Mechanical and vibro-acoustic aspects of composite sandwich cylinders

Proefschrift

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Summary

Designing a fuselage involves many considerations such as strength and stability, fatigue, damage tolerance, fire and lightning resistance, thermal and acoustic insulation, production, inspection, maintenance and repair. In the background of the application of composite sandwich structures on the aircraft fuselage, the focus of the thesis is to investigate the vibration and acoustic behaviours of sandwich structures.

As a preliminary design of aircraft fuselages, a sizing work of sandwich cylinders was conducted with respect to the strength and stability. FE models for the buckling prediction of the sandwich cylinder were validated with the analytical expressions. Under a typical flight loading, the sizing results of a sandwich cylinder and a laminated cylinder were compared and it was found that the mechanical efficiency of the sandwich cylinder is comparable to that of the traditional stiffened cylinder.

Subjected to the diffuse acoustic field, the sound transmission loss (TL) of composite sandwich cylinders was investigated using an analytical method and the Statistical Energy Analysis (SEA) method at 100-16000 Hz. The SEA method showed a good agreement with the analytical method. The parameters, including the fibre orientation, facing materials, cylinder geometry, core thickness, sandwich layup and core shear stiffness, were studied for their influences on the TL of cylindrical structures. A uniform laminated, a stiffened and a sandwich cylinder with the equivalent mass were compared for the sound insulation performance. The laminated cylinder had the largest TL below the coincidence frequency and the sandwich cylinder had the largest TL above the coincidence frequency.

The structural velocities and noise reductions of laminated and sandwich cylinders were experimentally tested at 1-4000 Hz under a point acoustic excitation, and a mechanical excitation respectively. The wave propagation in the sandwich structure was compared with that in the laminated structure, as an explanation of the noise reduction difference of the two structures. As the coincidence frequency plays an important role on the sound transmission, influence parameters of the coincidence frequency of sandwich structures were also studied.

To investigate the vibro-acoustic performance of sandwich structures under different kinds of external excitations, the FEM/BEM numerical method was used to analyze the noise reduction of sandwich cylinders at low frequencies. Under a force excitation, some parameters including the core shear stiffness, sandwich layup, core thickness and facing orientation were studied for their influences on the sound transmission. Results showed that there exist optimal values for these parameters to achieve the best sound insulation performance. Therefore, an efficient optimization technique using the acoustic transfer vector (ATV) and the genetic algorithm (GA) was applied to optimize a typical sandwich cylinder for the best noise insulation. In addition, taking a fuselage section as
an example, a multi-objective optimization (weight & noise insulation) was conducted considering the mechanical constraints under flight load cases.

The noise control treatment such as the addition of absorption layers is one of the common methods for the noise control of the transport vehicles. Thus the sound transmission of sandwich panels with open-cell foam was studied. The transfer matrix method (TMM) was used for the TL prediction of sandwich panels with porous foams. This method was validated by experimental results. A sensitivity study of the flow resistivity, tortuosity and porosity on the TL of sandwich panels was conducted. Then four kinds of absorption materials were studied for their influences on the TL of sandwich cylinders. Finally the TLs of a stiffened cylinder and a sandwich cylinder were compared in case of addition of absorption layers.

As the damping plays an important role on the vibro-acoustic behaviors of sandwich structures, the damping properties of composite sandwich structures were studied using the modal strain energy (MSE) method and experimental measurements. The hysteresis method and the half power method were used for the damping measurement. Compared to the facing, the cores usually have much higher damping and they make the main contribution on the sandwich damping. Therefore the material damping properties of two kinds of foams (PMI & PVC) were measured at low frequencies using the hysteresis method. The measured results have been validated by numerical models. The damping of the PVC foam were also measured using the half power method and results showed a good agreement with those measured using the hysteresis method. For the damping prediction of sandwich structures, the MSE method was verified by the measurements using the half-power method. Finally, the effects of the core thickness and core properties on the damping of sandwich structures were studied.
Samenvatting

Het ontwerpen van een romp omvat een groot aantal overwegingen, zoals sterkte en stabiliteit, vermoeiing, schadetolerantie, brand- en bliksemgedrag, thermische en akoestische isolatie, productie, inspectie, onderhoud en reparatie. Met de toepassing van composiet sandwich constructies in een vliegtuigromp in gedachte, is de focus van dit proefschrift het onderzoeken van de trillingen en het akoestische gedrag van sandwich constructies.

In een voorontwerp van een vliegtuigromp is de dimensionering van sandwich cilinders uitgevoerd met de nadruk op sterkte en stabiliteit van de constructie. FE modellen voor de knikvoorspelling van de sandwich cilinder werd geverifieerd met analytische uitdrukkingen. Onder normale vluchtbelastingen werden het dimensioneringsresultaat van een sandwich cilinder vergeleken met die van een gelamineerde cilinder. Het bleek dat het mechanisch rendement van de sandwich cilinder vergelijkbaar is met die van de traditionele verstijfde gelamineerde cilinder.

Onderworpen aan een diffuus akoestisch veld, lopend van 100 tot 16000 Hz, werd het verlies van geluidsoverdracht van composiet sandwich cilinders onderzocht met behulp van een analytische methode en met de Statistische Energie Analyse (SEA)-methode. De SEA-methode toonde een goede overeenkomst met de analytische methode. Parameters, zoals vezeloriëntatie, bekledingsmaterialen, cilinder geometrie, sandwich lay-up, kerndikte en kernafschuifstijfheid, werden onderzocht op hun invloed op het Transmissie Verlies (TL) van cilindrische constructies. Een uniform gelamineerde cilinder, een verstijfde cilinder en een sandwich cilinder met onderling equivalente massa’s werden vergeleken op geluidsisolerende eigenschappen. De gelamineerde cilinder had het grootste TL onder de coincidence frequentie en de sandwich cilinder had het grootste TL boven de coincidence frequentie.

De constructie en de geluidsreductie van gelamineerde en sandwich cilinders werden experimenteel gemeten tussen de 1 en 4.000 Hz onder een punt-akoestische excitatie en een mechanische excitatie. De golfvoortplanting van trillingen in de sandwichconstructie werd vergeleken met die in de gelamineerde constructie. Deze verschillen worden gebruikt voor de verklaring in het verschil in reductie tussen de twee constructies. Daar de coincidence frequentie een belangrijke invloed heeft op de geluidsoverdracht werden parameters die de coincidence frequentie van sandwich constructies beïnvloeden onderzocht.

Om de vibro-akoestische prestaties van sandwich constructies onder verschillende soorten externe excitaties te onderzoeken, werd in de lage frequentie bereik de FEM / BEM methode gebruikt om de geluidsreductie van sandwich cilinders te analyseren. Onder een exciterende kracht werden een aantal parameters waaronder de kernafschuifstijfheid, de kerndikte, de sandwich lay-up volgorde en de lay-up oriëntatie
bestudeerd op hun invloeden op de geluidsoverdracht. Resultaten toonden aan dat er optimale waarden voor deze parameters zijn om de beste geluidsisolatie te verkrijgen. Daarom werd een efficiënte, op ATV & GA gebaseerde optimalisatietechniek uitgevoerd om een representatieve sandwich cilinder te optimaliseren voor de beste geluidsisolatie. Daarnaast werd voor een rompdeel een multi-objective optimalisatie (gewicht en geluidsisolatie) uitgevoerd waarbij op stijfheid en sterkte gebaseerde randvoorwaarden door vluchtbelastingen werden meegenomen.

De geluidsbeperkende methodieken zoals de toevoeging van geluidsabsorberende lagen is een algemene methode voor de geluidsisolatie van transportvoertuigen. Daarom werd de geluidsoverdracht van sandwichpanelen met open-celschuim bestudeerd. De transformatie matrix methode (TMM) werd gebruikt voor de voorspelling van het TL van sandwichpanelen met poreuze schuimen. Dit model werd gevalideerd aan de hand van experimentele resultaten. Een gevoeligheidsanalyse met parameters zoals stroomweerstand (flow resistivity), tortuositeit en porositeit werd op het TL van sandwichpanelen uitgevoerd. Eveneens werden vier soorten absorberende materialen onderzocht op hun invloed op het TL van sandwich cilinders. Tenslotte werden de TLs van een verstijfde cilinder en een sandwich cilinder vergeleken rekeninghoudend met het geval van toevoeging van absorptielagen.

Daar de demping een belangrijke rol speelt bij het vibro-akoestische gedrag van sandwichconstructies zijn de dempende eigenschappen van composiet sandwich constructies bestudeerd met behulp van de modal vervormingsenergie methode (MSE) en experimentele metingen. De hysteresis methode en de half-vermogen methode werden gebruikt voor de characterisatie van de demping metingen. In vergelijking met vezelversterkte lagen geven de kernen meestal een veel hogere demping en zijn ze bepalend voor de belangrijkste bijdrage aan de sandwich demping. Daarom zijn de materiaaldempingseigenschappen van twee soorten schuim (PMI en PVC) gemeten bij lage frequenties volgens de hysteresis methode. Numerieke modellen zijn via deze gemeten resultaten gevalideerd. De demping van het PVC-schuim werd gemeten met de halve-vermogen methode en de resultaten toonden een goede overeenkomst met de waarden die gemeten werden via de hysteresis methode. Voor de dempingsvoorspelling van sandwichconstructies werd de MSE methode gevalideerd door metingen met half-vermogen methode. Tenslotte werden de effecten van de kerndikte en de kerneigenschappen op van de demping van sandwichconstructies bestudeerd.
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<tr>
<td>A</td>
<td>membrane stiffness matrix</td>
<td>MPa.m</td>
</tr>
<tr>
<td>B</td>
<td>bending-extensional coupling matrix</td>
<td>Pa.m²</td>
</tr>
<tr>
<td>( B_t )</td>
<td>bending stiffness of facings</td>
<td>Pa.m³</td>
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<tr>
<td>( c_b )</td>
<td>bending wave speed of the sandwich panel</td>
<td>m/s</td>
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<tr>
<td>( c_{bf} )</td>
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<td>D</td>
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<td>Pa.m³</td>
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<td>bending stiffness of the laminated plate</td>
<td>Pa.m³</td>
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<td>( E_c )</td>
<td>core Young's modulus</td>
<td>Pa</td>
</tr>
<tr>
<td>( E_f )</td>
<td>facing Young's modulus</td>
<td>Pa</td>
</tr>
<tr>
<td>( f )</td>
<td>frequency</td>
<td>Hz</td>
</tr>
<tr>
<td>( f_c )</td>
<td>coincidence frequency</td>
<td>Hz</td>
</tr>
<tr>
<td>( f_R )</td>
<td>ring frequency</td>
<td>Hz</td>
</tr>
<tr>
<td>( G_c )</td>
<td>core shear stiffness</td>
<td>Pa</td>
</tr>
<tr>
<td>( G_e )</td>
<td>core shear stiffness considering damping</td>
<td>Pa</td>
</tr>
<tr>
<td>( I )</td>
<td>moment of inertia of the sandwich beam</td>
<td>Pa.m³</td>
</tr>
<tr>
<td>( k )</td>
<td>wavenumber in the fluid, ( 2\pi/\lambda )</td>
<td>m⁻¹</td>
</tr>
<tr>
<td>( k_b )</td>
<td>wavenumber in the structure</td>
<td>m⁻¹</td>
</tr>
<tr>
<td>( l )</td>
<td>length inside the cylindrical cavity</td>
<td>m</td>
</tr>
<tr>
<td>( M )</td>
<td>the mass per unit area of sandwich</td>
<td>kg/m</td>
</tr>
<tr>
<td>( M_l )</td>
<td>mass per unit area of the laminated plate</td>
<td>kg/m</td>
</tr>
<tr>
<td>( r )</td>
<td>radius inside the cylindrical cavity</td>
<td>m</td>
</tr>
<tr>
<td>( t_c )</td>
<td>core thickness</td>
<td>m or mm</td>
</tr>
<tr>
<td>( t_f )</td>
<td>facing thickness</td>
<td>m or mm</td>
</tr>
<tr>
<td>( t_{f_{\text{olv}}} )</td>
<td>facing layer thickness</td>
<td>mm</td>
</tr>
<tr>
<td>( \gamma )</td>
<td>incident angle of the acoustic plane wave</td>
<td>rad</td>
</tr>
<tr>
<td>( \lambda )</td>
<td>wavelength</td>
<td>m</td>
</tr>
<tr>
<td>( \rho_1, \rho_2 )</td>
<td>air density of external or internal cavity</td>
<td>kg/m³</td>
</tr>
<tr>
<td>( \rho_c )</td>
<td>core density</td>
<td>kg/m³</td>
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<tr>
<td>( \rho_f )</td>
<td>facing density</td>
<td>kg/m³</td>
</tr>
<tr>
<td>( \sigma )</td>
<td>flow resistivity of the absorption materials</td>
<td>N.s /m⁴</td>
</tr>
</tbody>
</table>
$\tau$  transmission coefficient
$\nu_{12}$  Poisson ratio of the facing
$\nu_c$  Poisson ratio of the core
$\phi$  Porosity of the foam
$\omega$  circular frequency, $\omega = 2\pi f$ Hz

**Abbreviations**

ANOVA  Analysis of Variance
ATV  Acoustic Transfer Vector
BEM  Boundary Element Method
CLT  Classical Lamination Theory
CFRP  Carbon Fiber Reinforced Polymer
DOF  Degree of Freedom
ECS  Environmental Control System
FAA  Federal Aviation Administration
FEM  Finite Element Method
FFT  Fast Fourier Transform
FSTD  First-Order Shear Deformation Theory
GA  Genetic Algorithm
MPC  Multi-point Connection method in FE software Patran
MSE  Modal Strain Energy
NR  Noise Reduction
PMI  Polymethacrylimide
PVC  Polyvinylchloride
RKU  Ross-Kerwin-Ungar
RMS  Root Mean Square
SEA  Statistical Energy Analysis
TL  sound Transmission Loss
TMM  Transfer Matrix Method

Note: some symbols or abbreviations may not be explained here and they are illustrated in the main text.
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1.1. Fundamental of sandwich structures

Composite sandwich structures are widely used in aerospace structures, ship building and infrastructures due to their high strength to weight ratio. The traditional sandwich structures are composed of two thin and stiff facings with a thick and light-weight core, as shown in Figure 1-1. The core materials can be mainly divided into four types [1]: corrugated, honeycomb, balsa wood and foams, see Figure 1-2. The sandwich composites can be manufactured by bonding pre-cured facings with a core (bonding) or co-curing the facing and the core in one step (co-cure). The co-cure process can be performed by the vacuum bagging, autoclave molding, liquid molding, etc.

![Figure 1-1 Typical sandwich structure: thin carbon/epoxy facing with thick honeycomb or foam core](image)

![Figure 1-2 Four types of cores: corrugated, honeycomb, balsa wood and foam](image)

The properties of primary interest for the facings of a sandwich structure are [2]:

- High stiffness giving high flexural rigidity
- High tensile and compressive strength
- Impact resistance
- Surface finish
- Resistance to environment
- Wear resistance
The properties of primary interest for the core of a sandwich structure are [2]:

- Low density
- High shear stiffness and shear strength
- High stiffness perpendicular to the facings
- Acoustic and thermal insulation

1.2. Fundamental of acoustics

Acoustic is closely linked with the human life and it is a very important tool that helps people to communicate with the outside world. In most cases, the sound that makes people comfortable is called music while the sound that annoys people is viewed as noise. If acoustics is viewed as a science defined as the generation, transmission and reception of energy as vibration waves in matter, the noise control is only one substructure of the science. The research in this thesis mainly concerns the improvement the sound insulation of sandwich structures to prevent the outside noise as much as possible. To facilitate reading the thesis, some basic knowledge about the acoustics is introduced here.

![Image](image.png)

Figure 1-3 A visible pattern of sound waves produced by a scanning technique [3]

Sound is the result of pressure variations or oscillations in an elastic medium (e.g. air, water and solid). It is generated by a vibrating surface or turbulent fluid flow and propagates in the form of waves. As shown in Figure 1-3, sound waves are made visible using a scanning technique, and they looks like the wind-caused waves on the water. Note that sound waves are longitudinal waves, which means that the particles vibrate back and forth in the same direction as the direction propagation. It is unlike bending waves on a beam, which are transversal waves in which the particles move back and forth in a direction perpendicular to the direction of propagation [4].
1.2.1. Types of sound wave and evaluation of sound

Sound waves can be classified into: (1) Spherical wave, the wavelength is much larger than the size of sound source and sound wave spreads to all direction as spherical wave; (2) Plane wave, waves have the same acoustical properties at any position on a plane surface drawn perpendicular to the direction of propagation of the wave [5].

The sound pressure, the difference between the instantaneous value of the total pressure and the static pressure, is usually used to evaluate the sound quantity we hear. It can be measured by special microphones. The decibels (dB), which is widely used to characterize the sound pressure, is defined as a ratio between the specific sound pressure to a reference sound pressure [4]:

$$L_p = 10 \log_{10} \left( \frac{p^2}{p_{ref}^2} \right)$$

(1.1)

where $p_{ref}$ is 20e-6 Pa for sound waves in air, corresponding roughly to the lowest audible sound at 1000 Hz.

![Figure 1-4 Equal loudness curves for human](http://hyperphysics.phy-astr.gsu.edu/%E2%80%8Chbase/sound/eqloud.html)

The human hearing ability is in the range of 20-20000 Hz, while the hearing sensibility of the human ear is not constant over the audible frequency range. Figure 1-4 shows that the ear is less sensitive to the sound at low frequencies and very sensitive the sound at frequencies around 3000-4000 Hz. For this reason, A-weighting is applied to instrument-measured sound levels in effort to account for the relative loudness perceived by the human ear. It is employed by arithmetically adding a table of values, listed by octave or third-octave bands, to the measured sound pressure levels in dB. The units of A-weighted values are written as dBA. The calculation method of A-weighting can refer to [4].
1.2.2. Terms explanation about the sound transmission

For an easier understand of the thesis, some terms that are frequently used in the sound transmission analysis are shortly introduced here.

Sound transmission loss and noise reduction

Sound transmission loss (TL) and noise reduction (NR) are used to evaluate the sound insulation of the sandwich structures in the thesis. The TL is usually used to characterize the sound insulation under a diffuse acoustic field where the sound power is equal at any places. However, many noise sources are not diffuse in practical applications, e.g., the noise from the aircraft engine. In this case, the NR is often used to characterize the sound pressure difference between the noise source and the receiver positions. Note that larger values of both TL and NR represent better sound insulation. The TL and NR can be calculated as:

\[
TL = -10 \log \left( \frac{W^T}{W^I} \right) = -10 \log \frac{\overline{P}_i^2}{\overline{P}_i^2}
\]

(1.2)

\[
NR = -10 \log_{10} \frac{1}{n} \left[ \left( \frac{p_1}{p_{out}} \right)^2 + \left( \frac{p_2}{p_{out}} \right)^2 + \ldots + \left( \frac{p_n}{p_{out}} \right)^2 \right]
\]

(1.3)

where \(W^T\) represents the transmitted sound power and it is proportional to the square of the spatially averaged sound pressure \(\overline{P}_i^2\); \(W^I\) denotes the incident sound power and it is proportional to the square of the spatially averaged sound pressure \(\overline{P}_i^2\). \(p_i\) denotes the sound pressure at receiver positions and \(p_{out}\) the sound pressure at the noise source. \(n\) is the number of the receiver positions.

Radiation efficiency

Radiation efficiency, a characteristic for the effectiveness of sound radiation by vibrating surfaces, is defined as [6]:

\[
\sigma_{rad} = \frac{P_{rad}}{\rho c A v^2}
\]

(1.4)

where, \(P_{rad}\) represents power radiated into the cavity from the panels. \(\rho\) denotes the density of fluid, \(c\) the speed of sound in the fluid, \(A\) the area of radiating face and \(v\) the Root Mean Square (RMS) velocity of the radiating face. Larger radiation efficiency and larger structural velocities lead to lower TL or NR.

Wavenumber

Wavenumber \(k\), denoting the number of waves that exist over a specified distance, is of vital importance to the mathematical representation of two- and three-dimensional wave fields. It has a relationship with the wave length \(\lambda\) as [6]:

\[
k = \frac{\pi}{\lambda}
\]
Chapter 1 Introduction

\[ k = \frac{2\pi}{\lambda} \]  
(1.5)

Ring frequency

The motion of a system is a superposition of its normal modes, where all parts of the system move sinusoidally with the same frequency and with a fixed phase relation. Each normal mode corresponds to a resonant frequency. As shown in Figure 1-5, the radial displacements of a cylinder exhibit different shapes at different resonant frequencies. The first mode shape is often named as the “breathing mode”, where all the points of the cylinder are moving in phase (all points go inside or go outside at the same time). The resonance frequency where the breathing mode occurs is referred to as the ring frequency. At the ring frequency, the quasi-longitudinal wavelength in the shell wall equals the shell circumference; the vibration amplitude is large and much sound can be transmitted through the cylindrical structures.

![Cross-sectional radial displacement mode shapes of a circular cylindrical shell](image)

**Figure 1-5** Cross-sectional radial displacement mode shapes of a circular cylindrical shell [6]

Coincidence frequency

At the coincidence frequency, the wavelength of flexural waves in the solid structure is equal to the wavelength of acoustic waves in the air; the vibration amplitude is much larger and more sound can be transmitted through the structure. There generally exist dips at the ring frequency and the coincidence frequency on the TL or NR curves. The calculations of ring frequency and coincidence frequency are introduced in Section 3.5.1.

Modal density

As discussed in the terms explanation for the ring frequency, each system or structure has its own resonant (natural) frequencies, where the normal (natural) modes occur. The modal density is a statistically based measure of the distribution of modal natural frequencies in the frequency domain. It can be interpreted as the expected number of natural frequencies per unit frequency. The natural frequencies can be theoretically estimated using analytical or FE methods, noting that the theoretical estimates are subject to increasing uncertainty as the frequency increases because they are increasingly sensitive to minor details of materials properties, geometry, connections and damping distribution. However, the average density of natural frequencies along the frequency axis (modal density) became increasingly less sensitive to the boundary conditions [6].
That is one reason why the SEA method, a statistically based method, is preferred for the sound transmission analysis at high frequencies rather than the FEM. Modal densities of the structure and the cavity are important parameters for the SEA calculation.

1.3. Application of composite sandwiches in the aircraft fuselages

Lightness has always been one of the most important goals for the aircraft design. Beukers and Hinte [7] elaborated the practical applications of low-density materials and the lightness design in engineering. The lightness design generally requires using materials and structures with high stiffness-to-mass ratio, e.g., the sandwich structures. As early as 1924, Von Karman and P. Stock were granted a patent for designing a glider plane with a sandwich fuselage. Since then great progress has been made in the design and production of sandwich structures, coupled with the advent of high-performance core materials, more and more sandwich composites have been successfully applied on the fuselage of aircrafts, e.g.: the ATEC 321 FAETA is an advanced, aerodynamically controlled, two-seat and low-wing aircraft. Its fuselage is made of a composite sandwich using a carbon fiber facing cored with a NOMEX® honeycomb and hardened foam. The Extra 500 (as shown in Figure 1-6), a six-seat business aircraft, has a sandwich fuselage with CFRP facings and an Aramid honeycomb core. The sandwich skin is integrally stiffened with frames in the vicinity of cut-outs and at the wing support [8]. In addition, the Beechcraft Starship has approximately 70% composite structures in structural weight. All the major components like main and forward wing, pressure cabin, control surfaces and topsails consist of composite sandwich constructions.

Designing a fuselage involves many considerations such as strength and stability, fatigue, damage tolerance, fire and lightning resistance, thermal and acoustic insulation, production, inspection, maintenance and repair. As shown in Figure 1-7, besides the well-known higher stiffness-to-mass ratio of sandwich structures, the lower maintenance cost, crash resistance because of the energy absorption of the core, lower vibration level, as well as promising thermal and acoustic insulation of the composite sandwich structures would bring the possibility of their applications on the fuselage of a large composite civil aircraft. In addition, the multifunctional structures are researched and developed increasingly, where the smart sensors including fiber optics, piezoelectric ceramics, magneto-rheological fluid can be embedded in or attached to the structure during the material processing and vehicle operations [9]. Compared to the traditional metal materials, the manufacture of composite structures can be integrated with the sensor installation and this would reduce the production cost and the assembly time largely.
Figure 1-6 The 6-seat Extra 500, both the skeleton and fuselage are made of composite sandwiches [7]

Figure 1-7 Advantages of sandwich structures in multidisciplinary design of pressurized fuselages [8]

1.4. Problems of the vibration and noise in current aircrafts

Noise can lead to harmful health consequences for people, such as hearing impairment, hypertension, ischemic heart disease, and sleep disturbance. A large-scale statistical analysis of the health effects of aircraft noise shows that aircraft noise clearly and
significantly impairs human health. For example, a day-time average sound pressure level of 60 decibel increases the incident of coronary heart disease by 61% in men and 80% in women [10]. The noise during the aircraft flight not only influences the residents nearby the airports but also make the passenger feel uncomfortable in the aircraft cabin. The control of the cabin noise is usually considered in the aircraft design. The noise sources and the noise levels inside typical aircraft fuselages are introduced as follows.

1.4.1. Noise sources of aircrafts

Aircraft cabin noise has many complex external and internal sources. Taking the commercial jet aircraft as an example, the major internal source is the Environmental Control System (ECS) or air conditioning system. The major sources of external noise are [11]:

- **Airborne noise sources:**
  1. turbulent boundary layer excitation, which causes broadband noise radiation inside the cabin and cockpit
  2. engine noise, particularly the jet noise (mainly broadband) in the rear part of the cabin and the fan noise (tone)
  3. propeller noise for a propeller-driven aircraft (tonal mainly but also broadband), with a directivity as a function of frequency
  4. local aerodynamic phenomena due to singularities, antennas and probes on an aircraft

- **Structure-borne noise sources:** mainly shaft engine (tonal).

The main external noise sources for a commercial passenger airplane are shown in Figure 1-8. In many case, the airplane is not subjected to only the airborne sources or only the structure-borne sources, but the interaction of the two kinds of sources. The interaction of airborne noise and structural borne noise is illustrated Figure 1-9. Besides the transmission of the airborne noise through the fluid/structure interface between the air and the fuselage, the turbulent boundary layer and the engine can also cause the fuselage to vibrate and make the fuselage structure radiate noise. As seen from the right subfigure of Figure 1-9, the structure-borne noise level is higher at low frequencies while the airborne noise dominates at high frequencies. The actual noise that human hears is caused by the combination of the two kinds of noise.
1.4.2. Noise levels

Current jet transport aircrafts require noise control treatments with minimal weight penalties to achieve comfortable interior noise levels in the cabin. The maximum A-weighted sound level anywhere in the fuselage interior is set as 80 dBA [14]. Average interior noise levels typically range from 75 to 80 dBA, the noise level at the area such as a window seat can be 2 to 5 dBA higher than other area [15]. The typical noise levels in a long-range aircraft cabin are shown in Figure 1-10. In turbofan aircrafts at high-speed cruise (Mach number = 0.85) at 9144 m (30,000 ft) altitude, the interior noise levels are governed by transmitted turbulent boundary layer noise. The typical exterior noise levels are 125 dB and a reduction of the order of 30 to 50 dB is needed below 1000 Hz. The interior sound pressure levels of an airplane at different cruise speeds are shown in Figure 1-11 and it is seen that the interior pressure levels increase as the cruise speed increases, especially at 400-4000 Hz. This is caused by the fact that the turbulent boundary layer noise increases as the cruise speed is increased. A computer program in
Mechanical and vibro-acoustic aspects of composite sandwich cylinders

[16] can be used to estimate of the surface pressure fluctuations in the turbulent boundary layer of a flight vehicle.

Figure 1-10 Noise level at pilot’s ear in a long-range aircraft cockpit under cruise conditions [11]

![Figure 1-10](image)

Figure 1-11 Airplane interior sound pressure levels for different flight Mach numbers

([■], M=0.8; ▲, M=0.65; ♦, M=0.55 [15])

1.5. Research purpose

The composite sandwich has a high stiffness to weight ratio, which is beneficial for the mechanical applications, however, structures with high stiffness to weight ratio usually have a poor acoustic insulation. Thus we set out to answer the following questions:
Regarding composite sandwich structures, can they be made with both high mechanical performance and good acoustic insulation? Is it possible to improve the acoustic insulation properties of sandwich structures by a smart design? Comparing uniform structures and stiffened structures, what are the disadvantages and advantages of sandwich structures in terms of static mechanical efficiency and vibro-acoustic performance?

The focus of this thesis is the mechanisms of structural vibration and sound propagation in composite sandwich structures under mechanical or acoustic excitations, with the aim of designing sandwich aircraft fuselages, or other vehicle structures, for high structural efficiency and good noise insulation. In addition, as damping plays an important role in the vibroacoustic behavior of sandwich structures, it is also studied in the thesis.

1.6. Thesis Outline

As shown in Figure 1-12, the thesis is structured as follows:

The mechanical design of sandwich composite cylinders in terms of buckling and materials failure is introduced in chapter 2.

Sound transmission loss in composite cylinders is predicted using the analytical method and the SEA method in chapter 3, and parameters influencing the sound transmission of the cylinder are investigated.

A sandwich cylinder is experimentally compared with a laminated cylinder under a force excitation and a point acoustic excitation respectively in chapter 4. The wave propagation mechanism is used to explain the sound transmission differences of the two cylinders.
In chapter 5, the FEM/BEM is used to analyze the noise insulation of sandwich cylinders at low frequencies. The efficient techniques using ATV & GA are developed to minimize the inner sound pressure of sandwich cylinders. The techniques are also extended to find the solutions of the multi-objective and multi-discipline optimization problems.

Sound transmission of sandwich structures with absorption foam is discussed based on Biot’s theory in chapter 6.

The damping properties of foams and sandwich structures are studied experimentally and numerically in chapter 7.

Finally, conclusions and recommendations are given in chapter 8.

1.7. References

Chapter 2 Mechanical design of composite sandwich fuselages

Mechanical properties are the most basic design requirements for aircraft fuselages and thus a preliminary sizing of a typical aircraft fuselage was discussed in this chapter. The theoretical buckling computations of sandwich cylinders under bending and torsion moments were discussed. In addition, FE models were built and the predicted buckling values were validated with the theories. The fuselage of the A320 aircraft was idealized a sandwich and a stiffened cylinder, respectively. The design parameters like stiffeners positions and facing layup were optimized using the verified FE models. Finally, the sandwich fuselage was optimized in more detail: in the first model, the fuselage was divided into four parts in the hoop direction; in the second model, the fuselage was divided into four parts in the hoop direction and ten parts in the axial direction.

2.1. Introduction

Mechanical design of an aircraft fuselage is one of the most complicated and expensive projects in reality. Many aspects are needed to be taken into consideration, such as weight and balance, aerodynamics, flight profiles and pressurization profiles. There are some textbooks having systematic and comprehensive introductions about the aircraft design [1-3]. As the focus of the thesis is not the structural sizing, only a short introduction about the mechanical design of aircraft fuselages is given here.

2.1.1. Research about mechanical design of composite fuselages

The rising price of fuel and ever stricter environment protection requirements have led to lighter composite materials to become increasingly a choice when designing new aircrafts. Till now, composites have gradually been applied on the airplane from secondary structure to the primary structure. The utility of composites materials on the primary structures of airplane are being hot topics. The principal considerations, including the structural consideration and the manufacturing consideration, associated with the design of a composite fuselage were summarized in [4]. NASA started its research program on the application of advanced composites on the fuselage for large transports [5]. Five different design concepts (Figure 2-1) were initially put forward and after the cost and weight estimations, honeycomb stabilized skin panels (Figure 2-1a) and I-section stiffened laminate skin panels (Figure 2-1b) were selected for further research and development.
As early as 1993, the damage tolerance of a full composite airplane fuselage was researched by Beijnen et al [6]. Since 2000, the Federal Aviation Administration (FAA) has conducted lots of testing work on the impact damages of composite sandwich panels for airframes [7-9]. The damage tolerance characteristics and failure mechanisms of six honeycomb sandwich composite fuselage panels were investigated in [10]. A fixture which was capable of applying pressurization, axial, hoop and shear loads to a fuselage panel was used. During the testing, besides the strain gages, a digital image correlation method [11] was used to measure the displacement and strain field. In addition, the acoustic emission method was adopted to monitor damage growth in real time and served as an early warning for imminent failure. A more detailed description of the acoustic emission method was introduced in another paper of the authors [12].

2.1.2. Load cases of fuselage

The loads placed on fuselages can be classified into three main categories: flight loads, ground loads and pressurization loads. In flight, maneuvers such as pitching, rolling and yawing cause bending and torsion loads. Ground loads are caused by landing, taxing, ground turning, and towing, etc. In order to determine the distributed loads on the whole fuselage structure, it is vital to make a reasonable estimate of the weight distribution in the early design [13]. How to estimate the weight distribution for an aircraft fuselage and wing is introduced in [14-15].

Figure 2-2 Ultimate bending moment and shear load envelopes for a fuselage during flight [5]

Figure 2-2 provides an example of a typical distribution for the bending moment and shear load. It is obvious that both the bending moment and the shear load have a largest value at the wing position. Fuselage pressurization is also an important loading since it
induces hoop and longitudinal stresses in the fuselage. During the flight, the cabin pressure is required to maintain at a certain level for the comfort of passengers. However, the outside pressure deceases as the flight altitude increases. Thus the pressurized fuselage should be designed to withstand the pressure difference between the cabin pressure and the outside pressure [1].

2.1.3. Mechanical design considerations of fuselage

Structural design of a composite fuselage is complex and many factors must be taken into account, such as the distribution of the loads, global and local stability, material failure, damage tolerance, impact resistance and the cut-out stress concentration. A discussion of the damage tolerances of several full-scale composite sandwich fuselage panels subjected to quasi-static pressurization and longitudinal loading is given in [10]. The design of joints and large cutouts for a composite fuselage is discussed in [16]. Van Tooren [13] studied the mechanical design, stress at cut-outs, damage tolerance of sandwich structures and concluded that sandwich cylindrical shells do not need additional stiffeners to obtain sufficient buckling strength. Krakers [17] compared the sizing results of sandwich cylinders and stiffened cylinders under a typical flight load. It was found that the carbon/epoxy stiffened fuselage showed a lower weight than the sandwich cylinder. Following the work of M. Van Tooren [13] and L.A. Krakers, the sizing of a composite sandwich cylinder using a mechanical design consideration is discussed in this chapter. As global stability and material failure can give an initial sizing for the fuselage, only the two criteria are considered for the mechanical design in the study.

2.2. Theoretical buckling criteria

At low stress regions, sandwich composites structures are usually assumed to exhibit a linear stress-strain relationship, which means that the structure can return to its original position once the applied load is removed, however, when external loads exceed a particular threshold, a structure will become unstable and continue to deflect without an increase in the magnitude of the applied loads. In this case, the structure will buckle. Therefore the term ‘instability’ is also called “buckling”. The primary design problem is to prevent buckling that leads to undesirable configurations, in particular, collapse, a compression buckling test of a cylindrical shell is shown in Figure 2-3. The critical buckling load of a structure generally depends on its geometric proportions, the boundary conditions, the manner in which it is stiffened, or the bending and extensional stiffness of its components. Examples of failure modes of sandwich plates are shown in Figure 2-4. A summary of the buckling criteria for thin cylindrical shells is given in [18]. More specifically, the theoretical buckling predictions under different loading cases are discussed in this section for the sandwich cylinder.
2.2.1. Facing wrinkling

Facing wrinkling is usually viewed as a local, short wavelength buckling phenomenon. The facing can deform symmetrically or anti-symmetrically. Most of the theoretical analyses for facing wrinkling are based on mathematical models of a flat sandwich strut under a uniaxial load. With respect to sandwich struts with thick cores, the mostly common theoretical expression for the wrinkling stress is [20]:

$$\sigma_{wr} = k_n \left( \frac{E_f E_c G_c}{3} \right)^{1/3}$$  \hspace{1cm} \text{(2.1)}

where $k_n$ is the knockdown factor which varies from 0 to 1 according to different theories and boundary conditions and the $k_n$ is equal to be 0.5 in this study. A comprehensive summary of the different mathematical models used to calculate $k_n$ is given in [21].
2.2.2. Bending buckling

Bending buckling is triggered by the compressive component of bending moment. With respect to the sandwich cylinder, the interaction between shear crimping and compressive buckling is taken into account because of the role of the core. The critical buckling stress under compression can be predicted as [22]:

\[ N_x = k_{xa} \frac{D_s \pi^2}{L^2} \eta_b \]  
\[ (2.2) \]

The allowable bending moment for buckling can be expressed in terms of \( N_x \) as:

\[ M = 2\pi R^2 N_x \]  
\[ (2.3) \]

where \( L \) and \( R \) denote the length and the radius of the cylinder respectively. \( D_s \) is the bending stiffness of the sandwich structure, \( \eta_b \) the knockdown factor for initial imperfections, and \( k_{xa} \) the compressive load coefficients. \( D_s, \eta_b, k_{xa} \) can be computed as:

\[ D_s = \frac{E_f}{1 - v_f^2} \left( -\frac{1}{6} t_f^3 + \frac{t_f}{2} h^2 \right) \]  
\[ (2.4) \]

\[ \eta_b = 1 - 0.731 \left( 1 - \frac{1}{16 \sqrt{R/t_f}} \right) \]  
\[ (2.5) \]

\[ k_{xa} = \begin{cases} 
\frac{1}{1 + r_a^2} + \frac{z_a^2}{\pi^2} & \frac{z_a}{\pi^2} \leq \frac{1}{1 + r_a} \\
\frac{z_a}{\pi^2} \left( 2 - \frac{z_a r_a}{\pi^2} \right) & \frac{1}{1 + r_a} < \frac{z_a}{\pi^2} < \frac{1}{r_a} \\
\frac{1}{r_a} & \frac{z_a}{\pi^2} \geq \frac{1}{r_a}
\end{cases} \]  
\[ (2.6) \]

where \( z_a \) is the curvature parameter and \( r_a \) is the ratio between the bending stiffness \( D_s \) and the shear stiffness \( D_Q \).

\[ z_a = \sqrt{\frac{2 t_f L^4 (1 - v_f^2)}{R^2 I_s}} \]  
\[ (2.7) \]

\[ r_a = \frac{D_s \pi^2}{D_Q L^2}, \quad D_Q = G_c t_c + 2 G_f t_f \]  
\[ (2.8) \]
2.2.3. Torsion buckling

An energy method to determine the shear buckling stress under the torsion is developed in [23], in which a small-deflection theory is used to find the solution of the energy equilibrium equation. The method fits a sandwich structure with composite facings and orthotropic cores. As a special case of the structural anisotropy, a simplified expression for isotropic facings and an isotropic core is given here. The more complex expressions for orthotropic sandwich structures can be found in the Mathematica program in Appendix F. Considering a sandwich structure with significantly thin skins, the critical shear stress can be evaluated using the following formula [23]:

\[
\tau_{cr} = K_f \eta E_f \frac{h}{R}
\]

(2.9)

where \( E_f \) is the Young’s modulus of the facing. \( \eta \) is the knockdown factor, here \( \eta = 0.8 \).

\( h = t_1 + t_c + t_2 \), and \( K_f \) is given by:

\[
K_f = \psi_1 + \psi_2
\]

(2.10)

in which:

\[
\psi_1 = \frac{J \rho^2}{4\pi^2 x} \left[ \frac{(x - \rho)^4}{[1 + (x - \rho)^2]^2} \right] + \frac{(x + \rho)^4}{[1 + (x + \rho)^2]^2}
\]

(2.11)

\[
\psi_2 = \frac{\pi^2}{(1 - \frac{t}{h})4J \rho^2 x} \left[ \frac{J}{h^4} \left( A_1 + 2A_2 + A_3 + (A_1A_2 - A_2^2) \frac{\pi^2}{J}\frac{1}{\rho^2} \frac{A_4}{A_4} \frac{S}{S} + \frac{S}{S} \right) - \frac{J}{h^4} \right]
\]

(2.12)

\[
J = \frac{L^2}{hR}, \quad \phi = \frac{t_c t_f}{2}, \quad \rho = \frac{\pi R}{nL}, \quad S = \frac{\phi E_f}{\lambda G_c R h}
\]

(2.13)

\[
A_1 = \frac{2}{\lambda} (x^4 + 6x^2 \rho^2 + \rho^4) + 2 \frac{G_f}{E_f} (x^2 + \rho^2), \quad A_2 = 2 \left( \frac{v_f}{E_f} + \frac{G_f}{E_f} \right) (x^2 + \rho^2)
\]

(2.14)

\[
A_3 = \frac{2}{\lambda} + 2 \frac{G_f}{E_f} (x^2 + \rho^2), \quad A_4 = x^2 + \rho^2
\]

(2.15)

where \( K_f \) is a function of arbitrary number \( x \), and a value of \( x \) exists which can make \( K_f \) the minimum. The minimum value of \( K_f \) can be substituted into Equation(2.9). Finally, the allowable torsion moment can be given by:

\[
T_{cr} = 2\pi R^2 (t_f + t_f) \tau_{cr}
\]

(2.16)
2.2.4. Shear buckling

The critical transverse shear stress under the shear force can be assumed as the same with the critical shear stress under the torsion moment. The shear force can be computed using the following formula:

$$Q_{cr} = \pi R (t_f + t_c) \tau_{crs}$$  \hspace{1cm} (2.17)

$$\tau_{crs} = K_f \eta E_f \frac{h}{R}$$  \hspace{1cm} (2.18)

where, \(K_f\) is the same as the one used for the torsion buckling computation shown in Equation (2.10). The analytical method was written in Wolfram Mathematica 7.0 for prediction of the buckling loads. The Mathematica program and input parameters can be found in Appendix F.

2.2.5. Effect of structural parameters on the buckling loads of sandwiches

The dependencies of critical buckling loads on the facing thickness \((t_f)\) and the core thickness \((t_c)\) are shown in Figure 2-5. The ranges of \(t_f\) and \(t_c\) are based on the common sandwich configurations for the aerospace engineering. It can be found that both the critical torsion moment and the bending moment have a nearly linear relationship with \(t_f\) and \(t_c\). Increasing both \(t_f\) and \(t_c\) leads to an improvement of the critical buckling loads.

Figure 2-5 Effect of skin thickness (a) and core thickness (b) on critical torsion and bending buckling moment
In addition, the effects of the core shear stiffness ($G_c$) and the facing stiffness ($E_f$) on the critical buckling loads are shown in Figure 2-6. The allowable bending moment has no difference as $G_c$ increases and this is due to the fact that $G_c$ is much lower than the facing stiffness and its contribution to the structure stability of sandwiches can be neglected under the compressive load. However, the allowable torsion moment increases to a certain extent as $G_c$ increases. This can be easily explained by the fact that the shear stress is mainly caused under the torsion and thus $G_c$ shows a significant influence. As can be seen from Figure 2-6b, the allowable bending moment and torsion moment increase linearly as the $E_f$ increases. Finally it is found that $E_f$ and $t_f$ have similar influences on the bending and torsion buckling moments.

