Zero-Dimensional States and Single Electron Charging in Quantum Dots

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We observe new transport effects in lateral quantum dots where zero-dimensional (0D) states and single electron charging coexist. In linear transport we see coherent resonant tunneling, described by a Landauer formula despite the many-body charging interaction. In the nonlinear regime, Coulomb oscillations of a quantum dot with about 25 electrons show structure due to 0D excited states as the bias voltage increases, and the current-voltage characteristic has a double-staircase shape.

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Transport through semiconductor quantum dots shows striking effects due to the electron wave nature and its finite charge. The first leads to the formation of zero-dimensional (0D) states with discrete energies in a system confined in all three directions, and the possibility of coherent resonant tunneling [2], as with photons in a Fabry-Pérot cavity. The latter induces Coulomb effects, which cause a strong shift in the dot energy upon addition of a single electron [3].

Experiments are beginning to be done on quantum dots where these effects coexist. McEuen et al. [4] used transport measurements to determine the magnetic field dependence of N-electron ground-state energies in such a system, and related it to the calculated energies of single-particle levels in the absence of charging. Similar issues have been addressed in double barrier resonant tunneling structures [5], and using capacitive [6] and optical [7] techniques. Here we report new results from dots where 0D states and charging coexist. We observe coherent resonant tunneling in the linear regime (low bias voltage), surprisingly well described by a Landauer formula (until now applied only to noninteracting systems) despite the electronic Coulomb repulsion. We also see the signature of combined 0D states and charging in nonlinear transport. The current-voltage characteristic has a double-staircase shape, and we can measure the tunnel spectroscopy of the excitations of a quantum dot containing about 25 electrons.

The two quantum dots of this work are defined by metal gates on top of a GaAs/AlGaAs heterostructure with a two-dimensional electron gas (2DEG) 100 nm below the surface. The ungated 2DEG has mobility $230 \text{ m}^2/\text{Vs}$, and electron density $1.9 \times 10^{13} \text{ m}^{-2}$ at 4.2 K. Applying $-300 \text{ mV}$ to the gates depletes the 2DEG under them, making a quantum dot coupled to large reservoirs via barriers at the center of two quantum point contacts (QPCs). Making the voltage on the QPC gates more negative (positive) decreases (increases) the barrier transmissions. The inset of Fig. 1 gives the geometry of dot 1, with pattern size $0.8 \mu \text{m} \times 1 \mu \text{m}$. The “finger” gate $F$ forms one side of the dot, while QPC gates 1 and 2, and center gate $C$ form the other side. With depletion, we expect this dot to be circular with a diameter of about 0.6 $\mu \text{m}$. The inset of Fig. 3 shows the layout for sample 2, with QPC gate pairs 1 and 2, and center gate pair $C$. The central region is $0.2 \mu \text{m} \times 0.6 \mu \text{m}$. For this sample, we apply a more negative voltage to the center gates (typically $-900 \text{ mV}$), enhancing the depletion region around them. The dot is again circular, but now with diameter 0.1 $\mu \text{m}$. Measurements were done in a dilution refrigerator at its base temperature below 20 mK.

We took data on sample 1 in a magnetic field of 7 T, when transport through the dot is via the lowest-energy edge channel, lying along the dot circumference, and is essentially one dimensional [2,8]. If a quantum particle of proper energy moves between two barriers without loss of phase memory, coherent resonant tunneling occurs through a 0D state formed by constructive interference of multiply reflected partial waves. As in an optical Fabry-Pérot cavity, the transmission probability can approach 1, even if each barrier alone is highly reflecting. At zero temperature in one dimension and in the absence of charging effects, the conductance of this interferometer is given by a Landauer formula [2]:

$$G_{\text{dot}} = \frac{e^2}{h} \frac{T_1 T_2}{1 + (1 - T_1)(1 - T_2) - 2[(1 - T_1)(1 - T_2)]^{1/2} \cos \varphi}.$$  

(1)

$T_1$ and $T_2$ are the barrier transmissions, and $\varphi$ is the phase acquired by a wave in one round trip between the barriers. Finite temperature leads to energy averaging by the derivative of the Fermi function, reducing the peak transmission. Büttiker [9] has described the transition from coherent to incoherent (sequential) tunneling using the Landauer-Büttiker formalism.

