LOW-SPEED, LOW-FREQUENCY AC DRIVE
LOW-SPEED, LOW-FREQUENCY AC DRIVE

Matrix-Čuk Converter in an AC Drive Application with Shaft Sensorless Vector Control

PROEFSCHRIFT

ter verkrijging van de graad van doctor
aan de Technische Universiteit Delft,
on gezag van de Rector Magnificus Prof. ir. K.F. Wakker,
in het openbaar te verdedigen ten overstaan van een commissie
door het College van Dekanen aangewezen,
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To Heleen, Vuk
my parents and Tamara
Preface

This research project started in the summer of 1990 with the objective to find and analyze novel approaches/ideas which would improve the performance of low-speed ac drives. The path of the research was full of unexpected turns and detours. It started with an idea to develop a novel three-phase converter topology suitable for low-speed ac drive applications. That part of the research was successfully completed by making and testing an experimental ac drive powered by the novel converter topology. The acquired experience and knowledge was also used for an industrial project.

The research took an unexpected direction in the summer of 1992. Based on the features of the developed converter topology, a new idea of a straightforward implementation of shaft and voltage sensorless vector control of ac drives came up. From that moment the struggle to finish the whole project on time became a fierce one. Primarily because the theory and implementation of the vector control of ac drives were for me at that time a new field of interest. By investing a lot of effort the project was finished in time with results matching the expectations.

During the four years of research, many people were of great help to me. From those, I would specially like to thank:

• Prof. W. Deleeroi and dr. J.B. Klaassens, who gave me much freedom in exploring my topic and encouraged me to exchange ideas at international conferences.
• The members of the committee, prof. J. Holtz, prof. E.M. Kamerbeck, prof. A.J.A. Vandenput and especially prof. J.A. Schot and prof. J.A. Melkebeek for their interest and the constructive criticism.
• Prof. V. Stefanović for new and supportive advices.
• Freddie de Beer, Peter van Duijzen, Andrei Rauta and Pavol Bauer for their comradeship in the struggle to reach our Ph.D's.
• Mrs. J.B. Zaat-Jones for correcting my English.
• The staff of the laboratory for Power Electronics and Electrical Machines who surrounded me with a friendly working atmosphere.

Finally, I would like to thank you Heleen for the support and understanding during the last four years.
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LIST OF SYMBOLS

**Parameters**

- $A_{Fe}$: cross section of the core [m$^2$]
- $A_p$: area product [m$^4$]
- $A_w$: window area [m$^2$]
- $C_{sn}$: snubber capacitor [F]
- $D$: diameter of the stator bore [m]
- $F_{ij}, D_{ij}$: coefficients
- $G_1, G_3$: variable factors
- $\dot{e}_g$: maximum voltage power grid $e_g$ [V]
- $h$: harmonic order
- $i_C$: maximum collector current [A]
- $i, j$: integers
- $J$: momentum of inertia [kgm$^2$]
- $k_s, k_r, k_m$: constants
- $k_h$: constant
- $L_1$: inductance of matrix-čuk converter [H]
- $L_r$: rotor inductance [H]
- $L_s$: stator inductance [H]
- $l_r$: rotor length [m]
- $l_w$: effective length of the winding [m]
- $\ell = \sigma L_s$: leakage inductance [H]
- $M$: mutual (main inductance) [H]
- $n$: index for stator windings ($n=1,2$ or $3$)
- $p$: number of pole pairs
- $p_1, p_2$: poles in complex plane
- $pf$: power factor
- $pf_i, pf_o$: input and output power factor
- $R_R$: rotor resistance [Ω]
- $R_1$: parasitic resistance of coil with inductance $L_1$ [Ω]
- $R_s$: stator resistance [Ω]
- $R_{sn}$: resistance of snubber resistor [Ω]
- $r$: rotor radius [m]
- $s$: Laplace operator $ddt$ [s]
- $T_R$: rotor time constant [s]
- $T_{sw}$: switching period [s]
- $t_f$: transistor fall time [s]
- $w$: number of winding turns
**List of Symbols**

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>( w_{snh} )</td>
<td>effective turns for ( h )-th harmonic of winding ( n )</td>
</tr>
<tr>
<td>( w_s, w_r )</td>
<td>number of stator and rotor winding turns</td>
</tr>
</tbody>
</table>

**Reference frames**

- \( a - b \) reference frame fixed to rotor
- \( m - n \) arbitrary reference frame rotating with arbitrary angular speed \( \omega_k \)
- \( x - y \) reference frame fixed to rotor flux
- \( \alpha - \beta \) reference frame fixed to stator

**Normalized parameters**

- \( f_r \) normalized rotor frequency
- \( f_s \) normalized stator frequency
- \( H \) normalized inertia \([s]\)
- \( r_R \) normalized rotor resistance referred to stator side
- \( r_s \) normalized stator resistance
- \( x_l \) normalized converter inductance
- \( x_s \) normalized stator inductance
- \( x_m \) normalized main inductance
- \( x_{s\sigma} \) normalized stator leakage inductance

**Matrices**

- \( A, A_p, A_s \) system matrices
- \( B, B_p, B_s \) system matrices
- \( I \) unity matrix
- \( K, K_k, K_n \) system matrices
- \( T \) complex transfer function matrix
- \( u \) input matrix
- \( V, V_k \) system matrices
- \( x \) matrix of state variables
- \( X \) system vector

**Space vectors**

- \( \vec{\varphi}_c \) ems vector induced by rotor flux \([V]\)
- \( \vec{\tau}_s, \vec{\tau}_r \) stator and rotor current space vectors \([A]\)
- \( \vec{u}_s \) space vector of stator voltage \([V]\)
- \( \Delta \vec{\varphi}, \Delta \vec{\tau} \) sum of flux and current harmonics \([Vs],[A]\)
- \( \vec{\varphi}_c \) calculated rotor flux \([Vs]\)
- \( \vec{\varphi}_m \) space vector of air gap linkage flux \([Vs]\)
- \( \vec{\varphi}_s, \vec{\varphi}_r \) stator and rotor space vector fluxes \([Vs]\)
- \( \vec{\varphi}_R \) rotor flux referred to stator side \([Vs]\)
### List of Symbols

#### Variables

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>$B$</td>
<td>magnetic induction</td>
<td>[T]</td>
</tr>
<tr>
<td>$d$</td>
<td>duty cycle</td>
<td></td>
</tr>
<tr>
<td>$e_x$, $e_y$</td>
<td>source voltage components in synchronous reference frame</td>
<td>[V]</td>
</tr>
<tr>
<td>$E$</td>
<td>power supply voltage</td>
<td>[V]</td>
</tr>
<tr>
<td>$F(\Theta,t)$</td>
<td>sheaving force</td>
<td>[Nm]</td>
</tr>
<tr>
<td>$f_{m}(\Theta,t)$</td>
<td>air gap MMF at point $\Theta$ and time $t$</td>
<td>[A]</td>
</tr>
<tr>
<td>$f_r$</td>
<td>rotor frequency</td>
<td>[Hz]</td>
</tr>
<tr>
<td>$f_s$</td>
<td>stator frequency</td>
<td>[Hz]</td>
</tr>
<tr>
<td>$f_{sw}$</td>
<td>switching frequency</td>
<td>[Hz]</td>
</tr>
<tr>
<td>$i_{sx}$, $i_{sy}$</td>
<td>stator current in synchronous reference frame</td>
<td>[A/m$^2$]</td>
</tr>
<tr>
<td>$i_{syc}$</td>
<td>calculated value of the torque current component</td>
<td></td>
</tr>
<tr>
<td>$J$</td>
<td>current density</td>
<td>[A/m$^2$]</td>
</tr>
<tr>
<td>$l_{g}^{-1}(\Theta,\rho)$</td>
<td>reciprocal air gap length function regarding slot harmonics</td>
<td>[l/m]</td>
</tr>
<tr>
<td>$n_r$</td>
<td>rotor speed</td>
<td>[s$^{-1}$]</td>
</tr>
<tr>
<td>$P_d$</td>
<td>dissipation snubber resistor $R_{sn}$</td>
<td>[W]</td>
</tr>
<tr>
<td>$pf$</td>
<td>power factor</td>
<td></td>
</tr>
<tr>
<td>$Q$</td>
<td>electrical charge of capacitor $C_{sn}$</td>
<td>[C]</td>
</tr>
<tr>
<td>$T_L$</td>
<td>load torque</td>
<td>[Nm]</td>
</tr>
<tr>
<td>$T_e$</td>
<td>electrical torque</td>
<td>[Nm]</td>
</tr>
<tr>
<td>$t$</td>
<td>time</td>
<td>[s]</td>
</tr>
<tr>
<td>$U$, $I$</td>
<td>average values of voltage and current</td>
<td>[V],[A]</td>
</tr>
<tr>
<td>$\hat{u}$, $\hat{i}$</td>
<td>maximum values of voltage and current</td>
<td>[V],[A]</td>
</tr>
<tr>
<td>$u_{a,b,c}$</td>
<td>output phase voltages of matrix converter</td>
<td>[V]</td>
</tr>
<tr>
<td>$u_c$</td>
<td>capacitor voltage</td>
<td>[V]</td>
</tr>
<tr>
<td>$u_{Cx}$, $u_{Cy}$</td>
<td>capacitor voltage components in synchronous reference frame</td>
<td>[V]</td>
</tr>
<tr>
<td>$u_s$</td>
<td>stator voltage</td>
<td>[V]</td>
</tr>
<tr>
<td>$W_L$</td>
<td>energy mechanical load</td>
<td>[J]</td>
</tr>
<tr>
<td>$W_m$</td>
<td>input energy</td>
<td>[J]</td>
</tr>
<tr>
<td>$w_{sn}(\Theta)$</td>
<td>winding function depending on electrical angle $\Theta$</td>
<td></td>
</tr>
</tbody>
</table>

#### Greek variables

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\delta$</td>
<td>effective air gap length</td>
<td>[m]</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>angle between stator current space vector and $x$ axis</td>
<td>[rad]</td>
</tr>
<tr>
<td>$\varepsilon$</td>
<td>error angle between real and calculated rotor vector flux</td>
<td>[rad]</td>
</tr>
<tr>
<td>$\zeta_{sh}$</td>
<td>stator slot factor</td>
<td></td>
</tr>
<tr>
<td>$\Theta$</td>
<td>angular displacement around stator periphery</td>
<td>[rad]</td>
</tr>
<tr>
<td>$\Theta_n$</td>
<td>position angle of stator winding $n$</td>
<td>[rad]</td>
</tr>
<tr>
<td>$\theta$</td>
<td>angle between stator voltage space vector and $\alpha$ axis</td>
<td>[rad]</td>
</tr>
</tbody>
</table>
List of Symbols

\[ \lambda \quad \text{coefficient} \]
\[ \Lambda \quad \text{parameter} \]
\[ \mu_0 \quad \text{magnetic permeability of air (vacuum)} \quad [\text{H/m}] \]
\[ \xi_{\text{sh}} \quad \text{winding factor} \]
\[ \nu \quad \text{angle between x axis and } \alpha \text{ axis} \quad [\text{rad}] \]
\[ \rho \quad \text{angular distance} \quad [\text{rad}] \]
\[ \sigma \quad \text{leakage coefficient} \]
\[ \zeta \quad \text{angle between stator voltage space vector and x axis} \quad [\text{rad}] \]
\[ \psi \quad \text{angle between arbitrary axis } m \text{ and stator axis } \alpha \quad [\text{rad}] \]
\[ \phi_{Rx}, \phi_{Ry} \quad \text{rotor flux components in synchronous reference frame } x-y \quad [\text{Wb}] \]
\[ \phi \quad \text{angle between } a \text{ axis and } \alpha \text{ axis} \quad [\text{rad}] \]
\[ \mathcal{Z} \quad \text{complex operator } \mathcal{Z} = e^{j2\pi/3} \]
\[ \omega_s \quad \text{stator angular frequency,} \quad [\text{rad/s}] \]
\[ \omega_2 \quad \text{slip frequency } \omega_2 = \omega_s - p \omega_m \quad [\text{rad/s}] \]
\[ \omega_m \quad \text{angular mechanical speed of rotor} \quad [\text{rad/s}] \]

Abbreviations

ac \quad \text{alternating current} \\
BPM \quad \text{brushless permanent magnet machine} \\
CSI \quad \text{current source inverter} \\
dc \quad \text{direct current} \\
IGBT \quad \text{insulated gate bipolar transistor} \\
MCT \quad \text{MOS controlled thyristor} \\
PWM \quad \text{pulse width modulation} \\
SRM \quad \text{switched reluctance motor} \\
VSI \quad \text{voltage source inverter} \\

Conventions

\[ + \quad \text{convention for the presentation of a voltage} \]
1. INTRODUCTION

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1. INTRODUCTION

1.1. Short history of electrical drives
Since the three-phase machine with the concept of the rotating magnetic field was developed, two basic designs have been made: the synchronous and the asynchronous machine. The rotating field machine gradually became the most significant work horse for industrial applications. The final proof of its importance was the winning of the technological race from its main counterpart, the direct-current (dc) machine. The operation of the contemporary induction machine drive equals or even exceeds performances of the dc drive. Unlike the dc machine for which the power level is limited by the commutation process, the power of the ac machine is virtually unlimited. The only difficulty is starting up of the machine connected directly to the power grid, when the stator currents can reach the value ten times higher than the rated value. To produce the rated torque during start up additional slip rings are required or a rotor with a double cage or a rotor with deep bars. However, in combination with the adjustable frequency converter the standard squirrel cage induction machine, when overloaded, generates a starting torque up to 400% of its rated value. Also in high-performance applications where the position and the speed have to be obtained very accurately (up to ±0.1% of the set-point), the induction machine is frequently in use. Additional superior performances attributable to the induction machine drive are: a controlled soft start, fast and smooth speed adjustment, limited inrush current at start-up, economical maintenance, safe operation in dangerous environments and a good specific power to weight ratio (W/kg).

For about one hundred years the original design of synchronous and asynchronous (induction) machines for fixed frequency feeding has dominated in industrial applications. Over the last decade, the initial concept has changed and ac machine technology is undergoing significant modifications. This is mainly attributed to the progress made in power electronics and the development of power semiconductors. Manufacturers now offer novel machine designs for all power ranges. In medium and high power applications, particular interest has been shown in the induction machine designed for inverter application with low rotor resistance and modified rotor slot geometry. This became possible with the development of power semiconductor devices over the last five years, such as:

- industrial grade IGBTs with ratings: 1700 V, 360 A,
- rugged GTOs with ratings: 8 kV and 4 kA,
- bipolar transistors with improved ratings: 1400 V, 600 A,
- power MOSFETs with ratings: 500 V, 60 A,
- MOSFET controlled thyristor (MCT) with ratings: 5 kV, 400 A.

In the last five years low-power applications brought expanded use of the brushless permanent magnet (BPM) machine, which is basically a synchronous machine. The BPM machine shows advantages over the induction machine in its higher efficiency and a simpler control. In addition, the use of special rotor magnetic materials (rare earth) enables them to be smaller. However, their considerable drawbacks are to be found in the limited power range due to the restricted size of the permanent magnet rotor,
position sensing for accurate phase commutation, magnet containment at high speeds and a quite expensive rotor manufacturing process.

Another two synchronous machine designs that do not seem to be mature enough for general-purpose industrial applications are the switched reluctance machine (SRM) with saliency on both the stator and rotor, and the synchronous reluctance machine with saliency on the rotor only. Both designs have an advantageous compact design compared to the induction machine: a better torque to inertia ratio and the possibility of high-speed operation up to 50000 s\(^{-1}\). However, they are both suited for inverter operation at low power applications. The SRM has as additional disadvantages the high torque ripple, significant machine noise and high rotor core losses.

The disadvantages of synchronous machine designs and the improvements in power converter topologies and novel control techniques, led to the extended industrial application of the induction machine based ac drives. This, in turn, enabled the design of a full range of converters with power ratings that extend up to several MW. Because of the low price of the induction machine, its simple design and advantageous features, it has taken the leading role in a wide power range of industrial applications such as traction, conveyors, printing presses, rolling mills, winders, lathes and other applications.

A large interest of the industry is presently focused on low-speed, all-purpose drives. Also in this application the ac machine has proved its competency. This study concentrates on low-speed drives, harmonic waveforms and analysis of unwanted effects appearing at low stator frequencies.

1.2. Induction machine-based low-speed drives

An ideal ac machine supplied with sinusoidal stator currents generates a constant torque. In special industrial low-speed (e.g. \(n_r=0.25\) s\(^{-1}\)) and low stator frequency applications (e.g. \(f_s=1\) Hz), where the ac machine is powered by a power converter, rippleless torque and the torque control are difficult to obtain. At rated speed applications this problem does not exist because the effect of the torque ripple is softened due to the high ripple frequency. In low-power, low-speed applications BPM and SRM machine drives have advantageous characteristics but their comparatively higher price and power limitation increase the need for other solutions. The induction machine is a good alternative, but the maturing technology still has to solve some problems like low-speed oscillations produced by the inherent light damping of the machine or produced by undesirable low stator frequency effects.

For the successful application of low-speed induction machine drives the control and the performance of the power converter are critical. Unlike the simple control of the dc machine, the control of the induction machine magnetic field and that of the electromagnetic torque are closely coupled. In the early seventies, Blaschke [1] and Hasse [2] proposed a new control method, field-oriented (vector) control, which basically solved this problem. This method proposed decoupled flux and torque control
by means of two orthogonal stator current components. The vector control introduced simple control algorithms for induction machine based drives, similar to the algorithms of dc drives.

The main problem for the vector control is to provide the information about the position and amplitude of the flux space vector in the machine. In time, variations of vector control appeared to overcome this problem. One approach is to sense the air gap flux with Hall sensors in the air gap or, simply, with sensing coils wound around stator pole pairs. Other methods are calculating flux by using a machine model and the measured slip frequency or other measurable quantities (stator currents and voltages). Some proposed methods have been successfully implemented in drives that have to achieve high precision in torque, speed or rotor position at all operating points. These, also called high-performance drives, are becoming the standard in high-tech applications, like robotics and precision machine tools where synchronous types of machines presently prevail. The present domination of synchronous machines in these high-tech applications is because their flux and torque control are inherently decoupled. For example, the BPM machine has the permanent magnet rotor that generates constant flux while the torque is controlled only by the stator current. In asynchronous machines, however, both the flux and torque have to be controlled by means of the stator current.

Beside the utilization of power converters, the low-speed operation of the induction machine may be also achieved by increasing the number of pole pairs and running it attached directly to the power grid. Thus, with the rated grid frequency, a fixed low-speed operation point can be realized. However, an increased number of pole pairs would enlarge the size of the machine and further, no speed or torque regulation would be possible. Therefore, the induction machine is suited for low-speed operation by making use of the electronic power converter.

The fast technological breakthrough in the vector control of induction machine drives is attributable to the extensive use of microprocessors. They have solved the problem of rendering the continuous coordination of the machine flux and torque parameters which are important in field-oriented control. Complex control algorithms have been introduced with machine parameter estimation, compensation and multiple feedback loops. Microprocessors have also enabled the implementation of shaft sensorless control techniques which utilize only the information about stator currents and voltages.

In practice, the induction machine drive, operating at low speeds and low frequencies, shows some weak points. Difficulties that arise are discussed in the following.

### 1.3. Power inverters

Power converters in their evolution have followed the development of power semiconductor devices. Important, recently developed power topologies have been designed to meet the following requirements:

- sinusoidal input and output waveforms,
- unrestricted output frequency variation,
Introduction

- unity input power factor,
- high voltage gain,
- bidirectional energy flow,
- short response time.

Conventional voltage source inverter
Most of the contemporary power ac-ac converters are based on PWM voltage source converters (VSI). This means a rectifier - dc capacitor - PWM controlled inverter topology. The dc voltage link usually contains also a dc inductor in addition to the dc capacitor to reduce ac line harmonics and improve the input power factor. Drives above 20 kW may use a half-controlled thyristor rectifier or a fully-controlled rectifier to limit the pre-charge currents of the dc link capacitor. Depending on the application, a VSI can operate in a voltage- or current-controlled mode.

Conventional current source inverter
The PWM current source inverter (CSI) has appeared as a successor to the autosequentially commutated CSI. It took advantage of the development of turn-off power devices operating at high switching frequencies. This inverter employs a dc link with an inductor which is a more reliable topology than the dc voltage link with electrolytic capacitors. With powerful GTO devices, the PWM CSI is used in drives rated above 500 kW. The concept of the CSI is also attractive because of its inherent four-quadrant nature. However, the recent breakthrough of high-speed turn-off devices gave the advantage to the VSI in the high-power applications as well.

Suppressed dc link rectifier-inverter topology
To eliminate cumbersome reactive elements in the dc link, there are some ideas of how to establish ac-ac conversion with a suppressed dc link. Ziogas proposed a rectifier-inverter topology with suppressed dc link components [13], [14]. Thus, the size of the topology is reduced and the system response is the fastest possible. In the standard topology with the capacitor bank, reactive energy that flows back from the inverter during regenerative load breaking, is normally stored in the dc link capacitor. Since the proposed topology with the suppressed dc link components uses the power grid as an alternate energy sink, its front-end has to provide a bidirectional power flow.

Matrix converter
The matrix converter [15], [16] is a direct frequency converter without a dc link and meets all mentioned requirements except one: the best possible voltage gain. Up to this moment, the maximum output voltage of 0.868 of the maximum input voltage is obtained by adding the third harmonic to the output voltage.

The major common drawback of both topologies (the matrix and the suppressed dc link) is the required number of semiconductor switches. Their successful applicability in the future depends on a breakthrough in the power semiconductor technology. This means the development of cost-effective, high-power four-quadrant switches.
Resonant converter
Series-resonant converters, including soft switching converter topologies, are another important power converter group. These converters feature negligible switching losses and enable ultrasonic operations. Finally, in comparison with the hard-switched converter, the $du/dt$ stress applied to the machine windings is reduced. Resonant converters meet all the mentioned requirements but, as a drawback, they have a power rating limitation because the energy has to be transferred through the resonant link.

New power devices (like bidirectional semiconductor switches) will certainly lead to novel power topologies. These will have advantageous characteristics in applications like low-speed drives. However, experimenting with such alternative topologies, which is the objective of this study, has already been made possible by the application of existing technology.

1.4. Problem definition of low-speed drives
Driving the induction machine at very low stator frequencies (a few percent of the rated frequency) is a very difficult task. There are two major problems:
- limitations imposed by the power converter attached to the induction machine,
- limitations imposed by the control concepts.

The first problem is related to the conventional rectifier - dc link - PWM inverter topology designed as the CSI (Fig.1.1) or the VSI (Fig.1.2).

Current source inverter
Current source inverters (Fig.1.1) with semiconductor switches that can be turned off (SCR with forced commutation, GTO, IGBT, BJT) have been in use particularly in high-power machine drive applications.

In this class of converter, the switch turns-off the currents flowing through the stator inductance of the connected machine. As a result, large absorbing commutation capacitors are necessary to suppress voltage spikes during the commutation. The worst problem, considering low stator frequency operation, is the presence of higher time
current harmonics in the stator windings of the machine. The torque ripple produces the rotor shaft cogging effect and reduces the performance of the drive. This happens especially at low speed when the frequency of the torque ripple is so low that the mechanical inertia of rotating masses is not able to smooth the amplitude of the ripple.

The CSI injects the current into the stator and stator voltages can easily exceed permitted values during transient states and damage the machine. Therefore, an ac drive based on this type of converter must utilize some kind of control with feedback.

Voltage source inverters
Voltage source inverters (Fig. 1.2) are today the most frequently applied representatives of the variable frequency drives family, especially in the low and medium power ranges. However, the following problems remain:

- the instantaneous output voltage contains "chopped" voltage blocks containing harmonic components. Although the stator current harmonics are damped by the machine leakage inductance, they are still troublesome for the torque at low stator frequencies,
- the output voltage waveform has high values of $du/dt$ that give rise to stresses on the machine insulation,
- increased harmonic losses.

![Fig.1.2 Voltage source inverter](image)

Oscillations produced by low stator frequency effects
Some unwanted effects appear at near zero stator frequencies, which are negligible at the rated frequency. The most significant effects are: voltage drop over semiconductor switches, dc offset in the system and system asymmetries. Their common feature is that they cause the torque ripple at low speeds, thus worsening the performance of an ac drive. CSI have less problems regarding these effects than VSI.

Instabilities of an ac machine caused by light damping
The steady state operation of an induction machine may become unstable even in situations when the machine is powered with a symmetrical, sinusoidal three-phase voltage supply. This is because an induction machine is typically lightly damped when operating at low stator frequencies, light loads and low relative rotor and stator
resistances [22]. This can result in motor instabilities during the steady state operation at low speeds.

Torque ripple
To reduce the undesirable torque ripple when using a CSI or a VSI at low speed, usually a high switching frequency is chosen. The purpose is to provoke torque harmonics with frequencies high enough to be damped by the inertia of the system. However, this gives rise to losses. Another possibility for improving the torque quality is the application of various PWM methods that eliminate specific stator current harmonics.

Some power topologies, like resonant-link topologies, have inherently the feature of generating only negligible harmonic components in addition to the fundamental stator current harmonic. This, however, has as a trade-off considerable control complexity and converter power rating limitations.

Control of low-speed drives
The second problem regarding low speed drives is their control. High-performance induction drives are currently based on field-oriented control that needs accurate information about the position of the flux space vector in the machine. This can be obtained in two basic ways:
1. indirect vector control method [2],
2. direct vector control method [1].

Indirect vector control
Fig.1.3 represents the block scheme of the first method, indirect vector control, defined in the rotor-flux-oriented reference frame. The control is applied to the current-controlled PWM inverter-fed induction machine.

![Fig.1.3 Indirect vector control in the rotor-flux-oriented reference frame](image)

This method, also known as flux-feedforward control, utilizes the monitored stator currents and the rotor speed to obtain the modulus and space angle of the flux-linkage
space vector. The space angle is obtained as the sum of the monitored rotor angle $\theta_m$ and the computed reference value of the slip angle $\theta_2$. This approach is very simple and has very good low-speed performance but is very sensitive to changes of the machine parameters, in particular the rotor resistance and the stator inductance. This is because the flux is not explicitly known and the flux orientation acts in an open loop. Besides, it requires the use of shaft encoders, which derate the mechanical robustness of the system and increase the cost of the drive.

Direct vector control

The second method, direct vector control, directly measures the the modulus and space angle of the flux-linkage (by using Hall-effect sensors, search coils or tapped stator windings). These values can be also calculated from a flux model as presented in Fig.1.4. The installation of Hall-effect sensors or sensing coils into the machine armature makes the direct vector control unattractive. An additional difficulty with search coils and tapped windings present low stator frequency applications because of the low induced ems.

![Diagram](image)

**Fig.1.4 Direct vector control in the rotor-flux-oriented reference frame**

This control principle is based on explicit flux information, and the field orientation is achieved by forcing the amplitude and the phase of machine input currents so that the required flux orientation is provided. Since the flux orientation is set up in the closed loop, the direct control is insensitive to machine parameter changes. The disability of being able to use standard, off the shelf machines is the prime obstacle to the wide use of this method.

Shaft sensorless control

Shaft sensorless control of ac drives is an interesting solution that employs the elements of the direct vector control. It utilizes the stator current and voltage measurements only and uses the machine model to estimate the rotor slip [3] or angular slip frequency [4]. The flux is also obtained from the machine model. It is an attractive solution since no shaft encoder is deployed. This means high operation reliability and, in commercial
terms, a lower drive price. As a trade-off, the method is sensitive to machine parameter changes and has serious problems while controlling the drive at low speed.

At low stator frequencies (\(\omega_s=0\)), the resistive component of the stator impedance is one order of magnitude larger than the inductive component. The low stator voltage, the rms value of which can be of the same order as the noise, is therefore hard to detect. Further, the calculation of the flux requires the numerical integration of the induced back emf that has both low frequency and low amplitude. This is a problem due to the drift of operational amplifiers in the integrator itself or in A/D converters. Because of the drift, integrators accumulate error and can cause the entire control to become unstable.

The problems regarding highly inaccurate control of low speed drives are not uniformly distributed throughout the complete power range of drives. Namely, driving high-power machines at low stator frequencies is much more difficult because their rated slip can be even less than 1 %. Small machines, with a rated slip around 15 %, have fewer problems at near-zero stator frequencies.

1.5. Review of PWM and vector control techniques
This section provides a review of what is available today in solving the stated difficulties indicated for ac drives. The major interest is focused on low-speed drives powered by a rectifier-inverter bridge converter and, in particular, various harmonic elimination techniques are presented. Also an overview of recent field-oriented control principles based on stator current and voltage measurements is presented.

Harmonic elimination
The solution of the torque ripple problem currently focuses on the application of PWM techniques that eliminate or mitigate higher stator current harmonics. A very popular group of PWM techniques is that which involves the simple direct comparison of various waveform "modulating" signals with a triangular "carrier" to produce the PWM switching patterns [5], [6]. The instantaneous intersection of these two signals detects the PWM switching instants by a process of "naturally sampled" PWM. Beside the sinusoidal modulating signal, also a trapezoidal signal can be used [9], [10].

Another group of PWM techniques is based on precalculated switching instants. These methods optimize a particular objective function such as to obtain minimum losses [11] or reduced torque pulsations [12]. Patel and Hoft have developed numerical techniques [7], [8] for calculating switching angles in the PWM process for eliminating higher harmonics. These techniques are suitable for microprocessor implementation and offer high-performance results at rated speeds. The lower stator frequency range has problems due to the large number of switching instants that have to be precalculated [23].

Precalculated methods exhibit several distinct advantages over the carrier-modulated methods:
- reduction of the switching frequency by 50% compared to the standard carrier-modulated PWM scheme,
- higher voltage gain because of obtainable overmodulation,
- lower losses due to the reduction of the switching frequency,
- small ripple in the dc link current,
- elimination of lower-order harmonics avoiding harmonic interference like resonance with external line filtering networks.

Field-oriented control
To tackle problems related to the low-speed control of the ac drive, an optimal solution was found in the use of control methods based on speed/position sensors attached to the machine rotor shaft. These methods provide excellent static and dynamic control behaviour. However, the shaft encoder remains a costly component of the drive, and further, in some industrial applications, the use of a mechanical sensor is either not possible or not desirable. For example, a polluted environment (chemical industry), or excessive vibrations (traction), do not allow the use of fragile shaft sensors and cabling to the control circuitry. Therefore, in last two decades there has been a steady growth of interest in shaft sensorless control techniques. Further, some slip estimating techniques based only on stator current and voltage information are mentioned here.

The technique derived by Abbondanti [17] in 1975 was suited for an analog slip estimating circuit with the machine variables given in the synchronous reference system. Baader's technique [18] maintains air gap flux control directly through commands that control the inverter state. The estimated machine torque and an estimated error signal are used to estimate the slip. Beck's technique [19] is based on the steady state equivalent circuit model but does not assume field-oriented control. It obtains the rotor slip from a theoretical relationship between the slip and the phase delay of the stator current compared with the voltage. It suffers from numerical problems and is useless with large slips. Joetten's technique [20] is based on a steady state model of the induction machine. The slip estimate is achieved by calculating the back electromotive force induced by the rotor flux. Flux is calculated from the integration of the electromotive force. Problems appear when the machine is running at low stator frequencies in a noisy environment and are caused by the difficulties with the stator voltage measurement and the integration process. Ohtani proposed a method [21] similar to Joetten's. It uses lagging compensation circuits for the flux calculation. Thus, the estimate of the flux and the torque control is possible even when the stator frequency is zero (ω_s=0).

1.6. Objective of this study
There are still many remaining problems related to low-speed induction machine drives like the induction machine torque ripple at low stator frequencies and limitations of the vector control. The objective of this study is twofold: to introduce a novel converter topology and to implement the shaft and voltage sensorless vector control, both suited for low-speed applications.

The proposed converter topology consists of a three-phase boost-buck converter
integrated with a matrix converter. It has the following features:
• the capability to inject stator currents that contain, beside the fundamental harmonic, only the switching frequency harmonic resulting in a negligible torque ripple,
• four-quadrant operation of the converter,
• voltage step-up as well as step-down operation,
• optional input power factor control,
• easy accomplishment of shaft and voltage sensorless vector control,
• ac and dc operation of the input and output part of the converter circuit.

Fig.1.5 represents the proposed block scheme of the vector control.

![Block scheme of the novel vector control in the synchronous reference frame]

The method is based on rotor slip estimation and has the following advantages:
• only information about stator currents is needed,
• reliable torque control during low-speed operation and even at zero stator frequency,
• robustness regarding machine parameter variations,
• avoidance of extra microprocessor power.

References


Introduction

1972, pp.60-66.


Introduction

pp.739-748.


2.

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2. INDUCTION MACHINE

2.1. Introduction
A range of spatial and time harmonics exists in the physical induction machine which affects its operation. Their influence is predominantly negative, causing the machine performance to deteriorate by increasing losses, accelerating the aging process of materials of windings and insulation and generating torque ripple. The main interest is directed at the influence of harmonics on the electromagnetic torque. Also, a suitable two-phase model equivalent to the three-phase symmetrical induction machine is presented which considers only the existence of the fundamental time and spatial harmonic.

