



Delft University of Technology

## emg3d: A multigrid solver for 3D electromagnetic diffusion

Werthmüller, Dieter; Mulder, Wim; Slob, Evert

### DOI

[10.21105/joss.01463](https://doi.org/10.21105/joss.01463)

### Publication date

2019

### Document Version

Final published version

### Published in

Journal of Open Source Software

### Citation (APA)

Werthmüller, D., Mulder, W., & Slob, E. (2019). emg3d: A multigrid solver for 3D electromagnetic diffusion. *Journal of Open Source Software*. <https://doi.org/10.21105/joss.01463>

### Important note

To cite this publication, please use the final published version (if applicable). Please check the document version above.

### Copyright

Other than for strictly personal use, it is not permitted to download, forward or distribute the text or part of it, without the consent of the author(s) and/or copyright holder(s), unless the work is under an open content license such as Creative Commons.

### Takedown policy

Please contact us and provide details if you believe this document breaches copyrights. We will remove access to the work immediately and investigate your claim.

# emg3d: A multigrid solver for 3D electromagnetic diffusion

Dieter Werthmüller<sup>1</sup>, Wim A. Mulder<sup>1, 2</sup>, and Evert C. Slob<sup>1</sup>

<sup>1</sup> TU Delft, Building 23, Stevinweg 1, 5048 CN Delft, NL <sup>2</sup> Shell Global Solutions International BV, Grasweg 31, 1031 HW Amsterdam, NL

DOI: [10.21105/joss.01463](https://doi.org/10.21105/joss.01463)

## Software

- [Review](#) ↗
- [Repository](#) ↗
- [Archive](#) ↗

Submitted: 03 May 2019

Published: 22 July 2019

## License

Authors of papers retain copyright and release the work under a Creative Commons Attribution 4.0 International License ([CC-BY](#)).

## Summary

Controlled-source electromagnetic (CSEM) surveys are a common geophysical investigation tool in the search for, amongst other, groundwater, hydrocarbons, and minerals. The numerical modelling of CSEM data requires the solution of the Maxwell equations. These can be simplified in the particular case of CSEM, as the frequencies used in surveys are usually sufficiently low to ignore any displacement currents. A diffusive problem remains, which has the resulting system of equations given in the frequency domain by

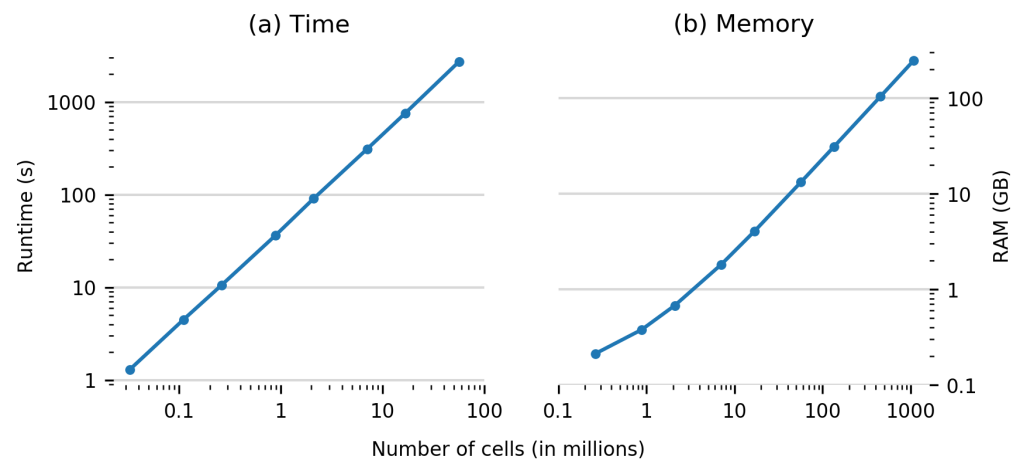
$$\eta \mathbf{E} - \nabla \times \mu_r^{-1} \nabla \times \mathbf{E} = -i\omega \mu_0 \mathbf{J}_s,$$

where  $\eta = i\omega \mu_0 (\sigma - i\omega \epsilon)$ . The electric field and the current source are denoted as  $\mathbf{E}$  and  $\mathbf{J}_s$ , respectively,  $\sigma$  is the conductivity,  $\omega$  is the angular frequency,  $\epsilon = \epsilon_0 \epsilon_r$  is the electric permittivity, and  $\mu = \mu_0 \mu_r$  is the magnetic permeability.