Figure 1(a) shows the conductance $G$ of dot 1 as a function of voltage $V_1$ on QPC gate 1, when $V_2$ is set so QPC2 has transmission $T_2 = 1$, and gates $F$ and $C$ are formed. Transport at $B = 7 \text{ T}$ is adiabatic over distances much larger than
the dot size, so $G$ measures $T_1$: $G = T_1 e^2/h$. When $V_2$ is more negative, so QPC$_2$ is a tunnel barrier ($T_2 \approx 0.02$), sweeping $V_1$ gives the periodic conductance peaks of Fig. 1(b). These are the Coulomb oscillations [3] of the charging regime, caused by sweeping a gate voltage that is capacitively coupled to the dot. In contrast to the usual experiments, here sweeping $V_1$ simultaneously changes $T_1$, the transmission of QPC$_1$. The peak height of the oscillations shows a dramatic modulation, correlated with $T_1$ [Fig. 1(a)], but in a nonclassical manner. Near $V_1 = -770$ and $-850$ mV, for example, the peak conductance is strongly suppressed, even though $T_1$ is at a maximum of 0.6. The classical, one-dimensional sequential tunneling prediction for the conductance maxima is shown by the dashed line in Fig. 1(b): $G_{cl} = (e^2/h)T_1 T_2/(T_1 + T_2 - T_1 T_2)$. The actual conductance peaks exceed this prediction by as much as a factor of 15.

In contrast, the peak conductance ($\cos \varphi = 1$) predicted by the quantum formula (1) agrees well with the data, when thermal averaging of about 50 mK is included [Fig. 1(b), heavy line], in line with the temperature and bias voltage ($5 \mu$V) of the experiment. In Eq. (1), $T_1$ and $T_2$ must match to have total transmission well above the sequential value. Since $T_2 \approx 0.02$ in Fig. 1(b), increasing the transmission of QPC$_1$ above 0.02 reduces the total transmission predicted by Eq. (1), just as in the data.

This is the first demonstration that coherent transport described by an independent-electron Landauer formula occurs despite the many-body charging interaction. Although initially surprising, this result is in agreement with the idea that transport in the linear regime occurs when the electrochemical potential of the dot is equal to that of the reservoirs [10]. Transport of the $N$th electron is an energy-conserving process, where phase coherence is maintained even though the other $N-1$ electrons experience a Coulomb energy change. Meir and Wingreen [11] have recently developed a Landauer-type formula for interacting systems.

Along with coherent resonant tunneling in the linear regime, combined 0D states and charging lead to novel nonlinear transport effects, clearly shown in experiments on the smaller dot 2. If a set of 0D states of energy $E_1, E_2, \ldots$ coexists with charging, the dot's electrochemical potential changes discontinuously as the number of electrons increases: $\mu_d(N+1) - \mu_d(N) = E_C + \delta E$ [10]. Here $E_C = e^2/C$ is the electrostatic energy cost of charging the quantum dot by one electron ($C$ is the total capacitance from the dot to ground), and $\delta E = E_{N+1} - E_N$ is the energy between 0D states, also the minimum energy needed to excite the $N$-electron ground state. With diameter $d = 0.1 \mu$m, dot 2 contains about $N = 25$ electrons at the bulk density. We estimate the charging energy $E_C = e^2/C \approx 1.54/\varepsilon_r \varepsilon_0 d = 3.5$ meV, where $\varepsilon_r \approx 13$ for GaAs, and excitation energy $\delta E = E_{N}/N = 300$ µeV. Both energies far exceed $k_B T$ at 20 mK. Although we refer here to separate charging and 0D state energies of an independent (uncorrelated) electron system, the concepts can be generalized: A correlated system also has a change in electrochemical potential upon adding one electron and a minimum excitation energy at a fixed number of electrons, the analogs of $E_C + \delta E$ and $\delta E$, respectively.