By introducing space vectors in the theory of electrical machines, the electrical torque of the induction machine is presented as the cross-product of various rotor and stator space vectors. However, in the following discussion, the relation between the rotor flux and stator current defined in (2.1) is used.

\[ \vec{T}_e = k q \vec{\Phi}_r \times \vec{i}_s \]  \hspace{1cm} (2.1)

Used symbols are:
- \( \vec{\Phi}_r \) - rotor flux space vector,
- \( \vec{i}_s \) - stator current space vector,
- \( k_q \) - constant,
- \( \vec{T}_e \) - the vector of electromagnetic torque commonly expressed as a scalar \( T_e \).

The idealized induction machine is characterized by:
- symmetrical machine with a smooth air gap (without stator and rotor slots),
- sinusoidal stator and rotor winding distribution,
- cylindrical rotor,
- linear magnetizing characteristic,
- no skin effect,
- no iron losses.

The operation of such a machine, when attached to a symmetrical and sinusoidal voltage or current source, is described by a straightforward mathematical model. Under these circumstances the produced torque is constant. This is an adequate approximation for most applications where rotation speed is close to the nominal speed and the inertia of rotating masses is large.

However, in a real induction machine, a multitude of effects exist caused by nonideal mechanical and geometrical characteristics on one hand and nonlinear magnetizing effects on the other hand. These effects generate so-called space harmonics.

Time harmonics are introduced by power converters, commonly used as variable frequency power supplies. The method of modulation for the stator voltage or the stator
current produces, beside the fundamental time harmonic, additional time harmonics in the machine.

Since harmonic effects on the torque are emphasized at low speed applications and might cause problems, it is useful to give a short analysis of both spatial and time harmonics. In addition, each harmonic group causes its own torque ripple, and the problem becomes even more difficult when the interrelation between various harmonic groups is to be studied. The last problem, however, exceeds the context of this study and is not further discussed.

2.2. Space and time harmonics

When considering the generalized squirrel cage machine with sinusoidal stator currents, two types of space harmonics are distinguished.

The first type is associated with harmonics that rotate at subsynchronous speeds with respect to the speed of the fundamental magnetomotive force (MMF) harmonic. These MMF harmonics are generated by two main factors:
- nonsinusoidal distribution of the machine windings [1],
- permeance variations caused by the slot openings [3].

The second type of MMF harmonics is caused by the saturation, in the machine [2]. This type is characterized by the third being the dominant harmonic component and which has the same synchronous speed and the same direction of the rotation as the fundamental air gap harmonic.

2.2.1. Space harmonics

It is known that the stator and rotor magnetomotive forces (MMF) of the induction machine show the presence of odd and sometimes even spatial harmonics [8], [9]. These harmonics are generated by the nonsinusoidal distribution of stator and rotor windings. Further discussion is valid under the following assumptions:
- uniform air gap,
- no saturation.

Fig. 2.1 gives a graphical representation of a typical nonsinusoidal winding function over one pole pair of the machine. The harmonic contribution of this function can be studied by following the theory of Schmitz and Novotny [4]. It is assumed that MMF space harmonics caused by the rotor are negligible because of the nearly uniform distribution of rotor bars along the rotor circumference. However, higher odd harmonics of the
stator MMF have to be taken into account because of the highly nonsinusoidal distribution of the stator windings.

For each of \( n \) stator phases, the winding function \( w_{sn}(\Theta) \) of the nonsinusoidally distributed stator windings can be defined as a number of conductors in the winding which is enclosed in an rotating contour. The Fourier expansion of the function is:

\[
w_{sn}(\Theta) = \sum_{h=1}^{\infty} w_{snh} \cos[h(\Theta - \Theta_n)]
\]

(2.2)

It can be shown that the self inductance \( L_{sm} \) of each stator winding of \( n \) system phases and the mutual inductance between two stator windings \( M_{sm} \) are described by:

\[
L_{smn} = \frac{\mu_0 r }{2\delta} \int_0^{l_\pi} \left[ w_{sn}(\Theta) \right]^2 d\Theta dz
\]

(2.3)

\[
M_{smn} = \frac{\mu_0 r }{2\delta} \int_0^{l_\pi} [w_{sn}(\Theta) w_{s(n\pm1)}(\Theta)] d\Theta dz
\]

(2.4)

In (2.2), (2.3) and (2.4) the following symbols are used:

- \( h \) - harmonic order,
- \( n \) - index denoting stator windings \( (n=1,2,3) \),
- \( w_{snh} \) - turns for the \( h \)-th harmonic of the winding \( n \),
- \( w_{sn}(\Theta) \) - winding function dependent on the electrical angle,
- \( \Theta \) - angular displacement around the stator or the air gap periphery,
- \( \Theta_n \) - angular position of the heart of the stator winding \( n \),
- \( \delta \) - effective air gap length [m],
- \( r \) - rotor radius [m],
- \( \mu_0 \) - magnetic permeability of the air (vacuum),
- \( l \) - lamination stack length [m],
- \( z \) - axial position.

The expression (2.2) shows that each harmonic component is independent of all other harmonics. It can be assumed that an idealized squirrel cage machine responds independently to each individual MMF harmonic component as produced by the stator. This "harmonic model" of the machine influences the torque by adding to it ripple components which at low-speeds cause the rotor cogging.

2.2.2. Slot harmonics

The existence of slots in rotating induction machines results in a variable air gap length. Fig.2.2 gives a symbolic presentation of the stator and rotor slot distribution and the resulting air gap.
Stator and rotor slots are the cause of a variable air gap flux density that generates MMF harmonics [3], [10]. The reciprocal air gap length function is defined by a Fourier series with two variables:

$$l_g^{-1}(\Theta, \rho) = \sum_{i=-\infty}^{\infty} \sum_{j=-\infty}^{\infty} \left( F_{ij} \cos(i\Theta+j\rho) + D_{ij} \sin(i\Theta+j\rho) \right)$$  \hspace{0.5cm} (2.5)

and is related to the magnetic induction $B(\Theta, \rho, t)$:

$$B(\Theta, \rho, t) = \frac{\mu_0 f(\Theta, t)}{l_g(\Theta, \rho)}$$  \hspace{0.5cm} (2.6)

In Fig.2.2, equations (2.5) and (2.6) the following variables and constants are introduced:

$\rho$ - angular distance from the reference coordinate on the rotor,

$B(\Theta, \rho, t)$ - maximum value of the magnetic induction (radial component),

$f(\Theta, t)$ - air gap MMF that influences the stator at point $\Theta$ and time $t$,

$l_g(\Theta, \rho)$ - air gap length function regarding slot harmonics,

$i, j$ - integers,

$F_{ij}, D_{ij}$ - coefficients.

If the permeance function is found the magnetic induction function can be computed by means of term-by-term expansion of equation (2.6). Consequently, the voltage induced in the stator windings and its harmonic content can be calculated. Again, harmonic components in the machine affect its torque by creating a torque ripple.

2.2.3. Saturation Harmonics

Saturation in the machine [2] is a nonlinear process that mainly takes place in the stator
and rotor teeth. The result is a decrement of the magnetic path permeability (increase of the reluctance). As the stator and rotor teeth become saturated, the resulting sinusoidal air gap induction distribution is flattened. Fig. 2.3 represents the fundamental, the third harmonic component and the resulting waveform of the air gap induction distribution $B(\Theta)$.

![Diagram of air gap induction distribution](image)

*Fig. 2.3 Distribution of the air gap induction and its harmonic components for saturated iron*

In this manner saturation generates synchronous odd space harmonics, including triplets. The third harmonic component is the dominant one. It is important to note that the third harmonic component of the air gap flux has always a constant zero phase relationship with respect to the fundamental flux component. If star connection of stator phases is considered, zero-sequence currents are not induced in the stator windings. However, the third harmonic air gap induction induces currents in the rotor windings which, as a consequence, generate a spatial sheaving force $F(\Theta,t)$ ripple. Therefore, the total electromagnetic sheaving force contains of two main components. The first one is the electrical torque $T_{e1}$ which is constant in time and is produced by the interaction between the fundamental component of the stator and rotor current.

$$T_{e1} = G_1(i_{s\beta}i_{r\alpha} - i_{s\alpha}i_{r\beta}) \quad (2.7)$$

The second component is the sheaving force $F(\Theta,t)$ which is a result of the interaction between the fundamental stator current wave component and the third order rotor current component.

$$F(\Theta,t) = G_3(i_{s\beta}i_{r\alpha3} - i_{s\alpha}i_{r\beta3}) \quad (2.8)$$

It is important to note that the average of the periphery force defined in (2.8) around the total periphery of the rotor is zero. This spatial ripple can well be the reason of noise produced by excitation of rotor teeth.
Expressions (2.7) and (2.8) are given in the stator reference frame \( \alpha-\beta \). Coefficients \( G_1 \) and \( G_3 \) are dependent on the level of the saturation; as the saturation level increases the factor \( G_1 \) decreases, reducing the torque component \( T_{e1} \). The factor \( G_3 \), on the contrary, increases its value thus giving a rise to the force component \( F(\Theta,t) \).

The usual method for modeling the saturation effect is to modify the air gap length as a function of the air gap flux position and its amplitude. The assumptions are that the permeability of the iron is constant and infinite.

### 2.2.4. Time harmonics

Beside the spatial harmonics which are a function of the position along the airgap of the induction machine, the time harmonics are an important group [8], [9]. The latter group is mainly generated by the power converter connected to the induction machine and as a consequence of the iron saturation in the machine. In the following discussion it is assumed that the characteristic of the output terminals of the converter is described by the characteristic of a voltage source.

The voltage imposed by the power converter contains normally a rich content of higher harmonics. When Fourier analysis is applied to the impressed stator voltage \( u_s \) the following expansion is found:

\[
u_s(t) = \sqrt{2} \left[ \hat{u}_{s1} \sin \omega_s t + \hat{u}_{s5} \sin 5\omega_s t + \hat{u}_{s7} \sin 7\omega_s t + \ldots + \hat{u}_{s(i6\pm1)} \sin (i6\pm1) \omega_s t \right]
\]  

(2.9)

In a three-phase symmetrical system the fundamental voltage component and \((i6+1)\) harmonics \((i \) is positive integer\) rotate in one direction while \((i6-1)\) harmonics rotate in the opposite direction. This results in the sixth harmonic and its multiples for the torque. Fig.2.4 represents a typical stator phase voltage, as modulated by a power converter, composed from a series of pulses with variable width. Similar to the space harmonic machine model, the nonsinusoidal excitation of the induction motor can be modeled by sinusoidal generators connected in series, each representing one term of the expansion (2.9).

Because of the symmetry, only odd voltage harmonics exist, except for the triplet harmonic components. Corresponding stator current harmonics are mainly softened by the stator leakage inductance. The magnetizing current (flux in the machine) can be assumed to be sinusoidal because of the large magnetizing time constant. The produced electro-magnetical torque is presented in terms of the total air gap linkage flux as:

---

**Fig.2.4 Pulse width modulation**
\[
\vec{T}_e = \frac{3}{2} p \left( \vec{\Phi}_m + \Delta \vec{\Phi}_m \right) \times \left( \vec{i}_s + \Delta \vec{i}_s \right) \tag{2.10}
\]

In equation (2.10) \( \vec{\Phi}_m \) and \( \vec{i}_s \) are space vectors for the linkage flux and the stator current and \( \Delta \vec{\Phi}_m \) and \( \Delta \vec{i}_s \) are the sums of the harmonic components of the linkage flux and stator currents respectively.

\[
\Delta \vec{\Phi}_m = \sum_{h>1} \vec{\Phi}_{mh} \tag{2.11}
\]
\[
\Delta \vec{i}_s = \sum_{h>1} \vec{i}_{sh} \tag{2.12}
\]

Pulsating torques are generated from the interaction between all the different flux and current harmonics (terms \( \Delta \vec{\Phi}_m \times \Delta \vec{i}_s \) are neglected) and have frequencies of multiples of six of the inverter output frequency \( \omega_s \):

\[
\Delta \vec{T}_e = \frac{3}{2} p \left[ \left( \vec{\Phi}_m \times \Delta \vec{i}_s \right) + \left( \Delta \vec{\Phi}_m \times \vec{i}_s \right) \right] \tag{2.13}
\]

For the conventional PWM voltage source inverter the sixth harmonic component of the electrical torque \( T_e \) is the most remarkable component.

-Example

Fig.2.5 presents the simulation of the induction machine connected to a nonsinusoidal voltage source and operating at low stator frequency \( f_s = 5 \) Hz.

Fig.2.5 a) the sixth harmonic component in the torque produced by \((5+1)\) and \((7-1)\) current harmonic currents; b) stator voltage \( u_s(t) \) containing 1st, 5th (rotates in opposite direction) and 7th harmonic components (stator frequency \( f_s = 5 \) Hz); c) resulting stator current \( i_s(t) \) with the same harmonic components.
The source includes only the first three terms of expression (2.9) at a low fundamental stator frequency \( f_s = 5 \text{ Hz} \). The harmonic amplitudes are \( \hat{u}_{s5} = \hat{u}_{s1}/5 \) and \( \hat{u}_{s7} = \hat{u}_{s1}/7 \).

When considering a standard bridge inverter topology, much attention has been paid to PWM methods for the reduction of higher harmonics in the machine. In particular, much effort is made to develop switching patterns that generate a low harmonic content in the stator voltage. The basic limitation for the application of sophisticated patterns is the required high converter switching frequency that limits the power ratings.

### 2.3. Three-phase machine model

Machine models that take into account the separate influence of space, slot, saturation and time harmonics have already been developed. The creation of the compound machine model that deals with all the mentioned effects is a very difficult task and is not further considered in this study. Some results in this area have been published [5], [6] and [7] they reveal the complexity of the problem.

Induction machine analysis in the following assumes a mathematical model of the three-phase machine that handles only the fundamental time harmonic. Fig.2.6 shows the machine schematically with three-phase inductors symbolizing stator and rotor windings.

![Fig.2.6 Schematic of the induction machine for the fundamental harmonic](image)

The following assumptions are valid for this model:
- symmetrical three-phase stator and rotor windings,
• the influence of mutual slotting is accounted for,
• saturation is accounted by an additional increase of the air gap,
• nonsinusoidal distribution of stator and rotor windings,
• skin effect is irrelevant because of the low fundamental frequency assumptions,
• iron losses are disregarded,
• the rotor is placed concentrically in the stator bore,
• symmetrical and sinusoidal voltage source.

The stator and rotor voltage equations in matrix form are:

\[
\begin{align*}
\mathbf{u}_{sabc} &= [R_{sabc}] \mathbf{i}_{sabc} + \frac{d\mathbf{\phi}_{sabc}}{dt} \quad (2.14) \\
\mathbf{u}_{rabc} &= [R_{rabc}] \mathbf{i}_{rabc} + \frac{d\mathbf{\phi}_{rabc}}{dt} \quad (2.15)
\end{align*}
\]

In equations (2.14) and (2.15) voltages, currents and fluxes of the stator are:

\[
\begin{align*}
\mathbf{u}_{sabc} &= [u_{sa} \ u_{sb} \ u_{sc}]^T \\
\mathbf{i}_{sabc} &= [i_{sa} \ i_{sb} \ i_{sc}]^T \\
\mathbf{\phi}_{sabc} &= [\phi_{sa} \ \phi_{sb} \ \phi_{sc}]^T
\end{align*}
\]

The rotor variables are presented as:

\[
\begin{align*}
\mathbf{u}_{rabc} &= [u_{ra} \ u_{rb} \ u_{rc}]^T \\
\mathbf{i}_{rabc} &= [i_{ra} \ i_{rb} \ i_{rc}]^T \\
\mathbf{\phi}_{rabc} &= [\phi_{ra} \ \phi_{rb} \ \phi_{rc}]^T
\end{align*}
\]

Matrices \([R_{sabc}]\) and \([R_{rabc}]\) contain the stator and rotor resistances:

\[
[R_{sabc}] = \begin{bmatrix}
R_s & 0 & 0 \\
0 & R_s & 0 \\
0 & 0 & R_s
\end{bmatrix} \quad \quad \quad
[R_{rabc}] = \begin{bmatrix}
R_r & 0 & 0 \\
0 & R_r & 0 \\
0 & 0 & R_r
\end{bmatrix}
\]

(2.18)

The fluxes through each stator and rotor winding are separated into the main, leakage and two mutual fluxes. Considering only stator fluxes, each flux is related to the corresponding inductance such as:

- the stator and rotor main inductance \(L_{sm}\) and \(L_{rm}\):

\[
L_{sm} = \frac{\mu_o 2 w_s^2 D l_w}{\pi p^2 \delta} \sum_h \frac{\zeta_{sh}^2}{h^2} \zeta_{sh} \quad (2.19)
\]

\[
L_{rm} = \frac{\mu_o 2 w_r^2 D l_w}{\pi p^2 \delta} \sum_h \frac{\zeta_{rh}^2}{h^2} \zeta_{rh} \quad (2.20)
\]
- mutual inductance between stator windings $M_{sm}$:

$$M_{sm} = \mu_o \frac{2 w_s^2 D l_w}{\pi p^2 \delta} \sum_h \frac{\xi_{sh}}{h^2} \xi_{srh} \cos \left( \frac{2}{3} h \pi \right)$$  \hspace{1cm} (2.21)

- maximum mutual inductance between stator and rotor windings $\hat{M}_{abch}$:

$$\hat{M}_{abch} = \hat{M}_{ch} = \mu_o \frac{2 w_s w_r D l_w}{\pi p^2 \delta} \xi_{sh} \xi_{srh}$$  \hspace{1cm} (2.22)

The total maximum mutual inductance between stator and rotor windings $\sum \hat{M}_{abch}$ can be presented with the aid of inductance $M_{sm}$:

$$\sum_h \hat{M}_{abch} = \sum_h \hat{M}_{ch} = M_{sm} \sum_h \frac{w_r}{w_s} \frac{\xi_{srh}}{\xi_{sh} \xi_{srh} h^2 \cos \left( \frac{2}{3} h \pi \right)}$$  \hspace{1cm} (2.23)

The same definition as (2.21) is valid for rotor parameter $M_{rm}$.

In equations (2.19) to (2.23) the following symbols are used:

- $h$ - harmonic order (1,3,5,7,...),
- $k_h$ - constant dependent on harmonic order,
- $\xi_{sh}$, $\xi_{srh}$ - stator and rotor winding factors,
- $\xi_{sh}$, $\xi_{srh}$ - stator and rotor slotting factors,
- $\delta$ - effective air gap,
- $l_w$ - effective length of the winding,
- $D$ - diameter of the stator bore,
- $w_s$, $w_r$ - number of stator and rotor windings.

Finally, the total stator and rotor flux are defined with the inductance matrices as:

$$\Phi_{sabc} = \left\{ [L_{smabc}] + [L_{s\sigma abc}] + [\Delta L_{sabc}] \right\} i_{sabc} + [M_{abc}] i_{rabc}$$  \hspace{1cm} (2.24)

$$\Phi_{rabc} = [M_{abc}]^T i_{sabc} + \left\{ [L_{rmabc}] + [L_{r\sigma abc}] + [\Delta L_{rabc}] \right\} i_{rabc}$$  \hspace{1cm} (2.25)

where:

- $[L_{smabc}]$ - matrix of stator magnetizing inductances,
- $[L_{s\sigma abc}]$ - matrix of stator leakage inductances,
- $[\Delta L_{sabc}]$ - pulsating part caused by the stator and rotor slotting,
- $[L_{rmabc}]$ - matrix of rotor magnetizing inductances,
- $[L_{r\sigma abc}]$ - matrix of rotor leakage inductances,
- $[\Delta L_{rabc}]$ - pulsating part caused by the stator and rotor slotting.
If the pulsating part is assumed to be negligible, the matrix of the stator self inductance is:

\[
\begin{bmatrix}
L_{sm} & M_{sm} & M_{sm} \\
M_{sm} & L_{sm} & M_{sm} \\
M_{sm} & M_{sm} & L_{sm}
\end{bmatrix} + \begin{bmatrix}
L_{s\sigma} & M_{s\sigma} & M_{s\sigma} \\
M_{s\sigma} & L_{s\sigma} & M_{s\sigma} \\
M_{s\sigma} & M_{s\sigma} & L_{s\sigma}
\end{bmatrix}
\]

(2.26)

and for the rotor:

\[
\begin{bmatrix}
L_{rm} & M_{rm} & M_{rm} \\
M_{rm} & L_{rm} & M_{rm} \\
M_{rm} & M_{rm} & L_{rm}
\end{bmatrix} + \begin{bmatrix}
L_{r\sigma} & M_{r\sigma} & M_{r\sigma} \\
M_{r\sigma} & L_{r\sigma} & M_{r\sigma} \\
M_{r\sigma} & M_{r\sigma} & L_{r\sigma}
\end{bmatrix}
\]

(2.27)

For a three-phase symmetrical machine the mutual inductance matrix \([M_{abc}]\) in (2.24) and (2.25) is defined as:

\[
[M_{abc}] = \begin{bmatrix}
\frac{2}{3} \dot{M} \cos(\tilde{\theta}) & \frac{2}{3} \dot{M} \cos(\tilde{\theta} + \frac{2}{3} \pi) & \frac{2}{3} \dot{M} \cos(\tilde{\theta} - \frac{2}{3} \pi) \\
\frac{2}{3} \dot{M} \cos(\tilde{\theta} - \frac{2}{3} \pi) & \frac{2}{3} \dot{M} \cos(\tilde{\theta}) & \frac{2}{3} \dot{M} \cos(\tilde{\theta} + \frac{2}{3} \pi) \\
\frac{2}{3} \dot{M} \cos(\tilde{\theta} + \frac{2}{3} \pi) & \frac{2}{3} \dot{M} \cos(\tilde{\theta} - \frac{2}{3} \pi) & \frac{2}{3} \dot{M} \cos(\tilde{\theta})
\end{bmatrix}
\]

(2.28)

where:

\[
\dot{M}^2 = L_{sm} L_{rm}
\] - mutual induction when the magnetic coupling is ideal,

\[
\tilde{\theta}
\] - mechanical angle between the rotor and the stator axis.

By using the flux-linkage components the expression for the torque \(T_e\) is now evaluated as:

\[
T_e = \frac{1}{2} \begin{bmatrix} i_{sabc}^T \\ i_{rabc}^T \end{bmatrix} \cdot \frac{d}{d\tilde{\theta}} \begin{bmatrix}
[L_{s\sigma abc}]+[L_{s\sigma abc}]+[\Delta L_{s\sigma abc}] & [M_{abc}] \\
[M_{abc}] & [L_{rmabc}]+[L_{r\sigma abc}]+[\Delta L_{rabc}]
\end{bmatrix} \begin{bmatrix} i_{sabc} \\ i_{rabc} \end{bmatrix}
\]

(2.29)

From the described mathematical model with included slot harmonics, the induction machine is presented by a set of machines.

**2.4. Two-phase machine model at low stator frequency**

Fig.2.7 presents some useful coordinate systems to which the machine model in this analysis can be referred.
In Fig. 2.7 the introduced indexation has the following meaning:

- \( \alpha - \beta \) reference frame fixed to the stator and consequently standing still,
- \( a - b \) reference frame fixed to the rotor rotating with speed \( \omega_m \),
- \( m - n \) arbitrary reference frame rotating with arbitrary angular speed \( \omega_k \),
- \( x - y \) reference frame fixed to the rotor flux rotating at speed \( \omega_s \),
- \( \beta \) angle between \( a \) axis of the rotor reference frame and stator \( \alpha \) axis,
- \( \nu \) angle between \( m \) axis of the arbitrary reference frame and stator \( \alpha \) axis,
- \( \nu \) angle between \( x \) axis of the synchronous frame and stator \( \alpha \) axis.

Since the mathematical analysis using the three-phase machine model is unnecessarily complicated, the two-phase dynamical equivalent model of the induction machine is commonly used. This means a significant simplification of the three-phase model with variable parameters as defined in section 2.3. Further simplification assumed in this application applies to a two-phase model with constant parameters. In the following elaboration, system transformations that start with the three-phase model and conclude with the two-phase model are assumed to be known.

### 2.4.1. Symmetrical T-scheme

The equations of the three-phase model are transformed into equations of a two-phase model and suitable normalization is done. The model is defined in an arbitrarily reference frame \( m-n \) which allows simple switching between other reference frames \( m-n=x-y, m-n=\alpha-\beta \) in the following chapters. The two-phase set of equations follows:

\[
\begin{bmatrix}
    u_{sm} \\
    u_{sn}
\end{bmatrix} = R_s \begin{bmatrix}
    i_{sm} \\
    i_{sn}
\end{bmatrix} + \frac{d}{dt} \begin{bmatrix}
    \phi_{sm} \\
    \phi_{sn}
\end{bmatrix} + \omega_k \begin{bmatrix}
    0 & -1 \\
    1 & 0
\end{bmatrix} \begin{bmatrix}
    \phi_{sm} \\
    \phi_{sn}
\end{bmatrix}
\]  

(2.30)

\[
\begin{bmatrix}
    u'_{Rm} \\
    u'_{Rn}
\end{bmatrix} = R'_R \begin{bmatrix}
    i'_{Rm} \\
    i'_{Rn}
\end{bmatrix} + \frac{d}{dt} \begin{bmatrix}
    \phi'_{Rm} \\
    \phi'_{Rn}
\end{bmatrix} + (\omega_k - p\omega_m) \begin{bmatrix}
    0 & -1 \\
    1 & 0
\end{bmatrix} \begin{bmatrix}
    \phi'_{Rm} \\
    \phi'_{Rn}
\end{bmatrix}
\]  

(2.31)
\[
\begin{align*}
\begin{bmatrix}
\phi_{sm} \\
\phi_{sn}
\end{bmatrix} &= L_{s\sigma} \begin{bmatrix} i_{sm} \\
i_{sn} \end{bmatrix} + L_m \begin{bmatrix} i_{sm} + i_{Rm}' \\
i_{sn} + i_{Rn}' \end{bmatrix} \\
\begin{bmatrix}
\phi_{Rm}' \\
\phi_{Rn}'
\end{bmatrix} &= L_m \begin{bmatrix} i_{sm} + i_{Rm}' \\
i_{sn} + i_{Rn}' \end{bmatrix} + L_{R\sigma} \begin{bmatrix} i_{Rm}' \\
i_{Rn}' \end{bmatrix}
\end{align*}
\]

(2.32) (2.33)

The electrical torque is presented as:

\[
T_e = \frac{3}{2} p L_m \begin{bmatrix} i_{sm} & i_{sn} \end{bmatrix} \begin{bmatrix} 0 & -1 \\
1 & 0 \end{bmatrix} \begin{bmatrix} i_{Rm}' \\
i_{Rn}' \end{bmatrix}
\]

(2.34)

where:

\begin{itemize}
  \item \(\phi_{sm}, \phi_{sn}, \phi_{Rm}', \phi_{Rn}'\) - stator and transformed rotor flux components,
  \item \(i_{sm}, i_{sn}, i_{Rm}', i_{Rn}'\) - stator and transformed rotor space vector currents,
  \item \(R_s, R_R'\) - stator and rotor resistance transformed to the stator side,
  \item \(L_s = L_m + L_{s\sigma}\) - stator inductance,
  \item \(L_R' = L_m + L_{R\sigma}\) - rotor inductance transformed to the stator side,
  \item \(\omega_m = d\phi/dt\) - mechanical angular rotor speed (in space rad.),
  \item \(\omega_k = dv/dt\) - angular speed of the arbitrary reference frame (in elec. rad.).
\end{itemize}

Fig.2.8 presents the two-phase symmetrical T-scheme of the induction machine based on equations (2.30) to (2.33). The rotor parameters are transformed to the stator side.

---

Fig.2.8 Equivalent symmetrical T scheme of the induction machine
From the model shown in Fig. 2.8 and presented in an arbitrary reference frame, the symmetrical T-scheme, which is commonly in use, is easily derived. By selecting the synchronous reference frame \( x-y (\omega_k=\omega_s) \) and assuming steady state operation of the machine with short circuited rotor circuit, the following equations are obtained from (2.30) and (2.31):

\[
\begin{bmatrix}
u_{sx} \\
u_{sy}
\end{bmatrix} = R_s \begin{bmatrix} i_{sx} \\
i_{sy}
\end{bmatrix} + \omega_s \begin{bmatrix} 0 & -1 \\
1 & 0
\end{bmatrix} \begin{bmatrix} \phi_{sx} \\
\phi_{sy}
\end{bmatrix}
\]

(2.35)

\[
\begin{bmatrix} 0 \\
0 \end{bmatrix} = R_R \frac{\omega_s}{\omega_2} \begin{bmatrix} i_{Rx}' \\
i_{Ry}'
\end{bmatrix} + \omega_s \begin{bmatrix} 0 & -1 \\
1 & 0
\end{bmatrix} \begin{bmatrix} \phi_{Rx}' \\
\phi_{Ry}'
\end{bmatrix}
\]

(2.36)

In (2.36) \( \omega_2=\omega_s-p\omega_m \) is the rotor slip frequency.

### 2.4.2. Asymmetrical T-scheme

It is advisable to avoid separate leakage inductions of the stator \( L_{s\sigma} \) and of the rotor \( L_{r\sigma} \) in the machine model due to the purposes of the field-oriented control. By means of concentrating both leakage inductances on the stator side, the machine equations becomes very simple. This simplification is achieved by applying the following transformations to the rotor variables:

\[
\begin{bmatrix}
u_{Rm} \\
u_{Rn}
\end{bmatrix} = \frac{\hat{M}}{L_r} \begin{bmatrix} i_{Rm}' \\
i_{Rn}'
\end{bmatrix}
\]

\[
\begin{bmatrix} i_{Rm} \\
i_{Rn}
\end{bmatrix} = L_r \begin{bmatrix} i_{Rm}' \\
i_{Rn}'
\end{bmatrix}
\]

\[
\begin{bmatrix} \phi_{Rm} \\
\phi_{Rn}
\end{bmatrix} = \frac{\hat{M}}{L_r} \begin{bmatrix} \phi_{Rm}' \\
\phi_{Rn}'
\end{bmatrix}
\]

(2.37)

where:

\( L_r = L_{rm} + L_{r\sigma} \) - rotor inductance,

\( L_s = L_{sm} + L_{s\sigma} \) - stator inductance,

\( \hat{M}^2 = L_{sm}L_{rm} \) - mutual inductance (ideal coupling between the stator and rotor).

and:

\[
R_R = (\hat{M}/L_r)^2 R_R'
\]

(2.38)

For purpose of generalization stator and rotor matrix equations are shown in an arbitrary reference frame \( m-n \):

\[
\begin{bmatrix}
u_{sm} \\
u_{sn}
\end{bmatrix} = R_s \begin{bmatrix} i_{sm} \\
i_{sn}
\end{bmatrix} + \frac{d}{dt} \begin{bmatrix} \phi_{sm} \\
\phi_{sn}
\end{bmatrix} + \omega_k \begin{bmatrix} 0 & -1 \\
1 & 0
\end{bmatrix} \begin{bmatrix} \phi_{sm} \\
\phi_{sn}
\end{bmatrix}
\]

(2.39)

\[
\begin{bmatrix}
u_{Rm} \\
u_{Rn}
\end{bmatrix} = R_R \begin{bmatrix} i_{Rm} \\
i_{Rn}
\end{bmatrix} + \frac{d}{dt} \begin{bmatrix} \phi_{Rm} \\
\phi_{Rn}
\end{bmatrix} + (\omega_k - p\omega_m) \begin{bmatrix} 0 & -1 \\
1 & 0
\end{bmatrix} \begin{bmatrix} \phi_{Rm} \\
\phi_{Rn}
\end{bmatrix}
\]

(2.40)
flux equations are:
\[
\begin{bmatrix}
\phi_{sm} \\
\phi_{sn}
\end{bmatrix} = \sigma L_s \begin{bmatrix}
i_{sm} \\
i_{sn}
\end{bmatrix} + (1 - \sigma) L_s \begin{bmatrix}
i_{sm} + i_{Rm} \\
i_{sn} + i_{Rn}
\end{bmatrix}
\]
(2.41)
\[
\begin{bmatrix}
\phi_{Rm} \\
\phi_{Rn}
\end{bmatrix} = (1 - \sigma) L_s \begin{bmatrix}
i_{sm} + i_{Rm} \\
i_{sn} + i_{Rn}
\end{bmatrix}
\]
(2.42)
the electrical torque is now defined as:
\[
T_e = \frac{3}{2} p (1 - \sigma) L_s \begin{bmatrix}
i_{sm} & i_{sn}
\end{bmatrix} \begin{bmatrix}
0 & -1 \\
1 & 0
\end{bmatrix} \begin{bmatrix}
i_{Rm} \\
i_{Rn}
\end{bmatrix}
\]
(2.43)
In equations (2.37) to (2.43) the following symbols are used:
\[
\phi_{sm}, \phi_{sn}, \phi_{Rm}, \phi_{Rn} \quad \text{- stator and rotor flux components referred to the stator,}
\]
\[
i_{sm}, i_{sn}, i_{Rm}, i_{Rn} \quad \text{- stator and rotor current components referred to the stator,}
\]
\[
R_s, R_R \quad \text{- stator and rotor resistance, both referred to the stator,}
\]
\[
\sigma = \frac{1 - \hat{M}^2}{L_s L_T} \quad \text{- leakage coefficient.}
\]
Fig.2.9 presents the equivalent scheme of the two-phase machine model based on equations (2.39) and (2.40).

The scheme shown in Fig.2.9 is the equivalent asymmetrical T-scheme where rotor parameters are transformed to the stator side and the lumped stator and rotor leakage inductances are concentrated on the stator side. This equivalent scheme is adopted as
a base model in the study on low-speed, low-frequency ac drives because this scheme is suited for the control methods based on the vector control. The model provides extremely simple analytical equations, which are easily applied in microprocessor implementation.