2.3. Verification of FE models for the buckling analysis

Three modeling techniques are widely used to simulate the structural behavior of sandwich structures: (1) modeling the sandwich using shell elements with laminated materials, (2) modeling the core with solid elements and the facing with shell elements, and (3) modeling both the facing and the core with solid elements. Since the facing thickness is small compared to the radius of a cylinder, solid elements are not necessary to model the facing. Thereby only the first two methods were used to simulate the sandwich cylinder. The predicted buckling loads will be compared with the analytical results in Subsection 2.3.3.

2.3.1. The computation method for the buckling in the FE analysis

The sandwich cylinder was modeled in MD Patran and the buckling analysis of the models was submitted to MD Nastran for computation. In Nastran, the buckling eigenvalue $\lambda_i$ can be obtained by solving the following equation:

$$\left[ K_a \right] + \lambda_i \left[ K_d \right] = \left[ 0 \right]$$

(2.19)
where $\text{||}$ stands for the determination of the matrix, and $\lambda_i$ denotes the $i$th eigenvalue, the total number of eigenvalue is equal to the degree of freedom of the FE model. $[K_a]$ is the system linear stiffness matrix. $[K_d]$ is the differential stiffness matrix which denotes the higher-order terms of the strain-displacement relationship. $[K_d]$ is a function of the geometry, the element type, and applied loads.

With the known $\lambda_i$, the buckling load can be obtained by:

$$P_{cr i} = \lambda_i P_a$$

(2.20)

where $P_a$ is the applied load. In general, the absolute value of the lowest eigenvalue should be $>1$ to prevent buckling happening.

### 2.3.2. Introduction of the FE model

A cylinder model with a length of 10 m and a radius of 2 m was built to simulate the fuselage of the A320 aircraft in MD Patran. As shown in Figure 2-7, the QUAD4 shell elements were used to model the whole sandwich cylinder in the FE model (a) and the shell elements were attributed with laminated properties. While in the FE model (b), QUAD4 and HEX8 elements were chosen to model the skins and the core respectively. Each skin consists of 500 QUAD4 elements and the core consisted of 500 HEX8 elements. Offsets of half the skin thickness were given when the QUAD4 elements were assigned the properties. Since this study was not aimed at investigating the bonding strength between core and skins, it was assumed that the adhesive agent was sufficiently strong to prevent delamination. Therefore the core shared its nodes with the two skins. An offset distance of half of the skin thickness was done when attributing the properties to the shell elements. This kind of modeling method greatly simplifies the modeling task, especially for the optimization work, as it allows redefinition of the skin thickness without a need to change the geometry and the mesh.

With respect to the boundary condition, a fixed displacement constraint was added at one end of the cylinder, and the multipoint connector RBE3 was used to distribute the bending or torsion moments to the nodes at the other end.
2.3.3. Verification of the FE model using analytical values

In order to verify the FE model with respect to the buckling analysis, the FE model (a) and FE model (b) were compared with the analytical method for the critical buckling loads. The computed critical buckling loads by the two FE models are compared with the analytical results in Table 2-1. It can be seen that the predicted bending moment and torsion moment of model (a) show good agreement with that of model (b). Moreover, their small differences with the analytical results are acceptable considering the boundary conditions of the FE models were not exactly the same as those of the analytical models.

Table 2-1 Critical buckling loads calculated by the FE models and the analytical model

<table>
<thead>
<tr>
<th></th>
<th>FE Model (a)</th>
<th>FE Model (b)</th>
<th>Analytical</th>
</tr>
</thead>
<tbody>
<tr>
<td>Predicted Bending moment</td>
<td>4.9e7 Nm</td>
<td>4.891e7 Nm</td>
<td>5.3e7 Nm</td>
</tr>
<tr>
<td>Predicted Torsion moment</td>
<td>2.5e7 Nm</td>
<td>2.39e7 Nm</td>
<td>2.8e7 Nm</td>
</tr>
</tbody>
</table>

Note: The boundary condition for the bending buckling is fixed in the radius and tangential direction, and is fixed in the radius direction for the torsion buckling.

2.4. Structural efficiencies of stiffened cylinders and sandwich cylinders

Stiffened and sandwich structures are widely used in current aircraft fuselage because of their high stiffness-to-mass ratios. In order to compare the structural efficiency of sandwich structures with stiffened structures, a stiffened cylinder and a sandwich cylinder were optimized, and the optimal designs of the two structures were compared in term of total weight of the structure.

The objective of the optimization was to minimize the total weight, providing that the structure could meet the requirements for global stability and the safety requirements of composite materials under combined flight loads. The Tsai-Wu criterion was used to evaluate the composite materials failure [24]:

\[
(\frac{1}{X_t} - \frac{1}{X_c})\sigma_1 + (\frac{1}{Y_t} - \frac{1}{Y_c})\sigma_2 + \frac{\sigma_1^2}{X_tX_c} + \frac{\sigma_2^2}{Y_tY_c} + 2F_{12}\sigma_1\sigma_2 + \frac{\sigma_{12}^2}{S^2} \leq 1 \tag{2.21}
\]

where \(X\) and \(Y\) represent the tensile/compressive strength along and perpendicular the fiber, respectively, and \(S\) is the shear strength. Subscripts \(t\) and \(c\) stand for the tension and compression respectively, and \(\sigma_1, \sigma_2, \sigma_{12}\) are the actual in-plane stress. The stress interaction \(F_{12}\) is a failure criteria property that is used only for the Tsai-Wu failure theory and is determined from biaxial tests. If \(F_{12}\) is unknown, then the following equation will be used: \(F_{12} = \frac{1}{2}\sqrt{F_{11}F_{22}}\) where \(F_{11} = 1/(X_tX_c)\) and \(F_{22} = 1/(Y_tY_c)\). The following condition must be satisfied: \(F_{11}F_{22} - F_{12}^2 \geq 0\) for numerical stability.
2.4.1. Models of the stiffened and the sandwich structures

The models of stiffened and the sandwich cylinders were also built using MSC patran, and the modeling of the two kinds of structures was introduced respectively.

Figure 2-8 Stiffened cylinder with C frames and Z stringers (a) and Sandwich cylinder with C frames (b)

With respect to the stiffened cylinder, the skins were modeled using linear QUAD4 elements. The stringer and frames were modeled using beam elements, as shown in Figure 2-8a. Since some geometry parameters such as the pitch of stringers and frames, the height of stringers and frames cannot be set as design variables in the Nastran analysis, they were respectively assigned three levels to form an analysis of variables (ANOVA) table with nine groups, as shown in Table 2-2. The sizing variables of each group include stringers thickness, frames thickness and the thickness of angle plies in the Nastran analysis.

With respect to the sandwich cylinder, the facings were modeled using linear QUAD4 elements. The core was modeled using HEX8 elements and the frames were modeled using beam elements, as shown in Figure 2-8b. Similar to the stiffened structures, the geometry parameters such as the core thickness, the frame pitch and the frame height were assigned three levels to form an ANOVA table, as shown in Table 2-3. The design variables included the frame thickness and the thickness of angle plies.
### 2.4.2. Optimization results and comparison

The optimization results for both the stiffened and the sandwich cylinders are analyzed and the two kinds of structures are compared in terms of the total weight.

#### Table 2-2 ANOVA of the stiffened composite cylinder

<table>
<thead>
<tr>
<th>Set</th>
<th>Frame pitch[m]</th>
<th>Stringer pitch[m]</th>
<th>Frame height[m]</th>
<th>Stringer height[m]</th>
<th>Total weight[kg]</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.5</td>
<td>0.1</td>
<td>0.05</td>
<td>0.02</td>
<td>881</td>
</tr>
<tr>
<td>2</td>
<td>0.5</td>
<td>0.2</td>
<td>0.1</td>
<td>0.03</td>
<td>802</td>
</tr>
<tr>
<td>3</td>
<td>0.5</td>
<td>0.3</td>
<td>0.08</td>
<td>0.04</td>
<td>874</td>
</tr>
<tr>
<td>4</td>
<td>0.1</td>
<td>0.1</td>
<td>0.1</td>
<td>0.04</td>
<td>1750</td>
</tr>
<tr>
<td>5</td>
<td>0.2</td>
<td>0.08</td>
<td>0.08</td>
<td>0.02</td>
<td>1658</td>
</tr>
<tr>
<td>6</td>
<td>1</td>
<td>0.3</td>
<td>0.05</td>
<td>0.03</td>
<td>2738</td>
</tr>
<tr>
<td>7</td>
<td>2</td>
<td>0.1</td>
<td>0.08</td>
<td>0.03</td>
<td>2149</td>
</tr>
<tr>
<td>8</td>
<td>2</td>
<td>0.2</td>
<td>0.05</td>
<td>0.04</td>
<td>3476</td>
</tr>
<tr>
<td>9</td>
<td>2</td>
<td>0.3</td>
<td>0.1</td>
<td>0.02</td>
<td>2058</td>
</tr>
<tr>
<td>Level I</td>
<td>853</td>
<td>1593</td>
<td>2365</td>
<td>1532</td>
<td></td>
</tr>
<tr>
<td>Level II</td>
<td>2049</td>
<td>1979</td>
<td>1560</td>
<td>1896</td>
<td></td>
</tr>
<tr>
<td>Level III</td>
<td>2561</td>
<td>1890</td>
<td>1537</td>
<td>2033</td>
<td></td>
</tr>
<tr>
<td>Extreme Level</td>
<td>Difference</td>
<td>1708</td>
<td>385</td>
<td>828</td>
<td>501</td>
</tr>
</tbody>
</table>

The ANOVA for the stiffened composite cylinder is shown in Table 2-2. It can be found that the second group has the smallest weight (802 kg). It should be noted that the total skin thickness of the second group is 2.79 mm if the materials failure constraint is considered, while it is 1.6 mm if the material’s failure constraint is not considered. In other words, the facing thickness is increased to prevent failure of the composite materials.

The influence of the four geometry parameters on the optimal weight was studied by comparing the extreme level differences in Table 2-2. It can be seen that the frame pitch plays the most important role on the total weight, followed by the frame weight. In this analysis, a decrease of the frame pitch from 2 m to 0.5 m leads to a decrease of the total weight, however, this trend does not continue when the frame pitch is further decreased. A stiffened cylinder with a frame pitch of 0.25 m was optimized in the same way, and its optimal weight is 923 kg, which is larger than that of the structures with a frame pitch of 0.5 m.

As an example, the history of the optimization design variables for the second group is shown in Figure 2-9. The design variables include the angle ply thickness and the beam properties of stringer and frames. In the initial design, the initial layup is [0/30/45/60/90]s and each ply has the same thickness. While in the final design cycle, the 90° direction (along the axial direction) has the largest ply thickness. This can be explained by the fact that the bending moment $M_y$ is the largest in the combined load cases, which causes the compressive stress to be dominant in the axial direction. For the beam dimension, the frame width (C_W) shows a significant increase from the initial design to the final design. In contrast, the stringer width (Z_W) decreases dramatically.
It demonstrates that increasing the frame width can improve the structural efficiency of the stiffened cylinder in this load case.

![Graph showing variable history of the stiffened cylinder](image)

**Figure 2-9 Variable history of the stiffened cylinder**

(Z_W, Z_H1, Z_t denote: the width, height and thickness of Z stringers; C_W, C_t, C_t1 denote: the width, thickness of the C frames, refer to Figure 2-8; ply_0, ply_30 etc. denote: layer thickness of the angle plies.)

The ANOVA results for the sandwich cylinder are shown in Table 2-3. Among the nine groups, the fourth group presents the lowest weight (867 kg) and the corresponding skin thickness is 1.34 mm. When analyzing extreme level differences, frame height and pitch have important effects on the weight of the cylinder. Increasing the frame height improves the structural efficiency of the sandwich cylinder (this is the same as that for a stiffened structure). Since local buckling is not considered, it is not clear whether local buckling of the frames occurs, however, frame height cannot be increased too much to prevent local buckling of the frames. In addition, it should be noted that the core thickness does not significantly influence the total weight. The reason could be that the total weight is increased by increasing the core thickness, in spite of the fact that an increase of core thickness leads to an increase of critical buckling loads in Figure 2-5. In other words, the structural efficiency cannot be improved by increasing the core thickness, particularly when the core thickness is assumed to be uniform, as excessive weight can be expected at the low-stress area. Thus the total weight may be further decreased if the sandwich cylinder is optimized with the division into different regions. This will be discussed in Section 2.5.
Table 2-3 ANOVA of the composite sandwich cylinder

<table>
<thead>
<tr>
<th>Experiment</th>
<th>Frame pitch/m</th>
<th>Core thickness/m</th>
<th>Frame height/m</th>
<th>Weight/kg</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.5</td>
<td>0.005</td>
<td>0.05</td>
<td>2151</td>
</tr>
<tr>
<td>2</td>
<td>0.5</td>
<td>0.01</td>
<td>0.1</td>
<td>912</td>
</tr>
<tr>
<td>3</td>
<td>0.5</td>
<td>0.02</td>
<td>0.08</td>
<td>1326</td>
</tr>
<tr>
<td>4</td>
<td>1</td>
<td>0.005</td>
<td>0.1</td>
<td>867</td>
</tr>
<tr>
<td>5</td>
<td>1</td>
<td>0.01</td>
<td>0.08</td>
<td>1297</td>
</tr>
<tr>
<td>6</td>
<td>1</td>
<td>0.02</td>
<td>0.05</td>
<td>1967</td>
</tr>
<tr>
<td>7</td>
<td>2</td>
<td>0.005</td>
<td>0.08</td>
<td>2743</td>
</tr>
<tr>
<td>8</td>
<td>2</td>
<td>0.01</td>
<td>0.05</td>
<td>3567</td>
</tr>
<tr>
<td>9</td>
<td>2</td>
<td>0.02</td>
<td>0.1</td>
<td>1524</td>
</tr>
<tr>
<td>Level I</td>
<td>1463</td>
<td>1920</td>
<td>2562</td>
<td></td>
</tr>
<tr>
<td>Level II</td>
<td>1377</td>
<td>1926</td>
<td>1789</td>
<td></td>
</tr>
<tr>
<td>Level III</td>
<td>2611</td>
<td>1606</td>
<td>1101</td>
<td></td>
</tr>
<tr>
<td>Extreme Level Difference</td>
<td>1234</td>
<td>320</td>
<td>1461</td>
<td></td>
</tr>
</tbody>
</table>

The variable history of the sandwich cylinder is shown in Figure 2-10. Similar to the stiffened cylinder, there are significant increases in the frame width (C_W) and the 90° ply thickness from the initial cycle to the final cycle. The difference from the stiffened cylinder is that the 0° ply thickness of the sandwich cylinder also has a large increase, which does not take place for the stiffened cylinder. This could be because the stringer is absent in the sandwich cylinder and more reinforcement is needed in the hoop direction.

![Figure 2-10 Variable history of the sandwich cylinder](image)

(C_W, C_t, C_t1 denote: the width, thickness of the C frames, refer to Figure 2-8; ply_0, ply_30 etc. denote: layer thickness of the angle plies)
In conclusion, a comparison of the stiffened cylinder and the sandwich cylinder shows that the optimal stiffened cylinder is 802 kg, which is slightly lighter than the optimal sandwich cylinder (867 kg).

2.5. Detailed sizing of composite sandwich cylinder

It is well known that the loading is not uniform in realistic fuselages, for example, if an aircraft fuselage is loaded with a bending moment, the crown is mainly subject to tension strain, the keel is mainly subject to compression strain, and the side is mainly subjected to the shear strain. Therefore, the cylinder is divided into 4 parts in the hoop direction: the crown, the keel and two side sections (side1 and side2). In addition, as mentioned before, the loading is varied with the fuselage station. The fuselage stations which are close to the wing usually have the largest loading in either the flight condition or the ground condition. Therefore, the cylinder is further divided into 10 parts in the axial direction for sizing, and 4 parts in the hoop direction. This results in a total 40 parts division (as shown in Figure 2-11). This approximation could prevent excessive thickness of the core or the facing at the low-stress regions of the sandwich cylinder.

![Figure 2-11 optimizations of the sandwich cylinder: (a) 4 parts divisions; (b) 40 parts divisions](image)

2.5.1. Model introduction

Similar to the model in Section 2.4, a cylinder with a length of 10 m and a radius of 2 m is used to simulate one section of the fuselage of A320. The facing and core materials used for the sandwich cylinder are shown in Appendix A. The load distribution in the axial direction of the 10m-length fuselage section is shown in Figure 2-12, which is generated based on the loading figures given in [13]. Spatial varied bending moments and shear forces were applied to the cylinder. The boundary condition is simply supported (transitionally fixed and rotational free) at the station of 0 m, as shown in Figure 2-13.

Global buckling and the strain levels provide the sizing constrains for the models. The design strain levels of graphite/epoxy structures are restricted by many considerations including: stress concentrations associated with cutouts, joints and splices; tolerance for impact damage; transverse cracking in the 90° fiber-oriented plies; and compatibility.
with adjacent aluminum strain levels. Thus these considerations restrict the ultimate
design strains to approximately fifty percent of the composite material failure strain [4].
Finally the structural sizing or the structural optimization problem can be expressed as:

\[
\text{Minimize } \quad N t_{f1\_layer} \rho_f + t_c \rho_e + N t_{f2\_layer} \rho_f
\]

\text{Subject to}

Global buckling: $|\lambda| > 1$

\[\epsilon_{\text{max, face}} < 0.4\%, \quad \epsilon_{\text{max, core}} < 3\% \quad \text{(2.22)}\]

With design variables shown in Table 2-4

Table 2-4 Design variables for two sizing models

<table>
<thead>
<tr>
<th>Variables</th>
<th>Variables range</th>
</tr>
</thead>
<tbody>
<tr>
<td>4-part division</td>
<td>Angle ply thickness $t_{\text{angle}}$, core thickness $t_c$, 16 variables</td>
</tr>
<tr>
<td>40-part division</td>
<td>Facing thickness $t_{\text{face}}$, core thickness $t_c$, 80 variables</td>
</tr>
</tbody>
</table>

Variables range:
- $0.1 \text{ mm} < t_{\text{angle}} < 8 \text{ mm}$
- $10 \text{ mm} < t_c < 50 \text{ mm}$
- $0.6 \text{ mm} < t_{\text{face}} < 5 \text{ mm}$
- $10 \text{ mm} < t_c < 50 \text{ mm}$

Figure 2-12 Bending moments and shear forces as the fuselage station increases

Figure 2-13 Application load cases onto the cylinder through the multi-point connection
2.5.2. Results and discussion

The calculation of objectives of the two optimizations shows that the lightest cylinder is 3521 kg with a 4-part division, while it is only 1312 kg with a 40-part division.

In the optimization of 4-part division, four sub-graphics representing the optimization history of 90° layer thickness, the 0° layer thickness, the 45° layer thickness and the core thickness are shown in Figure 2-14. The results indicate that the thickness of the 90° layer and 45° layer is the largest at the side2 part, the larger thickness of 90° layer is because the hoop strain at this part is the largest under the combine loads, and 45° layer is increased to reduce the shear strain at this part. In addition, the 0° layer thickness is the largest at the crown part, which results from the largest strain. The core thickness at the keel part is the largest since buckling is prone to occur at this part when the bending moment is the dominant. Finally, it is found that the core strain constraint is not as critical as the facing strain constraint by observing the strain levels at the final design cycle. Compared with the facing, the strain level in the core is low while the allowable strain level is much higher than the composite facing, as shown in Equation(2.22).

Figure 2-14 History of four kinds of variables in the 4-part optimization: (a) layer thickness of 90°; (b) layer thickness of 0°; (c) layer thickness of 45°; (d) core thickness

The distribution of skin thickness and core thickness in the 40-part optimization along the fuselage station (the axial direction) is shown in Figure 2-15. It can be found that the core thickness has an overall increase as the fuselage station increases, because the
Mechanical and vibro-acoustic aspects of composite sandwich cylinders

buckling first occurs at the position where the loads are applied (fuselage station=10), and the core thickness increases in order to improve the stability. In contrast, the facing thickness has an overall decrease as the fuselage station increases. This is due to the higher strain level at the restrained position (fuselage station=0).

Figure 2-15 Core thickness (a) and facing thickness (b) distribution along the fuselage station

2.6. Conclusions

The sizing of sandwich cylinders was conducted considering global buckling and materials failure. The predicted buckling loads obtained from numerical FE models were verified using the analytical results. Using FE models, the differences in structural efficiency between stiffened cylinders and sandwich cylinders are compared, and more detailed optimizations are conducted for the sandwich cylinder. The following conclusions can be drawn from the research:

The optimal stiffened cylinder with frames and stringers is slightly lighter than the optimal sandwich cylinder with frames. The frame pitch plays the most important role in the stability of both sandwich and stiffened cylinders. It was found that the optimal frame pitch was about 0.5 m for stiffened cylinders and about 1 m for sandwich cylinders. The core thickness does not have a significant influence on the final weight for the sandwich cylinder. One reason is that the core thickness was assumed to be uniform overall the cylinder in the optimization, and an increase of core thickness caused excessive weight increases for the low-stress regions.

It was found for the 4-part division sizing that the 90° layer and 45° layer dominated in lamination layup at the side panels, and 0° layer thickness dominated at the crown part, while the core thickness was the largest at the keel part. In addition, it was found that the core material failure was not as critical as the facing materials for sizing, because its strain level was much lower than its allowable strain. The strain level in the facing rather than the global buckling was the critical constraint for the sizing. The division along the axial direction has lead to a significant decrease of the total weight of the sandwich
Chapter 2 Sound transmission loss prediction of composite cylinders

cylinder: the cylinder weight in the 4-part division model was 3521 kg, while it was only 1312 kg in the 40-part division model.

To summarize, the composite sandwich cylinder showed equivalent structural efficiency with the stiffened cylinder in terms of global stability and material failure, and thus it could be a good option for the application as aircraft fuselages.

2.7. References

Chapter 3  Sound transmission loss prediction of composite cylinders

Taking the fuselage of an aircraft as a composite sandwich cylinder with infinite length, analytical models for prediction of sound transmission loss are introduced in this chapter. In addition, a statistical energy analysis (SEA) model was built to predict sound transmission loss (TL) for a finite-length cylinder in a broad frequency range. The TL results of the analytical models and the SEA model were compared. Finally, the influences of different parameters on the TL of cylinders were studied.

3.1. Introduction

Acoustic transmission insulation is one of the principal design drivers for composite aircraft fuselages [1]. Therefore being able to predict sound transmission loss (TL) for a fuselage structure is very important. There are two major sources of noise when an airplane is in flight: turbulent boundary layer noise and the engine noise. Engine noise can further be divided into airborne noise and structure borne noise. As a theoretical study, the fuselage is usually idealized as a cylinder and its TL is studied under a diffuse acoustic field.

3.1.1. Review of analytical models of sound transmission

A large number of investigations have been devoted to predicting fuselage TL under airborne noises. The published models mainly consider three classical types of structures: monolithic, stiffened and sandwich.

Koval [2] studied the TL of infinite monolithic cylinders taking into account the external airflow and internal shell pressurization. The impedance of the shell and its content were used to determine the TL. He also developed a theory to investigate the TL of orthotropic and laminated composite shells [3-4]. The analytical method developed by Koval is based on the thin shell theory and thus it is not suitable for thick structures. With respect to the stiffened structures, the space harmonic expansion method developed by Mead et al. [5] is usually used to model the effects of periodic stiffeners. Sound transmission loss of an infinite long cylinder subjected to a plane wave incidence was developed by Lee and Kim [6]. A program to predict noise transmission in a stiffened aircraft fuselage was developed in [7]. The properties and dimensions of stringers and frames can be easily varied and the effects of noise treatment layers are also considered in the program.

With respect to sandwich structures, Dym and Lang [8-11] worked extensively on sound transmission through sandwich panels with isotropic and orthotropic cores and
they introduced the concepts of symmetric (dilatational) and antisymmetric motions of sandwich panels. The total displacement of the skins and core can be written in terms of the symmetric and antisymmetric motions. Tang [12] developed an analytical method to predict sound transmission of sandwich fuselage. The shear and rotation effects were taken into account using the first order shear deformation theory. Results show that the shear waves of sandwich cylinders transmit sound through the shell, resulting in a decrease of TL at high frequencies. Following the works of Koval and Tang, Daneshjou et al. [13-14] generated two more explicit analytical models with thin shell and first order deformation theory respectively. Li [15] developed a mathematical model that can be used to study the sound transmission into a finite sandwich cylindrical structure. The end caps were assumed to be rigid and only the radial motion of the cylindrical structure excited the acoustic cavity.

All the above mentioned analytical methods for predicting TL of different types of structures are developed based on certain ideal assumptions. Therefore these analytical methods cannot accurately describe the realistic structures, and numerical models have been developed for the TL prediction of more complex structures with more complicated boundary conditions.

3.1.2. Review of numerical models (SEA) of sound transmission

The transmission loss of elastic structures can be numerically estimated using finite element methods (FEM), boundary element methods (BEM) and statistical energy analysis (SEA). The FEM and BEM can model the vibroacoustic behavior of structures more accurately than the SEA method; however, they require extensive computational resources when used for large structures and or at high frequencies. For example, over half a million degrees of freedom are needed to capture accurately the modes of vibration which fall below 225 Hz for a 2 m length of the fuselage of the Fokker F27 aircraft [16]. Moreover, at high frequencies, the wavelength of the structural modes is much smaller than the overall dimensions of the structure. It is generally recognized that the FE model is not suitable because excessive number of degrees of freedom are required to capture the short wavelength deformation. It is for this reason that the SEA method has been widely adopted to predict sound transmission for large structures or normal structures at high frequencies. As only the SEA model is used for the parametric study discussed in this chapter, to facilitate reading, the FEM/BEM models will not be discussed further here, they will be introduced in detail in Chapter 5.

As the name suggests, SEA involves a statistical analysis of the vibration energy that stored in fluids or structures. The primary variable of interest in a SEA analysis is energy. Other parameters, such as average vibration and sound pressure levels, are obtained from the energy. SEA does not attempt to predict the detailed spatial pattern of the response of a structure at every single frequency of excitation. Rather the method predicts the average response in three senses:
Chapter 3 Sound transmission loss prediction of composite cylinders

- Ensemble averaging
- Time averaging or Frequency band averaging
- Spacing averaging

In SEA, the structure can be divided into different subsystems. For example, an aircraft fuselage consists of crown panels, side panels and keel panels, windows, and frames subsystems. Each subsystem can also be represented by three parts: out-of-plane motion, in-plane stretching and shear motion, in other words, the wave motion of a given subsystem can consist of flexural, extensional, and shear modes. There are some guidelines as to how the subsystems should be selected [16]:

1. Each subsystem should contain a minimum number of modes for any particular band. The exact value of the minimum number is the subject of some debate, but the general view is that it lies somewhere between three and seven.

2. Each mode should contribute more or less equally to the energy of the subsystem, so that no single mode or small groups of modes dominate the response. i.e., it is sometimes advantageous to consider the bending modes and the in-plane modes to be separate subsystems, because they are unlikely to support an equal amount of energy.

3. The subsystems should ideally be weakly coupled. This means that if only one particular subsystem is subjected to excitation, then the response of that subsystem should be significantly greater than that of any other subsystems.

Weryk [17] introduced the SEA methods in marine applications, and presented the examples about how to divide a ship structure into subsystems such as beams, rods, shells and cavities.

A power balance is derived in each analysis band for steady-state conditions. In each band, the input power to the subsystems is either dissipated within the subsystems or coupled to other subsystems. The energy is distributed among the subsystems to establish a balance in the power flow into and between subsystems. It should be noted that energy resides only in resonance modes, so that the more modes a subsystem has, the greater the capacity of the subsystem to accept and store energy [18]. In SEA, the accurate prediction of the energy of the subsystems mainly depends on accurate estimates of (1) the modal density, (2) the internal loss factor and (3) the coupling loss factor. The measurement of modal density, total loss factors, radiation loss factor and internal loss factors of sandwich panels are introduced in [19].

There are two approaches that can be used when applying the SEA methodology: the first, ‘mode approach’, is based on modeling each subsystem as resonant responses. This approach is used by most of the researchers referenced here. The second, ‘wave approach’, is based on modeling each subsystem as a superposition of waves travelling throughout the subsystem. Ghinet and Atalla [20] used this method to predict the TL of curved laminates and sandwich panels, and found a good agreement between the numerical and the experimental results. Note that the modal approach and the wave
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approach are not fundamentally different, in the sense that a mode can be considered to be a set of standing waves, and the equivalence of the two views is sometimes referred to as “wave-mode duality” [16]. Take a fuselage window as an example, from the modal viewpoint, a subsystem is the bending modes or in-plane shear modes of the fuselage window; while from the wave viewpoint, a subsystem is the bending waves, the shear waves or the longitudinal waves in the fuselage window.

An outline of the steps involved in SEA is given below [16]:

1. Specify the frequency bands for the analysis
2. Define the subsystems
3. Calculate the subsystem properties loss factor: \( \eta_i \), modal density: \( n_i(\omega) \), and the coupling loss factors: \( \eta_{ij} \)
4. Determine the external power input to each subsystem \( p_{i,in} \)
5. Formulate the power balance equations:
   \[
   \sum_{i=1}^{N} \omega \eta_i n_i \frac{E_i}{n_i} - \frac{E_j}{n_j} = 0
   \]
6. Solve the equations to obtain the average energy \( E_i \) within each subsystem
7. Convert average subsystem energies into desired response quantities

Some commercial software such as AutoSEA® and VA one® is used successfully to deal with the SEA computation of complex structures. Rajesh Arjunan [21], for example, used VA one® to performed a parametric analysis of curved honeycomb composites panels in his master thesis. Zhuang Li [22] used AutoSEA® to predict the TL in sandwich composites, using a two-room method experiment to verify the modeling results. The predicted and measured results show good agreement. Besides the commercial software, Johansson and Connell [23] developed a free and open source SEA code (named SEAlab) in their master study, which was implemented in Matlab. The TL prediction using SEAlab of three structures, plate-plate, single wall and double wall, were validated using AutoSEA®.

Although the SEA method is efficient for solving acoustic problems of complex structures or simple structures at high frequencies, it is difficult to understand from in a physical perspective. Thus a comparison study between the analytical method and the SEA is given in this chapter. In addition, the parameters that may influence the sound insulation of a sandwich cylinder are still not clear yet, therefore, a parametric study on the TL of sandwich cylinders is presented as well.

3.2. Analytical model without consideration of shear deformation

As shown in Figure 3-1, it is assumed that a plane wave with an incident angle \( \gamma \) hits the cylinders. Because of the cylindrical structure, all analyses are conducted in cylindrical coordinates with \( z, \varphi, r \) denoting the axial, circumferential and radial
direction respectively. \( c_1, c_2 \) represent the sound speed in the cabin air and the ambient air, and \( \rho_1, \rho_2 \) are the densities of the cabin air and the ambient air respectively.

\[
\begin{align*}
\text{Figure 3-1 Schematic diagram of incident acoustic plane wave onto the fuselage}
\end{align*}
\]

In this section, the thin shell theory is used to build a mathematical model based on the following assumptions:

1. The ratio of the shell thickness to the cylinder radius is \( << 1 \);
2. The effect of rotational inertia is negligible;
3. The layer deformation complies with the Kirchhoff hypothesis;
4. There is no relative displacement at the interface of layers;

3.2.1. The sound transmission loss

The TL of the cylindrical shell is defined as the ratio of the transmitted power \( W^T \) and the incident power \( W^I \) per unit length of the cylinder:

\[
TL = 10 \log \left( \frac{W^I}{W^T} \right) \tag{3.1}
\]

\[
W^I = \frac{\cos(\gamma) P_0^2}{\rho_1 c_1} R \tag{3.2}
\]

\[
W^T = \frac{1}{2} \text{Re} \left[ \int_0^{2\pi} P_2^T r \frac{\partial}{\partial t}(w^*) r \, d\varphi \right] \tag{3.3}
\]

where \( \gamma \) is incident angle of the acoustic wave, \( P_0 \) is the amplitude of the incident wave(constant value), \( R \) is the radius of the cylinder, \( \rho_1 \) and \( c_1 \) are density and sound speed of the external air. \( P_2^T \) is the transmitted pressure as a function of time and frequency, \( \varphi \) the angle around the cross section, \( w \) the shell displacement in radial direction. The superscript * denotes the complex conjugate of the argument.
As can be seen in Equation (3.3), if $P_2^T$ and $w$ are known, the TL can be solved. Thus solutions of the two parameters will be discussed step by step below.

3.2.2. The governing equation of sound waves

The governing equations of sound waves are dissimilar for the inside and outside of cylinders. These governing equations lead to the expression for the acoustic pressure and the shell displacements.

**Inner cavity**

For the internal cavity of the cylinder, it is supposed that the shell interior is totally absorptive and we assume that only an inward traveling wave exists. The acoustic pressure of the inner cavity satisfies [24]:

$$c_1^2 \nabla^2 P_2^T - \frac{\partial^2}{\partial t^2} P_2^T = 0$$  \hspace{1cm} (3.4)

where, $\nabla^2 = \frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial}{\partial r} \right) + \frac{1}{r^2 \sin \phi} \frac{\partial^2}{\partial \phi^2} + \frac{\partial^2}{\partial z^2}$, is the Laplace operator expressed in cylindrical coordinates, and $c_2$ is the sound speed in the cylinder.

**Outer cavity**

As the airflow outside the cylinder is a combination of the incident wave and the reflected wave, the outflow of the cylinder (the external flow and the reflected sound wave) must satisfy the convected wave equation [24]:

$$c_1^2 \nabla^2 (P_1^I + P_1^R) - \left( \frac{\partial}{\partial t} + \mathbf{V} \cdot \nabla \right)^2 (P_1^I + P_1^R) = 0$$  \hspace{1cm} (3.5)

where $\mathbf{V}$ is the velocity of the external airflow, where $\mathbf{V} = \langle V_x, V_y, V_z \rangle$. In this case, because the external airflow propagates in axial direction, $V_x = 0, V_y = 0$. $P_1^I$ is the pressure of the incident wave and $P_1^R$ is the pressure of the reflected wave.

3.2.3. Equations of motion

The equations of motion are given here for expressing the shell displacements as a function of time [25]:

$$\partial_z N_z + \frac{1}{R} \partial_\phi N_{z \phi} - m \partial_{tz} u = q_z$$  \hspace{1cm} (3.6)

$$\frac{1}{R} \partial_\phi N_\phi + \partial_z N_{z \phi} + \frac{1}{R} \partial_\phi M_\phi + \frac{1}{R} \partial_z M_{z \phi} - m \partial_{tz} v = q_\phi$$  \hspace{1cm} (3.7)

$$-\frac{N_\phi}{R} + \partial_{zz} M_z + \frac{2}{R} \partial_{z \phi} M_{z \phi} + \frac{1}{R^2} \partial_\phi M_\phi - m \partial_{tz} w = q_r$$  \hspace{1cm} (3.8)
where \( q_z, q_{\phi}, q_r \) are the external forces per unit area in the axial, circumferential and radial directions respectively. \( N \) and \( M \) are the forces and moment resultants. \( u, v, w \) are the shell displacements of the neutral face in the axial, circumferential and radial directions respectively, \( m = \rho h \), is the surface density of the cylinder.

It is assumed that the pressure is only varied in the radial direction, \( q_{\phi} = q_z = 0 \), \( q_r = P_1^I + P_1^R - P_3^T \). The expression for the incident wave pressure \( P_1^I \), the reflected wave pressure \( P_1^R \) and the transmitted wave pressure \( P_3^T \) will be introduced later.

The forces \( (N) \) and moments \( (M) \) can be calculated by:

\[
\begin{bmatrix}
N_z \\
N_{\phi} \\
N_{z\phi} \\
M_z \\
M_{\phi} \\
M_{z\phi}
\end{bmatrix}
= 
\begin{bmatrix}
A & B \\
B & D
\end{bmatrix}
\begin{bmatrix}
\varepsilon_z \\
\varepsilon_{\phi} \\
\varepsilon_{z\phi} \\
k_z \\
k_{\phi} \\
k_{z\phi}
\end{bmatrix}
\tag{3.9}
\]

where \( A, B, D \) are the stiffness matrix, the expressions of which can be seen in Appendix B. The strains and curvatures for Equation(3.9) can be written as [25]:

\[
\varepsilon_z = \partial_z u; \varepsilon_{\phi} = \frac{1}{R}(\partial_{\phi} v + w); \varepsilon_{z\phi} = \partial_z v + \frac{1}{R}\partial_{\phi} u;
\tag{3.10}
\]

\[
k_z = -\partial_z w; k_{\phi} = \frac{1}{R^2}(\partial_{\phi} v - \partial_{\phi,\phi} w); k_{z\phi} = \frac{1}{R}\partial_z v - \frac{2}{R}\partial_{\phi,\phi} w
\tag{3.11}
\]

Substitution of Equation(3.9) into Equations(3.6) to (3.8) leads to the expressions for the shell displacements \(<u, v, w>\) as a function of time \( t \).

3.2.4. Boundary conditions at the fluid–structure interfaces

At the interface between the internal and external shell surfaces and the air, the acoustic speed in the radial direction have to be equal to the normal velocity of the shell [13]. Applying Newton’s second law, the equations which describe the effect of acoustic pressure on the motion of the shell can be derived as:

\[
\left. \frac{\partial}{\partial r} \left( P_1^I + P_1^R \right) \right|_{r=R} = -\rho_1 \left( \frac{\partial}{\partial t} + v_z \frac{\partial}{\partial z} \right)^2 w
\tag{3.12}
\]

\[
\left. \frac{\partial}{\partial r} P_2^I \right|_{r=R} = -\rho_2 \frac{\partial^2}{\partial t^2} w
\tag{3.13}
\]

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3.2.5. Solutions

To obtain the solution of Equations (3.4) and (3.5), the incident wave $P_1^I$ can be expressed as

$$P_1^I = P_0 \sum_{n=0}^{\infty} \xi_n (-i)^n J(n, r k_{1r}) e^{-i(ax - k_{1z}z - n\phi)}$$

(3.14)

and the reflected wave $P_1^R$:

$$P_1^R = \sum_{n=0}^{\infty} A_n H_2(n, r k_{1r}) e^{i(ax - k_{1z}z - n\phi)}$$

(3.15)

where $P_0$ is the amplitude of the incident wave, $n$ is the order of the circumferential mode, $\omega = 2\pi f$ is the frequency and $A_n$ is the unknown constant. $J$ is the Bessel function of the first kind of integer order $n$ ($n = 0, 1, 2, 3\ldots$). $H_2$ is the Hankel function of the second kind with integer $n$, which represents the outgoing wave from the cylinder [13].

$\xi_n$ is the Neumann factor, which is given by:

$$\xi_n = \begin{cases} 1 & n = 0 \\ 2 & n \geq 1 \end{cases}$$

(3.16)

and

$$k_{1z} = k_1 \cos(\gamma); \quad k_{1r} = k_1 \sin(\gamma)$$

(3.17)

$k_1$ is the wavenumber for the region outside the cylinder and $\gamma$ is the incident wave angle, as shown in Figure 3-1. Substitution of Equations (3.14) to (3.17) into Equation (3.5) gives the following expression for $k_1$:

$$k_1 = \frac{\omega}{c_1} \left( \frac{1}{1 + M_1 \cos(\gamma)} \right)$$

(3.18)

where $M_1 = \nu_2 / c_1$ is the Mach number of the external flow. The transmitted wave $P_2^T$ is:

$$P_2^T = \sum_{n=0}^{\infty} B_n H_1(n, r k_{2r}) e^{i(ax - k_{2z}z - n\phi)}$$

(3.19)

where $B_n$ is a temporally unknown complex amplitude factor; $k_{2r}$ and $k_{2z}$ are the wavenumbers in the radial and circumstantial direction respectively. $H_1$, representing the incoming wave for the cylinder, is the Hankel function of the first kind with integer $n$.

The travelling waves in the cylinder and outside the cylinder are both driven by the incident-travelling wave, and because of this the wavenumber in the $z$ direction should match throughout the system, therefore $k_{1z} = k_{2z}$. With:
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\[ k_2^2 = k_{2z}^2 + k_{2r}^2; k_2 = \omega / c_2 \]  
(3.20)

\[ k_{2r} \text{ can be described as:} \]
\[ k_{2r} = \sqrt{k_2^2 - k_{1z}^2} \]  
(3.21)

The displacements can now be expressed by:

\[ u = i \sum_{n=0}^{\infty} U_n e^{i(\alpha - k_{1z}z - np)} \]  
(3.22)

\[ v = i \sum_{n=0}^{\infty} V_n e^{i(\alpha - k_{1z}z - np)} \]  
(3.23)

\[ w = \sum_{n=0}^{\infty} W_n e^{i(\alpha - k_{1z}z - np)} \]  
(3.24)

where \( U_n, V_n, W_n \) are the unknown parameters which will be determined in the following section; they are dependent on the frequency \( \omega \) and the order of the circumferential mode \( n \).

3.2.6. Results

Collecting the equations of motion, Equations(3.6) to (3.8), and boundary condition, Equations(3.12) and (3.13), a system of five equations consisting of \( P_0, U_n, V_n, W_n, A_n \) and \( B_n \) is obtained. Finally the solutions for \( <U_n, V_n, W_n, A_n, B_n> \) vector are expressed by \( P_0 \) which can be eliminated later.

Substitution of Equations(3.19) and (3.24) into Equation(3.3) results in:

\[ W^T = \sum_{n=0}^{\infty} \frac{\pi R}{\xi_n} \text{Re}[B_n H_1(n, k_2, R)(i\omega W_n)^{*}] \]  
(3.25)

Finally, the transmission coefficient \( \tau \) can be predicted by substituting Equations(3.2) and (3.25) into Equation(3.1):

\[ \tau(\gamma) = \frac{W^T}{W^T} = \sum_{n=0}^{\infty} \frac{\text{Re}[B_n H_1(n, k_2, R)(i\omega W_n)^{*}]\rho_1 c_1 \pi}{\xi_n \cos(\gamma) P_0^2} \]  
(3.26)

\[ TL = -10 \log[\tau(\gamma)] \]  
(3.27)

where \( \tau(\gamma) \) is the transmission coefficient for a particular incident angle \( \gamma \). For the diffuse sound field (the wave hits the cylinder in all directions with equal sound intensity), the average sound transmission coefficient, \( \tau_{av} \), is given as:

\[ \tau_{av} = \int_0^{\pi/2} \sin(2\gamma)d\gamma \]  
(3.28)
\[ TL_{\text{av}} = -10 \log(\tau_{\text{av}}) \] (3.29)

3.3. Analytical model with consideration of shear deformation

As the shell thickness increases, the transverse shear deformation cannot be neglected anymore. Thus a refined analytical model considering the transverse shear deformation is discussed. In this analytical model, the first order shear deformation theory is used to describe the displacement fields.