Calculations exist for nonlinear transport [12], but the dot potential energy landscape of Fig. 2(a) gives us a qualitative understanding. At zero temperature, states of the left (right) reservoir are fully occupied up to $\mu_L$ ($\mu_R$) and empty at higher energies. Solid lines in the dot show $\mu_d(N)$ and $\mu_d(N+1)$ characterizing the $N$- and $(N+1)$-electron ground states, while dashed lines are discrete 0D excited states of the dot. Suppose $0 < \mu_L - \mu_R < \mu_d(N+1) - \mu_d(N)$, so at most one charge level lies between $\mu_L$ and $\mu_R$. When the transport condition $\mu_L > \mu_d(N) > \mu_R$ is satisfied, current flows as electrons tunnel one by one from left to right via states in the dot with energy between $\mu_L$ and $\mu_R$. If, on the other hand, $\mu_d(N+1) > \mu_L, \mu_R > \mu_d(N)$, no current flows due to the Coulomb blockade. Changing the center gate voltage $V_C$ shifts the conduction-band bottom and with it all charge levels $\mu_d(N)$, producing Coulomb oscillations in the current as the transport condition is alternately satisfied and not satisfied. In the metallic limit, when the broadening of the 0D states is much larger than the splitting $\delta E$, the dot excitation spectrum is continuous. As the bias voltage $V = (\mu_L - \mu_R)/e$ increases, the Coulomb oscillations broaden and grow in amplitude, but remain featureless.

This is not true when discrete 0D states exist. At small bias voltage $eV \ll E_C, \delta E$, the number of states in the allowed energy range between $\mu_L$ and $\mu_R$ changes from 0 to 1 to 0 as $V_C$ is varied, giving a smooth oscillation.
FIG. 2. (a) Potential energy landscape (left) and Coulomb oscillation with 0D shoulders (right) for a quantum dot with bias voltage $V = \mu_L - \mu_R = 1.8 \delta E$. Solid lines in the dot are the electrochemical potentials $\mu_d(N)$ and $\mu_d(N+1)$. Dashed lines show excitations with splitting $\delta E$. The number of states available for transport, noted by the peak, changes as $V_C$ varies. (b) Evolution of 0D shoulders with increasing bias voltage in dot 2. The curves are offset for clarity. From the bottom, the bias voltages are 100, 400, and 700 $\mu$V. The magnetic field is 4 T.

When $eV$ is of order $\delta E$, however, this model predicts the appearance of “0D shoulders” in the oscillations. Suppose $\mu_d(N)$ is just less than $\mu_R$, and we scan $V_C$ so that $\mu_d(N)$ and the 0D state energies increase. The current is first zero (Coulomb blockade). When $\mu_d(N) = \mu_R$ current flows via the $p+1$ 0D states with energy between $\mu_L$ and $\mu_R$, where $p < eV/eE < p+1$ (assuming the same energy $\delta E$ separates all 0D states). As $V_C$ varies further, one 0D state first becomes unavailable for transport as its energy exceeds $\mu_L$ (leaving $p$ current-carrying states); next a second becomes available as its energy rises above $\mu_R$ ($p+1$ current-carrying states again). More allowed states give a larger transition rate and more current. After $p$ such cycles, $\mu_d(N)$ exceeds $\mu_L$, and the current drops to zero. So when $V > \delta E/e$, the Coulomb oscillations acquire structure, 0D shoulders, due to the discrete spectrum of 0D excitations. This is shown in Fig. 2(a) for $eV \sim 1.8 \delta E$ ($p=1$). The number of states available for transport changes as $V_C$ varies between 0-2-1-2-0.

The appearance of 0D shoulders with increasing bias voltage in the Coulomb oscillations of dot 2 is shown in Fig. 2(b). The magnetic field is 4 T. Starting with the bottom curve, the bias voltage is 100, 400, and 700 $\mu$V, and current flows via a maximum of one, two, and three 0D states ($p=0, 1, 2$), respectively. Above the shoulders we show the number of states contributing to transport. We can determine the typical 0D splitting $\delta E$ by noting that since two or three 0D states appear in 700 $\mu$V, then $270 \mu V < \delta E < 350 \mu V$. This measured $\delta E$ is in good agreement with that given above based on the dot size, and confirms that the dot contains about $N = E_F/\delta E = 25$ electrons.

