Prediction of Tail Downwash, Ground Effect and Minimum Unstick Speed of Jet Transport Aircraft
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R. Slingerland

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PREDICTION OF TAIL DOWNWASH, GROUND EFFECT AND MINIMUM UNSTICK SPEED OF JET TRANSPORT AIRCRAFT

PROEFSCHRIFT

ter verkrijging van de graad van doctor aan de Technische Universiteit Delft,
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Summary

One of the requirements to be imposed on the horizontal tail surface of a subsonic transport airplane is the so-called minimum unstick speed. This speed is determined during a dynamic maneuver that has to prove to lowest speed at which the airplane can safely and repeatedly lift off the ground. On one hand this is a safety issue and therefore part of the certification procedure (FAR/JAR 107(d)), on the other hand it has economic significance. At low speeds there is a risk of flow separation on the wing that reduces lift substantially and increases drag drastically. Consequently, the aircraft requires a much longer take-off field length or can not take-off at all. However, at higher speeds a longer field length is required, with possibly a reduction in payload or fuel. Therefore, it is of vital importance to obtain the lowest unstick speed feasible, but still safe. Sufficient rotation capacity must be generated by the horizontal tailplane. This can be a limiting requirement for the horizontal tail area.

With regard to the weight and drag of the horizontal tailplane it is important to be able to calculate this speed in an early design phase. This requires a fast and flexible computation tool, suited for parameter variation and based on a minimum of aerodynamic and geometrical data. Therefore, the following aerodynamic characteristics in ground effect must be determined: lift, drag and pitching moment. They have been split in aircraft characteristics tail-off on one hand and the downwash at the tail on the other hand.

For the purpose of the downwash an aerodynamic model has been developed that utilizes Prandtl's classical lifting-line theory. The power of this model is that it is composed of closed-form analytical expressions to determine the downwash in free-air. In ground effect these expressions must be integrated numerically. This model is innovative in the sense that it includes the vortex sheet relaxation. For that purpose the vortex sheet vertical displacement as well as its amount of rolling-up are taken into account. Also innovative is that wing sweep and dihedral, the interaction of the wing- and flap vortices and the contribution of the rear fuselage-mounted nacelles have been modelled. The same model is used to determine the downwash at the engine intake at the rear fuselage. The change in lift, drag and pitching moment due to ground effect are also determined using lifting-line theory, but without vortex sheet relaxation. This tool is also used for the downwash computation at the intake of wing-mounted engines.

This aerodynamic model has been extensively validated against experimental data. Overall, the developed method appears to be a considerable improvement compared to existing handbook methods such as ESDU and DATCOM. Especially the free-air
downwash is predicted well. In ground effect the accuracy is good and better than for ESDU with flaps retracted, but less satisfactory and not better than ESDU with flaps fully deflected. The lift in ground effect is also more accurate than existing handbook methods. The drag in ground effect is estimated reasonably well, except for high wing sweep and at high angles of attack. The pitching moment exhibits the least accuracy of the aerodynamic coefficients considered.

For the undercarriage no generic model has been developed. Several existing undercarriages have been added as a module to the tool. Engine tables that were available have been used to model the propulsion.

The aforementioned modules have been used in dynamic simulations of the various manoeuvres that constitute the so-called minimum unstick certification procedure. De simulations run from the nosewheel lift-off speed and the scheduled rotation speed up to lift-off. The scheduled rotation speed has been determined empirically using a large number of aircraft. A control law has been devised for the elevator to prevent the tail striking the ground hard. The stabilizer incidence angle, necessary for a safe take-off, is determined within this program as well. These simulations have been implemented in a fully operational automated horizontal tail sizing computer program, called HOT, that determines center of gravity limitations for varying horizontal tail areas.

The computed minimum unstick speeds appear to be within ±5% compared to certified data for a number of 100-seater aircraft. In addition, the tool appears to indicate correctly whether an aircraft is stall-, geometry- or elevator power-limited with respect to its minimum unstick speed. It should be noted however, that the stall limitation in ground effect is highly inaccurate.

The accuracy in the computed minimum unstick speed enables sizing the horizontal tail phase integrally with the undercarriage and rear-fuselage in the preliminary design for optimum take-off performance. A case study for the Fokker 100 yields a center of gravity limitation within 1% of the certified forward take-off limit. Moreover, it is demonstrated that this model can easily be extended to predict the scheduled rotation speed and that it improves the take-off performance prediction. It should be stressed that no empirical nor aircraft-specific data have been incorporated into the method, with the exception of the undercarriage characteristics of the Fokker 100 and Airbus A330.
Samenvatting

Een van de eisen die gesteld moeten worden aan het horizontaal staartvlak van een subsonic transportvliegtuig is de zogenaamde minimale lostrekkingsnheid. Deze wordt bepaald middels een dynamische manoeuvre, waarmee aangetoond moet worden bij welke laagste haalbare snelheid het vliegtuig veilig en herhaaldelijk losgetrokken kan worden van de grond. Enerzijds heeft dit een veiligheidsaspect en is daardoor een onderdeel van de certificatie geworden (FAR/JAR 107(d)), anderszijds heeft dit een economische betekenis. Het risico bij zeer lage snelheden is dat de strömning over de vleugel loslaat, waardoor de draagkracht aanzienlijk verminderd en de weerstand drastisch toeneemt. Daardoor heeft het vliegtuig een veel langere startbaanlengte nodig of kan zelfs in zijn geheel niet loskomen. Bij hogere snelheden echter is vanzelf een langere baanlengte nodig, wat tot gevolg kan hebben dat het vliegtuig minder betalende lading of brandstof mee kan nemen. Het is dus van belang om een zo laag mogelijke, maar nog steeds veilige lostrekkingsnheid te realiseren. Er moet dan wel voldoende rotatievermogen door het horizontaal staartvlak geleverd worden. Dit kan een maatgevende eis voor het staartvlakoppervlak zijn.

Gezien het gewicht en de weerstand van het horizontale staartvlak is het van belang deze snelheid reeds in een vroege ontwerp stadium te kunnen berekenen. Dit vraagt om een snelle en flexibele rekenmethode, geschikt voor parameter variatie en gebaseerd op een minimum aan aërodynamische en geometrische gegevens. Daarvoor moeten de volgende aërodynamische eigenschappen in grond effect bepaald worden: de draagkracht, de weerstand en het moment om de dwarsas. Deze zijn gesplitst in de eigenschappen van het vliegtuig zonder staart enerzijds en de neerstoming ter plekke van het stabilo anderszijds.

Voor deze neerstoming is een aërodynamisch model ontwikkeld, dat gebruik maakt van de klassieke dragende-lijn theorie van Prandtl. De kracht van het model is dat het is opgebouwd uit gesloten analytische uitdrukkingen voor de bepaling van de neerstoming in de vrije vlucht. In grond effect moeten deze uitdrukkingen numeriek geïntegreerd worden. Het vernieuwende van dit model is dat het de vervorming van het vervelvlak meeneemt. Daartoe worden zowel de zakking als de mate van oprollen van het vervelvlak in rekening gebracht. Eveneens vernieuwend is dat ook de pijlhoek, de V-stelling, de interactie van vleugel- en klepervels en de bijdrage van motor- gondels achteraan de romp zijn gemodelleerd. Met dezelfde methodiek wordt ook de neerstoming ter plekke van motorinlaten achteraan de romp bepaald. De veranderingen van draagkracht, weerstand en langsmoment in grond effect worden ook bepaald met de dragende lijn theorie, maar dan zonder vervorming van het vervelvlak. Deze
methodiek wordt ook toegepast bij het berekenen van de neerstroming ter plekke van de inlaat van motorgondels onder de vleugel.

Dit aerodynamische model is uitgebreid gevalideerd aan de hand van experimentele gegevens. Het ontwikkelde model blijkt in het algemeen een aanzienlijke verbetering op te leveren ten opzichte van de huidige handboekmethoden zoals de ESDU en DATCOM. Met name de neerstroming in de vrije vlucht wordt goed voorspeld. In grond-effect is de nauwkeurigheid goed en beter dan ESDU met kleppen ingetrokken, maar minder bevredigend en niet beter dan ESDU met kleppen volledig uitgeslagen. De draagkracht in grond-effect is ook nauwkeuriger dan de bestaande handboekmethoden. De weerstand wordt redelijk goed geschat, behalve bij grote pijnhoeken van de vleugel en bij grote invalshoeken. Het langsmoment vertoont de minste nauwkeurigheid van de beschouwde aerodynamische coëfficiënten.

Voor het onderstel zelf is geen generiek model ontwikkeld. Enkele bestaande onderstellen zijn als module aan de methodiek toegevoegd. Voor de voorstuwing is gebruik gemaakt van beschikbare motortabellen.

De voornoemde modules zijn gebruikt in dynamische simulaties van de verschillende manoeuvres, waaruit de zogenaamde minimale losreksnelheid certificatie procedure is opgebouwd. De simulaties lopen vanaf de neuswiellosreksnelheid en vanaf de voorgeschreven rotatiesnelheid tot aan de loskomsnelheid. De voorgeschreven rotatiesnelheid is empirisch bepaald aan de hand van een groot aantal vliegtuigen. Er is een regelwerk ontwikkeld voor de aansturing van het hoogteroer, die moet voorkomen dat de staart hard tegen de baan slaat. Ook de instelhoek van het stabilo, benodigd voor een veilige start, wordt met dit programma bepaald. Deze simulaties zijn ondergebracht in HOT, een volledig operationeel geautomatiseerd horizontaal startvlak-dimensionerings computerprogramma, dat zwaarte- en motortabellen bepaalt voor variërende horizontale startoppervlakken.

De berekenende minimale losreksnelheden blijken voor een aantal vliegtuigen in de categorie van 100-zitters een nauwkeurigheid te hebben van ±5% ten opzichte van de gecertificeerde waarden. Ook blijkt de methodiek correct aan te geven of een vliegtuig overtrek gelimiteerd is in zijn minimale losreksnelheid of geometrisch of hoogteroer capaciteit-gelimiteerd. Daarbij moet aangetekend worden dat de overtreklimiet in grond-effect een grote mate van onzekerheid in zich heeft.

De nauwkeurigheid in de berekenende minimale losreksnelheid maakt het mogelijk al in het voorontwerpstadium het horizontaal startvlak integraal met het onderstel en het rompontwerp de dimensieering voor optimale startprestaties. Een voorbeeld berekening voor de Fokker 100 leidt tot een zwaarte- en motortabellen behorend bij de minimale losreksnelheid die minder dan 1% MAC afwijkt van de gecertificeerde voorste limiet in de start. Bovendien wordt aangetoond dat dit model eenvoudig kan worden uitgebreid om de geplande rotatiesnelheid te voorspellen en dat het de voorspelling van de startprestaties verbetert. Tenslotte is opgemerkt dat de ontwikkelde methodiek HOT geen empirische noch vliegtuig-specifieke gegevens gebruikt, afgezien van de onderstellarkarakteristieken van de Fokker 100 en de Airbus A330.
# Nomenclature

## Symbols

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A$</td>
<td>$b^2/S$ aspect ratio</td>
<td>[-]</td>
</tr>
<tr>
<td>$a_x$</td>
<td>horizontal acceleration</td>
<td>[m/s²]</td>
</tr>
<tr>
<td>$a_z$</td>
<td>vertical acceleration</td>
<td>[m/s²]</td>
</tr>
<tr>
<td>$b$</td>
<td>wing span</td>
<td>[m]</td>
</tr>
<tr>
<td>$B$</td>
<td>Boost Ratio in longitudinal control system</td>
<td>[-]</td>
</tr>
<tr>
<td>$b_f$</td>
<td>flap span</td>
<td>[m]</td>
</tr>
<tr>
<td>$c$</td>
<td>local wing chord</td>
<td>[m]</td>
</tr>
<tr>
<td>$\bar{c}$</td>
<td>mean aerodynamic chord</td>
<td>[m]</td>
</tr>
<tr>
<td>$C_D$</td>
<td>drag coefficient</td>
<td>[-]</td>
</tr>
<tr>
<td>$C_{D_o}$</td>
<td>zero-lift drag coefficient</td>
<td>[-]</td>
</tr>
<tr>
<td>$c_g$</td>
<td>S/b geometric chord</td>
<td>[m]</td>
</tr>
<tr>
<td>$c_l$</td>
<td>profile lift coefficient</td>
<td>[-]</td>
</tr>
<tr>
<td>$C_L$</td>
<td>lift coefficient</td>
<td>[-]</td>
</tr>
<tr>
<td>$C_{L,a}$</td>
<td>lift curve slope</td>
<td>[1/deg]</td>
</tr>
<tr>
<td>$C_{L_{n.b}}$</td>
<td>tail lift curve slope</td>
<td>[1/deg]</td>
</tr>
<tr>
<td>$C_{L_{n.b}}$</td>
<td>tail lift derivative to $\delta_e$</td>
<td>[1/deg]</td>
</tr>
<tr>
<td>$C_m$</td>
<td>pitching moment coefficient</td>
<td>[-]</td>
</tr>
<tr>
<td>$C_{m_o}$</td>
<td>aerodynamic moment coefficient</td>
<td>[-]</td>
</tr>
<tr>
<td>$D_{ram}$</td>
<td>ram drag</td>
<td>[N]</td>
</tr>
<tr>
<td>$e$</td>
<td>Oswald factor</td>
<td>[-]</td>
</tr>
<tr>
<td>$g$</td>
<td>acceleration due to gravity</td>
<td>[m/s²]</td>
</tr>
<tr>
<td>$G$</td>
<td>Gearing, ratio elevator deflection to stick travel</td>
<td>[rad/m]</td>
</tr>
<tr>
<td>$h$</td>
<td>height above runway (ground)</td>
<td>[m]</td>
</tr>
<tr>
<td>$i$</td>
<td>incidence</td>
<td>[degr]</td>
</tr>
<tr>
<td>$I_{yy}$</td>
<td>moment of inertia around y-axis</td>
<td>[kgm²]</td>
</tr>
<tr>
<td>$k_f$</td>
<td>artificial flap span increase factor</td>
<td>[-]</td>
</tr>
<tr>
<td>$k_{ra}$</td>
<td>correction factor for rolling-up out of the vortex sheet plane</td>
<td>[-]</td>
</tr>
<tr>
<td>$k_{ra0}$</td>
<td>correction factor for rolling-up in the vortex sheet plane</td>
<td>[-]</td>
</tr>
<tr>
<td>$K_z$</td>
<td>downwash factor</td>
<td>[-]</td>
</tr>
<tr>
<td>$K_{z\alpha}$</td>
<td>downwash factor including sweep</td>
<td>[-]</td>
</tr>
<tr>
<td>$k_q$</td>
<td>gain constant</td>
<td>[-]</td>
</tr>
<tr>
<td>$k_q$</td>
<td>gain constant</td>
<td>[-]</td>
</tr>
<tr>
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<td>Description</td>
<td>Unit</td>
</tr>
<tr>
<td>--------</td>
<td>-------------</td>
<td>------</td>
</tr>
<tr>
<td>$k_0$</td>
<td>gain constant</td>
<td>[-]</td>
</tr>
<tr>
<td>$l$</td>
<td>length</td>
<td>[m]</td>
</tr>
<tr>
<td>$l_s$</td>
<td>sectional lift</td>
<td></td>
</tr>
<tr>
<td>$l_h$</td>
<td>length between aerodynamic centers wing and tail</td>
<td>[m]</td>
</tr>
<tr>
<td>$L$</td>
<td>lift</td>
<td>[N]</td>
</tr>
<tr>
<td>$m$</td>
<td>$z/(b/2)$ dimensionless z-coordinate</td>
<td>[-]</td>
</tr>
<tr>
<td>$m_{sh}$</td>
<td>$z/(b/2)$ dimensionless vertical distance between a flat vortex sheet and its image</td>
<td>[-]</td>
</tr>
<tr>
<td>$m_{fus}$</td>
<td>$z/(b/2)$ dimensionless vertical distance flat vortex sheet to tail</td>
<td>[-]</td>
</tr>
<tr>
<td>$m_{vd}$</td>
<td>$z/(b/2)$ dimensionless vertical displacement of flat vortex sheet and the ground</td>
<td>[-]</td>
</tr>
<tr>
<td>$N$</td>
<td>normal force</td>
<td>[N]</td>
</tr>
<tr>
<td>$NUC$</td>
<td>undercarriage normal force</td>
<td>[N]</td>
</tr>
<tr>
<td>$q$</td>
<td>dynamic pressure</td>
<td>[N/m$^2$]</td>
</tr>
<tr>
<td>$q_0$</td>
<td>dynamic pressure within wake</td>
<td>[N/m$^2$]</td>
</tr>
<tr>
<td>$q_{em}$</td>
<td>dynamic pressure at wake center</td>
<td>[N/m$^2$]</td>
</tr>
<tr>
<td>$r$</td>
<td>$x/(b/2)$ dimensionless x-coordinate</td>
<td>[-]</td>
</tr>
<tr>
<td>$S$</td>
<td>reference wing area</td>
<td>[m$^2$]</td>
</tr>
<tr>
<td>$s$</td>
<td>$b/2$ semispan</td>
<td>[m]</td>
</tr>
<tr>
<td>$T$</td>
<td>thrust</td>
<td>[N]</td>
</tr>
<tr>
<td>$TOFL$</td>
<td>take-off field length</td>
<td>[m]</td>
</tr>
<tr>
<td>$TOW$</td>
<td>take off weight</td>
<td>[N]</td>
</tr>
<tr>
<td>$V$</td>
<td>true air speed</td>
<td>[m/s]</td>
</tr>
<tr>
<td>$V_{sm,n}$</td>
<td>stalling speed, minimum speed during stall</td>
<td>[m/s]</td>
</tr>
<tr>
<td>$V_{s,g}$</td>
<td>stalling speed, speed at g-break during stall</td>
<td>[m/s]</td>
</tr>
<tr>
<td>$u$</td>
<td>induced horizontal speed</td>
<td>[m/s]</td>
</tr>
<tr>
<td>$v$</td>
<td>induced sideways speed</td>
<td>[m/s]</td>
</tr>
<tr>
<td>$w$</td>
<td>induced vertical speed</td>
<td>[m/s]</td>
</tr>
<tr>
<td>$w_e$</td>
<td>wake semi-width</td>
<td>[m]</td>
</tr>
<tr>
<td>$W$</td>
<td>weight</td>
<td>[N]</td>
</tr>
<tr>
<td>$x$</td>
<td>x-coordinate (body axis system)</td>
<td>[m]</td>
</tr>
<tr>
<td>$\bar{x}$</td>
<td>$(x/c)_g$, dimensionless center of gravity</td>
<td>[-]</td>
</tr>
<tr>
<td>$y$</td>
<td>y-coordinate (body axis system)</td>
<td>[m]</td>
</tr>
<tr>
<td>$\bar{y}$</td>
<td>y-coordinate of mean aerodynamic chord</td>
<td>[m]</td>
</tr>
<tr>
<td>$z$</td>
<td>z-coordinate (body axis system)</td>
<td>[m]</td>
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<tr>
<td>$\bar{z}$</td>
<td>undercarriage extension</td>
<td>[m]</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>angle of attack</td>
<td>[degr]</td>
</tr>
<tr>
<td>$\alpha_{max}$</td>
<td>stall angle of attack</td>
<td>[degr]</td>
</tr>
<tr>
<td>$\alpha_{gr,max}$</td>
<td>maximum rotation angle on the ground</td>
<td>[degr]</td>
</tr>
<tr>
<td>$\beta$</td>
<td>$1/(\pi A_e)$ induced drag factor</td>
<td>[-]</td>
</tr>
<tr>
<td>$\beta$</td>
<td>side wash angle</td>
<td>[degr]</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>strength of distributed vortex along the chord</td>
<td>[m/s]</td>
</tr>
<tr>
<td>$\gamma_2$</td>
<td>second segment climb angle</td>
<td>[rad]</td>
</tr>
<tr>
<td>$\Gamma$</td>
<td>strength of concentrated vortex at the quarter-chord</td>
<td>[m$^2$/s]</td>
</tr>
<tr>
<td>$\Gamma$</td>
<td>dihedral</td>
<td>[degr]</td>
</tr>
<tr>
<td>Symbol</td>
<td>Description</td>
<td>Unit</td>
</tr>
<tr>
<td>--------</td>
<td>--------------------------------------------------</td>
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</tr>
<tr>
<td>$\delta_e$</td>
<td>elevator deflection</td>
<td>[degr]</td>
</tr>
<tr>
<td>$\delta_f$</td>
<td>flap deflection</td>
<td>[degr]</td>
</tr>
<tr>
<td>$\Delta_{ge}$</td>
<td>change due to ground effect</td>
<td>[-]</td>
</tr>
<tr>
<td>$\Delta t$</td>
<td>time step</td>
<td>[s]</td>
</tr>
<tr>
<td>$\Delta z$</td>
<td>change in z-direction</td>
<td>[m]</td>
</tr>
<tr>
<td>$\Delta_{ge-c}$</td>
<td>downwash change at the wing quarter-chord due to camber</td>
<td>[rad]</td>
</tr>
<tr>
<td>$\Delta_{ge-w}$</td>
<td>downwash change at the wing quarter-chord due to upwash</td>
<td>[rad]</td>
</tr>
<tr>
<td>$\varepsilon$</td>
<td>downwash angle</td>
<td>[rad]</td>
</tr>
<tr>
<td>$\zeta$</td>
<td>y/c relative height within wake</td>
<td>[-]</td>
</tr>
<tr>
<td>$\zeta_w$</td>
<td>w/c relative wake semi-width</td>
<td>[-]</td>
</tr>
<tr>
<td>$\eta$</td>
<td>y/(b/2) dimensionless spanwise coordinate</td>
<td>[-]</td>
</tr>
<tr>
<td>$\eta_{ho}$</td>
<td>relative loss in dynamic pressure at the wake center</td>
<td>[-]</td>
</tr>
<tr>
<td>$\theta$</td>
<td>pitch angle</td>
<td>[degr]</td>
</tr>
<tr>
<td>$\Lambda$</td>
<td>quarter-chord sweep angle</td>
<td>[rad]</td>
</tr>
<tr>
<td>$\lambda$</td>
<td>$c_t/c_r$ taper ratio</td>
<td>[-]</td>
</tr>
<tr>
<td>$\xi$</td>
<td>x/c, relative x-coordinate along chord</td>
<td>[-]</td>
</tr>
<tr>
<td>$\mu$</td>
<td>rolling friction coefficient</td>
<td>[-]</td>
</tr>
<tr>
<td>$\rho$</td>
<td>air density</td>
<td>[kg/m$^3$]</td>
</tr>
<tr>
<td>$\varphi$</td>
<td>integration parameter for wing</td>
<td>[rad]</td>
</tr>
<tr>
<td>$\Psi$</td>
<td>stream function</td>
<td>[m$^2$/s]</td>
</tr>
</tbody>
</table>

**Subscripts**

- $ac$ aerodynamic center
- $ave$ average
- $bs\,sh$ between sheets, i.e. the free-air and its image counterpart
- $c$ including effects due to chordwise pressure distributions
- $c/A$ quarter-chord point in plane of symmetry
- $cg$ center of gravity
- $damp$ undercarriage damping force
- $df$ due to flap deflection
- $ell$ elliptical
- $eng$ engine
- $exp$ experimental
- $ext$ at maximum extension of UC
- $ext$ extended chord due to flap extension
- $f$ due to flap deflection
- $fl$ flapped part of the wing
- $fme$ fuselage-mounted engines
- $fus$ fuselage
- $fus$ flat vortex sheet
- $fwd$ forward fuselage ahead of wing
- $ge$ due to ground effect
- $gr$ on the ground
**Nomenclature**

<table>
<thead>
<tr>
<th>Abbreviation</th>
<th>Definition</th>
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<tr>
<td>gross</td>
<td>thrust at exhaust</td>
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<tr>
<td>h</td>
<td>horizontal tail</td>
</tr>
<tr>
<td>i</td>
<td>induced drag, time step</td>
</tr>
<tr>
<td>i + 1</td>
<td>next step in numerical integration</td>
</tr>
<tr>
<td>ige</td>
<td>in ground effect</td>
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<tr>
<td>im</td>
<td>image counterpart</td>
</tr>
<tr>
<td>int</td>
<td>intake</td>
</tr>
<tr>
<td>j</td>
<td>wing-fuselage junction</td>
</tr>
<tr>
<td>jet</td>
<td>jet exhaust</td>
</tr>
<tr>
<td>lbo</td>
<td>lower integration boundary</td>
</tr>
<tr>
<td>lemac</td>
<td>leading edge of mean aerodynamic chord</td>
</tr>
<tr>
<td>LOF</td>
<td>lift-off</td>
</tr>
<tr>
<td>lv</td>
<td>lifting vortices</td>
</tr>
<tr>
<td>m</td>
<td>median of the wing wake</td>
</tr>
<tr>
<td>mac</td>
<td>mean aerodynamic chord</td>
</tr>
<tr>
<td>min</td>
<td>minimum</td>
</tr>
<tr>
<td>MU</td>
<td>minimum unstick</td>
</tr>
<tr>
<td>muc</td>
<td>main undercarriage</td>
</tr>
<tr>
<td>nac</td>
<td>nacelle</td>
</tr>
<tr>
<td>nuc</td>
<td>nose undercarriage</td>
</tr>
<tr>
<td>melof</td>
<td>nosewheel lift-off</td>
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<tr>
<td>oge</td>
<td>out of ground effect</td>
</tr>
<tr>
<td>P</td>
<td>at computation point P</td>
</tr>
<tr>
<td>plos</td>
<td>plane of symmetry</td>
</tr>
<tr>
<td>pruv</td>
<td>partially rolled-up vortex sheet (in plos)</td>
</tr>
<tr>
<td>R</td>
<td>rotation</td>
</tr>
<tr>
<td>r</td>
<td>root chord</td>
</tr>
<tr>
<td>rear</td>
<td>rear part of the fuselage behind the wing</td>
</tr>
<tr>
<td>ruv</td>
<td>fully rolled-up vortices (in tip plane)</td>
</tr>
<tr>
<td>ruwd</td>
<td>fully rolled-up vortices displacement relative to trailing edge</td>
</tr>
<tr>
<td>ruvs</td>
<td>fully rolled-up vortex sheet (in plos)</td>
</tr>
<tr>
<td>S</td>
<td>stall</td>
</tr>
<tr>
<td>shsv</td>
<td>single horse-shoe vortex</td>
</tr>
<tr>
<td>spr</td>
<td>undercarriage spring force</td>
</tr>
<tr>
<td>stat</td>
<td>at static extension of undercarriage (aircraft at standstill)</td>
</tr>
<tr>
<td>t</td>
<td>horizontal tail</td>
</tr>
<tr>
<td>tar</td>
<td>target value</td>
</tr>
<tr>
<td>thec</td>
<td>tail bumper contact</td>
</tr>
<tr>
<td>te</td>
<td>trailing edge</td>
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<tr>
<td>TO</td>
<td>Tail-Off</td>
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<td>to</td>
<td>take-off</td>
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<td>tr</td>
<td>trimmed</td>
</tr>
<tr>
<td>tv</td>
<td>trailing vortices</td>
</tr>
<tr>
<td>ubo</td>
<td>upper integration boundary</td>
</tr>
<tr>
<td>uc</td>
<td>due to undercarriage extension, undercarriage</td>
</tr>
<tr>
<td>vsd</td>
<td>vortex sheet displacement</td>
</tr>
</tbody>
</table>
$w$ wake
\n$w$ wing

$wfair$ wing-fuselage fairing

$wlf$ wing-slat-flap, the total at the plane of symmetry

Acronyms

$ADB$ Aerodynamic Data Base

$AEQ$ All Engines Operating

$AND$ Aircraft Nose Down

$ANU$ Aircraft Nose Up

$DATCOM$ DATA COMpendium, American handbook

$DNW$ German-Dutch co-owned wind tunnel in the Netherlands

$ESDU$ Engineering Science Data Unit (formerly RAeS), European handbook

$EX$ EXPERimental

$HOT$ Horizontal Tail sizing, present tool

$HTAIL$ Horizontal TAIL sizing, previous tool

$IGE$ In Ground Effect

$MAC$ Mean Aerodynamic Chord

$NPLS$ Numerical Planar Lifting Surface, panel code

$OEI$ One Engine Inoperative

$OGE$ Out of Ground Effect

$PROPER$ PROgram for PERformance, computer program used at Fokker Advanced Design Department

$TB$ TorenBeek method

$RPM$ Revolutions Per Minute

$TSACL$ Tail Strike Alleviation Control Law

$WT$ Wind Tunnel
Chapter 1

Introduction

1.1 Historical background

The minimum unstick speed $V_{MU}$ was first found to be significant during operations with the first civil jet transport the DeHavilland Comet in the early fifties. Two crashes occurred due to wing stall in ground effect (IGE). Due to flow separation the loss in lift coefficient was about 30% and required a 15% higher lift-off speed, whereas the drag increased considerably and reduced the forward acceleration. Both effects prohibited the aircraft from lifting off and finally led to runway overrun and a crash. Subsequent investigation revealed that the danger of over-rotation is typical for jet aircraft as they lack the beneficial effects of the propeller slipstream on lift and flow separation on the wing. As a result, a test sequence was set up to establish the lowest calibrated speed at and above which the aircraft can safely lift off the ground, and continue the takeoff. This became the minimum unstick speed and certification item FAR/JAR 25.107(d). The Comet underwent modification to its wing leading edge to postpone separation and was fitted with a tailbumper to prevent over-rotation and damage in case of a tailstrike.

Even today tail-strikes occur every now and then (Figure 1.1). Tailstrike may be the result of operational errors such as erroneously calculated weight, center of gravity or rotation speed $V_R$ or incorrect stabilizer incidence, and may also follow after an engine failure or tyre burst. Whichever the cause, the current system of certification ensures safe operations in case of a tailstrike by demanding proof that the aircraft can fly away safely.

This was achieved by linking the $V_{MU}$ to the $V_R$ via the lift-off speed $V_{LOF}$ through a set of margins in between. The safety barrier is twofold: first these margins reduce the likelihood of a tailstrike by providing room for operational deviations. Second, if a tailstrike does happen nevertheless, it ensures that the aircraft can get away with it. However, there might be a price to pay. The $V_R$ is initially determined from the FAA/JAA 25.107(e) requirement that stipulates that after engine failure at $V_R$ the aircraft has to arrive at 35 ft screen-height with at least 1.2 $V_{spmin}$. In addition, it has to demonstrate a minimum climb gradient after gear retraction and out of ground effect of 2.4, 2.7 or 3.0% for 2, 3, or 4-engined aircraft respectively.
(FAA/JAA 25.121(b)). If the $V_{MU}$ after flight testing appears to be so high that the prescribed margins with respect to the $V_R$ are not attained, the $V_R$ has to be increased. The all engines operating (AEO) take-off field length (TOFL) will go up as a consequence. So an increase in $V_{MU}$ to the extent that leads to an increase in $V_R$ results in a performance penalty, however, making the $V_{MU}$ an issue of safety as well as economy. Thus it can be highly attractive to drive down the $V_{MU}$ by increasing the coefficient of lift. The limitation thereof does not necessarily have to be the wing stall but can also be a pitch limit due to the tail touching the ground (see the cover). Alternatively, a lack of elevator power might result in a take-off without the first two scenarios happening. Here the horizontal tail sizing comes in. Depending on the type of $V_{MU}$ limit, an aircraft designer might want to adapt the location and layout of the undercarriage, the rear fuselage and the horizontal tail.

Recent developments in undercarriage design by the two world-players Airbus and Boeing underline the importance of the $V_{MU}$ with regard to field performance. Airbus has developed a dedicated articulating bogey undercarriage for its A330-340 programme intended to increase the rotation angle of the aircraft with the wheels still touching the ground through an ingenious bogey pitch trimmer (Figure 1.2). In addition, the main leg features a shortening mechanism. Airbus claims that this reduces the $V_{MU}$ by some 5 Kts. Boeing claims that their competitive system on the B777-300ER together with heavier brakes and a tailstrike protection system reduces the take-off field distance by an impressive 1000 ft.

1.2 Motivation of the present research

In the early nineties the author was involved in tail sizing studies of the Fokker 70, a shortened 80-seat version of the Fokker 100 twin jet. As a result of shortening its tail moment arm the center of gravity range was decreased to a lesser extent, thereby demanding more longitudinal control capacity. The effects were investigated at the Advanced Design Department of Fokker using the computer program HTAIL. This
1.2 Motivation of the present research

![Diagram of an articulating bogey]

Figure 1.2. Articulating bogey of the Airbus A330 main undercarriage

The program computed center of gravity (cg) limits stemming from a large number of longitudinal stability and control requirements. The program was set up for automated horizontal tail sizing studies during preliminary design. Most requirements have been derived from the airworthiness requirements but some from specific Fokker experience. Some requirements were related to free air and others to in ground effect conditions, such as:

- with advised stabilizer setting and the most forward center of gravity (cg) a 3 deg/s pitch rate shall be achievable
- with advised stabilizer setting and the most aft cg location no auto-rotation may occur
- with a fully mistrimmed stabilizer and the most forward cg the aircraft must be able to take-off safely, i.e. the nose must start to raise soon enough after elevator deflection at $V_R$ to avoid forcing the pilot to abort the take-off
- with a fully mistrimmed stabilizer and aft cg the aircraft is not allowed to enter the stall after lift-off with the elevator fully deflected to aircraft nose-down
- the aircraft must be controllable up to the 1-g stall during the landing flare

The program had been validated extensively against the Fokker 100 simulator data package [49]. This revealed that whereas the free-air cg limits were accurate within several percent $\tilde{c}$, the aforementioned in ground effect limits were significantly erroneous. The aerodynamic coefficients in ground effect were estimated from the free air
properties using the ESDU item for ground effect 72023 [14]. In addition, a known deficiency in HTAIL was the lack of the minimum unstick speed requirement.

One of the requirements imposed upon the longitudinal control system is the ability to execute the minimum unstick manoeuvre without unduly restricting the scheduled rotation speed. A comprehensive tail sizing study revealed that the reduced longitudinal control capacity could affect the $V_{MU}$ and drive up the rotation speed $V_R$ and take-off distance. Due to the lack of a suitable model for the calculation of the minimum unstick speed $V_{MU}$, however, no certainty could be given as to the potential deficiency. Several alternative solutions were designed to remedy this situation, which took time and man-hours. Later in the development program flight testing proved these measures to be not necessary as the aircraft exhibited better than anticipated characteristics.

In order to avoid these additional burdens draining attention, people and money away from an already demanding task, a research effort was initiated aimed at developing a tool for estimating the $V_{MU}$ in as early a design phase as possible, preferably as part of an automated horizontal tail sizing tool such as HTAIL. As a consequence, an improvement over the ESDU ground effect module was needed, which would also yield more accurate normal and abused take-off cg limits. Such a tool should enable the designer to integrate the design of the horizontal tailplane, the rear fuselage and undercarriage for optimum take-off performance. This would also yield more accurate $V_R$ and take-off field length as a by-product.

The development of the tool came to an abrupt halt with Fokker’s bankruptcy. But in his new job as an assistant professor at the Royal Netherlands Military Academy the author was granted permission to devote some of his time to convert the research into a doctoral research. That work really gained impetus when he switched to the Faculty of Aerospace Engineering of the Technical University Delft. A new tool was developed, called HOT for HOrizontal Tail sizing, that included a module for downwash \(^1\) and ground effect and the minimum unstick speed.

Just as the horizontal tail is at the rear of conventional aircraft configurations, so its design is often at the end of the preliminary design process, almost as an afterthought. Yet its importance for stability, control and performance warrants attention right from the start: for transport aircraft it contributes some 4% to structure weight and about 10% to total wetted area [36, pp.1.2-7].

The impact on aircraft economics can easily be estimated. If there is no field length limitation, an increase in $V_R$ due to $V_{MU}$ does no harm. But every aircraft meets a runway limitation, depending on airport and ambient conditions. When an aircraft is field length limited for a thrust-to-weight ratio where it also has a $V_{MU}$ limitation, the take-off weight penalty would have been less had it not been $V_{MU}$ limited. Let us indicate the original situation with index 1 and the situation without a $V_{MU}$ limitation by index 2. Without this $V_{MU}$ limitation the lift-off speed can be reduced. For a given take-off field length this can be exploited by an increase in take-off weight $W_2$. If we keep the lift-off speeds in both situations equal we get:

$$V_{LOF_1} = \left( \frac{V_{LOF_1}}{V_s} \right) \sqrt{\frac{W_1}{S \rho \frac{1}{C_{L_{max}}}}} = V_{LOF_2} = \left( \frac{V_{LOF_2}}{V_s} \right) \sqrt{\frac{W_2}{S \rho \frac{1}{C_{L_{max}}}}}$$  \hspace{1cm} (1.1)

\(^1\)based upon [48]
If we assume that for both situations the relation between \( V_{MU} \) and \( V_{LOF} \) remains the same this leads to

\[
W_2 = W_1 \left( \frac{V_{LOF}}{V_{LOP_2}} \right)^2 \approx W_1 \left( \frac{V_{MU}}{V_{mug}} \right)^2
\]

It follows that for 1% drop in lift-off speed the take-off weight benefits 2%. For short-haul aircraft with a payload fraction of 25% this equates into an 8% improvement in payload and thus revenues and for long-range aircraft with 10% payload fraction it yields an impressive 20% improvement in revenues. Of course, the take-off weight improvement can also be used for fuel to increase range instead of payload. It is evident that the minimum unstick speed is highly relevant to aircraft economics, albeit not as dominating as for example the stall speed.

1.3 Minimum unstick speed characteristics

The main obstacles for such a tool are the aerodynamics IGE, the landing gear characteristics and the highly dynamic nature of the minimum unstick maneuver itself. In addition, many design parameters are likely to change during the preliminary design phase and only a small amount of geometric detail is available. This almost precludes the use of CFD that typically needs a high degree of geometric detail and a model with its grid for each geometric design and a lot of CFD experience. Most of the preliminary designers do not qualify as a real CFD expert. Moreover, CFD can only tackle the ground effect on viscous drag through the more complicated Navier-Stokes type of CFD tools. On the other hand it appeared that current analytical handbook methods such as ESDU [14], DATCOM [9], Torenbeek [55], Obert [35], Roskam [40] and Rayner [38] do not provide sufficient accuracy in aerodynamics IGE for the present purpose. The inaccuracy is partly due to the simplifications that have been imposed in order to facilitate closed form expressions for the sake of analytical integration along the wing span. The fuselage-wing combination, for example, is always modelled as an uninterrupted wing and it should be investigated whether that is responsible for the high inaccuracy of \( 0.1 \Delta_{ge} C_L/C_L \) as mentioned in [14]. Furthermore, the origins of these tools can be traced back to the early forties when they were validated against straight-winged propeller driven aircraft such as the Short Shetland and Bristol Brabazon. It seems that the use of state of the art personal computers could bypass the need for these simplifications using numerical solving techniques. Also, wing sweep should be taken into account as well as fuselage-mounted nacelles and a validation against current swept-wing jet transports should be carried out.

The \( V_{MU} \) is influenced by the relation between lift coefficient and angle of attack and by the angle of attack at lift-off. Due to the nature of the unstick maneuver it is not known in advance whether the \( V_{MU} \) will be geometry, stall or elevator power limited. Also, the vertical speed of the wing and horizontal tail can not be calculated. As a result, the angle of attack is undetermined. The common approach is to assume the static case with maximum geometric pitch angle with the main leg fully extended at the point of lift-off and neglect the difference between angles of attack and pitch. Although in some cases this will be not too far off, there are also cases with a high
vertical speed or with the main legs not fully extended at lift-off. Indeed it has been
reported by flight test engineers that they sometimes heard the undercarriage leg hit
its extension socket after lift-off. And when the $V_{MU}$ is elevator power or stall limited,
the pitch angle at lift-off is always smaller than the maximum geometric angle. Again,
this angle can not be calculated from a static case as it is statically undetermined.
Pitch angles at lift-off have been observed of more than 2° smaller than the maximum
value. This leads to a $C_L$ more than 0.2 smaller, which is about 10% of the maximum
$C_L$ at lift-off and thus 5% of $V_{MU}$. This deadlock calls for dynamic simulations,
enabling the designer to determine what limitations the design will encounter.

1.4 Objectives and outline of this thesis

1.4.1 Objectives

The intention of the present research is to develop a tool that helps an aircraft designer
to include the minimum unstick requirement into the horizontal tail sizing process,
and to predict the ground run. Therefore the tool must include:

1. downwash at the tail in ground effect
2. lift tail-off change due to ground effect
3. drag change due to ground effect
4. pitching moment tail-off change due to ground effect
5. a numerical simulation of the minimum unstick manoeuvre
6. an optimizer to obtain the most forward center of gravity that meets $V_{MU}$
   requirements

On one hand the focus shall be on the minimization of the horizontal tail area but
on the other hand the optimization of the take-off distance is highly lucrative as well
and should preferably be available too. The method must fit in the preliminary de-
sign phase with only the aircraft’s major dimensions and characteristics available. In
addition, the tool should be very fast, allowing for parameter variation for optimization
purposes. The tool shall pay great attention to the longitudinal aerodynamics
IGE, the undercarriage characteristics and the dynamics of the unstick manoeuvre
itself. Finally, the tool shall be validated against preferably experimental data from
actual aircraft or else windtunnel or CFD data. In addition it shall be compared to
the aforementioned analytical tools to determine whether it gives an improvement in
accuracy or not.

The required accuracy governs the tool selection and amount of detail. Because
the $V_{MU}$ is directly linked to the stall speed the latter provides some guidance. Per-
formance engineers aim to determine the stall speed during flight testing with an
accuracy of 0.1 KTS because of its economic relevance. This is nowhere near an
indication of the state of the art capabilities for predicting the stall speed however,
which at best is several percentage points. The same holds for flight testing $V_{MU}$. 
1.4 Objectives and outline of this thesis

Somewhat arbitrarily we aim for ± 2.5%. The consequence for \( \Delta_{\rho} C_{L_{TO}} \) is ± 5% \( C_{L_{max}} \) or about ± 0.1. From pitching moment equilibrium at lift-off we can derive that the required equivalent accuracy in downwash in ground effect is then ± 1°. This is also true for variable-incidence tails, as their take-off setting is determined such as to achieve trimmed flight during the second segment climb. According to the airworthiness regulations the aircraft’s configuration is not allowed to be changed below 400 feet (FAR/JAR 25.111c.(4)).

