Predicting the failure behavior of off-axis composite laminates subjected to dynamic compression loads

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by

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Abstract

Rapid release of large amounts of energy in loading events such as blasts and impacts cause severe damage in a material. The necessity for lightweight and smart materials that can withstand and mitigate such extreme loading conditions is growing every day. In recent times, composite materials are widely applied in the construction industry because of their superior properties compared to traditional materials like steel and concrete. The use of composite materials in defense and infrastructure protective applications to resist high rate dynamic loading conditions has been gaining the interest of a lot of researchers over the previous decade. The limitations involved in studying the behavior of composites when subjected to high rate dynamic loads experimentally generated a need for numerical models that could simulate such complex material behavior. These numerical models are also highly useful in situations where it is difficult to perform direct measurements in experiments. Modelling extreme loading conditions such as blasts and impacts is a highly challenging task due to their complicated nature. Hence, comparatively moderate and simple dynamic loading conditions are considered in this thesis for initial trials.

The idea of this thesis originated from an aim to study the behavior of a TNO developed composite laminate with alternate $0^\circ$ and $90^\circ$ ply layup when subjected to out of plane blast loads. Considering the complexities involved in such a loading situation, a simplified test setup that can approximately replicate stress conditions in the composite laminate due to out of plane blast load has been proposed to be designed. This simplified test setup is prepared by performing an angled cut from the composite laminate thus making the plies off-axis with respect to the global co-ordinate system of its cross-section. The motivation behind choosing such a test setup is that the interface/s can be loaded by a combination of compressive and shear stresses with axial loading on the specimen. The behavior of the simplified test setup with off-axis angles $30^\circ$, $45^\circ$ and $60^\circ$ when subjected to dynamic compression loads with different rates ranging from quasi-static to high is analyzed in this thesis using the finite element (FE) method. The plies in the simplified test setup are modelled as continua using the orthotropic material model and the interfaces are modelled using mixed mode cohesive laws coupled with friction.

The initial parts of the study focus on analyzing the quasi-static failure behavior of the off-axis angled composite specimens with a single critical interface at the center modelled using the rate-independent cohesive law coupled with friction adapted by Van der Meer et al. [1] from Zou et al. [2]. The effort required to model the complete composite specimen with all the plies and one critical interface at the center are reduced by applying a smearing technique. Parametric studies are carried out to analyze the influence of varying the interface material properties on the overall quasi-static behavior of the composite specimens with different off-axis angles. The changes in the quasi-static model response when possible failure of multiple interfaces is included is also investigated. Moreover, for the model with multiple interfaces the influence of the ply thickness is analyzed. The later parts of the study focus on analyzing the dynamic failure behavior of the composite specimens. The rate-independent cohesive law with friction is improved by adding a rate-dependency feature based on a Johnson-Cook law similar to Liu et al. [3]. Finally, the
dynamic behavior of different off-axis angled composite specimens with critical interface/s modelled using the rate-independent and the rate-dependent cohesive laws with friction when subjected to compression loads with different rates is analyzed.

The results obtained from the study indicate that the failure behavior of a discrete laminate configuration, with single critical interface, can be replicated with its smeared configuration counterpart with only minor differences. Parametric studies on the quasi-static behavior of different off-axis angled composite specimens reveal that an increase in the mode-II cohesive strength, mode-II cohesive fracture energy, and interface friction coefficient lead to an increase in the peak load carried by the laminates. An increase in the thickness of the plies of an off-axis angled composite specimen results in a decrease in its peak load and changes its governing failure mechanism. There will be a dominant contribution of local inertia and wave propagation effects in dictating the failure behaviors of the specimens when subjected to compression loads with higher rates. The rate-dependent cohesive law with friction developed in this thesis seemingly captured the rate effects in the off-axis angled composite specimens when loaded in compression at different rates. Finally, it is suggested to design the simplified test setup by performing a 45° cut from the original composite laminate to simulate delamination due to compression loads of different rates effectively. But all three simplified setups have to be tested to derive the unique set of interface parameters that dictate the failure behaviors of the specimens.
## Contents

List of Figures xiii

List of Tables xiv

1 Introduction 1
   1.1 Background & Motivation .............................................. 1
   1.2 Research Questions .................................................. 3
   1.3 Research Methodology ................................................ 3
   1.4 Thesis outline ....................................................... 4

2 Literature Review 6
   2.1 Blast loading and composites ........................................ 6
      2.1.1 Blast loads: General ............................................. 6
      2.1.2 Use of composites against blast loading ......................... 7
   2.2 Cohesive Zone Modelling ............................................. 7
      2.2.1 Delamination failure & CZM ..................................... 8
      2.2.2 Rate independent mixed mode cohesive law ....................... 8
      2.2.3 Cohesive laws coupled with friction ............................ 12
      2.2.4 Incorporating rate dependency in the cohesive law ............. 14
   2.3 Dynamic behaviour of composites .................................. 16
      2.3.1 Failure mechanisms in composites due to dynamic loads ........ 16
      2.3.2 Modelling matrix cracks and fiber failure ...................... 21

3 Simplifying the FE model 25
   3.1 Overview of the problem & initial approach ........................ 25
   3.2 FE modelling details .................................................. 26
   3.3 Interface tractions & load-displacement plots ....................... 28
      3.3.1 Interface tractions ............................................. 28
      3.3.2 Load-Displacement plots ..................................... 32
   3.4 Interface Shear to Pressure ratios .................................. 34

4 Parametric studies (quasi-static behavior) 39
   4.1 Updated material properties ......................................... 39
   4.2 Specimen behavior with updated properties ........................ 40
   4.3 Strength variation for interface cracking .......................... 42
   4.4 Mesh dependence study .............................................. 44
   4.5 Parametric studies on mode-II strength ............................ 44
   4.6 Parametric studies on mode-II fracture energy ..................... 47
   4.7 Parametric studies on friction coefficient .......................... 49
List of Figures

1.1 Laminate configuration before cutting (left) and composite specimen after cutting (right) ................................................. 2
2.1 Different delamination stress states ........................................ 8
2.2 Fracture process zone representation ....................................... 9
2.3 The traction-separation law .................................................. 9
2.4 The mixed mode cohesive law used by Liu et al. .......................... 11
2.5 The mixed mode cohesive law used by Yamazaki et al. ................. 11
2.6 Cohesive law with friction adapted by Van der Meer et al. from Zou et al. .................................................. 14
2.7 Failures in carbon fiber reinforced composite laminate subjected to blast load .................................................. 16
2.8 Failure patterns of composite laminate subjected to underwater blast load 17
2.9 Strain band formation in 15° off-axis specimen subjected to quasi-static load 18
2.10 Strain localisation in 45° off-axis specimen subjected to quasi-static load 19
2.11 Failure in 15° off-axis specimen subjected to dynamic load ............ 20
2.12 Failure in 45° off-axis specimen subjected to dynamic load ............ 20
2.13 In plane shear failure in 30° off-axis specimen .......................... 21
3.1 Laminate configurations chosen for simplifying the FE model: (a) Discrete (DIS) specimen (b) Smeared (SM) specimen. .......... 27
3.2 Typical FE meshes used in case of 15 mm x 5 mm specimen size for laminate configuration: (a) DIS (b) SM. ............................. 27
3.3 Typical FE meshes used in case of 30 mm x 10 mm specimen size for laminate configuration: (a) DIS (b) SM. ............................. 27
3.4 Shear and Normal tractions along the critical interface at damage initiation point in case of 15 mm x 5 mm specimen size and all three off-axis angles for laminate configuration: (a) DIS (b) DIS (c) SM (d) SM. ............................. 28
3.5 Shear and Normal tractions along the critical interface at a random point on the failure branch in case of 15 mm x 5 mm specimen size and all three off-axis angles for laminate configuration: (a) DIS (b) DIS (c) SM (d) SM. 30
3.6 Shear and Normal tractions along the critical interface at damage initiation point in case of 30 mm x 10 mm specimen size and all three off-axis angles for laminate configuration: (a) DIS (b) DIS (c) SM (d) SM. 31
3.7 Shear and Normal tractions along the critical interface at a random point on the failure branch in case of 30 mm x 10 mm specimen size and all three off-axis angles for laminate configuration: (a) DIS (b) DIS (c) SM (d) SM. 32
3.8 Load-Displacement plots in case of 15 mm x 5 mm specimen size for both laminate configurations and all three off-axis angles ............................. 33
3.9 Load-Displacement plots in case of 30 mm x 10 mm specimen size for both laminate configurations and all three off-axis angles .......... 33
3.10 SP ratio of 15 mm x 5 mm DIS specimen with 30° off-axis angle at: (a) Damage initiation (b) crack initiation (c) random point on failure branch. 34
3.11 SP ratio of 15 mm x 5 mm DIS specimen with 45° off-axis angle at: (a) Damage initiation (b) crack initiation (c) random point on failure branch. 35
3.12 SP ratio of 15 mm x 5 mm DIS specimen with 60° off-axis angle at: (a) Damage initiation (b) crack initiation (c) random point on failure branch. 35
3.13 SP ratio of 30 mm x 10 mm DIS specimen with 30° off-axis angle at: (a) Damage initiation (b) crack initiation (c) random point on failure branch. 36
3.14 SP ratio of 30 mm x 10 mm DIS specimen with 45° off-axis angle at: (a) Damage initiation (b) crack initiation (c) random point on failure branch. 36
3.15 SP ratio of 30 mm x 10 mm DIS specimen with 60° off-axis angle at: (a) Damage initiation (b) crack initiation (c) random point on failure branch. 37

4.1 Typical FE mesh used in case of 45° off-axis angled SM specimen with size 60 mm x 20 mm. .................................................. 40
4.2 Shear tractions along the critical interface of the specimen with updated material properties at different points in the equilibrium path for off-axis angles: (a) 30° (b) 45° (c) 60°. .................................................. 41
4.3 Evolution of damage parameter along the critical interface with loading in case of 60° off-axis angled specimen .......................... 42
4.4 Evolution of damage parameter in case of 60° off-axis angled SM specimen by keeping all interface properties same as updated ones except the mode-II strength is increased by: (a) 25% (b) 50%. ....................... 43
4.5 Load-displacement plots obtained in case of different mesh sizes for specimen off-axis angles: (a) 30° (b) 45° (c) 60°. ................. 45
4.6 Specimen peak loads obtained for different interface element sizes in the case of specimen off-axis angles: (a) 30° (b) 45° (c) 60°. .......... 45
4.7 Load-displacement plots for different mode-II interface strengths in case of specimen off-axis angle: (a) 30° (b) 45° (c) 60°. ............... 46
4.8 Variation of specimen peak load with the mode-II interface strength for all three specimen off-axis angles. .............................. 46
4.9 Shear tractions along the critical interface at the DIP upon varying the mode-II interface strength for off-axis angles: (a) 30° (b) 45° (c) 60°. .... 47
4.10 Shear tractions along the critical interface at a random point on the failure branch of the equilibrium path upon varying the mode-II interface strength for off-axis angles: (a) 30° (b) 45° (c) 60°. ................. 47
4.11 Load-displacement plots for different mode-II interface fracture energies in case of specimen off-axis angle: (a) 30° (b) 45° (c) 60°. .......... 48
4.12 Variation of specimen peak load with the mode-II interface fracture energy for all three specimen off-axis angles. ....................... 48
4.13 Shear tractions along the critical interface at a random point on the failure branch of the equilibrium path upon varying the mode-II interface fracture energy for off-axis angles: (a) 30° (b) 45° (c) 60°. ................. 49
4.14 Load-displacement plots for different static friction coefficients in case of specimen off-axis angle: (a) 30° (b) 45° (c) 60°. ............... 50
5.13 Damage in the interfaces of a 45° off-axis angled MIM of the composite specimen when subjected to quasi-static load at: (a) DIP (b) failure.

5.14 Damage in the interfaces of a 45° off-axis angled MIM of the composite specimen at failure when subjected to: (a) dynamic load case 1 (b) dynamic load case 2 (c) dynamic load case 3.

5.15 Load-displacement plots obtained when the MIMs of the composite specimens with all three off-axis angles are subjected to dynamic load case 4.

5.16 Contours of $\sigma_{xx}$ in the case of 45° off-axis angled MIM of the composite specimen when subjected to dynamic load case 4 at simulation times: (a) $1 \times 10^{-7}$ sec (b) $1.1 \times 10^{-6}$ sec (c) $2.1 \times 10^{-6}$ sec (d) $6.65 \times 10^{-6}$ sec.

5.17 Damage in the interfaces for dynamic load case 4 in the case of MIM of the composite specimens with off-axis angles: (a) 30° (b) 45° (c) 60°.

5.18 Load pulses at the left and right boundaries of the 45° off-axis angled MIM of the composite specimen when subjected to: (a) dynamic load case 3 and (b) dynamic load case 4.

6.1 One dimensional version of the rate-independent and the rate-dependent cohesive laws with constant friction $\tau_f$, constant displacement jump rate and positive rate effects.

6.2 Typical FE mesh used to analyze the SIM of a composite specimen with cross-section size 30 mm x 10 mm and the critical interface modelled using the rate-dependent cohesive law.

6.3 Load-displacement plots obtained for different time step size trials when the SIM of the composite specimens with rate-dependence parameters as in Pcase2 and off-axis angles: (a) 30° (b) 45° (c) 60° are subjected to dynamic load case 1.

6.4 Load-displacement plots obtained for different parameter cases when the SIMs of the composite specimens with RDCZM and off-axis angles: (a) 30° (b) 45° (c) 60° are subjected to dynamic load case 1.

6.5 Load-displacement plots obtained for different parameter cases when the SIMs of the composite specimens with RDCZM and off-axis angles: (a) 30° (b) 45° (c) 60° are subjected to dynamic load case 2.

6.6 Increase (%) in the peak loads of different off-axis angled composite specimens when compared to Pcase1 due to the incorporation of rate-dependency in dynamic load case 2.

6.7 Load-displacement plots obtained for different parameter cases when the SIMs of the composite specimens with RDCZM and off-axis angles: (a) 30° (b) 45° (c) 60° are subjected to dynamic load case 3.
List of Tables

3.1 Mechanical properties of the composite plies (bulk material properties) . . 26
3.2 Mechanical properties of the interface . . . . . . . . . . . . . . . . . . . . . 26
3.3 Averaged mechanical properties of the smeared bulk material . . . . . . . 26

4.1 Updated mechanical properties of the composite plies . . . . . . . . . . . . 39
4.2 Updated mechanical properties of the interface . . . . . . . . . . . . . . . . 40
4.3 Updated average mechanical properties of the smeared bulk material . . . 40

5.1 Load cases considered for dynamic analysis of the composite specimens . . 58

6.1 Rate-dependence parameters considered for modelling the critical interface
with the rate-dependent cohesive law . . . . . . . . . . . . . . . . . . . . . . . 75
6.2 Load cases considered for dynamic analysis of the composite specimens
with the rate-dependent cohesive law . . . . . . . . . . . . . . . . . . . . . . . 76
1 Introduction

1.1 Background & Motivation

Accidental explosions, impacts and detonations impose highly intensive dynamic loads on the structures close to their vicinity [4]. Explosions and detonations are a release of huge amounts of energy in a short interval of time. These include breaching of structural integrity in pressure vessels, heavy explosive detonations and vapor cloud explosions. The necessity to protect people and make structures blast and impact resistant has increased over the last decade. According to the United States State department, the number of people that died due to explosions in terrorist attacks was more than 87,000 in a five year period [5]. Not only civil engineering structures but also aerospace structures which are increasingly made using composite materials are subjected to high rate dynamic loads such as sonic boom pulses and gust [6]. Hence it is necessary to design smart materials, which are light in weight and could absorb high amounts of energy releases from events such as explosions. The material to be designed has to withstand and mitigate damage caused by these high rate dynamic loads [7].

Fibre Reinforced Polymer (FRP) materials are widely used in retrofitting of structures to improve their resistance towards more adverse loading conditions than they were actually designed for. The main advantage of FRP materials are their mechanical properties such as high impact resistance, high strength to weight ratio and durability [4, 7]. One very important failure mechanism in composite materials is delamination in which an interfacial crack separates the plies attached to each other. Hence the crack cannot change its direction but has to follow a particular pre-determined path [3, 8]. Cohesive zone modelling is one of the popular tools used to simulate delamination in composite materials. The effect of through-thickness stress is not considered in most of the existing cohesive laws on the damage initiation and propagation in the laminate which produces inaccurate results. The through-thickness stress affects the contact/friction traction development in the damaged area. Friction in the damaged area of the interface has to be considered for accurately capturing the failure behavior of the laminate [2].

The approach developed by Zou et al. [2] couples interface damage with friction based
on a cohesive zone energy parameter. But this Cohesive Zone Model (CZM) is only applicable to predict the failure behavior of interface when subjected to quasi-static loads. As the loading rate increases in a composite laminate from a quasi-static to a dynamic range, the propagation of stress waves, complex mixed mode failure mechanisms and complicated stress states make the delamination failure even more difficult to simulate. Also, the material properties of the laminate vary as the loading rate increases [3–5]. Hence the CZM for simulating such a dynamic failure behavior should be rate-dependent.

This study is based on a TNO developed composite laminate with a given fibre type, matrix, fibre volume ratio, ply thickness, and alternate 0 and 90° layup, which has to be tested for out of plane blast loads. The laminate is designed to dissipate the blast energy majorly through the delamination failure mechanism. But the stress states at the interfaces of the laminate will be very complicated due to the reflections and conversions of the incident normal blast wave. Considering the complexity of this case both experimentally and numerically, it is decided to first investigate a simplified test setup under moderate dynamic loading conditions for its design feasibility. The simplified test setup is prepared by performing an angled cut from the original composite laminate with alternate 0 and 90° layup. This cutting makes the plies off-axis with respect to the global co-ordinate system of the specimen (loading direction). The advantage of this simplified setup is that it is relatively easier to load the specimen and also to simulate delamination at different loading rates. The problem that has to be addressed now is the angle of cut which is not yet decided and will be finalized based on the failure behaviors obtained from the simulations. For this purpose, composite specimens with different off-axis angles have to be investigated to study the failure of the interfaces under varying shear to pressure ratios.

The laminate configuration before cutting and the composite specimen (simplified test setup of original problem) obtained after cutting is as shown in Figure 1.1. The interfaces in the composite specimens, due to their off-axis nature, are loaded by combined shear and normal stresses (especially high compressive stresses) in quasi-static up to high-loading rates. Due to the compressive stresses on the interface, friction traction develops and it
will have a considerable effect on the delamination mechanism. The in-plane ply properties can be estimated using the ply-composition and constituent mechanical properties. But the interface properties are unknown. In a finite element (FE) modelling framework, ply will be modelled as a continuum and the interface with a cohesive law. Rate-independent and the rate-dependent cohesive laws coupled with friction will be used to model the interface/s in different off-axis angled composite specimens to analyze their delamination failure behaviors when subjected to compression loads with rates ranging from quasi-static to high.

1.2 Research Questions

The main goal of this research is to simulate the delamination failure in different off-axis angled composite specimens with alternate $0^\circ$ and $90^\circ$ layup at different loading rates in compression. Based on the simulation results obtained, general suggestions/recommendations have to be given regarding the simplified test setup. To accomplish the goal of this study, the rate-independent cohesive law coupled with friction adapted by Van der Meer et al. [1] from Zou et al. [2] is made more general by incorporating rate dependency.

The following research questions have been identified as a formulaic approach to achieve the main goal of this thesis:

1. How do composite specimens with alternate $0^\circ$ and $90^\circ$ layup and with three different off-axis angles $30^\circ$, $45^\circ$, and $60^\circ$ between the load and ply orientation behave in compression at different loading rates?

2. Can the interface properties be derived from the global specimen response?

3. How does changing the ply thickness affect the overall specimen behavior?

4. Will the rate-dependent cohesive law coupled with friction be able to describe the delamination failure when the interface is loaded by combined normal stress, shear stress, and friction at different rates?

5. How should the test setup be designed to get most information out of it?

1.3 Research Methodology

The objectives of this thesis involve modelling the failure behavior of off-axis composite specimens at different loading rates ranging from quasi-static to high in compression. In the initial parts of the thesis, quasi-static failure behaviors of different off-axis angled composite specimens will be investigated using the rate-independent cohesive law with friction adapted by Van der Meer et al. [1] from Zou et al. [2]. Parametric studies will be carried out to study the influence of varying the interface properties and ply thickness on the overall quasi-static behavior of the composite specimens. The later parts of the work will focus on studying the dynamic behavior of the specimens when subjected to compression loads of different rates. To begin with, the critical interface/s of the composite specimens for analyzing their dynamic behavior will be modelled using the rate-independent cohesive law with friction. But to simulate such a failure of the interface for different loading rates effectively, a rate-dependent cohesive law coupled with friction has to be developed.
Incorporating rate-dependency in the cohesive law is theoretically grounded and hence an extensive literature review should be carried out to describe the behavior of composites at high loading rates both physically and mathematically. The rate-dependent cohesive law uses mathematical formulations from literature to add a rate-dependency feature to the rate-independent cohesive law developed by Zou et al. [2]. The rate-dependent cohesive law will be implemented based on the FE method by programming in C++ in the JemJive modeling environment. The existing FE framework with friction coupled rate-independent cohesive law should be adapted and the rate-dependent cohesive law has to be programmed for implementation. The rate-dependent cohesive law with friction will then be used to model the interfaces of different off-axis angled composite specimens when subjected to compressive loads with rates ranging from quasi-static to high. Finally, based on the observations regarding the behavior of different off-axis angled composite specimens, suggestions are to be given about the design of a simplified test setup from which most of the failure information can be obtained.

1.4 Thesis outline

The report is divided into six chapters and structured logically such that the research questions can be addressed at the end. Chapter-2 contains a literature review about the general aspects of blast loading, the behavior of composites against blast loading, cohesive zone modeling approaches for interface failure, and the dynamic behavior of composites. The simplification of the FE model from a discrete configuration model to a smeared configuration model reducing the cumbersome work of modeling all the plies in the composite laminate is dealt with in chapter-3. Chapter-4 discusses the effects of varying the interface material properties on the quasi-static behavior of the composite specimens with off-axis angles $30^\circ$, $45^\circ$, and $60^\circ$. The effects of changing the ply thickness on the quasi-static behavior of the composite specimens are also discussed in chapter-4. The dynamic behavior of different off-axis angled composite specimens with a single and multiple critical interface/s modelled using a rate-independent cohesive law with friction when subjected to compression loads of different rates is analyzed in chapter-5. In chapter-6, the main aspects of the rate-dependent cohesive law with friction and the dynamic behaviors of different off-axis angled composite specimens with a single critical interface modelled using the rate-dependent cohesive law when subjected to compression loads of different rates are discussed and analyzed respectively. Finally, chapter-7 summarises the main findings from this thesis and the related recommendations for further research and development.
2

Literature Review

2.1 Blast loading and composites

This section discusses the general aspects of blast loading, why and how are composite materials used in blast loading scenarios and the complications involved in modeling blast loads. In this section, only the behavior of air blast loading i.e. blast pressure waves propagating through air is addressed and other variants of blast such as water blast loading are not considered. Even though blast loads are not a part of this thesis study, understanding the difficulties involved in modelling such complex load situations provides the context for why the simplified version of the test setup was chosen over the actual one to carry out the initial experiments and numerical simulations.