The analysis of applications with low stator frequency, which is the objective of the study, does not benefit particularly from the use of the asymmetrical T-scheme. However, there are a couple of basic differences between the applications with low stator frequencies and the applications with nominal frequency. Considering ideal sinusoidal stator currents, the stator resistance $R_s$ plays a dominant role in stator equations at low frequencies. The voltage drop over $R_s$ is at low stator frequencies much larger than the voltage drop over the stator leakage inductance $\sigma L_s$. In practice, the stator voltage $u_s$ has to be increased to maintain the nominal flux in the machine. The participation of the leakage inductance $\sigma L_s$ compared to the main inductance $(1-\sigma)L_s$ in the stator inductance $L_s$ is minor. The consequence is that the stator flux is nearly equal to the linkage flux which is in the case of the asymmetrical T-scheme equal to the rotor flux. Hence:

$$
\begin{bmatrix}
\phi_{sm} \\
\phi_{sn}
\end{bmatrix} = \begin{bmatrix}
\phi_{Rm} \\
\phi_{Rn}
\end{bmatrix}
$$

(2.44)

2.5. Conclusions

Phenomena that take place in the real induction machine have been discussed, such as:
- nonsinusoidal winding distribution around the stator circumference,
- effects of saturation,
- effects of slotting.

These undesirable phenomena are the cause of higher harmonics in the machine which affect the torque at near-zero stator frequencies (near-zero rotor speed). Due to the low-frequency of the torque ripple the inertia of the rotor with load is not sufficient to smooth the rotor cogging.

Beside these effects (saturation, slotting, etc.) which are all functions of the spatial position in the machine, there are also time harmonics injected by power converters as induction machine power supplies. A large content of time harmonics is generated by discontinuous stator voltages.

Although all the mentioned harmonic groups (spatial and time harmonics) provoke multiple effects in the machine (extra Joule losses, voltage stresses, etc.) only their influence on the electrical torque is discussed as being important for the purposes of this study. Further, only the separate influence of each mentioned harmonic group on the torque is described. The analyses which would show the combined effect of all harmonic groups on the torque is far too complex and exceeds the purpose of this work.
The basic theory developing the model of the induction machine which incorporates phenomena like winding distribution, saturation in the machine and slotting is presented. The further development of such a model, that would encounter all of the discussed effects and their influence on low stator frequency operation of the machine, is meant to be the objective of a future study.

Finally, for the practical application of this study which concerns low stator frequency (low-speed) operation of the machine, a two-phase equivalent model is adopted. This model neglects all space and time harmonics and considers only the fundamental time harmonic. Moreover, it allows the analysis of low-speed effects produced by the semiconductor voltage drop, dc offset and asymmetries in the machine which are among the objectives of this study. It will be shown that the use of this simple machine model for the shaft sensorless vector control provided satisfactory results in the laboratory setup.

It should be noted that at low stator frequencies the stator resistance \( R_s \) dominates the stator impedance and causes proportionally the largest voltage drop. Therefore, when machine parameters are determined, the stator resistance must be measured particularly accurately.

References


3.

INTEGRATED MATRIX-ČUK TOPOLOGY

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3. INTEGRATED MATRIX-ČUK TOPOLOGY

3.1. Introduction
Most contemporary power converters which are used to feed ac machines are voltage-source converters. The power converter topology is usually based on the combination of an input rectifier, a low-frequency LC filter and an inverter bridge. The output voltage of the inverter is modulated by chopping the dc link voltage. The output voltage contains a broad harmonic spectrum and, consequently, current harmonics are injected in the stator of the machine. They are the cause of the torque ripple which is a significant problem for the application of the induction machine drive. The problem is particularly emphasized while the machine operates at low stator frequencies, when the cogging of the low-speed rotor shaft deteriorates the performance of the drive. Since the torque ripple is predominantly generated by time harmonics, much of the research effort has been made to search for possible ways to eliminate or mitigate them. There are two main approaches to this problem.

1. A range of time harmonics are eliminated by the application of PWM techniques.
2. Operating the power converter at a high switching frequency will reduce the amplitude of the harmonic components. A drawback of this method is the reduction of the total system efficiency due to the increased switching losses. Soft-switching techniques are a possible solution to this problem.

A power topology is proposed that successfully solves the problem of higher harmonic waveforms. Additionally, the topology enables a simple implementation of shaft-sensorless control of ac drives.

3.2. Integrated matrix-čuk converter
At this point the constituent parts of the topology, the matrix and the čuk converter, are described separately; the matrix converter as a frequency changer and the čuk converter as an electronic transformer. The study resumes then with their integrated operation which allows four-quadrant operation.

3.2.1. Matrix converter
Basic principles
The matrix converter [4]...[7] is a direct ac-to-ac frequency converter which does not utilize intermediate energy storage elements. This topology differs from the conventional converter topology mainly by the avoidance of large reactive elements in the dc or high-frequency link. This is beneficial because the intermediate energy storage ruins all valuable information about the input voltage waveform and input phase shifting. Therefore, a bidirectional power flow is difficult to provide with indirect converters. The matrix converter without intermediate coupling is susceptible to subharmonics in the output waveforms as well. They are induced by the difference between the input and output frequency. Further, the output waveform generation is usually not very accurate which has as a result that input voltage fluctuations appear on the output side as well. However, the matrix converter performs a true four-quadrant operation.
Fig.3.1 presents the basic configuration of the matrix converter. Nine bidirectional switches $S_{ij}$ ($i=1..3, j=1..3$) link each output phase with each input phase. Switches are closed sequentially following a cyclical pattern of pulse width modulation. Each of the three sinusoidal output voltages $u_{o_j}$ ($j=1,2,3$) can have any frequency independent of the input. By adding third harmonic components to the target output waveform, the output voltage amplitude can be up to 0.866 of the input voltage amplitude [4]. The input set and the desired output set of voltages are:

$$e_1(t) = \hat{e}_g \cos(\omega_i t)$$
$$e_2(t) = \hat{e}_g \cos(\omega_i t + \frac{2}{3} \pi)$$
$$e_3(t) = \hat{e}_g \cos(\omega_i t + \frac{4}{3} \pi)$$

(3.1)

$$u_{o_1}(t) = \hat{u}_o \cos(\omega_o t + \xi_o) + \hat{e}_g \frac{1}{4} \cos(3\omega_o t) - \hat{u}_o \frac{1}{6} \cos(3\omega_o t + 3\xi_o)$$
$$u_{o_2}(t) = \hat{u}_o \cos(\omega_o t + \frac{2}{3} \pi + \xi_o) + \hat{e}_g \frac{1}{4} \cos(3\omega_o t) - \hat{u}_o \frac{1}{6} \cos(3\omega_o t + 3\xi_o)$$
$$u_{o_3}(t) = \hat{u}_o \cos(\omega_o t + \frac{4}{3} \pi + \xi_o) + \hat{e}_g \frac{1}{4} \cos(3\omega_o t) - \hat{u}_o \frac{1}{6} \cos(3\omega_o t + 3\xi_o)$$

(3.2)

with:

- $\omega_i, \omega_o$ - angular frequencies of the input and output voltages,
- $\hat{e}_g$ - amplitude of the input voltages,
- $\hat{u}_o$ - amplitude of the matrix phase output voltages.

In this application an inductive load is assumed to be connected to the output. A short circuit on the input side and an open circuit on the output side are not permitted states. During each switching sequence three switches $S_{ij}$ ($i=1,2,3$) successively connect one output phase $j$ at a time to all three input phases. Those basic control rules are imposing the following constraints:
\[ \begin{align*}
  d_{11} + d_{21} + d_{31} &= 1 \\
  d_{12} + d_{22} + d_{32} &= 1 \\
  d_{13} + d_{23} + d_{33} &= 1 \\
  0 < d_{ij} < 1 & \quad (i = 1, 2, 3, \quad j = 1, 2, 3) 
\end{align*} \] (3.3)

where \( d_{ij} \) denotes duty cycle of the corresponding switch \( S_{ij} \).

**Three-phase input to single-phase output voltage synthesis**

Fig.3.2 presents the synthesis of the output waveform of a single phase matrix converter.

During the \( k \)-th switching sequence, the duty cycle \( d_i \) of the corresponding switch \( S_i \) is determined by the control algorithm of the switch. The switching interval \( T_{sw} \) is constant for each sequence and the duty cycle \( d_i \) corresponds to the on-state of the switch \( S_i \) during time \( d_i T_{sw} \). Switches \( S_1, S_2 \) and \( S_3 \) are closed sequentially and cyclically according to constraints (3.3) and (3.4). The switching period \( T_{sw} \) is always constant while duty cycles are varied to obtain the proper voltage waveform. The resulting output voltage \( u_o \) is a discontinuous function consisting of segments of the three input voltages \( e_1, e_2 \) and \( e_3 \) assembled successively. The contribution of the individual sources for the \( k \)-th switching interval is determined by the duty cycles \( d_1, d_2 \) and \( d_3 \). The average value of the output voltage \( U_o(k) \) for a constant switching interval is:

\[ U_o(k) = d_1 e_1 + d_2 e_2 + d_3 e_3 \] (3.5)

The resulting phase output voltage \( u_o(t) \) tracks the reference voltage by controlling the average voltage \( U_o(k) \) for each individual switching sequence \( k \). For a symmetrical three-phase system at the source side of the converter two simple modes of operation are defined by the application of expression (3.5):

1. If for each switching sequence \( k \) is valid that \( d_1 = d_2 = d_3 \) then the average output voltage \( U_o(k) \) during each sequence is zero.
2. If for each switching sequence \( k \) is valid that \( d_1, d_2, d_3 \) are constant, the generated output voltage vector is fixed with respect to the input voltage vectors. The output voltage \( u_o \) is sinusoidal with the same frequency as the source voltage \( e_1 \). The amplitude and phase of the output voltage \( u_o \) is controlled via \( d_1, d_2, d_3 \).
The Fourier spectrum of the output voltage \( u_o \) is related to the input voltage \( e_i \), the switching frequency \( f_{sw} = 1/T_{sw} \) and the switching pattern. At low frequencies of the output voltage the most significant influence on the harmonic spectrum has the average output voltage \( U_o(k) \). Other harmonic components are insignificant since they are in the neighbourhood of the switching frequency \( f_{sw} \) which is much higher than the frequency of the output voltage \( f_o = \omega_o / 2\pi \):

\[
f_{sw} \gg f_o
\]

(3.6)

This also means that if \( 1/T_{sw} \rightarrow \infty \) holds, all switching functions with the same average value during time intervals \( T_{sw} \) tend to have the same spectrum regardless of the pulse modulation method.

*Three-phase input to three-phase output voltage synthesis*

The desired three-phase output voltage set (3.2) has to be constructed from the set of input voltages (3.1). A matrix function \( D(t) \) is introduced which determines the process of low-frequency modulation [5]:

\[
\begin{bmatrix}
  u_{o1}(t) \\
  u_{o2}(t) \\
  u_{o3}(t)
\end{bmatrix} = \begin{bmatrix}
  e_1(t) \\
  e_2(t) \\
  e_3(t)
\end{bmatrix} = D(t) \begin{bmatrix}
  d_{11}(t) & d_{21}(t) & d_{31}(t) \\
  d_{12}(t) & d_{22}(t) & d_{32}(t) \\
  d_{13}(t) & d_{23}(t) & d_{33}(t)
\end{bmatrix} \begin{bmatrix}
  e_1(t) \\
  e_2(t) \\
  e_3(t)
\end{bmatrix}
\]

(3.7)

where \( D(t) \) is defined as:

\[
D(t) = \frac{1}{3} \begin{bmatrix}
  (1 + \beta_1 m_+(1) + \beta_2 m_- (1)) & (1 + \beta_1 m_+(2) + \beta_2 m_- (3)) & (1 + \beta_1 m_+(3) + \beta_2 m_- (2)) \\
  + \gamma_1 n_+(1) + \gamma_2 n_- (1) & + \gamma_1 n_+(2) + \gamma_2 n_- (3) & + \gamma_1 n_+(3) + \gamma_2 n_- (2) \\
  + \delta_1 q_+(1) + \delta_2 q_- (1) & + \delta_1 q_+(2) + \delta_2 q_- (3) & + \delta_1 q_+(3) + \delta_2 q_- (2)
\end{bmatrix}
\]

(3.8)

The means of the coefficients in (3.8) are as follows:

\[
m_+(j) = \cos \left( (\omega_o + \omega_i) t + \theta_o - (j-1) \frac{2}{3} \pi \right)
\]

\[
m_-(j) = \cos \left( (\omega_o - \omega_i) t + \theta_o - (j-1) \frac{2}{3} \pi \right)
\]

(3.9)
\[ n_+(j) = \cos \left[ 4\omega_1 t - (j-1) \frac{2\pi}{3} \right] \]
\[ n_-(j) = \cos \left[ 2\omega_1 t - (j-1) \frac{2\pi}{3} \right] \]  
(3.10)

\[ q_+(j) = \cos \left[ (3\omega_o + \omega_i) t + 3\theta_0 - (j-1) \frac{2\pi}{3} \right] \]
\[ q_-(j) = \cos \left[ (3\omega_o - \omega_i) t + 3\theta_0 - (j-1) \frac{2\pi}{3} \right] \]  
(3.11)

\[ \beta_1 = \frac{\hat{a}_o}{\hat{a}_i} \left[ 1 - \frac{\tan \text{(input pf angle)}}{\tan \text{(output pf angle)}} \right] \]
\[ \beta_2 = \frac{\hat{a}_o}{\hat{a}_i} \left[ 1 + \frac{\tan \text{(input pf angle)}}{\tan \text{(output pf angle)}} \right] \]  
(3.12)

\[ \gamma_1 = -\frac{\hat{a}_o}{\hat{a}_i} \frac{1}{6\sqrt{3}} \quad \gamma_2 = +\frac{\hat{a}_o}{\hat{a}_i} \frac{7}{6\sqrt{3}} \]  
(3.13)

\[ \delta_1 = -\frac{\hat{a}_o}{\hat{a}_i} \frac{1}{6} \quad \delta_2 = -\frac{\hat{a}_o}{\hat{a}_i} \frac{1}{6} \]

The input current of the converter is sinusoidal (with additional switching frequency harmonic) with the same frequency as the input voltage. Respecting constrains (3.3) and (3.4) the current transfer through the converter is described with the use of the transpose of \( D(t) \) as:

\[
\begin{bmatrix}
i_{11}(t) \\
i_{12}(t) \\
i_{13}(t)
\end{bmatrix} \begin{bmatrix}
d_{11}(t) & d_{12}(t) & d_{13}(t) \\
d_{21}(t) & d_{22}(t) & d_{23}(t) \\
d_{31}(t) & d_{32}(t) & d_{33}(t)
\end{bmatrix} \begin{bmatrix}
io_1(t) \\
io_2(t) \\
io_3(t)
\end{bmatrix}
\]  
(3.14)

The switching function \( D(t) \) allows the control of the power factor of the input terminals. Coefficients \( \beta_1 \) and \( \beta_2 \) defined in (3.12) generate the desired output power factor \( pf_o \) in combination with unity input power factor \( pf_i=1 \) [5].

Fig.3.3 shows the synthesized output voltage \( u_{0i} \) with the introduced switching function \( D(t) \). The maximum output to input voltage ratio \( \hat{u}_o/\hat{e}_i=0.866 \) is obtained for desired output to input frequency ratio \( \omega_o/\omega_i=1.5 \) by adding the third harmonic component of the input and output frequencies to the desired output voltage.

In spite of all the mentioned benefits, the matrix converter still fails to provide an output voltage amplitude higher than 0.866 of the input voltage. This imposes limitations when this converter is applied in a standard ac drive.
3.2.2. AC-AC Ćuk converter

Step-up, step-down and step-up/step-down dc-dc power converters can be considered as electronic transformers. Depending on the duty cycle they link different dc input and dc output voltage levels. These topologies have found some extended applications. The first is the improvement of the power quality by interfacing single-phase ac-dc systems described in [2] and [3]. Two important aspects of power quality improvement are thus successfully fulfilled:

1. reduction of the harmonic distortion of the current drawn from the ac grid,
2. correction of the power factor at the ac utility side.

The second novel application of dc-dc converters is linking ac-ac power grids directly and thus providing controllable bidirectional power flow [1]. Beside the amplitude control of the output voltage, also the unity input power factor can be achieved. The use of these ac-ac electronic transformers, instead of common transformers with a magnetic core, has advantages. Their specific power is higher, which enables a more compact design. The output to input voltage transfer ratio is easily modified by means of the duty cycle.

These characteristics are used in the novel power converter topology. In particular, the boost-buck (ćuk) converter topology ([12], [13]) has been implemented. Fig.3.4 shows the basic topology of the boost-buck dc-dc converter. At each time instant, one of the switches $S_1$ and $S_2$ is closed enabling a free current path. A simultaneous on-state of both switches would cause a short circuit of the capacitor $C$ and is therefore not allowed. Because the inductors $L_1$ and $L_1$ are placed in series with the voltage source
E and the load $R_L$, the source and the load current ($i_1$ and $i_L$) are continuous.

![Fig.3.4 Basic boost-buck (čuk) dc/dc topology](image)

A topology similar to the one shown in Fig.3.4 can be used as an ac-ac converter which operates as an electronic transformer. In other words, the frequency of the output voltage waveform is the same as the frequency of the voltage of the input terminals. The output to input voltage ratio is regulated with the converter duty cycle $d$. The output converter voltage can be boosted to a multiple of the input voltage amplitude.

Fig.3.5 shows the basic čuk ac-ac one-phase topologies.

![Fig.3.5 Basic ac-ac čuk topologies](image)

Topology a) is directly achieved from the dc-dc topology as shown in Fig.3.4 by substituting the dc voltage source $E$ with the ac voltage source $e_g$. In topology b) the voltage source $e_g$ is placed in the switching branch. Unfortunately, the source current is discontinuous but the topology has some practical advantages. The usefulness of topology b) is elaborated in the following section. The control of the stator voltage amplitude is one task that the converter used for an ac drive has to perform.

However, the converter has to vary the output frequency as well. The topologies shown in Fig.3.5 are not able to operate as a frequency changer. The matrix converter on the one hand and the čuk converter on the other hand, do have complementary features. This idea led to the design of the integrated matrix-čuk power topology that overcomes the disadvantages of each individual circuit.
3.2.3. Integrated matrix-čuk topology

Fig. 3.6 presents the novel three-phase power converter topology. It is based on the three-phase ac-ac boost-buck (čuk) converter and the matrix converter.

The proposed topology accomplishes the following important objectives.

- Currents injected in the stator have reduced harmonic content even at low switching frequencies. The torque ripple is reduced and a smooth rotor rotation at low stator frequencies is accomplished.
- Any ratio of the momentary values of the output and input voltage can be achieved.
- Implementation of a high performance shaft and voltage sensorless control of the induction motor is possible as is described in the following.
- Four-quadrant converter operation is inherent.
- The input power factor is controllable.

For this application, as shown in Fig.3.5, scheme b) has been implemented, where the three-phase voltage source is replaced by the matrix converter in the switching branch of the ċuk converter. This enables the matrix converter to carry out the multiple task:

- symmetrical three-phase output voltage generation at its output terminals (a, b, c) within the limitations of the matrix converter,
- controllable duty-cycle of the three-phase ċuk converter,
- correction of the input power-factor of the converter.
The selection of this topology has as a consequence that the grid currents are discontinuous and therefore an input filter is necessary. An alteration in the topology by placing the matrix converter in series with the inductors $L_{1,3}$ has an advantage that the source currents become continuous. As a trade-off, three additional switches have to replace the matrix converter in the switching branch, which increases the total number of switches to 15.

Fig.3.6 reveals another difference regarding the basic Čuk topology shown in Fig.3.4: the absence of the inductor $L_L$ at the load side. This inductor is now a part of the induction machine itself. The stator leakage inductance of the machine is large enough to take over the role of the external inductor and enable a continuous and low-ripple stator current. Considering that inductor $L_L$ is a part of the load the new topology belongs to the VSI group. The Čuk converter in Fig.3.6 serves as a three-phase electronic transformer capable of boosting up or stepping down the output voltage of the matrix converter. For low stator frequency operation around $f_s=1$ Hz, the step-down operation of the Čuk converter is crucial and ensures a greater modulation depth of the matrix converter. A better utilization of the digital control resolution is provided and ensures a more accurate stator voltage modulation. For higher values of output frequencies the duty cycle of the Čuk converter is kept constant while the matrix converter converts the frequency and controls the voltage improving the dynamical response of the total system [8], [9].

Fig.3.7 shows a simplified, one-phase presentation of the three-phase converter with a one-phase equivalent circuit of the three-phase induction machine. In the further analysis, for the sake of simplicity, the load resistance $R_s$ represents the sum of the stator and rotor resistance $R_s+R_R$ of the induction machine.

![Fig.3.7 Single-phase presentation of the three-phase matrix-Čuk converter and the induction machine](image)

The waveforms shown in Fig.3.8 illustrate the principle of synchronization between the matrix and the Čuk converters as presented in Fig.3.7. The action of the switches $S_{11}, S_{21}, S_{31}$ with which they modulate the voltage $u_{o1}=u_{r}$ is shown for one phase of the matrix converter. The action of the switch $S'_{21}$ with which the duty cycle is controlled is also shown.
As can be observed, the matrix converter and the ĉuk converter are not operating independently. The matrix converter generates the voltage waveform of the desired frequency and amplitude. It also determines the duty cycle \( d \) and \((1-d)\) within the switching period \( T_{sw} \) of the ĉuk converter.

During the period equivalent to the duty cycle \( d \) of the ĉuk converter, the operation of the matrix converter consists of consecutive switching sequences of the same duration. The following condition is satisfied:

\[
d = d_{11} + d_{21} + d_{31}
\]  

(3.15)

During each switching sequence three switches \( S_{i1} \) \((i=1,2,3)\) successively connect one output phase \( j \) at a time to all three input phases. During the period equivalent to \((1-d)T_{sw}\), all switches in the matrix converter are opened. In the same time period, switch \( S'_{21} \) on the load side is closed thus creating the path for the free-running stator current \( i_s \). The average stator voltage over the switching sequence \( k \) is:

\[
-U_s(k) = d \left( U_a - U_C \right)
\]

(3.16)

It is important to synchronize an integer number of the matrix converter switching sequences with the interval \( dT_{sw} \) of the ĉuk converter. Only then the correct voltage will be generated at the stator terminals. Moreover, since in all three converter phases intervals \( dT_{sw} \) and \((1-d)T_{sw} \) occurs simultaneously, switches \( S'_{21} \) \((i=1,2,3, \text{Fig.3.6})\) are opened or closed at the same time. When closed, stator terminals are short circuited and all stator currents \( i_{s1..s3} \) are free-running.

The input power factor \( pf \) of the converter is also controllable. In the literature has been shown that this control is one of the merits of the matrix converter [5]. The
following study shows that the ĉuk converter in the presented topology can improve the input power factor of the system as well.

3.3. Conclusions
The presented integrated matrix-ĉuk topology has the following advantages:
1. currents injected in the induction machine stator have very low harmonic content eliminating the torque ripple at low stator frequencies,
2. the stator voltage can be boosted up to multiple values of the input nominal voltage,
3. four-quadrant operation [7],
4. control of the input power factor [5],
5. in the following text will be shown that a high-performance vector control method of the induction machine without the use of a shaft sensor and stator voltage measurement can be implemented. Torque control can be performed even at close to zero stator frequencies.

Drawbacks are:
1. 12 bidirectional switches are necessary,
2. capacitor C in the current path limits the application of the topology for higher power ranges.

References


4.

MODELING THE MATRIX-ČUK CONVERTER

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4. MODELING THE MATRIX-ČUK CONVERTER

4.1. Introduction
The mathematical tools for the modeling and analysis of ac-ac converters are continuously developing. To analyze the properties of the proposed three-phase matrix-čuk converter topology, the first step is the development of a mathematical model. The method of state-space averaging is a mathematical method commonly used for small-signal analysis and modelling of dc-dc switching converters. It is proved that the same method can be a tool for the analysis of a three-phase matrix-čuk topology [7]. The method is valid under the condition that the modulation frequency is less than half of the switching frequency. If this is not true, the accuracy of the method becomes too low [1], [2], [3], [4]. For practical applications, where the modulation frequency is about $f_s=50$ Hz and the switching frequency is in the order of two kilohertz, this requirement is easily met.

4.2. Theoretical background
The choice of modeling techniques introduces here some problems. The most popular modeling techniques like state-space averaging [6] and absorbed-injected-current methods [5] are well suited for a process of small-signal linearization. This is a proper approach to study the characteristics of dc-dc switching converters. In ac-ac applications the small-signal assumption is violated because of large-signal fluctuations of ac waveforms. In this application, where the dc-dc converter is used for ac-ac conversion, the small-signal assumption is satisfied because of negligible signal variations during a single switching period compared to the signal variations during a whole period of the modulated waveform [7].

Discrete modeling techniques have been proven to be accurate for the whole range of modulating frequencies up to the theoretical limit of one-half of the switching frequency [1], [2], [3], [4]. However, these methods are inevitably complex. If the modulating frequency is lower than the theoretical limit of one-half of the switching frequency a good approximation can be achieved if averaging techniques are applied. The results of computer simulation of the integrated matrix-čuk converter are in good agreement with the results achieved with the linearized model. This statement holds particularly when the converter operates at a low modulating frequency ($f_s=0$ Hz).

4.3. Modeling and analysis
To simplify the analytical model of the converter, the following is assumed:
- a single-phase equivalent model instead of the three-phase model is discussed,
- the matrix converter is presented as an ideal voltage source defined as the averaged three-phase input voltage $e_g(k)=U_0(k)$ over one switching cycle $k$, as shown in (3.5),
- the asynchronous machine is replaced by an inductive passive load; the back electromotive force $e_r$ induced by the rotation of the rotor flux is neglected in the following analysis,
the ĉuk converter is studied without magnetic coupling of both inductances.

Fig.4.1 shows the single-phase circuit diagram of the matrix-ĉuk power converter loaded with a single-phase induction machine. Due to the simplicity of the following analysis it is assumed that the sum of the stator and rotor resistance $R_s + R_R$ are compound in the resistance $R_s$, as shown in Fig.4.1.

![Fig.4.1 One-phase presentation of the three-phase converter](image)

Since the matrix converter is a direct frequency changer without passive energy storage elements, its operation can be easily linearized by a process of averaging of the internal waveforms over one switching period. In Fig.4.2 the voltage source $e_g$ represents the averaged output voltage of the matrix converter. Since the voltage drop over the resistance $R_s$ dominates at low stator frequencies the back electromotive force of the machine $e_r$ is omitted in the following discussion.

![Fig.4.2 One phase of the matrix-ĉuk converter where the matrix converter is represented as an ideal voltage source $e_g$](image)

From (3.5), the source voltage $e_g(k)$ over one single switching cycle $k$ equals:

$$e_g(k) = d_1 e_1 + d_2 e_2 + d_3 e_3$$

(4.1)

The scheme shown in Fig.4.2 is the basic diagram to be used for the following mathematical analysis. The control of the duty cycle $d$ is established for synchronous operation of switches $S_1$ and $S_2$. Fig.4.3 presents both possible subcircuits of the
converter in Fig. 4.2 where $R_1$ denotes the parasitic resistance of the inductor $L_1$. Parameters $\sigma L_s$ and $R_s$ represent the simplified inductive load (induction machine). Both the inductor currents $i_1$ and $i_s$ are continuous.

![Diagram of converter circuit]

**Fig. 4.3 Two subcircuits of the ac Čuk converter:**
- a) p-mode: switch $S_1$ closed, $S_2$ opened
- b) s-mode: switch $S_2$ closed, $S_1$ opened

For each mode of operation presented in Fig. 4.3 a second-order, linear time-invariant system is defined which after the state-space averaging becomes the third order system.

During the p-mode, as shown in Fig. 4.3 a) the switch $S_1$ is closed and switch $S_2$ opened. The differential equations describing this mode of operation are:

\[
\begin{align*}
e_g &= R_1 i_1 + L_1 \frac{d i_1}{dt} \\
e_g &= R_s i_s + \sigma L_s \frac{d i_s}{dt} - u_C \\
C \frac{d u_C}{dt} &= -i_s
\end{align*}
\] (4.2)
The state-equation for the p-mode is presented as:
\[
\dot{x} = A_p x + B_p u
\]  
(4.3)

or in the matrix form:
\[
\begin{bmatrix}
\frac{d}{dt} i_1 \\
\frac{d}{dt} i_s \\
\frac{d}{dt} u_C
\end{bmatrix}
= \begin{bmatrix}
-\frac{R_1}{L_1} & 0 & 0 \\
0 & -\frac{R_s}{\sigma L_s} & \frac{1}{\sigma L_s} \\
0 & 0 & -\frac{1}{C}
\end{bmatrix}
\begin{bmatrix}
i_1 \\
i_s \\
u_C
\end{bmatrix}
+ \begin{bmatrix}
1 \\
0 \\
1
\end{bmatrix} e_g
\]  
(4.4)

- During the s-mode of operation as shown in Fig.4.3 b) the switch \(S_1\) is opened and switch \(S_2\) is closed. The differential equations describing this mode of operation are:
\[
0 = R_1 i_1 + L_1 \frac{di_1}{dt} + u_C
\]
\[
0 = R_s i_s + \sigma L_s \frac{di_s}{dt}
\]  
(4.5)

\[
C \frac{du_C}{dt} = i_1
\]
The state-equation for the s-mode of operation is presented as:
\[
\dot{x} = A_s x + B_s u
\]  
(4.6)

or in the matrix form:
\[
\begin{bmatrix}
\frac{d}{dt} i_1 \\
\frac{d}{dt} i_s \\
\frac{d}{dt} u_C
\end{bmatrix}
= \begin{bmatrix}
-\frac{R_1}{L_1} & 0 & -\frac{1}{L_1} \\
0 & -\frac{R_s}{\sigma L_s} & 0 \\
1 & 0 & 0
\end{bmatrix}
\begin{bmatrix}
i_1 \\
i_s \\
u_C
\end{bmatrix}
\]  
(4.7)

In equations (4.3) and (4.6) the following notation is used:
- \(x\) - state vector of circuit variables in the time domain,
- \(u\) - scalar input,
- \(A_p, A_s\) - matrices with constant coefficients,
- \(B_p, B_s\) - matrices with constant coefficients.
4.3.1. Averaged model of the ċuk converter

According to the state-space-averaging technique [6] the averaged model of the converter topology shown in Fig.4.2 over a single switching period is presented as:

$$\dot{x} = (dA_p + (1-d)A_s)x + (dB_p + (1-d)B_s)u = Ax + Bu$$

(4.8)

where matrices $A$ and $B$ are:

$$A = \begin{bmatrix}
-\frac{R_1}{L_1} & 0 & -(1-d) \frac{L_1}{L_1} \\
0 & -\frac{R_s}{\sigma L_s} & \frac{d}{\sigma L_s} \\
(1-d) \frac{C}{C} & -\frac{d}{C} & 0
\end{bmatrix}$$

(4.9)

also:

$$\dot{x} = \frac{d}{dt}[i_1 \ i_s \ u_C]^T \quad x = [i_1 \ i_s \ u_C]^T \quad u = e_g$$

(4.10)

From the equation (4.8) the low-frequency equivalent scheme of the ċuk converter is found and presented in Fig.4.4.

![Fig.4.4 Low-frequency equivalent scheme of the single-phase averaged ċuk converter model](image)

**Example**

Fig.4.5 and Fig.4.6 present simulation results of the switched (Fig.4.2) and the averaged model (Fig.4.4) of the single-phase ċuk converter. Parameters of both models are identical: $L_1=5$ mH, $R_1=0.3$ $\Omega$, $C=250$ $\mu$F, $R_s=2$ $\Omega$, $\sigma L_s=10$ mH, $d=0.3$, $f_s=30$ Hz, $e_g=35$ V, $f_{sw}=1.5$ kHz. The averaged model shows a good agreement with the actual switching converter model in spite of the low switching frequency ($f_{sw}=1.5$ kHz).
Fig. 4.5 Switching model simulation of the single-phase ĉuk converter (scale: current ±10 A, voltage ±60 V, $f_s=30$ Hz)

Fig. 4.6 Averaged model simulation of the single-phase ĉuk converter (scale: current ±10 A, voltage ±60 V, $f_s=30$ Hz)

These two simulation results justify the application of small signal analysis in this particular large signal ac-ac utilization.
4.3.2. Linearization - a steady-state ac model of the ĉuk converter

In a common dc-dc converter, the small-signal behaviour of the time-varying system, presented in (4.8), is linearized by using perturbation techniques around the steady-state operation point. Assuming that the parameters of the converter are constant, small perturbation vectors $\Delta x$, $\Delta u$ and a small perturbation of the duty cycle $\Delta d$ around the dc steady state values $X$, $U$, $D$ of vectors $x$, $u$ and of the duty cycle $d$ are defined:

$$x = X + \Delta x$$

$$d = D + \Delta d$$  \hspace{1cm} (4.11)$$

$$u = U + \Delta u$$

Substituting terms (4.11) into (4.8), the dc and ac components of the linearized model are obtained:

- dc component:

$$0 = AX + BU$$  \hspace{1cm} (4.12)$$

- ac component:

$$\Delta \dot{x} = A \Delta x + B \Delta u + \Delta d [(A_p - A_s)X + (B_p - B_s)U]$$  \hspace{1cm} (4.13)$$

In ac-ac applications of the ĉuk converter the dc model component does not exist ($X=0$, $U=0$) if the dc offset is assumed negligible. As it is shown later in the study, the duty cycle $d$ is set to be constant which results in $\Delta d=0$. What remains from the linearized converter model (4.13) is the ac component:

$$\Delta \dot{x} = A \Delta x + B \Delta u$$  \hspace{1cm} (4.14)$$

The transformation of (4.14) into the frequency domain, followed by suitable matrix transformations, deduces the following result:

$$x(s) = (sI - A)^{-1} B u(s) = T(s) u(s)$$  \hspace{1cm} (4.15)$$

where:

$I$ - unity matrix 3x3,

$s=j\omega_s$ - Laplace operator,

$x(s)$, $u(s)$ - state vectors in the frequency domain,

$T$ - matrix of complex transfer functions (1x3).

