THE BIOMORPHIC MODEL OF THE HUMAN PILOT

by

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SUMMARY

This report discusses a mathematical model of the overt behaviour of a human operator, when controlling an aircraft-like dynamic element in a closed loop situation.

Because the aim has been to achieve a rather close correspondence between the physiological and psychological processes going on in the actual human controller and their mathematical expression in the model, the latter has been named the "biomorphic" model.

In its initial and relatively simple version described in this report, the biomorphic model refers only to single-display, single-axis control situations. A mathematical description and the underlying psychophysical justification are given. Furthermore, a computer program selecting the free parameters in the model and calculating the latter's various characteristics is briefly discussed in the report.

Quantitative applications and extensions, in particular those that will permit portraying certain mental processes in the brain leading to the pilot's judgement of the aircraft's handling qualities, will be described in subsequent reports.
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SYMBOLS

$a_1$, $a_2$  control law parameters
A  amplitude of impulse fed to the neuromuscular system
$\Delta A$  amplitude of impulse due to motor noise
$[A]$  system matrix, see (6-2)
$b_1$, $b_2$  control law parameters
B  amplitude of incremental step fed to the neuromuscular system
$\Delta B$  amplitude of incremental step due to motor noise
$[B]$  control distribution matrix, see (6-2)
$\tilde{c}$  vector, see (7-5)
c_{11}$, c_{12}$, c_{21}$, c_{22}$  elements of the forcing function matrix, $[C_1(t)]$
c_{4}$  parameter controlling part of the motor noise, see (5-4)
$[c]$  observation matrix, see (6-5)
$[C_{f1f2}]$  covariance matrix of the characteristic features $f_1$ and $f_2$
$[C_X(t)]$  covariance matrix of the state vector $\tilde{x}$ at time $t$
$[C_1(t)]$  forcing function matrix at time $t$
$[C_X]$  time-averaged covariance matrix of the state vector $\tilde{x}(t)$
$\tilde{d}_1$, $\tilde{d}_2$, $\tilde{d}_3$, $\tilde{d}_4$  vectors required in the calculation of $[C_X(T)]$
$[D_L]$  diagonal matrix, the elements of which on the main diagonal are the eigenvalues of $[L']$
$[D_F]$  diagonal matrix, the elements of which on the main diagonal are the eigenvalues of $[F]$
\[ D_F \]
diagonal matrix, the elements of which on the main diagonal are the eigenvalues of \([F']\]

\[ D \]
matrix defined in (A5-1)

\[ DF(j\omega) \]
describing function of the biomorphic model

\[ e(t) \]
control system tracking error

\[ \bar{e}_1, \bar{e}_2, \bar{e}_3, \bar{e}_4 \]
vectors required in the calculation of \([C_X(T)]\)

\[ [E_j] \quad j = 1, \ldots, 9 \]
matries required in the calculation of \([C_X(T)]\)

\[ E( ) \]
expectancy operator, ensemble average

\[ f_1, f_2 \]
characteristic features

\[ \Delta f_1, \Delta f_2 \]
estimation error of the characteristic features

\[ [F] \]
system matrix, see (7-9)

\[ [F'] \]
system matrix, see (7-23)

\[ g_1^T, g_2^T \]
transposed vectors required in the calculation of \([C_X(T)]\)

\[ [G] \]
matrix defined in (8-38)

\[ h_1^T, h_2^T, h_3^T, h_4^T \]
transposed vectors required in the calculation of \([C_X(T)]\)

\[ H \]
entropy

\[ H_X, H_Y \]
entropy of the variables \(x(t)\) and \(y(t)\) respectively

\[ H_i(j\omega) \]
transfer function of the input shaping filter

\[ i(t) \]
control system input signal or forcing function

\[ I \]
transmitted information

\[ I \]
indefinite integral, see (7-11)

\[ I' \]
definite integral, see (7-19)

\[ [IS] \]
initial condition required to calculate \(S\)

\[ J \]
cost function
\[ k_1^2, k_2^2 \] factors determining the observation noise
\[ k_3^2, k_4^2 \] factors determining the motor noise
\[ [K] \] matrix, see (7-13)
\[ K_i \] gain factor of \( H_i(j\omega) \)
\[ K_c \] gain factor of the controlled element
\[ K_v \] gain factor of the internal model
\[ [L] \] matrix, see (7-14)
\[ [L'] \] matrix, see (7-26)
\[ [M_L] \] matrix, the columns of which are derived from the eigenvalues of \([L']\)
\[ n \] order of the system describing the control system in which the biomorphic model is the controlling element
\[ o(t) \] control system output signal
\[ P_i \] probability of a discrete stochastic variable having the value \( i \)
\[ p(x) \] probability density function of a continuous stochastic variable \( x \)
\[ s(t) \] control manipulator deflection
\[ S \] deterministic basis of the costfunction \( J \)
\[ t \] time
\[ T \] sample interval
\[ [T] \] transformation interval
\[ \Delta t_i \] initiation interval
\[ \Delta t_o \] observation interval
\[ \Delta t_u \] uncommitted interval
\[ [V] \] intensity matrix of the white noise signal \( x_i(t) \)
$W(t)$ weighting function

$x$ independent variable

$x(t)$ state vector

$x'(t)$ state vector of the system with shifted eigenvalues

$x_i(t)$ white noise signal

$x_j(t)$ ($j = 1, \ldots, n$) state variable

$y$ observed variable

$y(t)$ vector of observed variables

$y_i$ ($i = 1, \ldots, 4$) observed variable

$[Y]$ nx1 matrix (vector) defined in (7-16)

$z(t)$ state vector, see (A3-3)

$z_i(t)$ ($i = 1, \ldots, n$) state variable

$z'(t)$ state vector of the system with shifted eigenvalues

$z_i'(t)$ ($i = 1, \ldots, n$) state variable

$\Delta(\omega)$ term in the biomorphic model's describing function, see (9-1)

$\varepsilon$ error

$\varepsilon_1, \varepsilon_2$ observation errors in the observed characteristic features

$\zeta_c$ damping ratio in the second order controlled element

$\zeta_d$ damping ratio of the desired response

$\zeta_i$ damping ratio in the input shaping filter

$\zeta_s$ damping ratio in the neuromuscular shaping network
\( \zeta_v \)  
\( \lambda_i \)  
\( \lambda_i' \)  
\( \nu \)  
\( \sigma_i \)  
\( \sigma_{ij} \)  
\( \tau_c \)  
\( \tau_d \)  
\( \tau_{ob1}, \tau_{ob2} \)  
\( \tau_v \)  
\( \Phi_{ii}(\omega) \)  
\( \Phi(t) \)  
\( \Phi_{x_1x_5}(j\omega) \)  
\( \Phi_{x_1y_1}(j\omega) \)  
\( \omega \)  
\( \omega_i \)  
\( \omega_{ie} \)  
\( \omega_i, \omega_k \)  
\( \omega_{oc} \)  
\( \omega_d \)  

damping ratio in the second order internal model  
eigenvalue  
shifted eigenvalue, see (A3-2)  
dummy time variable  
standard deviation of the variable i  
covariance of the variables i and j  
time constant of the first order controlled element  
time constant of the time delay  
time constants of the observation process  
time constant of the first order internal model  
power spectral density function of \( i(t) \)  
transition matrix  
cross-power spectral density function of \( x_1(t) \) and \( x_5(t) \)  
cross-power spectral density function of \( x_1(t) \) and \( y_1(t) \)  
circular frequency  
bandwidth of a periodic forcing function  
effective bandwidth of a filtered noise forcing function  
frequency of the \( i^{th} \) and \( k^{th} \) sinusoidal component of the periodic forcing function  
undamped natural frequency of the second order controlled element  
undamped natural frequency of the desired response
$\omega_{oi}$  
undamped natural frequency of the input shaping filter

$\omega_{os}$  
undamped natural frequency of the neuromuscular shaping network

$\omega_{ov}$  
undamped natural frequency of the second order internal model

$\Omega_s$  
circular frequency of the sampling process, see (9-4)

$\hat{\cdot}$  
expected value

$\circ$  
observed value
1. INTRODUCTION

Mathematical models of human controller behaviour in closed loop manual control systems have been studied for over thirty years (1) and a multitude of models have been proposed (2-6). The subject is one of continuing interest (7) and mathematical models of pilot behaviour in particular find many applications (8-10).

The principal aims of the human pilot model presented in this report are twofold. In the first place, the model should be able to portray the overt physical actions of the pilot, in particular the actuation of the controls, in certain well defined and more or less "classical" closed loop control situations. This means that the model should show in the prescribed situations a control behaviour, as expressed by the describing function, cross-over frequency, remnant power etc., which is comparable to the behaviour of actual pilots.

In the second place, the model should - in an appropriately expanded form - express certain more covert mental activities involved in manual control, such as information handling and decision making. By including in the model both the overt physical and the covert mental activities, it is hoped that the model will be a useful tool in the understanding of what constitute good handling qualities of aircraft. The model should allow conclusions to be drawn about the level of difficulty a well-trained human pilot would experience in controlling a given aircraft configuration to the same level of performance, subject to the same level of external disturbances. This facility of the model should provide a link with the pilot's subjective opinion and thus with some of the factors making up the pilot workload. Ultimately it may even be possible with the aid of such a model to express desirable handling qualities of aircraft in terms of the behaviour, capabilities and limitations of the human pilot.

Because of these ultimate aims of the model is it understandable that
relatively much attention is given in this report to physiological and psychological aspects of the pilot's task, in particular to the processing of sensed data and to the workings of the neuromuscular system. Hence also the name, the biomorphic model.

The initial and simplified form of the model presented in this report, has rather more restricted aims. The purpose here is merely to describe the pilot's overt behaviour in certain aircraft-like closed loop manual control systems. Achieving the wider aims with the biomorphic model required further expansions, which would be beyond the scope of this report.

The closed loop control systems considered in this report are characterized by a number of restrictive features, see Fig. 1.1.

1. The system has only one input signal, or forcing function, \( i(t) \), which is stochastic or least appears stochastic to the human operator.

2. Continuous and full operator's attention is assumed in the performing of a continuous compensatory tracking task. This latter condition implies that the operator is presented only with the error signal, \( e(t) \), on a single display, see Fig. 1.1.

3. The human operator senses the displayed variable visually (foveally) only and he derives measures of both the magnitude, \( e \), and its rate of change, \( \dot{e} \), from the display.

4. The operator reacts on the sensed data by means of deflections, \( s(t) \), of a single control manipulator, see Fig. 1.1.

5. The controlled element - i.e. the aircraft or a dynamically similar element - is assumed to be linear in its reactions to the control deflections.

It should be remarked here, that in experiments it is not always easy to fulfill the requirement of attracting and holding the operator's continuous and full attention. In many cases it leads to a forcing function of rather higher bandwidth than would occur in actual flight.
For reasons to be explained later in this report, the biomorphic model operates in a sampling way, rather than in a continuous one like many human operator models. In line with modern control theory, the model is described primarily in the time domain.

The description of the biomorphic model in the following Sections is based on the well known three aspects of human activity in manual closed loop control:

1. observation
2. decision making
3. output generation

These three subjects are discussed in the Sections 3, 4 and 5. In Section 6 the complete set of equations representing the behaviour of the closed loop in which the biomorphic model is the controller, is assembled. The next three Sections 7, 8 and 9 discuss the determination of the control law in the model, the covariance matrix of the state variables and the describing function of the model respectively. These matters do not further expand the biomorphic model but they are of importance when working with the model. The final Section 10 presents a brief overview of a computer program which is based on the previous three Sections. It calculates the various items discussed in these Sections and selects the free parameters in the biomorphic model via an optimization procedure discussed in Section 10.
2. INPUT SIGNAL

Before starting the discussion of the biomorphic model proper, the input signal to the closed loop system, i.e. the forcing function $i(t)$, will be defined. It is the logical first step in building up the total matrix formulation describing the operation of the biomorphic model.

The input signal to the control loop, $i(t)$ see Fig. 1.1., is a continuous, stochastic, normally distributed, band-limited signal. This signal is derived by passing normally distributed white noise, $x_i(t)$, through a low-pass second-order filter characterized by a transfer function $H_i(j\omega)$:

$$H_i(j\omega) = \frac{K_i}{1 + 2\zeta_i \cdot j \frac{\omega}{\omega_i} + \left(j \frac{\omega}{\omega_i}\right)^2} \quad (2-1)$$

where:

$$K_i = 1$$
$$\zeta_i = 1$$
$$\omega_i = \text{variable, chosen according to the experiment}$$

In matrix notation the equation for the input shaping filter is:

$$
    \begin{bmatrix}
        \dot{x}_1 \\
        \dot{x}_2 
    \end{bmatrix} =
    \begin{bmatrix}
        0 & 1 \\
        -\omega_i^2 & -2\omega_i
    \end{bmatrix}
    \begin{bmatrix}
        x_1 \\
        x_2 
    \end{bmatrix}
    +
    \begin{bmatrix}
        0 \\
        \omega_i^2
    \end{bmatrix}
    \cdot x_i \quad (2-2)
$$

where:
\( x_1 = \) forcing function, \( i(t) \)
\( x_2 = \) time derivative of \( x_1 \)
\( x_4 = \) normally distributed white noise signal

The power spectral density function of this forcing function, \( i(t) \), is shown in Fig. 2.1.

In many experiments with human pilots or pilot models, use is made of a harmonic function which is the sum of say-10 sine waves of different frequencies.
The frequencies are chosen such that:
  a. an integral number of periods at each frequency is contained in the record length to be analyzed
  b. no two frequencies are integral multiples of one another.

Such a forcing function, although strictly periodic, appears nevertheless stochastic to the human pilot, due to a suitable choice of the composing frequencies\(^{(11)}\).

The amplitudes belonging to the lowest - for example 6 - frequencies are equal and significantly larger than the amplitudes belonging to the remaining higher frequencies. The bandwidth of such a periodic forcing function is characterized by the highest frequency, \( \omega_1 \), having the larger amplitude. The spectrum of this periodic forcing function is shown in Fig. 2.2.

In the analysis of measurements, the periodic forcing function made up from discrete frequencies, offers certain advantages over true stochastic signals having a continuous frequency content.

As mentioned already, the human pilot will not be able to distinguish the periodic forcing function from a true stochastic signal, provided
both signals are normally distributed, have the same variance and have
the same effective bandwidth. It is, therefore, of some importance
to know the magnitude of the undamped natural frequency \( \omega_o \) of the
input shaping filter, required to generate a true stochastic signal
having a bandwidth that matches the bandwidth \( \omega_i \) of a periodic signal
on which certain experimental results are based. According to (12),
the effective bandwidth, \( \omega_{ie} \), of filtered noise forcing functions can
be best derived by:

\[
\omega_{ie} = \frac{\int_{0}^{\infty} \left[ \phi_{i1}(\omega) \right]^2 \, d\omega}{\int_{0}^{\infty} \left[ \phi_{i1}(\omega) \right]^2 \, d\omega}
\]

where:

\( \phi_{i1}(\omega) = \text{power spectral density function of the forcing function} \)

It can be shown, that for the input shaping filter given by (2-1) or
(2-2) the result is:

\[
\omega_{ie} = \frac{2\pi}{5} \cdot \omega_o
\]

or

\[
\omega_{ie} = 1.26 \cdot \omega_o
\]

where \( \omega_{ie} \) and \( \omega_o \) have been defined in the above text.
3. OBSERVATION PROCESS

3.1. Introduction

In the following a mathematical model is discussed, describing the process of visually observing a single instrument display, as performed by the human pilot.

In the situations under consideration, the pilot performs a compensatory control task. This means that he visually observes the error or difference signal, $e(t)$, between the input signal, $i(t)$, and the output signal of the control loop, $o(t)$, which is fed back according to Fig. 1.1. Usually this output signal is assumed to be an attitude angle of an aircraft, either the angle of pitch or the angle of roll. The error signal is presented to the pilot on a visual display. On the basis of his visual observations, the pilot generates control deflections aimed at reducing $e$ to zero in a certain optimal way.

The observation model to be described is limited to the case where the pilot looks foveally at the display indicating the error signal, $e$, while his attention is directed continuously at the display. The model of the visual observation process is stochastic, not only because the forcing function of the control loop is stochastic, but above all because in the course of the observation process, observation noise is introduced. This leads to observed values containing stochastic observation errors.

One could imagine that this Section might begin with a brief description of physiological processes involved in visual perception, based on the extensive knowledge available on this subject in physiology. In this fascinating field many exciting new insights have been obtained in recent years and significant progress is still being made.
Nevertheless, important gaps in available knowledge still remain, particularly as regards the functioning of higher centers in the brain in those cases of particular interest to the present study, where the visually perceived scene changes more or less quickly in time, rather than being stationary.

Even greater hiatuses in physiological knowledge appear to exist when situations are considered where the human observer can partially predict what he will observe, because he acts as the controller in a closed loop control system. And those situations are precisely the subject of this report.

The notion of an "internal model", to be discussed in par. 3.3. is of the utmost importance to those closed loop control situations and it seems to have found increasing acceptance in recent years in cognitive psychology\(^{33-37}\). Yet it appears that physiologists are generally somewhat reluctant to accept the notion\(^{38}\), partly because this internal model resides in memory. A fully satisfactory explanation of various aspects of "memory" in terms of physiology is well beyond present capabilities\(^{39-42}\).

The foregoing may explain why no attempt is made here at a description of the physiology of vision. Reference is made again to the many excellent textbooks on this subject. The above makes it also clear that several aspects of the observation model, to be discussed in this Section, are of necessity somewhat speculative.

### 3.2. Sampling process

For various reasons the observation model discussed here is a sampling model. Sampling models of pilot observation and control behaviour have appeared more or less regularly during the years that pilot models were studied. Ref. (6) gives a short account of these developments. The main reason to propose sampling models was usually the desire to improve the correspondence between human operator response data and the
results from the model (43-47).
It should be remarked, however, that the results of careful studies, e.g. (48), have not substantiated the assumption of a sampling operation in the human, at least not an operation at a fixed sample interval. As observed in (49), and confirmed in (48), sampling models and continuous models generally seem to be able to describe the human operator's behaviour studied here equally faithful.

The reasons for choosing a sampling operation in the present observation model are only partly the same as those of authors in the past. In particular the existence, noted in (50-53), of standard output patterns in the human operator behaviour points in the direction of intermittent operation, at least at a certain level in the Central Nervous System. At the input side of the human operator, the extraction of certain characteristic features from sensed data, as discussed in (17, 18, 21, 31, 53) and contained in the present observation model, may also be described more conveniently and perhaps even more convincingly in a sampling model.

