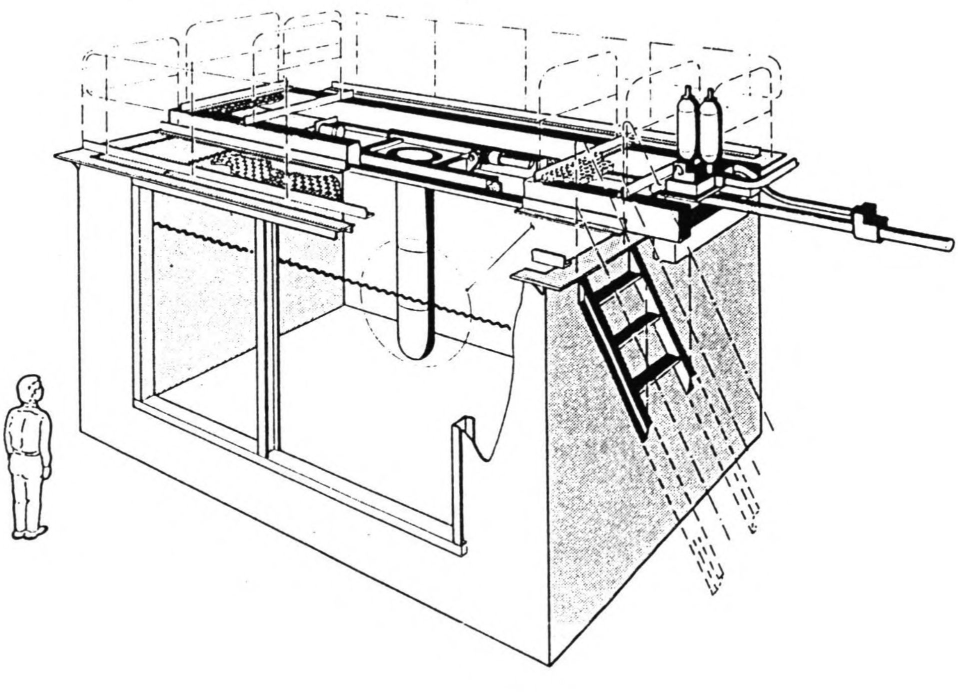
DRAG FORCES ON OSCILLATING CYLINDERS

appendices



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DRAG FORCES ON

OSCILLATING CYLINDERS

appendices

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APPENDIX A DIMENSIONAL ANALYSIS

The force per unit length of the fluid on the cylinder is a function of 7 variables:

$$F = f(t, T, D, \hat{U}_0, \rho, \nu, ks) \tag{A.1}$$

Expressed in the elementary quantities M (mass), L (length) and T (time):

t	:	$[T]$
T	:	$[T]$
D	:	$[L]$
$\hat{U}_0$	:	$[L*T^{-1}]$
ρ	:	$[M*L^{-3}]$
ν	:	$[L^2*T^{-1}]$
ks	:	$[L]$
F	:	$[M*T^{-2}]$

Combination of these terms gives the following dimensionless quantity Π:

$$\Pi = [T]^{k1}*[T]^{k2}*[L]^{k3}*[L*T^{-1}]^{k4}*[M*L^{-3}]^{k5}* [L^2*T^{-1}]^{k6}*[L]^{k7}*[M*T^{-2}]^{k8} \tag{A.2}$$

thus:

$$\Pi = M^{k5+k6}*L^{k3+k4-3*k5+2*k6+k7}*T^{k1+k2-k4-k6-2*k8} \tag{A.3}$$

All three exponents have to be equal to zero:

$$k5 + k8 = 0 \tag{A.4}$$

$$k3 + k4 - 3*k5 + 2*k6 + k7 = 0 \tag{A.5}$$

$$k1 + k2 - k4 - k6 - 2*k8 = 0 \qquad (A.6)$$

According to the Buckingham theorem 8 variables and 3 elementary numbers supply 8 - 3 = 5 dimensionless numbers:

	k1	k2	k3	k4	k5	k6	k7	k8
	t	T	D	$\hat{U}_0$	ρ	ν	ks	F
Π_1	1	-1	0	0	0	0	0	0
Π_2	0	1	-1	1	0	0	0	0
Π_3	0	0	1	1	0	-1	0	0
Π_4	0	0	-1	-2	-1	0	0	1
Π_5	0	0	-1	0	0	0	1	0

table (A.1)

According to table (A.1), equations (A.4)-(A.6) can be solved if:

$$\Pi_1 = \frac{t}{T} \qquad (A.7)$$

$$\Pi_2 = \frac{\hat{U}_0 T}{D} \qquad (A.8)$$

$$\Pi_3 = \frac{\hat{U}_0 D}{\nu} \qquad (A.9)$$

$$\Pi_4 = \frac{ks}{D} \qquad (A.10)$$

$$\Pi_5 = \frac{F}{\rho \hat{U}_0^2 D} \tag{A.11}$$

Now the dimensionless force can be expressed as a function of four dimensionless quantities:

$$\frac{F}{\rho \hat{U}_0^2 D} = f\left(\frac{t}{T}, \frac{\hat{U}_0 T}{D}, \frac{\hat{U}_0 D}{\nu}, \frac{ks}{D}\right) \tag{A.12}$$

in which

$\frac{t}{T}$: nondimensional time

$\frac{\hat{U}_0 T}{D}$: Keulegan-Carpenter number $\left(= \frac{2\pi A}{D}\right)$

$\frac{\hat{U}_0 D}{\nu}$: Reynoldsnumber

$\frac{ks}{D}$: nondimensional roughness

The Keulegen-Carpenter number K is the ratio of the convective term in the momentum equation to the inertia term. The Reynolds number Re is the ratio of the convective term to the viscous term. K and Re can be combined to the oscillatory Reynolds number $\beta = \frac{Re}{K} = \frac{fD^2}{\nu}$.

APPENDIX B DRAG FORCES IN LAMINAR FLOW

The drag force exerted on a cylinder oscillating in a fluid is caused by two phenomena:

1. wall shear stress,
2. pressure in phase with the velocity.

First, the part of the drag force caused by the shear stress will be calculated.