1.4.2 Outline

It is customary to split the aerodynamics for aircraft equipped with a variable-incidence tail into aircraft tail-off (TO) coefficients and the tail-related coefficients. The downwash and dynamic pressure at the tail IGE is captured with a new method that does not rely on a correction of the free-air coefficients, contrary to what most methods do. Instead, it is developed for the free-air case first and validated, as reported in Chapter 2. A paper has been presented as well [30]. First a straight wing is considered and the downwash is determined including the effect of vortex sheet relaxation (vertical displacement and deformation), followed by the addition of sweep and high-lift devices. Also the contribution of rear fuselage-mounted nacelles to the downwash at the tail is investigated. The influence of the pressure distributions on the intake angle of attack is taken into consideration [52]. The next step was to extend this method to the ground effect and validate it as described in chapter 3 [44].

The ground effect on the aircraft tail-off lift and pitching moment is treated in chapter 4 [43, 45]. Special emphasis is placed on the fuselage contribution. A comparison with handbook methods is made. Subsequently the zero-lift and induced drag tail-off is discussed in chapter 5 as well as how it holds up against existing tools. The tail contribution to induced drag is also reviewed.

Once the ground effect on the aerodynamics has been determined, the minimum unstick simulation itself is addressed in chapter 6 [46, 47]. First the airworthiness regulations and advisory circulars for the test sequence are discussed, including the decision tree to determine the type of \( V_{MU} \) limit. The physical model is discussed, which consists of the addition of the undercarriage and propulsion model, and is followed by the equations of motion and the numerical simulation itself. Special emphasis is placed on the lift-off criterion. The module to compute the stabilizer setting required for the take-off is presented. Subsequently elevator control laws are developed to simulate the flying technique in order to prevent tail damage due to severe tailstrike. The comparison of the calculated \( V_{MU} \)'s with certified data for a given center of gravity is shown. Finally the ultimate goal of the research is presented: the center of gravity limits stemming from the \( V_{MU} \) case are demonstrated and integrated in the tail sizing process. In chapter 7 the conclusions of the previous chapters are summarized.
Introduction
Chapter 2

Downwash behind wings in free air

2.1 Introduction

The presence of the ground has a significant effect on the downwash at the tail. The downwash at ground level will be zero, so the downwash at the tail will be reduced compared to free air, yielding a positive lift change on the tail. Because of that the pitching moment of the aircraft changes in the direction aircraft nose down (AND). That requires a larger elevator deflection to maintain the same angle of attack, changing lift in a downward direction until the elevator hits its stop. Moreover, the lift on other aircraft parts changes as well and that also affects the pitching moment. That will be covered in chapter 4. The increased elevator deflection IGE might explain why the horizontal tail size can be governed by the take-off or landing case. This underline the value of the downwash IGE.

Ground effect can be treated by employing the technique of mirroring the wing relative to the ground. In doing so, the boundary condition of zero normal flow at the ground is obeyed. According to Prandtl’s lifting-line theory a wing can be replaced by a set of horse-shoe vortices. The bound vortices are concentrated at the wing’s quarter-chord line and their strength is distributed in spanwise direction so as to resemble the spanwise lift distribution. The trailing vortices descend according to their mutual induced vertical velocities. But near the ground the downwash will be very small and the trailing vortices may be considered horizontal. The tail is far downstream of the wing and will probably not experience the simplification of concentrating all the bound vortices at the wing’s quarter-chord. What results is a system of two identical horizontal opposite single horse-shoe vortices (shsv), one above and one below the ground. Most handbook methods estimate the downwash IGE by calculating the ratio of the downwash caused by this set and by one of the two, reflecting the IGE and free-air case, respectively. This ratio is then multiplied by the free-air downwash from some other, more accurate source such as a CFD computation or a windtunnel or flight test (exp):
\[ \varepsilon_{ige} = \left( \frac{\varepsilon_{ige}}{\varepsilon_{oge}} \right)_{shsv} \varepsilon_{oge_{exp}} \] (2.1)

The idea behind this is that the ratio of the induced velocities of the two mirrored sets of vortices does not alter significantly with the absolute values of the downwash of the constituents. In other words, an inaccuracy in the downwash out of ground effect (OGE) will be present in the downwash IGE also, but their ratio will be more accurate. The advantage of this approach is its combination of simplicity and accuracy, but it needs the free air downwash first. The assumption of the trailing vortices being horizontally aligned is valid only very close to the ground and definitely not in free air, especially not with flaps deployed. This renders the OGE downwash part of the ratio (the denominator in (2.1)) less accurate than the IGE part (the nominator). For take-off performance calculations one needs to know the ground effect right up to screen height and beyond, when the trailing vortices will not be horizontal anymore. Since the present tool is intended to be part of a horizontal tail sizing tool with many other requirements [49], including take-off performance, it would be very valuable if this tool could predict the downwash accurately at any height.

The free air downwash is an asset in itself because of its significance for stability and control. For horizontal tail sizing purposes it is paramount to predict the downwash at the tail accurately. However, this is often difficult to attain with handbook methods. Moreover they can be quite cumbersome due to all the charts that have to be interpolated such as in [14]. The inapplicability of CFD has already been mentioned. In addition, the author had already investigated free air downwash at canards in the framework of his master thesis [48]. Therefore it was decided to tackle the downwash IGE by finding a suitable model for the downwash OGE first and extend that to ground effect later.

Thus there is a need for an alternative, quick yet accurate estimation of the free air tail downwash for a clean configuration and with high-lift devices deployed, since the size of the horizontal tail is often dictated by conditions with flaps deflected. The tool has to be very efficient in order to be suitable for parameter variations so often required during the preliminary design phase. First the downwash for the clean configuration will be dealt with, as a stepping stone for the addition of high-lift devices and nacelles. Subsequently a validation for the clean and landing configuration will be discussed, being the two extreme cases with take-off in between. In this way the applicability may be checked to other design requirements for tail sizing purposes such as the capability of raising the nose right up to the stall in free air and during the landing flare.

The downwash at the intake position will be considered as well because the intake angle of attack is considered a significant parameter. The nacelles effect not only the downwash at the tail but also the lift change due to ground effect. This is caused by the change of the downwash at the intake due to ground effect, especially for rear fuselage-mounted engines. In addition, this lift change has an effect on the pitching moment change in ground effect. The following treatment assumes that the lateral dimensionless coordinate \( \eta = 2y/b = 0 \) which often does not hold for the engine intake. This will be addressed in section 2.8.
2.2 Downwash behind straight wings without vortex sheet relaxation

Since downwash has such a significance, publications on this topic are numerous and date back as early as the origins of aviation. A selection is given in [16]. Because downwash is directly linked to lift generation it is really a matter of spanwise lift distribution. Prandtl laid down the foundations for its calculation by his famous lifting line theory. Some highly complicated schemes have been developed that include the effects of taper, twist, sweep [12] and profile changes in spanwise direction. For minimum induced drag the lift distribution should be elliptical and the downwash infinitely downstream at the vortex sheet is uniform. This greatly simplifies the calculations.

In spite of this, minimum induced drag is not necessarily an optimum goal in itself. Elliptical lift distribution yields a wing bending moment that is greater than the minimum, creating a balance between minimum drag and its effect on fuel weight on one hand and minimum structure weight on the other hand. Shifting the lift more inboard decreases the bending moment and thus wing weight, but also increases induced drag and thus fuel weight. This balance is different for short-haul than for long-haul transport aircraft. Short-haul aircraft carry about 25% of their take-off weight in fuel and long-haul about 45%. That requires a complete optimization including an accurate estimation of wing structure weight and induced drag. But that issue is outside the scope of the current research and has been addressed elsewhere [26]. Furthermore, a wing’s lift distribution consists of a basic contribution at $C_L=0$, generated by the wing twist, and an additional one, dependant amongst others upon $C_L$. Because wing twist is generally negative outboard (wash-out) to tailor the stalling characteristics, an elliptical lift distribution is feasible at one $C_L$ only, usually the cruise $C_L$. As a consequence, at other values of $C_L$ the lift distribution will not be elliptical. Yet in order to prevent the necessity of estimating the lift distribution for a given wing design, we adhere to the commonly used elliptical lift distribution. This greatly simplifies our model and will be the cornerstone for obtaining an efficient computing routine.

First a straight wing will be modelled with lifting-line theory. This model is well known from the work by Prandtl but is presented here as a foundation for subsequent extensions. For high aspect ratios it is allowed to concentrate the lifting lines at the quarter-chord line because the effects of moving these in streamwise direction are negligible. Initially it is assumed that the vortex sheet remains quasi-flat: the lifting vortices will be regarded as horizontal lines in lateral direction and the trailing vortices as horizontal lines with negligible curvature viewed downstream (Figure 2.1). Later on this will be rectified. As a reminder the dimensionless vertical distance from the lifting vortices to a point of interest is indicated as $m_w$ and the one of the trailing vortices by $m_v$. The induced velocities can now be found using Biot-Savart’s law for each horseshoe vortex and integrating in spanwise direction. The unknown vortex strength $\Gamma_0$ can be found from the total lift generated. The lifting and trailing vortex geometry has been sketched in Figure 2.2. Throughout this chapter a coordinate system will be used with the x-axis parallel to the velocity vector and the z-axis positive downward. We now obtain for downwash generated by the lifting vortices:
\[\varepsilon_{i,0} = 2 \int_{0}^{b/2} \frac{dw_i}{V}\]  
\[dw_i = \frac{x}{\sqrt{x^2 + z_i^2}} dV_i = \frac{r}{\sqrt{r^2 + m_{i,v}^2}} dV_i\]  
\[dV_i = \frac{d\Gamma}{4\pi \sqrt{r^2 + m_{i,v}^2}} (\cos \theta_A - \cos \theta_B)\]  
\[\Gamma = \Gamma_0 \sin \varphi\]  
\[C_L = \frac{1}{2} \rho V^2 S = 2 \int_{0}^{b/2} \rho VT dy\]  
\[\eta = \frac{y}{b/2}\]  
\[\cos \theta_A = \frac{\eta}{\sqrt{r^2 + \eta^2 + m_{i,v}^2}}\]  
\[\cos \theta_B = \frac{\eta - \cos \varphi}{\sqrt{r^2 + (\eta - \cos \varphi)^2 + m_{i,v}^2}}\]

Figure 2.1. Quasi-flat vortex sheet behind an elliptically loaded wing in horizontal flight
2.2 Downwash behind straight wings without vortex sheet relaxation

Figure 2.2. Induced velocities due to a horse-shoe vortex

Applying these equations similarly to the trailing vortices yields:

\[ dv_i = \frac{\cos \varphi - \eta}{\sqrt{(\cos \varphi - \eta)^2 + m_{wv}^2}} dV_i \]  \hspace{1cm} (2.2i)  
\[ dV_i = \frac{d\Gamma}{4\pi \sqrt{(\cos \varphi - \eta)^2 + m_{wv}^2}} (\cos \theta_A - \cos \theta_B) \]  \hspace{1cm} (2.2j)  
\[ \cos \theta_A = \frac{\sqrt{r^2 + (\cos \varphi - \eta)^2 + m_{wv}^2}}{r} \]  \hspace{1cm} (2.2k)  
\[ \cos \theta_B = \cos \theta_0 = 1 \]  \hspace{1cm} (2.2l)

First \( \Gamma_0 \) must be determined. For this derivation we may state \( y = b/2\cos \varphi \) because our point of interest coincides with the lifting vortex itself. This should not be confused with (2.2f), which represents another point of interest, i.e. for the downwash.
Then $y$ and $b/2 \cos \varphi$ are not equal, see Figure 2.2(a). We may write:

$$
2 \int_0^{\pi/2} \rho \nu T dy = 2 \rho V \frac{b}{2} \Gamma_0 \int_0^{\pi/2} \sin \varphi d \cos \varphi = \rho V b \Gamma_0 \int_0^{\pi/2} \sin^2 \varphi d \varphi
$$

$$
= \rho V b \Gamma_0 \int_0^{\pi/2} \left( 1 - \cos 2 \varphi \right) d \varphi = \rho V b \left( \frac{b}{2} \Gamma_0 \left( \varphi - \frac{1}{2} \sin 2 \varphi \right) \right)_{0}^{\pi/2}
$$

$$
= \rho V \frac{b^2}{2} \Gamma_0 = \rho V \frac{\pi}{4} \Gamma_0 = C_L \frac{1}{2} \rho V^2 S
$$

and hence

$$
\Gamma_0 = \frac{4}{\pi} C_L V S = 2b \frac{C_L V}{\pi A}
$$

(2.4)

Combining this result with the other expressions in (2.2) we obtain:

$$
\varepsilon_x = K_x \frac{C_L}{\pi A}
$$

(2.5)

with the downwash factor defined as

$$
K_x = \frac{1}{\pi} \int_0^{\pi} \left\{ \frac{r}{r^2 + m_{tv}^2} \left[ \frac{\eta}{\sqrt{r^2 + \eta^2 + m_{tv}^2}} - \frac{\eta - \cos \varphi}{\sqrt{r^2 + (\eta - \cos \varphi)^2 + m_{tv}^2}} \right] + \right.
$$

$$
\frac{\cos \varphi - \eta}{(\cos \varphi - \eta)^2 + m_{tv}^2} \left( 1 + \frac{r}{\sqrt{r^2 + (\cos \varphi - \eta)^2 + m_{tv}^2}} \right) \cos \varphi d \varphi
$$

$$
- \frac{r}{r^2 + m_{tv}^2} \frac{1}{\sqrt{r^2 + \eta^2 + m_{tv}^2}} \frac{1}{\pi} \int_0^{\pi} \cos \varphi d \varphi + \right.
$$

$$
- \frac{1}{\pi} \int_0^{\pi} \left\{ \frac{r}{r^2 + m_{tv}^2} \frac{\cos \varphi - \eta}{\sqrt{r^2 + (\cos \varphi - \eta)^2 + m_{tv}^2}} \right.
$$

$$
+ \frac{\cos \varphi - \eta}{(\cos \varphi - \eta)^2 + m_{tv}^2} \left( 1 + \frac{r}{\sqrt{r^2 + (\cos \varphi - \eta)^2 + m_{tv}^2}} \right) \cos \varphi d \varphi
$$

$$
- \frac{1}{\pi} \int_0^{\pi} \left\{ \frac{r}{r^2 + m_{tv}^2} \frac{\cos \varphi - \eta}{\sqrt{r^2 + (\cos \varphi - \eta)^2 + m_{tv}^2}} \right.
$$

$$
+ \frac{\cos \varphi - \eta}{(\cos \varphi - \eta)^2 + m_{tv}^2} \left( 1 + \frac{r}{\sqrt{r^2 + (\cos \varphi - \eta)^2 + m_{tv}^2}} \right) \cos \varphi d \varphi
$$

(2.6)

The first term in (2.6) calculates the contribution of the bound vortices and the second one of the trailing vortices. This integral can not be solved analytically as it is of the elliptical type of the second order. Solving it numerically we can explore several
2.2 Downwash behind straight wings without vortex sheet relaxation

Interesting solutions, for example in the plane of symmetry (plos). Then \( \eta=0 \) and (2.6) simplifies into:

\[
K_\varepsilon = \frac{2}{\pi} \int_0^{\pi/2} \left\{ \frac{r}{r^2 + m_{tv}^2 \sqrt{r^2 + (\cos \varphi)^2 + m_{tv}^2}} \frac{1}{(\cos \varphi)^2 + m_{tv}^2} \left( 1 + \frac{r}{\sqrt{r^2 + (\cos \varphi)^2 + m_{tv}^2}} \right) \right\} \cos \varphi \, d\varphi \tag{2.7}
\]

![Graph of \( K_\varepsilon \) vs. \( m \) and \( r \)]

**Figure 2.3.** Downwash constant \( K_\varepsilon \) in the plane of symmetry

The solution is shown in Figure 2.3 with \( m_{tv} = m_{tv} = m \). As expected, the downwash increases towards the vortex sheet due to the diminishing distance to it. It reaches a maximum for some x-coordinate, caused by the balance between the increase in the contribution from the bound vortices with decreasing distance on one hand and the more horizontally inclined induced velocity vector \( \vec{V}_i \) on the other hand. The other cause is that the lifting vortices do not generate downwash at \( r=0 \) because their induced velocities are horizontally oriented here, nor at \( r \to \infty \) due to the infinite distance to the bound vortex. The downwash factor \( K_\varepsilon \) infinitely downstream at the x-axis is 2, which is in line with the well-known Prandtl result for a flat, non-rolled up vortex sheet. The contributions by the lifting and trailing vortices at the x-axis can be found by inserting \( m = 0 \) into (2.7):

\[
K_\varepsilon = \frac{2}{\pi} \int_0^{\pi/2} \left\{ \frac{1}{r} \frac{\cos^2 \varphi}{\sqrt{r^2 + (\cos \varphi)^2}} + 1 + \frac{r}{\sqrt{r^2 + (\cos \varphi)^2}} \right\} \, d\varphi \tag{2.8a}
\]
Downwash behind wings in free air

\[
K_z = \frac{2}{\pi} \int_0^{\pi/2} \left( 1 + \frac{1}{r} \sqrt{r^2 + (\cos \varphi)^2} \right) d\varphi
\]
\[
= 1 + \frac{2}{\pi} \int_0^{\pi/2} \left\{ \frac{1}{r} \sqrt{r^2 + (\cos \varphi)^2} \right\} d\varphi
\]

(2.8b)

Figure 2.4. Lifting and trailing vortex contributions to \(K_z\) along the x-axis

This elliptical integral must be solved numerically. Figure 2.4 depicts the total downwash as well as the separate contributions from the lifting vortices (the first term in (2.8a)) and the trailing vortices (the other terms). The contribution of the trailing vortices at \(r = 0\) is half the value infinitely downstream. Hence the downwash factor at \(r = 0\) is \(K_z = 1\), i.e. half the value infinitely downstream. This result is also well known from Prandtl and leads to the famous expression of the induced drag when multiplied with the coefficient of lift:

\[
C_{D_{\text{ind}}} = C_L \frac{C_D}{\pi A} = \frac{C_L^2}{\pi A}
\]

(2.9)

Also visible is the increasing contribution by the lifting vortices moving upstream due to the smaller distance with a singularity for \(r = 0\).

Another area of interest is the Trefftz-plane, the vertical plane infinitely downstream. Then \(r \to \infty\), the first term in (2.6) \(\to 0\) and the fractions with roots in the
denominator in the third term become 1:

\[ K_\varepsilon = \frac{2}{\pi} \int_0^\pi \frac{\cos\varphi - \eta}{(\cos\varphi - \eta)^2 + m^2_{\eta,0}} \cos\varphi d\varphi \]  \hspace{1cm} (2.10)

This result is shown in Figure 2.5. Clearly visible is the downwash inboard and the upwash (negative \( K_\varepsilon \)) outboard of the tip. In the vortex plane, where \( m = 0 \), \( K_\varepsilon = 2 \) along the span. Of course, this is a theoretical result that only holds when the vortex sheet is considered completely flat.

![Figure 2.5. Induced velocity factor \( K_\varepsilon \) in the Trefftz-plane](image)

To improve the previous result, one might regard the flow as a two-dimensional one with the vortex sheet as a finite horizontal line descending vertically through the air. This view has led to an entirely different approach using the technique of conformal mapping. It is derived in appendix A. It yields exactly the same solution in the Trefftz-plane but is mentioned here because it will be required later on. Figure A.2 shows the same downwash as in Figure 2.5 but presented in a different way. For negative values of \( m \), below the vortex sheet, the mirrored image is obtained. In general tail spans are about 30% of wing spans and the tail is often less than \( m = 0.25 \) above the vortex sheet. Also, the local tail \( c_t \) drops to 0 towards the tip and experiences less influence from the downwash than the root section. Therefore we restrict ourselves initially to the downwash in the plane of symmetry. In the Trefftz-
plane \((r \to \infty)\) we then find by inserting \(\eta = 0\) into (2.10):

\[
K_\varepsilon = 4 \pi \int_0^{\pi/2} \frac{(\cos \varphi)^2}{(\cos \varphi)^2 + m_{tv}^2} d\varphi = 2 - \frac{4}{\pi} \int_0^{\pi/2} \frac{m_{tv}^2}{(\cos \varphi)^2 + m_{tv}^2} d\varphi
\]

\[
= 2 - \frac{4m_{tv}}{\pi \sqrt{1 + m_{tv}^2}} \left[ \arctan \left( \frac{\sqrt{1 + m_{tv}^2} - 1}{m_{tv}} \right) + \arctan \left( \frac{\sqrt{1 + m_{tv}^2} + 1}{m_{tv}} \right) \right]
\]

\[
= 2 - \frac{4m_{tv}}{\pi \sqrt{1 + m_{tv}^2}} \left[ \alpha + \beta \right] \quad (2.11)
\]

The analytical integration has been performed by the mathematical program Maple®, that could not further simplify the resulting expression. Still, the terms between brackets hint at a graphical construction, which is made in Figure 2.6. It can be proven that the angle \(\delta\) in the top of the triangle is equal to \(\alpha\):

\[\tan \delta = \frac{m}{\sqrt{1 + m^2} + 1} = \frac{m(\sqrt{1 + m^2} - 1)}{(\sqrt{1 + m^2} + 1)(\sqrt{1 + m^2} - 1)} = \frac{m(\sqrt{1 + m^2} - 1)}{m^2} = \frac{\sqrt{1 + m^2} - 1}{m} = \tan \alpha \quad (2.12)\]

Hence it follows that \(\alpha + \beta = \pi/2\) and that reduces (2.11) into:

\[
K_\varepsilon = 2 \left[ 1 - \sqrt{\frac{m_{tv}^2}{1 + m_{tv}^2}} \right] = k_{m_{tv}} \quad (2.13)
\]
This otherwise well-known result (see (A.13)) has hereby been achieved in a new way and is shown in Figure 2.7. Obviously, when moving away from the vortex sheet, the downwash decreases. It features a distinct kink at the vortex sheet for \( m_{vl} = 0 \), that is typical for potential flow but never present in subsonic viscous flow. This matter will be addressed later on. For \( m = 0 \) we obtain the classical result of 2.

![Figure 2.7. Induced velocity factor in the Trefftz-plane infinitely downstream of elliptically loaded straight wings](image)

Sofar we have only walked trodden paths. The integral for \( K_v \) (2.7) for \( \eta = 0 \) has been solved numerically, but a closed form solution is sought. The reason behind this is that the vortex sheet does not remain horizontal nor flat due to rolling-up, especially with increasing \( C_L \) and flap deflection. This subject will be discussed in section 2.4. The vertical displacement and rolling-up of the vortex sheet, also called relaxation, is caused by its own induced velocities. This implies that the relaxation can be modelled by integrating the downwash over the vortex sheet, from the trailing edge to the tail. In order to do so we need a very fast and preferably analytical tool to prevent double numerical integrations. Therefore, we will try to find an approximation of (2.7).

The trailing vortex term in (2.7), the last term, is built up from two factors. The first of these, without \( r \), represents the solution infinitely downstream as given by (2.13). The solution of this integral without the second factor is thus given by half the value of (2.13). Further inspection of (2.7) reveals that the downwash variation with \( m \) is only slightly influenced by \( r \). This may be deduced from the other part of the last term containing \( r \) because the root contains two other terms beside \( m \) that will be larger (a tail will normally lie within \( m = 0.1 \text{ to } 0.3 \)). This means that the downwash in the plane of symmetry due to the trailing vortices can be thought of as the product of two almost independent relations: one in horizontal and one in vertical direction, the latter given by half the value of (2.13). Furthermore, the cosine term in the roots is the integration variable, disappearing in the final solution. This is an indication it might be represented as a constant for certain ranges of \( r \) and \( m \). Two
points of interest have been defined that represent typical locations of a horizontal T-tail and the intake of a rear fuselage-mounted nacelle: \( r=1, m=0.3 \) respectively \( r=0.4, m=0.07 \). Bearing this in mind and building upon the structure of (2.6), the following preliminary solution was suggested:

\[
K_e = \frac{r}{r^2 + m_{iv}^2} \sqrt{\frac{a}{r^2 + b + m_{iv}^2}} + \left[ 1 + \left( \frac{r^2}{r^2 + c + d m_{iv}^2} \right) \right] \left( 1 - \sqrt{\frac{m_{iv}^2}{1 + m_{iv}^2}} \right) (2.14)
\]

The constants \( a \) through \( e \) are numerical constants, selected in such a way that they generate a solution that is close to the numerical solution of (2.6). The five constants were solved by splitting the lifting and trailing parts first. The constants \( a \) and \( b \) can simply be found by filling in the values for the two aforementioned collocation points in the first part of (2.7) (the lifting vortex contribution) and in (2.14) and equating the expressions. This yields two equations for which the two unknowns \( a \) and \( b \) can be solved. For the trailing vortex part it is slightly more complicated for there are 3 constants to be determined. The idea behind the constant \( d \) is as follows. For \( r=0 \) the lifting lines do not contribute and the downwash by the trailing vortices is modelled perfectly by the last term in brackets, being the exact solution as shown by (2.13). The same is true for \( r \to \infty \) for again there is no downwash caused by the lifting vortices. For \( m=0 \) (2.14) can come very close to the exact solution when \( e=0.5 \) and \( c \) is chosen appropriately. So we can have an accurate solution on the \( x \)-axis and above and below the \( x \)-axis for \( r=0 \) and \( \infty \). What is left is the area between the \( x \) and \( z \)-axis, for moderate values of \( r \) and \( m \), right where the tail is. That is taken care of by the factor \( d \), that does not alter the solution described so far. First \( m \) is set to 0 and \( r \) to 1 and 0.4 because we want this approximation to yield accurate downwash also in the vortex plane itself. These points represent two new collocation points. Subsequently only the earlier tail collocation point \((1.0, 0.3)\) is substituted into (2.7) and the last term of (2.14), yielding a total of three equations. By using three collocation points we force the approximation into a shape that will yield accurate solutions in their vicinity. Solving this set of 5 equations the following highly accurate approximation was found:

\[
K_e = \frac{r}{r^2 + m_{iv}^2} \sqrt{\frac{0.4876}{r^2 + 0.6319 + m_{iv}^2}} + \left[ 1 + \left( \frac{r^2}{r^2 + 0.79 + 5.0734 m_{iv}^2} \right)^{0.3113} \right] \left( 1 - \sqrt{\frac{m_{iv}^2}{1 + m_{iv}^2}} \right) (2.15)
\]

The error due to this approximation has been determined by calculating the ratio of the downwash according to (2.15) and (2.7) for a number of combinations of \( r \) and \( m \) (Figure 2.8). It is observed the inaccuracy is less than 1% within the normal range of tail and intake positions behind a wing. Note that the collocation points can be recognized on the lines for error 0: \((1.0, 0.3), (1.0, 0.0)\) and \((0.4, 0.07)\).

We can now conclude that (2.15) is a very fast and highly accurate approximation for the downwash in the plane of symmetry, generated by a vortex sheet without
relaxation behind a straight elliptically loaded lifting surface. The addition of sweep will be dealt with in the next section.

2.3 Swept wings

The subsequent step is to incorporate sweep into the downwash calculation. This has been implemented by adding sweep to the downwash along the $x$-axis only. The downwash in the plane of symmetry is then calculated by multiplication of (2.15) by the ratio of the downwash along the $x$-axis including and excluding sweep. In this way the task of finding an approximation for the extremely complex equivalent of (2.7) but including sweep is bypassed. Similarly to what has been derived in (2.2) we now develop for $\eta = m = 0$, see Figure 2.9:

\[
\varepsilon_{i_s} = 2 \int_0^{b/2} \frac{dw}{V} \\
\Delta w = dw_{3 \rightarrow 4} + dw_{4 \rightarrow \infty} \\
dw_{3 \rightarrow 4} = \frac{d\Gamma}{4\pi x\cos \Lambda} [\cos \theta_A - \cos \theta_B] \\
dw_{4 \rightarrow \infty} = \frac{d\Gamma}{4\pi y} (1 + \cos \theta) \\
\theta_A = \frac{\pi}{2} + \Lambda - \theta \\
\theta_B = \frac{\pi}{2} + \Lambda
\]
Figure 2.9. Horseshoe vortices on a swept wing

\[ \sin \theta = \frac{y}{\sqrt{(x - y \tan \Lambda)^2 + y^2}} \]  

(2.16g)

\[ \cos \theta = \frac{x - y \tan \Lambda}{\sqrt{(x - y \tan \Lambda)^2 + y^2}} \]  

(2.16h)

\[ \Gamma = \Gamma_0 \sin \varphi \]  

(2.16i)

\[ C_L \frac{1}{2} \rho V^2 S = 2 \int_0^{b/2} \rho V \Gamma \, dy \]  

(2.16j)

\[ \cos \varphi = \frac{y}{b/2} \]  

(2.16k)

Note that the sweep is not reflected in (2.16j) because the lift depends only on the integration of the circulation perpendicular to the free air velocity. With

\[ \cos(\alpha + \beta) = \cos \alpha \cos \beta - \sin \alpha \sin \beta \]  

(2.17a)

\[ \cos(\alpha - \beta) = \cos \alpha \cos \beta + \sin \alpha \sin \beta \]  

(2.17b)

we get:

\[ dw = \frac{\Gamma_0 \cos \varphi d\varphi}{4\pi} \left\{ -\frac{\sin \Lambda - \theta}{x \cos \Lambda} + \frac{1 + \cos \theta}{y} \right\} \]  

(2.18)
and with
\[ \sin(\alpha - \beta) = \sin \alpha \cos \beta - \cos \alpha \sin \beta \quad (2.19) \]
we obtain:
\[
dw = \frac{\Gamma_0 \cos \varphi d\varphi}{4\pi} \left\{ \frac{-\sin \Lambda \cos \theta + \cos \Lambda \sin \theta + \sin \Lambda + 1 + \cos \theta}{x \cos \Lambda} \right\}
\]
\[
= \frac{\Gamma_0 \cos \varphi d\varphi}{4\pi} \left\{ \frac{\tan \Lambda (1 - \cos \theta) + \sin \theta + 1 + \cos \theta}{x} \right\}
\]
\[
= \frac{\Gamma_0 \cos \varphi d\varphi}{4\pi} \left\{ \frac{1}{y} + \frac{\tan \Lambda}{x} + \frac{\sin \theta}{x} + \cos \theta \left( \frac{1}{y} - \frac{\tan \Lambda}{x} \right) \right\} \quad (2.20)
\]
Substituting (2.16g) and (2.16h) into (2.20) leads to
\[
dw = \frac{\Gamma_0 \cos \varphi d\varphi}{4\pi} \left\{ \frac{1}{y} + \frac{\tan \Lambda}{x} + \frac{\sin \theta}{x} + \frac{x - y \tan \Lambda \left( \frac{1}{y} - \frac{\tan \Lambda}{x} \right)}{\sqrt{(x - y \tan \Lambda)^2 + y^2}} \right\} \quad (2.21)
\]
This equation can be integrated over the span to yield the downwash angle \( \varepsilon_x \) using (2.16a), (2.16i), (2.4) as the solution of (2.16j) and finally (2.16k):
\[
\varepsilon_{\text{in}} = 2 \int_0^{b/2} \frac{dw}{V} \frac{\pi}{2b}
\]
\[
= 2 \frac{\pi}{V} \int_0^{\pi/2} \frac{\Gamma_0 \cos \varphi d\varphi}{4\pi} \left\{ \frac{1}{y} + \frac{\tan \Lambda}{x} + \frac{\sin \theta}{x} + \frac{x - y \tan \Lambda \left( \frac{1}{y} - \frac{\tan \Lambda}{x} \right)}{\sqrt{(x - y \tan \Lambda)^2 + y^2}} \right\}
\]
\[
= 2 \frac{2b}{\pi} \frac{C_L V}{\pi A} \int_0^{\pi/2} \left\{ \frac{1}{y} \cos \varphi + \frac{\tan \Lambda}{r} + \frac{\cos \varphi}{r} + \frac{(r - \tan \Lambda \cos \varphi) \left( \frac{1}{\cos \varphi} - \frac{\tan \Lambda}{r} \right)}{\frac{\pi}{2} \sqrt{(r - \tan \Lambda \cos \varphi)^2 + \cos^2 \varphi}} \right\} \cos \varphi d\varphi
\]
\[
= \frac{2}{\pi} \left\{ \int_0^{\pi/2} \frac{\tan \Lambda}{r} + \frac{\cos \varphi}{r} + \frac{(r - \tan \Lambda \cos \varphi) \left( \frac{1}{\cos \varphi} - \frac{\tan \Lambda}{r} \right)}{\sqrt{(r - \tan \Lambda \cos \varphi)^2 + \cos^2 \varphi}} \right\} \cos \varphi d\varphi \frac{C_L}{\pi A}
\]
\[
\varepsilon_{i_z} = \left[ 1 + \frac{2}{\pi r} \int_0^{\pi/2} \left\{ \tan \Lambda \cos \varphi + \frac{\cos^2 \varphi (1 + \tan^2 \Lambda) + r^2 - 2r \tan \Lambda \cos \varphi}{\sqrt{(r - \tan \Lambda \cos \varphi)^2 + \cos^2 \varphi}} \right\} d\varphi \right] \frac{C_L}{\pi A}
\]

\[
= \left[ 1 + \frac{2}{\pi r} \int_0^{\pi/2} \left\{ \tan \Lambda + \frac{\sqrt{(r - \tan \Lambda \cos \varphi)^2 + \cos^2 \varphi}}{\sqrt{(r - \tan \Lambda \cos \varphi)^2 + \cos^2 \varphi}} \right\} d\varphi \right] \frac{C_L}{\pi A}
\]

\[
= \left[ 1 + \frac{2}{\pi r} \int_0^{\pi/2} \sqrt{(r - \tan \Lambda \cos \varphi)^2 + \cos^2 \varphi} d\varphi \right] \frac{C_L}{\pi A}
\] \hspace{1cm} (2.22a)

It can be easily verified that for \( \Lambda = 0 \) equation (2.8b) is obtained. Again, we prefer to derive an approximation to enable analytical integration for the vortex sheet vertical displacement later on. This has been achieved by searching a formula that reflects the behavior observed in Figure 2.4 and forcing this through two collocation points (\( r=0.4 \) and \( r=1 \)) for several sweep angles. Thus we obtain for \( \eta = 0 \) and \( m = 0 \):

\[
\varepsilon_x = \left( \frac{0.1124 + 0.1265\Lambda + 0.1766\Lambda^2}{r^2} + \frac{0.1024}{r} + 2 \right) \frac{C_L}{\pi A}, \text{ with } \Lambda \text{ in radians}
\]

\[
= K_{\varepsilon \Lambda} \frac{C_L}{\pi A}
\] \hspace{1cm} (2.23)

Figure 2.10. The downwash factor \( K_{\varepsilon \Lambda} \) in the vortex sheet behind swept wings along the x-axis
2.4 Vortex sheet roll-up

The results for downwash are identical to figure 2 from [16] in the plane of the vortex sheet and shown in Figure 2.10 for various values of $r$ and $\Lambda$. Three factors have been plotted: the solution from (2.22a), the approximation according to (2.23) and the more advanced solution (2.15) which is only valid and plotted for $\Lambda=0$. It is obvious that the approximation of (2.22a) by (2.23) is quite good for moderate sweep angles. A compromise had to be struck in order to obtain a satisfactory approximation over a wide range of sweep angles and relative $x$-locations $r$. The difference between (2.22a) and (2.15) is so small that it can not be seen. It should be recalled that (2.15) includes vertical distance but excludes sweep, whereas (2.23) excludes vertical distance but includes sweep. They have been constructed such that they match very well for the case without sweep and vertical distance. So this is used as a common basis and the effects of vertical distance including sweep are now approximated by:

$$
\varepsilon_x = \frac{K_{\varepsilon_A}}{K_{\varepsilon_A=0}} \left( \frac{r}{r^2 + m_v^2} \frac{0.4876}{\sqrt{r^2 + 0.6319 + m_v^2}} + \left[ 1 + \left( \frac{r^2}{r^2 + 0.7915 + 5.0734m_v^2} \right)^{0.3113} \right] \left\{ 1 - \frac{m_v^2}{1 + m_v^2} \right\} \right) \frac{C_L}{\pi A} \quad (2.24)
$$

This concludes the addition of sweep to the treatment of a flat, horizontal vortex sheet. The effects of rolling-up of the vortex sheet will be discussed next.

2.4 Vortex sheet roll-up

As stated before, the trailing vortices do not remain horizontal but deform under their own induced velocities. The outer trailing vortices curve upwards and inwards relative to the inner ones and vice versa. In the end all vortices behind one wing half coalesce into one completely rolled-up concentrated tip vortex, see Figure 2.11. The effect on the downwash field is significant and is the topic in this section.

This process of rolling-up is extremely complex and has received much research attention. The first scientific publication is from Kaden [23], which models the rolling-up directly behind the trailing edge. However, his theory is not valid any longer at larger distances and needs correction, as indicated by [30, 53]. They have derived that the process of rolling-up may be considered 99% complete for elliptical lift distribution at

$$
r_{rw} = 0.56 \frac{A}{C_L} = 0.56 \frac{q}{W/b^2} \quad (2.25)
$$

Consequently, this phenomenon depends on lift coefficient and aspect ratio, i.e. on span loading $W/b^2$ and dynamic pressure $q$. One may thus expect rolling-up to be completed earlier with increasing angle of attack and flap deflection. This underscores its importance to the present application of tail sizing. It must be remarked that (2.25) is not an exact solution and a gross simplification of the physics. Several publications have criticized this relation [22] and some experiments suggest that the process of rolling-up is already 60% complete at the trailing edge [58]. Inasmuch as these views may be correct, they do not offer a usable theory and (2.25) will be retained for lack of a better alternative. Also, as the rolling-up takes place at some distance downstream
of the wing, the impact of the rolling up of the trailing vortices on the downwash at the location of the lifting vortices is negligible. Therefore elliptical lift distribution is maintained irrespective of the amount of rolling-up.

For the flat vortex sheet this has been handled by taking the ratio of the induced velocities between a pair of completely rolled-up trailing vortices and a flat vortex sheet infinitely downstream in the plane of symmetry. This ratio is then applied to any calculated downwash. It is assumed that the error due to the change in this ratio moving from infinity upstream to a tail location is negligible. The flat vortex sheet has already been described in the previous sections and the rolled-up case will now be addressed.

The spanwise location $b_{raw}/2$ of the fully rolled-up tip vortices can be found by equating their lift to the lift generated by a flat vortex sheet:

$$2 \int_0^{b/2} \rho VT dy = \rho VT_{raw} b_{raw}$$

(2.26)

Because all of the circulation is present in the two concentrated tip vortices, we can also state $\Gamma_{raw} = \Gamma_0$, that is the circulation in the plane of symmetry. The left-hand side of (2.26) has already been solved in (2.3):

$$\rho VT_0 \pi b = \rho VT_{raw} b_{raw}$$

(2.27)

and we find the well-known result

$$b_{raw} = \frac{\pi}{4} b$$

(2.28)
2.4 Vortex sheet roll-up

In order to find the vertical induced velocity by the two tip vortices at an arbitrary point in the Trefftz plane we apply Biot-Savart to an infinitely long vortex:

\[
w_{\text{raw}} = \left[ \frac{\Gamma_0}{4\pi \frac{1}{2} \sqrt{(\frac{\pi}{4} - \eta)^2 + (m_{\text{raw}})^2}} \right] + \frac{\frac{\pi}{4} - \eta}{\sqrt{(\frac{\pi}{4} - \eta)^2 + (m_{\text{raw}})^2}} \left( \frac{1}{(1 - (-1))} \right) (2.29)
\]

so

\[
\varepsilon_{\text{raw}} = \frac{w_{\text{raw}}}{V} = \frac{2}{\pi} \left[ \frac{\frac{\pi}{4} - \eta}{(\frac{\pi}{4} - \eta)^2 + (m_{\text{raw}})^2} + \frac{\frac{\pi}{4} + \eta}{(\frac{\pi}{4} + \eta)^2 + (m_{\text{raw}})^2} \right] \frac{C_L}{\pi A} (2.30)
\]

which is also a classical result and shown in Figure 2.12 together with the lines for the flat vortex sheet from Figure A.2. In the plane of symmetry (\(\eta = 0\)) this equation evolves into:

\[
\varepsilon_{\text{raw}} = \frac{1}{(\frac{\pi}{4})^2 + m_{\text{raw}}^2} \frac{C_L}{\pi A} (2.31a)
\]

\[
= k_{m_{\text{raw}}} \frac{C_L}{\pi A} (2.31b)
\]

which is plotted in Figure 2.13.

For \(m = 0\) we get the classical solution \(16/\pi^2\) which is directly comparable to the value 2 for the non rolled-up vortex sheet as shown in Figure 2.3 and 2.4. The case for complete roll-up shows the physical solution for the discontinuity mentioned on page 19. It is noted that only close to the vortex sheet the downwash is less for the fully rolled-up case than for the flat case, compared for the same vertical distance to the vortices. While (2.13) and (2.31) relate to the downwash at infinity only, their ratio is applied to (2.23) from the distance (2.25) behind the wing where roll-up is complete, and further aft. At the wing trailing edge and further forward no correction is applied as it is assumed that rolling-up starts at the trailing edge. Between the wing trailing edge and the location according to (2.25) a linear ratio between (2.13) and (2.31) has been assumed (Figure 2.14). As already stated, this approach is questionable and the linear relation is entirely motivated by the desire to keep the model as simple as can be. The rolling-up process is already under way at the wing itself and does not start at the trailing edge. But it is extremely difficult to devise a theory to model the rate of rolling-up and no suitable model seems available. Nonetheless, the current approach is an improvement over most existing methods as these assume either a flat
Figure 2.12. The downwash factor due to fully rolled-up vortices in the vertical plane at infinity behind an elliptically loaded wing.