Various open-source codes exist to model CSEM responses for a layered Earth, e.g., *DIPOLE1D* (Key, 2009) and *empymod* (Werthmüller, 2017), and for a two-dimensional Earth, e.g., *MARE2DEM* (Key & Oval, 2011). Open-source modellers for a three-dimensional (3D) Earth only recently became available, notably *SimPEG* (Cockett, Kang, Heagy, Pidlisecky, & Oldenburg, 2015), *PETGEM* (Castillo-Reyes, Puente, & Cela, 2018), and *custEM* (Rochlitz, Skibbe, & Günther, 2019). SimPEG is a framework that not only includes CSEM but also other geophysical methods and can model them on various types of regular grids. It currently primarily uses the direct solver *PARDISO* (Schenk & Gärtner, 2004). PETGEM and custEM use finite elements with the *FEniCS* solver (Alnaes et al., 2015). All three codes require substantial memory and are not easily run on a laptop for models with several million cells.

Mulder (2006) has shown that the multigrid method (Briggs, Henson, & McCormick, 2000), with its optimal scaling for both runtime and memory consumption as shown in Figure 1, works fine for diffusive CSEM problems. This was later also confirmed by others (Jaysaval, Shantsev, Kethulle de Ryhove, & Bratteland, 2016). However, the multigrid CSEM codes discussed in these publications are proprietary. The code *emg3d* is a multigrid solver for 3D CSEM diffusion with tri-axial electrical anisotropy using a staggered grid (Mulder, 2006, 2007). It can act as a solver on its own, or be used as a preconditioner for various Krylov subspace methods. Multigrid solvers can struggle to converge with strong grid-stretching or strong anisotropy. We implemented *semicoarsening* (coarsening the grid only in some coordinate directions) and *line relaxation* (solving for the values of a whole gridline simultaneously in some directions) inside the multigrid technique to deal with these issues (Jönsthövel, Oosterlee, & Mulder, 2006).

The code is written completely in Python using the NumPy/SciPy stack (Jones, Oliphant, Peterson, & others, 2001; Walt, Colbert, & Varoquaux, 2011), where the most time- and



**Figure 1:** Example showing the optimal scaling of emg3d for both (a) runtime and (b) memory consumption. The model is a homogeneous fullspace of 1 S/m and frequency is 1 Hz (running as a single thread on an Intel(R) Xeon(R) CPU @ 2.50GHz).

memory-consuming parts are sped up through jitted functions using Numba (Lam, Pitrou, & Seibert, 2015). It can currently be used as a stand-alone modeller or as a solver in conjunction with the SimPEG-framework.

## Acknowledgements

This code was developed within the Gitaro.JIM project funded through MarTERA as part of Horizon 2020, a funding scheme of the European Research Area (ERA-NET Cofund, <https://www.martera.eu>).

