Elastodynamic Marchenko Focusing, Green's Function Retrieval and Imaging

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SUMMARY

Building on acoustic autofocusing in 1D media, we previously proposed acoustic Marchenko imaging for 1D and 3D media. Recently, the first steps have been set towards extending the single-sided Marchenko method to the elastodynamic situation. Here we discuss the extension of single-sided Marchenko focusing, Green's function retrieval and imaging to the elastodynamic situation. With numerical examples in a horizontally layered medium we show that, at least in principle, a true amplitude image can be obtained, free of artefacts related to multiple reflections and wave conversions. The method can be extended to 3D situations, in a similar way as we extended the acoustic 1D method to the 3D situation.
Introduction

Building on work of Rose (2002) and Broggni and Snieder (2012) on acoustic autofocusing in 1D media, we proposed acoustic Marchenko imaging for 1D and 3D media (Slob et al., 2014; Wapenaar et al., 2014). Marchenko imaging can be summarised as follows. Given the reflection response at the surface (after surface-related multiple elimination) and an estimate of the direct arrival of the Green’s function between a virtual-source point in the subsurface and a receiver at the surface, an iterative solution of the 3D single-sided Marchenko equation gives the complete Green’s functions (including the correct internal multiples) between the virtual source in the subsurface and receivers at the surface. Subsequently, these Green’s functions are used to redatum the reflection response from the surface to the depth of the virtual sources in the subsurface. This redatumed reflection response, which is free of disturbances by the overburden (including the internal multiples), is subsequently used for imaging.

Recently, Wapenaar and Slob (2014) and da Costa et al. (2014) made the first steps towards extending the single-sided Marchenko method to the elastodynamic situation. Here we review the single-sided elastodynamic Green’s function representations (Wapenaar, 2014) and discuss some aspects of elastodynamic Marchenko focusing. Next we discuss how to use the retrieved elastodynamic Green’s functions for redatuming and imaging.

Single-sided Green’s function representations

For simplicity we consider horizontally layered media, so that all our expressions can be given in the \((p, \tau)\)-domain, where \(p\) is the ray parameter and \(\tau\) the intercept time. Note, however, that our expressions can also be formulated in the space-time domain for 3D inhomogeneous media (Wapenaar and Slob, 2014). Consider the following elastodynamic Marchenko-type Green’s function representations

\[
G^{−+}(p, z_0, z_i, \tau) + F^+_1 (p, z_0, z_i, \tau) = \int_{−\infty}^{\tau} R(p, z_0, \tau − \tau') F^+_1 (p, z_0, z_i, \tau') d\tau',
\]

\[
−G^{−−} (p, z_0, z_i, \tau) − F^-_1 (−p, z_0, z_i, −\tau) = −\int_{−\infty}^{\tau} R(p, z_0, \tau − \tau') F^-_1 (−p, z_0, z_i, −\tau') d\tau'.
\]

The boldface symbols are \(2 \times 2\) matrices, hence

\[
R(p, z_0, \tau) = \begin{pmatrix} R_{PP} & R_{PS} \\ R_{SP} & R_{SS} \end{pmatrix} (p, z_0, \tau),
\]

etc. Here, for example, \(R_{PS}(p, z_0, \tau)\) is the plane-wave reflection response at the acquisition level \(z_0\) in terms of upgoing \(P\)-waves in response to downgoing \(S\)-waves. \(G^{−+}(p, z_0, z_i, \tau)\) and \(G^{−−}(p, z_0, z_i, \tau)\) are Green’s functions, i.e., responses to a virtual source at depth level \(z_i\), observed at \(z_0\). Here the second superscript refers to the radiation direction of the virtual source at \(z_i\) (+ for downward and − for upward radiation), whereas the first superscript (−) denotes that the observed field at \(z_0\) is upward propagating.

The elements are organized similar as in equation (3). For example, \(G^{−+}_{PS}(p, z_0, z_i, \tau)\) represents an up going \(P\)-wave observed at \(z_0\), in response to a virtual source for downgoing \(S\)-waves at \(z_i\), \(F^+_1\) and \(F^-_1\) are focusing functions in a reference medium, which is identical to the real medium above the focus level \(z_i\) and homogeneous below this level. \(F^+_1 (p, z_0, z_i, \tau)\) is defined such that, when emitted from \(z_0\) into the medium, it focuses at \(\tau = 0\) at \(z_i\). The superscript + denotes that it propagates downward (i.e., in the positive \(z\)-direction) at \(z_0\). Its elements are organized in a similar way as in equation (3). Hence, the elements in the left column, \(F^-_{PP}(p, z_0, z_i, \tau)\) and \(F^-_{SP}(p, z_0, z_i, \tau)\), represent the downward propagating \(P\)- and \(S\)-waves at \(z_0\), respectively, which give rise to a \(P\)-wave focus at \(z_i\). Similarly, the elements in the right column cause an \(S\)-wave focus at \(z_i\). \(F^-_1 (p, z_0, z_i, \tau)\) is the upward (−) reflected part of the focusing function at \(z_0\) (see Wapenaar et al. (2014) for a more detailed discussion).