Figure 2.13. The downwash factor variation with vertical distance in the plane of symmetry for the flat vortex sheet and fully rolled-up tip vortices.
vortex sheet \([16, 18]\) or completely rolled-up vortices \([10]\). The linear relation yields:

\[
\varepsilon_x = k_m \begin{cases} 
    k_{\alpha=0} \frac{r}{r^2 + m_{0v}^2} \sqrt{r^2 + 0.6319 + m_{0v}^2} \\
    \left[ 1 + \left( \frac{r^2}{r^2 + 0.79 + 5.0734m_{0v}^2} \right)^{0.3113} \right] \left( 1 - \sqrt{\frac{m_{0v}^2}{1 + m_{0v}^2}} \right) \frac{C_L}{\pi A} 
\end{cases} \tag{2.32}
\]

with

\[
k_m = \begin{cases} 
    \frac{k_{m_{RWV}}}{k_{m_{WS}}}, & \text{if } r \geq r_{RWV}, \\
    1, & \text{if } r \leq r_{TE}, \\
    1 - \left(1 - \frac{k_{m_{RWV}}}{k_{m_{WS}}} \right) \frac{r - r_{TE}}{r_{RWV} - r_{TE}}, & \text{if } r_{TE} < r < r_{RWV}.
\end{cases} \tag{2.33}
\]

The constants \(k_{m_{RWV}}\) and \(k_{m_{WS}}\) are given by \((2.31)\) and \((2.13)\) respectively.

We have now obtained the downwash in the plane of the vortex sheet including sweep and rolling-up, but the vortex sheet is still horizontal. In the next section the vertical movement of the vortex sheet will be treated.

### 2.5 Vortex sheet vertical displacement in the plane of symmetry

The vertical displacement of the vortex sheet is handled by integration of the downwash along the r-axis from the wing trailing edge to the tail quarter-chord, see Figure 2.15:

\[
m_{VS,d} = \int_{r_{TE}}^{r_{RWV}} \tan \varepsilon dr \tag{2.34}
\]
Figure 2.15. The vertical displacement of the quasi-flat vortex sheet

Unfortunately, (2.15) can not be integrated analytically. Therefore, the more simple approximation (2.23) is used instead. Assuming \( \tan \varepsilon \approx \varepsilon \) for small values of the downwash angle we get for the vertical wake displacement from the wing trailing edge up to the horizontal tail:

\[
m_{v,s,d} = \frac{C_L}{\pi A} \int_{r_s}^{r} k_{m_o} \left( \frac{0.1124 + 0.1265 \Lambda + 0.1766 \Lambda^2}{r^2} + \frac{0.1024}{r} + 2 \right) dr \quad (2.35)
\]

The rolling-up affects the downwash not only directly through \( k_m \) in (2.32), but also indirectly by altering the vertical displacement of the vortex sheet through \( k_m \) in (2.35). This effect can be found by substituting \( m=0 \) into (2.13) and (2.31) and using these for the determination of \( k_m \) in (2.35). Note that we have now assumed that the vortex sheet and the rolled-up tip vortices lie in one horizontal plane. This does not really hold but is necessary to find an analytical solution. Also, this simplification is limited to the determination of \( k_{v,ws} \) only. In order to ease integration for the purpose of obtaining the vertical displacement the set of equations is rewritten slightly different from (2.33). This yields the correction factor \( k_{m_o} \) to be used only for the calculation of the vertical displacement of the vortex sheet:

\[
k_{m_o} = \begin{cases} 
  \frac{k_{v,ws}}{k_{f,ws}} & \text{if } r \geq r_{v,ws}, \\
  1 - k_{pr,ws} (r_{v,ws} - r_{t,c}) & \text{if } r_{t,c} \leq r < r_{v,ws}.
\end{cases} \quad (2.36)
\]

\[
k_{pr,ws} = \left( 1 - \frac{k_{v,ws}}{k_{f,ws}} \right) \frac{1}{r_{v,ws} - r_{t,c}} \quad (2.37)
\]
2.5 Vortex sheet vertical displacement in the plane of symmetry

\[ k_{\text{raw}} = \frac{16}{\pi^2} \quad (2.38) \]

\[ k_{fv} = 2 \quad (2.39) \]

The vertical displacement of the partially rolled-up vortex sheet can now be solved analytically by combining (2.35) through (2.39). Whence, if \( r \leq r_{\text{raw}} \):

\[
m_{v,sd} = \left\{ (1 + k_{\text{raw}} r_{te}) \times \left[ -k_\lambda \left( \frac{1}{r} - \frac{1}{r_{te}} \right) + 0.1024 \ln \left( \frac{r}{r_{te}} \right) + 2 (r - r_{te}) \right] - k_\tau \left[ k_\lambda \ln \left( \frac{r}{r_{te}} \right) + 0.1024 (r - r_{te}) + r^2 - r_{te}^2 \right] \right\} \frac{C_L}{\pi A} \tag{2.40a}
\]

or, if \( r > r_{\text{raw}} \):

\[
m_{v,sd} = \left\{ (1 + k_{\text{raw}} r_{te}) \left[ k_\lambda \left( \frac{1}{r_{te}} - \frac{1}{r_{\text{raw}}} \right) + 0.1024 \ln \left( \frac{r_{\text{raw}}}{r_{te}} \right) + 2 (r_{\text{raw}} - r_{te}) \right] - k_{\text{raw}} \left[ k_\lambda \ln \left( \frac{r_{\text{raw}}}{r_{te}} \right) + 0.1024 (r_{\text{raw}} - r_{te}) + r_{\text{raw}}^2 - r_{te}^2 \right] + \frac{k_{fv}}{k_{fv}} \left[ k_\lambda \left( \frac{1}{r_{\text{raw}}} - \frac{1}{r} \right) + 0.1024 \ln \left( \frac{r}{r_{\text{raw}}} \right) + 2 (r - r_{\text{raw}}) \right] \right\} \frac{C_L}{\pi A} \tag{2.40b}
\]

For both cases we write:

\[ m_{v,sd} = k_{m_{v,sd}} \frac{C_L}{\pi A} \quad (2.41) \]

Clearly the vertical displacement factor \( k_{m_{v,sd}} \) of the vortex sheet is predominantly a function of \( C_L \) and \( \lambda \) (through \( r_{te} \)) and the sweep angle \( \Lambda \) (through \( k_\lambda \)). The dimensionless \( x \)-coordinate \( r_{te} \) of the trailing edge has also some influence through the wing aspect- and taper ratio:

\[ r_{te} = \frac{x_{te}}{b/2} = \frac{0.75c_r}{b/2} = \frac{3}{A(1 + \lambda)} \quad (2.42) \]

A series of plots for different values of sweep angle \( \Lambda \), taper ratio \( \lambda \) and aspect ratio \( A \) would have to be produced. However, for the present purpose we restrict ourselves by merely assuming \( r_{te} = 0.4 \). The factor \( k_{m_{v,sd}} \) is plotted in Figure 2.16 for sweep angle 0° and 40°. The lower lines for each sweep angle are equal to the non-rolled up case (\( k_{m_{v,sd}} = 0 \)). It can clearly be seen that the effect of rolling-up on the vertical displacement of the vortex sheet in the plane of symmetry is only minor but still worthwhile. The cause for this is that the rolling-up only comes into play during downstream development of the vortex sheet. At \( r \rightarrow r_{\text{raw}} \) the self-induced velocity has reduced to (2.38) from (2.39) at the trailing edge as can be seen in Figure 2.13. As a result, the downwards movement is reduced. The most upstream location where rolling-up is complete is \( r = 1.4 \) for \( A/C_L = 2.5 \). This is only applicable to full landing
flaps, as will be shown in section 2.7. Therefore in most cases (2.40a) will be needed instead of (2.40b). The calculation scheme so far consists of two main steps: first calculate the vortex sheet displacement \( m_{\text{vd}} \) with (2.40a) or (2.40b), and subsequently the downwash by (2.24). The figures 2.10 and 2.16 may be used for convenience.

### 2.6 Tip vortex relaxation

In this section the horizontal and vertical displacement of the tip vortices will be addressed simultaneously, opposed to the treatment of those of the trailing vortices. The expression "tip vortex" in this context stands not only for the fully rolled-up vortex far downstream, but also for most outboard trailing vortex directly behind a wing. This subject is much more complicated than for the vortex sheet in the plane of symmetry because the effects of rolling-up are much more pronounced. Contrary to a popular misconception, the downwash in the vortex plane behind an elliptical lift distribution is not uniform in spanwise direction at any location of the vortex sheet. This is only true infinitely downstream, in the Trefftz-plane, but not directly behind the lifting surface. As a consequence, the vortex sheet is double-curved. The two separate contributions by the bound and trailing vortices in Figure 2.17(a) and 2.17(b) reveal this clearly. They have been calculated using (2.6). It follows that the lifting vortices are the biggest contributors to non-uniform downwash in spanwise direction. By contrast, the trailing vortices soften this to some extent by their increasing downwash towards the tip. There is a singularity at the edge of the vortex sheet. Their combined downwash is given in Figure 2.17 and shows a uniform downwash in the Trefftz-plane. This is the same solution as we have already observed in Figure 2.5. But a slight decrease in downwash towards the tip region directly behind the lifting surface is also clearly visible. From this figure we learn that the outboard trailing vortices will descend less than the more inboard ones. And once they emerge above
Figure 2.17. The downwash factor at the vortex sheet, non-rolled up
the others, they experience also an inboard component of the induced velocities. Vice versa, the inboard ones are being moved down as well as outboard due to the more outboard vortices. That is how the rolling-up starts and the edge of the vortex sheet starts to curl upward and inward. The process of rolling-up is initially caused by the contribution of the lifting vortices to the downwash at the vortex sheet itself. It is concluded that the process of rolling-up is initially caused by the contribution of the lifting vortices to the downwash at the vortex sheet itself.

Our equations that have been stipulated for a flat vortex sheet no longer hold for a relaxed vortex sheet. The work by Kaden [23] provides a way out by given expressions for the vertical and horizontal displacement of the tip vortex although his theory is only valid directly behind the wing. The solution at infinity has already been derived in the previous sections. A connection between the two with respect to the inward shift has been proposed in [53]:

\[
\eta_{r_{tw}} = 1 - \left(1 - \frac{\pi}{4}\right) \left[\tanh\left(\frac{r}{r_{tw}}\right)\right]^2 \quad (2.43)
\]

and is shown in Figure 2.18. Note that the x-axis depicts the ratio of the actual r-

![Figure 2.18. The effect of rolling-up on the lateral tip vortex position](image)

cordinate and the value where roll-up is considered complete, i.e. \(r_{tw}\). This solution is identical to Kaden’s solution directly behind the wing and is within 4% of the value at the location \(r_{tw}\). This relation is disputed in [12] because it yields too far inward tip vortex locations with increasing sweep of the wing’s trailing edge. However, the effect of rolling-up becomes significant with flap deployment which often reduces the trailing edge sweep. Therefore (2.43) is retained.

The vertical displacement is treated slightly differently. According to [53] this problem can best be tackled by considering a so-called center of gravity of the trailing
vortices. The downwash at this equivalent center of gravity and the trailing edge is then derived to be:

\[
\varepsilon_{\text{vtx}} = (1 - \frac{\pi}{4}) \left( \frac{\Gamma_0}{\beta V} \right) = 2\left(1 - \frac{\pi}{4}\right) \frac{C_L}{\pi A} \approx 0.43 \frac{C_L}{\pi A}
\]  

(2.44)

The downwash at a concentrated tip vortex for \( r \to \infty \) can be determined by inspection of (2.38). This expression yields the downwash due to the two tip vortices in the plane of symmetry at the Trefftz-plane. For the current application however we must consider that their mutual distance is double the value to the plane of symmetry and that each vortex is induced only by its counterpart and not by itself. Because the distance is in the denominator according to (2.2b) we now obtain a quarter of (2.38):

\[
k_{\text{raw}} = \frac{4}{\pi^2} \approx 0.405
\]  

(2.45)

which is also evident in Figure 2.12. Again, this result is well known. Because the two values in (2.44) and (2.45) are so close the downwash to be used for the determination of the vertical movement is considered constant and equal to (2.45):

\[
m_{\text{raw}} = \int_{r_{\text{c}}}^{r} \varepsilon dr = \int_{r_{\text{c}}}^{r} \frac{4 \cdot C_L}{\pi^2 \cdot \pi A} dr = \frac{4}{\pi^2} (r - r_{\text{c}}) \frac{C_L}{\pi A}
\]  

(2.46)

It is noteworthy that this solution is entirely different from the case excluding roll-up. Then the downwash at the tip vortices would follow directly from (2.2). A calculation for \( \eta = 1 \) and \( m = 0.001 \) (to avoid the singularity) yields an almost uniform downwash along the edge of the vortex sheet, that is where the tip vortex is supposed to be (Figure 2.19). This value is almost 5 times bigger than the one for the case including rolling-up.

![Figure 2.19. The downwash factor along the tip vortex excluding rolling-up effects](image)

The calculation scheme ends up quite complicated. The scheme follows two lines as shown in Figure 2.20: one for the vortex sheet and one for the tip vortices. It
Figure 2.20. Schematic of the downwash computation

Features three vertical distances: $m_{lv}$ for the lifting vortices, which is fixed; $m_{vsl}$ for the vortex sheet in the plane of symmetry and finally $m_{rot}$ for the concentrated tip vortices. In addition, a connection must be added between the vertical movement of the vortex sheet and the tipvortices relative to the wing’s trailing edge on one hand and the distance towards the point of interest (for example the tail) on the other hand, see Figure 2.21:

$$m_{fvs} = m_{vsl} + (m_t - m_{le}) \cos \alpha - (r_t - r_{le}) \sin \alpha \quad (2.47)$$

and

$$m_{rot} = m_{rot} + (m_t - m_{le}) \cos \alpha - (r_t - r_{le}) \sin \alpha \quad (2.48)$$

A qualitative comparison with the standard handbook methods is at place here. The improvement relative to [16] is not in the downwash in the vortex sheet, as they are identical, but above or below it. In [16] the vertical displacement is not estimated but roughly approximated by the assumption that the vortex sheet remains in the zero-lift angle of attack reference line. Additionally, a fixed variation of the downwash with the vertical distance to the vortex sheet is assumed, including a fixed amount of
distortion of the vortex sheet. This may be observed in the difference of the change in downwash above and below the vortex sheet. The American standard tool \cite{10} assumes complete roll-up and excludes sweep \cite{11}. Of all methods, \cite{41} seems superior but is labor intensive and lacks a correction for rolling-up and sweep. Its quality is apparent as this method is the basis for \cite{18}. In the present method however, the distortion of the vortex sheet is implicitly incorporated by differentiating between the vertical distance of the lifting and the trailing vortices \eqref{2.24}. Moreover, the extent of rolling-up is taken into account in the vortex sheet development as well as in the downwash itself. The latter feature enables application to flaps, as will be shown in the next section.

2.7 High-lift devices

2.7.1 Location of vortex sheet and tip vortices

High-lift devices are categorized as full-span leading edge slats and partial-span trailing edge flaps. It is assumed that slat deflection at constant lift coefficient does not alter the shape of the spanwise and chordwise lift distribution to the extent of appreciable changes in tail downwash. As a consequence, lift due to slats is treated as wing lift and downwash likewise. But this does not hold for partial-span flaps. As flaps deflect, they generate lift on themselves and on the wing in front and outboard due to their upwash, thereby altering the shape of the total lift distribution. This issue is managed by splitting the total lift distribution into separate wing and flap lift, both assumed elliptical over their own respective spans (Figure 2.22). In this manner, flaps are treated as a wing with flap span, co-located with the main wing. This does not take into account yet any lift carry-over due to flap lift at the wing outboard of the flap tip, but this effect will be treated separately. Likewise, the flap lift distribution is not elliptical by design, as it is the mere outcome of the design lift distribution over the wing and the chord extension due to flap deflection. Furthermore, it is assumed that the chordwise lift distribution over the flap and wing does not have an appreciable effect on tail downwash. In addition, the flat vortex sheets due the wing lift and due to flap lift are both assumed to originate and thus to coincide at the flap trailing
edge. As a consequence, the vertical displacement of the vortex sheet due to wing lift is added to the one due to flap lift (Figure 2.23 and solid horizontal lines in 2.24).

The second vertical displacement, the one of the wing tip vortices, is not corrected for flap lift as the upwash outboard of the flap span decreases rapidly for span ratios of 1.4 and higher (Figure A.2). The third displacement, the vertical displacement of the flap tip vortices, is taken as the summation of their own contribution and the one due to wing lift in the plane of symmetry (see lower tip vortex in Figure 2.24). The reason behind the latter is that the downwash due to wing lift at the location of the flap tip vortex does not alter significantly towards the plane of symmetry, as can be seen in Figure A.2. Moreover, if the downwash at the actual and moving lateral position of the flap tip vortex would be taken, an analytical solution would no longer be viable. Note that the lateral position is outboard of the geometrical flap tip as a consequence of the artificial flap span increase. Note also that the flap tip vortex is assumed to originate at the quarter-chord of the flap tip chord, in contrast to the wing tip vortex that starts at the trailing edge. The rationale behind this is the swept wing tip opposed to the square flap tip. It is reasoned that for a square tip separation of the trailing vortices begins where the pressure differential is largest, which is around the quarter-chord. This is confirmed by the vapor trails as observed during flights.
under humid conditions. With these three vertical displacements, the downwash at the tail due to wing lift and flap lift is determined and added to each other.

The horizontal movement of the tipvortices of the wing and flap vortex systems remain unchanged from the isolated case, i.e. they are treated similarly as discussed in section 2.6. Of course, the wing tipvortex is pulled inboard by the lower flap tipvortex and the flap tipvortex is pulled outboard by the wing tipvortex (Figure 2.24), but this effect can not be calculated analytically. This introduces an error and the validation later on will reveal whether that is allowable or not.

2.7.2 Flap lift carry-over effect

So far this approach is identical to [41], except that there the span-wise lift distributions including lift carry-over effects have been approximated using Fourier series. The flap trailing vortices are weakened due to the reduced slope of the span-wise circulation distribution and also move more outboard (Figure 2.22). This reduces the downwash at the tail. To mimic this effect in the present tool we have increased the flap span artificially by a factor $k_f$ (dashed line in Figure 2.22). Subsequently the design charts in [41] for downwash due to elliptical lift distribution have been recalculated with the present method and varying $k_f$ until the contours for constant downwash in the plane of symmetry coincided. This is shown in Figure 2.25 where our method is overlaid over the righthand top plot in figure 13 from [41] for $C_L = 1.0$. The downwash contours for flap-to-wing span-ratios of 0.4 and 0.7 in [41] could closely be matched using span-ratios of 0.6 and 0.875 respectively. It appeared that varying the aspect ratio from 6 to 12 had no discernible effect. This equates to a flap span increase of 50 and 25% respectively. Obviously no increase is applicable for a span-ratio of 1, leading to:

$$k_f = 1 + \frac{5}{6} \left(1 - \frac{b_f}{b}\right)$$  \hspace{1cm} (2.49)

Nevertheless [41] is based on simple split flaps, whereas today’s aircraft mainly
use Fowler flaps or single, double or even triple slotted flaps. As the flap deflection increases, so does the chordwise flap extension. More lift will be on the flap itself where no lift carry-over occurs as there is no wing outboard of it. This might lead to higher downwash angles than according to the flapspan increase. The validation in section 2.12 will prove whether this requires a separate modelling of the lift on the flap for higher flap angles.

2.7.3 Averaged vortex sheet

Finally another correction is added to incorporate dihedral as well as the distortion of the vortex sheet in spanwise direction. The model so far relies on two distinct pillars. The first is the downwash in the plane of symmetry with the trailing vortices equally curved downwards along the x-axis but straight along the y-axis. The second is the downwash at and induced by the concentrated tip vortices. The amount of roll-up determines their blending. Of course this is a rather crude model, considering the extremely complicated flow around a wing, especially with high-lift devices deployed. Our mathematical model does not easily allow for a vortex sheet that is curved when viewed along the y-axis (see curved lines in Figure 2.24). Nonetheless, this may be accounted for by averaging the vertical movement of the vortex sheet over the value at the plane of symmetry and at the tip vortices (Figure 2.26). To this effect equations have been formulated similarly to (2.2) for a straight but skewed trailing vortex plane inclined through the vortex sheet in the plane of symmetry and through
the tip vortices in the tip plane:

\[
K_v = \frac{4}{\pi} \int_0^{\pi/2} \frac{(\cos \varphi)^2}{(\cos \varphi)^2 + m_{tv}^2} d\varphi
\]

\[
= \frac{4}{\pi} \int_0^{\pi/2} \frac{(\cos \varphi)^2}{(\cos \varphi)^2 + (m_{wolf} (1 - \cos \varphi) + m_{run} \cos \varphi)^2} d\varphi
\]

\[
= \frac{4}{\pi} \int_0^{\pi/2} \frac{(\cos \varphi)^2}{(\cos \varphi)^2 + (m_{wolf} (1 - k_{mvw}) + m_{run} k_{mvw})^2} d\varphi
\]

\[
= 2 \left[ 1 - \sqrt{\frac{(m_{wolf} (1 - k_{mvw})^2}{1 + (m_{wolf} (1 - k_{mvw})^2} \right]
\]

(2.50)

The last equation has been obtained using (2.13). The factor $k_{mvw}$ denotes the averaged height of the vortex sheet. For $k_{mvw} = 0$ we obtain the height of the combined sheets in the plane of symmetry ($m_{wolf}$) and for $k_{mvw} = 1$ the height of the tip vortices ($m_{run}$). By equating these two expressions we obtain the averaged vertical position of the horizontal system that would generate the same downwash as the skewed system. This vertical position can be regarded as a weighted position of a vortex sheet that is straight and horizontal when viewed in y-direction. It was found that for common tail locations $k_{mvw} = 0.5$ i.e. the midpoint between the z-coordinate of the vortex sheet in the plane of symmetry and of the tip vortices:

\[
m_{fus,v} = 0.5(m_{fus} + m_{run})
\]

(2.51)

The vertical location of the wing related and the flap related trailing vortices are both treated this way, leading to the final model (Figure 2.27). This consists of two separate vortex sheets and their concentrated tip vortex. The direct implication is that the vortex sheets no longer coincide. This marks a break from our ambition to stay as close to the physical world as can be, but it is the result of the limitations of
the mathematical tools that we have imposed upon ourselves for the sake of creating a fast and flexible tool.

It should be remarked that dihedral is a function of aerodynamic loading for flexible wings. This is certainly the case with today’s high-aspect ratio wings. The effect of wing bending is that the tip vortices move upward. For a low-set tail in the clean configuration the distance between the vortex sheet and the tail will be relatively small. It can be seen in Figure 2.12 that for small values of \( m \) the downwash variations near the plane of symmetry are small. However for a T-tail or deflected flaps the impact might be significant. Therefore care should be taken in the determination of the wing tip height. For our computations however we have neglected this effect.

A final note must be made on the computation of the aspect ratio \( A \) related to the flap contribution to downwash. In many equations this aspect ratio is required, in combination with the lift coefficient \( C_L \). Let us consider the flapped part of the wing and call the related area the flapped area \( S_{fl} \) and the lift increase due to flap deflection expressed in that area \( \Delta C_{L,fl} \). The lift increase due to flap extension can now be expressed as:

\[
\Delta C_{L,fl} S_{fl} = \Delta \delta f C_{L_w} S_w
\]

(2.52)

The downwash can now be expressed as

\[
\varepsilon = K_c \frac{\Delta C_{L,fl} S_{fl}}{\pi A_{fl}} = K_c \frac{\Delta C_{L,fl} S_{fl}}{\pi b_f^2} = K_c \frac{\Delta \delta f C_{L_w} S_w}{\pi b_f^2} = K_c \frac{\Delta \delta f C_{L_w}}{\pi A_{\delta f}}
\]

(2.53)

with

\[
A_{\delta f} = \frac{b_f^2}{S_w}
\]

(2.54)

Because it is customary to use the wing area as reference area, we use the last expression in (2.53) and (2.54). As a result this "flapped wing" aspect ratio can be very small which seems odd. Of course, being in the nominator (\( C_L \)) and denominator (\( A \)) the reference area cancels out in the downwash calculations, leaving the flap’s span loading as driving parameter for downwash.
2.8 Nacelles

For an application to complete aircraft configurations, the effect of nacelles must be incorporated as well. As nacelles yield lift they may be regarded as small wings of low aspect ratio. These equivalent wings have a span equal to the nacelle span and are located in the horizontal plane of maximum width. They reflect a biplane with the upper and lower wing joined at the tips. Because of the tip-losses their lift will be less than for a wing of the same span. For wing-mounted engines it was assumed that they have no effect on the wing lift distribution. Moreover, their downwash at the tail is negligible, as the distance to the tail is large compared to the nacelle span. But nacelles mounted on a pylon at the rear fuselage can have an appreciable effect on tail downwash. First of all their combined span with the fuselage and pylons in between is considerable, and they are located slightly ahead of the tail instead of beside it (Figure 2.28). Moreover, the angle of attack at the nacelle intake is strongly affected by the downwash behind the wing, which may create large negative angles and lift and thus deliver upwash at the tail. However, the T-tail that usually accompanies rear fuselage-mounted engines is high above the nacelle-pylon combination and that reduces their effect. Also, the nacelle and fuselage act as mirror plates for the pylon and increases its effective span a little.

![Figure 2.28. Modelled lift distribution on the fuselage-pylon-nacelle combination for rear fuselage-mounted engines](image)

The engine intake angle of attack is calculated with the present method as well, the nacelle lift with [15], and the resultant downwash at the tail again with the present method.

The lift on the nacelle-pylon-fuselage combination according to [15] is:

\[
C_{L_{nac}} = 2.4 \frac{N_{nac}(C_{L_n}b)_{nac} + b_{pylon}^2}{S} \left( \alpha - \varepsilon_{int} + \varepsilon_{nac} \right)
\]

with the lift curve slope of the nacelles \(C_{L_{nac}}\) given by figure 1 in [15]. This relation is based on windtunnel test data. The factor 2.4 accounts for the lift carry-over effect of the fuselage-pylon-nacelle combination as well as for the effect of the fuselage on
the local angle of attack. The latter occurs when the flow at the nacelle highlight is not parallel with the fuselage centerline. Then the fuselage alters the velocity normal to the fuselage just like a circle does in a parallel flow, see Figure 2.29. The formula

\[
K_J = 1 + \frac{\eta^2 - m^2}{(\eta^2 + m^2)^2}
\] (2.56)

and the intake angle of attack is found from:

\[
\alpha_{\text{int}} = \alpha + i_{\text{nac}} - \varepsilon_{\text{int}} \quad (2.57a)
\]

\[
= (1 + K_J)(\alpha - \varepsilon_{\text{int},w}) + i_{\text{nac}} \quad (2.57b)
\]

in which \(\varepsilon_{\text{int},w}\) stands for the downwash that would have occurred in the plane of symmetry had there been no fuselage. This downwash angle follows directly from the method as outlined in the previous sections. Equating (2.57a) and (2.57b) yields:

\[
\varepsilon_{\text{int}} = (1 + K_J)\varepsilon_{\text{int},w} - K_J\alpha
\] (2.58)

Differentiation of (2.58) with respect to \(\alpha\) learns that the intake downwash gradient reduces due to the fuselage upwash. The result is an increase in the local flow angle, which contributes to the factor 2.4 being higher than 1. This fuselage induced effect would only have to be modelled separately when comparing intake angles of attack with for example windtunnel data. The ESDU item requires only the downwash at the intake without fuselage effect and incorporates this effect by the factor 2.4.

The engine being close behind the wing’s trailing edge experiences the chordwise pressure distributions on its intake angle of attack. Until now the wing has been modelled as a set of lifting lines, concentrated at the quarter-chord. The real pressure distribution over the chord induces a different downwash in the vicinity of the wing. The pressure peak is further away from the intake, at the wing leading edge, but weaker lifting lines are closer to it. Although the factor 2.4 includes this as well, the ESDU item does not explicitly incorporate it by a lengthwise coordinate. So the number 2.4 should be regarded as an average value over the range of intake positions.
However, if we want to compare the calculated intake angle of attack with computed or experimental data, we need to include the chordwise effect explicitly. Besides, it can also be applied to the tail downwash to investigate whether the impact is negligible or not. This effect is described in the next section. Moreover, it might even be possible to determine the constituents of the factor 2.4 by combining the effects of fuselage-induced upwash, the chordwise effect on the downwash and the mirroring effects of the fuselage and nacelle on the pylon. However, the effects of pylon lift on the fuselage and nacelle require integration of the mirrored vortices along their circumference. In addition, there is also an impact on the flow over the wing’s trailing edge which is more difficult to model. Therefore it was decided to stick with (2.55) and correct only the flow inclination at the inlet.

To summarize it can be concluded that for nacelle lift and its effect on the downwash at the tail the fuselage- and chordwise effects on the intake angle of attack do not have to be computed separately. Only for validation purposes of the intake angle of attack these effects must be computed explicitly.

The vortex sheet’s vertical displacement is not computed according to its own induced velocities, as these are insignificant. Due to the low aspect ratio of the fuselage-pylon-nacelle combination the tip vortex can be considered to be rolled-up instantly. The resultant tipvortices have very little displacement as indicated by (2.46). But the trailing vortices are all immersed in the flow field of the wing- and flap vortex systems. Therefore, their vertical displacement is calculated by computing the downwash generated by these vortex systems at the location of the nacelle tip vortices using the expression for the flat plate (A.11).

2.9 Chordwise pressure distribution

Another feature that should be included is the effect that chord-wise pressure distributions on the wing and high-lift devices have on the downwash at the engine intake. The similar effect on the downwash at the tail has been neglected because the tail is considered sufficiently behind the wing. Therefore a correction is needed [52].

The distribution of lift in chordwise direction has an impact on the location of the lifting as well as trailing vortices. With regard to the latter, it can be said that moving them forward or rearward has less effect on the $\theta_A$ in (2.2c) when they are outboard and more effect when they are inboard. Also, the inboard trailing vortices are much weaker than the outboard ones. Based on this it was decided to neglect the influence of chordwise pressure distributions on the downwash in the plane of symmetry induced by the trailing vortices. However, this argument is not valid for the bound vortices, as they are increasing in strength moving inboard. As a result, the correction factor for the effect of chordwise pressure distributions on downwash is developed only for the lifting vortices. A further simplification is added by assuming that the change in the cosine terms in (2.2c) for the lifting vortices are negligible. This seems allowable because these changes are larger going outboard, where the lifting vortices are much weaker. This reduces the error caused by an assumed unmodified cosine term. We can now set up a scheme for the correction. In our present model we have concentrated all the bound vortices onto the wing’s quarter-chord line. First we
will write out the downwash generated by a distributed vorticity at an arbitrary point. Then we will do the same for the downwash near a concentrated lifting vortex. Their ratio, effectively a two-dimensional correction, is then applied to the lifting vortex contribution to the downwash as obtained from our three-dimensional work so far.

Figure 2.30. Distributed circulation on a flat plate with flap

The theoretical solution for the pressure distribution on a flat plate under angle of attack has been studied extensively [3]. Referring to Figure 2.30 we may write for the vorticity of a thin, cambered airfoil:

$$\gamma(\theta) = 2V \left( A_0 \frac{1 + \cos \theta}{\sin \theta} + \sum_{n=1}^{\infty} A_n \sin(n\theta) \right)$$

(2.59)

$$\Gamma = \int_0^\alpha \gamma(x) dx = \pi c V \left( A_0 + \frac{1}{2} A_1 \right)$$

(2.60)

with

$$\theta = \arccos(1 - 2\xi)$$

(2.61)

$$\xi = \frac{x}{c_{ext}}$$

(2.62)

$$A_0 = \alpha - \frac{1}{\pi} \int_0^\pi \frac{dz}{dx} d\theta_0$$

(2.63)

$$A_n = \frac{2}{\pi} \int_0^\pi \frac{dz}{dx} \cos(n\theta_0) d\theta_0$$

(2.64)

and

$$c_p = \frac{2\gamma}{V}$$

(2.65)

$$c_t = \int_0^1 c_p d\xi = \int_0^1 c_p d\xi$$

(2.66)

in which the notations in [3] have been maintained. The Fourier constants $A_0$ and $A_n$ can be evaluated by formulating the mean camberline $dz/dx$. For our research
we assume the wing profile has little camber so we can treat it as a flat plate, i.e. $dz/dx=0$. This assumption has been checked for several modern airfoils. This leaves only $A_0$ and the vorticity is expressed in one term with a singularity at the leading edge.

A wing with flap may be considered as a highly cambered airfoil. For the flap we state that:

$$\frac{dz}{dx} = -\delta_f$$

(2.67)

It is important however to remark that the vorticity is located on the straight, extended chord line as sketched in Figure 2.30 and not on the flap itself. This introduces an error, but otherwise the underlying integral would be extremely hard to solve as it literally turns the corner at the flap leading edge. As a result this approach is valid only for small flap deflection angles. The consequence is that for high flap angles the downwash in the vicinity of the flap is inaccurate. As long as the nacelle is not directly behind or above the flap, this is acceptable.

The chord extension by the flap can be modelled by considering this a translating hinged flap and multiplying the chord by the chord extension factor. Empirical factors from the Fokker 100 double-slotted Fowler flap system have been used:

$$c_{ext} = 0.64 + 0.245 \frac{\delta_f}{\delta_{fmax}} + 0.36 \cos \delta_f$$

(2.68)

Since it is a continuous expression it does not prescribe the number of flap elements. The average flap angle is taken from the rear of the fixed wing to the trailing edge. Effectively this means that it is a single flap, which is another simplification and cause for error. Nonetheless, it is believed that this is negligible. The average flap angle and chord extension are more important than variations of the local camber at the flap.

Substituting the $dz/dx$ of this schematized wing with flap into the equations above we can solve the integral analytically:

$$A_0 = \alpha + \frac{1}{\pi} \delta_f (\pi - \theta_1)$$

(2.69)

$$A_n = -\frac{2}{n\pi} \delta_f \left( \sin(n\pi) - \sin(n\theta_1) \right) = \frac{2}{n\pi} \delta_f \sin(n\theta_1)$$

(2.70)

with $\theta_1$ related to the leading edge position of the flap:

$$\theta_1 = \left( 0.64 + 0.245 \frac{\delta_f}{\delta_{fmax}} \right) \frac{1}{c_{ext}}$$

(2.71)

Here again the chord extension must be used in order to regard the extended profile with flap as a cambered airfoil with unit length 1. The vorticity can now be written as:

$$\gamma(\theta) = 2V \left\{ \alpha + \frac{1}{\pi} \delta_f (\pi - \theta_1) \frac{1 + \cos \theta}{\sin \theta} + \sum_{n=1}^{\infty} \frac{2}{n\pi} \delta_f \sin(n\theta_1) \sin(n\theta) \right\}$$

(2.72)
Note that in (2.72) \( \theta \) is the variable indicating the position along the chord at which \( \gamma \) is computed, whereas \( \theta_1 \) is a constant and refers to the leading edge of the flap.

A check on the model so far is made by a comparison with windtunnel data of the pressure distribution on the profile of the Fokker F28 Mk4000 with the vane and flap fully deployed. The flap is deflected over 42° and the vane 10° downward relative to the flap. The flap stretches 27% and the vane 9% of the chord. The first six Fourier terms of the vorticity are shown in Figure 2.31. Figure 2.32 shows the computed cumulative pressure differential distributions for the first 9 and 70 terms as well as the experimental data from the windtunnel test. The shape of the pressure distribution is correct, apart from the singularity at the leading edge of the wing, vane and flap. It can clearly be seen how much lift the vane and flap develop ahead of them due to their upwash. The vane carries disproportionately much lift (11%), the flap 9% and the wing itself the remaining 80%. In order to verify the total lift coefficient the integral (2.66) must be multiplied by the chord extension because \( c_l \) refers to the unextended chord. For full flaps this value is 1.15 according to (2.68). The computed lift increase of 3.60 at \( \alpha = 0° \) is an 35% overestimation of the measured \( c_l 2.70 \). In itself this is a gross overestimation but not uncommon as indicated on page 27.3 in [33], albeit that refers to an aircraft instead of to a profile. This error is partially due to the lack of viscous effects in the theoretical model, which occur predominantly at the vane and flap themselves. The consequence of viscosity is a reduction in lift due to reduced effective camber. This can clearly be observed from the computed excess lift at the vane and flap compared to the experimental data. In addition there is the intrinsic error of enforcing the boundary condition of parallel flow at the extended chordline instead of at the vane and flap chordlines. The result is an even higher downwash then really necessary, leading to an also higher circulation and thus lift. Increasing the number of Fourier terms leads the calculated result away from the measurements, because the theoretical solution has a singularity at the vane.

![Figure 2.31. Fourier terms to vorticity distribution over a flat plate with 10° vane and 42° flap deflection at \( \alpha = 0° \)](image-url)
2.9 Chordwise pressure distribution

![Graph showing chordwise pressure distribution]

Figure 2.32. Comparison of pressure distribution over the F-28 Mk 6000 airfoil with 10° vane and 42° flap deflection at $\alpha = 0^\circ$, between HOT and wind tunnel data [33, pp. 25-7]

and flap kink where it meets the wing (see the two peaks at the right hand side in Figure 2.32). Therefore it was decided to use the first 9 terms only, which yield a better match with the test data. This bears no effect on the computed lift as that is built up from the first two terms only.

The induced downwash generated by this vorticity distribution can then be written as (see Figure 2.30):

$$\varepsilon_c = \int_0^1 \frac{\gamma(\xi)}{2\pi} \left[ \frac{z_P}{\xi - \xi_c} \cos \alpha + \frac{z_P}{\xi - \xi_c} \sin \alpha \right] d\xi$$

Note that the chord extension only affects the relative distance between point $P$ and the vorticity in the computation of the downwash generated by the chordwise pressure distribution in (2.73). A note should be made on the reference chord used. As far as downwash is concerned all distances are related to the semispan $s$ or $b/2$ and the origin lies at the quarter-chord line. But for chordwise effects the chord is what really matters and we have chosen the chord’s leading edge as the origin of the axis system. That greatly simplifies the expressions. The new dimensionless coordinates are called $\xi_P$ and $\gamma_P$. They are referenced to the junction chord where the wing meets the fuselage. This is chosen because the intake is directly behind the inboard part of the wing where the bound vortices are strongest.

The downwash generated in point $P$ by a concentrated infinite vortex at the
quarter-chord is obtained from Biot-Savart’s law:

\[
\varepsilon_{e/4} = \frac{1}{2\pi V} \left[ \frac{x_p - 0.25c}{(x_p - 0.25c)^2 + z_p^2} \cos \alpha + \frac{z_p}{(x_p - 0.25c)^2 + z_p^2} \sin \alpha \right]
\]

\[
= \frac{1}{2} (A_0 + A_1) \left[ \frac{\bar{x}_p - 0.25}{(\bar{x}_p - 0.25)^2 + \bar{z}_p^2} \cos \alpha + \frac{\bar{z}_p}{(\bar{x}_p - 0.25)^2 + \bar{z}_p^2} \sin \alpha \right]
\] (2.74)

The correction factor is the ratio of the two downwash angles (2.73) and (2.74):

\[
k_e = \frac{\varepsilon_e}{\varepsilon_{e/4}}
\] (2.75)

This factor consists of two components: one related to camber at \( \alpha = 0 \) and one related to lift. It would therefore seem appropriate to linearize \( k_e \) in order to save precious computing time:

\[
k_{\alpha} = \frac{\varepsilon_{e}(0)}{\varepsilon_{e/4}(0)}
\] (2.76)

\[
k_{\alpha} = \frac{\varepsilon_{e}(\alpha) - \varepsilon_{e}(0)}{\varepsilon_{e/4}(\alpha) - \varepsilon_{e/4}(0)}
\] (2.77)

However, for the flaps retracted case there is a singularity for \( \alpha = 0 \). This is handled by performing the calculation of \( \varepsilon(0) \) not for exactly 0 but for \( 10^{-6} \). Obviously the chord correction increases with increasing flap deflection. In order to determine the need for this correction values for \( k_{\alpha} \) and \( k_{\alpha} \) have been determined for typical intake positions (\( \bar{x}_p = 1.5, \bar{z}_p = 0.25 \)) and tail locations (\( \bar{x}_p = 2.5, \bar{z}_p = 1.0 \)). These are \( k_{\alpha} = 1.5, k_{\alpha} = 1.4 \) and \( k_{\alpha} = 1.24, k_{\alpha} = 1.22 \) respectively. It is concluded that it is worthwhile to apply this correction to downwash at the intake as well as at the tail. It must be remembered that although these numbers seem substantial they apply to the lifting vortex contribution to downwash only, which is already quite small at the tail. Plots of \( k_{\alpha} \) and \( k_{\alpha} \) are shown in Figure 2.33 for the clean and full flaps case. For the clean case there is no camber and the \( k_{\alpha} \) and \( k_{\alpha} \) are equal. With flaps deflected the difference between \( k_{\alpha} \) and \( k_{\alpha} \) is clearly visible. The most forward line is the contour with the value 1. The effect of inclusion of the chord correction is that the downwash increases directly behind and decreases ahead of the wing’s quarter-chord line. The singularity at \( \bar{x}_p = 0 \) is not shown as this area is of no practical interest. The lines for \( k_{\alpha} \) for the flaps 42° case converge at the leading edge of the flap due to the kink there. The correctness of the other three plots is confirmed by the convergence of the contours at the 75% x/c collocation point.

The chordwise correction is finally implemented in the following way:

\[
\varepsilon(\alpha) = k_{\alpha}\varepsilon_{t\alpha}(0) + k_{\alpha}(\varepsilon_{t\alpha}(\alpha) - \varepsilon_{t\alpha}(0)) + \varepsilon_{t\alpha}
\] (2.78)

A final note on \( k_{\alpha} \) and \( k_{\alpha} \) needs to be made, as these are not really constant. This is due to the sine and cosine terms in (2.73) and (2.74). Therefore these constants need to be re-computed for every angle of attack.
2.10 Downwash in the wing wake

Until now only potential flow has been utilized. The viscous effects appear to be restricted to the wake that separates from the trailing edge. Within the wake the dynamic pressure can be severely reduced leading to less tail lift (see Figure 2.34). In addition the downwash increases above and decreases below the wake center. That can be explained by the rise in airspeed at the median of the wake from the trailing edge downstream. Due to this speed increase the massflow increases as well, causing the streamlines to converge while the wake widens [13], see Figure 2.35. These effects depend on the profile coefficient and are small with high-lift devices stowed and for attached flow [25]. Yet it remains valuable to know when the tail enters and leaves the wake, for this is likely to be an area of reduced longitudinal stability and control. For T-tails it is even more important due to their inherent deep-stall characteristic, albeit that occurs at extreme angles of attack with fully separated flow unsuitable for analytical modelling. That phenomenon is outside the scope of our present research.

