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DEPARTMENT OF AEROSPACE ENGINEERING

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Part A: Text

FOMABILITY:
relation between workhardening,
anisotropy an forming limit diagram

by

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DELT - THE NETHERLANDS

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1. SUMMARY

In the past little attention was paid in the literature to formability of sheet metal. The last 10-15 years this has changed; the importance of predicting the formability is recognized more and more.

This report deals with some basic questions related to formability. Part I is related to general considerations, as; definition of formability and methods of measuring formability. Also some attention is paid to the analysis of stress-strain relations and to plastic instability.

In Part II two aspects of plastic behaviour under uniaxial loading are discussed: workhardening exponent and the anisotropy values are reviewed. The influence of coldworking and annealing is discussed. Finally the results of experiments are analyzed.

Part III is dealing with plastic behaviour under biaxial loading. An experimental procedure for the determination of forming limit diagrams is described. The question of a failure criterion is very important in this respect. Forming limit diagrams are determined; the results are analyzed and discussed.

Finally the plastic behaviour under uniaxial and biaxial loading is compared and analyzed. Some suggestion for further research are presented.

In the experiments three materials were used: commercially pure aluminium and two aluminium alloys.
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3. NOTATIONS

$A_0$  
original area of a cross section

$A$  
final area of a cross section

$l_0$  
original gauge length

$l$  
final gauge length

$P$  
load

$\sigma'$  
engineering stress

$\sigma$  
true stress

$\bar{\sigma}$  
effective stress

$e$  
engineering strain

$\bar{e}$  
natural strain

$e$  
strain
effective strain

$C$  
specific stress

$n$  
workhardening exponent

$E$  
elastic modulus

$\sigma_p$  
proportional limit

$\sigma_y$  
yield stress

$\sigma_T$  
ultimate stress

$\varepsilon_{\text{inst}}$  
effective strain at the beginning of instability

$x$  
stress ratio $\frac{\sigma_2}{\sigma_1}$

$l$  
length

$t$  
thickness

$v$  
volume

$w$  
width

$r$  
factor of plastic anisotropy

$\Delta x$  
planar anisotropy

$r$  
normal anisotropy

Indices

1, 2, 3  
principal directions of stress and strain

0, 45, 90  
direction with respect to rolling direction (degrees)

0  
before deformation

l  
length direction

$\tau$  
thickness direction

$v$  
volume

$w$  
width direction
4. MATERIALS

For all experiments of this investigation the following three materials were used:
- commercially pure aluminium: Al;
- an aluminium magnesium alloy (3.5% Mg): 5154 (NA54S);
- an aluminium copper alloy (3.8 - 4.9% Cu, 1.2 - 1.8% Mg): 2024 (plated).

The material was ordered in sheets with dimensions of 6 x 3 feet, and a nominal thickness $t_0$ of 1/8 (0.125) in; 2024 was ordered in the T3 condition. The sheets were cut into pieces of approximately 14 x 7 in.

One third of these pieces was cold rolled to a final thickness of $t_1$, and another third to a final thickness of $t_2$; in the case of commercially pure aluminium $t_1$ and $t_2$ were 0.08 in and 0.04 in respectively (approximately); for the alloys these values were 0.10 in and 0.08 in. The rolling always occurred in the original rolling direction without any intermediate annealing.

After machining of the final test specimens, one half of the number of specimens was annealed. The aim of this annealing was to obtain recrystallization and to reduce the effect of cold working. The specimens were evenly spaced in a rack and loaded in an air circulating furnace when the desired temperature had been reached. Commercially pure aluminium and the aluminium-magnesium alloy were annealed at 350°C for 4 hour. The specimens were then taken from the furnace and air cooled. The aluminium-copper alloy was annealed at 420°C ± 5°C for 30 minutes and subsequently furnace cooled with a cooling rate of about 20°C/hour until a temperature of about 200°C followed by cooling in air.

As a result each material was available in 6 different conditions: three thicknesses $t_0$, $t_1$ and $t_2$ and each thickness with and without annealing.
5. FORMABILITY

5.1. Definition of formability
For a production process based on plastic deformation four questions are important:
1. How large is the amount of deformation a material can withstand without failure?
2. How accurate are the dimensions of the product?
3. How large is the load required for the production process?
4. What are the various properties of the final product?

In this report, the attention is primarily paid to the first question: "How large is the amount of deformation a material can withstand without failure?". The formability of one material is larger than another material if this amount of deformation is higher. Another important aspect of formability is: the probability of failure, or the safety factor against failure during the production. If two materials are formed into the same product the amount of deformation is the same. The material with the highest safety factor (the smallest chance of failure) has a better formability.

In the following definition both aspects, amount of the deformation before failure as well as safety factor against failure are included: "The formability of a material is its capacity to be submitted to a technical forming process without failure".

ad 2
The accuracy of a formed product does not depend on the material properties only, but also on the variables of the forming process. This aspect can be evaluated only if both the material properties and the forming process are considered.

ad 3
The loads which are required to form a certain product, are very important for the machine and tool manufacturers. These loads are closely related to the strength of the material and thus to the required strength of the product.

ad 4
The properties of the final product (strength, fatigue strength, etc.) are influenced by the material properties before forming and by the forming process. These properties are affected by the forming process as a result of: internal stress, changes in texture, forming anisotropy, work-hardening.

5.2. Determination of formability

5.2.1. General
The prediction or evaluation of the formability of sheet metal is very important for the production. If it is impossible to predict the formability, the successful forming of a new product is a matter of trial and error. As a result the try-out period is time-consuming and expensive. A formability test should show whether or not a certain material is suitable for the forming of a particular product. Two different methods for determining the forming capability of sheet metal are:
(1) measuring the fundamental mechanics properties;
(2) testing the forming properties by means of formability simulation tests.
General aspects of both methods are discussed below (see also ref. 1). A more detailed analysis of the stress-strain relation and plastic instability is given in chapters 6 and 7.
5.2.2. Mechanical properties and formability

5.2.2.1. The uniaxial tensile test

The determination of the fundamental mechanical properties in a tensile test is a well established procedure. Because the formability cannot be derived directly from tensile test, the mechanical properties must be translated into general forming properties. However, if these relationships can be derived, the formability of a sheet metal can be determined with a simple, well-defined test. Obviously this should be a great advantage.

The tensile test reveals a lot of information about the fundamental mechanical properties of sheet metal. Moreover it will be shown later that these properties can be translated into forming properties successfully! In a standard tensile test the load and the elongation are measured simultaneously. Form the load elongation curve, the engineering stress-strain curve can be derived. The engineering stress $\sigma'$ and the engineering strain $e$ are defined as:

$$
\sigma' = \frac{P}{A_o} \quad e = \frac{l - l_o}{l_o}
$$

$P$: load;
$A_o$: original area of a cross section;
$l$: final gauge length;
$l_o$: original gauge length.

The general form of the engineering stress-elongation curve is given in figure 1:

![Engineering stress-elongation curve](image)

Figure 1: Engineering stress-elongation curve

In chapter 6 it will be shown that the results of an uniaxial tensile test can be translated to any other deformation by employing the effective stress $\bar{\sigma}$ and the effective strain $\bar{e}$.

Initially the stress and strain are proportional, the ratio between stress and strain is the elastic modulus: $E$. The deformation is fully elastic. When the proportional limit $\sigma'_{p}$ has been reached, plastic deformation will start and the stress increases slower than the strain.

The material still continues to deform uniformly (see also chapter 7). The slope of the stress-strain curve becomes smaller with increasing deformation. After the stress reaches a maximum, $\sigma'_{u}$, the ultimate tensile stress, localized necking starts to develop. Subsequent deformation is no longer uniform. It is concentrated at one particular cross-section. At the same time the stress reduces until fracture occurs. The strain at
fracture is called the total elongation.
If the stress condition applied to the material allows localized necking, the uniform strain can be used as a measure for the formability. However, in many forming processes the stress condition prevents localized necking; then the total strain is more significant.

In forming operations there must be plastic deformation, otherwise forming is impossible. This means that the elastic part of the stress-strain curve, and thus the elastic modulus, is not very important. The proportional limit indicates the stress at which plastic deformation is starting and therefore this stress is sometimes used to evaluate the formability. In general the formability increases as the proportional limit decreases. However, this limit alone cannot be a very accurate measure for formability. When the proportional limit is exceeded the material starts to workharden. The workhardenning is the ability of the material to resist further deformation. This process continues until the ultimate tensile stress is reached. The total amount of workhardening is dependent on the ratio of the ultimate tensile stress to the proportional limit. Therefore this ratio might give a better indication of the formability than the yield stress alone.

In chapter 6 the engineering stress and strain will be replaced by the effective stress $\bar{\sigma}$ and the effective strain $\bar{\varepsilon}$. The plastic part of the effective stress-strain curve for many metals can be approximated by the power relation:

$$\bar{\sigma} = C \bar{\varepsilon}^n.$$

$n$ is called the workhardening coefficient. $C$ may be called the specific stress; it represents the effective stress $\bar{\sigma}$ when the effective strain $\bar{\varepsilon}$ equals 1.

The value of $n$ has an important influence on the uniformity of the strain distribution in the presence of a stress gradient. This effect is shown in figure 2 schematically. Assume that for an initial strain value of $P$, the two materials have the same stress level $A$. At some adjacent point in the product the stress has the value $B$: a stress-increase $\Delta \bar{\sigma}$.

The strain value for the high $n$ material increases to $Q$: a strain increase $\Delta \bar{\varepsilon}_1$. The strain of the low $n$ material becomes $R$: an increase of $\Delta \bar{\varepsilon}_2$. It is obvious from figure 2 that $\Delta \bar{\varepsilon}_2$ is greater than $\Delta \bar{\varepsilon}_1$. The strain distribution of the low $n$ material is more nonuniform than of the high $n$ material, which will have an unfavourable effect on the formability.

It will be shown later that theoretically the value of $n$ equals the strain at the onset of localized necking.

\[\begin{array}{c}
\bar{\sigma} = C \bar{\varepsilon}^n \\
\text{Figure 2: Effect on } n \text{ on the uniformity of the strain distribution}
\end{array}\]
For a material with a low n value necking starts at a lower strain value. Necking creates a very nonuniform strain distribution.

It is concluded that the workhardening coefficient n will be an useful property in describing the formability of sheet metal. A high n value is desirable if local regions with a high deformation (a nonuniform strain distribution) should be avoided; this is the case in stretchforming operations and in deep-drawing, particular in the walls of deepdrawn cups. In most bending operations it is desirable that the material deforms locally only (a nonuniform strain distribution); a low n value is favorable in this case.

5.2.2.2 Anisotropy
In evaluating the results of a tensile test it must be kept in mind that the mechanical properties of most sheet metals are dependent on the orientation with respect to the rolling direction. This effect, called directionally or anisotropy, has two aspects: mechanical fiber, which causes a fibrous structure of the material, and plastic anisotropy.

Mechanical fiber is present in varying degrees in all wrought materials as a result of the alignment of particles, pores or weak interfaces. During fabrication of sheet metal the material is rolled from an ingot to a sheet. All inclusions, porosities and other imperfections are oriented and elongated parallel to the rolling direction. Another consequence is that residual stresses are present in wrought materials. In rolled materials these stresses will act in the rolling directions and they can be neglected transverse to this direction. The residual stresses must balance across the thickness. Although these stresses will effect the start of plastic deformation, they will be less important for subsequent plasticity.

The largest influence of mechanical fiber is on the termination of the stress-strain curve and on ductile fracture in general. The effect will be minimal when the maximum stress or strain is imposed along the rolling direction.

In contrast to mechanical fiber, plastic anisotropy is crystallographic in origin. Each metal has one or several preferred directions with respect to deformation and slip. In an isotropic polycrystalline material the orientation of the crystal lattices of the various grains is fully random (no texture). In most wrought materials, however, certain orientations are clearly aligned with the directions of prior working. Due to this, the slip systems of the grains do not have a random orientation: deformation in some directions is easier than in other directions. This creates plastic anisotropy: the mechanical deformation properties in the rolling direction are different from those in the transverse directions.

Plastic anisotropy affects the overall shape of the stress-strain curve and therefore the mechanical properties as: yield stress, ultimate tensile stress, amount of workhardening and the value of the coefficient n.

For sheet metals the plastic anisotropy is characterized by the mechanical index r. In a tensile test both the width w and the thickness t will become smaller. Employing the natural strain instead of the engineering strain, the plastic anisotropy index r is defined by:

\[ r = \frac{\varepsilon_w}{\varepsilon_t} = \frac{\ln(w/w_0)}{\ln(t/t_0)} \]

The index o refers to the dimensions before deformation. It should be recognized that r may vary during a tensile test. Obviously r will loose its meaning beyond necking.

The determination of the r-value requires the measurements of two plastic strains. Unfortunately the measurements of a thickness increment of thin sheets can lead to large errors of the thickness strain.
However, bearing in mind that the volume remains constant during plastic deformation, the formula for $r$ can be rewritten as:

$$r = \frac{\varepsilon_w}{-(\varepsilon_1 + \varepsilon_w)} = \frac{\ln(w_w/w)}{\ln(-1/w_0)}$$

In studying the anisotropy of sheet metals the $r$-value is determined for different directions in the sheet. Usually these directions are at 0, 45 and 90 degrees to the rolling direction, see fig. 3. When the material is isotropic, the $r$ values in all directions are equal to unity. However, for all rolled materials (and in general for all wrought materials) the $r$-value varies with the direction: the material is anisotropic. The variation of $r$ within the plane of the sheet is called planar anisotropy $\Delta r$.