For a laminar, attached boundary-layer, the velocity profile can be calculated from the following differential equation (see section 4.4):

$$\frac{\partial u}{\partial t} = \frac{\partial U}{\partial t} + \frac{1}{\rho}\frac{\partial \tau_{xy}}{\partial y} \qquad (B.1)$$

with

$$\tau_{xy} = \rho\nu \frac{\partial u}{\partial y} \qquad (B.2)$$

The boundary-conditions are:

$$\begin{aligned} &u = 0 \quad \text{at} \quad y = 0 \\ &u = U \quad \text{at} \quad y\text{, just outside the boundary-layer} \\ &\text{with } U = \hat{U}(\theta)\cos\omega t \end{aligned} \qquad (B.3)$$

and the coordinates x and y are attached to the moving cylinder.
After integration of eq. (B.1) and application of the boundary-conditions the velocity profile is obtained:

$$u = \hat{U}(\theta)\cos\omega t\{1-\exp(-(1+i)\frac{y}{\delta})\} \qquad (B.4)$$

with $\delta = \sqrt{(2\nu/\omega)}$ (see Batchelor [4] and Lighthill [17]).
The shear stress at the cylinder surface is:

$$\tau_0 = \rho\nu \left(\frac{\partial u}{\partial y}\right)_{y=0} = \rho\nu(1+i)\frac{\hat{U}(\theta)}{\delta}\cos\omega t \tag{B.5}$$

with $\hat{U}(\theta) = 2\hat{U}_0\cos\theta$, with $\hat{U}_0$ the amplitude of the cylinder velocity.

To calculate the drag force caused by the shear stress, the component of the wall shear stress which has the direction in which the cylinder is moving should be taken (see fig. 1):

$$\rho\nu \left(\frac{\partial u}{\partial y}\right)_{y=0} \cos\theta = \rho\nu(1+i)\frac{2\hat{U}_0\cos^2\theta}{\delta}\cos\omega t \tag{B.6}$$

Only the part of this component of the wall shear stress which is in phase with the velocity can cause drag. Integration around the cylinder of eq. (B.6) gives the following drag force:

$$F_{dls} = 2\pi\rho\nu\frac{\hat{U}_0}{\delta}R\cos\omega t \tag{B.7}$$

In an irrotational flow field, the net work done on the body by the pressure is zero, because the pressure from the inviscid fluid on the body is 90° out of phase with the cylinder velocity. Therefore no work can be done by the pressure field on the body (see Batchelor [4]).

In this study however, we consider a fluid with a certain viscosity, and there is a drag force caused by a part of the normal stress which is in phase with the velocity. To calculate this drag force, a close analysis of the Stokes theory (see Stokes [27]) is needed.

Stokes solved equation (B.1) and calculated the total force on a cylinder by integrating the normal and shear stresses over the cylinder-surface. He showed that the drag forces caused by the shear stresses (F_{dls}) and the normal stresses (F_{dlp}) have the same magnitude, so that the total drag force equals:

$$F_{d1} = 4\pi\rho\nu\frac{\hat{U}_o}{\delta}R\cos\omega t \qquad (B.8)$$

APPENDIX C DISPLACEMENT THICKNESS

In this Appendix the principle of the displacement thickness will be explained.

The displacement thickness will be calculated for a fixed cylinder in oscillating flow. This is different from most of the other derivations in this report, in which the cylinder oscillates. For the physical understanding of the principle of displacement thickness, this does not make any difference.

Within the boundary-layer on the cylinder, there is a defect of volume flow, relative to the volume flow outside the boundary-layer (see fig. 40). This defect of volume flow for a laminar boundary-layer is, using cylindrical coordinates (see Lighthill [17]):

$$\int_0^{\infty}(U-u)dr = (\rho i\omega)^{-1}\left(-\frac{\partial p}{r\partial\theta}\right)\sqrt{\left(\frac{\nu}{i\omega}\right)} \qquad (C.1)$$

in which $U = \hat{U}\,e^{i\omega t}$ is the external flow outside the boundary-layer and $\frac{\partial p}{r\partial\theta}$ is the pressure gradient, that oscillates the flow. For u, the laminar boundary-layer velocity profile has been used.

In order to simplify the calculations on a cylinder with a boundary-layer with rotational, visous flow, the principle of the displacement thickness can be used (see Lighthill [17]). Thus one works with an enlarged cylinder with radius $R+\delta_1$, which is assumed to move in an irrotational, inviscid fluid. The total force on the enlarged cylinder is the same as the force on the original cylinder. In order to satisfy this definition, δ_1 is given by:

$$\int_0^{\infty}(U-u)dr = U\delta_1 \qquad (C.2)$$

The external flow also equals:

$$U = (\rho i\omega)^{-1}\left(-\frac{\partial p}{r\partial\theta}\right) \tag{C.3}$$

Using equations (C.1) through (C.3) gives δ_1 as:

$$\delta_1 = \surd\left(\frac{\nu}{i\omega}\right) = (1-i)\surd\left(\frac{\nu}{2\omega}\right) \tag{C.4}$$

Note that δ_1 is complex. Thus the interpretation of displacement thickness, valid for steady flow, namely that the influence of the viscous boundary-layer on the flow is as if irrotational flow is flowing along a displaced solid boundary is not valid anymore. Mathematically however, the solution for δ_1 is still correct. The imaginary part of δ_1 causes a component of the cylinder velocity in phase with the pressure. Therefore, the pressure can do work and contributes to the drag force.

Since we assume potential flow around the enlarged cylinder, there is no shear stress in the calculation. The drag force on the cylinder is only caused by the pressure. The total force on the cylinder is:

$$F = 2 \int_0^\pi \rho\frac{\partial\phi}{\partial t}R\cos\theta d\theta \tag{C.5}$$

(C.5) shows the stream function according to Bernoulli, integrated along a streamline around the cylinder. In this equation:

$$\frac{\partial\phi}{\partial t} = \hat{U}_0 i\omega e^{i\omega t} 2(R+\delta_1)\cos\theta \tag{C.6}$$

$\hat{U}_0$ is the amplitude of the cylinder velocity.

Thus the amplitude of the force is (with eq. (C.4) for δ_1):

$$\hat{F} = 2\pi\rho\omega\hat{U}_0 2R\surd(\nu/2\omega) + 2i\pi\rho\omega\hat{U}_0(R^2 + 2R\surd(\nu/2\omega)) \tag{C.7}$$

Note that δ_1^2 is negligible in comparison with R^2 and $2R\delta_1$.