From equations (4.15) and (4.9) three basic transfer functions are gleaned:

$$\frac{i_l(s)}{e_g(s)} = \frac{d(s^2 C \sigma L_s + s CR_s + d)}{s^3 C \sigma L_s L_1 + s^2 C (L_1 R_s + \sigma L_s R_1) + s^2 [CR_1 R_1 + d^2 L_1 + \sigma L_s (1-d)^2] + d^2 R_1 + R_s (1-d)^2}$$  \hspace{1cm} (4.16)$$

$$\frac{i_s(s)}{e_g(s)} = \frac{d(s^2 C L_1 + s CR_1 + 1-d)}{s^3 C \sigma L_s L_1 + s^2 C (L_1 R_s + \sigma L_s R_1) + s^2 [CR_1 R_1 + d^2 L_1 + \sigma L_s (1-d)^2] + d^2 R_1 + R_s (1-d)^2}$$  \hspace{1cm} (4.17)$$
\[
\frac{u_C(s)}{\mathcal{E}_g(s)} = \frac{d}{s^3 \sigma L_s L_1 + s^2 C(L_1 R_s + \sigma L_s R_1) + s[CR_1 R_s + d^2 L_1 + \sigma L_s (1-d)^2] + d^2 R_1 + R_s (1-d)^2}
\]

(4.18)

Two more important transfer functions are derived:

- the first is the output to input voltage transfer function \( h(s) \) which is, following the linearized circuit in Fig.4.4, presented as:

\[
h(s) = \frac{u_s(s)}{\mathcal{E}_g(s)} = \frac{d e_g(s) + d u_C(s)}{\mathcal{E}_g(s)} = d \left( 1 + \frac{u_C(s)}{\mathcal{E}_g(s)} \right)
\]

(4.19)

or:

\[
h(s) = \frac{d}{s^3 \sigma L_s L_1 + s^2 C(L_1 R_s + \sigma L_s R_1) + s[CR_1 R_s + d^2 L_1 + \sigma L_s (1-d)^2] + d^2 R_1 + R_s (1-d)^2}
\]

(4.20)

- the second one is, according to the same figure, the input impedance \( Z_{\text{in}} \) of the \( \text{čuk} \) converter defined as:

\[
Z_{\text{in}} = \frac{\mathcal{E}_g}{d (i_1 + i_s)}
\]

(4.21)

From equations (4.16) and (4.17) follows:

\[
Z_{\text{in}}(s) = \frac{s^3 \sigma L_s L_1 + s^2 C(L_1 R_s + \sigma L_s R_1) + s[CR_1 R_s + d^2 L_1 + \sigma L_s (1-d)^2] + d^2 R_1 + R_s (1-d)^2}{d^2 [s^2 C(L_1 + \sigma L_s) + s C(R_1 + R_s) + 1]}
\]

(4.22)

4.3.3. Low stator frequency considerations

For low stator (output) frequencies that approach the dc-dc operation (\( f_s = 0 \)) and neglecting the parasitic resistance \( R_1 \), the transfer functions become simpler. These functions resemble those known from the modeling of the dc-dc \( \text{čuk} \) converter. For example, for the applied converter (Fig.4.2) with the voltage source \( \mathcal{E}_g \) in the switching branch holds:

\[
\frac{h}{\mathcal{E}_g} = \frac{1}{1-d} \quad \text{and} \quad \frac{u_C}{\mathcal{E}_g} = \frac{d}{1-d}
\]

(4.23)

For the basic dc-dc \( \text{čuk} \) converter topology (Fig.3.4) the corresponding transfer functions are:

\[
\frac{h'}{\mathcal{E}_g} = \frac{d'}{1-d'} \quad \text{and} \quad \frac{u_C'}{\mathcal{E}_g} = \frac{1}{1-d'}
\]

(4.24)

From (4.23) and (4.24) differences between the basic \( \text{čuk} \) converter (Fig.3.5 a) and the one applied in this study (Fig.4.2), for low stator frequency, can be emphasized:

- the applied converter has the same output to input voltage transfer function \( h \) as it
is for the standard ĉuk topology ($h'$),
- the voltage over the capacitor $u_C$ in this application is lower than the voltage $u_{C'}$ in the standard topology,
- the current through the voltage source $d(i_1+i_s)$ is discontinuous. This generates higher harmonics in the power grid.

4.4. Simulation results

The defined transfer functions in 4.3.2 characterize features of the novel converter. The following figures represent graphically the obtained analytical expressions.

Equation (4.20) defines the output to input voltage transformation for step-up and step-down operation. Fig. 4.7 represents the modulus of the transfer function $h=|h(s)|$ for various stator frequencies $0 \leq f_s \leq 30$ Hz and duty cycles $0 < d < 1$.

![Fig.4.7 Modulus of the complex transfer function $h$ for various stator frequencies $f_s$ and duty cycles $d$](image)

The converter operates as the well-known step-up, step-down topology with the transfer function $h=d/(1-d)$ except for $d=1$. For the frequency $f_s=0$ Hz and the duty cycle $d=0.5$ the modulus of the transfer function is $h=1$. The transfer function $h$ reaches its maximum when the duty cycle approaches $d=0.85$.

At low-frequency application ($f_s=0$) the duty cycle is $d<0.5$ the converter operates in the step-down mode. As the frequency $f_s$ increases the impedance of the converter circuit becomes a part of the load. It alters the characteristic of the circuit and consequently the rate of amplification $h$ decreases. The amplification depends strongly on the set of converter parameters. Basically, the whole circuit behaves like a low pass filter having the highest amplification factor $h$ for low stator frequencies.
Equation (4.22) characterizes the complex input impedance $Z_{in}$ of the applied ĉuk converter topology. The argument of this function defines the input power factor $pf_i$ of the ĉuk converter, which is an important practical issue. This suggests that it is possible to correct the input power factor not only by means of the matrix converter, as pointed in chapter 3 but also by means of the duty cycle $d$ of the ĉuk converter topology. The following equation defines the power factor of the ĉuk converter in the time domain:

$$\begin{align*}
 pf_i &= \frac{1}{T} \int_0^T i(t) u(t) \, dt \\
 &= \frac{1}{T} \int_0^T i(t) u(t) \, dt \\
 &= \frac{1}{T} \int_0^T i(t) u(t) \, dt
\end{align*}$$

(4.25)

where:

- $i(t)$, $u(t)$ - instant values of the current and voltage,
- $I_{\text{rms}}$, $U_{\text{rms}}$ - effective values of the current and voltage.

The input power factor $pf_i$ of the linearized system is defined as:

$$pf_i = \text{Arg} \left[ \frac{u_C}{e_s} \right]$$

(26)

Fig.4.8 is a graphical representation of the power factor $pf_i$ for various stator frequencies and duty cycles. It also reveals that at lower stator frequencies $pf_i=1$ because of dominant ohmic resistance in comparison with the inductive impedance.

![Fig.4.8 The converter input power factor $pf_i$ as a function of duty cycle $d$ and frequency $f_s$.](image)
4.5. Conclusions

The design and construction of the proposed power converter topology required its accurate mathematical model in order to reveal advantages and limitations of the topology. The objective of this study was to develop a simple, linearized steady-state model of the highly discontinuous switching processes in the converter, a model which is suitable for computer simulation purposes.

The mathematical method used for analysis of this ac-ac topology is "borrowed" from the small signal analysis of dc-dc converters which considerably simplifies the model. The simulation results in Fig.4.5 and Fig.4.6 are provided in the time domain to show the accuracy of the linearized model at low stator frequencies. In Fig.4.7 and Fig.4.8 numerical results are shown representing properties of the proposed converter topology important for driving the induction machine. There are two important characteristics.

- The input to output voltage transfer function of the ċuk converter $h$ shows that the step-up and step-down operation is possible. The numerical results imply that the power converter based on the ac-ac matrix-ćuk topology eliminates the inherent limitation of the matrix converter. The amplitude of the stator voltage can exceed 0.866 of the nominal input voltage amplitude. At near-zero frequencies the step-down mode of operation of the proposed topology enables better utilization of the digital control than the standard converter topology does.

- The power factor control becomes an important issue for ac drives. Besides the possibility to control the input power factor by means of the matrix converter the results of this study show that it is possible to control the power factor by means of the duty cycle $d$ of the ċuk converter as well. However, no attempt was made in this study to implement this kind of control.

References


5.

SHAFT SENSORLESS VECTOR CONTROL

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5. SHAFT SENSORLESS VECTOR CONTROL

5.1. Introduction
The problem is the control of the induction machine at near zero stator frequencies. At the first glance, it seems that the converter topology and the control of the machine are separate topics. However, hereafter it is shown that a shaft and voltage sensorless control technique is a logical choice for the proposed topology. This class of control enables design of robust and reliable industrial ac drives and is therefore a very attractive solution.

The presented power converter topology enables the generation of sinusoidal stator currents with insignificant content of higher harmonics even at low switching frequencies ($f_{sw} = \pm 1 \text{ kHz}$). This is an important issue in low-speed drives and a solid basis for implementation of high-performance control methods.

5.2. Basic vector control methods
Since the vector control of the induction machine was introduced by Blaschke [1] and Leonhard, it has been successfully implemented in various applications. It gained tremendous popularity because, in this way, the torque and the field of the induction machine is controlled independently, just as in case of a controlled dc machine. Classical field orientation is always performed with respect to the rotor flux vector because simple decoupling of the machine equations is realized. These machine equations, although simple, depend on the rotor time constant which is in practice hard to measure. The rotor time constant is also subject to continuous changes due to various operation conditions of the machine. Therefore, recently a stator flux field-oriented control that is independent of the rotor time constant, has been developed. However, this controller utilizes a more complicated machine model where the torque and the flux are not decoupled [9], [10].

5.2.1. Direct vector control method
In the last two decades, two major control methods, for the rotor-flux-oriented control, have been developed. The first is the direct vector control where the position and the magnitude of the rotor flux vector are either directly measured or derived from the measurements of the stator voltages and currents.

- Direct flux measuring employs sensing coils, tapped stator windings, Hall sensors or by means of a so-called flux model. However, a major construction problem is the installation of sensors in the machine armature.
- The rotor flux can also be calculated by using an estimator that is based on the applied machine model. This method is highly dependent on machine parameters.

Low stator frequency applications are particularly troublesome no matter whether the rotor flux is measured or calculated. All types of direct rotor flux methods (except the one using Hall sensors) fail to obtain a continuous torque (flux) control at zero stator frequency. This is because the flux is sensed by induced voltages that are zero at zero
stator frequency. Also, if the flux calculation is obtained via the measured stator voltages and currents, the numerical integration of the electromotive force induced by the flux over long periods, becomes inaccurate. Initial conditions, drift and dc offset of the analog data acquisition devices are additional problems.

5.2.2. Indirect vector control method
The second type of field-oriented control is the indirect vector control. For this control, the information about the rotor slip is essential and it requires measurements of the rotor position or speed and the stator currents. The slip is computed from the reference values of the slip and flux. This method obtains torque control even at zero speed but is highly dependent on machine parameters, especially on the rotor time constant. Also, the use of an additional shaft speed and position encoder is inevitable, reducing the reliability and increasing the price of the drive.

5.3. Shaft sensorless control
Recent trends are in favour of shaftless control of the induction machine because a shaft encoder is highly undesirable in almost any industrial application [2]...[8]. First, it increases the price of electrical drives, especially for the low power applications. Secondly, in many industrial applications, the use of the fragile, commonly optical sensor, is either not possible or not desirable. To obtain the information about the slip, without the use of the encoder, the shaftless controller comprises the slip estimation from the machine model and utilizes only measurements of stator currents and voltages.

By definition, the slip frequency $\omega_2$ is the difference between the converter output frequency $\omega_s$ and the electrical shaft speed $p\omega_m$ ($p$ number of pole pairs):

$$\omega_2 = \omega_s - p\omega_m$$

(5.1)

Whenever feed-back control of the motor speed or torque is required, the information concerning the slip is fundamental in order to compensate the natural speed drop caused by the load. Knowing simultaneously the slip frequency and the stator current, it is possible to uniquely determine the flux level and the torque magnitude independent of the load, voltage and frequency. In other words, the information on the stator current and the slip frequency is sufficient to calculate motor conditions precisely. Consequently, the slip-frequency-based control principles improve the static, and in particular the dynamic performance compared to other methods. For low-frequency applications (near $f_s=0$ Hz) this method is usually the best choice since it contains no operation of integration of currents and voltages which becomes inaccurate when frequency decreases.

The control proposed hereafter presents a modification of the shaftless method and combines elements of the direct and indirect vector control. Its main advantage is that it requires only measurements of stator currents and the control enables torque and flux control even at zero stator frequency. The proposed method is a step forward in the simplification of control schemes and is based on previous work [6].
5.4. Applied shaft-sensorless control

The field-oriented control is based on induction machine equations presented in the x-y coordinate system (see Fig.2.7) rotating synchronously with the rotor flux. The model in the x-y reference frame is the basic model also in this application. For the particular purposes of the applied shaft-sensorless control the machine model in the α-β coordinate system is also defined and used in the following elaboration.

5.4.1. Machine model in a synchronous reference frame

The electro-mechanical equations (5.2) and (5.3) of the system are presented in the x-y synchronous reference frame using the stator currents and rotor flux as state variables. The matrix equation of the electrical part of the system is:

\[
\begin{bmatrix}
    u_{sx} \\
    u_{sy} \\
    0 \\
    0 
\end{bmatrix} =
\begin{bmatrix}
    R_s + s\sigma L_s & -\omega_s\sigma L_s & s & -\omega_s \\
    \omega_s\sigma L_s & R_s + s\sigma L_s & \omega_s & s \\
    -R_R & 0 & s + \frac{R_R}{(1-\sigma)L_s} & -\omega_2 \\
    0 & -R_R & \omega_2 & s + \frac{R_R}{(1-\sigma)L_s} 
\end{bmatrix}
\begin{bmatrix}
    i_{sx} \\
    i_{sy} \\
    \phi_{Rx} \\
    \phi_{Ry} 
\end{bmatrix}
\]

(5.2)

The mechanical equation is given as:

\[
T_e = k_q (\phi_{Rx} i_{sy} - \phi_{Ry} i_{sx})
\]

(5.3)

where:

- \( \sigma = 1 - M^2/(L_s L_r) \) - leakage coefficient,
- \( s = d/dt \) - differential operator
- \( k_q \) - constant.

Fig.7.1 represents the two-phase asymmetrical T-scheme of the induction machine that is used in this study.

\[ Fig.7.1 \text{ Asymmetrical T-scheme of the induction machine in the synchronous reference frame} \]
Remarkable simplification of system (5.3) is achieved when the x-axis of the chosen reference frame x-y coincides with the orientation of the space vector of the rotor flux $\Phi_R$. Additionally, since the machine magnetizing inductance $(1-\sigma)L_s$ is much larger than the stator leakage inductance $\sigma L_s$, variations of the amplitude of the rotor flux are much slower than changes of the stator current. Therefore, the derivative of the rotor flux, for the purposes of the control, can be considered to be zero. The consequences of these assumptions are mathematically formulated in:

$$\Phi_{Rx} = \Phi_R \quad \Phi_{Ry} = 0 \quad \frac{di_{sx}}{dt} = 0 \quad \frac{d\Phi_R}{dt} = 0$$

(5.4)

By combination of (5.3) and (5.4) the following system of equations is obtained:

$$\begin{bmatrix}
    u_{sx} \\
    u_{sy} \\
    0 \\
    0
\end{bmatrix} =
\begin{bmatrix}
    R_s + s\sigma L_s & -\omega_s\sigma L_s & 0 \\
    \omega_s\sigma L_s & R_s + s\sigma L_s & \omega_s \\
    -R_R & 0 & R_R \\
    0 & -R_R & \omega_2
\end{bmatrix}
\begin{bmatrix}
    i_{sx} \\
    i_{sy} \\
    i_R
\end{bmatrix}$$

(5.5)

The torque equation is:

$$T_e = k_q \Phi_R i_{sy}$$

(5.6)

If the rotor time constant is denoted with $T_R$ then two simple relations for the rotor flux $\Phi_R$ and the rotor slip frequency $\omega_2$ follow from the rotor equation (5.5):

$$\Phi_R = (1-\sigma)L_s i_{sx}$$

(5.7)

$$\omega_2 = R_R \frac{i_{sy}}{\Phi_R} = \frac{1}{T_R} \frac{i_{sy}}{i_{sx}}$$

(5.8)

Equations (5.6) and (5.7) show that the control of the electric torque $T_e$ and the rotor flux $\Phi_R$ are decoupled, similar to the control of the dc machine. The rotor flux $\Phi_R$ depends only on the x-component of the stator current $i_{sx}$ and the torque $T_e$ depends on the y-component of the stator current $i_{sy}$ since the rotor flux can be considered to be constant. The slip frequency $\omega_2$ for the point of operation is also provided.

Fig.5.2 represents graphically equations (5.5) by using a space vector presentation for the system variables. In this figure the following symbols are introduced:

- $\vec{u}_s$ - space vector of the stator voltage,
- $\vec{e}_r$ - space vector of the electromotive force induced by the rotor flux,
- $\nu$ - angle between the x axis and the $\alpha$-axis,
- $\theta$ - angle between the stator voltage space vector and the $\alpha$-axis,
- $\varsigma$ - angle between the stator voltage space vector and the x-axis,
\[ \gamma \] - angle between the stator current space vector and the \( x \)-axis,
\[ \omega_s \] - synchronous frequency.

\[ \begin{align*}
\alpha & \quad \frac{di_s}{dt} \\
\omega_s & \quad \alpha L_s i_s \\
R_s i_s & \quad u_s \\
\theta & \quad \phi_R \\
\beta & \quad i_s \beta \\
\zeta & \quad i_s y \\
\gamma & \quad i_s x \\
\end{align*} \]

*Fig. 5.2 Vector diagram of the rotor-field-oriented control*

The applied field-oriented control uses the advantage of this simple presentation of the machine model. Its performance depends mainly on the accuracy of the rotor time constant \( T_R \) which can be measured or estimated. Other machine parameters are related to the stator and can be easily measured.

### 5.4.2. Machine model in the stator \( \alpha-\beta \) reference frame

Application of the proposed control principle requires also a machine model defined in the reference frame \( \alpha-\beta \) fixed to the stator in order to allow the calculation of crucial information done by the flux estimator. Based on the asymmetrical T-scheme shown in Fig.5.1, the following system of equations is derived:

\[
\begin{bmatrix}
    u_{s\alpha} \\
    u_{s\beta} \\
    0 \\
    0
\end{bmatrix}
= \begin{bmatrix}
    R_s + s\sigma L_s & 0 & s & 0 \\
    0 & R_s + s\sigma L_s & 0 & s \\
    -R_R & 0 & s + \frac{R_R}{(1-\sigma)L_s} & p\omega_m \\
    0 & -R_R & -p\omega_m & s + \frac{R_R}{(1-\sigma)L_s}
\end{bmatrix}
\begin{bmatrix}
    i_{s\alpha} \\
    i_{s\beta} \\
    \phi_{R\alpha} \\
    \phi_{R\beta}
\end{bmatrix}
\]

(5.9)

From the stator equations the space vector of the electromotive force \( \vec{e}_r \) induced by the rotor flux vector \( \vec{\Phi}_R \) is defined via its \( \alpha \) and \( \beta \) components:
\[
\begin{bmatrix}
  e_{r\alpha} \\
  e_{r\beta}
\end{bmatrix} =
\begin{bmatrix}
  d \frac{\phi_{R\alpha}}{dt} \\
  d \frac{\phi_{R\beta}}{dt}
\end{bmatrix} =
\begin{bmatrix}
  u_{s\alpha} \\
  u_{s\beta}
\end{bmatrix} -
\begin{bmatrix}
  (R_s + \frac{d}{dt} \sigma L_s)
\end{bmatrix}
\begin{bmatrix}
  i_{s\alpha} \\
  i_{s\beta}
\end{bmatrix}
\]

(5.10)

5.4.3. Strategy for the proposed control

Fig.5.3 depicts the presentation of the stator current \(i_s\) with its ripple generated by switching events of the matrix-\(\text{cuk}\) converter. The switching period \(T_{sw}\) and the duty cycle \(d\) are set to be constant. During the period corresponding to the duty cycle \((1-d)\) the stator terminals are short circuited \((u_s=0)\) creating a freewheeling path for the stator current \(i_s\). During the period \((1-d)T_{sw}\) the decay of the stator current \(di_s/dt\) is determined only by the parameters \(R_s, \sigma L_s\) and the electromotive force \(e_r\) induced by the rotor flux \(\phi_R\). The stator equation (5.11) from the machine model provides now a tool for the calculation of the vector of the rotor emf \(\vec{e}_c\):

\[
\vec{e}_c = -(R_s + \frac{d}{dt} \ell) \; \vec{i}_s
\]

(5.11)

where \(\ell = \sigma L_s\) is the leakage inductance of the machine.

![Fig.5.3 Presentation of the stator current \(i_s\)](image)

If the stator current is sampled at fixed sampling instants \((m_i, n_i\) in Fig.5.3) the measurement of the stator voltage becomes redundant since \(u_s=0\). Besides the instant value of the current, it is necessary to know its derivative in order to calculate equation (5.11). During the interval \((1-d)T_{sw}\) the derivative of the stator current is assumed to be constant since the switching period \(T_{sw}\) is much shorter than the stator time constant. From two successive samples \(m_i\) and \(n_i\) (Fig.5.3) follows:

\[
\frac{di_s}{dt} \approx \frac{\Delta i_s}{\Delta t}
\]

(5.12)

In practice this approximation allows simple and accurate method of calculation of the voltage drop produced by the leakage inductance.
The block scheme of the proposed method is presented in Fig. 5.4. A three-phase induction machine is connected to the matrix-čuk voltage source inverter. The controller receives only information about the stator currents. This information is sufficient for estimating the rotor flux vector $\Phi_c$ and the torque component of the stator current $i_{sy}$. The calculations take place in the flux estimator (shown in the figure) using the model of the induction machine. Although the proposed method requires data about the rotor slip frequency, the information is not measured. The data is calculated, or better, estimated from the calculated current component $i_{sy}$ and the reference (command) value of the same current $i_{sy}^*$.

Feed-back loops ensure that the actual values of the rotor flux $\phi_R$ and the torque generating current component $i_{sy}$ are equal to their reference $\phi_R^*$ and $i_{sy}^*$.

- The first feed-back loop is actually a part of the current controller that regulates the amplitude of the current via the modulus of the stator voltage space vector $U_{max} = |\overline{U}_s|$. It is a fast loop where the controller (PI-1 in Fig. 5.4) has a time constant of about $T_{pi} = 60$ ms and proportional amplification of $k_{pi} = 40$. The controller PI-1 commissions the current control of the voltage source inverter.

- The second feed-back loop is the frequency (torque) feed-back loop. The calculated slip frequency $\omega_2$ is integrated and added to the integrated output of the PI-2 controller resulting in the angle $\theta$. This angle is the reference position angle of the stator voltage space vector $\overline{U}_s$ generated by the VSI. The input of the PI-2 controller is proportional to the desired rotor acceleration, thus the output of the controller is proportional to the mechanical speed of the rotor $\omega_m$, assuming a machine with one pair of poles ($p = 1$). In the stator reference frame $\alpha\beta$ and for
steady state operation condition, the increment of the angle $\theta$ defines the synchronous speed $\omega_s$ of the voltage vector $\bar{u}_s$. The rotor flux $\bar{\Phi}_R$ (the flux angular position is $\nu$) rotates with the same angular speed $\omega_s$.

$$\omega_s = \frac{d\theta}{dt} = \frac{dv}{dt}$$  

(5.13)

During transients caused by the instant change of input or output values, the instant angular speeds of vectors $\bar{u}_s$ and $\bar{\Phi}_R$ are not equal. Consequently, the reference flux value $\bar{\Phi}_R^*$ will not coincide with the real flux vector $\bar{\Phi}_R$ and the controller generates an error. The transient error block $\zeta$ is introduced to compensate this error. Since the transient angle $\zeta$ between space vectors $\bar{u}_s$ and $\bar{\Phi}_R$ is constant during steady state operation it does not contribute to the synchronous speed $\omega_s$ since $d\zeta/dt=0$. During a transient, this angle increments or decrements which affects the final value of $\omega_s$ as presented in equation (5.14).

$$\omega_s = \frac{d\theta}{dt} - \frac{d\zeta}{dt}$$  

(5.14)

For high-performance motor drives, it is important to include this block in the control scheme since transient states seriously influence the control.

With current source inverters the transient angle $\gamma$ is the phase difference between the current space vector $\bar{I}_s$ and the flux space vector $\bar{\Phi}_R$ (see Fig.5.2). It is then easy to obtain its value from the values of the currents $i_{sx}^*$ and $i_{sy}^*$:

$$\gamma = \arctan\left(\frac{i_{sy}^*}{i_{sx}^*}\right)$$  

(5.15)

With a voltage source converter, like the matrix-čuk, the voltage is imposed to the stator. The transient angle $\zeta$ is then the angle between the reference stator voltage space vector and the flux vector (see also Fig.5.2):

$$\zeta = \arctan\left(\frac{u_{sy}^*}{u_{sx}^*}\right)$$  

(5.16)

Because the field-oriented control makes use of the stator current $i_s$ in the $x-y$ reference frame, it is necessary to calculate angle $\zeta$ in terms of $i_{sx}^*$ and $i_{sy}^*$. From matrix equation (5.3) with $\Phi_R=0$ the stator equations provide the linkage between the stator voltage and the stator current:

$$u_{sx}^* = R_s i_{sx}^* + \sigma L_s \frac{di_{sx}^*}{dt} - \omega_s \sigma L_s i_{sy}^* + \frac{d\Phi_R^*}{dt}$$  

(5.17)

$$u_{sy}^* = R_s i_{sy}^* + \sigma L_s \frac{di_{sy}^*}{dt} + \omega_s \sigma L_s i_{sx}^* + \omega_s \Phi_R^*$$  

(5.18)

Fortunately, some assumptions can be made for a simplification of the calculation of $\zeta$. In equations (5.17) and (5.18) the derivative terms of the rotor flux $d\Phi_R^*/dt$
and the current component $di_{sx}/dt$ are assumed zero. Due to simplicity the derivative of the torque current component $di_{sy}/dt$ is also assumed to be zero. Finally, the stator voltage equations are reduced to:

$$u_{sx}^* = R_s i_{sx}^* - \omega_s \sigma L_s i_{sy}^*$$  \hspace{1cm} (5.19)

$$u_{sy}^* = R_s i_{sy}^* + \omega_s \sigma L_s i_{sx}^* + \omega_s \phi_R^* = R_s i_{sy}^* + \omega_s \sigma L_s i_{sx}^* + e_r^*$$  \hspace{1cm} (5.20)

- The third feed-back loop is the rotor flux loop. The modulus of the calculated rotor flux $\phi_c$ is compared to the command value $\phi^*$ and the difference is fed into the input of the PI-3 controller. The output of the controller PI-3 is the flux component of the stator current $i_{sx}^*$. This improves the dynamic behaviour of the system, in particular the flux control during transient states.

### 5.4.4. Flux estimator

The efficiency of the proposed method depends on the accuracy of the calculated current component $i_{sy_{yc}}$ and the calculated amplitude of the rotor flux $\phi_c$ with respect to their actual values $i_{sy}$ and $\phi_R$. There are two fundamental problems.

- One problem is the sensitivity of motor parameters like stator and rotor resistance in relation to temperature changes. The saturation of inductances is another problem. For the low speed range the stator resistance creates a serious problem.

- The rotor flux is usually obtained by integrating the induced electromotive force $e_r$ provided by the measured stator voltages and currents according to equation (5.10). This conventional method is not appropriate for low stator frequencies. For a low amplitude and frequency of $e_r$, the integration is performed over a long period and consequently, the calculation becomes unreliable. If the stator frequency is zero, integration is unstable and practically difficult to obtain.

Based on results presented in [6], [7] a particular flux estimator is proposed that gives solutions to the stated problems. Fig.5.5 presents its block scheme where all space vectors are defined in the stator reference frame $\alpha-\beta$.

![Flux estimator diagram](image)

**Fig.5.5 Flux estimator**

The flux estimator calculates the emf vector $\vec{e}_c$ induced by the rotor flux using
equation (5.11). The reference value of the rotor flux vector \( \vec{\phi}^*_R \) and the calculated induced electromotive force vector \( \vec{e}^*_c \) are then imposed on a filter circuit. The calculated rotor flux vector \( \vec{\phi}^*_{c} \) is obtained from the addition of two terms:

\[
\vec{\phi}^*_{c} = \frac{T_c}{1 + T_c s} \vec{e}^*_{c} + \frac{1}{1 + T_c s} \vec{\phi}^*_R = \vec{\phi}^*_{c1} + \vec{\phi}^*_{c2}
\]  

(5.21)

where \( T_c \) is the time constant of filter circuit. At low stator frequencies the second term \( \vec{\phi}^*_{c2} \) dominates while at higher frequencies \( \vec{\phi}^*_{c1} \) dominates.

The induced voltage vector \( \vec{e}^*_{c} \) is calculated with respect to the possible errors between the existing values for \( R_s \) and \( \sigma L_s \) and the values applied in the model \( R^*_s \) and \( \sigma^* L^*_s \):

\[
\vec{e}^*_{c} = \vec{u}^*_{s} - (R^*_s + \ell^* s) \vec{i}^*_{s} = \vec{e}^*_{r} + (\Delta R_s + \Delta \ell s) \vec{i}^*_{s} = \frac{d\vec{\phi}^*_R}{dt} + (\Delta R_s + \Delta \ell s) \vec{i}^*_{s}
\]  

(5.22)

where:

- \( \vec{e}^*_{r} = j\omega_s \vec{\phi}^*_{R} \) - actual induced electromotive force,
- \( \vec{e}^*_{c} \) - calculated induced electromotive force,
- \( \Delta R_s = R_s - R^*_s \) - error stator resistance,
- \( \ell = \sigma L_s \) - leakage inductance,
- \( \Delta \ell = \ell - \ell^* \) - error leakage inductance.

For the steady state operation after transformation to the frequency domain equation (5.21) becomes:

\[
\vec{\phi}^*_{c} = \frac{T_c}{1 + j\omega_s T_c} \vec{e}^*_{c} + \frac{1}{1 + j\omega_s T_c} \vec{\phi}^*_R
\]  

(5.23)

Inserting equation (5.22) into (5.23) results in:

\[
\vec{\phi}^*_{c} = \vec{\phi}^*_R - \mathcal{R} \vec{\phi}^*_R + (\Delta R_s T_c - \Delta \ell) \mathcal{R} \vec{i}^*_{s} + \Delta \ell \vec{i}^*_{s} + \mathcal{R} \vec{\phi}^*_R
\]  

(5.24)

where:

\[
\mathcal{R} = (1 + j\omega_s T_c)^{-1} = \left[ 1 + (\omega_s T_c)^2 \right]^{-\frac{1}{2}} e^{-j\gamma_k}
\]

\[\gamma_k = \arctan(\omega_s T_c)\]

Equation (5.24) is discussed with the use of Fig.5.6. The comparison is performed with the standard flux calculation method that does not use the compensation \( \vec{\phi}^*_{c2} \) in (5.21).

**Standard method**

The standard method applies only one phase-delay circuit and imposes \( \vec{e}^*_{c} \) to its action as presented in (5.25).
\[
\phi_c = \frac{T_c}{1 + j\omega_s T_c} \varepsilon_c \tag{5.25}
\]

Even when the motor parameters are correctly estimated \((\Delta\ell=0, \Delta R_s=0)\), the phase error \(\varepsilon_o = \arctan(1/\omega_s T_c)\) as well as the amplitude error are large at low stator frequencies \(\omega_s\). The error becomes smaller and the control realizes better performance for larger values of \(T_c\). Unfortunately, the value of \(T_c\) is limited for practical reasons (thermal drift of the integrator, etc.).

**Proposed method**

The proposed method compensates the error \(\varepsilon_o = \arctan(1/\omega_s T_c)\) of the phase-delay circuit with the second term in expression (5.23) containing the reference rotor flux vector \(\Phi_R^*\). For a phase error \(\varepsilon=0\) and in case of \(\Delta\ell=0\) and \(\Delta R_s=0\) equation (5.23) becomes:

\[
\phi_c = \frac{j\omega_s T_c}{1 + j\omega_s T_c} \Phi_R + \frac{1}{1 + j\omega_s T_c} \Phi_R^* \tag{5.26}
\]

The calculated and the actual rotor flux coincide also at low speeds (low stator frequencies) and, theoretically, at zero stator frequency.