![Figure 3: Planar and normal anisotropy](image)

There are several definitions of planar anisotropy. The definition:

$$\Delta r = \frac{1}{4} \left( r_0 - 2r_{45} + r_{90} \right)$$

is used frequently. As a result of planar anisotropy ears occur in deepdrawing operations at the high $r$-value directions.

For the definition of the planar anisotropy $\Delta r$, it will be clear that it cannot give a complete picture of the plastic anisotropy. Isotropy would require all $r$-values to be equal to 1, which yields $\Delta r = 0$. However, $r_0 = r_{45} = r_{90} \neq 1$ also gives $\Delta r = 0$ and the same is true for $r_0 + r_{45} = 2r_{45}$. Nevertheless experience has shown that $\Delta r$ gives a useful indication of the planar variation of $r$.

Another indication about the anisotropy is obtained by calculating:

$$\bar{r} = \frac{1}{2} \left( r_0 + 2r_{45} + r_{90} \right)$$

Since this is a kind of an average $r$-value it will not reflect a planar variation of $r$. However, a sheet metal which exhibits a larger resistance to thinning than to width reduction will give high $r$-values in all directions. This will lead to a high $\bar{r}$-value ($>1$), although it may leave $\Delta r$ unaffected. For this reason $\bar{r}$ was labelled as the normal anisotropy or thickness anisotropy.

Anisotropy has a very clear influence on deep drawing operations, where a flat circular blank is drawn into a flat-bottom cylindrical cup. To avoid earing planar anisotropy must be absent, that means $\Delta r$ must be zero. During drawing the flange is stretched along the radial direction and compressed along the circumferential direction. Assuming that the thickness remains constant, the blank deforms in its plane only. A large deformation in its plane is necessary anyhow. To facilitate this deformation without the risk
of premature failure, a high normal anisotropy is desirable, because a high value of $r$ means that the blank has a preference to deform in this plane. In the cup wall straining in the circumferential direction is precluded by the punch. Therefore, elongation of the cup wall is accompanied by a thickness reduction. For this deformation a high $r$-value means a great resistance to thinning. The wall can support a larger drawing load without failure. A high normal anisotropy also means that the thickness reduction decreases, resulting in a final thickness more nearly the original thickness. This is important to ensure a minimum thickness variation.

It may be concluded that in deep drawing operations the formability is increased with increasing $r$-value. But unfortunately a high $r$-value is usually accompanied with a high $t$-value, which is undesirable with respect to earing.

5.2.3. Formability tests

5.2.3.1. Simulation tests

In most forming operations the deformation conditions are very complex. For this reason it is almost impossible to predict and to evaluate the plastic behaviour of materials under the complex stress and strain conditions from the results of uniaxial tensile tests. Therefore formability (or forming) simulation tests were developed.

For several types of basic forming operations, various simulation tests can be adopted. Deep drawing is simulated in the Swift flat-bottom cup test and the Sach's wedge-draw test. On the other hand the Erichsen and Olsen tests simulate stretch forming. A combination of deep drawing and stretch forming is simulated in the Swift round-bottom cup tests and the Fukuji conical cup test. Sheet bending can be simulated in a (pure) bending test.

In practice several types of basic forming operations are combined in one product. The relative amount of the basic operations can vary at different locations within one product and also with time at the same location.

One simulative test will evaluate only one of the many combinations of forming operations. Therefore it is impossible to obtain a generally applicable and reliable formability index from a simulative test. Thus simulative tests have a limited value for the evaluation of sheet metal formability.

5.2.3.2. Forming limit diagrams

The forming limit diagram is used as a tool to predict failure. It is assumed that failure depends on the plastic strains only, i.e. on the principal strains in the plane of the sheet. The principal strains at the onset of failure are plotted in a diagram (fig. 4). The curve through the failure points, obtained in tests with different strain situations, is called the critical strain level.

The critical strain level is a single characteristic curve for each material; it separates failure and non-failure conditions. Above the critical strain level the material fails, under this level the materials does not fail; at the critical strain level the material may fail. Thus the critical strain level can be used to predict the proximity to failure.
In a situation corresponding to point A, fig. 4, the product will fail. Without changing the maximum strain level, the unsafe condition A can be moved to the safe condition B by increasing the minor principal strain. It is obvious from the diagram that the point with the highest strain needs not always be the failure site; compare points A and C.

The higher the critical strain level the better the formability of the material. However, this picture is not complete. The question is now to obtain a strain distribution as uniform as possible under the critical strain level. When the strain distribution is less uniform, the peak strain will be higher, assuming that the same total deformation is obtained. The higher the peak strain the smaller the safety factor, the strain difference between the maximum strain and the critical strain level. In figure 2 it was illustrated that, comparing two materials with different n-values, the material with the highest n-value has the most uniform strain distribution. It is also obvious that the strain distribution is more uniform, when the stress gradient is smaller. However, this is not a material property, but a question of product-design.

As a result of cold working, the formability of a material decreases. First of all the amount of usable strain before failure is reduced, resulting in a lower critical strain level. Secondly, the strain distribution becomes less uniform. Thus the critical strain level and the peak strain approach each other.

Summarizing, the formability of a material is higher when:
- the critical strain level is higher;
- the strain distribution is more uniform, or:
  - the n-value is higher;
  - the stress gradient is smaller.
6. EFFECTIVE STRESS AND EFFECTIVE STRAIN

6.1. Introduction
In evaluating the formability of sheet metals, it is important to have a relationship between stress and strain. In the literature various theories of plasticity have been proposed, both for isotropic materials (ref. 2 and 3) and for anisotropic materials (ref. 4 and 5). These theories are based on a particular definition of stress and strain, i.e., the effective stress and the effective strain.

In this chapter the significance of the use of the effective stress and the effective strain, in describing a relationship between stress and strain, is discussed. The well-known uniaxial tensile test (see also ref. 6) is chosen as a starting point.

6.2. The stress
Usually the tensile stress in a tensile specimen is defined as:

\[ \sigma' = \frac{P}{A_0} \quad \text{(engineering stress)} \]  \hspace{2cm} (1)

P : load;
A₀: original area of a cross-section.
The magnitude of this stress is lower than the actual stress (see fig. 5); the cross-section of the specimen decreases when the axial stress and strain increase. The true stress \( \sigma \), defined as:

\[ \sigma = \frac{P}{A} = \frac{P}{A_0} (1 + e) \]  \hspace{2cm} (2)

gives a correct value, provided that necking of the specimen has not yet occurred.
A: final area of a cross-section;
e: engineering strain.

In a tensile test there is only one stress in the axial direction; the stress condition is uniaxial. This stress is uniformly distributed across a cross-section and is constant in the axial direction. When necking has occurred this condition does not hold any longer.

In general, plastic deformation will start when a specified function of the stress condition has reached a critical level. Frequently the criterion of Von Mises is used for this function:

\[ 2\sigma^2 = (\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2 = 2(I_1^2 - 3I_2^2) \]  \hspace{2cm} (3)

\( \bar{\sigma} \) is called: the effective stress; \( \sigma_1, \sigma_2 \) and \( \sigma_3 \) are the principal true stresses; \( I_1 \) and \( I_2 \) are the first and the second invariant of the stress tensor (ref. 7).
This criterion is applicable only when the following requirements are satisfied:

- the material is isotropic and homogeneous;
- a change of the sign of the stress condition does not influence the magnitude of the deformation; this
means for example that the tension and compression curves are congruent and that the Beuschinger effect is neglected;
- a hydrostatic pressure does not influence the deformation.

Originally the yield criterion of Von Mises was used only to predict the beginning of plastic deformation. In that case the effective stress equals the yield stress (in uniaxial deformation).

When it is assumed that plastic deformation after yielding is governed by the same relation between the principal stresses as yielding, eq. (3) can be used throughout the plastic range. The effective stress, $\bar{\sigma}$, does not equal the yield stress, but the true stress $\sigma$, found from an uniaxial tensile test.

In an uniaxial tensile test, without necking, there is only one principal stress in the axial direction. Thus:

$$\sigma_1 = \sigma$$

$$\sigma_2 = \sigma_3 = 0$$  \hspace{1cm} (4)

Substitution of (4) into (3) leads to:

$$\bar{\sigma} = \sigma = \frac{P}{A_0} (1 + e)$$  \hspace{1cm} (5)

When necking has occurred the deformation continues in the necking area only; the stress condition is no longer uniaxial and thus equation (5) does not hold anymore. In 1943 Bridgman derived theoretically a correction factor $B$ for the effective stress in the smallest cross-section (ref. 8). With this factor it is possible to interpret the stress, up to fracture, in an unequivocal manner.

![Stress-strain curve](image)

**Figure 5: Stress-strain curve**

6.3. The strain

The most commonly used strain in describing the stress-strain relation is the engineering strain $e$. For an uniaxial tensile test this strain (in the axial direction) is given by:
\[ e = \frac{1 - \frac{L}{L_0}}{L_0} \] \hspace{1cm} (6)

\( L_0 \): original gauge length;
\( L \): final gauge length.

In general, the engineering strain is not equal to the true strain: \( e \) is an average strain along the gauge length \( L_0 \). The strain in a particular point can be different from this average strain. When necking has occurred inside the gauge length, the average engineering strain is certainly an impractical measure for the true strain. A stress-strain curve, based on the average engineering strain, depends not only on the material but also on the gauge length and the point of necking. Such a curve is not useful.

In plastic deformation the volume is constant with a high degree of accuracy. Thus the following relation must hold:

\[ l_0 A_0 = l A \] \hspace{1cm} (7)

Combining eq. (6) and eq. (7):

\[ e = \frac{1}{L_0} - 1 = \frac{A_0}{A} - 1 \] \hspace{1cm} (8)

Therefore it is possible to calculate the strain along a very small gauge length \( L_0 \) from the measurement of the cross-section before and after deformation. Equation (8) gives the strain in a point and not the average strain along the gauge length.

Although the engineering strain \( e \) is used frequently, another strain definition \( \varepsilon \), called the natural strain, has many advantages. This strain is defined as:

\[ \varepsilon = \int_{L_0}^{L} \frac{dL}{L} = \ln \frac{L}{L_0} = \ln (e + 1) \] \hspace{1cm} (9)

The advantages of this definition are discussed briefly.

- It is assumed that during plastic deformation the volume remains constant. It can easily be proved that in this case the sum of the natural principal strains is zero:

\[ \varepsilon_1 + \varepsilon_2 + \varepsilon_3 = 0 \] \hspace{1cm} (10)

This equation does not hold for engineering strains. In a tensile test the axial direction is a principal direction. When the material is homogeneous and isotropic the principal strains perpendicular to the axial direction are equal. Thus:

\[ \varepsilon_2 = \varepsilon_3 = -\varepsilon = -\frac{1}{2} \ln \frac{L}{L_0} = -\frac{1}{2} \ln \frac{A_0}{A} \] \hspace{1cm} (11)
Therefore all principal strains are known after measurement of one of them, due to the constant ratio.

Consider a cylindrical bar with a length $l_0$. In one case the cylinder is extended to a length $l = a l_0$ and in the other case compressed to a length $l = \frac{l_0}{a}$ $(a > 1)$. It is expected that in both cases the absolute value of the deformation is the same. This is true when the natural strain is used:

**extension:**\[ e = \frac{1 - l_0}{l_0} = a - 1; \quad \varepsilon = \ln \frac{1}{l_0} = \ln a \]

**compression:**\[ e = \frac{1 - l_0}{l_0} = \frac{1 - a}{a}; \quad \varepsilon = \ln \frac{1}{l_0} = - \ln a \]

As a result the tension and compression curves will be congruent only when the natural strain is used. Thus it is correct only to use the plasticity criterion of Von Mises (eq. 3) in combination with the natural strain.

On account of the definition of the natural strain (eq. 9) the following relation must hold:

\[ \varepsilon = \sum_{i=1}^{n} \ln \frac{l_i}{l_{i-1}} \]

The total natural strain is equal to the sum of the independent partial strains; this does not hold for the engineering strain.

In a similar manner as the effective stress $\bar{\sigma}$, the effective strain $\bar{\varepsilon}$ can be defined as:

\[ \varepsilon^{-2} = \frac{2}{9} \left( (\varepsilon_1 - \varepsilon_2)^2 + (\varepsilon_2 - \varepsilon_3)^2 + (\varepsilon_3 - \varepsilon_1)^2 \right) = \frac{4}{9} \left( J_1^2 - 3 J_2^2 \right) \]

$\varepsilon_1$, $\varepsilon_2$, and $\varepsilon_3$ are the principal natural strains; $J_1$ and $J_2$ are the first and second invariant of the strain tensor (ref. 7). This equation is symmetric with respect to extension and compression. Substitution of eq. (11) into eq. (13):

\[ \bar{\varepsilon} = \varepsilon \]

6.4. Conclusion

From the equations (5) and (14) it follows that the tensile curve $\sigma(\varepsilon)$ is identical with the effective workhardening function $\bar{\sigma}(\bar{\varepsilon})$. For an isotropic material the relation between stress and strain can be a function between invariants of the stress and strain tensor only. Otherwise this relation should depend on the arbitrary chosen coordinate system, and this is unlikely to be true for an isotropic material. The effective stress and strain are invariants of the two tensors (see equation (3) and (13)).