2.1.1 Blast loads: General

Blasts, explosions and detonations are rapid release of huge amounts of energy in short interval of time. Other forms of such dynamics loads include gusts and sonic boom pulses in aerospace applications such as supersonic/hypersonic flights [5, 6]. These explosions can be mechanical or chemical in origin. When an explosion happens in the air, a shock wave is generated inside the exploding material which rapidly compresses the air in the surrounding medium thus creating a short duration, high pressure blast wave [5].

The velocity of these propagating waves is directly proportional to the pressure change and the nature of detonating material which may be linear or non-linear (plastic) state depending on its failure type during explosion. This material behaviour governs the pressure/stress waves generated from the detonation which may be normal sound waves in case of linear material behaviour and shock waves in case of non-linear material behavior [9].

The intensity of the generated blast wave determines the destruction caused to the objects along its path and losses caused in terms of life, economy or environment [5]. The characteristics of the blast wave depend various factors such as: stand-off distance (distance between explosion and object), source of explosion, surrounding medium, degree of confinement and ground proximity. It can obviously be understood that increasing the
stand-off distance causes a decrease in the magnitude of the pressure wave and vice-versa. Based on the stand-off distance, blasts/explosions are classified into two types: 1. if the stand-off distance of the blast is short, it is termed as near-field explosion 2. if the stand-off distance increases and reaches a certain range, they are termed as far-field explosions. Near field explosion blasts are very complicated in nature to simulate using simple closed forms because of their complex pressure profiles [6].

2.1.2 Use of composites against blast loading

Composite materials in the recent times gained an increased demand for use in structural applications. These materials have a lot of advantages over traditional construction materials such as steel and concrete namely high strength and stiffness properties, lesser weight, better energy absorption and mitigation of impact and blast loads, superior corrosion and thermal resistance and the flexibility in their design i.e. they can be fabricated based on the application requirements by carefully adjusting the parameters such as fiber orientations, ply thicknesses etc. Composites are currently used in the design of various load bearing structures in aerospace industry such as aeroplane fuselage sections. They also find application in the manufacture of hybrid composites such as fiber metal laminates and foam-core sandwich panels [4–7,10–12]. Carbon Fiber Reinforced Polymer (CFRP) composites are used to retrofit existing protective structures for improving their performance against blast loads [4]. CFRP composites are extensively used in applications such as armored vehicles, air crafts, naval ships etc. which are at a high risk of attack using explosives, sonic boom pulses and gusts. The advancements in the design of composite materials led to increase in the interest of researchers to explore their non-linear dynamic behavior when subjected to blast loads, time-dependent pulses etc [6,12].

The deformation of composite laminates when subjected to impulses due to an explosive shock wave depends on various factors such as mass of explosive charge, explosive charge stand-off distance, laminate dimensions, geometry, boundary conditions and material properties such as failure strength, stiffness and interlaminar fracture toughness. The extent of damage caused by an explosion depends on a large number of factors such as fiber properties, fiber-matrix interfacial strength and matrix type etc. There are many experimental procedures performed to test the blast resistance of composite materials such as ballistic pendulum, shock tubes, small, intermediate and large-scale explosion tests. But there are certain problems to carry out these experiments accurately like the validity of the results only for specific test conditions and the results obtained depend on boundary conditions and geometry of the test specimen. These experiments are potentially dangerous, time-consuming and very expensive to perform [12]. Also, the results from these tests cannot be used to derive the individual material properties, even though they provide reliable data for the integral material and structural response for complex loading conditions. Therefore performing numerical analyses to study the behaviour of composite laminates subjected to impact and blast loads will definitely help in minimizing the potential danger and efforts involved in the experiments.

2.2 Cohesive Zone Modelling

This section discusses about cohesive zone modelling (CZM) in general, its applications, its use in modelling composite interlaminar failure, different delamination failure modes
possible and various types of cohesive laws used to model delamination.

### 2.2.1 Delamination failure & CZM

Delamination is one of the crucial failures in composite laminates mostly due to weak interlaminar strengths. Delamination is separation between two adjacent layers in a composite laminate. There are four causes of delamination in which the first one is due to matrix cracks and the second is because of curved sections. These failures occur because of high normal and shear stresses in the adjacent plies, thereby leading to loss of adhesion and eventually an interlaminar crack [13]. The third reason is because of abrupt changes in cross-sections and the fourth is due to temperature and moisture effects. The difference between the thermal coefficients of matrix and the fiber materials during the curing process causes residual stresses in the composite which may give rise to delamination [13].

Considering the micro-mechanics of delamination, the damage zone ahead of the crack tip leads to the growth of an interlaminar crack. The shape and size of the damage zone depend on the stress state and the toughness of the resin material. In general, there are three modes of crack growth depending on the separation type occurring at the interface as illustrated in Figure 2.1. Failure mode-I,II and III referred to as Tensile opening, in-plane shear and anti-plane or out-of-plane shear respectively. Combination of these three pure modes might also result in the delamination failure which is called a mixed mode failure [13].

![Figure 2.1: Different delamination stress states](image)

CZM is one of the most widely used tools to model failure in composite laminates particularly for delamination (interlaminar failure), fiber-matrix debonding and adhesive bonding joint failure [2]. The idea behind a cohesive zone model is that a fracture process zone ahead of the visible crack (illustrated in Figure 2.2) can be collapsed onto the plane ahead of the crack. The traction across this plane is determined from the traction-separation relation as shown in Figure 2.3. Two parameters are generally required to describe the cohesive behaviour: strength of the interface and the fracture energy of the interface or work of separation [15]. A dummy stiffness parameter is used to describe the initial stiffness of the interface which is generally very high when compared with the surrounding material so that the cohesive material doesn’t contribute to any considerable deformations in the elastic phase of the composite material.

### 2.2.2 Rate independent mixed mode cohesive law

i. Pure mode loading
In pure mode I, II or III loading cases, the damage initiation happens when the interlaminar traction reaches the maximum interfacial strength. The delamination crack propagation happens when the total energy released at that particular point is equal to the interfacial fracture toughness in that respective mode [13].

ii. Mixed mode loading

Damage initiation criterion

In case of mixed mode loading, coupling effects between different modes of loading I, II and III have to be taken into account. There are three major components to be chosen for complete formulation of a cohesive law. They are: damage initiation criterion, crack propagation criterion and shape of the traction-separation relation. Damage initiation in an interface happens when a failure criterion is reached [7,13]. A simple quadratic failure
criterion is used by many researchers in the literature [1–4, 8, 10–12, 16–21]:

\[
 f_{\text{initiation}} = \left( \frac{\langle \tau_1 \rangle}{\tau_1^0} \right)^2 + \left( \frac{\tau_2}{\tau_2^0} \right)^2 + \left( \frac{\tau_3}{\tau_3^0} \right)^2 - 1 = 0 \quad (2.1)
\]

Where,

- '1' is the direction perpendicular to the delamination surface.
- \( \langle \cdot \rangle \) is the MacAuley operator.
- \( \tau_1^0, \tau_2^0 & \tau_3^0 \) are maximum interfacial strengths in modes I, II and III respectively.
- \( \tau_1, \tau_2 & \tau_3 \) are momentary tractions on the interface in modes I, II and III respectively.

**Crack propagation criterion**

The second component of cohesive law is the crack propagation criterion. Crack propagation criterion is generally formulated independently from the damage initiation criterion. This failure initiation criterion is considered by different authors in the literature in different ways. The first form of the generally used crack propagation criteria is as follows:

\[
 f_{\text{propagation}} = \left( \frac{G_I}{G_{IC}} \right)^\alpha + \left( \frac{G_{II}}{G_{IIC}} \right)^\beta + \left( \frac{G_{III}}{G_{IIIC}} \right)^\gamma - 1 = 0
\]

Where,

- \( \alpha, \beta & \gamma \) are the parameters used to fit the experimental data.
- \( G_{IC}, G_{IIC} & G_{IIIC} \) are interfacial fracture toughnesses in modes I, II and III respectively.
- \( G_I, G_{II} & G_{III} \) are individual components of energy release rates in modes I, II and III respectively.

The values of \( \alpha = \beta = \gamma = 1 \), a linear failure criterion or 2, a quadratic failure criterion are generally chosen if no experimental data is available [13]. A linear crack propagation criterion is used by Gargano et al. [12], Yao et al. [16] and Karagiozova et al. [17] and a quadratic criterion is used by Tran et.al [7] and May et al. [20].

The second generally used crack propagation criterion is the one proposed by Kenane and Benzeggagh which fits the experimental data more accurately. The form of this expression is as follows:

\[
 f_{\text{propagation}} = \left( \frac{G_T}{G_C} \right) - 1 = 0
\]

Here \( G_C \) is the mixed mode fracture energy defined as:

\[
 G_C = G_1^0 + (G_2^0 - G_1^0)B^\eta \quad (2.2)
\]

Where, \( \eta \) is a parameter obtained from fitting Equation 2.2 to the experimental data,  \( B \) is considered as a measure for the mode mixity, ranging from 0 for mode-I to 1 for mode-II or mode-III and \( G_T \) is the total energy release rate calculated as sum of energy release rates in all the three modes of failure \( (G_1 + G_2 + G_3) \).

Zou et al. [2] used the following crack propagation criterion:

\[
 f_g = \frac{G_1 - G_{1o}}{G_{1c} - G_{1o}} + \frac{G_2 - G_{2o}}{G_{2c} - G_{2o}} = 1
\]
2.2. Cohesive Zone Modelling

Where,

$G_{1c}$ and $G_{2c}$ are cohesive fracture energies in modes I and II respectively.

$G_{1o}$ and $G_{2o}$ are energy release rates in modes I and II at damage initiation respectively.

Alternative expressions/methods have been proposed by Shojaei et al. [18], Gozulkulu et al. [8], Yamazaki et al. [19], Hou et al. [11] and Liu et al. [3] as the crack propagation criteria. The mixed mode cohesive law used to model delamination in a Double Cantilever beam specimen by Liu et al. [3] is as shown in the Figure 2.4 and the one used by Yamazaki et al. [19] to model dynamic failure in cylindrical CFRP laminate is as shown in Figure 2.5.

Figure 2.4: The mixed mode cohesive law used by Liu et al. [3]

Figure 2.5: The mixed mode cohesive law used by Yamazaki et al. [19]

Traction-Separation relations

The third and the last aspect that needs to be defined is the shape of the traction-separation relation. There are two tasks to be carried out in this component which include choosing the softening shape of the traction-separation law and therefore formulating the right traction-separation relations and the damage parameters. Liu et al. [3] chose a bilinear softening law for modelling the mode-I delamination crack. The traction-
separation relation and the damage parameters that they have used are as given in Equations 2.3 and 2.4 respectively. May et al. [20] used a similar bilinear traction separation softening relation as in Equation 2.3 but with a different damage parameter as given in Equation 2.5. Tran et al. [7], Wei et al. [10] and Gargano et al. [12] also used a bilinear traction-separation relation to model the interlaminar damage in composites but no information regarding the calculation of damage parameter is provided.

\[ t_1 = K \left( 1 - d \frac{\delta_1}{\delta_1} \right) \quad \& \quad t_2 = K(1 - d)\delta_2 \]  
\[ d = \begin{cases} 
0 & \delta \leq \delta_0 \\
\frac{\delta_f(\delta - \delta_0)}{\delta_f(\delta_f - \delta_0)} & \delta_0 < \delta < \delta_f \\
1 & \delta \geq \delta_f 
\end{cases} \]  

Where, 
'1' is the direction normal to the crack plane and '2' is the shear direction. 
K is the initial dummy stiffness of the interface. 
\( \delta_1 \) and \( \delta_2 \) are mode-I and mode-II displacement jumps across the interface respectively. 
\( t_1 \) and \( t_2 \) are normal and shear tractions along the interface respectively.

\[ d = \frac{\delta_m - \delta^c_m}{\delta^f_m - \delta^c_m} \]  

Where, 
\( \delta_m, \delta^c_m \) & \( \delta^f_m \) are the mixed mode displacement jumps at arbitrary load step, damage initiation and crack propagation respectively (see May et al. [20] for detailed formulations).

### 2.2.3 Cohesive laws coupled with friction

Zou et al. [2] developed a cohesive law that combines the damage at the interface with the friction. This cohesive interface model considers the effect of through-thickness compression on the shear resistance of the interface. The main advantage of this model when compared with other existing cohesive laws coupled with friction in the literature is that it solves the problem of the dependence of friction development on the initial dummy stiffness.

In this model, the mode-II cohesive strength and fracture energy are enhanced as given in Equation 2.6 and 2.7 to consider the effect of through-thickness compression on the shear resistance of the interface.

\[ \tau_{2n}^0 = \tau_2^0 \left( 1 + \zeta \frac{\langle -\tau_1 \rangle}{\tau_2^0} \right) \]  
\[ G_{2n}^0 = G_2^0 \left( 1 + \zeta \frac{\langle -\tau_1 \rangle}{\tau_2^0} \right) \]  

Where, 
\( \tau_{2n}^0 \) and \( G_{2n}^0 \) are the enhanced cohesive shear strength and fracture energy of the interface. 
\( \zeta \) is a material property called the enhancement factor.
The enhanced shear strength of the interface is used to formulate a quadratic cohesive damage initiation criterion as given in Equation 2.8. The cohesive traction-separation relations are given in Equations 2.9 and 2.10. Friction doesn’t effect the cohesive behaviour of the interface but resists the sliding deformation. Hence friction traction is added to the shear traction of the interface. It is assumed that friction is only developed in the damaged part of the interface.

$$f_s = \left( \frac{\tau_1}{\tau_1^0} \right)^2 + \left( \frac{\tau_2}{\tau_2^0} \right)^2 - 1 = 0 \quad (2.8)$$

$$\tau_1 = \begin{cases} k_1\delta_1 & \delta_1 < 0 \\ (1-\omega)k_1\delta_1 & \delta_1 > 0 \end{cases} \quad (2.9)$$

$$\tau_2 = \begin{cases} (1-\omega)k_2\delta_2 & \delta_1 < 0 \\ (1-\omega)k_2\delta_2 + A\tau_f & \delta_1 > 0 \end{cases} \quad (2.10)$$

Where,
$A$ is the damaged part of representative elementary area of the interface which is updated incrementally based on the energy dissipation concept (see Zou et al. [2] for detailed formulations).
$\omega$ is the damage parameter updated using an incremental damage evolution law (see Zou et al. [2] for details).
$k_1$ and $k_2$ are initial dummy normal and shear stiffnesses of the interface.
$\tau_f$ is the frictional stress developed along the interface.

The friction traction $\tau_f$ is obtained using the formulations given in Equation 2.11.

$$\tau_f = \begin{cases} k_2(\delta_2 - \delta_s) & k_2|\delta_2 - \delta_s| + \mu\tau_1 < 0 \quad (sticking occurs) \\ -\mu\tau_1\frac{\delta_2 - \delta_s}{|\delta_2 - \delta_s|} & k_2|\delta_2 - \delta_s| + \mu\tau_1 \geq 0 \quad (sliding occurs) \end{cases} \quad (2.11)$$

Where,
$\mu$ is the coefficient of friction.
$\delta_s$ is the frictional sliding displacement initially equal to zero and incremented using Equation 2.12.

$$d\delta_s = \begin{cases} 0 & k_2|\delta_2 - \delta_s| + \mu\tau_1 < 0 \\ \left( |\delta_2 - \delta_s| + \frac{\mu\tau_1}{k_2} \right) \frac{\delta_2 - \delta_s}{|\delta_2 - \delta_s|} & k_2|\delta_2 - \delta_s| + \mu\tau_1 > 0 \end{cases} \quad (2.12)$$

Adapted by Van der Meer et al. [1] from Zou et al. [2]

The version of shear traction-separation relation adapted from Zou et al. [2] in the cohesive zone model with friction developed by Van der Meer et al. [1] is defined as given in Equation 2.13. $d$ and $D$ are the damage parameters in the cohesive law which represent loss of stiffness and relative energy dissipation respectively.

$$t_2 = (1 - d)K\delta_2 + Dt^{fric} \quad (2.13)$$

Where,
$t_2$ is the shear traction along the interface.
$\delta_2$ is the displacement jump in the shear direction.
$t^{\text{fric}}$ is the Coulomb friction traction.

K is the initial dummy stiffness of the interface.

The friction traction $t^{\text{fric}}$ formulation is considered similar to one developed by Zou et al. [2] given in Equations 2.11 and 2.12. The damage parameters $d$ and $D$ are defined as given in Equations 2.14 and 2.15 respectively. The one dimensional version of the cohesive law with constant friction developed based on all these formulations will look as shown in the Figure 2.6.

\[
d = \frac{\delta^f (\delta_{eq} - \delta^0)}{\delta_{eq} (\delta^f - \delta^0)} \tag{2.14}
\]

\[
D = \frac{\delta_{eq} - \delta^0}{\delta^f - \delta^0} \tag{2.15}
\]

Where,

$\delta_{eq}$ is the equivalent displacement jump across the interface.

$\delta^0$ and $\delta^f$ are mixed mode equivalent displacement jumps at damage initiation and crack initiation respectively.

Figure 2.6: Cohesive law with friction adapted by Van der Meer et al. [1] from Zou et al. [2]

### 2.2.4 Incorporating rate dependency in the cohesive law

The interlaminar properties of the composite laminate vary with the rate of loading. Especially high rate loading situations such as blast highly influence the interlaminar properties [4]. Researchers in the literature found an increase in mode-I and mode-II fracture energies of the interface due to increase in rate of loading. On a contrary note, some researchers also found a decrease in interlaminar fracture toughness with rate of loading. [18,22,23].

There is no particular trend observed with regards to change in fracture toughness with the loading rate. There are various reasons reported in the literature which include material composition, geometry of the test specimen, problems with experimental measurements and data reduction scheme etc. There are four approaches in general to model rate dependencies in composite laminates: dynamic intensity factor (DIF) models,
damage-delay models, viscoplasticity models, and the viscoelasticity models. The first approach assumes that properties such as fracture toughness are functions of displacement jump rate [3].

In the second approach, the damage evolution equation is modified into a rate form which limits the damage rate to a certain maximum. The third approach uses an over-stress function along with the rate-independent softening law and combines components of viscoelasticity with the damage in rate-independent cohesive law [3]. Rate dependency is incorporated in the cohesive law by Liu et al. [3] and Wei et al. [10] with a Johnson-Cook law following the DIF approach for both cohesive strengths and fracture energies of independent modes of failure. The rate dependent cohesive strengths and fracture energies are as follows:

\[
\tau_i(\dot{\delta}) = \begin{cases} 
\tau_0^i & \dot{\delta} \geq \dot{\delta}_{i\text{ref}} \\
\tau_0^i \left(1 + c_i \ln \left(\frac{\dot{\delta}}{\dot{\delta}_{i\text{ref}}^r}\right)\right) & \dot{\delta} < \dot{\delta}_{i\text{ref}}
\end{cases}
\] (2.16)

\[
G_i(\dot{\delta}) = \begin{cases} 
G_0^i & \dot{\delta} < \dot{\delta}_{i\text{ref}}^r \\
G_0^i \left(1 + m_i \ln \left(\frac{\dot{\delta}}{\dot{\delta}_{i\text{ref}}^r}\right)\right) & \dot{\delta}_{i\text{ref}}^r \leq \dot{\delta} \leq \dot{\delta}_{inf}^i \\
G_{inf}^i & \dot{\delta} > \dot{\delta}_{inf}^i
\end{cases}
\] (2.17)

Where,
'\(i\)' is the mode number (\(i = 1 \& 2\)).
\(\tau_0^i\) is the quasi-static cohesive strength.
\(G_0^i\) is the quasi-static fracture energy.
\(G_{inf}^i\) is the parameter introduced to bound the fracture energy.
\(c_i\) & \(m_i\) are the rate sensitivity constants of strength and fracture energy respectively.
\(\dot{\delta}_{i\text{ref}}^r\) is the reference displacement jump rate.
\(\dot{\delta}_{inf}^i\) is the limit displacement jump rate used to define the fracture energy bound (see Liu et al. [3] for details).

May et al. [20] observed from the experiments carried out on DCB specimens that the mode-I fracture toughness of composite material with increasing loading rate. Hence a piece-wise function (similar to DIF approach) for fracture toughness is introduced depending on the crack growth velocity. Also a lower threshold fracture toughness is added into the formulation to avoid negative fracture toughness at low velocities. The mode-I strength is introduced as a function of strain rate along the interface. The strength evolution equation is derived by fitting a parabolic function to the data representing strength vs strain rate on a semi-logarithmic scale. A reference strain rate is introduced in the formulations which defines the quasi-static conditions. Shojaei et al. [18] introduced rate dependence again following an approach similar to DIF but the cohesive strengths are a function of applied strain rate, reference strain rate and temperature (assumed to be homogeneous) instead of the displacement jump rate.
2.3 Dynamic behaviour of composites

In this section, failure mechanisms in composite materials when subjected to dynamic loads are discussed. Brief information about modelling the failures in composites such as matrix cracking and fiber failure is also provided.

2.3.1 Failure mechanisms in composites due to dynamic loads

Various experiments have been conducted by researchers to investigate the failure mechanisms of composite materials subjected to dynamic loads with high rates such as blast. Most of them have reported three major failure processes: delamination, matrix cracking and penetration. The resistance to delamination of composite laminates decreased with an increase in thickness and density. The composite laminates with low thickness are tested with small PE4 charges (charge mass: 0.5-1.0 g) and stand-off distances of 54 to 500mm. Delamination and fiber fracture are observed at the edges of the composite panel due to effect of boundary conditions. The damage developed in clamped laminates is considerably higher than the damage in its adhesively bonded counterparts. Lesser fiber damage is observed at the supports for carbon fiber composites when compared with glass fiber composites [5].

Glass and Carbon fiber reinforced composite laminates with alternate 0° and 90° arrangement with 50% fiber volume fraction are subjected to blast loading with PE4 detonations with a stand-off distance of 90 mm. With the use of low charge masses, small fiber cracks are observed on rear surface while an array of fine cracks on loaded surface. Rear surface fiber fracture, front surface fiber buckling and extensive shear damage were observed when mass of charges is increased (pictures of failures in Figure 2.7) [5].

![Figure 2.7: Failures in carbon fiber reinforced composite laminate subjected to blast load](image)

Upon using the maximum mass of charges for testing in the study produced top surface fiber fracture, small amounts of delamination and fiber fracture in bottom most plies due to flexural response of the laminate. The top surface fiber fracture extended through the laminate thickness and a small delamination region is observed in the mid plane. It is a surprise that only small delamination regions are observed but this is due to high interlaminar fracture toughness. This is not always good especially in situations like the blast loading because it reduces the energy absorbing capabilities of the laminate. It
is also observed that fiber damage propagates immediately after a certain threshold for onset of damage is reached [5].