The choice of the time constant \(T_c\) is very important. A minimum phase error \(\varepsilon = \arctan(\Phi_{Ry}/\Phi_{Rx})\) in the most critical operation (rotor at standstill, \(\omega_2 = \omega_s\)) is the criterion for determining \(T_c\). Considering \(\Delta\ell\neq0\), \(\Delta R_s\neq0\) and \(\Delta R_s \Delta \ell\) in expression (5.24), it follows that the third term is the most important term for the error caused by \(\Delta\ell\) and \(\Delta R_s\). Rewriting the third term on the right side of (5.24) yields:

\[
(\Delta R_s T_c - \Delta \ell) \bar{r}_s \equiv (\Delta R_s T_c - \Delta \ell) \kappa i_{sx} e^{j(\gamma - \gamma_k)} \tag{5.27}
\]

where for the rotor stand still \((\omega_m = 0\) and \(\omega_2 = \omega_s)\):

\[
\bar{r}_s = i_{sx}/\cos(\gamma),
\]

\[
\gamma = \arctan(\omega_2 T_R),
\]

\[
\gamma_k = \arctan(\omega_2 T_c).
\]

From (5.27) follows that the phase error reaches its minimum value for \(\gamma = \gamma_k\). In other words, the control has its best performance if the phase-delay circuit time constant \(T_c\) equals the rotor time constant \(T_R\).
Example
The simulated operation of the estimator is depicted in Fig.5.7.
- The linear rise of the flux with dc currents (the reference flux value is 0.5 Wb),
- Low stator frequency operation \( f_s = 0.95 \text{ Hz} \).

![Diagram](image)

Fig. 5.7 Operation of the rotor flux estimator; 0.4 Wb/div

In Fig.5.7 trajectory a) represents the first term in expression (5.23) being the vector of the rotor flux \( \Phi_R^* \) imposed to the phase-delay circuit. Graph b) represents the second term in expression (5.23) that is the reference value of the rotor flux \( \Phi_R^* \) imposed on the phase-delay circuit. Graph c) is the resulting vector \( \Phi_c^* \) in expression (23). At low stator frequencies \( (\omega_s = 0) \) the first term in (5.23) is negligible and the second term is dominant. When the frequency increases the first term dominates over the second term. The resulting calculated flux represents always the actual value of the rotor flux for any frequency. This simulation shows that the flux can be calculated even for dc operation of the machine what is actually the goal of this study.

However, the estimated flux value is sensitive to the variation of the machine parameters. In the next chapters, the sensitivity of the control to motor parameter variations will be discussed in detail and simulation and experimental results will show the dynamical performance of this control.

The flux estimator shown in Fig.5.4 calculates also the torque component of the stator current \( i_{sy_c} \) needed for the speed estimation. This calculation is obtained from the \( \alpha \) and \( \beta \) components of the calculated rotor flux vector \( \Phi_c^* \) and the reference value of the rotor flux \( \Phi_R^* = \Phi_{R \alpha}^* = \Phi_{R \beta}^* \):

\[
i_{sy_c} = \frac{\phi_c \alpha i_{sy_c} - \phi_c \beta i_{sy_c}}{\phi_R^*}
\]

(5.28)

5.5. Conclusions
The applied control opens new prospects on the implementation of induction motor
field-oriented control. It minimizes the number of needed sensors in the ac motor drive that consequently has a couple of favourable effects:
• increased reliability of the drive,
• reduction of the motor drive total price,
• better dynamical response of the control since there are fewer measurements errors.

This control presents the natural choice regarding the proposed matrix-čuk topology because stator terminals are simultaneously short-circuited during the switching period. Important advantage of the applied control method is that it enables accurate estimation of the rotor flux at all frequencies which makes this control suitable for low-frequency, shaft-sensorless applications. The performance of the control depends on the accurate measurement of rotor parameters which can be done by on-line identification methods.

An interesting subject of a research would be the control application into the conventional bridge converter (VSI) which makes the same operation condition possible. With particular voltage PWM principles, like space-vector modulation the proposed control method is implementable [11]. The space-vector modulation generates 6 nonzero and 2 zero-voltage vectors in order to create the desired stator voltage pattern. Zero-voltage vectors present switching states when the stator terminals are short-circuited which is the basic requirement for this class of control.

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6.

SENSITIVITY FOR PARAMETER ERRORS OF THE SHAFT SENSORLESS VECTOR CONTROL AT LOW STATOR FREQUENCIES

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6. SENSITIVITY FOR PARAMETER ERRORS OF THE SHAFT SENSORLESS VECTOR CONTROL AT LOW STATOR FREQUENCIES

6.1. Introduction

The flux estimator which is the essential part of the vector controller is presented in Fig.6.1. The calculated vector of electromotive force \( \vec{e}_c \) and the reference value of the rotor flux \( \vec{\Phi}_R^* \) are imposed on the phase-delay circuit. The result is the calculated vector of the rotor flux \( \vec{\Phi}_c \) that gives a reliable approximation of the actual flux in the machine in all operating points. The introduction of \( \vec{\Phi}_R^* \) as a compensation term in equation (6.1) solves the phase error problem and enables a good performance of the control method also at low stator frequencies.

\[
\begin{align*}
\vec{u}_s & \rightarrow \text{VSI} \rightarrow \text{Induction machine} \\
\vec{u}_s & \rightarrow \vec{t}_s \rightarrow -R_s - \sigma L_s \vec{s} \rightarrow \vec{e}_c \rightarrow T_c (1 + T_c s)^{-1} \rightarrow \Sigma \rightarrow \vec{\Phi}_c \\
\omega_s \rightarrow \phi_R^* \rightarrow \text{rotator} \rightarrow \vec{\phi}_R \rightarrow (1 + T_c s)^{-1} \\
\end{align*}
\]

*Fig.6.1 Block scheme of the flux estimator*

The calculated vector of the rotor flux in the \( \alpha-\beta \) reference frame is (superscript * designates reference values):

\[
\vec{\Phi}_c = \vec{\Phi}_R^* + T_c \vec{e}_c = \vec{\Phi}_R^* + T_c s \vec{\Phi}_R = \vec{\Phi}_R^* + \frac{\vec{\Phi}_R^* - \vec{\Phi}_R}{1 + T_c s} \tag{6.1}
\]

where space vectors are defined as complex values:

\[
\begin{align*}
\vec{\Phi}_R^* &= \phi_{R\alpha} + j \phi_{R\beta} \\
\vec{e}_c &= e_{c\alpha} + j e_{c\beta} \\
\vec{\Phi}_R &= \phi_{R\alpha} + j \phi_{R\beta} \\
\vec{i}_s &= i_{s\alpha} + j i_{s\beta} \\
\vec{\Phi}_c &= \phi_{c\alpha} + j \phi_{c\beta} \\
\vec{e}_c &= \vec{u}_s - (R_s^* + \ell^* s) \vec{i}_s \\
\end{align*}
\tag{6.2}
\]

and:

\[
\begin{align*}
\ell^* &= L_s^* M^* / L_t^* & \text{leakage inductance} \\
s &= j \omega_s & \text{Laplace operator} \\
T_c & \text{time constant of the phase -delay circuit} \\
\end{align*}
\tag{6.3}
\]

Equation (5.26) shows that for a controlled machine and for \( \Delta R_s = 0, \Delta \ell = 0 \) flux \( \vec{\Phi}_R^* \) becomes equal to \( \vec{\Phi}_R \) and the error caused by the phase-delay circuit is zero. Hence, the
real flux in the machine is accurately approximated with the estimated flux value $\Phi_c$ over the whole range of stator frequencies including the zero frequency operation.

An important problem related to the field-oriented control is the machine parameter variation caused by temperature changes, saturation in the iron, skin effect, etc. This problem appears in particular when the reference values $R_s^*$ and $\ell^*$ used in the model differ from the actual values $R$ and $\ell$. The study of the control sensitivity gives a clue to the extent to which the stator and rotor parameter variations deteriorate the performance of the vector control.

The calculated electromotive force $\tilde{\varphi}_c$ for the steady state operation is:

$$\tilde{\varphi}_c = \tilde{e}_r + (\Delta R_s + \Delta \ell \ell) \bar{\ell}_s$$

(6.4)

where $\Delta R_s$ and $\Delta \ell$ are introduced error terms. Inserting equation (6.4) into (6.1) the calculated flux $\Phi_c$ becomes:

$$\Phi_c = \Phi_R + (\Phi_c^* - \Phi_R) \kappa e^{-j\gamma \kappa} + (T_c \Delta R_s - \Delta \ell \bar{\ell} \bar{\ell}_s \kappa e^{-j\gamma \kappa} + \Delta \ell \bar{\ell}_s$$

(6.5)

In (6.4) and (6.5) the following symbols are used:

- $\kappa = [1 + (\omega_s T_c)^2]^{-1/2}$ - modulus of the vector $\vec{\kappa}$,
- $\gamma \kappa = \arctan (\omega_s T_c)$ - phase shift produced by the filter circuit,
- $\Delta R_s = R_s - R_s^*$ - stator resistance error,
- $\Delta \ell = \ell - \ell^*$ - stator inductance error,
- $\tilde{\varphi}_r$ - actual electromotive force induced in the machine,
- $\tilde{\varphi}_c$ - calculated electromotive force induced in the machine.

The first term in equation (6.5) is the actual value of the machine rotor flux, the second term represents the difference between the reference and the real rotor flux. The third and the fourth terms are influenced by the resistance and the leakage inductance error $\Delta R_s$ and $\Delta \ell$.

### 6.2. Derivation of the phase error

The operation of the flux estimator is studied in the $x$-$y$ synchronous reference frame and with the presentation in Fig.6.2. If the calculated flux value $\Phi_c$ and the orthogonal $x$-axis coincide with the reference rotor flux value $\Phi_R^*$ follows:

$$\Phi_c = \Phi_{cx} = \Phi_R^* = \Phi_{Rx}$$

(6.6)

$$\Phi_{Rx} = 0 \quad \Phi_{cy} = 0$$

(6.7)
However, the real flux in the machine $\vec{\phi}_R$ in nonideal circumstances leads or lags with respect to the x-axis in consequence of the terms $\Delta R_s$≠0 and $\Delta \ell$≠0 in equation (6.5). The phase error angle $\varepsilon$ is:

$$\varepsilon = \arctan \left( \frac{\phi_{Ry}}{\phi_{Rx}} \right)$$  \hspace{1cm} (6.8)

The components of the vector $\vec{\phi}_c$ are obtained from expression (6.5). The first is the x-component of the calculated flux vector:

$$\phi_{cx} = \phi_{Rx} + \phi_{Rx}^* \left( \frac{\Delta \ell}{M^* \omega_s} + \frac{\Delta R_s}{\omega_s} \right) i_{sx}^* = \phi_{Rx} + \Delta \ell i_{sx}^* + \frac{\Delta R_s}{\omega_s} i_{sy}^*$$  \hspace{1cm} (6.9)

Under the assumption that the second calculated flux component is $\phi_{cy}$=0, the following equation is obtained from expression (6.5):

$$\frac{\phi_{Ry}}{\phi_{Rx}} = \frac{\phi_{Rx}^*}{\phi_{Rx}} \left[ 1 - \frac{\phi_{Rx}}{\omega_s T_c} \left( 1 - \frac{\phi_{Rx}}{M^* \omega_s T_c} \right) \right] - \frac{\phi_{Rx}^*}{\phi_{Rx}} \Delta \ell \left( \frac{1}{\omega_s T_c} i_{sx}^* - \frac{i_{sy}^*}{M^* \omega_s T_c} i_{sx}^* \right)$$  \hspace{1cm} (6.10)

The reference flux $\phi_{Rx}^*$ is presented as:

$$\phi_{Rx}^* = M^* |i_s^*| \cos(\gamma^*)$$  \hspace{1cm} (6.11)

By introducing the phase error $\varepsilon$ the modulus of the real flux is presented as:

$$|\vec{\phi}_R| = M |i_s^*| \cos(\gamma^* - \varepsilon)$$  \hspace{1cm} (6.12)

and its quadrature x-y components:
\[ \phi_{Rx} = |\phi_R| \cos(\varepsilon) \]  
\[ \phi_{Ry} = |\phi_R| \sin(\varepsilon) \]  
\[ \text{If } M^* = M \text{ and } \gamma^*_s = \gamma^*_s \text{ then from (6.11) and (6.13) it follows:} \]

\[ \frac{\phi_{Rx}}{\phi_{Rx}^*} = \frac{1}{2} \left[ 1 + \frac{\cos (\gamma^* - 2\varepsilon)}{\cos \gamma^*} \right] \]  

With \( \Gamma = \phi_{Ry}/\phi_{Rx} \), \( \gamma = \phi_{Rx}/\phi_{Rx}^* \) and by introducing the following normalized quantities:

- \( \mu^* = i^*_{sy}/i^*_{sx} \) - specific torque current,
- \( \omega_m = \omega_s - \mu^*/T_R^* \) - reference mechanical speed,
- \( \lambda^* = \omega_m T_R^* \) - normalized speed,
- \( \Delta r_s = \Delta R_s/R_s^* \) - variation of primary resistance,
- \( k_t = R_s^*/R_R^* \) - ratio primary to secondary resistance,
- \( \Delta \sigma = \Delta \theta M^* \) - variation leakage inductance,
- \( \chi = T_c/T_R^* \) - normalized time constant.

The equation (6.10) is rewritten as:

\[ \Gamma = \frac{1}{\gamma} \left[ \frac{1 - \gamma}{\chi (\lambda^* + \mu^*)} \right] + \frac{1}{\gamma} \left[ \frac{k_r \Delta r_s}{\chi} \frac{\chi \lambda^* + \mu^*(\chi - 1)}{(\lambda^* + \mu^*)^2} \right] - \frac{\Delta \sigma}{\chi} \left[ \frac{1}{\chi (\lambda^* + \mu^*)} + \mu^* \right] \]  

The phase error angle (6.8) is now presented as:

\[ \varepsilon = \arctan(\Gamma) \]  

The following paragraphs discuss in details the influence of this error on the accuracy of the estimation of the flux, speed and torque.

All presented error functions in the following graphics are shown as a function of the specific torque current \( \mu^* = i^*_{sy}/i^*_{sx} \). The value of \( \mu^* \) varies from 0 to 5 which corresponds to torque values from zero to approximately 200% of the nominal torque and assumes that the flux current component \( i^*_{sx} \) is kept constant.

### 6.3. Parameter variations and phase error

The phase error \( \varepsilon \), defined by (6.15) and (6.16), is built up from three components:

\[ \varepsilon = \varepsilon_o + \varepsilon_t + \varepsilon_1 \]  

- The first term \( \varepsilon_o \) is produced by the difference between \( \phi_{Rx}^* \) and \( \phi_{Rx} \) which is in practice small. Since \( \gamma = 1 \) the first term \( \varepsilon_o \) can be neglected.
The second term $\varepsilon_r$ depends on the stator resistance error $\Delta R_s$. In particular, the term $\varepsilon_r$ becomes insensitive to changes in $\Delta R_s$ for $\chi=1$ and $\lambda^*=0$ (for $\omega_m^*=0$ and any value of $\mu^*$). This term vanishes also when $\chi=\mu/ (\lambda^*+\mu^*)$. Fig.6.3 presents the values of $\lambda^*$ and $\mu^*$ for various values of $\chi$ for which the term $\varepsilon_r=0$.

**Fig.6.3 Specific speed $\lambda^*$ versus specific torque current $\mu^*$ for which $\varepsilon_r=0$**

The calculations of the phase error $\varepsilon_r$ as a function of $\mu^*$ for values of $\Delta R_s=0.2$, $k_r=1$ and various values of $\lambda^*$ are presented in Fig.6.4.

**Fig.6.4 Phase error $\varepsilon_r$ resulting from the stator resistance error $\Delta R_s$ and for parameter values $\lambda^*=0.5$ and $\lambda^*=1$ respectively**

From expression (6.15) can be also concluded that for any value of $\chi$ the influence of $\varepsilon_r$ becomes significant for small values of $\lambda^*$ (low-speed) and small values $\mu^*$ (low-torque).

- The third term $\varepsilon_l$ presents the influence of the leakage inductance variation $\Delta \sigma$ on the phase error which vanishes as the specific speed $\lambda^*$ increases. Effects of
parameters variations on $\varepsilon_l$ are shown in Fig.6.5 for $\Delta \sigma = -0.02$ and -0.05 ($\chi = 1$). 

Fig.6.5 Phase error $\varepsilon_l$ resulting from the leakage inductance $\Delta \sigma = -0.02$ and $\Delta \sigma = -0.05$ respectively ($\chi = 1$)

From Fig.6.5 it can be concluded that for values of $\mu^* < 0.5$ (low torque applications) and for low $\lambda^*$ the error $\varepsilon_l$ decreases exponentially. For large $\lambda^*$ or for large values of $\mu^*$ the error $\varepsilon_l$ increases linearly.

Fig.6.6 Phase error $\varepsilon_l$ resulting from the leakage inductance $\Delta \sigma = -0.02$ and $\Delta \sigma = -0.05$ respectively ($\chi = 0.7$)

By comparing Fig.6.6 with Fig.6.5 it can be concluded that the normalized time constant $\chi$ does not influence significantly the phase error $\varepsilon_l$, particularly not for values of $\mu^* > 1$ (high torque applications).

6.4. Machine parameter variations and torque estimation

When an induction machine operates under the conditions implied by vector control ($\Phi_R = \phi_{Rx}$) the torque of an induction machine with two pair of poles ($p = 2$) is expressed as:
\[ T_e = \frac{3}{2} p \ i_{sy} \Phi_{Rx} = 3 M i_{sy} i_{sx} = \frac{3}{2} M |i_s|^{2} \sin(2\gamma) \] (6.18)

The absolute error of the electrical torque caused by variations of the phase error \( \gamma = \gamma^* \pm \epsilon \) is presented as:

\[ \Delta T_e = \Delta \gamma \left( \frac{\partial T_e}{\partial \gamma} \right)_{\gamma = \gamma^*} = 3 M i_{sx}^{\ast 2} (1 - \mu^{\ast 2}) \epsilon \] (6.19)

where \( \Delta \gamma = \epsilon \) is a small displacement around the steady state \( \gamma^* \). More convenient is the presentation of the normalized torque error \( \Delta T_e^* \) where the normalization is done by using the value of the torque reference \( T_e^* \):

\[ \Delta T_e \ [\text{p.u.}] = \frac{\Delta T_e^*}{T_e^*} = \frac{i_{sx}^{\ast 2} (1 - \mu^{\ast 2}) \epsilon}{i_{sy}^{\ast} i_{sx}^{\ast}} = \frac{(1 - \mu^{\ast 2}) \epsilon}{\mu^{\ast}} \] (6.20)

Since both components \( \epsilon_r \) and \( \epsilon_l \) of the phase error angle \( \epsilon \) in equation (6.17) are dominant, the relative torque error is divided into two components \( \Delta T_e(\epsilon_r) \) and \( \Delta T_e(\epsilon_l) \). The influence of the phase error \( \epsilon_r \) on the relative torque error \( \Delta T_e \) for various values of \( \chi \) and two values of the specific speed \( \lambda^\ast \) is shown in Fig.6.7.

![Fig.6.7 Torque error resulting from the phase error \( \epsilon_r \); stator resistance error is \( \Delta r_s = 0.2 \); parameter \( \lambda^\ast = 0.5 \) and \( \lambda^\ast = 1 \) respectively](image)

From these figures can be concluded that the torque error \( \Delta T_e(\epsilon_r) \) diverges towards infinity when \( \mu^\ast < 1 \), for \( \mu^\ast = 1 \) the torque error \( \Delta T_e(\epsilon_r) = 0 \) and for the values of \( \mu^\ast \gg 1 \) the torque error reaches a constant value.

Fig.6.8 represents the influence of the phase error component \( \epsilon_l \) on the relative torque error. The results are provided for two values of the leakage inductance error \( \Delta \sigma \) while \( \lambda^\ast \) is the parameter.
Fig. 6.8 Torque error resulting from the phase error $\varepsilon_i$: the leakage inductance error $\Delta \sigma = -0.02$ and $\Delta \sigma = -0.05$ respectively ($\chi = 1$)

In Fig. 6.8 the torque error $\Delta T_e(\varepsilon_i)$ increases with the applied torque ($\mu^*$) and with the leakage inductance error $\Delta \sigma$. It is clear from the figures that the influence of the mechanical speed is insignificant.

Fig. 6.9 Torque error resulting from the phase error $\varepsilon_i$: the leakage inductance error $\Delta \sigma = -0.02$ and $\Delta \sigma = -0.05$ respectively ($\chi = 0.7$)

Fig. 6.9 shows the phase error $\varepsilon_i$ when $\chi = 0.7$. Compared with Fig. 6.8 this result shows that $\chi$ has for low torque applications only a minor effect on $\Delta T_e(\varepsilon_i)$.

6.5. Machine parameters and speed estimation

In the proposed control algorithm the mechanical speed of the rotor $\omega_m$ is a matter of estimation. Assuming that the number of machine pole pairs $p = 1$ the rotor speed is represented as:

$$\omega_m = \omega_s - \omega_2 = \omega_s - \frac{R_R}{M} i_{sy} = \omega_s - \frac{1}{T_R} \tan \gamma$$  \hspace{1cm} (6.21)

where the symbols are:
\( \omega_s \) - synchronous angular speed,

\( \omega_2 \) - slip frequency,

\( T_R \) - rotor time constant.

The accuracy of the estimated rotor speed \( \omega_m \), defined by (6.21), is sensitive to variations of the rotor resistance \( R_R \) and the stator current phase angle \( \gamma \). For the sake of clarity, each effect is analyzed separately.

Introducing small variations of the rotor resistance \( \Delta R_R \) around reference values \( R_R^* \), the absolute error of the estimated rotor speed is:

\[
\Delta \omega_m(\Delta R_R) = \Delta \omega_2(\Delta R_R) = \Delta R_R \left( \frac{\partial \omega_2}{\partial R_R} \right)_{R_R = R_R^*, \gamma = \gamma^*} = \frac{\Delta R_R}{T_R^*} \mu^* \quad (6.22)
\]

Similarly to the analysis of the torque error, small variations of the phase error angle \( \Delta \gamma = \varepsilon \) around angle \( \gamma^* \) are introduced. According to equation (6.17) the influence of the composite phase error \( \varepsilon \) on the rotor speed error \( \Delta \omega_m \) can be separated into two additive components shown in (6.23) and (6.24):

\[
\Delta \omega_m(\varepsilon_r) = \Delta \omega_2(\varepsilon_r) = \Delta \gamma \left( \frac{\partial \omega_2}{\partial \gamma} \right)_{R_R = R_R^*, \gamma = \gamma^*} = \frac{\varepsilon_r}{T_R^*} (1 + \mu^*2) \quad (6.23)
\]

\[
\Delta \omega_m(\varepsilon_l) = \Delta \omega_2(\varepsilon_l) = \Delta \gamma \left( \frac{\partial \omega_2}{\partial \gamma} \right)_{R_R = R_R^*, \gamma = \gamma^*} = \frac{\varepsilon_l}{T_R^*} (1 + \mu^*2) \quad (6.24)
\]

where \( \Delta \gamma = \varepsilon \), \( \Delta r_R = \Delta R_R/R_R^* \), \( T_R^* = M/R_R^* \) and \( \mu^* = \tan(\gamma^*) \).

The following equations are obtained by introducing normalized rotor speed error terms where the normalization is done by using the reference mechanical speed \( \omega_m^* \):

\[
\Delta \omega_m(\Delta r_R) \text{ [p.u.]} = \frac{\Delta \omega_m(\Delta r_R)}{\omega_m^*} = \frac{\Delta r_R \mu^*}{\omega_s^* T_R^* - \mu^*} \quad (6.25)
\]

and

\[
\Delta \omega_m(\varepsilon_r) \text{ [p.u.]} + \Delta \omega_m(\varepsilon_l) \text{ [p.u.]} = \frac{\Delta \omega_m(\varepsilon_r) + \Delta \omega_m(\varepsilon_l)}{\omega_m^*} = \frac{1 + \mu^*2}{\omega_s^* T_R^* - \mu^*} (\varepsilon_r + \varepsilon_l) \quad (6.26)
\]

Graphical presentation of the relative mechanical speed error component \( \Delta \omega_m(\Delta r_R) \) in (6.25) is presented in Fig.6.10 at a low stator frequency \( (\omega_s^* = 0.032 \text{ p.u.}) \). The rotor resistance variations \( \Delta r_R \) is chosen as a parameter. This result reveals exponential increase of the speed error \( \Delta \omega_m(\Delta r_R) \) for low values of \( \mu^* \).
Fig. 6.11 represents the component \( \Delta \omega_m (\varepsilon_1) \) defined in (6.26) at a low stator frequency \((\omega_s^* = 0.032 \text{ p.u.})\). The specific time constant \( \chi \) is chosen as a parameter.

**Fig. 6.10 Rotor speed error \( \Delta \omega_m \) for various rotor resistance errors \( \Delta r_R \)**

**Fig. 6.11 Rotor speed error \( \Delta \omega_m \) due to the stator resistance error \( \Delta r_s = 0.2 \)**

Fig. 6.12 shows the relative variation of the speed error component \( \Delta \omega_m (\varepsilon_1) \) as a function of \( \mu^* \). The effect of two relative leakage error values \( \Delta \sigma \) is observed.

**Fig. 6.12 Rotor speed error \( \Delta \omega_m \) due to the leakage inductance error \( \Delta \sigma = -0.02 \) and \( \Delta \sigma = -0.05 \) respectively \((\chi = 1, \omega_s^* = 0.032 \text{ p.u.})\)**

In both examples shown in Fig. 6.12 the error functions have their minimum for \( \mu^* = 1.5 \). The actual rotor speed, represented via the specific speed \( \lambda^* \), does not influence significantly this error.

Fig. 6.13 compared to Fig. 6.12 shows the minor influence of the normalized time constant \((\chi = 0.7)\) on the same speed error component \( \Delta \omega_m (\varepsilon_1) \).
Fig. 6.13 Rotor speed error $\Delta \omega_m$ due to the leakage inductance error $\Delta \sigma = -0.02$ and $\Delta \sigma = -0.05$ respectively ($\chi = 0.7$, $\omega_s^* = 0.032$ p.u.)

6.6. Parameter changes and flux estimation
The rotor flux in the machine, in the synchronous x-y reference frame, is defined as:

$$\phi_{Rx} = \phi_R = M i_{sx} = M i_s^* \cos(\gamma)$$  \hspace{1cm} (6.27)

The flux presented in equation (6.27) depends only on the phase angle $\gamma$ between the x-component of the stator current $i_{sx}$ and the stator current space vector $i_s^*$. Assuming small angle variations $\Delta \gamma = \varepsilon$ and the phase error $\varepsilon = \varepsilon_r + \varepsilon_l$, two flux error components are recognized:

$$\Delta \phi_R(\varepsilon_r) = \Delta \gamma \left( \frac{\partial \phi_R}{\partial \gamma} \right)_{\gamma = \gamma^*} = -\varepsilon_r M i_{sy}^*$$  \hspace{1cm} (6.28)

$$\Delta \phi_R(\varepsilon_l) = \Delta \gamma \left( \frac{\partial \phi_R}{\partial \gamma} \right)_{\gamma = \gamma^*} = -\varepsilon_l M i_{sy}^*$$  \hspace{1cm} (6.29)

The relative flux errors assuming $\Delta \gamma = \varepsilon_r - \varepsilon_l$ is normalized in (6.30) and (6.31):

$$\Delta \phi_R(\varepsilon_r)_{[p.u.]} = \frac{\Delta \phi_R(\varepsilon_r)}{\phi_R^*} = \frac{M i_{sy}^*}{M i_{sx}^*} \varepsilon_r = \mu^* \varepsilon_r$$  \hspace{1cm} (6.30)

$$\Delta \phi_R(\varepsilon_l)_{[p.u.]} = \frac{\Delta \phi_R(\varepsilon_l)}{\phi_R^*} = \frac{M i_{sy}^*}{M i_{sx}^*} \varepsilon_l = \mu^* \varepsilon_l$$  \hspace{1cm} (6.31)

The influence of the error in the stator resistance $\Delta r_s$ on the flux error $\Delta \phi_R$ is presented in Fig.6.14 and for various values of the specific speed $\lambda^*$ and specific time constant $\chi$. 
Fig. 6.14 Rotor flux error $\Delta \Phi_R$ resulting from the stator resistance error $\Delta r_s = 0.2$ normalized time constant $\chi$ is parameter; $\lambda^* = 0.5$ and $\lambda^* = 1$ respectively

The growth of the specific torque current $\mu^*$ beyond the nominal value does not influence further increase of the flux error. Besides, these two figures reveal that the mechanical speed has a small influence on the flux error $\Delta \Phi_R(\varepsilon_i)$.

The influence of the stator leakage inductance $\Delta \sigma$ on the estimated flux error component $\Delta \Phi_R(\varepsilon_i)$ is shown in Fig. 6.15.

Fig. 6.15 Rotor flux error $\Delta \Phi_R$ resulting from the leakage inductance error $\Delta \sigma = -0.02$ and $\Delta \sigma = -0.05$ respectively ($\chi = 1$)

Compared to the flux error $\Delta \Phi_R(\varepsilon_i)$, the error $\Delta \Phi_R(\varepsilon_i)$ produced by the leakage inductance $\Delta \sigma$ appears to be much more significant due to the exponential grow of the error with the specific torque current $\mu^*$. The mechanical speed shows also here insignificant influence on the flux error $\Delta \Phi_R(\varepsilon_i)$. Also the relative time constant $\chi$ has a minor influence as shown in Fig. 6.16.
6.7. Conclusions

The parameters of the induction machine change during operation due to physical phenomena like temperature variation, iron saturation, skin effect, etc. Therefore, field-oriented control methods using a mathematical model for calculating flux, torque and speed result in errors of these. The results of this study are presenting graphically quantitative relations between particular error coefficients used by the control. For the complete range of variations of the specific torque current $\mu^*$ (from zero to 200 % of the rated torque) it is observed that the error can be significant for the various parameters applied in the model. For the applied shaft sensorless control at low stator frequencies (low-speed) is therefore the accurate parameter identification important in order to maintain tolerable values of these errors.
7.

UNWANTED EFFECTS AT LOW STATOR FREQUENCIES

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7. UNWANTED EFFECTS AT LOW STATOR FREQUENCIES

7.1. Introduction
Two major issues are recognized in an ac drive operating at low-speeds. In the first place, the converter part must be capable to generate stator currents with extremely low harmonic components. This is an issue with respect to torque harmonics, to avoid the shaft cogging at low rotor speed. In the second place, ac drive control methods often get into well-known difficulties at near zero stator frequencies.

There are also a number of unwanted effects inherent to all families of ac drives that can be neglected at nominal stator frequency (nominal speed) and which are commonly disregarded. In applications which require the operation of the drive at near zero stator frequencies these effects become very important for the performance of the drive and require attention from the designer to avoid them. The effects are related to:
- nonlinear voltage drop over semiconductor switches,
- dc offset in the system,
- asymmetries in the system.

These effects superimpose additional torque harmonic components and significantly deteriorate the performances of the low-speed drive.

7.2. Distortion of the stator current
Since each switch in the converter topology is in practice a serial connection of a diode and an IGBT, the composite \( u_d^{-i_d} \) characteristic of any switch in the topology is a highly nonlinear function. This introduces higher harmonics in the stator current and creates further difficulties in the use of the field-oriented control at low modulating frequencies. Fig.7.1 represents an experimentally provided voltage-current characteristic \( u_d^{-i_d} \) of one switch and the fitted exponential curve. The following relation is valid:

\[
i_d = (e^{3u_d} - 1) \times 10^{-3.05} \quad (7.1)
\]

The expression (7.1) is an approximation of a simple diode characteristic. However, it will be used in the further analysis as the compound characteristic of the switch.
### 7.2.1. Analysis of the switch voltage-drop

The voltage drop over a semiconductor switch is studied with a simplified one-phase converter model. Fig. 7.2 shows the scheme of the applied model.

![Fig. 7.2 One-phase model of the converter including nonlinear switch voltage-drop effect](image)

In practice, the switch consists of an IGBT and a diode connected in series. Therefore, the appropriate experimental voltage-current characteristic shown in Fig. 7.1 combines the superimposed property of each of them. The further analysis includes the simplified model provided by equation (7.1).

The matrix converter is replaced by a variable-frequency voltage source $e_g$. Current-controlled voltage sources $u_{d1}(i_d)$ to $u_{d4}(i_d)$ are emulating the nonlinear voltage characteristic of switches $S_1$ and $S_2$. In the applied scheme, all switches and diodes are assumed to be ideal.

