summation of each mass moment of inertia of the individual spacecraft parts. The mass moments of inertia of each part are summarized in Table D.2. The dimensions of the satellite are estimated from [21], and illustrated in Figure D.2.

<table>
<thead>
<tr>
<th>Spacecraft part</th>
<th>(I_{xx}) [kgm²]</th>
<th>(I_{yy}) [kgm²]</th>
<th>(I_{zz}) [kgm²]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Telescope tube</td>
<td>30.6</td>
<td>14.6</td>
<td></td>
</tr>
<tr>
<td>Primary mirror</td>
<td>13.4</td>
<td>18.6</td>
<td></td>
</tr>
<tr>
<td>On-board instruments</td>
<td>57.3</td>
<td>99.8</td>
<td></td>
</tr>
<tr>
<td>Sunshield</td>
<td>167</td>
<td>282</td>
<td></td>
</tr>
<tr>
<td>Total mass moment of inertia</td>
<td>268.3</td>
<td>415</td>
<td></td>
</tr>
</tbody>
</table>

Table D.2: The mass moments of inertia of the spacecraft parts with respect to the center of mass of the total spacecraft.
D.3 Equations of motion for translation and rotation of a satellite due to an impact

1. Translational motion
   It is assumed that immediately after the particulate impact, the FEEPs are activated. However, during the impact itself no FEEPs are turned on yet. This means that during the impact the linear momentum is conserved except for the momentum enhancement factor $e$ [56]. Once the particulate has struck the spacecraft, its mass is neglected with respect to the spacecraft mass. At the moment of impact holds:

$$ e \cdot (m v)_p + 0 = 0 + m_{sc} v_{sc} \quad (D.15) $$

The particulate impact is concerned to be an impulsive force, which means that the spacecraft will get a certain velocity, but that its acceleration due to the impact is zero after impact. The velocity of the spacecraft after impact is

$$ v_{sc,0} = \frac{e \cdot (m v)_p}{m_{sc}} \quad (D.16) $$

When the FEEPs are turned on, the spacecraft also gets an acceleration that is equal to (Newton’s law):

$$ m_{sc} a_{sc} = -F_T \quad (D.17) $$

or:

$$ a_{sc} = -\frac{F_T}{m_{sc}} \quad (D.18) $$

The displacement of the spacecraft is:

$$ x = \frac{1}{2} a_{sc} t^2 + v_{sc,0} t + x_{sc,0} \quad (D.19) $$

where $x_{sc,0}$ is equal to 0. Filling in the values for $a_{sc,0}$ and $v_{sc,0}$ yields:

$$ x_{sc} = \frac{1}{m_{sc}} \left( -\frac{1}{2} F_T t^2 + n \cdot (m v)_p t \right) \quad (D.20) $$
Differentiating this equation with respect to the time $t$ and equaling it to zero gives us the time $t$ for which the deviation $x$ has a maximum. This time is equal to:

$$t = \frac{e \cdot (mv)_p}{F_T} \quad (D.21)$$

Filling in equation D.21 in equation D.19 yields

$$x_{\text{max}} = \frac{1}{m_{sc}} \frac{(e \cdot mv)^2_{\text{imp}}}{2F_T} \quad (D.22)$$

which is the same as Equation 4.5 in Section 4.3.1.

2. Rotational motion

In the same manner Equation 4.9 can be proven starting from the angular velocity the spacecraft gets after an impact:

$$e \cdot d \cdot (mv)_p + 0 = 0 + I_{G,sc}\omega_{sc} \quad (D.23)$$

The angular velocity of the spacecraft after impact is:

$$\omega_{sc,0} = \frac{e \cdot d \cdot (mv)_p}{I_{G,sc}} \quad (D.24)$$

When the FEEP’s are turned on, the spacecraft also gets an angular acceleration that is equal to (Newton’s law):

$$I_{G,sc}\alpha_{sc} = -M_T \quad (D.25)$$

or:

$$\alpha_{sc} = -\frac{M_T}{I_{G,sc}} \quad (D.26)$$

The angular displacement of the spacecraft is:

$$\theta = \frac{1}{2}\alpha_{sc} t^2 + \omega_{sc,0} t + \theta_{sc,0} \quad (D.27)$$

where $\theta_{sc,0}$ is equal to 0. Filling in the values for $\alpha_{sc,0}$ and $\omega_{sc,0}$ yields:

$$\theta_{sc} = \frac{1}{I_{G,sc}} (-\frac{1}{2}M_T t^2 + e \cdot d \cdot (mv)_p t) \quad (D.28)$$

