Application of Powell’s analogy for the prediction of vortex-pairing sound in a low-Mach number jet based on time-resolved planar and tomographic PIV

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This paper describes an experimental investigation by time-resolved planar and tomographic PIV on the sound production mechanism of vortex pairing of a transitional water-jet flow at \( Re=5000 \). The shear layer is characterized by axisymmetric vortex rings which undergo pairing with a varicose mode. Three-dimensional measurements show the presence of longitudinal pairs of counter-rotating vortices inducing vortex azimuthal instabilities prior to the breakdown of the vortices. Based on Powell’s aeroacoustic analogy, flow structures responsible for noise generation are characterized by the second-time-derivative of the Lamb vector field, which is directly evaluated by planar and tomographic PIV. The analysis of the dynamics of such structures shows peak activity in correspondence of the vortex cores during the leapfrogging, vortex-azimuthal instabilities and vortex breakdown mechanism. Under the hypothesis of axisymmetric flow, far field acoustic produced by vortex pairing is predicted by directly applying time-resolved planar PIV data to Powell’s acoustic analogy. Pronounced acoustic emission is found during the coalescence of the vortices.

Introduction

The idea of an involvement of coherent flow structures in the region next to the jet nozzle exit in the sound generation mechanism\(^1\)–\(^3\) brought into question the theory of a random distribution of compact convected quadrupoles producing noise. Michalke & Fuchs\(^4\) analytically showed the presence of vortex-ring like structures that, later, Dahan et al.\(^5\) demonstrated to be the cause of 50% of the far-field sound energy. Also wavy-wall type instabilities happening along the potential core were indentified as mechanism of noise production.\(^6\) Moreover, an empirical modal decomposition (Most Observable Decomposition) which distills the noisy and quiet modes showed that while 12% is related to convected axisymmetric vortex-rings, over 48% of the sound generated in jet at \( M=0.9 \) and \( Re=3,600 \) is due to the breakdown of coherent structures in the transition region.\(^7\)

A direct application of experimental data to aeroacoustic analogy was done by Bridges & Hussain\(^8\) who, on a low-Mach number jet, performed vortex-sound based predictions of vortex-pairing by means of conditional-phased-averaged hot-wire measurements founding a favorable comparison with microphone measurements and proving that, in vortex pairing, asymmetry of vortex motion is an important feature causing noise. A direct approach was also taken in the inspiring work by Schram et al.\(^9,10\) who studied the mechanism of vortex pairing in an incompressible acoustically excited air jet flow applying phase-locked planar PIV data to Moehring’s aeroacoustic analogy.\(^11\) Resulting less sensitive to experimental errors, the same authors proposed a conservative formulation of the vortex-sound theory for axisymmetric vortical flows reporting a vortex-pairing sound prediction in good agreement with both numerical simulations and microphone measurements.

Time-resolved PIV technique has been recently used for noise investigation by Schröder\(^12\) et al. who investigated the time-space correlation between the trailing-edge velocity fluctuations and the far field noise.

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measured by microphones. Sound predictions based on time-resolved PIV were first attained by Haigermoser\textsuperscript{13} who studied cavity noise making use of Curle’s analogy.\textsuperscript{14} Based on a similar approach, the investigation of rod-airfoil noise by Lorenzoni\textsuperscript{15} reported a favorable comparison for the tonal component between acoustic prediction and the microphone measurements.

The mechanism of noise production during vortex pairing has been investigated under simplified assumptions due to the experimental method limited in time and dimension resolution. In the work of Schram et al.,\textsuperscript{9} for example, it was hypothesized axisymmetry of the flow field, so that predictions could be based on planar-PIV measurements. Moreover, it was used acoustical excitation of the jet to obtain spatial and temporal periodicity of the pairing phenomenon. Considering in fact that the available time-resolution of the PIV equipment was insufficient to measure the temporal evolution of the velocity and vorticity field, the periodic behavior made possible the use of phased-locked PIV method. Similarly, by using an acoustically excited jet, Bridges & Hussain\textsuperscript{8} could apply conditional phase average to hot-wire measurements.

In this work, time-resolved PIV technique is applied with kilo-hertz based rate to study the unsteady flow patterns during the pairing of two vortices that are shed by a non-excited water jet at $Re=5000$. The inner profile of the jet is similar to that used by Schram et al.\textsuperscript{9} Flow phenomena most contributing to the production of sound are identified by means of vorticity fields and Powell’s analogy.\textsuperscript{16} The dynamics of the previous quantities are visually compared to investigate the mechanism of noise generation. Furthermore, as the knowledge of the unsteady flow properties together with those three-dimensional may represent a fundamental requirement to identify physical phenomena most contributing to the production of sound, particularly in non axisymmetric flows, time-resolved tomographic PIV\textsuperscript{17} experiments of the jet are performed at high repetition rate on the region of vortex pairing. Noise sources are characterized by Powell’s analogy and their dynamics and three-dimensional features are visually compared to those of large fluid structures.