The 0D excitations also cause structure in $I$-$V$ characteristics, as predicted in Ref. [12]. Suppose in the energy landscape of Fig. 2(a) that the bias voltage $V$ is swept so $\mu_L$ increases from $\mu_L = \mu_R$ with $\mu_R$ fixed. At small bias, the Coulomb blockade suppresses current until $\mu_L > \mu_d(N) > \mu_R$. As $\mu_L$ increases from this point, the current grows in small steps as the window between $\mu_L$ and $\mu_R$ expands to include additional 0D states one by one, each contributing to transport. Eventually an extra charge level $\mu_d(N+1)$ is included between $\mu_L$ and $\mu_R$. There is a larger current jump at this point, since transport can now occur two electrons at a time. The $I$-$V$ characteristic has a double-step structure, with small 0D excitation steps and larger steps of the Coulomb staircase familiar from the metallic regime.

This double-step structure is clearly visible in the $I$-$V$ characteristics of Fig. 3, taken at zero magnetic field. The curves are for different values of the center gate pair voltage ranging from $V_C = -920$ mV (large Coulomb blockade, bottom curve) to $V_C = -905$ mV (zero Coulomb blockade, top curve). The typical spacing between smaller 0D state steps is 300 $\mu$V, in good agreement with the above estimates of $\delta E$ and $N$. Regions of negative differential resistance (NDR), not predicted by usual theories, appear at both positive and, more clearly, negative bias voltage. These can be caused by 0D states that for some reason (e.g., dopant-induced irregularities in the dot confining potential) are more weakly coupled to the reservoirs than the other levels [13]. If an electron tunnels into this state, further transport is blocked by the
charging energy until the state empties. Increasing $V$ making this extra 0D state available for transport, reduces the current, causing a NDR.

Current steps in the $I$-$V$ characteristic caused by 0D states cause peaks in the differential conductance $dI/dV$. Tunnel spectroscopy of the 0D levels is possible, although complicated by the capacitances between the electrodes and the quantum dot, which shift the energy levels as the bias voltage $V$ is scanned. Figure 4 shows traces of $dI/dV$ vs $V$ for magnetic fields from 0 (top curve) to 2.5 T (bottom curve). At $B=0$ the center gate voltage is tuned to a conductance maximum. Since the largest blockade in $I$-$V$ characteristics was 3.5 mV, at zero field peaks in $dI/dV$ at $|V| < 3.5$ mV correspond to excitations of the dot at fixed electron number. These measurements let us track the field dependence of the discrete excitation energies of the quantum dot in the charging regime. This field evolution at times resembles results of theories of confined, noninteracting electrons [14], for example, the behavior of the peak near $V = -1$ mV at $B=0$ (top curve in Fig. 4). As $B$ increases, this peak moves towards the peak at $V=0$, then near $B = 1.5$ T it reverses direction and moves back to more negative bias, similar to what is expected for noninteracting electrons in the intermediate-field regime. In general, however, the excitation energies do not evolve as predicted by the usual theories. A full discussion will appear in a later publication.

In summary, small lateral quantum dots show the combined effect of 0D states and single electron charging in linear and nonlinear transport. Coherent resonant tunneling occurs, described by a Landauer formula despite the charging interaction. Coulomb oscillations and $I$-$V$ characteristics show extra structure due to the 0D excitation spectrum.

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FIG. 1. (a) Conductance of QPC1 vs gate voltage $V_1$ for dot 1. $V_2$ is set so $T_2 = 1$. (b) Coulomb oscillations of dot 1 as $V_1$ is swept. $T_2$ is about 0.02. The maximum peak height calculated with Eq. (1) at a temperature of 50 mK is shown by the heavy line. The dashed line is the classical prediction. The magnetic field is 7 T. Inset: Gate geometry for dot 1.
FIG. 3. Zero-field $I$-$V$ curves at various center gate voltages for dot 2, showing the double-staircase structure. From the bottom, the center gate voltage is $-920$, $-910$, $-907$, and $-905$ mV. The curves are offset for clarity; all traces have $I = 0$ when $V = 0$. Inset: Sample 2 gate geometry. Transport is from left to right through QPCs 1 and 2.