Differentiating this equation with respect to the time $t$ and equaling it to zero gives us the time $t$ for which the deviation $\theta$ has a maximum. This time is equal to:

$$t = \frac{e \cdot d \cdot (mv)_p}{M_T} \quad (D.29)$$

Filling in equation D.29 in equation D.27 yields:

$$\theta_{\text{max}} = \frac{1}{I_{G,sc}} \frac{(e \cdot d \cdot mv)^2_{\text{imp}}}{2M_T} \quad (D.30)$$

which is the same as equation 4.9 in Section 4.3.2.

D.4 Surface area of the free flyer satellites

The total surface area of the satellite is calculated. Table D.3 lists the surface areas of the free flyers.
### Table D.3: The surface area of a telescope free flyer.

<table>
<thead>
<tr>
<th>Spacecraft part</th>
<th>Equation</th>
<th>Surface area [m²]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sunshield, downside</td>
<td>(\pi r^2)</td>
<td>38.5</td>
</tr>
<tr>
<td>Primary mirror + sunshield, upside</td>
<td>equal to the downside of the sunshield</td>
<td>38.5</td>
</tr>
<tr>
<td>Telescope tube</td>
<td>(2\pi rh)</td>
<td>9.42</td>
</tr>
<tr>
<td>Total surface area</td>
<td>Sum of the previous ones</td>
<td>86.4</td>
</tr>
</tbody>
</table>

### D.5 The micrometeoroid environment in LEO

In Low Earth Orbit, the particulate environment consists of man-made space debris and micrometeoroid particulates (both the isotropic flux and micrometeoroid streams). Most of the particulates between 5 and 500 \(\mu\)m are micrometeoroids, while most of the particulates smaller than 5 \(\mu\)m or larger than 500 \(\mu\)m are space debris [20].

For the micrometeoroid environment in LEO, the same model as in \(L_2\) can be used (Grün, [31]). After using this model, a few corrections must be made for the Earth attraction, the Earth shielding, the drag on the particulates in the atmosphere and directional effects.

- **Earth attraction**
  
The amount of micrometeoroids in the vicinity of the Earth increases due to the attraction of the Earth. The flux as predicted by the interplanetary model can simply be modified by multiplying it with an additional factor for the Earth attraction. This factor is equal to [22]:

\[
G_E = 1 + \frac{R_E}{r} \tag{D.31}
\]

where \(R_E\) is the mean Earth radius and \(r\) is the orbit radius. In case of the FFDEM orbit (561 km) this factor is equal to 1.9.

- **Shielding**
  
The presence of the Earth also causes the particulate flux to be decreasing, as a result of geometrical shielding. In this case the flux can also be corrected by multiplying it with a factor \(S_f\). This factor depends on the altitude above the Earth surface, and on the orientation of the surface normal with respect to the Earth direction [22]:

\[
S_f = \frac{1 + \cos \eta}{2} \tag{D.32}
\]

with

\[
\sin(\eta) = \frac{R_E + 100}{R_E + h} \tag{D.33}
\]

In the FFDEM case, \(S_f = 0.68\) and \(\eta \approx 69°\). \(\eta\) is illustrated in Figure D.3.

- **Directional effect**
  
The Earth shielding effect also introduces a directional distribution different from the isotropic distribution assumed in the Grün model. Most of the impacts will occur on the surfaces of the spacecraft that are oriented towards free space. The Earth faced surface and the trailing surface of the satellite will experience a flux of one tenth the normal interplanetary flux [73].

\[
F_{\text{dir}} = \frac{1}{10} \tag{D.34}
\]
The ramside of the spacecraft experiences a flux reduction of [73]:

\[ F_{\text{dir}} = 1.8 + \sqrt{1 - \left(\frac{R_E+100 \text{ km}}{R_E+r}\right)^2} \]  

(D.35)

In our case \( F_{\text{dir}} \) is equal to 0.54.

- **The total flux**
  The total meteoroid flux is now equal to [22, 73]:

\[ F_{\text{met}} = F_{\text{met,0}} \cdot G_E \cdot S_f \cdot F_{\text{dir}} \]  

(D.36)

From this equation it is concluded that the influence of the presence of the Earth is a decrease in meteoroid flux with a factor of 0.7 for the ramside of the satellite, and 0.13 for the other spacecraft surfaces.