3.3.1. Displacement-strain relationship

In the first order shear deformation theory, it is assumed that the displacement field at an arbitrary layer is the function of its distance from the middle plane. In the cylindrical coordinate, the strains and curvatures can be expressed as:

\[
\begin{align*}
\epsilon_{0z} &= \partial_z u_0; \epsilon_{0\phi} = (\partial_\phi v_0 + w_0) / R; \epsilon_{0z\phi} = \partial_z v_0; \epsilon_{0\phi\phi} = \partial_\phi u_0 / R \\
\kappa_z &= \partial_z \psi_1; \kappa_\phi = \partial_\phi \psi_2 / R; \kappa_{z\phi} = \partial_z \psi_2; \kappa_{\phi\phi} = \partial_\phi \psi_1 / R \\
\gamma_{0z} &= \partial_z w_0 + \psi_1; \gamma_{0\phi} = \partial_\phi w_0 / R - v_0 / R + \psi_2
\end{align*}
\] (3.30)

where \( \psi_1 \) and \( \psi_2 \) denote the rotational angle around the \( z \) and \( \phi \) directions respectively, which can be expressed as:

\[
\psi_1 = i \sum_{n=0}^{\infty} \psi_{1n} e^{i(nx-k_1z-n\phi)}
\] (3.31)

\[
\psi_2 = i \sum_{n=0}^{\infty} \psi_{2n} e^{i(nx-k_1z-n\phi)}
\] (3.32)

The force and the moment acting on the cylinder can be expressed as:

\[
\begin{bmatrix}
N_z \\
N_\phi \\
N_{z\phi} \\
N_{\phi z} \\
M_z \\
M_\phi \\
M_{z\phi} \\
M_{\phi z}
\end{bmatrix} =
\begin{bmatrix}
\bar{A}_{11} & A_{12} & \bar{A}_{16} & A_{16} & \bar{B}_{11} & B_{12} & \bar{B}_{16} & B_{16} \\
A_{12} & \bar{A}_{22} & A_{26} & \bar{A}_{26} & B_{12} & \hat{B}_{22} & B_{26} & \hat{B}_{26} \\
\bar{A}_{16} & A_{26} & \bar{A}_{66} & A_{66} & \bar{B}_{16} & B_{26} & \bar{B}_{66} & B_{66} \\
A_{16} & \hat{A}_{26} & A_{66} & \hat{A}_{66} & B_{16} & \hat{B}_{26} & B_{66} & \hat{B}_{66} \\
\bar{B}_{11} & B_{12} & \bar{B}_{16} & B_{16} & \bar{D}_{11} & D_{12} & \bar{D}_{16} & D_{16} \\
B_{12} & \hat{B}_{22} & B_{26} & \hat{B}_{26} & D_{12} & \hat{D}_{22} & D_{26} & \hat{D}_{26} \\
\bar{B}_{16} & B_{26} & \bar{B}_{66} & B_{66} & \bar{D}_{16} & D_{26} & \bar{D}_{66} & D_{66} \\
B_{16} & \hat{B}_{26} & B_{66} & \hat{B}_{66} & D_{16} & \hat{D}_{26} & D_{66} & \hat{D}_{66}
\end{bmatrix} \begin{bmatrix}
\epsilon_{0z} \\
\epsilon_{0\phi} \\
\epsilon_{0z\phi} \\
\epsilon_{0\phi\phi} \\
\kappa_z \\
\kappa_\phi \\
\kappa_{z\phi} \\
\kappa_{\phi\phi}
\end{bmatrix}
\] (3.33)

The transverse shear force can be expressed as:
\[
\begin{bmatrix}
Q_z \\
Q_\phi
\end{bmatrix}
= 
\begin{bmatrix}
A_{55} & A_{45} \\
A_{45} & A_{44}
\end{bmatrix}
\begin{bmatrix}
\gamma_{0z} \\
\gamma_{0\phi}
\end{bmatrix}
\]

(3.34)

The formulations for \( A_{ij} \), \( B_{ij} \), \( D_{ij} \) can be found in Appendix B.

### 3.3.2. Equations of motion

Building on the equations of motion in the previous section, the transverse shear force \( Q_z \) and \( Q_\phi \) are introduced:

\[
\frac{1}{R}\partial_z N_z + \frac{1}{R}\partial_\phi N_{z\phi} - I_1\partial_{t,t}u_0 - I_2\partial_{t,t}\psi_1 = q_z
\]

(3.35)

\[
\frac{1}{R}\partial_\phi N_{\phi} + \partial_z N_{z\phi} + \frac{Q_\phi}{R} - I_1\partial_{t,t}v_0 - I_2\partial_{t,t}\psi_2 = q_\phi
\]

(3.36)

\[
-\frac{N_\phi}{R} + \partial_z Q_z + \frac{1}{R}\partial_\phi Q_\phi - I_1\partial_{t,t}w_0 = q_r
\]

(3.37)

\[
\frac{1}{R}\partial_\phi M_\phi + \partial_z N_{z\phi} - Q_\phi - I_2\partial_{t,t}v_0 - I_3\partial_{t,t}\psi_2 = 0
\]

(3.38)

\[
\frac{1}{R}\partial_\phi M_\phi + \partial_z N_{z\phi} - Q_\phi - I_2\partial_{t,t}v_0 - I_3\partial_{t,t}\psi_2 = 0
\]

(3.39)

where \( R \) denotes the radius of the cylinder, \( t \) represents the time. \( q_z \), \( q_\phi \) and \( q_r \) are the forces acting on the cylinder in three directions. Here \( q_z = q_\phi = 0 \), \( q_r = P_1^1+P_1^R-P_3^T \), and

\[
I_i = (I_i + \frac{I_{i+1}}{R}), \; i=1,2,3; \quad I_i = \sum_{k=1}^{N} \int_{h_{i-1}}^{h_i} \rho_k h^{-i} \; dh, \; i=1,2,3,4
\]

(4.40)

where \( I_i \) is the mass inertia term and \( \rho_k \) is the mass density of the \( k \)-th layer.

Finally, combing the five equations of motion (3.35-3.39) and two boundary condition Equations (3.12) and (3.13), the solutions of seven unknown parameters \( U_{in}, V_{in}, W_{in}, A_{in}, B_{in}, \psi_{1n} \) and \( \psi_{2n} \) can be obtained. Using the solved \( W_n \) and \( B_n \), the transmission coefficient \( \tau \) can be found according to Equation(3.26), and the TL in the diffuse acoustic field can be predicted using Equation(3.29).

The derivations of the two analytical methods were performed using Mathematica 7.0, and the programs are shown in Appendix G.

### 3.4. SEA model

As shown in Figure 3-2, the SEA method consists of the source SEA acoustic cavity (outer cavity), the junction area and the receiving SEA acoustic cavity (inner cavity).
Figure 3-2 SEA model consists acoustic cavities and junction

The TL is calculated using the following equation [26]:

$$ TL = 10 \log_{10} \left[ \frac{A \omega}{8 \pi^2 n_1 c_1^2 \eta_2} \left( \frac{E_1}{E_2} \cdot \frac{n_1}{n_2} \right) \right] $$

(3.41)

$$ E_i = \begin{cases} m_i \langle v_i^2 \rangle & \text{structural} \\ V_i \langle p_i^2 \rangle / \rho c_i^2 & \text{acoustic} \end{cases} $$

(3.42)

where $A$ is the junction area, $E_1$ is the energy of the source cavity, and $E_2$ is the energy of the receiving cavity. $n_1$ and $n_2$ are the mode density in source and receiving cavities respectively. $c_1$ is the acoustic wave velocity in the source cavity. $\eta_2$ is the loss factor in the receiving cavity. Equation (3.42) shows the relationship between the subsystem energy $E_i$ and the spatial averaged velocity $\langle v_i \rangle$ (for structural) or pressure $\langle p_i \rangle$ (for acoustic). $m_i$ is the mass of the structural subsystem, and $V_i$ is the volume of the acoustic cavity. In the following computation, $\eta_2 = 0.01$, $c_1 = 343$ m/s.

High radiation efficiency is used to denote the capability of a structure to radiate maximum acoustic energy. The radiation efficiency $\sigma_{\text{rad}}$ can be calculated by the expression:

$$ \sigma_{\text{rad}} = \frac{P_{\text{rad}}}{\rho c A v^2} $$

(3.43)

where, $P_{\text{rad}}$ represents power radiated into the cavity from the panels. $\rho$ denotes the density of fluid, $c$ the speed of sound in the fluid, $A$ the area of radiating face and $v$ the Root Mean Square (RMS) velocity of the radiating face.
3.5. Comparison of the results computed by different methods

The TL results of two types of cylinders, a thin laminated cylinder and a composite sandwich cylinder, were predicted using different methods. The TL results were compared in the frequency range of 100-10000 Hz.

In the analytical models, the radius for the cylinder was 2 m and the length was infinite for the laminated and the sandwich cylinder. The sandwich cylinder had the same facing materials with the laminated cylinder, i.e. the Carbon/Epoxy. The core of the sandwich cylinder was made of Rohacell 200. The layup of the laminated cylinder was [0/90/45/-45/0], the ply thickness was 0.159 mm. The layup of the sandwich cylinder was [0/90/45/-45/0/core/0/-45/45/90/0], the ply thickness was also 0.159 mm and the core thickness is 10 mm. $c_1 = c_2 = 343 \text{ m/s}, \rho_1 = \rho_2 = 1.29 \text{kg/m}^3$. The materials properties is shown in Table 3-1.

In the SEA model, the cylinder length was 10m and the radius was 2 m. The diffuse acoustic field was applied on the outer cavity. The material properties and composites layup of the cylinders were the same as those used in the analytical model.

<table>
<thead>
<tr>
<th>Table 3-1 The properties of the unidirectional Carbon/Epoxy materials and Rohacell 200 foam</th>
</tr>
</thead>
<tbody>
<tr>
<td>Material</td>
</tr>
<tr>
<td>Carbon/Epoxy</td>
</tr>
<tr>
<td>Rohacell 200</td>
</tr>
</tbody>
</table>

3.5.1. Comparison of TL of the laminated cylinder

TL results of a laminated cylinder predicted by the analytical, the SEA method and the hybrid FE&SEA are shown in Figure 3-3. For the hybrid model, the FE method was applied in the low frequency range (100~800 Hz) and the SEA method was used for middle and high frequencies. For the analytical method, Equations (3.28) and (3.29) were used for the analytical method to obtain the TL of the diffuse acoustic field by averaging the TL of different incident angles. It was assumed that the external acoustic speed was zero for these comparisons.
As can be seen from Figure 3-3, there are two characteristic frequencies for the cylindrical shells. The first one corresponds to the ring frequency $f_R$ and the second to the coincidence frequency $f_c$. $f_R$ is the special characteristic for sound transmission of the cylinder. At $f_R$, the longitudinal wavelength in the cylindrical shell is equal to the circumference wavelength and an axisymmetric resonance occurs [24]. The dip of TL nearby $f_R$ is caused by the interaction of bending forces and membrane forces in the curved shell, which lead to the convergence of resonance. The convergence not only increases the modal density of the curved panel around the ring frequency, but also increases the sound radiation efficiency of these modes by shifting them to a relatively higher frequency [27]. $f_c$ is an universal characteristic for sound transmission of all kinds of structures. At $f_c$, the wavelength of the forced flexural wave in the fuselage structure equals the wavelength of the bending acoustic wave. When this takes place, the intensity of the transmitted wave approaches the intensity of the incident wave. The bending waves become "acoustically fast" and the panel radiates sound from its whole area instead of just from its edges or corners [28-29]. Radiation efficiency approaches a value of 1 and there is a dip for the TL curve. The equations used to estimate $f_R$ and $f_c$ of the infinite orthotropic cylindrical shells are [30]:

$$f_R = \frac{1}{2\pi R} \sqrt{\frac{E_o \alpha}{\rho(\alpha - \mu^2)}}$$

(3.44)

$$f_c = \frac{c^2}{2\pi} \sqrt{\frac{M}{D}}$$

(3.45)

where $\alpha = E_z/E_\phi$, $E_z$ and $E_\phi$ are the composites modulus of elasticity, $D$ is the bending stiffness matrix in the hoop direction, $c$ is the sound speed through the cylinder, and $M$ is
the mass per area. According to Equations (3.44) and (3.45), \( f_R = 541 \text{Hz} \) and \( f_c = 8026 \text{Hz} \) for the laminated cylinder, which show good agreements with the results shown in the Figure 3-3. Three frequency ranges can be classified using \( f_R \) and \( f_c \) with respect to the sound transmission mechanisms, as shown in Table 3-2.

![Modal density of the cylindrical structural used for SEA computation](image)

**Figure 3-4 Modal density of the cylindrical structural used for SEA computation**

With respect to the comparison among the TLs of different methods in Figure 3-3, at lower frequencies, the TL predicted by SEA is higher than that predicted by the analytical method and the FE method. The FE method has been proven that it can predict the sound transmission accurately at lower frequencies (see Chapter 5). The low accuracy of the SEA method is due to the low modal density at lower frequencies, as shown in Figure 3-4. However, the four curves start to show good agreement as the frequency increases. They show consistent TL values at the mass-controlled and damping controlled area, and the positions of \( f_R \) and \( f_c \) are nearly the same.

**Table 3-2 Different frequency regions for sound transmission**

<table>
<thead>
<tr>
<th>Stiffness control ((f &lt; f_c))</th>
<th>Stiffness dominates the vibration and sound transmission</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mass control ((f_c &lt; f &lt; f_c))</td>
<td>Mass dominates, the mass law can be applied in this region</td>
</tr>
<tr>
<td>Damping control ((f &gt; f_c))</td>
<td>Damping makes a significant influence in this region.</td>
</tr>
</tbody>
</table>

### 3.5.2. Comparison of TL of the sandwich cylinder

The TL of the sandwich cylinder predicted by CLT, FSTD and SEA methods are compared in Figure 3-5. The three methods are in good agreement at 200-8000 Hz. The prediction of the CLT and the FSTD are consistent below 200 Hz. The predicted TL using the FSTD differs from that using the CTL above 8000 Hz. Compared with Figure 3-3, it can be found that difference is more significant for the thick sandwich structures than that for the thin laminated shell. In other words, the shear deformation in the thickness direction cannot be neglected for relatively thick structures. Moreover, the difference between the analytical methods (CLT&FSTD) and the SEA method indicates
that some experimental tests need to be done to discern which method can make a better prediction.

![Figure 3-5 TL of a sandwich cylinder predicted by CTL, FSTD, and SEA](image)

3.6. Parametric study on composite cylinders at middle and high frequencies

The theoretical comparison between two analytical methods and the SEA method show that they show good agreement to predict the TL at middle and high frequencies (1000-8000 Hz). Because of its high computation efficiency, the SEA method was used in the following section to study the influence parameters on the TL of composites cylinders. The parameters include: fiber orientation, facing materials, cylinder radius and length, core thickness and sandwich layup.

3.6.1. Effect of the fiber orientation

One of the advantages of composite materials over isotropic materials is they can be structural tailored. In order to evaluate the effect the layup has on sound transmission loss, four different arrangements were chosen, as shown in Table 3-3. In the comparison, the cylinders were composed of Graphite/Epoxy laminates with 10 layers, and each layer was 0.159 mm thick. The structural damping loss factor was given as 1% and the properties of Graphite/Epoxy are shown in Table 3-4.
Table 3-3 Mechanical properties of different layups

<table>
<thead>
<tr>
<th>Layup</th>
<th>$E_z$ [Pa]</th>
<th>$E_\phi$ [Pa]</th>
<th>B</th>
<th>$D_{11}$ [Pa]</th>
</tr>
</thead>
<tbody>
<tr>
<td>$[90]_{10}$</td>
<td>1.25E11</td>
<td>1.01E10</td>
<td>0 0 0</td>
<td>3.4</td>
</tr>
<tr>
<td>$[0]_{10}$</td>
<td>1.01E10</td>
<td>1.25E11</td>
<td>0 0 0</td>
<td>42</td>
</tr>
<tr>
<td>$[0/90/45/-45/0]_s$</td>
<td>6.95E10</td>
<td>4.6E10</td>
<td>0 0 0</td>
<td>25</td>
</tr>
<tr>
<td>$[0/10/20/30/40/50/60/70/80/90]$</td>
<td>6.8E10</td>
<td>6.8E10</td>
<td>-4.2E4 1.1E4 9.5E3</td>
<td>25.7</td>
</tr>
</tbody>
</table>

Figure 3-6 TL of cylinders with different fiber orientations

The TL differences between the layup $[0]_{10}$ and the layup $[90]_{10}$ are the most significant among the four fiber orientations. The first difference is the position of $f_R$ and $f_c$ in the frequency domain. As shown in Figure 3-6, the curve representing $[90]_{10}$ shows the lowest $f_R$ and highest $f_c$. On the contrary, the curve representing $[0]_{10}$ shows the highest $f_R$ and the lowest $f_c$. When the fibers are all along the axial direction, which is represented by $[90]_{10}$, the extensional stiffness $E_\phi$ is the smallest and the bending stiffness $D$ is the largest. According to Equations 3.42-3.43, the $f_R$ of the cylinder with $[90]_{10}$ should be the lowest and the $f_c$ should be the highest. The second difference is that the TL of $[0]_{10}$ is larger than that of $[90]_{10}$ except in the mass-control range. In other words, the fiber layup in hoop direction shows an advantage compared to that in the axial direction in terms of sound insulation. This can be explained that the layup of $[0]_{10}$ has the highest stiffness in the hoop direction, which leads to a lower vibration amplitude in the radial direction. In addition, the layup $[0/10/20/30/40/50/60/70/80/90]$ presents a lower TL compared to the layup $[0/90/45/-45/0]_s$. This is because the layup is an
asymmetrical and its bending-extensional coupling matrix $\mathbf{B}$ is not zero, see Table 3-3, which could cause additional out-plane vibration waves from the in-plane waves.

### 3.6.2. Effect of the facing materials

The material choice is an important issue for composite fuselage design. Thus three different kinds of composites were used to evaluate their influence on the TL. To get a representative comparison, the models of the different kinds of materials had the same layup $[0]_{10}$ and the same thickness of 1.59 mm. The three different kinds of materials used in the analysis are shown in Table 3-4.

#### Table 3-4 Properties of three kinds of materials

<table>
<thead>
<tr>
<th></th>
<th>Graphite/Epoxy</th>
<th>Fiberglass/Epoxy</th>
<th>Aramid/Epoxy</th>
</tr>
</thead>
<tbody>
<tr>
<td>Density $[\text{kg/m}^3]$</td>
<td>1600</td>
<td>1900</td>
<td>1500</td>
</tr>
<tr>
<td>$E_1[\text{Pa}]$</td>
<td>1.25E11</td>
<td>5.6E10</td>
<td>7.6E10</td>
</tr>
<tr>
<td>$E_2[\text{Pa}]$</td>
<td>1E10</td>
<td>1.3E10</td>
<td>5.5E9</td>
</tr>
<tr>
<td>$\nu_{12}$</td>
<td>0.25</td>
<td>0.26</td>
<td>0.34</td>
</tr>
<tr>
<td>$G_{12}[\text{Pa}]$</td>
<td>5.9E9</td>
<td>4.2E9</td>
<td>2.3E9</td>
</tr>
</tbody>
</table>

As shown in Figure 3-7, Graphite/Epoxy has the largest TL at low frequencies and at high frequencies because of its highest elastic modulus. Below the $f_R$, sound transmission is dominated by the resonance of the structural shell and the air cavity. At the mass controlled range the curve which represents the Fiberglass/Epoxy composites has the largest TL because of its highest density. In addition, the mass controlled range of the Fiberglass/epoxy laminate is broader due to its lower $f_R$ and higher $f_c$, which are dependent on its elastic modulus and bending stiffness, see Equations (3.44) (3.45). In general, the Fiberglass/epoxy materials show the best TL in a broad frequency range (400-8000 Hz). On the other hands, its highest density is a disadvantage when the light weight structures are needed in the practical applications.
3.6.3. Effect of radius and length

The effect of the radius \( R \) and the length \( L \) on the TL of cylinders was studied. The \( R \) and \( L \) were kept varied and the other parameters like materials properties and atmospheric conditions were kept constant. Both the thin laminated cylindrical shells and the thick sandwich cylindrical shells were investigated. For the thin laminated cylinders, the materials used were Graphite/Epoxy, and the layup was \([0]_{10}\) with a thickness of 1.59 mm. For sandwich cylinders, the facing materials were the glass fiber/epoxy and the core material was the Tycor\(^\circledR\) foam. The layup was \([0/0/0/core/0/0/0]\), the thickness of each facing layer, \( t_{\text{ply}}=0.53 \text{ mm} \), and the thickness of the core thickness, \( t_c=15 \text{ mm} \).

The effects of \( L \) and \( R \) on the TL of thin cylindrical shells are shown in Figure 3-8. It can be seen that \( L \) had little influence on the TL. An increase of \( R \) lead to a decrease of the ring frequency \( f_R \), which can be explained by Equation (3.44). Moreover, the cylinder with a larger \( R \) exhibited a lower TL below the \( f_R \). Finally it is worth noting that \( R \) had no effect on the TL at high frequencies. In conclusion, both \( R \) and \( L \) have little influence on the TL above the \( f_R \).

The effects of \( L \) and \( R \) on the TL of sandwich cylinders are shown in Figure 3-9. It can be found that \( L \) did not affect the TL at all frequencies and increasing \( R \) lead to a decrease of the \( f_R \), which was similar to the finding for the thin laminated cylinders. However, it should be noted that TL dips at \( f_R \) and \( f_c \) are not as significant as those for the laminated cylinder, especially at \( f_c \). In addition, the \( f_c \) of sandwich cylinders is about 2000 Hz, which is lower than 4000 Hz of the laminated cylinders. The reason is that the M/D of the laminated cylinder is larger than that of the sandwich cylinder (see Equation 3.45).

---

**Figure 3-8 TL of thin laminated cylinders with different length (a) and with different radius (b)**

The effects of \( L \) and \( R \) on the TL of sandwich cylinders are shown in Figure 3-9. It can be found that \( L \) did not affect the TL at all frequencies and increasing \( R \) lead to a decrease of the \( f_R \), which was similar to the finding for the thin laminated cylinders. However, it should be noted that TL dips at \( f_R \) and \( f_c \) are not as significant as those for the laminated cylinder, especially at \( f_c \). In addition, the \( f_c \) of sandwich cylinders is about 2000 Hz, which is lower than 4000 Hz of the laminated cylinders. The reason is that the M/D of the laminated cylinder is larger than that of the sandwich cylinder (see Equation 3.45).
Figure 3-9 TL of sandwich cylinders with different length (a) and with different radius (b)

As the \( R \) and \( L \) have little influence on the TL of cylinders above \( f_R \), the TL investigation of the aircraft fuselage can be scaled down a smaller cylinder at high frequencies. In addition, the effects of the ring frequency on the TL of sandwich cylinders are not as significant as that of thin laminated cylinders, especially when the \( R \) is small. When the \( R \) is too small, the frequency range between \( f_R \) and \( f_c \) that is mass controlled becomes narrow or even difficult to discern.

3.6.4. Effect of the core thickness of sandwich cylinders

As shown in the introduction, core thickness plays a vital influence on sound transmission in sandwich structures. Three levels of core thickness, \( t_c=10 \text{mm}, t_c=20 \text{mm}, t_c=30 \text{mm} \), were chosen for a comparison. In order to avoid the influence of the mass influence, the facing thicknesses for the sandwich cylinder were varied with \( t_c \) to keep the total mass constant at each level of \( t_c \). The structural damping loss factors were all set as 3%, and the fluid damping as 0.1% in the three groups.

A comparison of TL results is given in Figure 3-10a. It can be seen that the sandwich cylinder with \( t_c=20 \text{ mm} \) had the highest TL above 1000 Hz. Either an increase or a decrease in the core thickness lead to a lower TL. As shown in Figure 3-10b, the radiation efficiency of the sandwich cylinder with \( t_c=20 \text{ mm} \) was higher than that of the cylinder with \( t_c=10 \text{ mm} \) at low frequencies, and it was lower than that of the cylinder with \( t_c=10 \text{ mm} \) at high frequencies. This partly explain the change in TL as the frequency increases for the two cylinders, as shown in Figure 3-10a. With respect to the cylinder with \( t_c=30 \text{ mm} \), it had the worst sound insulation at all frequencies. At low frequencies, the sandwich bending stiffness influences the sound transmission of sandwich cylinders. Decreasing the bending stiffness causes more shell vibration and thus decreases the sound insulation. Therefore, the poor sound insulation of the cylinder with \( t_c=30 \text{ mm} \) at low frequency was due to its bending stiffness being the lowest, see in Table 3-5. It should be noted that although the core thickness has been increased to 30 mm, the total facing thickness was reduced only 0.5 mm to keep the cylinder weight constant, which leads to the low bending stiffness. In other words, the sandwich bending
stiffness does not increase constantly as the core thickness increases if a constraint of constant mass is given. The poor sound insulation of the cylinder with $t_c = 30$ mm at high frequency was caused by its high radiation efficiency, see Figure 3-10b.

![Figure 3-10 Effect of the core thickness on the TL (a) and on the radiation efficiency (b)](Facing material is carbon/epoxy and core material is Rohacell 200, the facing thickness is changed to keep the weight of sandwich cylinders constant)

Bending waves play important roles on the sound transmission of sandwich structures and the bending stiffness influence the TL to a large extent. At high frequencies, the sandwich configuration with $t_c = 20$ mm has the largest bending stiffness and thus the surface velocity is lowest. Moreover, it can be seen from Figure 3-10b that a higher bending stiffness leads to higher radiation efficiency below the coincidence frequency; however, it leads to lower radiation efficiency above the coincidence frequency. In addition, the phenomenon that the configuration with $t_c = 20$ mm has the lowest $f_c$ indicates that the coincidence frequency decreases as the bending stiffness increases. Therefore a possible way for the noise control is to transfer the coincidence frequency to lower frequencies by increasing the bending stiffness.

| Table 3-5 Bending stiffness of the sandwich structures |
|---------------------------------|-----------------|-----------------|-----------------|-----------------|
| $t_c$=10mm, $t_f$=2.87mm        | $t_c$=20mm, $t_f$=1.59mm | $t_c$=30mm, $t_f$=0.5mm |
| Bending stiffness matrix $D$    | 6543            | 1324            | 53              | 12165           | 2514           | 30              | 5547            | 1123            | 1.57            |
|                                | 1324            | 4538            | 53              | 2514           | 8276           | 30              | 1123            | 3965            | 1.57            |
|                                | 53              | 53              | 1506            | 30              | 30             | 2892           | 1.57            | 1.57            | 1454            |

### 3.6.5. Effect of the sandwich layup

An asymmetric sandwich layup was compared to a symmetric sandwich layup, as shown in Figure 3-11. It was found that the asymmetric sandwich cylinder showed a higher TL than the symmetric sandwich cylinder at frequencies below 500 Hz, and a lower TL above 1050 Hz. This is due to the bending-extensional coupling of the asymmetric sandwich layup, which is similar to the phenomenon in Subsection 3.6.1.
Mechanical and vibro-acoustic aspects of composite sandwich cylinders

3.6.6. Effect of the core shear stiffness

The shear deformation of the core plays an important role in vibration energy dissipation and thus affects sound transmission loss. Therefore three sandwich cylinders with different kinds of cores were compared for their TL. The cylinders had a length of 10 m and a radius of 2 m. The layup of all the sandwich cylinders were [0/90/45/-45/0/core/0/-45/45/90/0]. The layer thickness of the facing was 0.159 mm and the core thickness was 20 mm. The properties of the three kinds of cores are shown in Table 3-6.

Table 3-6 Properties of three different cores

<table>
<thead>
<tr>
<th>Core name</th>
<th>Type</th>
<th>Density[kg.m⁻³]</th>
<th>$E_c$[MPa]</th>
<th>$G_{L}$[MPa]</th>
<th>$G_{W}$[MPa]</th>
<th>$G_{12}$[MPa]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Aluminum honeycomb</td>
<td>3/16 – 5052 –</td>
<td>70</td>
<td>999.7</td>
<td>468.8</td>
<td>206.8</td>
<td>10</td>
</tr>
<tr>
<td>Tycor® foam</td>
<td>Glass fibers/PVC</td>
<td>70</td>
<td>151</td>
<td>68.7</td>
<td>68.7</td>
<td>68.7</td>
</tr>
<tr>
<td>Rohacell® foam</td>
<td>71A/ PMI</td>
<td>75</td>
<td>92</td>
<td>29</td>
<td>29</td>
<td>29</td>
</tr>
</tbody>
</table>

The TL results for the three kinds of structures are shown in Figure 3-12a. It can be seen that below the coincidence frequency (about 700 Hz), the Tycor® sandwich had the highest TL. The shear deflection of the sandwich decreased as the $G_c$ increased, however, the radiation efficiency also increased as the $G_c$ increased, see Figure 3-12b. At this frequency range ($<f_c$), the Tycor® sandwich achieved the best balance between structural deflection and radiation efficiency of cylindrical shells, and thus presented the best sound insulation. Above the $f_c$, the radiation efficiency of the aluminum honeycomb sandwich was the lowest, where the velocity was nearly equal to the two other sandwich cylinders. Therefore the TL of the aluminum sandwich cylinder was the largest at high frequencies. Finally, it should be noted that the $G_c$ has a smaller effect on the TL of sandwich cylinders compared to other structural parameters such as the core thickness or
the facing thickness. This is because the $G_c$ is much low compared to the facing stiffness and thus its value variance leads a small change of the sandwich bending stiffness.

![Figure 3-12 TL (a) & Radiation efficiency and surface velocity (b) of three cylinders with different kinds of core](image)

### 3.7. TL comparisons of sandwich, stiffened and uniform laminated cylinders

As the comparison among uniform, stiffened and sandwich structures has seldom been mentioned to the date, TLs of a uniform, a stiffened, and a sandwich cylinder were compared under a diffuse acoustic field. The geometries of the three cylinders were all radius 2 m and length 10 m. For the stiffened cylinder, the materials of both the skin and the stiffeners were quasi-isotropic Carbon fabric /Epoxy. The skin thickness was 4.5 mm, and the geometry and pitch of stringers and frames are shown in Figure 3-13. For the sandwich cylinder, the skin material was also Carbon fabric /Epoxy, and the core was Rohacell 200. The thickness of both the facings was 1 mm and the core thickness was 10.5 mm. For the uniform laminated cylinder, the material was Carbon fabric/Epoxy and the lamination thickness was 4.5 mm. All the above materials properties can be found in Appendix A. With respect to the total weight, the stiffened cylinder was about 1067 kg, the sandwich cylinder was about 1070 kg and the uniform laminated cylinder was 1041 kg. The three cylinders had nearly equal masses so that the effect of mass on the TL difference could be neglected.

As shown in Figure 3-14a, the uniform cylinder performed the best for sound insulation below 1500 Hz and this phenomenon can be explained by looking at Figure 3-14b: although the uniform cylinder exhibits the largest structural velocity, its radiation efficiency is the lowest and the difference of the radiation efficiency is more significant than the structural velocity. Thus the uniform cylinder had the largest TL in this frequency range. At high frequencies above 1500 Hz, the sandwich cylinder presented the lowest structural velocity and its radiation efficiency was low. This leads to that the sandwich cylinder had the highest TL at high frequencies. The stiffened cylinder had an overall low TL at most frequencies. At low frequencies, as the stiffened cylinder has a
larger global bending stiffness and the global vibration modes were smaller compared to the laminated cylinder, it had a lower surface velocity. However, the radiation efficiency was the highest because of the stringers and the frames. The interaction between the surface velocity and the radiation efficiency lead to the lowest TL of the stiffened cylinder at low frequencies. At high frequencies, the local vibration modes of un-stiffened thin area on the stiffened cylinder contributed to the high structural response. Therefore, the stiffened cylinder also exhibited the lowest TL at high frequencies.

As the damping properties for the stiffened cylinder was not available (the bolts on the stiffened panels might have had complex influence on the damping which should be specifically measured), the damping loss factors of all the three cylinders were assumed to be 0.01 in the models. In practice, the sandwich cylinder generally has a higher loss factor than the other two cylinders. Thus the sandwich cylinder could actually perform better if the damping difference is taken into consideration.

Figure 3-13 Dimensions of the C frame and the Z stringer for the stiffened cylinder
(the C frame pitch is 0.5 m and the Z stringer pitch is 0.2 mm)

Figure 3-14 TL(a) & Structural velocity and radiation efficiency (b) of the laminated, stiffened and sandwich cylinder
Finally an overview of the TL simulation results of the three kinds of cylinder is given in Table 3-7. It is apparent that the non-stiffened cylinder gives the best noise insulation at 20-1500 Hz and the sandwich cylinder shows the highest TL at 1500-20000 Hz. The stiffened cylinder exhibits the worst insulation performance over the entire frequency range.

**Table 3-7 Summary of the transmission loss of different structure types**

<table>
<thead>
<tr>
<th></th>
<th>Non-stiffened cylinder</th>
<th>Sandwich cylinder</th>
<th>Stiffened cylinder</th>
</tr>
</thead>
<tbody>
<tr>
<td>TL at 20–1500 Hz</td>
<td>highest</td>
<td>middle</td>
<td>lowest</td>
</tr>
<tr>
<td>TL at 1500–20000 Hz</td>
<td>middle</td>
<td>highest</td>
<td>lowest</td>
</tr>
</tbody>
</table>

**3.8. Conclusions**

The focus of the investigation presented here was on the prediction of sound transmission loss (TL) of composite cylindrical shells in a broad frequency range. The analytical methods, based on the superposition of natural modes of the cylinder structure and its enclosed cavity, were used to predict the TL of infinite-length cylinders. In addition, the statistical energy analysis (SEA) method was applied to predict the TL of finite-length cylinders. The two methods showed good agreement on predicting TL of cylinders at high frequencies. Moreover, it was found that shear deformation of the core should be taken into consideration for thick cylindrical shells in the analytical models.

The SEA model showed a much higher computation efficiency, which could be 10 times, than the analytical method for predicting the TL. Thus the SEA method was used for studying the influence of geometric and material parameters on the TL of cylinders. The following parameters were studied: laminate layup, fiber material, cylinder dimensions, core thickness and sandwich layup.

(1) It was found that typical asymmetric layups cause bending-extensional coupling of laminates, which is not beneficial for the sound insulation of at high frequencies. The layup in the hoop direction, as compared to a layup comprising fibers mainly in axial direction, showed a higher TL at most frequencies in that its high stiffness in the hoop direction reduces the radial vibration amplitude. (2) Fiberglass/Epoxy laminates exhibited the best insulation performance at 400-8000 Hz compared to Graphite/Epoxy and Aramid/Epoxy laminates, which can be explained by that its density is the highest and it has the widest mass controlled frequency range. (3) The length of the considered cylinders had little influence on the TL while increasing the radius decreased the ring frequency and the TL at low frequencies. (4) Increasing the core thickness or facing thickness can improve the sound insulation, however, given the total mass is constant and if a core thickness is chosen from 10 mm, 20 mm, and 30 mm. The sandwich cylinder with a core thickness of 20 mm presented the best sound insulation performance above 1000 Hz. This can be explained by that its bending stiffness is the highest, leading to the lowest structural velocity at higher frequencies. (5) Similar to the asymmetric
layup of the facing, asymmetric sandwich layups also cause the bending-extensional coupling and its TL was lower than the symmetric layup.

A uniform laminated cylinder, a stiffened cylinder and a sandwich cylinder, of a comparable mass, were compared together for the sound insulation performance. It was found that the uniform cylinder performed the best below 1500 Hz because its radiation efficiency was the lowest. The sandwich cylinder performed the best above 1500 Hz because its surface velocity was the lowest and its radiation efficiency was low as well.

To sum up, the coincidence frequency is an important characteristic for the sound transmission. Above the coincidence frequency, a higher bending stiffness leads to lower radiation efficiency and a higher TL. Below the coincidence frequency, a higher bending stiffness leads to higher radiation efficiency but a lower structural velocity, therefore there exists an optimal bending stiffness which can make a good compromise between the structural response and the radiation efficiency.

3.9. References

Chapter 4  Experimental comparison of sound transmission between a laminated cylinder and a sandwich cylinder

As discussed in Chapter 3, the sound transmission loss of a laminated and a sandwich cylinder was compared using the SEA method. Following the previous work, experimental measurements of the noise insulation of a sandwich cylinder and a laminated cylinder are present in this chapter. The experimental measurement under an acoustic excitation and a force excitation are introduced in Section 4.2. Section 4.3 shows the noise reduction comparison between the two cylinders under the acoustic excitation and Section 4.4 presents the inner pressure of the two cylinders subjected to the force excitation. In Section 4.5, the wave propagation in the sandwich structure is compared with that in the uniform laminated structure, as an explanation of the noise reduction difference of the two structures. Finally, following the work in Section 4.6, influence parameters of the coincidence frequency of sandwich structures are studied.

4.1. Introduction

The common experimental method to characterize the sound transmission is to compare the sound power of a source room and a receiving room which are separated by the sample [1]. The source is usually a reverberation room with diffuse acoustic field. The receiving room is anechoic which approximates a free-field. A rigid wall is positioned between the two rooms, which can contain the specimen and divide the two cavities. Arjunan [2] measured the sound transmission loss of curved honeycomb panels with this method. H. Wal [3] also adopted this technique to investigate sound transmission loss of the composite and metal fuselage. However, a drawback of the above mentioned method is that it needs a special source room and a special receiving room, which is an expensive investment. In addition, this method is not suitable for analyzing the contribution of the fluid cavity, which is important for the sound transmission of the enclosed structures like the aircraft fuselage. There exists another experimental method to characterize the sound insulation of enclosed structures, and the noise reduction (NR) is measured for the experimental method. In this approach, one or more microphones are placed inside the cylinder and some microphones are placed outside the cylinder. The sound pressure difference between the inner microphones and the outer microphones are calculated as NR. Pope et al. [4], Krakers [5] and Li [6] adopted this method to characterize the sound transmission of aluminum cylinders.
The TL comparison between the uniform and the stiffened panels was performed by some researchers. Krakers [5] tested the noise reductions of three aluminum cylinders including a non-stiffened cylinder, and a cylinder with 6 stringers and a cylinder with 12 stringers. The results showed that the non-stiffened cylinder had larger noise reduction than the two stiffened cylinders. Val and Nilsson [3] compared the TL at the window area and the stringer area of two fuselage panels. They found that the window area has a larger TL than the stringer area and the stringers’ influences were larger for the composite panels than for the aluminum panels. Liu [7] studied sound transmission of the curved panels with frames and stringers, and he concluded that the frames had little influence on the sound transmission while the stringers resulted in significant deterioration of TL above the ring frequency.

The sound insulation of sandwich panels and uniform panels were also researched in the past. Huang and Ng [8] found that the TL of a fuselage panel can be increased as 2-13 dB at 63-630 Hz by bonding the honeycomb on the panel. The TL comparison among four composite sandwich panels and one aluminum panel is studied in the PhD thesis of Yu [9]. It was found that increasing the core thickness improve the TL at low frequencies and high frequencies, increasing the face thickness improves the TL at low frequencies, and the aluminum plate has an overall larger TL than the sandwich panels.

Although there are some research about sound transmission of sandwich structures and laminated structures, it is still not clear about the reason that leads to the sound transmission difference between the two kinds of structures. In addition, the experimental investigation on the sound transmission loss of sandwich cylinders has not been done yet. Thus in this chapter, a laminated cylinder and a composite sandwich cylinder were produced and their sound insulation was measured. The difference of sound transmission between the laminated cylinder and the sandwich cylinder under acoustic excitations was illustrated in terms of wavenumber. Finally, a parametric study on the coincidence frequency of the sandwich structures is discussed.

4.2. Experimental measurement under an acoustic and a force excitation

In this section, the manufacturing of composite cylinders, the noise reduction test under an acoustic excitation and the inner pressure test under a force excitation are introduced.

4.2.1. Production of composite cylinders

A fiberglass/epoxy composite cylinder was made by filament winding. The glass fiber roving (PPG 2400 tex) was immersed in a resin bath filled with Epoxy L1100+ hardener EPH 294 (weight ratio=100:31). The impregnated tow was then wound around a steel mandrel in a controlled pattern. The pattern (Computer Numerical Control - CNC code) was written with a Mathematica program based on the shape of the mandrel (generation
Chapter 4 Experimental comparison of sound transmission between a laminated cylinder and a sandwich cylinder

of CNC codes is introduced in Appendix C). The wet winding took about 4 hours. The wound cylindrical composite panel was cured for 15 hours at room temperature and then post-cured for 12 hours at 60 °C in an oven. The final laminated cylinder was composed of six plies of unidirectional fiberglass/epoxy with fiber orientation in the hoop direction. The total shell thickness of the laminated cylinder was 2.9 mm.

Using the similar winding method, a sandwich cylinder with the layup [0/0/0/Tycor® /0/0/0] was also produced. Three layers of wet fiberglass/epoxy were wounded on the steel mandrel and then the core was placed on top of the three layers of fiberglass/epoxy. Finally another three layers of fiberglass/epoxy were placed on the core. The total sandwich thickness was 18.18 mm and the core thickness was 15 mm. The mechanical properties of the composite materials and the Tycor® foam are shown in Table 4-1. The inner radius and the length of the two cylinders were 0.25 m and 1 m respectively. The weight of the sandwich cylinder was 11.1 kg and the weight of the laminated cylinder was 7.1 kg. With respect to the layup, the sandwich cylinder had an additional core compared with the laminated cylinder.

Table 4-1 Materials properties for the laminated and the sandwich cylinder

<table>
<thead>
<tr>
<th></th>
<th>$E_1$[Pa]</th>
<th>$E_2$[Pa]</th>
<th>$\nu_{12}$</th>
<th>$G$[Pa]</th>
<th>$\rho$[kg/m$^3$]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Glass/epoxy</td>
<td>3.66e+010</td>
<td>5.4e+009</td>
<td>0.3</td>
<td>4.2e+009</td>
<td>1800</td>
</tr>
<tr>
<td>Foam(Tycor®)</td>
<td>1.51e+008</td>
<td></td>
<td></td>
<td>6.87e+007</td>
<td>70</td>
</tr>
<tr>
<td>Wood for end caps</td>
<td>6e+009</td>
<td>6e+009</td>
<td>0.25</td>
<td>2.4e+009</td>
<td>700</td>
</tr>
</tbody>
</table>

4.2.2. The sound insulation test

Two kinds of noise sources were used to excite the two cylinders: one was a point acoustic source generated by a speaker and the other was a point force generated by a shaker.