The glass fiber reinforced laminates showed better resistance to blast loading when compared carbon fiber reinforced laminates because the former ones exhibited a delamination damage at high shock pressures and the latter ones failed in a brittle way under similar loading conditions. Coating composite surfaces with materials like Poly urea or including additives such as metallic fillers, nanoclays, carbon nanotubes, urea formaldehyde etc. improves their blast and ballistic impact resistance (Poly urea is an elastomeric polymer). Pressure dependent high compressive stiffness, high dissipation, low tensile stiffness and residual stiffness at high deformations are the typical properties of elastomeric materials which serve for their advantage [5,7].

In cohesive zone models (discussed in section 2.2), the effect of compressive stress in the through-thickness direction on damage initiation and crack propagation is generally ignored. This might lead to errors in the results obtained because it is proven that the interlaminar strength and fracture toughness are enhanced under compression. The direct effect of this compressive stress on the interface is the development of friction which delays the delamination process by dissipating the applied energy [2].

Composite laminates containing four unidirectional plies made out of vinylester resin and Devold glass fibers are subjected to underwater blast loading and the damage patterns such as fiber pull-out, interlaminar debonding and matrix cracking can be seen in Figure 2.8 [7].

![Figure 2.8: Failure patterns of composite laminate subjected to underwater blast load](image)

The effect of changing the fiber types on the quasi-static to high rate dynamic response of composite laminate is studied by using two different fiber types: glass and carbon fibers. It is observed that glass fiber reinforced laminates exhibited a progressive damage and carbon fiber reinforced composites failed catastrophically. Upon studying the blast response of glass-fiber sandwich panels with real explosives at varying stand-off distances and fixed at edges, it is observed that the damage initiated at the front skin leading to local delamination and shear induced failure at the core. Small damaged areas in the interface between front skin and core is also observed. The tests conducted to study the strain rate dependence of glass fiber and carbon fiber reinforced polymers at varying strain rates from $10^{-3}$ s$^{-1}$ to 450 s$^{-1}$ revealed that dynamic strength of the fiber reinforced polymers increased with an increase in the strain rate while strain to failure reduced [4].

High strength composites such as polymethylmethacrylate have unsatisfactory strain energy absorption capabilities due to their insufficient plastic deformation capability. This
The study to explore the pure transverse compression and pure in-plane shear behaviour of unidirectional AS4/PEEK composite and neat PEEK resin at quasi-static to medium rates of loading revealed that increase in transverse compression and in-plane shear strength of composite is similar to increase in the same properties of neat resin [25]. An optical study of loading rate effects on the fracture behavior of carbon-epoxy T800/3900-2 reported an increase in material fracture toughness with increase in dynamic loading rate and decreased with increase in degrees of anisotropy for quasi-static and dynamic loading [25].

Koerber et al. [25] conducted experiments to characterize the high strain rate off-axis, transverse compression and in-plane shear behaviour of unidirectional carbon-epoxy IM7-8552 composite using digital image correlation. It is observed that the yield strength and the failure strength had a considerable influence from the change in rate of loading but the modulus of elasticity is less influenced in all three loading cases transverse compression, in-plane shear and the combination of both. Different failures observed in the off-axis specimens subjected to transverse compression are documented. The observed failure behavior of specimens is almost same at all strain rates. All specimen sizes used in the testing are 20 mm x 10 mm x 4 mm. The kink band formation in 15° off-axis specimen, strain localisation in 45° off-axis specimen and dynamic failure mode of 15° and 45° specimens are shown in Figures 2.9, 2.10, 2.11 and 2.12 respectively.

Figure 2.9: Kink band formation in 15° off-axis specimen subjected to quasi-static load [25]

Tsai et al. [26] worked on the strain rate effects on the in-plane shear strength unidirectional composite laminates and concluded that in-plane shearing is a dominant failure
2.3. Dynamic behaviour of composites

Figure 2.10: Strain localisation in 45° off-axis specimen subjected to quasi-static load [25]

mode in 30° and 45° off-axis angled composite specimens for strain rates up to 600 s⁻¹. There is negligible effect of transverse normal stress on in-plane shear strength. It is also observed that the shear strength of the laminate increased with an increase in the strain rate. The in-plane shear failure in 30° off-axis specimen is shown in Figure 2.13.

In case of low-velocity impacts, delamination started at mid thickness but is within limited area. But due to high-velocity impacts, delamination started at mid thickness and propagated towards the rear end of the laminate. Carbon/epoxy composite laminates tested for their drop weight impact and compression after impact behaviours with different thicknesses revealed that laminates with relatively smaller thickness exhibited elastic instability regardless of the fiber orientations and overall dimensions. Through-thickness position and delamination size affect the compressive strength of composites [11]. Sandwich panels are widely used now-a-days against blast loading because of their crushable core that can dissipate large amounts of energy, could attenuate impulse transmitted to back-side face sheet preventing a catastrophic failure of the composite panel. Four monolithic solid panels tested for varying range of impulses exhibited different crucial failure mechanisms including delamination, fiber and matrix cracking in plies and foam crushing [10].

The longitudinal compressive strength comparatively lesser when compared with the longitudinal tensile strength and this fact is used in the design of composite structures. Longitudinal tensile strength of composites has almost no effect of strain rate on it but the longitudinal compressive strength does show strain rate dependency. With an increase in the strain rate, an increase in the longitudinal compressive strength is observed [27].

Standard Double Cantilever Beam (DCB) test procedure to used to find the mode-I
interlaminar fracture toughness of composite laminate but this procedure is subject to asymmetric loading on the arms at relatively high loading rates and flexural inertial effects. Wedge-Insert Fracture (WIF) method can be used instead of DCB to mitigate dynamic effects and mixed mode crack propagation problems. Modified Wedge-Insert Fracture (MWIF) method can be used instead of simple WIF method which introduces rollers attached to the specimen arms eliminating the problem of friction between the crack surfaces and the wedge. The rate sensitivity of unidirectional CFRP is dominated by the toughness of the matrix. A matrix which is brittle had less effect of loading rate than composites with tough matrix. There is no particular trend observed in the fracture toughness of composite with increasing loading rate due to the complexity involved in dynamic testing of composites but the crack initiation toughness increased with the loading rate. With the increase in applied velocity, the brittleness of resin increased [28].

Fiber Metal Laminates (FMLs) consist of multiple components such as aluminium alloy (typically) and alternate glass/carbon fiber reinforced polymer material. Due to presence of these components, there is a possibility of damage modes such as plastic deformation of metal, matrix cracking, fiber failure interlaminar and interfacial delamination. THE low velocity impact tests on FMLs revealed that metal layers prevent impactor penetration and delamination propagation. FMLs with carbon fibers majorly absorbed energy through penetration and perforation and glass fiber FMLs dissipated energy through plastic deformation and delamination. The fiber orientation in FMLs have negligible effect on the size and shape of delamination. FMLs fabricated with alu-
minium sheets having higher yield strengths can improve their impact resistance. There are a bunch of other factors that influence the behaviour of FMLs such as fiber type, size effects, metal constituents, ply stacking sequence etc [16].

Tests on dynamic delamination of an L-shaped composite laminate used in air-craft wings revealed that when the crack tip speed in the material exceeds the shear wave velocity, the propagating delamination becomes *intersonic*. Propagation of intersonic delamination is found to produce shear mach waves in the material. The interfacial fracture toughness and strength logarithmically increase with crack tip speed/crack tip separation velocity [8]. Several failure mechanisms occur in ultra-high molecular weight polythene (UHMWPE) composites during ballistic impact such as shear plugging, delamination and bulge formation at rear end of the panel. UHMWPE are used in manufacturing protective vests such as helmets. Rate effects can be captured in impact applications if cohesive interface elements are used [20].

FMLs predominantly behave elastically and hence their response to deformations and damage differs from other composite materials. FMLs can be divided into two categories based on the number of layers and ply thickness. The thinner FMLs behave like metal plates and thicker ones exhibited diamond shaped back face debonding. In FMLs subjected to localised blast loading, high transverse velocities are induced causing compression in through thickness direction, leading to inelastic deformation of metal layers and failure of composite layers. The main damages observed are debonding between the metal layers and blocks, fiber breakage (in composite layers) and petalling of metal layers [17].

### 2.3.2 Modelling matrix cracks and fiber failure

The Hashin’s failure criteria [29] is used by Tran et al. [7], Wei et al. [10], Hou et al. [11] and Gargano et al. [12] to model matrix crack and fiber failure initiations. Hashin’s
model provides four material damage mechanisms: fiber failure in tension, fiber failure in compression, matrix damage in tension and matrix damage in compression. Note that the Hashin’s failure criteria are not only specific to dynamic failures but can be used in all loading situations including both quasi-static and dynamic loads to model the damage initiations in matrix and fiber. In all the equations for modelling these failures, the x-axis is in the fiber direction and y-axis is perpendicular to it (a local co-ordinate system of the ply). The damage initiation initiation equations for all four modes of failure are given in Equations 2.18 to 2.21 [10].

Fiber failure in tension \((\sigma_{11} \geq 0)\):

\[
\left( \frac{\sigma_{11}}{X_t} \right)^2 + \alpha \left( \frac{\sigma_{12}}{S_L} \right)^2 \geq 1 \quad (2.18)
\]

Fiber failure in compression \((\sigma_{11} \leq 0)\):

\[
\left( \frac{\sigma_{11}}{X_c} \right)^2 \geq 1 \quad (2.19)
\]

Tensile damage of matrix \((\sigma_{22} \geq 0)\):

\[
\left( \frac{\sigma_{22}}{Y_t} \right)^2 + \left( \frac{\sigma_{12}}{S_L} \right)^2 \geq 1 \quad (2.20)
\]

Compressive damage of matrix \((\sigma_{22} \leq 0)\):

\[
\left( \frac{\sigma_{22}}{2S_T} \right)^2 + \left[ \left( \frac{Y_c}{2S_T} \right)^2 - 1 \right] \left( \frac{\sigma_{22}}{Y_c} \right) + \left( \frac{\sigma_{12}}{S_L} \right)^2 \geq 1 \quad (2.21)
\]

Where,

\(\sigma_{11}, \sigma_{22} \& \sigma_{12}\) are components of the stress tensor.

\(X_t \& X_c\) are longitudinal (fiber direction) tensile and compressive strengths respectively.

\(Y_t \& Y_c\) are transverse tensile and compressive strengths respectively.

\(S_L \& S_T\) are longitudinal and tensile shear strengths respectively.

Phadnis et al. [4] modelled the fiber failure initiation using the Hashin’s failure criteria but the matrix damage using the Puck’s failure criterion. Puck’s criterion is chosen because the failure of epoxy matrix is better estimated in this case when compared with the Hashin’s criteria. The Puck’s failure criterion equation for modelling matrix failure initiation is given in Equation 2.22.

As the matrix is subjected to high strain rates during a blast event, Phadnis et al. [4] modelled the properties of the polymer matrix material vary with the strain rate/loading rate. This primarily effects the transverse direction of the ply where most of the load is taken by the polymer matrix. The in-plane elastic modulus and shear strength of the matrix are represented in terms of the applied strain rate by constructing regression curves using the experimental data as given in Equation 2.23 and 2.24 respectively.

\[
\left[ \left( \frac{\sigma_{11}}{2X_{tt}} \right)^2 + \left( \frac{\sigma_{22}}{X_{2t} \cdot X_{2c}} \right)^2 + \left( \frac{\sigma_{12}}{S_{12}} \right)^2 \right] + \sigma_{22} \left( \frac{1}{X_{2t}} + \frac{1}{X_{2c}} \right) \geq 1 \quad (2.22)
\]
2.3. Dynamic behaviour of composites

Where,

\( X_1 \), \( X_2 \) \& \( X_2c \) are tensile failure stress in fiber direction, tensile failure stress and compressive failure stress in transverse direction respectively.

\( S_{12} \) is the shear failure stress of the unidirectional ply.

\[
E(\dot{\varepsilon}) = E(\dot{\varepsilon}_0) \left[ m_e \log \left( \frac{\dot{\varepsilon}}{\dot{\varepsilon}_0} \right) + 1 \right] \quad (2.23)
\]

\[
F(\dot{\varepsilon}) = F(\dot{\varepsilon}_0) \left[ m_f \log \left( \frac{\dot{\varepsilon}}{\dot{\varepsilon}_0} \right) + 1 \right] \quad (2.24)
\]

Where,

\( \dot{\varepsilon}_0 \) is the reference strain rate.

\( m_e \) \& \( m_f \) are the curve fitting parameters.

After matrix damage and fiber failure initiations are predicted the degradation of properties of the matrix and fiber material is modelled by Phadnis et al. [4], Tran et al. [7] and Wei et al. [10] using a Continuum Damage Modelling (CDM) approach. The effective stress tensor \( \hat{\sigma} = M\sigma \) is calculated using the nominal stress tensor \( \sigma \) and damage operator \( M \) defined as follows [10]:

\[
M = \begin{bmatrix}
\frac{1}{1-d_f} & 0 & 0 \\
0 & \frac{1}{1-d_m} & 0 \\
0 & 0 & \frac{1}{1-d_s}
\end{bmatrix}
\]

Where,

Fiber damage variable, \( d_f = \begin{cases} 
\frac{\sigma_{11}}{d^f_t}, & \text{fiber tensile damage variable} \\
\frac{\sigma_{11}}{d^c_f}, & \text{fiber compressive damage variable}
\end{cases} \quad \sigma_{11} \geq 0 \\
\begin{cases} 
\frac{\sigma_{11}}{d^f_c}, & \text{fiber tensile damage variable} \\
\frac{\sigma_{11}}{d^c_c}, & \text{fiber compressive damage variable}
\end{cases} \quad \sigma_{11} < 0
\]

Matrix damage variable, \( d_m = \begin{cases} 
\frac{\sigma_{22}}{d^t_m}, & \text{matrix tensile damage variable} \quad \sigma_{22} \geq 0 \\
\frac{\sigma_{22}}{d^c_m}, & \text{matrix compressive damage variable} \quad \sigma_{22} < 0
\end{cases}
\]

Shear damage variable, \( d_s = 1 - (1 - d^f_t)(1 - d^c_f)(1 - d^t_m)(1 - d^c_m) \)

The fracture toughness in each of the failure modes are considered, to evaluate the damage variables by assuming a bilinear softening model during damage evolution [10].
3

Simplifying the FE model

In this chapter, the changes in the overall behavior of the composite specimens with alternate 0° and 90° ply arrangement and off-axis angles 30°, 45° and 60° due to a simplification from the cumbersome discrete laminate configuration model to a smeared laminate configuration model are discussed in detail.

3.1 Overview of the problem & initial approach

The thesis work started with investigating the quasi-static delamination failure behaviour of the composite specimen used in the simplified blast loading test setup. It can be seen from Figure 1.1 that the simplified test specimen after being cut from the original specimen, consists of alternate 0 and 90° (fiber angle with respect to the longitudinal direction of ply) ply layup at an angle. The angled cut makes the plies off-axis with respect to the global co-ordinate system of the specimen. The two important aspects of the test specimen that need special attention are the off-axis nature based on the angle of cut and friction development due to compressive traction on the interface.

The cross-section size of the simplified specimen that will be experimented is approximately chosen as 60 mm x 20 mm. The thickness of individual plies used for fabricating the simplified test specimen is 1 mm approximately. For the preliminary investigations, only the interface at the center of the specimen is modelled in such a way that it fails due to delamination and all other interfaces remain intact. The mechanical properties of the composite plies (bulk material) and the interface for this particular study are taken from [30] and [31] as mentioned in Tables 3.1 and 3.2 respectively.

In the initial stages, the research work is focused on reducing the cumbersome efforts required to model individual plies in the laminate. For that purpose, initial trials are carried out by scaling down the model size to 15 mm x 5 mm and 30 mm x 10 mm which are equal to a quarter and half of the actual specimen size (60 mm x 20 mm) chosen for the experimental studies. The FE models are developed by assuming a 2D plane-strain situation of cross-section of the simplified test specimen.
Table 3.1: Mechanical properties of the composite plies (bulk material properties)

<table>
<thead>
<tr>
<th>Property</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Elastic modulus $E_1$</td>
<td>51580 MPa</td>
</tr>
<tr>
<td>$E_2 = E_3$</td>
<td>12770 MPa</td>
</tr>
<tr>
<td>Poisson’s ratio $\nu_{12} = \nu_{31}$</td>
<td>0.29</td>
</tr>
<tr>
<td>$\nu_{23}$</td>
<td>0.37</td>
</tr>
<tr>
<td>Shear modulus $G_{12} = G_{31}$</td>
<td>7030 MPa</td>
</tr>
<tr>
<td>$G_{23}$</td>
<td>4660 MPa</td>
</tr>
<tr>
<td>Density $\rho$</td>
<td>1893 kg/m$^3$</td>
</tr>
</tbody>
</table>

Table 3.2: Mechanical properties of the interface

<table>
<thead>
<tr>
<th>Property</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cohesive strength $f_{2t}$ (Mode-I)</td>
<td>50 MPa</td>
</tr>
<tr>
<td>$f_{12}$ (Mode-II)</td>
<td>200 MPa</td>
</tr>
<tr>
<td>Fracture energy $G_{Ic}$ (Mode-I)</td>
<td>2.5 N/mm</td>
</tr>
<tr>
<td>$G_{IIc}$ (Mode-II)</td>
<td>10 N/mm</td>
</tr>
<tr>
<td>Coefficient of friction $\mu_{static}$</td>
<td>0.3</td>
</tr>
<tr>
<td>$\mu_{dynamic}$</td>
<td>0.1</td>
</tr>
</tbody>
</table>

Two laminate configurations are chosen for the modelling simplification purpose: Discrete (DIS) specimen and Smeared (SM) specimen. In the DIS specimen, all plies of the composite laminate with alternate 0 and 90° layup are explicitly modelled. In the SM specimen, the plies adjacent to the critical interface (the one at the center of the specimen) are modelled explicitly and the remaining plies are modelled as combined bulk material smeared over the region. The properties of the smeared bulk material are found by assuming an alternate infinite stack up of 0 and 90° plies and averaging their stiffness. The averaged mechanical properties of the smeared bulk composite material are given in Table 3.3. It is made sure that the stacking sequence of plies in both laminate configurations is similar so that there can be a proper comparison of the results obtained. The two laminate configurations chosen for simplifying the FE model are as shown in Figure 3.1.

Table 3.3: Averaged mechanical properties of the smeared bulk material

<table>
<thead>
<tr>
<th>Property</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Elastic modulus $E_1 = E_3$</td>
<td>34252 MPa</td>
</tr>
<tr>
<td>$E_2$</td>
<td>14411 MPa</td>
</tr>
<tr>
<td>Poisson’s ratio $\nu_{12} = 0.1765$ and $\nu_{31} = 0.1780$</td>
<td>$\nu_{23} = 0.4024$</td>
</tr>
<tr>
<td>Shear modulus $G_{12} = G_{23}$</td>
<td>5846 MPa</td>
</tr>
<tr>
<td>$G_{23}$</td>
<td>7030 MPa</td>
</tr>
</tbody>
</table>

3.2 FE modelling details

In a 2D plane-strain situation of cross-section of the simplified test specimen, the 0° ply in both laminate configurations is modelled as an isotropic material due to absence of the in-plane load carrying contribution of fibers. The 90° ply is modelled as a transversely orthotropic material with fiber orientation equal to the off-axis angle of the specimen. The combined bulk material in the SM configuration is modelled as a fully orthotropic material with material properties (given in Table 3.3) derived by averaging the stiffness and fiber orientation equal to the off-axis angle of the specimen.

The critical interface at the center of specimens in both configurations is modelled using the cohesive law with friction adapted by Van der Meer et al. [1] from Zou et al. [2]. A cohesive law with friction is used in this study to model delamination failure of the interface because the compressive traction on the interface generates friction which will affect the failure process. The mathematical formulations involved in this cohesive law are
3.2. FE modelling details

Figure 3.1: Laminate configurations chosen for simplifying the FE model: (a) Discrete (DIS) specimen (b) Smeared (SM) specimen.

given in section 2.2.3 of the literature review. The numerical analysis of the specimens is carried out based on the FE method. FE models of the simplified test specimen as shown in Figure 1.1 are developed. The composite plies and combined bulk materials in the laminate configurations are meshed using the three-noded triangular finite elements. The interface is meshed with four-noded interface elements consisting of two coinciding nodes two-noded line elements.

Figure 3.2: Typical FE meshes used in case of 15 mm x 5 mm specimen size for laminate configuration: (a) DIS (b) SM.

Figure 3.3: Typical FE meshes used in case of 30 mm x 10 mm specimen size for laminate configuration: (a) DIS (b) SM.

The X-translational degree of freedom of the left edge (X-direction is along the maximum dimension of the specimen) in all the FE models is fixed. The Y-translation degree of freedom of the bottom node in the left edge is also fixed to prevent rigid body motion. A uniform negative displacement is applied to the right side edge to simulate compressive loading. The typical FE meshes used to analyze both laminate configurations and in case of scaled down cross-section sizes 15 mm x 5 mm and 30 mm x 10 mm are as shown in Figures 3.2 and 3.3 respectively (meshes shown are used for 45° off-axis angled composite laminate). The mesh size scaling factors while generating the meshes are chosen to be the same for both configurations belonging to a particular specimen size which would facilitate a better comparison of results. The dissipation based arc-length solver with a Newton-Raphson iterative procedure is used for following the quasi-static equilibrium path. As there are no deliberately introduced damages in the specimens (such as notches),
arc-length based solver is used in these problems because a snap-back type of behaviour is expected in the equilibrium path. The FE analyses are carried out by programming in C++ using the JemJive modelling environment.

### 3.3 Interface tractions & load-displacement plots

#### 3.3.1 Interface tractions

The FE models of the simplified test specimen with different off-axis angles and laminate configurations are developed as explained in section 3.2. The off-axis angles of specimens chosen for the current investigation are 30°, 45° and 60°. The shear and normal tractions at damage initiation point along the critical interface obtained after running the FE analyses in case of all three off-axis angles and 15 mm x 5 mm cross-section specimen size are as shown in Figure 3.4. The x-axis in all the traction plots represents the x-coordinate of points on the interface. In every traction plot, there are three sub-plots which are the shear/normal tractions along the critical interface for different specimen off-axis angles as indicated in the figures.

![Figure 3.4](image)

Figure 3.4: Shear and Normal tractions along the critical interface at damage initiation point in case of 15 mm x 5 mm specimen size and all three off-axis angles for laminate configuration: (a) DIS (b) DIS (c) SM (d) SM.

It can be observed from Figures 3.4 (a) and (c) that the shear traction distribution in 30° off-axis angled specimen for both DIS and SM laminate configurations is non-uniform
and the difference between the top and the bottom points of the interface is high (close to 175 MPa) due to lower off-axis angle. The bottom most point of the interface is subjected to higher shear traction, hence the interface damage starts from this point. The damage initiation at the bottom point happens when the shear traction at that point reaches 200 MPa which is equal to the mode-II cohesive strength of the interface. Hence it can be understood that the delamination failure type in this problem is purely mode-II in nature.