**Example**

Fig. 7.3 and Fig. 7.4 present the simulation results for two different low modulating frequencies $f_s=5$ Hz and $f_s=20$ Hz and an internal switching frequency of $f_{sw}=2$ kHz. The inductive load is attached to the output terminal of the converter. The converter parameters are: $L_j=5$ mH, $R_j=0.3$ Ω, $C=300$ μF, $L_L=10$ mH, $R_L=2$ Ω, $f_{sw}=2$ kHz, $d=0.3$. The results presented in Fig. 7.3 and Fig. 7.4 show the load current distortion caused by the $u_d_i_d$ characteristic of the switch which becomes important for lower output frequencies. This is because of the decrease of the stator impedance, which makes the part of the voltage drop over the switch relatively large in comparison to the excitation voltage. The current distortion also gets worse when the load impedance $Z_L$ decreases (current through switches $i_d$ remain nominal).
Fig. 7.3 Load current \( i_s \) and source voltage \( e_g \):
\[ \text{stator frequency } f_s = 20 \text{ Hz} \]

Fig. 7.4 Load current \( i_s \) and source voltage \( e_g \):
\[ \text{stator frequency } f_s = 5 \text{ Hz} \]

The basic principle of the phenomena is shown in Fig. 7.5. It considers the worst case when the load impedance \( Z_L \) is zero \( (R_L = \omega_L L_L = 0) \) and the current is shaped only by the nonlinear resistance of the switches. The voltage \( u_d \) is equal to the sinusoidal power supply voltage \( e_g \) and the result is a heavily distorted current through a switch \( i_d \). The number and the amplitude of induced higher current harmonics are directly related to the shape of the \( u_d-i_d \) characteristic as shown in Fig. 7.5. A lower voltage amplitude causes a higher distortion of the current \( i_d \) and of the stator current \( i_s \).
Fig. 7.5 Current distortion because of the $u_d$-$i_d$ characteristic of the switch

Fig. 7.6 presents the FFT of the worst case diode current $i_d$ (shown in Fig. 7.5) for a one-phase converter model. Since for the practical three-phase application the motor star point is isolated, triplet harmonics do not exist. The results show a strong relationship between the stator frequency and the current distortion.
The assumption that the motor control maintains the stator current and the flux at a constant level throughout the range of stator frequencies \( \omega_\mathrm{s} \) is graphically shown in Fig.7.7. The analytical expression is given by expression (7.2).

![Graph](image)

**Fig.7.7 Ratio between the resistive stator voltage drop and the total stator voltage drop**

Equation (7.2) shows the ratio between the voltage drop over the equivalent stator resistance which includes also the resistance of the switches and the back electromotive force \( E_\mathrm{R} \) induced by the rotor flux. The back electromotive force is a linear function of the stator frequency and it increases much faster than the voltage drop caused by the stator resistance if the source frequency increases.

\[
\frac{R_\mathrm{s}I_\mathrm{s}}{E_\mathrm{R}} = \frac{R_\mathrm{s}I_\mathrm{s}}{\omega_\mathrm{s} \phi_\mathrm{R}}
\]  \hspace{1cm} (7.2)

### 7.2.2. Effects on the machine torque

Current harmonic components generate a pulsating torque \( \Delta T_\mathrm{c} \) which is obtained from the asymmetrical T-scheme presented in the synchronous \( x-y \) reference frame:

\[
\Delta T_\mathrm{c} = 3 \phi_\mathrm{R} \sum_{n=1}^{\infty} i_{\mathrm{sy}(6n\pm1)}
\] \hspace{1cm} (7.3)

Assuming a constant rotor flux \( \phi_\mathrm{R} \), the torque pulsations are produced by the combined action of stator current harmonic components \( \sum_{n=1}^{\infty} i_{\mathrm{sy}(6n\pm1)} \). Fig.7.8 gives the graphical presentation of equation (7.3), encountering only the fifth and the seventh current harmonic as the most important. In the \( x-y \) reference frame the fundamental harmonic \( \vec{\tau}_{\mathrm{s1}} \) rotates synchronously with the flux vector \( \vec{\phi}_\mathrm{R} \). The fifth harmonic rotates at the speed \( 6\omega_\mathrm{s} \) and in the opposite direction from the first harmonic. The seventh harmonic rotates at the speed \( 6\omega_\mathrm{s} \) and in the same direction with respect to the first harmonic \( \vec{\tau}_{\mathrm{s1}} \). Since the tip of the resulting current space vector \( \vec{\tau}_\mathrm{s} \) follows an elliptic trajectory,
the torque-producing current component \( i_{sy} = i_{sy1} + i_{sy5} + i_{sy7} \) oscillates periodically with the frequency \( 6\omega_s \). Assuming a constant rotor flux, the oscillation of the current component \( i_{sy} \) generates a sixth torque harmonic.

![Fig.7.8 Stator current components in x-y frame](image)

- **Example**

Fig. 7.9 shows the experimental rotor speed and stator phase current at low-speed of the system with open control loops. The FFT of the stator current is shown in Fig. 7.10. The additional fifth and the seventh harmonic in the stator current are capable of producing a speed ripple at low speed.

![Fig.7.9 One phase stator current \( i_s \) (2 A/div) and rotor speed \( n_r \) (20 rpm/div), stator frequency \( f_s = 0.5 \) Hz](image)
From the low frequency measurement of the stator current FFT ($f_s=0.5$ Hz), it is anticipated that the amplitude of the fifth harmonic is about 100 times and the seventh harmonic is nearly 200 times lower than the fundamental harmonic. The following values are assumed for the machine with two pole pairs ($p=2$), as it was used in the laboratory setup:

- nominal flux in the machine,
- magnetizing inductance,
- rotor inertia.

The measured quantities are:

- stator angular frequency,
- amplitude of the stator current vector,
- flux-producing component of the current,
- torque-producing component of the current.

Because the y-components of the fifth and the seventh harmonic have amplitudes one hundred and two hundred times smaller than the fundamental and considering the worst case (both harmonic components are in phase) follows:

- fifth harmonic

$$i_{sy5} = 0.01 i_{sy} \sin(6 \omega_s t) = 0.024 \sin(18.8 t)$$

(7.4)

- seventh harmonic

$$i_{sy7} = 0.005 i_{sy} \sin(6 \omega_s t) = 0.012 \sin(18.8 t)$$

(7.5)

The combination of the fifth and the seventh current harmonic generates the sixth torque harmonic with a maximum amplitude:

$$\Delta T_{e6} = 3 \Phi_R (i_{sy5} + i_{sy7}) = 0.077 \sin(18.8 t)$$

(7.6)
Neglecting the mechanical load and considering only the fifth and the seventh current harmonic the mechanical equation of the machine is:

\[ J \frac{d\omega_m}{dt} = \Delta T_{e6} \]  (7.7)

Finally, for the fundamental frequency \( f_s = 0.5 \) Hz the ripple of the mechanical frequency produced by the sixth torque harmonic is:

\[ \Delta \omega_m = \int_{0}^{0.2} \frac{\Delta T_{e6}}{J} \, dt = 0.43 \text{ rad/s} \]  (7.8)

which in terms of the rotor speed ripple \( \Delta n_r \) gives:

\[ \Delta n_r = \frac{60}{2\pi} \Delta \omega_m = 4.1 \text{ s}^{-1} \]  (7.9)

This result is in a good agreement with the experimental results shown in Fig.7.9.

7.3. Additional effects on system performances
The experimental results presenting the minimum controllable torque, also revealed additional torque and speed harmonics beside the sixth harmonic component. This occurs only at stator frequencies lower than \( f_s < 1.5 \) Hz \( (n_t = 40 \text{ rpm or in per-unit system } 40/1500 = 0.026 \text{ p.u.}) \). In this section the problems which provoke additional torque oscillations and negatively influence the performance of the torque at low stator frequencies are discussed. These problems are:

- dc offset in the system,
- asymmetry in the induction machine due to asymmetrical stator and rotor parameters,
- asymmetries of the converter and semiconductor switches which have various dynamical and steady-state performance (turn-on time, turn-off time, on-resistance).

7.3.1. System dc offset
The stator current in the experimental setup is monitored by current transformers. Because they contain operational amplifiers, these suffer from a well-known problem, the dc offset. The acquisition card which samples analog stator current information and converts it into a digital signal contributes its own dc offset. The final effect has a cumulative character and ends up, due to the closed loop control, with torque harmonics having a dominant role at low stator frequencies (low rotor speed). This influence is particularly big because the inertia of the rotating masses is not large enough to damp the speed oscillations due to the torque ripple at low stator frequency.

- Example
Fig.7.11 represents simulation results when 4% dc offset is superimposed on the output signal of the current transformer. The vector control obtains the current information with the superimposed error and generates oscillations of the calculated torque current \( i_{sync} \) and, consequently, the oscillations of the torque itself. Fig.7.11
points out the offset between measured currents $i_{sa}$ and $i_{sb}$. The frequency of the generated oscillations is the same as the stator current frequency. The rest of the drive components have been assumed ideal. Chosen operating point is $T_e = 0.1$ p.u., $\phi_R = 1$ p.u., stator frequency $f_s = 0.02$ p.u., rotor speed $n_r = 0.014$ p.u. (reference values are $T_e = 10$ Nm, $\phi_R = 0.1$ Wb, $f_s = 50$ Hz, $n_r = 1500$ s$^{-1}$).

![Graph showing $n_r$, $\Delta n_r$, $i_{sa}$, $i_{sb}$, $\Delta i_s$, $T_e$ over time](image)

*Fig. 7.11* Effects of 4% current dc offset on the torque, $f_s = 0.02$ p.u.

Fig. 7.12 represents the amplitude of the speed ripple $\Delta n_r$ versus the rotor speed $n_r$, both in p.u. system, for 2% and 4% dc offset values of the stator current as parameters.

![Graph showing $\Delta n_r$ versus $n_r$ with 2% and 4% dc offset](image)

*Fig. 7.12* Speed ripple $\Delta n_r$ versus rotor speed $n_r$ (p.u.)
The strong dependence of the ripple on the rotor speed is evident. All values are given in the per-unit system.

Fig. 7.13 shows graphically the effect of the dc offset on the electrical torque. In the synchronous reference frame the dc offset current vector component $\vec{I}_{dc}$ rotates clockwise with the synchronous speed $\omega_s$. The tip of the measured stator current space vector $\vec{I}_s + \vec{I}_{dc}$ (with the superimposed dc offset) follows the circle trajectory with the same speed $\omega_s$. Although the actual flux $\vec{\phi}_R$ and the actual current $\vec{I}_s$ rest in the x-y reference frame, the calculated rotor flux $\vec{\phi}_c$ becomes asymmetrical because of the error in the current reading. From the space-vector presentation shown in Fig. 7.13 it is clear that the calculated torque $T_{ec} = 3(i_{sy} + i_{dcy})\phi_c$ has a ripple produced by the oscillating torque current component $i_{dcy} = i_{dc} \sin(\omega_s t)$. In the close-loop vector control this effect produces the actual torque ripple presented in Fig. 7.11.

![Fig. 7.13 Space vector presentation of the current dc offset effect](image)

**7.3.2. Asymmetries of the asynchronous machine**

Although often assumed symmetrical, in reality the parameters of the three-phase induction machine differ from phase to phase. This happens because of the manner in which the windings are designed and wound. For example, small machines with a single-layer winding show larger variations in phase inductances and resistances than larger machines with a two-layer winding.

At low stator frequencies the stator resistance dominates the machine’s total input impedance. Therefore, in the following elaboration only stator resistance variation between machine phases is considered.

- **Example**

  The simulation result shown in Fig. 7.14 represents the effect of machine asymmetries on the torque control at low stator frequencies.
It is assumed that the stator resistance of one phase differs 5% from the resistance of the other two phases which is a common case. The resulting torque (speed) oscillation has a frequency twice as high as the fundamental stator frequency.

From Fig.7.15 it is clear that the amplitude of oscillations grows nearly exponentially with the lowering of the stator frequency (rotor speed $n_r$). This is for two reasons. In the first place, the frequency of the torque ripple is low so that the speed ripple cannot be smoothed by inertia of rotating masses, secondly, the participation of stator resistance voltage drop in all three phases is proportionally much larger than the voltage drop over the stator reactances.

Considering low-speed operation, the generation of the torque harmonic with the frequency twice higher than the stator fundamental frequency can be proved analytically. This can be done by presenting the asymmetrical three-phase machine via its direct, indirect and zero component symmetrical subsystem.
- Example

For the sake of simplicity, it will be assumed that stator reactances are negligible compared to stator resistances and the converter line voltages are symmetrical and sinusoidal. The asymmetrical resistive load (machine) is connected in star. The set of symmetrical line voltages is:

\[ U_{ab} = U \quad U_{bc} = \mathfrak{Z}^2 U \quad U_{ca} = \mathfrak{Z} U \]

where \( \mathfrak{Z} = e^{j2\pi/3} \).

Phase admittances and corresponding direct, indirect and zero components are:

\[ Y_{sa} = 1/R_{sa} = 1/2.5 = 0.4 \, S \]
\[ Y_{sb} = 1/R_{sb} = 1/2.388 = 0.41 \, S \]
\[ Y_{sc} = 1/R_{sc} = 1/2.388 = 0.41 \, S \]

\[ Y_{sa} = 1/3(Y_{sa} + Y_{sb} + Y_{sc}) = 0.412 \, S \]
\[ Y_{sd} = 1/3(Y_{sa} + 3Y_{sb} + 3^2Y_{sc}) = -0.00623 \, S \]
\[ Y_{si} = 1/3(Y_{sa} + 3^2Y_{sb} + 3Y_{sc}) = -0.00623 \, S \]

Direct indirect and zero phase voltage components are:

\[ U_{sd} = 1/3 (U_{ab} - \mathfrak{Z}^2 U_{bc}) = \sqrt{3}/3 \, U \, e^{j\pi/6} \]
\[ U_{si} = 1/3 (U_{ab} - \mathfrak{Z} U_{bc}) = 0 \]
\[ U_{s0} = -(Y_{si}/Y_{s0}) U_{sd} - (Y_{sd}/Y_{s0}) U_{si} = 0.0087 \, U \, e^{j\pi/6} \]

Finally, asymmetrical phase current components are:

\[ I_{sd} = Y_{sd} U_{s0} + Y_{s0} U_{sd} + Y_{si} U_{si} = 0.23 \, U \, e^{j\pi/6} \]
\[ I_{si} = Y_{si} U_{s0} + Y_{sd} U_{sd} + Y_{s0} U_{si} = -0.036 \, U \, e^{j\pi/6} \]
\[ I_{s0} = Y_{s0} U_{s0} + Y_{si} U_{sd} + Y_{sd} U_{si} = 0 \]

With this, the symmetrical components of phase currents have been found. As expected, the zero current component \( I_{s0} = 0 \) due to the isolated motor star point.

Fig.7.16 gives the space-vector presentation of symmetrical stator current components in the synchronous reference frame \( x-y \).

*Fig.7.16 Symmetrical current components as vectors*
The space vector of the direct component of the stator current $\vec{i}_{sd}$ rotates synchronously with the rotor flux $\phi_R$. In the steady-state reference frame $\alpha-\beta$ the space vector of the inverse current component $\vec{i}_{sj}$ rotates in the opposite direction from the direct current component with the same frequency $\omega_s$. Consequently, in the synchronous reference frame $x-y$ vector $\vec{i}_{sd}$ rests and vector $\vec{i}_{sj}$ rotates with the speed $2\omega_s$. The product of the flux and the direct stator current component $i_{sdy}$ renders the torque which is constant. The product of the flux and the inverse stator current $i_{sy}=i_{sy} \sin(2\omega_s t)$ generates the torque ripple shown in Fig. 7.14.

7.3.3. Asymmetries of the converter part

Fig. 7.17 represents the influence of the converter asymmetries on the performance of the drive. Considering low stator frequency applications, we can conclude that resistive elements in the converter circuit dominate reactive elements. Therefore, as a parameter the serial (parasitic) resistance $R_1$ (Fig. 4.1) on the converter side is chosen. The ratio between the error and accurate values is taken to be unrealistically large ($R_{1o}/R_1=2$) due to the insignificant influence on the speed harmonics. The oscillation generated in this way shows the same behaviour as the oscillation generated by asymmetries in the machine and the same graphical explanation is valid. All values are given in per-unit system.

Simulation shows that the effect of converter asymmetries is far less important than effects of the dc offset and asymmetries in the induction machine. Fig. 7.17 shows nearly exponential dependence of the relative speed ripple that does not exceed $\Delta n_r=0.001$ p.u. This result is reached in the unrealistic case when the resistance $R_{1o}$ of one converter phase is twice as high as the resistance in the other two phases. Therefore, the asymmetries of the converter part do not have practical importance.

7.4. Conclusions

The influence of some effects on the low-frequency operation of the induction motor drive has been discussed. The effects are related to:

- voltage drop over semiconductor switches,
- dc offset in the control system,
• parameter variations between phases of the induction machine,
• parameter variations between phases of the power converter.

The influence of the voltage drop over semiconductors at low speeds has to be considered when designing a high-performance vector control. Experiments show that due to the nonlinear voltage-current characteristic of semiconductor switches the spectrum of odd current harmonics is injected into the stator. This is an important unwanted effect which creates difficulties at low frequencies also in other families of converter types (e.g. the bridge converter based on PWM modulation). The experimental setup shows that the utilization of a simple current controller involves serious problems at near-zero stator frequencies. The controller, which is actually based on the tracking of the modulus of the stator current space vector, loses the valuable information about the instant value of each phase current. Only more sophisticated current controllers that monitor each stator current independently can detect and suppress undesirable harmonic components.

The effect of dc offset, as present in the drive, has also been elucidated. Simulations show that it might affect rotor speed by generating oscillations with the same frequency as the stator current. Therefore, during the design of the control of the low-speed ac drive, special attention has to be paid to the dc offset in the system.

The effect of parameter variation between the phases of the machine or converter is not detected during the experimental work thus proving their minor importance. However, this effect has been elaborated using simulation methods and supported with numerical results. It is proved that the machine and converter asymmetries have negligible influence on the ac drive performance as shown by varying the stator resistance $R_s$ and converter resistance $R_1$.

At low-voltage applications the voltage drop over semiconductor switches has the most important effect.
8.

SYSTEM STABILITY AT LOW FREQUENCIES

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8. SYSTEM STABILITY AT LOW FREQUENCIES

8.1. Introduction
The steady-state operation of an induction machine can become unstable even in situations where the machine is powered with a symmetrical, sinusoidal three-phase voltage supply. Considerable research has already been done indicating that an induction machine is typically lightly damped when operating at low stator frequencies and light loads [1]...[4]. This can result in motor instabilities at low speeds.

An additional difficulty is that contemporary induction machines are commonly powered by variable frequency converters. A filter is usually placed between the rectifier and inverter part (VSI) or a capacitor is placed between the motor terminals and the output terminals of the inverter (CSI). The exchange of energy between the filter components and the magnetic field of the machine can also cause the ac drive to become dynamically unstable while operating at low stator frequencies [5].

Hereafter the stability of the proposed converter-machine system is analyzed for two different sets of machine parameters. The first set represents the machine with a considerable damping (machine A) while the second set of parameters belongs to the machine with light damping (machine B). The purpose of this stability study is twofold:
- first, to prove that the particular oscillations observed at low stator frequencies during experiments with machine A are produced by effects other than the light damping of the converter and induction machine as a system,
- second, to show that the controlled converter-machine system performs a stable operation even when a lightly damped machine (machine B) is used.

The generalized theory of linearization, showing the influence of various machine parameters on the damping, is assumed to be known [1]...[5]. The model is valid only under the assumptions of small-signal analysis, which means that this theory gives inaccurate results for large transient states and therefore can not be used.

In this particular application each phase of the proposed matrix-čuk converter behaves as a second-order active filter having a complex dynamical relation with the induction machine. Due to the system complexity (the system is of the ninth order) only the numerical results considering the characteristic equation of the system are presented. The final stability analysis applies the root-locus method.

8.2. Definitions and system equations
The discussion about stability assumes an idealized symmetrical induction machine with the following characteristics:
- uniform air gap,
- linear magnetic circuit,
- sinusoidal distribution of stator windings,
- arrangement of rotor windings producing smooth rotor MMF,
- temperature dependence of motor parameters is negligible.
The idealized matrix-čuk converter part has the following characteristics:
- the matrix converter is treated as an ideal sinusoidal voltage source,
- all parasitic resistances and inductances related to the semiconductor switches and passive components are neglected.

Fig.8.1 presents a two-phase equivalent model of the averaged model of the converter and the induction machine. The model is defined in the synchronous x-y reference frame, thus steady-state variables are defined via constant dc values enabling a linearization technique based on small perturbations. The machine is presented as an asymmetrical T-scheme where the stator and rotor leakage inductances are concentrated on the stator side ($\sigma L_s$).

The total system of state equations in the synchronous reference frame is presented in matrix form as:

$$\dot{x} = Kx + Vu$$  \hspace{1cm} (8.1)

where the state variable vector $x$ and the input vector $u$ are defined as:

$$x^T = [i_{1x} \quad i_{1y} \quad u_{Cx} \quad u_{Cy} \quad i_{sx} \quad i_{sy} \quad \phi_{Rx} \quad \phi_{Ry}]$$  \hspace{1cm} (8.2)

$$u^T = [e_x \quad e_y]$$  \hspace{1cm} (8.3)

Matrix $K$ is presented by:
The additional mechanical equation is:

\[ J \dot{\omega}_m = k_q (i_{sy} \phi_{Rx} - i_{sx} \phi_{Ry}) - T_L \]  

(8.5)

Matrix \( V \) is defined as:

\[
V_T = \begin{bmatrix}
\frac{d}{L_1} & 0 & 0 & \frac{d}{\sigma L_s} & 0 & 0 \\
0 & \frac{d}{L_1} & 0 & 0 & \frac{d}{\sigma L_s} & 0 \\
0 & 0 & \frac{1-d}{C} & \omega_s & \frac{d}{C} & 0 \\
0 & 0 & \frac{1-d}{C} & -\omega_s & \frac{d}{C} & 0 \\
\frac{R_R + R_R}{\sigma L_s} & \omega_s & \frac{R_R}{\sigma (1-\sigma)L_s^2} & \omega_s & \frac{R_R}{\sigma L_s} & \omega_s \\
0 & -\omega_s & \frac{R_R}{\sigma L_s} & \omega_s & \frac{R_R}{\sigma (1-\sigma)L_s^2} & \omega_s \\
0 & 0 & 0 & 0 & 0 & -\omega_s \\
0 & 0 & 0 & 0 & 0 & -\omega_s \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix}
\]

(8.6)

Hence, the complete system is of the ninth order and is a highly nonlinear system. As the analytical solution of the nonlinear system is not available, a linearization of the system must be performed.

### 8.3. System linearization

Small displacement equations valid for small excursions around the steady-state operating point are established by introducing the following replacement terms:

\[ x = X + \Delta x \]  

(8.7)

In the matrix equation (8.7), which includes also \( \omega_m = \omega_{m0} + \Delta \omega_m \), \( X \) presents the vector of steady-state variable values and \( \Delta x \) defines the small displacement vector.
The linearization finally renders the following matrix equation which is provided after eliminating the steady-state terms:

\[
\begin{bmatrix}
\Delta \dot{x} \\
\Delta \dot{\omega}_m
\end{bmatrix} = K_k \begin{bmatrix}
\Delta x \\
\Delta \omega_m
\end{bmatrix} + V_k \begin{bmatrix}
\Delta u \\
\Delta T_L
\end{bmatrix}
\]  
(8.8)

The column vector \([\Delta u \; \Delta T_L]^T\) presents the forcing functions of the system and is assumed to be zero for the stability analysis. Then, the solution of the equation (8.8) can be found as:

\[
\begin{bmatrix}
\Delta x \\
\Delta \omega_m
\end{bmatrix} = e^{K_k t} \begin{bmatrix}
\Delta x(0) \\
\Delta \omega_m(0)
\end{bmatrix}
\]  
(8.9)

The last column vector in (8.9) represents an arbitrary set of initial conditions. The transition matrix exponential function \(e^{K_k t}\) defines the unforced response of the system. The system matrix \(K_k\) is the linearized matrix \(K\) defined in equation (8.4) after the small-signal theory is applied on expression (8.1).

\[
K_k = \begin{bmatrix}
0 & \omega_s & -\frac{1-d}{L_1} & 0 & 0 & 0 & 0 & 0 & 0 \\
-\omega_s & 0 & 0 & -\frac{1-d}{L_1} & 0 & 0 & 0 & 0 & 0 \\
\frac{(1-d)}{C} & 0 & 0 & \omega_s & -\frac{d}{C} & 0 & 0 & 0 & 0 \\
0 & \frac{(1-d)}{C} & -\omega_s & 0 & 0 & -\frac{d}{C} & 0 & 0 & 0 \\
0 & 0 & \frac{d}{\sigma L_s} & 0 & -R_s \frac{R_R}{\sigma L_s} & \omega_s & \frac{R_R}{\sigma(1-\sigma)L_s} & \frac{p\omega_m}{\sigma L_s} & \frac{p\omega_m}{\sigma L_s} & \frac{\Phi_{Ryo}}{\sigma L_s} \\
0 & 0 & \frac{d}{\sigma L_s} & -\omega_s & -R_s \frac{R_R}{\sigma L_s} & \omega_s & \frac{R_R}{\sigma(1-\sigma)L_s} & \frac{p\omega_m}{\sigma L_s} & \frac{p\omega_m}{\sigma L_s} & \frac{\Phi_{Rxo}}{\sigma L_s} \\
0 & 0 & 0 & 0 & R_R & 0 & -\frac{R_R}{(1-\sigma)L_s} & (\omega_s - p\omega_m) & -\Phi_{Ryo} & \Phi_{Rxo} \\
0 & 0 & 0 & 0 & 0 & R_R & -\Phi_{Ryo} & \frac{R_R}{(1-\sigma)L_s} & (\omega_s - p\omega_m) & \Phi_{Rxo} \\
0 & 0 & 0 & 0 & 0 & 0 & -k_q \frac{\Phi_{Ryo}}{f} & k_q \frac{\Phi_{Rxo}}{f} & k_q \frac{i_{syo}}{f} & -k_q \frac{i_{xio}}{f} & 0
\end{bmatrix}
\]  
(8.10)

The analysis is further simplified by introducing the normalized terms:

\[
K_n, \; V_n \quad - \text{system matrices } K \text{ and } V,
\]

\[
\Delta x_n, \; \Delta u_n \quad - \text{small displacement vectors } \Delta x, \Delta u,
\]

\[
f_s = \frac{\omega_s}{\omega_n} \quad - \text{stator angular frequency},
\]
\[ f_s = (\omega_n - p\omega_m)/\omega_n \] - rotor angular frequency,
\[ r_s = R_s/Z_n \] - stator resistance,
\[ r_R = R_R/Z_n \] - rotor resistance seen from the stator side,
\[ x_l = L_1\omega_n/Z_n \] - reactance \( L_1 \),
\[ x_c = 1/(Z_nC\omega_n) \] - capacitive reactance \( C \),
\[ x_m = (1-\sigma)L_s\omega_n/Z_n \] - main reactance,
\[ x_{s\sigma} = \sigma L_s\omega_n/Z_n \] - leakage reactance,
\[ i_{sx} = i_{sx}/I_n \] - stator current \( x \) component,
\[ i_{sy} = i_{sy}/I_n \] - stator current \( y \) component,
\[ \phi_{Rx} = \omega_n\phi_{Rx}/U_n \] - rotor flux \( x \) component,
\[ \phi_{Ry} = \omega_n\phi_{Ry}/U_n \] - rotor flux \( y \) component,
\[ H = J(\omega_n/p)^2/1.5U_nI_n \] - inertia [s],

where:
\[ U_n, I_n, \omega_n \] - nominal stator voltage, current and frequency,
\[ Z_n = U_n/I_n \] - nominal impedance,
\[ p \] - number of pole pairs.

By replacing normalized terms in matrix \( K_k \) the normalized matrix \( K_n \) is obtained:

\[
K_n = \begin{bmatrix}
0 & f_s & -\frac{1-d}{x_l} & 0 & 0 & 0 & 0 & 0 & 0 \\
-f_s & 0 & 0 & -\frac{1-d}{x_l} & 0 & 0 & 0 & 0 & 0 \\
(1-d)x_c & 0 & 0 & f_s & -dx_c & 0 & 0 & 0 & 0 \\
0 & (1-d)x_c & -f_s & 0 & 0 & -dx_c & 0 & 0 & 0 \\
0 & 0 & \frac{d}{x_{s\sigma}} & 0 & -\frac{r_s+r_R}{x_{s\sigma}} & f_s & \frac{r_R}{x_m x_{s\sigma}} & \frac{f_s-f_R}{x_{s\sigma}} & \phi_{Ry} \\
0 & 0 & 0 & \frac{d}{x_{s\sigma}} & -f_s & -\frac{r_s+r_R}{x_{s\sigma}} & \frac{f_s-f_R}{x_{s\sigma}} & \frac{r_R}{x_m x_{s\sigma}} & -\phi_{Rx} \\
0 & 0 & 0 & 0 & r_R & 0 & -\frac{r_R}{x_m} & f_r & -\phi_{Ry} \\
0 & 0 & 0 & 0 & 0 & r_R & -f_r & -\frac{r_R}{x_m} & \phi_{Rx} \\
0 & 0 & 0 & 0 & -k_q \frac{\phi_{Ry}}{H\omega_n} & k_q \frac{\phi_{Rx}}{H\omega_n} & k_q \frac{i_{sy}}{H\omega_n} & -k_q \frac{i_{sx}}{H\omega_n} & 0
\end{bmatrix}
\]

(8.11)

In the per-unit system equation (8.9) becomes:
\[
\begin{bmatrix}
\Delta x_n \\
\Delta \omega_m / \omega_n
\end{bmatrix} = e^{\omega_n K_n t} \begin{bmatrix}
\Delta x_n(0) \\
\Delta \omega_m(0) / \omega_n
\end{bmatrix}
\]

(8.12)

In matrix \( K_n \) all parameters are dimensionless (except \( H \) [s]) and presented in the per-unit system. At this point conventional methods of establishing the system stability can be applied like the Routh-Hurwitz or Nyquist method. This study employs the root-locus method.

### 8.4. Stability study

The local stability of the system of the matrix-ćuk converter and machine is assured if all elements of the transition matrix \( e^{K_n t} \) approach zero as the time approaches to infinity. Parameters of the system are presented in table I. Asymptotic behaviour of all matrix elements occurs when all roots of the characteristic equation have negative real parts. Normalization of the roots of the characteristic equation to the nominal frequency \( \omega_n (\omega_n = 314 \text{ \text{rad/s}}) \) is realized by introducing the parameter \( \Lambda' \). The roots of the characteristic equation are presented for those values of \( \Lambda' = \Lambda / \omega_n \) for which the determinant (8.13) is zero:

\[
|\Lambda' I + K_n| = 0
\]

(8.13)

The system contains nine eigenvalues. The stability study is based on checking the trajectories of all eigenvalues in the \( \Lambda' \) complex plane which are provided by variation of the normalized inertia \( H \) in matrix \( K_n \) from 0 to \( \infty \). The trajectories provided are identical to those obtained by the well-established root-locus method, where the parameter \( 1/H \) corresponds to the conventional root-locus gain parameter [4], [5].

The stability study of the converter-machine system is presented for two machines: machine \( A \) with strong damping and machine \( B \) with light damping.

#### 8.4.1. System with strongly damped machine

Table I represents parameters of a system with a strongly damped machine (machine \( A \)) which was used in the experimental setup.

### TABLE 1 - nominal parameters of the system with the machine A

<table>
<thead>
<tr>
<th>( x_f )</th>
<th>( x_c )</th>
<th>( d )</th>
<th>( r_s )</th>
<th>( r_R )</th>
<th>( x_{s\sigma} )</th>
<th>( x_m )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.062</td>
<td>0.42</td>
<td>0.5</td>
<td>0.094</td>
<td>0.13</td>
<td>0.31</td>
<td>3.37</td>
</tr>
</tbody>
</table>

The stability is tested for various converter parameters (table II) which were actually used in the experimental setup. It is shown in [3], [4], [5] that a particular combination of machine parameters \( r_s, r_R, x_{s\sigma}, x_m \) and load exists where no regions of instability are found over a normal range of frequency (0-100 Hz).
The graphical presentation of numerically obtained trajectories are presented in the following figures. Each trajectory of nine poles of the characteristic equation is shown in the complex plane at low stator frequencies and for light loads.

Fig.8.2 shows the root-loci of one representative of each complex pair of system eigenvalues (total of nine eigenvalues) for two different values of stator frequency $f_s$ and nominal parameter values of machine $A$. The root-locus trajectories are obtained by varying the relative inertia $H$ from 0 (denoted in all figures as $O$) to $\infty$ (denoted in all figures as $\times$). All roots $\Lambda'$ of the characteristic equation have negative real parts. All terms in the state transition matrix will consequently decrease in time.

Fig.8.3 presents the root-loci of all nine eigenvalues for two values of the converter capacitance $x_c$ ($x_c=0.35$, $x_c=0.57$). Generally, the shape of the trajectories are similar to those shown in Fig.8.2. The imaginary part of eigenvalues decreases when the parameter $x_c$ decreases. In both figures for frequency $f_s=0.01$ poles $p_1...p_4$ are fixed while $p_5$ and $p_6$ are moving in the complex plane. Pole $p_5$ is important because it provides light damping at very low stator frequencies and nominal values of inertia $H$. 

<table>
<thead>
<tr>
<th>Fig.</th>
<th>$x_l$</th>
<th>$x_c$</th>
<th>$d$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fig.8.2</td>
<td>0.062</td>
<td>0.42</td>
<td>0.5</td>
</tr>
<tr>
<td>Fig.3</td>
<td>0.062</td>
<td>0.57</td>
<td>0.5</td>
</tr>
<tr>
<td>Fig.4</td>
<td>0.062</td>
<td>0.35</td>
<td>0.5</td>
</tr>
<tr>
<td>Fig.4</td>
<td>0.026</td>
<td>0.42</td>
<td>0.5</td>
</tr>
<tr>
<td>Fig.5</td>
<td>0.087</td>
<td>0.42</td>
<td>0.5</td>
</tr>
<tr>
<td>Fig.5</td>
<td>0.062</td>
<td>0.42</td>
<td>0.3</td>
</tr>
<tr>
<td>Fig.5</td>
<td>0.062</td>
<td>0.42</td>
<td>0.7</td>
</tr>
</tbody>
</table>
Pole $p_4$ starts to move for higher values of the stator frequency ($f_s=0.2$).