There is, however, yet another and overriding argument. In the cockpit in actual flight, sampled observation of the various instruments, resulting in more or less discontinuous responses, is the rule rather than an exception. Conversely, paying continuous and full attention to a single display for any length of time appears to be quite an extraordinary situation in actual flight.

Experimental data on display scanning (46), has shown that the actual visual sampling occurring in multi-display tasks is never perfectly periodic. The average interval differs for each instrument and depends on the flight task.

Apart from the sample interval one has to distinguish a dwell-time for each instrument in the display. It is the time during which the pilot
fixates foveally on that particular instrument. Like the sample interval, the dwell-time varies about an average value for a given instrument. More complex and higher bandwidth displays require larger dwell-times. The sample interval and the dwell-time also appear to depend very significantly on the individual test subject.

In a sense, the case of a continuous attention, single-display control task considered here, can be taken as an extreme of a sampled-operation. It is the case where the dwell-time for the single display equals the entire sample interval. If the difficulty of the control task so requires and at the same time the lay-out of the display so allows, the sample interval may for a well-trained operator become as short as the duration of the saccadic eye motions, i.e. some 200 to 300 ms. As Bekey has shown (47), even relatively small fluctuations in the sample interval completely mask the sampling operation from any experimental evidence taken from the human operator in such a continuous attention, single-display task.

In order to avoid undue complications in the present observation model, the sample interval, T, is constant. The actual observation is assumed to be made at the end of the interval, which coincides with the beginning of the next interval, see Fig. 3.1.

On the basis of extensive discussions in the existing literature on seeing and vision (26, 31, 53), it is hypothesized that the first step in the observation process relevant to a control task is the extraction of so called "characteristic features". This means that the observer takes account only of the most essential elements in the visual scene which is the subject of his attention. In the present situation these elements - the characteristic features - are considered to be the error, e, and its rate of change, ë, at the end of the sample interval.

It should be remarked here, again, that from a study of the physiological literature on the visual sensory channel it becomes clear that many
questions concerning the visual observation process, especially
where the observed variable are time-varying, are still wide open for
further study and explanation.

The foregoing can be summarized by saying that the model makes an
observation at the discrete instants in time marked 0 in Fig. 3.1.,
where observed values of e and  are obtained.

3.3. Internal model

An important element in the mathematical description of the observation
process as used in the biomorphic model, is the use of an explicit
"internal model", which is assumed to reside in the human observer's
memory. The notion of an internal model can be elucidated as follows.

A pilot flying an aircraft knows, or rather expects, in a general sense
what will happen next. This expectancy stems in the first place from his
general flying experience and from the flight plan he made before taking
off. On a more limited time scale, the pilot expects certain motions
from the aircraft in the next few moments, because of his previous
observations of the aircraft's motion and also because he remembers
his most recent control deflections.

From this simple example follows very generally, that an expectancy
or estimate of what will be observed in the very near future, is
obtained from knowledge residing in memory. It is due to the internal
representation in the mind of the world around us. This internal
representation is commonly called the "internal model". Obviously,
the term "internal model" is used here in an abstract sense, rather
than in a pictorial sense. Observing the continuously changing world
around us can thus be described as entailing the construction and
regularly updating of a predictive internal model, using various parts
of memory to incorporate past experience.
The idea of internal models is not all new. It was proposed as early as 1943 by Craik (54). The concept is gaining more and more acceptance, see (51, 53-58).

In the more particular case of a trained pilot flying an aircraft fully familiar to him, an internal model can be said to exist in his brain for each of his sensory channels. These internal models provide a prediction or estimate for each characteristic feature the pilot is paying attention to. The various partial models combined make up the total internal model mentioned above. The internal model is thus-subconsciously employed to predict the impending motion of the aircraft.

Observations in the form of observed characteristic features serve to update the state variables of this predicting internal model. Each subsequent observation updates the internal model anew.

The purpose of the internal model in the description of the visual observation process thus is: to provide an estimate of the characteristic features for the next observation to be made.

The dynamic characteristics of the internal model should, of course, match those of the controlled element as closely as possible, as the result of a previous learning or training process. This learning process will be modelled separately.

The internal model is represented here by a set of linear differential equations. When the controlled element is of zero or first order, the internal model is of the same order. In all other cases the internal model is of second order, irrespective of the order of the controlled element whose dynamic behaviour the internal model has to match. This is due to the fact that only two independent observed values are available to update the state variables of the internal model.

Taking account of other elements of the pilot model, yet to be
discussed in subsequent Sections, the internal model is written as:

\[
\begin{bmatrix}
\dot{x}_7 \\
\dot{x}_8
\end{bmatrix} =
\begin{bmatrix}
0 & 1 \\
-\omega_v & -2\zeta_v\omega_v
\end{bmatrix}
\begin{bmatrix}
x_7 \\
x_8
\end{bmatrix}
+ 
\begin{bmatrix}
0 \\
K_v\omega_v
\end{bmatrix}
\cdot x_5
\]

(3-1)

where:

\(x_5\) = total control deflection, to be discussed in Section 5

\(x_7\) = estimate of visually observed variable

\(x_8\) = time derivative of \(x_7\)

At the beginning of a new sample interval the calculations in this internal model are restarted, using the most recent observed characteristic features as initial conditions. Later in this Section, these observed characteristic features are written as \(\overset{\circ}{f}_1\) and \(\overset{\circ}{f}_2\). Using this result already here, the initial conditions of the internal model at \(t = 0\) are then:

\[
\begin{bmatrix}
x_7(0) \\
x_8(0)
\end{bmatrix} = 
\begin{bmatrix}
\overset{\circ}{f}_1 \\
\overset{\circ}{f}_2
\end{bmatrix}
\]

The internal model receives in addition as an input signal the continuously varying control deflection, represented by \(x_5\) in (3-1). In the present version of the pilot model, this variable is assumed to be observed via the tactile and proprioceptive senses, both continuously and without error, see also par. 5.5.
It is to be noted, that the internal model is not directly subjected to the forcing function, $i(t)$, of the control loop because this variable is not accessible to observation by the pilot in the compensatory control task, see Fig. 1.1.

At this stage of the discussion, a refinement in the notation should be introduced. In contrast to the state variables of the biomorphic model, $x_j$ $(j = 1, \ldots, n)$, the observed variables and their estimated counterparts produced by the internal model are denoted by $y_i$ $(i = 1, \ldots, m)$. The characteristic features, to be written as $f_k$ $(k = 1, \ldots, l)$ are the values of the observed variables at the instant of observation, $t = T$.

Running-again-slightly ahead of the further discussion and assuming a second order controlled element, having $x_9(t)$ and $x_{10}(t)$ as its state variables, the controlled element, see also Fig. 1.1., is described by:

\[
\begin{bmatrix}
\dot{x}_9 \\
\dot{x}_{10}
\end{bmatrix}
= \begin{bmatrix}
0 & 1 \\
-\omega_c^2 & -2\zeta_c\omega_c
\end{bmatrix}
\begin{bmatrix}
x_9 \\
x_{10}
\end{bmatrix}
+ \begin{bmatrix}
0 \\
k_c\omega_c^2
\end{bmatrix}
\cdot x_5
\]

where:

$x_9$ = controlled variable

$x_{10}$ = time derivative of $x_9$

The observed variables, indicated earlier as $e$ and $\dot{e}$, see Fig. 1.1., are now written as:

\[
y_1 = e = x_1 - x_9
\]

\[
y_2 = \dot{e} = x_2 - x_{10}
\]
The estimates of these observed variables are:

\[ y_3 = x_7 \]  \hspace{1cm} (3-4)

\[ y_4 = x_8 \]  \hspace{1cm} (3-5)

Using this new notation for the sensed and estimated values of the observed variables, the characteristic features are written as:

\[ f_1 = y_1(T) \]  \hspace{1cm} (3-6)

\[ f_2 = y_2(T) \]  \hspace{1cm} (3-7)

and the estimated characteristic features are:

\[ \hat{f}_1 = y_3(T) \]  \hspace{1cm} (3-8)

\[ \hat{f}_2 = y_4(T) \]  \hspace{1cm} (3-9)

3.4. Observation noise

A further essential element in the observation process is the observation noise. The basic idea behind the modelling of the observation noise in the biomorphic model is the empirical fact that it takes a human being some time to make a decision. In the present situation the decision to be made concerns the magnitude of the observed variable and its time derivative. The more precise the observation has to be, the more time it takes to arrive at the decision.
Formulated in another way: man has a limited capacity to handle information. The word "information" is used here in the sense given by Shannon and Weaver (59), meaning a measure of entropy or uncertainty, whereas "handling information" means: reducing the uncertainty.

This subject is dealt with in some detail in Appendix 1. There it is indicated that the main variable of interest in the present discussion is the so-called "transmitted information", I, usually expressed in bits. As stated in Appendix 1 it is this transmitted information which provides a measure of the limited information handling capacity of the human observer.

For simple tasks and given the continuous attention of the test subject, experimental data indicate a remarkably linear relationship between the number of bits of transmitted information the brain handles and the time it takes to do (6, 60, 61). This important empirical fact was discovered as early as 1885 by Merkel (62), although described in slightly different terms at the time.

It has been noted already that every observed value is predicted by the internal model which provides the estimated values. As a consequence, the uncertainty in a variable to be observed resides, before the observation, in the difference, $\Delta f$, between the sensed value, $f$, and the estimated value, $\hat{f}$, provided by the internal model. This difference is the so-called "estimation error":

\[
\Delta f_1 = f_1 - \hat{f}_1 \\
\Delta f_2 = f_2 - \hat{f}_2
\]

where $f_1$, $f_2$, $\hat{f}_1$, and $\hat{f}_2$ have been introduced before in (3-6) - (3-9).
After the observation, the remaining uncertainty in an observed value lies in the residual observation error, \( \varepsilon \). This residual error is assumed to be stochastic, uncorrelated with \( \hat{f} \) or \( \tilde{f} \), and Gaussian. It constitutes the observation noise in the biomorphic model.

Denoting, finally, the observed value of a characteristic feature by \( \hat{f} \), the following relations hold:

\[
\begin{align*}
\hat{f}_1 &= f_1 + \varepsilon_1 \quad (3-12) \\
\hat{f}_2 &= f_2 + \varepsilon_2 \quad (3-13)
\end{align*}
\]

The statement made earlier, that the human observer has a limited information handling capacity, can now be further interpreted by saying that the observer needs some time, \( \Delta t_0 \), to reduce the observation error from the initial value which is equal to the estimation error, \( \Delta f \), to the smaller residual value, \( \varepsilon \). The observation interval, denoted by \( \Delta t_0 \), is taken to be the final part of the sample interval, see Fig. 3.2.

Note, that in this more refined presentation of the sample interval, as compared to Fig. 3.1., the actual observation takes an interval \( \Delta t_0 \) to come about, whereas in the model of the observation process it is still assumed that the instant of the formal observation is situated at the end of the sample interval, coinciding with the beginning of the next one.

It is relatively easy to show that the linear relationship between the number of bits to be handled and the time to do so, as mentioned before in this Section, leads to an exponential decrease of the variance of the observation error from that of the initial estimation error to a final, reduced value, in the time \( \Delta t_0 \) during which attention is given to the observation:
\[ \sigma_{\varepsilon_1}^2 = k_1^2 \cdot \sigma_{\Delta f_1}^2 \]  
(3-14)

\[ \sigma_{\varepsilon_2}^2 = k_2^2 \cdot \sigma_{\Delta f_2}^2 \]  
(3-15)

where:

\[ k_1^2 = e^{-\Delta t_o / \tau_{ob1}} \]  
(3-16)

\[ k_2^2 = e^{-\Delta t_o / \tau_{ob2}} \]  
(3-17)

Using (A1-2) and (A1-3), it is easy to derive from (3-16) or (3-17) a relation between \( \tau_{ob} \) and the rate of transmitted information, \( \hat{I} \):

\[ \tau_{ob} = \frac{1}{2} \cdot \frac{1}{\ln 2} \cdot \frac{1}{\hat{I}} \approx \frac{0.72}{\hat{I}} \]  
(\( \hat{I} \) in bits/s)

With these final formulas, the presentation of the observation model is complete.

A block diagram of the model is given in Fig. 3.3. This diagram summarizes the previous description. The observation model clearly needs as continuous inputs the displayed error, \( e(t) \), and error rate, \( \dot{e}(t) \), of the closed loop. In addition the control deflection, \( x_5(t) \), is needed as a continuous input. The output of the model are the observed characteristic features, \( \hat{f}_1 \) and \( \hat{f}_2 \), which become available periodically, at the beginning of every new sample interval.

From the discussion of the observation model, a possible trade-off becomes apparent when this model is used in a closed loop control
situation. On the one hand, a long observation interval, $\Delta t_0$, and as a consequence – a long sample interval, $T$, would – within limits – lead to accurate observations enabling a more accurate control to be exercised. On the other hand, however, the longer intervals between successive control deflections based on these observations, would necessarily reduce the accuracy of the control loop. Clearly, an optimum in the observation interval has to be found where, for given characteristics of the forcing function, the best accuracy of the closed loop control is obtained, as expressed for instance in a minimum variance of the error signal of the control loop, $e$. 
4. DECISION PROCESS

4.1. Introduction

A central difficulty in the discussion of the decision process lies in the following fact. Although the decision process expresses the seemingly willful and conscious decisions of the pilot about his control actions, the processes in the central nervous system involved in the decision making as well as in the generation of the subsequent control actions occur beyond the pilot's conscious control and without the possibility of his interference with the processes.

As a consequence, the modelling of what the pilot decides and how he arrives at the decisions can hardly be based on what he consciously thinks or verbally expresses about these processes. After an initial learning period these actions mostly occur to the pilot "just naturally".

The following brief discussion of the decision process, therefore, has two main characteristics:

- the process is based on external observations of the pilot's overt behaviour, rather than on the pilot's own introspection;
- the following mathematical formulation is inevitably a limited, arbitrary and extremely simplified picture of reality, given:
  a) the richly nuanced possibilities the pilot has to arrive at his decisions, and
  b) our relatively incomplete knowledge of what goes on in the brain as far as these decision processes are concerned.

Justification of the choices made in the following can lie only in the ultimate results. The behaviour of the biomorphic model must correspond in its essential aspects with actual pilot behaviour over as wide a range of control situations as possible.
4.2. Control law

On the basis of the results produced by the observation process described in Section 3, a pilot generates control deflections, aimed at reducing the observed error in the aircraft's attitude.

In Section 5 it will be argued that the pilot's control deflections generally exhibit certain more or less fixed patterns. The biomorphic model reflects this characteristic. Of all the possible types of response patterns, the biomorphic model uses only a very small number, namely two. They are based on the impulse function and on an incremental step. Those two time functions form the idealized incremental control deflections the pilot is supposed to have in mind. His actual control deflections are shaped after these idealized patterns. This subject will be discussed in greater detail in Section 5.

It is thus assumed, that following an observation, at the beginning of a new sample interval, the pilot makes a decision regarding the next intended incremental control deflection. On the basis of the values of the observed characteristic features $\bar{f}_1$ and $\bar{f}_2$ just obtained, he decides how large the intended magnitudes of impulse $\hat{A}$ and incremental step $\hat{B}$ shall be.

In order to distinguish between those intended control deflections and the actual ones, the symbols $\hat{A}$ and $\hat{B}$ are used to indicate the intended magnitudes of $A$ and $B$.

The relations between $\hat{A}$ and $\hat{B}$ on the one hand and $\bar{f}_1$ and $\bar{f}_2$ on the other hand form the control law in the biomorphic model. Assuming linear relations, the control law can be expressed as follows:

\[
\begin{bmatrix}
\hat{A} \\
\hat{B}
\end{bmatrix} = \begin{bmatrix}
a_1 & a_2 \\
b_1 & b_2
\end{bmatrix} \cdot \begin{bmatrix}
\bar{f}_1 \\
\bar{f}_2
\end{bmatrix}
\]

(4-1)
The four elements $a_1$, $a_2$, $b_1$ and $b_2$ in this control law are obtained as the result of an optimization procedure, which is the representation of a learning process occurring in reality. The required optimization procedure is described in Section 7 of this report. A block diagram of the decision process just described, is given in Fig. 4.1.
5. NEUROMUSCULAR SYSTEM

5.1. Introduction

The neuromuscular system is the element in the closed loop control system, generating the overt output actions of the pilot when he controls the aircraft, see Fig. 1.2.

The input to the neuromuscular system is assumed to be a central command in certain parts of the central nervous system (C.N.S.), generated by the decision process elsewhere in the C.N.S., probably in the association cortex, as will be discussed in par. 5.2.2. Such a command gives rise to a signal flow in other parts of the nervous system, ultimately leading to a force exerted on the aircraft's control manipulator. This results in a displacement of the pilot's hand or foot which is in direct contact with the control manipulator. As a consequence of this control displacement the aircraft's motion changes, in the sense of the pilot's intention when he generates the initiating command in the C.N.S.

The principal elements of the neuromuscular systems are considered to be:

- various structures in the brain and the spinal chord,
- the skeletal muscles.

These elements are briefly discussed in the following paragraphs, before the generation of movement is considered. The neurophysiological data thus presented from the background for a mathematical formulation of a model of the neuromuscular system.

It should be mentioned, however, that in particular the anatomy and operation of the supraspinal structures of the C.N.S. are a very complex subject, with respect both to the macroscopic organization and to the fine structure of the individual components.
A corresponding discussion of the physiological and psychological aspects of seeing, vision and observing was not given in Section 3, simply because the required most relevant knowledge in those areas - which proves to be highly specialized even for physiologists and psychologists - is even less existent than that on the subject of the neuromuscular system.

5.2. Structures in the nervous system related to the neuromuscular system

5.2.1. Nerve cells

The elementary building blocks of the entire nervous system are the nerve cells or neurons. Before embarking on a description of certain elements of the nervous system, it seems appropriate to pay some attention to these fundamental units (14, 15, 16, 24, 27, 63).

Neurons exist in the nervous system in many forms and in large quantities. It is estimated that the human brain contains well over $10^{10}$ neurons. A typical neuron from a mammalian spinal chord is shown in Fig. 5.1. Each neuron has a cell body or soma, from which protrude a number of small branches, the dendrites. From the neural body also protrudes a long fibre called the axon. This axon links the nerve cell to other neurons. Neural body and dendrites from the major input area of the neuron.

Axon and dendrites normally divide at their ends in branches. The tips or end bulbs of the axonal branches are the output area of the neuron, since they make contact with other neurons or muscles. The junction of an axonal ending with another neuron is called a synapse. The ending of an axon on a skeletal muscle fibre is a neuromuscular end plate or neuromuscular junction; it will be mentioned again in par. 6.3.