After damage initiation at the bottom most point, the shear traction at the same point starts to reduce. At crack initiation (also at the bottom most point of the interface), the shear traction at the bottom most point of the interface reaches zero which means that this point of the interface completely failed (or cracked). After the bottom most fails completely, the points on the interface adjacent to it start to crack. Even after the initiation of crack in the interface, the shear traction doesn’t go to zero but friction develops along the interface which delays the delamination failure. The development of friction in 30° DIS and SM specimens after cracking of interface can be visualized from Figures 3.5 (a) and (c) respectively. The distribution of shear and normal tractions in case of 30° off-axis angled specimen at damage initiation as shown in Figure 3.4 is non-uniform and irregular in case of DIS configuration and comparatively smoother in SM configuration. From Figures 3.5 (b) and (d), it can be observed that the normal traction on the interface can be seen even after cracking and this is due to the effect of applied load on the specimen.

Upon looking at Figures 3.4 (a) and (c), it can be observed that the shear traction distributions in 45° off-axis angled DIS and SM specimens are non-uniform and the shear traction difference between the top and bottom points of the interface reduced (close to 75 MPa) when compared with their 30° counterparts due to increase in the off-axis angle. The bottom point of the interface in case of 45° specimen is subjected to higher shear traction and hence the damage initiates from the bottom. In case of the 60° off-axis angled specimens, the shear traction difference between the top and bottom points of the interface is close to 125 MPa but the damage initiates from the top point of the interface. The reason behind this change of damage initiation point is because of higher shear traction at the top of the interface which is in turn due to the off-axis angle of the specimen being greater than 45°. The process of delamination failure in case of 45° and 60° specimens is completely similar to the 30° specimen but in the former cases, damage initiates in major part of the interface (uniformly damaging interface) before crack propagation starts. But in case of 30° specimens, higher traction differences cause the failure of the bottom most point even before damage in the other parts of the interface starts to initiate. The delamination failure type in case of 45° and 60° specimens is also purely mode-II in nature. Development of friction after cracking of the interface in 45° and 60° specimens can also be seen in Figures 3.5 (a) and (c).

Figures 3.4 (a) and (b) reveal that the shear and normal traction distributions of DIS configuration in case of 45° and 60° off-axis angles are irregular when compared with their SM configuration counterparts in Figures 3.4 (c) and (d). Hence it can be concluded that one reason behind this irregular traction distributions of DIS specimen is the laminate configuration. But this irregular traction distribution nature becomes comparatively smoother as the off-axis angle is increased. So the off-axis angle of the specimen also has an effect on the traction distributions. In 45° and 60° off-axis angled specimens, the damage and crack initiations are delayed when compared with the 30° specimens again due to higher off-axis angle which results in a lower shear traction difference.
Figure 3.5: Shear and Normal tractions along the critical interface at a random point on the failure branch in case of 15 mm x 5 mm specimen size and all three off-axis angles for laminate configuration: (a) DIS (b) DIS (c) SM (d) SM.

The traction plots in case of both DIS and SM laminate configurations for different off-axis angles look similar qualitatively and quantitatively except that the traction distributions in case of SM configuration are comparatively smoother than their DIS configuration counterparts along with some other minor differences. These differences generally arise due to the difference in stress concentrations along the interface (different laminate configurations). By observing Figures 3.4 (a) and (c), it can be noted that the traction difference between the top and bottom point of the interface reduces as the off-axis angle increases from 30° to 45° and completely changes in case of 60° specimens.

The traction distributions in case of 30° and 45° specimens cause the damage and crack initiations from the bottom of the interface and from the top in case of 60° specimens. Also the traction distribution completely flips when the off-axis angle changes from 45° to 60°. Figures 3.5 (a) and (c) show that the magnitude of friction developed along the interface increases as the off-axis angle of the specimen increases due to increase in the magnitude of normal traction on the interface which is directly related to the friction (see Figures 3.5 (b) and (d)). There are not many differences that can be observed between the shear and normal tractions of DIS and SM laminate configurations for different off-axis angles but some irregularities in DIS specimens and other negligible variations. Hence it can be concluded that in case of 15 mm x 5 mm specimen size, the tractions along the
3.3. Interface tractions & load-displacement plots

interface for both laminate configurations are same qualitatively and quantitatively with some negligible differences.

To analyze the effect of size on the delamination failure behaviour, cross section dimensions of the specimen are doubled. The size of the simplified test specimen that is analyzed in this section is 30 mm x 10 mm. Again the three off-axis angles that are considered for this analysis are 30°, 45° and 60°. FE models are developed by following the same procedure as explained in section 3.2. These specimens also have only one critical interface at the center of the specimen which fails during the loading process and all other interfaces remain intact.

![Graphs showing shear and normal tractions along the critical interface at damage initiation point in case of 30 mm x 10 mm specimen size and all three off-axis angles for laminate configuration: (a) DIS (b) DIS (c) SM (d) SM.](image)

Figure 3.6: Shear and Normal tractions along the critical interface at damage initiation point in case of 30 mm x 10 mm specimen size and all three off-axis angles for laminate configuration: (a) DIS (b) DIS (c) SM (d) SM.

The shear and normal traction plots obtained in case of 30 mm x 10 mm specimen size for off-axis angles 30°, 45° and 60° and both DIS and SM laminate configurations at damage initiation and a random point on failure branch of the equilibrium path are as shown in Figures 3.6 and 3.7. It can be observed from these figures that similar irregularities in traction distributions, damage and crack initiations in the specimen, development of friction, failure process and behaviour etc. observed and discussed in case of 15 mm x 5 mm specimen size are also applicable for the 30 mm x 10 mm specimens for both laminate configurations. So also in case of 30 mm x 10 mm specimen size, tractions along the interface in for DIS configuration are qualitatively and quantitatively closer.
to the SM configuration except for some minor differences. Based on observations from the trends of shear and normal tractions along the critical interface, it can be concluded that the delamination failure behaviour of DIS configuration can be replicated using SM configurations if minor (negligible) differences in their behaviors can be accepted.

Figure 3.7: Shear and Normal tractions along the critical interface at a random point on the failure branch in case of 30 mm x 10 mm specimen size and all three off-axis angles for laminate configuration: (a) DIS (b) DIS (c) SM (d) SM.

3.3.2 Load-Displacement plots

The load-displacement plots obtained in case of specimen sizes 15 mm x 5 mm and 30 mm x 10 mm for both laminate configurations and all three off-axis angles are as shown in the Figures 3.8 and 3.9 respectively. It can be observed from Figure 3.8 that the initial stiffness of the 30° off-axis angled DIS specimen with 15 mm x 5 mm cross-section size is close to the 30° SM specimen. On the other hand, the peak load in case of SM specimen is slightly lower (difference around 3% which can be neglected) than the DIS specimen. But the initial stiffness in case of 45° and 60° SM specimens are 6.89% and 16.27% higher than their DIS specimen counterparts. There is again a negligible difference of around 1% between the peak loads in case of 45° and 60° DIS and SM specimens. The common point that can be observed in the plots in Figure 3.8 is that the peak load of SM configuration for a particular off-axis angle is lower than that its DIS configuration counterpart. The
lower peak load in case of SM configuration is due to higher stress concentration along the critical interface.

Figure 3.8: Load-Displacement plots in case of 15 mm x 5 mm specimen size for both laminate configurations and all three off-axis angles

Figure 3.9: Load-Displacement plots in case of 30 mm x 10 mm specimen size for both laminate configurations and all three off-axis angles

The difference between the initial stiffness of DIS and SM configurations increases as the off-axis angle of the specimen increases. The reason behind this observation is found in the variation in strain distribution of the specimen due to the change in laminate configuration. There is an increase of 6.4% in the peak load of the specimen when the off-axis angle increased from 30° to 45°. But the peak load suddenly increased by 49.1% when the off-axis angle increased from 45° to 60°. It can also be observed that the initial stiffness of the laminate in a particular configuration (either DIS or SM) decreases with
an increase in the off-axis angle of the specimen. This is due to reduction in the stiffness of the plies with increase in the specimen off-axis angle. The post-peak load-displacement responses of both the configurations and various off-axis angles of the specimen all look similar. The snap-back behaviour observed in the specimens becomes sharper as the off-axis angle increases. Based on this observation, it can be concluded that the damage and crack initiations happen faster in the lower off-axis angled specimens but the damage development and crack propagation processes slow down relatively when compared with higher off-axis angles.

Similar observations and discussions as in case of 15 mm x 5 mm sized specimens are also applicable to 30 mm x 10 mm specimens. No special size effects in terms of the failure behaviours are observed upon doubling the cross-section size of the specimen. The peak loads in case of 30 mm x 10 mm specimen size increased when compared with their 15 mm x 5 mm counterparts. The peak load increased by 4.2% when the off-axis angle increased from 30° to 45° and by 48.7% when the angle increased from 45° to 60°. It can also be observed that load-displacement graphs in case of lower angled specimens (example 30°) in both specimen sizes look bulky due to higher energy dissipation through friction than through crack propagation (linked to larger interface lengths). From all these observations, it can be concluded that the DIS configuration behaviour in case of all three off-axis angles can be replicated by SM configuration if the differences in initial stiffness can be neglected.

3.4 Interface Shear to Pressure ratios

![Figure 3.10: SP ratio of 15 mm x 5 mm DIS specimen with 30° off-axis angle at: (a) Damage initiation (b) crack initiation (c) random point on failure branch.](image)

The Shear to Pressure (SP) ratio is defined as the ratio of shear traction to normal traction on the interface. This metric gives an idea about the generation of shear traction along the interface when compared with the normal traction and also about the development of friction during different phases of loading. The SP ratios at interface damage initiation point, interface crack initiation point and a randomly chosen point on the equilibrium path (same point chosen for presenting traction plots) in case of 15 mm x 5 mm DIS specimens along the interface with off-axis angles 30°, 45° and 60° are as shown in Figures 3.10, 3.11 and 3.12 respectively. It can be observed from Figures 3.10 (a), 3.11 (a) and 3.12 (a) that the SP ratios along the interface at damage initiation in case of all the off-axis angled specimens are non-uniform and irregular due to the shear and normal tractions being non-
uniform and irregular. These plots become more regular as the off-axis angle increases because of improved regularity in the normal traction plots.

Figure 3.11: SP ratio of 15 mm x 5 mm DIS specimen with 45° off-axis angle at: (a) Damage initiation (b) crack initiation (c) random point on failure branch.

Figure 3.12: SP ratio of 15 mm x 5 mm DIS specimen with 60° off-axis angle at: (a) Damage initiation (b) crack initiation (c) random point on failure branch.

There are peaks that can be observed in Figures 3.10 (b) and (c), 3.11 (b) and (c) and 3.12 (b) and (c) which can be due to high shear tractions or low normal tractions or both high shear and low normal tractions. By looking at the traction plots in Figures 3.4 to 3.7, it can be seen that the effect of the occurrence of high shear traction (since it is a pure Mode-II type failure) is dominant when compared with low normal traction. Also the combination of high shear and low normal tractions occurred in some cases. Hence these peaks occur at a point on the interface which is going to fail because of high shear traction. There is a very high peak in the SP ratio of 30° off-axis angled specimen which can be observed in Figure 3.10 (c). The region around this peak indicates the location of the cohesive zone at that particular point on the equilibrium path. The presence of the cohesive zone leads to an occurrence of very high shear traction and very low normal traction at the same time, resulting in a very high peak in the SP ratio.

It can be observed from Figures 3.10, 3.11 and 3.12 that the SP ratios change as the off-axis angle changes. The increase in normal tractions is higher when compared with the shear tractions along the interface as the off-axis angle of the specimen increases. Hence the SP ratios reduce with an increase in the off-axis angle. Looking closely at Figures 3.10 (c), 3.11 (c) and 3.12 (c) reveals that the value of SP ratio for a particular length of the interface is equal to 0.3 which is the static coefficient of friction. So after complete cracking what remains along the interface is the pure frictional traction.
Chapter 3. Simplifying the FE model

Figures 3.13, 3.14 and 3.15 are the SP ratios of 30 mm x 10 mm DIS specimens with off-axis angles 30°, 45° and 60° at same points on the equilibrium path chosen in case of 15 mm x 5 mm specimens. It can be observed from Figures 3.13 (a), 3.14 (a) and 3.15 (a) that the extent of values in these SP ratio plots are equal to their counterparts in case of 15 mm x 5 mm size specimens at damage initiation. The reason behind this observation is that the interface doesn’t play any role in the behaviour (elastic) of the specimen before damage initiation and the SP ratio is only governed by off-axis angle of the specimen. But the plots are non-uniform irregular and become smoother as the off-axis angle increases due to the same reason that the shear and normal tractions are non-uniform and irregularly distributed. Unlike being very irregular, the SP ratios of a particular length of interface at the center are constant due to normal and shear tractions being almost constants at the same points. This is due to the increase in the length of interface because of the increase in specimen size. The observations regarding the effect of off-axis angles on the SP ratios and pure friction development after cracking in case of 15 mm x 5 mm specimens can also be seen in 30 mm x 10 mm specimens. Figures 3.13 (b) & (c), 3.14 (b) & (c) and 3.15 (b) & (c) reveal that the SP ratios in case of 30 mm x 10 mm are lesser than their 15 mm x 5 mm counterparts. This is due to change in the trend of traction plots at the center of interface because of increase in the interface length. It can be concluded from this observation that size of the specimen influences the traction distributions and in turn the SP ratios along the interface.

Figure 3.13: SP ratio of 30 mm x 10 mm DIS specimen with 30° off-axis angle at: (a) Damage initiation (b) crack initiation (c) random point on failure branch.

Figure 3.14: SP ratio of 30 mm x 10 mm DIS specimen with 45° off-axis angle at: (a) Damage initiation (b) crack initiation (c) random point on failure branch.

Figure 3.15: SP ratio of 30 mm x 10 mm DIS specimen with 60° off-axis angle at: (a) Damage initiation (b) crack initiation (c) random point on failure branch.
Figure 3.15: SP ratio of 30 mm x 10 mm DIS specimen with 60° off-axis angle at: (a) Damage initiation (b) crack initiation (c) random point on failure branch.
Parametric studies (quasi-static behavior)

The possibility of deriving the interface properties accurately from the overall quasi-static behavior of the simplified test setup can be assessed by performing the parametric studies. Hence the effects of varying the interface material properties such as strength, fracture energy, and friction coefficient on the quasi-static behavior of composite specimens with alternate 0° and 90° ply arrangement and off-axis angles 30°, 45° and 60° are analyzed in this chapter. The changes in the quasi-static behavior of the laminates with multiple critical interfaces due to a change in the thickness of the plies are also discussed.

4.1 Updated material properties

The simplified test setup of the special composite laminate that will be tested for its response against blast load (high rate dynamic loads in general) is fabricated for initial trials at the TNO laboratory. Before proceeding with actual experimentation, basic material properties of the composite such as the Young’s modulus, shear modulus and Poisson’s ratio of the fiber and resin materials, and the interface properties such as the Mode-I and Mode-II strengths and fracture energies along with the friction coefficients (both static and dynamic) are tested. It is observed that the material properties obtained from experiments and the ones taken from [30] and [31] which are considered for simplifying the FE model in chapter 3 are not the same.

Table 4.1: Updated mechanical properties of the composite plies

<table>
<thead>
<tr>
<th>Property</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Elastic modulus</td>
<td>$E_1 = 51580$ MPa, $E_2 E_3 = 9560$ MPa</td>
</tr>
<tr>
<td>Poisson’s ratio</td>
<td>$\nu_{12} = \nu_{31} = 0.29$, $\nu_{23} = 0.37$</td>
</tr>
<tr>
<td>Shear modulus</td>
<td>$G_{12} = G_{31} = 4670$ MPa, $G_{23} = 3490$ MPa</td>
</tr>
<tr>
<td>Density</td>
<td>$\rho = 1893$ kg/m$^3$</td>
</tr>
</tbody>
</table>

Because changing the material properties would affect the behavior of both discrete and smeared laminate configurations equally (verified), the updated material properties are used in the simulations from this point onwards i.e. from chapter 4 of the thesis.
Table 4.2: Updated mechanical properties of the interface

<table>
<thead>
<tr>
<th>Property</th>
<th>Value (Mode-I)</th>
<th>Value (Mode-II)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cohesive strength</td>
<td>$f_{2t}$</td>
<td>$f_{12}$</td>
</tr>
<tr>
<td></td>
<td>12 MPa</td>
<td>45 (67.50) MPa</td>
</tr>
<tr>
<td>Fracture energy</td>
<td>$G_{Ic}$</td>
<td>$G_{IIc}$</td>
</tr>
<tr>
<td></td>
<td>0.52 N/mm</td>
<td>2.3 N/mm</td>
</tr>
<tr>
<td>Coefficient of friction</td>
<td>$\mu_{static}$</td>
<td>$\mu_{dynamic}$</td>
</tr>
<tr>
<td></td>
<td>0.3</td>
<td>0.1</td>
</tr>
</tbody>
</table>

The updated mechanical properties of the composite plies and the interface are given in Table 4.1 and 4.2 respectively. The updated average mechanical properties (due to change in the basic material properties) of the smeared bulk composite material that will be used in modelling the SM laminate configuration are given in Table 4.3.

Table 4.3: Updated average mechanical properties of the smeared bulk material

<table>
<thead>
<tr>
<th>Property</th>
<th>Value (Mode-I)</th>
<th>Value (Mode-II)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Elastic modulus</td>
<td>$E_1 = E_3$</td>
<td>$E_2$</td>
</tr>
<tr>
<td></td>
<td>32216 MPa</td>
<td>10838 MPa</td>
</tr>
<tr>
<td>Poisson’s ratio</td>
<td>$\nu_{12}$</td>
<td>$\nu_{31}$</td>
</tr>
<tr>
<td></td>
<td>0.1388</td>
<td>0.1408</td>
</tr>
<tr>
<td></td>
<td>$\nu_{23}$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>0.3976</td>
<td></td>
</tr>
<tr>
<td>Shear modulus</td>
<td>$G_{12} = G_{23}$</td>
<td>$G_{31}$</td>
</tr>
<tr>
<td></td>
<td>4080 MPa</td>
<td>4670 MPa</td>
</tr>
</tbody>
</table>

4.2 Specimen behavior with updated properties

The change in properties of the composite material will definitely affect the behavior of the specimen. It can be observed from Tables 4.1 and 4.2 that the Young’s modulus perpendicular to the fiber direction and shear modulus of the composite ply have reduced when compared with the old properties (see Tables 3.1 and 3.2). This change will affect the stress concentration along the critical interface of the specimen. The interface strengths and fracture energies in modes-I and II have reduced (almost by four times) which leads to drastic changes in the delamination failure of the specimen. Hence, the failure behavior of the special composite laminate with updated material properties is studied to get an idea about the changes in the behavior of the specimen due to change in the properties.

Figure 4.1: Typical FE mesh used in case of 45° off-axis angled SM specimen with size 60 mm x 20 mm.

The quasi-static behavior of the composite specimen i.e. specimen behavior when subjected to quasi-static load will be investigated in this chapter. The cross-section size of specimen considered for all investigations in this chapter is 60 mm x 20 mm which is the actual specimen size used for performing the experiments. The thickness of plies is considered as 1 mm and delamination only occurs at the interface in the center of the
4.2. Specimen behavior with updated properties

specimen. The FE models of the specimen are developed by assuming a 2D plane-strain situation of its cross-section. The SM laminate configuration is adopted for modelling the specimen instead of an actual DIS configuration (which is rather tedious in this case) because it is concluded from investigations in chapter 3 that the behavior of a DIS configuration model can be replicated using its SM configuration counterpart except for some minor differences. The material properties given in Tables 4.1, 4.2 and 4.3 are used and the procedure explained in section 3.2 of chapter 3 is followed for developing the FE models of the specimen with off-axis angles 30°, 45° and 60°. The typical FE mesh used for analyzing the 45° off-axis angled SM specimen with size 60 mm x 20 mm is as shown in Figure 4.1.

Figure 4.2: Shear tractions along the critical interface of the specimen with updated material properties at different points in the equilibrium path for off-axis angles: (a) 30° (b) 45° (c) 60°.

The shear tractions along the specimen’s critical interface obtained after the FE analyses with updated material properties in case of off-axis angles 30°, 45° and 60° at different points in the equilibrium path are as shown in Figure 4.2. Different points on the equilibrium paths of the specimens mentioned in the figures are Damage Initiation Point (DIP), Crack Initiation Point (CIP), Failure Branch Point (FBP) and Uniformly Damaging Point (UDP). UDP 1 and 2 are two points on the equilibrium path where the interface damages uniformly. It can be observed from Figure 4.2 (a) and (b) that damage initiation in the 30° and 45° off-axis angled specimens happens from the bottom most point of the interface followed by crack initiation from the same point. The crack then propagates along the interface and also the development of pure friction can be visualized. The cohesive zone (also called as damage zone or fracture process zone) size in case of 45° specimen is close to the length of the interface which results in a uniform damage after a certain point of loading.

Figure 4.2 (c) reveals that the damage in 60° off-axis angled specimen starts from the top most point of the interface. The damage initiation in the specimen is followed by a uniform damage of the interface. This failure condition is named as “Uniformly Damaging Interface (UDI)” which is due to the cohesive zone size being greater than or equal to the length of the interface. UDI generally occurs due to a combination of small specimen size, low interface strength and high fracture energy. There is no crack growth in an interface that fails due to UDI condition but a uniform damage followed by a sudden cracking failure. There is no scope for the development of pure friction in such type of failures. It can be concluded from these observations that change in material properties might change the governing failure mechanism of the specimens. The governing failure
mechanism in a specimen is not only affected by the material properties but also by the off-axis angle because specimens with same size and different off-axis angles have different cohesive zone sizes and interface lengths.

The UDI failure can be better visualized by looking at the evolution of damage parameter along the critical interface with loading as shown in Figure 4.3. UDP 1, 2, 3 and 4 are four different points in the equilibrium path where the interface damages uniformly. It can be seen from Figure 4.3 that the damage parameter at all points along the interface when the damage initiates is equal to '0'. After damage initiation, the top most point of the interface seemingly has a higher damage parameter value in the initial stages of damage. But as the loading progresses, all points on the interface slowly start to damage with the damage parameter values at every point getting close to the neighbouring ones. The entire interface suddenly cracks when the damage parameter values at all the points on the interface reach '1'.

![Figure 4.3: Evolution of damage parameter along the critical interface with loading in case of 60° off-axis angled specimen](image)

The main focus in this thesis is placed on studying the failure mechanisms which involve delamination of the composite specimen due to cracking of the interface. It can be observed from these results that if the 60° specimen fails due to cracking, then the interface failure in 30° and 45° specimens definitely happens due to cracking. The reason behind this statement is that the cohesive zone size and interface length in case of the 60° specimen are the smallest among all the three off-axis angles 30°, 45° and 60°. As the material properties influence the governing failure mechanism of the specimen, the effect of varying the material properties will be studied in the subsequent sections to achieve the desired failure mechanism (cracking of the interface).

### 4.3 Strength variation for interface cracking

Generally when the interface strengths and fracture energies are tested from experiments, strength measurements are less unambiguous when compared with the energy measurements. It is stated by Turon et al. [32] and Ural et al. [33] that the cohesive zone size $l_{cz}$
can be predicted by the following expression:

\[ l_{cz} = ME \frac{G_c}{(\tau_0)^2} \]  \hspace{1cm} (4.1)

Where,
E is the Young's modulus of the material.
\( G_c \) is the critical energy release rate.
\( \tau_0 \) is the maximum interfacial strength.
M is a parameter that depends on the cohesive zone model.