The amplitude of the inertia force, which is in phase with the acceleration of the cylinder, equals:

$$\hat{F}_i = \pi\rho\omega\hat{U}_0 R^2 (1+2(\frac{1}{\sqrt{(\pi\beta)}})) \tag{C.8}$$

In this equation, the first part is the mass-inertia of the volume of the cylinder, and the second part is the added-mass inertia of the fluid. The amplitude of the drag force, which is in phase with the cylinder velocity, equals:

$$\hat{F}_d = \pi\rho\omega\hat{U}_0 R^2 (\frac{4}{\sqrt{(\pi\beta)}}) \tag{C.9}$$

in which β is the oscillatory Reynolds number.

Comparison of eq. (C.9) and (B.8) shows that the principle of the displacement-thickness is correct, and that it is much easier to use in drag force calculations. than --- ? ?

APPENDIX D WORK METHOD

The total work done by the flow around the cylinder can be calculated from the momentum equations.

If U is the velocity in x-direction just outside the boundary-layer, the momentum equation in x-direction outside the boundary-layer is:

$$-\frac{\partial p}{\partial x} = -\rho\frac{\partial U}{\partial t} = \rho i\omega U \qquad (D.1)$$

In this equation U is, using potential flow:

$$U(t) = 2\hat{U}_0\cos\theta e^{i\omega t} \qquad (D.2)$$

Regarding the flow up to $y = \xi$ with ξ larger than the boundary-layer thickness (see fig. 2), and using the displacement-thickness principle, ~~this~~ the momentum equation becomes (see appendix C):

$$-\frac{\partial p}{\partial x}\xi = \tau_0 + \rho i\omega \int_0^{\xi} u\,dy = \tau_0 + \rho i\omega U(\xi-\delta_1) \qquad (D.3)$$

Subtracting eq. (D.1), integrated over $\xi-\delta_1$:

$$-\frac{\partial p}{\partial x}(\xi-\delta_1) = \rho i\omega U(\xi-\delta_1) \qquad (D.4)$$

yields:

$$(-\frac{\partial p}{\partial x})(-\delta_1) = \tau_0 \qquad (D.5)$$

The rate of doing work in the boundary-layer (per unit surface area) is:

$$W(t) = \int_0^{\xi}(-\frac{\partial p}{\partial x})u\,d\xi = (-\frac{\partial p}{\partial x})U(\xi-\delta_1) = U\tau_0 \qquad (D.6)$$

Note that to calculate this rate of doing work, only the real parts of the equations should be taken. The mean rate of doing work is:

$$\overline{W} = \frac{1}{T}\,{}_{0}\!\int^{T} W(t)dt = \overline{U\tau_0} \tag{D.7}$$

To calculate the mean rate of doing work per unit length of the cylinder, this expression should be integrated around the cylinder surface:

$$\overline{W}^{*} = {}_{0}\!\int^{2\pi} \overline{W}Rd\theta \tag{D.8}$$

This expression should be equal to the mean rate of doing work, per unit length of the cylinder, of the drag force:

$$\overline{W}^{*} = \overline{F_d U_0} \tag{D.9}$$

Now the drag force can be calculated.

The derivation made above, is valid for laminar and (partly) turbulent boundary-layers. We ~~control~~ check the expressions in the case of a laminar boundary-layer. The expression for the shear stress then is:

$$\tau_0 = (1+i)(\sqrt{(2\nu\omega)})\hat{U}_0\cos\theta\cos\omega t \tag{D.10}$$

Substituting the real parts of eq. (D.2) and (D.10) in eq. (D.8) gives:

$$W^{*} = {}_{0}\!\int^{2\pi}(\sqrt{(2\nu\omega)}\rho\hat{U}_0^2 \cos^2\theta)Rd\theta \tag{D.11}$$

In the case of a laminar boundary-layer, eq. (D.9) becomes:

$$\overline{W}^{*} = 0.5\hat{F}_{d1}\hat{U}_0 \tag{D.12}$$

or

$$\hat{F}_{d1} = \frac{2\bar{W}^*}{\hat{U}_0} \tag{D.13}$$

Substituting eq. (D.11) in eq. (D.13) yields for the drag force:

$$\hat{F}_{d1} = 2\sqrt{(2\nu\omega)}\rho\pi R\hat{U}_0 \tag{D.14}$$

This expression is the same as the one obtained in Appendix B

The work method presented here is based on eq. (D.7) which later will be used to calculate the drag force on an oscillating cylinder including laminar, partly laminar partly turbulent, and completely turbulent boundary-layer flow (see Appendix E).

APPENDIX E WORK AND DRAGFORCE CALCULATIONS

As explained in section 5.3, the flow around the cylinder is laminar between $\theta = \theta_{crl}$ and $\theta = \pi/2$. In order to calculate the mean rate of doing work of that part of the cylinder which has a laminar boundary-layer, the integration limits are θ_{crl} and $\pi/2$:

$$\overline{W}_l^* = 4 \int_{\theta_{crl}}^{\pi/2} \overline{\tau_0 U}\, R d\theta \tag{E.1}$$

Integration yields the following expression:

$$\overline{W}_l^* = 2\rho\sqrt{(2\nu\omega)}\hat{U}_0^2 R(\pi/2 - 0.5\sin 2\theta_{crl} - \theta_{crl}) \tag{E.2}$$

This equals:

$$\overline{W}_l^* = \overline{F_{dl} U} \tag{E.3}$$

This yields for the drag force caused by the laminar part of the boundary-layer flow:

$$\hat{F}_{dl} = 2\overline{W}_l^*/\hat{U}_0 \tag{E.4}$$

The result for this part of the dragforce is therefore:

$$\hat{F}_{dl} = 4\sqrt{(2\nu\omega)}\rho\hat{U}_0 R(\pi/2 - 0.5\sin 2\theta_{crl} - \theta_{crl}) \tag{E.5}$$

The same method will be applied to calculate the drag force caused by the turbulent part of the boundary-layer flow. Now a quadratical relation between the shear stress and the velocity will be used (see section 5.2):

$$\tau_0 = 0.5\rho f_w(2\hat{U}_0\cos\theta)^2|\cos(\omega t+\psi)|\cos(\omega t+\psi) \tag{E.6}$$