During plastic deformation the tensile curve can be proceeded along in one direction only: a certain amount of workhardening cannot be cancelled by changing the sign of the load-increase. The natural strain has a geometrical meaning: this strain is defined by the quotient of two lengths (eq. (9)). When a material is extended from a length $l_0$ and afterwards compressed to its original length $l_0$, the total natural strain is
zero (eq. (12)). It is concluded that the natural strain in a certain direction is reversible and therefore not useful in describing the relation between stress and strain in the plastic region. The effective strain has no geometrical meaning; in principal it is a mathematical expression only. Therefore it is allowed to define that a change of the effective strain is always an increase; this strain is not reversible.

When the effective stress and the effective strain are defined as stated above, it must be concluded that the effective tensile curve \( \bar{\sigma}(\bar{\varepsilon}) \), in contrast with the \( \sigma(\varepsilon) \) curve, is unequivocal. On fundamental grounds the effective tensile curve is preferable in describing the material properties. In principle it is possible to extrapolate the material properties of the tensile test to every other deformation; the relation between effective stress and effective strain is independent from both the stress and strain state.

Often it is desirable to have a mathematical formulation for the relationship between stress and strain. In many cases the function:

\[
\bar{\sigma} = C \bar{\varepsilon}^n
\]

(15)

described the effective tensile curve very accurate (but in the plastic region only; in the elastic region Hooke's law is valid).
7. INSTABILITY OF PLASTIC DEFORMATION

7.1. Uniaxial tension

Plastic flow starts in a cross-section which for some reason is the weakest one. The area of this cross section reduces. However, the stress increases, because $\frac{d\sigma}{dc} > 0$. Due to workhardening the strength of the decreasing cross section increases. Plastic flow continues in another cross section and so on. The deformation is apparently uniform. When the deformation increases, the workhardening rate decreases. At a given moment, the geometrical softening (reduction in load due to the reducing cross section) will become greater than the workhardening (an increased load is required to produce each additional increment of elongation). Instability starts; from now on further deformation can be obtained by a decreasing load. In a usual tensile test set-up, this will be an unstable phenomenon. Moreover the deformation will be concentrated in a particular cross section, the necked down area. The remaining part of the specimen does not deform anymore.

The instability phenomenon can also be described mathematically.

$$P = \sigma A$$

($P =$ load; $A =$ area of the cross section; $\sigma =$ true stress). $P$ has a maximum when:

$$d (\sigma A) = \sigma dA + A d\sigma = 0$$

or:

$$\frac{dA}{A} = \frac{d\sigma}{\sigma}$$

Volume variance requires that:

$$dV = d(Al) = ldA + Adl = 0$$

($V =$ volume; $l =$ length)

or:

$$\frac{dA}{A} = \frac{dl}{l} = dc$$

(2)

From eq. (1) and eq. (2):

$$\frac{d\sigma}{dc} = \sigma$$

(3)

Instability occurs at the point on the stress-strain curve at which the slope of the curve $\frac{d\sigma}{dc}$ is equal to the value of the true stress $\sigma$ at that point.

When the power equation:

$$\sigma = C \epsilon^n$$

(4)

is introduced, eq. (3) may be written as:

$$\frac{d\sigma}{dc} = C \epsilon^n$$

(5)
Differentiation of eq. (4):

$$\frac{d\gamma}{dc} = C_n \epsilon^{(n-1)}$$  \hspace{1cm} \text{(6)}

From eq. (5) and eq. (6) it follows that at the beginning of instability the following equation must hold:

$$\epsilon_{\text{inst}} = n$$  \hspace{1cm} \text{(7)}

7.2. General biaxial stress system

In most forming operations the stress in the thickness direction of the sheet ($\sigma_3$) is zero or negligible compared to the stresses in the plane of the sheet ($\sigma_1$ and $\sigma_2$). The material is subjected to a biaxial stress system. When the stress ratio $x = \frac{\sigma_2}{\sigma_1}$ is introduced the effective stress $\bar{\sigma}$ can be written as:

$$\bar{\sigma} = \sigma_1 \left\{1 - x + x^2\right\}^{1/2}$$  \hspace{1cm} \text{(8)}

When the elastic strains are neglected, the Saint-Venant equations give the following relation between the strain increment ratio $\frac{d\varepsilon_2}{d\varepsilon_1}$ and the stress ratio $x$:

$$\frac{d\varepsilon_2}{d\varepsilon_1} = \frac{2x - 1}{2 - x}$$  \hspace{1cm} \text{(9)}

When the volume remains constant during deformation ($d\varepsilon_3 = - (d\varepsilon_1 + d\varepsilon_2)$) the effective strain increment $d\bar{\varepsilon}$ can now be written as:

$$d\bar{\varepsilon} = \frac{2d\varepsilon_1}{2 - x} \left\{1 - x + x^2\right\}^{1/2}$$  \hspace{1cm} \text{(10)}

If $P_1$ ($P_2$) is defined as the load per unit length in the $\sigma_1$ ($\sigma_2$) direction, and $t$ is the thickness of the specimen, then:

$$P_1 = \sigma_1 t$$  \hspace{1cm} \text{(11)}

$$P_2 = \sigma_2 t$$

Instability occurs when the larger load reached a maximum. When the $\sigma_1$ direction is chosen as the direction of the larger load, $x$ can vary between 0 and 1.

At the point of instability the following relation must hold:

$$dP_1 = d(\sigma_1 t) = \sigma_1 dt + t d\sigma_1 = 0$$  \hspace{1cm} \text{(12)}
or:
\[
\frac{d\sigma_1}{\sigma_1} = -\frac{dt}{t}
\]

(13)

From the volume invariance condition, it follows that:

\[
l \frac{dt + t \, dl}{dt} = 0 \quad \text{or} \quad \frac{dt}{t} = \frac{dl}{l} = \frac{dc_1}{c_1}
\]

(14)

(V = lt; l = length in the \(\sigma_1\) direction; the width equals 1)

Combining eq. (13) and eq. (14):

\[
\frac{d\sigma_1}{\sigma_1} = dc_1
\]

(15)

From equation (8) it follows that for a certain (constant) value of the stress ratio \(x\):

\[
\frac{d\sigma_1}{\sigma_1} = \frac{d\sigma}{\sigma}
\]

(16)

From eq. (10):

\[
dc_1 = \frac{d\sigma}{Z}
\]

(17)

with:

\[
z = \frac{2}{2 - x} \left\{ 1 - x + x^2 \right\}^{\frac{1}{2}}
\]

(18)

Substituting eq. (16) and eq. (17) into eq. (15):

\[
\frac{d\sigma}{\sigma} = \frac{dc}{Z}
\]

(19)

From the equations (4), (6) and (19) it follows that at the beginning of instability under a general biaxial stress system, the following equation must hold:

\[
\frac{c}{c_{\text{inst}}} = Z \, n
\]

(20)

(In the equations (4) and (6) the true stress and natural strain have to be replaced by the effective stress and the effective strain respectively.)

The strain at the onset of instability depends on the stress ratio \(x\). For a few values of \(x\), table I gives the corresponding values of \(Z\).
<table>
<thead>
<tr>
<th>Stress system</th>
<th>stress ratio</th>
<th>Z (from (18))</th>
<th>$\varepsilon_2$ (from (19))</th>
</tr>
</thead>
<tbody>
<tr>
<td>Balanced biaxial tension</td>
<td>1</td>
<td>2</td>
<td>$\varepsilon_1$</td>
</tr>
<tr>
<td>General biaxial tension</td>
<td>3/4</td>
<td>1.444</td>
<td>0.4 $\varepsilon_1$</td>
</tr>
<tr>
<td>Plane strain tension</td>
<td>1/2</td>
<td>1.155</td>
<td>0</td>
</tr>
<tr>
<td>General biaxial tension</td>
<td>1/4</td>
<td>1.031</td>
<td>-0.286$\varepsilon_1$</td>
</tr>
<tr>
<td>Uniaxial tension</td>
<td>0</td>
<td>1</td>
<td>-0.5$\varepsilon_1$</td>
</tr>
</tbody>
</table>

Table I

7.3. Evaluation

Assuming that the unstable behaviour of a material is a similar phenomenon under any stress condition, the instability criterion for uniaxial tension was extended to cover a general biaxial stress system. Instability in uniaxial tension is associated with the onset of localized necking. The necessary condition for necking is that along the trough of the neck $d\varepsilon_2 \leq 0$ (ref. 2). From eq. (9) it follows that this condition only holds for $x \leq \frac{1}{2}$. Therefore localized necking under a general biaxial stress system can only be expected as long as the stress ratio is smaller than $\frac{1}{2}$. When the stress ratio is larger the form of instability is often called: "diffuse necking".

Although instability in uniaxial tension and instability under a general biaxial stress system may be described by the same mathematical relation, the behaviour is quite different in both cases.
8. WORKHARDENING EXPERIMENTS

(Determination of the workhardening exponent $n$ and the specific stress $C$ from uniaxial tensile tests.)

8.1. Experimental procedures

8.1.1. Specimens

The dimensions of the specimens are given in fig. 6. These dimensions are according to British Standard Specifications 18. To prepare these specimens, strips with dimensions of $7 \times 1$ in. were cut from the sheet material and clamped in a tensile jig with the desired shape. The shaping was carried out on a "Tensilcut" machine: a double-edged cutter, rotating at 20,000 rev/min. with an adjustable guide. The jig was moved against the guide across the face of the cutter. By moving the guide towards the cutter, the specimens were given the desired shape. Finally the edges of the specimens were smoothed with emery paper.

The tensile specimens were taken at three orientations to the rolling direction: 0, 45 and 90 degrees. Each material was available in 6 different conditions (see chapter 4). For each condition the tensile tests were duplicated. This means that for each material 36 tests were needed (see tables 1a, 1b and 1c).

8.1.2. The testing machine

The uniaxial tensile tests were carried out on an "Introm" testing machine, a mechanical tensile test machine, with a maximum tensile force of 20,000 lb. The fixed (upper) crosshead of the machine contains an electronic load cell. The lower crosshead can be moved mechanically by means of two screw spindles at various speeds. The machine can be used in combination with a 2 in. gauge length strain gauge extensometer. Commercially pure aluminium was strained at a crosshead speed of 0.2 in./min. and the two alloys at a crosshead speed of 0.1 in./min.

The signals from the load cell and the extensometer were amplified to record load-extension curves on an X-Y plotter with maximum plotting ranges of 10 in. Various amplification factors can be selected on the plotter.

8.1.3. Determination $n$ and $C$

During each test a load-extension curve was recorded. The strain signal and the load-signal were amplified in such a way as to exploit the full X and Y-range of the plotter as far as possible. The fracture strain and the ultimate load were determined in preliminary tests. According to the results of these tests the required settings of the amplifiers were determined.

The general form of the recorded load-extension curve is given in fig. 7 schematically.
The true stress $\sigma$ and the natural strain $\varepsilon$, defined as respectively:

$$
\sigma = \frac{P}{A_0} (1 + \varepsilon) \quad \varepsilon = \ln (1 + \varepsilon)
$$

can be calculated from the load-extension curve. The original cross section $A_0$ was determined by measuring the original thickness and width of the specimens before deformation. A micrometer with a reading accuracy of 0.001 in. was used. The thickness and width are given in the tables: la, lb and lc.

It was shown before that for a uniaxial tensile test in which necking has not yet occurred, $\sigma$ and $\varepsilon$ are equal to the effective stress $\bar{\sigma}$ and the effective strain $\bar{\varepsilon}$. It was assumed that the relation between $\bar{\sigma}$ and $\bar{\varepsilon}$ in the plastic range can be represented by the formula:

$$
\bar{\sigma} = C \bar{\varepsilon}^n
$$

or:

$$
\ln \bar{\sigma} = \ln C + n \ln \bar{\varepsilon}
$$

From the load-extension curve the magnitude of the load can be read at a constant strain interval throughout the total range above the proportional limit and below necking. Theoretically necking starts when $dP$ equals zero. In our tests, however, there was not a single point at which $dP = 0$, but in almost every test, a region from $M$ to $M'$ (fig. 7) in which the load was constant. Therefore it is difficult to determine precisely the point at which necking starts. To be sure that necking does not influence the results it is advisable to consider data points up to point $M$ only.

The values of $n$ and $C$ can be calculated from any two points with the formula:

$$
n = \frac{\ln \bar{\sigma}_1}{\ln \bar{\varepsilon}_1} - \frac{\ln \bar{\sigma}_2}{\ln \bar{\varepsilon}_2}
$$

$$
C = \frac{\bar{\sigma}}{\bar{\varepsilon}^n}
$$
When this calculation is carried out for pairs of subsequent points \((i\text{ and } i+1,\text{ for } i = 1, 2, \ldots, \text{no} - 1;\text{ no} = \text{total number of points})\) an indication is obtained about the variation of \(n\) (and \(C\)) along the stress-strain curve. However, this method is not suitable for the determination of the best approximation of \(n\) and \(C\). The method is very sensitive to reading inaccuracies.