From Equation 4.1, it can be understood that increasing the strength of the interface would result in a decrease in the cohesive zone size and increasing the fracture energy of the interface results in an increase in the cohesive zone size. As the composite specimens in this study undergo a pure mode-II type failure, two cases of increase in the strength of the interface are studied. In the first and second case, the mode-II interface strength is increased by 25% and 50% respectively and all other interface properties are same as given in Table 4.2. The increase in the strength of interface reduces the cohesive zone size which might lead to a switching of the failure mechanism from uniformly damaging interface to cracking interface failure (desired mechanism) especially in the 60\(^\circ\) specimen.

The evolution of damage parameter along the critical interface when the mode-II interface strength is increased by 25% and 50% of its original value and keeping all other interface properties same as updated ones in case of a 60\(^\circ\) off-axis angled SM specimen is as shown in Figure 4.4. FBP1, FBP2, FBP3 and FBP4 are four different points chosen on the failure branch of the equilibrium path to present the evolution of damage parameter when the mode-II interface strength is increased.

![Figure 4.4: Evolution of damage parameter in case of 60\(^\circ\) off-axis angled SM specimen by keeping all interface properties same as updated ones except the mode-II strength is increased by: (a) 25 \(\%\) (b) 50\(\%\).](image)

It can be observed from Figure 4.4 that in both cases of increasing the mode-II strength of interface by 25% and 50%, the failure mechanism of the specimen changes from a uniformly damaging interface to a cracking interface. The reason behind this change being the cohesive zone size in these cases is smaller than the interface length due to an increase in the mode-II interface strength. Upon looking closely at the damage parameter
evolution in case of increasing the mode-II strength by 25% in Figure 4.2 (a), it can be observed that the interface fails due to cracking up to half of its length approximately and then starts to damage uniformly.

But Figure 4.2 (b) shows that by increasing the mode-II strength by 50%, almost three-fourth of the interface cracks and the rest of it damages uniformly. This difference is mainly due to the cohesive zone size being smaller in case of increasing the mode-II interface strength by 50% when compared with the case in which it is increased by 25%. So it can be concluded from all these observations that the desired interface cracking failure mechanism can be achieved in both cases of increasing the mode-II interface strength by 25% and 50%. But considering an added advantage of extra cracking region in case of the increase by 50%, the mode-II interface strength is finalized as 67.50 MPa (updated value indicated in ( ) in Table 4.2). This increased mode-II interface strength value will be used as the final updated property in simulations from this point onwards in the study.

4.4 Mesh dependence study

With the newly updated mode-II interface strength, a mesh sensitivity study is carried out before proceeding with the actual parametric studies to check if there is any change in the behavior of the composite laminate due to change in the mesh size. Three different mesh sizes are chosen for meshing the FE models of the specimens with SM configuration and off-axis angles 30°, 45° and 60°. The load-displacement plots and specimen peak loads obtained in case of different mesh sizes for all three specimen off-axis angles are as shown in Figures 4.5 and 4.6 respectively. The representation of mesh sizes used in the legends on these figures is of the form "x/y". "x" in the representation means the finite element size in mm used to mesh the interface and the plies adjacent to it and "y" means the finite element size in mm for the smeared bulk material part in the SM configuration.

It can be observed from Figure 4.5 that the load-displacement plots obtained in the case of three different mesh sizes look very similar for specimen off-axis angles 30°, 45° and 60°. Figure 4.6 also shows that changes in the element size used to mesh the interface and the plies don’t have a considerable effect on the specimen peak load in the case of all three off-axis angles. Also, the specimen peak loads seem to approach a particular value upon mesh refinement. Hence, it can be concluded that the FE models of the composite specimen with different off-axis angles do not suffer from the mesh dependence problem and so a mesh size of "0.050/0.50" is chosen for carrying out the subsequent FE analyses.

4.5 Parametric studies on mode-II strength

In this part of the study, the effect of varying the mode-II strength of the critical interface on the quasi-static behavior of the composite specimens with different off-axis angles is investigated. For this purpose, four different mode-II strengths are considered: 56.25 MPa, 67.50 MPa, 78.75 MPa and 90.00 MPa which are equal to the actual strength of interface (= 45 MPa) increased by 25%, 50%, 75% and 100% respectively. All other interface properties remain same as the updated properties in Table 4.2. It is made sure that the interface fails due to cracking (desired failure mechanism) and not due to any other governing failure mechanism in different cases of the strength variation and for all the specimen off-axis angles.
4.5. Parametric studies on mode-II strength

The load-displacement plots obtained upon varying the mode-II cohesive strengths in case of specimen off-axis angles 30°, 45° and 60° are as shown in Figure 4.7. It can be observed from Figure 4.7 that the load-displacement plots obtained for different mode-II interface strengths in the case of all three specimen off-axis angles look the same except for the specimen peak load which increases with an increase in the strength. Figure 4.8 shows the variation of specimen peak load with the mode-II interface strength for specimen off-axis angles 30°, 45° and 60°. It is evident from Figure 4.8 that the specimen peak load increases linearly with an increase in the mode-II interface strength in case of all specimen three off-axis angles. This linear trend changes due to a change in the governing failure mechanism of the specimen.

The shear tractions along the critical interface at the DIP and at a random point on the failure branch of the equilibrium path (chosen to show friction development) obtained by varying the mode-II interface strength in case of specimen off-axis angles 30°, 45° and
Figure 4.7: Load-displacement plots for different mode-II interface strengths in case of specimen off-axis angle: (a) 30° (b) 45° (c) 60°.

Figure 4.8: Variation of specimen peak load with the mode-II interface strength for all three specimen off-axis angles.

60° are as shown in Figure 4.9 and 4.10 respectively. It can be observed from Figure 4.9 that the shear tractions along the critical interface at the DIP shifted to a higher range of values but their distribution remained the same due to an increase in the mode-II cohesive strength in case of all the specimen off-axis angles. In Figure 4.10, the continuous lines (highlighted regions) represent the cohesive zones or the damage zones or the fracture process zones. From Figure 4.10, it can be noticed that the cohesive zone size decreases with an increase in the mode-II cohesive strength in case of specimen off-axis angles 30°, 45° and 60°. This observation is in agreement with the statements made by Turon et al. [32] and Ural et al. [33] in their respective studies. It can also be observed that there is
a higher increase in the magnitude of pure friction due to an increase the mode-II interface strength in case of the composite specimen with higher off-axis angle.

Figure 4.9: Shear tractions along the critical interface at the DIP upon varying the mode-II interface strength for off-axis angles: (a) 30° (b) 45° (c) 60°.

Figure 4.10: Shear tractions along the critical interface at a random point on the failure branch of the equilibrium path upon varying the mode-II interface strength for off-axis angles: (a) 30° (b) 45° (c) 60°.

4.6 Parametric studies on mode-II fracture energy

The changes in the quasi-static behavior of composite specimens with off-axis angles 30°, 45° and 60° upon varying the mode-II fracture energy of the critical interface are studied in this section. Five different mode-II interface fracture energies are chosen for this analysis: 0.58 N/mm, 1.15 N/mm, 2.30 N/mm, 3.45 N/mm and 4.60 N/mm which are equal to 25%, 50%, 100%, 150% and 200% of the actual fracture energy (= 2.3 N/mm) respectively. All other interface properties remain same as the updated properties given in Table 4.2 except the mode-II cohesive strength which is the newly updated strength equal to 67.50 MPa. The load-displacement plots obtained for different mode-II interface fracture energies in case of specimen off-axis angles 30°, 45° and 60° are as shown in Figure 4.11.

It can be observed from Figure 4.11 that the specimen peak load and the total dissipated energy by the specimen (in the entire loading phase) increase with an increase in the mode-II fracture energy of the interface in the case of all three specimen off-axis angles. Figure 4.12 shows the variation of specimen peak load with the mode-II interface fracture energy in case of specimen off-axis angles 30°, 45° and 60°. A two parameter
Figure 4.11: Load-displacement plots for different mode-II interface fracture energies in case of specimen off-axis angle: (a) 30° (b) 45° (c) 60°.

A power function is used to fit the specimen peak load vs. mode-II interface fracture energy data in the case of all three off-axis angles. Also, these trends in the case of all three off-axis angles reach a plateau which might be due to change in the governing failure mechanisms of the specimens.

Figure 4.12: Variation of specimen peak load with the mode-II interface fracture energy for all three specimen off-axis angles.

The shear tractions along the critical interface at a random point on the failure branch of the equilibrium path due to variation in the mode-II interface fracture energy in the case of all three specimen off-axis angles are as shown in Figure 4.13. It can be observed from Figure 4.13 that the cohesive zone size increases with an increase in the mode-II interface fracture energy.
interface fracture energy in case of all the specimen off-axis angles. This observation is again inline with the statements by Turon et al. [32] and Ural et al. [33]. The magnitude of pure friction developed after cracking of the interface also increased with an increase in the mode-II interface fracture energy in the case of all specimen off-axis angles. The increase in magnitude of pure friction due to the increase in mode-II interface fracture energy is higher in case of specimens with higher off-axis angle. In Figure 4.13 (c), it can be noticed that interface in the case of 60° off-axis angled specimen with mode-II fracture energy equal to 4.6 N/mm damages uniformly. This implies a change of delamination failure mechanism of the specimen from cracking interface to a uniformly damaging interface which explains why the specimen peak load vs. mode-II fracture energy trend lines for all specimen off-axis angles in Figure 4.12 are reaching a plateau. So it can be concluded that an increase in mode-II interface fracture energy to a value greater than 4.6 N/mm will lead to a UDI in case of 60° off-axis angled specimen and might also in case of 30° and 45° off-axis angled specimens.

Figure 4.13: Shear tractions along the critical interface at a random point on the failure branch of the equilibrium path upon varying the mode-II interface fracture energy for off-axis angles: (a) 30° (b) 45° (c) 60°.

4.7 Parametric studies on friction coefficient

The effect of varying the static friction coefficient of the critical interface on the quasi-static behavior of the composite specimens with off-axis angles 30°, 45° and 60° is analyzed in this particular section. Five static friction coefficient values are considered for this study: 0.075, 0.15, 0.30, 0.45 and 0.60 which are equal to 25%, 50%, 100%, 150% and 200% of the actual friction coefficient (= 0.30) respectively. The rest of the interface properties are the same as in Table 4.2 except the mode-II cohesive strength which is equal to 67.50 MPa (the newly updated strength). Only four friction coefficient values 0.075, 0.15, 0.30, 0.45 are considered to present the behavior of 60° specimen because a static friction coefficient of 0.60 resulted in a completely different failure mechanism which cannot be analyzed by assuming a single critical interface. The load-displacement plots obtained by varying the static friction coefficient of the interface for specimen off-axis angles 30°, 45° and 60° are as shown in Figure 4.14.

Figure 4.14 reveals that increasing the static friction coefficient of the interface leads to an increase in the specimen peak load and also a negligible increase in the total energy dissipated by the specimen. The effect of increase in the dissipated energy due to an increase in the friction coefficient is more pronounced in the 30° specimen when compared
Figure 4.14: Load-displacement plots for different static friction coefficients in case of specimen off-axis angle: (a) 30° (b) 45° (c) 60°.

Figure 4.15: Variation of specimen peak load with the static friction coefficient of the interface for all three specimen off-axis angles.

with the other two off-axis angles because the SP ratios of lower off-axis angled specimens are high and hence they dissipate more energy through friction. Figure 4.15 shows the variation of specimen peak load with the static friction coefficient of the interface for all specimen three off-axis angles. In Figure 4.15, a two parameter exponential function is used to fit the specimen peak load vs. static friction coefficient data in the case of all three off-axis angles.

The increasing trend of specimen peak load with the increase in friction coefficient drastically changed from a 45° to a 60° off-axis angle. The increase of specimen peak
4.8 Variation in the ply thickness

The changes in the quasi-static behavior of the composite specimens with off-axis angles 30°, 45° and 60° due to a variation in the ply thickness are analyzed in this part. At the start of this thesis, initial trials for studying these changes are performed with the DIS and SM laminate configurations of the composite specimens but no variations in the quasi-static behaviors are observed. Therefore, multi-interface models (MIMs) are adopted in this part of the study to analyze the changes in the quasi-static behavior of the specimens when the ply thickness is changed. A MIM is defined as the FE model of a composite specimen in which all the interfaces are critical i.e. all of them can fail depending on the stress distribution in the specimen. All the composite plies of the laminate in a MIM are modelled explicitly and can only exhibit a linear elastic behavior. The cross-section size

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Figure 4.16: Shear tractions along the critical interface at a random point on the failure branch of the equilibrium path upon varying the static friction coefficient for off-axis angles: (a) 30° (b) 45° (c) 60°.

The shear tractions along the critical interface of the specimen at a random point on the failure branch of the equilibrium path with different static friction coefficients for specimen off-axis angles 30°, 45° and 60° are as shown in Figure 4.16. It can be observed from Figure 4.16 that the cohesive zone size is unaffected due to variation in the static friction coefficient of the interface but the magnitude of pure friction developed after interface cracking, increased with an increase in the friction coefficient in case of all three specimen off-axis angles. The increase in the magnitude of pure friction is more pronounced in the 60° than the other two because of high normal tractions and this is the reason why a static friction coefficient value of 0.60 in the 60° off-axis angled specimen leads to complicated stress states.
of the specimens adopted in this study is 30 mm x 10 mm. Three different ply thicknesses 0.5 mm, 1 mm and 2 mm are chosen to study the changes in the quasi-static behavior of the specimens.

The 2D plane strain MIMs of the composite specimens are developed by following the same procedure as explained in section 3.2 and the interfaces are modelled using the rate-independent cohesive law with friction adapted by Van der Meer et al. [1] from Zou et al. [2]. The material properties given in Tables 4.1 and 4.2 are used to model the composite plies and the interfaces after updating mode-II interface shear strength. A dissipation based arc-length control solver with Newton-Raphson iterative is used to trace the quasi-static equilibrium paths of the specimens with different ply thicknesses. The typical FE meshes used to analyze the MIM of a composite specimen with cross-section size 30 mm x 10 mm and ply thicknesses 0.5 mm and 2 mm are as shown in Figure 4.17. The load-displacement plots obtained by subjecting the MIM of the composite specimens with different ply thickness and all three off-axis angles to a quasi-static load are as shown in the Figure 4.18.

It can be observed from Figure 4.18 that there is a slight increase in the initial stiffness and decrease in the peak load of composite specimens with all three off-axis angles due to an increase in the thickness of the plies. Increase in the ply thickness results in a decrease in the number of critical interfaces in the composite specimen thereby reducing the energy dissipation capacity of the material. With more critical interfaces, there are multiple energy dissipation locations before the failure of a critical interface is triggered and hence a specimen with smaller ply thickness dissipates more energy than the one with larger ply thickness causing a delay in its overall failure. Also, larger deformation energy (before failure) is stored in thicker plies which results in a faster failure of the interfaces after damage initiation. These are the main reasons why the peak loads in case of specimens with smaller ply thickness are higher. By observing the area under the load-displacement graphs, it can be noted that the specimens with smaller ply thickness indeed dissipate more energy than the ones with larger ply thickness with an exception in the case of 30° off-axis angled specimen.

The damage in the interfaces of a 45° off-axis angled MIM of the composite specimens due to a quasi-static load upon varying the ply thickness at failure is as shown in Figure 4.19. It can be observed from these figures that the quasi-static failure behaviors of 45° (all off-axis angled specimens in general) off-axis angled MIM of the composite specimens with different ply thicknesses are governed by different mechanisms. From these figures, it also looks like the total energy dissipated by the specimen is only due to the failure of a single interface which is completely cracked. This is actually not the case
4.8. Variation in the ply thickness

Figure 4.18: Quasi-static load-displacement behaviors obtained in the case of MIM of the composite specimens with different ply thicknesses and off-axis angles: (a) 30° (b) 45° and (c) 60°.

Figure 4.19: Damage in the interfaces of a 45° off-axis angled MIM of the composite specimen when subjected to quasi-static load at failure with ply thickness: (a) 0.5 mm (b) 1 mm (c) 2 mm.

because the other interfaces which are partially damaged and cannot be visualized clearly in these figures also dissipate a part of the total energy. Hence having more interfaces that can damage in a specimen is beneficial in terms of the total energy dissipated and so the
specimens with lower ply thickness display a better quasi-static behavior when compared with their higher ply thickness counterparts. This may not always be the case in reality because the failures associated with matrix and the fiber materials are not considered in these analyses.
Dynamic behavior with rate-independent cohesive law

To understand the failure behavior of the simplified test setup subjected to dynamic compression loads, the rate-independent cohesive law with friction adapted by Van der Meer et al. [1] is used for initial trials. This chapter describes the dynamic behavior of composite specimens with alternate 0° and 90° ply arrangement and off-axis angles 30°, 45° and 60° when excited with compressive loads of different rates. The composite specimens contain single and multiple critical interface/s, which are analyzed as two separate cases and modelled using the rate-independent cohesive law coupled with friction.

5.1 Dynamic behavior of single interface model

In this study, a single interface model (SIM) is defined as the FE model of a composite specimen which has only one critical interface at the center that can fail due to delamination. All other entities in the SIM can only exhibit a linear elastic behavior. Hence all the FE models analyzed up to now in this thesis can be categorized as SIMs. The two different laminate configurations of the SIMs dealt in this thesis were: Discrete (DIS) and Smeared (SM) configurations (see section 3.2). It is concluded in chapter 3 that the behavior of a composite specimen with any off-axis angle and DIS configuration can be replicated with its SM configuration counterpart except for some minor differences in the linear elastic response and post-peak response.

Therefore in this part of the study which aims at analyzing the dynamic behavior of composite specimens with a single critical interface, the SM configuration is adopted for modelling the laminates with different off-axis angles. The actual cross-section size of the specimen chosen for the analysis and experimentation at the start of this thesis was equal to 60 mm x 20 mm. But the cross-section width and height of the specimens are changed to 30 mm and 10 mm respectively in the later part of the experimental studies. So the cross-section size of simplified test setup of the composite specimens that will be investigated from this point onwards in the thesis is 30 mm x 10 mm. The thickness of
plies in the composite laminates for this part of the study is considered as 1 mm (the ply thickness chosen in experiments).

The 2D plane strain SIMs of the composite specimens with different off-axis angles are developed by following the same procedure as explained in section 3.2. The critical interface is modelled using the rate independent cohesive law coupled with friction adapted by Van der Meer et al. [1] from Zou et al. [2]. The material properties given in Tables 4.1, 4.2 and 4.3 are used for modelling different entities in the composite specimens except that the mode-II shear strength of the interface is considered as 67.50 MPa in order to make sure that the interface fails by cracking and not by damaging uniformly. The static friction coefficient of the interface is used for the analysis till this point in the thesis but it is later realized that the frictional traction develops only when there is a sliding displacement along the interface. So instead of the static friction coefficient, the dynamic friction coefficient is used in the analyses hereafter.

It is assumed that the specimens are at rest before the starting the loading process. Hence the initial conditions of all the nodes in the analyses are set to zero. The X-translation of left end of the specimen is fixed and the right end is excited with a chosen displacement pulse. The Y-translation of bottom left node of the specimen is fixed to avoid rigid body motions. An second-order adaptive Newmark time integration solver with a Newton-Raphson iterative procedure is adopted to trace down the equilibrium paths of the composite specimens. The value of $\gamma$ in the Newmark time integration scheme which governs numerical damping in the problem is set to 1.5 in all the simulations. Such a high value for $\gamma$ is chosen to eliminate erratic numerical vibrations. It is confirmed from the simulations that increasing $\gamma$ would just suppress the sudden jumps in the behavior but doesn’t change the underlying physics of the problem. A typical FE mesh used to analyze the dynamic behavior of the SIM of a composite specimen with cross-section size 30 mm x 10 mm is shown in Figure 5.1.

![Figure 5.1: Typical FE mesh used to analyze the SIM of a composite specimen with cross-section size 30 mm x 10 mm](image)

The composite specimens should be subjected to compressive loads of different rates. Therefore, the specimens are excited with a velocity (loading rate) signal as shown in Figure 5.2. This signal comprises a monotonically increasing part with a constant slope initially which means that the specimens are subjected to constant acceleration for a certain amount of time which is considerably shorter when compared with the total simulation time. Exciting the specimens initially with a constant acceleration for a short duration ensures that they are steadily set into motion which might otherwise lead to unrealistic numerical vibrations. The velocity becomes a constant after increasing linearly for a short duration of time and then continues to be a constant thereafter. The velocity
Figure 5.2: Typical velocity signal chosen to excite the composite specimens

upon integration over time with zero initial conditions gives the displacement signal which is used in the FE analysis of composite specimens.

Four different load cases as given in Table 5.1 are chosen for analyzing the dynamic behavior of SIM of the composite specimens with different off-axis angles. Two parameters $V_{\text{const}}$ and $T_{\text{shift}}$ are introduced in Table 5.1 which indicate the magnitude of constant part in the velocity signal and the time at which velocity (loading rate) becomes a constant respectively. The value of $V_{\text{const}}$ is increased by a factor of 10 in each case when compared its previous case and $T_{\text{shift}}$ is decreased by a factor of 10 because as the loading rate increases, the specimens fails faster and so the total simulation time decreases. Therefore, the value of $T_{\text{shift}}$ is decreased when the loading rate is increased to make sure that the shift time is considerably smaller when compared with the total simulation time. This ensures that the initial constant acceleration imparted to the specimen within the time $T_{\text{shift}}$ in a particular load case, will not have a major influence on the dynamic behavior of the specimen. It is confirmed from simulations that small changes in $T_{\text{shift}}$ for a given load case will not affect the specimen behavior by a great extent. But a value of $T_{\text{shift}}$ comparable to the total simulation time might have a considerable effect of the specimen behavior. So in this study, the parameter $T_{\text{shift}}$ is restricted to a value which is smaller than the total simulation time at least by a factor of five. It is observed that $T_{\text{shift}}$ values less than this limit will not have a considerable effect on the simulation results but only set the specimens into motion to minimize unrealistic numerical vibrations.

Table 5.1: Load cases considered for dynamic analysis of the composite specimens

<table>
<thead>
<tr>
<th>Dynamic load case</th>
<th>$V_{\text{const}}$ (m/sec)</th>
<th>$T_{\text{shift}}$ (sec)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.1</td>
<td>0.001</td>
</tr>
<tr>
<td>2</td>
<td>1</td>
<td>0.0001</td>
</tr>
<tr>
<td>3</td>
<td>10</td>
<td>0.00001</td>
</tr>
<tr>
<td>4</td>
<td>100</td>
<td>0.000001</td>
</tr>
</tbody>
</table>

The load-displacement plots obtained for dynamic load cases 1, 2 and 3 along with the quasi-static response obtained in the case of 30 mm x 10 mm sized SIM of the composite specimens with off-axis angles 30°, 45° and 60° are as shown in Figure 5.3. It can be observed that the load-displacement plots up to the peak load of all three off-axis angled specimens when subjected to dynamic load cases 1 and 2 almost match with the quasi-
5.1. Dynamic behavior of single interface model

static response of the specimen. As the off-axis angle increases, the difference between the peak loads obtained for dynamic load cases 1 and 2 and the quasi-static case also increases. These differences are in the range of 4.20 % to 6.15 % in the case of all three off-axis angles which are not considerably large and therefore can be ignored. They can be eliminated by performing the analyses with smaller time steps than the ones considered in here.