The derivation of the method is given in appendix B. Assuming an average friction coefficient of 0.003 and a shape factor 1.08 for a wing we get a profile drag coefficient of 0.0065 for attached flow. At high lift coefficients this may increase substantially.
With flaps deflected we may expect a $c_{dp}$ for the flapped section between 0.04 and 0.09, which is part of the input file. Furthermore an average distance between the tail and the wing’s trailing edge of 2.5 chord lengths is assumed. For these values we find in Figure 2.36 the associated change in downwash. Along the $x$-axis is the height above the wake’s centerline expressed in chord length. Clearly the downwash change is negligible for the clean case and is limited to less than 0.4° for flaps fully deflected. It is hard to make a definite statement for the clean case at high lift coefficient because that mainly depends on the specific airfoil characteristics. Also visible is the limitation of this model: outside the wake the downwash change does not drop to zero but remains constant. From our aforementioned discussion of the inflow we have already concluded that the downwash change away from the wake should diminish to zero. This drawback is the consequence of our mathematical model that only prescribes the velocity’s magnitude but not its direction. That would require a much more complicated approach. Some effort has been undertaken by prescribing a function of dipoles that generates a body with the same contour as the replacement thickness [8]. This method is still under development and can not yet be included in HOT. Therefore the present model can only be used inside the wake; outside the wake the effect of viscosity is set to 0. This represents a jump in downwash over the edge of the wake, which is unrealistic. Still we can conclude from this model that for the clean configuration the downwash change due to wake effects can be neglected altogether and for the full flaps case is smaller than 0.4°. With flaps fully deflected the wake descends so fast that it is highly unlikely that even a low-set tail approaches the wake at all. When that occurs the angle of attack is so high that by then the flow is separated anyway and can not be treated anymore by analytical tools.
Figure 2.36. Effect of wake on downwash angle for $\xi=2.5$

The wake does not follow the vortex sheet automatically. Although the wake separates at the trailing edge it sinks below the vortex sheet due to the lower dynamic pressure in combination with the same vertical induced velocities as the vortex sheet (Figure 2.34). This effect has been observed from experimental data [42, fig. 20] but has never been explained this way. This change in downwash angle is fundamentally different from the one due to inflow. The former is nil at the wake edges and always positive within the wake with a maximum at the median. The latter is positive above and negative below the wake and nil at the median. As a consequence, the wake does not retain its symmetrical shape. The line connecting the locations with lowest dynamic pressure descends towards the wake’s lower boundary.

It appeared not to be possible to find an analytical solution for the wake’s vertical displacement. When needed, it can be found by numerical integration of (2.35) divided by the square root of (B.2). This is an approximation because once the wake drops below the vortex sheet (2.35) is not exactly valid anymore, but it is a good estimate.

In view of these considerations it was decided to model the viscous effects in free air as follows:

- the effects of inflow and reduced dynamic pressure on the downwash within the wake are neglected, leading to the co-location of the wake center and the vortex sheet,
- the effect of reduced dynamic pressure on tail lift is taken into account as a separate factor $q_h/q$.

The factor $q_h/q$ has not been validated due to lack of available experimental data. In potential flow the contours for constant downwash feature a sharp kink at the vortex
sheet itself, see Figure 2.13. In real subsonic flow however these discontinuities do not exist. The present addition takes care of that, together with the effects of rolling-up. Even so, the downwash changes markedly when passing the vortex sheet. When the tail approaches the vortex sheet with increasing angle of attack, the downwash increases not only due to increasing lift, but also due to crossing the contours for constant downwash. Once the tail passes the vortex sheet, the reverse happens: a sharp decrease in downwash gradient, sometimes leading to even a decrease in downwash.

The tail is also affected by the wake from the fuselage. It is assumed that this is accounted for in the tail’s aerodynamic coefficients or by the dynamic pressure ratio $q_{w,m}/q_{\infty}$ in the input file.

### 2.11 Miscellaneous effects and limitations

Several further limitations of the present tool should be mentioned specifically. No influence on downwash of an extended undercarriage is modelled. The undercarriage induces flow separation at the lower fuselage and wing surface, thereby reducing the lift. As a result the downwash for a given lift is expected to increase somewhat. But the amount is probably small and extremely difficult to estimate. The lift on the fuselage is assumed to fill the gap in the lift distribution due to the wing being covered by the fuselage. An explicit computation of this lift and its associated downwash has been covered extensively in literature on missiles [37], but is only applicable to bodies with a blunt rear-end, quite unlike fuselage tailcones. Even so, this body lift only becomes significant at higher angles of attack. However, flight conditions at higher angles of attack do not appear to be significant for horizontal tailplane design. Also the upsweep of the rear fuselage is not taken into account. This upsweep tends to locally induce changes in downwash. Initial estimates by integration of (A.17) over the tail’s span indicate this represents less than several percent downwash for low-set tails and almost zero for T-tails. Finally no spanwise variation of the downwash over the tailplane has been taken into account. From Figure 2.12 it can be concluded that these variations are quite small for a flat vortex sheet as well as for fully rolled-up vortices, except very close to these concentrated vortices. According to [41] this corresponds to less than 0.5% of the downwash in the plane of symmetry and is therefore neglected. The cause behind this is the reduction in lift distribution towards the tip that picks up less effect of downwash variations.

### 2.12 Application and validation

#### 2.12.1 Downwash behind a clean wing with aspect ratio 6

The present analysis has been implemented as a dedicated downwash module in the computer program HOT that has been mentioned in the introduction. This program computes center of gravity limits stemming from several longitudinal stability and control requirements for a given horizontal tail size. It uses free air aerodynamic
coefficients from its input file but can also compute downwash in free air using this special module.

Throughout this research the free-air lift curve is not generated by our own tool but obtained from experimental data instead. The rationale behind this is that in the horizontal tail sizing design stage the lift curve is often already known. Also, the lifting-line theory that is utilized in our investigation lends itself well to adopting an existing lift curve, as it only requires a coefficient of lift. Furthermore, this choice prevents differences between a theoretical and an experimental lift curve blurring the comparison of theoretical and experimental downwash later on.

The downwash method is verified by a comparison with a numerical application [42] to a tapered wing with a U.S.A. 45 airfoil with aspect ratio 6 and taper 0.5. The angle of attack $\alpha$ is 11.5° and the coefficient of lift $C_L$ is 1.175. In that report the wing has been subjected to calculations with the method as described in [41]. The results are compared with windtunnel tests which will be discussed later on. The calculation scheme is as follows:

1. the method as laid down in [41] first calculates the downwash in the plane of symmetry induced by flat vortices such as calculated by HOT in Figure 2.37(a),

2. then the position of the vortex sheet is calculated according to the downwash angles for $z=0$ (Figure 2.37(b)),

3. subsequently in HOT the new downwash contours are calculated using the distance of the trailing vortices to any grid point in the plane of symmetry (Figure 2.37(c)),

4. the equivalent result from [42] is shown in Figure 2.37(c) together with the HOT results.

It should be noted that in HOT the lifting vortices' contribution to downwash is calculated using $m_{tv}$ in (2.32) as opposed to $m_{tv}$ for the trailing vortices. This means that only the contribution by the trailing vortices is affected by the vertical movement of the vortex sheet, which is correct. By contrast, in [41] the same distance $m_{tv}$ is used for the lifting as well as trailing vortices. Implicitly the bound vortices are displaced as well, which is incorrect and decreases the downwash above the vortex sheet. Effectively the downwash contours move vertically with the vortex sheet, which is precisely what is graphically done in [41].

The first observation is that the vortex sheet locations are almost identical. This implies that the downwash at the vortex sheet itself for both computations are in good agreement. The overall conclusion on downwash is that the agreement is good, especially away from the vortex sheet and further downstream. The reason for the slight discrepancies near the vortex sheet lies in the discrete number of 6 horse-shoe vortices used in [42]. As a result, the calculated downwash slightly above and below the trailing vortices will be inaccurate. The deviation for small values of $r$ was found to be attributable to an erroneous vertical displacement of the lifting vortices in [41]. As the downwash contours of Figure 2.37(a) were displaced along with the vortex sheet, the bound vortices were unintentionally moved along as well because of their contribution to those contours. Obviously only the trailing vortices descend vertically, contrary to the bound vortices. This can be corrected in two ways:
Figure 2.37. Comparison of computed downwash from [42, figure 17] and HOT, USA 45 airfoil, A=6, α=11.5°, \( C_L = 1.175 \)

1. either the contributions of the lifting and trailing vortices should be separated and the vortex sheet vertical displacement used for the latter only,

2. or the coordinate system should be rotated about the y-axis by the downwash angle, in order to roughly coincide with the local vortex sheet.

Using these corrected coordinates, the downwash could be determined using one vertical distance for both vortex types and then converted back to the original coordinate system. The results of both methods are nearly equal and provide about 4% improvement in accuracy for usual tail locations. Also for other locations like rear fuselage mounted engine intakes, the error would mount up to 5%, necessitating this correction. The error is only small due to the small contribution of the bound vortices to downwash compared to the trailing vortices. In our method the first option has been applied by using different vertical separations for the lifting and trailing vortices in (2.15) as indicated by \( m_{lv} \) and \( m_{tv} \). In order to quantify the error \( m_{tv} \) and \( m_{tv} \) have
been set equal in our present method and shown in the lower curves in Figure 2.38, by contrast to the upper lines without the error, taken from Figure 2.37(c). The lower lines are now almost identical to [42], which confirms the systematic error made in [41] and [42]. The result is an underestimation above the vortex sheet and an overestimation below it due to the larger distance by moving the lifting vortices downwards. Moving downstream away from the lifting vortices the error diminishes. It should be

![Diagram of lifting vortices and downwash](image)

**Figure 2.38.** Effect of erroneous vertical displacement of lifting vortices on downwash in [42], simulated by HOT computations

remarked that as the ESDU method [18] is based on [41], it suffers from this 5% error as well.

An example of the effect of rolling-up is given in Figure 2.39 and Figure 2.40 for the case in [42]. Hardly visible in Figure 2.39 is the upward curving of the vortex sheet relative to the quasi-flat case (dotted line slightly above solid line). The location of the tip vortices is given only for the case of rolling-up, as it is not significant otherwise. Indeed they do emerge from the vortex sheet. Also apparent in Figure 2.40 is the increase in downwash at some distance above the vortex sheet, due to the tip vortices above it. Near the vortex sheet the smaller downwash, according to (2.36), starts to prevail as indicated by the increased curvature of the downwash contours. Below the vortex sheet there is a definite decrease in downwash.

The change in the downwash above and below the vortex sheet can also be shown by an intersection of this plot, as shown in Figure 2.41. The line including roll-up (solid line) is slightly more curved above the sheet than below, whereas the line excluding roll-up (dotted line) is symmetrical.

Now that all separate elements have been reviewed, the comparison between the experimental downwash contours and our final results can be made as well in Figure 2.42. The overall agreement is satisfactory, apart from the local increase in the measured downwash right in the middle of usual tail locations. Also, the area directly behind and above the trailing edge shows some deviations. It is unclear whether this
Figure 2.39. Impact of rolling-up on the vortex sheet position

Figure 2.40. Impact of rolling-up on downwash
Figure 2.41. Effect of rolling-up on downwash, $r=1$, USA 45 airfoil, $A=6$, $\alpha=11.5^\circ$, $C_L=1.175$

Figure 2.42. Downwash validation on USA 45 wing, $A=6$, $\alpha=11.5^\circ$, $C_L=1.175$, [42, figure 14]
is typical for this particular profile or a general phenomenon. Obviously, the effects on downwash of other phenomena such as profile drag have not been modelled, but these are not expected to be restricted to these particular areas. The vertical position of the vortex sheet, however, seems to be predicted quite accurately. In addition, the asymmetry caused by rolling up is evident in the measured as well as computed downwash contours.

2.12.2 Downwash behind a flapped wing with aspect ratio 6

A similar validation has been undertaken for a flapped configuration in the same reference. It features a 2 by 12 foot rectangular wing with a 70% -span split flap deflected over 60°. The HOT computation and the experimental data have been combined in Figure 2.43. The overall agreement is good, albeit that HOT slightly overestimates the downwash. As a result the vortex sheet descends a little too much. It is believed this is caused by the elliptical lift distribution in the computation as opposed to the more outboard real lift distribution associated with a rectangular wing. It should be noticed that the strange hump in Figure 2.42 is not repeated in this case.

2.12.3 A collection of vortex sheet positions

The foregoing analysis has been applied to a number of windtunnel tests [1]. The experimental location of the vortex sheet is difficult to assess. A sudden change in lateral velocity over the sheet may be noticed but is hard to determine with sufficient accuracy. Another indication might be the location of the contour with lowest dynamic pressure, but then the wake would be assumed to coincide with the vortex sheet which is not always the case, especially not with flaps deflected. Yet another
indicator might be the vertical tangent to the contours for constant downwash angle. But these contours sometimes demonstrate a highly irregular pattern. In addition, several mistakes in data handling were found in the literature used, causing erroneous plots in wake and vortex sheet location.

The comparison between the experimental vortex sheet displacement and the HOT data for the clean configuration is given in Figure 2.44 and for split flaps in Figure 2.45. The overall conclusion is that the correlation with flaps retracted is good, but

![Graphical comparison: non-flapped airfoil](image)

**Figure 2.44. Validation of the vortex sheet vertical displacement, flaps retracted [1]**

with flaps deflected rather poor with an overestimation of the vortex sheet vertical displacement. This might be caused by the thick wake behind the split flaps with the vortex sheet roughly at the wake center, whereas our tool HOT uses the trailing edge of a Fowler-type flap as the starting point for the vortex sheet. In addition, the viscous effects on downwash for split flaps are significant and tend to increase the downwash above the vortex sheet. Depending on how the vortex sheet position has been determined, this might lead to a misinterpretation of its position.

No such elaborate validation was done for the position of the tip vortex core. Some comparisons indicated that the vertical and lateral position for straight wings with flaps retracted were in good agreement with experimental data. But in [58] an upward displacement of the tip vortex for a swept wing has been observed. The validation of the downwash itself in the next subsections will reveal whether the vortex sheet and tip vortex position is determined with sufficient accuracy.
2.12.4 Downwash at the tail of the Fokker 100, clean configuration

Until now the downwash behind a wing in the aircraft plane of symmetry has been reviewed. It is now time to turn our attention to complete aircraft configurations. Hence a comparison has been made with the results of the lifting surface program NPLS (Non Planar Lifting Surface) for the downwash at the horizontal tail of the Fokker 100. Its Fokker 100 model comprises the wing, fuselage, horizontal and vertical tail, nacelles and pylons. It does not incorporate the landing gear. This lifting surface program does not incorporate vortex sheet relaxation, i.e. the orientation of the trailing vortices has to be prescribed. Within the present application they remain horizontal. Therefore two sets of calculations have been performed with HOT: one without relaxation, in order to imitate NPLS, and one including relaxation. The comparison is shown in Figure 2.46. The markers indicate a computed result for a certain angle of attack, ranging from 0 to 12 with increments of $2^\circ$. This is maintained for all subsequent figures. The agreement between NPLS and HOT is excellent. This implies that for this aircraft the assumption of elliptical lift distribution holds and the analytical approach is valid. Also, both lines curve upwards, due to the tail approaching the vortex sheet with increasing $\alpha$. The slight difference in the marker positions is caused by differences in the relation of $C_L$ versus $\alpha$. NPLS calculates this itself, whereas HOT uses the relation from the Fokker 100 aerodynamic database (ADB). In addition, the downwash data from the ADB and the HOT results including
relaxation have been plotted. These agree very well too. There is a slight difference in slope, which is caused by the downwash being linearized when processed into the ADB. From the original test data however this upward curve can not clearly be discerned due to the low number and wide scatter of the data points. It should be realized that HOT computes the downwash in the plane of symmetry, whereas the experimental data are effectively averaged over the tailplane span. Furthermore it can be concluded that the effect of relaxation is to decrease the downwash and the downwash gradient. In other words: thanks to relaxation the downwash behaves more linear with $\alpha$ than without relaxation. The explanation is that due to the increasing vertical descent of the vortex sheet with $C_L$ it moves further away from the tail, thereby decreasing the downwash. The effect of rolling-up is negligible for small lift coefficients, as found from calculations with the vertical descent included and the rolling-up switched off and on. This effect being small is the result of the relatively low span loading of the wing and the high position of the T-tail (see Figure 2.13). As a whole Figure 2.46 in all aspects confirms the validity of the present tool.

2.12.5 Downwash at the tail of the Fokker 100, full flaps configuration

For the full flaps landing configuration the same excellent agreement is found as for the clean case (Figure 2.47). The difference between HOT and NPLS is almost nil and between HOT and the ADB less than $0.5^\circ$. Moreover this plot confirms the
validity of the approach with artificially increased flap span to capture the lift carry-over effect. The effect of relaxation is much more pronounced than in the clean case due to increased vertical descent and rolling-up of the vortex sheet. This in turn is caused by the high span loading of the fully deflected flaps. The almost constant slope confirms our earlier conclusion that vortex sheet relaxation is responsible for the linear relation between downwash and $C_L$. In addition it is concluded that for downwash calculations with flaps deflected the incorporation of vortex sheet relaxation is mandatory.

The next topic to be validated is the contribution of the nacelles and pylons. Figure 2.48 shows results from HOT computations for the Fo100 with and without nacelles. It is striking that without nacelles the downwash at the tail is almost linear with the coefficient of lift and independent of the flap setting. Apparently the influence of the smaller flap span upon the downwash is nullified by the larger vertical descent of the combined vortex sheet. But with nacelles the picture is different. In the clean configuration the downwash at the nacelle intake is smaller than the angle of attack, resulting in positive nacelle lift and related downwash at the tail. Due to flap deflection however the intake angle of attack is negative. The negative nacelle lift results in upwash at the tail. The addition to downwash by the nacelles is 0.7° at most, provided the flow remains attached. The reason is that the tail is so high above the nacelles. Of course, at extreme angles of attack such as during deep stall, the nacelles have a dramatic effect on the flow at the tail plane because then their wake covers the horizontal tail. That is however outside the scope of our research and our
method should not be applied to such conditions.

![Graph showing the nacelle contribution to downwash at the Fokker 100 tail](image)

**Figure 2.48.** Nacelle contribution to downwash at the Fokker 100 tail

### 2.12.6 Downwash at the intake of the Fokker 100, clean configuration

The chordwise effects on the nacelle intake angle of attack have been investigated separately and are shown in Figure 2.49 for the clean configuration. Several methods have been included: the Aerodynamic DataBase (ADB) which is based on experience with earlier products and windtunnel tests; O216, a more accurate panel code; NPLS, although the intake angle of attack was not computed explicitly and was estimated by dividing the nacelle lift by the lift curve slope from (2.55); and HOT computations with and without chord effects. It is apparent that chord effects have a significant impact on the nacelle intake angle of attack indeed for rear fuselage-mounted engines, even if they are located as far downstream as on the Fokker 100. Inclusion of the chord effects improves the intake angle of attack prediction considerably. The accuracy is hard to determine as all the other sources are somewhat apart, but HOT is very close to the ADB and the higher fidelity O216. The slope for the HOT data with chord effects is higher than for the ADB and NPLS, but in good agreement with O216. Although this angle must not be used for the determination of nacelle lift and its downwash at the tail as explained before, it does help in establishing the range of angles of attack that the intake has to cope with without flow separation. It is concluded the HOT prediction inaccuracy is less than 1°.
2.12.7 Downwash at the intake of the Fokker 100, full flaps configuration

The similar validation for the full flaps landing configuration is shown in Figure 2.50. The chord effects increase the nacelle intake angle of attack to an even greater extent and are in good agreement with the ADB. This time the NPLS results indicate somewhat lower values. The fact that they are close to the HOT data without chord effects is purely coincidental. No firm conclusions can be drawn on the accuracy, other than that it is better than several degrees. This agreement is all the more satisfactory considering the large local angles in this area of the aircraft. Without the inclusion of the fuselage effect the computed slope of the intake angle of attack would be much higher.

A separate analysis revealed that the chordwise effects on the downwash at the tail are small, because they are opposed by the chordwise effects on the downwash at the nacelle [52]. As explained before, an additional downwash at the intake due to chordwise effects leads to a negative downwash at the tail. This upwash counteracts the increased downwash at the tail that is directly generated by the chordwise effects, apart from the presence of the nacelles.

A much more comprehensive research on the downwash at the nacelle intake has been reported in [28]. That study involved the application of the present tool HOT to several windtunnel models with different nacelle positions, corresponding to the positions for the Fokker 100 and the Cessna Citation II. The downwash at the nacelle intake and tail were compared with the results from a CFD tool developed by
Fig. 2.50. Downwash at the Fokker 100 nacelle intake, full flaps configuration.

Jan Willem van Staveren and with windtunnel data. The overall conclusions are in line with the present ones, albeit there were complicating non-linear trends in the measured downwash.

2.12.8 Downwash at the tail for various flap settings

The final comparison between the calculated and experimental downwash at the tail of the Fokker 100 for all flap settings is shown in Figure 2.51. The agreement is excellent with an error band of about $0.75^\circ$ only. The mean error is even much smaller. The slope is a little bit off with increasing $C_L$ except for full flaps. This a commonly observed trend that will be discussed in the next section. It is suspected this is caused by the linearized downwash versus the angle of attack in the aerodynamic database or by the variation along the tailplane span. It is evident that the variation in the accuracy with flap deflection is very little. For this reason, and because the downwash data for the intermediate flap settings are often hard to get, we will mainly focus on the extremes of clean and full flaps. In order to prove that this is not an incidental result, we hereby show the equivalent for a low-set tail configuration, the A330-300 (Figure 2.52). The quality of the match is equally excellent, although there is a non-linear variation with lift coefficient in the experimental data for small flap settings that is not captured by the computations. This will be addressed in the next section.
Figure 2.51. Fokker 100 tail downwash validation for all flap settings

Figure 2.52. A330-300 tail downwash validation for all flap settings
2.12 Application and validation

2.12.9 Downwash collection for civil airliners

Finally a validation has been undertaken for other aircraft using data from [31, 35] for two different configurations: one with wing-mounted engines and a low tail and the other with rear fuselage-mounted engines and a T-tail. The data in [31] come from many sources such as wind-tunnel and flight tests and engineering performance manuals. These data have been used to generate input files for HOT to perform the downwash computations. The same computations and analysis as reviewed in the previous section for the Fokker 100 have been carried out for this wide range of aircraft.

The results for the clean configurations (Figure 2.53) show good overall agreement between the actual and calculated downwash. At first sight this may not seem to be

![Figure 2.53. Tail downwash validation for clean configurations](image)

- one explanation might be that the actual aircraft data have been linearized for
low angle of attack. It has been observed for several aircraft that windtunnel-
and flight test data do show this increase in downwash gradient with lift, whereas
their aerodynamic databases incorporate a linear relation. The intention behind
this seems to be to provide a ballpark number for the downwash gradient for
usual operational flight conditions as opposed to the edge of the flight envelope,

• another possibility is that in the present method the rolling-up has not been
modelled correctly. With increasing lift coefficient the rolling-up increases, lead-
ing to decreased downwash (and thus gradient). But this feature is not likely to
be significant for the clean configuration due to the relatively low span loading.
In addition, it does not hold for T-tailed aircraft, as the rolled-up downwash is
only smaller for small values of \( m \) (Figure 2.13),

• another possible source for error is the determination of the tail lift curve slope.
It is common practice not to measure the downwash angle in flight, but to derive
it from the difference between windtunnel complete-aircraft and tail-off pitching
moments extrapolated to full-scale. Alternatively, in-flight strain-gage measure-
ments can be used. In both cases the tail lift curve slope is needed and in the
latter case the elevator lift curve slope as well. The latter is subject to short-
comings in full-scale prediction from windtunnel data and CFD computations
according to [39].

Further analysis suggests the deviation for the A330-300 to be caused by the tail
being in the vicinity of the wing wake, as described above. It is the only aircraft where
this could occur, given its extremely high aspect ratio (≈10) and its tail location close
to the wing wake. Moreover, this non-linear increase does not occur with landing flaps
as the vortex sheet is further away from the tail, confirming our hunch. Modelling
of the wake does not explain this problem, as the computed order of magnitude is
10 times lower than the 1° we are looking for. However, our computations indicate
that the tail enters the wake at angles of attack where the non-linear increase of the
downwash starts to occur and passes the wake center right where the increase changes
into a decrease (see Figure 2.36).

The full flaps configuration results show a comparable agreement with the ex-
perimental data as for the clean configuration (Figure 2.54). The mean error in
downwash thus obtained is about 0.25° and the standard deviation is 0.5°. But this
time the B747-400 exhibits a much higher experimental than computed downwash
gradient whereas the A330-300 displays an excellent match. The actual downwash
for the B747-400 seems to be far off compared to other aircraft of the same general
arrangement, see Figure 2.55. Note that solid lines are for T-tailed and dashed lines
for low-tailed aircraft. The only explanation at hand would be that whereas the
database does simulate the overall aircraft’s behavior, that does not imply that all
ingredients thereof are necessarily accurate. Indeed it has been discovered that the
downwash for the landing configuration exhibits two distinct kinks versus the angle
of attack just outside Figure 2.55, for which no physical explanation seems feasible.
More refined computations with a CFD tool developed by Jan Willem van Staveren
did not shed any light on this issue due to the lack of vortex sheet relaxation in that
tool. For this reason it was decided to leave the B747 out of the validation. Also visi-
2.12 Application and validation

Figure 2.54. Tail downwash validation for full flaps configurations

Figure 2.55. A collection of manufacturer’s tail downwash data in free air, full flaps configuration
ble is the extremely small downwash for the A300, only obtained by T-tailed aircraft. Therefore it was decided was not to take the A300 into account as well.

No differences in accuracy for slatted versus non-slatted aircraft can be discerned. It seems that the complicated set of assumptions and simplifications that have been imposed to model the effect of high-lift devices on downwash strikes a good balance.

Finally a comparison with the DATCOM method is made, which contains a special addition for flap deflection. No effort has been undertaken to do this for ESDU as well because of its time consuming process. The mean error and standard deviation for HOT and DATCOM are plotted in Figure 2.56(a) for the clean configuration and in Figure 2.56(b) for the full flaps landing configuration. The standard deviation is computed by

\[
\sqrt{\frac{n \sum (\Delta \varepsilon)^2 - \left( \sum \Delta \varepsilon \right)^2}{n^2}}
\]  

where \( \Delta \varepsilon \) stands for the difference in downwash angle compared to the experimental data.

It is concluded that for the clean configuration HOT yields a slightly better mean downwash but no improvement in the deviation. However, for the full flaps landing configuration HOT is about 1° better in the mean downwash and more than 0.5° in its deviation.

### 2.13 Conclusions

A new combination of well-known elements from classical aerodynamic theory has been developed into closed-form expressions for calculating the downwash angle at horizontal tail and nacelle intake locations behind wings, including:

- wing sweep
- vortex sheet relaxation
- high-lift devices
- rear fuselage-mounted nacelles
- chordwise pressure distribution

The tool is very efficient, making it especially useful for the preliminary design phase when geometry is not yet known and parameter variations like aspect ratio and wing loading require fast yet accurate tools with as little input as needed.

Validation with theoretical and experimental tail downwash data proved the usefulness of the tool for clean configurations and with high-lift devices extended. A mean error of -0.5° and a standard deviation of 0.5° for the clean configuration is attained based on manufacturer’s data for 10 aircraft. Similarly the numbers for the landing configuration are 0.25 and 0.5°, based on 9 aircraft. The downwash prediction for the Fokker 100 nacelle intake exhibits an inaccuracy of less than 1° for the clean configuration and better than several degrees for the full flaps case, depending on which validation source is considered more reliable.
Figure 2.56. Tail downwash comparison with DATCOM
The effect of vortex sheet roll-up on downwash is much larger with flaps deployed than for the clean configuration due to increased span loading over the flapped part of the wing. It is concluded that the concept of artificial flap span increase and the interaction between the wing and flap vortex systems is viable and that incorporation of vortex sheet relaxation is a prerequisite for accurate downwash estimation. The effects of rear fuselage-mounted nacelles appeared to be small. Chordwise effects are significant for the intake angle of attack for such configurations but less so for the downwash at the tail. Viscous effects are negligible for the clean configuration and smaller than 0.4° for the full flaps case. Therefore, they have not been included in the present method.

The method appears to be superior to the DATCOM method, especially with flaps deployed. It should be stressed that neither empirical nor aircraft-specific data have been incorporated into the method. The only empirical data used are for the nacelle lift [15] and for the wake [42].

This chapter may seem disproportionately big at first sight. However, downwash in free air is not only a necessity to obtain the downwash in ground effect but also a highly valuable asset in itself with regard to

- all other design requirements used in horizontal tail sizing,
- stability and control analysis,
- performance items such as trim drag.
Chapter 3

Downwash behind wings in ground effect

3.1 Introduction

The need for a detailed knowledge of the effect of the ground on lift and drag is obvious: it affects aircraft field performance. The determination of ground effect has been extensively dealt with during the first half of the 20th century. Especially the influence on section and wing lift has been addressed, but complete aircraft have received less attention. Also ground influence on the downwash at the horizontal tail has received less attention: the method employed by ESDU [14] and DATCOM [9] stems from the early forties. It uses a single horseshoe vortex and its mirrored counterpart to model a wing with flaps in ground effect. This seems rather crude because it replaces the actual lift distribution by a rectangular one. In addition it assumes that roll-up is complete, irrespective of the coefficient of lift and aspect ratio. Furthermore it neglects the influence of the ground on the vertical displacement of the trailing vortices. Unfortunately part of the derivation of the method is unpublished. By contrast, the method by Katzoff and Sweberg [25] is an extension of their free-air method and includes the vertical movement of the vortex sheet. However, it lacks any rolling-up which is the opposite side of the spectrum. Moreover, the method is quite laborious just like its free air counterpart. As wings were straight in those days, no sweep is included. Obviously, jet engines behind the wing have not been incorporated as well.

The downwash at the tail in ground effect seems to deserve more attention than it has received so far. This undervaluation may be caused by the fact that downwash at the tail in ground effect has only an indirect impact on field performance. But due to ground effect there is a nose-down pitching moment and the resulting lack of elevator power may limit the minimum unstick speed and thus \( V_R \) and take-off performance. In addition, the landing flare capability may be affected, especially with large negative (nose-heavy) pitching moments that can restrict the lift coefficient and thus deteriorate landing performance [54]. In recent years much work has been devoted to
lifting surface models and CFD applications. However, within the preliminary design phase one would rather not turn to CFD tools because of the many numerical models required, each with their own grid, due to design parameter variations. As a consequence, the preliminary designer is still stuck with outdated handbook methods such as the aforementioned ones. In order to improve this situation, the present research was initiated to estimate the downwash at the tail including the effects of vortex sheet relaxation and sweep for clean configurations and with high lift devices deployed and fitted with rear fuselage-mounted jet engines.

3.2 Analytical approximation

3.2.1 Method

The method as outlined in chapter 2 will now be extended to ground effect using the well-known technique of mirroring as first employed by Wieselberger [59]. Two vortex sheets, called the original and the image, now determine the tail downwash (Figure 3.1). The vertical displacement $m_{vsd}$ of both sheets may be calculated by

$$m_{vsd} = m_{v} - m_{g}$$

![Figure 3.1. Geometry of a tail in ground effect](image)

applying (2.32) to a sheet itself ($m_{tu} = m_{tv} = 0$) as done in chapter 2, and to the image sheet. However, the latter calculation needs the vertical distance between the sheets and the vertical displacement is part thereof. This head-tail deadlock requires a numerical computation from the trailing edge downstream which may be avoided by assuming a constant average distance between the two vortex sheets, say equal to half the displacement at the tail x-coordinate. This choice seems reasonable because the ground reduces the vertical movements of the vortices. The vertical displacement of the vortex sheets can then be established using (2.40) but with the addition of the last term in (2.15) as a constant to account for the influence of the mirrored sheet on the original. This introduces an error as this vertical correction factor for the trailing
vortices is now also applied to the contribution of lifting vortices. Detailed analysis however has proven this to be acceptable due to the diminishing contribution of the lifting vortices to downwash. This simplification greatly facilitates the calculation scheme. We may now write, see Figure 3.1:

\[
m_{vsd} = k_{m_{vsd}} \left\{ 1 - \left[ 1 - \frac{2(m_{ac} - m_{e} - 0.5m_{vsd})}{\sqrt{1 + 4 \left( m_{ac} - m_{e} - 0.5m_{vsd} \right)^2}} \right] \frac{C_L}{\pi A} \right\} (3.1)
\]

The first term between accolades in (3.1) is the vertical displacement of the original and the second one of the image vortex sheet. The latter, between brackets, is the vertical correction factor from (2.15), the double value of which is also shown at infinity in Figure 2.7. Equation (3.1) cannot be solved for \( m_{vsd} \) analytically, but closer inspection reveals that \( m_{vsd} \) may be neglected in the denominator with respect to 1 as its value is much smaller than 1. Then with

\[
m_{fusgr} = m_{ac} - m_{e}
\]

(3.2)

(3.1) can be written as:

\[
m_{vsd} = k_{m_{vsd}} \frac{2m_{fusgr}}{k_{m_{vsd}} \frac{C_L}{\pi A} + \sqrt{1 + 4m_{fusgr}^2}} \approx k_{m_{vsd}} \frac{2m_{fusgr}}{\sqrt{1 + 4m_{fusgr}^2}} \frac{C_L}{\pi A} \quad (3.3)
\]

Thus we have found an analytical expression in closed form for the vertical displacement of the vortex sheet in ground effect, including sweep and rolling-up. This descent must be determined using (3.3) for the wing and flap lift separately and summing these contributions:

\[
m_{vsd_{wef}} = \left( k_{m_{vsd}} \frac{2m_{fusgr}}{\sqrt{1 + 4m_{fusgr}^2}} \frac{C_L}{\pi A} \right) + \left( k_{m_{vsd}} \frac{2m_{fusgr}}{\sqrt{1 + 4m_{fusgr}^2}} \frac{C_L}{\pi A} \right) \frac{b_f}{b_w} \quad (3.4)
\]

The last ratio is required to express the vertical displacement relative to the wing span. Of course it can also be expressed in the flap span by multiplying the first term by the inverse ratio. It is reminded that the downwash induced by the lift due to slat deflection is treated similarly to the component by the wing because of the same span. Effectively, wing lift coefficient and slat lift coefficient are totalled at the start of the computations. The distance of the mirrored lifting vortices to the tail is given by:

\[
m_{tv} = m_{tv} + 2m_{ac}
\]

and of the image vortex sheet to the tail by:

\[
m_{fusim} = m_{tv} + 2m_{ac} - m_{ev} - m_{vsd}
\]

Substituting these displacements into (2.15) for both the original and the mirrored slat/wing and flap vortex systems leads to four downwash calculations. These downwash contributions of the original and mirrored systems are totalled to yield the final downwash in ground effect.
The features that lack in [14] and have been added in our tool are the effect of the ground on the trailing vortices’ displacement, elliptical lift distribution, sweep, dihedral and weighted rolling-up. The present method can also yield the downwash in free air by substituting a large height above the ground or simply by not adding the mirrored systems. By contrast, [14] only gives ground effect as a correction to the free air value that has to be taken from another source such as [16, 18], CFD, wind tunnel or flight test data. This is deemed a great advantage. Yet it must be added that at greater heights the negligence of \( m_{\text{inj}} \) in (3.1) is no longer small and leads to an error. The present method is complicated but analytical and compact, demonstrated by the fact that a Maple\textsuperscript{®} implementation requires only one A4-size page. This Maple\textsuperscript{®} sheet will be used for application in the next section.

3.2.2 Validation

In order to validate the present method a comparison is made with the method in [25]. Both methods follow the same lines, albeit that [25] uses a discrete number of 6 horseshoe vortices to compute the spanwise lift distribution. The vertical descent of the vortex sheet is estimated by a numerical integration of the downwash at three locations of the vortex sheet. It does, however, not include rolling-up. The aspect ratio of the test case chosen is 6 in order to generate a reasonable number of downwash contours. The taper is 1/3 and the lift coefficient is 1.5. The relative height \( z/c \) of the mean aerodynamic chord above the ground is 0.5. However, a small mistake was discovered in [25] as this height was inconsistent with the height of the trailing edge of the root chord, being the origin of the wake, expressed as fraction of the wing semi-span. Their relation may be written as:

\[
\left( \frac{h}{b/2} \right) = \frac{8}{3} \frac{(1 + \lambda + \lambda^2) z/c}{(1 + \lambda)^2 A}
\]  

(3.7)

It was found that a taper of 1 had been used erroneously in [25], probably because this had been used in the experimental part of the study. In our calculations the \( z/c \) was calculated from (3.7) such that the correct height of the trailing edge as expressed in semi-span was realized. This could be checked using the contour for zero downwash, as this had to coincide with the ground. This yields for \( 2h/b=0.167 \), a \( z/c=0.4615 \), see Figure 3.2.

Comparison with [25] shows some differences, especially close to and above the vortex sheet. They may be attributed to the small number of horseshoe vortices, but also to the erroneous vertical shifting of the lifting surfaces in [25] as explained on page 55. Their effect is the convex shape of the contours instead of the concave one in the present method. Also, in the HOT computations the effects of rolling-up have been included, in contrast to the method in [25]. It has been noticed in [25] that the experimental downwash contours exhibited a concave shape instead of their calculated convex shapes. This was attributed to chordwise pressure distributions. However, the experiments were conducted with a wing of taper 1, which also tends to curve the contours. The effect of rolling-up is very weak as the vertical displacement of the vortex sheet is so small that it cannot be altered much by rolling-up. No comparison with experimental data is made because only tests with a rectangular wing have been
Figure 3.2. Computed (HOT) and experimental downwash contours [25] in ground effect, 70% span flap, $A=6$, $z/c=0.4615$, $C_{L_v}=1.2$, $C_{L_d}=1.2$

performed, which yields a rather different lift distribution. The conclusion is that the present method is in overall agreement with the method given in [25].

3.3 Numerical extension

3.3.1 Introduction

One drawback of the tool thus far that has not been addressed yet is that the sideways displacement of the trailing vortices due to ground effect is neglected. In free air the tip vortices emerge above the vortex sheet and are drawn inboard. This effect has been covered in 2.6. But in ground effect the image trailing vortices generate a sideways velocity at the original trailing vortices and vice versa (Figure 3.3). As a result the trailing vortices shift outboard, thereby reducing the downwash at the tail. This is especially true with flaps deflected due to the stronger influence of the rolled-up tip vortices than for the clean configuration. This outboard movement is difficult to capture with analytical expressions. The problem is not connected with the direct opposite image trailing vortices, because they move outboard together and their position relative to each other does not change. But as the flap tip vortices move outboard they approach the wing tip vortices and their mutual interference is extremely complicated. Some of that nature is found at the edge of the wing in Figure A.1(f) where a strong curvature around the edge of the uniform vortex sheet is observed. Because of the relative movement of the tip vortices no analytical expression for the outboard shift can be formulated. In addition the analytical method in the previous section incorporates an error that grows with height above the ground as already pointed out. This may become a nuisance when this tool would be utilized to complete the minimum unstick maneuver up to screen height. Therefore a radically different approach is chosen than followed so far, i.e. a numerical one.
3.3.2 Vortex sheet vertical displacement in plane of symmetry

The first particular to be addressed is similar to the one in free air (section 2.5. The most accurate determination would be obtained by application of (2.32) to the vortex sheet and to the image one. In order to simplify the expressions however (2.23) is taken instead with the addition of the term that corrects for vertical distance to the vortex sheet. The integration runs from the flap trailing edge towards the tail-mounted engine intake, depending on the application. The integral (3.9b) is evaluated numerically with the starting value for the relative height \( m_{vus,gr} \) of the vortex sheet to the ground equal to the geometric height of the trailing edge. Thus we find, see Figure 3.4:

\[
m_{vusd} = \sum_{l} \Delta m_{vusd} \quad \text{(3.8)}
\]

with

\[
\Delta m_{vusd} = \epsilon \Delta r
\]

\[
= k_m \left( \frac{0.1124 + 0.1265 \lambda + 0.1766 \lambda^2}{r^2} + \frac{0.1024}{r} + 2 \right) \frac{C_L}{\pi A} \times \left\{ 1 - 1 - \sqrt{\left( \frac{2m_{vus,gr}}{2m_{vus,gr}} \right)^2} \right\} \Delta r \quad \text{(3.9a)}
\]
\[ \Delta m_{\text{vsw}} = k_m \left( \frac{0.1124 + 0.1265 \Lambda + 0.1766 \Lambda^2}{r^2} + \frac{0.1024}{r} + 2 \right) \frac{C_L}{\pi A} \times \sqrt{\frac{(2m_{fus,gr})^2}{1 + (2m_{fus,gr})^2}} \Delta r \]  

(3.9b)

and \( m_{fus,gr} \) is updated each integration step by

\[ m_{fus,gr_{i+1}} = m_{fus,gr_i} - \Delta m_{\text{vsw}} \]  

(3.10)

With flaps deflected this computation must be carried out twice for each integration step: once for the combined wing and slat lift and once for the flap lift. The correction factor \( k_m \) serves the same purpose as (2.36) but must have a setup more like (2.33) due to the presence of the image vortices. Because of the latter, we now have to compute the effect of the rolled-up vortices on a location out of their own horizontal plane, which is exactly what (2.33) represents. The factor \( k_m \) is not different from (2.36) but the factors \( k_{fus} \) and \( k_{vsw} \) are slightly different than in the free air treatment as given by (2.13) and (2.31). From (3.9b) we can directly see that \( k_{fus} \) must be:

\[ k_{fus} = \sqrt{\frac{(2m_{fus,gr})^2}{1 + (2m_{fus,gr})^2}} \]  

(3.11)

which represents the downwash at the original vortex sheet due to the original and image flat vortex sheets. From Figure 3.5 follows for \( k_{vsw} \):

![Figure 3.5](image-url)

**Figure 3.5.** Determination of downwash factor at the vortex sheet in the plane of symmetry due to concentrated tip vortices
\[ k_{\text{raws}} = \frac{2b}{4\pi^2} \left[ \frac{1}{\sqrt{\eta_{\text{raw}}^2 + (m_{f_{\text{us}} - m_{\text{raw}}})^2}} \sqrt{\eta_{\text{raw}}^2 + (m_{f_{\text{us}} - m_{\text{raw}}})^2} - \frac{\eta_{\text{raw}}}{\eta_{\text{raw}}^2 + (m_{f_{\text{us}} - m_{\text{raw}}})^2} \right] \left[ 1 - (-1)^2 \right]

= \frac{4}{\pi} \left[ \eta_{\text{raw}}^2 + (m_{f_{\text{us}} - m_{\text{raw}}})^2 - \eta_{\text{raw}}^2 + (m_{f_{\text{us}} + m_{\text{raw}}})^2 \right]

(3.12)\]

The term between brackets accounts for the infinitely long vortices and the factor 2 for the left hand and right hand vortices. The first term represents the original vortices and the second the image ones. The circulation has been substituted according to (2.4). A difference with the treatment in section 2.5 is that there the flat vortex sheet and the tip vortices were assumed to lie in the same horizontal plane, whereas presently they are separated. The underlying cause for this is the numerical approach that allows much more freedom in detail as opposed to the analytical solutions for the free air situation. In addition the actual lateral position of the tip vortices is now used opposed to the \(\pi/4\) location in the free air case. The computation of their vertical and lateral location will be addressed in the next sections.