When all point are used, the problem of finding the best values of \(n\) and \(C\) may be solved using the criterion of least squares. Equation (2) can be written as:

\[y = a + bx, \text{ with } a = \ln C \text{ and } b = n\]

Linear regression calculations with \(x = \ln \varepsilon\) as the independent variable and \(y = \ln \sigma\) as the dependent variable give the best estimates of \(a\) and \(b\) and thus \(n\) and \(C\). In linear regression calculations the correlation coefficient \(R_C\) is a measure for the correlation between \(y\) and \(x\) and as a consequence also for the accuracy of the relation between \(y\) and \(x\) as approximated by a straight line. For a straight line \(R_C\) equals 1.

### 8.2. Results and discussion

#### 8.2.1. General remarks

A survey of the tensile test specimens for the three materials is given in the tables 1a, 1b and 1c. In some cases the test was not carried out correctly. The results of these specimens, marked with \(*\), are omitted. When fracture occurs outside the extensometer, the recorded load-elongation curve is still correct in the range before necking, but not after necking. For the calculation of \(n\) and \(C\) only points below necking are considered. Therefore a fracture outside the extensometer is not serious for the present purpose.

#### 8.2.2. The calculated values of \(n\) and \(C\)

In the graphs 1a, 1b and 1c examples of the observed values of stress and strain are compared with the calculation curve: \(\sigma = C \varepsilon^n\). The same comparison is made in the graphs 2a, 2b and 2c on double logarithmic paper. The agreement is rather good in all cases. Therefore it is concluded that the results of the tensile tests can be described by the formulae: \(\sigma = C \varepsilon^n\).

The tables 2a, 2b and 2c give the calculated values of \(n\) and \(C\) and the value of the correlation coefficient \(R_C\) for all specimens. In all cases the correlation coefficient \(R_C\) is high. The average values of \(R_C\) are respectively: 0.9831, 0.9922 and 0.9884. The smallest values are: 0.9267, 0.9321 and 0.9403 (these values refer cold worked specimens). It is concluded that \(\sigma = C \varepsilon^n\) represents the test results very good.

In the tables 2a, 2b and 2c the values of \(n\) and \(C\) are given up to point \(M\) and up to points \(M_1\). The values are not the same for the two points. It was tested statistically whether or not the values of \(n\) for both points differ significantly. The null hypothesis \((p = 1/2)\) was tested with the sing-test and a probability level of 5%. For all three materials the null hypothesis has to be rejected. The values of \(n\) up to point \(M\) are significantly higher than the values up to point \(M_1\).

To be sure that necking does not influence the results, it is advisable not to go further than point \(M\) anyway. Therefore these values of \(n\) and \(C\) are summarized in the tables 3a, 3b and 3c.

#### 8.2.3. Variation of \(n\) along the stress-strain curve

For one specimen of each material the variation of \(n\) along the stress-strain curve was investigated (tables 4a, 4b and 4c).

In column (1) of the tables the values of \(n\) are given when a very small strain interval between two subsequent points \(i\) and \(i+1\) is used. As expected, the scatter is rather large, due to the effect of the reading inaccuracies on the small strain interval. The effect of reading inaccuracies and therefore the scatter becomes smaller, if two points but a larger strain interval is used (column (2)).
This also holds if the n-values, obtained in small intervals, are averaged over a larger interval (column (3)). The difference between the values of column (2) and (3) is rather small and it can be explained partly by the reading inaccuracies. Therefore it is concluded that n does not vary considerably along the stress strain curve. Apparently the value of n may be considered as a constant for a particular material in a particular condition.

The conclusion does not hold for the initial part of the stress-strain curve for commercially pure aluminium (table 4a) and the aluminium-copper alloy (table 4c).

However the conclusion still hold when the first 9 points in table 4a and the first 18 points in table 4c are neglected. Investigation of the results of the other specimens reveals that this initial effect is present in almost every case for all three materials. It is concluded that for the initial part of the stress-strain curve n is not constant, but may vary. However, this effect does not influence the conclusion of chapter 8.2.2. that \( \bar{\sigma} = C \bar{\sigma}^N \) represents the test results very well.

In the columns (4) and (5) of the tables the n values are given when the first and the last point of the total range are used, respectively the n values of the small intervals are averaged over the total range.

In both cases the n-values are different from the value obtained with linear regression, but the difference is rather small. Therefore it is considered that a good approximation of the n value is obtained when a strain interval as large as possible is used.

8.2.4. Relation between n and \( \bar{\varepsilon}_{\text{neck}} \)

Theoretically the value of n must be equal to the strain at the beginning of necking, \( \bar{\varepsilon}_{\text{neck}} \). In our experiments it was difficult to decide precisely at which point necking starts: somewhere between the points \( M \) and \( M^\prime \). For this reason it was expected that n will equal the strain at some point between \( M \) and \( M^\prime \) (\( \bar{\varepsilon}_N < \bar{\varepsilon}_{\text{neck}} < \bar{\varepsilon}_{M^\prime} \)). In the graphs 3a, 3b and 3c n is plotted as a function of \( \bar{\varepsilon}_N \) and in the graphs 4a, 4b and 4c as a function of \( \bar{\varepsilon}_{M^\prime} \). It must be concluded that the expectation does not hold generally as illustrated by a summary of the results given below.

**Commercially Pure Aluminium**
- annealed: \( \bar{\varepsilon}_N < n < \bar{\varepsilon}_{M^\prime} \)
- as received: \( n < \bar{\varepsilon}_N (\bar{\varepsilon}_N < \bar{\varepsilon}_{M^\prime}) \)
- cold worked: \( \bar{\varepsilon}_{M^\prime} < n \)

**Aluminium-Magnesium Alloy (5134)**
- all conditions: \( (\bar{\varepsilon}_N < n < \bar{\varepsilon}_{M^\prime}) \)

**Aluminium-Copper Alloy (2024)**
- as received: \( \bar{\varepsilon}_N < n < \bar{\varepsilon}_{M^\prime} \)
- annealed: \( \bar{\varepsilon}_{M^\prime} < n \)
- cold worked: \( \bar{\varepsilon}_{M^\prime} < n \)

An explanation of the discrepancies would require further investigation.

8.2.5. The effect of cold working

As a result of cold working, the value of both n and \( \bar{\varepsilon}_{\text{neck}} \) will decrease; this is clearly shown in the graphs 3a, 3b and 3c. The values seem to be the same for \( t = 0.08 \) and \( t = 0.04 \) (graphs 3a and 4a) and \( t = 0.10 \) and \( t = 0.08 \) (graph 3b, 3c, 4b and 4c). It was expected that n decreases with decreasing thickness (more deformation by cold working).
Therefore in the graphs 5a, 5b and 5c examples of the workhardening curves: \( \sigma = C \varepsilon^n \) are plotted for the three thicknesses of each material (in the rolling direction). It seems that as a result of the first thickness reduction (from \( t = 0.125 \) to \( t = 0.08 \) or \( t = 0.10 \)) \( n \) decreases. The effect of the second reduction (from \( t = 0.08 \) to \( t = 0.04 \), or from \( t = 0.10 \) to \( 0.08 \)) on \( n \) is very small (sometimes \( n \) decreases and other times \( n \) increased). The specimens with an angle to the rolling direction of 90 and 45 degrees behave in the same way (tables 3a, 3b and 3b).

The effect of the first thickness reduction on \( n \) was as expected. The effect of the second thickness reduction cannot be explained; it was expected that after the second reduction the value of \( n \) should be smaller than after the first reduction (more deformation by cold working). Perhaps a critical amount of cold working exists after which the \( n \) value is not more affected. To investigate this aspect small amounts of succeeded thickness reductions should be used.

8.2.6. The effect of annealing

The aim of annealing is to reduce the effect of cold working. This means that both \( n \) and \( C \) should return to their original values. From that graphs 3a and 3b (see also tables 3a and 3b) it is obvious that this holds roughly as far as \( n \) is concerned. Based on the graphs 5a and 5b (and the tables 3a en 3b) the same conclusion holds with respect to \( C \). The \( n \) values of the annealed material are close together for all three thicknesses. Besides these values are higher than the \( n \) value in the as delivered condition. Therefore it is concluded that at the end of the rolling process, the material is cold worked to the ultimate thickness; the material in the as delivered condition is not completely virginal.

In case of the Aluminium-Copper Alloy there is another point to be considered. Annealing of this material not only eliminates the effect of cold working but also the effect of the dispersion hardening. After annealing the material is in the soft condition and the value of \( C \) is much lower than in the aged condition (graphs 5c and table 3c). This means that the forces for deformation and also the ultimate stress are lower (expected). The \( n \)-value in the soft condition is higher than in the aged condition (graphs 5c and table 3c). The annealed material with \( t = 0.10 \) has a considerable lower \( n \) and \( C \) value than the material with \( t = 0.125 \) and \( t = 0.08 \). For \( t = 0.10 \) the \( n \)-value looks more like that of the aged condition. The annealing treatment was apparently not fully complete.

8.2.7. The effect of the rolling direction

The effect of the rolling direction on \( n \) and \( \varepsilon_{\text{neck}} \) is discussed only for the as received and the annealed condition. Both \( n \) and \( \varepsilon_{\text{neck}} \) in the cold worked condition are so small (see tables 3 and 5) that differences between both values, at different angles to the rolling direction, could be contributed to inaccuracies.

In chapter 8.2.4. the relation between \( n \) and \( \varepsilon_{\text{neck}} \) was discussed. Although the expectation: \( n \) equals \( \varepsilon_{\text{neck}} \) does not hold generally, it is still very likely that the direction with the greatest \( n \) value and the greatest value of \( \varepsilon_{\text{neck}} \) will coincide. As illustrated by a summary of the results given below, this appears not to be true in all cases.

Commercially Pure Aluminium (graph 6a)
- except for the annealed condition, \( t = 0.04 \) in., both \( n \) and \( \varepsilon_{\text{neck}} \) are the greatest in the 45°-direction; in the 0°- and 90°-direction the differences are rather small.
- in the annealed condition, \( t = 0.04 \), the value of \( \varepsilon_{\text{neck}} \) is still the greatest in the 45°-direction; the \( n \)-value is the greatest in the 0°-direction but the difference with the other directions is small.

Aluminium-Magnesium Alloy (5154) (graph 6b)
- for the annealed condition, \( t = 0.125 \) in. \( n \) and \( \varepsilon_{\text{neck}} \) are the greatest in the 90°-direction,
- for the annealed condition, \( t = 0.08 \) in., \( n \) and \( \varepsilon_{\text{neck}} \) are the greatest for the 45°-direction; however, there is very little difference in the \( n \)-value for the three directions,
- for the as recieved condition, the $n$-value is the greatest for the $45^\circ$-direction; the difference in the value of $\varepsilon_{\text{neck}}$ for the $90^\circ$- and the $45^\circ$-direction is small,
- for the annealed condition, $t = 0.10$ in., the $n$-value is the greatest in the $0^\circ$-direction, and the value of $\varepsilon_{\text{neck}}$ in the $45^\circ$-direction.

Aluminium-Copper Alloy (2024) (graphs 6c)
- for the annealed condition, $t = 0.08$ in., $n$ and $\varepsilon_{\text{neck}}$ are the greatest in the $90^\circ$-direction,
- for the as recieved condition, both $n$ and $\varepsilon_{\text{neck}}$ have nearly the same value for the $45^\circ$- and $90^\circ$-direction,
- for the annealed condition, $t = 0.125$ in., $n$ is the greatest in the $0^\circ$-direction and $\varepsilon_{\text{neck}}$ in the $90^\circ$-direction; the difference in the $n$-values for all three directions is small,
- for the annealed condition, $t = 0.10$ in., $n$ is the greatest in the $90^\circ$-direction and $\varepsilon_{\text{neck}}$ in the $0^\circ$-direction; the difference in the values of $\varepsilon_{\text{neck}}$ for all three directions is small.
9. ANISOTROPY EXPERIMENTS

9.1. Experimental procedures

9.1.1. Specimens and testing machine

The specimens were prepared in the same way as in the case of the workhardening experiments (see chapter 8.1.1.); the shape of the specimens was also the same (see fig. 6). The specimens were provided with line markings for strain measurements (see fig. 7).

Figure 7: Line markings on the specimen

The specimens were taken at three orientations to the rolling direction: 0, 45 and 90 degrees. Each material was tested in 6 conditions. In each condition two specimens were tested. This means that for each material the total number of specimens was 36 (see tables 6a, 6b and 6c).

The tests were carried out on an Instron tensile testing machine (see chapter 8.1.2.). During each test a load-elongation curve was recorded, however without employing a strain gauge extensometer. In all cases a chart speed of 2 in./min., and a crosshead speed of 0.1 in./min., were used.

9.1.2. Determination of the anisotropy values

The anisotropy factor \( r \) can be determined with either of the formulae:

\[
\frac{\epsilon_w}{\epsilon_t} = \frac{\ln w/w_o}{\ln t/t_o} \quad (1)
\]

or:

\[
\frac{\epsilon_w}{-(\epsilon_1 + \epsilon_1')} = \frac{\ln w/w_o}{\ln w/w_o} \quad (2)
\]

\( l \): length
\( w \): width
\( t \): thickness

index o: before deformation

Mostly formulae (2) is used, because a higher accuracy could be obtained. This is correct only when the volume remains constant during plastic deformation. To check this hypothesis the three strains were determined, independently.