![Figure 5.3: Load-displacement plots obtained for different loading conditions in the case of SIM of the composite specimens with off-axis angles: (a) 30° (b) 45° (c) 60°.](image)

It can also be noted that with an increase in loading rate or \( V_{\text{const}} \) from 0.1 to 1 m/sec , resulted in an increase of 3.20% in the peak load in the case of 30° off-axis angled specimen. But this increase cannot consistently be observed in the case of 45° and 60° off-axis angled specimens. In fact there is a decrease of 0.2% in peak load in the case of 60° off-axis angled specimen when the velocity increased. But these differences are again very small and hence they can be neglected. The specimens fail immediately after attaining the peak load when subjected to dynamic load cases 1 and 2. So the load-displacement responses obtained for dynamic load cases 1 and 2 can be considered almost same except for some minor differences.

Figure 5.4 shows the shear traction distribution along the critical interface at (close to) the Damage Initiation Point (DIP) of SIM of the composite specimens with cross-section size 30 mm x 10 mm and all three off-axis angles. It can be observed from this Figure 5.4 that the shear traction distributions along the critical interface at (close to) DIP in the case of the quasi-static response and when subjected to dynamic load cases 1 and 2 are similar for all the specimen off-axis angles. Based on all these observations, it can be concluded that the behavior of SIM of the composite specimens with any of the three off-axis angles when subjected to dynamic load cases 1 and 2 match with their
respective quasi-static behavior up to the peak load and fail immediately after it. The shear tractions along the critical interface at DIP in the quasi-static case are slightly lower than the dynamic load cases 1 and 2 which might be due to the difference in numerical procedures adopted for solving the problems.

![Shear tractions along the critical interface at DIP in the case of SIM of the composite specimens with all three off-axis angles for load cases: (a) quasi-static (b) dynamic load case 1 (c) dynamic load case 2.](image)

Figure 5.4: Shear tractions along the critical interface at DIP in the case of SIM of the composite specimens with all three off-axis angles for load cases: (a) quasi-static (b) dynamic load case 1 (c) dynamic load case 2.

Upon looking at Figure 5.3, it can be noted that the behavior of composite specimens with all three off-axis angles when subjected to dynamic load case 3 ($V_{const}$ or loading rate = 10 m/sec) is completely different when compared with the behavior of specimens when subjected to quasi-static load and dynamic load cases 1 and 2. By studying the behavior of composite specimens excited with different loading rates, their response towards compression loads of different rates are broadly classified into two types in this study: 1. Failure Dominated Response (FDR) and 2. Inertia Dominated Response (IDR). In a FDR type of response, the dominant failure mode of the composite specimen governs its behavior and the inertial effects due to loading are negligible.

Composite specimens display FDR type responses at low compression loading rates. From Figure 5.3, it can be observed that the behavior of composite specimens when subjected to dynamic load cases 1 and 2 can be categorized as FDRs. With an increase in the loading rate, the prominence of inertial effects at structural and material level (wave phenomenon) in deciding the failure behavior of a composite specimen increases. In an IDR type response, the behavior of the composite specimen is governed by inertial effects and the actual dominant failure mechanism has a comparatively smaller role to play in governing the dynamic response of the specimen. It can be observed from Figure 5.3 that the load-displacement response of composite specimens with all three off-axis angles when subjected to dynamic load case 3 initially deviates from the FDR due to inertial effects and then follows it after a certain amount of time due to the specimen getting adapted to the loading situation.

Hence this response is not completely a FDR or an IDR but is a combination of both to a certain extent. The behavior of the composite specimens when excited with higher rate compressive loads is also dictated by other factors which include the compressive stress waves generated in the material due to stress localization caused by inertial effects. When the composite specimen is subjected to a higher rate compressive load, its response towards loading situation on a whole takes time due to local inertia which results in stress gradients and in turn the development of stress waves. As the loading rate increases, larger stress gradients start to develop in the material. The amplitude of these stress waves and
their propagation through the specimen determine it’s dynamic behavior especially when excited with higher loading rates.

![Contours of $\sigma_{xx}$ in the case of 45° off-axis angled SIM of the composite specimen when subjected to dynamic load case 3 at simulation times: (a) $1 \times 10^{-6}$ sec (b) $1.1 \times 10^{-5}$ sec (c) $3.1 \times 10^{-5}$ sec (d) $4.4 \times 10^{-5}$ sec.](image)

Figure 5.5: Contours of $\sigma_{xx}$ in the case of 45° off-axis angled SIM of the composite specimen when subjected to dynamic load case 3 at simulation times: (a) $1 \times 10^{-6}$ sec (b) $1.1 \times 10^{-5}$ sec (c) $3.1 \times 10^{-5}$ sec (d) $4.4 \times 10^{-5}$ sec.

It can be observed from Figure 5.3 that subjecting composite specimens with all three off-axis angles to dynamic load case 3 generates major changes in their response when compared with their quasi-static response and response to dynamic cases 1 and 2. When subjected to dynamic load case 3, the peak loads carried by the composite specimens with all off-axis angles increase and a steady delamination failure can be observed when compared with the responses obtained in the case of dynamic load cases 1 and 2 due to inertial effects. The evolution of $\sigma_{xx}$ with time in the case of 45° off-axis angled SIM of the composite specimen when subjected dynamic load case 3 is as shown in Figure 5.5. The development of stress gradients, generation of stress waves and inertial effects in the specimen for rather a short duration of time can be visualized from Figures 5.5 (a) and (b). The load-displacement responses of the composite specimens don’t match with the FDRs after the specimens get adapted to the inertial effects because of the reflections of stress waves from the fixed boundary of the specimens and their interactions with other propagating stress waves which cannot be visualized properly from these contours.

In dynamic load case 3, a state of stress equilibrium as obtained in dynamic load cases 1 ad 2 cannot be achieved before damage initiation in the specimen which results in the development of stress gradients and in turn the stress waves. The reflections from the boundaries of the specimens are due to the impedance difference across the interface. The failure mechanisms of the composite specimens with different off-axis angles when subjected to dynamic load cases 1, 2 and 3 are exactly same as their respective failure mechanisms observed when subjected to a quasi-static load. Except that the failure mechanisms observed in dynamic load cases 1 and 2 are sudden and considerably steady
in dynamic load case 3. The specimens with different off-axis angles finally fail due to delamination typically as shown in Figure 5.5 (d). It can also be observed from Figure 5.5 that the speed of the stress wave increases when it travels in the 90° ply due to higher stiffness in ply direction. The propagation of stress waves through the interface can also be visualized by comparing the traction plots shown in Figures 5.4 and 5.6 as a disturbance propagating through the material. The change in traction distributions due to the incoming stress waves can be observed from Figure 5.6 (a) and the distortion in the traction distributions due to wave propagation can be observed from Figure 5.6 (b). The tractions developed along the interface due to loading and propagating stress waves cannot be differentiated completely but can be visualized as distortions in the traction distributions.

The load-displacement plots obtained when the SIM of the composite specimens with all three off-axis angles are subjected to dynamic load case 4 ($V_{cont}$ or loading rate = 100 m/sec) are as shown in the Figure 5.7. Comparing the load-displacement responses in Figure 5.3 and Figure 5.7, it can clearly be observed that the behavior of the specimens when subjected to dynamic load case 4 is completely different when compared with their quasi-static response and response to dynamic load cases 1, 2 and 3. The peak load carried by the specimens when compared with their quasi-static responses increased drastically by more than three times, their initial stiffness increased by approximately a 100 times and a completely different post-peak responses can be observed. The reason behind these changes can be understood by looking at the evolution of normal stress in time in the case of 30° off-axis angled SIM of the composite specimen when subjected to dynamic load case 4 as shown in Figure 5.8.

The generation of stress waves due to stress localization in the composite specimens with any of three off-axis angles can be visualized from Figures 5.8 (a) and (b). The dominant inertial contribution in the load equilibrium resulting in the development of stress gradients in a region of the material causes a drastic increase in the peak load and the initial stiffness of the specimens. The amplitude of these stress waves gradually increase with time due to a drawback of the SIM that there is only one critical interface at the center of the specimen from which energy can be dissipated. So by time stress waves reach the critical interface, their amplitude is very high (can be seen in Figure 5.8 (e)) which results in an immediate damage and cracking of the interface. The propagation of stress waves through the interface and the time they take to travel back to the loaded edge
5.1. Dynamic behavior of single interface model

Figure 5.7: Load-displacement plots obtained when the SIMs of the composite specimens with all three off-axis angles are subjected to dynamic load case 4 are the reasons for a gradual post-peak response instead of a sudden drop as observed when the specimens are subjected to dynamic load cases 1 and 2 and a comparatively steeper drop when subjected to dynamic load case 3.

Figure 5.8: Contours of $\sigma_{xx}$ in the case of 30° off-axis angled SIM of the composite specimen when subjected to dynamic load case 4 at simulation times: (a) $1.1 \times 10^{-6}$ sec (b) $2.1 \times 10^{-6}$ sec (c) $4.1 \times 10^{-6}$ sec (d) $7.1 \times 10^{-6}$ sec.

By comparing Figures 5.5 and 5.8, it can be noted that the response of the composite specimens slows down comparatively when subjected to a higher loading rate due to inertial effects. The interface of composite specimens with all the off-axis angles typically fails as shown in Figure 5.8 (d). The failure of the interface in case of 30° and 45° off-axis angled composite specimens when subjected to quasi-static load and dynamic load
cases 1, 2 and 3 actually initiates from bottom most point of the interface. But when these specimens are subjected to dynamic load case 4, the failure of the interface initiates from the top most point of the interface (see Figure 5.8 (c)). Hence this response can be characterized as a pure IDR because the inertia of the material results in a stress localization which in turn generates stress waves that govern the dynamic behavior and failure mechanism of the composite specimens when subjected to dynamic load case 4.

Figure 5.9: Load pulses at the left and right boundaries of the 45° off-axis angled SIM of the composite specimen when subjected to: (a) dynamic load case 3 and (b) dynamic load case 4.

The load pulses at the left (fixed) and right (loaded) boundaries of the 45° off-axis angled SIM of the composite specimen when subjected to dynamic load cases 3 and 4 are as shown in Figure 5.9. In Figure 5.9, the time taken by the stress wave to reach the left boundary of the specimen in dynamic load cases 3 and 4 can clearly be observed. The changes in the nature of load pulses at the right boundary of the specimen when the stress waves reached the left boundary can also be observed. The reflection of these waves from the left (fixed) boundary can be seen from the change in the nature of load pulses at the same boundary. Upon looking closely at Figure 5.9, it can be noted that the time taken by the stress waves to reach the left boundary of the specimens is approximately same in both the dynamic load cases (because the compressive stress wave speed is a material property) but their amplitude is higher in load case 4 than in load case 3 due to an increase in the loading rate.

5.2 Dynamic behavior of multi-interface model

As mentioned briefly in section 5.1, the SIM of the composite specimens suffers from some major drawbacks. First of all, considering only one critical interface might not always be a realistic assumption because delamination and subsequent failure of the specimen can be triggered by any interface depending on the stress distribution. When subjected to a higher loading rate, the generation of stress waves in the specimen decides its failure (discussed in 5.1) and so the presence of critical interfaces in the propagation path of the stress waves might influence the specimen’s dynamic behavior. Taking these drawbacks of the SIM into account and with an intent of getting closer to the reality, multi-interface models (MIMs) are adopted to analyze the dynamic behavior of different off-axis angled composite specimens with the friction coupled rate-independent cohesive law.
Detailed explanation about the MIMs can be found in section 4.8. For this part of the study, the cross-section size and ply thickness of the specimens chosen for the analysis is 30 mm x 10 mm and 1 mm respectively. The 2D plane strain MIMs of the composite specimens with different off-axis angles are developed by following the same procedure as explained in section 3.2. All the interfaces in the composite specimens are modelled using the rate-independent cohesive law coupled with friction adapted by Van der Meer et al. [1] from Zou et al. [2]. The material properties given in Tables 4.1 and 4.2 are used for modelling the composite plies and the interfaces between them respectively except that the mode-II interface shear strength is updated to avoid uniformly damaging interfaces.

Figure 5.10: Typical FE mesh used to analyze the MIM of a composite specimen with cross-section size 30 mm x 10 mm

The boundary conditions, initial conditions and the solver adopted in MIMs are exactly same as the ones adopted in SIMs (explained in section 5.1) for dynamic analysis. For the quasi-static analysis, same boundary conditions and a dissipation based arc-length control solver with Newton-Raphson iterative procedure is used. The value of $\gamma$ in the Newmark time integration scheme is set to 1.5 to suppress unrealistic numerical vibrations. A typical FE mesh used to analyze the dynamic behavior of the MIM of a composite specimen is as shown in Figure 5.10. The same velocity signal and load cases (see Table 5.1) used for analyzing the SIMs are also used to study the dynamic behavior of MIMs of the composite specimens with off-axis angles $30^\circ$, $45^\circ$ and $60^\circ$ when subjected to compression loads of different rates.

The load-displacement responses obtained in case of MIM of the composite specimens with all three off-axis angles when subjected to a quasi-static load, dynamic load case 1, 2 and 3 are as shown in Figure 5.11. Comparing load-displacement responses in Figures 5.3 and 5.11, it can be observed that there are major differences in the quasi-static behaviors of SIM and MIM composite specimens with all the three off-axis angles. It can be noted from Figure 5.11 that the increase in specimen off-axis angle from $30^\circ$ to $45^\circ$ resulted in an 8.44% decrease in the peak load and an increase of 14.69% when the off-axis angle increased from $45^\circ$ to $60^\circ$. These trends in the peak loads are a consequence of specimen off-axis angles and material properties of the interface.

The initial stiffnesses of the specimens decreased with an increase in the specimen off-axis angle. The post-peak responses of the specimens with different off-axis angles when subjected to quasi-static load are completely different. Especially the $60^\circ$ off-axis angled MIM of the composite specimen attains a plateau after reaching the peak load. This result can be explained by observing the evolution of damage in the interfaces over the plateau region in the quasi-static load-displacement response of the $60^\circ$ off-axis angled
Chapter 5. Dynamic behavior with rate-independent cohesive law

Figure 5.11: Load-displacement plots obtained for different loading conditions in the case of MIM of the composite specimens with off-axis angles: (a) 30° (b) 45° (c) 60°.

Figure 5.12: Evolution of damage with loading in the interfaces over the plateau region in the quasi-static load-displacement response of the 60° off-axis angled MIM composite specimen as shown in Figure 5.12. Three Figure 5.12 (a), (b) and (c) are shown which represent the damage in the plies at the start, middle and end of the plateau region in the quasi-static load-displacement response of the specimen. It can be observed from these figures that the damage evolution in critical interfaces of the specimen over the
plateau region slowed down (almost halted) and there is a rather slow growth of damage in the other interfaces.

Figure 5.13: Damage in the interfaces of a 45° off-axis angled MIM of the composite specimen when subjected to quasi-static load at: (a) DIP (b) failure.

This phenomenon of slowing down of the damage in the interfaces happens only in the 60° off-axis angled MIM of the composite specimen and that is the reason why there is a plateau in their load-displacement responses. Figure 5.13 shows the damage in the interfaces of a 45° off-axis angled MIM of the composite specimen when subjected to quasi-static load at DIP and failure. By observing these figures and damage evolution in the other off-axis angled specimens, few points regarding their behavior can be generalized. The initiation of damage in the composite specimens with any off-axis angle happens at multiple interfaces depending on the stresses at those locations (see Figure 5.13 (a)). The location of initiation of the damage in an interface of the specimen is not fixed as observed in the case of a SIM but depends on the traction distributions along that interface. Depending on the rate of damage growth, only some interfaces among the ones in which damage initiation occurred will govern the overall failure behavior of MIM of the composite specimens (see Figure 5.13 (b)).

Figure 5.14: Damage in the interfaces of a 45° off-axis angled MIM of the composite specimen at failure when subjected to: (a) dynamic load case 1 (b) dynamic load case 2 (c) dynamic load case 3.
As observed in the case of SIM of the composite specimens, the load-displacement plots in Figure 5.11 show that the responses of MIM of the composite specimens with all three off-axis angles when excited with dynamic load cases 1 and 2 match with their quasi-static counterparts till the peak load and suddenly fail after it. The adaptive dynamic solver used in these analyses cannot trace the snap-back behavior of the specimens and hence a sudden drop in the load-displacement responses can be observed. The differences in peak loads when the MIM of the composite specimens with all three off-axis angles are subjected to dynamic load cases 1 and 2 are rather minimal ranging from 0% to 0.4% which can be neglected. The post-peak responses of the specimens with all three off-axis angles for dynamic load cases 1 and 2 differ possibly due to the initially prevalent inertial effects due to increase in the loading rate from 0.1 to 1 m/sec. The responses of MIM of the composite specimens when excited with dynamic load cases 1 and 2 can be classified as FDR. The reason behind classifying them as FDRs can typically be observed by looking at the damage in the interfaces of a 45° off-axis angled MIM of the composite specimen at failure when subjected to dynamic load cases 1, 2 and 3 as shown in Figure 5.14 and comparing with it’s quasi-static equivalent in Figure 5.13.

It can be observed from Figure 5.13 (b) that the failure of 45° and also the other off-axis angled specimens in general is governed by only a few critical interfaces among all of them in the specimen which are the cracked interfaces (yellow colored with d = 1) in the figure with negligible inertial effects and hence these responses can be categorized as FDRs. Upon looking closely at Figures 5.14 (a) and (b), it can be observed that the governing failure mechanisms of the composite specimens when subjected to quasi-static load, dynamic load case 1 and 2 are different. The fully cracked interface which governs the quasi-static behavior in the case of 45° off-axis angled MIM of the composite specimen is not same as the ones in dynamic load cases 1 and 2 (but failure occurred merely around the same region of the specimen). In fact the governing failure mechanisms for dynamic load cases 1 and 2 are also not exactly the same.

But the load-displacement responses of MIM of the composite specimens subjected to quasi-static load and dynamic load cases 1 and 2, match up to the peak load because dominant failure mechanisms generally govern the post-peak responses of the specimens. The responses up to the peak load are mostly decided by the ply arrangement, specimen dimensions, specimen off-axis angle and also the loading conditions. Because in the dynamic load cases 1 and 2, the loading rates are not too high for the generation of stress waves due to inertia, the specimens responded as their quasi-static equivalents when subjected to dynamic load cases 1 and 2 up to the peak load. One of the main reasons why their post-peak responses are different when excited with dynamic load cases 1 and 2, is because of the difference in their governing failure mechanisms. Therefore, it can be concluded from these observations that the governing failure mechanism of MIM of the composite specimens with all three off-axis angles will depend on the applied compressive loading rate.

Looking at Figure 5.11, it can be observed that the dynamic behaviors of MIM of the composite specimens with all three off-axis angles when subjected to dynamic load case 3 are completely different when compared with their dynamic load case 1 and 2 counterparts as observed in the SIMs. The reason behind this behavior of the MIMs is same as explained in the case of SIMs and this response can be classified as a combination of IDR initially and FDR after a certain duration of time. The dynamic behavior of MIM of the composite specimens with all three off-axis angles is initially governed by the
5.2. Dynamic behavior of multi-interface model

inertia of the specimen due to increase of the loading rate from 1 m/sec to 10 m/sec which results in the generation of stress waves. The propagation of these stress waves and their properties decide the dynamic behavior of the specimens. But as observed in the case of SIMs, the peak load carried by the composite specimens didn’t always increase due to an increase in the loading rate. In case of subjecting the 30° and 45° off-axis angled MIM of the composite specimens to dynamic load case 3, their peak loads decreased by 10.19% and 3.26% when compared with their corresponding quasi-static peak loads where as an increase of 1.07% is observed in the 60° off-axis angled specimen. The failure mechanisms of MIM of the composite specimens with all three off-axis angles when subjected to dynamic load case 3 is also not same as their quasi-static and dynamic load cases 1 and 2 counterparts because of the change in the loading rate (see Figure 5.14 (c)). A steady failure of MIMs of the composite specimens with all three off-axis angles when subjected to dynamic load case 3 can be observed on contrary to the sudden failure in dynamic load cases 1 and 2 which is also the case in the SIMs.

![Figure 5.15: Load-displacement plots obtained when the MIMs of the composite specimens with all three off-axis angles are subjected to dynamic load case 4](image)

The load-displacement plots obtained when the MIMs of the composite specimens with all three off-axis angles are subjected to dynamic load case 4 are as shown in Figure 5.15. By comparing Figures 5.11 and 5.15, it can be observed that the initial stiffnesses and peak loads of the composite specimens with all three off-axis angles increased drastically when subjected to dynamic load case 4. The peak loads carried by the specimens when subjected to dynamic load case 4 decrease with an increase in the off-axis angle. Higher peak load in the case of the 30° off-axis angled specimen when compared with the other specimens is because of the time delay in the propagating stress wave caused by its longer interfaces. Also the failure mechanisms of the specimens changed from a completely brittle mode (such as in dynamic load cases 1 and 2) to a comparatively ductile mode. These responses can be categorized as pure IDRs because the generation of the compressive stress waves and the evolution of their properties govern the specimen’s failure behavior. The reason behind these changes in the dynamic behavior is due to the generation of stress waves in the specimens due to their inertia as explained in the case of SIMs.

The propagation of these stress waves can be visualized from the evolution of $\sigma_{xx}$ with time in the case of 45° off-axis angled MIM of the composite specimen when subjected
to dynamic load case 4. Similar phenomena as observed in these contours in the case of 45° off-axis angled specimen can also be observed in other specimens. The localisation of stresses in the specimens which resulted in the generation of stress waves in the material and a growth in its amplitude with time can be observed from Figures 5.16 (a) and (b). Because of the propagation of such high amplitude compressive stress wave, all the interfaces of the specimen in the propagation path of the stress wave start to damage and crack as shown in Figure 5.16 (c). Figure 5.16 (d) shows the complete failure of the specimen due to delamination of the critical interfaces and a subsequent failure of the entire specimen even though a lot of interfaces remain intact.

![Image](a)

![Image](b)

![Image](c)

![Image](d)

Figure 5.16: Contours of $\sigma_{xx}$ in the case of 45° off-axis angled MIM of the composite specimen when subjected to dynamic load case 4 at simulation times: (a) $1 \times 10^{-7}$ sec (b) $1.1 \times 10^{-6}$ sec (c) $2.1 \times 10^{-6}$ sec (d) $6.65 \times 10^{-6}$ sec.