### 3.3.3 Tip vortex vertical displacement

In free air we assumed that the wing tip vortex vertical displacement was solely due to the wing trailing vortices and not influenced by the flap's tip vortices. Similarly, for the flap we took their own contribution but added to that the vertical displacement in the plane of symmetry due to wing lift. For our present application, however, these simplifications no longer hold because the trailing vortices are moved outboard by the image vortices. Because the flap vortices are closer to the ground than the wing vortices they experience a stronger sidewash. As a consequence the distance between the wing and flap vortices decreases and their interference can no longer be treated in the same manner as in free air. An entirely different approach also enables us to include the contributions from the image vortices. The treatment of the vertical displacement of tip vortices due to their own lift remains unchanged, but we will now develop a tool to model the interaction between tip vortices of different vortex systems, that is between wing and flap and between original and image vortex systems.

From our analysis in section 2.6 we may conclude that the vortex sheet without taking into account concentrated tip vortices may initially be considered flat (Figure 2.17). However, we must realize that the vortex sheet is not really flat when viewed in y-direction. This has already been discussed in section 2.6 and is clearly visible in Figure 2.17(c) and Figure 2.19. Equation (2.23) effectively is equal to the downwash for \(\eta=0\) but the values for \(\eta=1\) are smaller in the region just behind the wing (lower lines in Figure 2.17). The curling tip is likely to reduce the local overspeeds and therefore (2.23) will overestimate the downwash at the tip. Therefore it was decided to reduce the downwash factor along the x-axis according to (2.23) for the present purpose to 2. This is equal to the value induced by the tip vortices only. This has only some effect just behind the wing or flap. Furthermore it is expected that the curling of the vortex sheet tip has only a local effect on the induced velocities, such as
on the tip downwash that is reduced by a factor 5 compared to the non-rolled up case.
The present application focusses however not on the downwash induced by a vortex
sheet onto itself, but onto another sheet like the image. Consequently the curling
effect on that downwash will be much smaller. All in all we can state that the current
approach is a gross simplification of the complex flow field but will hopefully yield a
reasonable accuracy at some distance away from the vortex sheet. The downside is
that the effects of sweep are no longer included in the calculation of the downwash at
the tip vortices.

This important assumption of a flat vortex sheet bears as a consequence that the
relation between the downwash at any point outside the plane of symmetry and at
the x-axis is independent of the x-coordinate. It allows us to split the downwash
formula into to parts: the relation between the downwash along the x-axis and at the
quarter-chord point, and the relation between the downwash at any point outside the
plane of symmetry and at the x-axis. The former relation has now been determined
at 2 and the latter relation in (A.11). Multiplication of (A.11) by this factor 2 yields
the downwash at any point behind a lifting surface:

\[ \varepsilon_{f_{vs}} = 2k_{in} \frac{k_{f_{vs}} C_L}{2 \pi A} \]  

with \( k_{in} \) as given in (2.36) and repeated here for convenience:

\[
k_{m_0} = \left\{ \begin{array}{ll}
  \frac{k_{m_{vs}}}{k_{f_{vs}}} & r \geq r_{raw} \\
  1 - k_{pruv} (r_{raw} - r_{ie}) & r_{ie} < r \leq r_{raw}
\end{array} \right. \]  

(3.14)

\[ k_{pruv} = \left( 1 - \frac{k_{pruv}}{k_{f_{vs}}} \right) \frac{1}{r_{raw} - r_{ie}} \]  

(3.15)

The factors \( k_{f_{vs}} \) and \( k_{pruv} \) are now more complicated than for the free air case because
they are now computed for the tip vortex instead of for the plane of symmetry. In
order to avoid confusion we refer to this point as P, opposed to the index ruv that
stands for the inducing rolled-up vortex. With reference to Figure 3.6 we get:

\[ k_{f_{vs}} = \frac{2}{\left[ 1 - \frac{(\eta^2 + m^2)^2 - (\eta^2 - m^2)^2 + (\eta^2 + m^2)\sqrt{(\eta^2 + m^2)^2 - 2(\eta^2 - m^2) + 1}}{2(\eta^2 + m^2)^2 - 2(\eta^2 - m^2) + 1} \right]} \]  

(3.16)

where \( \eta \) is the lateral position of a tip vortex of one vortex system (the wing for
example) expressed in the semi-span of another vortex system (the flap for example)
and \( m \) is the vertical separation between the two divided by the same semi-span:

\[ m = m_P - m_{f_{vs}} \]  

(3.17)

The factor \( k_{f_{vs}} \) is used twice: once in (3.13) and once in (2.36). The factor 1/2 in
(3.13) is necessary because of the use of \( k_{f_{vs}} \); this factor is the value of the flat vortex
Figure 3.6. Geometric definitions for the induced velocity generated by a lifting system at the tip vortex location of another lifting system

Sheet at infinity and must be halved to obtain the correction of height to the vortex sheet. The factor \( k_{ruv} \) is similar to the one in (3.12) but now refers to the tip vortex location outside the plane of symmetry:

\[
k_{ruv} = \frac{2h}{4\pi} \left[ \frac{1}{\sqrt{(\eta_P + \eta_{ruv})^2 + (m_P - m_{ruvd})^2}} \frac{\eta_P + \eta_{ruv}}{\eta_P - \eta_{ruv}} \right] - \frac{1}{\sqrt{(\eta_P - \eta_{ruv})^2 + (m_P - m_{ruvd})^2}} \frac{\eta_P - \eta_{ruv}}{\eta_P + \eta_{ruv}} \right]^2
\]

There are some distinct differences notable with (3.12). First of all the present factor represents one tip vortex system only; its image counterpart is not included. The two different terms reflect the left hand and right hand vortices. That is also why the factor 2 at the end of the middle line in (3.12) is lacking in (3.18). It implies that the formulas in this section must be applied to every vortex system individually.

The vertical displacement of any tip vortex is computed by a numerical integration of the downwash in streamwise direction as in the previous section. The downwash at the tip vortices consists of four contributions. The first is its own induced movement as given by (2.45). The second is caused by its image system, the third by the other original system (be it either the wing or flap lift) and the fourth by that image system. The last three contributions are computed by the formulas given above. In the end the vertical displacement of the tip vortex is computed by a numerical integration of the downwash using:

\[
m_{tip} = m_{tip} + \sum_{r} \varepsilon_{tip} \Delta r
\]

Finally it should be noted that the tip vortex vertical displacement due to wing bending increases significantly during the take-off rotation due to the wide range of associated lift coefficients. Nevertheless our computations are limited to a rigid wing. A separate module is required to estimate the wing tip bending using the wing structural layout.
3.3 Numerical extension

3.3.4 Tip vortex sideways displacement

In a similar way we can now derive the sidewash induced by a descending flat vortex sheet:

$$\beta_{fus} = 2 k_{fus} \frac{k_{fus}}{2} \frac{C_L}{\pi A}$$

(3.20)

with

$$k_{fus} = 2 \sqrt{- \frac{(\eta^2 + m^2)^2 - (\eta^2 - m^2)^2 - (\eta^2 + m^2)\sqrt{(\eta^2 + m^2)^2 - 2(\eta^2 - m^2) + 1}}{2((\eta^2 + m^2)^2 - 2(\eta^2 - m^2) + 1)}}$$

(3.21)

and

$$k_{russ} = \frac{2b}{4\pi \frac{A}{2}} \left[ \frac{1}{\sqrt{(\eta_p + \eta_{russ})^2 + (m_p - m_{russ})^2}} \frac{m_p - m_{russ}}{\sqrt{(\eta_p - \eta_{russ})^2 + (m_p - m_{russ})^2}} \right]$$

$$= \frac{2}{\pi} \left[ \frac{m_p - m_{russ}}{(\eta_p + \eta_{russ})^2 + (m_p - m_{russ})^2} - \frac{m_p - m_{russ}}{(\eta_p - \eta_{russ})^2 + (m_p - m_{russ})^2} \right]$$

(3.22)

Note the negative sign in (3.21) indicating inflow above the vortex sheet. Because this sign is not changed automatically below the vortex sheet this must be done explicitly to obtain outflow. The sidewash is also obtained from a numerical integration:

$$\eta_{tip} = 1 + \sum_{r_s} \beta_{tip} \Delta r$$

(3.23)

3.3.5 Downwash and dynamic pressure in ground effect

This concludes the computation of the development of four vortex systems:

1. the original combined wing and slat
2. the mirrored combined wing and slat
3. the original flap
4. the mirrored flap

Once their locations relative to the tail or intake have been determined, the downwash at such a position can be obtained using (2.32). For rear fuselage-mounted configurations this downwash at the nacelle intakes leads to two additional vortex systems:

1. the original nacelle
2. the mirrored nacelle
Those six systems are then used to compute the downwash at the tail.

The downwash and dynamic pressure at the intake of wing-mounted nacelles is required because of the associated lift increase due to ground effect. Because the nacelles are ahead of the bound vortices the model that has been developed in chapter 2 is not applicable nor easily adaptable. There is no quasi-uniform upflow ahead of the wing such as the quasi-uniform downwash behind it. Therefore the spanwise downwash distribution bears no such relation with the downwash in the plane of symmetry. Alternatively a solution was found by applying exactly the same formulas for computing the downwash and dynamic pressure at any location of an original wing due to its mirrored counterpart to the nacelle intake. These formulas will be derived in section 4.2.

The downwash belonging to fully rolled-up vortices in ground effect is different than in free air because of their altered span. In ground effect this span is no longer constant. Therefore we have to modify also the factor \( k_{\text{ruw}} \). This is achieved by first computing the vertical distance between the tip vortices and the tail using (2.48) and substituting \( \eta_P = 0 \) into (3.18):

\[
k_{\text{ruw}} = \frac{4}{\pi} \frac{\eta_{\text{ruw}}}{\eta_{\text{ruw}}^2 + m_{\text{ruw}}^2}
\]  

(3.24)

The difference with (2.31) is that we now use the actual spanwise location of the tip vortex instead of \( \pi/4 \). Note that for \( \eta = \pi/4 \) we obtain again (2.31).

So far we have only covered downwash at the tail and intakes due to elliptical lift distributions. It has been found however that the ground effect itself alters the lift distribution in spanwise direction due to higher induced upwash inboard (see Figure A.2). This non-elliptical lift distribution is expected to increase the downwash at the tail for the clean configuration because of its trailing vortices being closer to the centerline than for an elliptical equivalent. With flaps deployed this effect may be reversed because of the reduction in inboard lift due to the drop in lift as a result of the loss in dynamic pressure (see section 4.2). This non-elliptic effect can be captured by formulating equations for the lift change due to ground effect and its downwash induced by a swept wing, similarly to (2.16a) but now out of its horizontal plane. Comparison of these equations shows by what factor the non-elliptic lift must be multiplied in order to obtain the downwash. The lift is given by

\[
C_{L_{\infty}} = \frac{2}{S} \int_0^{b/2} (c_t c_p) \, dy = \int_0^1 \left( \frac{c_t c_p}{c_g} \right) \, g \, d\eta
\]  

(3.25)

and with

\[
l = c_t c_e \frac{1}{2} \rho V^2 = \gamma \rho V
\]  

(3.26)
\[ \varepsilon_{non\,elt} = \frac{2}{V} \int_{0}^{b/2} \left\{ \frac{1}{4\pi \sqrt{(x\cos\Lambda)^2 + z^2}} \right. \\
\left. \frac{x\sin\Lambda}{\sqrt{x^2 + z^2}} + \frac{\frac{y}{\cos\Lambda} - x\sin\Lambda}{\sqrt{(x - y\tan\Lambda)^2 + y^2 + z^2}} \right. \\
\left. + \frac{1}{4\pi \sqrt{y''^2 + z^2}} \frac{y}{\sqrt{y''^2 + z^2}} \left[ 1 + \frac{x - y\tan\Lambda}{\sqrt{(x - y\tan\Lambda)^2 + y^2 + z^2}} \right] \right\} \, d\gamma \\
= \frac{1}{2\pi b} \int_{0}^{1} \left\{ \frac{r\sin\Lambda}{(r\cos\Lambda)^2 + m^2} \left[ \frac{r\sin\Lambda}{\sqrt{r^2 + m^2}} + \frac{\frac{\eta}{\cos\Lambda} - r\sin\Lambda}{\sqrt{(r - \eta\tan\Lambda)^2 + \eta^2 + m^2}} \right] \\
+ \frac{\eta}{\eta^2 + m^2} \left[ 1 + \frac{r - \eta\tan\Lambda}{\sqrt{(r - \eta\tan\Lambda)^2 + \eta^2 + m^2}} \right] \right\} (c_{c})_{ge} \, d\eta \\
= \frac{1}{2\pi b} \int_{0}^{1} K_{non\,elt}(c_{c})_{ge} \, d\eta = \frac{1}{2\pi A} \int_{0}^{1} K_{non\,elt} \left( \frac{c_{c}}{c_{g}} \right)_{ge} \, d\eta \quad (3.27) \]

Combining (3.25) and (3.27) reveals that when lift is computed by numerical integration (see section 4.2), its related downwash can be determined simultaneously by multiplying the lift integral steps by the factor

\[ \frac{c_{g}}{2\pi b} K_{non\,elt} = \frac{1}{2\pi A} K_{non\,elt} \quad (3.28) \]

This way the complicated expressions for the lift change in ground effect (3.27) and the computations are accelerated. In addition it must be remembered that there is also a mirrored lift increment that induces downwash. Therefore (3.28) is applied twice: first for the original wing and then for its image, each with their respective vertical distances to the tail.

In ground effect the wake descends only a little, thereby increasing the chance of a fuselage-mounted tail entering the wake. Due to the numerical set-up of the computation of the vortex sheet development, the wake vertical displacement can now be computed including the effect of reduced dynamic pressure on the downwash at the wake center (see Figure 2.34):

\[ \varepsilon_{w} = \arcsin \left( \frac{\varepsilon}{\sqrt{1 - \eta_{\varepsilon}}} \right) \quad (3.29) \]

Due to a lack of experimental data for the dynamic pressure at the horizontal tail in ground effect this item could not be validated.

### 3.4 Validation

#### 3.4.1 A collection of vortex sheet positions

This method as been implemented in our tool HOT. Similar to the free air situation a comparison has been made between computed and experimental vortex sheet vertical
displacements. The results for the fewer available cases are given in Figure 3.7(a) and 3.7(b). The conclusions are the same as in the free air case: with flaps retracted the vortex sheet position is predicted reasonably well (although there is more scatter now) and with split flaps the vertical displacement is consistently overestimated.

3.4.2 Location of vortex sheet and tip vortex of the Fokker 100

In this section the vertical displacement of the vortex sheet and the vertical and sideways displacement of the tip vortex will be shown. An example of the results of a computation as described above is given in Figure 3.8 for the Fokker 100 with flaps retracted at $\alpha = 12^\circ$ in free air and with the wheels touching the ground. Unfortunately no experimental data are available to compare with. In this figure the vertical position of the averaged vortex sheet with respect to the quarter-chord point at the plane of symmetry is shown for free air and in ground effect. All distances have been divided by the wing’s semispan. The reference point for the coordinates is the intersection of the wing quarter-chord line with the plane of symmetry. The lines start at the wing tip’s trailing edge. It is obvious that the ground effect reduces the vertical movement considerably. Also the vertical location of the tip vortices is shown. Note that the tip vortices start at a higher location than the vortex sheet due to the wing’s dihedral. In free air they move downwards over about a fifth ($4/\pi^2$ opposed to 2) of the distance as the vortex sheet in the plane of symmetry, whereas in ground effect they do not descend noticeably. The inward movement of the tip vortices is included as well. It is clearly visible that the free air inboard movement is practically not present in ground effect. Because the vertical and sideways displacement of the tip vortices is very small it can be concluded that modelling these vortices as horizontal horseshoe vortices such as in [14] is very reasonable indeed.

Figure 3.9 shows the same results for the flaps fully extended case. Only the positions of the wing vortices are shown but including the effect of the flap lift. The vortex sheet starts at a lower position than for flaps retracted due to the lowered flap trailing edge, whereas the tip vortices’ origin has remained the same. The averaged vortex sheet in free air descends much more than for flaps retracted due to the high span loading of the flapped part of the wing. In ground effect it even floats upward because it is the average of the sheet in the plane of symmetry and the tip vortices. The wing tip vortices go up instead of down due to the flap tip vortices, in free air as well as in ground effect. The upward movement is stronger in free air because there are no mirror flap vortices. Apparently this effect is more powerful than the outboard movement of the flap vortices which brings them closer to the wing tip vortices. The inboard shift of the wing tip vortices is more linear in free air than without flap deflection due to the inward pulling by the flap tip vortices. Again, they move outboard in ground effect.

The position of the flap vortices is shown in Figure 3.10. The flap vortex sheet trails from a slightly lower position than the wing’s sheet in Figure 3.9 because the flap tip vortices are more inboard and thus at a lower height due to dihedral and because they are supposed to originate at the flap quarter-chord point.\(^1\) The vertical descent

\(^1\)It should be kept in mind that the vortex sheet positions shown are the ones averaged over the values at the plane of symmetry and at the tip plane as explained in section 2.6.
Figure 3.7. Validation of the vortex sheet vertical displacement [1]
Figure 3.8. Fokker 100 wing vortex positions in free air and in ground effect, flaps retracted, $\alpha = 12^\circ$, HOT computation

Figure 3.9. Fokker 100 wing vortex positions in free air and in ground effect, flaps extended, $\alpha = 12^\circ$, HOT computation
3.4 Validation

![Graph showing flap vortex positions in free air and in ground effect. Flaps extended, α = 12°, HOT computation.](image)

Figure 3.10. Fokker 100 flap vortex positions in free air and in ground effect, flaps extended, α = 12°, HOT computation

of the flap tip vortices is less in ground effect than in free air, as expected. Clearly visible is the strong outboard movement of the flap tip vortices in ground effect. The kink is due to the flap tip vortices dropping below the wing trailing vortices and being carried outboard by their sidewash. Just as for the clean case, straight vortices seem a good alternative for simulating the trailing vortices.

3.4.3 Downwash at the tail of the Fokker 100

A comparison of downwash at the tail is made with wind-tunnel data and the lifting surface code NPLS that does not include vortex sheet relaxation. In order to provide a comparison with this code, the rolling-up has been switched on and off in HOT. Also, ESDU is included in the validation. For a fair comparison with HOT the ESDU method has been applied to the free air HOT downwash instead of the manufacturer’s data (ADB) in computing the ground effect. It must be remembered that ESDU does not yield the free air downwash easily and the accuracy is not very high anyway. Therefore HOT seems a better basis. It is recalled that HOT does not utilize any downwash from external sources but acts as an independent downwash data generator, although it needs the free air lift-curve tail-off. The height is chosen such that the main wheels with the landing gear leg fully extended just touch the ground.

The results are shown for the clean configuration in Figure 3.11. The downwash is highest according to the wind-tunnel, followed by the panel code, the present method and ESDU. The HOT results including relaxation are in very good agreement with NPLS data. But this is considered coincidental because the downwash excluding
relaxation are less in agreement. It is unclear why the wind-tunnel data are even higher than for the methods without relaxation. It was expected that the NPLS downwash would be highest because of the horizontal and non rolled-up vortex sheet, yielding the smallest distance to the tail. This discrepancy can not be caused by the lack of moving belt because the ratio of lift coefficients to relative height $h/b=0.18$ is less than the limit value 20 as indicated in [56]. Rather, slots in the wind-tunnel wall intended to suppress wall interference with respect to wing lift might lead to an increase in downwash at the tail. This matter raises some suspicion as to the accuracy of the wind-tunnel data. A comparison with other manufacturer’s data in Figure 3.12 confirms that the Fokker 100 wind-tunnel downwash slope is rather high. It can also be seen that NPLS overestimates the lift for a given angle of attack, whereas HOT predicts almost the same amount of lift as the wind tunnel test data.

The ESDU data are (somewhat) lower than the others. It is concluded that there is a wide scatter in the Fokker 100 downwash from various sources without any one being really reliable. This prevents definite conclusions as to the accuracy of our method.

The comparison for the landing configuration is shown in Figure 3.13. The general trend is the same with the exception of HOT data that are relatively smaller. The amount of rolling-up is larger due to the high flap span loading according to (2.25). Contrary to our expectations the difference in HOT downwash between relaxation switched off and on is very small. Closer examination revealed this is caused by two opposite effects. The first is that due to ground effect the trailing vortices do not
Figure 3.12. A collection of manufacturer’s tail downwash data in ground effect, flaps retracted, gear down

Figure 3.13. Fokker 100 tail downwash in ground effect validation, full flaps, gear down
descend by far as much as in the free air case. This increases the downwash. The second effect is that the trailing vortices move outboard, which decreases the downwash. Apparently these effects cancel each other out. This is not generally true and can be quite different for low-tailed aircraft as the effects of rolling-up near the vortex sheet are different, see Figure 2.13. But the downwash from our computations without relaxation is quite smaller than from NPLS. Detailed investigation of the NPLS output brought to light several inconsistencies therein, which could not be resolved. Unfortunately, we are left again without any solid foundation for drawing conclusions with regard to the accuracy of HOT. It should be pointed out that the big difference between the Fokker 100 downwash as computed with ESDU and the wind-tunnel data was one of the main drivers for the development of the ground effect module within HOT. But having collected data from various sources that difference seems smaller now than it did then. Even so, HOT represents a much closer resemblance to the physics than ESDU does because it distinguishes the wing lift related and flap lift related vortices. This enables us to understand the many and complicated interactions that generate the non-linear downwash in ground effect.

When comparing with other T-tailed aircraft wind-tunnel data in Figure 3.14, the Fokker 100 data seem more realistic than for the clean configuration. The comparable Douglas DC9-10 exhibits a smaller downwash slope, whereas the DC9-30 features a strong reduction in slope unlike any other T-tailed aircraft. It is suspected that the data has been tailored to match the simulator characteristics to the real aircraft characteristics. The aircraft with a fuselage-mounted tail have a much lower downwash.

![Graph showing downwash data](image-url)

Figure 3.14. A collection of manufacturer’s tail downwash data in ground effect, full flaps, gear down
3.4 Validation

3.4.4 Downwash at the nacelle intake of the Fokker 100

Just as for the free air case, the intake of the Fokker 100 has been analyzed extensively. The underlying reason is the suspicion that the change in intake angle of attack due to ground effect might lead to a significant increase in nacelle lift. As already mentioned in section 2.12.4, NPLS had not been programmed to generate the intake angle of attack explicitly. The only way to obtain at least one set of validation material was to derive the angle of attack from the NPLS lift output. There were two ways of doing so, because it was unknown how the ESDU nacelle lift slope had been derived from experimental data: one with the total lift change due to nacelles, and the other with the lift on the nacelles only. Both lift contributions have been divided by the estimated nacelle lift curve and are shown in Figure 3.15. In sharp contrast to the free

![Graph showing downwash at the nacelle intake of the Fokker 100](image)

**Figure 3.15.** Fokker 100 intake downwash in ground effect, flaps retracted

air case the correlation does not seem good at all. The HOT results yield a downwash far too small. The addition of chordwise effects help only a little. However, it was observed that in free air the NPLS downwash was too large. The downwash in free air and in ground effect are related to each other. This can be visualized by extrapolation of the downwash to \( C_L = 0 \). This downwash value of about 4° coincides with the free air value in Figure 2.49.\(^2\) The consequence is that the NPLS overestimation of downwash in free air automatically implies an overestimation in ground effect. This prohibits firm conclusions on the HOT inaccuracy. It remains unclear why NPLS would overestimate the downwash at the nacelle intake.

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\(^2\)Also visible in Figure 3.18.
The case for full flaps (Figure 3.16) is quite better which is somewhat surprising, given the fact that this case is in general the more difficult one. The chordwise effect shifts the non-chord solution to the proper magnitude. Yet the slope has the wrong sign: a negative downwash gradient. This has also been observed for the clean configuration.

Considerable effort has been devoted to solve this mystery but to no avail. The intriguing question is why the agreement is so good in free air but somewhat disappointing in ground effect for flaps retracted. One important element is the fuselage-induced upwash that accounts for a 5° decrement of the intake downwash at \( \alpha = 12^\circ \) (see (2.58) with \( K_f \approx 0.32 \)). But no cause could be found for this upwash to be so much different in ground effect than in free air.

The consequence is that especially for flaps retracted the nacelle lift change due to ground effect might be overestimated. For \( \alpha = 12^\circ \) the HOT computed intake angle of attack will be around 12° whereas NPLS yields around 6° which implies an overestimation by a factor 2. The nacelle is not intended to be a good lift producer but still this may be significant, as discussed in chapter 4.

3.4.5 Downwash at the tail of the A330-300

As for the free air case we now focus on a low-tailed aircraft as well, the A330. Computed data is compared with data taken from its ADB, for which a moving ground belt was used in the wind-tunnel tests at the DNW. The comparison for all flap settings is shown in Figure 3.17 with an inaccuracy of \( \pm 1^\circ \). The trend in
downwash error is identical to the one in free air: an underestimation by the present method for the clean configuration at high lift coefficients and angle of attack, and an overestimation for the full flaps configuration for all lift coefficients, see Figure 3.18. Moreover it appears clearly from this figure that the downwash in ground effect has a maximum for a certain angle of attack, which is generally not observed for T-tailed aircraft such as the Fokker 100. The explanation is that when a low tail approaches the descending vortex sheet with increasing angle of attack, the tail passes contours for constant downwash as may be noticed in Figure 3.2. This effect is responsible for the more than linear increase of downwash in free air as indicated by the slight upward curvature of the lines in Figure 2.53. But that also holds for the mirrored vortex sheet, which moves in the opposite direction (upward) as the tail. Thus this mirrored sheet approaches the tail much faster than the original sheet. There is an angle of attack where the latter influence overrides the former one, thereby decreasing the downwash.

For a T-tailed aircraft this angle of attack is much higher and outside the operating range of angles of attack, as the tail is further away from the mirrored sheet. Therefore these configurations do not exhibit such a reduction in downwash with increasing lift coefficient. An additional conclusions is that the downwash in ground effect, being a small difference between two large contributions (by the original vortex system and the image one) is extremely sensitive and therefore difficult to be estimated accurately. In other words: an error in the location of the trailing vortices is magnified by the mirrored trailing vortices.

The variation of the A330 downwash with angle of attack and altitude as computed
Figure 3.18. A330 tail downwash versus lift coefficient, free air and in ground effect, clean and full flaps. Comparison of HOT and wind tunnel test data.

Figure 3.19. Computed downwash at the tail as a function of altitude and α for the Airbus A330, flaps retracted
by HOT is shown in Figure 3.19. It is apparent that the linear relation between lift coefficient and downwash in free air alters into a non-linear one. Again it is very clear how sensitive the downwash in ground effect is, but now to variations in lift coefficient or $\alpha$ and height.

### 3.4.6 Downwash collection civil airliners

A comparison of the manufacturer’s and computed tail downwash for various aircraft in ground effect is depicted in Figure 3.20. The experimental data have been taken mainly from [35]. For the A320 no data for the clean configuration was available and a take-off setting has been chosen instead. That may be recognized in Figure 3.12 as the A320 lift coefficient is relatively high.

![Figure 3.20. Tail downwash in ground effect validation, flaps retracted](image)

The inaccuracy is about $\pm 1^\circ$. It can be concluded that the match is good, especially considering the sensitivity of downwash in ground effect. In addition, there are many difficulties in measuring the actual downwash in the wind-tunnel as well as in flight testing. Large discrepancies between wind-tunnel and flight test data have been found such as discussed for the Fokker 100. Also different trends in downwash in ground effect have been observed between various aircraft of similar configuration and size, raising questions about the accuracy thereof. An example thereof is the DC9-10 versus the DC9-30 in Figure 3.20. It is especially odd that the latter’s downwash behaves somewhat as for a low-set tail configuration. Besides, one can easily imagine the challenge a test pilots faces flying in steady flight, slightly above the ground at various constant angles of attack for all flap deflections. Often these test are carried
out at somewhat greater heights and only used to verify computer models, that are then used for extrapolation to the height corresponding with wheels in ground contact [5, 20].

![Graph](image_url)

**Figure 3.21.** Tail downwash in ground effect validation, flaps extended

The next comparison is for the full flaps configuration (Figure 3.21). This agreement is worse than for the flaps retracted case, with an inaccuracy of ±2°. That is a trend break compared to the free air situation, where the accuracy was independent of the flap setting. However, as concluded earlier on, ground effect downwash is much more sensitive and with flaps deflected rolling-up effects are stronger causing larger vertical displacements. The only comfort is that the aircraft that have been subjected to the most advanced testing techniques, the A320 and A330, have an error of only ±1°.

Finally a comparison with ESDU is made in Figures 3.22(a) and 3.22(b). Because the DC9-30 downwash data seem flawed they are discarded for this comparison. The comparison basis thus consists of 6 aircraft, 4 T-tailed and 2 low-tailed. For the clean configuration the HOT average error is almost constant with angle of attack and -0.5° with a deviation of about 0.5°. The ESDU error is about -0.5° larger and the deviation slightly better than for HOT. The kink in the average at α = 8° is caused by the lack of data for the F28Mk1000 at α ≥ 10° due to wing stall, leaving a smaller number of aircraft and thereby increasing the average. For full flaps the HOT average error is again about -0.5° and comparable to ESDU. The HOT deviation however of about 1° is inferior to ESDU’s 0.5°

In general it was expected that ESDU would yield too small downwash because it uses straight and horizontal horseshoe vortices, thereby underestimating the distance
Figure 3.22. Tail downwash comparison with ESDU
between the vortices and the tail, especially in free air. This yields an overestimation of the free air downwash. Because ESDU relies on the ratio of the in ground effect and free air downwash, applied to free air downwash from another source, this would then lead to an underestimation of the downwash in ground effect. For the flaps retracted configurations this happens to be the case. Nevertheless it is truly remarkable how well the simple ESDU model performs compared to our much more complicated method, especially with flaps fully deflected. This is supposedly attributable to the stronger rolling-up effect of the trailing vortices with flaps fully deflected, which is more in line with the ESDU model of two concentrated vortices than for the clean configuration. Unfortunately, the foundation for the derivation thereof remains unpublished. In addition, ESDU places the straight and horizontal trailing vortices with flaps deflected even inboard of the flap tip, which increases the downwash in ground effect and compensates for the overestimation of the downwash in free air. This explains why ESDU performs so well in ground effect with flaps deflected.

The major advantage of HOT is twofold: it also generates the downwash in free air, for which ESDU has a very time consuming item with allegedly less accuracy and a different computational model; and it helps understand the build-up of the downwash as it is much closer to the real physics by staying with separate wing and flap vortices.

3.5 Conclusions

The set of closed form expressions for the downwash in free air has been extended to include ground effect. The results for flapped wings are in good agreement with other theoretical tools such as [25]. However, the sideways movement of the trailing vortices could not be modelled in closed form expressions. These are considered essential for deployed flaps due to the large contribution of the rolled-up vortices. Therefore the same set of equations has been converted into a numerical tool that also includes sideward. Special care has been given to the interaction of the wing-lift and flap-lift trailing vortices. The required input has remained the same. Computations last several seconds only on a state of the art PC.

Again an extensive validation has been performed. For the Fokker 100 the computed downwash seems consistently too small, especially with increasing angle of attack. The agreement for the A330 downwash however is within ±1° for all flap settings. The overall mean accuracy on basis of 6 aircraft (2 low-set tail, 4 T-tailed) is ±0.5° with an 0.5 standard deviation for the flaps retracted configuration. For the full flaps case this is ±0.5 and 1° respectively.

The downwash at the nacelle intake for the Fokker 100 was predicted reasonably, but the downwash gradient was negative as opposite to the NPLS prediction. There are some doubts about the NPLS accuracy however, based on its free air prediction.

The distinct difference in downwash curve with lift coefficient between low set- and T-tails is explained by the closure rate of the image trailing vortices towards the tail. Likewise, the downwash in ground effect is particularly difficult to predict because the image vortex sheet amplifies the downwash error associated with the original vortex sheet.
3.5 Conclusions

The ESDU method performs surprisingly well compared to HOT. For flaps retracted HOT offers a clear improvement over ESDU, but not so with flaps fully deflected. The major advantage of HOT is twofold: it also generates the downwash in free air, for which ESDU has a very time consuming item with allegedly less accuracy; and it helps understand the build-up of the downwash as it is much closer to the real physics by adhering to separate wing and flap vortices.
Downwash behind wings in ground effect
Chapter 4

Ground effect on lift and pitching moment

4.1 Introduction

The problem of the change in wing lift due to ground effect has been dealt with extensively since the twenties, when Wieselsberger employed the technique of mirroring [39]. Often Prandtl’s famous biplane theory is used representing a wing and its image. Although there is a considerable amount of literature on ground effect, most of it is devoted to airfoils, such as [7], some of it to wings and very little to complete aircraft configurations including the fuselage and nacelles. A comprehensive outline has been given in [14].

As stated in the introduction (section 1.4.1) an accuracy for the change in lift coefficient due to ground effect of 0.1 is aimed for. The available methods seem to be insufficient in accuracy for this particular purpose. In for example ESDU [14] an accuracy in $\Delta_y C_L$ of $-0.1/C_L$ is claimed, whereas $\Delta \alpha C_L$ itself is of the same order of magnitude. It follows that for a $C_L \alpha$ of 2.0 at lift-off the error in $C_L$ can be as high as 0.2. Assuming a $C_L \alpha = 0.09/\alpha$ this error must be made up for by an change of 0.2/0.09 = 2° in angle of attack. The resultant error in center of gravity can be found from the pitching moment equilibrium. If we assume that the center of gravity coincides with the aerodynamic center the pitching moment error around the center of gravity only contains the tail contribution. This error must be cancelled out by a shift in center of gravity according to:

$$\Delta C_m = -\Delta x_{cg} C_L = -C_{L \alpha}(1 - \frac{d\alpha}{d\alpha}) \Delta \alpha \frac{S_h l_h}{S \bar{c}}$$

(4.1)

so

$$\Delta x_{cg} = \frac{C_{L \alpha}}{C_L}(1 - \frac{d\alpha}{d\alpha}) \Delta \alpha \frac{S_h l_h}{S \bar{c}}$$

(4.2)

For a typical tail lift gradient of 0.06/°, a downwash gradient of 0.35 and a tail volume of about 1, $\Delta \alpha = 2^\circ$ leads to 4% of the MAC when calculating the allowable center of gravity limits stemming from stability and control considerations. This is a significant
part of a total center of gravity range of typically 25 to 30% \( \bar{c} \), especially since there are many contributors to the pitching moment which is therefore hard to estimate accurately. It must be realized that this is the maximum error one can expect and the mean error is likely to be half this value, as will be shown later on in section 4.7.1.

Taper, sweep, dihedral, wing-fuselage lift carry-over, lift on the nacelles mounted at the rear-fuselage and partial-span flaps are all being addressed. As these are not explicitly incorporated in [9, 14, 55], a considerable improvement in accuracy is hoped for.

The effect of a mirrored wing on the real wing consists of four components:

1. there is an increase in lift due to the vertical component of the velocities induced by the trailing vortices (and by the lifting vortices of the opposite half of the mirrored wing if it is swept),

2. another increase in lift stems from an artificial camber, generated by the lifting vortices as the vertical orientation of the induced velocity vector varies along the chord,

3. the forward component of the induced velocities by the lifting vortices decreases dynamic pressure and thus lift,

4. often thickness can be neglected, unless the wing is very close to the ground \( (h/c<0.5) \) and the wing is very thick [14].

The first two contributions are linear with lift coefficient and the third one is primarily squared as will be shown later on. The second one is the smallest.

The situation at the fuselage is entirely different.

1. First, the vortices of a wing in free air induce velocities at the fuselage as well, thereby extending the pressure at the wing root over the fuselage belly and crown [29]. For a low-winged aircraft the distance between the lifting vortices and the center section’s belly is shorter than to the fuselage crown and for a high-winged aircraft it is the other way around. As a consequence one might expect that the lift carry-over at the lower surface of a low-set wing is higher than at the top surface of the fuselage and vice versa for a high-mounted wing. In general the lift acting on the wing’s upper surface is several times larger than the one on the lower surface. Therefore it is also expected that a high-mounted wing generates more lift than a low-set wing. Likewise, the ground effect is stronger for a low-set wing than for a high-set wing. In ground effect the induced horizontal velocities will be smaller at the fuselage upper half than at its lower half. The resulting effect on the fuselage is a smaller reduction of the dynamic pressure at the top than at the bottom, which translates into a lift increase linear with lift coefficient. As the center section area represents about 20 to 30% of the wing area, this is a significant contribution.

2. Second, the usually pronounced curvature of the upper fuselage lobe behind the cockpit yields additional lift that is larger than the negative counterpart caused by the upsweep of the rear fuselage. An isolated fuselage does not produce lift in parallel potential flow because then these two contributions are equally
large and opposite. But the downwash induced by the wing at the rear fuselage reduces the negative lift there, resulting in a net lift due to the positive lift on the upper front lobe. In ground effect this downwash at the rear fuselage is strongly reduced, causing the fuselage lift to decrease. This fuselage lift can be computed using the method laid out in [2].

3. Third, the influence of the presence of the fuselage on the wing lift consists of a local increase in angle of attack at the inboard wing due to the accelerated vertical flow when the fuselage is at an angle of attack (see Figure 2.29).

The first and third term can be regarded as a two-dimensional problem of a quasi-ininitely long fuselage, whereas the second takes the finite length of the fuselage into account.

None of these effects are incorporated in handbook methods commonly used for ground effect. Typically, there the fuselage is treated like it is in free air, i.e. as if it were an extension of the wing to the plane of symmetry. In other words: the lift carry-over at the fuselage is estimated to be equal to the lift generated by that part of the isolated wing that is in reality covered by the fuselage plus the lift on the fuselage upper front lobe. In addition, the handbook methods assume constant induced upwash and constant induced reduction in dynamic pressure along the span with the wing quarter-chord in the plane of symmetry as calculation point. Because this point represents also the maximum value, this leads to an overestimation of the lift gain due to upwash and the lift loss due to reduced dynamic pressure. The net effect is that the lift change due to ground effect is underestimated with increasing lift coefficient, as will be shown later on.

One way of improving the estimation of ground effect on lift would be to do a much more detailed analysis of the effects along the wing and fuselage as described above. Unfortunately the author did not succeed in implementing all these effects in time. What has been programmed so far is the ground effect on the wing from the tip to the fuselage junction chord, and the lift on the isolated fuselage caused by the pressure differential between the upper and lower surface. The elements that are still lacking are the interference between the wing and fuselage lift, and the lift due to the finite fuselage length. However, it is expected that the main effects have been captured and that this approach is an improvement over the existing handbook methods.

4.2 Wing lift

4.2.1 Introduction

It is assumed that the image wing lift distribution is elliptical, and remains so, even in ground effect. The latter assumption is deemed viable because the change in lift due to ground effect is generally much smaller than the lift itself. The effect on wing lift may be obtained from spanwise integration of the local changes in lift using elliptical lift distribution:

\[
(e_0 e_0 dy = \frac{4}{\pi} C_L \frac{S}{b} \sqrt{1-\eta^2} dy
\]  

(4.3)
The change in lift due to ground effect may be expressed as

\[
\Delta_{ge} C_{Lw} = C_{Lw_{ge}} - C_{Lw_{lag}}
\]

\[
= \frac{2}{S} \int_{b_{flw}/2}^{b/2} \alpha_a (\alpha - \varepsilon - \Delta_{ge} \varepsilon) c(y) \left( 1 + \frac{\Delta_{ge} V}{V} \right)^2 dy - \frac{2}{S} \int_{b_{flw}/2}^{b/2} \alpha_a (\alpha - \varepsilon) c(y) dy
\]

\[
+ \frac{2}{S} \int_{b_{flw}/2}^{b/2} \alpha_a \Delta_{ge} \varepsilon c(y) \left[ 1 + 2 \frac{\Delta_{ge} V}{V} + \left( \frac{\Delta_{ge} V}{V} \right)^2 \right] dy
\]

(4.4)

By differentiating (4.3) to angle of attack, substitution into (4.4) and neglecting the last squared term in (4.4) we get:

\[
\Delta_{ge} C_{Lw} = \frac{4}{\pi} \int_{b_{flw}/2}^{b/2} \left\{ C_L \left[ 2 \frac{\Delta_{ge} V}{V} + \left( \frac{\Delta_{ge} V}{V} \right)^2 \right] - C_{L_{a}} \Delta_{ge} \varepsilon \left[ 1 + 2 \frac{\Delta_{ge} V}{V} \right] \right\} \times \sqrt{1 - \eta^2} d\eta
\]

(4.5)

The additional terms in (4.5) compared to [14, 55] are the second and fourth term. When the speed reduction is large, such as with full flaps, these terms are no longer negligible. Because the speed change and upwash are linear with \( C_L \), the drop in lift due to reduced dynamic pressure (the first term between brackets) goes mainly with \( C_L^2 \) and somewhat with \( C_L^3 \). The increase due to upwash and camber (last term) go mainly with \( C_L \) and a little with \( C_L^2 \). In order to improve the accuracy the change in lift is added to the lift in free air when computing the induced dynamic pressure, upwash and camber by means of a loop. The expression (4.5) can be solved by numerical integration once the changes in horizontal speed (\( \Delta_{ge} V/V \)) and downwash (\( \Delta_{ge} \varepsilon \)) are known. These can be calculated with a set of formulas as shown below.