Finally, to discuss the feasibility of sound-predictions based on the evaluation of Powell’s analogy from time-resolved PIV, for axisymmetric flows, far-field predictions of the sound scattered during vortex pairing are compared to that reported by Schram et al.\textsuperscript{9} in terms of trend and peak values.

Direct numerical simulations of the jet flow at $Re=5000$ have been performed by Moore et al.\textsuperscript{18} to compare the TR-TOMO PIV experiments presented in this study.

### Sound prediction from Powell’s acoustic analogy

Powell’s aeroacoustic analogy is a different formulation of Lighthill’s analogy\textsuperscript{19} that explicitly shows the role of the vorticity in the generation of sound. It is therefore a convenient framework in which to investigate the connection between the hydrodynamic flow properties and the sound emission in vortical flows, such as transitional jet flows. In an incompressible flow, which is the case of low-Mach number flows, Powell’s analogy reads as

$$p'(x, t) = \frac{1}{4\pi\rho_0} \int_V \left[ \frac{1}{|x - y|} \nabla \cdot (\omega \times \mathbf{v}) \right] \, dV(y) + \frac{1}{4\pi\rho_0} \int_V \left[ \frac{1}{|x - y|} \nabla^2 \left( \frac{1}{2}|\mathbf{v}|^2 \right) \right] \, dV(y), \tag{1}$$

where $(\omega \times \mathbf{v})$ is the so-called Lamb vector $L$, $(\frac{1}{2}|\mathbf{v}|^2)$ is the kinetic energy, $t^* = t - \frac{|x - y|}{c_p}$ is the retarded time where $x$ and $y$ are respectively the listener and source position vectors. From (1) it results that the Lamb vector divergence $(\nabla \cdot L)$ and the Laplacian of the kinetic energy $\nabla^2 \left( \frac{1}{2}|\mathbf{v}|^2 \right)$ are the terms contributing to the noise production. However, when the Mach number is very small, under the assumption of compact source, the second term of (1) can be neglected.\textsuperscript{16} In far-field conditions, equation (1) is written\textsuperscript{20} as

$$p'(x, t) = \frac{\rho_0}{4\pi c_0^2 |x|^3} \frac{\partial^2}{\partial t^2} Q \tag{2}$$
where, being the integral a linear operator,

$$\frac{\partial^2}{\partial t^2} Q(y) = \int_V (x \cdot y) x \cdot \frac{\partial^2}{\partial t^2} L \, d^3 y$$  \hspace{1cm} (3)$$

saying that the noise production is related to the second time derivative of the Lamb vector $\frac{\partial^2}{\partial t^2} L$. Under the assumption of axisymmetric flow, $x$, $y$ and $L$ become function of the radial position $r$ and the axial position $z$ and equation 3 may be reset as

$$\frac{\partial^2}{\partial t^2} Q(r, z) = 2\pi \int (x \cdot y) x \cdot \frac{\partial^2}{\partial t^2} L \, dr dz.$$  \hspace{1cm} (4)$$

In the above equations, the physical quantities are normalized with respect to the jet diameter $D$ and the axial velocity at the jet exit $V_{exit}$:

$$x = \frac{x_0}{D}, \quad y = \frac{y_0}{D}, \quad t = \frac{t_0 V_{exit}}{D}$$  \hspace{1cm} (5)$$

$$v = \frac{v_0}{V_{exit}}, \quad \omega = \frac{\omega_0 D}{V_{exit}}$$  \hspace{1cm} (6)$$

Prediction of vortex-pairing sound by Powell's analogy

The prediction of the vortex-pairing sound is done following the approach proposed by Schram et al.\textsuperscript{9} Powell's analogy for axisymmetric flows (equation 2 and 4) is evaluated from time-resolved planar PIV data over a rectangular region that is centered on the vortices. The integration window moves downstream with constant velocity which is an average of the vortex-convection velocities. Its extent, which remains constant in time, is such to include the significant levels of the second time derivative of the Lamb vector

$$\mathcal{L} = \mathcal{L}_{xx} = \mathcal{L}_{yy} = \mathcal{L}_{zz},$$

that correspond to the vortices.