### D.6 The space debris environment in LEO

In Low Earth Orbit not only micrometeoroids play a role in the impact risk of satellites, but also man-made space does. The large particulates (>10 cm) that really form a danger for spacecrafts in orbit, are usually tracked by radars on Earth. The smaller particulates that cannot be tracked are predicted in space debris models, that are based on statistics of spacecraft impacts. The flux and characteristics of this particulate population is different from the micrometeoroid environment. The main differences and general characteristics of space debris are listed below:

- In general, the average impact velocity is about 10 km/s which is lower than the average micrometeroid impact velocity.
- Impacts mainly happen on the ramside of the spacecraft [73].
- The space debris flux depends on the solar cycle. When the Sun is active, the atmosphere responds to that by expansion. This has an effect of a decrease of debris particulates due to the increased drag force, that causes them to re-enter the atmosphere.
- The amount of space debris continues to grow, while the meteoroid flux is a more or less constant particulate flux.
• The average mass density of space debris is higher than the meteoroid mass density.

### D.6.1 Model of the space debris flux

An analytical model is used to derive an order of magnitude of the space debris flux in LEO, called the Kessler model. This model gives the accumulated flux of space debris, on a randomly tumbling plate in a circular orbit around the Earth with a height lesser than 2000 km above the Earth. The flux is not directly related to the mass of the particulates, but to their minimum diameter and is described by [32]:

\[
F(d, h, i, t, S) = kH(d)f(h, S)y(i)[F_1(d)g_1(t) + F_2(d)g_2(t)]
\]  
(D.37)

where \( F \) is the cumulative flux of space debris particulates with diameter bigger than \( d \), per unit surface per year, \( d \) is the minimum diameter of particulate taken into account (in cm), \( h \) is the altitude of the orbit (< 2000 km), \( i \) is the inclination of the orbit, \( t \) is the time (year) and \( S \) is the 13-month smoothed 10.7 cm wavelength solar flux, measured in the year before the actual flight time (in \( 10^4 \) Jy).

In equation D.37 the functions are defined as [32]:

- \( H(d) = \sqrt{10 \exp[-(\log_{10} d - 0.78)^2/0.63^2]} \)
- \( f(h, S) = \frac{f_1(h, S)}{f_1(h, S) + f_2(h, S)} \)
- \( f_1(h, S) = 10^{\frac{\log_{10} S - 1.5}{100}} \)
- \( F_1(d) = 1.22 \times 10^{-5} d^{-2.5} \)
- \( F_2(d) = 8.1 \times 10^{10} (d + 700)^{-6} \)
- \( g_1(t) = (1 + q)(t - 1988)^{-1} \) for \( t < 2011 \)
- \( g_1(t) = (1 + q)^{23}(1 + q')(t - 2011) \) for \( t > 2011 \)
- \( g_2(t) = 1 + [p(t - 1988)] \)

\( p \) is the annual growth rate of mass in orbit equal to 0.05, \( q \) is the fragment growth rate before 2011 equal to 0.02 and \( q' \) is the fragment growth rate after 2011 equal to 0.04 [32, 70].

\( y(i) \) is a function that represents the dependence of the flux on the inclination of the orbit, given in a table for a few inclinations in [32, 70]. For \( i = 90^\circ \) this function is constant and equal to 1.37.

### D.6.2 Space debris characteristics

#### Mass density \( \rho \)

In the Kessler model, a mass density of 4 g/cm\(^3\) is assumed for particulates with a diameter below 0.62 cm. For particulates bigger than 0.62 cm, a mass density of \( 2.8 \cdot d^{-0.74} \) g/cm\(^3\) is used, with \( d \) the diameter of the particulate in cm [32, 73, 70]. In our case the threshold masses will be far below 0.5 g, that corresponds to a diameter of 0.62 cm (the particulate is assumed to be spherical). The influence on the space debris flux of the big particulates with different mass density is very small compared to the large population of small particulates. For these two reasons we will assume the mass density to be constant at a value of 4 g/cm\(^3\) in our calculations.