In Figure 5.15, a sharp drop after the peak load in the load-displacement responses of MIMs of the composite specimens when subjected to dynamic load case 4 on contrary to a gradual drop in SIMs is because in the SIMs failure starts only when the stress wave reaches the critical interface. But in a MIM (as any interface can damage), as soon as the amplitude of stress waves reaches the strength of the interface at a particular location, damage starts to initiate and grow. Figure 5.17 shows the governing failure mechanisms of different off-axis angled MIMs of the composite specimens which can be visualized by looking at extent of damage in the interfaces after a complete failure of the specimen. The extent of damage in the interfaces of different off-axis angled MIMs of the composite specimens reveal that generation and propagation of a high amplitude stress wave (generally at higher loading rates), leads to a complete damage and cracking of the interfaces in its propagation path which leads to failure of the specimens even when the other interfaces remain intact. It can also be observed that most of the interface failure happened in the 60° off-axis angled specimen when compared with the other two off-axis angles and that is why the 60° off-axis angled specimen exhibited more ductile load-displacement response. The load pulses at the left and right boundaries obtained in the case of 45° off-axis angled MIM of the composite specimens in Figure 5.18 reveal that the nature of load pulse at the right boundary in dynamic load case 3 is governed by the
5.2. Dynamic behavior of multi-interface model

Figure 5.17: Damage in the interfaces for dynamic load case 4 in the case of MIM of the composite specimens with off-axis angles: (a) 30° (b) 45° (c) 60°.

wave reflections from the fixed boundary. In dynamic load case 4, the specimen already failed before the stress wave reached the left boundary and hence no reflection effects can be observed on the nature of the load pulse at the right boundary.

Figure 5.18: Load pulses at the left and right boundaries of the 45° off-axis angled MIM of the composite specimen when subjected to: (a) dynamic load case 3 and (b) dynamic load case 4.
6 Dynamic behavior with rate-dependent cohesive law

There will be a considerable influence of variation in the displacement jump rates across the interfaces of the simplified test setup with the applied loading rates on the interface strengths and fracture energies. Ideally, a rate-dependent cohesive law can capture these effects more effectively than a rate-independent cohesive law. Therefore, rate-dependency is incorporated in the rate-independent cohesive law with friction adapted by Van der Meer et al. [1]. In this chapter, the main aspects of the rate-dependent cohesive law coupled with friction developed in this thesis are discussed. The dynamic behaviors of different off-axis angled composite specimens with a single critical interface modelled using the rate-dependent cohesive law when subjected to compression loads with rates ranging from quasi-static to high are also analyzed.

6.1 Rate-dependent cohesive law with friction

One of the main objectives of this thesis is to improve the rate-independent (RI) cohesive law coupled with friction adapted by Van der Meer et al. [1] from Zou et al. [2] by incorporating the rate-dependency (RD) feature. There are different approaches as mentioned in section 2.2.4 to model the rate-dependencies in composite materials. In this thesis, the DIF approach similar to Liu et al. [3] is followed to develop the rate-dependent cohesive zone model with friction. The advantage of this phenomenological approach over the other approaches is that it can accommodate both positive and negative rate effects of strength and fracture energy [3]. The rate-dependency for cohesive strengths and fracture energies is introduced into the cohesive zone model independently in pure modes-I and II based on a Johnson-Cook law. It is assumed that the mixed mode formulations adopted in the rate-independent cohesive law with friction by Van der Meer et al. [1] are valid even after the rate-dependency is activated.

The dependencies of the mode-I and II cohesive strengths and fracture energies on the displacement jump rates across the interface are given in equations 2.16 and 2.17. The parameters $G_{1}^{inf}$ and $G_{2}^{inf}$ are introduced in the rate-dependency formulations to
keep the fracture energies from becoming very high or very low when positive or negative rate-sensitivity constants are used respectively. The rate-dependent cohesive law is implemented based on the FE method by programming in C++ in the JemJive modeling environment. An implicit framework of analysis is adopted in this study and hence the consistent tangent matrix for the rate-dependent cohesive law with friction is derived as detailed in Appendix A. The changes that occur in the rate-independent cohesive law with friction adapted by Van der Meer et al. [1] due to the incorporation of rate-dependency (with positive rate effect) can be visualized from Figure 6.1. It can be observed from Figure 6.1 that adding the rate-dependency feature to the rate-independent cohesive law with friction leads to a delay in the damage and crack initiations and a completely different evolution of the damage parameters \( d \) and \( D \). These changes amplify and give rise to a more complicated rate-dependent cohesive law when the displacement jump rates across the interface increase and vary with time.

![Figure 6.1](image)

Figure 6.1: One dimensional version of the rate-independent and the rate-dependent cohesive laws with constant friction \( \tau_f \), constant displacement jump rate and positive rate effects

### 6.2 Analysis of SIMs with the rate-dependent CZM

The initial trials for testing the rate-dependent cohesive law coupled with friction developed in this thesis are performed on the single interface models (SIMs) of the composite specimens with off-axis angles 30°, 45° and 60°. Detailed description about the SIMs are given in section 5.1 of this report. The cross-section size and ply thickness of the composite specimens chosen for these analyses is 30 mm x 10 mm and 1 mm respectively. The 2D plane strain SIMs of the composite specimens are developed by following the same procedure as detailed in section 3.2. The critical interface in the composite specimens is modelled using the rate-dependent CZM with friction and the bulk material (plies and the smeared material) is modelled using the orthotropic material model. The linear elastic properties of the bulk material and the quasi-static properties of the interface used in these analyses are given in Tables 4.1, 4.2 and 4.3. Six different rate-dependence parameter cases are chosen in this study for analyzing the dynamic behavior of SIMs with the
rate-dependent cohesive law as given in Table 6.1. The rate-dependence parameters in mode - I and II are considered to be the same because the delamination failure of the composite specimens in this thesis is predominantly mode - II in nature. In Pcase1, the rate-sensitivity constants $c_1s$ and $m_1s$ are set to zero, which results in the dynamic behaviors of the composite specimens with the rate-independent cohesive law (as described in chapter 5).

Table 6.1: Rate-dependence parameters considered for modelling the critical interface with the rate-dependent cohesive law

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<td>0.1</td>
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<td>0.02</td>
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</tr>
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<tr>
<td>Pcase6</td>
<td>0.2</td>
<td>0.05</td>
<td>0.02</td>
<td></td>
</tr>
</tbody>
</table>

Figure 6.2: Typical FE mesh used to analyze the SIM of a composite specimen with cross-section size 30 mm x 10 mm and the critical interface modelled using the rate-dependent cohesive law

Due to the lack of experimental data in this thesis, the values of $c_1s$, $m_1s$ and $\dot{\delta}_{\text{ref}}$ (quasi-static reference displacement jump rates) in Pcase2 are chosen as 0.2, 0.1 and 0.02 m/s respectively which are approximately the same values considered by Liu et al. [3]. The values of these parameters are influenced by the material composition in a composite specimen and hence an experimental calibration would be an ideal approach. But the main aim of this part of the study is to test the rate-dependent cohesive law with friction and therefore the parameter values from literature are adopted. In each of the subsequent parameter cases Pcase3, Pcase4, Pcase5 and Pcase6, $c_1s$ and $m_1s$ are considered as double and half the values as in the second parameter case to analyze their influence on the overall dynamic behavior of the composite specimens but $\dot{\delta}_{\text{ref}}$ are chosen to be the same in all the parameter cases. Only the rate-dependence parameters that give rise to positive rate effects are chosen for the analyses. In this study, the lower and upper bounds on the mode - I and mode - II fracture energies are considered to be 50% and 200% of their quasi-static values respectively in all the simulations. Based on these bounds, the maximum reference
displacement jump rates and the limit fracture energies in the rate-dependent cohesive law with friction are computed. The boundary conditions, initial conditions and the solver adopted in the analyses are same as the ones mentioned in section 5.1. The typical FE mesh used for analyzing the dynamic behavior of SIMs of the composite specimens with the critical interface modelled using the rate-dependent cohesive law with friction is as shown in Figure 6.2. The element size chosen for generating the FE mesh in Figure 6.2 is twice the element size used for FE meshes in section 5.1. It is confirmed that the FE models of different off-axis angled composite specimens do not suffer from mesh sensitivity issues and hence a relatively coarser mesh is employed in this part of the study to reduce the computational efforts. The same velocity signal as mentioned in section 5.1 is used for the dynamic analysis of different off-axis angled SIMs of the composite specimens with the rate-dependent cohesive law but three different load cases as given in Table 6.2 are adopted. Detailed information about $V_{\text{const}}$ and $T_{\text{shift}}$ can be found in section 5.1.

**Table 6.2:** Load cases considered for dynamic analysis of the composite specimens with the rate-dependent cohesive law

<table>
<thead>
<tr>
<th>Dynamic load case</th>
<th>$V_{\text{const}}$ (m/sec)</th>
<th>$T_{\text{shift}}$ (sec)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>$10^{-4}$</td>
</tr>
<tr>
<td>2</td>
<td>10</td>
<td>$10^{-5}$</td>
</tr>
<tr>
<td>3</td>
<td>100</td>
<td>$10^{-6}$</td>
</tr>
</tbody>
</table>

### 6.2.1 Time step size issues in capturing the rate effects

The FE analyses using the rate-dependent cohesive law with friction are initiated by subjecting the SIM of the composite specimens to dynamic load case 1 (see Table 6.2). The dynamic solver used in this thesis requires three inputs: the minimum time step size, the maximum time step size and the current time step size to decide the actual time step size that will be used in a particular time step. The actual time step size will always be within the minimum and the maximum time step sizes and the current time step size is used to start the analysis. If the Newton-Raphson process is unable to find an equilibrium point with the required precision, the solver automatically reduces the current time step size based on the reduction factor until convergence is achieved or the minimum time step size is reached. The solver then starts to increase the time step size until the maximum time step size is reached if there is no convergence upon successive reduction to the minimum time step size. If the convergence is still not achieved, then the solver gives up and declares that the problem doesn’t converge with the specified input parameters.

The load-displacement plots obtained for different trials of input parameters in the dynamic solver when the SIM of the composite specimens with rate-dependence parameters as in Pcase2 and all three off-axis angles are subjected to dynamic load case 1 are as shown in Figure 6.3. The trials of input parameters to the dynamic solver in this case are grouped into two categories: larger time step sizes and accurate time step sizes. In Figure 6.3, larger time step sizes and accurate time step sizes are the trails of input parameters in which the minimum time step sizes are less than and exactly equal to $10^{-11}$ sec respectively. The current and maximum time step sizes in both larger time step size and
accurate time step size trials are considered to be the same equal to $10^{-5}$ and $5 \times 10^{-5}$ sec respectively. RICZM and RDCZM represent the analyses in which the critical interface of the composite specimens is modelled using the rate-independent and the rate-dependent cohesive zone models with friction respectively. It can be observed from Figure 6.3 (a) that the peak load carried by the $30^\circ$ off-axis angled composite specimen with RDCZM and larger time step size trials increases by 4.30% approximately when compared to its RICZM counterpart but decreases by 2.25% approximately with RDCZM and accurate time step size trial. This observation is surprising because the reason behind a higher peak load in RDCZM and larger time step size trials when compared with RICZM might be due to the introduction of the positive rate effects. Upon using a smaller time step as in RDCZM and accurate time step size trial, the peak load has to increase or at least stay the same instead a reduction is observed.

![Load-displacement plots](image)

Figure 6.3: Load-displacement plots obtained for different time step size trials when the SIM of the composite specimens with rate-dependence parameters as in Pcase2 and off-axis angles: (a) $30^\circ$ (b) $45^\circ$ (c) $60^\circ$ are subjected to dynamic load case 1.

When the equilibrium path of the specimen approaches the damage initiation point, the displacement jump rates across the interface fluctuate a lot because of which the dynamic solver requires a sufficiently small time step size to achieve convergence and capture
Chapter 6. Dynamic behavior with rate-dependent cohesive law

the rate effects. If the time step size required for capturing the rate effects is smaller than the minimum time step size given as an input to the solver, then the actual equilibrium path can never be traced. In the case of the 30° off-axis angled specimen with RDCZM and larger time step size trials, the solver jumps directly from the damage initiation point to a random equilibrium point which seems like the actual peak load but not exactly. Upon using a smaller minimum time step size (10^{-11} sec) as in RDCZM and accurate time step size trial, the actual equilibrium path close to the damage initiation point is traced but due to the high stress concentrations and fluctuations in the displacement jump rates along the interface in the 30° off-axis angled specimen, a lower peak load is obtained because of the sudden failure. The result in the case of 30° off-axis angled specimen with RDCZM and accurate time step size trail can be improved by adopting a minimum time step size smaller than 10^{-11} sec in the dynamic solver but is not done in this thesis due to the lack of time.

From Figures 6.3 (b) and (c), it can be observed that the peak loads carried by the 45° and 60° off-axis angled specimens with RDCZM and larger time step size trials decreased by 6.51% and 3.60% approximately when compared with their RICZM counterparts. This is due to a smaller time step size requirement in the dynamic solver for tracing the right equilibrium path than the ones used in RDCZM and larger time step size trials. But with RDCZM and accurate time step size trials, the equilibrium paths for both the off-axis angled specimens surpass the peak loads from the RICZM and RDCZM with larger time step size trials. This observation clearly shows the effect of including rate-dependency on the dynamic behavior of the composite specimens. The fluctuations in the displacement jump rates across the interface reflect in the load-displacement responses which can be visualized from the zoomed-in images in Figures 6.3 (b) and (c). The analyses are not continued after a certain point of capturing the rate effects because of the very high computational time required. The main reasons behind the requirement of very small time step sizes in the order of 10^{-11} sec to capture the rate effects in these specimens are the high stress-concentrations along the interface, huge fluctuations in the displacement jump rates across the interface and the brittle modes of failure. Taking these reasons into account, it can be concluded that the time step size issues mainly arise due to the design of the simplified test setup.

6.2.2 SIMs with the rate-dependent CZM and dynamic load case 1

Figure 6.4 shows the load - displacement responses obtained for different parameter cases when the SIMs of the composite specimens with the rate-dependent cohesive law and different off-axis angles are subjected to dynamic load case 1. The reader is requested to ignore the post-delamination specimen responses in the load-displacement plots that will be presented from here on in this chapter. The legend of the plots that will be presented from here on consists of "(x,y)" representation of the rate-sensitivity constants. 'x' in this representation is the value of the strength rate-sensitivity constants (c_s) and 'y' is the value of the fracture energy rate-sensitivity constants (m_s) used in the analysis. The influence of local inertia and wave propagation effects on the failure behaviors of all three off-axis angled composite specimens for different rate-dependence parameters when subjected to dynamic load case 1 is negligible due to the attainment of stress equilibrium. It can be observed from Figure 6.3 (a) that the peak load carried by the 30° off-axis angled composite specimen decreased due to the incorporation of rate-dependency with
6.2. Analysis of SIMs with the rate-dependent CZM

parameters as in Pcase2 when compared with its Pcase1 counterpart. This behavior definitely cannot be physical because positive rate effects should increase the interface strengths and fracture energies, thereby increasing the specimen peak load. There is an increase in the specimen peak load upon employing the rate-dependence parameters as in Pcase3, Pcase4, Pcase5 and Pcase6 when compared with the peak load for Pcase1 (with the rate-independent cohesive law).

A higher specimen peak load is expected when the parameters in Pcase3 are employed when compared with Pcase4 because of higher $c_i$s in Pcase3 but this is not observed. The 30° off-axis angled composite specimens in Pcase2, Pcase5 and Pcase6 are not analyzed till their failure due to the lack of time. So the maximum loads that can be noticed in these parameters cases may not be the actual specimen peak loads. More accurate results can be obtained by continuing the analyses with an increased number of time steps (for a longer duration). The reasons behind this peculiar behavior of the 30° off-axis angled specimen are the time step size issues encountered in capturing the rate effects as explained in section 6.2.1. The peak load obtained in Pcase2 is lower than in Pcase1 because a comparatively higher minimum time step size is used than the one required for capturing the actual behavior. Upon reducing $c_i$s as in Pcase4, a better behavior of the specimen could be captured with the same minimum time step size. But increasing $c_i$s as

Figure 6.4: Load-displacement plots obtained for different parameter cases when the SIMs of the composite specimens with RDCZM and off-axis angles: (a) 30° (b) 45° (c) 60° are subjected to dynamic load case 1.
in Pcase3, resulted in predicting an incorrect specimen behavior because the peak load in Pcase4 cannot be higher than in Pcase3 (higher $c_i$s).

Taking all these observations into consideration, it can be concluded that reliable results cannot be obtained in the case of a specimen with low off-axis angle (higher stress concentration) that exhibits a brittle failure behavior when a comparatively larger minimum time step size is used. It can be noticed from Figure 6.4 (b) and (c) that the rate effects are captured more-efficiently with the same minimum time step size in 45° and 60° off-axis angled specimens when compared with the 30° off-axis angled specimen when subjected to dynamic load case 1. Also, as the specimen off-axis angle increases, the convergence problems due to the time step size issues reduce relatively. This is due to the reduction in stress concentrations across the interface because of an increase in the off-axis angle. The maximum loads obtained in Pcase2 to Pcase6 are higher than in Pcase1 for both the 45° and 60° off-axis angled specimens when subjected to dynamic load case 1 indicating the influence of rate effects on their failure behavior.

![Figure 6.5: Load-displacement plots obtained for different parameter cases when the SIMs of the composite specimens with RDCZM and off-axis angles: (a) 30°(b) 45°(c) 60°are subjected to dynamic load case 2.](image)

Similar convergence problems observed in 30° off-axis angled specimen are also observed in 45° and 60° off-axis angled specimens when higher $c_i$s and $m_i$s as in Pcase3 and Pcase5 are used due to the brittle failure behaviors of the specimens. Due to the lack
of time, the actual peak loads of specimens with off-axis angles 45° and 60° for all the parameter cases could not be determined in this thesis. Finally, a few load-displacement responses presented in this section are not reliable due to the time step size issues and the behaviors in some other cases are not completely explored due to the lack of time. These results can definitely be improved by adopting smaller minimum time step sizes with an increased number of time steps to capture the accurate behaviors of the rate-dependent delamination behavior of the composite specimens when subjected to dynamic compression loads.

6.2.3 SIMs with the rate-dependent CZM and dynamic load case 2

The load-displacement plots for different parameter cases upon subjecting the SIMs of the composite specimens with the rate-dependent CZM and different off-axis angles to dynamic load case 2 are as shown in Figure 6.5. Upon comparing Figures 6.4 and 6.5, it can be observed that the dynamic behaviors of different off-axis angled composite specimens for all the parameter cases when subjected to dynamic load case 2 are completely different when compared with their responses in dynamic load case 1. This is due to a considerable contribution of local inertia and wave propagation effects in dictating the failure behaviors of the specimens in dynamic load case 2. It can also be noted that the rate effects for different parameter cases and off-axis angles are captured when the specimens are subjected to dynamic load case 2 by choosing similar displacement steps as in dynamic load case 1. The reason behind this difference is because of a change in the failure mechanism of the specimens from a very brittle mode to a comparatively ductile mode. The increase (%) in the peak loads of different off-axis angled composite specimens when compared with Pcase1 due to the incorporation of rate-dependency in dynamic load case 2 are shown in Figure 6.6.

![Figure 6.6: Increase (%) in the peak loads of different off-axis angled composite specimens when compared to Pcase1 due to the incorporation of rate-dependency in dynamic load case 2](image)

It can be observed from Figure 6.6 that the increase (%) in the peak loads when all the parameter cases are considered in general, is highest in the 30° off-axis specimen followed
by the 45° off-axis specimen. The increase (%) in peak loads is least in the 60° off-axis specimen and these trends are obtained due to the longer interface lengths in the 30° off-axis specimen, which causes a delay in the overall failure. A common point that can be noted in all the three off-axis angled specimens when subjected to dynamic load case 2 is that the variation in $c_i$s has a greater influence on their behavior than the variation in the $m_i$s. Furthermore, it can be observed from Figure 6.5 and 6.6 that with an increase in the off-axis angle, the behaviors of the composite specimens when subjected to dynamic load case 2 for Pcase2, Pcase5, and Pcase6 resemble and get closer to each other with less than 3% difference in their peak loads. This observation reveals that the specimen with highest off-axis angle (ex: 60°) when subjected to dynamic load case 2 will have a least effect on its behavior due to the variation in $m_i$s. It can also be noticed that the decrease in $c_i$s has a greater influence on the dynamic behaviors of all the three off-axis angled composite specimens rather than an increase in them. Also, the specimen with highest off-axis angle has a least effect of increasing $c_i$s on its overall delamination behavior when subjected to dynamic load case 2.

Figure 6.7: Load-displacement plots obtained for different parameter cases when the SIMs of the composite specimens with RDCZM and off-axis angles: (a) 30° (b) 45° (c) 60° are subjected to dynamic load case 3.
6.2.4 SIMs with the rate-dependent CZM and dynamic load case 3

The load-displacement plots for different parameter cases when the composite specimens with the rate-dependent cohesive law and different off-axis angles are subjected to dynamic load case 3 are as shown in Figure 6.7. The dynamic behaviors of the specimens for different parameter cases with all the three off-axis angles subjected to dynamic load case 3 are completely different when compared with the behaviors of their dynamic load case 1 and 2 counterparts because of the stronger inertia and wave propagation effects. A steady failure of the specimens in dynamic load case 3 reduced the scope for convergence problems as observed in dynamic load cases 1 and 2 (to some extent), thereby increasing the ease of capturing the rate effects.

The effects of incorporating rate-dependency with the parameters in Pcase2 to Pcase6 on the behavior of the composite specimens with all the three off-axis angles when subjected to dynamic load case 3 are visualized in Figure 6.7. As the off-axis angle increases, the variations in the behavior of the composite specimens with rate-dependence parameters in Pcase2 to Pcase6 when subjected to dynamic load case 3 reduce substantially. Similar to the observations in dynamic load case 2, it can be observed from Figure 6.7 that the change in $c_i$s has a considerable influence on the overall behavior of the composite specimens predominantly on the ones with lower off-axis angle (ex: $30^\circ$).

The variations observed in the behaviors of all the three off-axis angled composite specimens for different values of $m_i$s as in Pcase2, Pcase5 and Pcase6 when subjected to dynamic load case 3 are minimal. The reasons behind these observations are the generation of high amplitude stress waves in the specimen before damage initiation and their reflections, which amplify the strength of the interface with an increase in $c_i$s. Because of the propagation of high amplitude stress waves, the damage and crack initiation events in the interface occur in a relatively short duration of time, resulting in the dissipation of all the available energy. Therefore, the possibility of a considerable amplification in the interface fracture energy with an increase in $m_i$s is less. That is why the variation in $m_i$s did not have a great influence on the behavior of the composite specimens when subjected to dynamic load case 3.
Conclusions and Recommendations

7.1 Conclusions

The main aim of this thesis is to suggest the best approach for designing the simplified test setup by analyzing its overall failure behavior when subjected to compression loads with rates ranging from quasi-static to high. The rate-independent cohesive law coupled with friction adapted by Van der Meer et al. [1] from Zou et al. [2] is initially used to study the quasi-static failure behavior of composite specimens with three different off-axis angles $30^\circ$, $45^\circ$, and $60^\circ$. Rate-dependency is incorporated in the rate-independent cohesive law with friction based on a Johnson-Cook law similar to Liu et al. [3]. Finally, the dynamic behaviors of the composite specimens with different off-axis angles due to compression loads of different rates are analyzed using the rate-independent and the rate-dependent cohesive laws with friction.

The main findings from this study for each research question are discussed below briefly.