Acoustic predictions are erroneously evaluated in case of spatial-domain truncation of the sound source.\textsuperscript{21} Thus, flat-Hann window function $w_s(t)$ is applied to $\frac{\partial^2}{\partial t^2} L$ field along the axial and the radial directions:

$$w_s(t) = \begin{cases} 
\frac{t}{\alpha W} - \frac{1}{2\pi} \sin \left[ 2\pi t / \alpha W \right] & 0 \leq t \leq \alpha W \\
1 & \alpha W \leq t \leq (1 - \alpha) W \\
(W - t)/\alpha W - \frac{1}{2\pi} \sin \left[ 2\pi (W - t) / \alpha W \right] & (1 - \alpha) W \leq t \leq W \\
0 & \text{elsewhere}
\end{cases}$$

where $W$ is the integration window length and $\alpha W$ is the damping length ($0 \leq \alpha \leq 1$). From the above and equations (2) and (4), it results that the prediction of vortex pairing sound is evaluated by

$$p'(r, z) = \frac{\rho_0}{2c_0^2 |x|^3} \int \int (x \cdot y) x \cdot \frac{\partial^2}{\partial t^2} L w_s(r, z) \, dr dz.$$  \hspace{1cm} (8)$$

When time-resolved three-dimensional velocity fields are concerned, on the other hand, the sound prediction of pairing vortices may be made in more general terms by equation (2) and (3). The approach is an extension to the three-dimensional domain of the one used for the axisymmetric case. The three-dimensional domain, which moves downstream with a constant velocity, contains the significant levels of $< \mathcal{L} >$ related to the vortices. Spurious noise generation due to source truncation is limited by applying the above windowing function along all the three directions of the domain. Thus, the formulation applied for vortex pairing noise prediction reads as:

$$p'(x, t) = \frac{\rho_0}{4\pi c_0^2 |x|^3} \int_V (x \cdot y) x \cdot \frac{\partial^2}{\partial t^2} L w_s(y) \, d^3 y.$$  \hspace{1cm} (9)$$

Time and space derivatives of velocity data are approximated by central difference scheme.
Experimental investigation

Experiments are performed in a water jet facility at the Aerodynamic Laboratories of TU Delft in the Aerospace Engineering Department. A round nozzle of exit diameter $D=10\text{mm}$ and contraction ratio of 56.25, emerges from the bottom wall of the facility, an octagonal water tank (600mm of diameter and 800mm of height) made of Plexiglas allowing full optical access for illumination and tomographic imaging. Nozzle contraction is shaped following the design of Schram et al. The water flow is hydrostatically driven to provide a stabilized supply in a range of exit velocity from 0.1 to 2m/s, corresponding to Reynolds numbers between 1,000 and 20,000 (figure 1a). Planar and tomographic experiments are performed for a nominal axial velocity at the jet exit of 0.5m/s yielding a Reynolds number of $Re=5,000$ based on the jet diameter.

Time-resolved planar PIV

Neutrally buoyant polyamide particles of 10µm of diameter are employed with a concentration of approximately 6 particles/mm$^3$. The illumination is provided by a solid-state diode-pumped Nd:YLF laser. Obtained through knife-edge slit, the laser-light sheet is placed coaxially with the nozzle. Two-CMOS-camera system is used to record the light scattered by the particles. Cameras are equipped with Nikon objectives of
105 mm focal length set at numerical aperture $f_\# = 2.8$ to allow focused imaging of the illuminated particles. The set up configuration is sketched in figure 2a. Sequences of images of tracer particles are recorded at frequency of 1.2kHz yielding to imaged-particle displacement of approximately 7 pixel along the jet axis. The field of view (FOV) of each camera is $4.7\text{mm} \times 4.7\text{mm}$, with a digital resolution of 20.7 pixels/mm. The two FOVs overlap of 2 mm and, when combined, form a measurement domain of $4.7\text{mm} \times 9.2\text{mm}$, as illustrated in figure 2b. The experimental settings are summarized in table 1.

Vector field computation

The image cross-correlation is performed by a Windows Deformation Iterative Multigrid (WIDIM) technique$^{23}$ with a final interrogation window of $17 \times 17$ pixels ($0.8 \times 0.8 \text{mm}^2$) with an overlap factor of 75% leading to a vector pitch of 0.2 mm. The velocity vector field is obtained by averaging the cross-correlation map obtained from three subsequent exposure-pairs within an overall recording time of 2.5 milliseconds. This approach is an extension of the method proposed by Meinhart et al.$^{24}$ for unsteady velocity fields that reduce the precision error without compromising the temporal resolution.$^{25}$ Noisy fluctuations of the velocity vectors are reduced by applying a space-time regression, a second-order polynomial least-square fit over a kernel of 15 points in time and 5$^2$ points in space,$^{26}$ corresponding to time length of 11.7ms and spatial domain of $0.8\times0.8 \text{mm}^2$. The choice of a time kernel of 15 points, which is longer than that used by Scarano & Poelma$^{26}$ for three-dimensional velocity fields, is discussed in the section of results where raw and filtered data are compared.