### 4.2.2 Estimation of horizontal speed at the wing

It has been found in [4] that the best correlation between calculation and experiment is obtained when the induced velocities are determined at the mid-chord point (instead of the quarter-chord point). However, the decrease in horizontal speed may be calculated at the quarter point, since the cosine of the angle towards the mid-chord point will be about 1. Consequently the change in horizontal speed at this point of a wing due to the mirrored image may then be written using lifting line theory, see Figure 4.1 and Figure 4.2:
\[ \frac{\Delta_{gr} V}{V} = \frac{1}{\pi} \frac{C_L}{\pi A} \left\{ \int_0^{\pi/2} \frac{\cos \Lambda}{m_{bsh}^2} K_1 \cos \varphi d\varphi + \int_{\pi/2}^{\pi} \frac{m_{bsh} \cos \Lambda}{(2\eta \sin \Lambda)^2 + m_{bsh}^2} K_2 \cos \varphi d\varphi \right\} \]

with

\[ K_1 = \sqrt{\left( \frac{\eta \cos \Lambda}{\cos \Lambda} \right)^2 + m_{bsh}^2} - \sqrt{\left( \frac{\eta \cos \varphi}{\cos \Lambda} \right)^2 + m_{bsh}^2} \]

and

\[ K_2 = \left[ \frac{\eta \cos 2\Lambda + \cos \varphi}{\cos \Lambda} \right] - \left[ \frac{\eta \cos 2\Lambda - \cos \varphi}{\cos \Lambda} \right] \]

These expressions may seem complicated but are mere applications of Biot-Savart’s law. The first term in (4.6) represents the contribution from the mirrored wing half.
Figure 4.2. Detailed geometry of a vortex system and its image
directly below the original wing half, say the right wing. The second term represents the contribution from the opposite, left wing half. The $K_1$ term is the development of $(\cos \Theta_A - \cos \Theta_B)$ in (2.2) for the right-hand wing half and $K_2$ for the left-hand wing half. The additional $\cos \Lambda$ term is needed to get the speed change component in flight direction instead of perpendicular to the quarter-chord line.

The vertical distance between the mirrored lifting vortex and the original mid-chord point can be calculated including the combined effects of angle of attack and sweep due to the numerical integration. A distinction must be made between the mirrored lifting vortices below and opposite to the original wing half over which the integration takes place (Figure 4.3). It follows that

\[ m_{sh_{iv}} = \left\{ \begin{array}{ll}
2z_\varphi + \frac{b}{2} \eta \sin [2\Gamma - \arctan(\Lambda \sin \alpha)] & \frac{2}{b} \text{ for } 0 \leq \varphi \leq \frac{k}{2},

2z_\varphi \frac{2}{b} & \text{ for } \frac{k}{2} \leq \varphi \leq \pi.
\end{array} \right. \tag{4.9} \]

where $z_\varphi$ stands for the height of the intersection point of the quarter chord line and the plane of symmetry above the ground. For the left hand and right hand trailing vortices the vertical distance is the same:

\[ m_{sh_{iv}} = \left\{ \begin{array}{ll}
2z_\varphi + \frac{b}{2} (|\cos \varphi| + |\eta|) \tan \Gamma - \tan \Lambda_{\varphi} \sin \alpha - \frac{c}{4} \sin \alpha & \frac{2}{b} \text{ for } \frac{k}{2} \leq \varphi \leq \pi.
\end{array} \right. \tag{4.10} \]

\[ \text{Figure 4.3. Vertical distance between original and image vortices} \]
4.2.3 Estimation of downwash at the wing

For the upwash, expressed as negative downwash, caused by the image vortices at the mid-chord point of the wing we may write, using Figures 4.1 and 4.2:

\[
\Delta_{ge} \varepsilon_{wu} = -\frac{1}{\pi} \frac{C_l}{\pi A} \left[ I_1 + I_2 \right]
\]

(4.11)

with

\[
I_1 = \int_{\pi/2}^{\pi} \left[ \frac{\eta \tan \Lambda + r_c/4}{\cos \Lambda} + \left( \frac{\eta}{\cos \Lambda} + \frac{(\eta - (\eta \tan \Lambda + r_c/4) \tan \Lambda) \sin \Lambda}{\sqrt{(\eta \tan \Lambda + r_c/4)^2 + \eta^2 + m_{bshw}^2}} \right) \cos \varphi d\varphi \right] \tan \Lambda - r_c/4
\]

(4.12)

\[
I_2 = \left[ 1 - \frac{((\cos \varphi - \eta) \tan \Lambda - r_c/4)}{\sqrt{(\cos \varphi - \eta)^2 + \tan \Lambda - r_c/4}^2 + (\cos \varphi - \eta)^2 + m_{bshw}^2}} \right] \cos \varphi d\varphi
\]

(4.13)

If the computations start at the right hand wing, the term \( I_1 \) represents the contribution by the left hand image lifting vortices, and \( I_2 \) the one by the right hand and left hand image trailing vortices. The absolute sign in (4.13) of the cosine terms in the computation of \( \cos \Theta_A - \cos \Theta_B \) is necessary to enable one formula for both the left hand and right hand trailing vortex.

For the induced camber (also expressed as negative downwash) the vertical component of the velocities induced by the lifting vortices is used (Figure 4.4):

\[
\Delta_{ge} \varepsilon_{wu} = -\frac{1}{\pi} \frac{C_l}{\pi A} \times \left\{ \int_0^{\pi/2} \frac{2\eta}{\sin 4m_{bshw}^2} K_1 \cos \varphi d\varphi + \frac{\eta}{\sin 4m_{bshw}^2} K_2 \cos \varphi d\varphi \right\}
\]

(4.14)

with \( K_1 \) and \( K_2 \) according to (4.7) and (4.8). Within HOT a straight-tapered wing has been implemented:

\[
c(\eta) = cr(1 - (1 - \lambda)\eta) = \frac{2}{1 + \lambda} (1 - (1 - \lambda)\eta)
\]

(4.15)
Finally the downwash due to induced camber must be added to the one from (4.11):

$$\Delta_{g\varepsilon} = \Delta_{g\varepsilon w_u} + \Delta_{g\varepsilon w_c}$$  \hspace{1cm} (4.16)

For each integration point at the wing the change in horizontal speed $\Delta_{g\varepsilon} V/V$ and in upwash $\Delta_{g\varepsilon}$ is calculated with a two-fold numerical integration: one over the image clean wing and one over the image flapped wing, see Figure 4.5. For clarity these two integrations are shown separately on the right and left wing respectively, but are carried out over both sides. Using Simpson’s integration scheme 20 integration intervals already satisfy the required accuracy. These integrations are used within the second integration loop with (4.5). This is a two-fold integration as well: over the clean wing and over the flapped part of the wing. This double numerical integration is still very fast on today’s PC’s.

4.3 Tail lift

Especially for fuselage-mounted tails the ground effect on tail lift can be significant at high angle of attack due to the close presence of the ground. The tail lift coefficient change due to ground effect is computed with the ESDU module 72023 [14], included in HOT, instead of the method as described in the previous section. The reason behind this is that it is expected that the fuselage contribution to tail lift will be smaller than to wing lift due to the rear fuselage tapering. For T-tails the simplification of an uninterrupted tail is valid anyway. Also, the elevator lift carry-over effect will be smaller than for the flap because the elevator span is usually only marginally smaller than the tailplane span. Therefore it seems that the simpler ESDU method is allowed. But future work may investigate this by applying the same formulas as for the wing and treating the tail lift derivative $C_{L_{\alpha}}$ as a function of elevator deflection.
4.4 Fuselage lift

As already discussed, the pressure differential due to the horizontally induced velocities at the fuselage top and belly is an important contribution to the change in lift (see Figure 4.6). The integration of pressures is performed along the fuselage keel line and crown. The induced velocities are calculated along these lines in the plane of symmetry only and multiplied with the projected horizontal area of the fuselage. This assumes a rectangular cross-section instead of a circular one, see the horizontal lines in Figure 4.7. This is deemed acceptable as the lift, being the force perpendicular to the fuselage surface, is concentrated at the top and bottom thereof. The error is of the order of several percent and thus neglected. This scheme requires only two calculations (i.e. at the top and bottom) for each x-coordinate instead of multiple ones along the fuselage circumference. It is assumed that the fuselage after-body is swept upwards according to the tangent from the undercarriage to the after-body. In addition, the fuselage diameter is decreased linearly from the rear end of the cylindrical part of the fuselage to the end of the fuselage. With these assumptions we may write:

$$\Delta_{ge}C_{L,fus} = -\frac{I_{fus}b_{fus}}{S} \int_0^1 \frac{\Delta_{ge}V_{fus}}{V} \left(2 + \frac{\Delta_{ge}V_{fus}}{V} \right) \left(\frac{x}{I_{fus}}\right) d\left(\frac{x}{I_{fus}}\right)$$

\[ (4.17) \]
Figure 4.6. Side-view of model for fuselage lift change in ground effect

with

$$
\frac{\Delta_g V_{fus}}{V} = -\frac{2 C_L}{\pi A} \frac{m_{fus} \cos \alpha}{(r_{fus} \cos \lambda)^2 + m_{fus}^2} \times \\
\int_0^{\pi/2} \left[ \frac{\cos \phi + r_{fus} \sin \lambda}{\sqrt{(\cos \phi \cos \lambda)^2 + 2 r_{fus} \tan \lambda \cos \phi + r_{fus}^2 + m_{fus}^2}} - \frac{r_{fus} \sin \lambda}{r_{fus}^2 + m_{fus}^2} \right] \cos \phi d\phi
$$

(4.18)

and

$$
m_{fus} = \frac{2}{6} \left[ h_w + h + z_{wffair} \cos \alpha + \left[ \left( \frac{l_{muc}}{c} - 0.25 \right) \bar{c} + \bar{y} \tan \lambda \right] \sin \alpha + \\
x_{fus} \tan \alpha \right]
$$

(4.19)

with the integration lower and upper bound for the $x$-coordinate

$$
x_{ubo} = (l_{fus,efus} + 0.25 c_j - 0.5 b_{fus} \tan \lambda) \cos \alpha
$$

(4.20)

$$
x_{lho} = -\left\{ \bar{g} \tan \Lambda + \left( \frac{l_{muc}}{c} - 0.25 \right) \bar{c} + \frac{z_{wffair}}{\tan \alpha g_{max,stat}} \right\} \cos \alpha
$$

(4.21)

because the reference point for all the formulas is the intersection of the wing quarter-chord line with the plane of symmetry.
A special note is at its place concerning the popular belief that the ground effect on wing lift is caused by air being trapped below the wing, thereby creating a cushion of air that the aircraft is resting on. The foregoing treatment clarifies why that is false: the static pressure increase due to the induced velocity in flight direction actually means reduced dynamic pressure and hence a lift decrement. The lift loss on the wing upper surface is much larger than the lift increase at the lower surface. In free air, the lift on a wing's upper surface is much larger than on its lower surface. Only thanks to the upwash, associated with the decreasing mass flow between the wing and the ground with increasing $C_L$, the lift on the wing increases. The "cushioning effect" lends itself to being visualized as opposed to the increased upflow, which easily leads to the popular explanation of ground effect on lift.

4.5 Nacelle lift

4.5.1 Rear fuselage-mounted nacelles

Nacelles generate another contribution. As the aircraft approaches the ground, the downwash at the engine intake changes considerably, especially for rear fuselage-mounted nacelles and flaps deflected. For the Fokker 100, for example, in ground effect at small angle of attack during the take-off run or the beginning of the landing flare, the downwash is modest at about 5°, whereas in free air it increases considerably to about 15°, see [32]. As a consequence, although the nacelle- pylons-fuselage combination is not much of a lift producer by itself, its lift increases significantly enough to form a substantial contribution to the total ground effect on lift and pitching moment. The calculation of the intake downwash and nacelle lift in free air has been covered by section 2.8. The extension of the downwash method to ground effect from chapter 3 is applied to the engine intake as well. The difference between the two yields the lift change on the nacelle due to ground effect. No correction on the downwash for the upwash due to the presence of the fuselage is applied, as it is assumed that the method in [15] takes this into account, amongst other phenomena, in a correction factor on
the nacelle- pylons lift of 2.4. Furthermore it is assumed that the change in dynamic pressure around the nacelle due to ground effect is negligible because the nacelles are sufficiently behind the wing.

4.5.2 Wing-mounted nacelles

As already stated in section 3.3.5 the downwash and dynamic pressure computation around the wing is entirely different than for rear fuselage-mounted engines. The same formulas as derived for the computation at the wing as in section 4.2 are being used. Since they are changes due to the mirrored systems, these contributions can be used directly to compute the lift change on the nacelle due to ground effect.

The total change in tail-off lift coefficient due to ground effect can now be summarized as:

$$\Delta C_{L_{TO}} = \Delta C_{L_w} + \Delta C_{L_{fus}} + \Delta C_{L_{nac}}$$  (4.22)

4.6 Pitching moment of swept wing due to ground effect tail-off

Most methods where ground effect is considered do not model the fuselage lift separately, and hence neglect the change in pitching moment tail-off in ground effect altogether as it is a magnitude smaller than the tail contribution. Shifts of the wing aerodynamic center are neglected as well since these are very small [4]. Although we include a chordwise pressure distribution correction in our downwash computations, we do not use these in the lift computations and neglect changes in chordwise pressure distributions. However, because more lift is generated inboard than outboard compared to an elliptical lift distribution this will affect the pitching moment for a swept wing and it is worthwhile to perform an integration along the span. The pitching moment of a wing-body combination can now simply be estimated by integration of the lift changes over the wing and fuselage multiplied by their respective moment arm. All wing lift changes are assumed at the local quarter-chord point, except for the lift due to induced camber, which is placed at 50% chord. It is assumed that the contributions of the drag changes to the pitching moment are negligible, which leads to the wing pitching moment change:

$$\Delta_{\rho e}C_{m_w} = \frac{4}{\pi} \int_0^{h/2} \left\{ \frac{L_{a_{\lambda_\rho e}} V}{V} \left[ 1 + 2 \frac{\Delta_{\rho e} V}{V} \right] \left[ 1 + \frac{\Delta_{\rho e} V}{V} \right] \right\} \times$$

$$(x - x_{cg}) \cos \alpha \sqrt{1 - \eta^2} d\eta$$  \hspace{1cm} (4.23)

Another element that is usually ignored, but taken into account here is the effect of reduced dynamic pressure on the zero-lift aerodynamic moment (see Figure 4.6). The moment about the aerodynamic center is scaled with the square of the horizontal speed (4.6) and the local chord (4.15):

$$C_{m_{nac}} = \frac{2}{5\epsilon} \int_0^{h/2} \bar{\epsilon} \bar{c}^2 \left[ 1 + \frac{\Delta_{\rho e} V}{V} \right]^2 V d\eta$$  \hspace{1cm} (4.24)
Ground effect on lift and pitching moment

The contribution of the fuselage consists of the difference in static pressure over the top and bottom surfaces due to the mirrored bound vortices, see also Figure 4.6:

\[
\Delta_{ge} C_{m_{fus}} = -\frac{L_{fus} b_{fus}}{S} \int_0^1 \frac{\Delta_{ge} V_{fus}}{V} \left( 2 + \frac{\Delta_{ge} V_{fus}}{V} \right) \left( \frac{x - x_{cg}}{c} \right) \frac{d}{d\left( \frac{x}{L_{fus}} \right)} \tag{4.25}
\]

The effects of vertical speed changes on the fuselage lift, especially ahead of and behind the wing, are neglected.

For rear fuselage-mounted nacelles, the ground effect on their lift (and pitching moment) can be significant, especially with flaps deflected. For the determination of the associated pitching moment change it is assumed that the lift is concentrated at the nacelle intake lip leading edge.

4.7 Application and validation

4.7.1 Lift tail-off

The method as outlined in the above has been included in a ground effect module, which forms part of the horizontal tail sizing computer program HOT. The main difference with respect to the methods in [14, 55] is in the inclusion of taper, sweep, dihedral, partial-span flap, lift on the rear fuselage-mounted nacelles and on the front and rear fuselage. In [14, 55] the flap span is set equal to the wing span, thereby underestimating the induced velocities. Furthermore, ESDU adds an apparently empirical correction factor N, whereas [55] employs the aforementioned induced camber. In order to test the present method, HOT has temporarily been stripped down by excluding the aforementioned features. Subsequently HOT, [14, 55] have been applied to 11 transport aircraft. HOT and [55] yield almost identical results for the flaps retracted configuration as well as for landing flaps. ESDU however gives higher values for the lift increase due to ground effect for the flaps retracted configuration. It is concluded that the correction factor N has been determined for the flaps down configuration. This application proved the correctness of the presented formulas and their implementation.

The next step was to switch the features on again and calculate their effect on \( \Delta_{ge} C_{L_{TO}} \). For the baseline aircraft the Fokker 100 was chosen as it represents a present-day transport aircraft and exhibits distinct taper, sweep, dihedral and rear-fuselage mounted engines. To create the baseline for investigating the effects of taper, sweep and dihedral these were set to 1, 0 and 0 respectively, the flapspan set equal to the wingspan and the engines moved under the wing (where their contribution to the ground effect is considered to be nil). The wing was pivoted around the mean aerodynamic quarter chord point when reducing the sweep to 0. The aircraft thus obtained is called the baseline aircraft.

Figure 4.8 shows the results when the features are being added one by one while retaining the earlier ones, as indicated by the + sign. For example, +dihedral actually means: the baseline aircraft including taper, sweep and dihedral. The baseline results are equal to the results obtained with [55] and the last one, +nacelles, is the Fokker
again. First of all, the linear increase and quadratic decrease of lift with lift coefficient is clearly visible. For high lift coefficients the latter becomes larger than the former contribution and the lift change becomes negative. The effects of taper, sweep and dihedral are very small, especially for the clean configuration. The reason for that is that the induced velocities do not alter much along the wing span and the chord. Moreover, the elliptical lift distribution does not change due to taper changes. Adding a flap span smaller than the wingspan has a minor effect and increases the curvature due to the higher induced velocities. The cumulative increment in $\Delta_{\alpha}C_{L_{TO}}$ due to increased accuracy so far (difference between lines without markers and with squared markers) is about 0.06 for the clean and 0.1 for the landing configuration. This is small, but still worthwhile pursuing. It was found through detailed analysis of the calculations, that the additional contributions in (4.6) that are not included in [14, 55] almost nullify each other, which also explains the small difference. However, the biggest improvement is caused by the rear-fuselage mounted engines, which are not being taken into account by [14, 55]. The effect of adding nacelles to the rear fuselage is shown in Figure 4.8 to be at the most 0.04 for the clean and 0.06 for the landing configuration. The total prediction improvement in $\Delta_{\alpha}C_{L_{TO}}$ is thus 0.06 for the clean and 0.1 for the landing configuration. These values should be regarded with caution, however. The downwash at the intake in ground effect for the flaps retracted configuration is suspiciously low, as indicated in Figure 3.15. The consequence is that the nacelle lift is overestimated, as already indicated at the end of section 3.4.2. For some unknown reason this is not the case for the full flaps configuration.

The next step was to apply HOT to the Fokker 100, the results of which are
shown in Figure 4.9. Results from various sources are compared: experimental data from the wind-tunnel (WT), ESDU, Torenbeek’s method (TB), the panel code NPLS and finally HOT. The experimental data are considered to be quite reliable, although by their nature they have some unique difficulties of their own, as will be discussed later on. As stated before, the results from the ESDU and Torenbeek methods differ considerably for the clean configuration, but almost coincide for the full flaps configuration. Considering the differences between the two methods, the agreement for the latter configuration is really remarkable. However, both significantly underestimate the change in lift. It must be remarked that both methods do not account for the rear fuselage-mounted nacelles. The panel code NPLS yields a better match with the experimental data than ESDU and Torenbeek. It yields higher lift changes than the experimental data for the clean configuration and lower with flaps fully extended. A complicating factor is that it overestimates lift for a given angle of attack, resulting in a different aircraft attitude for a given lift. This is clearly visible in Figure 4.9 as the markers indicate the same angles of attack with 2° intervals. This raises the question what the calculated lift change would be at the correct attitude. It is expected that the geometric attitude is of less importance than the lift, indicating that if NPLS would be adapted so as to yield the correct lift, the results would not be much different. Overall HOT shows the best agreement with the experimental data, although being the only one overestimating the lift increase with flaps down and at small angle of attack. The relative error can be viewed in Figure 4.10 where the lift curve in free air as well as in ground effect is given according to wind-tunnel tests and HOT. The largest error in total lift in ground effect occurs at low \( \alpha \) and full flaps and
is 4%, which is considered acceptable.

Figure 4.10. The Fokker 100 lift curve in free air and in ground effect

Subsequently this analysis was extended for all transport aircraft for which ground effect data was available. Data on ground effect was gathered from various sources [31] and is shown in Figure 4.11 for 11 aircraft (5 wing- and 6 rear fuselage-mounted engine configurations). Some of these originate from windtunnel data, others from open literature or engineering manuals. A fundamental problem with this kind of data is that it never was intended for the present purpose, but rather for simulating the aircraft’s stability and control characteristics as well as performance analysis. Because these particulars are built up of many components, their only reliability lies in their combination and not necessarily in each constituents’ absolute numerical value. However, because of lack of a better alternative, these data have been used as a benchmark to compare HOT with. Figure 4.11 does show some of this problematic nature. The following observations have been made:

- some aircraft seem to exhibit a rather different lift increase behavior due to ground effect than others. The B747-100 for example shows a lift decrement in the clean configuration at small angles of attack,

- some similar aircraft within one family show very different lift changes, like the DC9-10 and the DC9-30, or the F-28 Mk1000 and Mk4000 aircraft,

- it seems strange that the effect of rear fuselage-mounted nacelles is not consistently evident,
Figure 4.11. Experimental data for $\Delta_g e C_{L,T0}$ for several transport aircraft

- similar aircraft but from different manufacturers show large differences, for example the DC9-10 and F-28 Mk1000 or the DC9-30 and the Fokker 100,

- it seems as if aircraft can be grouped in their behavior by manufacturer, suggesting that each manufacturer represents a school of thought in test procedure and data analysis.

Because the DC9-10 deviates so much from all other aircraft it was decided to drop this aircraft from the validation.

The HOT calculation of the 10 example aircraft for all flap settings (typically 6) and the whole range of operational lift coefficients takes only several seconds. The difference between the HOT results and the experimental data from Figure 4.11 is given in Figure 4.12. Obvious is the wide scatter, especially with increasing lift coefficient and thus high lift devices deployed. Compared to the actual lift change due to ground effect the error is still quite large, despite the complexity and detail of the method. Most lines fall within a $6\% C_{L,T0}$ deviation band, which is an improvement over ESDU’s 10%. There is a clear parabolic trend with curves going up with $C_{L,T0}$. This is an indication that the computed linear and quadratic influence of $C_{L,T0}$ is too little. That may be due to the lack of modelling the wing-fuselage lift interference, which is expected to boost lift. For higher $C_L$ a change in the slope of the curves may be discerned, i.e. the underestimation of the lift increase diminishes. That might be attributed to the omission of the loss in lift at the rear fuselage due to the decrease in downwash. Also evident is the large difference with flaps deployed between the F-28 Mk1000 and F-28 Mk4000. On top of that the B747-100 is much different from all
others, especially in the clean configuration at small angles of attack. These aircraft were also identified in Figure 4.11 as probably being exceptional.

In order to establish the improvement being sought with HOT, the ESDU method has been applied to these aircraft as well. Figure 4.13 shows the difference between the ESDU results and the experimental data from Figure 4.11. The resemblance with the HOT results in Figure 4.12 is evident with a clear shift in the negative direction. This is partly attributable to the omission of rear fuselage-mounted nacelles in the ESDU method. The same aircraft tend to exhibit large discrepancies compared to the experimental data. The largest errors are related to the landing configuration at high angle of attack and come from the DC9-30, F-28 Mk4000 and Mk4000. It is noteworthy that these errors are in excellent agreement with the inaccuracy as claimed by ESDU itself, i.e. $+/-0.1$ in $\Delta_{ge}C_{L_{TO}}$ with most aircraft in the negative half.

Figure 4.12 and Figure 4.13 can more easily be compared using their mean errors and standard deviations, see Figure 4.14. In this figure the Torenbeek method has been added as well. For the flaps up configuration HOT is only slightly better than ESDU and Torenbeek. The Torenbeek method performs inferior but it does not contain empirical factors such as ESDU does. It was expected that the ESDU deviation error with respect to the experimental data would be improved by HOT because so much more detail was incorporated, as discussed above. The aim of this research was amongst others to decrease the scatter in the ESDU results. This is clearly not the case, leading to the very cautious suggestion that this deviation might be caused by scatter in flight test data more than by inaccuracies of the calculation methods. This
scatter is probably the result of the data acquisition and analysis process as governed by a certain school of thought and experience. The ground presence itself poses a significant problem, as discussed earlier on. Bypassing this by wind-tunnel tests brings in other problems of accuracy that are not easily solved.

For flaps down the situation is quite different (Figure 4.14(b)). Now HOT is definitely superior to the ESDU and Torenbeek methods in the mean error whilst delivering the same deviation. The improvement is about 0.07 in $C_L$. Part of the improvement is due to the inclusion of rear fuselage-mounted nacelles, but inspection of the wing-mounted engine configuration results reveals these aircraft exhibit a better match as well. It is remarkable how well the Torenbeek method holds its own against ESDU.

The improvement in accuracy translates into an adjustment in angle of attack of about 0.7° or a shift of 1.5% of in center of gravity limits stemming from stability and control requirements using HOT. On a total cg range of about 25 to 30% of this is a small but still worthwhile improvement. For performance however the improvement in lift is much more important. It is equivalent to half the lift increase going from a single to a double slotted flap.

4.7.2 Pitching moment tail-off

For the Fokker 100 pitching moment NPLS results were available as well as windtunnel data that have been extrapolated to full-scale Reynolds number. A comparison with the windtunnel data is shown in Figure 4.15. It is evident that HOT is off by a
4.7 Application and validation

Figure 4.14. Validation of Methods for Lift Change due to Ground Effect
wide margin and NPLS to a lesser extent. Closer inspection of the NPLS pressure distributions reveals that only a few inner wing panels contribute to the large nose-up moment. This throws some doubt on the NPLS data.

Figure 4.15. Fokker 100 pitching moment change due to ground effect

The contribution of the nacelles to the difference with the windtunnel data has been investigated by comparing NPLS and HOT results for the configuration without nacelles. No wind-tunnel data for this configuration were available. It appears that the difference between NPLS and HOT does not change appreciably due to the omission of the nacelles. This indicates that the changes in downwash at the nacelle intake, found for the flaps up configuration, have little impact on the pitching moment and do not explain the difference with the wind-tunnel data.

As stated before, the ESDU, Torenbeek and DATCOM methods neglect the pitching moment change altogether. In view of Figure 4.15 that appears to be a better choice than trying to compute it.

An overview of pitching moment in ground effect for other aircraft in the flaps up configuration is given in Figure 4.16. Only data for aircraft with rear fuselage-mounted engines was found. A consistent aircraft nose down error in the pitching moment is generated by HOT. The error is uncomfortably large, between -0.02 and -0.1. This equates for a $C_L$ = 2 to an error in center of gravity of 5% $x/c$.

The similar validation for the full flaps configuration is shown in Figure 4.17. The overall error is relatively better and more evenly spread around 0.

To put the error into perspective, the experimental changes in pitching moment tail-off have been collected in Figure 4.18 for the flaps in and full flaps configurations. It is striking that some similar aircraft show different trends with flap deflection, for
Figure 4.16. Pitching moment in ground effect, flaps up; comparison of HOT with windtunnel data

Figure 4.17. Pitching moment in ground effect, flaps down; comparison of HOT with windtunnel data
example the Fokker 100 and DC9-30. It is suspected that the omission of viscous effects and aerodynamic detail (such as flap type and gaps between the flaps) in our tool is the main cause for the lack of accuracy in the estimation of the pitching moment. In addition, the lack in our model of the wing-fuselage lift interference and of the change in front and rear fuselage lift due to downwash changes is likely to be significant. As already indicated in the introduction, a loss in lift might be expected at the rear fuselage and that would generate an aircraft nose-up pitching moment. According to Figures 4.16 and 4.17 the computed results would improve, especially with flaps retracted.

![Graph](image)

**Figure 4.18. A collection of manufacturer’s pitching moment data in ground effect, gear down**

It is concluded that the estimation of pitching moment in ground effect remains disappointing and elusive.

### 4.8 Conclusions

A set of analytical expressions has been derived from classical lifting line theory to model the velocities induced by an elliptical lift distribution mirrored by the ground. These are integrated numerically over the wing, fuselage and nacelles to yield the change in lift. Taper, sweep, dihedral, partial-span flaps and rear fuselage-mounted nacelles have been included as well. However, the interference between the lift change on the wing and on the fuselage has not been included yet. Also the fuselage lift change due to the change in downwash is still lacking. A very limited amount of geometrical
and aerodynamic data is required for input, making the method especially suitable for preliminary design.

Validation of the lift in ground effect by comparison with experimental data for 10 transport aircraft of various configurations (5 wing- and 5 rear fuselage-mounted engines) showed the method is sufficiently accurate in lift for preliminary design purposes for aircraft with moderate to high aspect ratio wings. An improvement of 0.07 in $\Delta g$, $C_L$ has been determined with respect to ESDU [14] in the mean error for full flaps compared to experimental data. No improvement however has been obtained in the standard deviation of these errors. This suggests that the scatter is caused by phenomena that are not included in ESDU, nor in HOT. The improvement is not limited to the rear fuselage-mounted engine configurations, indicating the inclusion of the other said effects is instrumental as well.

The pitching moment prediction appears to give no conclusive positive results, probably due to the lack of viscous effects, the aforementioned omissions and the lack of more refined aerodynamic detail in our model.
Ground effect on lift and pitching moment
Chapter 5

Ground effect on drag

5.1 Introduction

The significance of drag in ground effect with respect to $V_{MU}$ is twofold.

- The lift-off itself is governed by the balance between horizontal and pitch acceleration for elevator power-limited aircraft (see chapter 6). Such an aircraft will lift-off at higher angles of attack when the drag increases.

- The drag also affects the field length, which plays a role for geometry-limited aircraft as explained in chapter 6.

Drag can change considerably due to ground effect. The drop in dynamic pressure reduces profile drag and the upwash lowers induced drag by canting the lift vector forward. ESDU uses the induced velocities at the wing quarter-chord point as a reference and applies that value to the complete aircraft. No dedicated attention is paid to the undercarriage that is in the area of reduced dynamic pressure behind and below this reference point. The effect is that the drag change is overestimated. The Torenbeek method exhibits close resemblance to ESDU but applies the speed change to the wing profile drag only. This seems reasonable because the speed reduction is caused by the image wing and is largest at the original wing's quarter-chord line, reducing rapidly upstream and downstream of it. DATCOM neglects the profile drag change altogether. In the present research the aim is to improve the accuracy by considering the wing, fuselage and undercarriage separately and by including the effects of sweep, dihedral and partial-span flaps into the expressions for induced velocities as developed in chapter 4.

5.2 Zero-lift drag of a wing-body

The reader is reminded that this section is concerned with wing-body, tail-off aircraft configurations only. The trim drag is added in section 5.5. The change in wing profile drag due to ground effect can be established by numerical integration along the span of the product of the change in speed (4.6) caused by the mirrored wing and the
local profile drag coefficient. The profile drag in free air is related to the local chord and supervelocities. Due to increasing relative thickness from tip to root, profile drag is likely to increase by more than the chord, i.e. more than linearly. On the other hand, the Reynolds number increases as well, reducing drag. This is confirmed by measurements indicating that overall the profile drag is linearly distributed over the span [6]. Accordingly the drag is modelled proportional to the chord distribution of a straight-tapered wing:

\[ C_{d_{pw,ge}}(\eta) = C_{D_{pw}} \frac{c(\eta)}{c_g} = C_{D_{pw}} \frac{2}{1 + \lambda} [1 - (1 - \lambda)\eta] \]  \hspace{1cm} (5.1)

with the wing drag split in the drag of the flaps up and flaps deflected contributions:

\[ C_{D_{pw}} = C_{D_{pw,\text{clean}}} + \Delta_{df} C_{D_p} \]  \hspace{1cm} (5.2)

The drag in ground effect can now be expressed as an function of the drag in free air:

\[ C_{D_{pw,ge}} = \int_{\eta_{fu}}^{1} C_{d_{pw,ge}}(\eta) \left( \frac{V + \Delta_{ge} V}{V} \right)^2 \ d\eta \]  \hspace{1cm} (5.3)

which yields for the drag change due to ground effect:

\[ \Delta_{ge} C_{D_{pw}} = C_{D_{pw,ge}} - C_{D_{pw,\text{clean}}} \]

\[ = \frac{2}{1 + \lambda} \left\{ C_{D_{pw,\text{clean}}} \int_{\eta_{fu}}^{1} \left[ \frac{2 \Delta_{ge} V}{V} + \left( \frac{\Delta_{ge} V}{V} \right)^2 \right] [1 - (1 - \lambda)\eta\eta_{fu} + \Delta_{df} C_{D_0} \int_{\eta_{fu}}^{1} \left[ \frac{2 \Delta_{ge} V}{V} + \left( \frac{\Delta_{ge} V}{V} \right)^2 \right] [1 - (1 - \lambda)\eta] d\eta_{df} \right\} \]  \hspace{1cm} (5.4)

Both lift distributions are considered elliptical. The first integral represents the integration over the clean wing, whose drag is assumed to be 0.0065 as a typical value. This is based on a friction coefficient of 0.003 for both the upper and lower wing surface, a shape factor to account for the supervelocities around the wing of 1.108. A more detailed analysis can improve this value as a function of aircraft size. Because the speed change is proportional to \( C_L \) the drag change is proportional to \( C_L \) and \( C_L^2 \). The second, quadratic term is lacking in the ESDU and Torenbeek methods and should provide a higher accuracy. The second integral is the integration over the flapped part of the wing. It is assumed that the drag associated with flap deflection in free air is known already and part of the input file.\(^1\) The change in speed is already available from the lift change computations.

The drag change of the fuselage can be obtained from integration along its keel and crown lines of the profile coefficient and induced velocity parallel to the fuselage

\(^1\)Large variations for various aircraft and flap types have been observed, ranging between 0.04 and 0.15 [34].
and converting this to the direction of flight:

$$\Delta_{g_f} C_{D_{fus}} = \cos \alpha \int_0^1 C_{d_{fus}} \left( \frac{\Delta_{g_f} V}{V} \right)^2 d\left( \frac{x}{l_{fus}} \right)$$

(5.5)

It is assumed that the product of local friction coefficient and shape factor is about 0.0020. This procedure follows the same lines as the integration of the lift changes: it is assumed that the main changes act on the upper and lower part of the fuselage, which are represented as flat panels. The velocity changes at the crown and keel lines are kept constant within each cross section and are applied along the lower respectively upper lobe of the fuselage. Therefore the drag computation is part of the same routine as the lift calculations, which speeds up the computations.

Detailed calculations have shown that although the horizontal and vertical tail represent about 50% of the clean wing wetted area the change in their profile drag can be neglected as well, as they are sufficiently far away from the image wing. The drag change of the nacelles can be neglected as well, even for wing-mounted nacelles that are close to the image wing. The undercarriage will be discussed in section 5.4.

5.3 Induced drag of a wing

The induced drag can be determined by referring to the induced upwash and horizontal speed component at the wing quarter-chord line. Both are already known from the calculation of the change in lift. Note that we stick to the positive definition of downwash for positive lift, so upwash will be expressed as negative downwash. Furthermore we need to compare the situation in ground effect with free air for the same lift coefficient in order to avoid mixing up the change in induced drag due to the lift change in ground effect. Therefore we write:

$$\Delta_{g_f} C_{D_{i}} = \int_0^1 c_{i}(\eta) \left( \frac{C_{L}}{\pi A e} + \Delta_{g_f} \epsilon \frac{V}{V + \Delta_{g_f} V} \right)^2 d\eta - \frac{C_{L}^2}{\pi A e}$$

(5.6)

Note that by contrast to (5.3) which reflects a situation for given angle of attack the speed change is now in the denominator, as the lift coefficient must be increased in order to compensate for the loss in dynamic pressure. The lift coefficient lacks a subscript because the free air and in ground effect $C_{L}$ are considered equal. Although we have assumed elliptical lift distribution until now, the Oswald factor $e$ has been included in the second term as part of the downwash. The reason behind this is that the drag change is computed as a fraction of the free air drag, which contains non-elliptical drag. The Oswald factor is part of the input file. Note also that for uniform downwash the $c_{i}(\eta)$ becomes $C_{L}$ and in free air the resultant change in induced drag becomes 0. The induced upwash $\Delta_{g_f} \epsilon \frac{V}{V} \epsilon \frac{V}{V + \Delta_{g_f} V}$ is already available from (4.11). Since the induced camber is an order of magnitude smaller and distributed over the chord it is

\[\text{---}\]

2Based on several Fokker studies, taking into account Reynolds number effects and fuselage fineness ratio
Ground effect on drag

not taken into account here. Once again assuming elliptical lift distribution we can substitute (4.3) into (5.6):

\[
\Delta_{ge} C_{D_i} = \int_0^1 \frac{4}{\pi} \frac{C_L c g}{c} \sqrt{1-\eta^2} \left( \frac{C_L}{\pi \alpha} + \Delta_{ge} \varepsilon_u \right) \left[ 1 - \frac{\Delta_{ge} V}{V + \Delta_{ge} V} \right]^2 d\eta - \frac{C_L^2}{\pi \alpha e}
\]

\[
\Delta_{ge} C_{D_i} \approx \int_0^1 \frac{4}{\pi} \frac{C_L c g}{c} \sqrt{1-\eta^2} \left( \frac{C_L}{\pi \alpha} + \Delta_{ge} \varepsilon_{w_u} \right) \left[ 1 - \frac{\Delta_{ge} V}{V} \right]^2 d\eta - \frac{C_L^2}{\pi \alpha e}
\]

\[
\approx \frac{4}{\pi} \frac{C_L^2}{\pi \alpha e} \left[ -2 \int_0^1 \frac{1}{c} \frac{\Delta_{ge} V}{V} \sqrt{1-\eta^2} d\eta + \int_0^1 \frac{1}{c} \left( \frac{\Delta_{ge} V}{V} \right)^2 \sqrt{1-\eta^2} d\eta \right] + C_L \left[ \int_0^1 \frac{1}{c} \Delta_{ge} \varepsilon_{w_u} \sqrt{1-\eta^2} d\eta - 2 \int_0^1 \frac{1}{c} \Delta_{ge} \varepsilon_{w_u} \Delta_{ge} \varepsilon_{w_u} \sqrt{1-\eta^2} d\eta \right] \right) \right] (5.7)
\]

This integral is calculated twice:

- once for the clean wing, using the speed \( \Delta_{ge} V \) and upwash \( \Delta_{ge} \varepsilon_{w_u} \) induced by the image wing and flap lift systems and the wing lift coefficient \( C_{L_w} \) and Oswald factor

- once for the flapped part of the wing, employing the speeds induced at the integration points over the flapped wing part and using the lift coefficient \( \Delta_{e, f} C_L \) due to flap extension

For the induced drag due to flap deflection in free air the difference is taken between the total induced drag and the contribution of the clean wing only as provided in the input file.

The drag change consists of three contributions which are discussed in the following, see Figure 5.1:

1. The first two terms together in (5.7) represent the rise in induced drag as the required local lift coefficient must be increased due to the reduction in local speed (note that \( \Delta_{ge} V/V < 0 \)). It is proportional to \( C_{L_w}^2 \) and \( C_{L_w}^4 \) as the speed change is linear with \( C_L \).

2. The third term indicates the drop in induced drag due to the forward canting of the local lift vector (note that \( \Delta_{ge} \varepsilon_{w_u} < 0 \)). It is proportional to \( C_{L_w}^2 \) because the upwash is linear with the lift coefficient. It is by far the largest of all three terms, causing the induced drag to decrease in ground effect.

3. The last term reflects the combination of both effects: the forward canting of the increase of the lift coefficient. This is a drag reduction as well, related to \( C_{L_w}^2 \), and about as large as the first term with opposite sign.
5.3 Induced drag of a wing

![Diagram of induced drag](image)

**Figure 5.1. Change in induced drag coefficient due to ground effect**

A major departure here from most handbooks is that the Oswald factor does not appear in the second and third term. It is common practice to perform a single integration over the wing using the total lift distribution [14, 55]. These terms are then written in the form of the free air induced drag, including the Oswald factor, multiplied by ground effect correction factors. However, the Oswald factor is only an indication of the change of the product of the local downwash and lift coefficient of the original wing, integrated over the span, relative to the minimum for elliptical lift distribution. This does not imply that it is also applicable to the product of the lift coefficient of the original wing and the upwash of the mirrored wing. So instead of a ground effect correction factor to be applied to the free-air induced drag including the Oswald factor, this product is calculated assuming perfect elliptical lift distribution. This approach bypasses the problem which Oswald factor should be used for wings with high-lift devices deployed, as the product of the downwash induced by the mirrored lift due to the flap and the lift coefficient of the clean wing is calculated and vice versa. This problem is inherent to methods that separate wing and flap lift.