#### Impact velocity

The Kessler model assumes a certain impact velocity distribution [70] but just like in the case of the micrometeoroid particulates, we will assume one constant value for the impact velocity. Collisions happen with impact speeds from nearly zero to 15.4 km/s [59] with an average speed of about 10 km/s.
• **Directional distribution**
The space debris population increases from 300 to 1000 km. This increase is a result of decreasing air drag with height. From 1000 to 2000 km the amount of man-made particulates is nearly constant. Apart from that, the population is strongly dependent on the popularity of the orbit of the satellite. GEO, sun-synchroneous and polar orbits are most often used, and therefore experience a peak in amount of space-debris. At 561 km in general there is fewer space debris than at higher heights or more popular heights, but the inclination of 90° in our case increases the collision risk with space debris particulates, since polar orbits are favoured for many missions.

**D.6.3 A worst-case scenario for the space debris flux**

For a worst-case scenario several assumptions can be made as summarized in the following:

- When the solar activity is high, the atmosphere expands and space debris will be slowed down and falling back to Earth due to the increased air drag much easier than at solar minimum. Therefore most particulates exist during solar minimum. For a worst-case, a value of \( S = 70 \) will be taken in accordance to the reference index values for \( F_{10.7} \) from [22]. An average value for \( S \) is 140.

- The worst-case impact velocity is set to 15 km/s. The Average impact velocity is 10 km/s.

- Space debris clusters might exist but are not taken in account in this study.

Using this information on the micrometeoroid and space debris environment in LEO, the particulate impact on FFDEM or another mission in LEO could be evaluated easily.
Appendix E

GRACE and CHAMP data analysis

E.1 GRACE data analysis steps

1. The Level 1B data are obtained from podaac.jpl.nasa.gov via FTP.
2. The data are converted from binary to ascii using a program called “bin2ascilevel1”. This program is available via the same FTP-site.
3. The ascii data are validated using the validation script available via FTP.
4. The ASC1B-data are read in an IDL\footnote{Interactive Data Language} programme. The next steps are also done in IDL.
5. Date conversion from GPS-time to time in seconds past the first measurement.
6. The mean of the data was removed in order to exclude the low-frequent component from the Fourier transform.
7. Removing the thruster spikes. This is done as described in Appendix E.2.
8. A Fast Fourier Transform (FFT) is taken from the acceleration data. This is allowed because the data are equally spaced.
9. Calculation of the PSD from the Fourier Transform.
10. The PSD is transformed into a onesided-PSD.
11. Calculation of the X-axis.
12. Calculation of the displacements PSD from the acceleration PSD (in nanometres).
13. Calculation of the ASD and DSD (square root of both PSD’s).
14. The satellite’s mass is scaled down to 120 kg in order to match for FFDEM (120 kg is worse than 480 kg). The cross-section surface of the satellite is supposed to be equal to the GRACE frontal surfaces in order to remain in a worst-case situation.
15. Taking the mean of the results of the ASD’s and DSD’s of different days.
16. The DSD is approximated by a straight line using Matlab. The Root Mean Square value of the displacements is calculated for $10^9$ – $10^3$ Hz.

\footnote{Interactive Data Language}
17. The results are checked by the CHAMP case and by using Parseval’s theorem which implies that the RMS-value of the data in the time-domain is equal to the RMS-value of the PSD of the data in the frequency domain:

\[ \sqrt{\frac{1}{T} \int_0^T x^2(t) \, dt} = \sqrt{\int_{f_{\text{low}}}^{f_{\text{up}}} \text{PSD} \, df} \]  

(E.1)

Some final remarks remain about the GRACE data analysis:

- The quality flags in the data were ignored because they were not consistent with any significant data error visible in the time-domain data.
- The bias of the data only has an effect on the low-frequent component of the disturbance forces. Therefore it was not necessary to implement it.
- The data scale factor was also not applied because it has almost no influence on the order of magnitude of the data. The scale factors are between 0.9 and 1.
- The mass decrease of the satellites is not taken into account. The total amount of fuel on-board was 34 kg for a mission of 5 years. The mass decrease does not contribute significantly to the general order of magnitude of the ASD.

E.2 Removing the spikes

At first the spikes in the data have been inspected by vision in the time-domain. What strikes the most is that the spikes occur at very regular intervals at the local maxima and minima of the data, and that they are most of the time pointed in the same direction. This pattern suggests that these spikes are a result of the orbit maintenance manoeuvres, that counteract the main disturbance forces. Another part of the spikes are a result of the AOCS manoeuvres. These manoeuvres are listed in the thruster data (THR1B-files). Also a possibility exists that some spikes are caused by other forces than the AOCS and orbit maintenance pulses.