Time-averaged velocity fields are computed following the approach of Meinhart et al.$^{24}$ with an interrogation window of $6 \times 6$ pixels ($0.29 \times 0.29 \text{mm}^2$) with an overlap factor of 50% leading to a vector pitch of 0.13mm.

Table 1. Experimental settings

<table>
<thead>
<tr>
<th></th>
<th>Planar PIV</th>
<th>TOMO PIV</th>
</tr>
</thead>
<tbody>
<tr>
<td>Seeding material</td>
<td>polyamide particles diameter [$\mu$m]</td>
<td>$\approx 10$</td>
</tr>
<tr>
<td></td>
<td>concentration [particles/mm$^3$]</td>
<td>6</td>
</tr>
<tr>
<td>Illumination</td>
<td>Quantronix Darwin-Duo Nd:YLF laser</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(2 × 25mJ@1 kHz)</td>
<td></td>
</tr>
<tr>
<td>Recording device</td>
<td>Photron Fast CAM SA1 cameras</td>
<td>$\times 2$</td>
</tr>
<tr>
<td></td>
<td>(1024 × 1024 <a href="mailto:pixels@5.4kHz">pixels@5.4kHz</a>)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>20$\mu$m pixel pitch</td>
<td></td>
</tr>
<tr>
<td>Recording method</td>
<td>double frame/single exposure</td>
<td></td>
</tr>
<tr>
<td>Optical arrangement</td>
<td>Nikon objectives ($f: f_#$)</td>
<td>105mm; 2.8</td>
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<tr>
<td></td>
<td>field of view</td>
<td>4.7D×9.2D</td>
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<tr>
<td>Acquisition frequency</td>
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<td>1kHz</td>
</tr>
<tr>
<td>Pulse separation</td>
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<td>1/1000s</td>
</tr>
<tr>
<td>Number of recorded images</td>
<td>4000</td>
<td>500</td>
</tr>
</tbody>
</table>

Time-resolved tomographic PIV

Neutrally buoyant polyamide particles of 56$\mu$m of diameter are employed with a concentration of 0.65 particles/mm$^3$. The illumination is provided by a solid-state diode-pumped Nd:YLF laser. The laser beam diameter of 6 mm diameter is expanded with a Linos beam expander to deliver a beam of 30 mm of diameter coaxial with the nozzle (figure 3a). The light scattered by the particles is recorded by a tomographic system composed of four CMOS cameras arranged along a different azimuthal directions in a horizontal plane. The cameras form a maximum angle of 90 degrees. *Nikon* objectives of 105 mm focal length are set with a numerical aperture $f_\# = 32$ to allow focused imaging of the illuminated particles. For the chosen illumination and imaging configuration the maximum particle image density (approximately 0.04 particles/pixel at the
jet axis) is obtained only on the line of sight intersecting the jet axis and decreases towards the sides. The details of the experimental set up are summarized in table 1.

The choice of a cylindrical domain of illumination results in the enhancement of the accuracy of the particle field as the particle image density does not change with the angle of sight and decreases moving from the axis to the periphery of the jet. Moreover, the tomographic imaging system does not suffer from any increase in the intersection of lines-of-sight with the observation angle (figure 3b).

Sequences of images of tracer particles are recorded at 1kHz resulting in high temporal resolution (imaged-particle displacement of approximately 8 pixel along the jet axis). The field of view is of 50×50 mm with a digital resolution of 21.5 pixels/mm and the measurement domain is 15mm off the nozzle exit and extends for 50mm in the axial direction and 30mm in the radial one (figure 3c).

**Tomographic reconstruction**

The volumetric light intensity reconstruction is performed by LaVision Davis 7.4 which processes the acquired images making use of volume-self-calibration procedure, followed by four iteration of MART algorithm with a diffusion parameter of 0.5 for the first three. The three-dimensional calibration function is corrected by the volume self-calibration to minimize the disparity fields, decreasing in the calibration error from a typical value of 0.5 to 0.05 pixel.

The detection of the region where the light signal is concentrated is performed by an a-posteriori evaluation: an object of x and z dimensions larger than the light beam diameter is reconstructed and, in a cross section of the volume, the particle average peak intensity is evaluated. It is found that the light signal is concentrated in a circular region of approximately 550 voxels of diameter (figure 4a), corresponding to 27.5mm. The tomographic reconstruction is therefore applied within the region defined by the red box whose x and y sizes are increased of 25 voxels to avoid any signal truncation (figure 4a). It results that volumes of approximately 30×30×50 mm³ are discretized with 600×600×1000 voxels applying a pixel to voxel ratio of 1. The resulting voxel pitch is 50 µm. The accuracy of the reconstructed object is improved applying image pre-processing with sliding minimum subtraction over windows of 31×31 pixels. Finally, the peak intensity profile is extracted along both x and y (figure 4b and c) where a signal-to-noise ratio (SNR) above 2 is found within 500 voxels in both directions. Within such a region of signal the velocity measurement is considered reliable.