The diameter of neural cell bodies is about 5 to 100 µm, the dendrites
can be several hundred μm long. The axons have a diameter of a few μm and their length can vary between some μm, to well over a meter in the case of certain neurons in the human body.

The two fundamental processes of neural activity are the conduction of impulses along the axons and the excitatory or inhibitory transmission of these impulses across the synapses.

Neurons are excited by a stimulus, which may be electrical, chemical or mechanical. The stimulus creates a disturbance, the impulse or action potential, which is transmitted along the axon. The impulse in a particular axon has a fixed shape and size. It is a brief, electrical event lasting about 1 ms. Stronger stimuli produce higher repetition frequencies of the impulse firing in the neuron. The impulse moves along the axon at a fixed speed of 1 to 100 m/s, depending on the axon diameter. Thicker axons have the higher conduction velocities.

Neurons influence each other at the synapses, either by excitation producing impulses in other neurons, or by inhibition preventing impulses from arising in other cells.

At the vast majority of synapses the presynaptic endbulb liberates a chemical transmitter substance in response to an action potential. At an excitatory synapse the chemical liberated by the presynaptic ending drives the postsynaptic cell toward the excitation threshold. At an inhibitory synapse the transmitter substance keeps the postsynaptic cell below that threshold.

5.2.2. Motor cortex

A number of brain structures, constituting a large proportion of the brain's mass, are involved in the generation and control movement. The more important motor centers in the brain are (14, 15, 16, 27):

- motor cortex,
basal ganglia,
cerebellum,
parts of the brainstem.
Fig. 5.2. indicates the approximate position of the motor centers in the brain and the spinal chord. All the motor centers are paired, i.e. each is represented in both the right and the left side of the brain.

The motor cortex, the first mentioned center in the above list, is part of the cerebral cortex, the outer layer of the brain. Because of the important functions of several areas of the cerebral cortex, it seems in order to briefly describe this part of the brain first.

The cerebral cortex, see Fig. 5.3., is a thin layer of neuronal tissue with a surface area of about 2200 cm$^2$ and a thickness varying between 1.3 and 4.3 mm in the various areas. It makes up about 40% of the total weight of the human brain and it contains $10^9$ to $10^{10}$ neurons, some 75% of all the neurons in the brain. The cortex is highly convoluted, its surface consisting of convex folds, or gyri, separated by furrows called sulci, or fissures, see Figs. 5.3. and 5.5.

The precentral gyrus and its surroundings are the motor cortex, see Fig. 5.2. The cortical regions in which the sensory pathways terminate (after relay in thalamic sensory nuclei) are termed the sensory cortex, or specific cortex as they are directly addressed by peripheral sensory inputs, see Fig. 5.4.

Not all sections of the cortex can be classified as either sensory or motor areas. Since it has been suspected that the remaining areas are mainly concerned with the interaction between motor and sensory cortical areas, they are usually called association cortex, or non-specific cortex. The association cortex takes up appreciably more space than the sensory and motor cortices.

The primary motor cortex is a strip lying immediately forward of the
central fissure, see Fig. 5.5. The skeletal muscles are represented in the motor cortex in an orderly, topographical pattern. The area of cortex devoted to particular muscle groups is more or less proportionate to their importance to the organism and the precision of movements which they are required to generate. This reflects the increased number of underlying cortical cells that are associated with the finer and more precise movements.

In addition to the primary motor area, there is a secondary and a supplementary motor area. The secondary area is situated in front of the primary motor area and may be concerned in part with learned motor activity of a complex and sequential nature. The supplementary motor area contains also a complete representation of the body musculature on the cortex. It lies mostly on the medial wall of the cortex, somewhat forward of the primary motor cortex.

5.2.3. Basal ganglia

The basal ganglia are a group of large nuclei lying in the central regions of the cerebral hemispheres, below the cerebral cortex, see Fig. 5.2. They are an important link between the motor cortex and the rest of the cerebral cortex. The associative cortex sends neural signals to, and the motor cortex receives such signals from the basal ganglia.

The functions of the basal ganglia involve participation in the conversion of the plans for movement arising in the associative cortex, into programs for movement, see Fig. 5.11.

5.2.4. Cerebellum

The cerebellum is situated at the back of the head, below the cerebrum, see Fig. 5.2. It is almost completely covered over by occipital lobes of the cerebral hemispheres. It consists itself also of two highly fissured hemispheres. The cortex of the cerebellum is even more
convoluted than that of the cerebrum, in order to achieve a large surface area with a small volume.

The cerebellum receives connections from the vestibular system, from spinal sensory fibers, from the auditory and visual systems, from various regions of the cerebral cortex and from the reticular formation. It sends efferent fibers to the thalamus, reticular formation and several other brainstem structures. The cerebellum eventually influences motorneurons supplying the skeletal musculature. It functions in the regulation of movement both before and during the movement.

The cerebellum and basal ganglia are centers of equal rank, see Fig. 5.11. They appear to be chiefly involved in the programming, coordination adjustment and smoothing of cortically initiated movement patterns. The precise way in which they influence the generation and control of movements is - unfortunately - not yet quite settled.

5.2.5. Brainstem

The brainstem, see Fig. 5.2., consists of the medulla, pons and midbrain. The three structures appear as a somewhat enlarged continuation of the turbular shaped spinal chord. Although each of the three regions has special features, they have certain fiber tracts in common, since the brainstem contains all fiber systems connecting higher brain structures and the spinal chord.

5.2.6. Spinal chord

The brain is one part of the C.N.S., the spinal chord is the other, see Fig. 5.6. Basically the spinal chord is a transmission cable with interconnections receiving and sending data between the periphery and the brain. All bodily sensations are sent up the spinal chord to the brain, whereas the cerebral cortex and other brain-structures controlling movement of the body convey activity down to motor neurons in the
spinal chord.

5.3. Skeletal muscles

In the human body muscles make up 40 to 50% of the total body weight. The main function of the muscles is to develop force and to contract. Muscles are generally divided into three categories: skeletal, cardiac and smooth muscles. Only the most important, quantitatively dominant part of the musculature, the skeletal muscles, is of concern here\(^{16, 27, 64, 65, 66}\).

A typical skeletal muscle consists mainly of a number of slender muscle fibers, see Fig. 5.7., each having a diameter of 10 to 100 \(\mu m\). These muscle fibers may be considered the basic elements of the muscular system, in the same sense as neurons are of the nervous system.

The fibers in a single muscle are organized into a hierarchy of bundles and these in turn are bound together to form the muscle. Most muscles are connected on both ends to the skeletal bones by tendons, which characteristically cannot contract. A tendon may be considered mechanically as a very stiff spring.

Each muscle contains a number of fibers that are thinner and shorter than the common muscle fiber. Usually 2 to 10 of these fibers are grouped together and enclosed in a connective tissue capsule. Such a structure is called a muscle spindle, see Fig. 5.8. The muscle spindles are situated in parallel with the rest of the muscle fibers and they are innervated such that they send signals to the C.N.S. proportional to the degree of stretching of the muscle. The muscle spindles thus signal changes in length of the muscle.

A second important receptor organ associated with muscles are the Golgi tendon organs. As the name implies, these sensors are situated in the tendon connecting the muscle to the skeletal bone. Each of the tendon
organs is associated with 3 to 25 muscle fibers, with which they are arranged in series. The chief function of the Golgi tendon organs is to monitor muscle tension.

The basic element for modification of skeletal muscle tension or length is the motor unit, see Fig. 5.9. This consists of a single motor neuron in the spinal chord, the axon through which it transmits impulses to the periphery and all muscle fibers to which the axon is connected. As the axon reaches the muscle it divides into multiple branches, each of which makes contact with a single muscle fiber, as mentioned in par. 5.2.1. Each muscle fiber is activated by only one axon. All the muscle fibers in one motor unit lie within the same muscle. There may be from about 10 to 10000 fibers in a single motor unit, and from rather few to several million units within a muscle.

The progressive development of tension in each motor unit, from minimal strength to maximum voluntary force level, is brought into action - at least in some muscles - at a rather precise and reproducible tension level. Once brought into the effort, the motor unit pulse frequency increases rapidly as over-all tension increases, to a maximum firing rate for the neuron. The tension range for the entire muscle, when the motor unit first enters and when it reaches its maximum rate, is rather narrow. There is considerable variability in the size of motor units in a given muscle. As a result the maximum tension which a motor unit is capable of generating varies from unit to unit.

It has been shown, that the order in which motor units are called into action may depend on the size of the motor unit, the smaller units are recruited first. This means that at low tensions the increments of tension are small, while at larger tensions the increments are larger. Apparently, this arrangement lends itself to a nearly constant relative fineness of control.

Skeletal muscles can only contract actively. As a consequence, movements
involving high-grade skill generally require coordinated groups of muscles. The simplest of these is the agonist/antagonist pair. Fig. 5.10. shows such a pair for a simple situation, similar to the biceps/triceps system. For rotation of the bones to occur, one muscle must contract while the other extends. If the opposing muscles each have a steady-state tension - or tonus - caused by some steady-state or average firing rate, rotation can be accomplished by increasing the firing rate for the contracting muscle, while simultaneously decreasing the firing rate in the antagonist by about the same amount.

The actual muscle system involved in almost any complex limb motion is seldom, if ever, as simple as that just described. However, the same principles hold for each agonist/antagonist pair involved. The efforts of the pairs contributing to the actual limb motion of interest are summed to produce the motion.

5.4. Generating motion

At the present time very little is known with certainty of the generation of the patterns of neuronal discharge that convert the initial conception of an action into a program for the required movements. The conversion of thinking and intent into cortical impulse patterns remains, for the time being, well beyond the limits of our understanding. It is thus not surprising that the higher motor functions represent an area of research in which established knowledge is mixed with an extraordinary amount of hypothesis and speculation.

A highly schematic picture indicating various elements of the neuromuscular system and their interconnections as they appear according to present knowledge is shown in Fig. 5.11. (26, 67).

There seems to be a general understanding that complex movements, such as required when performing complex skills, come about in several stages, as briefly indicated in the upper part of Fig. 5.11. Initially there is
the urge to act and to generate a movement. The second stage entails the generation of a program for the desired action and in the third and final stage the desired and programmed motion is actually executed. Needless to say, that this concept of three stages is a gross simplification of reality. Also, the distinction between the stages does not reach the conscious mind.

The lower part of Fig. 5.11. goes into some more detail in assigning the various motor centers and the muscles discussed in the previous paragraphs, to the three stages involved in generating movement. It will be seen that in the light of present knowledge it appears reasonable that ideas or commands for fine movements arise in the association areas of the cortex.

At the next stage it seems probable that movement patterns or programs for movements are generated in structures such as the basal ganglia and the cerebellum. These patterns are influenced and modified in the cortex by feedback signals provided by efferents from the somatic sensors in the periphery. Certain parts of the cerebellar cortex in particular receive a massive feedback from proprioceptors throughout the body. They appear to regulate movement that is underway. Thus, these parts of the cerebellum function as a stabilizing comparator circuit that, by continuous comparison of the motor program to the actual performance, adjusts motion so that is is smooth and precise.

Note: the signals from basal ganglia and cerebellum are conducted to the motor cortex, mostly through a subcortical structure called the thalamus, as indicated in Fig. 5.11. The thalamus, however, is not usually considered a motor center in the sense as those discussed in par. 5.2. It is also a relay station for sensory pathways, as mentioned in par. 5.2.2.

The motor cortex appears to be the final supraspinal station for conversion of the designs for movements arising in the associative cortex,
into programs for movement. At the same time, as is also shown in Fig. 5.11., it is the beginning of the chain of structures responsible for the execution of movement.

It is known, however, that many sequences of movement involve only subcortical structures. The neurons in the spinal chord are interconnected in such a way that they can generate rather complex motor actions on their own, when appropriate signals arrive from the periphery or from higher centers of the nervous system. Such spinal reflexes can be regarded as a set of elementary "subroutines" which the organism can use as required.

Human motor performance can thus be seen to be organized about patterns of movement. These patterns are of different levels, ranging from the simplest reflex to the most complex coordinated skill. The more complex human operator response programs incorporate simpler motion patterns, without the necessity for consciously planning in detail the individual movements that make up each pattern. As a consequence, a selection of output motion patterns made at a sufficiently high level in the brain, need not involve specification of the details of execution down to the muscular level.

5.5. A mathematical model of the neuromuscular system

5.5.1. Shaping network

After the discussion in the previous paragraphs of a number of physiological details of the human neuromuscular system, a mathematical model of the system can now be drawn up. Such a model might take several forms, depending on the specific application. The situations for which the present model is to be used are those where closed loop control of aircraft-like dynamic systems are studied.

Mathematical models of the neuromuscular system for such situations
have been presented for instance in \((6, 68)\). If known or estimated quantitative values of the various constants \((69, 70)\) are substituted in such a model, it turns out that the neuromuscular system should generally have a bandwidth of more than about 50 rad/s. Carefully executed measurements on two widely differing neuromuscular/manipulator systems, reported in \((71)\), showed in both cases a bandwidth of only 10-11 rad/s. This fact clearly indicates, that the bandwidth of the neuromuscular system is not determined primarily by the masses, spring constants and internal dampings of limb and manipulator, although these play a certain role. It appears, that the bandwidth or speed of response of the neuromuscular system is set in the first place by rather severe restrictions in higher centers of the brain. The schematic diagram shown in Fig. 5.11. and the accompanying discussions make it clear that from the neurophysiological point of view such a situation is quite plausible.

In view of the above, the approach taken here is somewhat different. It is explained by referring back to Fig. 5.11. Although the skilled human operator has a large repertoire of motion programs at his disposal, the biomorphic model uses only the very restricted number of two, as mentioned already in par. 4.2. Herein lies one of the major simplifications in the present mathematical model of the neuromuscular system. As a consequence, however, the decision process discussed in par. 4.2. need produce only two independent quantities, labelled \(A\) and \(B\) respectively, each being representative of the desired magnitude of one of the two possible output programs. This process is considered to be the equivalent of the first stage in generating movement, discussed in the previous paragraph.

When it comes to chosing the general shapes of the two desired output programs, it should be noted that in many cases the control output time histories of human operators can be approximated as being made up of only two basic elements: pulse-type patterns and small incremental steps. In many situations the human operator uses combinations of these
two basic types of responses. Furthermore, experimental data indicate that the actual mixture of the two basic types of response patterns used in a particular closed loop control situation strongly depends on the dynamics of the controlled element. A pure integrator may appear to be controlled almost entirely by small incremental steps, whereas a double integrator would evoke predominantly pulse-type responses. Clearly, individual differences between different human operators may manifest themselves also in their selection of these standard output patterns.

The two output programs chosen for the present model are represented mathematically by the responses of a second order low pass "shaping filter", to either an impulse of magnitude A or to an incremental step of size B. For the time being, the undamped natural frequency of the shaping filter, \( \omega_0 \), is taken at 10 rad/s and the damping, \( \zeta \), is 0.7 of critical damping. The generation of these desired output programs is considered to be the equivalent of the second stage in Fig. 5.11.

5.5.2. Motor noise

The third and final stage in Fig. 5.11. is the translation of the desired output programs into actual movements. A further simplification is made here. It is assumed that the shape of the actual output pattern is identical to that of the desired program. The only differences allowed between the two are due to a certain form of motor noise. This motor noise is implemented by letting the desired or intended magnitudes of impulse, \( \hat{A} \), and incremental step, \( \hat{B} \), to be different from the actual values, A and B respectively. Mathematically this is expressed as:

\[
\begin{bmatrix}
A \\
B
\end{bmatrix} = \begin{bmatrix}
\hat{A} \\
\hat{B}
\end{bmatrix} + \begin{bmatrix}
\Delta A \\
\Delta B
\end{bmatrix}
\]

(5-1)

It is assumed that the motor noise components \( \Delta A \) and \( \Delta B \) are elements of
two discrete and uncorrelated, Gaussian stochastic processes.

As regards $\Delta A$, it is assumed that the variance $\sigma_{\Delta A}^2$ is related to the variance of $\hat{A}$, by:

$$\sigma_{\Delta A}^2 = k_3^2 \cdot \sigma_{\hat{A}}^2 \tag{5-2}$$

As regards the variance of $\Delta B$, two different assumptions are made, which may apply separately or in combination:

a. $\sigma_{\Delta B}^2$ is related to $\sigma_B^2$ in a way similar to (5-2):

$$\sigma_{\Delta B}^2 = k_4^2 \cdot \sigma_B^2 \tag{5-3}$$

b. $\sigma_{\Delta B}^2$ has a constant value, independent of $\sigma_B^2$. This second assumption will apply in situations where control takes place mainly through impulses rather than through incremental steps. Then:

$$\sigma_{\Delta B}^2 = C_4 \tag{5-4}$$

The latter expression can also be written in the form (5-3), if $k_4^2$ is expressed as:

$$k_4^2 = \frac{C_4}{\sigma_B^2} \tag{5-5}$$

In the following, only (5-3) will be used to calculate the variance of $\Delta B$. This covers either of the two above assumptions.
Returning to Fig. 5.11, it is seen, that the extensive feedback circuitry shown in the lower part of Fig. 5.11 is not modelled explicitly in this mathematical model of the neuromuscular system. Implicitly, however, the feedback signals are taken into account by the assumption that the desired and the actual output motions have identical shapes.

5.5.3. Time delay

The mathematical description of the neuromuscular system is rounded off by the inclusion in the model of a representation of at least part of the time taken by the neuronal signals to progress from the association cortex, where the decision process is assumed to take place, to the muscles that move the manipulator. From encephalographic recordings it is known\(^\text{(27, 72)}\), that neuronal activity starts well before the beginning of an overt voluntary movement. Intervals as long as 800 ms have been recorded. Such long intervals are, however, not necessarily applicable to closed loop control situations.

Although the time delay could be modelled as a pure dead time, it is approximated here by a simple first order lag, having a time constant, \(T_d\). For the time being, \(T_d\) is taken as 50 to 100 ms, which seems to be reasonably representative of the duration of the so called "premotor potential", see \((27)\).