The usual technique for determining the \( r \) value is to apply line markings on the specimen surface for measuring \( \epsilon_1 \). The line distance between two markings is measured with a travelling microscope, before and after deformation. The width strain, \( \epsilon_w \) (and in this investigation also \( \epsilon_t \)) can be determined with a micrometer. However, it is also possible to adapt line markings for determination of \( \epsilon_w \). This will have advantages...
when the sides of the specimens are not fully parallel or when the sides are rough. For this reason a width of 5/16 in. approximately was marked on the specimens.

When the deformation is homogeneous along the length of the specimen, it should be expected that the $r$-value is independent of the gauge length $l_0$. To check this point two gauge lengths $l_0$ were marked on the specimens, one of approximately 1\(\frac{1}{4}\) in. and one of 1 in.

In the literature it is often stated that the $r$-value is independent of the magnitude of the strain in the length direction. To test this statement two specimens were used. One specimen was given a large deformation (but still below necking), and the other one a much smaller deformation (above yielding).

The distances between the line markings were measured with an accuracy of $\pm 0.001$ mm (using a travelling microscope). The width and thickness of the specimens were measured with an accuracy of $\pm 0.0001$ in. (using a micrometer) at the points -B, -A, 0, A and B (fig. 7); these measurements were averaged.

The planar anisotropy $\Delta r$ and the normal anisotropy $\bar{r}$ were calculated with the well known formulae:

$$
\Delta r = \frac{r_0 + r_{90} - 2r_{45}}{2}
$$

$$
\bar{r} = \frac{r_0 + r_{90} + 2r_{45}}{4}
$$

9.2. Results and discussion

9.2.1. General remarks

9.2.1.1. Accuracy of the strains

In the tables 6a, 6b and 6c the calculated strains in the length-, width- and thickness-direction for all three materials are shown. The accuracy of these strains depends on several factors. First of all: the reading accuracy of the micrometer and the travelling microscope. When the travelling microscope is used, a distance is determined as the difference of two measurements. Thus the absolute accuracy is twice the reading accuracy: $\pm 2 \times 0.001$ mm. The absolute reading accuracy of the micrometer is: $\pm 0.0001$ in.

In general the total accuracy at which distances can be determined will be lower than the reading accuracies, for instante due to: sharpness of the line marking, surface roughness, sides of the specimens not parallel. To obtain an idea about the total accuracy in a few cases the same distance was measured several times. When the micrometer was used, the total accuracy appeared to be the same as the reading accuracy. In the case of the travelling microscope the total accuracy was about twice the reading accuracy.

When it is now supposed that the dimensions before and after deformation are the same, the absolute total accuracies of the strains may be calculated. The results is shown in table II.
<table>
<thead>
<tr>
<th>ε₁</th>
<th>l₀ = 1½ in.</th>
<th>± 0.0002</th>
<th>0.0040</th>
<th>0.0020</th>
</tr>
</thead>
<tbody>
<tr>
<td>l₀</td>
<td>1 in.</td>
<td>0.00032</td>
<td>0.0064</td>
<td>0.0032</td>
</tr>
<tr>
<td>εₘ</td>
<td>microscope (5/16 in.)</td>
<td>0.0010</td>
<td>0.0200</td>
<td>0.0100</td>
</tr>
<tr>
<td></td>
<td>micrometer (1/2 in.)</td>
<td>0.0004</td>
<td>0.0080</td>
<td>0.0040</td>
</tr>
<tr>
<td>εₜ</td>
<td>t = 0.125 in.</td>
<td>0.0016</td>
<td>0.0320</td>
<td>0.0160</td>
</tr>
<tr>
<td></td>
<td>t = 0.100 in.</td>
<td>0.002</td>
<td>0.0400</td>
<td>0.0200</td>
</tr>
<tr>
<td></td>
<td>t = 0.080 in.</td>
<td>0.0025</td>
<td>0.0500</td>
<td>0.0250</td>
</tr>
<tr>
<td></td>
<td>t = 0.040 in.</td>
<td>0.0051</td>
<td>0.1000</td>
<td>0.0500</td>
</tr>
</tbody>
</table>

* values of the strain when the total relative accuracy equals 5 and 10% respectively.

Table II: Accuracy of the strains

The table II also shows the values of the strains when the total relative accuracy equals 5 and 10%. When these accuracies are compared with the strain given in the tables 6a, 6b and 6c the following conclusion may be drawn:

- With respect to ε₁:
  . for almost all specimens the accuracy is better than 10%, in most cases even better than 5%.
  . the aluminium-copper alloy in the cold worked condition (t = 0.08 in.), specimens numbers 23, 29 and 35 are exceptions (accuracy in the order of magnitude of 20%).

- With respect to εₘ:
  . for all three materials in the as-received and the annealed conditions the accuracy is better than 5%.
  . for all three materials, in the cold worked conditions, with exception of the specimens 3, 9 and 12 of the two alloys (t = 0.100 in, high value of ε₁) the accuracy is worse than 10%.

- With respect to εₜ:
  . for all three materials in the as-received and the annealed condition, with exception of the specimens 9, 15, 6, 24 and 30 for commercially pure aluminium, the accuracy is better than 10%, for a lot of specimens even better than 5%, particularly for the thicker specimens.
  . for all three materials in the cold worked conditions the accuracy is worse than 10%.

These conclusions have affected the analysis of incompressible material and the r-values. Due to the poor accuracy of εₜ in the cold worked condition, these specimens are omitted from the analysis of the volume change. With exception of the specimens 3, 9 and 12 of the two alloys, the cold worked specimens are also neglected in the analysis of the r-values.

9.2.1.2. Results for commercially Pure Aluminium

The plastic elongation of the specimens of commercially pure aluminium caused a bending of the cross-section. In lit. 9 it is stated that curling about the tensile axis is presumable due to through-thickness texture variations. The texture variation is a result of different variables of the rolling process: friction between sheet and rolls, draught, temperature of rolling and the rolling sequence, either unidirectional or reversed between passes (lit. 10).
A possible through-thickness texture variation of the commercially pure aluminium used in our experiments was not investigated. In table 6a the radius of the cross-section of the specimens, assuming that the shape is a circle segment measured in the middle of the specimens, is shown. It appears that there is no relation at all between the value of the radius and factors as: cold working, annealing, magnitude of $\varepsilon_1$.

When the cross-section is curved, the measurement of the width particular with the micrometer, becomes less accurate. For this reason table 6a gives the width strain determined with the microscope only.

A curved cross-section means that the strain in the width direction and therefore the anisotropy values, is not constant through the thickness. If the curvature is constant along the length of the specimens, the variation through the thickness of the strain and the anisotropy values can be calculated. However, the curvature was not constant. It had a maximum in the middle, towards the ends the curvature decreased. For these reasons the meaning of the calculated r-values is questionable, they only can represent an "average".

9.2.2. Incompressible material
The calculated change of volume is shown in the tables 7a, 7b and 7c. In the graphs 7a, 7b and 7c the volume strain is plotted as a function of the length strain.

From the graphs 7b and 7c it is obvious that the volume-change of the two alloys is small, in almost every case smaller than 1%. The volume-change of commercially pure aluminium is much higher, up to 11%; perhaps this high magnitude was caused by the curved cross-section, discussed in par. 9.2.1.2.

When the deformation is in the elastic region the volume-change is positive. For this reason it was expected that in the plastic region the volume-change should be also positive (but very small) or zero.

When the averaged volume-change is considered the expectation holds for commercially pure aluminium (positive) and the aluminium-copper alloy (zero) but not for the aluminium-magnesium alloy (negative).

From the above results it is concluded that the assumption of incompressible material is valid for the two alloys. This means that the formulae $r = \frac{\varepsilon_w}{\varepsilon_t}$ can be used instead of $r = \frac{\varepsilon_w}{\varepsilon_t}$; it will give a better relative accuracy of the r-value.

It is difficult to decide from the present results whether or not commercially pure aluminium is incompressible. For this reason the r-values were calculated with the formulae: $\frac{\varepsilon_w}{\varepsilon_t}$.

9.2.3. The most reliable method for the determination of r
(Influence of $\varepsilon_w$, $l_0$ and $\varepsilon_1$).

The calculated r-values for the three materials are summarized in the tables 8a, 8b and 8c. It was tested statistically whether a significant influence on the calculated r-values (a and b only for the alloys) could be indicated depending on:

a. different ways in which the width is measured;

b. different gauge length $l_0$;

c. different magnitudes of the length-strain $\varepsilon_1$.

The null hypothesis is that a significant influence of the variables does not exist (p = 1%), which was tested with the sign-test and a probability level of 5%. In all cases the null hypothesis holds. However, on subjective reasons, it was decided that for all three variables a preference exist.

ad a
The absolute accuracy of the width-measurement is higher for the micrometer than for the microscope (see table II, par. 9.2.1.1.). For this reason $\varepsilon_w$ determined with the micrometer is used.
When the gauge length \( l_o \) increases, the accuracy of the length strain (see table II, par. 9.2.1.1.), and thus the \( r \) value, increases too. For this reason a gauge length of 1\( \frac{1}{2} \) in. instead of 1 in. is used in the following paragraphs. To investigate the question of a homogeneous deformation in more detail, it is advisable to apply a very fine screen on the specimens. This screen should be measured before and after deformation.

With the same absolute accuracy, the relative accuracy increases with an increasing value of the strain. For this reason a high value of \( \varepsilon_t \) is advisable in determining the \( r \) values. When during deformation both \( \varepsilon_t \) and \( \varepsilon_w \) are recorded, the influence of the magnitude of the length strain on the \( r \) value can be investigated in more detail.

In the table 9a, 9b and 9c the most reliable \( r \) values, and the calculated values of \( \Delta r \) and \( \bar{r} \) are summarized. The values are plotted in the graphs 8a, 8b and 8c.

9.2.4. The effect of the rolling direction

For an anisotropic material the \( r \) value depends on the direction in the plane of the sheet. This directionality is clearly shown by the planar anisotropy \( \Delta r \). When \( \Delta r = 0 \) directionally does not occur; when \( \Delta r > 0 \), or \( \Delta r < 0 \), the average of \( r_0 \) and \( r_{90} \) is greater, resp. smaller, than \( r_{45} \).

There is an obvious difference between commercially pure aluminium (\( \Delta r > 0 \)) and the two alloys (\( \Delta r < 0 \)). In all conditions, except the aluminium-copper alloy \( t = 0.080 \text{ in.} \), annealed (table 9c, specimen numbers 6, 12 and 18, \( \Delta r > 0 \)). This different behaviour may be caused by a mistake in the determination of \( \varepsilon_t \) for specimen 18, in comparison with the specimens 6 and 12 (table 6c, column \( l_o = 1\frac{1}{2} \text{ in.} \)). When the specimens 24, 30 and 36 are used (table 6c, column \( l_o = 1\frac{1}{2} \text{ in.} \)) \( \Delta r \) becomes negative (table 9c), in agreement with the other conditions.

The \( \Delta r \) value for commercially pure aluminium, \( t = 0.040 \text{ in.} \), annealed, is extremely high (table 9a, specimen 6, 12 and 18). This is caused by a very high value of \( r_0 \) (specimen 6). Inspection of table 6a reveals that for specimen 6 \( \varepsilon_t \) is low, just like specimen 12, 24 and 30. All this specimens have a thickness of \( t = 0.040 \text{ in.} \) and are annealed. The values of \( \varepsilon_t \) of the annealed specimens with a thickness of \( t = 0.125 \text{ in.} \) and \( t = 0.080 \text{ in.} \), are much higher. The different behaviour of \( t = 0.40 \text{ in.} \) can hardly be explained: it is unlikely that in the four cases the same fault in the determination of \( \varepsilon_t \) was made. However, when the formula \( r = \frac{\varepsilon_w}{(\varepsilon_t + \varepsilon_w)} \) is used instead of \( r = \frac{\varepsilon_w}{\varepsilon_t} \) the values of both \( \bar{r} \) and \( \Delta r \) seem to be very reasonable (table 9a).

9.2.5. The effect of cold working

The effect of cold working is only discussed for the two alloys and for a thickness reduction from \( t = 0.125 \text{ in.} \) to \( t = 0.100 \text{ in.} \). In the other cases the strains are too inaccurate (see par. 9.2.1.1.).

It was expected that the anisotropy of a material after cold working would be more pronounced. The results of both alloys confirm this (see graphs 8b and 8c). Both \( \Delta r \) and \( \bar{r} \) are higher in the cold worked condition. It appears that as a result of cold working the change of \( r_{45} \) is stronger than the change of \( r_0 \) and \( r_{90} \) articularly for aluminium magnesium alloy.

In the present investigation a particular method of cold working was used: rolling. It is unknown whether or not the influence of another method of cold working would give the same results. The effect of cold working on the anisotropy is a result of the change in the preferred direction with respect to deformation. It is expected that this change will not be the same for different methods of cold working. It is also advisable to use small subsequent amounts of cold working.
9.2.6. The effect of annealing

The effect of annealing of the as received condition is different for the three materials. For commercially pure aluminium the effect is rather pronounced (table 9a and graph 9a). Both $\Delta r$ and $\bar{T}$ decrease; $r_{90}$ also decreases. There is only a very slight influence on $r_0$ and $r_{45}$. The aluminium magnesium alloy, hardly shows any influence of annealing on the anisotropy (table 9b and graph 8b). In the case of the aluminium copper alloy annealing particularly effects the value of $r_{90}$ (table 9c and graph 8c). The value of $r_{90}$ decreases considerably; as a result $\bar{T}$ decreases also. The other values are roughly the same as in the as-received condition. However it must be kept in mind that before annealing the material is in the aged condition, and after annealing in the soft condition.