**3D-Vector field computation**

Three-dimensional particle field motion is computed by Volume Deformation Iterative Multigrid (VODIM) technique with a final interrogation volume of 40×40×40 voxels (2×2×2 mm³) with an overlap between adjacent interrogation boxes of 75%, leading to a vector pitch of 0.5 mm. Velocity vector fields are obtained by averaging the cross-correlation map over three subsequent object-pairs (Meinhart et al., 2000), which

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[Figure 3. Tomographic PIV experiments: a) schematic of the system and b) detail of the illumination and recording system; c) measurement-domain schematic.]

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Figure 4. a) Reconstructed particle peak intensity distribution averaged along $y$-axis (the red box indicates the reconstructed region); normalized particle peak intensity profile along $x$ (b) and $y$ (c) axis, to indicate the signal-to-noise ratio.

totally corresponds to a recording time of 3 milliseconds.

At the given particle concentration, 6.5 particles are counted, on average, within the interrogation box. Data processing is performed in parallel on dual quad-core Intel Xeon processors at 2.83 GHz with 8 GB RAM memory requiring, respectively, 22 min and 70 min for the reconstruction of a pair of objects and 3D cross-correlation.

Noisy fluctuations of the velocity vectors are reduced by applying a space-time regression, a second-order polynomial least-square fit over a kernel of 15 points in time and 5 points in space corresponding to a time length of 14 ms and a volumetric domain of $2 \times 2 \times 2$ mm$^3$. The choice of such a time kernel, which is longer than that used by Scarano & Poelma, is discussed in the section of results where raw and filtered data are compared.

Results

The axial velocity at the jet exit $V_{\text{exit}}$ is 0.41 m/s, meaning of a Reynolds number $Re$ based on the jet diameter of 4176. A total of 100 vortex-shedding cycles are measured by planar PIV. Tomographic acquisition are instead limited to 10 subsequent cycles. On average, vortices are shed with a frequency of 30 Hz leading to a Strouhal number of 0.735 based on the nozzle diameter $D$ and $V_{\text{exit}}$.

A visual example of the spatial resolution of planar PIV with respect to shed vortices is given in figure 5a which shows a detail of an instantaneous of vorticity field. The leading vortex of size $\Delta y=0.46$ and $\Delta x=0.3$, is respectively sampled by 23 and 15 velocity vectors. Spatial resolution, in contrast, decreases by a factor 2.5 in tomographic fields as illustrated in figure 5b.

Jet characterization

The flow regime at the exit of the nozzle is laminar (figure 8a and b) and is wrapped by a shear layer whose turbulent kinetic content becomes 50% of the peak values at $y=1.5$ (figure 6a) where axisymmetric Kelvin-Helmholtz instabilities are detected. From plot of velocity statistics (figure 6a and b) it is observed that the shear layer is responsible for the thinning of region of large axial velocity. This is result of flow entrained by Kelvin-Helmholtz instabilities of the shear layer both from the jet column and the quite surrounding.

The profile of mean axial velocity and turbulent intensity $TI$ along the cross-section of the jet exit is sampled by a total 77 vectors along the jet diameter (figure 7a and b). It is extracted 0.1 jet diameter off the nozzle to avoid noisy vectors resulting from cross-correlation of intensity levels affected by the nozzle. Trends and peak values are in agreement to those obtained by Liepmann & Gharib. Momentum thickness at $y=0.1$ is 0.018 (figure 7).
Vortex pairing dynamics

Between 1 and 3 jet diameters off the nozzle the flow exhibits Kelvin-Helmholtz instabilities which feature vortex-ring like topology, as shown by vorticity contours in figure 8b and, in figure 13, by the three-dimensional visualization with isosurfaces of $\lambda_2$-criterion.\(^{31}\)

As illustrated in figure 8a between $y=2.5$ and $y=3$, the flow field within the vortex ring features axial velocity ($V=1.2$) larger than the region between the vortices ($2<y<2.5$), the so-called braid region,\(^{30}\) where the velocity is $V=0.7$. Such a flow field is characteristic of a pulsatile motion which results from the shedding of coaxial vortices (figure 8a, $1<y<2$).