The data have been filtered according to the time-tags of thruster-activation as observed in the thruster files. Unfortunately this did not lead to a thorough filtering of the spikes, or at least not the big spikes. At certain moments some time-tags of the thruster activation did correspond to the spikes in the acceleration, but most of the time they were slightly shifted with respect to each other or absolutely no correlation could be found. The filtering resulted in a noise reduction as shown in Figure E.1. This Figure represents the one-day ASD for the cross-track direction on 01-08-2002. Filtering the data using the knowledge of the thruster activation times of the AOCS and the orbit maintenance manoeuvres would be the best way to start filtering these spikes, but a more sophisticated filtering algorithm would be necessary.

Instead, a filter has been used that removes the spikes by inspection, using a criterium according to the spike magnitude. The data are first smoothed for determining an approximation of the data without the peaks. The original dataset and the smoothed data were subtracted from each other and the remains are the data spikes themselves, with on the bottom the left-overs of the real signal. In theory a spike is “selected” if it is bigger than the difference between the smoothed and real data summed by the magnitude of the noisy signal on the maxima and minima of the time-series of the data. In practice however, the criterium was utilized in combination with visual inspection of both the time-domain data as the ASD’s. The criteria used for the filters are:

- soft filter: \([2 \cdot 10^{-8}, 4 \cdot 10^{-8}, 1.5 \cdot 10^{-8}]\)
- hard filter: \([1 \cdot 10^{-8}, 5 \cdot 10^{-9}, 4 \cdot 10^{-9}]\)
- overdone filter: \([1 \cdot 10^{-8}, 2 \cdot 10^{-9}, 1 \cdot 10^{-9}]\)
Figure E.1: The ASD after filtering, using the AOCS-data. Many of the real AOCS spikes are not filtered yet, due to a slight and unstable shift in the time-tag of the thruster activation compared to the time tag of the acceleration spikes.

### E.3 CHAMP data analysis steps

1. The Level 1 data are obtained from [http://www.gfz-potsdam.de/pbl/op/champ/index_CHAMP.html](http://www.gfz-potsdam.de/pbl/op/champ/index_CHAMP.html) via the Delft University of Technology, (DEOS²).

2. The acceleration data are unpacked, converted from binary to ascii and a bias and scale correction is made. The program used is based on the programme written by Ir. P.N.A.M. Visser, originally written for the Geodyn software. In this programme for instance the bias and scale factor are applied ($a_{\text{corr}} = (a_{\text{uncorr}} - \text{bias}) \cdot \text{scalefactor}$) although they are not of importance in this study.

3. The data are read in an IDL programme which is basically the same as for GRACE, except for the time conversion, and except that no filtering is done. The next steps are also done in IDL.

4. Time conversion from [yr, mt, day, hr, min, sec] to time in seconds past the first measurement.

The other steps are the same as for GRACE, except that no spikes were removed.

As a final remark on the CHAMP data analysis, the effect of the malfunctioning of the accelerometer in the radial direction on the ASD is shown in Figure E.2. Even the first peak at the orbital frequency of $1.8 \cdot 10^{-4}$ Hz is not visible in the SD of this time sample. The accompanying time-series of the data is shown in Figure E.3.

---

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Figure E.2: The ASD for the radial direction of the accelerometer measurements, that did not function properly.

Figure E.3: The time-series of the CHAMP accelerometer data in the radial direction. No clear orbital frequency can be seen.
Appendix F

Special topics belonging to the external forces in L\textsubscript{2}

F.1 Mutual gravity forces between the satellites

Very often when calculating forces and displacements to the nanometer level, one cannot rely on estimates based on common sense anymore, due to the extremely small scales. For illustrating this, the order of magnitude of the gravity forces between the Darwin satellites is calculated in this Appendix. Usually these forces are neglected. When working with nanometer-level disturbances, this force, which looks on the first sight negligible, becomes of larger importance than most people would have estimated on the first sight. This illustrates that prejudice during research in the nanometer-level world, based on common sense, could lead easily to mistakes and ignored forces.

The gravity forces exerted on and from the satellites themselves can be easily calculated with Newton’s law:

\[ F = -G \frac{M \cdot m}{r^2} \]  \hspace{1cm} (F.1)

The parameters used in equation F.1 are shown in Table F.1, while the mutual distances between the free flyers are given in Table F.2.