Annealing of the cold worked material, $t = 0.100$ in. results for both alloys in a decreasing value of $\bar{T}$ and a decreasing absolute value of $\Delta r$ (graph 8b and 8c). For the aluminium magnesium alloy particularly $r_{45}$ is effected. As a result $\Delta r$ change very strongly. Annealing of the aluminium copper alloy effects $r_0$, $r_{45}$ and $r_{90}$ in the same way: all values decrease considerably. As a result $\bar{T}$ decreases considerably too, and the effect on $\Delta r$ is much smaller.

For all three materials the anisotropy values of the annealed as-received condition and the annealed cold worked conditions differ considerably. Annealing of commercially pure aluminium (graph 9a) results in a slightly increasing value of $\bar{T}$ ($\bar{T}$ becomes closer to 1) and a decreasing value of $\Delta r$ ($\Delta r$ becomes closer to 0) with an increasing amount of cold working. For an isotropic material $\bar{T}$ equals 1, and $\Delta r$ equals 0. This could lead to the conclusion that the material becomes more isotropic. However, this conclusion does not hold: for a more isotropic material $r_0$, $r_{45}$ and $r_{90}$ also should be more equal to 1, but the results do not show this tendency.

For the two alloys (graph 8b and 8c) annealing, after an increasing amount of cold working, results in an increase or $\bar{T}$ and also an increase of the absolute value of $\Delta r$. The value of $r_{45}$ increases, but the values of $r_0$ and $r_{90}$ are hardly effected.
10. FORMING LIMIT DIAGRAMS

10.1. Experimental procedures

10.1.1. Specimens
The critical strain level was determined, using two types of blanks: elliptical and circular ones. The elliptical blanks had principal axes of about 5.4 in and 4.1 in. The angle between the major axis and the rolling direction was 45 degrees. The circular blanks had a diameter of about 5.5 in.

The form of the blanks was scribed on strips; for the elliptical blanks a template was used for the circular blanks a pair of dividers. The blanks were roughly cut to shape on a guillotine; afterwards sharp edges were removed.

For the determination of the critical strain level, the surface strains have to be measured. For this purpose a circular grid was etched on the blanks. The grid consists of 0.1 in diameter circles in 0.25 in squares. The etching procedure is described in Appendix.

The tests with the elliptical blanks were duplicated, with exception of thickness of 0.125 in; the tests with the circular blanks were not duplicated (see tables 10a, 10b and 10c).

10.1.2. The tests
The tests were carried out on a "Hille Engineering" 35 tons cupping press. The press was capable of applying a blankholder pressure up to a maximum of 800 lb/in². Various blankholders were available: one with a smooth ground surface and another serrated one were used. It was also possible to apply different punches. A X-Y recorder was fitted to the press for the recording of punch load vs punch displacement. All tests were carried out with a low speed, up to a maximum of 1 in/min.

With a cupping press it is possible in principle to cover both the tension-tension and the tension-compression part of a forming limit diagram. In the walls of deep-drawn products a tension-compression strain-state will be present. However, it is difficult to obtain failure in this area. Usually failure occurs in the area of the sheet which is in contact with the punch. In this area the strain state is tension-tension.

In order to investigate the tension-tension part deep drawing and stretch forming were simulated. By varying the form of the blank, lubricants and lubrication conditions, punch configuration and blankholder pressure, different test conditions were obtained. The conditions are summarized in table III. The purpose of having different conditions was to cover a broad range of the forming limit diagram, i.e. different ratios of $c_2/c_1$. 
<table>
<thead>
<tr>
<th>form of specimen</th>
<th>deep drawing</th>
<th>stretch forming</th>
</tr>
</thead>
<tbody>
<tr>
<td>blankholder pressure</td>
<td>120 lb/in²</td>
<td>800 lb/in² ¹</td>
</tr>
<tr>
<td>punch configuration</td>
<td>hemispherical; 2 in diameter</td>
<td>hemispherical and flat; 2 in diameter</td>
</tr>
<tr>
<td>lubricants</td>
<td>- mineral oil</td>
<td>- mineral oil</td>
</tr>
<tr>
<td></td>
<td>- polyethylene film</td>
<td>- polyethylene film</td>
</tr>
<tr>
<td></td>
<td>- PVC film</td>
<td>- PVC film</td>
</tr>
<tr>
<td></td>
<td>- natural rubber ²</td>
<td>- natural rubber ²</td>
</tr>
<tr>
<td>lubrication condition</td>
<td>- blankholder and punch</td>
<td>- punch only</td>
</tr>
<tr>
<td></td>
<td>- punch only</td>
<td></td>
</tr>
</tbody>
</table>

1) 475 lb/in² in the case of commercially pure aluminium
2) a plug of natural rubber on top of a shortened punch

Table III: Test conditions

In order to obtain different amounts of material-flow in different direction from under the blankholder to the punch, the elliptical blanks were used in the deep-drawing experiments. The blankholder pressure was a compromise: the pressure must be high enough to prevent wrinkling and low enough to allow moving of the material. For the latter fact the blankholder with the smooth ground surface was used.

In the case of the stretch forming experiments, it was the intention to prevent the flow of the material from under the blankholder to the punch completely. For this reason the circular blanks were used, in combination with a high blankholder pressure and the blankholder with the serrated ground surface.

During each test a load-displacement curve was recorded; a typical curve is shown in fig. 8. After a gradual increase, the load reaches a maximum and then decreases very sharply. At this point the test was stopped.

![Figure 8: A load-displacement curve](image)

After the test the grid was measured with a binocular microscope. The microscope was fitted with a cross-scale graticule; the smallest division on the graticule was 0.1 mm. A low magnification (2x or 3x) was used to accommodate the maximum deformation within the field of vision. Moreover at larger magnifications the boundary of the grid became very diffuse.
10.1.3. Failure criterion

For the determination of a forming limit diagram, it is necessary to have a criterion for the decision whether or not the material has failed. In a uniaxial tensile test two criteria can be used: fracture of the material or instability.

Up to the point of instability the deformation of the material is uniform with a gradual increase of the load for further deformation and a gradual increase of the true stress in the material. When the point of instability is passed the deformation it is no longer uniform, but it is concentrated at a particular gross section. At the same time the load required to induce each additional deformation decreases, although the true stress still increases (see chapter 6 and 7). In forming operations it is important to have an uniform deformation and an increasing load for each additional deformation. For this reason in a tensile test the point of instability is more useful as a criterion for failure than the actual fracture point.

Theoretically it is possible to extend the instability criterion for uniaxial tension to a general biaxial stress system, assuming that the failure behaviour of a material is the same under any stress condition. However, this assumption is not valid in all circumstances. In an uniaxial tension test the point of instability is associated with the onset of localized necking. However, under a general biaxial stress system localized necking can not occur: the form of necking can be described as "diffuse": the load continues to increase, although unstable flow has developed (see also chapter 7.3).

In fig. 9 the maximum surface strain $\varepsilon_1$ of a point at some distance from the pole (the "highest" point on the punch) in a deep drawing test is plotted versus the punch travel.

To construct this figure the punch must be stopped periodically after which the strain can be measured. One could think of considering point B as the point at which the material fails. However, to find the onset of unstable flow, one has to measure each time the strain at several possible failure points, since it is impossible to predict at which particular point the material will fail. This is very-time consuming and therefore it is an impracticable criterion.

A more practicable, although theoretically a less exact criterion can be found from the load displacement curve (fig. 8). After a gradual increase, the load reaches a maximum and starts to decrease. At this same moment some form of instability has been initiated. This will be defined as the beginning of failure. This point is unique for each curve and therefore it is a very useful failure criterion. For practical reason (it is difficult to stop the press at the maximum exactly) a point somewhat below the maximum is used (point A, fig. 8).
Now the beginning of failure has been defined, another practical problem arises: which ellipse represents the failure behaviour of the material? When the test is stopped at the point mentioned above, the material usually will have fractured along a certain length. As a consequence several ellipses can be selected for measuring the failure strain. Four alternatives can be indicated:
- a "fractured" ellipse;
- an ellipse along the edge of the fracture;
- an ellipse at the end of the fracture;
- the ellipse at the equivalent point.

In a successful forming operation, fracture of the material is not allowed. For this reason it would be essentially incorrect to use the strains of a "fractured" ellipse for the construction of a forming limit diagram. At the same time there are more practical disadvantages. The measurement of the length of the axis of a fractured ellipse will be very inaccurate because one has to measure across a (very rough) fracture line. Secondly the amount of instability, incorporated into the measured strains, depends on the location at which the ellipse is intersected by the fracture line. For different specimens this location will not be the same in general and consequently the amount of instability will vary from specimen to specimen.

This last disadvantage holds also if an ellipse along the edge or at the end of the fracture line is measured due to different distances of the ellipse to the fracture line. Apart from this practical disadvantage, it is questionable whether the ellipses along the edge of the fracture line could represent the surface strains at the onset of failure. This could be true only for the ellipses adjacent to the point where the fracture has initiated and it is difficult in most cases to find this point afterwards.

Apparently there are sufficient reasons to reject the first three alternatives. One has to look for an ellipse which can represent the strains shortly before failure. In lit. 11 the "equivalent" point was introduced.

The concept of the equivalent point is based on the fact that in a deep drawing test, the stress and strain system over a hemispherical punch has a mirror image symmetry about the pole of the pressing. For every point on the surface of the blank there is an equivalent point, equi-spaced, on the far side of the pole, where the stress and strain system is identical. Failure may start, with equal probability, from either of the two equivalent points where the limiting strain of the material is reached. Therefore, the equivalent point, corresponding to the fracture initiation, represents the strains of the material just before fracturing. These strains are not affected by different amounts of instability; therefore these strains should be more representative than the strains based on the other criterions.

![Figure 10: The equivalent point](image_url)
The concept of the equivalent point is illustrated in fig. 10. The length of the largest axis of the ellipses is plotted against the location of the ellipses along a line passing through the pole. The length has a maximum at location 3. This ellipse may be considered as the fractured ellipse 7, mirrored with regard to the pole ellipse 5.

The equivalent point corresponds to the fracture initiation point. It was mentioned before that it is difficult to find the point where the fracture was initiated. For this reason the largest ellipse near to the equivalent point should be chosen.

10.2. Results and discussion

10.2.1. Accuracy of the strain determination

The accuracy with which the strain can be determined depends on:

a. the accuracy of the undeformed grid on the surface of the specimen;

b. the accuracy of the deformed grid;

c. the accuracy with which the dimensions of the deformed grid are measured.

ad a. The undeformed grid

First of all the accuracy of the undeformed grid depends on the accuracy of the stencil which is used for imprinting the grid on the sheet. Three factors are important: the difference between the actual and the nominal diameter of the circles, the variation in the line thickness and the sharpness of the lines. The accuracy will decrease when the stencil has been used for a longer time; with successive use the lines become coarser and their boundaries more diffuse.

Secondly the accuracy depends on the accuracy with which the grid of the stencil is imprinted on the surface of the sheet. Due to leakage of the electrolyte beneath the stencil the lines on the sheet are coarser and thicker than the lines of the stencil. Therefore the accuracy of the grid on the sheet is less than the accuracy of the stencil.

For our purpose it is assumed that the overall accuracy of the undeformed grid on the surface of the sheet is within about ± 4%.

ad b. The deformed grid

When the specimen is deformed, the grid on the specimen deforms too. The surface roughness shall increase during deformation; this effect is more pronounced with a coarse-graind material and with increasing deformation. Also during deformation the variation in line-thickness will increase and the boundaries of the lines become more diffuse. As a result the accuracy decreases; the decrease is estimated to be + 2% (in some cases the boundaries of the lines were so diffuse that it was impossible to measure the deformed grid at all).

ad c. The measurement accuracy

The relative accuracy with which the dimensions of the deformed grid can be measured depends both on the actual dimensions of the grid and on the magnification of the microscope. When a nominal length of 2.5 mm is measured (this is the diameter of the undeformed circles) using a magnification of 2 (mostly the magnification was set to this value and with a smallest detectable division on the graticule measuring scale of 0.1 mm, a relative accuracy of ± 2% (± 0.1 / 2 x 2.5 x 100) should be possible. In most cases the dimensions of the deformed grid were greater than 2.5 mm, so in general the measuring accuracy will have been slightly better.

Now the accuracy of the strains can be determined. The true strain is defined as: \( \varepsilon = \ln \frac{l}{l_0} \). The accuracy of \( l_0 \) is ± 4%; (see a) the accuracy of \( l \) is the sum of the accuracies mentioned at b and c, also ± 4%. From these two values the absolute accuracy of the strain can be calculated; the accuracy is within ± 0.08.
Apparently the method used for the determination of the strains, and thus for the determination of the critical strain level, is rather inaccurate. The influence of this inaccuracy and methods to increase the accuracy will be discussed in chapter 10.2.5.