The present method assumes perfect elliptical lift distribution for both the original and mirrored lift systems, although it is evident that the ground effect does alter the lift distribution. This is the consequence of lower induced vertical velocities moving outboard at some distance above or below the vortex sheet. It is assumed that the effects of the change in lift distribution of the image wing on induced velocities at the original wing are negligible for drag purposes.
5.4 Undercarriage drag

As undercarriage drag is a function of $C_L$ due to the dynamic pressure reduction below the wing it is often expressed in $C_{D_{puc}}$ and $\beta_{uc}$. The drag reduction is expressed as:

$$\Delta_{ge} C_{D_{puc}} = C_{D_{puc}} \left[ 2 \frac{\Delta_{ge} V_{uc}}{V} + \left( \frac{\Delta_{ge} V_{uc}}{V} \right)^2 \right]$$

(5.8)

and

$$\Delta_{ge} \beta_{uc} = \beta_{uc} \left[ 2 \frac{\Delta_{ge} V_{uc}}{V} + \left( \frac{\Delta_{ge} V_{uc}}{V} \right)^2 \right]$$

(5.9)

The main gear drag consists of contributions by the wheels, the leg itself, the folding mechanism and the landing gear bay cavity. The first three contributions are assumed to be the largest. Often wheel-well doors are closed again to cover this cavity when the undercarriage is extended. Because the computed speed change at the wing itself has already been obtained the $z$-coordinate where the main leg enters the wing is taken as reference. Again, the undercarriage drag coefficient in free air must be known in this stage and part of the input file as well. Typically the $C_{D_{puc}}$ is around 0.022 with flaps retracted and 0.0090 with flaps fully deployed [34]. A typical value for $\beta_{uc}$ is -0.0035.

The aforementioned formulas are only applied to the main gear as the nose gear is sufficiently ahead of the wing to neglect its drag change in ground effect. Therefore the nose gear drag is subtracted from the total undercarriage drag using formulas presented in chapter 10 of [55].

Finally we obtain:

$$\Delta_{ge} C_D = \Delta_{ge} C_{D_{puc}} + \Delta_{ge} C_{D_{fus}} + \Delta_{ge} C_D$$

(5.10)

and

$$\Delta_{ge} \beta = \frac{\Delta_{ge} C_D}{C_L^2} + \Delta_{ge} \beta_{uc}$$

(5.11)

5.5 Trim drag due to ground effect

Complementary to the drag tail-off is the tail contribution that can be derived as:

$$C_{D_{te}} = \left[ \frac{C_{L_{h}}^2}{\pi A} + C_{L_{h} e} \frac{S_{h}}{S} \right]$$

(5.12)

with

$$C_{L_{h}} = C_{L_{h} a} \alpha_h + C_{L_{h} \delta} \delta$$

(5.13)

and

$$\alpha_h = \alpha + \iota - \varepsilon$$

(5.14)

Evidently at this stage the difference in stabilizer incidence and elevator deflection between free air and ground effect can not be determined. Therefore they are considered
constant for the present analysis of trim drag change only. Applying these equations for the same \( \alpha, i_h \) and \( \delta_e \) in ground effect and free air conditions and subtraction leads to:

\[
\Delta_{ge} C_{D_i} = C_{L_{na}} \frac{S_h}{S} \left\{ (\alpha + i_h - \varepsilon_{ige}) \left[ \frac{C_{L_{na}}}{\pi A} (\alpha + i_h - \varepsilon_{ige}) + \varepsilon_{ige} \right] \right.
\]
\[
\left. - (\alpha + i_h - \varepsilon_{oge}) \left[ \frac{C_{L_{na}}}{\pi A} (\alpha + i_h - \varepsilon_{oge}) + \varepsilon_{oge} \right] \right\}
\]
\[
= C_{L_{na}} \frac{S_h}{S} \left\{ (\alpha + i_h)(\varepsilon_{ige} - \varepsilon_{oge}) \left[ 1 - 2 \frac{C_{L_{na}}}{\pi A} \right] + (\varepsilon_{ige}^2 - \varepsilon_{oge}^2) \left[ \frac{C_{L_{na}}}{\pi A} - 1 \right] \right\}
\]
\[
= C_{L_{na}} \frac{S_h}{S} (\varepsilon_{ige} - \varepsilon_{oge}) \left\{ (\alpha + i_h) \left[ 1 - 2 \frac{C_{L_{na}}}{\pi A} \right] \varepsilon_{ige} + \varepsilon_{oge} \left[ \frac{C_{L_{na}}}{\pi A} - 1 \right] \right\}
\]

(5.15)

Analysis of this relation shows that this ground effect on trim drag increases with increasing downwash and thus wing lift coefficient, decreasing tail incidence and angle of attack. Inserting typical values for the Fokker F-28 Mk6000 shows this amounts to several drag counts only. The largest value was obtained for flaps 42, \( i_h = -5.5^\circ \), \( \alpha = 12^\circ \), \( \varepsilon_{oge} = 7.7^\circ \), \( \varepsilon_{ige} = 4.2^\circ \) and yields \( \Delta_{ge} C_{D_{ir}} = 0.0050 \). This is considered to be negligible in the light of the large errors between calculations and experimental data that are encountered in the next section. In addition, the trim drag is a function of the center of gravity which is often taken at a mid cg-range value. However, the minimum unstick maneuver must be performed for the critical cg position, which is the most forward one. Because the minimum unstick estimation is part of the horizontal tail sizing process the cg range is unknown yet. This complicates the addition of trim drag to our drag analysis. Therefore no trim drag calculations will be performed in the simulations later on.

5.6 Validation

5.6.1 Fokker 100

As has become clear in the previous sections, the reductions in the profile drag coefficient \( C_{D_p} \) and induced drag factor \( \beta \) are a function of higher-order terms of \( C_L \). Hence the total drag in ground effect can not be expressed as a drag polar anymore. In addition, a reduction in profile drag at a certain \( C_L \) will be interpreted as a reduction in induced drag. Therefore the \( C_{D_{ir}} \) and \( \beta \) in ground effect can not be used for validation and the accuracy of the tools used can only be analyzed on basis of the total change in drag itself.

The Fokker 100 is chosen as test case since dedicated tests were set up for the determination of drag in ground effect. The flight tests have been performed with the nosewheel on the ground and hence \( \alpha \approx 0^\circ \). The aerodynamic coefficients from the windtunnel tests have been adjusted to the height with the oleo legs fully extended and the wheels touching the ground, as customary within the Fokker ADB. For comparison, the ESDU [14] and Torenbeek method [55] have been applied as well. For ESDU the full aircraft drag has been used as baseline but for the Torenbeek method
only the wing and flaps profile drag in accordance with their respective instructions. Since DATCOM neglects the zero-lift drag reduction [9] no comparison is possible. NPLS lacks viscous effects, being a panel code, and was not applicable as well. A much more complicated Navier-Stokes CFD tool is required to determine the ground effect on viscous drag. Unfortunately no such application was at hand for the Fokker 100.

Figure 5.2. Fokker 100 drag in and out of ground effect, various sources, gear down

The total drag coefficient is depicted in Figure 5.2. The computations with the ESDU, Torenbeek en HOT methods are all based on the free-air drag polar from the ADB. It is obvious that the ground effect on drag is significant, especially with flaps deflected. The ESDU and Torenbeek method underestimate the drag reduction with flaps deflected. Although it is hard to fully understand the ESDU method because most of its derivation has not been published, it is evident that it uses the induced horizontal velocity at the wing’s quarter-chord point in the plane of symmetry instead of an integration over the wing’s span, just like the Torenbeek method. In addition, it refers to the zero-lift drag of the complete aircraft, thereby assuming that the aforementioned induced velocity remains constant over the entire aircraft, by contrast to the Torenbeek method. These shortcomings would overestimate the drag reduction. On the other hand, no distinction is made to the flap deflection and hence the span loading is based on the wing span instead of the flap span. As a result the induced velocities are underestimated, leading to a too small drag reduction. In order to compensate for these anomalies, empirical factors have been used, by contrast to the Torenbeek method [53]. It should be noted these factors have been determined in the early 40’s, thereby excluding wing sweep. The overall result is that the ESDU
and Torenbeek method are quite close.

![Graph showing drag reduction in ground effect for the Fokker100, gear down.](image)

**Figure 5.3. Drag reduction in ground effect for the Fokker100, gear down**

To get a clearer view, Figure 5.3 shows the HOT, ESDU and Torenbeek calculated total change in drag coefficient for the flaps up and down configuration plotted against the flight test data. For the clean configuration all three methods perform very well, HOT in particular with an almost 100% match.

For the landing configuration HOT outperforms both methods. Even so, the largest error is still approximately -80 drag counts or -14% for low \( \alpha \)'s. The error switches sign for higher \( \alpha \)'s and decreases to 3%. Because of the influence that \( C_L \) has on both \( C_{D_0} \) and \( \beta \) it is difficult to say which of the two is the largest contributor to the error. It is striking that ESDU with all its empirical factors is only marginally better than the Torenbeek method.

An indication of the relative impact of the error in the drag change is given by the comparison of the total drag in ground effect (Figure 5.4). The error for the landing configuration is some 7% for low \( C_L \)'s and reduces to 3% for high \( C_L \)'s.

### 5.6.2 Collection of civil airliners

The analysis as described for the Fokker 100 has been applied to 8 transport aircraft (including the Fokker 100) whose drag data in ground effect were available. Five of these are T-tailed and 3 low-tailed but that does not seem significant for the present application. More importantly, two are non-slatted (F-28 Mk1000 and Fokker 100) and three feature little wing sweep (the Fokkers). The ratio of the drag in ground effect and the drag in free air at the same coefficient of lift according to HOT is
Figure 5.4. Fokker 100 total drag in ground effect, gear down

Figure 5.5. Validation of total drag ratio between in ground effect and free air
compared to the experimental ratio in Figure 5.5. The $C_L$ was kept constant in order not to change the induced drag due to a change in lift as a result of ground effect. It must be stressed that part of the error is due to the parabolic drag format, in which the drag in ground effect is given for some aircraft. In the previous sections it has been derived that the ground effect is a function of higher orders of $C_L$, leading to an error when using $C_L^2$ only.

It is striking that the agreement deteriorates with increasing wing sweep and angle of attack or lift coefficient. The only exceptions to this are the A300B1 that exhibits a very good match in contrast to the other similarly configured aircraft; and the F-28 Mk6000 that is the only slatted Fokker. It is unclear however whether the slat is responsible for the overestimation of the drag reduction or, in other words, the additional real drag. For the other less swept aircraft, which happen to be Fokkers, the agreement is quite good and the error is less than 10%. It should be stressed here that the present tool contains no aircraft-specific data, so this division must have some physical explanation. For the aircraft with large sweep the error amounts up to -30% or some 800 drag counts. Considering this huge error, early flow separation induced by the presence of the ground seems a likely cause. This phenomenon has already been hinted at in [33, pp. 45.14], see Figure 5.6. It was first suspected that

![Separated Flow](image)

(a) High sweep, high drag  
(b) Low sweep, low drag, oge (upper), ige (lower)

Figure 5.6. Variation of drag in ground effect with wing sweep [33, pp. 45.14]

the drag increase is caused by the blocking effect of the trailing edge that approaches the ground faster with increasing sweep as the pitch angle increases. The resulting reduction in massflow under the wing would cause upwash at the leading edge of
the wing or flap, inducing separation. But that would lead to loss in lift as well, which is not noticeable and renders this view questionable. It is also remarkable that early issues of the B747-100 aerodynamic database contained values quite close to ESDU, whilst later versions showed increases up to 800 drag counts. This suggests that Boeing was taken by surprise as well. The final conclusion is that no satisfactory explanation has been found for this drag increase. This puts to end our quest for improved drag in ground effect estimation. A positive note is that the drag increase occurs at higher angles of attack and thus not during the longest part of the take-off run. It is expected that the effect on the $V_{MU}$ during the unstick manoeuvre is very small.

Also striking is that the Boeing 747-100 and Airbus A310-200 have a slight drag reduction in the take-off configuration and at low coefficients of lift, whereas the A320 and A330-300 show a slight increase (top right hand corner in Figure 5.5). This seeming contradiction is at least partially due to the drag data extrapolation to the same values of $C_L$ for free air and in ground effect. The method of Least Squares has been applied to fit curves of $C_L^2$ to the drag data in ground effect. The error is indeed in the order of the deviations observed in this corner of the plot.

Detailed analysis revealed that the order of magnitude of the change in aircraft drag according to (5.10) and (5.11) is $-0.1 C_L D_0 - 0.5 \delta C_L^2$.

Another source for error might lie in the tail contribution to trim drag. Unfortunately it was not known for most of the aircraft whether the experimental drag data was tail-on or tail-off. It is expected that for most cases it was tail-on (at least for the flight test data), whereas the calculations are tail-off. However, it was found in section 5.5 that the maximum error that comes from this omission is 50 counts at most.

Finally HOT is compared to the ESDU and Torenbeek methods in terms of relative error in drag in ground effect in Figure 5.7. These errors have been determined by establishing the error with respect to the manufacturer’s data and dividing this error by that value. All three methods show the same trends, that is the mean error is mostly negative (too low drag) and increases with $C_L$. For flaps up HOT has a slightly better overall mean error, although the Torenbeek method performs better in the low $C_L$ regime. As far as the standard deviation is concerned there is not much to choose as the methods perform equally well.

For the full flaps configuration HOT and the Torenbeek method start with the same mean error, superior to ESDU, but for higher $C_L$ values the Torenbeek method is better. However, HOT is better in terms of the deviation. The overall conclusion is that HOT is an improvement in some, but not all areas of the drag curve in ground effect with respect to the ESDU and Torenbeek methods. This outcome is somewhat discouraging, considering the amount of detail in HOT compared to the ESDU and Torenbeek methods. The logical explanation is that because the latter methods do not discriminate flap span, they underestimate the induced velocities caused by flap lift. This drag reduction underestimation compares favorably with the real drag increase due to flow separation. Indeed, when the higher-swept aircraft are left out of the comparison, HOT improves significantly compared to the other methods. Still that is little comfort. It must be remarked that the added complexity in HOT compared to the ESDU and Torenbeek methods is not noticeable to the user as it requires very
few additional input data.

The effect of the drag underestimation by HOT is that aircraft reach their lift-off speeds earlier during the dynamic simulations of the unstick run. While this does not affect the minimum unstick speed itself, it might underestimate the required take-off distance which is also part of the certification. This will be discussed in the next chapter.

5.7 Conclusions

Most existing methods consider only the wing for drag purposes or apply the reduced dynamic pressure and upwash at the wing’s quarter-chord point to the drag of the entire aircraft. By contrast the present lifting line method integrates the local dynamic pressure and upwash as determined by lifting line theory along the wing, fuselage and undercarriage for the drag determination. Also, the tool takes into account higher order terms, sweep, dihedral and partial-span flaps. It was found that profile drag change in ground effect is primarily dependent upon \( C_L \) and induced drag change upon \( C_L^2 \). Therefore their contributions are more difficult to split in the drag analysis and a parabolic approximation is less appropriate over the entire range of lift coefficients than in free air. The order of magnitude of the change in aircraft drag is \(-0.1 C_L \, C_{Dh} - 0.5 \beta C_L^2\).

The method has been validated extensively against flight test data and compared to the ESDU and Torenbeek methods. Application to 8 transport aircraft (6 slatted, of which 5 with moderately to highly swept wings) showed a reasonable estimation for
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the clean configuration but an underestimation with increasing sweep and $C_L$. The error is less than 10% for the non-slatted Fokker aircraft with little sweep but increases with increasing sweep and $C_L$ to up to 30% or 800 drag counts. No satisfactory explanation has been found for this drag increase. This effect probably increases the mean error and deviation.

HOT is an improvement over the ESDU and Torenbeek methods in most, but not all areas of the drag curve in ground effect. While HOT is much more complicated, it achieves a higher accuracy without having to revert to empirical factors. It is also concluded that the assumption in DATCOM to neglect the zero-lift reduction is rather crude.
Chapter 6

Minimum unstick speed

6.1 Regulations and test sequence

Now that the aerodynamics in ground effect have been determined we have finally set the stage for dealing with the minimum unstick manoeuvre itself. The $V_{MU}$ certification flight test requires a comprehensive set of test runs. The outcome of some will require the execution or cancellation of others, depending on the type of $V_{MU}$ limitation. The $V_{MU}$ may be governed by

1. a geometrical limitation when the rear fuselage hits the ground
2. wing stall
3. the elevator power available

For each type of limitation the FAA and JAA prescribe a specific margin to be used for the relation between $V_{MU}$ and $V_R$. Figure 6.1 shows a typical example.

The process starts with a certain $V_R$ as determined from a normal take-off rotation technique with an engine failure at $V_R$. The $V_R$ must be chosen such that the aircraft arrives at 35 ft. screenheight with a speed of at least $1.2V_{S_{min}}$ or $1.13V_{S_{gl}}$. This is indicated by steps no.1 and 2 in Figure 6.1(a). The lift-off speed attained during this take-off is indicated by step 3. The next step is to perform take-offs with a different rotation technique, that is with a maximum rate of rotation. In this way the lift-off speed will be lower and is called the minimum lift-off speed, $V_{LOF_{min}}$, indicated as step 4. Subsequently the actual minimum unstick test is performed, step 5. For the sake of safety this $V_{MU}$ is increased by 4% in the OE1 case, step 6 and 8% in the AEO case. Only the OE1 case is shown in the figure to prevent the figure from becoming cluttered. It is the only case mentioned from now on. The yardstick is that the $V_{LOF_{min}}$ may not be less than the 1.04 or 1.08$V_{MU}$. If it is less, the $V_R$ has to be increased to the extent that $1.04V_{MU}$ equals $V_{LOF_{min}}$. This minimum value $V_{R_{min}}$ can be achieved by subtracting the speed increment between $V_R$ and $V_{LOF_{min}}$ obtained with the maximum rate of rotation from $1.04V_{MU}$, step 7. Figure 6.1(b) shows the consequences: with a higher $V_R$ (step 8) not only the $V_{LOF_{min}}$ increases to the value of $1.04V_{MU}$ (step 9) but in addition the $V_{LOF}$ after a normal rate of rotation will go...
Figure 6.1. Relations between various take-off reference speeds (one engine inoperative)
up as well (step 9), as will the $V_{2_{min}}$ (step 10). In the present example the inevitable result is that the take-off field length increases, underscoring the relevance of $V_{MU}$ for an aircraft’s performance and thus economy.

Three types of tests may have to be performed:

1. the attitude governed test is mandatory to establish the maximum geometric angle of attack and thus lift coefficient.

2. the geometry limit proof test is required if it appears that the $V_{MU}$ is limited by the aircraft’s geometry. If the aircraft passes this test the JAA allows to apply margins of 8\% between the $V_{MU}$ and $V_{LOF_{min}}$ in the AEO case. If the aircraft fails the criteria 10\% has to be applied instead of 8\% in the AEO case en 5\% instead of 4\% in the OEI case. Here is a difference between the JAA and the FAA: the latter does not allow the 4\% instead of 8\% in the OEI case. These smaller margins are allowed because the geometry limit safeguards the aircraft against over-rotation. During this test at maximum take-off weight and low $T/W$ ratio, the tail must be in contact with the ground for at least 50\% of the time from a speed 96\% of the lift-off speed until lift-off [19]. This requirement ensures that the tail does not only accidently bounce against the ground but is in firm contact with the ground. In addition, the aircraft’s attitude at 96\% $V_{LOF}$ must be within 5\% of the tail-dragging attitude to the point of lift off. There are more rules [19] for being allowed to use the smaller margins, such as a take-off field length no more than 105\% of the scheduled one, but these are not incorporated in the tool yet because that would require an estimate of the $V_R$ as accurate as the $V_{MU}$. That work is under way but not finished yet, as will be discussed later on. When one of the requirements within the simulation is not met, a flag is raised and the simulation is repeated with the adapted speed margins.

3. the elevator power limit assurance test is necessary when the elevator yields insufficient control power to push the tail against the ground before the aircraft lifts off or the wing stalls. This is more likely to happen for aircraft without slats because of the larger pitching moment caused by the flaps for a given lift coefficient. Furthermore, non-slatted aircraft do not need to rotate as far as slatted aircraft because the slats delay the stall to higher angles of attack. In this case a third test, the elevator power limit assurance test, must be performed to check that no problems will arise in daily operations due to the low elevator control power. In case the slow rotation causes the $V_{MU}$ to be less than 5 Kts below the lift-off speed after a take-off with a normal pitch rate, the $V_R$ must be increased until this 5 Kts difference is attained according to the FAA [19]. However, the JAA has a different ruling: they stipulate the 5 Kts margin shall be applied to the nosewheel lift-off speed after an early rotation and compared to the value following a scheduled rotation.

All these scenarios have been implemented in our tool HOT. The geometry and elevator power limited cases can be simulated readily. But the stall in ground effect is much more difficult as the ground effect tends to alter the chordwise as well as spanwise pressure distribution, thereby drastically changing the flow separation pattern.
on the wing. And it is already extremely difficult to estimate the stall limit in free air. Therefore the stall in ground effect is treated rather simply. From several flight test data series the maximum angle of attack is found to be about 2° smaller than the free air value and that is used in the present tool. What is left is a warning not to expect a high accuracy of this tool when the $V_{MU}$ is found to be stall limited. However, as most aircraft today employ slats and will therefore be either geometry or elevator power limited, this will usually not be applicable. The decision tree required for the determination of the type of $V_{MU}$ limit has been implemented in the tool (Figure 6.2).

**Figure 6.2. Decision tree for type of $V_{MU}$ limitation**
6.2 Physical Model

6.2.1 Fokker 100 undercarriage model

One important item to be modelled is the landing gear, as it influences the maximum rotation angle on the ground as it extends during rotation over a height $\Delta z_{\text{muc}}$ from the static to the maximum extension (see Figure 6.3):

$$\theta_{\text{gr, max}} = \theta_{\text{gr, stat}} + \frac{\Delta z_{\text{muc}}}{l_b}$$  \hspace{1cm} (6.1)

![Figure 6.3. Relation of undercarriage geometry and tail-clearance](image)

The change in angle of attack with oleo leg extension is large enough to yield a significant increase in lift coefficient and thus decrease in $V_{\text{MU}}$. Since the vertical equilibrium is statically undetermined, a dynamic simulation is warranted.

It proved to be difficult to find generalized expressions for an undercarriage’s spring- and damping force. Therefore empirical relations were used, such as between the extension $z_{\text{muc}}$ and main undercarriage spring force $N_{\text{muc, spr}}$ for the Fokker 100. These relations have been generalized by modelling the ratio of undercarriage normal force and the aircraft maximum take-off weight against the stroke. The stroke is part of the input and can be adjusted to match other aircraft than the Fokker 100. Hence follows:

$$z_{\text{muc}} = \begin{cases} z_{\text{muc, ext}} - 0.02 - 0.4558 \left( \frac{N_{\text{muc, spr}}}{\text{MTOW}} - 0.2 \right)^{0.319} & \text{if } \frac{N_{\text{muc, spr}}}{\text{MTOW}} \geq 0.2, \\ \frac{N_{\text{muc, spr}}}{\text{MTOW}} 0.02 & \text{if } \frac{N_{\text{muc, spr}}}{\text{MTOW}} < 0.2 \end{cases}$$  \hspace{1cm} (6.2)

This relation has been determined by computing and summing the deflection of the two main legs and four tires under a given load $N_{\text{muc, spr}}$. The top equation represents compressed tyre and main oleo leg and the bottom equation holds for compressed tyre
only. Conversely we find:

\[
N_{\text{muc,pr}} = \begin{cases} 
MTOW \left[ \frac{z_{\text{muc,ext}} - 0.02 - z_{\text{muc}}}{0.4558} \right]^{3.1348} + 0.2 & \text{if } z_{\text{muc}} \geq 0.02, \\
\frac{0.2}{0.02} MTOW & \text{if } z_{\text{muc}} < 0.02.
\end{cases}
\]  

(6.3)

Figure 6.4. The relation between undercarriage normal force and extension for the Fokker 100, both main legs together

This relation is plotted in Figure 6.4. At the upper left hand corner can be seen that initially only the tires compress until the break-out force of the main legs is reached. The maximum stroke \( z_{\text{muc,ext}} \) can be obtained from an analysis of the maximum vertical descent case. For the Fokker 100 it is 0.59 m. The required static \( N_{\text{muc}} \) is found from the pitching moment equilibrium (see Figure 6.5):

\[ N_{\text{muc,pr}} = TOW \frac{x_{cg} - x_{\text{muc}}}{x_{\text{muc}} - x_{\text{muc}}} \]  

(6.4)

The oleo leg damping and friction force are more difficult to model and have been constructed as a function of the stroke rate \( \Delta z_{\text{muc}} \) multiplied by constants with some typical values for each aircraft size. For example for the Fokker 100 we have for both legs together [21]:

\[ N_{\text{muc,damp}} = k_{\text{damp}} \Delta z^2_{\text{muc}} \]  

(6.5)

and for compression

\[ k_{\text{damp}} = 70000[N s^2 m^{-2}] \]  

(6.6)

and for extension

\[ k_{\text{damp}} = -3.6 \cdot 10^6[N s^2 m^{-2}] \]  

(6.7)
For the Fokker 100 internal friction is added as well:

\[ N_{muc_{fric}} = 8000 \varepsilon_{muc} \leq 2000[N] \]  \hspace{1cm} (6.8)

But no generally valid relation has been established. The total main undercarriage force is found from

\[ N_{muc} = N_{muc_{pr}} + N_{muc_{damp}} + N_{muc_{fric}} \]  \hspace{1cm} (6.9)

In addition, similar modules have been added for the Fokker F28 Mk 1000, Mk 4000 and Mk 6000 and for the Airbus A330. It is stressed that no tailoring of the tool HOT has been applied to these undercarriage models in order to improve the results.

### 6.2.2 Propulsion model

The propulsion was covered using look-up tables for a wide variety of engines that enabled interpolation during the take-off run for each value of Mach and altitude. All calculations have been performed at sea level and standard ISA temperature.

### 6.2.3 Equations of motion

The simulation of take-off runs are governed by the next set of equations of motion on a horizontal runway, zero wind. The horizontal and vertical force equilibrium and the pitching moment equation are stipulated using Figure 6.5:

![Figure 6.5. Equilibrium during take-off](image)

\[ a_x = \left[ \frac{T - (C_{D_{xr}} - \mu (C_{L_{xw}} + C_{L_{vvl}}))}{W} \frac{1}{2} \rho V^2 S - \mu \right] g \]  \hspace{1cm} (6.10)

\[ C_{D_{xr}} = C_{D_{a}} + C_{D_{aue}} + (\beta + \beta_{aw}) C_{L_{aw}}^2 \]  \hspace{1cm} (6.11)
Minimum unstick speed

\[
C_{L\text{sr}} = C_{L\text{TO}} + C_{L_h} \frac{S_h}{S} + C_{L_{en\alpha}}
\]

\[
C_{L\text{TO}} = C_{L_{a TO}} (\alpha - \alpha_{0\text{TO}})
\]

\[
C_{L_h} = C_{L_{a h}} \alpha_h + C_{L_{h\delta}} \delta_c
\]

\[
\alpha_h = \alpha + i - \varepsilon_h
\]

\[
C_{L_{en\alpha}} = \frac{T_{max} \sin(\alpha + \bar{i}_{mac} + i_{jet}) - D_{ram} \sin(\varepsilon_{int})}{\frac{1}{2} \rho V^2 S}
\]

\[
a_z = \frac{(C_{L\text{sr}} + C_{L_{en\alpha}}) \frac{1}{2} \rho V^2 S - W + N_{mac} g}{W}
\]

\[
\Delta \dot{z}_{mac} = \frac{h_{cg}}{\cos \alpha} - (\ddot{x}_{mac} - \ddot{x}_{cg}) \bar{\theta} \bar{e}
\]

\[
\Delta \dot{z}_{mac} = \frac{h_{cg}}{\cos \alpha} - (\ddot{x}_{mac} - \ddot{x}_{cg}) \theta \bar{e}
\]

\[
\ddot{\theta} = \frac{\ddot{e}}{I_{yy}} \left\{ \left[ C_{L_{TO}} (\ddot{x}_{cg} - \ddot{x}_{ac}) \cos(\alpha) - C_{L_h} \frac{S_h}{S} \left( \ddot{h}_h \cos(\alpha) + \ddot{h}_h \sin(\alpha) - \ddot{x}_{cg} \cos(\alpha) \right) + C_{m_h} - C_{m_{en\alpha}} \right] \frac{1}{2} \rho V^2 S - N_{mac} \text{arm} \right\}
\]

\[
\text{arm} = (\ddot{I}_{mac} - \ddot{x}_{cg})(\cos \alpha + \mu \sin \alpha) + \ddot{h}_{cg}(\mu \cos \alpha - \sin \alpha)
\]

\[
\alpha = \theta - \frac{w_{cg}}{V}
\]

The inclination angle \( \varepsilon_{int} \) of the ram drag \( D_{ram} \) in (6.16) has already been calculated as intake downwash for the purpose of the lift in ground effect. The inclination \( i_{jet} \) of the gross thrust is the angle over which the exhaust is canted downward relative to the nacelle centerline.

The rotation speed \( V_r \) is difficult to determine as it is the outcome of an iterative procedure for the analysis of the complete take-off, including the motion after lift-off, which is not a part of the present analysis. The aim is to arrive with one engine failed at the screen height of 35 ft at \( V_{e_{nn}} \) by rotating as late as possible in order to reduce the take-off field length. The piloting technique is difficult to model as it is not physically prescribed. Instead of extending the analysis to the air distance, an empirical relation has been derived by the author from an unpublished analysis of a multitude of aircraft of various configurations and sizes. The speed \( V_2 \) is fixed.
because it must be at least 1.2\(V_S\). The ratio \(V_R/V_S\) is related to 1.2 by the excess power \(T/W - D/L\). Because for most aircraft the thrust and lift to drag ratio with one engine inoperative were not available, the flight path angle at the start of the second segment \(\gamma_2\) was used instead, in accordance with:

\[
\gamma_2 = \frac{T}{W} - \frac{D}{L}
\]  

(6.23)

In the plots of \(V_R/V_S\) against \(\gamma_2\) two separate groups or aircraft could be discerned: slatted and non-slatted aircraft. Based on the multitude of lines two mean lines were drawn. For non-slatted aircraft:

\[
\frac{V_R}{V_S} = 1.2 - 0.6745(\gamma_2 + 0.01);
\]  

(6.24)

and for slatted aircraft:

\[
\frac{V_R}{V_S} = 1.16 - 0.875(\gamma_2 + 0.01);
\]  

(6.25)

The \(\gamma_2\) is calculated from the equilibrium out of ground effect with one engine failed, undercarriage up:

\[
\gamma_2 = \frac{T}{W} - \frac{D}{L}
\]  

(6.26)

including the effects of windmilling drag and deflected control surfaces.

### 6.2.4 Numerical simulation

The integration of the accelerations and velocities was tested with several schemes. The simple Euler method with a time interval of 0.01 sec appeared to work fine. The time interval had to be chosen very short due to the high frequency of the undercarriage oscillations. The equations are as follows:

\[
\dot{h}_{c,g_i} = h_{c,g_i} + \dot{a}_{z_i,1} \Delta t
\]  

(6.27)

\[
h_{c,g_i,1} = h_{c,g_i} + \dot{h}_{c,g_i,1} \Delta t
\]  

(6.28)

\[
\dot{\theta}_{i,1} = \dot{\theta}_i + \ddot{\theta}_{i,1} \Delta t
\]  

(6.29)

\[
\theta_{i,1} = \theta_i + \dot{\theta}_{i,1} \Delta t
\]  

(6.30)

\[
\left[ \frac{V}{V_s} \right]_{i+1} = \left[ \frac{V}{V_s} \right]_i + \frac{a_x \Delta t}{V_s}
\]  

(6.31)

The \(x\)-coordinate is not computed because the covered distance is not required. The simulation starts at the moment that the nosewheel can be lifted off the ground with the elevator fully deflected. During real tests the pilots deflect the elevator already
at lower speeds. This nosewheel lift-off speed is obtained from pitching moment equilibrium around the center of gravity:

\[
C_{m_a \alpha} = C_{m_c} + \Delta_{\alpha} C_m + C_{L_{\alpha \tau_0}} (\alpha - \alpha_0)(x_{cg} - x_{ac}) - \frac{N_{muc} \text{arm}}{\frac{1}{2} \rho V^2_{\text{nuelo}}}
\]

\[
C_{L_h} \frac{S_h q_h}{S} \left( l_h - \bar{x}_{cg} \right)
\]

(6.32)

and the vertical force equilibrium

\[
\frac{1}{2} \rho V^2_{\text{nuelo}} S \left[ C_{\tau_0} + \Delta_{\alpha} C_L + C_{L_h} \frac{S_h q_h}{S} \right] + N_{muc} - W = 0
\]

(6.33)

with

\[
W = C_L \frac{1}{2} \rho V^2_{\text{nuelo}} S = C_{L_{\text{max}}} \frac{1}{2} \rho V^2_{\text{S}}
\]

(6.34)

Substitution of (6.32) through (6.34) and rearrangement yields

\[
\frac{V_{\text{nuelo}}}{V_s} = \frac{C_{L_{\text{max}}} \text{arm}}{C_{m_c} + \Delta_{\alpha} C_m + K_1 - K_2 + \Delta_{\alpha} C_L \text{arm}}
\]

(6.35)

with

\[
K_1 = C_{\tau_0} (\bar{x}_{cg} - \bar{x}_{ac} + \text{arm})
\]

(6.36)

\[
K_2 = C_{L_h} \frac{S_h q_h}{S} \left( l_h - \bar{x}_{cg} - \text{arm} \right)
\]

(6.37)

The simulations end at lift-off because the airborne part is outside the framework of our research as its only impact on \( V_{M_{2u}} \) is through the 105% distance requirement. The determination of the moment of lift-off requires special attention. At first sight it seems evident that lift-off is defined by the wheels lifting off the ground. But it is very hard to determine when exactly the tyre and ground surface lose contact, as it depends on a lot of parameters, amongst others the extension rate of the undercarriage.\(^1\) The latter is extremely difficult to model. The next indication of lift-off would be full extension of the undercarriage and hence the normal force becoming equal to 0. It was found that due to numerical limitations of our undercarriage model and the integration scheme, sometimes spikes in the normal force occur that momentarily generate a normal force equal to 0 because of the aircraft’s inertia. As a result, the program assumes lift-off whereas the aircraft is not ready to fly. Moreover there are situations where the undercarriage is not fully extended at lift-off and hence the normal force is not equal to 0. After a lot of trial and error this problem was finally solved by equating lift and weight at lift-off. Starting from the vertical force equilibrium:

\[
ma_z = L + N_{muc} - W
\]

(6.38)

\(^1\)During flight testing often several parameters are monitored, such as the distance of the fuselage lower surface mid-point between the main legs and the ground, tyre speed and oleo leg pressure.
and substituting

$$W - L = 0$$  \hspace{1cm} (6.39)

we get

$$ma_z = N_{mue}$$  \hspace{1cm} (6.40)

The consequence is that if the undercarriage is fully extended at lift-off and hence the normal force equals 0, the vertical acceleration equals zero as well. In reality this does not need to be the case. On the other hand the program now allows for situations with the normal force not being equal to zero at lift-off, i.e. when the undercarriage is not fully extended. This criterium is far from ideal but it appears to work fine. Inspection of the $V_{MU}$ in the detailed HOT output at the moment that $N_{mue}$ equals zero (and skipping the spikes) revealed that the error in $V_{MU}$ associated with this simplification is negligible. It should be noted that (6.39) is also the exact phraseology used in [19].

Another interesting issue is tail-ground contact. It is not unlikely for a geometry-limited aircraft to maintain tail-ground contact after the wheels have lifted off the ground, as can be seen in the front cover photo. This raises the question whether the aircraft has lifted-off or not because not all the weight is supported by lift. Nevertheless wheel-ground contact is usually taken as the yardstick for lift-off.

All the relevant requirements mentioned in section 6.1 have been implemented in HOT. Whenever one of the requirements is not met a flag is raised internally and the margins between $V_{MU}$ and $V_{LOF_{max}}$ are set at the higher values of 5 and 10% instead of 4 and 8%. This has no impact on the $V_{MU}$ itself but only on the associated center of gravity as explained in section 6.4.

### 6.2.5 Stabilizer setting

An important parameter for any take-off with a variable incidence tail is the stabilizer setting. No written airworthiness requirements exist for the stabilizer setting but it is customary to select the stabilizer setting such that the aircraft is trimmed in the climb-out at 1.2$V_s$ with one engine failed. The rationale behind this is that now the pilot does not have to bother about the aircraft attitude and can devote all his attention to handling the engine failure. Alternatively, the all-engine case can be used for determining the trim setting because that is the daily operational condition. The trim setting thus becomes a function of the center of gravity, see Figure 6.6.\(^2\) Furthermore, the take-off rotation characteristics must be observed with the stabilizer at the correct setting. Within the Fokker Advanced Design Department the design rules stipulated an average rate of rotation of 3°/s for the forward center of gravity. For the rearmost location no auto rotation may occur below the stall speed. Because most aircraft are not equipped with a center of gravity indicator, the maximum center of gravity error should be taken into account, i.e.:

- a take-off with the most forward center of gravity but the stabilizer set for the rearmost center of gravity,

\(^2\)Weight and flap setting have a negligible impact on the Fokker 100 trim setting.
a take-off with the rearmost center of gravity but the stabilizer set for the most forward center of gravity

For these cases different rules apply:

- for the forward center of gravity the maximum delay between elevator deflection and perceivable reaction by the aircraft is 2.5 seconds,
- for the rearmost center of gravity the auto rotation at the stalling speed may be countered by maximum aircraft nose down deflection of the elevator.

This set of in-house rules acts as a kind of rubber band to which the stabilizer setting is suspended. First the setting is determined from the climb-out condition and checked for the take-off rotation requirements. If the latter are not met the stabilizer setting can be adjusted to a certain degree. HOT contains dedicated modules to perform these computations. Test pilots should be consulted whether the residual stick-forces during climb-out are acceptable. The range of settings between the ones for the most forward and rearmost center of gravity is called the green band, because any take-off with setting within this range is considered a safe take-off, provided the center of gravity is within the certified forward and aft limit. The HOT program computes the stabilizer settings from the pitching moment equilibrium:

\[ C_m = C_{m_c} + \Delta_{eng} C_m + C_{L_{TO}} (\bar{x}_{rg} - \bar{x}_{ac}) - C_{L_h} \frac{S_{h_l} q_h}{S} = 0 \]  

(6.41)

and the vertical force equilibrium with the flight path angle negligibly small:

\[ \frac{C_{L_{max}}}{(V/V_g)^2} = C_{L_{TO}} + C_{L_h} \frac{S_q q_h}{S} + \Delta_{eng} C_L \]  

(6.42)

\[ \text{Several aircraft such as the Fokker 100 have a so-called take-off warning system, that prevents the throttles being moved forward of a position corresponding to a certain engine RPM with the undercarriage oleo legs compressed and the stabilizer outside the green band.} \]
6.2 Physical Model

Combination of (6.41), (6.42) and (6.14) and rearrangement yields

\[
\delta_c = \frac{1}{C_{L_h}} \left\{ \frac{C_{mae} + \Delta_{eng} C_m + \left[ \frac{C_{lima}}{(V/V_b)} - \Delta_{eng} C_L \right] (\tilde{x}_{cg} - \tilde{x}_{ac})}{\frac{S}{2} (\tilde{I}_h + \tilde{x}_{cg} - \tilde{x}_{ac})} \right\} - C_{L_h \alpha} \alpha_h \tag{6.43}
\]

and

\[
C_{L_{to}} = \left[ \frac{C_{lima}}{(V/V_b)} - \Delta_{eng} C_L \right] \frac{\tilde{I}_h - C_{mae} - \Delta_{eng} C_m}{\tilde{I}_h + \tilde{x}_{cg} - \tilde{x}_{ac}} \tag{6.44}
\]

The angle of attack follows from (6.13) and the tailplane angle of attack \(\alpha_h\) from (6.15). Now the stickforce can be determined for a reversible and boosted longitudinal control system [49] such as for the Fokker 100:

\[
F_e = \frac{G}{B} \left( C_{L_h} \alpha_h + C_{h_\delta_c} \right) \frac{1}{2} \rho \frac{V^2}{q} \frac{q_h}{S_e \bar{e}} \tag{6.45}
\]

The stabilizer setting is varied by an standard zeroing algorithm until the stickforce equals zero, which is the desired trim setting.

The airworthiness requirements allow some additional aircraft nose-up trim to be used for the determination of \(V_{MU}\). If more than 2° additional trim setting is required to obtain the maximum altitude, the additional elevator-power assurance test must be performed in accordance with [19, 10(b)(5)(vii)]. Therefore in our computations the green band plus 2° aircraft nose-up trim is used by default.

During the subsequent computations the stabilizer setting is calculated in this manner unless it is known from certified data. In this light it is noteworthy that the original Fokker 100 green band had to be modified as an outcome of the \(V_{MU}\) flight tests. It surfaced that the elevator capacity was insufficient to raise the nose timely and the rotation speed had to be increased in order to obtain the required \(V_{LOF_{min}}\). This was rectified by adjusting the green band for the forward center of gravity from -3 to -5.5°, which necessitated a new series of test flights. During the Fokker 100 development no resources were available to predict the green band nor the \(V_{MU}\) accurately. It is exactly this kind of occurrences that our present research aims to avoid.

6.2.6 Elevator deflection control law

Analysis of flight test data shows that the elevator deflection is adapted by the pilot to prevent the tail from slamming against the ground, causing damage to the aircraft. Typically, for geometry limited aircraft the elevator is deflected against its stop to raise the nose as early as possible. Then the control yoke is released to a certain extent until the tail has touched the ground, followed by a gentle pull to ensure ground contact. This ground contact might allow the use of smaller margins between the \(V_{MU}\) and the \(V_{LOF_{min}}\). A test pilot will use his experience to determine when and how much to relieve the back pressure on the control column.
In the present tool this is simulated by predicting the time left until tailstrike and estimating the required average pitch deceleration from the moment of reduced deflection to the moment of maximum attitude with a prescribed pitch rate. This prescribed pitch rate is built up of two components:

1. the pitch rate that is due to the extension speed of the undercarriage leg with the tail scraping continuously along the ground.