In a different way:

$$\tau_0 = \hat{\tau}_0|\cos\psi\cos\omega t-\sin\psi\sin\omega t|(\cos\psi\cos\omega t-\sin\psi\sin\omega t) \tag{E.7}$$

f_w is the turbulent smooth or turbulent rough friction factor, f_{ws} or f_{wr}, ψ is the phase difference between the outer flow and the wall shear stress and $\hat{\tau}_0$ is the amplitude of the turbulent shear stress at the wall, which can be derived from equation (E.6). As can be seen from figure 10, the work done by the turbulent part of the boundary-layer, with a relatively smooth surface, has to be calculated by integrating from $\theta = 0$ to $\theta = \theta_{cr2}$. The work done by the turbulent part of the boundary-layer, with a relatively rough surface, can be calculated by integrating between $\theta = \theta_{cr2}$ and $\theta = \theta_{cr1}$. Working this out gives for the work done by the turbulent part of the boundary-layer with smooth surface:

$$\bar{W}^*_{ts} = 4\int_0^{\theta_{cr2}} \hat{U}\hat{\tau}_0\overline{\cos\omega t|\cos(\omega t+\psi)|\cos(\omega t+\psi)}Rd\theta \tag{E.8}$$

with a friction factor f_{ws}.

This is for $\cos(\omega t+\psi) > 0$:

$$\bar{W}^*_{ts} = 4\int_0^{\theta_{cr2}} \hat{U}\hat{\tau}_0\overline{(\cos^2\psi\cos^3\omega t-2\cos\psi\sin\psi\cos^2\omega t\sin\omega t+\sin^2\psi\sin^2\omega t\cos\omega t)}Rd\theta \tag{E.9}$$

and for $\cos(\omega t+\psi) < 0$:

$$\bar{W}^*_{ts} = 4\int_0^{\theta_{cr2}} \hat{U}\hat{\tau}_0\overline{(-\cos^2\psi\cos^3\omega t+2\cos\psi\sin\psi\cos^2\omega t\sin\omega t-\sin^2\psi\sin^2\omega t\cos\omega t)}Rd\theta \tag{E.10}$$

Solving $\cos(\omega t+\psi) = 0$, yields for given values of ω and ψ, $t = t^*$. Then the solution of eq. (E.9) and (E.10) is:

$$\bar{W}^*_{ts} = 32/\pi_0 \int^{\theta_{cr2}} \hat{U}\hat{\tau}_0 \Big| \cos^2\psi [\sin\omega t]^{t^*}_{t^*-T/2} + 2/3\cos\psi\sin\psi[\cos^3\omega t]^{t^*}_{t^*-T/2}$$

$$+ 1/3\sin^2\psi - 1/3\cos^2\psi)[\sin^3\omega t]^{t^*}_{t^*-T/2} \Big| R d\theta \tag{E.11}$$

The parts between [] are integrals from t^* to $t^* - T/2$ in which T is the period of the motion.

The same as above can be done for the rough surface. The difference is, that θ_{cr2} and θ_{cr1} are the integration limits and that f_{wr} has to be used. The contribution to the drag force can be calculated from:

$$\bar{W}^*_{ts} = \overline{F_{dts} U_0} \tag{E.12}$$

Now a different expression for the drag force has to be used, because in a turbulent boundary-layer flow, the drag force is quadratically related to the velocity of the cylinder:

$$\hat{F}_{dts} = \frac{3\pi \bar{W}^*_{ts}}{4\hat{U}_0} \tag{E.13}$$

For $\bar{W}^*_{ts}$, expression (E.11) should be used.

For the rough surface, exactly the same can be done.

The total drag force equals:

$$\hat{F}_d = \hat{F}_{dl} + \hat{F}_{dts} + \hat{F}_{dtr} \tag{E.14}$$

The subscripts dl, dts and dtr, denote drag laminar, drag turbulent smooth and drag turbulent rough.

APPENDIX F FLAT PLATE FRICTION FACTORS

This analysis on friction factors is restricted to oscillating flow on flat beds. When applying this theory to flow around a cylinder, it is not valid directly. However, in this study a momentum equation is used which describes the flow as if the surface of the cylinder locally is a flat bed. Therefore, the flat bed friction factors may be used, though they have to be determined locally.

Note that the next part of this appendix will deal with oscillating flow over a fixed bed. In most parts of this report the flow caused by an oscillating cylinder in a still fluid is described. Physically however, there is no difference, whether the flow oscillates, or the bed, if coordinates are used fixed to the surface.

The friction factor f_w for oscillating flow over a flat bed can be defined as:

$$\hat{\tau}_0 = 0.5 f_w \rho \hat{U}^2 \tag{F.1}$$

The momentum equation for the boundary-layer is (see section 4.4):

$$\frac{\partial u}{\partial t} = \frac{\partial U}{\partial t} + \frac{1}{\rho}\frac{\partial \tau_{xy}}{\partial y} \tag{F.2}$$

The solution for the shear stress in case of an oscillating laminar flow is given by Stokes (see appendix B):

$$\tau = \sqrt{2}\rho\nu\beta\hat{U}\exp(-\beta y)\cos(\omega t-\beta y+\frac{\pi}{4}) \tag{F.3}$$

in which $\beta = fD^2/\nu$.

Therefore, $\hat{\tau}_0 = \sqrt{2}\rho\nu\beta\hat{U}$ (F.4)

which yields $f_{wl} = \frac{2}{\sqrt{Re}}$ (F.5) with $Re = \frac{\hat{U}a}{\nu}$ (F.6)

in which a is the local amplitude of motion.
In the laminar case, the wall shear stress is linearly related to the velocity according to eq. (F.3), so the expression for the wall shear stress, τ_0 at y = 0 is:

$$\tau_0 = 0.5\rho\frac{2}{\sqrt{Re}}\hat{U}^2\cos(\omega t+\frac{\pi}{4}) \tag{F.7}$$

This expression has no restriction with respect to the roughness of the surface.

In case of a turbulent boundary-layer flow, there is no analytical description of the wall shear stress and friction factor.

There are, however, a few models available which describe the velocity profile in the boundary-layer under certain assumptions, and supply friction factors that can be used in eq. (F.1). Furthermore, empirical formulations for the friction factor are available which are directly obtained from experiments.