5.5.4. Complete model

On the basis of the foregoing, the mathematical model of the neuromuscular system could now be written in the following matrix form. The indices of the state variables are chosen such, that they fit into the total biomorphic model to be assembled in Section 6.
\[
\begin{bmatrix}
\dot{x}_4 \\
\dot{x}_5 \\
\dot{x}_6
\end{bmatrix} =
\begin{bmatrix}
-1/\tau_d & 0 & 0 \\
0 & 0 & 1 \\
\omega_s^2 & -\omega_s^2 & -2\tau_s \omega_s
\end{bmatrix}
\begin{bmatrix}
x_4 \\
x_5 \\
x_6
\end{bmatrix}
\]

where:

\(x_4\) = variable of the time delay
\(x_5\) = control deflection, output of the shaping filter
\(x_6\) = time derivative of \(x_5\)

In view of the manner in which the biomorphic model operates in the time domain, the input to the neuromuscular model is given by initial conditions, representing the impulse, \(A\), and incremental step, \(B\). To this end, the input impulse is simply replaced by an equivalent initial condition:

\[x_4(0) = \frac{A}{\tau_d}\]

Neglecting the incremental step for a moment, the initial conditions for the three variables of the neuromuscular model would be:

\[
\begin{bmatrix}
x_4(0) \\
x_5(0) \\
x_6(0)
\end{bmatrix} =
\begin{bmatrix}
x_4(T) + \frac{A}{\tau_d} \\
x_5(T) \\
x_6(T)
\end{bmatrix}
\]
In this specification of the initial conditions, $t = 0$ signifies — as in Section 3 — the beginning of the sample interval and $t = T$ is the end of that interval. The end of one sample interval coincides with the beginning of the next one.

The incremental step, $B$, requires some further detailed consideration. In the biomorphic model, the variable representing the output of the neuromuscular system, $x_5$, has to be the equivalent of the total control deflection. As a consequence, the input to the model also has to be the sum of all incremental steps. This means that the step at time $t = 0$ has to be added first to the sum of all previously generated incremental steps. This is conveniently achieved by introducing in the model a dummy integrator, acting as a summer. The result is the final model of the neuromuscular system:

$$
\begin{bmatrix}
\dot{x}_3 \\
\dot{x}_4 \\
\dot{x}_5 \\
\dot{x}_6
\end{bmatrix} =
\begin{bmatrix}
0 & 0 & 0 & 0 \\
1/\tau_d & -1/\tau_d & 0 & 0 \\
0 & 0 & 0 & 1 \\
0 & \omega_0^2 & -\omega_0^2 & -2\tau_0\omega_0
\end{bmatrix}
\begin{bmatrix}
x_3 \\
x_4 \\
x_5 \\
x_6
\end{bmatrix}
$$

(5-6)

with initial conditions:

$$
\begin{bmatrix}
x_3(0) \\
x_4(0) \\
x_5(0) \\
x_6(0)
\end{bmatrix} =
\begin{bmatrix}
x_3(T) + B \\
x_4(T) + \frac{A}{\tau_d} \\
x_5(T) \\
x_6(T)
\end{bmatrix}
$$

where:
$x_3 =$ auxiliary variable

$x_4 =$ variable of the time delay

$x_5 =$ total control deflection

$x_6 =$ time derivative of $x_5$

This completes the description of the model of the neuromuscular system. By way of a summary, Fig. 5.12. shows a block diagram of the neuromuscular model, as discussed in this Section.
6. EQUATIONS REPRESENTING THE CLOSED LOOP IN WHICH THE BIOMORPHIC MODEL IS THE CONTROLLING ELEMENT

In the previous Sections of this report the various elements of the biomorphic model were discussed and mathematical expressions for their operation in the time domain were derived. It is now possible to assemble the parts and arrive at the equations representing the complete biomorphic model.

The elements to be put together are:
1. the input shaping filter changing the white noise variable, \( x_i \), into the stochastic, band-limited forcing function, \( x_i(t) \), see (2-2),
2. the model of the neuromuscular system, see (5-6),
3. the controlled element and the corresponding internal model in the pilot's brain. Like in Section 3, a second order controlled element is assumed. Other controlled elements are presented in Appendix 2.

The various sub-matrices are combined into a single \( 10 \times 10 \) matrix, as shown on page 49.

In an abbreviated notation this expression (6-1) can be written as:

\[
\dot{x} = [A] \cdot \bar{c} + [B] \cdot x_i \tag{6-2}
\]

The initial conditions of (6-1) or (6-2) are:

\[
x(0) = \begin{bmatrix}
x_1(0) \\
x_2(0) \\
x_3(0) \\
x_4(0) \\
x_5(0) \\
x_6(0) \\
x_7(0) \\
x_8(0) \\
x_9(0) \\
x_{10}(0)
\end{bmatrix} = \begin{bmatrix}
x_1(T) \\
x_2(T) \\
x_3(T) + B \\
x_4(T) + \frac{A}{\tau_d} \\
x_5(T) \\
x_6(T) \\
 \frac{q_{F_1}}{O_1} \\
 \frac{q_{F_2}}{O_2} \\
x_9(T) \\
x_{10}(T)
\end{bmatrix} \tag{6-3}
\]
\[
\begin{bmatrix}
\dot{x}_1 \\
\dot{x}_2 \\
\dot{x}_3 \\
\dot{x}_4 \\
\dot{x}_5 \\
\dot{x}_6 \\
\dot{x}_7 \\
\dot{x}_8 \\
\dot{x}_9 \\
\dot{x}_{10}
\end{bmatrix}
= 
\begin{bmatrix}
0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
-w_0^2 & -2w_0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1/\tau_d & -1/\tau_d & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & w_0^2 & -w_0^2 & -2\zeta w_0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & K_v w_0 & 0 & -w_0^2 & -2\zeta w_0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\
0 & 0 & 0 & 0 & 0 & K_c w_0 & 0 & 0 & -w_0^2 & -2\zeta w_0
\end{bmatrix}
\begin{bmatrix}
x_1 \\
x_2 \\
x_3 \\
x_4 \\
x_5 \\
x_6 \\
x_7 \\
x_8 \\
x_9 \\
x_{10}
\end{bmatrix}
+ 
\begin{bmatrix}
0 \\
\omega_0^2 \\
0 \\
0 \\
0 \\
0 \\
0 \\
0 \\
0 \\
0 \\
0 \\
0 
\end{bmatrix}
= 
\begin{bmatrix}
x_1 \\
x_2 \\
x_3 \\
x_4 \\
x_5 \\
x_6 \\
x_7 \\
x_8 \\
x_9 \\
x_{10}
\end{bmatrix}
\]
where:

- $x_1$ = normally distributed, white noise
- $x_1$ = external disturbance, forcing function
- $x_2$ = time derivative of $x_1$
- $x_3$ = auxiliary variable
- $x_4$ = variable of the time delay
- $x_5$ = total control deflection
- $x_6$ = time derivative of $x_5$
- $x_7$ = estimate of visually observed variable
- $x_8$ = time derivative of $x_7$
- $x_9$ = controlled variable
- $x_{10}$ = time derivative of $x_9$

In order to formulate the appropriate initial conditions for the internal model, the observed variables have to be noted, according to (3-2)-(3-5):

\[
\begin{bmatrix}
    y_1 \\
    y_2 \\
    y_3 \\
    y_4
\end{bmatrix} =
\begin{bmatrix}
    1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 \\
    0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 \\
    0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\
    0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0
\end{bmatrix}
\begin{bmatrix}
    x_1 \\
    x_2 \\
    x_3 \\
    x_4 \\
    x_5 \\
    x_6 \\
    x_7 \\
    x_8 \\
    x_9 \\
    x_{10}
\end{bmatrix}
\]
where:

\[ y_1 = \text{visually observed variable} \]
\[ y_2 = \text{time derivative of } y_1 \]
\[ y_3 = \text{estimate of visually observed variable} \]
\[ y_4 = \text{time derivative of } y_3 \]

The above expression (6-4) can be written in a shorter notation:

\[ \bar{y} = [C] \cdot \bar{x} \quad (6-5) \]

From the observed variables, \( \bar{y} \), the characteristic features are obtained at \( t = T \), see also (3-6) and (3-7):

\[
\begin{bmatrix}
  \hat{f}_1 \\
  \hat{f}_2
\end{bmatrix} =
\begin{bmatrix}
  y_1(T) \\
  y_2(T)
\end{bmatrix} \quad (6-6)
\]

and the estimated characteristic features, see (3-8) and (3-9):

\[
\begin{bmatrix}
  \hat{\hat{f}}_1 \\
  \hat{\hat{f}}_2
\end{bmatrix} =
\begin{bmatrix}
  y_3(T) \\
  y_4(T)
\end{bmatrix} \quad (6-7)
\]

The estimation errors are, see (3-10) and (3-11):

\[
\begin{bmatrix}
  \Delta f_1 \\
  \Delta f_2
\end{bmatrix} =
\begin{bmatrix}
  \hat{f}_1 \\
  \hat{f}_2
\end{bmatrix} -
\begin{bmatrix}
  \hat{\hat{f}}_1 \\
  \hat{\hat{f}}_2
\end{bmatrix} \quad (6-8)
\]

And the final observation errors, \( \varepsilon_1 \) and \( \varepsilon_2 \), described as random, uncorrelated Gaussian variables having variances, given by:
\[ k_1 = e^{-\Delta \tau_1/\tau_{ob_1}} \]
\[ k_2 = e^{-\Delta \tau_2/\tau_{ob_2}} \]  

see (3-14) - (3-17).

The observation errors, \( \varepsilon_1 \) and \( \varepsilon_2 \), are needed to arrive at the observed characteristic features, according to (3-12) and (3-13):

\[
\begin{bmatrix}
\hat{O}_1 \\
\hat{O}_2
\end{bmatrix} =
\begin{bmatrix}
f_1 \\
f_2
\end{bmatrix} +
\begin{bmatrix}
\varepsilon_1 \\
\varepsilon_2
\end{bmatrix}
\]  

which serve as the initial conditions for the internal model at \( t = 0 \).

To complete the expressions for the internal model, the control law and the motor noise have to be added. According to (4-1), the intended impulse and incremental step are:

\[
\begin{bmatrix}
\hat{A} \\
\hat{B}
\end{bmatrix} =
\begin{bmatrix}
a_1 & a_2 \\
b_1 & b_2
\end{bmatrix} \cdot
\begin{bmatrix}
\hat{O}_1 \\
\hat{O}_2
\end{bmatrix}
\]  

and the motor noise is expressed as, see (5-1):

\[
\begin{bmatrix}
A \\
B
\end{bmatrix} =
\begin{bmatrix}
\hat{A} \\
\hat{B}
\end{bmatrix} +
\begin{bmatrix}
\Delta A \\
\Delta B
\end{bmatrix}
\]  

where \( \Delta A \) and \( \Delta B \) are uncorrelated, random, Gaussian variables with variances given by, see (5-2) and (5-3):

\[
\begin{bmatrix}
\sigma_{\Delta A}^2 \\
\sigma_{\Delta B}^2
\end{bmatrix} =
\begin{bmatrix}
k_3 & 0 \\
0 & k_4
\end{bmatrix} \cdot
\begin{bmatrix}
\sigma_A^2 \\
\sigma_B^2
\end{bmatrix}
\]  

\[ (6-14) \]
Collecting the assumptions made in Section 4 and 5, it is now possible to distinguish in the sample interval, $T$, three sub-intervals: $\Delta t_i$, $\Delta t_u$ and $\Delta t_o$, where:

$\Delta t_i = \text{initiation interval, corresponding to the time delay in the neuromuscular model,}$

$\Delta t_u = \text{remaining uncommitted interval,}$

$\Delta t_o = \text{observation interval.}$

These intervals are related as follows:

$$\Delta t_i = \tau_d \quad (6-15)$$

as introduced in par. 5.5., and:

$$T = \Delta t_i + \Delta t_u + \Delta t_o \quad (6-16)$$

This completes the summing up of the expressions representing the biomorphic model for a second-order controlled element. Fig. 6.1. shows the resulting block diagram of the complete system of which the biomorphic model is the controlling element. Appendix 2 gives the corresponding formulas for a few other simple controlled elements of first and second order.
7. DETERMINING THE CONTROL LAW

7.1. Introduction

The theme to be discussed in this Section is, in a strict sense, not part of the description of the biomorphic model itself. It models the training or learning process the human operator has to absolve, before he can skillfully manage the output of the controlled element, under the disturbances imposed on the control loop by the forcing function. The outcome of the training process is a certain "reflex-like" behaviour of the skilled operator (60, 73). The behaviour is characterized by control deflections which are appropriately related to the operator's observations. Modelling that process implies determining the control law.

The basic assumption on which the following discussion rests, concerns the logic of selecting the appropriate control response for a given set of observations. It is assumed that the operator's skilled behaviour can be represented in the biomorphic model by applying to the neuromuscular model an impulse and an incremental step input, linearly related to the observed characteristic features. The linear relations - the control law - are such that the estimated response of the controlled element - represented by the response of the internal model - to the incremental control deflections, with the given observed characteristic features as initial conditions, resembles in a certain optimum way the free response of a well-damped second order system to the same initial conditions. Apart from the initial conditions, the entire control loop is assumed to be otherwise at rest at time t = 0. The external forcing function is assumed to be absent.

In order to aboid undue complications in the following optimization procedure, observation noise and motor noise are also assumed to be absent in the modelling of this training process.
It was remarked already in Section 4, that this formulation of the decision process is arbitrary to a certain extent. It can find its justification only a posteriori in the fact that the biomorphic model adapts to various, widely differing controlled elements in a way comparable to a skilled human operator.

7.2. Desired response

As described in the previous paragraph, the desired response is expressed as the response of the two state variables, $x_1$ and $x_2$, of a second order system:

$$\begin{bmatrix}
\dot{x}_1 \\ \dot{x}_2
\end{bmatrix} = \begin{bmatrix}
0 & 1 \\ -\omega_d^2 & -2\zeta_d\omega_d
\end{bmatrix} \begin{bmatrix}
x_1 \\ x_2
\end{bmatrix}$$

(7-1)

to the initial condition:

$$\begin{bmatrix}
x_1(0) \\ x_2(0)
\end{bmatrix} = \begin{bmatrix}
f_1 \\ f_2
\end{bmatrix}$$

where:

$\omega_d$ = undamped natural frequency, determining the desired speed of response

$\zeta_d$ = damping constant, arbitrarily set at 0.7, to obtain a well-damped response

Note the use of $f_1$ and $f_2$ as initial conditions, rather than $\ddot{f}_1$ and $\ddot{f}_2$ because observation noise has been assumed to be absent.
7.3. Costfunction

The ultimate choice of the four elements $a_1$, $a_2$, $b_1$ and $b_2$ of the control law, see par. 4.2., is the result of an optimization procedure, in which a costfunction, $J$, is to be minimized. This costfunction is based on a quadratic measure of the difference between the estimated response of the internal model and the desired response. The latter was expressed as $x_1(t)$ in the previous paragraph. The estimated response of the internal model has been denoted as $y_3(t)$ in par. 3.3. and in Section 6. In principle the costfunction could thus be based on an expression like:

$$\int_0^\infty (y_3 - x_1)^2 \cdot dt$$

but a certain refinement has to be added.

As the biomorphic model is a sampling model, a new incremental control deflection is generated at the beginning of every sampling interval, i.e. after every $T$ seconds. As a consequence, the difference between $y_3(t)$ and $x_1(t)$, as it is influenced by the control deflection at the beginning of a certain sample interval, is of importance mainly during the first $T$ seconds, after which a new correcting control deflection will be initiated. This fact is accounted for by incorporating in the integrand of the costfunction a weighting function, $W(t)$, assumed to be of exponential form:

$$W(t) = e^{-t/T_p}$$

where the time-constant $T_p$ is a measure of the prediction interval for which the difference between $y_3(t)$ and $x_1(t)$ is to be primarily minimized by one control deflection. Typically, $T_p$ might be chosen to be
equal to T:

The cost function is thus based on the function S:

\[ S = \int_{0}^{\infty} (y_3 - x_1)^2 \cdot W \cdot dt \]  \hspace{1cm} (7-4)

The variable in (7-4), expressing the difference between the estimated and the desired response, is written as:

\[ y_3 - x_1 = c^T \cdot x \]  \hspace{1cm} (7-5)

where the transposed vector \( c^T \) is given in detail in Appendix 4.

The time histories of \( y_3(t) \) and \( x_1(t) \) are functions of the characteristic features \( f_1 \) and \( f_2 \). The latter are stochastic variables during the actual operation of the model, and their relations are expressed by a covariance matrix, see par. 8.5.:

\[
[C_{f_1f_2}] = \begin{bmatrix}
\sigma_{f_1}^2 & \sigma_{f_1f_2} \\
\sigma_{f_1f_2} & \sigma_{f_2}^2
\end{bmatrix}
\]

This stochastic character of some variables in the problem causes the cost function, \( J \), to be a stochastic function as well: it is the ensemble average of \( S \), defined in (7-4):

\[ J = E\{S\} \]  \hspace{1cm} (7-6)

where the operator \( E\{\} \) signifies taking the ensemble average.
The outcome of this discussion is the following expression for the cost-function, $J$:

$$J = E \left\{ \int_0^\infty (y_3 - x_1)^2 \cdot W \cdot dt \right\}$$  

(7-7)

7.4. Equations for the biomorphic model in the training process for the control law

It is now possible to assemble the equations expressing the biomorphic model's behaviour in the optimization process for the control law. They are derived from the equations (6-1) presented in Section 6, by exchanging the noise filter generating the forcing function, with the second order system (7-1) describing the desired response. This causes the state variables $x_1$ and $x_2$ to obtain a new meaning. The second change to the equations (6-1) consists of deleting the white-noise input variable, since it is not needed in the optimization procedure.

The third change, finally, consists of also deleting the controlled element since the human operator has only available an estimate of the internal model on which he can base his - subconscious - choice of the control law.

The resulting system of equations is given below, again for the case of a second order internal model.

$$\begin{bmatrix}
\dot{x}_1 \\
\dot{x}_2 \\
\dot{x}_3 \\
\dot{x}_4 \\
\dot{x}_5 \\
\dot{x}_6 \\
\dot{x}_7 \\
\dot{x}_8
\end{bmatrix} =
\begin{bmatrix}
0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
-\omega_d^2 & -2\zeta_d \omega_d & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & \omega_s^2 & -\omega_s^2 & -2\zeta_s \omega_s & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\
0 & 0 & 0 & 0 & 0 & 0 & K_v \omega_v^2 & -\omega_v^2 & -2\zeta_v \omega_v
\end{bmatrix}
\begin{bmatrix}
x_1 \\
x_2 \\
x_3 \\
x_4 \\
x_5 \\
x_6 \\
x_7 \\
x_8
\end{bmatrix}$$

(7-8)
In a short notation this system (7-8) can be written as:

\[
\dot{\bar{x}} = [F] \cdot \bar{x}
\]  

(7-9)

where the system matrix \([F]\) follows at once from (7-8). The same holds for the state vector \(\bar{x}\).