10.2.2. Lubricants and lubrication conditions

In the deep drawing tests the distribution of the surface strains and therefore the strains at failure, is affected by the interaction between the material and the punch and between the material and the blankholder. In the case of stretch forming the distribution is only affected by the interaction between the material and the punch, provided that the blankholder pressure is high enough to prevent flow of material from under the blankholder to the punch completely (this condition was satisfied in most tests).

The interaction depends on the lubricants and the lubrication condition. When various lubricants and lubrication conditions are used (see table III), the strains at failure (ζ₁ and ζ₂) will vary. To cover the forming limit diagram the strain ratio ζ₁/ζ₂ has to vary from 0 (plain strain) to 1 (balanced biaxial tension). From the graph 10a, 10b and 10c it is obvious that, although not the full range from 0 to 1 is covered, the range is still rather wide for all materials in all conditions. Therefore it is concluded to be worthwhile to continue this development of the method used for the determination of forming limit diagrams.

A disadvantage of the method is the fairly poor reproducibility. Investigation of the tables 10a, 10b and 10c reveals that the strains ζ₁ and ζ₂ for duplicated specimens differ considerably, even when the same lubricant and lubrication conditions are used. The rather inaccurate determination of the strains (see chapter 10.2.1.) will certainly contribute to this.

10.2.3. Fracture orientation

In the tables 10a, 10b and 10c the angle between the fracture direction and the rolling direction is indicated. The angles were observed at the fractured specimens. However it must be mentioned that it was difficult to determine the fracture direction at the fracture initiation point in many cases; in the tables this is indicated by more than 1 direction. The fracture orientations are summarized in table IV.

<table>
<thead>
<tr>
<th>t (in)</th>
<th>Commercially Pure Aluminium</th>
<th>Aluminium-Magnesium Alloy</th>
<th>Aluminium-Copper Alloy</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>annealed</td>
<td>annealed</td>
<td>annealed</td>
</tr>
<tr>
<td>0.125</td>
<td>DD</td>
<td>45° 2)</td>
<td>90°</td>
</tr>
<tr>
<td></td>
<td>SF</td>
<td>0°</td>
<td>0° , 90°</td>
</tr>
<tr>
<td>0.10</td>
<td>DD</td>
<td>0°</td>
<td>90°, 45° 1)</td>
</tr>
<tr>
<td>(0.08)SF</td>
<td>0°</td>
<td>0° , 90°</td>
<td>90°</td>
</tr>
<tr>
<td>0.08</td>
<td>DD</td>
<td>45° 1)</td>
<td>90°</td>
</tr>
<tr>
<td>(0.04)SF</td>
<td>0°</td>
<td>0°</td>
<td>90°</td>
</tr>
</tbody>
</table>

Table IV: Fracture orientation

1) parallel to minor axis  DD: deep drawing
2) parallel to major axis  SF: stretch forming

In the stretch forming tests employing circular blanks the strain distribution has a radial symmetry about the pole. Therefore the material is free to fall in any direction, so it will fall in the "weakest" direction (table IV).
However, the elliptical blanks had an imposed stress system, with the maximum stress in the direction of the major axis. In analogy with a tensile test it is expected also that the material has a tendency to fail parallel to the minor axis of the blank, perpendicular to the direction of the largest stress. Due to fact that the major and minor axis of the elliptical blanks made an angle of $45^\circ$ to the rolling direction, the angle between the rolling direction and the fracture direction is also $45^\circ$. This results in a conflict between the material preference to fail in its weakest direction and the direction imposed by the stress system (table IV). Sometimes the material shall fall in its weakest direction and other times parallel to the minor axis. Table IV shows that this holds in almost every case.

To avoid the above mentioned discrepancy the weakest direction must coincide with the minor axis of the elliptical blank, resulting that all specimens will fail in the same direction.

10.2.4. Amount of instability and failure criterion
In order to exclude the influence of instability or unstable flow on the strains to be measured at the moment of failure, a specific criterion for failure was developed in chapter 10.1.3. The criterion was related to a point slightly below the maximum of the load-displacement curve. For the same purpose the equivalent point was introduced there.

After failure the specimens were investigated visually whether or not "instability" had occurred. In the tables 10a, 10b and 10c the visual amount of instability is indicated with: no, low, moderate or high. It is obvious that the attempt to exclude instability was not very successful and that the amount of instability is not the same for different specimens.

When the amount of instability is not the same for all specimens, the "failure condition" varies from specimen to specimen. This means that the points in the forming limit diagrams are not comparable, resulting in large scatter.

10.2.5. Forming limit diagrams
To forming limit diagrams for the three materials are shown in the graphs: 10a, 10b and 10c. The scatter is rather high, resulting in a wide band for the critical strain level. Methods to reduce this scatter to an acceptable magnitude will be discussed below. Secondly some general tendencies are derived from the constructed critical strain levels.

10.2.5.1. The scatter
Several factors may contribute to scatter, for example:
- inaccuracy of the strain determination
- fracture orientation
- amount of instability
- failure criterion
- variation in material properties

Methods to reduce the contribution of each of these factors to the total scatter will be discussed.

a. Inaccuracy of the strain determination
One method to increase the accuracy of the strain determination is to measure the undeformed grid after imprinting the grid on the specimens. In this way it is possible to compensate in principle for variations in the nominal dimensions of the grid. To obtain a higher accuracy indeed, the measurement accuracy must be high, even higher than the sum of the accuracies of the stencil and of imprinting the grid on the specimen. A disadvantage of this method is that the method is very time-consuming: at many points the grid has to be measured, since it is impossible to predict at which particular point the material will fail.
Another method is to increase the accuracy of the undeformed grid itself. This concerns both the "stencil" and the way of imprinting the grid on the surface. It is believed that electrochemical marking is not suitable to increase the accuracy considerably. More promising techniques seem to be photo-etching or photo-gridding.

The relative accuracy of the measurement may be increased by increasing the nominal dimensions and by using a higher magnification. However, when the diameter of the circles becomes too large, it is impossible to detect steep strain-gradients. Therefore it is not recommendable to use rather large circles; a diameter of 0.1 in appears to be a reasonable compromise. A high magnification has two disadvantages. First of all the boundary of the grid becomes more diffuse. Perhaps this effect may be reduced when another gridding technique is used and when the contrast between the material and the grid is increased. Secondly when the magnification is too high the ellipse is no longer within the field of vision of the microscope, causing difficulties in determining the major and minor axis. Therefore it is advisable to use a low magnification for the determination of these axis. The actual length of the axis can be measured using a higher magnification. For this the specimen has to be placed on a travelling table. The moving distance can be measured with a micrometer, attached to the table with an accuracy of about 0.001 mm.

b. Fracture orientation
The failure direction of the elliptical specimens was not always the same due to the fact that the "weakest" direction of the material did not coincide with the failure direction induced by the imposed stress system (see chapter 10.2.3.). When the two directions coincide it is possible that the scatter decreases. However, it is very unlikely that the magnitude of the surface strains depend on the angle between the fracture direction and the rolling direction. Lit. 11 (p. 56) reports that the principal strains depend on the orientation with respect to the rolling direction. To confirm this several specimens with different angles between the major axis of the elliptical blank and the rolling direction should be tested.

c. Amount of instability and failure criterion
The amount of unstable flow incorporated in the measured strains has a very important influence on scatter. The amount of instability can be reduced however, if the very time-consuming procedure of determining point B in figure 13 (chapter 10.1.3) is used. The amount can be reduced also when the press is stopped automatically at the moment that the load starts to decrease (see chapter 10.1.3.).

d. Variation in material properties
A last point that may cause scatter is the variation in material properties. This variation is inevitable; this means that it is impossible to reduce the scatter in a forming limit diagram completely.

10.2.5.2. The critical strain level
As a result of the large scatter it is not allowed to derive firm conclusions from the forming limit diagrams. Only general tendencies can be indicated.

The concept of a critical strain level as a measure for the formability can be very useful. It is possible to cover all strain ratios \( \varepsilon_1/\varepsilon_2 \) from 0 to 1. The critical strain level, not a line but a band with certain width, separates failure and non-failure conditions. In order to determine the level with an acceptable accuracy, necessary for a measure for the formability, the scatter has to be reduced; it is believed that this is possible (see preceding paragraph).

An assumption behind the concept of the critical strain level is that failure of the material depends on the strain state only, and for instance not on the form and thickness of the specimen or the applied deformation process. The validity of this assumption has to be checked, both theoretical and experimental.
It was expected that as a result of cold working the critical strain level should be lowered and that annealing after cold working should bring the level back to its original value. This is confirmed by the results obtained.
11. EVALUATION: SUGGESTIONS FOR FURTHER RESEARCH

11.1. Relation between stress ratio \(x\) and effective strain \(\bar{\varepsilon}\), both theoretical and experimental

In chapter 7 the following, theoretical, relation was derived:

\[
\bar{\varepsilon}_{\text{inst}} = \frac{Z}{n} \tag{7.20}
\]

with:

\[
Z = \frac{2}{2 - x} \left\{1 - x + x^2\right\}^{\frac{1}{2}} \tag{7.18}
\]

The formulae employ a theoretical relation between \(x\) and \(\bar{\varepsilon}_{\text{inst}}\) for a given value of the workhardening exponent \(n\).

In the tables 3a, 3b and 3c \(n\) values for the three materials are given for three different orientation of the specimens. From table IV the "weakest" direction can be found (see chapter 10.2.3.). The \(n\) value of this direction is substituted in (7.20). The theoretical relation between \(x\) and \(\bar{\varepsilon}_{\text{inst}}\) thus obtained have been plotted in the graphs 10a, 10b and 10c, for the three materials in the different conditions (the differences between the curves for the different conditions are a result of the differences between the \(n\) values; the reason for this were discussed in the chapters 8.2.6. and 8.2.7.).

The experimental relation between \(x\) and \(\bar{\varepsilon}_{\text{inst}}\) (\(\bar{\varepsilon}_{\text{inst}}\) is defined as the effective strain at the moment the material fails) can be found from the results of the forming limit experiments. The values of \(x\) and \(\bar{\varepsilon}_{\text{inst}}\) are given in the tables 10a, 10b and 10c and plotted in the graphs 10a, 10b and 10c.

The following formulae were used in computing \(x\) and \(\bar{\varepsilon}_{\text{inst}}\):

\[
\frac{\varepsilon_2}{\varepsilon_1} = \frac{2x - 1}{2 - x}
\]

\[
\bar{\varepsilon}_{\text{inst}} = \frac{2\varepsilon_1}{2 - x} \left\{1 - x + x^2\right\}^{\frac{1}{2}}
\]

(this are the formulae (7.9) and (7.10) respectively, in integrated form).

The theoretical and experimental curves differ very much. There are several reasons for the difference. The theoretical curve is found by extending the instability criterion for uniaxial tension (localised necking) to a general biaxial stress system (chapter 7.3). In the experiments the stress ratio \(x\) was higher than \(\frac{1}{2}\) in almost every case (graph 10a, 10b and 10c); consequently localised necking can not occur. In chapter 10.1.3. a particular failure criterion was introduced (point A, Fig. 8); also the equivalent point concept was discussed (fig. 10). There is a certain relation between the strains measured at the equivalent point and some form of instability, but the form of instability has nothing to do with localised necking.

The scatter in the experimental relation is very large. The reasons for this large scatter are the same as the reasons discussed in chapter 10.2., particular 10.2.4. and 10.2.5.

In investigating the difference between the theoretical and the experimental curves, further research is needed. Several ways are possible:

- experiments with a stress ratio smaller than \(\frac{1}{2}\)
- a theoretical treatment of instability under a general biaxial stress system, not based on the onset of localised necking
- the development of a more realistic and a more exact failure criterion; a criterion based on the onset of diffuse necking (point B, fig. 9) looks promising.

11.2. Fracture orientation (weakest direction) in relation to the r-value

In chapter 10.2.3. it was mentioned that in the case of the stretchforming experiments the material will fail in the "weakest" direction. These "weakest" directions are summarized in table V (derived from table IV).

Based on the r-values, it is also possible to define a "weakest" direction: the directions of the lowest r-value (see chapter 5.2.2.2.). In this direction the effect of thinning has a maximum and therefore it is expected that the material shall fail in this direction. The directions with the lowest r-values are summarised in table V (derived from the graphs 8a, 8b and 8c).

<table>
<thead>
<tr>
<th></th>
<th>Commercially Pure Aluminium</th>
<th>Aluminium-Magnesium Alloy</th>
<th>Aluminium-Copper Alloy</th>
</tr>
</thead>
<tbody>
<tr>
<td>t (in) w.d.</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.125</td>
<td>0°</td>
<td>0°, 90°</td>
<td>0°</td>
</tr>
<tr>
<td>0.10</td>
<td>45°</td>
<td>0°, 90°</td>
<td>0°</td>
</tr>
<tr>
<td>0.08</td>
<td>0°</td>
<td>0°, 90°</td>
<td>0°</td>
</tr>
<tr>
<td>0.04</td>
<td>45°</td>
<td>0°, 90°</td>
<td>0°</td>
</tr>
<tr>
<td>l.r.</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

w.d.: weakest direction
l.r.: direction with lowest r-value

Table V: Comparison of weakest direction and direction with lowest r-value

With exception of the aluminium-copper alloy the two directions do not coincide. For the aluminium-magnesium alloy the difference between r_0 and r_90 is rather small; perhaps this could explain the difference between the two directions. The explanation can not hold for commercially pure aluminium: the difference between r_0 and r_45 is very pronounced.