## References

- Alnaes, M. S., Blechta, J., Hake, J., Johansson, A., Kehlet, B., Logg, A., Richardson, C., et al. (2015). The FEniCS project version 1.5. *Computing in Science & Engineering*, 3. doi:[10.11588/ans.2015.100.20553](https://doi.org/10.11588/ans.2015.100.20553)
- Briggs, W., Henson, V., & McCormick, S. (2000). *A Multigrid Tutorial, Second Edition*. Society for Industrial and Applied Mathematics. doi:[10.1137/1.9780898719505](https://doi.org/10.1137/1.9780898719505)
- Castillo-Reyes, O., Puente, J. de la, & Cela, J. M. (2018). PETGEM: A parallel code for 3D CSEM forward modeling using edge finite elements. *Computers & Geosciences*, 119, 123–136. doi:[10.1016/j.cageo.2018.07.005](https://doi.org/10.1016/j.cageo.2018.07.005)
- Cockett, R., Kang, S., Heagy, L. J., Pidlisecky, A., & Oldenburg, D. W. (2015). SimPEG: An open source framework for simulation and gradient based parameter estimation in geophysical applications. *Computers & Geosciences*, 85, 142–154. doi:[10.1016/j.cageo.2015.09.015](https://doi.org/10.1016/j.cageo.2015.09.015)
- Jaysaval, P., Shantsev, D. V., Kethulle de Ryhove, S. de la, & Bratteland, T. (2016). Fully anisotropic 3-D EM modelling on a Lebedev grid with a multigrid pre-conditioner. *Geophysical Journal International*, 207(3), 1554–1572. doi:[10.1093/gji/ggw352](https://doi.org/10.1093/gji/ggw352)
- Jones, E., Oliphant, T., Peterson, P., & others. (2001). SciPy: Open source scientific tools for Python. Retrieved from <http://www.scipy.org>

- Jönsthövel, T. B., Oosterlee, C. W., & Mulder, W. A. (2006). Improving multigrid for 3-D electro-magnetic diffusion on stretched grids. *European Conference on Computational Fluid Dynamics*.
- Key, K. (2009). 1D inversion of multicomponent, multifrequency marine CSEM data: Methodology and synthetic studies for resolving thin resistive layers. *Geophysics*, *74*(2), F9–F20. doi:[10.1190/1.3058434](https://doi.org/10.1190/1.3058434)
- Key, K., & Owall, J. (2011). A parallel goal-oriented adaptive finite element method for 2.5-D electromagnetic modelling. *Geophysical Journal International*, *186*(1), 137–154. doi:[10.1111/j.1365-246X.2011.05025.x](https://doi.org/10.1111/j.1365-246X.2011.05025.x)
- Lam, S. K., Pitrou, A., & Seibert, S. (2015). Numba: A LLVM-based Python JIT Compiler. In *Proceedings of the second workshop on the llvm compiler infrastructure in hpc, LLVM '15* (pp. 7:1–7:6). New York, NY, USA: ACM. doi:[10.1145/2833157.2833162](https://doi.org/10.1145/2833157.2833162)
- Mulder, W. A. (2006). A multigrid solver for 3D electromagnetic diffusion. *Geophysical Prospecting*, *54*(5), 633–649. doi:[10.1111/j.1365-2478.2006.00558.x](https://doi.org/10.1111/j.1365-2478.2006.00558.x)
- Mulder, W. A. (2007). A robust solver for CSEM modelling on stretched grids. *EAGE Technical Program Expanded Abstracts*, D036. doi:[10.3997/2214-4609.201401567](https://doi.org/10.3997/2214-4609.201401567)
- Rochlitz, R., Skibbe, N., & Günther, T. (2019). custEM: Customizable finite element simulation of complex controlled-source electromagnetic data. *Geophysics*, *84*(2), F17–F33. doi:[10.1190/geo2018-0208.1](https://doi.org/10.1190/geo2018-0208.1)
- Schenk, O., & Gärtner, K. (2004). Solving unsymmetric sparse systems of linear equations with pardiso. *Future Generation Computer Systems*, *20*(3), 475–487. doi:[10.1016/j.future.2003.07.011](https://doi.org/10.1016/j.future.2003.07.011)
- Walt, S. van der, Colbert, S. C., & Varoquaux, G. (2011). The NumPy array: A structure for efficient numerical computation. *Computing in Science & Engineering*, *13*(2), 22–30. doi:[10.1109/MCSE.2011.37](https://doi.org/10.1109/MCSE.2011.37)
- Werthmüller, D. (2017). An open-source full 3D electromagnetic modeler for 1D VTI media in Python: empymod. *Geophysics*, *82*(6), WB9–WB19. doi:[10.1190/geo2016-0626.1](https://doi.org/10.1190/geo2016-0626.1)