The initial condition \(\bar{x}(0)\) of this system can be derived from (6-3) and the text in the previous paragraphs. At time \(t = 0\), the desired response starts at the values of the characteristic features: \(x_1(0) = f_1\) and \(x_2(0) = f_2\). The impulse and incremental step lead to the initial conditions as in (6-3): \(x_3(0) = B\) and \(x_4(0) = A_\frac{1}{T_d}\). Otherwise, the neuromuscular model is assumed to be at rest at \(t = 0\), which means: \(x_5(0) = 0\) and \(x_6(0) = 0\). The estimated response of the internal model starts at the values of the characteristic features: \(x_7(0) = f_1\) and \(x_8(0) = f_2\). The resulting initial condition of the entire system thus is:

\[
\bar{x}(0) = \begin{bmatrix}
    x_1(0) \\
    x_2(0) \\
    x_3(0) \\
    x_4(0) \\
    x_5(0) \\
    x_6(0) \\
    x_7(0) \\
    x_8(0)
\end{bmatrix} = \begin{bmatrix}
    f_1 \\
    f_2 \\
    B \\
    A_\frac{1}{T_d} \\
    0 \\
    0 \\
    f_1 \\
    f_2
\end{bmatrix}
\]  

(7-10)

where the meaning of the various elements of \(\bar{x}\) will be clear from the above text.

As is apparent from (7-8) and (7-9), the above equations (7-8) express the open-loop response of a system to certain initial conditions, rather than the closed-loop response to an external forcing function as repre-
sented by the equations (6-1) and (6-3).

Expressions similar to (7-8) and (7-9), for the case of a first order internal model, are given in Appendix 4.

7.5. Integration of the system equations by means of a Liapunov-function

The system equations having been set up and the costfunction defined, it is now possible to calculate J, by integrating the differential equations.

Neglecting the weighting function, W(t), for a moment the costfunction (7-7) is based on the integral, S, of a quadratic function of state variables of the system. An elegant method exists to calculate such an integral. This method is based on the use of a so-called Liapunov-function \(^{(74, 75)}\). The integration method is discussed in the following.

A linear system is given, described by the equation:

\[ \dot{x} = [F] \cdot \bar{x} \]  

(7-9)

where \( \bar{x} \) is an \( n \)-dimensional state vector, and \([F]\) is a constant, \( n \times n \) matrix.

Suppose a variable \( y(t) \) is given as a linear combination of the state variables:

\[ y = c^T \cdot \bar{x} \]

where \( \bar{x} \) is an \( n \)-dimensional vector. It is then asked to calculate the integral:

\[ I = \int y^2 \cdot dt \]  

(7-11)
The answer to this question proceeds as follows. The integrand in (7-11), $y^2$, can be written as:

$$y^2 = \vec{x}^T \cdot \vec{c} \cdot \vec{c}^T \cdot \vec{x}$$

$$= \vec{x}^T \cdot [K] \cdot \vec{x}$$  \hspace{1cm} (7-12)

where $[K]$:

$$[K] = \vec{c} \cdot \vec{c}^T$$  \hspace{1cm} (7-13)

is an $n \times n$ symmetric matrix. Evidently, $y^2$ is a quadratic function of the state vector $\vec{x}$.

It is simple to demonstrate that the integral, $J$, is also a quadratic function of $\vec{x}$. Suppose:

$$I = \vec{x}^T \cdot [L] \cdot \vec{x}$$  \hspace{1cm} (7-14)

where $[L]$ is an $n \times n$ symmetric matrix, like $[K]$. The relation between $[K]$ and $[L]$, which exists if (7-14) is true, is derived by differentiating $I$ in (7-11) with respect to time:

$$\dot{I} = y^2$$  \hspace{1cm} (7-15)

Also, from (7-14):

$$\dot{I} = \vec{x}^T \cdot [L] \cdot \dot{\vec{x}} + \vec{x}^T \cdot [L] \cdot \vec{x}$$  \hspace{1cm} (7-16)

From (7-9) follows:

$$\dot{\vec{x}} = \vec{x}^T \cdot [F]^T$$  \hspace{1cm} (7-17)

Substituting (7-9) and (7-17) in (7-15):
\[ \dot{x} = x^T \cdot [L] \cdot [F] \cdot \ddot{x} + \ddot{x}^T \cdot [F] \cdot [L] \cdot \dddot{x} \]

And according to (7-15) and (7-12):
\[ \dot{i} = \ddot{x}^T \cdot [K] \cdot \dddot{x} \]

It thus follows that the relation between \([K]\) and \([L]\) is:
\[ [L] \cdot [F] + [F]^T \cdot [L] = [K] \]  
\[ (7-18) \]

Consider now the definite integral, \(I'\), which is similar to the integral \(S\) in the costfunction \(J\):

\[ I' = \int_0^\infty y^2 \, dt \]  
\[ (7-19) \]

\[ = \int_0^\infty (\ddot{x}^T \cdot [K] \cdot \dddot{x}) \, dt = \]
\[ = \ddot{x}^T \cdot [L] \cdot \dddot{x} \bigg|_0^\infty \]  
\[ (7-20) \]

where \([L]\) can be derived from \([F]\) and \([K]\), according to (7-18). If the system given by (7-9) is stable, the state vector \(\dddot{x}(t)\) will go to zero as time goes to infinity. Therefore:
\[ \ddot{x}^T(\infty) \cdot [L] \cdot \dddot{x}(\infty) = 0 \]  
\[ (7-21) \]

if the system is stable. In that case, \(I'\) can be written as:
\[ I' = \ddot{x}^T(0) \cdot [L] \cdot \dddot{x}(0) \]  
\[ (7-22) \]

This result can be applied at once to the integral \(S\), given by (7-4),
the weighting function still being neglected. Here, the vector $\vec{c}$, expressing $(y_3 - x_1)$ as a function of the state vector is given by (7-5). From this vector $\vec{c}$, the matrix $[K]$ is derived according to (7-13). Once $[K]$ is known, matrix $[L]$ is calculated by means of (7-18). Now $S$ can be obtained by using (7-22), provided the correct initial condition is used. This latter matter is discussed in par. 7.7.

The above integration method is valid only, if the system is stable. If the controlled element in (7-9) is unstable, the method would not apply because (7-21) would be violated. It is shown in the next paragraph that, by using the weighting function $W(t)$, an unstable controlled element can nevertheless be handled.

According to (7-22), the function $S$ in the cost function can be obtained only by using the proper initial condition. This is the reason why the control inputs $A$ and $B$ are represented in the system equations by equivalent initial conditions.

### 7.6. Fitting in the weighting function

The function $S$, appearing in the cost function $J$, was given in (7-4) as:

\[
S = \int_{0}^{\infty} (y_3 - x_1)^2 \cdot W \cdot dt
\]

where $W(t)$ is given by, see (7-2):

\[
W = e^{-t/\tau_p}
\]

This means that $S$ can be written as:

\[
S = \int_{0}^{\infty} (y_3 \cdot e^{-t/2\tau_p} - x_1 \cdot e^{-t/2\tau_p})^2 \cdot dt
\]

Using (7-5):
\[ y_3 \cdot e^{-t/2\tau} p - x_1 \cdot e^{-t/2\tau} p = \bar{c}^T \cdot \bar{x} \cdot e^{-t/2\tau} p \]

It is shown in Appendix 3, that a system can be derived:

\[ \dot{\bar{x}}' = [F'] \cdot \bar{x}' \]  \hspace{1cm} (7-23)

where \([F']\) is an \(n \times n\) constant, real matrix, related to \([F]\), such that by using the initial condition:

\[ \bar{x}'(0) = \bar{x}(0) \]  \hspace{1cm} (7-24)

the solution of the new system (7-23) is:

\[ \dot{\bar{x}}'(t) = \bar{x}(t) \cdot e^{-t/2\tau} p \]  \hspace{1cm} (7-25)

The required relation between \([F']\) and \([F]\) is simply, see (A3-10):

\[ [F'] = [F] - \frac{1}{2\tau} \cdot [I] \]  \hspace{1cm} (A3-10)

where \([I]\) is the \(n \times n\) unity matrix.

Using this modified system matrix, a new matrix \([L']\) is obtained, through (7-18):

\[ [L'] \cdot [F'] + [F']^T \cdot [L'] = [K] \]  \hspace{1cm} (7-26)

where the matrix \([K]\) remains unchanged, given by (7-13). Finally, the function \(S\), given by (7-22) is found as, see also (7-24):

\[ S = -x^T(0) \cdot [L'] \cdot \bar{x}(0) \]  \hspace{1cm} (7-27)

In this way the weighting function, \(W(t)\), is accommodated in the integrating method. At the same time, (7-25) shows that the response
\( \ddot{x}(t) \) is "stabilized" by the factor \( e^{-t/2\tau_p} \). The effect of this factor is a shift of all eigenvalues:

\[
\lambda_i \quad (i = 1, \ldots, n)
\]

of \([F]\) by an amount \(-\frac{1}{2\tau_p}\), i.e. the eigenvalues \(\lambda_i'\) of \([F']\) are related to those of \([F]\) by:

\[
\lambda_i' = \lambda_i - \frac{1}{2\tau_p} \quad \text{(A3-2)}
\]

Choosing \(\tau_p\) - and thus the sample interval \(T\), see (7-3) in par. 7.3. - sufficiently small, all eigenvalues of \([F']\) can be given a negative real part, even if the (internal model of) the controlled element in \([F]\) is unstable.

It is thus seen, that the weighting function \(W(t)\) enables the Liapunov integration method to be used for stable as well as for unstable controlled elements.

7.7. Initial condition of the integration

The initial condition \(\ddot{x}(0)\) of the integration was given in par. 7.4. by (7-20):

\[
\ddot{x}(0) = \begin{bmatrix}
\begin{array}{c}
f_1 \\
f_2 \\
B \\
A \\
\tau_d \\
0 \\
0 \\
f_1 \\
f_2
\end{array}
\end{bmatrix}
\quad \text{(7-10)}
\]
This initial condition can be further elaborated by using in (7-10) the control law, see (6-12):

\[
\begin{bmatrix}
A \\
B
\end{bmatrix} = \begin{bmatrix}
a_1 & a_2 \\
b_1 & b_2
\end{bmatrix} \cdot \begin{bmatrix}
f_1 \\
f_2
\end{bmatrix}
\]  

(7-11)

Note, that the absence of both observation noise and motor noise allows the use of \(f_1\) and \(f_2\) rather than \(\hat{f}_1\) and \(\hat{f}_2\), as well as \(A\) and \(B\) instead of \(\hat{A}\) and \(\hat{B}\) in (7-11).

Substituting (7-11) in (7-10), results in:

\[
\begin{bmatrix}
1 & 0 \\
0 & 1 \\
b_1 & b_2 \\
a_1/T_a & a_2/T_a \\
0 & 0 \\
0 & 0 \\
1 & 0 \\
0 & 1
\end{bmatrix} = [IS] \cdot \begin{bmatrix}
f_1 \\
f_2
\end{bmatrix}
\]  

(7-12)

The matrix \([IS]\), largely determining the initial condition \(x(0)\), depends on the order of the (internal model of the) controlled element. A similar expression for the initial condition in the case where the controlled element is of first order, is given in Appendix 4.

7.8. Calculation of the costfunction

The costfunction, \(J\), can now finally be obtained, using (7-6):

\[J = E\{S\}\]
Substituting (7-27) gives:

\[ J = E\{ -\bar{x}^T(0) \cdot [L'] \cdot \bar{x}(0) \} \]

and with (7-12):

\[ J = E\{ -[f_1 \ f_2] \cdot [IS]^T \cdot [L'] \cdot [IS] \cdot \begin{bmatrix} f_1 \\ f_2 \end{bmatrix} \} \] (7-13)

In this latter expression, \([L']\) is an \(n \times n\) symmetric, real matrix which can be diagonalized by writing:

\[ [L'] = [M_L]^T \cdot [D_L] \cdot [M_L] \] (7-14)

where:

\[ [D_L] = n \times n \text{ diagonal matrix, the elements of which on the main diagonal are the eigenvalues of } [L'] \]

\[ [M_L] = n \times n \text{ normalized matrix, the columns of which are given by the eigenvectors of } [L'] \]

Substituting (7-14) in (7-13) results in:

\[ J = E\{ -[f_1 \ f_2] \cdot [IS]^T \cdot [M_L]^T \cdot [D_L] \cdot [M_L] \cdot [IS] \cdot \begin{bmatrix} f_1 \\ f_2 \end{bmatrix} \} \] (7-15)

A new \(n \times 1\) matrix, \([Y]\), i.e. an \(n\)-vector, is now introduced:

\[ [Y] = [M_L] \cdot [IS] \cdot \begin{bmatrix} f_1 \\ f_2 \end{bmatrix} \] (7-16)
Using (7-16), \( J \) is written as:

\[
J = E\{-[Y]^T \cdot [D_L] \cdot [Y]\} \\
\text{(7-17)}
\]

Since \([D_L]\) is a diagonal matrix and \([Y]\) a vector, the following relation holds:

\[
[Y]^T \cdot [D_L] \cdot [Y] = \text{Trace} \left( [D_L] \cdot [Y] \cdot [Y]^T \right)
\]

The costfunction (7-15) can now be written as:

\[
J = E\{-\text{Trace} \left( [D_L] \cdot [Y] \cdot [Y]^T \right)\}
\]

or:

\[
J = E\{-\text{Trace} \left( [D_L] \cdot [M_L] \cdot [IS] \cdot \begin{pmatrix} f_1 \\ f_2 \end{pmatrix} \cdot [f_1 f_2] \cdot [IS]^T \cdot [M_L]^T \right)\}
\]

The operator \( E\{ \cdot \} \), indicating the process of taking the ensemble average, see par. 7.3., relates only to the characteristic features \( f_1 \) and \( f_2 \), which are stochastic variables. The calculation of the covariance matrix of \( f_1 \) and \( f_2 \) will be discussed in par. 8.5., see (8-39):

\[
E \left\{ \begin{pmatrix} f_1 \\ f_2 \end{pmatrix} \cdot [f_1 f_2] \right\} = [C_{f_1 f_2}]
\]

This leads to the final expression for the costfunction:

\[
J = -\text{Trace} \left( [D_L] \cdot [M_L] \cdot [IS] \cdot [C_{f_1 f_2}] \cdot [IS]^T \cdot [M_L]^T \right) \\
\text{(7-18)}
\]

For completeness sake, the meanings of the four different matrices occurring in (7-18) are briefly recapitulated here:
[D_L] = see (7-14), n x n diagonal matrix, whose elements on the main diagonal are the eigenvalues of [L'], where the latter matrix is obtained according to (7-26);

[M_L] = see (7-14), n x n normalized matrix, whose columns are given by the eigenvectors of [L'];

[IS] = see (7-12), n x 2 matrix determining the initial condition of the integration on which the costfunction, J, is based. [IS] varies with the order of the (internal model of the) controlled element, see also (A4-5);

[C_{f_1 f_2}] = see (8-39), covariance matrix of the characteristic features f_1 and f_2.

From the fact that in the calculation of the costfunction the covariance matrix of f_1 and f_2 is required, it follows that the optimization procedure leading to a minimum value of J, is a double iterative process. Starting with an initial, guessed covariance matrix of f_1 and f_2, the minimum of J is derived iteratively by varying in [IS] the four elements of the control law. Using the control law thus obtained, an improved covariance matrix of f_1 and f_2 is determined, according to the method discussed in Section 8. The new covariance matrix is used in (7-18) and the iterative determination of the elements of the control law, minimizing J, is repeated, and so on until convergence of both the covariance matrix and the control is attained. This optimization process is discussed in greater detail in Section 10.
8. COVARIANCE MATRIX OF THE VARIABLES IN THE BIOMORPHIC MODEL

8.1. Introduction

One of the elementary characteristics of the biomorphic model that can be obtained from the model described in the previous Sections, is the covariance matrix of the state variables. Such a covariance matrix is an ensemble characteristic. It varies during the sample interval and it is periodic, the period being the sample interval.

The covariance matrix at time \( t = T \) is derived in the following paragraphs for the case where the controlled element is of second-order. Appendix 5 gives the relevant expressions for the case of first-order controlled elements.

8.2. Covariance matrix at the end of the sample interval, \([C_x(T)]\)

The system equation representing the biomorphic model, (6-1), is written in the short notation of (6-2):

\[
\dot{x} = [A] \cdot \bar{x} + [B] \cdot x_i
\]  

(6-2)

where \( \bar{x}(t) \) is the state vector, \( x_i(t) \) is - as before - Gaussian white noise and the system matrices \([A]\) and \([B]\) can be read from (6-1).

The covariance matrix \([C_x(t)]\) of \( \bar{x}(t) \) at any time \( t \) can be written as (76):

\[
[C_x(t)] = \Phi(t) \cdot [C_x(0)] \cdot \Phi^T(t) + \\
\int_0^t \Phi(t - \nu) \cdot [B] \cdot [V] \cdot [B]^T \cdot \Phi^T(t - \nu) \cdot d\nu
\]  

(8-1)

where:
\[
\Phi(t) = \text{transition matrix of the system}
\]

\[
[C_X(0)] = \text{covariance matrix at time } t = 0
\]

\[
[v] = \text{intensity matrix of } x_i(t)
\]

By introducing the forcing function matrix \([C_i(t)]\):

\[
[C_i(t)] = \int_0^t \phi(t-\nu) \cdot [B] \cdot [v] \cdot [B]^T \cdot \phi^T(t-\nu) \cdot d\nu
\]

the expression (8-1) for \([C_X(t)]\) is simplified to:

\[
[C_X(t)] = \Phi(t) \cdot [C_X(0)] \cdot \phi^T(t) + [C_i(t)]
\]  \hspace{1cm} (8-2)

At the end of the sample interval, when \(t = T\), the covariance matrix is:

\[
[C_X(T)] = \Phi(T) \cdot [C_X(0)] \cdot \phi^T(T) + [C_i(T)]
\]  \hspace{1cm} (8-3)

The calculation of this \([C_X(T)]\) is based on the following principle.

It can be seen from (8-3) that \([C_X(T)]\) could be obtained if the covariance matrix \([C_X(0)]\) at time \(t = 0\) were known, since \([C_i(T)]\) can be derived independently, see par. 8.3. In the foregoing Sections it has been stated repeatedly that time \(t = 0\) coincides with \(t = T\) of the previous sample interval. The difference between the two is, that at \(t = 0\) the (noisy) observed values \(\bar{f}_1^0\) and \(\bar{f}_2^0\) have been obtained and the (noisy) control inputs A and B have been applied. This enables the state of the system at \(t = 0\) to be expressed as a function of the state at the previous time \(t = T\).