In figure 12a, a sequence of instantaneous vorticity contours show the dynamics of vortex "1" and "2", whose cores are tracked based on the maximum of $\lambda_2$-criterion.\(^{32}\) The axial evolution of the cores along the jet axis is illustrated in figure 9. Two cores are detected until $t=30$ ms (figure 12a), that is the last instant at which they are sufficiently far apart to be distinguished by the $\lambda_2$ method. Furthermore, from the plot of axial core evolution, vortex convection velocity along the jet axis can be inferred. As illustrated in figure (8a), the pursuer vortex ("2"), which, during the motion, contracts within the region of laminar flow, is convected along the jet axis with a speed of 0.487 reaching and later merging with the fore one ("1"). This, in the meanwhile, widening in core radius is convected with a lower velocity 0.426. Such a phenomenon, referred to as vortex pairing,\(^{33}\) typically takes place between $y=2.5$ and 3.5 and leads to the formation of vortex "3" (figure 9 and 12a). This is convected with a velocity of 0.46.

At $Re=5000$ vortex pairing is mainly driven by the so-called varicose mode,\(^{34}\) during the motion vortex planes stay parallel to that of the jet nozzle (figure 8b). The frequency of pairing is 14.5 Hz, half of that of shedding, yielding a Strouhal number of 0.35 based on exit velocity and nozzle diameter.

It is observed that, prior to pairing, vortices have axisymmetric coherence, as deducible from planar and tomographic velocity fields (figure 8b and 13a between $t=0$ ms and 15 ms). After pairing, instead, azimuthal instabilities grows along the ring-like structure with characteristic wavelength of $1D_v$, where $D_v$ is the local vortex diameter (see "$\lambda$" in figure 13a, $t=60$ ms).

Consequently to the pulsatile motion, the axial acceleration yields secondary coherent structures characterized by a streamwise pattern "s", as illustrated in figure 13a between $y=4$ and 6. Streamwise vortices are organized in counter-rotating pairs which entrain the flow towards the jet axis with an angle of approx-
Figure 6. Planar PIV measurements: contour of mean-axial velocity and b) turbulence kinetic energy TKE.

Figure 7. Planar PIV measurements. Cross-sectional profile of a) mean-axial velocity and b) turbulence intensity at jet exit $y = 0.1$; c) momentum thickness along jet axis.

Imately 45 deg. They are already detected at $z=3$ (figure 10) where they are responsible for the rise of azimuthal instabilities the vortex rings.\textsuperscript{30} Between $t=60$ms and $t=90$ms, the amplitude of the instabilities grows of a factor of 2 (figure 13a) leading to the breakdown of the vortex ring. In general terms, this mechanism, referred to as collapse of the potential core, reduces the axisymmetric coherence of the ring until the breakdown into smaller structures.\textsuperscript{35}
Figure 8. Planar PIV measurements: a) contours of velocity axial component; b) detail: contours of vorticity (vectors are shown every 3 and 2 along y and x respectively and are plotted in a convective frame of velocity $V_{conv}=0.47$).

Figure 9. Evolution of vortex cores during paring.

Figure 10. Instantaneous of axial-vorticity contours on plane $z=3$ extracted from TOMO-PIV data.
Effect of cross-correlation averaging and time filtering

The averaging effect of cross-correlation averaging over 3 subsequent exposure-pairs on the velocity field is here discussed. Consider that at $Re=5000$ the characteristic convection velocity of shed vortices ranges between 0.42 and 0.48 corresponding in physical units to a range between 0.172m/s and 0.2m/s. In the case of planar PIV data, the vortex position results averaged over displacement of 0.5mm at maximum, which is however smaller than the size of the interrogation window (0.8mm). Therefore, as the size of the pairing vortices at $Re=5000$ is typically 5 times larger than that of the interrogation window (figure 5a), the dynamics of the vortices is not biased by the cross-correlation average. Similar conclusion may be withdrawn for tomographic velocity data: the cross-correlation average is done over a maximum displacement of 0.6mm, which is shorter than the interrogation box size (2mm), which is in fact half of the vortex size (figure 5b).

For the further use to evaluate the second time derivative of the Lamb vector (see equation 8 and 9), space-time regression is applied to PIV data (see section 2) with a kernel of 15 points in time. The time history of the raw and filtered axial velocity at point $P(x=0.4; y=1.5)$ and $Q(x=0.4; y=0; z=1.5)$, which are representative for the shear layer respectively for the planar and tomographic case (figure 11). Respectively, the applied time filtering corresponds to 35% and 42% of the shedding period $T$ of 33.3ms. In case of planar data the standard deviation of the residual computed between the raw and filtered velocity is 0.025 pixel, which is, however, smaller than the PIV accuracy (0.1 px, Willert & Gharib). At point $P$, where the mean axial velocity is 5.5 pixels, the filtered velocity is therefore affected by a relative precision error of 2%, similarly to that of raw data. On the other hand, in case of tomographic measurements, the standard deviation of the velocity residual at point $Q$ rises to 0.2 voxels. This, being larger than the tomographic PIV precision error (0.1 voxel, Scarano & Poelma), leads to a relative precision error of the filtered velocity of 3%, based on mean axial velocity of 6.5 pixels.