Acoustic excitation

The noise reduction (NR), which denotes the sound pressure difference between the inner cavity and the outer cavity, was used to characterize the sound insulation under the acoustic excitation. The experimental setup for the NR test is shown in Figure 4-1. The cylinders were closed with two wooden bulkheads. The bulkheads had a thickness of 45 mm and their edges were milled to have a perfect fit with the cylinder. An aluminum rod with a radius of 5 mm was inserted into the center of wooden ends to support the inner microphone, and the assembled structure was supported by two wooden stands. During the testing, a white noise digital signal was generated by a generator (National Instrument), and then connected to a power amplifier. Finally, the amplified signal was transmitted to a speaker which was covered with a cone in order to simulate a point source, as shown in Figure 4-1. Two microphones were used to measure sound pressure inside the cylinder and outside cylinder respectively. The inner microphone was
connected to the aluminum rod and it could move in three directions. The NR measurement was performed with a sampling frequency of 8291 Hz. The measured signals were collected by an input module, and then recorded by the time data recorder of the commercial software PULSE (Brüel & Kjær). The recorded signals were finally analyzed with Matlab.

![Figure 4-1 Setup for the noise reduction test of cylinders under an acoustic excitation](image)

To obtain an accurate measurement for the NR, it is necessary to firstly check whether the two microphones detect the same signal when they are placed at the same angle and at the same distance from the speaker. In the calibration, both microphones were 50 cm away from the speaker. Results show that the difference of signals obtained from the two microphones is less than 1 dB at all frequencies.

The structural response of the cylinders was measured with an accelerometer, and the accelerometer was located at the point: \( r=0.25 \text{ m}, \ l=0.5 \text{ m}, \ \phi=0 \). The microphone outside the cylinder was fixed at the speaker position: \( r=0.27 \text{ m}, \ l=0.5 \text{ m}, \ \phi=0 \). The microphone inside the cylinder was at 2/5, 4/5 part of the radius, at 1/7, 3/7, 5/7 part of the half axis, and at 0°, 90°, 180°, 270° in the tangential direction respectively. The spatially NR is calculated by:

\[
NR = -10 \log_{10} \frac{1}{n} \left[ \left( \frac{p_1}{P_{out}} \right)^2 + \left( \frac{p_2}{P_{out}} \right)^2 + \cdots + \left( \frac{p_n}{P_{out}} \right)^2 \right]
\]  

(4.1)

where \( p_i \) is the measured inner pressure at different positions of the inner cavity, \( P_{out} \) is the pressure measured by the outer microphone, \( n \) is the total position of the inner microphone and here \( n=24 \).

**Force excitation**
Chapter 4 Experimental comparison of sound transmission between a laminated cylinder and a sandwich cylinder

The spatially averaged inner pressure was used to characterize the sound insulation under the force excitation. The experimental measurement setup is shown in Figure 4-2. The signal generation and signal collection systems were similar with that used for the acoustic excitation and thus they are not repeated here. A point force excitation was applied at the center of the cylinder surface using a shaker. Two microphones were placed inside the cylinder to measure the inner sound pressure. The two microphones were fixed on a support at 2/5, 4/5 part of the radius. The position of the support was at 1/7, 3/7, 5/7 part of the half axis, and at 0°, 90°, 180°, 270° in the tangential direction respectively (as illustrated in Figure 4-2). There were totally 24 measurement points. Finally the transferred pressure levels were spatially averaged by:

$$p_{av} = \sqrt{\frac{1}{n}(p_1^2 + p_2^2 + \ldots + p_n^2)}$$  (4.2)

where $p_{av}$ denotes the average sound pressure (Pa) and $p_1, p_2, p_n$ denotes the sound pressure at the specific point respectively, $n=24$.

![Figure 4-2 Setup for the inner pressure measurement of cylinders subjected to a force excitation](image)

Finally, Table 4-2 shows the types of the instruments that were used to conduct the sound transmission measurements under the force and the acoustic excitation.

<table>
<thead>
<tr>
<th>Instruments name</th>
<th>Type</th>
</tr>
</thead>
<tbody>
<tr>
<td>Shaker</td>
<td>4809 (Brüel &amp; Kjær)</td>
</tr>
<tr>
<td>Power amplifier</td>
<td>LING Electronics (stage accompany SA800)</td>
</tr>
<tr>
<td>Input module</td>
<td>Lan-Xi 3050-B-040 (Brüel &amp; Kjær)</td>
</tr>
<tr>
<td>Charge amplifier</td>
<td>Model 2721A (Endevco)</td>
</tr>
<tr>
<td>Force transducer</td>
<td>8200 (Brüel &amp; Kjær)</td>
</tr>
<tr>
<td>Accelerometer</td>
<td>4397 (Brüel &amp; Kjær)</td>
</tr>
<tr>
<td>Signal generator</td>
<td>NI 9264(National Instrument)</td>
</tr>
<tr>
<td>Microphone</td>
<td>multi-field 1/4&quot;, 4961 (Brüel&amp; Kjær)</td>
</tr>
<tr>
<td>Speaker</td>
<td>stage accompany</td>
</tr>
</tbody>
</table>
4.3. Results under an acoustic excitation

The NRs of the laminated cylinder and the sandwich cylinder under the acoustic excitation are compared in Figure 4-3. It can be seen that the two cylinders have a comparable NR at 1-2000 Hz and the sandwich cylinder has a significant larger NR above 2000 Hz. The coincidence frequencies of the two cylinders are labeled in Figure 4-3b, and it can be seen that the coincidence frequency of the sandwich cylinder is lower than that of the laminated cylinder.

![Figure 4-3 Measured NRs of a laminated cylinder and a sandwich cylinder subject to the acoustic excitation (a) a step of 1Hz; (b) 1/3 octave step](image)

For a comparison, the TLs of the two cylinders were predicted using the SEA method in a broad frequency range (100-10000 Hz). It should be mentioned that although the value of TL could not be equivalent as that of NR at each frequency, their trends as the frequency increases should be same. As shown in Figure 4-4, the laminated cylinder has a slightly higher TL below 1150 Hz, and has a much lower TL than the sandwich cylinder above 1150 Hz. In addition, as the laminated cylinder was 7.1 kg and the sandwich cylinder was 11.1 kg in the experiment, another laminated cylinder of 11.1 kg...
was also studied using the SEA model (its layup was the same with the laminated cylinder of 7.1 kg except that the layer thickness was increased). The laminated cylinder of 11.1 kg showed a better TL than the laminated cylinder of 7.1 kg, however, its TL was lower than the sandwich cylinder above 2000 Hz.

In a word, both the experimental results and the theoretical results demonstrate that the sandwich cylinder does not show an advantage on the sound insulation at low frequencies while has a good insulation performance at high frequencies under the acoustic excitation. In addition, as the cylinders radius is relatively small compared with the shell thickness, the effect of the ring frequency on the NR is not significant.

As the sound power of the cylinder cavity is determined by the structural surface velocity and the radiation efficiency, see Equation(3.43), the surface velocity and the radiation efficiency of the two cylinders are calculated and shown in Figure 4-5. The laminated cylinder has a similar surface velocity with the sandwich cylinder below 300 Hz. However, the velocity of the laminated cylinder is higher and higher than that of the sandwich cylinder as the frequency increases. On the other hand, the radiation efficiency of the laminated cylinder is lower than that of the sandwich cylinder. The radiation efficiency gap reaches the maximum around 2000 Hz. Thus the higher radiation efficiency and the lower structural response cause that the sandwich cylinder has a similar sound insulation with the laminated cylinder at low frequencies (< 2000 Hz in this case). Above 2000 Hz, the radiation efficiency difference between the sandwich cylinder and the laminated cylinder is decreasing as the frequency increases while the velocity difference is larger and larger. This causes that the sandwich cylinder has a larger and larger TL than the laminated cylinder at high frequencies.

In order to have a closer look at sound transmission at low frequencies, both the measured transferred acceleration (acceleration/outer pressure) and the measured NR at 0-1000 Hz are shown in Figure 4-6. It can be seen that the laminated cylinder has more peaks in the transferred acceleration curve than the sandwich cylinder at 0-400 Hz. This is because of low bending stiffness of the laminated cylinder. As a result, the NR curve of the laminated cylinder has more dips than that of the sandwich cylinder below 400 Hz. This demonstrates that the stiffness controls the sound transmission at 0-400 Hz. At 400-1000Hz, although the sandwich cylinder has lower acceleration amplitudes and lower structural modal density than the laminated cylinder, its NR is similar with the laminated cylinder. This can be explained not only by the previously mentioned radiation efficiency but also by the domination of fluid cavity resonances over the sound transmission in this range. As labeled in Figure 4-6, there are some common dips on the NR curves of the two cylinders. With the computation of the resonance frequencies of the fluid cavity, it is found that these common dips represent the resonance frequencies of the cylindrical cavity. Some resonance mode shapes of the cylindrical cavity are shown in Figure 4-7.
Figure 4-5 Calculated surface velocity and radiation efficiency of the sandwich cylinder and the laminated cylinder.

Figure 4-6 Experimental results subject to a point acoustic excitation with a step of 1 Hz, the labeled points are the cavity modes.

Figure 4-7 Modes shapes of the cylindrical cavity which lead to the noise reduction dips, cavity geometry: r=0.25 m, l=1 m.
4.4. Results under a force excitation

The transferred inner pressure (spatially average pressure/excitation force) of the sandwich and the laminated cylinders is used to characterize the sound insulation. If the transferred inner sound pressure is high, it means that the sound insulation of the cylinders is poor. Similar to the case under the acoustic excitation, the inner pressure of the cylinders under the force excitation is also presented in two frequency ranges: one is in a linear step of 1 Hz and the other is in a third octave step. In Figure 4-8a, it can be found that there are many peaks in the inner pressure curves of the two cylinders. It is apparent that the laminated cylinder has more peaks than the sandwich cylinder overall the frequency range. This is because the sandwich cylinder has a higher stiffness than the laminated cylinder. A clearer inner pressure comparison of the two cylinders is shown in Figure 4-8b and it can be seen that the sandwich cylinder has a lower inner pressure than the laminated cylinder at most frequencies. The exception is at 300-1000 Hz, where the sandwich cylinder has a comparable or higher inner pressure than the laminated cylinder. This phenomenon can be explained by their accelerations and radiation efficiencies. As shown in Figure 4-9, the sandwich cylinder has a comparable acceleration in this frequency range. In addition, Figure 4-5 shows that the radiation efficiency of the sandwich cylinder is much higher than the laminate cylinder in this frequency range.

With respect to the structural response (acceleration), it can be found that the sandwich cylinder has lower structural response than the laminated cylinder at all frequencies. This is because that the sandwich cylinder has a much higher bending stiffness than the laminated cylinder. In addition, the sandwich cylinder has a higher damping loss factor than the laminated cylinder due to the contribution of the core damping. However, this phenomenon did not occur under the acoustic excitation. One possible reason could be that the acoustic excitation cannot directly cause the structure to vibrate in that a fluid-structure interaction is needed. In this case, the vibration amplitudes of the two structures are low and they do not have significant difference. By comparing Figure 4-5 and Figure 4-9, it was found that the acceleration amplitude under the acoustic excitation was at 0.01-10 m/s$^2$ while the acceleration amplitude under the force excitation was at 0.1-100 m/ s$^2$. 
The structural response (velocity) and inner pressure of the two cylinders were computed using the SEA method. In addition, as the laminated cylinder was 7.1 kg and the sandwich cylinder was 11.1 kg, another laminated cylinder with a mass of 11.1 kg was also studied. As shown in Figure 4-10, the laminated cylinder of 7.1 kg had an overall higher velocity and inner pressure than the sandwich cylinder. Although the simulation results had some difference with the experiment results, they had a similar trend with the experimental results. The laminated cylinder of 11.1 kg also had a larger inner pressure than the sandwich cylinder at most frequencies. This excluded possibility that the higher TL of the sandwich cylinder is due to its higher mass. In conclusion, both the experimental and theoretical results showed that the sandwich cylinder had an advantage of the sound insulation over the laminated cylinder under the force excitation.
Chapter 4 Experimental comparison of sound transmission between a laminated cylinder and a sandwich cylinder

4.5. Wave propagation in sandwich and laminated structures

It was found in Section 4.3 that the sandwich cylinder did not show an advantage of the sound insulation at low frequencies over the laminated cylinder, while its insulation was much better at high frequencies under the acoustic excitation. As the wave propagations play a vital role on the sound transmission, the sound wave propagation inside the sandwich and the laminated structures was studied in order to probe this phenomenon.

4.5.1. Wavenumber of sandwich and laminated structures

E. Nilsson and A. C. Nilsson [10] developed an expression for calculating the wavenumber of sandwich plates, as shown in Equation (4.3):

\[
2D_f k_e^6 + \frac{2D_f k_e^4 \omega^2}{D} - (M + \frac{2D_f M}{D} + \frac{IG_f t_c}{D})k_e^2 \omega^2 + G_e t_c (k_e^4 - \frac{M \omega^2}{D}) + \frac{IM \omega^4}{D} = 0
\]  

\[
I = \rho_e t_e^3 / 12 + \rho_f (t_e^2 t_f / 2 + t_e t_f^2 + 2t_f^3 / 3)
\]

\[
G_e = G_c (1 + \eta_c)
\]

\[
D = E_f t_f (t_e + t_f)^2 / 2
\]

\[
M = 2t_f \rho_f + t_e \rho_c
\]

\[
D_f = E_f t_f^3 / [12(1 - v_{12}^2)]
\]
where \( I \) denotes the moment of inertia of the sandwich beam, \( G_c \) is the core shear stiffness considering damping \( \eta_c \). \( D \) denotes the bending stiffness of sandwich structures, \( M \) is the mass per unit area of sandwich structures, and \( D_f \) is the bending stiffness of facings. The wavenumber of the sandwich panel can be obtained by solving Equation(4.3).

For a uniform laminated plate, the wavenumber can be expressed as [11]:

\[
k_i = \sqrt{\frac{\omega^2 M_i}{B_i}} \quad (4.9)
\]

\[
M_i = t_i \rho_i \quad (4.10)
\]

\[
B_i = E_i t_i^3 / [12(1 - \nu_{12}^2)] \quad (4.11)
\]

where \( M_i \) is the mass per unit area and \( D_i \) is the bending stiffness of the laminated plate. \( t \), \( \rho \), \( E_i \), and \( \nu_{12} \) are respectively thickness, density, Young’s modulus, and Poisson’s ratio of the plate.

4.5.2. Relationship between the TL and the wavenumber

There exist some relationships between the TL and the vibration wavenumber of structures. Supposing a flat panel is excited by an acoustic plane wave, its TL can be computed by [11]:

\[
\tau = \frac{[2 \rho_0 c / (\omega M)]^2 \sec^2 \gamma}{[2 \rho_0 c / (\omega M) \sec \gamma + (k / k_b)^4 \eta \sin^4 \gamma]^2 + [1 - (k / k_b) \sin^4 \gamma]^2}
\]

\[
TL = -10 \log_{10} \tau \quad (4.13)
\]

where \( \gamma \) is the incidence angle of the plane wave, \( \rho_0 \) the fluid density, \( c \) the sound speed, \( M \) is the mass per area, \( k \) the wavenumber in the fluid, \( k_b \) the wavenumber in the structure, \( \eta \) the damping loss factor and \( \omega \) the frequency. A simple example is given here to show how \( k_b \) influences the TL.

4.5.3. Case study

According to Equation(4.3) and Equation(4.9), the wavenumber of a sandwich panel and a laminated panel were compared. The mechanical properties of two panels were the same with the sandwich cylinder and the laminated cylinder in Section 4.2.1. The two panels were supposed to have the same mass per unit area. According to Equation(4.13), the relationships between the TL and the \( k_b \) of the two panels were calculated at 300 Hz and 3000 Hz as an example. The parameters of Equation(4.12) were defined as: \( \rho_0 = 1.29 \) kg/m\(^3\), \( c = 343 \) m/s, \( M = 4.1 \) kg/m\(^2\), \( \eta = 0.01 \), \( k = \omega / c \), \( \gamma = 45^\circ \).
Chapter 4 Experimental comparison of sound transmission between a laminated cylinder and a sandwich cylinder

Figure 4-11 Wavenumber of a sandwich beam and a laminated beam

Figure 4-12 Influence of the wavenumber on the TL at 300 Hz (a) and 3000 Hz (b)

The wavenumber of the two cylinders is shown in Figure 4-11 and the relationships between the wavenumber and the TL are shown in Figure 4-12. At 300 Hz, the wavenumber of the sandwich structure is lower than the laminated one and the TL is increased as the wavenumber increases. Therefore the TL of the sandwich structure is lower than the laminated structure. At 3000 Hz, although increasing wavenumber firstly leads to a decrease and then an increase of TL (the critical point is approximately 20), the wavenumber of both the laminated and the sandwich structure is larger than 20 at 3000 Hz (Figure 4-11) and the TL increases as the wavenumber increases in this wavenumber range. Hence the sandwich structure has a larger TL than the laminated one.

Although this case study only showed the influence of wavenumber on the TL at 300 Hz and 3000 Hz, the relationship between the TL and the wavenumber at other frequencies were also studied. It was concluded that the wavenumber in the structure has a positive influence on the TL. This theory can illustrate why the sandwich cylinder has
a worse performance than the laminated cylinder at low frequencies but a better performance at high frequencies under acoustic excitations.

4.6. Parametric study of the coincidence frequency of sandwich structures

It is known that the TL is low at the coincidence frequency \( f_c \) where the wave speed in the structure is equal to the wave speed in the fluid. Thus it is better to transfer the \( f_c \) to be beyond the frequency range of interests. The \( f_c \) can be transferred by modifying the stiffness and the mass of structures. The effects of the core thickness, facing thickness, core density and core shear stiffness on the sandwich coincidence frequency were studied as follows.

4.6.1. Determination of the coincidence frequency

Kurtze [12] derived a governing equation of the sound wave speed \( c \) in sandwich structures, as shown in Equation(4.14). This equation is similar with Equation(4.3) except that it is expressed in terms of wave speed. In this equation, the facing is viewed as thin and isotropic. The wave speed in sandwich structures was compared with that in uniform laminated structures. The sound wave speed in the laminated panel \( c_L \) can be expressed as Equation(4.18).

\[
\frac{c_s^2}{c_b^4} c^6 + c^4 - c_s^2 c^2 - c_{bf}^4 = 0 \quad (4.14)
\]

\[
c_b = \sqrt{\omega^2 D/M} \quad (4.15)
\]

\[
c_s = \sqrt{G t_e / M} \quad (4.16)
\]

\[
c_{bf} = \sqrt{\omega^2 2D_t / M} \quad (4.17)
\]

\[
c_l = \sqrt{\omega^2 D_l / M_l} \quad (4.18)
\]

where \( c_b \) is the bending wave speed of the sandwich panel, \( c_s \) the shear wave speed, \( c_{bf} \) the bending wave speed of the facing. The definitions of other parameters refer to Nomenclature. The bending waves and shear waves of a sandwich beam can be seen in Figure 4-13.
Chapter 4 Experimental comparison of sound transmission between a laminated cylinder and a sandwich cylinder

Figure 4-13 Bending waves and shear wave of the sandwich panels, (a) and (b) are kinds of longitudinal or compression waves.

Using the material properties in Appendix A, the wave speeds of the sandwich and the laminated structures were calculated and they are shown in Figure 4-14. It can be found that:

At low frequencies, the sandwich panel behaves like an equivalent uniform panel in bending, and the bending wave of the sandwich structure dominates the sound propagation. At middle frequencies, the wave speed is mainly controlled by shear deformation within the core. According to Equation (4.16), a core with lower shear modulus leads to a smaller wave speed. At high frequencies, the wave speed agrees well with the bending wave speed of the individual facings. The strain energy of the structure is concentrated in the skins and is associated with separate bending wave propagation in each skin.

The coincidence frequency can be calculated by making the wave speed in the structure equal to the sound wave speed in the air (c_air). As shown in Figure 4-14, there are two intersection points which determine the coincidence frequencies: the intersection between the wave speed in the laminated structure (c_laminate) and c_air (f_cr_laminate=1540 Hz); the intersection between the wave speed in the sandwich structure (c_sand) and c_air (f_cr_sand=410 Hz).
Figure 4-14 the transverse wave speed in the laminated and the sandwich panels, 
\[ t_{\text{f1}}=t_{\text{f2}}=3 \text{mm}, \ t_{\text{c}}=20 \text{mm}, G_{\text{c}}=100 \text{MPa} \]

4.6.2. Effect of core thickness and facing thickness

As shown in Figure 4-15, \( f_c \) is decreasing as the core thickness increases. On the contrary, \( f_c \) is increasing as the core thickness increases in Figure 4-16. This is because that the intersection between \( c_{\text{air}} \) and \( c_{\text{sand}} \) occurs at different positions for different \( G_c \) and \( E_f \). When \( G_c \) is not too low compared with \( E_f \), the intersection occurs at a low frequency, and \( f_c \) can be approximated using Equation (4.19). According to this equation, the ratio \( M/B \) decreases as the core thickness increases. When \( G_c \) is much lower than \( E_f \), the intersection occurs at a high frequency, and \( f_c \) can be approximated according to Equation (4.20). In this case, an increase of core thickness leads to an increase of \( M \) but has nothing with \( D_f \), and thus \( f_c \) increases.

\[
\text{at low frequencies: } f_c = \frac{1}{2\pi} \sqrt{\frac{M c_{\text{air}}^4}{D}} \quad (4.19)
\]

\[
\text{at high frequenies: } f_c = \frac{1}{2\pi} \sqrt{\frac{M c_{\text{air}}^4}{2D_f}} \quad (4.20)
\]

The effect of the facing thickness on \( f_c \) is shown in Figure 4-17. Two cases were studied: \( E_f=49 \text{ GPa} \) and \( G_c=50 \text{ MPa} \); \( E_f=490 \text{ GPa} \) and \( G_c=50 \text{ MPa} \). It can be seen that the \( f_c \) firstly increases and then decreases as \( t_f \) increases no matter the values of \( E_f \) and \( G_c \). There is a peak at \( t_f=2 \text{ mm} \) in both cases. The fact that the relationship between \( t_f \) and \( f_c \) cannot be expressed as linearly is caused by that \( t_f \) nonlinearly influences both the \( M/D \) in Equation (4.19) and \( M/D_f \) in Equation (4.20).
Chapter 4 Experimental comparison of sound transmission between a laminated cylinder and a sandwich cylinder

Figure 4-15 $G_c=50$MPa, $E_f=49$GPa, the intersection between $c_{air}$ and $c_{sand}$ happens at low frequencies, the coincidence frequency decreases as core thickness increases.

Figure 4-16 $G_c=50$MPa, $E_f=490$GPa, the intersection between $c_{air}$ and $c_{sand}$ happens at high frequencies, the coincidence frequency increases as core thickness increases.

Figure 4-17 Influence of the facing thickness on the coincidence frequency $f_c$
4.6.3. Effect of core density and core shear stiffness

The effect of $G_c$ on the coincidence frequency of sandwich structures is shown in Figure 4-18a. It can be found that $f_c$ decreases sharply when $G_c$ is increased from 10 MPa to 80 MPa. This is the case that the intersection between $c_{air}$ and $c_{sand}$ occurs at low frequencies. According to Equation(4.19), as the $G_c$ increase, the bending stiffness of the sandwich structure $B$ increases, and thus $f_c$ decreases. On the other hand, if $E_f$ is much higher than the $G_c$ and Equation(4.20) fits the situation, it is apparent that $G_c$ has no influence on $f_c$.

The influence of the core density on $f_c$ is shown in Figure 4-18b. It can be seen that an increase of the core density leads to an increase of $f_c$. This phenomenon can be easily explained using both Equation(4.19) and Equation(4.20).

![Figure 4-18 Coincidence frequency of sandwich structures with varied core shear stiffness (a) and core density (b)](image)

4.7. Conclusions

A sandwich cylinder and a laminated cylinder were produced using the filament winding method. Their sound insulation was compared under both an acoustic and a force excitation.

Under the acoustic excitation, the noise reduction was used to characterize the sound insulation. The sandwich cylinder had a comparable structural response and noise reduction with the laminated cylinder below 2000 Hz. However, the sandwich cylinder had a much better sound insulation than the laminated cylinder above 2000 Hz. In addition, it was found that the structural resonances contributed to the sound transmission at 0-400 Hz. While the cavity resonances lead to some common dips on the noise reduction curves of both the sandwich and the laminated cylinders at 400-1000 Hz.

Under the force excitation, the structural response of the sandwich cylinder was much lower compared to the laminated cylinder at 1-4000 Hz. The sandwich cylinder had lower inner pressure than the laminated cylinder at most frequencies. The maximum
pressure difference between the two cylinders was about 20 dB. The experimental results have been validated by the theoretical prediction with the SEA method.

The different performances of the two cylinders under different kinds of excitation denote that noise controls should specifically applied on structures under various noise sources.

Compared with the laminated cylinder, the sandwich cylinder had a poor insulation performance at low frequencies but had a good performance at high frequencies under acoustic excitations. This was illustrated by the frequency-dependent wavenumber of the two structures. It was found that the wavenumber has positive influences on the TL under an acoustic excitation. Moreover, at low frequencies, the sandwich structure has lower wavenumber than the laminated structure; at high frequencies, the sandwich structure has higher wavenumber than the laminated structure.

One way to improve the sound insulation of sandwich structures is to transfer their $f_c$ to beyond the interested frequency range. Therefore the methods to compute the coincidence frequency ($f_c$) of sandwich structures were discussed. The core thickness, core shear stiffness ($G_c$), core density and facing thickness were studied for their influences on the $f_c$ of sandwich structures. The following conclusions can be drawn from the parametric study:

- $f_c$ decreases as the core thickness increases, however, the contrary trend works if $G_c$ is extremely low compared with the facing stiffness. It should be noted that the latter case rarely happens for the practical sandwich structures with mechanical functions.
- When $G_c$ is much lower than the facing stiffness, $G_c$ has no influence on $f_c$, or else, $f_c$ decreases as $G_c$ increases.
- $f_c$ increases as the core density increases.
- $f_c$ firstly increases and then decreases as the facing thickness increases.

4.8. References

Chapter 5  Vibroacoustic study of composite sandwich cylinders at low frequencies

The FEM/BEM numerical method was used to predict the noise reduction of sandwich cylinders at low frequencies. The predicted results and the experimental results are in agreement under the acoustic excitation and the force excitation respectively. Some parameters including the core shear stiffness, the sandwich layup, the core thickness and the facing orientation were studied for their influences on the sound transmission. An efficient optimization technique using acoustic transfer vector (ATV) & Genetic Algorithm (GA) was applied to find an optimum sandwich structure for the best noise insulation performance. Finally a multi-objective optimization (weight & noise insulation) was conducted for a fuselage section considering the mechanical requirements.

5.1. Introduction

The finite element method coupled with the boundary element method (FEM/BEM) is widely used to investigate the vibroacoustic behavior of structures. Buehrle [1] developed a FEM/BEM model to predict the vibroacoustic response of a curved honeycomb sidewall panel. The model was validated by a modal test and a sound transmission test. Fernholz and Robinson [2] studied the coupled fluid/structure vibration of a composite cylinder using FEM. They also studied the influence of lamination angles on the internal noise level of a composite aircraft in [3]. Cheng [4] investigated the influence of the shell-plate joints on the inner pressure of a plate-ended cylindrical shell and found that changing the joint condition cannot lead to a systematic improvement of noise insulation if the excitation occurs on the shell, while reducing the joint stiffness proves to reduce the cavity noise if the plate is excited.

In addition, the noise insulation optimization of composite sandwich structures is also a hot topic in recent years. A review about the structural-acoustic optimization of composite sandwich structure is shown in [5]. The search algorithms including the GA are widely used for the optimization. The parameters for the optimization include the material properties such as the stiffness, damping, Poisson’s ratio; the sandwich layup such as the layer thickness, the reinforcement orientation and the stacking sequence; the geometry and topology such as the microstructure of the core and the shape. The core geometry has been shown to be one of the most versatile and effective design parameters to enhance the multi-functionality of composite sandwich structures. Liu and Lu [6] conducted an optimization of the truss-core sandwich panel with respect to the geometry of all struts. Scarpa and Tomlinson [7] found that transverse shear modulus is affected
by the unit cell geometry. It is possible for cell geometries with a negative Poisson’s ratio to obtain higher shear modulus than a regular hexagonal honeycomb.

Concerning cylindrical structures, some works about the minimization of the inner noise are introduced here. The fuselage of the aircraft Beech Starship was modeled with FEM and the lamination angles were optimized for a reduction of the internal noise level. Point forces were used to simulate the structure borne noise produced by the engines. The optimization was performed over a frequency range of 185 Hz to 210 Hz. Finally the peak acoustic response in the cabin was reduced by 4 dB through the optimization in [3]. Johnson [8] introduced a design tool incorporating the FEM/BEM techniques for structural acoustic optimization of a composite cylindrical shell. The objective was to minimize the sum of the squared pressure amplitudes within the enclosed air cavity by optimizing the lamination angles. The shell was excited by two external acoustic monopole sources at a single frequency. A 2 dB reduction was obtained for the average inner sound pressure level at this excitation frequency. The layup of the shell was shifted from symmetric at the initial stage to unsymmetrical at the final stage. Denli and Sun [9] introduced a coupled FE/BE model to minimize the interior sound in cylinder shells under acoustic excitations. They found that the lamination angles were effective to minimize interior noise of a cylindrical shell.

In the past, numerical studies on sound transmission of composite sandwich cylinders have seldom been verified by experimental tests. In addition, the multi-objective optimization of sandwich cylinders in multi-discipline has seldom been studied. Therefore, the numerical prediction by the FE/BE model is validated by the experimental results in this chapter. Meantime, a multi-objective optimization is done for a sandwich cylinder considering the mechanical requirement, the weight and the acoustic response. The chapter is organized in the following manner: Firstly the FE/BE numerical model and the acoustic transfer vector (ATV) concepts are introduced in Section 5.2. Secondly the predicted and the measured results are compared and discussed in Section 5.3. Thirdly the structural parameters which may influence inner pressure of sandwich cylinders are studied in Section 5.4. The inner sound pressure of a sandwich cylinder is minimized in Section 5.5. Finally, a case study for the multi-objective optimization of a fuselage section is presented in Section 5.6.

5.2. FE/BE numerical model

In this section, the computational model for the inner pressure prediction with FEM/BEM is clarified. Section 5.2.1 shows the FE model considering assembly of stiffness, shape and mass matrices. Section 5.2.2 clarifies the BE model and Section 5.2.3 shows the combination of the FE and BE model into a matrix representation. Finally the sound pressure at field points and the ATV method are presented in Section 5.2.4.
5.2.1. FE model for the structural shell

The first-order theory was used to define the deformation of cylindrical shells. According to the theory, the displacements in the axial, tangential, and radial direction \((u_z, u_\phi, u_r)\) can be expressed respectively as:

\[
\begin{align*}
    u_z &= u_z^0 + h \cdot \psi_z \\
    u_\phi &= u_\phi^0 + h \cdot \psi_\phi \\
    u_r &= u_r^0
\end{align*}
\]  

(5.1)

where \(u^0\) denotes the displacement at the mid-plane of the laminate. \(h\) denotes the normal distance between the mid-plane and the considered layer. \(\psi_z, \psi_\phi\) represents the rotational angle around the axial and tangential plane respectively.

The strain-displacement relationship can be expressed as:

\[
\begin{bmatrix}
    \varepsilon_z \\
    \varepsilon_\phi \\
    \varepsilon_{z\phi} \\
    \varepsilon_{zr} \\
    \varepsilon_{r\phi}
\end{bmatrix} =
\begin{bmatrix}
    \partial_z & 0 & 0 & h\partial_z & 0 \\
    0 & 1 / R\partial_\phi & 1 / R & 0 & h / R\partial_\phi \\
    1 / R\partial_\phi & \partial_z & 0 & h\partial_\phi / R & h\partial_z \\
    0 & 0 & \partial_z & 1 & 0 \\
    0 & -1 / R & 1 / R\partial_\phi & 0 & 1
\end{bmatrix}
\begin{bmatrix}
    u_z \\
    u_\phi \\
    u_r \\
    \psi_z \\
    \psi_\phi
\end{bmatrix}
\]  

(5.2)

where \(\varepsilon\) represents the strain and \(R\) is the radius of the cylinder.

Stiffness matrix

For the elements with the laminated properties, the stiffness matrix of each finite element can be computed by [10]:

\[
K^e = K_{mm}^e + K_{mf}^e + K_{fm}^e + K_{ff}^e + K_{ss}^e
\]  

(5.3)

where \(K_{mm}^e\) is the membrane part of the stiffness matrix, \(K_{mf}^e\) and \(K_{fm}^e\) are the membrane-bending coupling components, \(K_{ff}^e\) the bending part and \(K_{ss}^e\) is the transverse shear part, which can be expressed respectively as:

\[
\begin{align*}
    K_{mm}^e &= \sum_{k=1}^{N} \int_A S_m^T A_k S_m dA; \\
    K_{mf}^e &= \sum_{k=1}^{N} \int_A S_m^T B_k S_f dA; \\
    K_{fm}^e &= \sum_{k=1}^{N} \int_A S_f^T B_k S_m dA; \\
    K_{ff}^e &= \sum_{k=1}^{N} \int_A S_f^T D_k S_f dA; \\
    K_{ss}^e &= \sum_{k=1}^{N} \int_A S_s^T A_k S_s dA;
\end{align*}
\]  

(5.4)

where \(N\) denotes the total layer number of the laminates, and \(A\) the area of the element. \(A_k\) is the membrane stiffness matrix, \(B_k\) the bending-membrane stiffness matrix, \(D_k\) the bending stiffness matrix, \(S_s\) the transverse shear stiffness matrix. The stiffness expressions can be seen in the Appendix B. \(S_m, S_f, S_s\) and \(S_k\) represent the
strain-displacement relationship for the membrane, bending and shear part respectively. Based on Equation (5.2), they are formulated as the following:

\[
S_m = \begin{bmatrix}
\partial_z N_i & 0 & 0 & 0 & 0 \\
0 & N_i / R \partial_\phi N_i & N_i / R & 0 & 0 \\
1 / R \partial_\phi N_i & \partial_z N_i & 0 & 0 & 0
\end{bmatrix}
\] (5.5)

\[
S_f = \begin{bmatrix}
0 & 0 & 0 & \partial_z N_i & 0 \\
0 & 0 & N_i / R & 0 & 1 / R \partial_\phi N_i \\
0 & 0 & 0 & 1 / R \partial_\phi N_i & \partial_z N_i
\end{bmatrix}
\] (5.6)

\[
S_s = \begin{bmatrix}
0 & 0 & \partial_z N_i & N_i & 0 \\
0 & -N_i / R & 1 / R \partial_\phi N_i & 0 & N_i
\end{bmatrix}
\] (5.7)

where \(N_i\) is the shape function of the finite element, which builds the relationship between the structural response \(v\) at the gauss points \((\xi, \eta)\) and that at the nodes \((x, y)\).

\[v(\xi, \eta) = Nv(x, y)\] (5.8)

The rectangle element with four nodes and the triangle element with three nodes are shown in Figure 5-1. Concerning the rectangle element, the shape function \(N\) can be expressed as [11]:

\[
N = \frac{1}{4} \begin{bmatrix}
(1-\xi)(1-\eta) \\
(1+\xi)(1-\eta) \\
(1+\xi)(1+\eta) \\
(1-\xi)(1+\eta)
\end{bmatrix}
\] (5.9)

With respect to the triangle element, the shape function \(N\) can be expressed as:

\[N(i) = a_i + b_i \xi + c_i \eta\] (5.10)

\[a_i = -\frac{1}{2A}(x_j y_k - x_k y_j)\]

\[b_i = -\frac{1}{2A}(y_j - y_k)\] (5.11)

\[c_i = -\frac{1}{2A}(x_k - x_j)\]

where \(i, j, k=1, 2, 3\) denotes the number of gauss points in the triangle element. Therefore, when \(i=1\), then \(j=2\) and \(k=3\); when \(i=2\), then \(j=3\) and \(k=1\); when \(i=3\), then \(j=1\) and \(k=2\). \(A\) denotes the area of the triangle element.
Chapter 5 Vibroacoustic study of composite sandwich cylinders at low frequencies

Figure 5-1 Rectangle and triangle elements and the position of their respective gauss points

Mass matrix:

\[ M^e = M_f^e + M_c^e \]  \hspace{1cm} (5.12)

where \( M_f^e \) and \( M_c^e \) denote the transitional mass matrix and the rotational mass matrix respectively, and they can be formulated by:

\[ M_f^e = N N^T (\rho_f t_{f1} + \rho_f t_{f2} + \rho_c t_c) \]  \hspace{1cm} (5.13)

\[ M_c^e = N N^T I_e = N N^T \left( \int_{-t_{f2}+t_{f1}}^{t_{f1}} \rho_f h^2 dh + \int_{-t_{f2}+t_{f1}}^{t_{f1}} \rho_c h^2 dh + \int_{t_{f2}}^{t_{f1}} \rho_f h^2 dh \right) \]  \hspace{1cm} (5.14)

where \( \rho_f \) and \( \rho_c \) represent the density of the skin and the core respectively, \( t_c, t_{f1}, t_{f2} \) are the thickness of the core, the upper skin and the lower skin respectively.

Finally the stiffness matrix and mass matrix can be computed as:

\[ K = \sum_{e=1}^{N_{ele}} K^e W_{gau} |J| \]  \hspace{1cm} (5.15)

\[ M = \sum_{e=1}^{N_{ele}} M^e W_{gau} |J| \]  \hspace{1cm} (5.16)

where \( N_{ele} \) denotes the total number of the elements. Their matrix sizes are dependent on the node number of each element and the degree of freedom. For example, for a four-node element with five degree of freedom at each node, the \( K^e \) and \( M^e \) are square matrices with sizes of \( 20 \times 20 \). \( W_{gau} = [1, 1, 1, 1] \), which represents the weight factor of each Gauss point. \( J \) represents the Jacobian matrix, which is used to transform the local coordinates to the global coordinates, can be expressed as:

\[ J = \begin{bmatrix} \frac{\partial x}{\partial \xi} & \frac{\partial y}{\partial \xi} \\ \frac{\partial x}{\partial \eta} & \frac{\partial y}{\partial \eta} \end{bmatrix} = [x] \times \left[ \begin{bmatrix} \frac{\partial N}{\partial \xi} \\ \frac{\partial N}{\partial \eta} \end{bmatrix} \right] \]  \hspace{1cm} (5.17)

where \( x, y \) represent the global coordinates and they are vectors for a rectangle element or for a triangle element, respectively. The formulation of the FE system becomes:
\[(K + i\omega C - \omega^2 M)w + LP_a = F\]  
\[C = a_1K + a_2M\]

where \(C\) is the damping matrix, \(a_1\) and \(a_2\) are the Rayleigh (proportional) damping factors. \(w\) and \(F\) represent the structural response and force vectors respectively. \(w = [u_z, u_\phi, u_\tau, \psi_z, \psi_\phi]^T\). \(P_a\) denotes the pressure of coupled BE nodes. \(L\) denotes the global coupling matrix which transforms the fluid pressure into mechanical forces that act on the finite elements, which can be expressed as:

\[L = \int_S N_j^T \hat{n} N_b dS\]

where \(\hat{n}\) represents the unit normal direction. \(N_f\) and \(N_b\) are the shape functions of finite elements and boundary elements. Here the FE mesh and the BE mesh are identical, thus both \(N_f\) and \(N_b\) can be expressed by Equation(5.9) or Equation(5.10).

5.2.2. BE model for the fluid

The BE formulation can be developed by discretizing the continuous Helmholtz integral equation to a discrete system. The Helmholtz integral equation is [12]:

\[c \cdot p(R) = \int_S (p(R_0) \frac{\partial g}{\partial n_0} - g(|R - R_0|) \frac{\partial p}{\partial n_0}) dS\]

\[g(|R - R_0|) = \frac{e^{-ik|R - R_0|}}{4\pi|R - R_0|}\]

\[v = \frac{i}{\rho_0 \omega} \frac{\partial p}{\partial n_0}\]

where \(g\) is the Green’s function, \(S\) the boundary surface, and \(k\) the wavenumber. \(R_0\) represents the points located at the boundary surface. \(R\) denotes the points set inside the boundary domain. \(p\) is the sound pressure at the points. \(v\) is the normal velocity of points on the boundary surface. \(c\) is a factor dependent on the position of \(R\): \(c=1\) when \(R\) is inside the domain, \(c=1/2\) when \(R\) is on the smooth boundary of fluid domain, \(c=\Omega/4\) when \(R\) is at the nonsmooth edge, \(\Omega\) is the edge angle. Considering a constant element on a smooth surface, whose \(p\) and \(v\) are assumed to be constant, \(c=1/2\), and the Equation(5.21) can be discretized as:

\[\frac{1}{2} p_j - \sum_{j=1}^{N} (\int_S g dS) p_j = i \rho_0 \omega \sum_{j=1}^{N} (\int_S g dS) v_j\]

where \(j\) refers to elements on the boundary surface, and \(i\) refers to receiver nodes, \(N\) is the total number of the elements. Since it is assumed that \(p\) and \(v\) are constant over each
element, they are labeled as \( p_j \) and \( v_j \) for element \( j \). It should be noted that non-unique solutions problems would occur in solving the Equation (5.24). The Burton-Miller formulation [13], which is a linear combination of the Helmholtz integral equation and its normal derivative equation, is usually used to avoid the non-unique problem.

For the computation convenience, two matrices are introduced:

\[
\begin{align*}
H_{ij} &= \int_{S_j} \hat{g} dS \\
G_{ij} &= i\rho_0 \omega \int_{S_j} g dS 
\end{align*}
\]  

(5.25)

By introducing

\[
H_{ij} = \frac{1}{2} \delta_{ij} - \hat{H}_{ij}
\]  

(5.26)

where \( \delta_{ij} \) is Kronecker delta, \( \delta_{ij} = 0 \) when \( i \neq j \), and \( \delta_{ij} = 1 \) when \( i = j \). Thus \( H_{ij} = \hat{H}_{ij} \) when \( i \neq j \), \( H_{ij} = 1/2 \) when \( i = j \) [12]. Equation (5.24) can be rewritten in matrix form as:

\[
Hp = Gv
\]  

(5.27)

By dividing the BE nodes into the FE-coupled nodes (set \( \mathbf{a} \)) and the uncoupled nodes (set \( \mathbf{b} \)), Equation (5.27) can be expressed as:

\[
\begin{bmatrix}
H_{11} & H_{12} \\
H_{21} & H_{22}
\end{bmatrix}
\begin{bmatrix}
p_a \\
p_b
\end{bmatrix}
= \begin{bmatrix}
G_{11} & G_{12} \\
G_{21} & G_{22}
\end{bmatrix}
\begin{bmatrix}
v_a \\
v_b
\end{bmatrix}
\]  

(5.28)

Implementation of \( H \) and \( G \) into the element results in:

\[
H_{ij}^e = \begin{cases} 
-\int_{S_j} (\nabla g)^T \hat{n} dS & \text{when } i \neq j \\
1/2 & \text{when } i = j 
\end{cases}
\]  

(5.29)

\[
G_{ij}^e = i\rho_0 \omega \int_{S_j} gN dS
\]  

(5.30)

\[
\nabla g^T = [-d_x, -d_y, -d_z]e^{-id} \frac{(ik + 1/d)}{4\pi d^2}
\]  

(5.31)

\[
d_x = x - x_g, d_y = y - y_g, d_z = z - z_g
\]  

(5.32)

\[
d = \sqrt{d_x^2 + d_y^2 + d_z^2}
\]  

(5.33)

where \( \hat{n} \) is the unit normal vector, \( g \) is the Green’s function (Equation (5.22)), \( N \) is the shape function (Equation (5.9) for the rectangle element and Equation (5.10) for the triangle element), \( \rho_0 \) is the density of the air.
5.2.3. The structure/fluid (FE/BE) coupling

The normal velocity of the coupled BE nodes, \(v_a\), can be expressed with the normal displacement as follows:

\[
v_a = i \omega T w
\]

where \(T\) is the transfer matrix which distills the normal displacement from the structural response \(w\). Finally, the coupled BE/FE equations can be obtained by combining Equation (5.18) and Equation (5.28):

\[
\begin{bmatrix}
K + i \omega C - \omega^2 M & L & 0 \\
-i \omega G_{11} T & H_{11} & H_{12} \\
-i \omega G_{21} T & H_{21} & H_{22}
\end{bmatrix}
\begin{bmatrix}
w \\
p_a \\
p_b
\end{bmatrix} =
\begin{bmatrix}
F \\
G_{12} v_b \\
G_{12} v_b
\end{bmatrix}
\]

(5.35)

where the subscript \(a\) denotes the uncoupled BE nodes and the subscript \(b\) denotes the coupled BE nodes.