The now following discussion aims at expressing \(\bar{x}(0)\) in \(\bar{x}(T)\) and the two observation and two motor noise values. It is then possible to express \([C_X(0)]\) as a function of \([C_X(T)]\) and the parameters of the four noise processes. Substituting this \([C_X(0)]\) into (8-3) gives an
expression for \([C_X(T)]\) from which the elements of this covariance matrix can be calculated.

The initial state \(\bar{x}(0)\) has been written in Section 6 as (6-3). It can now be expressed in the following form:

\[
\bar{x}(0) = [D] \cdot \bar{x}(T) + \bar{d}_1 \cdot A + \bar{d}_2 \cdot B + \bar{d}_3 \cdot \bar{f}_1 + \bar{d}_4 \cdot \bar{f}_2
\]  \hspace{1cm} (8-4)

where the matrix \([D]\) and the vectors \(\bar{d}_1, \bar{d}_2, \bar{d}_3\) and \(\bar{d}_4\) are spelled out in Appendix 5.

In (8-4) is:

\[
A = \hat{A} + \Delta A = \\
= a_1 \cdot f_1 + a_2 \cdot f_2 + a_1 \cdot \varepsilon_1 + a_2 \cdot \varepsilon_2 + \Delta A
\]  \hspace{1cm} (8-5)

and equally:

\[
B = \hat{B} + \Delta B = \\
= b_1 \cdot f_1 + b_2 \cdot f_2 + b_1 \cdot \varepsilon_1 + b_2 \cdot \varepsilon_2 + \Delta B
\]  \hspace{1cm} (8-6)

Substituting (8-5) and (8-6) into (8-4) results in:

\[
\bar{x}(0) = [D] \cdot \bar{x}(T) + \bar{e}_1 \cdot f_1 + \bar{e}_2 \cdot f_2 + \bar{e}_1 \cdot \varepsilon_1 + \bar{e}_2 \cdot \varepsilon_2 + \\
+ \bar{e}_3 \cdot \Delta A + \bar{e}_4 \cdot \Delta B
\]  \hspace{1cm} (8-7)

where:

\[
\bar{e}_1 = a_1 \cdot \bar{d}_1 + b_1 \cdot \bar{d}_2 + \bar{d}_3
\]

\[
\bar{e}_2 = a_2 \cdot \bar{d}_1 + b_2 \cdot \bar{d}_2 + \bar{d}_4
\]
\[ \tilde{e}_3 = \tilde{g}_1 \]
\[ \tilde{e}_4 = \tilde{g}_2 \]

The characteristic features, \( f_1 \) and \( f_2 \), have been defined in (6-6) and (6-4) as:

\[ f_1 = y_1(T) = \]
\[ = x_1(T) - x_9(T) = \tilde{g}_1^T \cdot \tilde{x}(T) \quad (8-8) \]

\[ f_2 = y_2(T) = \]
\[ = x_2(T) - x_10(T) = \tilde{g}_2^T \cdot \tilde{x}(T) \quad (8-9) \]

The vectors \( \tilde{g}_1^T \) and \( \tilde{g}_2^T \) have been written out in Appendix 5.

The matrix \([E_1]\) is now introduced:

\[ [E_1] = [D] + \tilde{e}_1 \cdot \tilde{g}_1^T + \tilde{e}_2 \cdot \tilde{g}_2^T \]

Substituting this \([E_1]\) in (8-7) yields:

\[ \tilde{x}(0) = [E_1] \cdot \tilde{x}(T) + \tilde{e}_1 \cdot \varepsilon_1 + \tilde{e}_2 \cdot \varepsilon_2 + \tilde{e}_3 \cdot \Delta \Lambda + \tilde{e}_4 \cdot \Delta \Lambda \]

which expresses \( \tilde{x}(0) \) in \( \tilde{x}(T) \) and the two observation and two motor noise values. In this expression, \( \tilde{x}(T) \), \( \varepsilon_1 \), \( \varepsilon_2 \), \( \Delta \Lambda \) and \( \Delta \Lambda \) are uncorrelated, discrete stochastic variables. This fact permits the covariance matrix of \( \tilde{x}(0) \) to be written as:

\[ [C_{\tilde{x}(0)}] = [E_1] \cdot [C_{\tilde{x}(T)}] \cdot [E_1]^T + \tilde{e}_1 \cdot \sigma_{\varepsilon_1}^2 \cdot \tilde{e}_1^T + \tilde{e}_2 \cdot \sigma_{\varepsilon_2}^2 \cdot \tilde{e}_2^T + \]
\[ + \tilde{e}_3 \cdot \sigma_{\Delta \Lambda}^2 \cdot \tilde{e}_3^T + \tilde{e}_4 \cdot \sigma_{\Delta \Lambda}^2 \cdot \tilde{e}_4^T \quad (8-10) \]
The variances of the four noise processes, occurring in (8-10) are now
further analyzed. By definition, the following relations hold, see
(6-9) and (6-14):

\[ \sigma_{e_1}^2 = k_1 \sigma_{\Delta f_1}^2 \]  \hspace{1cm} (8-11)

\[ \sigma_{e_2}^2 = k_2 \sigma_{\Delta f_2}^2 \]  \hspace{1cm} (8-12)

\[ \sigma_{\Delta A}^2 = k_3 \sigma_{\tilde{A}}^2 \]  \hspace{1cm} (8-13)

\[ \sigma_{\Delta B}^2 = k_4 \sigma_{\tilde{B}}^2 \]  \hspace{1cm} (8-14)

The estimation errors \( \Delta f_1 \) and \( \Delta f_2 \) have been written as, see (6-8) and
(6-7):

\[ \Delta f_1 = y_1(T) - y_3(T) \]
\[ = x_1(T) - x_7(T) - x_9(T) = \tilde{h}_1^T \cdot \tilde{x}(T) \]  \hspace{1cm} (8-15)

\[ \Delta f_2 = y_2(T) - y_4(T) \]
\[ = x_2(T) - x_8(T) - x_{10}(T) = \tilde{h}_2^T \cdot \tilde{x}(T) \]  \hspace{1cm} (8-16)

The vectors \( \tilde{h}_1^T \) and \( \tilde{h}_2^T \) have been given in Appendix 5.

From (8-15) and (8-16) follows:

\[ \sigma_{\Delta f_1}^2 = \tilde{h}_1^T \cdot [C_{\tilde{x}}(T)] \cdot \tilde{h}_1 \]

\[ \sigma_{\Delta f_2}^2 = \tilde{h}_2^T \cdot [C_{\tilde{x}}(T)] \cdot \tilde{h}_2 \]

Substituting these expressions in (8-11) and (8-12) respectively,
results in the variances \( \sigma_{e_1}^2 \) and \( \sigma_{e_2}^2 \):
\[ \sigma_{\varepsilon_1}^2 = k_1^2 \cdot \hat{h}_1^T \cdot [C_X(T)] \cdot \hat{h}_1 \] (8-17)

\[ \sigma_{\varepsilon_2}^2 = k_2^2 \cdot \hat{h}_2^T \cdot [C_X(T)] \cdot \hat{h}_2 \] (8-18)

A similar analysis for the motor noise processes goes as follows:

\[ \hat{A} = a_1 \cdot \bar{f}_1 + a_2 \cdot \bar{f}_2 \]

\[ = a_1 \cdot f_1 + a_2 \cdot f_2 + a_1 \cdot \varepsilon_1 + a_2 \cdot \varepsilon_2 \]

\[ = (a_1 \cdot \bar{g}_1^T + a_2 \cdot \bar{g}_2^T) \cdot \bar{x}(T) + a_1 \cdot \varepsilon_1 + a_2 \cdot \varepsilon_2 \]

Introducing:

\[ \hat{h}_3^T = a_1 \cdot \bar{g}_1^T + a_2 \cdot \bar{g}_2^T \]

where \( \hat{h}_3^T \) is given in detail in Appendix 5, results in:

\[ \hat{A} = \bar{h}_3^T \cdot \bar{x}(T) + a_1 \cdot \varepsilon_1 + a_2 \cdot \varepsilon_2 \]

Since \( \bar{x}(T), \varepsilon_1 \) and \( \varepsilon_2 \) are - again - uncorrelated, discrete stochastic variables, the variance of \( \hat{A} \) can be written as:

\[ \sigma_{\hat{A}}^2 = \hat{h}_3^T \cdot [C_X(T)] \cdot \hat{h}_3 + a_1^2 \cdot \sigma_{\varepsilon_1}^2 + a_2^2 \cdot \sigma_{\varepsilon_2}^2 \] (8-19)

In a similar way the variance of \( \hat{B} \) is derived:

\[ \sigma_{\hat{B}}^2 = \hat{h}_4^T \cdot [C_X(T)] \cdot \hat{h}_4 + b_1^2 \cdot \sigma_{\varepsilon_1}^2 + b_2^2 \cdot \sigma_{\varepsilon_2}^2 \] (8-20)

where:

\[ \hat{h}_4^T = b_1 \cdot \bar{g}_1^T + b_2 \cdot \bar{g}_2^T \]
and $\vec{h}_4^T$ is again given in detail in Appendix 5.

The variances of $\Delta A$ and $\Delta B$ can now be expressed, using (8-13), (8-14), (8-19), (8-20), (8-17) and (8-18):

$$
\sigma_{\Delta A}^2 = k_3^2 \cdot \vec{h}_3^T \cdot [C_x(T)] \cdot \vec{h}_3 + k_4^2 \cdot k_3^2 \cdot a_1^2 \cdot \vec{h}_1^T \cdot [C_x(T)] \cdot \vec{h}_1 + \\
+ k_4^2 \cdot k_3^2 \cdot a_2^2 \cdot \vec{h}_2^T \cdot [C_x(T)] \cdot \vec{h}_2
$$

$$
\sigma_{\Delta B}^2 = k_4^2 \cdot \vec{h}_4^T \cdot [C_x(T)] \cdot \vec{h}_4 + k_4^2 \cdot k_3^2 \cdot b_1^2 \cdot \vec{h}_1^T \cdot [C_x(T)] \cdot \vec{h}_1 + \\
+ k_4^2 \cdot k_3^2 \cdot b_2^2 \cdot \vec{h}_2^T \cdot [C_x(T)] \cdot \vec{h}_2
$$

Returning now to the expression (8-10) for $[C_x(0)]$, the covariance of $\vec{\bar{x}}(0)$ can be written as:

$$
[C_x(0)] = \sum_{j=1}^{9} [E_j] \cdot [C_x(T)] \cdot [E_j]^T
$$

where, by using (8-17), (8-18), (8-21) and (8-22), the matrices $[E_j]$, $j = 1, \ldots, 9$ are:

$$
[E_1] = [D] + \vec{e}_1 \cdot \vec{e}_1^T + \vec{e}_2 \cdot \vec{e}_2^T
$$

$$
[E_2] = k_1 \cdot \vec{e}_1 \cdot \vec{h}_1^T
$$

$$
[E_3] = k_3 \cdot \vec{e}_2 \cdot \vec{h}_2^T
$$

$$
[E_4] = k_3 \cdot \vec{e}_3 \cdot \vec{h}_3^T
$$

$$
[E_5] = k_1 \cdot k_3 \cdot a_1 \cdot \vec{e}_3 \cdot \vec{h}_1^T
$$

$$
[E_6] = k_2 \cdot k_3 \cdot a_2 \cdot \vec{e}_3 \cdot \vec{h}_2^T
$$

$$
[E_7] = k_4 \cdot \vec{e}_4 \cdot \vec{h}_4^T
$$

\[ [E_8] = k_1 \cdot k_4 \cdot b_1 \cdot \vec{e}_4 \cdot \vec{n}_1^T \] (8-31)

\[ [E_9] = k_2 \cdot k_4 \cdot b_2 \cdot \vec{e}_4 \cdot \vec{n}_2^T \] (8-32)

Substituting (8-23) in (8-3) results in \([C_X(T)]\) being written as:

\[ [C_X(T)] = \sum_{j=1}^{9} \phi(T) \cdot [E_j] \cdot [C_X(T)] \cdot [E_j]^T \cdot \phi^T(T) + [C_1(T)] \] (8-33)

This expression (8-33) permits the elements of \([C_X(T)]\) to be calculated, provided the elements of the forcing function matrix \([C_1(T)]\) are known. This latter matrix is presented in the following paragraph.

### 8.3. Forcing function matrix, \([C_1(T)]\)

With four exceptions only, all 100 elements of the 10x10 symmetric matrix \([C_1(T)]\) are equal to zero. The only four non-zero elements are those of the 2x2 sub-matrix in the upper left hand corner:

\[ c_{11} = \sigma_{x_1^2}(T) = \]

\[ = \sigma_{x_1^2}^{(*)} \cdot \left\{ 1 - e^{-2\omega_1^T \cdot \left( 1 + 2\omega_1 \cdot T + 2\omega_1^2 \cdot T^2 \right)} \right\} \] (8-34)

\[ c_{12} = c_{21} = \sigma_{x_1 \cdot x_1}(T) = \]

\[ = 2\sigma_{x_1^2}^{(*)} \cdot e^{-2\omega_1^T} \cdot \omega_1^2 \cdot T^2 \] (8-35)

\[ c_{22} = \sigma_{x_1^2}(T) = \]

\[ = \sigma_{x_1^2}^{(*)} \cdot \omega_1^2 \cdot \left\{ 1 - e^{-2\omega_1^T \cdot \left( 1 - 2\omega_1 \cdot T + 2\omega_1^2 \cdot T^2 \right)} \right\} \] (8-36)

In many calculations, the variance of \(x_1, \sigma_{x_1^2}^{(*)}\), can be given a nominal value:

\[ \sigma_{x_1^2}^{(*)} = 1 \]
8.4. Covariance matrix during the sample interval, \([C_X(t)]\)

The covariance matrix \([C_X(t)]\) at any time during the sample interval has already been given in (8-2). Substituting the expression (8-23) for \([C_X(0)]\) results in \([C_X(t)]\) being written as:

\[
[C_X(t)] = \sum_{j=1}^{9} \Phi(t) \cdot [E_j] \cdot [C_X(T)] \cdot [E_j]^T \cdot \Phi^T(t) + [C_1(t)]^{(8-37)}
\]

where the matrices \([E_j], j = 1, \ldots, 9\) have been given in (8-24)-(8-32), the covariance matrix \([C_X(T)]\) has been obtained according to (8-33), and the forcing function matrix \([C_1(t)]\) follows from par. 8.3. by exchanging in (8-34)-(8-36) the time parameter \(T\) with the chosen value of \(t\).

The covariance matrix of \(\bar{x}(t)\) can also be obtained in principle by averaging the various elements of the matrix - i.e. the variances and covariances - in the time domain. Denoting this time-averaged covariance matrix as \([C_X]\), the relation with the above result (8-37) in the ensemble domain is:

\[
[C_X] = \frac{1}{T} \int_{0}^{T} [C_X(t)] \cdot dt
\]

(8-38)

8.5. Covariance matrix of the characteristic features, \([C_{f1f2}]\)

Once the covariance matrix of the state variables of the biomorphic model has been obtained for \(t = T\), various other covariance matrices may be derived. As an example, the covariance matrix of the two characteristic features, \(f_1\) and \(f_2\), is given, since it is mentioned in Section 7, where the derivation of the control law is discussed.

According to (8-8) and (8-9), \(f_1\) and \(f_2\) are written as:

\[
f_1 = \bar{g}_1^T \cdot \bar{x}(T)
\]

(8-8)
\[ f_2 = g_2^T \cdot \vec{x}(T) \]  \hspace{1cm} (8-9)

or:

\[
\begin{bmatrix}
  f_1 \\
  f_2
\end{bmatrix} = [G] \cdot \vec{x}(T) \tag{8-38}
\]

where the newly introduced matrix \([G]\) is given in detail in Appendix 5.

Denoting the covariance matrix of \(f_1\) and \(f_2\) by \([C_{f_1 f_2}]\), this matrix is obtained directly from (8-38) as:

\[
[C_{f_1 f_2}] = [G] \cdot [C_{\vec{x}(T)}] \cdot [G]^T =
\]

\[
= \begin{bmatrix}
  \sigma_{f_1}^2 & \sigma_{f_1 f_2} \\
  \sigma_{f_1 f_2} & \sigma_{f_2}^2
\end{bmatrix} \tag{8-39}
\]

8.6. Variance of \(\bar{B}\)

In certain situations, the variance of \(\bar{B}\) is needed, to express the variance of the motor noise component \(\Delta B\), according to (5-3), (5-4) and (5-5). \(\sigma_B^2\) is easily derived from (8-22):

\[
\sigma_B^2 = \bar{h}_4^T \cdot [C_{\vec{x}(T)}] \cdot \bar{h}_4 + k_1^2 \cdot b_1^2 \cdot \bar{h}_1^T \cdot [C_{\vec{x}(T)}] \cdot \bar{h}_1 +
\]

\[
+ k_2^2 \cdot b_2^2 \cdot \bar{h}_2^T \cdot [C_{\vec{x}(T)}] \cdot \bar{h}_2 \tag{8-40}
\]
9. DESCRIBING FUNCTION OF THE BIOMORPHIC MODEL

9.1. Describing function

In the study of closed loop manual control systems the behaviour of the human operator has traditionally been expressed in terms of the operator's describing function \((6, 12)\), and a stochastic remnant. In order to find a connection with the existing literature, a describing function has been derived also for the biomorphic model. Using the describing function, the behaviour of the operator and the entire control loop can be described in the frequency domain, with all the attendant possibilities of further analysis, see also par. 9.2.

The describing function of a system relates the output variable to the input of the system, in the case when the latter is a sine wave of constant frequency and amplitude \((77)\). The describing function expresses the amplitude ratio and the relative phase of that part of the output variable which is linearly related to the input signal. If the system under consideration is continuous, linear and time invariant, the describing function is identical to the transfer function.

In \((48)\) a detailed and careful derivation has been given of the describing function of the biomorphic model. In the situation considered in \((48)\), the input consisted of the sum of a number of discrete sine waves, a type of input signal discussed in Section 2. No observation noise and motor noise were assumed to be present in the biomorphic model.

In terms of the equations (6-1), describing the behaviour of the biomorphic model in the time domain, the describing function relates the output of the model, i.e. the total control manipulator deflection \(x_3(t)\), to the input which is the observed tracking error, \(y_1(t)\). As a consequence, in the notation of the present report the describing function is written as:
$$DF(j\omega) = \frac{x_5(j\omega)}{y_1(j\omega)}$$

Due to the fact that the describing function expresses the behaviour of the system as if it were a continuous, linear and noise-free system, the internal model in the biomorphic model finds no representation in the describing function.

Without repeating the analysis given in (48), the results obtained are presented here, using again the notation adopted in the present report.