![Root-locus trajectories for various values of the capacitor value $x_c$: relative inertia $0<H<\infty$; $\alpha:H=0$; $x:H=\infty$](image)

**Fig. 8.3** Root-locus trajectories for various values of the capacitor value $x_c$; relative inertia $0<H<\infty$; $\alpha:H=0$; $x:H=\infty$

Fig. 8.4 presents the behaviour of the converter-machine $A$ system when the converter reactance $x_1$ varies. The increase of the parameter $x_1$ results in the decrease of the imaginary part of $p_5$.

![Root-locus trajectories for various values of the inductor value $x_1$: relative inertia $0<H<\infty$; $\alpha:H=0$; $x:H=\infty$](image)

**Fig. 8.4** Root-locus trajectories for various values of the inductor value $x_1$; relative inertia $0<H<\infty$; $\alpha:H=0$; $x:H=\infty$

For each stator frequency the trajectories of pole $p_5$ and $p_5'$ converge towards the same point at the real axis when inertia $H \to \infty$ (denoted as $x$). As in Fig. 8.3, when the value of the mechanical inertia is large it becomes the dominant parameter. Poles $p_1$ to $p_4$ are fixed and thus not susceptible to changes of the inertia $H$, while poles $p_5$ and $p_6$ move in the complex plane far from the critical region.

The behaviour of system eigenvalues, when the duty cycle $d$ is changing, is presented in Fig. 8.5. This parameter is important for the dynamical behaviour of the Čuk
converter and therefore it is necessary to analyze its influence on the complete system. For larger values of the duty cycle $d$ the imaginary part of poles $p_5$ and $p_6$ decreases while the real part becomes more negative enabling positive damping of the system. All other poles for both values of the stator frequency are fixed for all values of $H$.

Fig.8.5 Root-locus trajectories for various values of the duty cycle $d$
relative inertia $0<H<\infty$; $a): H=0$; $x): H=\infty$

Fig.8.6 shows a simulation of the transient behaviour for an open loop control. The system performs strong damping as predicted by the root-locus stability study.

Fig.8.6 Simulation of the system (machine A) response on step changes of the frequency $f_s$; rotor frequency $f_r=0$ p.u., total time $t=2$ s, $H=0.1$ s

As indicated in the Fig.8.6 b), the stator frequency has been changed in steps of $\Delta f_s=0.06$ p.u. resulting in transient states showing that the system of the matrix converter and machine A does not suffer from instabilities. Moreover, for a stator frequency $f_s=0.24$ p.u. the frequency of free oscillations ($f_o=0.1$ p.u.) can be recognized.
in Fig.8.2 on the trajectory of the poles $p_4$ and $p_5$. The imaginary part of these poles represents a relative frequency of the oscillation $\omega_\circ=2\pi f_\circ=0.65$ p.u. This simulation result of the actual experimental drive justified its stability due to the strong damping of the applied induction machine.

8.4.2. System with lightly damped machine

The first induction machine discussed in paragraph 8.4.1, with parameters presented in table I, has a rather low efficiency. The machine has a large leakage inductance $x_{s\sigma}$ as well as large stator and rotor resistances ($r_s$, $r_R$). However, such high values of the stator and rotor resistances improve the stability of the machine itself and, in turn, the stability of the whole system.

Next, table III presents parameters of the proposed system consisted of the converter and machine $B$ with machine parameters being considerably smaller than those of machine $A$ (table I), thus machine $B$ is lightly damped.

<table>
<thead>
<tr>
<th>TABLE III - nominal parameters of the system with the machine $B$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$x_1$</td>
</tr>
<tr>
<td>0.062</td>
</tr>
</tbody>
</table>

Results of the stability analysis are obtained by applying the same linearized model of the system derived in section 8.3. with adapted machine parameters. Results of numerical calculations for various stator frequencies $f_s$ are shown in the following.

Fig.8.7 represents trajectories of the poles of the converter-machine system for stator frequencies $f_s=0.01$ and $f_s=0.1$ (per-unit), respectively.

![Fig.8.7 Root-locus trajectories of the system (machine B)](image)

The inertia $H$ varies from 0 (denoted as $\circ$) to $\infty$ (denoted as $\times$). In Fig.8.7 for $f_s=0.01$ three of the presented poles have complex pairs ($p_4$, $p_5$, $p_6$) while three poles are real and negative ($p_1$, $p_2$, $p_3$). For $f_s=0.1$ four poles are complex ($p_3$...$p_6$) and one is real ($p_1$). However, for both examples of low stator frequency operation ($f_s=0.01$ and $f_s=0.1$) each pole has a negative real part providing a positive damping.
Fig. 8.8 represents the trajectories of the poles when the stator frequency is $f_s=0.5$ and $f_s=0.8$. Four poles are complex ($p_3...p_6$) and the fifth pole $p_1$ is a real one.

![Root-locus contours of the system (machine B)](image)

*Relative inertia $0<H<\infty$; $\circ:H=0$; $x:H=\infty$*

The influence of the complex pole $p_3$ with a minimum imaginary part is remarkable in both situations. When the parameter $H$ is between the values $100<H<150$ s the system reaches the stability limit and becomes unstable. The area of instability increases when the stator frequency increases. Theoretically, this particular area of instability is troublesome but in practice this is not a problem whatsoever. This is because the positive damping for $f_s=0.8$ (instability region) occurs for unrealistically high values of inertia ($H=150$ s) which is in practice around the value $H=0.1$ s.

Fig. 8.9 shows the simulation result of the transient behaviour after a step change of the stator frequency $f_s$.

![Simulation of the response of the system (machine B) on the step change of $f_s$](image)

*Fig. 8.9 Simulation of the response of the system (machine B) on the step change of $f_s$; rotor frequency $f_r=0$ p.u., total time $t=10$ s, $H=0.1$ s*
The converter-machine system operates in the open loop control mode for the parameters presented in Table III. The simulation shows that the system is lightly damped at a stator frequency around $f_s=0.38$, when the frequency of free oscillation is $f_o=0.13$ ($\omega_o=2\pi f_o=0.816$, nominal frequency $f_n=60$ Hz). The frequency of the free oscillation can be recognized in Fig.8.8 ($f_s=0.5$) where the trajectory of pole $p_4$ has an imaginary part for $H=0.1$ s approximately equal to $\omega_o=0.816$.

The simulation result for the closed loop speed control mode is presented in Fig.8.10. The speed command is changed in steps $\Delta f_s=0.15$ resulting in variations of the stator frequency from $f_s=0.43$ to $f_s=0.37$. The high torque fluctuation is needed to compensate the speed ripple. However, the system under closed loop speed control shows now a very good dynamical response.

![Fig.8.12 Simulation of the system (machine B) with closed loop control; stator frequency $f_s=0.4$ p.u.; rotor frequency $f_r=0$ p.u., total time $t=1$ s, inertia $H=0.1$ s](image)

### 8.5. Conclusions

When an asynchronous machine operates at low-speed, low-frequency and light load, it might start oscillating. This phenomenon has been studied in this particular application by means of the method of linearization. The characteristic equation of the converter-machine system has been provided and then, the roots of the characteristic equation has been studied by means of the root-locus method. The trajectories of the poles are presented in the complex plane. The applied method of linearization is only valid for small disturbances around the steady state point of operation.

The stability of the proposed drive in an open loop control mode has been studied for two different machines with different losses. The results of simulations with a strongly damped machine are presented in a numerical form and indicate that there are no regions of instability.
The simulation and the analytical results of the system containing a machine with low damping, show that the converter (in an open loop application) does not significantly influence the dynamical behaviour of the lightly damped machine (Fig.8.9). The regions of instability are still present and similar to the ones which appear when the same induction machine is directly connected to a low-frequency grid [4]. However, the results of simulation for this system, operating under a closed loop control, has shown a very good dynamical behaviour.

References


9.

SIMULATION RESULTS
OF THE LOW-SPEED DRIVE

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9. SIMULATION RESULTS OF THE LOW-SPEED DRIVE

9.1. Introduction

Computer simulations play an important role in the research on low-frequency, low-speed ac drives from its very beginning. Each phase of the program is accompanied by the use of computer models: testing the ac-ac operation of the cuk topology, proving the linearization method for analysis and, finally, the simulation of the proposed shaft and voltage sensorless control. Also, simulations gave us hints to solve problems during the process of building up the experimental drive the implementation of the control.

The presented results of the simulation show the ideal working conditions of the proposed drive, ignoring effects such as nonlinear voltage-current characteristic of semiconductor devices, system asymmetries, iron saturation, stator and rotor parameter variation, etc. A linearized converter model has been used for simulation purpose. Only low-frequency operation, up to \( f_s = 5 \) Hz has been considered. The results include:
- the dynamical characteristics of the drive,
- the steady-state sensitivity for parameter changes of the proposed control.

9.2. Dynamical behaviour of the controlled drive

9.2.1. Drive parameters

Machine A and the converter parameters used in the computer model are chosen to fit the actual parameters found in the experimental setup. Thus, simulations present the approximate performance of the real drive. The parameters of the converter model are:

- \( L_1 = 7 \) mH,
- \( C = 300 \) μF,
- \( f_{sw} = 1800 \) Hz.

Actual data on the machine name plate are:
1.7 kW; 220/380 V; 8.7/5 A; 50 Hz; \( \cos \varphi = 0.7 \); 1390 s\(^{-1}\)

Experimentally measured machine parameters used for the computer model:

- \( R_s = 2.388 \) Ω (0.0542 p.u.)
- \( L_s = 297 \) mH (2.12 p.u.)
- \( R_R = 3.308 \) Ω (0.075 p.u.)
- \( L_R = 272 \) mH (1.94 p.u.)

9.2.2. Simulation results

The machine has to be controlled by the converter in such a way that the rotor flux \( \phi_R \) is kept constant and the electrical torque \( T_e \) at the desired level. In each of the following experiments, during the start up of the drive, the flux is built up first followed by the torque command.
Fig. 9.1 shows the building up of the rotor flux of the machine $\phi_R$ and torque $T_e$ with a blocked rotor. The flux command is presented by its flux current component $i_{sx}^*$. The calculated torque in the flux estimator is represented by the calculated torque current component $i_{syn}$. The actual torque of the machine model is presented as $T_e$. Both the flux and the torque are built up consecutively with ramp-like functions.

![Diagram showing flux and torque](image)

*Fig. 9.1 Building up the rotor flux $\phi_R$ and the torque $T_e$ with a blocked rotor (scale for all waveforms: ±1 p.u.; total simulation time $t=4$ s)*

For a better event observation, the $x$, $y$, $\alpha$ and $\beta$ components of machine variables are depicted versus time. The maximum value of the electrical torque $T_e$ is 0.5 p.u. while the mechanical speed $\omega_m$ is kept zero. The maximum value for the stator current components $i_{s\alpha}$ and $i_{s\beta}$ is 0.7 p.u. The command of the flux-creating current $i_{sx}^*$ has a maximum value 0.3 p.u. The maximum of the calculated torque current component $i_{syn}$ is 0.62 p.u. The simulation confirms that even at zero stator frequency (torque command is zero) the proposed control enables the calculation of the rotor flux. The current component $i_{sx}^*$, being the output of the flux comparator, shows a small disturbance during the build up of the torque. Consequently, the flux vector is also not constant during the transient states in the machine. The simplified machine model used to calculate the transient angle $\zeta$ is the main cause for this small disturbances because this model ignores the derivative terms in the machine equations. A similar behaviour is also present in the following simulations. However, the steady state performs a stable operation.

Fig. 9.2 shows the same conditions as for Fig. 9.1 except that the rotor is free. At the end of the simulation the rotor speed reaches $\omega_m=0.08$ p.u. The current component $i_{sx}^*$ suffers from transients due to the simplified calculation of the transient angle $\zeta$. 
Fig. 9.2 Building up the flux and torque in a loaded machine
(scale: speed ±0.2 p.u., other waveforms ±1 p.u.; simulation time t=4 s)

The response on a step change of the torque command from 0.5 p.u. to 0.7 p.u. is shown in Fig. 9.3. A settling time of 250 ms is observed.

Fig. 9.3 Step change response on the change of the torque command
(scale: speed ±0.2 p.u.; other waveforms ±1 p.u.; simulation time t=4 s)
Beside the calculated value of the current component $i_{sy}$ the same figure shows the command value of the torque current $i_{sy}^\ast$.

Fig.9.4 shows the process of breaking and the change of the direction of the rotor motion. The torque $T_e$ is changed from +0.8 p.u. to -0.8 p.u. The speed $\omega_m$ changes from +0.1 p.u. to -0.1 p.u. Current amplitudes of $i_{s\alpha}$ and $i_{s\beta}$ are 0.8 p.u. Total simulation time is 4 seconds.

![Graph showing $T_e$, $\omega_m$, $i_{s\alpha}$, $i_{s\beta}$, $i_{sx}^\ast$, and $i_{syc}$ over time](image)

*Fig.9.4 Breaking and changing the direction of rotation (scale: speed $\pm0.2$ p.u.; other waveforms $\pm1$ p.u.; simulation time $t=4$ s)*

The simplified machine model, used in this application, with neglected derivative terms in stator and rotor equations, results also here in worsening the dynamical behaviour of the system. In this simulation is the flux current $i_{sx}^\ast$ oscillating as a result of the inaccurate estimation of the rotor flux $\Phi_c$ during the transient.

This simulation shows a better, smooth transition of the torque around the zero value which proves the versatility of the method.

The system response on the step change of the load at low stator frequency is shown in Fig.9.5. The mechanical speed $\omega_m$ changes from 0.07 p.u. to 0.01 p.u. The current component $i_{sx}^\ast$ equals 0.2 p.u. and $i_{syc}$ 0.5 p.u. The result shows a smooth transient that lasts a half of a second.
9.3. Steady-state sensitivity of the control

For a controlled low-speed drive the results of simulation are presented for the steady-state point of drive operation with the applied shaft-sensorless control. It is the objective to consider the sensitivity of the control to variations of the stator and rotor resistances and the stator leakage inductance.

Nominal values for the torque and flux are:

\[ T_{e\, nom} = 10 \text{ Nm} \quad (1 \text{ p.u.}) \]
\[ \Phi_{R\, nom} = 0.7 \text{ Wb} \quad (1 \text{ p.u.}) \]

The following symbols are used:

- \( T_R \) - rotor time constant,
- \( T_c \) - time constant of phase-delay circuits,
- \( \chi = \frac{T_c}{T_R} \) - ratio of the filter time constant and the rotor time constant,
- \( \Delta r_s = R_s^* - R_s \) - stator resistance error,
- \( \Delta \ell = \ell^* - \ell \) - leakage inductance error.

The most critical control condition is considered at standstill of the rotor. The results of simulation are presented in the following figures.

Fig.9.6, Fig.9.7 and Fig.9.8 present the deflection of the rotor flux \( \Phi_R \), torque \( T_e \) and synchronous frequency \( \omega_s \) with respect to the reference values caused by the stator resistance error \( \Delta r_s \). Computations are presented for various values of the parameter \( \chi \). All values are presented in the per unit system.
Considering the low-speed operation, the rotor speed $\omega_m$ for all three presented results is 0. The stator resistance error $\Delta r_s$ varies between 0 and 8% which is a realistic assumption. The parameter $\chi$ varies from 0.2 to 2 showing the influence of the variation of the rotor resistance $R_R$ on the flux, torque and synchronous frequency. For $\chi=1$ the rotor resistance error is zero.

Fig.9.9 and Fig.9.10 show the dependence of the torque $T_e$ and the rotor flux $\phi_R$ on the stator resistance error $\Delta r_s$ differently. The actual torque differs from its command value for $\Delta r_s \neq 0$. As shown in Fig.9.10 the flux in the machine becomes coupled to $i_{sy}$ and its space vector is no longer in parallel to the x-axis of the x-y frame.
Simulation Results

Fig. 9.9 Torque $T_e$ versus command torque current component $i_{sy}^*$

Fig. 9.10 Flux $\phi_R$ versus command torque current component $i_{sy}^*$

Fig. 9.11, Fig. 9.12 and Fig. 9.13 present the steady-state sensitivity of the frequency $\omega_s$, electrical torque $T_e$ and rotor flux $\phi_R$ on the leakage inductance error $\Delta l$.

Fig. 9.11 Stator frequency $\omega_s$ versus leakage inductance error $\Delta l$

Fig. 9.12 Electrical torque $T_e$ versus leakage inductance error $\Delta l$

Fig. 9.13 Rotor flux $\phi_R$ versus leakage inductance error $\Delta l$
Apparently, the synchronous frequency $\omega_s$ and the electrical torque $T_e$ increase their value with the error. Results are presented for two values of the parameter $\chi$. The value of the rotor flux $\phi_R$ decreases with the leakage inductance error $\Delta l$. The results show that the control is much less sensitive for the error of the leakage inductance $\Delta l$ than for the same error of stator resistance.

Fig.9.14, Fig.9.15 and Fig.9.16 present the sensitivity of the electrical torque $T_e$, rotor flux $\phi_R$ and calculated value of the mechanical speed $\omega_m$ on the rotor resistance variations $\Delta r_r$.

These variations have a minor effect on the accuracy of the torque and flux of the machine, what was expected from the analysis of the control block scheme (Fig.5.4 chapter 5). Namely, the outer flux and torque loops are independent of the rotor resistance. However, the parameter $\Delta r_r$ affects the calculation of the slip frequency $\omega_2$ and, consequently, the rotor speed.
As can be observed from Fig.9.16 the relation between the slip frequency \( \omega_2 \) and the rotor time constant \( T_R \) is linear:

\[
\omega_2 = R_R \frac{i_{sy}^*}{\phi_R^*} = \frac{1}{T_R} \frac{i_{sy}^*}{i_{sx}^*}
\]

(9.1)

**9.4. Conclusions**

Presented simulations are proving the versatility of the proposed topology with the implemented control. It is shown that the complete system performs a stable operation at the most critical operating points: control of the torque at zero speed, smooth change of the direction of the rotation and a step change of the torque or load.

The model used for these simulations does not include all physical phenomena present in the experimental drive like saturation, time and space harmonics, noise, etc. The common feature of these phenomena is that they all derate the performance of the drive.

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10. APPLICATION

10.1. Introduction
In order to prove the theoretical concept of the drive, an experimental 2 kW laboratory setup was designed and constructed. Hereafter, the practical application of the complete drive with the applied shaft sensorless vector control is presented.

10.2. The matrix-čuk converter topology
In this section the design and the construction of the 2 kW converter prototype are discussed, included the PWM modulation control. For the sake of simplicity, in this application two dc power supplies are used in stead of a three-phase power supply. This operation requires no additional alterations of the PWM controller.

10.2.1. Construction of the power part

Switches
Fig.10.2 represents the total scheme of the power part of the matrix-čuk converter. Since the converter topology performs a four-quadrant operation the switches must conduct in both directions. For this purpose, among other possibilities, scheme c) depicted in Fig.10.1, is used because of simple connection to a gate driver.

In practice, this means that the total number of transistors needed for this converter topology is 24, which is the main disadvantage of the concept. However, there are signs that the semiconductor technology will develop bidirectional semiconductor switches in the future. Thus the number needed for the whole topology will be reduced to 12. In the experimental setup, switches are assembled from TOSHIBA IGBT (15 A, 1000 V) and THOMSON fast-recovery epitaxial diodes (20 A, 1000 V, recovery time \( t_r = 35 \) ns). Each switch is controlled with a galvanically isolated IGBT driver stage. The main problem regarding the design of switches is the short-circuit and open-circuit protection. The duty cycle control of the matrix converter part and the čuk part should prevent those problems. However, the imperfection of logic circuits and power semiconductors (e.g. the time delay of the digital control circuitry, and the variable turn-on/turn-off times of IGBT’s) create short periods (in the order of hundreds of nanoseconds) of short-circuit events.
Fig.10.2 Electrical scheme of the power part
On the other hand, the nonoptimal programming of the duty cycle logic control is a cause of short instances of open circuits (all switches $S_{ij}$ and $S_{2j}$ $i=1,2,3$ $j=1,2,3$ are opened for a short time instance of 500 µs). Since the inductances $L_1$, $L_2$, $L_3$ and the stator inductances contain a considerable amount of energy, a sudden cut of current paths can be a cause of dangerous overvoltage peaks across the switches.

Two measures have been taken to prevent the short-circuit and open-circuit events:

- Each switch in the topology has connected in series a tiny ferrite core which adds a small inductance $L_o=2.5$ µH (Fig.10.3) to the parasitic inductance of the layout. This suppresses excessive short-circuit collector currents by decreasing $di_C/dt$. Considering the worst case, when e.g. switches $S_{11}$ and $S_{31}$ in Fig.10.2 are short-circuiting two input phases, follows:

$$\frac{di_C}{dt} = \frac{1}{2L_o} \frac{2E}{E} = \frac{E}{L_o} \quad (10.1)$$

- To prevent dangerous overvoltage peaks across the switches caused by sudden open switch states, $RC$ snubbers are applied as in Fig.10.3. Elements $C_{sn}$ and $R_{sn}$ are chosen as:

$$C_{sn} = \frac{Q}{E} = \frac{i_C t_f}{2E} \quad (10.2)$$

$$R_{sn} = \frac{E}{i_C} \quad (10.3)$$

The dissipation of the resistor $R_{sn}$ is:

$$P_R = \frac{2}{3} C_{sn} E^2 f_{sw} \quad (10.4)$$

In equations (10.2) to (10.4) the following symbols are used:

- $R_{sn}$, $C_{sn}$ - snubber resistor and capacitor respectively,
- $E$, $i_C$ - supply voltage and the maximum of the collector current respectively,
- $Q$ - electrical charge of the capacitor $C_{sn}$,
- $P_R$ - power dissipation of the resistor $R_{sn}$,
- $t_f$, $f_{sw}$ - transistor fall time and the switching frequency respectively.

**Power supply**

In the experimental setup two central tapped dc power supplies $E=100$ V have been used in place of the three-phase ac supply as presented in Fig.10.2. This is done for the purpose of measurement. The PWM controller of the matrix converter generates the output voltage regardless of the input voltage. Since the power grid is connected to the switching branch of the Čuk converter, this topology has disadvantageous injection of
the switching harmonic into the power grid. To overcome the problem of nonideal supply, which has nonzero impedance, in parallel to each power supply $E$ is a filter capacitor ($C_f=3600 \ \mu F$) connected.

**Inductor design**

In the power topology, the inductors $L_1$, $L_2$, $L_3$ and the capacitors $C_1$, $C_2$, $C_3$ are designed to match the power demands of the converter. Considering, in the first place, inductor design there are a couple of steps that should be made:

- The basic criterion for the inductor design is to keep the current ripple within ±20% boundaries of the average inductor current. Assumptions are:
  - maximum dc current through the inductor $I=10$ A, maximum ripple $\Delta i=2$ A,
  - time duration of the duty cycle $d$ state $t_d=100 \ \mu s$,
  - input voltage $E=150$ V.

The inductance $L=L_1=L_2=L_3$ is:

$$L = \frac{E t_d}{\Delta i} = 7.5 \ \text{mH} \quad (10.5)$$

- Copper current density $J$ is typically:
  $$250 \cdot 10^4 < J < 1000 \cdot 10^4 \ \text{A/m}^2$$

- By introducing $A_{Fe}$, the cross-section of the core, the number of winding turns $w$ is calculated by:

$$w = \frac{L I}{B A_{Fe}} \quad (10.6)$$

- Area-product approach $A_p$ of the inductor design gives:

$$A_p = A_{Fe} A_w = \frac{L I^2}{B J k} = 1.25 \ \times 10^{-6} \ \text{m}^4 \quad (10.7)$$

where the definitions of the window area $A_w$ and the core cross-section $A_{Fe}$ are shown in Fig.10.4. Also:

- $k$ - empirical window utilization factor (between 0.3-0.7),
- $I$ - average current through the inductor,
- $B$ - magnetic induction ($\pm 1$ T).

- The core with the area product $A_p$ bigger than calculated in (10.7) is chosen.

- Finally, the inductor air gap $l_g$ is calculated as:

$$l_g = \frac{\mu_0 A_{Fe} w^2}{L} \quad (10.8)$$

where:

- $l_g$ is the air gap length,
- $\mu_0$ is the magnetic permeability of the vacuum,
- $w$ is the number of inductor winding turns.
The core chosen for inductor $L_1$ ($L_2$ and $L_3$) is oversized, as well as the air gap length to prevent saturation of the core in excessive working conditions.

**Choice of capacitors**
The basic criteria for the choice of the capacitors $C_1$, $C_2$, $C_3$ is that each of them must have an energy content large enough to provide a nearly constant voltage $U_C$ over one switching period of the converter ($T_{sw}=500 \mu s$). Assuming the maximum voltage ripple $\Delta u_C=10$ V and that the current through the capacitor $I_C$ is constant the following equation provides a method for calculating the capacitance $C$:

$$\Delta u_C = \frac{1}{C} I_C T_{sw} \quad (10.9)$$

From (10.9) the capacitance $C=300 \ \mu F$ is found as sufficient when the switching frequency is $f_{sw}=2 \ kHz$ and assuming the maximum capacitor voltage $U_c=150 \ V$.

**10.3. Application of the field-oriented controller**
The field-oriented controller is implemented with a EISA-486DX Personal Computer (PC) with a system clock frequency of 33 MHz.

**10.3.1. PC environment**
Two additional A/D cards and an RS232 serial communication interface link the PC and the power converter. Fig.10.5 represents symbolically the information flow between the PC, the converter and the induction machine. The control algorithm uses only the measurement of stator currents $i_s$ in time instants when stator terminals are short circuited. In practice that means that the PC data sampling has to be synchronized with the matrix-čuk converter duty cycle $d$. The
synchro signal (Fig.10.5) initiates data sampling and the calculation process. At the end of each calculation cycle, command values of the stator voltage $u^*_s$ and the stator frequency $\omega_s$ are updated. Digital values of the voltage and frequency are converted into analog dc values via a MIO-16 D/A card and passed to the converter.

**Measurements of the stator current amplitude**

Stator current measurements are a very important part of the control process. Here, beside the instant amplitude, the current derivative in the same instant must be known. As presented in Fig.10.5, three LEM modules are used to sense the stator currents. The output from a LEM is passed to the National Instrument EISA-2000 A/D card with 12 bit resolution. This card, installed into the PC, has four differential simultaneously sampled analog inputs which is an important issue. All three stator phases can thus be monitored in the same instant. The sampling speed is 100 kilosamples per second.

Fig.10.6 represents the synchronization signal synchro and the signal calc that reveals the time distribution of the control algorithm implemented in the PC. During $t_2$ EISA-2000 samples stator currents simultaneously in all three phases and provides the values of averaged stator currents for the needs of the controller. Since this action takes place in the middle of the freewheeling converter operation (1-d state) the sampled currents represent the averaged stator current in each phase throughout one switching period $T_{sw}$.

**Measurements of the stator current derivative**

A peculiar practical problem is the measurement of the stator current derivative $di_s/dt$. The first applied solution was the use of the same EISA-2000 card for this purpose. From two successive current data samples ($i'_s$ and $i''_s$) during the interval $(1-d)T_{sw}$ state and knowing the time between sampling $\Delta t$, the derivative was found. With the assumption of a linear current drop during the period $(1-d)T_{sw}$ holds:

$$\frac{di_s}{dt} = \frac{\Delta i_s}{\Delta t} = \frac{i'_s - i''_s}{\Delta t}$$

(10.10)
However, due to the relatively small difference between two sampled current values (large leakage inductance) and the short time $\Delta t$ (typically 100 $\mu$s) the calculated derivative is often an unstable function with a lot of noise. The direct consequence is the improper calculation of the flux.

In Fig.10.6 the timing diagram of the $di_s/dt$ measurement that employs another A/D card (MIO-16) is represented. During the period $t_1$ MIO-16 for the first time samples three stator currents successively. During the period $t_2$ MIO-16 obtains for the second time current samples delayed for a constant period $\Delta t$ after $t_1$. From the difference between the current values sampled in $t_1$ and $t_3$ the controller calculates the derivative of the stator current $(10.10)$ needed for the flux calculation routine.

To improve the resolution of this sensitive measurement, ac coupling is applied (only the current ripple is recorded without the stator current dc component) and before the A/D conversion the analog ac signal is amplified with the internal MIO-16 amplifier. In this way better results of the flux calculation are achieved. The MIO-16 is a remarkably slower card than the EISA-2000. For this reason, in Fig.10.6 the time difference between $t_1$, $t_3$ on the one side and $t_2$ on the other is so large.

**The shaft and voltage sensorless control algorithm**

Fig.10.7 represents the simplified block diagram of the applied vector control. The control program begins with the initialization procedure of the MIO-16 and the EISA-2000 cards. Next, a very important step is initializing the phase angle $\theta$ (see Fig.5.2) of the reference stator voltage $\hat{u}_s^*$ that has to coincide with the initial phase angle value (normally $\nu=0$) of the calculated rotor flux vector $\Phi_r$. In contrast, the error angle $\varepsilon$ between the real and the calculated rotor flux value deteriorates the performance of the control or even generates unstable operation of the system.

The following step is data sampling. In each control cycle (Fig.10.6) this is done three times ($t_1$, $t_2$ and $t_3$) during the interval of short-circuited stator terminals. Data collected during periods $t_1$ and $t_2$ by MIO-16 are used for the $di_s/dt$ calculation. The EISA-2000 samples data during period $t_2$. Data sampling is followed by three- to two-phase transformation of the stator currents by the means of the software.

Further on, the calculation process begins. From Fig.10.6 it is clear that one calculation loop occurs during two switching periods $T_{sw}$. This is allowed because the dc output command values do not change significantly within one switching period. Therefore, updated values of $u_s^*$ and $\omega_s$ do not necessarily have to be calculated for each period $T_{sw}$. Time $t_4$ is needed by the controller for the calculation of the flux, torque and to update output command values $u_s^*$ and $\omega_s$. The electromotive force $\vec{e}_c$ is calculated from the machine model, estimated value of the rotor speed $\omega_m$ and using the information about $i_s$ and $di_s/dt$. Applying a simple integration method, the calculated rotor flux $\Phi_r$ is obtained. The updated command of dc values of $u_s^*$ and $\omega_s$ are obtained and passed to the analog outputs of MIO-16. Finally, the program stops when the control counter reaches its preset value.
When a PC is utilized for the vector control of an ac drive, one has to consider particular risks. Namely, the PC uses interrupts for its own internal purposes. During interrupts the microprocessor holds the execution of the user program. This results in missing the synchronization signal that triggers the execution of the control algorithm. Over a longer time period the integration error is accumulated and results in a phase error between the reference and the calculated value of the flux.

10.4. Asynchronous Machine
The basic concepts of the proposed topology and the novel control method are proved
by experimenting with an asynchronous machine in laboratory conditions. For better insight, the results of the measurement of the machine parameters are shown.

**10.4.1. Measured machine parameters**
The machine used in the drive is a three-phase induction machine with a wound rotor. The name plate contains the following data:

\[ P_{\text{mech}} = 1.7 \text{ kW} \]
\[ U = 220/380 \text{ V} \]
\[ I = 8.7/5 \text{ A} \]
\[ f_s = 50 \text{ Hz} \]
\[ \cos(\varphi) = 0.7 \]
\[ n = 1390 \text{ s}^{-1} \]

From the name plate data is calculated:

- active electrical input power,
  \[ P_e = \sqrt{3} I U \cos(\varphi) = \sqrt{3} \times 5 \times 380 \times 0.7 = 2303 \text{ W} \]  
  \[ (10.11) \]

- apparent input power,
  \[ S_e = \sqrt{3} I U = \sqrt{3} \times 5 \times 380 = 3291 \text{ VA} \]
  \[ (10.12) \]

- efficiency,
  \[ \eta = \frac{P_{\text{mech}}}{P_e} = 0.7 \]
  \[ (10.13) \]

Measurements with dc and ac current provided the following stator phase resistances and impedances of three input stator phases (the rotor circuit disconnected):

| \( R_{s1} \) [Ω] | 2.388 |
| \( R_{s2} \) [Ω] | 2.388 |
| \( R_{s3} \) [Ω] | 2.39 |
| \( R_s \) [Ω] | 2.388 |

| \( Z_{s1} \) [Ω] | 93.46 |
| \( Z_{s2} \) [Ω] | 94.16 |
| \( Z_{s3} \) [Ω] | 92.90 |
| \( Z_s \) [Ω] | 93.50 |

The information about the stator input impedance \( Z_{si} \) is provided by measuring stator currents and voltages of each phase with a 50 Hz power supply. Bearing in mind the asymmetrical T-scheme of the induction machine, the stator inductance \( L_s \) is:

\[ L_s = \sqrt{(Z_s^2 - R_s^2) / \omega_s^2} \]
  \[ (10.14) \]

Knowing \( R_s \) and \( Z_s \) and placing them in (10.14) the stator inductance is:
\[ L_s = 0.297 \text{ H} \]

From the third series of measurements with power supply frequency 50 Hz (rotor terminals short circuited) the rotor resistance \( R_R \) and the leakage inductance \( \sigma L_s \) are provided:

\[ R_R = 3.308 \text{ \Omega} \]
\[ \sigma L_s = 0.025 \text{ H} \]

The nominal input impedance of the machine is:

\[ Z_n = \frac{U}{I} = \frac{220}{5} = 44 \text{ \Omega} \] \hspace{1cm} (10.15)

In the per-unit system the normalized values of the machine parameters are:

- normalized stator resistance

\[ r_s = \frac{R_s}{Z_n} = 0.0542 \] \hspace{1cm} (10.16)

- normalized rotor resistance

\[ r_R = \frac{R_R}{Z_n} = 0.075 \] \hspace{1cm} (10.17)

- normalized stator inductance

\[ x_s = \frac{L_s \omega_s}{Z_n} = 2.12 \] \hspace{1cm} (10.18)

- normalized stator leakage inductance

\[ x_{s\sigma} = \frac{\sigma L_s \omega_s}{Z_n} = 0.178 \] \hspace{1cm} (10.19)

- normalized stator magnetizing inductance

\[ x_m = \frac{(1-\sigma)L_s \omega_s}{Z_n} = 1.94 \] \hspace{1cm} (10.20)

- normalized inertia

\[ H = \frac{J \omega_s^2}{p^2 \frac{3}{2} U_n I_n} \text{ s} \] \hspace{1cm} (10.21)

Nominal values of the electrical torque and the nominal flux are:
\[ T_e = \frac{P_{\text{mech}}}{\omega_m} = 11.7 \, \text{Nm} \]  

(10.22)

\[ \phi = \frac{U}{\omega_s} = 0.7 \, \text{Wb} \]  

(10.23)

where \( p \) is the number of pole pairs.