5.2.4. Sound pressure at the interior field points and ATV

The structural response \((v_j=i\omega w)\) and the pressure \((p_j=[p_a, p_b]^T)\) at the boundary surface element can be solved from Equation (5.35), and the pressure at the interior field points can be obtained by:

\[
p_i = \sum_{j=1}^{N} H_{ij} p_j + i \rho_0 \omega \sum_{j=1}^{N} G_{ij} v_j
\]

(5.36)

Note that the subscript \(i\) denotes the field points and the subscript \(j\) denotes the FE/BE nodes. According to Equation (5.27), \(p_j\) can be expressed in terms of \(v_j\) as the following:

\[
p(\omega) = H^{-1}(\omega)G(\omega)v(\omega)
\]

(5.37)

Combining Equation (5.36) and Equation (5.37), \(p_i\) can be expressed in terms of \(v_j\) as the following:

\[
p_i(\omega) = i \rho_0 \omega \left[ H_{ij} H_{ij}^{-1}(\omega)G_{ij}(\omega) + G_{ij}(\omega) \right] v_j(\omega)
\]

(5.38)

Let \(ATV = i \rho_0 \omega \left[ H_{ij} H_{ij}^{-1}(\omega)G_{ij}(\omega) + G_{ij}(\omega) \right]^T\), the pressure at field points \(p_f\) (matrix composed of \(p_i\)) can be determined in terms of the surface velocities \(v\) (matrix composed of \(v_j\)) as:

\[
p_f(\omega) = ATV^T(\omega) \cdot v(\omega)
\]

(5.39)

ATV links the input of the structural velocity of the radiating surfaces and the sound pressure levels at the desired output field points. It depends on the following system
parameters: geometry of the vibrating surfaces, acoustic treatment of the surfaces (impedance or admittance), microphone (field point) location, frequency, physical properties of the acoustic medium (speed of sound & density). However, it is independent of the operational structural response and the loadings. This represents an enormous advantage in that loading and design parameters can be varied without having to run original solvers again, as long as the acoustic model is not changed. Thus the ATV conception is used in this chapter to minimize the inner pressure of a cylinder by varying the structural parameters.

5.3. Validation of the FEM/BEM model for the sandwich cylinder

The experimental sound insulation performances of a sandwich cylinder under an acoustic excitation and that under a force excitation are introduced in Chapter 4. In this section, the experimental results at low frequencies are used to validate the FEM/BEM model for predicting the sound transmission performances of the sandwich cylinders at low frequencies.

5.3.1. Numerical model parameters

The cylinder in the numerical model has the same geometry with the one in the experimental test, which has a radius of 0.25 m and a length of 1 m. As shown in Figure 5-2, since the cylinder was excited by a point source at the symmetry plane in the axial direction, only a quarter of its geometry was modeled. Shell elements with laminated properties represented the sandwich cylindrical structure. The cylindrical surface was meshed with rectangle elements, and the two end caps were meshed with triangle elements. The mesh of the BE model was identical to that of the FE model. The material properties referred to Table 4-1 and the damping loss factor of the sandwich cylinder was set as 0.02. For the acoustic excitation case, a point source of white noise was used to simulate the speaker in the experimental test. The application point of the point source was at [-0.27, 0, 0]. For the force excitation case, a point force was applied on the cylinder outer surface in the normal direction. The application point was at [-0.25, 0, 0]. The mesh of the field points are the same with the measurement position of the microphones (see Section 4.2.2). There are 18 field points in the model as shown in Figure 5-2.
5.3.2. Comparison under a point acoustic excitation

Under the acoustic excitation, the noise reduction between the outer pressure and the inner pressure was used to characterize the sound insulation in the numerical model. As shown in Figure 5-3, the predict NR is consistent with the measured NR at most frequencies in the range of 0-1000 Hz.
Figure 5-4 Predicted displacement and noise reduction of a laminated cylinder and a sandwich cylinder (a step of 5 Hz)

Similar to the comparison between the experimental structural response and the experimental NR of the laminated cylinder and the sandwich cylinder in Figure 4-6, the predicted structural response and the predicted NR of the two cylinders are present in Figure 5-4. It can be found that although the displacement curve of the laminated cylinder shows much more peaks than that of the sandwich cylinder, their NRs do not show large difference. Many structural resonances of the laminated cylinder do not contribute to the sound transmission at 1-600 Hz. This is caused by the fact that the radiation efficiency is low in this frequency range. In addition, although the sandwich cylinder had a larger weight than the laminated cylinder, its NR was similar with the laminated one. This confirms that the mass has little influence on the sound transmission at low frequencies. At 600-1000 Hz, the resonance peaks of the displacement curve have more significant influences on the NR curves and many dips of the NR curves correspond to the peaks in the peaks of the displacement curves.

As labeled in Figure 5-4, the NR of the laminated cylinder and the NR of the sandwich cylinder share common dips and these dips correspond to the resonances of the cylindrical cavity. This implies that the cavity resonances play important roles on the sound transmission of the cylinders. Li [14] found that attachment of some acoustic resonators on the inner surface cylinder could improve the NR at the dips. The acoustic resonators usually expose very low acoustic impedance at their opening and make the sound level drop in the vicinity. In addition, the rather high impedance at the closed end of the resonator can make the sound level inside the resonator goes up and the energy is dissipated into heat. With the location of the resonators at the place corresponding to the acoustic cavity mode of the cylinder, the NR was increased by 2-9.17 dB at the dips because of cavity resonances.
5.3.3. **Comparison under a point force excitation**

The sandwich cylinder was modeled using FEM/BEM and its spatially averaged inner pressure under a force excitation was predicted. The geometry, properties and loading case were the same with the experiment. The predicted and the experimental results were compared to each other, as shown in Figure 5-5. It can be found that the coupled FE/BE model give a satisfactory prediction on the inner sound pressure of the sandwich cylinder at most frequencies. There is a discrepancy of the first peak at low frequencies (at approximately 200 Hz). The difference could be due to the mount conditions in the experimental test.

![Figure 5-5 Spatially averaged inner sound pressure of the sandwich cylinder in a narrow-band frequency range](image)

*(glass/epoxy facing, 90/90/90/Tycor® /90/90/90, \( t_f=3.18\text{mm}, t_c =15\text{mm} )* 

5.4. **Influence parameters of the inner sound pressure**

At low frequencies, the structural properties show less significant influence on the sound transmission under a point acoustic excitation than that under a force excitation. Thus a parametric study on the sound insulation of the sandwich cylindrical shell was conducted under a force excitation. The validated FE/BE model in Section 5.3.3 was used for study. The parameters included the core type, the core thickness, the sandwich layup and the fiber orientation. The interested frequency range was from 400 to 2000 Hz, in 1/6 octave steps.

5.4.1. **The effect of core types**

The core shear stiffness \( G_c \) plays a vital role on the dynamic response of sandwich structures. Therefore three different kinds of core were chosen to investigate the influence of mechanical properties of the core on the sound insulation. As shown in Table 5-1, the mass densities of the four kinds of cores are nearly equal so that the
influence of the mass on the sound transmission can be neglected. Among the three kinds of cores, the aluminum honeycomb has the largest \( G_c \) and the Rohacell® foam has the smallest \( G_c \). In Figure 5-6, the results show that the sandwich structure with the Ty cor® foam core has the best sound insulation at frequencies below 1000 Hz, and sandwich with Aluminum honeycomb has the best insulation at 1000-2000 Hz. The phenomenon can be explained by the following:

The dynamic deflection decreases as \( G_c \) increases. On the other hands, it is found that the radiation efficiency increases as \( G_c \) increases below the coincidence frequency (around 1000 Hz). The Ty cor® sandwich has an optimal balance between the structural response and the radiation efficiency, and thus it has the best insulation below 1000 Hz. While at frequencies above coincidence frequency, the radiation efficiency decreases as the \( G_c \) decreases and the aluminum honeycomb sandwich has the best insulation.

**Table 5-1 Properties of three different kinds of cores**

<table>
<thead>
<tr>
<th>Name</th>
<th>Type</th>
<th>Density([kg/m^3])</th>
<th>( E_c )[MPa]</th>
<th>( G_{L} )[MPa]</th>
<th>( G_{W} )[MPa]</th>
<th>( G_{12} )[MPa]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Aluminum honeycomb</td>
<td>3/16 – 5052 – .0015</td>
<td>70</td>
<td>999.7</td>
<td>468.8</td>
<td>206.8</td>
<td>10</td>
</tr>
<tr>
<td>Ty cor® foam</td>
<td>glass fibers/PVC foam</td>
<td>70</td>
<td>151</td>
<td>68.7</td>
<td>68.7</td>
<td>68.7</td>
</tr>
<tr>
<td>Rohacell® foam</td>
<td>71A/ PMI</td>
<td>75</td>
<td>92</td>
<td>29</td>
<td>29</td>
<td>29</td>
</tr>
</tbody>
</table>

Note: for the honeycomb, \( E_c \) denotes the compressive modulus, \( G_{L} \) and \( G_{W} \) denotes the transverse shear stiffness in the length and width direction, here is the length direction is the axial of the cylinder, the in-plane shear stiffness \( G_{12} \) of the honeycomb is neglected in most cases, here \( G_{12}=10 \) MPa is used for the computation

![Figure 5-6 Spatially and frequency averaged inner sound pressure for various core types in 1/6th octave bands](image)

(five frequencies averaged in each band, the facings are all glass/epoxy with layup 90/90/90/core/90/90/90, \( t_f =3.18 \)mm, \( t_c =15 \)mm)
5.4.2. The effect of core thickness

The core thickness ($t_c$) was changed in three levels (5 mm, 15 mm and 25 mm) to form three sandwich cylinders. The skin thickness was varied accordingly to keep the total sandwich weight constant. The skin thickness and stiffness matrices of the three sandwiches are shown in Table 5-2. It can be seen that the membrane stiffness $A$ has a little change and bending stiffness $D$ increases as the core thickness increases. The spatially averaged sound pressure inside the three sandwich cylinders are shown in Figure 5-7. It can be seen that the inner sound pressure of the cylinder with $t_c = 15$ mm is the lowest below 1200 Hz and the cylinder with $t_c = 25$ mm has the lowest inner sound pressure above 1200 Hz. The cylinder with $t_c = 5$ mm has the worst sound insulation at most frequencies because of its low bending stiffness. The lowest bending stiffness leads to a high structural response of the cylindrical structure.

Table 5-2 Mechanical properties of the different sandwich structures

<table>
<thead>
<tr>
<th></th>
<th>$A$ [MPa.m]</th>
<th>$B$ [Pa.m$^2$]</th>
<th>$D$ [Pa.m$^3$]</th>
</tr>
</thead>
<tbody>
<tr>
<td>$t_c$=5mm</td>
<td>20.4 6.09 0</td>
<td>232 69 0</td>
<td>69 1560 0</td>
</tr>
<tr>
<td></td>
<td>6.09 133 0</td>
<td>0 0 172</td>
<td></td>
</tr>
<tr>
<td>$t_c$=15mm</td>
<td>20 5.9 0</td>
<td>1250 373 0</td>
<td>0 0</td>
</tr>
<tr>
<td>(symmetrical)</td>
<td>5.9 120 0</td>
<td>373 8190 0</td>
<td>0 0</td>
</tr>
<tr>
<td></td>
<td>0 0 14 0</td>
<td>0 0 916</td>
<td></td>
</tr>
<tr>
<td>$t_c$=25mm</td>
<td>19 5.7 0</td>
<td>2880 859 0</td>
<td>0 0</td>
</tr>
<tr>
<td></td>
<td>5.7 108 0</td>
<td>859 18300 0</td>
<td>0 0</td>
</tr>
<tr>
<td></td>
<td>0 0 13 0</td>
<td>0 0 2080</td>
<td></td>
</tr>
<tr>
<td>$t_c$=15mm</td>
<td>20 5.9 0</td>
<td>-8.4E4 -2.5E4 0</td>
<td>1160 348 0</td>
</tr>
<tr>
<td>(unsymmetrical)</td>
<td>5.9 120 0</td>
<td>-2.5E4 -5.8E5 0</td>
<td>348 7570 0</td>
</tr>
<tr>
<td></td>
<td>0 0 14 0</td>
<td>-6.3E4 0 0</td>
<td>848 0</td>
</tr>
</tbody>
</table>

Figure 5-7 Spatially and frequency averaged inner sound pressure for various core thickness in 1/6th octave bands

(five frequencies averaged in each band, the facings are all glass/epoxy with layup 90/90/90/Tycor®/90/90/90)
Below 1200 Hz, as $t_c$ was increased from 15 mm to 25 mm, the sound insulation performance was not further improved although the bending stiffness was increased. It is because that the radiation efficiency increases as the $t_c$ increases, thereby counteracting the sound insulation effect. The radiation efficiencies of the three sandwich cylinders were calculated using the SEA method, as shown in Figure 5-8. It can be found that radiation efficiency of the sandwich cylinder with $t_c=25$ mm is the largest below 2000 Hz. In addition, the coincidence frequency decreases as the core thickness increases, i.e., it is about 6000 Hz for $t_c=5$ mm, and about 2000 Hz for $t_c=25$ mm. The reason is that the coincidence frequency is proportional to the mass-to-stiffness ratio, and an increase of $t_c$ leads to a decrease of the mass-to-stiffness ratio.

Above 1200 Hz, the radiation efficiency and of the cylinder with $t_c=25$ mm is the lowest, and its surface velocity is the lowest because of the highest bending stiffness. Therefore the cylinder with $t_c=25$ mm has the lowest sound pressure in this frequency range.

Figure 5-8 Radiation efficiency of sandwich cylinders with different core thickness

5.4.3. The effect of sandwich layup

An unsymmetrical sandwich layup was compared with a symmetrical layup, and their mechanical properties are shown in Table 5-2. Comparing the unsymmetrical layup to the symmetrical layup, it can be found that A is not changed and D decreases slightly. The most significant change is that the bending-extensional coupling matrix B is not zero anymore. This leads to a significant decrease of the sound insulation, as shown in Figure 5-9. In most cases, if B is non-zero, an in-plane wave causes an out-plane wave as well, which is not beneficial for vibration and noise control. Whitney [15] studied the effect of bending-extensional coupling of laminated plates and concluded that the coupling could increase the maximum deflection by as much as 300%. In addition, the
bending-extensional coupling may also cause some unexpected damage to the structure. Thus the unsymmetrical layup is usually avoided in the engineering design.

![Figure 5-9 Spatially and frequency averaged inner pressure for different sandwich layups in 1/6th octave bands](image)

(five frequencies averaged in each band, the facings are all glass/epoxy, $t_f=3.18\text{mm}$, $t_c=15\text{mm}$)

5.4.4. The effect of fiber orientation in the facings

The fiber orientations in the facings were varied to investigate its influence on the sound insulation of sandwich cylinders. Simulations were performed for three layups as shown in Figure 5-10. The value of 0 denotes that the fibers are along the hoop direction and 90 denotes along the axial direction of the cylinder.

![Figure 5-10 Spatially averaged inner pressure for different fiber orientation in 1/6th octave bands](image)

(five frequencies averaged in each band, the facings are all glass/epoxy, $t_f=3.18\text{mm}$, $t_c=15\text{mm}$)
Results showed that the curve representing the layup of 0/10/20/core/70/80/90 presented more fluctuations than the other two curves. This could attribute to the bending-extensional coupling of the unsymmetrical laminated layup. With respect to the comparison of fiber orientation, the 0 degree showed a similar trend with the 90 degrees below 1024 Hz. However, it is difficult to determine which orientation shows better sound insulation than the other in a broad frequency range (400-2000 Hz). In spite of this phenomenon, the fiber orientation has a large influence on the sound insulation in a narrow band frequency. For example, the transferred pressure difference between the 0/0/0/core/0/0/0 and the 0/10/20/core/70/80/90 is about 15 dB at 1093 Hz. Thus the fiber orientation could be a useful parameter for sound optimization in a narrow frequency range, while it is not effective in situations with broad band excitations.

5.5. Minimization of inner pressure of the sandwich cylinder

An optimization tool which integrated the FEM/BEM, ATV method and the Genetic Algorithm (GA) method was used to minimize the inner pressure of the sandwich cylinder. GA was used here as the search technique, because it is most powerful when the objective function and constraints are highly nonlinear, and contain many local optima and discontinuities in the design domain. In such situations, gradient based techniques can fall into a local optima near the starting configuration, while GA can avoid it and reach a more global optimum [16].

5.5.1. Introduction of GA

GA is a searching/selection method based on the biological evolution according to Darwin's genetic selection and natural selection. As shown in Figure 5-11, a set of Population is randomly generated and the search process starts from their initial solutions. Each individual in the Population is a solution of the problem, known as Chromosome. A chromosome is a string of symbols, such as a binary string. These chromosomes are evolving in subsequent iterations and the evolution can be done through Crossover or Mutation among previous generation. The Fitness function is used to evaluate whether the next generation of chromosomes is good or bad. Since the population size is constant, chromosomes with high fitness values have a higher probability to be selected. Finally the algorithm converges to the best chromosome (solution) after several generations.
5.5.2. The optimization

The objective of the optimization was to minimize inner pressure of the sandwich cylinder that mentioned in Section 5.3. The pressure at the field point with $x=0.1$, $y=0.1$, $z=0.1$ was chosen as a point of interest. The frequency range was from 400 Hz to 1000 Hz with a third octave step. Six variables were defined: the fiber orientation in each layer ($\text{angle}$), the core thickness ($t_c$), the facing layer thickness ($t_{f\_ply}$) and the core shear stiffness ($G_c$). The fiber angles were symmetric with respect to the core layer and the thickness of each facing layer was the same. An overview of the optimization is given as follows:

\[
\text{Minimize} \quad \sqrt[5]{\frac{1}{5}(p_1^2 + p_2^2 + p_3^2 + p_4^2 + p_5^2)}
\]

\[
\text{Subject to} \quad \text{mass} = N t_{f\_ply} \rho_f + t_c \rho_c < 8 \text{ Kg/m}^2
\]

- $-90 < \text{angle}_1, \text{angle}_3, \text{angle}_5 < 90$, $10 \text{ MPa} < G_c < 100 \text{ MPa}$
- $10 \text{ mm} < t_c < 50 \text{ mm}$, $0.1 \text{ mm} < t_{f\_ply} < 1 \text{ mm}$,

where the $p_i$ is the pressure at different frequencies. $N$ denotes the layer number for the facings, here $N=3$.

The optimization of the inner pressure was performed with the following procedures. The flowchart of the GA optimization is shown in Figure 5-12. Firstly the ATV matrix which describes the relationship between the sound pressure and the cylindrical surface.

![Flowchart of Genetic Algorithm (GA)](image-url)
velocity was obtained by BEM. The surface velocities were computed with FEM using Patran/Nastran. Finally the GA optimizer conducted the modification of the material properties and the computation of the velocity iteratively. The Matlab codes for the optimization method are shown in Appendix E.

By adopting the described optimization method a large amount of computation time can be saved because the ATV matrix is solved only once. In this optimization example, the computation took about 1 minute using the Intel i7-2637M dual 1.7Hz CPU for each iteration. When the ATV concept was not used, the computation of the full FEM/BEM model took about 30 minutes for each iteration. Note that the mentioned computation efficiency difference is only for a simple cylinder model and much more time would be saved for more complex models in reality.

Two optimizations with different GA parameters were computed. In the first optimization, the population size was ten at each generation and there was nine generations. In the second optimization, the population size was twenty and there was five generations. Figure 5-13 shows the progress of the objectives of the two optimizations at each generation. The curve labeled with ‘best’ denotes the fittest individual in the population and the other curve labeled with ‘average’ represents the average value of the population. In Figure 5-13a, the best one is much lower than the average value at the first generation. This means that the selection among the population is effective. In addition, for the best individual, no significant progress is found as the generation increases. This implies that there is no need to run further generation. As the more generations the more computation time, the population size is increased while the generation is decreased in the second optimization. To increase the possibility of the evolution among the generations, the crossover factor was increased from 0.4 to 0.7 and

![Figure 5-12 Flowchart of the optimization of inner pressure](image-url)
the mutation factor was increased from 0.005 to 0.01. The population size was increased from 10 to 20 for a broader selection among the population at each generation. As shown in Figure 5-13b, there is a larger difference between the average value and the best value compared to the first optimization. This can be explained by the increased crossover factor and the increased population size. However, the best value of the two optimizations are much agreeable, both of which are close to 80 dB at the final generation.

Figure 5-13 History of the objective in two GA optimizations
(a: population size is 10, crossover factor=0.4, mutation factor=0.005. b: population size is 20, crossover factor=0.7, mutation factor=0.1)

Figure 5-14 The inner pressure of three cases with different parameters

Table 5-3 Values of six design parameters after two optimizations

<table>
<thead>
<tr>
<th></th>
<th>angle1</th>
<th>angle2</th>
<th>angle3</th>
<th>t1 [mm]</th>
<th>Gc [MPa]</th>
<th>tf [mm]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Origin</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>15</td>
<td>68</td>
<td>3.1</td>
</tr>
<tr>
<td>First optimization</td>
<td>-85</td>
<td>52</td>
<td>58</td>
<td>17</td>
<td>25</td>
<td>4</td>
</tr>
<tr>
<td>Second optimization</td>
<td>69</td>
<td>68</td>
<td>48</td>
<td>15</td>
<td>39.5</td>
<td>5.8</td>
</tr>
</tbody>
</table>
The inner pressure at different frequencies is compared for three different cases: the original, the first optimization and the second optimization. The parameters and the results of the three cases are shown in Figure 5-14. The values of the six design parameters after the optimization are compared with those in the original one in Table 5-3. It can be found that the core thickness $t_c$ is not changed too much in the two optimizations (15 mm VS 17 mm), which proved that there is an optimum $t_c$ for the best sound insulation. In addition, compared to the original layup, the core shear stiffness is decreased and the facing thickness is increased after optimizations.

Although the objective value of the first optimization and the second optimization is nearly the same, their inner pressures show a distinct difference at different frequencies, especially at 400 Hz and 976 Hz. This is caused by that the objective was the root mean square (RMS) value of pressure at the five frequencies. Compared with the original case, the first optimization exhibits a significant decrease at 500 Hz and 781 Hz while it has a larger value at 400 Hz and 976 Hz. The second optimization has a larger value at 400 Hz while much lower value at 781Hz and 976 Hz. It is difficult to conclude which one of the two optimizations is better than the other in that different criteria exist at different circumstances. If the main purpose is to control the highest level of all the frequencies to be, i.e., $< 85$ dB, then the first optimization is better. On the other hand, if the main purpose is to reduce the noise level at all frequencies, the second optimization show a better performance. In conclusion, the optimization tool ATV&GA has been proved to be efficient and robust for minimizing the inner pressure of sandwich cylinders.

5.6. Multi-objective optimization of a fuselage section: a case study

A multi-objective optimization was conducted for a sandwich fuselage section. The Objectives included the weight and the inner sound pressure. The constraints were buckling and materials failure under a typical flight load. The optimization tool integrated the FEM/BEM and the multi-objective GA method.

5.6.1. Introduction of the optimization algorithms

In most real-life optimization situations, there is no unique optimal solution (Pareto solution) for the multiple-criteria or multi-objective problems. The function `gamultiobj()`, which is an optimization tool based on the Non-dominated Sorting Genetic Algorithm II (NSGA-II) in Matlab, was used for the multi-objective optimization. The `gamultiobj` solver attempts to create a set of Pareto front for a multi-objective minimization [17]. The Pareto front, which denotes the set of all Pareto solutions, gives a trade-off limit as an aid of decision making for the complex optimization. The flow chart of the optimization system is shown in Figure 5-15. The global buckling and the strain were computed using MD Nastran during the optimization. The input data and output data of Nastran were read by Matlab. The inner pressure was calculated in Matlab using
the ATV and the structural velocity from Nastran, according to Equation. In this way, the `gamultiobj` in Matlab could control the optimization loop for both the mechanical analysis and the acoustic analysis.

![Flowchart of the multi-objective and multi-discipline optimization in Matlab](image)

**Figure 5-15 Flowchart of the multi-objective and multi-discipline optimization in Matlab**

### 5.6.2. Case description

As shown in Figure 5-16, a fuselage section composed of sandwich composites was studied for the optimization. The facing material was carbon/epoxy and its mechanical properties are: $E_1=1.81\times10^9\,\text{Pa}$, $E_2=1.03\times10^9\,\text{Pa}$, $G_{12}=7.17\times10^8\,\text{Pa}$, $\rho_f=1600\,\text{kg/m}^3$, $\nu_{12}=0.28$.

The core material was rohacell200 and its mechanical properties are: $E_c=3.5\times10^8\,\text{Pa}$, $G_c=1.5\times10^8\,\text{Pa}$, $\rho_c=205\,\text{kg/m}^3$. It should be noted that $G_c$ was set as one of the design variables, and $E_c$ was also changed according to $E_c=2G_c(1+\nu_c)$, where $\nu_c$ was kept as 0.3.

The sandwich layup was [90/0/45/core/45/0/90]. The layer thickness in the top facing $t_{f1\_layer}$, the core thickness $t_c$ and the layer thickness in the bottom facing $t_{f2\_layer}$ were design variables.

For the mechanical analysis, the fuselage was subjected to a combined loading case including a bending moment, a torsion moment, a shear force and an inner pressure. A
simply supported boundary condition was applied at one end of the cylinder, and the other end was free. The combined loading was applied at the free end of the cylinder. The global buckling and the material failures were set as the design constraints. For the computations of the buckling and the materials failures of composite sandwich cylinders, the reference was made to Chapter 2.

For the acoustic analysis, the cylinder was subjected to two symmetric point force excitations at 60Hz, because the mechanical excitation from the shaft engine is usually tonal and it is 60Hz in a turbo-prop aircraft according to [18]. The boundary condition was the same as the mechanical analysis. There were six seats in the fuselage section, and only three of them were chosen to observe the noise level because the cylinder was symmetric in the x direction. Three points were used to simulate the passengers’ ears, pressure $p_1$, $p_2$ and $p_3$, their position is shown in Figure 5-16. The spatial average noise level at 60 Hz was set as one of the objectives.

$$f_1_{layer} + f_2_{layer} + f_{cc} + f_{Nt}$$

where $N$ denotes the layer number for the facings, here $N=3$.

$$0.1 \text{ mm} < t_{f1\_layer} < 1 \text{ mm}, 20 \text{ mm} < t_c < 100 \text{ mm}, 0.1 \text{ mm} < t_{f2\_layer} < 1 \text{ mm},$$

$$0.38 \text{ MPa} < G_c < 384 \text{ MPa},$$

Global buckling: $|\lambda| > 1$
facing strain: $|\varepsilon_{11}|, |\varepsilon_{22}|, |\varepsilon_{12}| < 0.4\%$, core strain :$|\varepsilon_c| < 3.5\%$

5.6.3. Results and discussion

Four optimizations with different population sizes and generations were studied and their optimization results are shown in Figure 5-17. The Pareto front results imply that the total weight is competing with the internal noise pressure level. In addition, the influences of the population sizes and generations on the optimization results can be found by comparing the Pareto fronts of the four optimizations.

![Figure 5-17 Pareto fronts of four multi-objective optimizations with different parameters. Pop = population size, Gen = maximum generations in the optimization.](image)

As seen in Figure 5-17, the third optimization (opti 3) has the worst optimization results in that the sound pressure is higher than other optimizations with a comparable mass. The reason should be that the generation number of the third optimization is too low, which is only two. The fourth optimization (opti 4) has the largest diversity in the Pareto fronts and this is a characteristic indicating a better optimization. In addition, the second optimization (opti 2) also shows a better optimization result because its Pareto front is the lowest when the total weight is at the range of 9-12 kg/m$^2$. It can be learned from the Pareto front analysis that a better NSGA-II optimization would be achieved if a larger population size and more generations are set for the optimization. Moreover, it is good to run different optimizations with varied parameters so that better compromises between the sound pressure and the total weight can be chosen from all the Pareto fronts.

In order to investigate some common characteristics of the good optimizations, four points representing the best optimization results were selected, see Figure 5-17. The design variables of the four points are shown in Table 5-4. It can be found that although the parameter $G_c$ has a wide range to vary in the design (0.38-384 Mpa), its final value is located at a relatively narrow range (175-355 MPa) after the optimization. This indicates
that the optimization algorithm makes a compromise between the acoustic property and the mechanical requirement. Similarly, the core thickness is also confined to a narrow range after the optimization, which is in the range of 26.6 mm to 36.8 mm. In addition, the layer thickness of the outer facing is larger than that of the inner facing and the scale factor is 1.5-2.6. Eventually it is demonstrated that a multi-discipline and multi-objective optimization can be done using the developed codes in Matlab. Based on the obtained Pareto front, a further compromise can be made between the weight and the acoustic performance according to the other design requirements of the aircraft, such as the operational cost and the thermal insulation.

### Table 5-4 design variables and the corresponding objectives of the four points in Figure 5-17

<table>
<thead>
<tr>
<th>Point</th>
<th>( t_{f1_layer} ) (mm)</th>
<th>( t_1 ) (mm)</th>
<th>( t_{f2_layer} ) (mm)</th>
<th>( G_c ) (Mpa)</th>
<th>Weight(Kg/m²)</th>
<th>Noise level(dB)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.37</td>
<td>26.6</td>
<td>0.16</td>
<td>230</td>
<td>7.99</td>
<td>55.55</td>
</tr>
<tr>
<td>2</td>
<td>0.34</td>
<td>30.5</td>
<td>0.22</td>
<td>306</td>
<td>8.94</td>
<td>50.44</td>
</tr>
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<td>3</td>
<td>0.31</td>
<td>36.8</td>
<td>0.14</td>
<td>355</td>
<td>9.76</td>
<td>44.55</td>
</tr>
<tr>
<td>4</td>
<td>0.85</td>
<td>32.7</td>
<td>0.32</td>
<td>175</td>
<td>12.30</td>
<td>40.97</td>
</tr>
</tbody>
</table>

### 5.7. Conclusions

A coupled FE/BE model was built to predict the inner pressure of the cylinder under the acoustic or the mechanical excitation. Shell elements were used to build the FE model and the laminate theory was used to compute the stiffness matrix. A parametric study was carried out on the sound insulation of sandwich cylinders and two optimizations were carried out to improve the sound insulation of sandwich cylinders.

The FE/BE models for predicting the sound transmission of sandwich cylinders at low frequencies were validated by experimental measurements. The validations include both the acoustic excitation and the force excitation cases. As compared to the force excitations, it was found that the modifications of the structural properties have less influence on the sound insulation under the acoustic excitations. Under the acoustic excitations, some modes shapes of the structural shell do not contribute to the sound insulation at 1-600 Hz, while the cavity resonances show significant influences on the sound insulation. However, the structure stiffness dominates the sound transmission at low frequencies in that it causes some dips on the noise reduction curve.

The following conclusions can be drawn based on the parametric study of sound insulation of the sandwich cylinder: The FE/BE models for predicting the sound transmission of sandwich cylinders at low frequencies have been successfully validated by experimental measurements. The influence parameters on the sound insulation of the sandwich cylinder were studied and it was found that: The coincidence frequency, \( f_c \), is an important characteristic for the sound transmission of sandwich structures. A higher bending stiffness leads to higher radiation efficiency below the \( f_c \) while it leads to lower radiation efficiency above the \( f_c \). Therefore a higher bending stiffness is beneficial for the
noise control at high frequencies ($f_c$). At low frequencies ($< f_c$), there exists an optimal bending stiffness which can make a good compromise between the structural response and the radiation efficiency. In other words, increasing the core thickness to a certain extent (there is an optimal core thickness for the largest bending stiffness in case of constant weight) or the core shear stiffness can improve the TL at high frequencies while right values of the core thickness or core shear stiffness exist for the best sound insulation at low frequencies. The unsymmetrical layup causes the extensional-bending coupling, increasing the vibration of the structure and is not beneficial for sound insulation. The sound insulation of composite cylinders in a narrow frequency range can be improved by changing the fiber orientation. However, it is difficult to find an optimal fiber orientation layup to have the best sound insulation in a broad frequency range.

The optimization system which integrated the FEM/BEM, acoustic transfer vector (ATV) and genetic algorithm (GA) method was used for solving the multi-variable nonlinear problems. The optimization system proved to be efficient in that the calculation of the inner pressure using the ATV method was 30 times faster than that using the traditional coupled FEM/BEM method. The optimization system proved to be robust in that two optimizations with different GA parameters showed good agreement on the best solution. Although the core thickness was varied from 10 mm to 50 mm, it was 15-17 mm for the optimal layup in the two optimizations. This means there is an optimum value of the core thickness for the best sound insulation of sandwich cylinders.

A multi-objective optimization was done for a sandwich fuselage section as a case study. The FEM/BEM and the multi-objective GA method were used to make a multi-discipline (both the mechanical and acoustic analysis) and multi-objective optimization (minimization of weight and minimization of inner noise). A series of possible configurations representing compromises among the weight, the mechanical performance and the acoustic properties were achieved by using the developed optimization system. These configurations can be guidance for the fuselage design considering different requirements. Although the optimization system is only demonstrated with a simple fuselage section structure, the essential algorithm can be extended for the optimization work of more complex structures, and the parameters in the `gamultiobj` function can be varied to find more reasonable solutions of the practical problems.

5.8. References

Chapter 6  Sound absorption of sandwich structures with soft porous cores

The noise control treatment such as the addition of absorption layers is one of the common methods for the noise control of the transport vehicles. Thus the sound transmission of sandwich panels with open-cell foam was studied in this chapter. The transfer matrix method (TMM) was used for the TL prediction of sandwich panels with porous foams. This method was validated by the experimental results. A sensitivity study of the flow resistivity, tortuosity and porosity on the TL of sandwich panels was conducted. Then four kinds of absorption materials were studied for their influences on the TL of sandwich cylinders. Finally the TLs of a stiffened cylinder and a sandwich cylinder were compared in case of addition of absorption layers.

6.1. Introduction

Absorbent materials are used in many industries to improve sound insulation or sound absorptions. As shown in Figure 6-1, the traditional fuselage is composed of a skin, stiffeners & frames, an insulation layer and an interior wall. The absorption materials such as the fabric and the soft foam are usually used in the insulation layer to improve the sound insulation. One principal function of absorbent materials in-between two panels is to suppress acoustic resonances of the cavities that would otherwise strongly couple the two panels. Another principal function is to decouple the trim sheet (interior wall) from the vibration field induced in the outside structural shell by various acoustic and mechanical sources [1]. The porous materials usually have two phases: the solid skeletons and the fluid inside the skeletons. For the open-cell foam, the fluids are connected with each other, which is beneficial for the sound wave dissipation during the propagation. The skeletons of the porous materials usually have relatively low elastic modulus and shear modulus [2].

In this chapter, the effect of the addition of the absorption layer on the sound insulation of sandwich composites was studied. The parameters based on the Biot theory were explained in Section 6.2. The TL of typical sandwich panels with of absorption foam was predicted using the TMM in Section 6.3. In Section 6.4, the TMM was validated by experimental results. Meanwhile, the effects of different acoustic foam on the TL of sandwich cylinder were discussed. Finally a sandwich and a stiffened cylinder with absorption layers were compared.
6.2. Overview of parameters in Biot’s theory

It is generally difficult to study the sound propagation on a microscopic scale because of the complicated geometries of the skeletons. Thus statistical average is applied on a macroscopic scale. Biot [3-4] published a theory to describe the propagation of elastic waves through a fluid filled medium. The theory is widely used to compute the sound wave propagation in the porous materials. In this theory, two compression waves and a shear wave propagate in the porous medium, and thus three incident and three reflected waves contribute to the sound propagation through a medium with finite thickness. The acoustic behavior will be completely known if the amplitudes of the six kinds of waves are made clear. In addition, the following microstructure parameters of the porous materials affect the sound propagation.

6.2.1. Porosity

Porosity is the ratio of interconnected void volume to total volume of a material, see Equation (6.1). The value for porosity is difficult to obtain by experimental test. For example, with respect to open cell foams, it is hard to determine which cells are really open and interconnected; with respect to fibrous absorbers, it is hard to measure the exact volume of a compressible sample. Porosities of 95-98% are usually used for fibrous absorbers [5].

\[
\phi = \frac{V_a}{V_t} \tag{6.1}
\]

where \( V_a \) is the air volume in porous medium and \( V_t \) is the total volume.

Some possible methods to measure the porosity are introduced in [6]. One measurement method is to fill the sample pores with a liquid and measure the volume.
Note that this method may contaminate the sample and preclude making other parameter measurements. Another popular method is based on Boyle’s law for the ideal gas. As shown in Figure 6-2, the porous medium is sealed in a container with a volume of $V_1$ at atmospheric pressure $P_1$. The container is attached via a valve to another container with a volume of $V_2$, containing gas at a known pressure of $P_2$. When the valve connecting the two volumes is opened slowly, the gas flows from the right container to the left container, and the gas pressure in the two volumes equalizes to $P_3$.

Boyle’s Law states that the pressure times the volume for a system is constant in the isothermal condition, thus the following equation can be written:

$$P_1V_0 + P_2V_2 = P_3(V_2 + V_t)$$  \hspace{1cm} (6.2)

$$V_1 = V_0 + V_s$$  \hspace{1cm} (6.3)

where $V_0$ is the volume of the total gas inside the left container, which includes the gas contained by the porous medium and the remained gas outside the porous medium. Then the volume of skeleton $V_s$ can be computed by:

$$V_s = V_1 - \frac{P_2 - P_3}{P_3 - P_1}V_2$$  \hspace{1cm} (6.4)

If the bulk volume of the porous medium is determined as $V_t$ before the experiment, the porosity can be obtained by:

$$\phi = \frac{V_t - V_s}{V_t}$$  \hspace{1cm} (6.5)

It should be noted that this method is only suitable for the porosity measurement of interconnected cell samples, where the air or gas can flow freely.

**Figure 6-2 Sketch about the porosity measurement employing Boyle’s law**

(Helium is used because it can penetrate even the smallest pores)

6.2.2. Flow resistivity

Flow resistivity is one of the materials parameters for Biot model which represent flow rate of the porous materials. Flow resistivity can represent the difficulties of the propagation of sound in the gap inside porous materials. Flow resistivity can be expressed as:
where $p_2 - p_1$ is the pressure difference between two sides of the porous medium, $v$ is the mean flow velocity of air per unit area of materials and $d$ is the thickness of the sample. The measurement of flow resistivity can be conducted according to [7]. A concept sketch for the laboratory measurement can be seen in Figure 6-3. Driven by the vacuum or the air supply, there is an airflow through the sample. The velocity of the airflow $v$ can be measured by the flow meter, and the pressure difference $p_2 - p_1$ can be measured by the pressure gauge.

![Apparatus for measuring flow resistivity](image)

### Figure 6-3 Apparatus for measuring flow resistivity

#### 6.2.3. Tortuosity

Tortuosity characterizes the “non-straightness” of the pore structure of the porous material. In other words, the larger its tortuosity is, the more time a wave is in contact with the absorbent material. One method for testing the tortuosity $\alpha$ is to saturate the sample with an electrically conducting fluid (i.e. the fluid can be water) and to measure the electrical resistivity of the saturated sample, $r_s$, as well as the resistivity of the fluid alone, $r_f$. The tortuosity $\alpha$ can be computed as Equation (6.7). A detailed discussion on the measurement of the tortuosity using electrical resistivity method is given in [8]. Other testing methods for $\alpha$ by measuring the transmitted waves or reflected waves in the ultrasonic measurement were introduced in [9-10].

$$\alpha = \frac{r_s}{r_f}$$  \hspace{1cm} (6.7)

#### 6.2.4. Characteristic lengths $\Lambda$ and $\Lambda'$

In 1987, Johnson et al. [11] defined the viscous characteristic length $\Lambda$ for more general micro-geometries as:
where \( v_i(r_w) \) is the velocity of the fluid on the pore surface and \( S \) is the pore surface area; \( v_i(r) \) is the velocity inside the pores and \( V \) is the volume of the pores. In 1992, Allard and Champoux [12] defined the thermal characteristic length \( \Lambda' \) as:

\[
\Lambda' = 2 \frac{\int dV}{\int dS} = \frac{2V_{\text{pore}}}{S_{\text{pore-wall}}}
\]

\( \Lambda' \) is the twice of the ratio between the volume and the surface area in connected pores. Its difference from \( \Lambda \) is that there is no weighting by the squared velocity. As the names suggest, the viscous and thermal characteristic lengths are average macroscopic dimensions of the cells related to viscous and thermal losses, respectively. The former may be seen as an average radius of the smaller pores, and the later as the average radius of the larger pores (as shown in Figure 6-4).

![Figure 6-4 Illustration of the thermal and viscous loss](http://www.mecanum.com/def_char_lengths.html)

Leclaire et al. [13] tested the \( \Lambda \) and \( \Lambda' \) using standard ultrasonic and vacuum equipment. Subsequently, Fellah et al. [14] and Kino [15] also developed ultrasonic methods for the characteristic length measurement.

When measured values are not available, \( \Lambda \) and \( \Lambda' \) can be estimated by the following equations [5]:

\[
\Lambda = \frac{1}{c} \left( \frac{8\alpha \eta}{\varphi \sigma} \right)^{1/2}; \Lambda' = \frac{1}{c_1} \left( \frac{8\alpha \eta}{\varphi \sigma} \right)^{1/2}; c_1 = 1 / c
\]

where \( \alpha \) denotes the tortuosity, \( \varphi \) the porosity, \( \sigma \) the flow resistivity, \( \eta \) the velocity of the gas fluid and \( c \) is a constant that depends on the pore geometry. For identical cylindrical pores, \( c=1 \) and \( \Lambda = \Lambda' \).
6.3. TL prediction of a sandwich panel with open-cell foam

The transfer matrix method (TMM) is an effective tool for predicting the TL of multilayered systems [5]. The method can be illustrated by an example of a sandwich structure.