Representations (1) and (2) provide the basis for retrieving the Green’s functions \(G^{−+}\) and \(G^{−−}\). The reflection response \(R(p, z_0, \tau)\) in these representations is obtained from measurements at the top boundary \(z_0\) only. Once the focusing functions \(F^+_1\) and \(F^-_1\) at \(z_0\) are known, the Green’s functions follow from equations (1) and (2).
Single-sided Marchenko focusing and Green’s function retrieval

Under certain conditions (Wapenaar, 2014), the focusing functions $F^+_1$ and $F^-_1$ can be obtained from the single-sided reflection response $R$ by solving equations (1) and (2) via an iterative Marchenko scheme in the time intervals where $G^{-,+}$ and $G^{-,+}$ are zero. This assumes not too complex media, so that the overlap in time of the functions on the left-hand sides in equations (1) and (2) is minimum. The scheme is initiated with the inverse of the direct and forward scattered parts of the transmission response of the medium between $z_0$ and $z_i$. Hence, this requires an estimate of the medium between these two depth levels. It is expected that the requirements for a model which explains forward scattering are less severe than the requirements for a model which explains backward scattered multiple reflections.

We illustrate the method for the horizontally layered model of Figure 1, in which $z_0 = 0$ m and $z_i = 1000$m. Figure 2 shows two components of the reflection response at the surface, $R(p, z_0, \tau)$, for a range of $p$-values. In the following we choose a fixed $p$-value of $p = 0.0002$ s/m (i.e., the right-most trace in both figures). The $\tau < 0$ part of the top trace in Figure 3a shows the focusing function $F^+_1(p, z_0, z_i, \tau)$ obtained from $R(p, z_0, \tau)$ after four iterations of the Marchenko scheme. As a matter of fact, it shows the superposition of the elements $F^+_1(p, z_0, z_i, \tau)$ and $F^-_1(p, z_0, z_i, \tau)$ in the right column of $F^+_1$, i.e., the downward propagating $P$- and $S$-waves at $z_0 = 0$, which give rise to an $S$-wave focus at $z_i$. The other traces in this figure show how this complex wave field propagates through the medium and causes a well-defined $S$-wave focus at $\tau = 0$ and $z_i = 1000$m. The focal point acts as a virtual source for downgoing $S$ waves, which, after propagation and scattering, reach the top surface $z_0$ in the form of upgoing $P$ and $S$-waves (the right column of $G^{-,+}$ in equation (1)). A similar experiment, with the left column of $F^-_1$ emitted from $z_0$ into the medium, yields a $P$-wave focus at $z_i$, which acts as a virtual source for downgoing $P$-waves, finally causing upgoing $P$- and $S$-waves at the top surface $z_0$ (the left column of $G^{-,+}$ in equation (1)). This is not shown because of space constraints.

Note that the result in Figure 3a lacks a virtual source for upgoing $S$-waves. Equation (2) suggests to emit the time-reversal of $-F^+_1$ (for opposite rayparameter) from $z_0$ into the medium, which gives $-G^{-,+}(p, z_0, z_i, \tau)$, i.e., the response at $z_0$ to a virtual source for upgoing waves at $z_i$. We combine the responses to $F^+_1$ (Figure 3a) and the time-reversal of $-F^-_1$. The superposition of the illuminating wave fields at $z_0$ reads (in simplified notation) $F^+_1(p, \tau) - F^-_1(-p, -\tau)$. The superposition of their responses at $z_0$ (i.e., the sum of the left-hand sides of equations (1) and (2)) gives $G^{-,+}(p, \tau) + F^-_1(p, \tau) -
The total field at $z_0$ is given by $\mathbf{H}(p, \tau) = \mathbf{F}_1^+ (p, \tau) + \mathbf{F}_1^- (p, \tau) - \mathbf{G}^{-+} (p, \tau) - \mathbf{G}^{++} (p, \tau)$. Hence, the reflection response at $z_0$ is resolved from these Green’s functions by inverting equation (4) for each $p$-value. This “redatumed” reflection response is shown in Figure 4. Despite the complexity of the reflection response in Figure 2, Figure 4 clearly shows the response of the single reflector below $z_l$ (the interface at $z = 1400$ m, see Figure 1). The $p$-dependent reflection coefficients are retrieved from these reflection responses after envelope detection. They are denoted by the blue marks in Figure 5. The green curves in this figure are the modeled $p$-dependent reflection coefficients of the interface at $z = 1400$ m. Note that for this idealized example the match is perfect. Repeating this procedure for all depth levels $z_l$ yields an image in the $(p, z)$ domain (not shown).

**Figure 3** Single-sided Marchenko focusing. (a) Illumination from above by $\mathbf{F}_1^+ (p, \tau)$. (b) Illumination from above by $\mathbf{F}_1^+ (p, \tau) - \mathbf{F}_1^- (-p, \tau)$, combined with its time-reversal.
Conclusions
We have discussed an extension of single-sided Marchenko focusing, Green’s function retrieval and imaging to elastic media. With numerical examples in a horizontally layered medium we have shown that, at least in principle, a true amplitude image can be obtained, free of artefacts related to multiple reflections and wave conversions. The method can be extended to 3D situations, in a similar way as we extended the acoustic 1D method to 3D. The representations for this 3D extension have been formulated (Wapenaar, 2014; da Costa et al., 2014). An important research question concerns the requirements with respect to the initial estimate of the Green’s functions, which in principle should contain the direct and forward scattered transmission response. Sampling issues also require further research.

References