2. the closure rate towards the ground at the moment of ground contact, which is estimated from flight test data at about 2°/s. This figure is important, as it will determine how hard the tail will contact the ground and how long it will take to do so. That in turn will affect the $V_{MU}$ directly.

The pitch deceleration was chosen as state variable because the elevator deflection angle results in a pitch acceleration and will therefore give the best convergence of the simulation. It is assumed that the reduced elevator deflection will instantly yield a constant pitch deceleration and that the pitch rate will therefore reduce linearly. Then the time interval elapsed until tail strike occurs can be expressed as (Figure 6.7):

\[ \Delta t_{ar} = \frac{\theta_{tar} - \theta}{0.5(\dot{\theta}_{tar} + \dot{\theta})} \geq 0 \]  

(6.46)

where $\theta_{tar}$ is the maximum pitch angle that the aircraft would attain with the undercarriage extended at that moment. The $\dot{\theta}_{tar}$ is found from the pitch rate that the

\[ 0 \leq \dot{\theta}_{tar} \leq \frac{\dot{\theta}}{2} \]
aircraft would have had if it had been scraping the ground with its tail with the actual extension rate of the undercarriage, plus 2°/s. This somewhat cumbersome construction is set up because it enables the evaluation of parameters that are available at any time step:

\[ \dot{\theta}_{tar} = \dot{z}_{muc} \frac{\theta_{gr_{max}} - \theta_{gr_{sat}}}{\Delta z_{muc}} + 2 \]  

(6.47)

The average pitch acceleration follows from

\[ \ddot{\theta}_{ave} = \frac{\dot{\theta}_{tar} - \dot{\theta}}{t_{tar}} \]  

(6.48)

and the corresponding change in elevator deflection until the tail strikes the ground is found from

\[ \Delta \delta_e = k_y (\bar{\theta} - \bar{\theta}_{ave}) \]  

(6.49)

The angles \( \Delta \delta_e \), \( \bar{\theta} \) and \( \theta_{ave} \) have been set in the same units, i.e. degrees. The dimensionless constant \( k_y \) was found by trial and error and was fixed at 0.1. The \( \Delta \delta_e \) is limited to a typical 50°/s deflection rate that stems from operational limits of control rates in general [33]. At the moment of ground contact the actual pitch rate is set equal to the value with the tail scraping along the ground:

\[ \dot{\theta} = \dot{z}_{muc} \frac{\alpha_{muc,\alpha} - \alpha_{s.dat}}{\Delta z_{muc}} \]  

(6.50)

Due to the still existing pitch deceleration (see Figure 6.7) the tail has the tendency to lift off the ground. In order to ensure firm ground contact the change in elevator deflection is then set equal to

\[ \Delta \delta_e = k_y (\bar{\theta} - 1) \]  

(6.51)

with \( k_y \) equal to 1.0. This control law effectively calls for a constant pitch acceleration of 1°/s². This value has been chosen somewhat arbitrarily, but appeared to work fine.

If the tail lifts off the ground despite the increased elevator deflection aircraft nose up the elevator control law is changed into

\[ \Delta \delta_e = k_y (\theta - \theta_{tar}) \]  

(6.52)

where \( k_y \) has been set at 100.

A problem with (6.46) is that negative times can result, when the actual pitch rate is positive and the actual pitch angle larger than the target angle. Or, similarly, when the actual pitch rate is negative and the actual pitch angle smaller than the target angle. In both cases the pitch angle is moving away from its target instead of towards it. What is required is for the elevator to move very fast and in the opposite direction. Hence the time must be of reversed sign and its magnitude decreased. The following control law is devised for cases when initially \( \Delta t_{tar} < 0 \):

\[ \Delta t_{tar} = \frac{0.01}{|t_{tar}|} \]  

(6.53)

In case of a stall limited \( V_{stall} \) a similar approach is followed but then with a constant target pitch equal to the stall angle of attack in free air minus 2° and thus
a target pitch rate of $0^\circ$/s. It proved impossible to use the actual angle of attack as state variable due to the high values of its derivatives. This is due to the fact that the angle of attack contains the vertical speed, see (6.22). As a result, the acceleration of the angle of attack contains the time change of the vertical acceleration that is highly unstable because of the undercarriage normal force oscillations. Therefore the pitch angle was used instead which leads to a very small and acceptable error but much improved numerical stability and accuracy. When the pitch angle is less than $1^\circ$ below the target angle, the control law is changed into:

$$\Delta \delta_c = k_\delta(\ddot{\theta} - \ddot{\theta}_{ave}) + k_\dot{\theta}(\dot{\theta} - \dot{\theta}_{ave}) + k_\theta(\theta - \theta_{tar})$$

(6.54)

with $k_\delta$ equal to 0.5, $k_\dot{\theta}$ equal to 0.1 and $k_\theta$ equal to 1.0.

Some remarks on the approach presented must be made. Equation (6.50) is a departure from the integration of the pitch acceleration into the pitch rate. But otherwise the exact characteristics of the tail bumper and its actuator would be required, as well as the fuselage bending. This actuator is often fitted to the prototype that is destined to perform the $V_{MU}$ certification tests in order to soften the impact when the tail hits the ground and thus prevent tail damage. Inclusion of that actuator with its spring and damping characteristics seems a very demanding but unnecessary refinement, as the end result would be the about same: the fuselage would follow the ground contour. Video footage has confirmed that in most cases the fuselage remains in contact with the ground after first contact. The reverse process, finding the required tail bumper spring and damper characteristics by means of this approach, seemed unviable.

The normal force exerted on the tail bumper has not been taken into consideration. Several attempts have been made to do so, but they resulted in a numerically less stable behavior and the end result would be about the same. Moreover, due to the present elevator deflection control laws the normal force on the tail bumper is only about 1% aircraft weight at maximum. This dwindles down to about zero at lift-off, leaving no improvement in $V_{MU}$ accuracy but a lot of numerical complications. Therefore the tail bumper force was deleted from the simulations.

The set of control laws as outlined in this section will from here on be referred to as Tail Strike Alleviation Control Law (TSACL). The TSACL is activated when the tail is less than 2 seconds away from its target, that is when $\Delta t_{tar}$ is less than 2 seconds. This value appeared to work properly. Once the TSACL is engaged it can not be de-activated anymore. Otherwise oscillatory behavior might occur with the TSACL switching on and off again. As the name says, it does not prevent tailstrikes, as they are intended and can be beneficial for attaining a low $V_{MU}$, but it reduces the impact against the ground. The significance to our present topic is that the altered elevator deflection does affect the $V_{MU}$ due to a different tail lift and due to the different pitch acceleration and hence angle of attack at lift-off.
6.3 Results and Discussion

6.3.1 Detailed results for Fokker aircraft

Figure 6.8 shows an intermediate result of the simulation of an attitude governed test for the Fokker 100. The elevator is assumed to be deflected to its maximum angle before the nose wheel can be lifted off the ground. The angle of attack and speed increase, and thereby lift. As a result the undercarriage extends and the maximum geometric angle of attack increases as well. The influence of the reduction in elevator deflection due to the TSACL coming in is clearly visible: the pitch acceleration and rate are reduced significantly and the tail approaches the ground at a reduced rate of rotation. Note that as the control laws are being evaluated each time step the required pitch acceleration and elevator deflection change continually. This is different from the assumption used in setting up the TSACL, but still it works fine. In this specific example with all engines operating the aircraft reaches lift-off before the tail hits the surface and before wing stall, indicating that it is elevator power-limited. The simulations stops at lift-off, which is hence indicated by the end of the lines in Figure 6.8.

![Graph showing Fokker 100 AEO $V_{MU}$ run pitch behavior](image)

Figure 6.8. Fokker 100 AEO $V_{MU}$ run pitch behavior

Figure 6.9 shows clearly that with one engine inoperative the tailstrike alleviation control law prevents the tail from slamming against the ground. The tail touches the ground softly and scrapes along the ground for some time until lift-off occurs. The higher pitch angle is attained due to the lower speed, caused by the smaller acceleration with a failed engine. It is a balance between the pitch and forward
Figure 6.9. Fokker 100 OEI $V_{MU}$ run pitch behavior

acceleration. This is confirmed by flight testing: with AEO the lift-off angles are smaller than with OEI.

The effect on the vertical acceleration (Figure 6.10) is caused by the reducing undercarriage normal force due to its extension and due to reduced tail lift. The tail lift is reduced because the slower sweeping motion of the tail due to the reduced elevator deflection yields a smaller tail angle of attack. In the first seconds an oscillation in the vertical acceleration can be discerned, that is not well damped and therefore probably not the well damped free-air short-period motion. More likely, it is an undercarriage bounce. However, no analysis to confirm this has been performed of the short period characteristics when excited by the elevator deflection and the undercarriage normal force, nor of the undercarriage itself.

The time history of the Fokker 100 undercarriage normal force and the weight-minus-lift term is shown in Figure 6.11. As long as the vertical acceleration is small, the two are equal, but during the tail-scraping part of the run the aircraft bounces on its main tyres with the legs fully extended (from $t = 6.1$ s, see Figure 6.9). The spikes in the normal force line are a clear indication why the normal force is not a suitable parameter for determining lift-off: it sometimes becomes zero while the weight minus lift is still positive, indicating that the aircraft is still not lifting off. Finally just prior to the calculated lift-off the undercarriage normal force becomes zero as the wheels leave the ground. But due to the calculated downwards acceleration not being taken into account in the lift-off criterium, the weight minus lift equals zero (indicating computed lift-off) only after the wheels have actually lifted off the ground. The associated error in $V_{MU}$ caused by the small time interval is negligible.
Figure 6.10. Fokker 100 AEO $V_{MU}$ run behavior along z-axis

Figure 6.11. Fokker 100 OEI undercarriage normal force during a $V_{MU}$ run
The time history for a geometry limited aircraft, depicted in Figure 6.12, shows that once the TSACL is activated the elevator movement according to (6.49) is initially limited to the maximum rate of $50^\circ$/s (constant slope). After ground-tail contact at 4.59 s as indicated by the discontinuous drop in pitch rate, the required elevator deflection remains large. This is driven by (6.52) to keep the tail against the ground despite the rise in pitch angle due to the extending undercarriage. Once the tail remains firmly established against the ground the elevator deflection is reduced to $-16.4^\circ$ at 4.86 s in accordance with (6.51), again at the maximum rate of $50^\circ$/s (constant slope). Such a time history is characteristic of the ones observed during flight testing and prove the validity of the control laws. Even the actual elevator deflections at lift-off are in good agreement with flight test data. An example is given in this figure by adding the flight test trace for the elevator deflection [32]. The stabilizer setting was set equal to the one used during the flight test. There are some differences between the measured and computed elevator deflections, but the overall trend is in fair agreement. Moreover, the differences between various flight test traces are larger than the differences observed here. Note that the computed peak in elevator deflection due to ground contact is not present in the flight test data, but the final elevator deflection is almost the same. The absence of this peak may be caused by the tail bumper shock absorber that was fitted to the test aircraft. Finally, it should be noticed that the pitch rate is reduced from the peak value of 7.7 to about 3.6 deg/s at tail-ground contact, showing that the TSACL works as intended.

Figure 6.13 proves that the TSACL prevents dynamic overshoot for a stall limited aircraft, in this case the Fokker F28 Mk4000. The control laws that are activated
automatically are (6.49) and (6.54). The direct effect of elevator deflection on pitch acceleration and on $C_L$ is clearly visible. The pitch rate rapidly converges to 0 once the stall angle is reached without any overshoot in $\alpha$.

These simulations can be called upon within the automated horizontal tail sizing tool HOT for a given center of gravity and various weights, flaps settings and for all engines-operating and one engine-out conditions. Such a result is depicted in Figure 6.14 together with the $V_{MU}$ as calculated from certified $C_{L_{max}}$ data for the Fokker F-28 Mk6000. The overall agreement is good, although the simulation does not show the trend that $V_{MU}/V_S$ has a maximum value for an intermediate flap angle. The reducing $V_{MU}$ with increasing $T/W$, caused by the increase in the vertical thrust vector component, is captured well. The gap between the left hand and right hand lines is caused by the change in thrust due to engine failure and the attainable operational take-off weight range.

The slight irregularities in the simulations are due to small movements of the elevator, that have a direct impact on lift. These are the result of numerical instabilities as can be observed in the previous figures. These irregularities are inherent to numerical simulations and cannot be avoided but are small enough to be acceptable. By contrast, the flight test data are linear because these are computed on basis of a $C_{L_{max}}$. That $C_{L_{max}}$ is determined by interpolation of the lift curve in ground effect for an averaged maximum pitch angle. This data reduction denies any difference between pitch angle and angle of attack and nullifies any vertical acceleration. These effects have been taken into account in the simulations which might on one hand increase the accuracy but on the other hand stimulates numerical instabilities. For the simu-
lated aircraft the effects of the difference between $\alpha$ and $\theta$ on $V_{MU}$ itself is negligible. The maximum value of $\gamma$ encountered in the simulations is less than 0.25°. If $\alpha$ is set equal to $\theta$ the simulation results become more stable and the computed lines are much more linear than now. Therefore the wisdom of inclusion of said effects is questionable. However, some people involved in the flight tests of the Airbus A330 maintain that the vertical acceleration absorbed so much lift that it effectively increased $V_{MU}$ by 5 Kts. It was one of the incentives to incorporate these effects in the present tool. But such a large impact on $V_{MU}$ has not been confirmed by our computations.

All of these simulations have been fully automated and are very fast, lasting only several seconds on a present state of the art personal computer. This makes the tool especially suited for preliminary design purposes as well as for in-depth stability and control and performance analysis.

6.3.2 End results for a collection of aircraft

$V_{MU}$ calculations have been performed for 6 aircraft in total: the aforementioned 3 Fokker aircraft with the addition of the Fokker F-28 Mk1000, the DC9-30 and Airbus A330-300. The undercarriage data and engine tables for the Fokker aircraft were available, but for the DC9-30 only the free air lift curve was known. Therefore its undercarriage and engine characteristics were estimated by scaling the Fokker 100 data. Airbus UK kindly provided aerodynamic, propulsion and undercarriage data for the A330-300. The special A330-300 undercarriage required a considerable modelling effort [60]. The engine model was created by scaling available engine tables for engines
of similar bypass ratio.

The results are depicted in Figure 6.15. In general the error is less than $5\%$ and for 3 out of 6 even less then $2.5\%$. Although this is a highly satisfactory achievement in view of the aforementioned lack of data, this number should be valued cautiously. If this accuracy would apply to a preliminary weight estimate, it would be amazing. But performance engineers are generally interested in an accuracy of tenths of a $K_t$ and it is known beforehand that the $V_{MU}$ will be somewhere between 1.05 and 1.2 $V_{S_{min}}$. So the $2.5\%$ should be weighed against a maximum error of about $15\%$. The second achievement is that the method correctly designates the type of $V_{MU}$ limitation: the F28 Mk1000 and 4000 are stall limited, the slatted Mk6000, DC9-30 and A330-300 are geometry limited and the stretched Fokker 100 is partially geometry, partially elevator power limited.

![Figure 6.15. Validation of $V_{MU}$ for various aircraft](image)

Some discussion of the results is required.

- One of the aircraft showing a larger error, the F-28 Mk1000, displays a peculiar behavior in its test data analysis [57]: the angle of attack is consistently $1.5^\circ$ larger than the pitch angle. That implies a constant flight path angle of $-1.5^\circ$, i.e. the aircraft is descending. Although not explicitly discussed, it appears from [57] that this might indicate an instrument error. Also it is indicated that the stall prohibits an accurate determination of the $V_{MU}$. Besides, the Mk1000 was Fokker’s first F-28 model and the certification procedures followed and equipment used were less refined than for later models. For instance, the T/W contribution was not properly included. The lower experimental $V_{MU}$
indicates additional lift compared to our simulations. An effort was undertaken to explain this difference in $C_{L_{\text{max}}}$ by comparing it with the difference found in chapter 4 but no correlation was found (which is also true for all other aircraft). It remains unclear what the cause is for the low experimental $V_{MU}$. Some comfort is given by the much better agreement for the Mk4000 that is also a stall limited aircraft.

- The other aircraft with a larger error is the DC9-30. The odd observation here is that the lines for varying T/W are far from parallel to the lines for 0% percentage error. That implies that there is a factor contributing to lift other than the T/W. The tail bumper is specifically mentioned to be fully extended, with the related $\alpha$ given, which is also true for the strut and tire compression as explanation. The only remaining source for error would then be a different relation between elevator deflection and T/W during flight testing than according to our simulations. However, there is no evident cause for that. The other error involved, the absolute value of $V_{MU}$ by contrast to the slope, may be caused by errors in the lift curve in free air, as several versions have been found that could not be reconciled. Again, the tool does not incorporate any empirical or aircraft-specific data, apart from the undercarriage characteristics. In other words: HOT has not been tailored to improve the results for Fokker aircraft. The fact that the DC9-30 exhibits a larger error than most of the Fokker aircraft is purely coincidental.

6.4 Horizontal tail sizing

The attained accuracy indicates that the tool is well suited for horizontal tail sizing in the preliminary design phase as will be illustrated here. The center of gravity limit pertaining to $V_{MU}$ can be determined by computing the $V_{MU}$ and the minimum lift-off speed $V_{\text{LOF}_{\text{min}}}$ following a rotation with a high pitch rate for various center of gravity locations. The center of gravity is iterated until the intersection is found using a standard zeroing algorithm. The kink in the $V_{MU}$ line in Figure 6.16 is the result of the geometry or stall limit switching into an elevator power limit for forward center of gravity. The increase of $V_{\text{LOF}_{\text{min}}}$ is the consequence of the reduced rate of rotation with center of gravity moving forward. The intersection of the two lines yields the forward center of gravity limit. In this way the most forward center of gravity is found that satisfies the requirements and the center of gravity range is maximized. This range must be attached to the right hand side of the two vertical dotted lines. Several situations are possible, as depicted in the figure. A problem with the right hand side intersection is that it may lead to extremely aft center of gravity positions. In addition, for more rearward center of gravity positions the $V_{\text{LOF}_{\text{min}}}$ becomes less than 1.04 (or 1.08) $V_{MU}$. But the airworthiness requirements only demand tests for the most forward center of gravity, thereby bypassing this problem. Moreover, both solutions of the center of gravity iterations are theoretically feasible but that only occurs in a schematized figure as this one, where secondary effects have been omitted. During simulations these problems have not been encountered.

The center of gravity computation discerns four scenarios:
Figure 6.16. Center of gravity variations and $V_{MU}$ limitations

1. The tail strikes the ground and the aircraft passes the geometry-limit proof test. Hence the reduced margins between $V_{LOF_{min}}$ and $V_{MU}$ are used. The cg is iterated until $V_{LOF_{min}}/V_{MU} = 1.04$ for OEI or 1.08 for AEO. The aircraft is labelled to be geometry limited with respect to $V_{MU}$.

2. The tail strikes the ground but the aircraft fails the geometry-limit proof test. Hence the reduced margins between $V_{LOF_{min}}$ and $V_{MU}$ are not allowed. The cg is iterated until $V_{LOF_{min}} = 1.05V_{MU}$ for OEI or 1.10 for AEO. The aircraft is geometry limited in a physical sense but not according to the airworthiness regulations.

3. When the aircraft is not labelled geometry limited it has to do the elevator power limit assurance test. If the margin between $V_{MU}$ and $V_{LOF}$ after a normal rotation is less than 5 Kts for the same cg that was found from the previous test, the cg is re-iterated until this margin becomes 5 Kts.

4. Whenever the stall limit is surpassed during the previous tests, the cg is re-iterated until $\alpha$ equals $\alpha_{max}$.

The variation of the center of gravity with horizontal tail size as calculated with the fully operational HOT program is shown in Figure 6.17. This figure shows the usual requirements, with the addition of the newly developed $V_{MU}$ requirement (thick line no.8). This at first sight simple addition exemplifies the significance of the research presented. It is evident that the horizontal tail area of 21.6 $m^2$ of the Fokker 100 is neatly designed to accommodate a take-off center of gravity range from 7 to 35% MAC (in-flight 5 to 37% MAC not shown for clarity). The computed center of gravity limit from the $V_{MU}$ requirement is within 1% of the actual forward take-off limit. Although this accuracy is reassuring, more applications to other aircraft have to be carried out.
to confirm the accuracy of HOT for tail sizing purposes. The $V_{MU}$ limit coincides with
other limitations and is not overly restricting the center of gravity. This is exactly
what our research is intending to contribute: to predict the $V_{MU}$ performance and
incorporate it into the tail sizing process instead of waiting until the flight tests show
whether the aircraft's take-off performance is compromised by the $V_{MU}$ limit.

The kink at 30 m² reveals that for larger tail areas the $V_{MU}$ becomes elevator power
limited, as the center of gravity lags behind where it might have been without this
limitation. This sounds paradoxical, but is simply due to the fact that with increasing
tail size the center of gravity moves forward. As a consequence, the aircraft rotates
slower and the tail does not touch the ground before lift-off.

## 6.5 Field performance

An extension of this method has been developed parallel to this study to determine the
$V_T$. It uses similar simulations but with different control laws for the rotation and it
includes the airborne phase up to screen height. That scheme yields the take-off field
length (unbalanced) as well, which is a highly valuable asset for the determination of
the aircraft's economic performance. Because the $V_T$ is considered a topic outside the
framework of this dissertation, it has not been included in this thesis but is reported
6.6 Conclusions

separately instead [27, 51]. An example of the agreement with the established Fokker method PROPER is given in Figure 6.18. The similar comparison of the take-off field length is given in Figure 6.19. The maximum errors are less than 5%. They are caused by the point-mass simulation in PROPER opposed to the much more refined 3° of freedom simulation in HOT. As a result the \( V_R \) according to HOT decreases for a large flap deflection configuration at low weight due to the reduced dynamic pressure and hence rotation capability. The landing performance estimation could be developed similarly.

From figures such as Figure 6.17 conclusions can be drawn on the impact of the \( V_{MU} \) requirement on the horizontal tail size. Alternatively, one may establish the effect of tail size on the \( V_{MU} \) and \( V_R \) using the numerical output. Then one can trade the increased wetted area and weight of a larger tail against an improvement in the \( V_{MU} \) using the take-off field length as a decisive parameter. If necessary, the undercarriage type may be changed from a simple straight legged into an articulating bogie, or the shape of the rear fuselage may be adapted accordingly to investigate its effect on the take-off field length. This tool is well suited not only for minimizing tail area but also for optimizing take-off performance. Likewise, it can easily be extended to do the same for landing performance.

6.6 Conclusions

A dynamic simulation of the \( V_{MU} \) run has been embedded in a simulation of the \( V_{MU} \) test sequence. An empirical model for the undercarriage has been set up and the
propulsion has been added by means of engines tables for several engines available. A so-called tail strike alleviation control law has been added to simulate the piloting technique. It contains several control laws, depending on the $V_{MU}$ limitation: geometric, elevator power or wing stall.

Extensive validation for 6 aircraft demonstrates that the tool correctly identifies the type of $V_{MU}$ limit. The $V_{MU}$ itself is estimated well within 5% accuracy which proves it is well suited for preliminary design purposes. The center of gravity limitation from the Fokker 100 tail sizing case is within 1% of the certified forward take-off limit. The simulations have been fully automated and are very fast, enabling not only parameter variations such as tail sizing studies but also detailed analysis and design of the undercarriage and rear fuselage.

The tool also generates downwash in free air and downwash, lift and drag in ground effect to fill aerodynamic databases until more accurate data from CFD, wind tunnel or flight tests become available. In addition, it yields an accurate prediction of unbalanced take-off field length.
Chapter 7

Conclusions

7.1 Downwash in free air

Validation of the present tool HOT with theoretical and experimental downwash data proved the usefulness of the tool for clean configurations and with high-lift devices extended. An mean error of -0.5° and a standard deviation of 0.5° for the clean configuration was attained based on manufacturer's data for 10 aircraft. Similarly the numbers for the landing configuration are 0.25° and 0.5°, based on 9 aircraft. The method appears to be superior to the DATCOM method, especially with flaps deployed. The downwash prediction for the Fokker 100 nacelle intake exhibits an inaccuracy of less than 1° for the clean configuration and better than several degrees for the full flaps case. The effect of vortex sheet roll-up on downwash is much larger with flaps deployed than for the clean configuration due to increased span loading over the flapped part of the wing. It is concluded that the concept of artificial flapspan increase and the interaction between the wing and flap vortex systems is viable and that incorporation of vortex sheet relaxation is a prerequisite for accurate downwash estimation.

7.2 Downwash in ground effect

The mean error in the downwash at the tail on basis of 6 aircraft (2 low-set tail, 4 T-tailed) is -0.5° with an 0.5° standard deviation for the flaps retracted configuration. For the full flaps case this is -0.5° and 1° respectively. The downwash at the Fokker 100 nacelle intake was predicted reasonably, but the downwash gradient was negative as opposite to the NPLS prediction. The distinct difference in downwash curve with lift coefficient between low set- and T-tails is explained by the closure rate of the image trailing vortices towards the tail. Likewise, the downwash in ground effect is particularly difficult to predict because the image vortex sheet amplifies the downwash error associated with the original vortex sheet. The ESDU method performs surprisingly well compared to HOT, especially with flaps fully deflected. The major advantage of HOT is that it also generates the downwash in free air.


7.3 Ground effect on lift and pitching moment

Validation by comparison with experimental data for 10 transport aircraft of various configurations (5 wing- and 5 rear fuselage-mounted engines) showed the method is sufficiently accurate in lift for preliminary design purposes for aircraft with moderate to high aspect ratio wings. An improvement of 0.07 in $\Delta g C_L$ has been determined with respect to ESDU [14] in the mean error for the landing configuration when compared to the experimental data. No improvement however has been obtained in the standard deviation of these errors, indicating that their scatter is caused by phenomena that are not captured by ESDU and the Torenbeek method, nor by the present method. The improvement is not limited to the rear fuselage-mounted engine configurations, indicating the inclusion of the other said effects is instrumental as well. The pitching moment appears to give no conclusive positive results.

7.4 Ground effect on drag

Application to 8 transport aircraft (6 slatted, of which 5 with moderately to highly swept wings) showed a reasonable estimation for the clean configuration but an underestimation with increasing sweep and $C_L$. The error is less than 10\% for the non-slatted Fokker aircraft with little sweep but increases with increasing sweep and $C_L$ to up to 30\% or 800 drag counts. No definite explanation has been found. HOT is an improvement in some, but not all areas of the drag curve in ground effect. While HOT is much more complicated than the ESDU and Torenbeek methods, it achieves a higher accuracy without having to revert to empirical factors. It is also concluded that the assumption in DATCOM to neglect the zero-lift reduction is rather crude.

7.5 Minimum unstick speed and horizontal tail sizing

Extensive validation for 6 aircraft demonstrates that the tool HOT correctly identifies the type of $V_{MU}$ limit. The $V_{MU}$ itself is estimated well within 5\% accuracy which proves it is very well suited for preliminary design purposes. The center of gravity limitation from the Fokker 100 tail sizing computation is within 1\% of the actual forward take-off limit. The simulations have been fully automated and are very fast, enabling not only parameter variations such as tail sizing studies but also detailed analysis and design of the undercarriage and rear fuselage.

The tool HOT also generates downwash in free air and downwash, lift and drag in ground effect to fill aerodynamic databases until more accurate data from CFD, wind tunnel or flight tests become available. Moreover, it has been demonstrated that by adding the airborne flight path simulation it yields an accurate prediction of the take-off performance and can easily be extended to do the same for the landing performance.

A very limited amount of geometrical and aerodynamic data is required for the HOT input file. The program is fully operational. By staying with analytical expressions the method prevents all the complications and computing capacity that are
typical for CFD computations and does not need the ultimate expert to generate reliable results. It should be stressed that no empirical nor aircraft-specific data have been incorporated into the method, with the exception of the undercarriage characteristics of the Fokker 100 and Airbus A330.
Bibliography


Appendix A

Downwash in the Trefftz-plane by conformal mapping

Classical aerodynamic theory has investigated vortex-induced flow extensively [24]. Our main interest concerns the flow behind an elliptically loaded lifting surface in particular. It has been derived in chapter 2 that such a lift distribution yields uniform downwash at the vortex sheet itself in the Trefftz-plane. Being so far downstream from the wing this flow can be regarded as being two-dimensional with the vortex sheet projected as a straight linepiece onto the y,z-plane. The flow around such a linepiece can be modelled using the technique of conformal mapping. This approach has been covered in [24, pp.146-149] for downwash but is repeated here because it is needed for sideward as well. Moreover, the following derivation is more transparent because it is expressed in y,z-coordinates instead of in elliptical coordinates such as in [24].

First a horizontal parallel flow is described by

\[ \chi = vx \]  \hspace{1cm} (A.1)  

with

\[ x = y + iz \]  \hspace{1cm} (A.2)  

This parallel flow in the x-plane (Figure A.1(b)) can be transformed into a horizontal flow past a circle with radius R in the \( \zeta \) plane by the conformal transformation

\[ x = \zeta + \frac{R^2}{\zeta} \]  \hspace{1cm} (A.3)  

yielding

\[ \chi = v\left(\zeta + \frac{R^2}{\zeta}\right) \]  \hspace{1cm} (A.4)  

This flow is visualized in Figure A.1(a). A horizontal linepiece with endpoints (-2R, 0) and (2R, 0) is transformed into the circle with radius R. The next step is to rotate
the flow clockwise over 90 degrees (Figure A.1(d)). The related transformation is:

\[ x = \zeta - \frac{R^2}{\zeta} \]  \hspace{1cm} (A.5)

leading to a vertical flow past a circle with velocity \( w_x \) at some \( x \)-coordinate as depicted in Figure A.1(c):

\[ \chi = i w_x \left( \zeta - \frac{R^2}{\zeta} \right) \]  \hspace{1cm} (A.6)

The final step is to transform this vertical flow past a circle using the first transformation (A.3) into a vertical flow around a straight, horizontal linepiece representing the vortex sheet. This combination is performed by raising (A.3) and (A.6) to the power 2, substitution and taking the root thereof, yielding:

\[ x^2 = \zeta^2 + 2R^2 + \frac{R^4}{\zeta^2} \]

and

\[ \chi^2 = (iw_x)^2 \left( \zeta^2 - 2R^2 + \frac{R^4}{\zeta^2} \right) = (iw_x)^2 \left( x^2 - 4R^2 \right) \]

so

\[ \chi = i w_x \sqrt{x^2 - 4R^2} \]  \hspace{1cm} (A.7)

This flow is shown in Figure A.1(f). Furthermore the wing span \( b \) equals 4\( R \) and we define \( b = 2s \). The complex velocity can now be written as

\[ \tilde{u} = v - iw = \frac{dx}{dx} = i w_x \frac{x}{\sqrt{x^2 - s^2}} \]  \hspace{1cm} (A.8)

Furthermore a vertical parallel flow has to be added in order to set the flow at infinity at standstill and force the linepiece to move downwards with the velocity \( w_x \):

\[ \tilde{u} = iw_x \left( \frac{x}{\sqrt{x^2 - s^2}} - 1 \right) \]  \hspace{1cm} (A.9)

The real vertical speed is extracted by first dividing by \( i \) which rotates the flow clockwise over 90 degrees. Then the complex speed and its complex conjugate must be added and divided by 2:

\[ w = \frac{1}{2} w_x \left\{ \frac{y + iz}{\sqrt{(y + iz)^2 - s^2}} - 1 + \left[ \frac{y - iz}{\sqrt{(y - iz)^2 - s^2}} - 1 \right] \right\} \]  \hspace{1cm} (A.10)

The coordinates \( y \) and \( z \) are divided by \( s \) and called \( \eta \) and \( m \). After extensive manipulation we get:

\[ w_i = w_x \ast \left[ 1 - \frac{(\eta^2 + m^2)^2 - (\eta^2 - m^2) + (\eta^2 + m^2)\sqrt{(\eta^2 + m^2)^2 - 2(\eta^2 - m^2) + 1}}{2 \left( (\eta^2 + m^2)^2 - 2(\eta^2 - m^2) + 1 \right)} \right] \]  \hspace{1cm} (A.11)
Figure A.1. Streamlines around various bodies
Infinitely downstream the downwash is twice the value at the quarter-chord as indicated by (2.39). Multiplying (A.11) by this factor yields the downwash at infinity which is depicted in Figure A.2. For \( m=0 \) we recognize the uniform downwash for lateral positions up to the wing span. For those values for \( \eta \) the root in the nominator may be written as

\[
\sqrt{\eta^4 - 2\eta^2 + 1} = \sqrt{(\eta^2 - 1)^2} = 1 - \eta^2
\]  

(A.12)

which renders the nominator to 0 and the total solution equal to 1. Also, the upwash outboard of the edge of the vortex sheet is clearly visible. For \( \eta=0 \) in (A.11) we obtain after some manipulation:

\[
w_i = w_{i*} \left[ 1 - \sqrt{\frac{m^2}{1 + m^2}} \right]
\]  

(A.13)

which is comparable to (2.13).

Likewise, by subtracting and halving we get the sideward factor:

\[
K_\beta = \frac{v_i}{w_{i*}} =
\]

\[
2 \sqrt{\frac{(\eta^2 + m^2)^2 - (\eta^2 - m^2)^2 - (\eta^2 + m^2)^2 \sqrt{(\eta^2 + m^2)^2 - 2(\eta^2 - m^2) + 1}}{2((\eta^2 + m^2)^2 - 2(\eta^2 - m^2) + 1)}}
\]  

(A.14)

which is shown in Figure A.3. It has a singularity at the edge of the vortex sheet (\( \eta = 1, m = 0 \)), is highest near the vortex sheet and slightly inboard of the tip and reduces above and below it.
In addition we can now determine the amount of upwash induced by an infinitely long fuselage at an angle of attack. The velocity $w_x$ is equal to the velocity perpendicular to the fuselage:

$$w_x = V \sin \alpha$$

(A.15)

The vertical velocity is acquired by taking the derivative of (A.6) to $\zeta$:

$$\tilde{u} = v - iw = \frac{dy}{d\zeta} = iV \sin \alpha \left(1 + \frac{R^2}{\zeta^2}\right)$$

(A.16)

and dividing that by $i$. This rotates the flow into a horizontal one with the implication that we are now dealing effectively with the sideway. The real part can be acquired by summing the additional complex vector to the original and halving the value. The $y$- and $z$-coordinate are divided by the semi-span, in this case the fuselage diameter, and expressed as $\eta$ and $m$, with the fuselage centerline being the origin of the axis:

$$w = \frac{1}{2}V \sin \alpha \left[1 + \frac{1}{(\eta + im)^2} + \left(1 + \frac{1}{(\eta - im)^2}\right)\right]$$

$$= V \sin \alpha \left[\frac{1 + \frac{1}{2}(\eta - im)^2 + (\eta + im)^2}{(\eta + im)^2(\eta - im)^2}\right]$$

$$= V \sin \alpha \left[\frac{1 + \frac{1}{2} \eta^2 - 2i\eta m + i^2 m^2 + \eta^2 + 2i\eta m + i^2 m^2}{[(\eta + im)(\eta - im)]^2}\right]$$

Figure A.3. Induced sideway in the Treftz-plane infinitely downstream of elliptically loaded straight wings
$$w = V \sin \alpha \left[ 1 + \frac{1}{2} \frac{2(\eta^2 - m^2)}{(\eta^2 - \bar{r} m^2)^2} \right] = V \sin \alpha \left[ 1 + \frac{\eta^2 - m^2}{(\eta^2 + m^2)^2} \right]$$ (A.17a)

This equation is used in section 2.8 as formula (2.56). The disturbance, the second term, is shown in Figure A.4, where only the righthand half of the symmetrical plot is depicted for clarity. The fuselage cross-section is represented by a semi-circle in the horizontal plane and the downwash is positive upwards. The peaks along its circumference indicate the maximum downwash at the top and bottom, where the flow is aligned with the fuselage. The upwash is maximum at the widest points, $(\eta, m) = (1,0)$ and $(-1,0)$. The magnitude of these peaks is equal to the velocity of the parallel flow at infinity, in this plot +1 or -1. At 3R from the centerline the effect has almost vanished.

![Figure A.4. Induced velocities due to an infinitely long fuselage at an angle of attack](image_url)
Appendix B

Downwash computation within a wake

The following is an abstract from a study performed under the author’s supervision [8]. The semi-width of the wake is related to the root-chord by an empirical formula [17]:

\[ \zeta_w = \frac{w}{c_r} = 0.55 \sqrt{C_{dp} \frac{x - x_{fe}}{c_r}} \]  \hspace{1cm} (B.1)

and the dynamic pressure at the wake center is given as

\[ \frac{q_{wm}}{q_\infty} = 1 - \eta_{wm} = 1 - 2 \sqrt{\frac{C_{dp}}{(x - x_{fe})/c_r} + \frac{C_{dp}}{(x - x_{fe})/c_r}} \]  \hspace{1cm} (B.2)

while the dynamic pressure above or below the wake center follows from

\[ \frac{q_\infty - q_w}{q_\infty - q_{wm}} = \left[ 1 - \left( \frac{z}{w} \right)^{1.75} \right]^2 = \left[ 1 - \left( \frac{\zeta}{\zeta_w} \right)^{1.75} \right]^2 \]  \hspace{1cm} (B.3)

The root chord has been chosen as reference because the horizontal tail is mainly influenced by the wake from the wing root. The velocity profile is shown in Figure 2.34. The downwash caused by the change in horizontal velocity can be found by the derivative of the flow potential function. But first this function must be adapted to reflect uniform velocity outside the wake. This is achieved by an adaption of an earlier version of (B.1) and (B.2) found in [42]. There (B.1) is replaced by

\[ \frac{w}{c_r} = 0.68 \sqrt{C_{dp}(\xi + 0.15)} \]  \hspace{1cm} (B.4)

Also, (B.2) is replaced by

\[ 1 - \eta_{wm} = 1 - 2.42 \frac{\sqrt{C_{dp}}}{\xi + 0.3} \]  \hspace{1cm} (B.5)
Downwash computation within a wake

where ξ is taken from the trailing edge (by contrast to the previous section):

\[ \xi = \frac{x - x_{te}}{c_r} \]  
(B.6)

The velocity profile in (B.3) is replaced by a function of e in order to attain a continuous function also applicable outside the wake’s boundary (see figure B.1):

\[ \frac{q_{\infty} - q_w}{q_{\infty} - q_{w_m}} = e^{-4.6837 \left[ \frac{\xi}{\xi_w}\right]^{2.6}} \]  
(B.7)

![Figure B.1. Dynamic pressure reduction within wake](image)

Combination of these expressions yields:

\[ \Psi = c_r \int_0^\zeta Vd\zeta = c_r V_\infty \int_0^\zeta \sqrt{\frac{q_w}{q_{\infty}}}d\zeta = c_r V_\infty \int_0^\zeta \sqrt{1 - \frac{q_{\infty} - q_w}{q_{\infty}}}d\zeta \]

\[ = c_r V_\infty \int_0^\zeta \sqrt{1 - \frac{q_{\infty} - q_w}{q_{\infty}} - \frac{q_{w_m} - q_w}{q_{\infty} - q_{w_m}}}d\zeta = c_r V_\infty \int_0^\zeta \sqrt{1 - \eta_{w_m} \frac{q_{\infty} - q_w}{q_{\infty} - q_{w_m}}}d\zeta \]

\[ = c_r V_\infty \int_0^\zeta \sqrt{1 - 2.42 \sqrt{\frac{C_{dz}}{\xi}} e^{-4.6837 \left[ \frac{\xi}{\xi_w}\right]^{2.6}}}d\zeta \]

\[ = c_r V_\infty \int_0^\zeta \sqrt{1 - 2.42 \sqrt{\frac{C_{dz}}{\xi + 0.3}} e^{-12.7663 \left[ \frac{\xi}{\xi_w} \eta_{w_m}\right]^{2.6}}}d\zeta \]  
(B.8)
From this expression we can obtain the downwash angle through the derivative to \(x\):

\[
\Delta_w \epsilon = \frac{1}{V_\infty} \frac{d\phi}{dx} = \int_0^\zeta \frac{2.42 \sqrt{C_{dp}}}{(\xi + 0.3)^{2.6}} \left( 1 - \frac{40.163 C_{dp}^{1.5} k_2 |\xi|}{(\xi + 0.3)(\xi + 0.15)^{1.4}} \right) e^{k_1 d\zeta}
\]

(B.9)

where

\[
k_1 = -12.7663 \left( \frac{|\xi|}{\sqrt{C_{dp} (\xi + 0.15)}} \right)^{2.6}
\]

(B.10)

\[
k_2 = \left[ \frac{|\xi|}{\sqrt{C_{dp} (\xi + 0.15)}} \right]^{1.6}
\]

(B.11)

Note that the chord has disappeared and only relative distances are present in (B.9). Obviously this integral cannot be solved analytically and must be treated numerically for certain values of the drag coefficient, as demonstrated in section 2.10.
Downwash computation within a wake
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Curriculum Vitae

The author was born in Woerden, the Netherlands, on 6 June 1961. In 1979 he obtained his High School diploma (Gymnasium) from the Christelijke Scholengemeenschap in Gouda. He obtained his MSc in 1986 from the Faculty of Aerospace Engineering, Delft University of Technology, in the section A2L Aircraft Design. His thesis was on the topic of canard configurations under the supervision of Prof.dr.ir. E. Torenbeek.

He served in the Royal Netherlands Air Force from 1987 until 1988 at its Headquarters, Department Operational Requirements, section Operational Test and Evaluation. His professional career as an aircraft designer started in 1988 at the Fokker Advanced Design Department, where he specialized in horizontal tail-sizing. After Fokker’s bankruptcy in March 1986 he was unemployed for almost a year, but in 1997 he became an assistant professor in Aircraft and Weapon Systems at the Royal Netherlands Military Academy in Breda. During his two year stay he successfully proposed to prof.dr.ir. E. Torenbeek to use his master thesis and the tail-sizing studies at Fokker as a basis for a doctoral research. In 1998 he returned to his old section at the Faculty of Aerospace Engineering as an assistant professor. He completed his doctoral research from 2001 to 2005.