Considering that vorticity precision error is defined as

$$\epsilon_\omega = \frac{\epsilon_v}{\Delta x}$$  \hspace{1cm} (10)

where $\epsilon_v=0.1$ pixel is the velocity precision error of the raw data and $\Delta x$ is the distance, in pixel units, between two subsequent measurement points. From the above equation it results that, at the considered points, which are representative of the shear layer, vorticity computed from raw velocity data are affected by precision error of 0.025 pixels and 0.01 voxels, respectively for the planar and tomographic case. Furthermore, it is evaluated that the standard deviation of vorticity residual is 0.01 pixel/pixel for planar PIV and 0.01 voxel/voxel for tomographic PIV. Such values are smaller than the precision errors committed in the respective cases. Thus, the time filtering do not cause any further aliasing of the vorticity, which, in relative terms, is affected by relative precision error of 30% based on half the minimum-to-maximum vorticity fluctuation (0.15 pixels).

![Figure 11. Time histories of raw (black) and filtered (red) axial velocity at point $P$ ($x=0.4, y=1.5$) from planar PIV (top) and at point $Q(x=0.4, y=0, z=1.5)$ from TOMO PIV (bottom).](image)

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Sound source dynamics

The sequence of instantaneous of vorticity contours showing the pairing between vortex "1" and "2", whose core positions have been tracked and illustrated in figure 9, is shown in figure 12a. In figure 9 the evolution of the vortex cores is also referred to a phase variable $\Phi$. This is assigned uniformly in time during the motion so that the pairing event of vortex "1" and "2" will be used as a reference for pairing phenomena. Between $\Phi=-6$ rad and $\Phi=-4.36$ rad (figure 12a), peak activity of vorticity is observed in correspondence of vortex "1" and "2", as well as of vortex "3" when merging is taking place ($-1.1<\Phi<1.1$ rad).

Figure 12b shows a sequence of instantaneous contours of $\langle \hat{L} \rangle$ on which iso-lines of vorticity $\omega_z=1$, $\omega_z=4$ and $\omega_z=7$ are drawn to facilitate the comparison with the vorticity field illustrated in figure 12a. It is observed that, during a pairing cycle, the contribution of the shear layer to the production of sound is never larger than 10% of the peak activity. By contrast, the flow region corresponding to the vortex "1", "2" and "3" are always characterized by $\langle \hat{L} \rangle$ activity larger than 30% of the peak. At $\Phi=-6$ rad the sound source is concetrated at the cores of the vortex "1" and "2". Between $\Phi=-4.36$ rad and $\Phi=-2.73$ rad, phases in which vortex "2" comes nearer "1", $\langle \hat{L} \rangle$ contours show that the region of noise production still concerns the cores. The source activity in the vortices increases between $\Phi=-1.1$ rad and $\Phi=1.1$ rad, phases in which the pursuer vortex is entrained by the leader around which it makes a rotational motion. Subsequently ($1.1<\Phi<4.36$ rad), during the formation and stabilization of the vortex "3", the peak intensity gradually drops of 50%.

Despite featuring a vortical motion weaker than that of vortex "1", "2" and "3" (figure 12a), the region of vortex collapse ($4<y<7.25$) is constantly characterized by high concentration of peak activity of $\langle \hat{L} \rangle$ (figure 12b).

Tomographic results are shown in figure 13 the instantaneous of $\langle \hat{L} \rangle$ isosurfaces corresponding to those of $\lambda_2$-criterion, to compare the dynamics and three-dimensional organization the sound source to those of the large flow structures. At $\Phi=-6$ rad the isosurface $\langle \hat{L} \rangle=35$, which is 30% of the peak activity, features axisymmetric coherence in the region corresponding to vortex "4" and "5". During pairing, $\langle \hat{L} \rangle$ gradually increases and at $\Phi=0$, where the vortex "6" is formed, reaches 60% of the peak activity coherently along the azimuthal direction. Subsequently, between $\Phi=0$ rad and $\Phi=6$ rad, rising azimuthal instabilities are detected along the isosurfaces of $\langle \hat{L} \rangle$. By visual analysis, it is observed that such instabilities feature the same wavelength, amplitude and phase observed in the corresponding $\lambda_2$ plot.