<table>
<thead>
<tr>
<th>Gravitational constant</th>
<th>( G )</th>
<th>6.67259 ( \times 10^{-11} ) ( m^3/(kg \cdot s^2) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mass of free flyer ( M_{FF} )</td>
<td>493 ( kg )</td>
<td></td>
</tr>
<tr>
<td>Mass of hub ( M_{hub} )</td>
<td>396 ( kg )</td>
<td></td>
</tr>
<tr>
<td>Mass of master ( M_{mstr} )</td>
<td>179 ( kg )</td>
<td></td>
</tr>
<tr>
<td>Smallest baseline ( r_{\text{min}} )</td>
<td>50 ( m )</td>
<td></td>
</tr>
<tr>
<td>Largest baseline ( r_{\text{max}} )</td>
<td>500 ( m )</td>
<td></td>
</tr>
<tr>
<td>Mean distance to master ( r_{mstr} )</td>
<td>200 ( m )</td>
<td></td>
</tr>
</tbody>
</table>

Table F.1: The parameters used in Newton’s law.

The results for the mutual gravity forces are shown in Table F.3. In these calculations care is taken for the directions of the force. For instance, the forces between the hub and the 6 free flyers is not surprisingly large, but still the hub will remain centered in the hexagonal shaped formation. This is because the force from each satellite on the hub cancels the one from the satellite on the opposite side of the Hub. As a result, the nett drift of the hub will be zero with respect to the hexagonal, but the distance between the free flyers and the
Mutual distance | Distances for $r_{\text{min}}$ | Distances for $r_{\text{max}}$
---|---|---
$r_{1-2} = r_{1-6}$ | $R$ | 50 m
$r_{1-3} = r_{1-5}$ | $\sqrt{3} \cdot R$ | 86.6 m
$r_{1-4}$ | $2R$ | 100 m

Table F.2: The mutual distances between the Free Flyers.

| Mass 1 | Mass 2 | Gravity force for $r_{\text{min}}$ | Gravity force for $r_{\text{max}}$
---|---|---|---
master | hub | $1 \times 10^{-10}$ N | $1 \times 10^{-10}$ N
one free flyer | one free flyer | $6 \times 10^{-9}$ N | $6 \times 10^{-11}$ N
one free flyer | hub | $5 \times 10^{-9}$ N | $5 \times 10^{-11}$ N
one free flyer | 5 free flyers | $11 \times 10^{-9}$ N | $11 \times 10^{-11}$ N
one free flyer | 5 FFs and hub | $16 \times 10^{-9}$ N | $16 \times 10^{-11}$ N

Table F.3: The mutual gravity forces between the satellites.

hub will still decrease. In the same manner some components of the forces between the free flyers will cancel each other and thus the nett force between the free flyers will be smaller than simply the sum of the forces between the mutual satellites. The force between one free flyer and the five other ones is given by:

$$F_{FF} = F_{1-4} + 2 \cdot F_{1-3} \cdot \cos(30^\circ) + 2 \cdot F_{1-2} \cdot \sin(30^\circ)$$

$$= -G \cdot M_{FF} \cdot M_{FF} \cdot \left( \frac{1}{(2R)^2} + \frac{\sqrt{3}}{(R \cdot \sqrt{3})^2 + \frac{1}{R^2}} \right)$$

where $R$ is equal to the baseline length.

For giving an indication, a satellite of 500 kg subject to a mutual gravity force of $16 \times 10^{-9}$ N, will reach a displacement of 1 nanometer in just 8 seconds.

F.2 The inter-satellite Coulomb forces

In the 18th century, Coulomb [1736-1806] experimented with two very small loaded masses in a vacuum and noticed that they exerted a force on each other that is equivalent with the square of their charges:

$$F_c = \frac{1}{4\pi\epsilon_0} \cdot Q \cdot \frac{q}{r^2} \cdot \frac{\hat{r}}{r}$$

where $\epsilon_0$ is the permittivity of free space ($\epsilon_0 = 8.854 \cdot 10^{-12}$ C$^2$/Nm$^2$). Sometimes the product $\frac{1}{4\pi\epsilon_0}$ is written in one constant: Coulomb’s constant $k_c$, equal to $9 \cdot 10^9$Nm$^2$/C$^2$ [10]. Coulomb's law implies that two small masses that are oppositely charged will attract each other, while two equally loaded masses will repel each other.