![Figure 6-5 Sandwich structure with a porous materials between two elastic plates](image)

Figure 6-5 depicts a plane wave incidence on a sandwich system, which is partly reflected, partly absorbed and partly transmitted. For the sandwich system, the transfer matrices for single layers include: the solid layer $T_{12}$, porous layer $T_{34}$ and solid layer $T_{56}$. The transfer matrices for the interfaces include: the fluid-solid interface $T_{01}$, solid-porous interface $T_{23}$, porous-solid interface $T_{45}$, and solid-fluid interface $T_{67}$. The matrices for the single layers and the interfaces can be coupled to form a global matrix, and then the pressure difference between $P_1$ and $P_2$ can be computed. The expressions of these matrices will be introduced in the section.

6.3.1. Transfer matrix of the porous layer

It is not straightforward to describe the acoustic field in a porous layer because of its complex microstructures and the fact that both forward and backward wave propagation coexist. In this situation, a matrix representation of sound propagation is widely used to predict the transmission loss or sound absorption of the porous layers [16-18]. Allard [19] defined six independent variables to describe the structural vibration and sound wave propagation of soft porous materials. The six variables are the velocity of the skeleton in $x_1$ and $x_3$ direction, $v_{1s}^s$ and $v_{3s}^s$; the velocity of the fluid in the porous materials in $x_3$ direction, $v_{3f}^s$; the stress of the skeleton, $\sigma_{33}^s$ and $\sigma_{13}^s$; and the stress of the fluid, $\sigma_{33}^f$. 
According to Biot’s theory, there are two compression waves and a shear wave in the porous medium. The skeleton displacement potentials of the two compression (longitudinal) waves can be written as:

\[
\varphi_1^s = A_1 \exp[i(\omega t - k_{13}x_3 - k_{1r}x_1)] + A_1^* \exp[i(\omega t + k_{13}x_3 - k_{1r}x_1)]
\]

\[
\varphi_2^s = A_2 \exp[i(\omega t - k_{23}x_3 - k_{2r}x_1)] + A_2^* \exp[i(\omega t + k_{23}x_3 - k_{2r}x_1)]
\]

(6.11) (6.12)

The skeleton displacement potentials of the shear (transverse) waves can be written as:

\[
\psi_2^s = A_3 \exp[i(\omega t - k_{33}x_3 - k_{3r}x_1)] + A_3^* \exp[i(\omega t + k_{33}x_3 - k_{3r}x_1)]
\]

(6.13)

The air (fluid) displacement potentials are related to the skeleton displacement potentials as

\[
\varphi_1^f = \mu_1 \varphi_1^s
\]

\[
\varphi_2^f = \mu_2 \varphi_2^s
\]

\[
\psi_2^f = \mu_3 \psi_2^s
\]

(6.14)

where \(\mu_1, \mu_2\) and \(\mu_3\) are the ratios between the fluid potentials and the skeleton potentials, which can be computed as:

\[
\mu_1 = \frac{P\delta_1^2 - \omega^2 \rho_{11b}}{\omega^2 \rho_{12b} - Q\delta_1^2}, \quad \mu_2 = \frac{P\delta_2^2 - \omega^2 \rho_{11b}}{\omega^2 \rho_{12b} - Q\delta_2^2}, \quad \mu_3 = \frac{-\rho_{12b}}{\rho_{22b}}
\]

(6.15)

\[
\rho_{11b} = \rho_a + j\sigma\phi^2 \frac{G(\omega)}{\omega}
\]

\[
\rho_{12b} = -\rho_a + j\sigma\phi^2 \frac{G(\omega)}{\omega}
\]

(6.16)

\[
\rho_{22b} = \rho_0 + \rho_a - j\sigma\phi^2 \frac{G(\omega)}{\omega}
\]

\[
\rho_a = \rho_0 \phi(\alpha - 1)
\]

(6.17)

\[
G(\omega) = (1 + \frac{4j\alpha^2 \eta \rho_0 \omega}{\sigma^2 \Lambda^2 \phi^2})^{1/2}
\]

(6.18)

where \(\sigma, \alpha, \phi\) indicate the flow resistivity, tortuosity, and porosity. \(\rho_0, \rho_a\) represent the air density and the skeleton density respectively. Six acoustic quantities are chosen to compute the matrix: two velocity components \(v_1^s, v_3^s\); one fluid velocity \(v_3^f\); two skeleton stresses \(\sigma_{33}^s, \sigma_{13}^s\); and one fluid stress \(\sigma_{33}^f\):

\[
v_1^s = \partial_t (\partial_{x_1}\varphi_1^s + \partial_{x_1}\varphi_2^s - \partial_{x_3}\psi_2^f)
\]

(6.19)
Mechanical and vibro-acoustic aspects of composite sandwich cylinders

\[ v^t_3 = \partial_t (\partial_{x_3} \varphi^t_1 + \partial_{x_3} \varphi^t_2 + \partial_{x_1} \varphi^t_2) \quad (6.20) \]

\[ v^f_3 = \partial_t (\partial_{x_3} \varphi^f_1 + \partial_{x_3} \varphi^f_2 + \partial_{x_1} \varphi^f_2) \quad (6.21) \]

The skeleton displacements caused by compression waves and shear waves are respectively given as:

\[ u^s_1 = \partial_{x_1} (\varphi^s_1 + \varphi^s_3) - \partial_{x_3} \psi^s_2 \quad (6.22) \]

\[ u^s_3 = \partial_{x_3} (\varphi^s_1 + \varphi^s_3) + \partial_{x_1} \psi^s_2 \quad (6.23) \]

The stress-strain relations of the porous foam materials are:

\[ \sigma^s_{33} = (P - 2N_s)\nabla u^s + Q\nabla u^f + 2N_s \partial_{x_3} u^s_3 \quad (6.24) \]

\[ \sigma^s_{13} = N_s (\partial_{x_3} u^s_1 + \partial_{x_1} u^s_3) \quad (6.25) \]

\[ \sigma^f_{33} = K \nabla u^f + Q\nabla u^s \quad (6.26) \]

where \( N_s \) represents the shear modulus of the foam skeleton. \( u^s \) and \( u^f \) are dilatations displacement of the skeleton and the air in the pores respectively. \( \nabla u^s \) and \( \nabla u^f \) can be expressed in terms of displacement potentials of the two compression waves as:

\[ \nabla u^s = \partial_{x_{1,3}} (\varphi^s_1 + \varphi^s_3) + \partial_{x_3} \psi^s_2 ; \quad (6.27) \]

\[ \nabla u^f = \partial_{x_{1,3}} (\mu_1 \varphi^s_1 + \mu_2 \varphi^s_2) + \partial_{x_3} \psi^s_2 ; \quad (6.28) \]

The parameters \( P, Q \) and \( K \) can be expressed as:

\[ P = \frac{4}{3} N_s + K_b + \frac{(1-\phi)^2}{\phi} K_f, \quad Q = K_f (1-\phi), \quad K = \phi K_f \quad (6.29) \]

\( K_b \) denotes the bulk modulus of the elastic skeleton, \( K_f \) the bulk modulus of the fluid. They can be computed as:

\[ K_b = \frac{2N_s (\nu_{foam} + 1)}{3(1 - 2\nu_{foam})} \quad (6.30) \]

\[ K_f = \frac{\gamma P_0}{\gamma - (\gamma - 1)(1 + \frac{8\eta}{jB^2\omega\rho_0} (1 + j\rho_0 \frac{\omega B^2 A^2_{1/2}}{16\eta})^{-1})}, \quad \gamma = \frac{c_p}{c_v} \quad (6.31) \]

where \( B \) is the Prandtl number of the air, \( c_p \) and \( c_v \) are heat capacity at constant pressure and volume respectively, \( \eta \) is the viscosity of the air. The wavenumber of the two compression waves and the shear wave in the \( x_3 \) direction are:

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\[ k_{13} = \text{Re}(\sqrt{\delta_{1}^2 - k_r^2}); k_{23} = \text{Re}(\sqrt{\delta_{2}^2 - k_r^2}); k_{33} = \text{Re}(\sqrt{\delta_{3}^2 - k_r^2}); \]  

\( k_r = \omega \sin \theta / c_0 \) is the wavenumber in the \( x_1 \) direction. The squared complex wavenumber of the two compression waves and the shear wave in the porous material

\[
\begin{align*}
\delta_1 &= \frac{\omega^2}{2(\rho_{11b} + 2Q\rho_{12b} + \sqrt{\Delta})} \\
\delta_2 &= \frac{\omega^2}{2(\rho_{11b} + 2Q\rho_{12b} - \sqrt{\Delta})} \\
\delta_3 &= \omega^2 \left( \frac{\rho_{11b}\rho_{22b} - \rho_{12b}^2}{N_s} \right) \\
\Delta &= (\rho_{22b} + K\rho_{11b} - 2Q\rho_{12b})^2 - 4(\rho_{11b}\rho_{22b} - \rho_{12b}^2)
\end{align*}
\]

A 6×6 matrix \( \Gamma_{ij}(x_3) \) can be computed by extracting the coefficients of the terms \([ (A_1 \pm A_{1b}), (A_2 \pm A_{2b}), (A_3 \pm A_{3b}) ]\) from Equations (6.11) to (6.13) and Equations (6.19) to (6.21). Finally the transfer matrix \( T_p \) for the porous layer can be given as:

\[
T_p = [\Gamma(-l_p)][\Gamma(0)]^{-1}
\]

where \( l_p \) is the thickness of the porous layer.

### 6.3.2. Transfer matrices for the solid layers

It is supposed that the porous layers are soft and their bending stiffness is negligible. Thus the sandwich panel is not treated as a whole panel, but the equations of motion of two facings (solid layers) are described individually.

An incident and a reflected longitudinal wave, an incident and a reflection shear waves propagate in an elastic solid [5]. Thus the displacement potentials of the two kinds of waves can be expressed as, respectively

\[
\begin{align*}
\varphi &= E x p[ j \omega t - jk_{13}x_1](A_{1s}E x p[-jk_{13s}x_3] + A_{2s}E x p[jk_{13s}x_3]) \\
\psi &= E x p[ j \omega t - jk_{13}x_1](A_{3s}E x p[-jk_{33s}x_3] + A_{4s}E x p[jk_{33s}x_3])
\end{align*}
\]

\[
k_{13} = \sqrt{\delta_{13}^2 - k_r^2}, \quad k_{33} = \sqrt{\delta_{33}^2 - k_r^2}
\]

where \( \varphi \) denotes the displacement potentials of the longitudinal wave, \( A_{1s} \) and \( A_{2s} \) the incident and reflected amplitude of the longitudinal wave respectively; \( \psi \) represents the displacement potentials of the shear wave, \( A_{3s} \) and \( A_{4s} \) the incident and reflected amplitude of the shear wave respectively. \( k_{13}^s \) and \( k_{33}^s \) are the \( x_3 \) components of the wavenumber.
vectors. \( \delta^2_{ls} \) and \( \delta^2_{3s} \) are the squares of the wavenumber of the longitudinal and shear waves, respectively. They can be given by

\[
\delta^2_{ls} = \frac{\omega^2 \rho_1}{\lambda_1 + 2G_1}, \quad \delta^2_{3s} = \frac{\omega^2 \rho_1}{G_1}
\]

\[
\lambda_1 = \frac{G_1(E_1 - 2G_1)}{3G_1 - E_1}
\]

where \( \rho_1, \lambda_1, G_1, E_1 \) respectively indicates the density, the first Lame coefficient, the shear modulus and the Young’s modulus of the first solid layer (face 1). Following Folds and Loggins [20], four mechanical variables, \( \{ v_1^s, v_3^s, \sigma^s_{33}, \sigma^s_{13} \} \), are chosen to express the sound propagation and they can be expressed by the displacement potentials as follows:

\[
v_1^s = j\omega(\partial x_1 \varphi - \partial x_3 \psi)
\]

\[
v_3^s = j\omega(\partial x_3 \varphi + \partial x_1 \psi)
\]

\[
\sigma^s_{33} = \lambda(\partial x_1 x_3 \varphi + \partial x_3 x_1 \varphi) + 2G_1(\partial x_3 x_3 \varphi + \partial x_1 x_3 \psi)
\]

\[
\sigma^s_{13} = G_1(2\partial x_1 x_3 \varphi + \partial x_3 x_1 \psi - \partial x_3 x_3 \psi)
\]

A 4×4 matrix \( \Gamma_{ij}(x_s) \) for the solid layer can be computed by extracting the coefficients of the terms \([A_{1s} \pm A_{2s}), (A_{3s} \pm A_{4s})]\) from Equations (6.40) to (6.43). Finally the transfer matrix \( T_s \) for the solid layers can be given as:

\[
T_s = [\Gamma(-l_s)][\Gamma(0)]^{-1}
\]

where \( l_s \) is the thickness of the solid layer. The derivation for the face 2 is the same as that for the face 1 so that it will not be discussed again.

6.3.3. Transfer matrices for solid-fluid interface

The continuity conditions for the solid-fluid interface can be given by

\[
\nu_3^f(M_0) = \nu_3^s(M_1)
\]

\[
-p(M_0) = \sigma^s_{33}(M_1)
\]

\[
0 = \sigma^s_{13}(M_1)
\]

Equation (6.45) can be written in terms of matrix as \( I_g V^s(M_1) + J_g V^f(M_0) = 0 \), with
6.3.4. Transfer matrices for solid-porous interface

The continuity conditions for the solid-porous can be given by

\[
\begin{align*}
\n \nu_1^s (M_2) &= \nu_1^f (M_3) \\
\nu_2^s (M_2) &= \nu_2^f (M_3) \\
\nu_3^s (M_2) &= \nu_3^f (M_3) \\
\sigma_{33}^s (M_2) &= \sigma_{33}^s (M_3) + \sigma_{33}^f (M_3) \\
\sigma_{13}^s (M_2) &= \sigma_{13}^s (M_3)
\end{align*}
\]

Equation (6.47) can be written as \( I_{sp} V^s (M_2) + J_{sp} V^s (M_3) = 0 \), with

\[
I_{sp} = \begin{bmatrix}
1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & 1 & 0
\end{bmatrix}, \quad J_{sp} = -\begin{bmatrix}
1 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 1 & 0
\end{bmatrix}
\]

6.3.5. Assembling the global transfer matrix

Coupling the transfer matrices for the single layers and the transfer matrices at the interfaces (see Figure 6-5), the following four equations can be obtained:

\[
\begin{align*}
J_{sf}^{01} V^f (P_1) + I_{sf}^{01} T_{12} V^s (M_2) &= 0 \\
I_{sp}^{23} V^s (M_2) + J_{sp}^{23} T_{34} V^p (M_4) &= 0 \\
J_{sp}^{45} V^p (M_4) + I_{sp}^{45} T_{56} V^s (M_6) &= 0 \\
I_{sf}^{67} V^s (M_6) + J_{sf}^{67} V^f (P_2) &= 0
\end{align*}
\]

The set of equations (6.49) can be rewritten in the form \( D_0 V_0 = 0 \), where

\[
D_0 = \begin{bmatrix}
J_{sf}^{01} & I_{sf}^{01} T_{12} & 0 & 0 & 0 \\
0 & I_{sp}^{23} & J_{sp}^{23} T_{34} & 0 & 0 \\
0 & 0 & J_{sp}^{45} & I_{sp}^{45} T_{56} & 0 \\
0 & 0 & 0 & I_{sf}^{67} & J_{sf}^{67}
\end{bmatrix}
\]

and
\[
V_0 = [V^f(P_1), V^s(M_2), V^p(M_4), V^s(M_6), V^f(P_2)]^T
\]  (6.51)

### 6.3.6. Computation of the transmission loss

There are two boundary conditions describing the relationship between the velocity and the pressure at the two sides of the sandwich system. The boundary conditions at the incident side can be given as:

\[
[-1 \ Z_s] V^f(P_1) = 0
\]  (6.52)

The boundary condition at the transmitted side is:

\[
[-1 \ Z_{P_2} / \cos \theta] V^f(P_2) = 0
\]  (6.53)

where \(Z_{P_2}\) is the impedance of the air, \(Z_{P_2}=415\). \(Z_s\) is an unknown parameter that needs to be solved. Adding Equation (6.52) and Equation (6.53) into the global matrix in Equation (6.50), a new square matrix (the size is 18×18 for the sandwich system) can be formed as:

\[
\begin{bmatrix}
-1 & Z_s & \cdots & 0 & 0 \\
\vdots & \ddots & \ddots & \ddots & \ddots \\
0 & 0 & \cdots & -1 & Z_{P_2} / \cos \theta \\
\end{bmatrix}
\]  (6.54)

The determinant of \(D\) is equal to zero and \(Z_{P_2}\) can be calculated by

\[
Z_{P_2} = -\frac{\det[D_1]}{\det[D_2]}
\]  (6.55)

where \(\det[D_1]\) is the determinant of the matrix obtained when the first column and the first line has been removed from \(D\); \(\det[D_2]\) is the determinant of the matrix when the second column and the first line has been removed from \(D\).

The transmission parameter \(T\) and the reflection coefficient \(R\) are related by

\[
\frac{P_1}{1 + R} - \frac{P_2}{T} = 0
\]  (6.56)

Combining the equation with the global matrix, as well as the boundary condition at the transmitted side, a new system is formed with the \(N\times N\) square matrix. \(N\) is the length of the vector \(V_0\) (N=18 for the sandwich system).

\[
\begin{bmatrix}
T & 0 & \cdots & -(1 + R) & 0 \\
\vdots & \ddots & \ddots & \ddots & \ddots \\
0 & 0 & \cdots & -1 & Z_{P_2} / \cos \theta \\
\end{bmatrix}
\begin{bmatrix}
V_0 \\
\vdots \\
\end{bmatrix} = 0
\]  (6.57)

The determinant of the matrix is equal to zero, and \(T\) is calculated by
\[ T = -(1 + R) \frac{\det[D_{N-1}]}{\det[D_1]} \]  

(6.58)

where \( \det[D_{N-1}] \) is the determinant of the matrix obtained when the \((N-1)\)th column and the first line has been removed from the matrix in Equation (6.57), and \( \det[D_1] \) is the determinant of the matrix obtained when the first column and the first line has been removed from the matrix in Equation (6.57).

For a plane wave of incidence \( \theta \), the transmission loss is defined by

\[ TL = -10 \log \tau(\gamma) \]  

(6.59)

where \( \tau(\gamma) = |T^2(\gamma)| \) is the transmission coefficient for the angle of incidence \( \gamma \). In case of a diffuse field excitation, the transmission loss is defined as:

\[ TL_d = \int_0^{\pi/2} |\tau(\gamma)|^2 \sin(2\gamma) d\gamma \]  

(6.60)

The derivation of the transfer matrices and the computation of TL of sandwich panels were performed in Mathematica and the source codes can be found in Appendix H. The TMM can also be used to describe the multilayered system which consists of the fluid layer. The description of transfer matrix for the fluid layer can refer to Allard [5].

6.4. TL of the sandwich cylinder with porous foam

The TMM model for predicting the TL of sandwich cylinders are verified by experimental results. The influences of the porosity, tortuosity and flow resistivity on the TL are presented. In addition, the TLs of sandwich cylinders with different noise control treatments are studied. Finally, a composite sandwich cylinder is compared with a stiffened cylinder concerning the noise control treatments.

6.4.1. Verification of the model

The TL prediction of sandwich panels using the TMM was firstly validated by an experimental measurement. The measurement results for sandwich panels with absorbing materials referred to [2]. The sandwich panel (a size of 1.14 m × 1.14 m) was composed of one layer of porous foam in-between two aluminum plates. The mechanical and acoustic properties of the sandwich panel are shown in Table 6-1.

The predicted TL using the TMM is compared with the experimental results in Figure 6-6. It can be found that the predicted TL is higher than the experimental results below 800 Hz. This may be because the flanking transmission was not considered in the numerical models which would happen in the experimental test. However, the TMM gave a satisfactory prediction of the TL above 1000 Hz. This indicates that the TMM can be an effective tool for the prediction of the TL of multilayered system which consists of porous layers.
Table 6-1 Material properties of a sandwich panels for the absorption study [2]

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Excited Panel</th>
<th>Foam</th>
<th>Radiating Panel</th>
</tr>
</thead>
<tbody>
<tr>
<td>Thickness, (m)</td>
<td>0.00127</td>
<td>0.027</td>
<td>0.000762</td>
</tr>
<tr>
<td>Porosity, ϕ</td>
<td></td>
<td>0.9</td>
<td></td>
</tr>
<tr>
<td>Flow resistivity σ (Nm²/s)</td>
<td></td>
<td>25000</td>
<td></td>
</tr>
<tr>
<td>Tortuosity, µ</td>
<td></td>
<td>7.8</td>
<td></td>
</tr>
<tr>
<td>Viscous charact. dimΛ (m)</td>
<td></td>
<td>226 x 10⁻⁶</td>
<td></td>
</tr>
<tr>
<td>Thermal charact. dimΛ (m)</td>
<td></td>
<td>226 x 10⁻⁶</td>
<td></td>
</tr>
<tr>
<td>Density of skeleton, (kg/m³)</td>
<td>2700</td>
<td>30</td>
<td>2800</td>
</tr>
<tr>
<td>Young’s Modulus, (E)</td>
<td>7.1 x 10¹⁰</td>
<td>800800</td>
<td>7.1 x 10¹⁰</td>
</tr>
<tr>
<td>Shear Modulus, (G)</td>
<td>2.67 x 10¹⁰</td>
<td>286000</td>
<td>2.67 x 10¹⁰</td>
</tr>
<tr>
<td>Poisson coefficient, ν</td>
<td>0.3</td>
<td>0.4</td>
<td>0.3</td>
</tr>
<tr>
<td>Structural damping, η</td>
<td>0.007</td>
<td>0.265</td>
<td>0.007</td>
</tr>
</tbody>
</table>

Figure 6-6 Experimental and predicted TL of the bonded and unbonded sandwich panels

Sometimes the absorption layer is isolated from facings by air gaps to improve the sound insulation. An unbonded case was also discussed here to see whether the TMM can predict the TL of multilayered systems having air gaps. There were two air gaps in the unbonded sandwich and the two gaps were respectively 1 mm and 6 mm thick. The predicted and measured TLs of the bonded sandwich panels and unbonded sandwich panels are shown in Figure 6-6. It can be found that the unbounded connection between the foam and the elastic panels have a lower TL than the bonded one below 400 Hz, because the stiffness dominates sound transmission at low frequencies and the bonded one has higher stiffness than the unbonded one. While above 400 Hz, the unbonded panel has a significantly higher TL than the bonded one. This is because the structural vibration waves are more effectively transmitted from one face to another through the foam skeletons in the bonded configuration. While in the unbonded configuration where
the foam layer is separated from the faces by the airspace, the structural wave of one face cannot directly transfer to the other face and thus the energy loss is larger in the foam layer. In general, the unbonded sandwich panel is preferred for noise control in a broad frequency range. However, if the noise control at low frequencies is the major concern, the bonded configuration could be taken into consideration.

6.4.2. Different noise control treatments on the sandwich cylinder

As introduced in Section 6.2, the sound absorption of the porous foam is affected by some parameters. Therefore the influences of the flow resistivity, tortuosity and porosity of the porous foam on the TL of the sandwich panel were studied. Properties of the sandwich panel referred to Table 6-1. It can be seen in Figure 6-7 that an increase of the flow resistivity of the foam material lead to an increase of the TL above 2000 Hz. This phenomenon can be easily understood in that an increase of the flow resistivity means more frictions or resistance as the acoustic waves propagate in the foams, and more energy can be dissipated during the frictions. In contrast, increasing the tortuosity caused a lower TL above 1000 Hz, see Figure 6-8. Knapen et al [21] experimentally measured the sound absorption of porous cement mortars with different tortuosity values. They found that an increase of the tortuosity lead to a decrease of the absorption coefficient above 2000Hz. The results from the model show good agreement with experimental results in [21]. In addition, the influence of the porosity on the TL is shown in Figure 6-9 and it can be seen that the TL of the sandwich cylinder increased as the porosity of the foam was increased from 0.5 to 0.9 above 1500 Hz. As the porosity increases, the contacting area between the acoustic waves and the skeleton surface is larger and more vibration energy can be converted into the heat energy.

![Figure 6-7 Influences of the flow resistivity on TL of a sandwich panel](image)
Four kinds of acoustic foams were investigated as the insulation layers attached to the inner surface of a sandwich cylinder, and their properties are shown in Table 6-2. The geometry of the sandwich cylinder is: length 10 m and radius 2m. The influences of the four different insulation layers on the TL of the sandwich cylinder were studied. The properties of the sandwich facing were: $E_1=1.25\times 10^{11}$ Pa, $E_2=1\times 10^{10}$ Pa, $\nu_{12}=0.4$, $G_{12}=5.9\times 10^{9}$ Pa, $\rho_f=1600$ kg/m$^3$. The properties of the core were: $E_c=1.51\times 10^8$ Pa, $G_c=6.87\times 10^7$ Pa, $\nu_c=0.3$, $\rho_c=70$ kg/m$^3$. Two cases of the comparison were conducted: (1) keep the thickness of all the insulation layers as 20 mm; (2) keep the weight increases of the insulation layers as the same, see Table 6-2.
Table 6-2 Properties of the foam for the noise control treatment study

<table>
<thead>
<tr>
<th>Foam</th>
<th>Polyurethane</th>
<th>Melamine</th>
<th>Cast</th>
<th>Typical Plastic</th>
</tr>
</thead>
<tbody>
<tr>
<td>Porosity, $\phi$</td>
<td>0.96</td>
<td>0.99</td>
<td>0.94</td>
<td>0.97</td>
</tr>
<tr>
<td>Flow resistivity, $\sigma$ (Nm$^{-4}$s)</td>
<td>5000</td>
<td>$1.09 \times 10^4$</td>
<td>$2.6 \times 10^4$</td>
<td>$8.8 \times 10^4$</td>
</tr>
<tr>
<td>Tortuosity, $\alpha$</td>
<td>1.24</td>
<td>1.02</td>
<td>1.3</td>
<td>2.52</td>
</tr>
<tr>
<td>Density of skeleton, $\rho$ (kg m$^{-3}$)</td>
<td>22</td>
<td>8.8</td>
<td>22</td>
<td>31</td>
</tr>
<tr>
<td>Young’s Modulus, $(E)$</td>
<td>$4.65 \times 10^4$</td>
<td>$8 \times 10^4$</td>
<td>$6.5 \times 10^4$</td>
<td>$1.43 \times 10^5$</td>
</tr>
<tr>
<td>Poisson coefficient, $\nu$</td>
<td>0.4</td>
<td>0.4</td>
<td>0.4</td>
<td>0.3</td>
</tr>
<tr>
<td>Structural damping, $\eta$</td>
<td>0.14</td>
<td>0.17</td>
<td>0.15</td>
<td>0.055</td>
</tr>
<tr>
<td>Case 1: weight increase for a 20 mm addition, %</td>
<td>11.1%</td>
<td>4.46%</td>
<td>11.1%</td>
<td>15.7%</td>
</tr>
<tr>
<td>Case 2: layer thickness with the same weight increase, mm</td>
<td>20</td>
<td>50</td>
<td>20</td>
<td>14</td>
</tr>
</tbody>
</table>

Figure 6-10 Predicted TL of a sandwich cylinder with different absorption foam layers, the same layer thickness

Figure 6-11 Predicted TL of a sandwich cylinder with different absorption foam layers, the same weight increase
In the first case, the four different insulation layers have the same thickness. Their TLs are compared in Figure 6-10 and it can be found that the plastic foam exhibited the largest increase of the TL of the sandwich cylinder, while the polyurethane foam showed the lowest. One possible reason is the flow resistivity of the polyurethane foam is the lowest and the plastic foam has the highest flow resistivity, see Table 6-2. Besides, the highest density (31 kg/m³) of the plastic foam could also contribute to the highest TL increase. On the other hand, the weight increase should be considered when the noise control layers are added. Thus the weight increases due to the addition of the foam layers were compared, as shown in Table 6-2. It can be found that the addition of 20 mm Melamine foam gave the least increase of the total weight.

In the second case, the TL of cylinders with the four absorption foams, having the same weight, were compared. As shown in Figure 6-11, the cylinder with the melamine foam has the largest TL while the cylinder with the polyurethane has the lowest TL at high frequencies. The difference could be explained by observing the microstructures of these two kinds of foam in Figure 6-12. It can be found that the melamine has a smaller skeleton size but a denser contact with air pores than the polyurethane. The smaller pore structure let the sound wave have more inelastic reflection and refraction contact with the skeleton surface, more vibration energy can be converted into the heat energy, which leads to a higher flow resistivity and a higher sound absorption efficiency. It should be noted that this does not mean that the smaller pore size, the better absorption. Actually when the pore size is too small to allow the acoustic wave to come into the pore, the sound absorption would be decreased dramatically.

![Figure 6-12 Electron microscope pictures of foams microstructures: a, polyurethane; b, melamine.](http://gtmma.sourceforge.net/aboutme/en/porous_media/index.html)

The comparisons in Figure 6-10 and Figure 6-11 indicate that the addition of the noise control treatment layer has little influence on the sound insulation below 500 Hz. However, the TL can be increased effectively by the addition of insulation layers above 500 Hz, i.e., the largest increase is about 10 dB in Figure 6-10. This phenomenon can be
explained by that the sound wavelength is large at low frequencies and thus the energy is low. After one time of the elastic reflection or refraction, the energy loss is small. While at higher frequencies, the sound wavelength is smaller and the acoustic vibration energy is higher, the wave is prone to have more times of reflection or refraction inside the pore of the foam cells. The possibility of inelastic contact between the sound wave and the skeleton surface increases, and thus the energy loss is high.

6.4.3. Noise control treatments on a sandwich and a stiffened cylinder

The influence of noise control treatment on the sound transmission loss of a stiffened cylinder and a sandwich cylinder was studied. The properties of the stiffened and the sandwich cylinder referred to the investigation in Section 3.7. A layer of 20mm-thick Melamine absorption foam was attached on inner surfaces the two cylinders.

The comparison results are shown in Figure 6-13. With respect to the two cylinders without noise control treatment, the sandwich cylinder showed a better sound insulation than the stiffened cylinder at most frequencies. This is because the structural response of the sandwich cylinder was lower than the stiffened cylinder, while the radiation efficiency was similar, as discussed in Section 3.7. With the addition of absorptive layers, the TL of the stiffened cylinder was comparable with that of the sandwich cylinder without absorption layers above 1000 Hz, but it was lower than the TL of the sandwich cylinder with the noise control treatment.

![Figure 6-13 Effect of noise control treatment on TL of the stiffened and sandwich cylinder](image)

For both the stiffened cylinder and the sandwich cylinder, the additions of a layer of melamine foam had little influence on the TL below 500 Hz. However, large improvements of TL above 1000 Hz can be seen. Above 1000 Hz, the addition of a 20mm-thick melamine layer lead to an improvement of TL and the improvement was more and more significant as the frequency increased. At 10000 Hz, the TLs of the
stiffened cylinder and the sandwich cylinder were approximately increased by 10 dB as a result of the addition of the melamine layer. Therefore, the noise problem of transport vehicles at high frequencies could be effectively resolved by addition of the absorption materials.

### 6.5. Conclusions

The transfer matrix method (TMM) was used to predict the sound transmission loss (TL) of multilayered systems. The Biot’s theory was adopted to describe the acoustic field inside the porous foam. The TMM has been validated by the experimental results.

The flow resistivity, tortuosity and porosity are important parameters of the porous absorption foam. A sensitivity study of these parameters indicated that they influence the TL of porous-foamed sandwiches at high frequencies as: a larger flow resistivity and porosity leads to a higher TL, while increasing the tortuosity decreases the TL. The positive effects of the flow resistivity and the porosity on the TL can be explained by that the acoustic waves have more time to contact and more friction with the foam skeleton when the values of the two parameters are larger, and thus more vibration energy can be dissipated. The negative influence of the tortuosity on the TL at high frequencies can be verified by the experimental results. However, the physical explanation is still unknown to the date.

Four different kinds of absorption foam were placed on the inner surface of a sandwich cylinder, and the TLs of cylinders with the four absorption layers were compared. A significant finding is that the absorbing materials showed little effect on TL at low frequencies (<500 Hz), however, they increased the TL to a large extent at high frequencies (>1000Hz). The reason is that the wavelength of the acoustic wave is large at low frequencies and there is little inelastic contact between the wave and the frame surface. Thus little vibration energy is converted into heat energy. While at high frequencies, the wavelength is small and the vibration energy is high, it is prone to have the inelastic contact and thus the vibration energy can be converted into the heat energy.

A comparison of the four absorptions foams having the same thickness was compared for their TL influence on cylinders. Results showed that the cylinder with the plastic foam layer had the largest TL because of the largest flow resistivity and the largest density. Although the plastic foam gave the best insulation performance, it also brought the largest weight increase to the total structure, which was about 15.7%. In contrast, the melamine made the least weight increase (4.5%).

Another comparison in which the four foams have different thickness but the same weight was conducted. It was found that cylinder with the melamine layer had the largest TL while the polyurethane had the lowest TL at high frequencies. This is due to the fact that the melamine foam has a smaller pore size than the polyurethane foam. The pore size has an important influence on the sound absorption, the pore size should be large...
enough to let the acoustic wave enter, and meanwhile it cannot be too large so that the acoustic wave can have more reflection or refraction inside the foam.

A stiffened and a sandwich cylinder were compared in case of the addition of an absorption layer. Compared with the sandwich cylinder, the stiffened cylinder gave a poorer sound insulation at most frequencies. The stiffened cylinder with an addition of melamine materials had a comparable TL with the sandwich cylinder without addition of absorption layers.

To sum up, the addition of the absorption layers can effectively improve the TL of cylinders at high frequencies. The analytical method using TMM can effectively predict the TL of sandwich panels with porous materials. The TMM was described for a typical sandwich structure in this chapter, and it can extend to predict the TL of arbitrary multilayered systems which consists of the porous layer, elastic layer, rigid wall and fluid (air) layer.

6.6 References

Chapter 7  Damping of composite sandwich structures

The damping properties of composite sandwich structures were studied using the modal strain energy (MSE) method and experimental measurements. With respect to the experimental techniques, the hysteresis method and the half power methods have their respective advantages and limitations, thus both of them are used for damping measurement. By means of the hysteresis method the material damping properties of two kinds of foams were measured in the frequency range from 0-100 Hz. The MSE method for the damping prediction of multi-layered structures was verified by the measurements using the half-power method. Finally, the effects of the core thickness and core properties on the damping of sandwich structures were studied.

7.1. Introduction

Damping properties play a vital role in the vibroacoustic behavior of composite structures, especially at frequencies above the coincidence frequency, where an increase of damping raises the sound insulation to a large extent. Compared with monolithic or stiffened structures, sandwich structures have a higher damping loss factor because the viscoelastic core has a high inherent damping capacity. When the beam or plate undergoes flexural vibration, the damped core is constrained to have a shear deformation. The shear deformation causes the vibration energy to be dissipated. Additionally, the normal-to-shear coupling between the core and face sheets reduces the vibration amplitude.

7.1.1. Classification of damping

According to the energy dissipation mechanism, the damping can be classified into three main categories [1]:

- Viscous damping (fluid or fluid-structure interactions), the energy is dissipated due to the resulting drag force when solid bodies move in a fluid medium.
- Coulomb or Dry friction damping (structural, at joints and interfaces), which is caused by friction between rubbing surfaces that are either dry or have insufficient lubrication. It is extremely difficult to develop a generalized analytical model that would satisfactorily describe structural damping. The most common method of estimating structural damping is by measurement.
- Material or Hysteretic damping (internal), for this kind of damping, the energy dissipations originate from microstructure defects, such as skeleton boundaries and impurities; thermo-elastic effects caused by local temperature gradients resulting from
non-uniform stresses, as in vibrating beams; eddy current effects in ferromagnetic materials; dislocation motion in metals; and chain motions in polymers. In some cases, the material damping can be characterized by mathematical expressions. For a linear viscoelastic material, the stress-strain relationship can be given by three commonly used models, which are:

**Kelvin Voigt**

\[
\sigma = E\varepsilon + E^* \frac{d}{dt}(\varepsilon) \tag{7.1}
\]

**Maxwell**

\[
\sigma + c_s \frac{d}{dt}(\sigma) = E^* \frac{d}{dt}(\varepsilon) \tag{7.2}
\]

**Standard linear solid**

\[
\sigma + c_s \frac{d}{dt}(\sigma) = E\varepsilon + E^* \frac{d}{dt}(\varepsilon) \tag{7.3}
\]

where \(E\) is the young’s modulus, \(E^*, c_s\) are viscoelastic parameters, which are all assumed to be time independent. \(\sigma\) denotes the stress and \(\varepsilon\) the strain. Figure 7-1 illustrates the difference among the three models: the Kelvin-Voigt model is a parallel connection between a spring and a dash; the Maxwell model is a series connection between a spring and a dash; and the standard linear solid model, a combination of the Kelvin-Voigt model and the Maxwell model, is the most accurate of the three. However, the Kelvin-Voigt model is adequate to describe the stress-strain relationship of viscoelastic materials for most practical purposes.

![Figure 7-1 Three models for viscoelastic materials: a, Kelvin-Voigt model; b, Maxwell model; c, standard linear solid model](image)

7.1.2. Overview of damping measurement methods

Three damping testing methods are widely used, which are the logarithmic method, the hysteresis loop method and the half-power method. Gibson [2] gave an overview of setups for the damping measurement of polymer composites.

**Logarithmic method**

Suppose a single degree of freedom (DOF) oscillatory system is excited by an impulse input, its vibration amplitude decays as the time increases, as shown in Figure 7-2. The loss factor can be computed according to Equation(7.4). It should be noted that this
method can only measure the damping loss factor at the first resonance frequency (or, said differently, at well isolated resonance frequencies, without interfering resonances in the ‘frequency neighborhood’).

\[ \eta = \frac{2(A_i - A_{i+1})}{\pi (A_i + A_{i+1})} \]  

(7.4)

**Hysteresis Loop method**

When a damped material or system vibrates cyclically, for example, with a sine wave form, there is a hysteresis loop between the stress and the strain, as shown in Figure 7-3. The area enclosed by the loop represents the dissipated vibration energy that has been transformed into heat (damping capacity). The loop area can be computed by the curve integral on the hysteresis loop, as given in Equation(7.5), and the loss factors can be computed from the hysteresis loop according to Equation(7.6).

\[ \Delta U = \oint \sigma d\epsilon \]  

(7.5)

\[ \eta = \frac{\Delta U}{\pi U_{\text{max}}^2} = \frac{ab}{\sigma_{\text{max}} \epsilon_{\text{max}}} \]  

(7.6)

where \( \Delta U \) is the dissipated energy, which is equal to the enclosed area of the hysteresis loop. The slope of the longer axis of the ellipse determines the stiffness of the measured
material. The hysteresis method is applied in many dynamic measurement machines for obtaining the dynamic modulus or damping properties, such as Dynamic Mechanical Analysis machine (DMA) and the Rheometric machine.

**The half-power method**

For a vibrating structure, its frequency response curve can be obtained by using the Fast Fourier Transform (FFT). As an example, a peak of the frequency response curve, which happens at one resonance frequency $f_0$, is shown in Figure 7-4. Three dB down from the peak there are two points, $f_1$ and $f_2$, corresponding to the half-power points. With the three characteristic frequencies, the damping loss factor can be computed according to Equation (7.7).

![Figure 7-4 Determination of damping loss factor using half-power method](image)

$$\eta = \frac{f_2 - f_1}{f_0} \quad (7.7)$$

Comparing the usage of the three methods shows that the half-power method is more widely applied. For example, Crane [4], Li [5], and Berthelot [6] used this method to measure the damping loss factor. In this method, various boundary conditions can be set during the vibration, such as free-free, fixed at one end, and fixed at two ends. In addition, the structure can be excited by a hammer or by a shaker.

**7.1.3. Damping treatment in the industry**

The viscoelastic materials can be used to enhance the damping of structures in three ways: free-layer damping treatment, constrained-layer damping treatment and tuned viscoelastic treatment, as shown in Figure 7-5. For the first method, the damping material is either sprayed or bonded on the structure. For the second method, the viscoelastic material is placed in between two outer elastic layers. The second method is more effective in increasing damping than the free-layer design since the viscoelastic core deforms in shear and more energy can be consumed and dissipated into heat. In addition, it is found that the symmetric configuration, in which the two elastic layers have the same thickness and stiffness, is the most effective design because the shear deformation in the core is maximized. For the third method, an appropriate mass in
combination with a viscoelastic damper is located at points of high displacements. In this way, the peak at an unwanted resonance can be divided into two peaks, one below and one above the original one. The vibration amplitudes of the new two peaks turn out to be smaller. The tuned damper method is generally applicable to reduce vibration or noise associated with a single frequency or a narrow frequency band. There are some practical examples about different damping treatments in automobiles and commercial airplanes in [7]. The second method, which using the sandwich concept for the damping improvement, will be discussed in this chapter.

Figure 7-5 Three different damping designs: free layer, constrained layer, and tuned viscoelastic damper [7]

7.1.4. Representation of damping in vibration analysis

It is well known that both the direct method and the modal method can be used to model the dynamic response problem of structures. Thus the representations of damping in the direct frequency response and that in the modal frequency response are introduced respectively.