It is observed that the source activity of a vortex that is just shed, such as vortex "4" between $\Phi=-2.73$ rad and $\Phi=2.73$ rad, is concentrated on the upper and inferior periphery. These two contributions grows along the motion and merge together during the pearing, as observed for vortex "4" and "5" between $\Phi=-6$ rad and $\Phi=0$ rad. If, instead the vortex do not pair, the source rings merge later under the action of azimuthal instabilities ($0<\Phi<6$ rad, vortex "7").

Intense sound-source activity is detected between $y=4.5$ and 6, which corresponds to the region of breakup of the vortex rings (figure 13, $-6$ rad $<\Phi<2.73$ rad). Regarding secondary flow structures, such as streamwise vortices, no evidence of $\langle \hat{L} \rangle$ production larger than 30% is found.
Figure 12. Planar PIV measurements. Sequence of instantaneous of two vortices that pair: a) vorticity contours (vectors which are shown one every 4 along both x and y, in a convective frame of velocity $V_{\text{conv}} = 0.47$); b) contours of $<L>$. Dashed box indicates the integration domain.
Figure 13. Tomographic PIV measurements. Sequence of instantaneous of two vortices that pair: a) isosurfaces of $\lambda_2 = -0.5$ and isocontours of axial velocity; b) isosurfaces of $< L > = 35$ (green) and 70 (blue). Red box identifies the analysed pair of vortices.
Vortex-pairing sound prediction

The sound produced by vortex pairing is predicted at one single point located 100 jet diameters far from the nozzle, at 90 deg with respect to the jet axis. Formulation of Powell’s analogy for axisymmetric flows of equation (8) is used for predictions based on time-resolved planar PIV data on half plane only (figure 12).

The integration window, which is centered in the region of the vortices, follows the two vortices with a constant speed $U_w = 0.487$, the same of the convection velocity of vortex “2”, and have a dimension of $1.5 \times 1.4$ (sketch in figure 12). This size is such that significant levels of second time derivative of the Lamb vector corresponding to the pairing vortices are included in the integration and, along the window boundaries, $\langle \dot{L} \rangle$ is smaller than 20% of the peak activity. In addition, flat Hann windowing function $w_\alpha$ is applied on the integration domain with $\alpha=0.2$ along the axial and radial direction.

In figure 14, it is shown the time-resolved prediction of the sound scattered during the pairing of vortex ”1” and ”2” between 0ms and 91.66ms. Results with and without windowing function are reported. The application of $w_\alpha$ results in a reduction of the predicted sound, especially observed between 0ms and 40ms when the truncation of the source is typically larger (figure 12b). Between $t=0$ms and $t=25$ms, period in which vortex ”2” comes closer ”1” (figure 12), no evident emission is detected (figure 14, see ”windowing” curve). In contrast, between $t=25$ms and $t=55$ms, that is the period where the pursuer vortex makes a rotatory motion around the leading one, the acoustic emission grows of three times. Subsequently, during the convection of vortex ”3” towards the region of breakdown, the acoustic emission reaches a minimum at $t=72$ms and rises up to 2.5 at $t=90$ms. By visually comparing the acoustic emission (figure 14) with the contours of $\langle \dot{L} \rangle$ (figure 12) during the merging of the vortices ($0ms < t < 55ms$), it is observed that a growth of the former may be associated to peak activity of the latter.

Other three pairs of vortices undergoing pairing with a dynamics comparable to that vortex ”1” and ”2” (figure 12a) are visually phase-matched and the scattered sound is predicted. Corresponding plots are illustrated in figure 14b: comparable trends characterized by peak values between $\Phi=-2$ rad and $\Phi=1.5$ rad are observed. Favorable comparison is found with the trend obtained in the work by Schram et al.

Conclusions

A time-resolved planar and tomographic PIV investigation has been conducted on a transitional water-jet flow at Re=5,000. The evolution of azimuthally-coherent flow structures have been described reporting events such as vortex shedding and vortex pairing. Furthermore, it has been given a three-dimensional description of the dynamics of vortex-ring azimuthal instabilities and counter-rotating pairs of streamwise vortices. The time-resolved planar and tomographic PIV data are used to evaluate the second time derivative of the Lamb vector by which flow structure generating noise are characterized. From the analysis of the dynamics of the latter, it has been observed that the largest contribution to the Powell’s analogy concerns the pairing of vortices rather than the shedding and the convection. In addition, peak activity is detected...
in the region of vortex breakdown on which, however, further investigations are needed. Finally, under the assumption of axisymmetric flow, predictions of vortex-pairing sound from time-resolved planar PIV data have been performed reporting trends in agreement with the literature. Further investigations on the windowing method are however needed to ensure that the predictions are not affected by source-truncation issues.

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References