The describing function was derived as the ratio of two cross-power spectral density functions (48):

$$DF(j\omega) = \frac{\phi_{x_1x_5}(j\omega)}{\phi_{x_1y_1}(j\omega)}$$

The result according to (48) is:

$$DF(j\omega) = \left[ \frac{1}{T} \cdot \frac{b_1 + (a_1 + b_2) j\omega + a_2(j\omega)^2}{j\omega} + \frac{\Delta(\omega)}{\phi_{x_1y_1}(j\omega)} \right] \cdot \frac{1}{1 + j\omega T_d} \cdot \frac{1}{1 + 2\zeta_s j \frac{\omega}{\omega_s} + (j \frac{\omega}{\omega_s})^2}$$

(9-1)

Due to the type of input signal considered, this describing function exists only for those discrete frequencies $\omega = \omega_k$ present in the input signal.

Two requirements for the validity of the above describing function are, according to (48):

a. $\frac{\omega_k + \omega_i}{\Omega_s} \neq$ a signed integer

(9-2)
b. \( \Omega_s > \omega_k \)  \hspace{1cm} (9-3)

where \( \omega_k \) and \( \omega_1 \) are any two of the discrete frequencies contained in the input signal, and:

\[
\Omega_s = \frac{2\pi}{T} \hspace{1cm} (9-4)
\]

is the circular frequency corresponding to the sample interval, \( T \).

It is remarked in (48), that: "if the biomorphic model's sampling rate is higher than twice the highest frequency component in the input signal, then (9-2) is identically satisfied". Obviously (9-3) is then satisfied as well. It is assumed in the following, that the above criteria are satisfied with a sufficient margin in all cases to be considered. This guarantees the validity of the describing function (9-1) for discrete input frequencies.

In the expression (9-1) the factors:

\[
\frac{1}{1 + j\omega t_d} \quad \text{and} \quad \frac{1}{1 + 2\zeta_s j\frac{\omega}{\omega_0} + \left(j\frac{\omega}{\omega_0}\right)^2}
\]

represent the time delay and the shaping filter respectively as discussed in par. 5.5., in the conventional way.

The factor between brackets:

\[
\left[ \frac{1}{T} \cdot \frac{b_1 + (a_1 + b_2) j\omega + a_2 (j\omega)^2}{j\omega} + \frac{\Delta(\omega)}{\phi x_1 y_1 (j\omega)} \right]
\]

expresses the combined effect of the sampling behaviour and the control law. In this expression the term:
\[ \frac{\Delta(\omega)}{\Phi_{X_1Y_1}(j\omega)} \]

attracts special interest. The cross-power spectral density function in the denominator relates the control system forcing function, \( x_1(t) \), to the observed error signal, \( y_1(t) \). The numerator \( \Delta(\omega) \) is given in explicit form in (48). It is an expression built up from summations over the various discrete frequencies contained in the signals circulating in the closed-loop control system. \( \Delta(\omega) \) is shown to tend to zero due to the low-pass filtering effect of the time delay, the shaping filter and the controlled element. As remarked in (48): "This is often assumed to be the case when describing functions are applied in the linearized analysis of nonlinear systems".

It is further stated in (48), that it is anticipated that a derivation of \( DF(j\omega) \) based upon a random input signal with a continuous power spectrum would lead to a formulation similar to (9-1). However the expression for \( \Delta(\omega) \) would involve integrals rather than summations. Similar findings about the validity of the simplified form of (9-1) when \( \Delta(\omega) \) is assumed to be negligible would be expected.

The foregoing shows, that under reasonable conditions the describing function of the biomorphic model can be written in the simplified form:

\[
DF(j\omega) = \left[ \frac{1}{T} \cdot \frac{b_1 + (a_1 + b_2) \cdot j\omega + a_2(j\omega)^2}{j\omega} \right] \cdot \frac{1}{1 + j\omega T_d} \cdot \frac{1}{1 + 2\zeta_s \cdot j \frac{\omega}{\omega_s} + \left( j \frac{\omega}{\omega_s} \right)^2} \]

(9-5)

Quoting again from (48): "It is seen from (9-1) that the parameters \( a_1 \) and \( b_2 \) perform a similar function and in the case where \( \Delta(\omega) = 0 \), only one or the other is required. The parameter \( a_2 \) is associated
with lead effects. The sampling period $T$ influences the gain of the describing function.

It is also shown and explained in (48), however, that in an extreme situation $a_1$ and $b_2$ do not have completely identical effects on the describing function, due to the remaining existence of the term containing $\Delta(\omega)$ in (9-1). Especially situations in which $a_i$ ($i = 1, 2$) is zero or very small and the corresponding $b_i$ is large, merit further experimental or theoretical study into the applicability of (9-4) rather than (9-1).

9.2. Stability of a closed loop system containing the biomorphic model, based on the approximate describing function

This subject has also been studied at some length in (48). Using the describing function in its simplified form (9-5), a first approximate stability analysis of the closed loop system can be made, by applying the Routh-Hurwitz criteria, see (78), to the characteristic equation of the closed-loop system.

Such an analysis was performed in (48), using a double integrator as the controlled element. Stability boundaries as functions of the elements of the control law were derived, based on the approximate describing function (9-5) and on the Routh-Hurwitz criteria. Quoting again from (48): "The predicted boundaries can (therefore) be characterized as conservative in the present case.

It has been found in the present study that decisions made concerning the biomorphic model parameters based on its describing function approximation have always led to the desired closed loop system characteristics. Thus it does not appear that the sampling nature of the control law has introduced any significant complications in the range of applications studied to date."
10. COMPUTER PROGRAM TO CALCULATE ENSEMBLE CHARACTERISTICS OF THE BIOMORPHIC MODEL

Two computer programs have been made to generate quantitative data on the behaviour of the biomorphic model. One of the two is a hybrid simulation program (79) to simulate on a real time basis the closed loop in which the biomorphic model is the controller. This simulation program will not be further discussed here.

The second program, "Biomorphic-human operator model" (80), has been written to quantitatively determine various ensemble characteristics of the model, as discussed in the previous Sections. Rather than going into the details of this program, an outline will be given of the way in which the different calculations, mainly those of the control law and the covariance matrices, are combined to obtain the desired ensemble characteristics of the biomorphic model.

The main program consists of six parts, labelled SUB0 to SUB5, as shown in Fig. 10.1. The principal functions of these parts are listed below.

SUB0 organizes the input data, amongst which are the characteristics of the controlled element and the bandwidth of the forcing function, \( \omega_1 \). A number of parameters which are not varied during the present optimization processes are given a fixed value here. These parameters are:

- motor noise constants, \( k_3^2 \), \( k_4^2 \) and \( c_4 \);
- observation noise time constants, \( \tau_{ob1} \) and \( \tau_{ob2} \);
- time intervals, \( \Delta t_1 \) and \( \Delta t_u \);
- undamped natural frequency of the shaping network, \( \omega_{08} \).

SUB1 calculates the four elements of the control law, \( a_1 \), \( a_2 \), \( b_1 \), \( b_2 \), according to the procedure discussed in Section 7. The cost function, \( J \), see (7-18), is minimized by varying the four elements. In the optimization process, the various constraint functions which arise from the requirement of stability of the control loop.
are respected.

SUB2 calculates the covariance matrix \([C_X(T)]\), as discussed in Section 8. The results of the calculations are the variance \(\sigma_e^2\), which is to be minimized, the variance \(\sigma_B^2\) and the covariance matrix \([C_{f_1}f_2]\), which is used as an input to SUB1, as indicated in Fig. 10.1.

SUB3 calculates some time-averaged output data which may be needed to compare the results obtained from the biomorphic model with results from other models as published in the literature.

SUB4 calculates the approximate describing function, \(DF(j\omega)\), according to Section 9. From \(DF(j\omega)\) the cross-over frequency and the phase margin are derived, again with the purpose of comparing results obtained from the biomorphic model with results given elsewhere.

SUB5 organizes the flow of the calculations between the parts SUB1 and SUB2.

The following indicates the optimization strategy used, to find the free parameters of the biomorphic model, such that a number of requirements imposed on the characteristics of a closed loop manual control system are satisfied. These requirements are formulated e.g. in (12). They can be summarized very briefly as follows.

1. The describing function of the human operator model has to meet certain criteria, depending on the type of the controlled element on the bandwidth of the forcing function.

2. If a forcing function having a continuous power spectral density is used, the variance of the error, \(\sigma_e^2\), has to be minimized.

Fig. 10.1. shows that the adjustment of the variable parameters in the biomorphic model takes place via a two-level optimization process. At the first level, the four coefficients in the control law are obtained in SUB1, by minimization of the costfunction, \(J\), for a given speed of the desired response, expressed by \(\omega_{\text{d}}\), and for a given covariance matrix \([C_{f_1}f_2]\). If the sample interval, \(T\), is known the control law fixes the only remaining free parameters which determine the describing
function of the biomorphic model.

The variance of the error signal sensed by the pilot, $\sigma_e^2$, which is to be minimized, depends not only on the control law, but also on a few more variables which can still be chosen, such as the observation interval, $\Delta t_o$, and the speed of the desired response, expressed by $\omega_0$. Furthermore, in Section 9 it is found that the coefficients $a_1$ and $b_2$ of the control law have the same effect on the describing function, see (9-5). The two coefficients differ, however, in their influence on the variance, $\sigma_e^2$, because the effects of observation noise and motor noise depend differently on the individual values of $a_1$ and $b_2$. As a consequence, one coefficient - say $b_2$ - can still be varied whilst $a_1 + b_2$ is kept constant.

This leads to the second level of the optimization process. This part of the optimization takes place in SUB5, see Fig. 10.1. In the process the variables $\Delta t_o$, $\omega_0$, and $b_2$ are changed - $a_1 + b_2$ being kept constant - and the variance $\sigma_e^2$ is minimized. The variance is calculated in SUB2. Since a variation of $\omega_0$ exerts its influence through changes in the coefficients of the control law, the first level of the optimization process can be regarded as an inner loop within the second level, or outer loop of the process, see Fig. 10.1. The inner loop is, of course, also influenced by variations in the covariance matrix $[Cf_1 f_2]$ obtained in SUB2, as a result of the calculations in the outer loop. A variation of $\Delta t_o$ has a direct influence on both the observation noise levels, $k_1^2$ and $k_2^2$, see (6-10), and on the sample interval, $T$, if $\Delta t_1$ and $\Delta t_u$ are kept constant, see (6-16). The entire process is run until both $J$ and $\sigma_e^2$ have arrived at their minimum values and the variables in the optimization procedures have become constant.

Given the above optimization strategy, the remaining parameters in the biomorphic model, earlier assumed to be constant, i.e. $k_3^2$, $k_4^2$, $c_4$, $\tau_{ob1}$, $\tau_{ob2}$, $\Delta t_1$, $\Delta t_u$ and $\omega_0$ have to chosen at such (constant) values that the biomorphic model adjusts itself by the above procedure, in
accordance with the criteria spelled out in (12), over a wide range of controlled elements and forcing function bandwidths.
11. CONCLUDING REMARKS

In this report an initial and relatively simple version of the biomorphic model of the human pilot has been described. Only the case of single-display, single-axis control, requiring continuous attention has been considered. In order to limit the scope and the size of the report, no quantitative applications of the biomorphic model in its present form have been added. Such examples will be the subject of a separate report.

The addition of further input signals to the biomorphic model, such as vestibular or peripheral visual cues, requires expansions of the model. This applies also the incorporation of multi-display situations. These expansions are to be discussed in subsequent studies.

Another important extension of the biomorphic model links the present model of the overt actions of the human pilot to the more internal, mental and covert processes going on in the brain when the pilot controls the aircraft. These matters have been deliberately neglected in the present study, although they are of great importance and are considered to form the basis of the pilot's judgement on the aircraft's handling qualities. An appropriate extension of the biomorphic model to encompass also these mental processes will be the subject of a separate report.
12. REFERENCES


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APPENDIX 1. A FEW ASPECTS OF INFORMATION THEORY

This Appendix briefly outlines a few basic concepts of information theory, see also (59,6). In its present precise mathematical formulation, the theory was developed in communication engineering.

Consider a set of \( n \) independent values of a discrete stochastic variable, \( x_i \), whose probability distribution is \( p_i \). The information contained in \( x_i \) is - by definition - expressed by the entropy of the set, \( H \), defined as:

\[
H = - \sum_{i=1}^{n} p_i \cdot \log p_i
\]

If the variable \( x \) is stochastic, but continuous rather than discrete, with probability density function \( p(x) \), the entropy is expressed as:

\[
H = - \int_{-\infty}^{+\infty} p(x) \cdot \log p(x) \cdot dx \quad \text{(A1-1)}
\]

If such a variable has in particular a Gaussian distribution with a standard deviation \( \sigma \), its entropy as derived from (A1-1) is:

\[
H = \log \sqrt{2\pi e} \cdot \sigma \quad \text{(A1-2)}
\]

Usually, in the above expressions for \( H \), the basis for the logarithms is taken as 2. The resulting values of entropy, or information, are expressed in "bits" (binary digits).

This entropy can be considered a measure of the uncertainty existing at a certain time when the value of \( x \) at that time is completely unknown. It is also a measure of the information gained, when \( x \) becomes exactly known.
Following this concept of entropy as a measure of information or uncertainty contained in a stochastic variable, another concept can now be introduced. This is the so-called "transmitted information", I.

Consider a situation close to the area of concern in this report. A stochastic variable, \( x \), is displayed such that it is available for visual observation. The entropy contained in \( x \) is \( H_x \). At a certain instant in time, \( x \) becomes known through visual observation. However, as is often the case, a certain inaccuracy remains after the observation has taken place. Suppose the observed value is called \( y \) and the remaining error is \( \varepsilon \). Then:

\[
y = x + \varepsilon
\]

The error, \( \varepsilon \), is assumed to be stochastic, uncorrelated with \( x \) and Gaussian with a standard deviation \( \sigma_\varepsilon \). The remaining uncertainty in the observed value, \( y \), embodied in the error, \( \varepsilon \), has an entropy according to (A1-2):

\[
H_y = \log \sqrt{2\pi e} \cdot \sigma_\varepsilon
\]

In such a situation, the concept of transmitted information, I, is defined as the reduction in uncertainty, or entropy:

\[
I = H_x - H_y
\]

Like entropy, \( H \), the transmitted information, \( I \), is usually expressed in bits. It is this transmitted information which provides a measure for the limited "information handling capacity" of the human observer.
APPENDIX 2. SYSTEM EQUATIONS OF THE BIOMORPHIC MODEL FOR SEVERAL
CONTROLLED ELEMENTS

In the following the system equations of the biomorphic model are given
for several controlled elements. Only the expressions are repeated which
differ from those given in Section 6 for the second order controlled
element. The additional controlled elements to be considered are:

1. single integrator;
2. first order element;
3. double integrator.

The single integrator

The system equation corresponding to (6-1) is:

\[
\begin{bmatrix}
\dot{x}_1 \\
\dot{x}_2 \\
\dot{x}_3 \\
\vdots \\
\dot{x}_8
\end{bmatrix}
= \begin{bmatrix}
0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
-\omega_0^2 & -2\omega_0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1/\tau_d & -1/\tau_d & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & \omega_0^2 & -\omega_0^2 & -2\tau_s\omega_0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & K_v & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & K_c & 0
\end{bmatrix}
\begin{bmatrix}
x_1 \\
x_2 \\
x_3 \\
\vdots \\
x_8
\end{bmatrix}
+ \begin{bmatrix}
0 \\
\omega_0^2 \\
0 \\
0 \\
0 \\
0 \\
0 \\
0
\end{bmatrix}
\begin{bmatrix}
x_1 \\
x_2 \\
x_3 \\
\vdots \\
x_8
\end{bmatrix}
\]

(A2-1)

with the initial condition:
\[
\begin{bmatrix}
    x_1(0) \\
    x_2(0) \\
    x_3(0) \\
    x_4(0) \\
    x_5(0) \\
    x_6(0) \\
    x_7(0) \\
    x_8(0)
\end{bmatrix}
= \begin{bmatrix}
    x_1(T) \\
    x_2(T) \\
    x_3(T) + B \\
    x_4(T) + \frac{A}{\tau_d} \\
    x_5(T) \\
    x_6(T) \\
    0 \\
    0
\end{bmatrix} \tag{A2-2}
\]

The observed variables are, corresponding to (6-4):

\[
\begin{bmatrix}
    y_1 \\
    y_2 \\
    y_3 \\
    y_4
\end{bmatrix} = \begin{bmatrix}
    1 & 0 & 0 & 0 & 0 & 0 & 0 & -1 \\
    0 & 1 & 0 & 0 & -K_c & 0 & 0 & 0 \\
    0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\
    0 & 0 & 0 & 0 & V & 0 & 0 & 0
\end{bmatrix}
\begin{bmatrix}
    x_1 \\
    x_2 \\
    x_3 \\
    x_4 \\
    x_5 \\
    x_6 \\
    x_7 \\
    x_8
\end{bmatrix} \tag{A2-3}
The first order element

The system equation corresponding to (6-1) is:

\[
\begin{bmatrix}
\dot{x}_1 \\
\dot{x}_2 \\
\dot{x}_3 \\
\dot{x}_4 \\
\dot{x}_5 \\
\dot{x}_6 \\
\dot{x}_7 \\
\dot{x}_8 \\
\end{bmatrix} = \begin{bmatrix}
0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
-\omega_0^2 & -2\omega_0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1/\tau_d & -1/\tau_d & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & \omega_0^2 & -\omega_0^2 & -2\tau_s \omega_0 & 0 & 0 \\
0 & 0 & 0 & 0 & K_v/\tau_v & 0 & -1/\tau_v & 0 \\
0 & 0 & 0 & 0 & K_c/\tau_c & 0 & 0 & -1/\tau_c \\
\end{bmatrix} \begin{bmatrix}
x_1 \\
x_2 \\
x_3 \\
x_4 \\
x_5 \\
x_6 \\
x_7 \\
x_8 \\
\end{bmatrix} + \begin{bmatrix}
0 \\
\omega_0^2 \\
0 \\
0 \\
0 \\
0 \\
0 \\
0 \\
\end{bmatrix}
\]

(A2-4)

The initial condition is identical to (A2-2) and the observed variables are, corresponding to (6-4):

\[
\begin{bmatrix}
y_1 \\
y_2 \\
y_3 \\
y_4 \\
\end{bmatrix} = \begin{bmatrix}
1 & 0 & 0 & 0 & 0 & 0 & 0 & -1 \\
0 & 1 & 0 & 0 & -K_c/\tau_c & 0 & 0 & 1/\tau_c \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & K_v/\tau_v & 0 & -1/\tau_v & 0 \\
\end{bmatrix} \begin{bmatrix}
x_1 \\
x_2 \\
x_3 \\
x_4 \\
x_5 \\
x_6 \\
x_7 \\
x_8 \\
\end{bmatrix}
\]