### 10.5. Conclusions

The practical implementation of the complete drive has been presented. The purpose of the experimental setup is to prove the validity of the basic idea of the matrix-\( \text{Ä} \)uk power topology and therefore the power ratings of the converter are low. It is shown that the proposed shaft and voltage sensorless control suitable for low stator frequencies.

The proposed topology is suitable only for low to medium power ac drives. A basic limitation is the serial capacitor link that limits the energy flow through the converter. Enlarging the nominal power would require increasing the capacitor bank or increasing the switching frequency according to (10.9). A second drawback is the number of semiconductor devices (24) that has a negative commercial effect and decreases the reliability of the system.

The novel control technique is a versatile solution. It is applicable to a wide range of generally used ac drives. A simple application requires only a PC and the use of a low-level programming language like C or Pascal.
11. EXPERIMENTAL RESULTS OF THE LOW-FREQUENCY DRIVE

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11.1. Introduction
An experimental system of a matrix-ćuk converter and an asynchronous machine (machine $A$) has been designed and constructed. Measurements on the experimental setup, primarily at low speed, have been done. The purpose is to prove the validity of the idea and the theoretical background of the proposed power converter topology in combination with a shaft-sensorless vector control. The first part of this section presents the results related to the matrix-ćuk converter operation. The second part presents results related to the shaft and stator voltage sensorless control.

11.2. Experimental results related to the power converter
The advantages of the proposed matrix-ćuk converter are:
- sinusoidal stator currents with excellent harmonic content,
- rippleless torque even at near-zero stator frequencies,
- four-quadrant operation,
- step-up and step-down converter operation,
- simple implementation of the shaft and stator voltage sensorless control.

The experimental waveform of the three-phase stator current $i_s$ is shown in Fig.11.1.

![Fig.11.1 Three-phase stator currents; 4 A/div, 0.2 s/div](image)

The converter switching frequency is $f_{sw}=2$ kHz and the stator frequency is $f_s=1$ Hz. The current waveforms are symmetrical, sinusoidal with the additional switching frequency ripple reduced by the stator leakage inductance $\sigma L_s$. The calculated torque ripple produced is small (3 %) and its effect on the shaft cogging is negligible.

The FFT of the stator current, as shown in Fig.11.1, is presented in Fig.11.2. Only fundamental harmonic and multiple switching frequency harmonics are present. The amplitude of the fundamental harmonic 1 Hz is 80 dB. The amplitude of the 2 kHz switching harmonic is 40 dB. The third peak at 4 kHz presents the first component of the multiple Nyquist frequency ($2nf_{sw}$, $n=1$). The small figure represents the enlarged
frequency range from 0 to 200 Hz. The FFT record proves the excellent harmonic content of the stator current.

![FFT of the stator current](image)

Fig.11.2 FFT of the stator current; 1kHz/div, 40 dB/div

Fig.11.3 shows the matrix phase current \( i_a \) and voltage \( u_a \) (low stator frequencies). The output matrix phase voltage is composed of voltage sequences \(+E, -E\) and zero over the time interval \( dT_{sw} \) (see also Fig.11.13). During the interval \((1-d)T_{sw}\) the capacitor voltage \( u_C \) in these particular figures is nearly zero \((u_C=0)\). The lower trace in the figure represents the matrix output phase current \( i_a \) over the time interval \( dT_{sw} \). The current \( i_a \) is unidirectional regardless of the polarity of the input voltage \( E \). This illustrates the unconstrained bidirectional energy flow and proves the four quadrant operation of the topology. During the time interval \((1-d)T_{sw}\) the current through the matrix converter is zero. Since the switches in the converter operate with hard switching, overvoltages over the switches and short circuit events between phases during commutations are observed in Fig.11.3. This is a serious reliability problem and extra protection measures should be taken. Experience shows that

![Matrix output voltage](image)

Fig.11.3 Matrix output voltage \( u_a \) and current \( i_a \); full time scale: \( t=2 \text{ ms} \)
sharp overvoltages are more dangerous to semiconductors than short circuit currents and can easily exceed the maximum permitted value of the semiconductor collector-emitter voltage of applied IGBT's. Also, hard switching conditions require robust semiconductors with a larger safe operating area (SOA).

Fig. 11.4 represents the same matrix phase voltages and currents but at switching instants with maximum modulation depth (minimum duration of \(-E\) or \(+E\)) and at low stator frequency \(f_s = 1\) Hz. This implies the nonzero capacitor voltage during \((1-d)T_{sw}\) interval. When the stator frequency is low, the amplitude of the capacitor voltage \(u_C\) is also low. In spite of this, the matrix phase current can have nominal values because of the low input impedance of the machine.

The power supply voltage \(E\) and the input phase current \(i_E\) of the matrix converter is represented in Fig. 11.5. The input matrix current is heavily distorted in spite of the filter capacitor \(C_f\). This is due to the chosen topology of the \(\text{čuk}\) converter including the voltage source in the switching branch (see Fig. 10.2). However, this choice is based on the reduction of the number of switches in the topology. The standard \(\text{čuk}\) converter topology with the voltage source in series with the inductance \(L_1\) \((L_2, L_3)\), having continuous input current, is also applicable. Unfortunately, three extra switches are necessary then.

Fig. 11.6 and Fig. 11.7 present the stator phase voltage \(u_s\) and current \(i_s\). In Fig. 11.6, during the interval \((1-d)T_{sw}\), only the voltage drop over the switch exists, while during
the interval $dT_{sw}$ the instant value of the capacitor voltage $u_C$ is superimposed on the value of the voltage of the matrix converter.

Fig.11.7 shows the sinusoidal envelope of the stator voltage caused by the capacitor voltage $u_C$. The stator current $i_s$ is sinusoidal and continuous with only a negligible switching frequency ripple.

![Fig.11.6 One-phase stator voltage $u_s$ and current $i_s$; total time $t=2$ ms](image)

![Fig.11.7 Phase stator voltage $u_s$ and current $i_s$; total time $t=1$ s, switching frequency $f_{sw}=1.8$ kHz](image)

11.3. Efficiency of the power converter

Efficiency measurement of the converter is based on the active electric power of the input and output terminals for a resistive load:

$$\eta = \frac{P_{out}}{P_{in}}$$ (11.1)

The results of the measurements, presented in Fig.11.8, show a maximum efficiency of nearly 90% at $P_{in}=2.2$ kW for a switching frequency $f_{sw}=2$ kHz and a duty cycle $d=0.5$. 
11.4. Unwanted effects

The experimental results demonstrating the consequences of the most significant unwanted effects related to the low stator frequency operation are presented below. All considered effects result in reduction of the converter efficiency, deterioration of the induction machine performance and degradation of the characteristics of the field-oriented control. Three major effects are shown:

- influence of the on-state resistance of the power switches,
- dc offset of the transducers (current transformers) and A/D converters,
- influence of asymmetries of the induction machine.

11.4.1. Distortion of the stator current

Experiments at low stator frequencies around \( f_s = 1 \) Hz reveal a distortion of the stator current which were not predicted by the simulation. The distortion exists in spite of the accurate sinusoidal modulation. Fig.11.9 presents a characteristic stator current measured at \( f_s = 2 \) Hz in the experimental drive. The distortion appears around the zero crossings and around the top of the waveform. Additionally, the experiments indicate that the distortion increases for lower values of the stator frequency.
Fig.11.10 shows the FFT of the stator current $i_s$ shown in Fig.11.9. A series of odd harmonics is present except the triplets which do not exist because of the star connection of the motor windings. The experiments show that lowering the supply voltage amplitude and frequency gives rise to distortion of the stator current. This happens, for example, at extremely low stator frequencies where practically only the stator resistance $R_s$ dominates the input machine impedance.

A careful study of the matrix converter modulation performance has not revealed any inconvenient event. Also, the reference signals are symmetrical and sinusoidal. The cause of the stator current distortion at low stator frequencies is found to be the voltage drop over the semiconductor switches of the converter. Fig.11.11 presents the matrix output phase voltage $u_a$ over the one switching period $T_{sw}$. Voltage $u_d$ denotes the dc offset caused by the voltage drop over semiconductor switches that is responsible for the distortion of the current.

-Example

At low stator frequencies, the input impedance of the machine is approximated by the stator resistance $R_s$. Therefore the nominal stator current $i_s$ at frequencies around $f_s = 1$ Hz is achieved at very low stator voltage $u_s$. The voltage is obtained by the step-down operation of the Čuk converter (duty cycle $d=0.3$). As shown in Fig.11.11, the peak matrix phase current of $i_a = 4.4$ A and the peak capacitor voltage $u_C = 6$ V have been reached using 50% of the modulation depth. The voltage drop over the switch is $u_d = 3$ V and creates the dc offset as indicated in the figure.
The stator voltage $u_s$, which includes the dc offset, can be averaged over the complete switching period $T_{sw}$. Assuming aforementioned values for $d$, $u_C$ and $u_d$ from the same figure (power supply $E=\pm90$ V) the stator voltage is:

$$u_s = d \left(-\frac{2}{9}E + \frac{4}{9}E + u_C - u_d\right) = 6.9 \text{ V} \quad (11.2)$$

where voltage $u_C$ is assumed to be constant over the switching period $T_{sw}$. Assuming on the other hand the ideal converter without losses in the semiconductor switches ($u_d=0$) the stator voltage $u_s'$ and the corresponding voltage error $\Delta u$ are:

$$u_s' = d \left(-\frac{2}{9}E + \frac{4}{9}E + u_C\right) = 7.8 \text{ V} \quad (11.3)$$

$$\Delta u = \frac{u_s' - u_s}{u_s} \times 100 = 11\% \quad (11.4)$$

The control of the proposed drive employs a relatively simple current controller. This controller is based on the tracking of the amplitude of the stator current space vector while the information concerning the instant values of each phase current is lost. The calculated error of $\Delta u=11\%$ is not compensated and is a potential source of distortions of the stator current.

11.4.2. Effect of the voltage drop on the torque

Fig.11.12, Fig.11.13 and Fig.11.14 show the experimental results presenting the influence of the higher harmonic components injected into the stator current. The effect on the motor torque is significant below a frequency of 5% of the nominal stator frequency. Measurements are performed with opened feed-back loops to present the uncompensated effect of the shaft cogging.

Fig.11.12 shows the low frequency ($f_s=2.6$ Hz) stator current $i_s$ and the measured rotor speed $n_r$ ($n_r=80$ s$^{-1}$). The sixth torque harmonic generates a slight oscillation disturbance of the rotor speed with a frequency six times the fundamental stator frequency. However, the inertia of the rotating masses is still capable of damping excessive rotor cogging. The following figures (Fig.11.13 and Fig.11.14) represent the experimental stator current and the rotor speed at very

![Fig.11.12 One phase stator current $i_s$ (2 A/div), rotor speed $n_r$ (20 s$^{-1}$/div), stator frequency $f_s=2.6$ Hz](image-url)
low stator frequency \((f_s=1 \text{ Hz and } f_s=0.5 \text{ Hz})\). The rise of the cogging effect is considerable. Besides the sixth harmonic of the torque produced by the combination of the fifth and the seventh current harmonic on one side and the fundamental wave of the flux on the other side, the presence of the higher-order slot harmonics is visible on the rotor speed as well.

![Graph](image1)

*Fig.11.13 One phase stator current \(i_s\) (2 A/div) and rotor speed \(n_r\) (20 s\(^{-1}\)/div), stator frequency \(f_s=1 \text{ Hz}\)*

![Graph](image2)

*Fig.11.14 Stator current \(i_s\) (2 A/div) and rotor speed \(n_r\) (20 s\(^{-1}\)/div), stator frequency \(f_s=0.5 \text{ Hz}\)*

**11.4.3. Harmonic effects on the control**

For practical application around the nominal operating point, the induction machine does not suffer from the detected distortion of the stator current. The influence of
higher current harmonics and the associated harmonic torque components is insignificant and therefore a smooth rotor motion is obtained. The high stator frequency and the inertia of rotating mass reduces the ripple of the speed. However, for low stator frequencies, distortion of the stator current causes problems for the implemented field-oriented control.

Fig.11.15 presents an experimental result where the flux and the torque feed-back loops are opened and the rotor is blocked (the stator frequency is about \( f_s = 1 \) Hz). This figure shows the estimated values of the rotor flux modulus \( |\phi_c| \) compared to its reference value \( \phi_R^* \) and the calculated torque-producing stator current component \( i_{syc} \) compared to its reference value \( i_{syc}^* \). The result reveals the significant ripple of the torque current component \( i_{syc} \), which is an unwanted effect generated by the presence of dc offset in the system. Additional current harmonics contribute to the noise level that exists in the controller. This results in a further reduction of the control dynamics because the gain of the PI controllers in the feed-back loop has to be reduced. Since the current distortion increases as the stator frequency decreases, the most critical operations are starting up the machine and its rotation at very low frequencies. The values of reference signals are then in the domain of the noise generated by the harmonics.

![Fig.11.15 Calculated flux \( \phi_c \), torque current \( i_{syc} \) (rotor is blocked)](image)

In order to enable the startup of the machine, feed-forward control is applied up to the stator frequency of \( f_s = 1 \) Hz. The consequence is the accumulation of the error. At a frequency \( f_s = 1 \) Hz feed-back control is enabled and the compensation of the flux error starts to operate. In combination with harmonic components produced by unwanted effects, the characteristic deviation of \( i_{syc} \) during the build up phase of the torque appears. This is called in the following experimental results the *startup effect*.

**11.4.4. Flux-frequency relation at low stator frequencies**

Fig.11.16 presents the voltage-frequency characteristic of the drive for the stator frequency range \( 0 < f_s < 0.025 \) p.u. for a variable load torque \( T_L \) as the parameter and a
constant rotor flux. As expected, all curves are asymptotically approaching the line $U_s/f_s=1$ for $f_s \rightarrow \infty$. Around the frequency $f_s=0$, the discrepancy between the curves becomes larger due to the increasing effect of stator resistance. For this range the drive is very sensitive to errors produced by the control. The machine becomes easily saturated which results in additional odd space harmonics in the air gap flux, including triplets. Once saturated, it is very difficult to restore the normal operation due to the nonlinear nature of the magnetization phenomena.

![Graph showing voltage-frequency characteristic](image)

*Fig.11.16 Voltage-frequency characteristic of the drive in p.u. ($T_{L1}$ zero load, $T_{L3}$ full load)*

11.5. **Experimental results of the drive under the applied control**

The following section presents the experimental results related to the applied field-oriented control without the shaft and stator voltage sensors. Experiments present the low-speed drive application, a particularly difficult task for this kind of control.

Fig.11.17 presents the results of an experiment showing the behaviour of the system with blocked rotor and opened torque feedback loop at low stator frequency ($f_s=1.5$ Hz). The results are presented in the per-unit system and give a realistic picture of the system performance.

The calculated torque component of the stator current $i_{syc}$ contains a considerable sixth harmonic ripple produced by the combined fifth and the seventh stator current harmonics. These harmonics are injected due to the nonlinear characteristic of the semiconductor switches and their influence becomes remarkable at near zero stator frequencies (the current components $i_{s\alpha}$ and $i_{s\beta}$ in Fig.11.17 are distorted).

The applied current controller, only based on the feedback of the stator current amplitude, cannot compensate this error. To solve this problem, it is necessary to apply a controller which should track instant values of all three stator currents simultaneously.

The calculated torque has within the first 2 seconds the maximum ripple. Under these conditions, starting up the machine with the torque feedback closed, is possible only with a substantial decrease in the gain of the PI controllers. The calculated flux $\phi_c$ is shown in the $x$-$y$ reference frame compared to its reference value.
Experimental Results

Fig. 11.17 Blocked rotor and opened torque loop experiment 
time in seconds, flux and currents in p.u., \( f_s = 1.5 \) Hz

Fig. 11.18 shows an experiment with a blocked rotor and a step change of the torque command \( i_{sy}^* \). In this example both the flux and the torque feedback loops are closed.

Fig. 11.18 Response on a fast change of a torque command \( i_{sy}^* \)
(rotor is blocked)
The sixth harmonic of the current component $i_{\text{syc}}$ is caused by the action of the torque feedback which is considerably smaller than that seen in Fig.11.17. In this experiment, the output of the control block PI-2 (estimated mechanical speed $\omega_m$) was kept at zero. This result shows a good system response.

The following experiments show the performance of the rotating machine.

Fig.11.19 presents the experimental results for a running machine. First, the flux is built up following a ramp-like command (calculated rotor flux $\phi_c$ is shown). After the flux has reached the steady state, the ramp-like torque command (presented as the current component $i_{\text{syc}}$) is applied. The rotor starts running from standstill and reaches the steady state showing a 10% speed ripple. The steady state stator frequency is $f_s=0.1$ p.u.

![Graph showing current components and flux](image)

*Fig.11.19 Building up the flux and the torque followed by steady state operation at low frequency*

The response on a ramp like change of the flux command is shown in Fig.11.20. The machine is loaded and running at low speed about $\omega_m=0.05$ p.u. (stator frequency $f_s=0.1$ p.u.). The speed waveform presents the calculated and the real (measured) speed which are in good agreement. The difference between the two of them is due to the discrepancy between the actual parameters and the parameters used in the machine model as implemented in the controller. All values are presented in the per-unit system.
The cause of the $i_{syC}$ current transient at the change of the flux command is mainly caused by the inaccurate value of the transient angle $\zeta=\arctan(u_{sy}^*/u_{sx}^*)$. Namely, the field-oriented controller uses the simplified algorithm for the calculation of the terms $u_{sy}^*$ and $u_{sx}^*$ in which the current and flux derivatives are disregarded. This results in a good steady state system performance but derates the system response when fast transients are introduced. Similar results are observed in computer simulations using the same simplified algorithm for the calculation of the transient angle $\zeta$.

Fig. 11.21 presents the system response on the combined ramp-like variation of the flux and the torque command at low stator frequencies for a loaded machine. The figure shows both the calculated and actual rotor speed $\omega_m$ (measured with an optical shaft encoder). Except for the time during the initial rise of the torque command $i_{syC}^*$, the calculated and actual rotor speed are in a good agreement. The difference in amplitudes is due to discrepancies between the actual machine parameters and those used in the machine model. The negative value of the calculated speed at the beginning of the torque command is the result of the 6th harmonic injected in the $i_{syC}$ component. This influence is particularly present at near-zero stator frequencies where the effect of the nonlinear switch resistance dominates. The speed waveforms and $i_{syC}$ show the presence of the fundamental and sixth harmonic components, indicating the existence of unwanted effects.
Experimental Results

Fig. 11.21 Response on a ramp-like flux and torque command

The response to a step-like change of the torque command is shown in Fig. 11.22 and Fig. 11.23. The current $i_{s\alpha}$ changes its amplitude and frequency to maintain a constant flux $\phi_c$ and produce the torque that complies with the command.

Fig. 11.22 System response to a step command of torque
Fig. 11.23 represents the system response on two consecutive step torque commands. The calculated flux $\phi_c$ remains constant throughout the transients, while the calculated current component $i_{\text{sync}}$ suffers from the sixth harmonic as a consequence of the nonlinear switch resistance.

![Graphs showing system response](image)

*Fig. 11.23 Response on a step-down and step-up torque command*

Fig. 11.24 presents the system response on a step change in the mechanical load connected to the machine.

![Graphs showing system response](image)

*Fig. 11.24 Response to a step change of the load*
In this experiment the discrepancy between the calculated and the reference value of $i_{syc}$ due to the start-up effect is particularly large. The controller resumes its stable operation if the error is compensated by the action of the feedback loop.

Fig.11.25 presents the minimum achieved controllable torque for an applied load and a rotor speed of $\omega_m=0.03$ p.u.

![Graph showing $i_{s\alpha}$, $\phi_c$, $i_{syc}$, and $\omega_m$ over time](image)

*Fig.11.25 Minimum controllable torque test*  

The additional fundamental harmonic ripple produced by the dc offset, as shown in the speed waveform, increases its amplitude when the speed decreases. The current waveform $i_{syc}$ contains, beside the fundamental harmonic component, also the sixth harmonic as a result of the nonlinear semiconductor resistance effect. This effect is particularly important when industrial low-speed drive applications are considered.
Fig.11.26 presents another experiment for the minimum achieved controllable torque for an applied load and a rotor speed $\omega_m=0.015$ p.u. The speed waveform shows an additional fundamental harmonic ripple, increasing its amplitude when the speed is decreasing and the frequency of the torque ripple enables the existence of the 6th speed harmonic. The torque producing current component $i_{syc}$ also shows clearly the fundamental and the 6th harmonic component. Fig.11.26 reveals clearly the importance of the fundamental harmonic produced by the dc offset on the $i_{syc}$ during the start-up.

**Fig.11.26 Minimum controllable torque experiment II**

### 11.6. Conclusions

Experimental results are presented to prove the concept of the drive with the shaft and stator voltage sensorless control. Results particularly emphasize the feasibility of low stator frequency (low-speed) operation, which is a difficult operating point for controlled ac drives. The experimental drive allows the rotor speed to go down to $n_r=20 \, s^{-1}$ (0.015 p.u.). For lower values of the speed the control signals reach the level of the noise and the control of the drive becomes inaccurate.

The experimental results prove the consequences of unwanted effects also resulting from the theoretical study and the computer simulations. The experimental and simulation results encounter mainly the presence of the 5th and the 7th stator current harmonic, produced by the nonlinear voltage drop over the semiconductor switches, which provoke a 6th harmonic torque component. Also, unwanted effects like current dc offset and system asymmetries produce undesirable harmonic components at near-zero stator frequencies.
Experimental Results

Theoretically the applied vector control enables a torque control even at zero stator frequency, as shown in the results of computer simulations. However, in the experimental low-speed drive the discrepancy with the theoretically obtained results cannot be neglected. Similar discrepancies between simulation and experimental results are obtained for changing the direction of rotation. The reason is found in the noise and the additional harmonic components in the system. When the reference value and the level of the noise are of the same order of amplitude, the system becomes inaccurate. As shown by the experimental results, the accuracy of the system gets worse when the reference values approach zero.
CONCLUDING REMARKS
Concluding Remarks

Contemporary converter-fed ac machines are being increasingly applied in low-speed industrial applications. This is a demanding two-fold task for an ac drive. On the one hand, the converter part must generate stator currents with reduced harmonic components. This is a necessary issue with respect to torque harmonics which will produce the shaft-cogging effect at low rotor speed. On the other hand, ac drive control methods often get into difficulties at near-zero stator frequencies. For example, the direct field-oriented control suffers from serious problems. This is because the flux calculation is obtained by the integration of the low-amplitude stator voltage over a long period. The noise in the system and the dc offset of analog devices can seriously distort the processed low voltage signal.

This thesis tackles low-speed problems in two conceptually different manners. One is the novel converter topology with an advantageous performance at low motor speeds. The generated torque is nearly constant thus enabling smooth shaft rotation. Secondly the low-speed problems are reduced by a modified vector control. It employs a minimum number of sensors to sample the information necessary for the calculation of the flux space vector. The method also enables (theoretically) the torque/speed control at zero stator frequency. Finally, the power topology and the proposed control are merged into an ac drive which should give good performance at low-speed (low stator frequencies).

When evaluating the presented concepts, one should make a difference between the results of academic and of pragmatic relevance. Both criteria can be used in the assessment of the results.

The proposed matrix-čuk power topology has, at present, chiefly academic significance because of its disadvantages.

- The number of semiconductor switches needed for the four-quadrant topology is unacceptably high (24) for commercial use. However, trends in semiconductor technology are foreseeing the arrival of the bidirectional semiconductor switch. This achievement would help the matrix-čuk topology to break the commercial barrier and to prove its advantages in practice.

- The presented topology has a limited power transfer because of the capacitor in the converter current path. Thus the proposed converter is suitable only for low to medium power ranges of ac drives up to 20 kW. Further, the location of the matrix converter in the topology is the cause of the injection of higher harmonic components into the mains. The solution to these problems is the use of the boost-buck in stead of the čuk topology. However, this results in even more semiconductor switches to be required and this would be a less lucrative solution.

In return, considering merely the pragmatic results, this thesis offers the following conclusions.

- For low-speed applications it is essential that the power supply provides sinusoidal stator current. In this particular case it is important because the torque calculation
is very sensitive to harmonic components which can result into an uncontrollable system. The introduced matrix-čuk converter generates virtually rippleless torque at all frequencies.

The use of the original dc-dc topology as a three-phase ac-ac converter is a conceptually new idea. The boost-buck (čuk) topology has led to a widespread converter in the field of dc power supplies. Many modifications are derived from the basic design like: super-boost, super-buck, quasi-resonant, etc. Recently published reports announced the use of the boost-buck topology as a single-phase ac-ac electronic transformer, active filter and power conditioner. However, so far the three-phase ac-ac boost-buck (čuk) topology has not yet been reported.

Further on, the integration/miniaturation of the proposed power converter, which is suited for low to medium power applications, is possible by using the integrated power modules technology.

○ The modeling of the dc-dc boost-buck converter has been a field thoroughly studied over the last decade. A couple of mathematical methods are developed to enable the simplified analysis of switching converters. Assuming the small signal analysis, the state space averaging and the current injection methods gained in popularity over discrete methods that are more accurate but inevitably complex. This thesis has shown that a simple method, like state space averaging, can be used as a method of linearization of ac-ac boost-buck (čuk) converters although large signal variations are considered and a dc component is not present.

○ Perhaps the most significant pragmatic contribution of the herewith presented work is the implementation of the shaft and voltage sensorless vector control of the asynchronous machine. Shaft sensorless control methods use only the information about stator currents and voltages to derive the flux space vector from the machine model. This technique is plausible in many drive applications where the use of the shaft sensor will give problems. An additional problem in a shaft sensorless method is the torque (speed) control at near-zero stator frequencies. The proposed vector control goes a step further. It only needs the information from the stator currents, while the voltage measurements are redundant. Thus the minimum number of sensors is used which, besides the positive commercial effect (lowering the costs), improves the total reliability and accuracy of the drive.

One more contribution of the proposed shaft and voltage sensorless control is its performance at low rotor speeds. Unlike most other shaft-sensorless methods, it provides torque (speed) control down to zero stator frequency as shown in the results of simulation. This is achieved by introducing a compensation term in the analytical expression that calculates the flux. With the experimental setup, shaft-sensorless control of the asynchronous machine is possible down to approximately 1 Hz (2%) with an error that does not exceed 10%. By improving the concept of the control (on-line parameter estimation, high-performance current sensors, µ-processor control, etc.) the torque (speed) control at zero stator frequencies can be achieved. These improvements are meant to be the object of a future research.

In this application, the proposed control is used in combination with the matrix-čuk converter topology but there are no obstacles to the application of the method with
any conventional converter. However, the PWM control of the converter should enable the generation of zero-voltage space vectors (Space Voltage Vector Modulation).

- At low-speeds (low stator frequencies) nonlinear voltage drop over semiconductor switches, dc offset and asymmetries in the system have very important influence on the operation of the drive. The study presented in this thesis is based on the matrix-ćuk converter topology which has inherently two semiconductors serially connected in one switch. These effects are inherent to all families of ac drives. Each converter topology should be carefully analyzed for low-speed applications.

All these effects superimpose additional harmonic torque components and significantly deteriorate the performance of the low-speed drive. The result is that the ac drive, operating at near zero speed, starts oscillating and, eventually, becomes uncontrollable. The cause of oscillations could be found in the light damping of the induction machine, which is a well-known problem at low-speeds and light loads. However, presented numerical results show that light damping in this particular application is not the cause of machine oscillations. Moreover, a stability study of the proposed converter-machine system, operating with an open loop control, has been extended on various parameters of the converter and the machine. The numerical and simulation results show the regions with light damping.

It is shown that the voltage drop over semiconductors has the most significant effect responsible for generation of harmonics in the machine. When designing the control scheme for a low-speed drive, particular care should be devoted to the elimination of harmonics produced by this effect. This can be done by means of the current controller. The simple one used in the experimental setup is suitable for implementation on a PC. However, this current controller monitors only the modulus of the stator-current space vector and neglects in the same time instant values of each phase current that carries the information about alien harmonics. For this reason, a more complex current controller that monitors all three currents independently is preferred. Thus undesirable harmonics can be detected and suppressed. The results of experiments, simulations and analysis are presented to provide a better understanding of the problems caused by side effects.

- The utilization of the PC for the on-line control of a low-speed ac drive, besides its user friendly environment, has a serious disadvantage. The frequent appearance of interrupts is the cause of nonequal sampling times that accumulates an error in cumulative processes such as an integration process. To avoid discretization errors when low frequency and amplitude signals are handled, at least 12- to 16-bit digital technology is needed.

The presented study considers strictly the low speed (low stator frequency) operation of the matrix-ćuk converter-based ac drive although it can operate at rated frequencies as well. Further interesting subject of research could be found in the performance of the drive at the nominal speed. Different boost-buck ac-ac topologies might bring up new, interesting applications.
Concluding Remarks

Considering the proposed vector control technique, its implementation on a standard bridge converter would be of particular interest. Industrial general-purpose drives could strongly benefit by the reduction of the number of sensors needed for the operation of the control.
Summary

Converter fed ac machines are being increasingly used in low-speed industrial applications which is a demanding twofold task for an ac drive. On the one hand, the converter part must generate stator currents with as few harmonic components as possible. This is necessary with respect to torque harmonics in order to prevent the shaft cogging effect at low rotor speeds. On the other hand, ac drive control methods often get into difficulties at near-zero stator frequencies. For example, a direct field-oriented control can suffer from serious problems e.g. because the flux calculation is obtained by the integration of the low-amplitude stator voltage over a long period. Additionally, the noise in the system and the dc offset of analog devices can seriously distort the processed low-voltage signal.

This study proposes an ac drive suited for low-speed (low stator frequency) operation. The drive is based on novel converter topology and the implemented shaft sensorless control applies the minimum number of sensors. The aim of this project is to prove the controllability of the machine torque at zero stator frequency (zero-speed), to analyze relevant problems and give solutions to them or hints for solutions.
Samenvatting

Door frequentieomzetter gevoede ac-machines worden steeds vaker gebruikt voor industriële lage-toerental aandrijvingen wat een veeleisende tweeledige taak is voor een wisselstroomaandrijving. Aan de ene kant moet de frequentieomzetter statorspanningen genereren met zo min mogelijk harmonische componenten. Dit is nodig met betrekking tot de harmonische koppelcomponenten om de rimpel van het toerental bij lage toerentalen te voorkomen. Aan de andere kant ondervinden besturingsmethoden van wisselstroomaandrijvingen vaak moeilijkheden bij zeer lage frequenties van de statorspanning. Een direkt-veldgeoriënteerde besturing kan serieuze problemen ondervinden, bijvoorbeeld omdat de flux-berekening verkregen is door de integratie van de statorspanning met een kleine amplitude over een lange periode. Bovendien kunnen de ruis in het systeem en de offset van analoge componenten het verwerkte lage spanningssignaal ernstig vervormen.

In deze studie staat het ontwerpen van een wisselstroomaandrijving die geschikt is voor lage toerentalen (lage statorfrequenties) centraal. De aandrijving is gebaseerd op een nieuwe omzettetopologie en de geïmplementeerde as-sensorloze besturing heeft een minimum aantal sensoren. Het doel van dit project is de bestuurbaarheid van het machinekoppelen aan te tonen bij een toerental gelijk aan nul en het analyseren en oplossen van relevante problemen.
Curriculum Vitae

Darko Antić was born in Vršac, Yugoslavia on February 17, 1963. After graduating from high school he went in 1981 to University of Beograd to study electrical engineering. He graduated from electrical engineering with a thesis on a series-resonant converter in 1987. From 1988 to 1990 he was employed with Energoinvest Sarajevo (Yugoslavia) as a design engineer. In 1990 he joined the department for Electrical Machines and Drives, University of Wuppertal, Germany, as a research fellow where he accomplished a project related to an advanced base driver of high-power bipolar transistors. From June 1990 he was working at the Laboratory for Power Electronics and Electrical Machines at Delft University of Technology, The Netherlands, as a Ph.D. student in the field of low-speed ac drives. He completed his thesis in October 1994. His fields of professional interest are power electronics and electrical drives.