In the direct frequency response analysis, there are generally two ways to describe the damping in a vibration analysis: by means of a complex stiffness matrix and by means of an additional damping matrix. For the case where the complex matrix is used, the equation of motion can be expressed as [8]:

\[
[ - \mathbf{M} \omega^2 + \mathbf{K}_1(\omega) + i \mathbf{K}_2(\omega) ] \mathbf{X}(\omega) = \mathbf{L}(\omega)
\]  

(7.8)

where \( \mathbf{K}_1, \mathbf{K}_2 \) denotes the stiffness matrices calculated using real and imaginary parts of the material properties, respectively. \( \mathbf{L} \) and \( \mathbf{X} \) represent the applied loads and the structural response, respectively. Alternatively, for the case where an additional damping matrix is used, the equation of motion can also be expressed as:

\[
[ \mathbf{M} ] \{ \ddot{\mathbf{X}}(\omega) \} + [ \mathbf{C} ] \{ \dot{\mathbf{X}}(\omega) \} + [ \mathbf{K} ] \{ \mathbf{X}(\omega) \} = \mathbf{L}(\omega)
\]  

(7.9)

where \([ \mathbf{M} ], [ \mathbf{C} ], [ \mathbf{K} ]\) denote the mass, damping and stiffness matrix, respectively. \( \ddot{\mathbf{X}}(\omega), \dot{\mathbf{X}}(\omega), \mathbf{X}(\omega) \) represent the vectors of nodal accelerations, velocities and displacements, respectively. Besides, the definition of damping can also be a combination of the damping matrix and the complex stiffness matrix. Taking the commercial FE software Nastran as an example, both the damping matrix and the complex stiffness matrix are used for damping characterization. It should be noted that \( \mathbf{C} \) is only generated when some special elements, such as viscous damper elements and the scalar dampers, are used in the FE model. In most cases, \( \mathbf{C} \) is not used and the damping is defined by \( \mathbf{K}^* \),
The stiffness matrix $k^*$ can be expressed in terms of the material damping $g_e$ and the global structural damping $g$ as follows [9]:

$$k^* = (1 + ig)[k] + ig_e[k_e]$$  \hspace{1cm} (7.10)

In the modal frequency analysis, the damped motion can be uncoupled into individual modes, and the equation of motion is expressed as:

$$\ddot{q}_r + \eta_r \omega_r \dot{q}_r + \omega^2_r q_r = \rho_r$$  \hspace{1cm} (7.11)

in which $q_r$ is the generalized coordinate of the $r$-th mode, $\omega_r$ is the undamped vibration frequency ($=\sqrt{k_r/m_r}$), and $\eta_r$ is the modal loss factor at each mode. In modal analysis, the damping matrix can be expressed as proportional damping (Rayleigh damping) [10]:

$$[c_r] = \alpha_r [m_r] + \beta_r [k_r]$$  \hspace{1cm} (7.12)

In many practical structural problems, the mass damping $\alpha_r$ which represents friction damping can be ignored ($\alpha_r=0$). In such case, the $\beta_r$ can be computed from the damping loss factor $\eta_r$ using the following relationship:

$$\eta_r = (\alpha_r + \beta_r \omega_r^2) / \omega_r$$  \hspace{1cm} (7.13)

Finally, it should be noted that damping may be represented by various parameters (such as the loss factor $\eta$, the Q-factor, the loss phase angle $\phi$, the specific damping capacity $\psi$ and damping ratio $\varsigma$). It is important to remember their relationships, which are [11]:

$$\eta = 2\varsigma = \frac{\psi}{\pi} = \frac{1}{Q} = \tan\phi$$  \hspace{1cm} (7.14)

7.1.5. Damping research of composite sandwich structures

Two methods are commonly used to predict the damping properties of sandwich structures: the Ross-Kerwin-Ungar (RKU) model and the Modal Strain Energy (MSE) method.

RKU model

As one of the first theories for the damping prediction of sandwich structures, the RKU model is introduced here [12-13]. Assuming that the facings and the core have uniform thickness; the extensional and flexural stiffness of the core is neglected, and the loss factors of the facings are neglected, damping loss factors of sandwich structures can be obtained by Equation(7.15). Because of the three assumptions, the RKU method is better suited as a damping indicator rather than a precise damping predictor when applied to complex, real world structures. In other words, the RKU model can be a


Chapter 7 Damping of composite sandwich structures

design guide for the selection of damping materials, constraining layers, and damping layer thickness to yield an optimal damping performance.

\[ \eta = \frac{\beta YX}{1 + (2 + Y)X + (1 + Y)(1 + \beta^2)X^2} \]  

\[ X = \frac{G_2b}{p^2H_2} S; Y = \frac{H_{13}^2}{E_iI_1 + E_3I_3} \left( \frac{1}{S} - \frac{1}{E_iA_i} + \frac{1}{E_3A_3} \right); \]  

\[ \frac{1}{p^2} = \left( \frac{\lambda}{2\pi} \right)^2 = \frac{1}{\omega\sqrt{\mu}} \]  

In Equations (7.15) to (7.17), \( X \) and \( Y \) respectively represents the shear and structural parameters. \( \beta \) is the damping in the viscoelastic layer or core. \( H_2 \) is the thickness of the viscoelastic layer, \( E_i, A_i \) and \( I_i \) denote the Young’s modulus, cross sectional area, and moment of inertia of component \( i \), respectively. \( H_{13} \) denotes the distances between the neutral axes of the two facings. \( G_2 \) represents the shear modulus (real part) of the viscoelastic materials, \( b \) the length of beam, \( p \) the wavenumber of the beam, \( \lambda \) the bending wavelength, \( B \) the flexural rigidity and \( \mu \) the mass per unit length of the composite beam. Two important conclusions can be drawn from the model: (1) if the facing is thinner than the viscoelastic damping layer, the damping has a maximum value when \( X \) has the value \( (X = 1/\sqrt{(1 + Y)(1 + \beta^2)}) \); (2) when a three-layer sandwich structure is symmetric about the neutral axis, its loss factor is maximal.

Li and Crocker [14] used the RKU model to analyze the damping of sandwich structures, and they studied the effects of the delamination and thickness of the facings and the core on the damping. It was concluded that an increase in the facing thickness lead to a decrease of damping in the low and high frequency ranges. On the other hand, the doubling of the core thickness increases the damping in the middle and high frequency ranges. In addition, the delamination introduces more friction and thus increases the damping of sandwich structures.

The MSE method

As the RKU model only fits structures with simple geometries and negligible facing damping, more advanced methods have been developed for predicting the damping of complex sandwich structures. The MSE method, which is based on the strain energy distribution among the facings and the core, is one of them, and will be introduced below. For thin facing layers consisting of orthotropic materials, the transverse strain energy can be neglected and the in-plane strain energy \( U_f \) of a finite element can be expressed as [15]:

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\[ U_{1f}^e = \sum_{k=1}^{N} \frac{1}{2} \int_{h_{k-1}}^{h_k} \int \sigma_{1k}^e \varepsilon_{1k}^e \, ds \, dh \]

\[ U_{2f}^e = \sum_{k=1}^{N} \frac{1}{2} \int_{h_{k-1}}^{h_k} \int \sigma_{2k}^e \varepsilon_{2k}^e \, ds \, dh \]  \hspace{1cm} (7.18)

\[ U_{6f}^e = \sum_{k=1}^{N} \frac{1}{2} \int_{h_{k-1}}^{h_k} \int \sigma_{6k}^e \varepsilon_{6k}^e \, ds \, dh \]

where \( \sigma_{1k}, \sigma_{2k} \) represent the stress along and perpendicular to the fiber orientation respectively. \( \varepsilon_{1k}, \varepsilon_{2k} \) represent the strain along and perpendicular the fiber orientation respectively. \( \sigma_{6k} \) and \( \varepsilon_{6k} \) are the shear stress and the shear strain respectively. The strain energy of the facing in the shear transverse direction is neglected because of the small thickness of the facings.

The MSE of the core is composed of the in-plane MSE and the transverse shear MSE. The in-plane MSE for a specific solid element is written as:

\[ U_{1c}^e = \frac{1}{2} \int \int \int \sigma_{1}^e \gamma_{1}^e \, dV \]

\[ U_{2c}^e = \frac{1}{2} \int \int \int \sigma_{2}^e \gamma_{2}^e \, dV \]  \hspace{1cm} (7.19)

\[ U_{6c}^e = \frac{1}{2} \int \int \int \sigma_{6}^e \gamma_{6}^e \, dV \]

The transverse shear MSE for a solid element can be defined:

\[ U_{4c}^e = \frac{1}{2} \int \int \int \sigma_{4}^e \gamma_{4}^e \, dV \]  \hspace{1cm} (7.20)

\[ U_{5c}^e = \frac{1}{2} \int \int \int \sigma_{5}^e \gamma_{5}^e \, dV \]

The total MSE can be expressed as the sum of the MSE in the facings and the MSE in the core:

\[ U = \sum_{e=1}^{M_f} (U_{1f}^e + U_{2f}^e + U_{6f}^e) + \sum_{e=1}^{M_c} (U_{1c}^e + U_{2c}^e + U_{6c}^e + U_{4c}^e + U_{5c}^e) \]  \hspace{1cm} (7.21)

The total dissipated MSE can be computed as:

\[ \Delta U = \sum_{e=1}^{M_f} (\lambda_{1f} U_{1f}^e + \lambda_{2f} U_{2f}^e + \lambda_{6f} U_{6f}^e) + \sum_{e=1}^{M_c} (\lambda_{1c} U_{1c}^e + \lambda_{2c} U_{2c}^e + \lambda_{6c} U_{6c}^e + \lambda_{4c} U_{4c}^e + \lambda_{5c} U_{5c}^e) \]  \hspace{1cm} (7.22)

Finally, the total loss factor of the sandwich structure is given by
where the subscript numbers 1, 2, 6 denote along the main axis, transverse the main axis, the in-plane shear. The subscript numbers 4, 5 denote the shear in the $xz$, and $yz$ direction respectively.

In the MSE method, the strain energy is usually calculated using the FE method. Johnson [16] predicted the damping of structures with constrained viscoelastic layers using the MSE method in MSC Nastran, and concluded that the MSE method was less computational expensive than the complex eigenvalue method. Hwang and Gibson [17] reviewed their research work on the damping in polymer composites at both macro-mechanical and micro-mechanical levels: at the micro-mechanical level, the effects of fiber interaction, fiber aspect ratio, and fiber/matrix interphase size were studied; at the macro-mechanical level, the effects of inter laminar stresses, fiber orientation, vibration coupling and constrained layer damping treatment were discussed. Berthelot [15] and Assarar [18] studied the damping properties of the composite sandwiches using MSE method.

Some investigations are conducted for the optimal design of sandwich structures showing the best damping properties. Araujo [19] made an optimization of the damping of composite sandwich structures using the active-passive method, in which the design variables include the core thickness, the constraining layer thickness and the orientation angles, as well as the position of the sensor and actuator pairs. An optimization tool for the damping of sandwich structures was introduced in [20], the damping was computed using the MSE method, and the optimization using the linear search algorithm in Matlab. Although the number of layers, layer thickness, fiber orientations and the stacking sentence were set as the variables in the optimization, their effects on the damping of sandwich structures were not discussed.

In the past research, the damping properties of viscoelastic core such as PVC foam has seldom been researched experimentally, despite the fact that this kind of materials are widely used in the industry. Thus it is needed to conduct experimental measurements to obtain the damping properties of the viscoelastic cores, which can be used for the vibration analysis. Two kinds of damping methods, the hysteresis and the half-power methods, are used for the damping measurement. The relevant work is introduced in the following way: Section 7.2 presents the damping of the foam core using the hysteresis method. The damping measurement of the foam using the half-power method is introduced in Section 7.3. Moreover, the MSE method is validated by the experimental results and it is used to study the parameters that may influence the damping of sandwich structures in Section 7.3.
7.2. Damping measurement of the foam core

Although the half-power method can measure the material damping at a broad frequency range with a good accuracy, it can only obtain the damping loss factor at the resonance frequencies. In many cases, the damping loss factor needs to be measured at specific frequencies, not necessarily at the resonance frequencies of the test specimen. For instances, when the composite sandwich structures are being used for the upper deck of a ship, its damping loss properties at a low frequency range (0-100 Hz) are of particular interest, because the propeller is the main source of low-frequency vibration (typically 18.5 Hz) that causes the ship structure to vibrate.

In order to obtain the damping loss factors of composite sandwich structures at arbitrary frequencies, the damping properties of core materials are measured using the hysteresis method. The measurement techniques exploit a tension-compression test using the Elastomer machine, as well as a shear test using the Elastomer machine and the Rheometric machine. Finally, the measured damping using the hysteresis method is compared with that using the half-power method.

7.2.1. Tension-compression measurement

A 10KN fatigue machine (MTS Elastomer®) was used to measure the damping loss factor in a dynamic tension-compression mode. As shown in the Figure 7-6, the bottom clamp of the sample was fixed, and the top clamp was controlled by a sine-wave vibration in terms of the displacement, the frequency of which sweeps from 1 Hz to 100 Hz with a step of 3 Hz. During the vibration, the force was measured using a load cell at the bottom, and the displacement was measured using a displacement sensor at the top of the machine.

![Figure 7-6 Tension-compression damping test for the foam using MTS elastomer machine](image-url)
The damping properties of the Rohacell-110 (PMI) foam were measured at different frequencies and strain amplitudes. As shown in Figure 7-7, as the frequency increases, the loss factor of Rohcell-110 has a slight decrease while the storage modulus (the real part of the complex modulus) shows an upward trend. In Figure 7-8a, the effect of the strain amplitude is present and it can be found that the loss factor increases significantly as the strain amplitude rises. On the other hand, increasing the strain amplitude leads to a nearly linear decrease of the storage modulus, as shown in Figure 7-8b.

Figure 7-7 The loss factor (a) and storage modulus (b) of the Rohacell 110 at 0-100 Hz, strain amplitude=0.7%

Figure 7-8 The loss factor (a) and storage modulus (b) of the Rohacell 110 at 0.1%-1.4%, frequency=10 Hz

In the static compression measurement, however, the Rohacell foam exhibits a linear elastic behavior as the strain amplitude is below 7% (Figure 7-9). This implies that the Rohacell foam’s performance under the static load is different from that under the dynamic load.
The phenomenon that the loss factor increases as the strain amplitude increases also occurs for other materials. Golovin and Sinning [21] measured the damping loss factor ($Q^{-1}$) of the aluminum foam and found that the loss factor increased as the vibration amplitude increased (as shown in Figure 7-10a). Konstantin and Vikram [22] found that the specific damping capacity ($\psi$) of the carbon foam increased as the vibration amplitude was above 0.01 mm (as shown in Figure 7-10b). Note that the relationship among $Q$ and $\psi$ is denoted in Equation (7.14).

### 7.2.2. Shear measurement

As the shear deformation usually takes place in the core of a sandwich structure, it is important to measure the damping loss factor of the core in shear loadings. The shear measurements were carried out using two different machines: the Elastomer machine and the Rheometric machines.
Elastomer machine

As shown in Figure 7-11, two PVC H60 foam blocks were bonded to a steel plate, and they were also bonded to a clamp with a steel U shape. The bonding material was the high strength epoxy (3M 2216 Gray®). The thickness of the steel plate was 2 mm, and the shell thickness of the U clamp was 4.5 mm. The bottom of the U clamp was fixed to the bottom of the machine, and the steel plate was driven as a tension-compression vibration in sine waves. In this setup, the foam’s shear stress was computed in terms of the measured force \( F \) and the contacting area between the foam and two steel plates \( (Ld) \):

\[
\tau = \frac{F}{2Ld}
\]  

(7.24)

where \( F \) is measured by a load cell at the bottom of the machine, and the contacting area is doubled because there two contacting interface. \( L \) and \( d \) denote the length and the depth of the foam respectively.

Figure 7-11 Test setup for the shear damping of PVC foam using MTS elastomer machine

The foam’s shear strain is computed in terms of the upper-clamp displacement of the machine \( (w) \) and the width of the foam \( (W) \).

\[
\gamma = \frac{w}{W}
\]  

(7.25)

For the foam blocks in the measurement, \( L=39 \) mm, \( d=20 \) mm and \( W=28 \) mm.

In order to investigate whether the bonding layer (between the foams and the steel plates) has influence on the damping measurement, the measurement setup was simulated in the FE model. In the model, the bonding layer was not simulated and the foam and the steel plate were directly connected by sharing common nodes. If the
predicted stress-strain curve from the FE calculation is consistent with the experimental results, it can be concluded that the bonding layer has negligible influence on the damping of the foam in the measurement.

![Deformation and shear strain in XY plane of the FE model for the shear damping system](image)

**Figure 7-12** Deformation and shear strain in XY plane of the FE model for the shear damping system

![Hysteresis loop of PVC foam from the numerical model and the experimental test at 1Hz](image)

**Figure 7-13** Hysteresis loop of PVC foam from the numerical model and the experimental test at 1Hz

The measured modulus and loss factor of the core, $G_c=10\text{MPa}$, $E_c=38\text{MPa}$, $\nu_c=0.4$, $\eta_c=0.0012$, were used as inputs for the numerical model. A transient response analysis was performed by MD Nastran, and the deformation of the FE model is shown in Figure
7-12. It can be found that the deformation of the steel plate and steel U clamp can be neglected as they are stiff enough. This means that the energy dissipation due to the vibration of the U steel clamp can be excluded. In addition, the shear strain of XY component of the foam is the largest at the center of each foam block and the shear strain is uniform at most area except the outer edges.

The measured hysteresis loop and the calculated hysteresis loop at 1 Hz are shown in Figure 7-13. The two hysteresis loops show good agreement. This implies that the bonding layer did not bring additional damping to the measured data, and the measured damping is the material damping of the foam.

**Rheometric machine**

In order to verify the damping loss factor measured by the Elastomer machine, the same PVC foam materials were measured using the Rheometric machine (RMS-800). As shown in Figure 7-14, the rectangular sample was clamped vertically. The lower fixture was fixed and the upper fixture was driven by a torque in sine waves.

During the cyclic rotation of the upper fixture with a form of sine waves, there is a phase angle $\phi$ between the torque and the strain. The complex shear modulus $G^*$, the storage shear modulus $G'$ and the loss shear modulus $G''$ can be expressed as [23]:

$$G^* = \frac{\tau}{\gamma}, \quad G' = G^* \cos \phi, \quad G'' = G^* \sin \phi$$  \hspace{1cm} (7.26)

The shear strain in torsion can be expressed as [23]:

---

**Figure 7-14 Torsion measurement system using Rheometric machine**

---
\[ \gamma = K_\gamma \theta, \quad K_\gamma = \frac{T}{L} \left[ 1 - 0.378 \left( \frac{T}{W} \right)^2 \right] \]  

(7.27)

The shear stress in torsion can be expressed as [23]:

\[ \tau = MK_\tau, \quad K_\tau = 1000 \left[ \frac{3 + 1.8 \left( \frac{T}{W} \right)}{WT^2} \right] G_{cons} \]  

(7.28)

where,

- \( T \) = Thickness of samples (mm)
- \( W \) = Width of samples (mm)
- \( L \) = Length of samples (mm)
- \( K_\gamma \) = Strain constant
- \( \theta \) = Actuator angular displacement (radians)
- \( M \) = Transducer torque (gram-cm)
- \( K_\tau \) = Stress constant
- \( G_{cons} = 98.7 \) (Pascals/gram)

The calculation methods of the shear stress and shear strain proved to be equivalent to the common computation method for the torsion of a rectangular beam as refer to [24], where the shear stiffness \( G \) is calculated in terms of the torsion moment as:

\[ G = \frac{ML}{K\theta} \]  

(7.29)

\[ K = \frac{1}{1e5} \frac{WT^3}{3} \left[ 1 - 0.63 \frac{T}{W} \left( 1 - \frac{T^4}{12W^4} \right) \right] = \frac{K_\gamma L}{K_\tau} \]  

(7.30)

**Measured damping results using Elastomer and Rheometric machines**

The shear damping loss factors of the PVC foam were studied at varied frequencies and vibration amplitudes using the elastomer and the Rheometric machine, respectively. Due to the respective restrictions of the two machines on vibration amplitudes and vibration frequencies, the measurement range was not the same on the two machines. For the elastomer machine, the loss factors were measured in the frequency range of 1-100 Hz in a log step, and in the strain amplitude of 0.3% to 3%; for the Rheometric machine, the loss factors were measured in the frequency of 1-16 Hz in a log step, and in the strain amplitude of 0.05% to 4%.
The influences of the strain amplitude on the loss factors are shown in Figure 7-15a and Figure 7-16a. It can be found that measured damping loss factors by the Elastomer and that by the Rheometric machines show good agreement when strain amplitude is below 1%, and the damping loss factors are approximately 0.01. However, when the strain amplitude is above 1%, the measured loss factor using the Rheometric machine increases significantly as the strain amplitude increases (Figure 7-16a), in contrast, the loss factor results from the Elastomer machine show little change (Figure 7-15a). The significant increase of the measured loss factor using the Rheometric machine may be caused by the inelastic behavior of the PVC foam when the strain is above 1%. Under the torsion, according to the membrane analogy developed by Prandtl [25], the shear stress distribution is not uniform on the rectangular cross-sections of the beam and the largest stress occurs at the middle point of the longer side, see Figure 7-18. In addition, the thickness of the beam is only 6 mm, and there might be a material failure or a material yield at the point with the largest stress under high strain amplitude vibrations. This reasoning can be proved by the measured $G_c$ as the vibration strain increases, see Figure 7-17a. It can be found that the measured $G_c$ from the Rheometric machine shows an apparent decrease as the strain is above 1%, in contrast, the measured $G_c$ from the
Elastomer machine shows much smaller changes at this area. This denotes that the stress-strain relationship of the materials is not linear when the vibration amplitude is above 1% in the torsion test. In other words, an inelastic behavior of the foam could happen. It is important to note that the strain amplitude is not allowed to be higher than 1% in vibrations of sandwich structures (for the safety of the facing materials), therefore the measured data below 1% is enough for engineering applications.

![Figure 7-17](image)

**Figure 7-17** Measured shear stiffness of the PVC foam as the strain amplitude increases (a), and as the frequency increases (b)

\[
\tau_{\text{max}} = \frac{3M}{8ab^2} \left[ 1 + 0.61 \frac{b}{a} + 0.89 \left( \frac{b}{a} \right)^2 - 1.8 \left( \frac{b}{a} \right)^3 + 0.91 \left( \frac{b}{a} \right)^4 \right]
\]

**Figure 7-18** Shear stress distribution of a rectangular beam under a torsion

With respect to the effect of the frequency (Figure 7-15b and Figure 7-16b) on the shear damping of the foam, the results from both the Elastomer machine and the Rheometric machine show that the shear loss factors of PVC foam have little dependency on the frequency at 1-100 Hz. Figure 7-17b shows that the \( G_c \) also shows little change (only a slight increase) as the frequency increases. It is of interest to compare the normal damping with the shear damping for the PVC foam (Table D-3 VS Table D-5 in Appendix D). It can be found that the shear damping compares well with the normal damping, both of which are close to 1%. This means that the damping of the PVC foam show little dependence on the load direction. In addition, a damping measurement of a PVC beam with a length of 1 m was conducted using the half-power
method. There are five resonance frequencies and the damping loss factors at the five resonance frequencies were also around 1% at 1-100 Hz.

7.3. Damping of sandwich beams

The FE models were built to predict and to analyze the damping of the sandwich beam using the MSE method. In addition, the experimental measurements using the half-power method were carried out to verify the FE model. Finally the influences of the core thickness and the core shear stiffness on the damping of sandwich beams were studied.

7.3.1. Experimental measurement introduction

The composite sandwich panels were manufactured with the vacuum infusion process (Figure 7-19a) and they were cut into beams using the Unitom® saw, as shown in Figure 7-19b.

![Figure 7-19 Production of sandwich panels by vacuum bagging (a); sandwich beams with different properties (b)](image)

The damping measurement was performed using the half-power method according to the standard ASTM E756-05 [26]. As shown in Figure 7-20a, the sandwich beams were excited at the center by a shaker. For light samples, the excitation point can also be at
one end of the beam. When the excitation point is at the end, more resonance frequencies can be measured, because the length for the resonance is the total length of the beam. While the support is at the center, only half-length is valid for the resonance. The vibration of the shaker was controlled by a white noise signal. The acceleration at the beam tip was measured with an accelerometer, and the force at the excitation point was measured with a force transducer. The signals were collected with a Bruel & Kjaer input module at a sampling rate of 8192 Hz and then they were FFT post-processed using a Hanning window. The types of used instruments are shown in Table 7-1.

Table 7-1 The used instruments for the free-free vibration test

<table>
<thead>
<tr>
<th>Instruments name</th>
<th>Type</th>
</tr>
</thead>
<tbody>
<tr>
<td>Shaker</td>
<td>4809 (Brüel &amp; Kjaer) 10Hz – 20kHz</td>
</tr>
<tr>
<td>Power amplifier</td>
<td>LING Electronics</td>
</tr>
<tr>
<td>input module</td>
<td>Lan-Xi 3050-B-040 (Brüel &amp; Kjaer)</td>
</tr>
<tr>
<td>Charge amplifier</td>
<td>Model 2721A (Endevco)</td>
</tr>
<tr>
<td>Force transducer</td>
<td>8200 (Brüel &amp; Kjaer)</td>
</tr>
<tr>
<td>accelerometer</td>
<td>4397 (Brüel &amp; Kjaer)</td>
</tr>
<tr>
<td>Signal generator</td>
<td>NI 9264(National Instrument)</td>
</tr>
</tbody>
</table>

During the measurement, it was found that a mechanical connection between the samples and the shaker had noisier signals than a bonded connection. Thus a fast-curing two-component adhesive was used for bonding the sample and the shaker (Figure 7-20b). This adhesive cured in two minutes, and the bonded specimen can be easily removed from the shaker after the testing.

7.3.2. Verification of MSE method

As the MSE method is commonly used to predict or to analyze the damping loss factors of sandwich structures, it was verified by the experimental results here and used for a parametric study in Section 7.3.3. A sandwich beam with the fiberglass/epoxy as the facings and the PVC foam as the core was used for the research. Before the production of the sandwich beam, the damping properties of the facing and the foam were measured using the half power method, which were used as the input for the MSE model. The measured damping loss factors of the facings and the core are shown in Figure 7-21. Then the damping loss factors of the sandwich beam were measured, which were used to compare measured results to the MSE predictions of the sandwich beams. The measured frequency response of the sandwich beam is shown in Figure 7-22. It was seen that the signals had little measurement noise and the resonance peaks were nicely distributed in the frequency range of 0-4000 Hz.

A FE model of the sandwich beam was built in Patran. The facings were modeled with QUAD4 shell elements and the core was modeled with HEX8 solid elements. At the interface between the facings and the core, the elements representing facings and the elements representing the core shared the common nodes.

The cross-ply fiberglass/epoxy (Hexcel4908) materials were used for the facings and their mechanical properties were:
$E_1=36.6$ GPa, $E_2=5.4$ GPa, $G_{12}=4.085$ GPa, $\rho_f=1800$ kg/m$^3$.

The PVC foam was used for the core and its mechanical properties were: $E_c=50$ MPa, $v=0.3$, $\rho_c=60$ kg/m$^3$.

**Figure 7-21** Measured damping loss factor of the core and the facing

**Figure 7-22** Measured frequency response of the sandwich beam with a free-free boundary condition

With respect to the sandwich beam model, the length was 444 mm, the width was 28 mm, the core thickness was 20 mm, and each facing thickness was about 0.6 mm. A normal modes analysis (SOL103) was conducted by MD Nastran and the strain energy percentage of the core and the facings can be calculated at each mode. With the measured damping loss factors of the facings and the core, the damping loss factors of
the sandwich beam can be computed using the MSE method (as introduced in Section 7.1.5). It should be noted that the facings were composed of cross-ply woven (0/90) and thus the effect of fiber orientation on the damping was not considered. In addition, the PVC foam was viewed as the isotropic material and its in-plane damping was supposed to be equal to the transverse damping.

![Mode shapes](image)

**Figure 7-23 FE predicted mode shapes of the sandwich beam**

(nine mode shapes corresponding to the resonance peaks in Figure 7-22, the calculated resonance frequencies are: 396, 809, 1225, 1616, 2000, 2367, 2730, 3081, 3428 Hz)

![Damping comparison](image)

**Figure 7-24 MSE predicted damping loss factor of a sandwich beam compared with the measurement results**

The mode shapes were calculated using Nastran and post processed in Patran, as shown in Figure 7-23. Most calculated resonance frequencies showed a good agreement with the measured resonance frequencies. There were some differences at the two
highest resonance frequencies (the position of measured and predicted resonance frequencies can be seen in Figure 7-24). This may be due to the influence of the connection between the shaker and the beam in the measurement. The measured loss factors and the MSE predicted loss factors of the sandwich beam are shown in Figure 7-24, and it can be found that they are nearly consistent at all the resonance frequencies. This proved the validity of the loss factor prediction of sandwich structures using the MSE method.

According to the MSE method, the influences of the core on the sandwich damping can be characterized by comparing the strain energy stored in the core to the total strain energy of the sandwich structure (strain energy percentage). The strain energy percentage of the core is presented in Figure 7-25. The core had an increasing influence on the damping of the sandwich structures as the frequency increased. The strain energy percentage of the core achieved to be 95% at 3428 Hz.

![Figure 7-25 Modal strain energy percentage of the core as the frequency increases](image)

### 7.3.3. Parametric study on the damping of sandwich structures

A paramedic study on the damping is useful for the optimal design of sandwich structures aiming at vibration controls. As previously introduced, there are several main parameters which may influence the damping of the sandwich structures, such as the layup of the sandwich, the fiber orientation in the composite facing, the core thickness, and the core shear stiffness. With respect to the layup of the sandwich, it has been found that the damping can be maximized if the two facings have the same thickness and the same stiffness. With respect to the fiber orientation, it is found that the damping is the largest at 45° or 30° [4, 27]. Since the core plays a vital role on the damping of the sandwich structures, especially at high frequencies, the influences of the core thickness ($t_c$) and the core shear stiffness ($G_c$) were studied.

Sandwich beam with different cores
The damping loss factors of three sandwich beams with different core thicknesses were measured. The three beams were all composed of glass/epoxy facings and Rohacell-110 (PMI) foam core. The facing thickness was 0.6 mm. The only difference among the three sandwich beams was the core thickness (2.52 mm, 5.15 mm and 9.5 mm respectively). For each kind of beam, three specimens were tested and their loss factors at each resonance frequency were averaged.

The measured damping loss factors of the first three natural modes are shown in Figure 7-26. It is obvious that as \( t_c \) increases, the resonance frequencies increase, and the damping loss factors increase as well. The fact that the resonance frequency increases as \( t_c \) increases can be easily explained by the increase of the ratio of the bending stiffness over the mass. By calculating the strain energy percentage of the core, it was found that the core strain energy percentage increases when \( t_c \) increases. As the core has higher damping loss factors than the facings, it is reasonable that the damping loss factors of the sandwich beam increase as \( t_c \) increases.

![Figure 7-26 Measured loss factors of three sandwich beam with different core thickness](image)

**Sandwich beam with different core stiffness**

In a FE model, \( G_c \) was varied from 10 MPa to 80 MPa in an octave step and the loss factors of sandwich beams with different \( G_c \) are shown in Figure 7-27. It can be found that \( G_c \) has a negative influence on the total damping of sandwich structures. This can be explained by the fact that when \( G_c \) increases, the shear deformation of the core decreases, and the percentage of shear strain energy in the core decreases.

**Table 7-2 Properties of three different foams**

<table>
<thead>
<tr>
<th>Foam Type</th>
<th>Density ((\text{kg/m}^3))</th>
<th>Elastic Modulus, ( E_c ) (MPa)</th>
<th>Shear Modulus, ( G_c ) (MPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Armacell 100/PET</td>
<td>100</td>
<td>105</td>
<td>25</td>
</tr>
<tr>
<td>Rohacell110WF/PMI</td>
<td>110</td>
<td>180</td>
<td>70</td>
</tr>
<tr>
<td>Rohacell200WF/PMI</td>
<td>205.2</td>
<td>350</td>
<td>150</td>
</tr>
</tbody>
</table>
In practice, it is difficult to obtain foam materials with only a different $G_c$ but having same other properties. Thus three kinds of foams with both different density and different $G_c$ were compared experimentally. Their mechanical properties are shown in Table 7-2. Sandwich beams with the three different cores were manufactured. The three sandwich beams had a core thickness of 15 mm, and all facings were composed of woven fiberglass /epoxy with a thickness of 0.6 mm. As both the density and $G_c$ were varied, the structural frequency responses of the three sandwich beams were compared instead of the damping loss factors. As shown in Figure 7-28, the properties of the core showed little influence on the value of the first resonance frequency (the fundamental frequency). However, the Rohacell-110 sandwich had less resonance peaks than the Armacell sandwich beam (modal density is lower). For this phenomenon, the effect of
mass can be excluded because the densities of the two foams were comparable. The
difference of modal density of the three sandwich beams were attributed to $G_c$. The
higher $G_c$ of the Rohacell-110 contributed to a higher stiffness of the sandwich beam and
thus lead to a lower modal density. Besides, it can be seen that the Armacell sandwich
beam had the lowest vibration amplitude at high frequencies, which could be due to the
lowest $G_c$ of the Armacell foam.

### 7.4. Conclusions

The study of Rohacell PMI foam showed that as the frequency increased from 1 Hz to
100 Hz the loss factor decreased approximately from 0.04 to 0.03 while the storage
modulus increased from 152 MPa to 164 MPa. On the contrary, it was found that as the
vibration strain amplitude increased from 0.1% to 4.2%, the loss factor increased
significantly from 0.021 to 0.036 while the storage modulus decreased from 162 MPa to
152 MPa.

The normal damping (tension-compression) and shear damping of PVC foam were
measured respectively on a fatigue machine (MTS Elastomer®). The measured normal
damping loss factors were nearly equal to the shear damping loss factors in the
frequency range of 0-100 Hz. In the shear damping test, an additional clamp was used to
transform the enforced tension-compression vibration to the shear deformation of the
PVC foam. By comparing the measured loss factor and modulus with the FE simulation
results, it was found that the bonding layer between the PVC foam and the clamp had
little influence on the measurement results. Moreover, the measured shear loss factors
from the Elastomer machine were compared with the results from the Rheometric
machines (no additional clamps were used for Rheometric machine), and they showed
good agreement for strain amplitudes < 1%. This proved the validity of the measured
data from Elastomer machines, at least for strain amplitudes < 1%. The damping of a
beam with a length of 1 m was measured using the half-power method. There were five
resonance frequencies and the damping at the five frequencies were also around 1%,
which was similar to that measured using the hysteresis method.

Compared to the Rohacell foam, the damping of PVC foam has little relationship with
the frequency at 1-100 Hz; it also has a limited change as the vibration strain amplitude
increases from 0.05% to 1%. Thus for the dynamic analysis, the PVC foam can be
viewed as an elastic material at low frequencies and vibration amplitudes, while the
Rohacell foam should be viewed as inelastic, i.e., an elastoplastic material.

The predicted loss factors using the MSE method were compared with the measured
loss factors of a sandwich beam. It was found that the MSE method reasonably predicted
the damping loss factors of sandwich structures. Once the damping loss factors of the
facings and the core are known, the damping of sandwich structures can be predicted
using the MSE method.
In addition, the effects of the core thickness \((t_c)\) and the core properties \((G_c)\) on the sandwich damping were studied by experimental measurements and the MSE method prediction. It was found that as \(t_c\) increases, the loss factor of sandwich beams increases. This is because the core has a larger loss factor than the facing and its strain energy percentage increases as \(t_c\) increases. As \(G_c\) increases, the damping is decreased because the shear deformation of the core is reduced.

Compared with the half-power method, the hysteresis method can measure the damping at arbitrary frequencies, and it is easier to measure or to obtain the relationship between the stress and the strain, which facilitates a physical understanding about the vibration and the damping. However, the hysteresis method can only measure the damping at low frequencies (<100Hz) in most cases, because it is difficult to obtain a good hysteresis loop at high frequencies. Generally speaking, the hysteresis damping method is suitable to measure the damping loss factors at low frequencies where structural vibration control is interested, while the half-power method is fit to measure the damping loss factors in a broad frequency range that the noise control are of interest.

Last but not the least, a good measurement technique is vital to obtain accurate and reliable results. With respect to the vibration test, some empirical suggestions are given here:

a) it is better to adopt the adhesive bonding rather than the mechanical connection to connect the sample with the shaker, because the mechanical/bolt connections cause larger noise signal in the measurement. Note that the adhesive should be high-strength and high-stiffness such as epoxy.

b) it is better to use wax to attach the accelerometer to the specimen than to use double adhesive tape, because the double adhesive tape has more viscosity, which may bring external damping at higher frequencies.

c) it is better to test each sample for more than 5 times and then make an average. By doing this, more smooth frequency response curves can be obtained. In addition, a filter window such as Hanning window is suggested to be used during the FFT signal processing.

d) for the safety of the force transducer, the specimen cannot directly be bonded to the force transducer. Thus an additional connector is needed. In this case, it is better to use a connector as light (and compact) as possible.

### 7.5. References


Mechanical and vibro-acoustic aspects of composite sandwich cylinders

Chapter 8  Conclusions and Recommendations

8.1. Conclusions and Highlights

In the application of composite sandwich structures in the aircraft fuselage, the composite sandwich fuselage was idealized as a cylinder. Its mechanical efficiency was compared with the traditional stiffened structure under the flight load. In addition, the sound insulation of composite sandwich cylinders was investigated: the sound transmission loss (TL) of sandwich cylinders were predicted using the analytical and the statistical energy analysis (SEA) methods at 100-16000 Hz. The influence parameters on the TL of cylindrical structures were studied. A laminated cylinder and a sandwich cylinder were produced using the filament winding method. The noise reductions of the laminated and sandwich cylinders were experimentally tested and compared with the FEM/BEM model at 0-1000 Hz. The optimization method, which adopted the acoustic transfer vector (ATV) and the genetic algorithm (GA), was developed to improve the sound insulation of sandwich cylinders at low frequencies, with constraints of mechanical stability. Moreover, the sound transmission of sandwich structures with open-cell foam and the damping properties of the foamed sandwich structures (mainly in the ship building application) were also studied. Similar to the thesis outline in Chapter 1, the conclusions are presented from three aspects: mechanical, sound transmission and damping.

8.1.1. Mechanical aspect

A preliminary design of the aircraft fuselage, in which only the global buckling and the materials failure were taken into consideration, showed that sandwich cylinders have comparable stiffness efficiency with stiffened cylinders. The frame has a significant influence on the structural efficiency of sandwich cylinders and adding some frames can effectively improve the stability of sandwich cylinders under a typical flight load.

With respect to the sandwich cylinders, increasing the facing thickness, core thickness and facing stiffness almost gives linear increases of the allowable torsion and bending moments. On the other hand, increasing the core shear stiffness has little influence on the bending buckling but leads to a slightly increase of the torsion buckling moment. This is caused by the fact that the core stiffness is much lower compared to the facing stiffness and thus its contribution on the bending stiffness is negligible.

Due to the nonuniform stress of the sandwich cylinder under external loads, it is recommended to divide the structures into different regions for the structural sizing, which can avoid the excessive weight at low-stress regions of the cylinder.
8.1.2. Sound transmission aspect

The predicted TL of sandwich cylinders using the SEA method was consistent with that using the analytical methods at most frequencies. However, it was found that the SEA method has higher computation efficiency than the analytical method. Therefore a parametric study was carried out using the SEA method and it was concluded that:

As a characteristic of the cylinder, the ring frequency, where the longitudinal wavelength in the cylindrical shell is equal to the circumference wavelength and an axisymmetric resonance occurs, causes a dip on the TL curve. However, it was found that the effect of the ring frequency on TL of the thick cylindrical shell was not as significant as that of thin shells. Increasing the cylinder radius leads to a decrease of the ring frequency while the cylinder length has little influence on the ring frequency.

The coincidence frequency, $f_c$, is an important characteristic for the sound transmission of sandwich structures. A higher bending stiffness leads to higher radiation efficiency below the $f_c$ while it leads to lower radiation efficiency above the $f_c$. Therefore a higher bending stiffness is beneficial for the noise control at high frequencies ($> f_c$). At low frequencies ($< f_c$), there exists an optimal bending stiffness which can make a good compromise between the structural response and the radiation efficiency. In other words, increasing the core thickness to a certain extent (there is an optimal core thickness for the largest bending stiffness in case of constant weight) or increasing the core shear stiffness can improve the TL at high frequencies while right values of the core thickness or core shear stiffness exist for the best sound insulation at low frequencies.

One way to improve the sound insulation of sandwich structures is to transfer their $f_c$ to be beyond the interested frequency range. However, as the coincidence of sandwich structures occurs at low frequencies (<2000 Hz), it is possible to achieve a better sound insulation by reducing the $f_c$. The reduction of the $f_c$ can be achieved by increasing the core thickness and the core shear stiffness, or by decreasing the core density for most practical configurations.

A sandwich cylinder and a laminated cylinder were compared experimentally for their sound insulation under the acoustic excitation and the force excitation respectively. The only difference between the two cylinders was that the sandwich cylinder had the additional foam core and the sandwich cylinder was heavier than the laminated cylinder. It was found from the comparison that:

The sandwich cylinder had a comparable noise reduction below 2000 Hz, and a much higher noise reduction above 2000 Hz compared to the laminated cylinder under the acoustic excitation. This is caused by that the sandwich cylinder has a similar structural response but higher radiation efficiency at low frequencies (below 2000 Hz); the sandwich cylinder had much lower structural velocity and thus its noise reduction was higher at high frequencies (above 2000 Hz). The phenomenon that the sandwich cylinder
has a lower TL at low frequencies but a higher TL at high frequencies could be explained in terms of the wavenumber: compared to the laminated structure, the sandwich structure has a lower wavenumber and a lower wavenumber leads to a lower TL at low frequencies. However, the sandwich structure has a higher wavenumber and a higher wavenumber leads to a higher TL at high frequencies, see Figures 4-11 and 4-12.

On the other hand, the sandwich cylinder had much lower structural response and lower inner sound pressure than the laminated cylinder at most frequencies under the force excitation. The maximum sound pressure difference between the two cylinders was about 20 dB. Compared to the acoustic excitation, more structural natural modes can be excited by the mechanical force and thus the bending stiffness has more significant influence on the structural response at low frequencies.

The experimental results of the laminated cylinder and the sandwich cylinder under the acoustic and the mechanical excitations were validated by the FEM/BEM simulations. Under the acoustic excitation, both the experimental and the predicted results showed that the cavity resonances make significant influence on the sound transmission besides the structural resonances at 0-1000 Hz. The sound insulation of a sandwich cylinder under the force excitation was parametrically studied at 400-2000Hz. It was found that there is an optimal bending stiffness for the best sound insulation of the sandwich cylinder at low frequencies, which is similar to the prediction results under the acoustic excitation using the SEA method.

The parametric study using the SEA method and the FEM/BEM method led to the following guidelines for designing sandwich structures with good sound insulation:

- Increasing core thickness or facing thickness can improve the sound insulation. However, given that the total weight is constant, there is an optimal core thickness that makes the sandwich structures perform the best sound insulation.
- The unsymmetrical sandwich layup should be avoided since this could cause the extensional-bending coupling and reduce the sound insulation above the $f_c$.
- Below the $f_c$, there exists an optimal core shear stiffness to achieve the best sound insulation. Above the $f_c$, increasing the core shear stiffness can improve the sound insulation. A possible way to improve the sound insulation of sandwich structures is to lower the $f_c$, which can be achieved through increasing the core thickness and the core shear stiffness, or decreasing the core density for most practical configurations.

As previously mentioned, there are right values of the core thickness and core shear stiffness for the best sound insulation of sandwich structures. The Acoustic Transfer Vector (ATV) method, transferring the structural velocity to the sound pressure at field points, and the Generic Algorithm (GA), a searching technique, were coupled in Matlab to optimize the sound insulation of sandwich cylinders at low frequencies. The optimization system proved to be efficient and robust for solve the multi-variable
nonlinear problems. Moreover, the optimization method was extended to perform a multidiscipline and multi-objective optimization of a fuselage section. It was found that the total weight of the fuselage section competed with the inner sound pressure of the fuselage section. The optimization tool provided different possible compromises between the weight and the inner sound pressure. These possible configurations above can be used as gridlines for a multidiscipline design of an aircraft fuselage.