(A2-5)
The double integrator

The system equation corresponding to (6-1) is:

\[
\begin{bmatrix}
\dot{x}_1 \\
\dot{x}_2 \\
\dot{x}_3 \\
\dot{x}_4 \\
\dot{x}_5 \\
\dot{x}_6 \\
\dot{x}_7 \\
\dot{x}_8 \\
\dot{x}_9 \\
\dot{x}_{10}
\end{bmatrix}
=
\begin{bmatrix}
0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
-\omega_1^2 & -2\omega_1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1/\tau_d & -1/\tau_d & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & \omega_s^2 & -\omega_s^2 & -2\tau_s \omega_s & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & K_v & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & K_c
\end{bmatrix}
\begin{bmatrix}
x_1 \\
x_2 \\
x_3 \\
x_4 \\
x_5 \\
x_6 \\
x_7 \\
x_8 \\
x_9 \\
x_{10}
\end{bmatrix}
\begin{bmatrix}
0 \\
\omega_1^2 \\
0 \\
0 \\
0 \\
0 \\
0 \\
0 \\
0 \\
0
\end{bmatrix}
\]

The initial condition is identical to (6-3). The observed variables are expressed by (6-4).
APPENDIX 3. DERIVING A SYSTEM MATRIX HAVING SHIFTED EIGENVALUES

A system is given, described by, see (7-9):

\[ \dot{\bar{x}} = [F] \cdot \bar{x} \]  \hspace{1cm} (7-9)

where \([F]\) is an \(n \times n\) constant, real matrix. It is desired to derive a new system, described by:

\[ \dot{\bar{x}}' = [F'] \cdot \bar{x}' \]  \hspace{1cm} (7-23)

where \([F']\) is again an \(n \times n\) constant, real matrix, such that by using the same initial conditions for both systems:

\[ \bar{x}'(0) = \bar{x}(0) \]

the solution of (7-23) is related to that of (7-9) by:

\[ \bar{x}'(t) = \bar{x}(t) \cdot e^{-t/2\tau_p} \]  \hspace{1cm} (A3-1)

The response of any element \(x_i\) of \(\bar{x}\) is the sum of \(n\) exponential functions, each belonging to one of the \(n\) eigenvalues \(\lambda_i\) \((i = 1, ..., n)\) of the system matrix \([F]\). The expression (A3-1) implies that all eigenvalues \(\lambda_i'\) \((i = 1, ..., n)\) of the new system matrix \([F']\) must have a real part which is changed an amount \(-\frac{1}{2\tau_p}\) relative to that of the original system:

\[ \lambda_i' = \lambda_i - \frac{1}{2\tau_p} \]  \hspace{1cm} (A3-2)

In order to obtain the matrix \([F']\), the given matrix \([F]\) is first diagonalized. The eigenvalues of the resulting diagonal matrix \([D_p]\) are the same as those of \([F]\):

\[ [D_p] = [T]^{-1} \cdot [F] \cdot [T] \]  \hspace{1cm} (A3-3)
where:

\[ [D_F] = n \times n \text{ diagonal matrix, the elements of which on the main diagonal are the eigenvalues of } [F], \]

\[ [T] = n \times n \text{ matrix, the columns of which are given by the eigenvectors of } [F]. \]

The matrix \([D_F]\) can be considered the system matrix of a new system, having the same eigenvalues as the original system, given by (7-9). The new system is:

\[ \dot{\tilde{z}} = [D_F] \cdot \tilde{z} \quad (A3-6) \]

The state vector \(\tilde{z}\) follows from \(\tilde{x}\) by:

\[ \tilde{z} = [T]^{-1} \cdot \tilde{x} \quad (A3-5) \]

For any initial condition \(\tilde{z}(0)\), the time response of an element \(z_i\) of \(\tilde{z}\) is given by:

\[ z_i = z_i(0) \cdot e^{\lambda_i t} \quad (A3-6) \]

It is now easy to establish a new diagonal matrix, \([D_F']\), whose eigenvalues \(\lambda_i'\) \((i = 1, \ldots, n)\) are related to those of \([D_F]\) according to (A3-2). The relation between \([D_F']\) and \([D_F]\) is:

\[ [D_F'] = [D_F] - \frac{1}{2T_F} \cdot [I] \quad (A3-7) \]

where \([I]\) is the \(n \times n\) unity matrix. Evidently, the eigenvalues of \([D_F']\) all satisfy the requirement that their real part has been changed by an amount of \(-\frac{1}{2T_F}\) relative to that of the corresponding eigenvalue of \([D_F]\).

The new matrix \([D_F']\) can again be considered as the system matrix of
a system, whose state vector is $\tilde{z}'$:

$$\dot{\tilde{z}}' = \left[D_{p}' \right] \cdot \tilde{z}'$$

The time-response of an element $z_i'$ of the state of this system to an initial condition $\tilde{z}'(0)$, is given by:

$$z_i'(t) = z_i'(0) \cdot e^{\lambda_i't}$$

If $\tilde{z}'(0)$ is chosen equal to $z(0)$ in (A3-5), it is seen, that due to (A3-2):

$$z_i'(t) = z_i \cdot e^{-t/2\tau_p}$$

This result is true for any element of $\tilde{z}'$ and $\tilde{z}$. As a consequence:

$$\tilde{z}'(t) = \tilde{z}(t) \cdot e^{-t/2\tau_p}$$ (A3-8)

if, again:

$$\tilde{z}'(0) = \tilde{z}(0)$$

Since the eigenvalues, $\lambda_i'$, of $\left[D_{p}' \right]$ already satisfy the condition (A3-2) which must hold also for the eigenvalues of the matrix $[F']$, the matrix $\left[D_{p}' \right]$ can be considered the result of the diagonalization of the – as yet unknown – matrix $[F']$. The transformation matrix, relating $\left[D_{p}' \right]$ to $[F']$ has to be determined only, in order to obtain $[F']$, the matrix of the system for which (A3-2) is true.

It follows from (A3-5):

$$\bar{x} = [T] \cdot \bar{z}$$

Using (A3-1) and (A3-8) it is found:
\[ \bar{x}' = [T] . \bar{z}' \]

Evidently, the transformation matrix \([T]\) not only diagonalizes \([F]\), but also \([F']\):

\[ [D_F'] = [T]^{-1} . [F'] . [T] \]  \hspace{1cm} (A3-9)

Substituting (A3-9) in (3-7) results in:

\[ [D_F] - \frac{1}{2T_p} [I] = [T]^{-1} . [F'] . [T] \]

Using (A3-3), the final result is the required relation between \([F']\) and \([F]\):

\[ [F'] = [F] - \frac{1}{2T_p} . [I] \]  \hspace{1cm} (A3-10)
APPENDIX 4. VECTORS AND MATRICES REQUIRED IN THE OPTIMIZATION PROCEDURE FOR THE CONTROL LAW

The optimization procedure leading to the control law described in Section 7, requires certain vectors and matrices which are given below, in so far as they have not been presented in Section 7.

The transposed vector, \( c^T \), required in (7-5) is:

for the single integrator and the first order controlled element:

\[
c^T = [-1 \ 0 \ 0 \ 0 \ 0 \ 0 \ 1]
\]

and for the double integrator and the second order controlled element:

\[
c^T = [-1 \ 0 \ 0 \ 0 \ 0 \ 1 \ 0]
\]

The system equation, corresponding to (7-8) is for a single integrator:

\[
\begin{bmatrix}
\dot{x}_1 \\
\dot{x}_2 \\
\dot{x}_3 \\
\dot{x}_4 \\
\dot{x}_5 \\
\dot{x}_6 \\
\dot{x}_7 \\
\end{bmatrix} =
\begin{bmatrix}
0 & 1 & 0 & 0 & 0 & 0 & 0 \\
-\omega_d^2 & -2\zeta_d\omega_d & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1/\tau_d & -1/\tau_d & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & \omega_s^2 & -\omega_s^2 & -2\zeta_s\omega_s & 0 \\
0 & 0 & 0 & K_v & 0 & 0 & 0 \\
\end{bmatrix}
\begin{bmatrix}
x_1 \\
x_2 \\
x_3 \\
x_4 \\
x_5 \\
x_6 \\
x_7 \\
\end{bmatrix}
\]

and the required initial condition is:
\[
\begin{bmatrix}
    x_1(0) \\
    x_2(0) \\
    x_3(0) \\
    x_4(0) \\
    x_5(0) \\
    x_6(0) \\
    x_7(0)
\end{bmatrix} = \begin{bmatrix}
    f_1 \\
    f_2 \\
    B \\
    \frac{A}{\tau_d} \\
    0 \\
    0 \\
    f_1
\end{bmatrix}
\]  

(A4-2)

The system equation, corresponding to (7-8) is for a first order element:

\[
\begin{bmatrix}
    \dot{x}_1 \\
    \dot{x}_2 \\
    \dot{x}_3 \\
    \dot{x}_4 \\
    \dot{x}_5 \\
    \dot{x}_6 \\
    \dot{x}_7
\end{bmatrix} = \begin{bmatrix}
    0 & 1 & 0 & 0 & 0 & 0 & 0 \\
    -\omega_d^2 & -2\zeta_d\omega_d & 0 & 0 & 0 & 0 & 0 \\
    0 & 0 & 0 & 0 & 0 & 0 & 0 \\
    0 & 0 & 1/\tau_d & -1/\tau_d & 0 & 0 & 0 \\
    0 & 0 & 0 & 0 & 0 & 1 & 0 \\
    0 & 0 & 0 & \omega_s^2 & -\omega_s^2 & -2\zeta_s\omega_s & 0 \\
    0 & 0 & 0 & 0 & K_v/\tau_v & 0 & -1/\tau_v
\end{bmatrix} \begin{bmatrix}
    x_1 \\
    x_2 \\
    x_3 \\
    x_4 \\
    x_5 \\
    x_6 \\
    x_7
\end{bmatrix}
\]

(A4-3)

whereas the initial condition is identical to (A4-2) for the single integrator.
The system equation, corresponding to (7-8) is for a double integrator:

\[
\begin{bmatrix}
\dot{x}_1 \\
\dot{x}_2 \\
\dot{x}_3 \\
\dot{x}_4 \\
\dot{x}_5 \\
\dot{x}_6 \\
\dot{x}_7 \\
\dot{x}_8
\end{bmatrix} =
\begin{bmatrix}
0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
-\omega_0^2 & -2\zeta_0 \omega_0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 1/\tau_d & -1/\tau_d & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & \omega_s^2 & -\omega_s^2 & -2\zeta_s \omega_s & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & k_v
\end{bmatrix}
\begin{bmatrix}
x_1 \\
x_2 \\
x_3 \\
x_4 \\
x_5 \\
x_6 \\
x_7 \\
x_8
\end{bmatrix}
\]

and the initial condition is identical to (7-10) for the second order element.

The matrix \([IS]\), required to express the initial condition, see (7-12), reads for the single integrator as well as for the first order element:

\[
[IS] =
\begin{bmatrix}
1 & 0 \\
0 & 1 \\
b_1 & b_2 \\
a_1/\tau_d & a_2/\tau_d \\
0 & 0 \\
0 & 0 \\
1 & 0
\end{bmatrix}
\]
The corresponding matrix \([IS]\) for the double integrator is identical to the one given in (7-12) for the second order element.
APPENDIX 5. VECTORS AND MATRICES RELEVANT TO THE CALCULATION OF THE
COVARIANCE MATRIX \([C_X(T)]\)

In Section 7 various vectors and matrices needed in the calculation of the covariance matrix \([C_X(T)]\) are mentioned. They are spelled out in detail in the following. A distinction has to be made between controlled elements of first and second order. Controlled elements of higher order, which are not considered in this report, would need slightly adapted forms of the various vectors and matrices, but the adaptations should be obvious.

First order elements

The matrix \([D]\):

\[
[D] = \begin{bmatrix}
1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 \\
\end{bmatrix}
\]  

(A5-1)

The vectors \(\tilde{d}_1, \tilde{d}_2, \tilde{d}_3, \tilde{d}_4\) are given in their transposed forms:

\[
\tilde{d}_1^T = [0 \ 0 \ 0 \ 1/\tau_d \ 0 \ 0 \ 0 \ 0]
\]

\[
\tilde{d}_2^T = [0 \ 0 \ 1 \ 0 \ 0 \ 0 \ 0 \ 0]
\]

\[
\tilde{d}_3^T = [0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 1 \ 0]
\]
\[ \tilde{\alpha}_4^T = [0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0] \]

The vectors \( \tilde{g}_1^T \) and \( \tilde{g}_2^T \), for a single integrator:

\[ \tilde{g}_1^T = [1 \ 0 \ 0 \ 0 \ 0 \ 0 \ -1] \]
\[ \tilde{g}_2^T = [0 \ 1 \ 0 \ 0 \ -K_c \ 0 \ 0 \ 0] \]

The vectors \( \tilde{h}_1^T, \tilde{h}_2^T, \tilde{h}_3^T \) and \( \tilde{h}_4^T \), for a single integrator:

\[ \tilde{h}_1^T = [1 \ 0 \ 0 \ 0 \ 0 \ 0 \ -1 \ -1] \]
\[ \tilde{h}_2^T = [0 \ 1 \ 0 \ 0 \ -K_c \ -K_v \ 0 \ 0 \ 0] \]
\[ \tilde{h}_3^T = [a_1 \ a_2 \ 0 \ 0 \ -a_2 K_c \ 0 \ 0 \ -a_1] \]
\[ \tilde{h}_4^T = [b_1 \ b_2 \ 0 \ 0 \ -b_2 K_c \ 0 \ 0 \ -b_1] \]

The matrix \([G]\), for a single integrator:

\[
[G] = \\
\begin{bmatrix}
1 & 0 & 0 & 0 & 0 & 0 & 0 & -1 \\
0 & 1 & 0 & 0 & -K_c & 0 & 0 & 0
\end{bmatrix}
\]

The vector \( \tilde{g}_1^T \) and \( \tilde{g}_2^T \), for a first order element:

\[ \tilde{g}_1^T = [1 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ -1] \]
\[ \tilde{g}_2^T = [0 \ 1 \ 0 \ 0 \ -K_c/\tau_c \ 0 \ 0 \ 1/\tau_c] \]

The vectors \( \tilde{h}_1^T, \tilde{h}_2^T, \tilde{h}_3^T \) and \( \tilde{h}_4^T \), for a first order element:

\[ \tilde{h}_1^T = [1 \ 0 \ 0 \ 0 \ 0 \ 0 \ -1 \ -1] \]
\[ h_2^T = \begin{bmatrix} 0 & 1 & 0 & 0 & -\frac{K_c}{\tau_c} & \frac{K_v}{\tau_v} & 0 & 0 & 1/\tau_v & 1/\tau_c \end{bmatrix} \]

\[ h_3^T = \begin{bmatrix} a_1 & a_2 & 0 & 0 & -\frac{a_2K_c}{\tau_c} & 0 & 0 & -a_1 + a_2/\tau_c \end{bmatrix} \]

\[ h_4^T = \begin{bmatrix} b_1 & b_2 & 0 & 0 & -\frac{b_2K_c}{\tau_c} & 0 & 0 & -b_1 + b_2/\tau_c \end{bmatrix} \]

The matrix \([G]\), for a first order element:

\[
G = \begin{bmatrix}
1 & 0 & 0 & 0 & 0 & 0 & 0 & -1 \\
0 & 1 & 0 & 0 & -K_c/\tau_c & 0 & 0 & 1/\tau_c
\end{bmatrix}
\]

**Second order elements**

The matrix \([D]\):

\[
[D] = \begin{bmatrix}
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1
\end{bmatrix}
\]

The vectors \(\dd_1, \dd_2, \dd_3, \dd_4\) are given in their transposed forms:
\( \vec{d}_1^T = [0 \ 0 \ 0 \ 1/\tau_d \ 0 \ 0 \ 0 \ 0 \ 0] \)

\( \vec{d}_2^T = [0 \ 0 \ 1 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0] \)

\( \vec{d}_3^T = [0 \ 0 \ 0 \ 0 \ 0 \ 1 \ 0 \ 0 \ 0] \)

\( \vec{d}_4^T = [0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 1 \ 0 \ 0] \)

The vectors \( \vec{g}_1^T \) and \( \vec{g}_2^T \):

\( \vec{g}_1^T = [1 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ -1 \ 0] \)

\( \vec{g}_2^T = [0 \ 1 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ -1] \)

The vectors \( \vec{h}_1^T, \vec{h}_2^T, \vec{h}_3^T \) and \( \vec{h}_4^T \):

\( \vec{h}_1^T = [1 \ 0 \ 0 \ 0 \ 0 \ 0 \ -1 \ 0 \ -1 \ 0] \)

\( \vec{h}_2^T = [0 \ 1 \ 0 \ 0 \ 0 \ 0 \ 0 \ -1 \ 0 \ -1] \)

\( \vec{h}_3^T = [a_1 \ a_2 \ 0 \ 0 \ 0 \ 0 \ 0 \ -a_1 \ -a_2] \)

\( \vec{h}_4^T = [b_1 \ b_2 \ 0 \ 0 \ 0 \ 0 \ 0 \ -b_1 \ -b_2] \)

The matrix \([G]\):

\[
[G] = \begin{bmatrix}
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0
\end{bmatrix}
\]
Fig. 1.1. Block diagram of compensatory manual control system.

Fig. 1.2. Main elements of human operator activities.
Figure 2.1. Power spectral density of stochastic input signal.

Figure 2.2. Amplitude spectrum of periodic input signal.
Fig. 3.1. The sampling interval.

Fig. 3.2. The sampling process in greater detail.
Fig. 3.3. Blockdiagram of the observation process.
Fig. 4.1: Block diagram of the decision process.

Fig. 5.1: Typical neuron.

Fig. 5.2: Motor centers in the brain and spinal chord.
Fig. 5.3: The cerebral cortex.

Fig. 5.4: The sensory or specific cortex and the nonspecific cortex.

Fig. 5.5: The motor cortex.
Fig. 5.6: Cross section of the spinal cord, sensory input and output to effectors.

Skeletal muscle

Fibre bundle in muscle

Fibre in fibre bundle

Fig. 5.7: Muscle fiber.

motor fiber   sensory fiber
stretch receptor   end plate   intrafusal muscle fiber
capsule

Fig. 5.8: Muscle spindle
Fig. 5.9: Motor unit.

Fig. 5.10: Agonist/antagonist
**Fig. 5.11.** Main functions and structures in the neuromuscular system.
Fig. 5.12. Block diagram of the neuromuscular system.