In case of a turbulent boundary-layer flow, there is a division between smooth and rough beds. First, we will examine the smooth beds.

Kajiura (see [26]) uses a so-called eddy-viscosity model to describe the shear stress:

$$\tau = \rho\epsilon\frac{\partial u}{\partial y} \tag{F.8}$$

with ϵ the eddy-viscosity. He presumes ϵ to be constant in time, though experiments show that ϵ is a function of time (Horikawa 1968, see [26]). Kajiura splits the flow into three layers with different eddy-viscosities and obtains an expression for the wall shear stress τ_0. His result in terms of friction factors is:

$$\frac{1}{8.1\sqrt{f_{ws}}} + \log\frac{1}{\sqrt{f_{ws}}} = -0.135 + \log\sqrt{Re} \tag{F.9}$$

with a shear stress that varies sinusoidally in time (see eq. (F.8)).

Jonsson (1967), see [26], has developed a model with a wall layer and a defect layer with the following conditions:

$$\text{wall layer} \qquad \frac{u}{u_*} = f\left(\frac{y}{ks}\right) \tag{F.10}$$

$$\text{defect layer} \qquad \frac{u-U}{u_*} = f\left(\frac{y}{\delta}\right) \tag{F.11}$$

with $u_* = \sqrt{(\frac{\tau}{\rho})}$ the shear stress velocity. Thus Jonsson presumes a wall region in which the velocity is determined by local conditions (eq. (F.10)) and a defect layer in which the velocity is independent of the viscosity ? (eq. (F.11)). With help from eq. (F.1) the following expression for the friction factor is obtained: in δ!

$$f_{ws} = 0.09 * Re^{-0.2} \tag{F.12}$$

In Jonsson's theory, the shear stress varies quadratically in time due to the use of the shear stress velocity.

Another model of the turbulent boundary-layer is the mixing length model (see section 3.1). This model also predicts the shear stress to vary quadratically in time:

$$\tau_0 = \rho l_m^2 \frac{\partial u}{\partial y}\left|\frac{\partial u}{\partial y}\right| \tag{F.13}$$

in which l_m is the mixing length.

Since the experiments show that a variation of the shear stress quadratically in time is more realistic than a linear relationship, we will use eq. (F.12) to compute the friction factor in a turbulent flow on smooth beds.

Kajiura (1968, see [26]) has developed his three layer model with eddy-viscosities for rough beds also. This resulted in the following expression for the friction factors:

$$\frac{0.98}{4\sqrt{f_{wr}}} + \log\frac{1}{4\sqrt{f_{wr}}} = -0.25 + \log\left(\frac{a}{ks}\right) \qquad (F.14)$$

Jonsson suggests the semi-emperical formula:

$$\frac{1}{4\sqrt{f_{wr}}} + \log\frac{1}{4\sqrt{f_{wr}}} = -0.08 + \log\left(\frac{a}{ks}\right) \qquad (F.15)$$

which is very similar to eq. (F.14).

By using a mixing length model for the boundary-layer, Bakker (see [3]) develops another equation for the friction factor that has the same character.

In view of the fact that the Jonsson model is used in the smooth turbulent case, because the shear stress varies quadratically in time, we will also use his equation (F.15) for the rough turbulent case. Swart has rewritten this equation in the following form:

$$f_{wr} = 0.00251*\exp\left[5.21*\left(\frac{a}{ks}\right)^{-0.19}\right] \qquad (F.16)$$

for $\frac{a}{ks} < 1.57 \quad f_{wr} = 0.3$

APPENDIX G THE CONVECTIVE TERM

The momentum equation in x-direction for a laminar boundary-layer on a flat surface is (see section 3.3):

$$\frac{\partial u}{\partial t} + u\,\frac{\partial u}{\partial x} + v\,\frac{\partial u}{\partial y} = -\,\frac{1}{\rho}\,\frac{\partial p}{\partial x} + \nu\,\frac{\partial^2 u}{\partial y^2} \qquad \text{(G.1)}$$

For the outer (potential) flow the shear stress vanishes, and the momentum equation reads:

$$\frac{\partial U}{\partial t} + U\,\frac{\partial U}{\partial x} = -\,\frac{1}{\rho}\,\frac{\partial p}{\partial x} \qquad \text{(G.2)}$$

Since the pressure is the same inside and outside the boundary-layer, equations (G.1) and (G.2) can be combined:

$$\frac{\partial u}{\partial t} + u\,\frac{\partial u}{\partial x} + v\,\frac{\partial u}{\partial y} = \frac{\partial U}{\partial t} + U\,\frac{\partial U}{\partial x} + \nu\,\frac{\partial^2 u}{\partial y^2} \qquad \text{(G.3)}$$

The integration of the equation of motion for the boundary-layer, eq. (G.3), cannot be done straight away because it includes non-linear convective terms. To estimate the influence of the convective terms on the flow field and the drag force, expansion series can be used to calculate successive approximations of the velocity-profile:

$$u(x,y,t) = u_0(x,y,t) + u_1(x,y,t) \qquad \text{(G.4)}$$

and of the shear stress:

$$\tau(x,y,t) = \tau_0(x,y,t) + \tau_1(x,y,t) \qquad \text{(G.5)}$$

Note that τ_0 is not the wall shear stress.

Though expansion series can give an idea of the importance of the convective terms, one should keep in mind that expansion series are only allowed when $u_1 << u_0$, thus when

$$u \frac{\partial u}{\partial x} << \frac{\partial u}{\partial t} \quad \text{or} \quad A/R << 1 \tag{G.6}$$

The first solution for u and τ, giving u_0 and τ_0, is the solution of the momentum-equation neglecting the convective terms. The second solution, giving u_1 and τ_1, is the solution of the momentum-equation including the convective terms and using the first solution u_0:

$$\frac{\delta u_0}{\delta t} = \frac{\delta U}{\delta t} + \nu \frac{\delta^2 u_0}{\delta y^2} \tag{G.7}$$

Using the following expression for the outer flow:

$$U = \hat{U}(x)\cos\omega t \tag{G.8}$$

yields for u_0:

$$u_0 = \hat{U}(x)\xi_0' \exp(i\omega t) \tag{G.9}$$

and yields for v_0 (by using the equation of continuity):

$$v_0 = - \frac{d\hat{U}}{dx} \delta\xi_0 \exp(i\omega t) \tag{G.10}$$

For ξ_0 and ξ_0' , which express the dependency of the amplitude of velocity on y, see Schlichting ([23]) p.429. This first solution is equal to the Stokes solution (see [27]).