1. How do composite specimens with alternate $0^\circ$ and $90^\circ$ layup and with three different off-axis angles $30^\circ$, $45^\circ$, and $60^\circ$ between the load and ply orientation behave in compression at different loading rates?

The behaviors of composite specimens with alternate $0^\circ$ and $90^\circ$ layup and off-axis angles $30^\circ$, $45^\circ$, and $60^\circ$ when subjected to compression loads with rates ranging from quasi-static to high are studied at different stages from chapters 3 to 6. In chapter 3, efforts are made to simplify the cumbersome discrete (DIS) laminate configuration of the composite specimens to smeared (SM) laminate configuration. It can be concluded from the results in this chapter that the failure behavior of a composite specimen with the DIS configuration and all the three off-axis angles can be replicated with its SM configuration counterpart if minor differences in their behaviors between them are accepted. The effects of varying the interface material properties on the quasi-static behavior of the composite specimens with a single critical interface are analyzed in chapter 4.
The results obtained revealed that there is an increase in the peak loads carried by the composite specimens with all the three off-axis angles due to an increase in the mode-II strength, mode-II fracture energy, and friction coefficient of the interface. The cohesive zone size in all the three off-axis angled composite specimens decreased with an increase in the mode-II interface strength and increased with an increase in the mode-II interface fracture energy but remains unchanged with an increase in the interface friction coefficient. This conclusion is inline with the statements by Turon et al. [32] and Ural et al. [33]. Also, if the cohesive zone size in a composite specimen is higher than the length of its interface due to comparatively high strength or low fracture energy, then the interface fails by damaging uniformly instead of stable crack growth.

In chapter 5, the dynamic behaviors of different off-axis angled composite specimens with single and multiple critical interface/s modelled using the rate-independent cohesive law with friction adapted by Van der Meer et al. [1] from Zou et al. [2] are analyzed. It is observed that the behaviors of the composite specimens when excited with the loading rates of 0.1 and 1 m/sec (lower loading rates) match with their quasi-static responses up to the peak load and fail immediately after it. The behaviors of the same composite specimens when subjected to the loading rates of 10 and 100 m/sec (higher loading rates) are completely different when compared with their quasi-static and lower loading rate counterparts. This is because of the attainment of stress equilibrium inside the specimen with the lower loading rates. But with the higher loading rates, stress equilibrium inside the specimen is not achieved before damage initiation due to local inertial effects, which result in the development of stress gradients.

These stress gradients are a manifestation of stress waves in the specimen. The propagation and reflections of the stress waves and their amplitudes determine the behavior of the composite specimens when subjected to higher loading rates, and hence completely different responses are obtained. The composite specimens with single and multiple critical interfaces behaved differently under both quasi-static (major differences) and dynamic loading (minor differences) due to the changes in their governing failure mechanisms. Therefore, the FE models with a single critical interface (SIMs) can be employed to get an initial understanding of the delamination behavior of the off-axis angled composite specimens. SIMs are advantageous in terms of simplicity and computational expenses, but their behavior might not always be close to reality (see section 5.2 for details). On the other hand, the FE models with multiple critical interfaces (MIMs) produce a more realistic behavior of the off-axis angled composite specimens but are computationally expensive.

The dynamic behaviors of different off-axis angled SIMs of the composite specimens with the critical interface modelled using the friction coupled rate-dependent cohesive law with various rate-dependence parameters and subjected to compression loads of different rates are analyzed in chapter 6. It is observed from the results in this chapter that the rate-dependent delamination behavior of a composite specimen can be captured effectively if and only if an accurate minimum time step size is chosen for the analysis in the dynamic solver. That is why the rate-dependent delamination behaviors for a loading rate of 1 m/sec, are incorrectly predicted in the 30° off-axis angled specimens for some of the analyzed cases. In the other parameter cases and off-axis angled specimens, the rate effects can be visualized clearly but the analyses are not completed up to final failure due to the lack of time in this thesis.
7.1. Conclusions

The rate-dependent delamination behaviors of different off-axis angled composite specimens with rate-dependence parameters in Cases 2 to 6 when subjected to loading rates of 10 and 100 m/sec are predicted with the rate-dependent cohesive law without any convergence problems due to a change in the failure mechanism of the specimens. It is observed that the influence of variation in rate-sensitivity constants of cohesive strengths on the overall rate-dependent delamination behavior of the composite specimens is greater than the variation in rate-sensitivity constants of cohesive fracture energies for both the loading rates of 10 and 100 m/sec. Also, with an increase in the off-axis angle, the sensitivity to the rate-dependence parameters reduces considerably.

2. Can the interface properties be derived from the global specimen response?

The parametric studies on the quasi-static behavior of different off-axis angled composite specimens in chapter 4 revealed that there is a considerable variation in the global specimen response due to a change in the interface strength, fracture energy, and friction coefficient. Therefore, deriving the interface properties based on the global quasi-static behavior of the composite specimens is not straightforward because these behaviors are sensitive to the change in all the properties of the interface that dictate the delamination failure behavior of the specimens. It is observed from dynamic analyses in chapters 5 and 6 that the local inertial effects play a crucial role in governing the failure behavior of the composite specimens especially when subjected to higher loading rates such as 10 and 100 m/sec. The propagation and the properties of the stress waves generated due to local inertia decide the delamination behavior of the composite specimens. But the behavior of every material point at a local level due to inertial effects will not always reflect in the global response of the specimens. Also, in some cases, the global specimen response is influenced by the reflections of stress waves from the fixed boundary. So the global specimen response not only reflects the delamination behavior but also the inertia response of the local material and the reflections of the stress waves. As the loading rates increase, the local inertial effects in the specimens become stronger, and deriving the interface properties from the global specimen response becomes more uncertain. Hence it can be concluded that the interface properties can be derived from the global specimen response in lower loading rate scenarios (ex: 0.1 and 1 m/sec) because of weak inertial effects and the attainment of stress equilibrium. Due to considerably strong local inertial effects and stress wave reflections in higher loading rate scenarios (ex: 10 and 100 m/sec), measurements based on the global specimen response provide less information on the interface properties.

3. How does changing the ply thickness affect the overall specimen behavior?

The effects of varying the ply thickness on the overall quasi-static behavior of the composite specimens with multiple critical interfaces and off-axis angles 30°, 45° and 60° are analyzed in section 4.8. It is observed from the quasi-static load-displacement responses of the specimens with all three off-axis angles that an increase in the ply thickness results in a slight increase in the initial stiffness and a decrease in the peak load. This is because of the larger deformation energy stored in thicker plies (in the elastic phase), which leads to an earlier failure of the interfaces after damage initiation. The quasi-static behaviors of the composite specimens with the same off-axis angle and different ply thicknesses are governed by different failure mechanisms. Hence it can be concluded that changing the ply thickness of a composite specimen with any off-axis angle will influence its quasi-static behavior due to a change in the governing failure
mechanism.

4. **Will the rate-dependent cohesive law coupled with friction be able to describe the delamination failure when the interface is loaded by combined normal stress, shear stress, and friction at different rates?**

In chapter 6, the rate-dependent cohesive law coupled with friction developed in this thesis is used to analyze the delamination behavior of 30°, 45° and 60° off-axis angled single interface models (SIMs) of the composite specimens when subjected to compression loads with rates 1, 10 and 100 m/sec. Rate effects in all the three off-axis angled composite specimens when excited with loading rates of 10 and 100 m/sec are seemingly captured with the rate-dependent cohesive law but with some exceptions for the loading rate of 1 m/sec due to the time step size issues (see section 6.2.1). The results in exceptional cases can be improved by reducing the minimum time step size to avoid convergence problems. Based on the behaviors of the specimens obtained for compression loads of different rates, it can be concluded that the rate-dependent cohesive law can describe rate-dependent dynamic delamination failure of the off-axis angled composite laminates. However, in order to be confident about, the model has to be validated with experimental data.

5. **How should the test setup be designed to get most information out of it?**

Based on the observations from the analysis in chapters 3 to 6, it can be concluded that the 30°, 45° and 60° off-axis angled SIMs of the composite specimens performed almost the same qualitatively in terms of their quasi-static behavior. But a distributed interface damage (damage in multiple interfaces) is observed only in the 60° off-axis angled MIM of the composite specimens when subjected to a quasi-static load and not in the other two off-axis angles. This behavior of the 60° off-axis angled specimen is disadvantageous in terms of measuring the crack growth because focusing on the failure of multiple interfaces in experiments is rather difficult, and no useful information can be derived from such a failure mechanism. In terms of the dynamic behavior, it is observed that capturing rate effects in the 30° off-axis angled specimen becomes relatively difficult due to a combination of high-stress concentrations and fluctuations in the displacement jump rates across their interface/s. As the specimen off-axis angle increases, rate effects can be captured with ease comparatively, and the variations in the failure behaviors reduce with the variations in the rate-dependence parameters. Therefore it is suggested to fabricate the simplified test setup by performing a 45° cut from the original composite laminate to simulate delamination due to compression loads of different rates effectively. But three different interface parameters cannot be calibrated from one single simplified test setup because of the sensitivity of specimen peak load to change in all the parameters. Therefore all three simplified setups considered in this thesis have to be tested, and the unique set of interface parameters that dictate the observed failure behaviors can be derived.
7.2 Recommendations

In this section, a few recommendations related to the extension and improvement of the work in this thesis are discussed for future research and development.

1. Calibration, validation and analysis of the MIMs

In this thesis, the rate-independent cohesive law with friction adapted by Van der Meer et al. [1] is improved by incorporating rate-dependency based on a Johnson-Cook law. The rate-dependency relations in pure modes - I and II involve different parameters that decide when the rate effects come into play and their influence on the cohesive strengths and fracture energies. These parameters have to be calibrated by performing trial simulations to find the best match with the experimental results. Due to the lack of experimental data, these parameters are set to the same values approximately as considered by Liu et al. [3] and a parametric study is carried out to study the influence of their variation. These parameters are affected by the behavior of matrix material in the composite [3] and hence calibrating them with experimental data for the laminate of interest is an ideal approach.

Therefore it is recommended to calibrate the parameters in the rate-dependent cohesive law with the experimental results. The behaviors of different off-axis angled composite specimens when subjected to compression loads with rates ranging from quasi-static to high are analyzed numerically in this thesis, but their deviations from the actual specimen behaviors are not studied. Hence an experimental validation is necessary to verify the accuracy of the FE models in predicting the failure behavior of the composite specimens. Due to the lack of time in this thesis, the dynamic analysis of the MIMs with the rate-dependent cohesive law with friction when subjected to compression loads of different rates is not performed. Therefore analysis of different off-axis angled MIMs of the composite specimens with the rate-dependent cohesive law (after calibration if possible) will result in a more realistic behavior of the laminates when subjected to dynamic compression loads.

2. Modelling the failure of the plies

The composite laminates comprise the plies (matrix and the fiber material) and the interfaces between them. In this thesis, the degradation of the interfaces in the simplified test setup of the composite specimens is modelled using the rate-independent and rate-dependent cohesive laws coupled with friction. But the failure of the plies which will play a major role in deciding the dynamic behavior of the specimens is not modelled. The behavior of the specimens thus obtained might not be close to reality, especially when the material properties of the plies and the interfaces are comparable to each other.

Hence it is recommended to model the failure of the plies i.e. failures of both matrix and fiber materials along with the interface failure so that a more realistic failure behavior can be obtained. This can be achieved by adopting a continuum damage approach in which the constitutive matrix of the plies is upgraded by incorporating damage in the matrix and fiber materials separately similar to the studies by Tran et al. [7], Wei et al. [10] and Hou et al. [11]. Along with the damage in the plies, the dependency of their moduli and strength on the strain rates can be captured based on empirical relations that can be formulated from experiments as adopted by Phadnis et al. [4]. A more challenging failure to model is cracking of the matrix material for which discrete crack modelling techniques such as XFEM can be used.
3. Adopting 3D FE models for the analyses

In this thesis, FE models of the simplified test setup of the composite specimens are developed based on the 2D plane-strain assumption. The reason for choosing the plane-strain FE models in this thesis is that they give satisfactory results for the chosen specimen dimensions with lesser computational efforts when compared with the more realistic 3D FE models. But this assumption might result in huge errors when the out of plane strains in the specimen due to Poisson’s effect cannot be neglected. Also, capturing the out of plane behavior of the specimens with the plane-strain FE models is not possible, and all failure modes of the materials which influence the overall specimen behavior cannot be incorporated.

Therefore, analyzing the composite specimens with 3D FE models will allow a better prediction of their failure behavior. The material models have to be upgraded accordingly to incorporate the other failure modes that arise due to the 3D situation. But using 3D FE models for the analyses will have consequences such as an increase in the human efforts in terms of pre- and post-processing, and a huge increase in the computational efforts. These consequences cannot be removed but can be reduced by adopting strategical modeling techniques.

4. A real composite laminate in a blast loading situation

The main idea of this thesis originated from an aim to study the behavior of a TNO developed special composite laminate with alternate 0° and 90° layup when subjected to out of plane blast loads. It is theorized that the special composite laminate after being subjected to a blast load resists it by dissipating the energy majorly through interply delamination. But the stress states at the interfaces of the laminate will be very complicated due to the reflections and conversions of the incident normal blast wave. So the test setup for studying the failure behavior of composite specimens when subjected to dynamic compression loads with high rates is simplified and analyzed for its design feasibility in this thesis.

Even though the simplified test setup of the composite specimens gives a rough idea about their dynamic delamination behavior, the actual behavior in a blast loading situation cannot be analyzed with this simplification. So the behavior of a composite specimen against blast loading based on the stand-off distance can be simulated based on the techniques mentioned in the study by Kazanci et al. [6]. The computational heaviness and experimental costliness of modelling and testing such a complex failure process make the study, a highly challenging task.
Bibliography


The rate-dependent cohesive law coupled with friction developed in this thesis is implemented in an implicit framework and hence its consistent tangent matrix is derived. The traction vs. separation relation employed in the rate-dependent cohesive law with friction is similar to the one used by Van der Meer et al. [1] as follows:

\[ t = K(1 - d)\delta + Dt^{fric} \]  

(A.1)

The consistent tangent matrix for a given time step \( \Delta t \) during the damage growth is defined as:

\[ D_{cons} = (1 - d)KI - K\delta\left(\frac{\partial d}{\partial \delta}\right)^T + t^{fric}\left(\frac{\partial D}{\partial \delta}\right)^T \]

(A.2)

where,
\( t \) is the traction vector.
\( \delta \) is the displacement jump or the separation vector.
\( t^{fric} \) is the Coulomb frictional traction vector.
d and D are two different damage variables which represent relative loss of stiffness and energy dissipation respectively.
K is the dummy stiffness.
I is the 2 x 2 identity matrix.

Detailed formulations of the traction-separation law with friction adopted in this study are given in section 2.2.3. The rate-dependency relations used in this study to improve the rate-independent cohesive law with friction are given in section 2.2.4. The derivatives of the damage variables d and D with respect to the displacement jump vector can be written as:

\[ \frac{\partial d}{\partial \delta} = \frac{\partial d}{\partial \delta_{eq}} \frac{\partial \delta_{eq}}{\partial \delta} + \frac{\partial d}{\partial \delta^0} \frac{\partial \delta^0}{\partial \delta} + \frac{\partial d}{\partial \delta^f} \frac{\partial \delta^f}{\partial \delta} \]

(A.3)
\[
\frac{\partial D}{\partial \delta} = \frac{\partial D}{\partial \delta_{\text{eq}}} \frac{\partial \delta_{\text{eq}}}{\partial \delta} + \frac{\partial D}{\partial \delta^0} \frac{\partial \delta^0}{\partial \delta} + \frac{\partial D}{\partial \delta^f} \frac{\partial \delta^f}{\partial \delta} \quad (A.4)
\]

The scalar derivatives in equations A.3 and A.4 can be written as:

\[
\frac{\partial d}{\partial \delta_{\text{eq}}} = \frac{\delta^f \delta^0}{\delta_{\text{eq}}^2 (\delta^f - \delta^0)} \quad (A.5)
\]
\[
\frac{\partial D}{\partial \delta_{\text{eq}}} = \frac{1}{\delta^f - \delta^0} \quad (A.6)
\]
\[
\frac{\partial d}{\partial \delta^0} = -\frac{\delta^f (\delta^f - \delta_{\text{eq}})}{\delta_{\text{eq}} (\delta^f - \delta^0)^2} \quad (A.7)
\]
\[
\frac{\partial D}{\partial \delta^0} = \frac{\delta_{\text{eq}} - \delta^f}{(\delta^f - \delta^0)^2} \quad (A.8)
\]
\[
\frac{\partial d}{\partial \delta^f} = \frac{\delta^0 (\delta_{\text{eq}} - \delta^0)}{\delta_{\text{eq}} (\delta^f - \delta^0)^2} \quad (A.9)
\]
\[
\frac{\partial D}{\partial \delta^f} = -\frac{\delta_{\text{eq}} - \delta^0}{(\delta^f - \delta^0)^2} \quad (A.10)
\]

The vector derivatives in equations A.3 and A.4 can be written/expanded as:

\[
\frac{\partial \delta_{\text{eq}}}{\partial \delta} = \frac{1}{\delta_{\text{eq}}} (\langle \delta^1 \rangle, \delta^2)^T \quad (A.11)
\]
\[
\frac{\partial \delta^0}{\partial \delta} = \frac{\partial \delta^0}{\partial B} \frac{\partial B}{\partial \delta} + \frac{\partial \delta^0}{\partial \tau_1} \frac{\partial \tau_1}{\partial \delta} + \frac{\partial \delta^0}{\partial \tau_2} \frac{\partial \tau_2}{\partial \delta} \quad (A.12)
\]
\[
\frac{\partial \delta^f}{\partial \delta} = \frac{\partial \delta^f}{\partial B} \frac{\partial B}{\partial \delta} + \frac{\partial \delta^f}{\partial \delta^0} \frac{\partial \delta^0}{\partial \delta} + \frac{\partial \delta^f}{\partial \delta^0} \frac{\partial \tau_1}{\partial \delta} + \frac{\partial \delta^f}{\partial \delta^0} \frac{\partial \tau_2}{\partial \delta} + \frac{\partial \delta^f}{\partial \delta^0} \frac{\partial \tau_1}{\partial \delta} + \frac{\partial \delta^f}{\partial \delta^0} \frac{\partial \tau_2}{\partial \delta} \quad (A.13)
\]

where, \(\delta^0\) and \(\delta^f\) are the displacement jumps at damage initiation in pure modes - I and II respectively. \(\delta^f\) and \(\delta^f\) are the displacement jumps at crack initiation in pure modes - I and II respectively.

The derivatives in equations A.12 and A.13 can be expanded/written as:

\[
\frac{\partial \delta^0}{\partial B} = \frac{\eta B^{n-1}}{\delta^0} ((\delta^0)^2 - (\delta^f)^2) \quad (A.14)
\]
\[
\frac{\partial \delta^f}{\partial B} = \frac{\eta B^{n-1}}{\delta^0} (\delta^0 \delta^0 - \delta^1 \delta^1) 
\]
\[
\frac{\partial B}{\partial \delta} = \left(\frac{\partial B}{\partial \delta^0}, \frac{\partial B}{\partial \delta^0}\right)^T = \left(\frac{-2\langle \delta^1 \rangle \langle \delta^2 \rangle^2}{\delta^2_{\text{eq}}}, \frac{2\langle \delta^1 \rangle^2 \delta^2}{\delta^4_{\text{eq}}}\right) \quad (A.16)
\]
\[ \frac{\partial \delta^0}{\partial \delta^0_1} = \frac{\partial \delta^f}{\partial \delta^f_1} = \frac{(1 - B^n) \delta^0}{\delta^0_1} \quad (A.17) \]
\[ \frac{\partial \delta^0}{\partial \delta^0_2} = \frac{\partial \delta^f}{\partial \delta^f_2} = \frac{B^n \delta^0_2}{\delta^0} \quad (A.18) \]
\[ \frac{\partial \delta^0}{\partial \tau_1} = \frac{\partial \delta^0_1}{\partial \tau_2} = \frac{1}{K} \quad (A.19) \]
\[ \frac{\partial \delta^f}{\partial \delta^0_1} = \frac{(1 - B^n) \delta^f_1}{\delta^0} \quad (A.20) \]
\[ \frac{\partial \delta^f}{\partial \delta^0_2} = B^n \delta^f_2 \quad (A.21) \]
\[ \frac{\partial \delta^f}{\partial \delta^0} = -\frac{\delta^f}{\delta^0} \quad (A.22) \]
\[ \frac{\partial \delta^f_1}{\partial G_1} = \frac{\partial \delta^f_1}{\partial \delta} \frac{\partial G_1}{\partial \delta} + \frac{\partial \delta^f_1}{\partial \tau_1} \quad (A.23) \]
\[ \frac{\partial \delta^f_2}{\partial G_2} = \frac{\partial \delta^f_2}{\partial \delta} \frac{\partial G_2}{\partial \delta} + \frac{\partial \delta^f_2}{\partial \tau_2} \quad (A.24) \]
\[ \frac{\partial \delta^f_1}{\partial G_1} = \frac{2}{\tau_1} \quad (A.25) \]
\[ \frac{\partial \delta^f_2}{\partial G_2} = \frac{2}{\tau_2} \quad (A.26) \]
\[ \frac{\partial \delta^f_1}{\partial \tau_1} = \frac{-2G_1}{(\tau_1)^2} \quad (A.27) \]
\[ \frac{\partial \delta^f_2}{\partial \tau_2} = \frac{-2G_2}{(\tau_2)^2} \quad (A.28) \]
\[ \frac{\partial \tau_1}{\partial \delta} = \begin{cases} \frac{c_1 v_0}{\delta_1 \Delta t} \left( \frac{\delta_1}{\delta_1}, 0 \right)^T, & \dot{\delta}_1 \geq \dot{\delta}^\text{ref}_1 \\ 0, & \dot{\delta}_1 < \dot{\delta}^\text{ref}_1 \end{cases} \quad (A.29) \]
\[ \frac{\partial \tau_2}{\partial \delta} = \begin{cases} \frac{c_2 v_0}{\delta_2 \Delta t} \left( 0, \frac{\delta_2}{\delta_2} \right)^T, & \dot{\delta}_2 \geq \dot{\delta}^\text{ref}_2 \\ 0, & \dot{\delta}_2 < \dot{\delta}^\text{ref}_2 \end{cases} \quad (A.30) \]
\[ \frac{\partial G_1}{\partial \delta} = \begin{cases} \frac{m_1 G_1}{\delta_1 \Delta t} \left( \frac{\delta_1}{\delta_1}, 0 \right)^T, & \dot{\delta}^\text{ref}_1 \leq \dot{\delta}_1 \leq \dot{\delta}^\text{inf}_1 \\ 0, & \text{otherwise} \end{cases} \quad (A.31) \]
\[ \frac{\partial G_2}{\partial \delta} = \begin{cases} \frac{m_2 G_2}{\delta_2 \Delta t} \left( 0, \frac{\delta_2}{\delta_2} \right)^T, & \dot{\delta}^\text{ref}_2 \leq \dot{\delta}_2 \leq \dot{\delta}^\text{inf}_2 \\ 0, & \text{otherwise} \end{cases} \quad (A.32) \]