The same holds for spacecrafts with a nett charge. In certain conditions, when flying through a plasma or when sunlit by the Sun, spacecrafts will acquire a nett charge. Typical charges are in the range of 1 V up to a maximum of 50 V, depending on the space environment where the satellites are positioned. In GEO, charges are usually larger and can become as large as 1 kiloVolt. A few example calculations are made to give an idea of the magnitude of the
inter-satellite coulomb forces. The same spacecraft parameters are used as in Appendix F.1 for the inter-satellite gravitational forces. In addition, it is assumed that the free-flying satellites are spheres with a radius of 1 m in order to calculate in terms of electric potential energy, instead of charges. The electric potential outside a sphere or point load is equal to:

\[ \bar{E} = \frac{Q}{4\pi\varepsilon_0 \cdot r^2} \cdot \frac{\bar{r}}{r} \]  

(F.5)

The electric potential is related to the electric field \( \bar{E} \) by [10]:

\[ \bar{E} = -\nabla V \]  

(F.6)

and thus:

\[ V(r) = \frac{Q}{4\pi\varepsilon_0 \cdot r} + V(\infty) \]  

(F.7)

We assume \( V(\infty) \) to be zero. The electric potential of a point mass, and the electric potential outside of a sphere is then equal to:

\[ V(r) = \frac{Q}{4\pi\varepsilon_0 \cdot r} \]  

(F.8)

For a spherical satellite with radius \( r_{sc} \), this means that the potential at its surface is equal to:

\[ V(\text{surface}) = \frac{Q_{sc}}{4\pi\varepsilon_0 r_{sc}} \]  

(F.9)

or:

\[ Q_{sc} = V(\text{surface}) \cdot 4\pi\varepsilon_0 \cdot r_{sc} \]  

(F.10)

Filling in Equation F.10 in Equation F.4 yields [44]:

\[ F_0 = 4\pi\varepsilon_0 \cdot \frac{r_{sc1} \cdot r_{sc2} \cdot V_{sc1} \cdot V_{sc2}}{d^2} \]  

(F.11)

where \( d \) is the distance between the satellites.

In Table F.4 some values of the Coulomb forces are given. They are based on a charge of respectively 1 Volt, 50 Volt and 1 kiloVolt. The baseline is assumed to be 50 m, and the distance between the hub and the master is assumed to be 200 m.

<table>
<thead>
<tr>
<th>sat 1</th>
<th>sat 2</th>
<th>( F_{1\text{V}} )</th>
<th>( F_{50\text{V}} )</th>
<th>( F_{1\text{kV}} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>master</td>
<td>hub</td>
<td>( 2.77 \cdot 10^{-15} )</td>
<td>( 6.94 \cdot 10^{-12} )</td>
<td>( 2.77 \cdot 10^{-9} )</td>
</tr>
<tr>
<td>one free flyer</td>
<td>one free flyer</td>
<td>( 4.44 \cdot 10^{-14} )</td>
<td>( 1.11 \cdot 10^{-10} )</td>
<td>( 4.44 \cdot 10^{-8} )</td>
</tr>
<tr>
<td>one free flyer</td>
<td>hub</td>
<td>( 4.44 \cdot 10^{-14} )</td>
<td>( 1.11 \cdot 10^{-10} )</td>
<td>( 4.44 \cdot 10^{-8} )</td>
</tr>
<tr>
<td>one free flyer</td>
<td>5 free flyers</td>
<td>( 1.29 \cdot 10^{-13} )</td>
<td>( 3.24 \cdot 10^{-10} )</td>
<td>( 1.29 \cdot 10^{-7} )</td>
</tr>
<tr>
<td>one free flyer</td>
<td>5 FF's and hub</td>
<td>( 1.74 \cdot 10^{-13} )</td>
<td>( 4.35 \cdot 10^{-10} )</td>
<td>( 1.74 \cdot 10^{-7} )</td>
</tr>
</tbody>
</table>

Table F.4: Coulomb forces between the Darwin satellites for a baseline of 50 m. No plasma Debye length is applied yet.

For a baseline of 500 m, the Coulomb forces will be 100 times smaller than in Table F.4 as a consequence of the squared distance in Equation F.4. For these calculations also a charge of the same polarity was assumed, which means that the satellites will repel from each other. In some cases for a spacecraft potential between 50 V and 1 kV, the Coulomb forces counteract the gravity forces between the satellites.