The second-order approximation can be derived by using u_0 and v_0 in the convective terms of the complete equation of motion:

$$\frac{\partial u_1}{\partial t} - \nu \frac{\partial^2 u_1}{\partial y^2} = U \frac{\partial U}{\partial x} - u_0 \frac{\partial u_0}{\partial x} - v_0 \frac{\partial u_0}{\partial y} \qquad \text{(G.11)}$$

This yields for the tangential velocity u_1:

$$u_1(x,y,t) = \hat{U}(x)\frac{d\hat{U}}{dx}\frac{1}{\omega}\{\xi_a \exp(2i\omega t)+\xi_b\} \qquad \text{(G.12)}$$

For ξ_a and ξ_b see Schlichting [23] p.429.
The solution for u_1 consists of two parts. The first part, with ξ_a, is a periodic contribution to u, u_{11}, and the second part, with ξ_b, is a steady-flow contribution, u_{12}.

If this solution is applied to the circular cylinder, with $x = r\partial\theta$, u_{11} depends on θ with $\cos 2\theta$, due to the term $\hat{U}(\theta)\frac{d\hat{U}(\theta)}{rd\theta}$. The magnitude of u_{11} is decreasing with increasing y, due to the form of ξ_a. At $y = \delta$, ξ_a damps u_{11} with exp(-1). u_{12} also depends on θ with $\cos 2\theta$. This steady-flow contribution to the flow around the cylinder is given by Schlichting (see fig. 3).

One can also split the expansion of the shear stress in two parts:

$$\tau_{11} = \rho\nu \frac{\partial u_{11}}{\partial y} \qquad \text{and} \qquad \tau_{12} = \rho\nu \frac{\partial u_{12}}{\partial y} \qquad \text{(G.13)}$$

both at $y = 0$.
If u_{11} and u_{12} are of the form $\cos 2\theta$, this is also the case for τ_{11} and τ_{12}.

Now that the influence of the convective terms on the velocity-profile and the shear stress is known, the influence on the drag force on the cylinder can be examined. For that purpose, a work method is used (see Appendix D). The mean rate of doing work of the total shear stress is:

$$\overline{W} = \overline{\tau U} = \overline{(\tau_0+\tau_{11}+\tau_{12})(U_0+U_{11}+U_{12})} \qquad \text{(G.14)}$$

U is the external inviscid flow just outside the boundary-layer. Both u_{11} and u_{12} contribute to the external flow. u_{12} is not zero at infinity and u_{11} is rapidly damping with increasing y, but is not yet zero at y = δ. Thus U will also consist, like u, of terms U_0, U_{11} and $U_{12}$and therefore, the total work is:

$$\begin{aligned}\overline{W} = \ & \overline{\tau_0\ U_0} + \overline{\tau_0\ U_{11}} + \overline{\tau_0\ U_{12}} + \\ & \overline{\tau_{11}U_0} + \overline{\tau_{11}U_{11}} + \overline{\tau_{11}U_{12}} + \\ & \overline{\tau_{12}U_0} + \overline{\tau_{12}U_{11}} + \overline{\tau_{12}U_{12}} \end{aligned} \qquad \text{(G.15)}$$

From the Stokes solution, it is known that τ_0 and u_0 depend on θ with cosθ. Above is showed that τ_{11}, τ_{12}, u_{11} and u_{12} vary with θ as cos2θ. Thus, since integrating cosθcos2θ over 2π radians gives 0:

$$\overline{W} = \overline{\tau_0U_0} + 0 + 0 + 0 + \overline{\tau_{11}U_{11}} + \overline{\tau_{11}U_{12}} + 0 + \overline{\tau_{12}U_{11}} + \overline{\tau_{12}U_{12}} \qquad \text{(G.16)}$$

This expansion-method is only admissible if $u_1 << u_0$ and $\tau_1 << \tau_0$, thus if

$$\begin{aligned} &\overline{\tau_{11}U_{11}} << \overline{\tau_0U_0} \quad \text{and} \quad \overline{\tau_{11}U_{12}} << \overline{\tau_0U_0} \quad \text{and} \\ &\overline{\tau_{12}U_{11}} << \overline{\tau_0U_0} \quad \text{and} \quad \overline{\tau_{12}U_{12}} << \overline{\tau_0U_0} \end{aligned} \qquad \text{(G.17)}$$

so the expression for the work gets:

$$\overline{W} = \overline{\tau_0 U_0} \tag{G.18}$$

From the former equation one can see that the second order term in the solution for the velocity-profile does not contribute to the drag force. Brouwer (see [8]) proved this for the second and third order term in a more complex manner.

Though the expansion series showed that the drag force does not change when the convective terms are included in the equation of motion, one should keep in mind that expansion series are only allowed if $u_1 << u_0$, thus if $A/R << 1$. In our model, drag forces are calculated for amplitude upon radius ratios up to about 2. This means that the use of expansion series to determine the influence of the convective terms is not allowed.

We have shown that for small A/R ratios the convective terms do not influence the drag force, as long as the boundary-layer flow is laminar. For a turbulent boundary-layer flow the same can be done, because then ν_t may be used in stead of ν. This will yield the same result as for a laminar boundary-layer.

For the time being, we assume that for larger values of A/R the convective terms will not influence the drag force either. If necessary, this assumption will be checked later, For example when theory and experiments show very different drag forces.

APPENDIX H FIXED VERSUS OSCILLATING CYLINDERS

When the force on a cylinder is measured, one should distinguish an oscillating cylinder in still water from a fixed cylinder in oscillating water. The measured forces for the above mentioned cases are different, due to inertia. Usually, investigators derive the drag and inertia coefficients C_d and C_m, as they are defined by the Morison equation (see section 3.2), from their measurements. They even do so when the flow is still attached to the cylinder. To obtain the right C_d and C_m values from the measurements, the following equations should be used:

$$\text{fixed cylinder} \quad : F = 0.25\ C_m \rho \pi D^2 \frac{\partial U_0}{\partial t} + 0.5\ C_d \rho D |U_0| U_0 \qquad \text{(H.1)}$$

$$\text{oscillating cylinder} : F = 0.25\ C_a \rho \pi D^2 \frac{\partial U_0}{\partial t} + M \frac{\partial U_0}{\partial t} + 0.5\ C_d \rho D |U_0| U_0 \qquad \text{(H.2)}$$

in which M is the mass of the cylinder.