Further research is needed to explain the observed facts. It is possible that the weakest direction in a biaxial stress system has noting to do with the direction of the lowest r-value, determined with an uniaxial tensile test. The reasons for this could be the same as the reasons for the different instability behaviour, mentioned in chapter 11.1.

11.3. Effect of n- and r-value on the forming limit diagram

In this research, different n- and r-values were obtained by cold working and annealing (see chapters 8.2.6. and 8.2.7., resp. 9.2.5. and 9.2.6.). The forming limit diagram is also affected (see chapter 10.2.5.2.). However, it is not possible to draw firm conclusions with respect to the influence of n- and r-values on the forming limit diagrams. There are several reasons for this.

First of all the scatter in the forming limit diagrams is too high. Reasons for this high scatter and methods to reduce the scatter were discussed in chapter 10.2. (particularly 10.2.5.1.). Secondly, the changes in n and r, as a result of very severe cold working, were too large. It is advisable to take much smaller steps. This has to be attended with very accurate method for the determination of n, r and the surface strains.
Cold working has two effects on the critical strain level: the level is lowered and the strain distribution becomes less uniform. The lowering of the level not contradicts with our results (see chapter 10.2.5.2.). The strain distribution was not determined, because this was too time-consuming.

In our experiments both \( n \) and \( r \) were changed. It was also impossible to separate the influence of \( n \) and \( r \) on the forming limit diagram. It is very difficult to change \( n \) of a particular material, without changing \( r \), and vice versa. Different methods of cold working have to be investigated. Another approach could be to look for different materials with different \( n \)-values and the same \( r \)-values and or different \( r \)-values and the same \( n \)-values. When these two methods are not successful, one could try to extrapolate to the same \( n \)-value in investigating the effect of \( r \), or to the same \( r \)-value in investigating the effect of \( n \).

Lit. 11 (p. 56) and lit. 12 report that, when elliptical specimens are used, the principal strains at failure depend on the angle between the principal axis and the rolling direction. The distribution of the principal strains should be the same as the distribution of the \( r \)-value. It is worthwhile to check this surprising statement.

Some points, which are important when the effect of \( n \) and \( r \) on forming limit diagrams is investigated, were mentioned. It seems that there are possibilities, but the research will certainly be difficult and time-consuming.
12. SUMMARY OF THE RESULTS AND CONCLUSIONS

1. Different properties of the following three materials were tested:
   - commercially pure aluminium: Al;
   - an aluminium magnesium alloy (3.5% Mg): 5154 (NA54S);
   - an aluminium copper alloy (3.8-4.9% Cu, 1.2-1.8% Mg): 2024.

   Three thicknesses: \( t_0, t_1 \) and \( t_2 \) from each material were tested; \( t_0 \) represents the as-received condition, the thicknesses \( t_1 \) and \( t_2 \) were obtained by cold working (rolling) the as-received condition. Each thickness was also tested in the annealed condition.

2. In order to obtain a relation between stress and strain, tensile tests were carried out. The tensile specimens were taken at three orientations with respect to the rolling direction: 0, 45 and 90 degrees.

   a. As a general result it turned out that the stress-strain relation in the plastic range for the three materials in all conditions was accurately described by the equation: \( \sigma = C \varepsilon^n \). Furthermore the variation of \( n \) along the stress strain curve seemed to be rather small.

   b. Theoretically the value of \( n \) must be equal to the strain at the beginning of necking. However this expectation does not hold generally for our experiments. The reason for this could be that it is very difficult to determine the beginning of necking in our experiments.

   c. As a result of the first thickness reduction (from \( t_0 \) to \( t_1 \)) \( n \) decreases; the effect of the second reduction (from \( t_1 \) to \( t_2 \)) on \( n \) is very small. To investigate the effect of thickness reduction in more detail, small amounts of succeeded thickness reductions should be used.

   d. The \( n \) values of the annealed as-received conditions are higher than the values of the as-received conditions. Therefore it is concluded that at the end of the rolling process, the material is cold worked to the ultimate thickness. For the three thicknesses the \( n \)-values of the annealed condition are close together.

   e. The values of \( n \) and \( \varepsilon_{\text{neck}} \) vary with the orientation of the specimens.

3. The anisotropy of the materials was determined in tensile tests, using three orientations of the specimens to the rolling direction: 0, 45 and 90 degrees.

   a. After straining the cross-section of the commercially pure aluminium specimens were curved. This could be caused by a through-thickness texture variation; however this was not investigated. The curvature of the cross-section was not constant along the length of the specimens. As a result the meaning of the \( r \)-values, particularly the values determined with the formula:

   \[
   r = \frac{\varepsilon_w}{-(\varepsilon_1 + \varepsilon_w)}
   \]

   is questionable.

   b. The calculated volume-change of the two alloys is rather small for all specimens, in almost every case smaller than 1%. This means that the formulae:
\[ r = \frac{\varepsilon_w}{-(\varepsilon_1 + \varepsilon_w)} \]

can be used instead of:

\[ r = \frac{\varepsilon_w}{\varepsilon_t} \]

The volume change of commercially pure aluminium was much higher, up to 11% (see also point 3a).

c. The results show no influence of the gauge length \( l_o \) and the magnitude of the length-strain \( \varepsilon_1 \) on the \( r \)-values.

d. The directionally in the plane of the sheet, was investigated by calculating the \( \Delta r \)-value: the planar anisotropy. There is an obvious difference in the planar anisotropy. There is an obvious difference in the planar anisotropy between commercially pure aluminium (\( \Delta r > 0 \)) and the two alloys (\( \Delta r < 0 \)).

e. As a result of cold working the anisotropy becomes more pronounced. Both the planar anisotropy \( \bar{r} \) and the normal anisotropy \( \bar{r} \) are higher in the cold worked condition than in the as-received condition. The change of \( r_{45} \) seemed to be stronger than the change of \( r_0 \) and \( r_{90} \).

(This was only investigated for the first thickness reduction of the two alloys, due to inaccuracies in the other cases.)

f. The aluminium-magnesium alloy hardly shows any influence of annealing of the as-received condition on the anisotropy values. For commercially pure aluminium the effect is rather pronounced: \( \Delta r \), \( \bar{r} \) and \( r_{90} \) decrease considerably, \( r_0 \) and \( r_{45} \) do not change much. For the aluminium copper alloy only \( r_{90} \) (and as a result also \( \bar{r} \)) decreases considerably.

g. Annealing of commercially pure aluminium, after an increasing amount of cold working, results in a decreasing value of \( \Delta r \), \( \bar{r} \) is only slightly influenced. The two alloys show the same behaviour; \( \bar{r} \), the absolute value of \( \Delta r \) and \( r_{45} \) increase, \( r_0 \) and \( r_{90} \) are hardly effected, as a result of annealing after an increasing amount of cold working.

4. Forming limit diagrams (plots of the principal strains in the plane of the sheet at the onset of failure) were determined for \( 0 \leq \varepsilon_1 / \varepsilon_2 \leq 1 \) by carrying out deep drawing and stretch forming tests and by varying the form of the blank, lubricants, lubrication conditions and punch figuration.

a. A practical failure criterion was defined, using the load-displacement curve. When after a gradual increase, the load reached a maximum and starts to decrease, the material has failed (in most tests the specimens actually has fractured along a certain length at this point). To determine the situation of the material just before fracture, the fracture initiation point was mirrored with regard to the pole; this point is called: the equivalent point.

b. With the method used for the determination of the forming limit diagram, a rather wide range of the strain ratio \( \varepsilon_1 / \varepsilon_2 \), between 0 and 1, is covered.

c. The reproductability of the strain ratio, under the same circumstances, is rather poor.
d. The fracture direction of the elliptical specimens was not always the same due to the fact that the "weakest" direction of the material did not coincide with the failure direction induced to by the imposed stress system.

e. One reason for the introduction of the equivalent point was to exclude the influence of instability on the strain. However, this attempt was not very successful.

f. The scatter in the forming limit diagrams is rather high, but there seems to be possibilities to reduce the scatter to an acceptable level. The contribution of the following factors to the scatter can be reduced:
   - inaccuracy of the strain determination;
   - fracture orientation;
   - amount of instability;
   - failure criterion.

g. Although the scatter was high, the method used for the determination of the forming limit diagrams looks very promising.

5. Theoretically the following relation between $\bar{\varepsilon}_{\text{inst}}$, the strain at the beginning of instability, and the stress ratio $x = \frac{\sigma_2}{\sigma_1}$ can be derived:

$$\bar{\varepsilon}_{\text{inst}} = \frac{2}{2 - x} \left(1 - x + x^2\right)^{\frac{1}{4}}$$

From the measured strains $\varepsilon_1$ and $\varepsilon_2$ in the forming limit diagram an experimental relation can be found. The theoretical and experimental curves differ very much. For a further investigation of this difference one could think of:
   - a theoretical treatment of instability under a general biaxial stress system, not based on the onset of localised necking;
   - the development of a more realistic and a more exact failure criterion.

6. In the stretchforming experiments (with circular specimens) the material will fail in the "weakest" direction.

Based on the r-values, the direction of the lowest r-value, may be defined as the "weakest" direction. Both weakest directions do not coincide.

7. Due to the following reasons the effect of n and r on the forming limit diagram was not investigated in detail:
   - the large scatter in the forming limit diagram;
   - the large changes in n and r, as a result of very severe cold working.

However it is possible to reduce the scatter and it is also possible to take much smaller steps in the cold working.

This has to be attended with very accurate methods for the determination of n, r and the surface strains.

It is also advisable to try to separate the influence of n and r on the forming limit diagram.
13. REFERENCES

1. The idea of the research reported here, was derived from 6 articles of: Keeler, S.P.: "Understanding sheet Metal Formability".


APPENDIX

VISIBLE STRAIN PATTERNS

When a grid is imprinted on a sheet before deformation, strain patterns are clearly visible after forming. The distribution of the strain may be determined from the measured distances between the lines of the deformed grid.

A very important requirement of any grid is proper spacing between lines. Any variation in strain from point to point between the lines is undetectable. Only an average strain value is obtained. Therefore, the lines must be sufficiently close to each other so that local differences in strain can be detected. Like the gridline spacing, the configuration and orientation of the grid is very important. When the grid system consists of squares it is only possible in two cases to measure the principal strains directly. When the sides of the square are oriented parallel to the principal strain directions, the square will deform into a rectangle. The principal strains can be calculated from the sides of the rectangle. If the diagonals of the square are oriented in the principal directions, the strains can be calculated from the diagonals of the deformed square.

However, the principal strain directions are usually unknown. Furthermore, the directions may vary from point to point. Thus in almost every case complex measurements of square dimensions and shear angle are necessary for the calculation of the principal strains. An other disadvantage of a square grid is the impossibility to observe the principal directions readily from the specimen. An ideal grid system should be nondirectional. The simplest nondirectional pattern consists of circles. A circle is always correctly oriented to furnish the magnitude and the direction of the principal strains directly from the specimen. When the specimen is deformed, the circle becomes an ellipse. The principal directions are indicated by the major and minor axis of the ellipse. The strains can be calculated directly from the measurement of the lengths of these axis.

Several patterns of circles are being used. The simplest pattern consists of circles and parallel lines (fig. 11). (In our research, this pattern was used.)

![Circle Grid](image)

The lines are not used in the strain measurement, but only for identification of the circles. Since each circle is separate, the strain pattern from point to point is easily seen. The spaces between the circles are a disadvantage of this pattern; in these spaces no strain measurement are made. In making the circle tangent, the space where no strain measured, is reduced. However, the tangent points reduce the accuracy of measurement.

It is possible to design patterns of overlapping circles so that all surface areas are within at least one circle. Due to the complexity of these patterns the strain distribution is hard to see. Another disadvantage is that some areas the strain is measured several times, which may be as bad as having areas where strain is not measured.

There are several methods of imprinting grid patterns on a sheet. For a very simple pattern hand scribing
may be used. The accuracy of this method is rather low. A very accurate method is photoprinting. The sheet is covered with a photosensitive emulsion. Then the emulsion is covered with a photographic negative of the pattern and exposed to ultraviolet light. Finally the latent image is developed. The resulting grid consists of very sharp and narrow lines. Disadvantages of the method are: the processing time (30 min. or even longer), the special equipment, which is necessary and the fact that the grid is removed by chemicals and rubbing.

Electrochemical marking is frequently used and has a reasonable accuracy in most cases.

![Diagram of electrochemical marking]

Figure 12: Electrochemical marking

In this process, a stencil with the desired pattern is placed on the decreased sheet. A felt pad, soaked with electrolyte is placed on top of the blank. The pad is covered by an electrode. Leads are attached to the electrode and the blank. Finally the electrode, with the pad and stencil, is firmly pressed against the sheet and an electric current is applied for some seconds. After etching, the solution on the blank must be neutralized. Since the pattern is etched, it cannot be removed by chemicals or ordinary rubbing. Three types of stencils are commonly used: PVC, plastic impregnated paper and plastic impregnated woven nylon. Each stencil can be used several hundred times.

In most cases the paper based stencils produce the sharpest and finest lines and is also preferable. The proper electrolyte depends on the material but there are electrolytes available which are suitable for a range of different materials.