However, the inter-satellite Coulomb forces are restricted by the so-called Debye-shielding
of the plasma in which the satellites are positioned. The Debye length of that plasma (see Section 3.4) is the parameter that describes the amount of Debye shielding, or the extent to which the Coulomb forces are “felt” by the satellites. Coulomb’s law, adapted for a plasma environment is \[44\]:

\[
F_{1,2} = \frac{1}{4\pi\epsilon_0} \cdot \left( \frac{q_1 q_2}{d^2} \right) \cdot e^{\frac{-d}{\lambda_D}} \quad \text{(F.12)}
\]

In Equation F.12, \(\lambda_D\) is the plasma Debye length, \(q_1\) and \(q_2\) are the charges of the satellites, and \(d\) is the distance between the two satellites. This Equation shows that whenever the distance between the satellites is smaller than the plasma Debye length, the Coulomb forces could be significant. For comparison, a few preliminary numbers of Coulomb forces are shown in Table F.5 as a function of the plasma Debye length and the inter-satellite distance. The Debye length is based on realistic numbers for a Geostationary Earth Orbit for this first estimation. The Debye length in GEO can range between 140 and 1400 m \[45\]. Again we assume spherical, charged satellites with a radius of 1 m. Filling in Equation F.12 in Equation F.4 yields:

\[
F_c = 4\pi\epsilon_0 \cdot \frac{V_{sc1} V_{sc2} r_{sc1} r_{sc2}}{d^2} \cdot e^{\frac{-d}{\lambda_D}} \quad \text{(F.13)}
\]

<table>
<thead>
<tr>
<th>sat 1</th>
<th>sat 2</th>
<th>(F_{1V})</th>
<th>(F_{50V})</th>
<th>(F_{1kV})</th>
<th>dist</th>
<th>(\lambda_D)</th>
</tr>
</thead>
<tbody>
<tr>
<td>one FF</td>
<td>one FF</td>
<td>(3.10 \cdot 10^{-14})</td>
<td>(7.77 \cdot 10^{-11})</td>
<td>(3.10 \cdot 10^{-08})</td>
<td>50</td>
<td>140</td>
</tr>
<tr>
<td>one FF</td>
<td>one FF</td>
<td>(4.28 \cdot 10^{-14})</td>
<td>(1.07 \cdot 10^{-10})</td>
<td>(4.28 \cdot 10^{-08})</td>
<td>50</td>
<td>1400</td>
</tr>
<tr>
<td>one FF</td>
<td>one FF</td>
<td>(1.24 \cdot 10^{-17})</td>
<td>(3.12 \cdot 10^{-14})</td>
<td>(1.24 \cdot 10^{-11})</td>
<td>500</td>
<td>140</td>
</tr>
<tr>
<td>one FF</td>
<td>one FF</td>
<td>(3.10 \cdot 10^{-16})</td>
<td>(7.77 \cdot 10^{-13})</td>
<td>(3.10 \cdot 10^{-10})</td>
<td>500</td>
<td>1400</td>
</tr>
</tbody>
</table>

Table F.5: Coulomb forces between the Darwin satellites for a baseline of 50 m.

For a small \(\frac{d}{\lambda_D}\)-ratio the influence of the plasma is minor. In LEO, where the Debye length is in the order of centimeters, the Debye length is much smaller than the distances between formation flying spacecrafts. Subsequently the Coulomb forces between spacecraft in LEO are negligible. As discussed in Section 3.4, in L_2 the plasma Debye length is in between 10 and 700 m. The interspacecraft Coulomb forces will hereby often be negligible in L_2, but also on certain moments not, i.e. when Darwin is situated in the lobes.

In reality the spacecraft charges will be different from the indications above. The satellites are not spherical, but in the shape of a tube with a flat sunshield attached to it. The sunshield will be illuminated and the rest of the telescope is in its shadow. When modeling the spacecraft potential, one will have to take care of effects like conductivity, geometry and shadowing. Also the influence of the FEEP-thrusters on the spacecraft charge and the plasma environment near the spacecraft will have to be looked at. Other parameters that can change the spacecraft charging and thus the inter-spacecraft Coulomb-forces are contamination on the spacecraft surfaces, eclipses and temporal variations in the plasma environment.