The forces are divided in an inertia-part in phase with the acceleration, and a drag-part in phase with the velocity (see eq. (H.1) and (H.2). In the eq. (H.1), C_m can be divided in an inertia coefficient for the added mass and an inertia coefficient for the fluid displaced by the cylinder: $C_m = C_a + 1$ (see Chakrabarti [9]).

It turns out that the equation for the force on a fixed cylinder contains an inertia term caused by the fluid displaced by the cylinder. The equation for the force on an oscillating cylinder contains an inertia term caused by the mass of the cylinder itself.

APPENDIX I WALL EFFECTS

To estimate the influence of the wall on the velocity profile in the tank, we assume a potential flow. The potential flow around an oscillating circular cylinder in still fluid is given by the following stream function:

$$\Phi = U\cos(\omega t)*r\sin(\theta) + U\frac{R^2}{r}\cos(\theta) \tag{I.1}$$

when the coordinate system is fixed to the cylinder (see [20]).
This yields

$$U_\theta = U\cos(\omega t)*\cos(\theta)(1 + \frac{R^2}{r^2}) \tag{I.2}$$

The maximum velocity during a cycle is

$$U_{max} = U(1 + \frac{R^2}{r^2}) \tag{I.3}$$

The velocity at the wall thus becomes 0.013 U with R = 0.2 m and r = 1.75 m. However, the wall will act as a mirror for the flow. At the wall, the undisturbed potential flow is U + 0.013 U. The disturbance of the wall will give a velocity at the wall, relative to the wall, of 0.02 U, in stead of 0.01 U.

Comparing this to the wall effects at the tests in a U-tube by Sarpkaya (see [21]), who used a tube of 1.42 m height and a cylinder with a diameter of 24 cm to achieve large β-values (about 11000), our deviation seems reasonable. The velocity at the wall in Sarpkaya's tests was 0.058 U, compared with 0.02 U in our case.

APPENDIX J STANDING WAVE

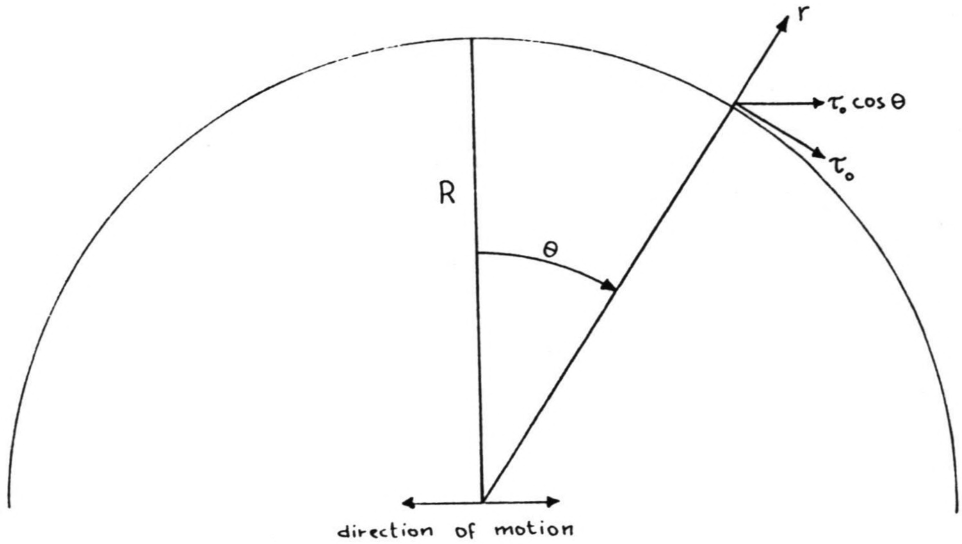
To calculate the natural frequency of the water in the tank, we calculate the case of a single standing wave in the tank, see fig. 4.
The wavelength is twice the length of the tank and thus 10 m, and the waterdepth is 2.5 m.
The velocity of the wave is given by

$$c = \left(\frac{g}{k}\tanh(kh)\right)^{0.5} \tag{C.2.1}$$

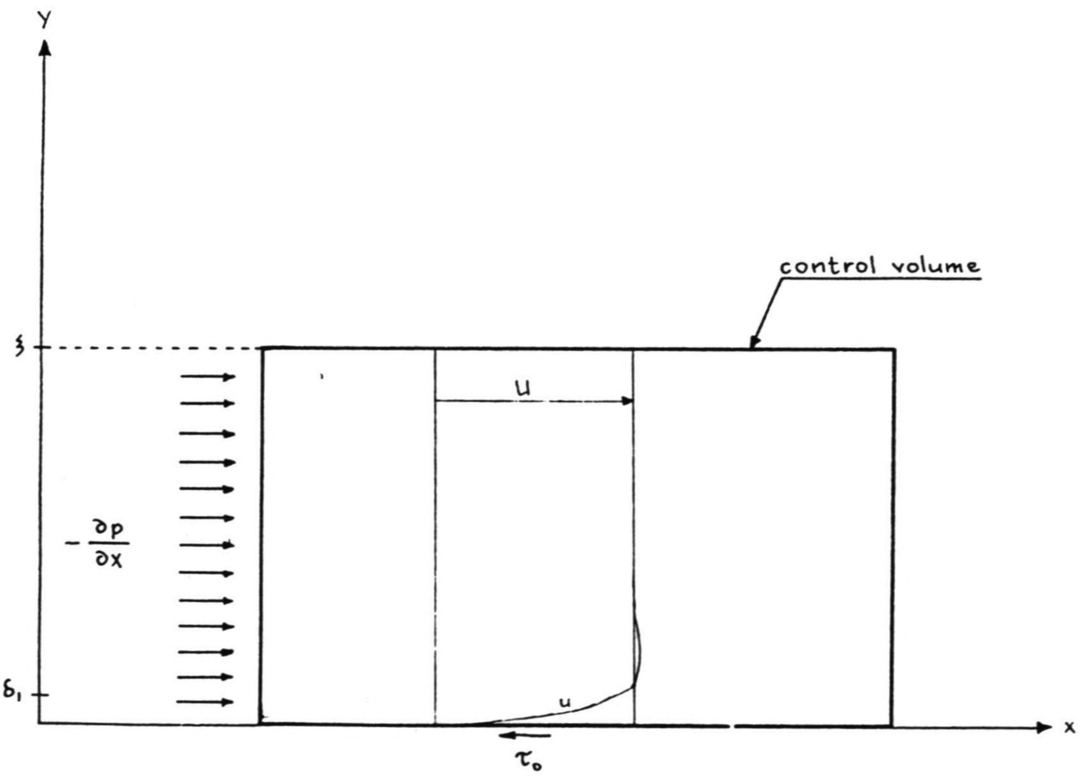
(see [5]). The lowest natural frequency is given by:

$$f_1 = \frac{c}{2l} \tag{C.2.2}$$

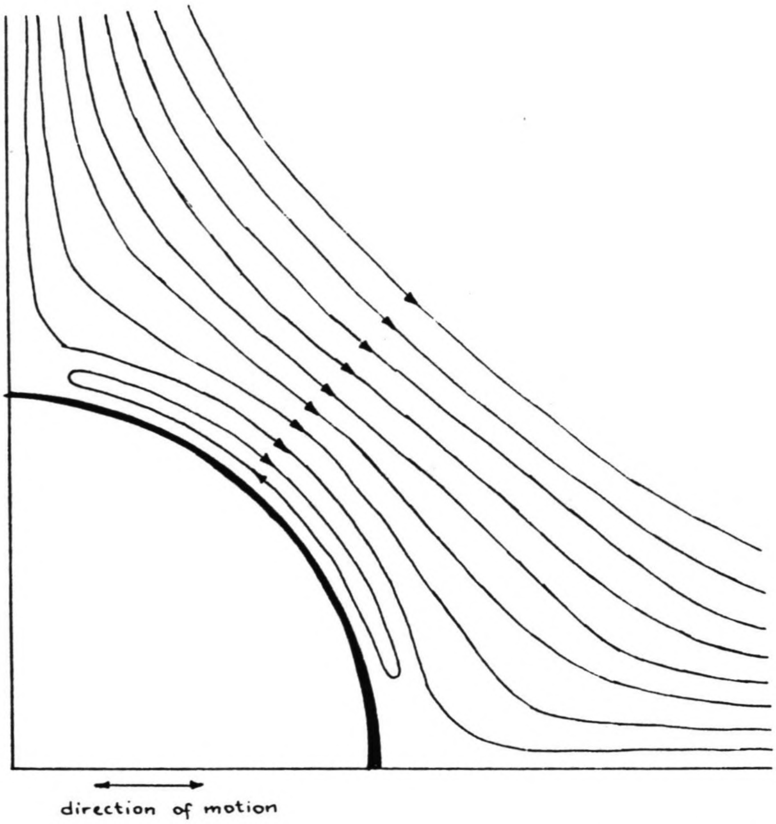
Using the correct values for our case yields c = 3.8 m/s and f_1 = 0.38 Hz.



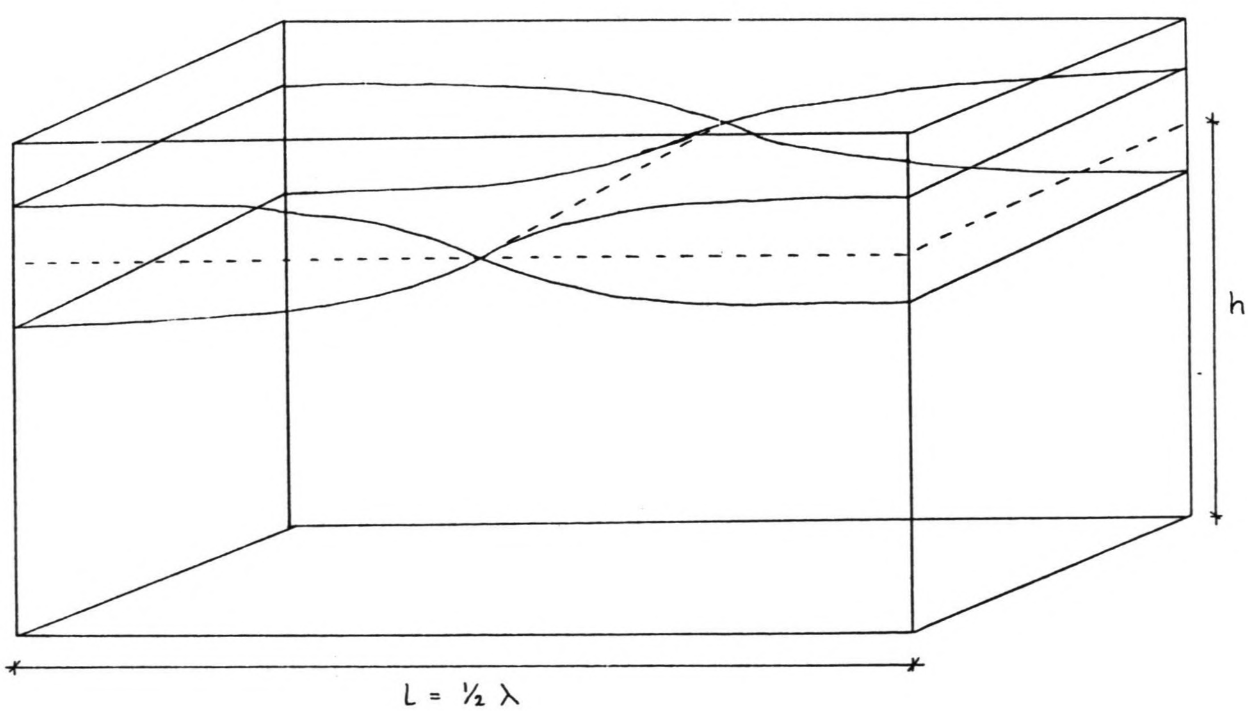
Direction of the wall shear stress and its component in the direction of motion Fig. 1



Control volume showing the flow field up to $y = \xi$ Fig. 2



Streamlines of the secondary motion (see Schlichting [27]) Fig. 3



Standing wave in tank Fig. 4