The Transfer Matrix Method (TMM), as an efficient TL prediction method for multi-layered structures, was used to study the TL of sandwich structures with porous absorption cores. The prediction model using the TMM was validated by experimental results. Using the model, the effect of absorption materials on the sound insulation was studied and it was found that the absorbing materials showed little effect on the TL at low frequencies (<500 Hz), however, they increased the TL to a large extent at high frequencies (>1000 Hz). This is because the acoustic wavelength is large and the vibration energy is low at low frequency so that it is prone to have elastic contact with the frame surface with no energy loss. While at high frequencies, the wavelength is small and the vibration energy is high, it is prone to having the inelastic contact and thus the vibration energy can be converted into heat energy. With respect to the characteristics of the absorption foam, a higher flow resistivity, a higher porosity, and a lower tortuosity lead to higher sound absorption. In addition, it was found that the cylinder with the melamine layer had the largest TL while the polyurethane had the lowest TL at high frequencies. This is due to the fact that the melamine foam has a smaller pore size than the polyurethane foam. The pore size should be large enough to let the acoustic wave enter, and meanwhile it cannot be too large so that the acoustic wave can have more reflection or refraction inside the foam. To sum up, the addition of absorption layers can effectively improve the TL of cylinders at high frequencies.

A sandwich, a laminated, and a stiffened cylinder with the same weight were compared with respect to the sound insulation in a diffuse acoustic field. It was found that the laminated cylinder performed the best at low frequencies while the sandwich cylinder had the highest TL at high frequencies. The stiffened cylinder had the worst performance at most frequencies. At low frequencies, the structural velocities of the three structures were similar under the acoustic field and the laminated cylinder had the lowest radiation efficiency. At high frequencies, the sandwich cylinder showed a much lower structural velocity because of the high bending stiffness, while the stiffened cylinder had the higher structural velocity because of the local vibration modes of the unstiffened thin area.

### 8.1.3 Damping aspect

A damping measurement system based on the hysteresis method was developed in the present work, which can characterize the damping properties of elastic cores at arbitrary frequencies in the frequency range of 0-100 Hz. The measurement was carried out on a fatigue machine and measured results were verified by the numerical model and the
measurement using a Rheometric machine. It was found that the measurement system could obtain the material damping of the foam core accurately.

The damping properties of PMI foam and the PVC foam were measured at different frequencies and vibration amplitudes. For the PMI foam, its loss factor decreases and its storage modulus increases as the frequency increases; its loss factor increases and its storage modulus decreases as the vibration amplitude increases. However, the PVC foam shows little difference as the frequency and the vibration amplitude increases (strain < 1%). Finally the loss factor of the PMI foam ranges from 0.03 to 0.04 while the loss factor of the PVC foam is around 0.01.

The damping of the PVC foam was also measured by the half-power method and the results from the half-power method were similar to the results from the hysteresis method. The modal strain energy method proved to predict the damping loss factor of sandwich structures. As the frequency increases, the core plays more and more important roles on the damping of sandwich structures.

8.1.4. Highlights

The CNC codes were developed using the Mathematica® and they were successfully applied to the filament winding of composite sandwich cylinders. The theory and programming method can be easily extended to manufacture other products with similar geometries.

It was found that the bending stiffness has an important influence on the sound transmission of cylinders under the diffuse acoustic field. Below the coincidence frequency, there exists an optimal value for the best sound transmission. Above the coincidence frequency, a higher bending stiffness leads to a better sound insulation.

It was found that an addition of the core has little influence on the sound transmission under the acoustic excitation at low frequencies while it can improve the sound insulation under the mechanical excitation by comparing the vibroacoustic behaviors of a sandwich cylinder and a laminated cylinder experimentally and numerically. However, the addition of a core can improve the sound insulation at high frequencies under both the mechanical and acoustic excitations.

A multidiscipline optimization system integrating the GA and ATV method were developed to improve the sound insulation of sandwich cylinders at low frequencies. For the calculation of the field point pressure, the adoption of ATV method proved to be much more computationally efficient than the traditional coupled FEM/BEM. The GA codes developed in Matlab reached a global optimal value and it can integrate the mechanical and acoustic analysis. Moreover, the optimization system has the possibility to be extended for the thermal analysis.
Compared to the stiffened cylinder, it was found that the sandwich cylinder has a comparable structural efficiency and a better sound insulation, especially at high frequencies above the coincidence frequency.

A damping measurement system was built which can measure the damping and stiffness at arbitrary frequency at 1-100 Hz. The measured system was validated by numerical model and other measurement techniques. The measured data contribute to building the numerical damping model which is used for the vibration control of transport vehicles like the ships.

8.2. Recommendations

In the thesis, a sandwich cylinder was experimentally compared with a laminate cylinder. It would be interesting to compare experimentally a composite sandwich cylinder with a composite stiffened cylinder.

Compared to the mechanical excitation, it was found that the inner pressure of a cylindrical cavity is less sensitive to modification of the cylinder stiffness at low frequencies under the acoustic excitation. Therefore, in this case, promising methods for the noise control could be the usage of acoustic resonators or the active control method, i.e. using speaker to produce a canceling wave in the acoustic cavity.

Although absorption materials can significantly increase the sound insulation at high frequencies, they cannot be used for structural purposes because of their low stiffness. An open-cell foam filled honeycomb sandwich structure would be a good alternative for noise control while meeting the mechanical requirement at the same time.

The damping test is one of the most complex measurements because of many influencing factors that cause errors. The developed measurement system using the hysteresis method can measure the damping properties at arbitrary frequencies with satisfactory results. However, there are some limitations, i.e., the measurement frequency range can only be 0-100 Hz, the Young’s modulus of the samples is confined to be the magnitude of the foam or elastomer. It is required to develop the measurement system to measure damping of high-stiffness materials at high frequencies.

Turbulent layer noise is one of the main external noise sources for current jet transport aircrafts and it can cause both the structural-borne and the acoustic borne noise. Due to the limitations of the experimental conditions, a point acoustic excitation and a point mechanical excitation rather than the turbulent layer noise were discussed in this thesis. It would be interesting to experimentally investigate the sound transmission in sandwich cylinders under simulated turbulent layer noises.

All the fiber placements in the facing were assumed to be straight in the research, however, with the advent of automated tow placement machines, unidirectional fibers can be aligned along curvilinear paths, fiber steering. This advanced layup would give
potential to improve the critical natural frequency of composite structures and to facilitate vibration and noise control in aircrafts.
Acknowledgments

Foremost, I would like to thank my promoter Prof. Adriaan Beukers for providing me with the opportunity to complete my PhD thesis at the Delft University of Technology. Prof. Adriaan Beukers, open minded and smiling, is always kindly and enthusiastic to help me during my PhD work. I especially want to thank my promoter Prof. N. Bert Roozen, whose support and guidance made my thesis work possible. He has been actively interested in my work and has always been available to advise me. I am very grateful for his patience, motivation, enthusiasm, and immense knowledge in vibration and acoustic fields. In addition, special thanks to my co-promoter Dr. Otto Bergsma. He has been a strong and supportive adviser to me throughout my PhD career, but he has always given me great freedom to pursue independent work.

Members of Aerospace Materials and Structure Laboratory also deserve my sincerest thanks, their friendship and assistance has meant more to me than I could ever express. They are Berthil Grashof, Hans Weerheim, Fred Bosch, Kees Sudmeijer. In addition, Cyril Wentzel, a very nice person, helps me to push the M2i project (related to damping) forward. Lisette Vollmer gives me many useful suggestions about the living matters in the Netherlands. Sotiris Koussios taught me a lot about the filament winding production of cylinders. The people that I should also appreciate include Prof. Rinze Benedictus and Mrs. Gemma Windt. Thanks sincerely for the contribution of Roger Groves on my thesis in that he read the thesis thoroughly and gave me lots of useful comments.

Completing my PhD work is the most challenging activity in my past life and it is not easy to get used to the Dutch life as an Asian person. I sometimes feel frustrated and fretful. Thanks to the friendship and the encouragement from my friends, I overcome the difficulties and see the final point eventually. Here thanks to Lei Zu, Qingshi Song, Huajun Fan, Yingxia Qu, Hui Yu, Hao Cui, Ping Liu, Huajie Shi, Lei Shi, Wandong Wang, LiaoJun Yao, Matteo, Marxes, Petter, Wauter etc.

Last but not least, I owe my deepest gratitude to my parents Xingsheng Yuan and Shaoyan Liu, who endlessly support and encourage me in my life. And my wife Ning Kang, who gives me lots of assistance during my PhD work, the most important is she gives me an angel --- my daughter Zhihe Yuan. Thanks to my little sister Liying Yuan in that she gives me lots of happiness and understanding when we grow up together.
Publications

Journal:


Conference:


Curriculum Vitae

Chongxin Yuan was born on 12-12-1984 in Fanxian, Henan, China. He started his junior school study in 1989 and graduated from high school in 2001. Then he entered the Civil Aviation University of China, Tianjin, in 2001. He studied in the field of materials science for 4 years and obtained the bachelor diploma in 2005. After that he continued his master study in Beihang University (Beijing University of aeronautics and astronautics). His main research was optimization of the manufacture of honeycomb sandwich materials. He got the master degree with honors in 2008. In the October of 2008, he entered the Delft University of Technology and started his PhD study in the group of Design and Production of Composite Materials. The main research is about the mechanical and vibroacoustic properties of composite sandwich structures.
Appendix A: Materials properties used in the thesis

1: Facing and core materials for the buckling computation

<table>
<thead>
<tr>
<th>Stiffness [GPa]</th>
<th>Strength [MPa], Ult. strain</th>
</tr>
</thead>
<tbody>
<tr>
<td>$E_1$ = 70</td>
<td>$X_1$ = 600, $\varepsilon_{\text{xc}}$ = 0.85%</td>
</tr>
<tr>
<td>$E_2$ = 70</td>
<td>$Y_1$ = 600, $\varepsilon_{\text{yc}}$ = 0.85%</td>
</tr>
<tr>
<td>$G_{xy}$ = 5</td>
<td>$X_1$ = 570, $\varepsilon_{\text{xc}}$ = 0.8%</td>
</tr>
<tr>
<td>$v_{12}$ = 0.1</td>
<td>$Y_1$ = 570, $\varepsilon_{\text{yc}}$ = 0.8%</td>
</tr>
</tbody>
</table>

$S$ = 90, $\varepsilon_{12}$ = 1.8\%

Core properties ($\rho_c$ = 205 kg/m$^3$); Rohacell 200WF foam

<table>
<thead>
<tr>
<th>stiffness [MPa]</th>
<th>strength [MPa]</th>
</tr>
</thead>
<tbody>
<tr>
<td>$E_c$ = 350</td>
<td>$\sigma_{\text{t,c}}$ = 5.8</td>
</tr>
<tr>
<td>$G_c$ = 150</td>
<td>$\sigma_{\text{c,c}}$ = -9.0</td>
</tr>
<tr>
<td>$\nu_c$ = 0.31</td>
<td>$\tau_{xy}$ = 5.0</td>
</tr>
</tbody>
</table>

Flight Loadings (KNm)

$T$ = 900, $M_y$ = 9500, $M_x$ = 2600, $P$ = 0.13 MPa

2: Unidirectional Carbon ud_t300_n5208 for the FEM analysis in Section 2.4

<table>
<thead>
<tr>
<th>$E_1$</th>
<th>$E_2$</th>
<th>$G_{12}$</th>
<th>$G_{13}$</th>
<th>$G_{23}$</th>
<th>$\nu_{12}$</th>
<th>$\rho$</th>
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</thead>
<tbody>
<tr>
<td>181000</td>
<td>10300</td>
<td>7170</td>
<td>5000</td>
<td>7170</td>
<td>0.28</td>
<td>1600</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>$X_1$</th>
<th>$Y_1$</th>
<th>$X_c$</th>
<th>$Y_c$</th>
<th>$S$</th>
<th>$F_{12}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1500</td>
<td>40</td>
<td>1500</td>
<td>246</td>
<td>68</td>
<td>50</td>
</tr>
</tbody>
</table>

3: Properties of core materials

Honeycomb:

<table>
<thead>
<tr>
<th>Materials name</th>
<th>$E_1$, Compression</th>
<th>$G_{12}$</th>
<th>$G_{23}$</th>
<th>$\rho$ [kg/m$^3$]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Al1/4-ACG-.003</td>
<td>1000</td>
<td>440</td>
<td>220</td>
<td>83.3</td>
</tr>
<tr>
<td>AramidHRH-10-1/8</td>
<td>621</td>
<td>120.69</td>
<td>75.86</td>
<td>144</td>
</tr>
<tr>
<td>Glassfiber HRP-3/16</td>
<td>1793</td>
<td>303</td>
<td>193</td>
<td>192</td>
</tr>
<tr>
<td>Al 5/32-5052-0.002</td>
<td>2411</td>
<td>930</td>
<td>372</td>
<td>129.6</td>
</tr>
</tbody>
</table>

Foam:

<table>
<thead>
<tr>
<th>Materials name</th>
<th>$E_{1,2,3}$</th>
<th>$G_{12,13,23}$</th>
<th>$\rho$ [kg/m$^3$]</th>
</tr>
</thead>
<tbody>
<tr>
<td>ROHACELL® 200 WF</td>
<td>350</td>
<td>150</td>
<td>205</td>
</tr>
<tr>
<td>ROHACELL® 110 WF</td>
<td>180</td>
<td>70</td>
<td>110</td>
</tr>
</tbody>
</table>

Notes: the unit for the modulus and strength for all materials considered here is MPa.

4: Properties of stringers and frames of the stiffened cylinder (aluminum 7075-T62)

$E$ = 7240 MPa, $\rho$ = 2700 kg/m$^3$, $\nu$ = 0.33
Appendix B: Expression of stiffness matrix for the laminates

The expressions to compute the stiffness matrix $A$, $B$, $D$ without concerning the transverse shear are as follows:

$$Q_{ij} = \begin{bmatrix} m^2 & n^2 & -2mn \\ n^2 & m^2 & 2mn \\ mn & -mn & m^2 - n^2 \end{bmatrix} ; Q_{ij}^k = \begin{bmatrix} m^2 & n^2 & -mn \\ n^2 & m^2 & mn \\ 0 & 0 & Q_{66} \end{bmatrix}$$

$$Q_{ij} = \begin{bmatrix} Q_{11} & Q_{12} & 0 \\ Q_{12} & Q_{22} & 0 \\ 0 & 0 & Q_{66} \end{bmatrix}^{-1}$$

$$A_{ij} = \sum_{k=1}^{N} Q_{ij}^k (h_{k+1} - h_k)$$

$$B_{ij} = \frac{1}{2} \sum_{k=1}^{N} Q_{ij}^k (h_{k+1}^2 - h_k^2)$$

$$D_{ij} = \frac{1}{3} \sum_{k=1}^{N} Q_{ij}^k (h_{k+1}^3 - h_k^3)$$

where $m = \cos \theta$, $n = \sin \theta$, and $\theta$ is the fiber angle. $k_1 = 5/6$ is the shear correction factor, $i,j=1,2,6$. When the transverse shear deformations are taken into consideration, the stiffness matrix $A$, $B$, $D$ can be expressed as:

$$F_{ij} = \frac{1}{4} \sum_{k=1}^{N} Q_{ij}^k (h_{k+1}^4 - h_k^4)$$

$$\overline{A}_{ij} = A_{ij} + B_{ij} / R; \overline{B}_{ij} = B_{ij} + D_{ij} / R; \overline{D}_{ij} = D_{ij} + F_{ij} / R$$

$$\hat{A}_{ij} = A_{ij} - B_{ij} / R; \hat{B}_{ij} = B_{ij} - D_{ij} / R; \hat{D}_{ij} = D_{ij} - F_{ij} / R$$

$$A_{44} = \sum_{k=1}^{N} K_1 K_2 G_{13} (h_{k+1} - h_k); B_{44} = \frac{1}{2} \sum_{k=1}^{N} K_1 K_2 G_{13} (h_{k+1}^2 - h_k^2)$$

$$A_{55} = \sum_{k=1}^{N} K_1 K_2 G_{23} (h_{k+1} - h_k); B_{55} = \frac{1}{2} \sum_{k=1}^{N} K_1 K_2 G_{23} (h_{k+1}^2 - h_k^2)$$

$$\hat{A}_{44} = A_{44} - B_{44} / R; \overline{A}_{55} = A_{55} + B_{55} / R$$
Appendix C: Generation of CNC codes for filament winding of composite cylinders

The composite cylinders for the noise tests were produced using the filament winding technique. The generation of the computer numerical control (CNC) codes for the filament winding machine is introduced. The filament winding machine in Delft University of Technology has four freedoms, which are the rotation of the mandrel, the movement of the feed eye in x and z direction, and the rotation of the feed eye.

C.1 Introduction of the mold

The mold is made of steel, which integrates one cylinder part and two cone parts. There is an adjustable gap/leakage on the cylinder surface, which is helpful for the mold release of the cured product. The geometry of the mold is shown in Figure C-1:

C.2 Fiber trajectory on the model

The cylinder part

The fiber starts from the junction between the cylinder and the cone because it is easier to position the fiber in practice. Firstly, the fiber orientation on the cylinder part can be expressed as the following the equation:

$$z_i = \cos[\text{Ang}] \cdot l + L_0$$  \hspace{1cm} (C.1)

$$\theta_i = \frac{\sin[\text{Ang}] \cdot l}{R}$$  \hspace{1cm} (C.2)

The cone part
Compared with the cylinder part, the fiber trajectory of the cone part is more difficult to determine since the radius is changing as the fiber placement. In spite of this, the relationship between the radius and the fiber placement length can be found using a finite element, as shown in Figure C-2.

\[ \phi = \text{ArcTan}\left[ \frac{R - R_0}{Lc} \right] \]  

\[ dl \cdot \cos[\alpha] = -dr / \sin[\phi] \]  

where \( \phi \) is the incline angle of the cone, \( \alpha \) is the fiber angle on the cone surface, it has a relationship with the fiber angle on the cylinder surface:

\[ R \sin[\text{Ang}] = r \sin[\alpha] \]  

By solving the Equations D.3-D.5 with the boundary condition at the junction where \( r_2[L_1] = R \), the radius of the fiber contact point can be expressed as the fiber length. In the same way, the relationship between the turn angle and the fiber placement length \( l \) can be computed as

\[ dl \cdot \sin[\alpha] = d\theta \cdot r_2 \]  

\[ \theta_2[L_1] = \theta_1[L] \bigg|_{L_1} \]  

where \( L_1 \) denotes the length of the placed fiber at the junction between the cylinder part and the cone part. The trajectory in the axial direction can be given by:

\[ z_2 = Lc + L_0 + (R - r_2) \cot[\phi] \]

Finally, the fiber trajectory on the cone 1 can be obtained when \( r_2, \theta_2 \) and \( z_2 \) are calculated by the changing fiber length \( l \). The computation of fiber trajectory on the cone 2 is similar with the cone 1. The details can be found in the attached Mathematica program and thus they are not introduced here.
C.3 Calculation of machine movement

The fiber placement is mainly controlled by three parameters: the rotation of the mandrel, the $x$ and $z$ movement of the feed eye. Thus the three important parameters of the filament winding should be figured out.

![Diagram of machine movement parameters]

**Figure C-3 illustration of determination of machine movement parameters**

**Method 1**

The $z$ position of the feedeye can be computed as:

$$Z_{b_1} = z_1 + L \cdot \cos[\text{Ang}]$$  \hspace{1cm} (C.9)

The $x$ position of the feedeye can be expressed as:

$$X_{b_1} = L \sin[\text{Ang}] \sin[\beta_1] + R \cos[\beta_1]$$  \hspace{1cm} (C.10)

The rotation angle of the mandrel is composed of two parts: the turn angle because of the fiber placement on the model surface ($\theta_1$), as well as the turn angle ($\beta_1$) to keep the distance $L$ constant.

$$\beta_1 = \arctan\left(\frac{L \sin[\text{Ang}]}{R}\right)$$  \hspace{1cm} (C.11)

$$\gamma_1 = \beta_1 + \theta_1$$  \hspace{1cm} (C.12)

The main idea is to find the position of the feed eye as the fiber length changes. The distance between the contact point on the cylinder surface and the feed eye is kept as constant.

For the cone part, the relationship between the fiber position on the model and the position of feed eye is determined by solving the geometric equations based on the directional derivatives.
\[ \beta_2 = \text{ArcTan}\left( \frac{L\sin[\alpha_i]}{L\cos[\alpha_i]\sin[\phi] + r_2} \right) \]  
(C.13)

\[ Xb_2 = L(\sin[\alpha_i]\sin[\beta_2] + \cos[\alpha_i]\sin[\phi]\cos[\beta_2]) + r_2\cos[\beta_2] \]  
(C.14)

\[ Zb_2 = z_2 + L\cos[\alpha_i]\cos[\phi] \]  
(C.15)

\[ \gamma_2 = \beta_2 + \theta_2 \]  
(C.16)

### Method 2

The method 2 is much easier to understand and to program than the first method. Its core idea is that the fiber trajectory is firstly discretized in terms of the fiber length \( \Delta l \); then the unit vector of the fiber direction is determined by using the two neighboring points of the fiber trajectory on the model, as shown in Equation (C.16). Finally, the position of the feed eye can be computed by multiplying the fiber length with the unit vector, as shown in Equation (C.17).

\[ Vec(i) = \frac{G_{\text{body}}(i + 1) - G_{\text{body}}(i)}{\text{Norm}[G_{\text{body}}(i + 1) - G_{\text{body}}(i)]} \]  
(C.16)

The relative position of the machine feed eye is:

\[ P_{\text{body}}(i) = G_{\text{body}}(i) + Vec(i)L \]  
(C.17)

Here both \( Vec \) and \( P_{\text{body}} \) are 3×1 vectors.

The absolute position of the machine feed eye can be expressed in the relative position as:

\[
\begin{bmatrix}
px(i) \\
0 \\
pz(i)
\end{bmatrix} =
\begin{bmatrix}
\sin[C(i)] & \cos[C(i)] & 0 \\
0 & \cos[C(i)] & -\sin[C(i)] \\
0 & 0 & 1
\end{bmatrix}
\begin{bmatrix}
P_{\text{body}}(1, i) \\
P_{\text{body}}(2, i) \\
P_{\text{body}}(3, i)
\end{bmatrix}
\]  
(C.18)

From the three equations in Equation (C.18), the three unknown parameters, \( px(i) \), \( pz(i) \), and \( C(i) \), can be solved. Here \( px \) is the position of the feed eye perpendicular to the mandrel, \( pz \) the position of the feed eye along the mandrel, and \( C \) the rotation angle of the mandrel.

The advantages of the second method are:

A, it is no need to treat the cylinder part and cone part individually.

B, it is easy to understand and to perform with mathematical program.
C, this computation of the machine movement is only dependent on the fiber trajectory rather than the geometry of the mode, which means that no extra analysis is needed if the geometry or shape of the mandrel is changed.

The simulated fiber trajectory on the mold (black line) and the movement trajectory of the machine feedeye (blue line) are shown in Figure C-4. The filament winding of dry glass fibers on the steel mold is shown in Figure C-5. The Mathematica program for the filament winding is attached in Appendix I.

**Figure C-4 Fiber trajectory (black) and the feedeye movement (blue), simulated in Mathematica**

**Figure C-5 Filament wind a composite cylinder using generated CNC codes, R=0.25 m, L=1.2 m**
Appendix D Measured damping data

*Rohacell foam in tension-compression using elastomer machine*

### Table D-1 Rohacell/PMI 110 foam VS strain amplitude at 10HZ, at 22°C

<table>
<thead>
<tr>
<th>Strain Amplitude</th>
<th>E'/G'</th>
<th>E''/G''</th>
<th>Loss factor</th>
</tr>
</thead>
<tbody>
<tr>
<td>mm/mm</td>
<td>N/mm²</td>
<td>N/mm²²</td>
<td></td>
</tr>
<tr>
<td>0.0013</td>
<td>164.5672</td>
<td>4.038386</td>
<td>0.024539</td>
</tr>
<tr>
<td>0.0027</td>
<td>163.4831</td>
<td>4.206055</td>
<td>0.025728</td>
</tr>
<tr>
<td>0.0040</td>
<td>162.2831</td>
<td>4.392205</td>
<td>0.027065</td>
</tr>
<tr>
<td>0.0054</td>
<td>161.2693</td>
<td>4.581592</td>
<td>0.02841</td>
</tr>
<tr>
<td>0.0067</td>
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</tr>
<tr>
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</tr>
<tr>
<td>0.0094</td>
<td>156.8061</td>
<td>5.137022</td>
<td>0.03276</td>
</tr>
<tr>
<td>0.0107</td>
<td>155.2886</td>
<td>5.317099</td>
<td>0.03424</td>
</tr>
<tr>
<td>0.0120</td>
<td>153.8603</td>
<td>5.565991</td>
<td>0.036176</td>
</tr>
<tr>
<td>0.0134</td>
<td>152.3227</td>
<td>5.781915</td>
<td>0.037958</td>
</tr>
</tbody>
</table>

### Table D-2 Rohacell/PMI 110 foam VS strain frequency, at 22°C

<table>
<thead>
<tr>
<th>Frequency</th>
<th>Strain Amplitude</th>
<th>E'/G'</th>
<th>E''/G''</th>
<th>Loss factor</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hz</td>
<td>mm/mm</td>
<td>N/mm²</td>
<td>N/mm²²</td>
<td>unitless</td>
</tr>
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<td>1</td>
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<td>152.3072</td>
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<tr>
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<td>0.040487</td>
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<tr>
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<tr>
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<td>0.038103</td>
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### PVC foam in tension-compression using elastomer machine

#### Table D-3 PVC H60 foam in tension-compression using elastomer, strain=1.5%, at 22°C

<table>
<thead>
<tr>
<th>Frequency (Hz)</th>
<th>E' (N/mm²)</th>
<th>E'' (N/mm²)</th>
<th>Loss factor</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>35.06959</td>
<td>0.463357</td>
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</tr>
<tr>
<td>6</td>
<td>35.42955</td>
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<tr>
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<td>16</td>
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</table>

#### Table D-4 PVC H60 foam in tension-compression using elastomer, frequency=10 Hz, at 22°C

<table>
<thead>
<tr>
<th>Strain Amplitude (mm/mm)</th>
<th>E' (N/mm²)</th>
<th>E'' (N/mm²)</th>
<th>Loss factor</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.007</td>
<td>36.52916</td>
<td>0.403132</td>
<td>0.011036</td>
</tr>
<tr>
<td>0.011</td>
<td>36.04811</td>
<td>0.410384</td>
<td>0.011384</td>
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<tr>
<td>0.014</td>
<td>35.52995</td>
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<td>0.011583</td>
</tr>
<tr>
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<td>34.97083</td>
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### PVC foam in shear using elastomer machine

**Table D-5 PVC foam VS frequency in shear using elastomer machine, at 22°C**

<table>
<thead>
<tr>
<th>Frequency (Hz)</th>
<th>Strain Amplitude (mm/mm)</th>
<th>$G'$ (N/mm$^2$)</th>
<th>$G''$ (N/mm$^2$)</th>
<th>Loss factor</th>
</tr>
</thead>
<tbody>
<tr>
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<td>0.014288</td>
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<tr>
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</tbody>
</table>

**Table D-6 PVC foam VS strain amplitude in shear using elastomer machine at 10 Hz, 22°C**

<table>
<thead>
<tr>
<th>Strain Amplitude (mm/mm)</th>
<th>$E'/G'$</th>
<th>$E''/G''$</th>
<th>Loss factor</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.003571</td>
<td>10.51969</td>
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<td>0.01428</td>
<td>10.3986</td>
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<tr>
<td>0.024999</td>
<td>10.17644</td>
<td>0.120918</td>
<td>0.011882</td>
</tr>
<tr>
<td>0.032154</td>
<td>9.991765</td>
<td>0.125803</td>
<td>0.012591</td>
</tr>
</tbody>
</table>

### PVC foam in torsion using Rheometric machine

**Table D-7 PVC H60 foam VS strain amplitude at 1.6Hz at 22°C**

<table>
<thead>
<tr>
<th>Strain amplitude (%)</th>
<th>$G'$ (Pa)</th>
<th>$G''$ (Pa)</th>
<th>Loss factor</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.048031</td>
<td>1.28E+07</td>
<td>1.45E+05</td>
<td>0.011353</td>
</tr>
<tr>
<td>0.06045</td>
<td>1.28E+07</td>
<td>1.37E+05</td>
<td>0.010743</td>
</tr>
<tr>
<td>0.076128</td>
<td>1.28E+07</td>
<td>1.39E+05</td>
<td>0.010865</td>
</tr>
<tr>
<td>0.095876</td>
<td>1.28E+07</td>
<td>1.35E+05</td>
<td>0.010621</td>
</tr>
<tr>
<td>0.12055</td>
<td>1.27E+07</td>
<td>1.34E+05</td>
<td>0.010498</td>
</tr>
<tr>
<td>0.15152</td>
<td>1.27E+07</td>
<td>1.29E+05</td>
<td>0.010132</td>
</tr>
<tr>
<td>0.19063</td>
<td>1.27E+07</td>
<td>1.30E+05</td>
<td>0.010254</td>
</tr>
<tr>
<td>0.23993</td>
<td>1.27E+07</td>
<td>1.28E+05</td>
<td>0.010071</td>
</tr>
<tr>
<td>0.30139</td>
<td>1.27E+07</td>
<td>1.25E+05</td>
<td>0.009888</td>
</tr>
<tr>
<td>0.37973</td>
<td>1.26E+07</td>
<td>1.27E+05</td>
<td>0.01001</td>
</tr>
<tr>
<td>0.47663</td>
<td>1.26E+07</td>
<td>1.29E+05</td>
<td>0.010254</td>
</tr>
<tr>
<td>0.59779</td>
<td>1.26E+07</td>
<td>1.29E+05</td>
<td>0.010254</td>
</tr>
<tr>
<td>Frequency</td>
<td>G'</td>
<td>G''</td>
<td>Loss factor</td>
</tr>
<tr>
<td>-----------</td>
<td>------</td>
<td>------</td>
<td>-------------</td>
</tr>
<tr>
<td>Hz</td>
<td>Pa</td>
<td>Pa</td>
<td>unitless</td>
</tr>
<tr>
<td>0.16</td>
<td>1.26E+07</td>
<td>1.63E+05</td>
<td>0.01294</td>
</tr>
<tr>
<td>0.25</td>
<td>1.26E+07</td>
<td>1.61E+05</td>
<td>0.012757</td>
</tr>
<tr>
<td>0.40</td>
<td>1.26E+07</td>
<td>1.62E+05</td>
<td>0.012865</td>
</tr>
<tr>
<td>0.63</td>
<td>1.26E+07</td>
<td>1.65E+05</td>
<td>0.01303</td>
</tr>
<tr>
<td>1.00</td>
<td>1.27E+07</td>
<td>1.51E+05</td>
<td>0.011919</td>
</tr>
<tr>
<td>1.59</td>
<td>1.27E+07</td>
<td>1.42E+05</td>
<td>0.011217</td>
</tr>
<tr>
<td>2.52</td>
<td>1.27E+07</td>
<td>1.49E+05</td>
<td>0.011719</td>
</tr>
<tr>
<td>4.00</td>
<td>1.27E+07</td>
<td>1.40E+05</td>
<td>0.010987</td>
</tr>
<tr>
<td>6.34</td>
<td>1.28E+07</td>
<td>1.50E+05</td>
<td>0.011719</td>
</tr>
<tr>
<td>10.05</td>
<td>1.28E+07</td>
<td>1.63E+05</td>
<td>0.012696</td>
</tr>
<tr>
<td>15.92</td>
<td>1.29E+07</td>
<td>1.60E+05</td>
<td>0.012437</td>
</tr>
</tbody>
</table>

Table D-9 PVC H60 foam VS frequency at strain=0.5% at 22°C

<table>
<thead>
<tr>
<th>Frequency</th>
<th>G'</th>
<th>G''</th>
<th>Loss factor</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hz</td>
<td>Pa</td>
<td>Pa</td>
<td>unitless</td>
</tr>
<tr>
<td>0.16</td>
<td>1.22E+07</td>
<td>1.47E+05</td>
<td>0.012025</td>
</tr>
<tr>
<td>0.25</td>
<td>1.23E+07</td>
<td>1.44E+05</td>
<td>0.011719</td>
</tr>
<tr>
<td>0.40</td>
<td>1.23E+07</td>
<td>1.41E+05</td>
<td>0.011475</td>
</tr>
<tr>
<td>0.63</td>
<td>1.23E+07</td>
<td>1.37E+05</td>
<td>0.011109</td>
</tr>
<tr>
<td>1.00</td>
<td>1.23E+07</td>
<td>1.32E+05</td>
<td>0.010743</td>
</tr>
<tr>
<td>1.59</td>
<td>1.23E+07</td>
<td>1.33E+05</td>
<td>0.010804</td>
</tr>
<tr>
<td>2.52</td>
<td>1.24E+07</td>
<td>1.36E+05</td>
<td>0.010987</td>
</tr>
<tr>
<td>4.00</td>
<td>1.24E+07</td>
<td>1.39E+05</td>
<td>0.011231</td>
</tr>
<tr>
<td>6.34</td>
<td>1.24E+07</td>
<td>1.44E+05</td>
<td>0.011597</td>
</tr>
<tr>
<td>10.05</td>
<td>1.25E+07</td>
<td>1.49E+05</td>
<td>0.011963</td>
</tr>
<tr>
<td>15.92</td>
<td>1.25E+07</td>
<td>1.49E+05</td>
<td>0.011902</td>
</tr>
</tbody>
</table>
Appendix E: Matlab codes

The following is the Matlab codes that used for the FEM/BEM analysis and optimizations in Chapter 5. The codes include:

1. Binary_Main_Chapter5.m  
Main program defining the parameter of GA

2. gaFitness1mCylinderSB.m  
Evaluate the fitness

3. compute_pressure_lunwen5B.m  
Compute the inner pressure

4. modify_bdf_paramatric_angle.m  
Modify the input file of Nastran

5. nastranRUN.m  
Call Nastran to run in Matlab

6. read_f06_sandwich.m  
Read the output file of Nastran in matlab

7. ga.m  
Different from the ga.m in the toolbox of Matlab

8. FEM_BEM_cylinderSUCCESS.m  
Calculate the inner pressure of the cylinder

---

1. Main program: **Binary_Main_Chapter5.m**

```matlab
% this program is to use GA to optimize the inner average sound pressure
% this is the main function to define the parameter of GA

clc
clear all
close all

% global bounds
rand('seed',0);
% Crossover Operators
xFns = 'simpleXover';
% xFns='linerorderXover';
% xFns = 'cyclicXover'
xOpts = [0.4]; % suggest 0.4~0.99
% Mutation Operators
mFns = 'binaryMutation';
mOpts = [0.01]; %0.005
% Termgaination Operators
termFns = 'maxGenTerm';
termOps = [9]; % 100 Generation step, here there are 20 steps
% Selection Function
% selectFn = 'tournSelect'
selectFn = 'roulette' %gamble
```
% selectFn = 'normGeomSelect'
% selectOps = [0.08];
selectOps = [];
% Evaluation Function
evalFn = 'gaFitness1mCylinderSB';
%evalFn = 'test';
evalOps = [];
% Bounds on the variables
bounds = [-90,90; -90, 90; -90, 90;1,30;10,50;1,10];
% GA Options [epsilon float/binar display]
gaOpts=[1e-6 0 1]; %precision=1e-6, second 0=binary, 1=float,
binary is better for the multi variables
startPop = initializega(10,bounds,evalFn,[],gaOpts);
%population(1st value) affect the results, but not mean that
increase will improve the optimization effect

[x endPop bestPop
trace]=ga(bounds,evalFn,evalOps,startPop,gaOpts,...
termFns,termOps,selectFn,selectOps,xFns,xOpts,mFns,mOpts);

% x is the best solution found
x
% endPop is the ending population
endPop;
% trace is a trace of the best value and average value of
generations
trace;
bestPop
clf
%plot the results
plot(trace(:,1),trace(:,2),'-ob',trace(:,1),trace(:,3),'-*r ');
legend('best','average');
title('optimization history of the genetic algorithm')
grid on

2. Fitness function: gaFitness1mCylinderSB.m

function [variable,y]=
gaFitness1mCylinderSB(variable)
modify_bdf_paramatric_angle(variable)
astraanRUN
pause(40)
averdB=compute_pressure_lunwen5B  %compute the inner
pressure using ATV method
weight=6*variable(6)*1e-4*1800+variable(4)*1e-3*70

if weight>8
   fprintf('weight too high, punish')
   y=1/(averdB+100)
% elseif maxpressure>55
%     fprintf('acoustic constraint happen, punish')
%     val=1/(weight+5000)*factor % punish
else
   y=1/averdB
end

3. Compute the inner pressure using ATV and surface velocity:
compute_pressure_lunwen5B.m

function averdB=compute_pressure_lunwen5B()
Nnode=784;  % the number of the BEM nodes
Nfreq=5;   % the number of frequency
[16,32,64,128,256,512,1024,20000]
Npoint=1;  %number of field points

ATV=load('D:\chongxinyuan\Dropbox\genetic_algorithm\optimization400to900\ATV400to976.mat');

CATV=ATV.CATV;
read_f06_sandwich
pause(2)
pressure=zeros(Nfreq,6);
% for k=1:Npoint %here only one field point is used, so k is disabled
for j=1:Nfreq
   pressureNode=zeros(1,6);
   for h=1:Nnode
      aa=[Cvelo(j,h,1).*CATV(j,h,1),Cvelo(j,h,2).*CATV(j,h,2),Cvelo(j,h,3).*CATV(j,h,3),Cvelo(j,h,4).*CATV(j,h,4),Cvelo(j,h,5).*CATV(j,h,5),Cvelo(j,h,6).*CATV(j,h,6)];
      pressureNode=pressureNode+aa;
   end
   %pressure(j)=20*log10(abs(pressureNode)/2e-5);
   pressure(j,:)=pressureNode;
   %Apressure(j,k)=20*log10(abs(Apressure(j,k))./2e-5);
end

pressure2(j)=(pressure(j,1)+pressure(j,2)*i)+(pressure(j,3)+pressure(j,4)*i)+(pressure(j,5)+pressure(j,6)*i);
% Apressure(j,k)=20*log10(abs(Apressure(j,k))./2e-5);
% end
% end
aver=sqrt((pressure2(1)^2+pressure2(2)^2+pressure2(3)^2+pressure2(4)^2+pressure2(5)^2)/5); %average for frequency
%transform to dB
pref=2e-5;
averdB=20*log10(abs(aver)/pref);

4. Modify the input file for Nastran: modify_bdf_paramatric_angle.m
function modify_bdf_paramatric_angle(variable)
    fid = fopen('D:\chongxinyuan\Dropbox\genetic_algorithm\optimization400to900\quater_cylinder_octavell1.bdf', 'r'); % acoustic force
    fid_n=fopen('D:\chongxinyuan\Dropbox\genetic_algorithm\optimization400to900\quater_cylinder_octavell1b.bdf','w');
    tf=variable(6);
    
    % for sandwich
    for k=1:1899
        tline=fgetl(fid);
        if k==33
            fprintf(fid_n, '   , 2, %1.1f-4 , %4.1f , YES , 2
, %1.1f-4 , %4.1f , YES
',[tf,variable(1),tf,variable(2)]);
        elseif k==34
            fprintf(fid_n, '   , 2 , %1.1f-4 , %4.1f , YES , 3
, %1.1f-3  , 0.   , YES
',[tf,variable(3),variable(4)]);
        elseif k==35
            fprintf(fid_n, '  ,  2, %1.1f-4 , %4.1f , YES , 2
, %1.1f-4   , %4.1f , YES
',[tf,variable(3),tf,variable(2)]);
        elseif k==36
            fprintf(fid_n, '   , 2 , %1.1f-4 , %4.1f , YES
',[tf,variable(1)]);
        elseif k==1024  %the core thickness and shear stiffness
            fprintf(fid_n,'MAT1  ,   2,      1.51+8,  %6.1e ,     ,70.\n',variable(5)*1e6)
        else
            fprintf(fid_n,'%s
',tline);
        end
    end
    fclose(fid);
    fclose(fid_n);
5. Call Nastran to run in Matlab: nastranRUN.m
% == run the bdf file for the acoustic pressure ==

!C:\Users\User\AppData\Roaming\MSC.Software\MD_Nastran\bin\mdnast2008w.exe
D:\chongxinyuan\Dropbox\genetic_algorithm\optimization400to900\quater_cylinder_octave111b.Bdf
!del
D:\chongxinyuan\Dropbox\genetic_algorithm\optimization400to900\quater_cylinder_octave111b.f04
!del
D:\chongxinyuan\Dropbox\genetic_algorithm\optimization400to900\quater_cylinder_octave111b.xdb
!del
D:\chongxinyuan\Dropbox\genetic_algorithm\optimization400to900\quater_cylinder_octave111b.log
fprintf('finish nastranRUN\n')

6. Read the output file from Nastran in Matlab: read_f06_sandwich.m

title_f06='D:\chongxinyuan\Dropbox\genetic_algorithm\optimization400to900\quater_cylinder_octave111b.f06';
Nfreq=5;
Nnode=784;
 fid = fopen(title_f06, 'r'); % no optimization
 Cv=zeros(10,3);
h=1;
while ~feof(fid)
    tline=fgetl(fid);
    if (length(tline)==114 || length(tline)==105) &&
        double(tline(28))>=48&&double(tline(28))<=57
%        fprintf(fid2,'%s
',tline);
    Cv(h,:)=[str2num(tline(26:40)),str2num(tline(40:55)),str2num(tline(56:70))];
    h=h+1;
end
fclose(fid);
% fclose(fid2)
% Velocity of every frequency from 100Hz to 1000Hz by 100Hz
Cvelo=zeros(Nfreq,Nnode,6); % 6 denotes Tx,jTx,Ty,jTy,Tz,jTz
for m=1:Nfreq
    st=1+(m-1)*2*Nnode; % starting point for each frequency
    en=m*2*Nnode; % ending point for each frequency
    for hl=st:2:en-1
Cvelo(m, (h1-st)/2+1,:)=[Cv(h1,1), Cv(h1+1,1), Cv(h1,2), Cv(h1+1,2), Cv(h1,3), Cv(h1+1,3)]; %Rvx, Imvx, Rvy, Imvy, Rvz, Imvz
end
end

fprintf('finish reading f06\n');

7. GA program (Note: this program is different from the one in the toolbox of Matlab)

**function** [x, endPop, bPop, traceInfo] = ga(bounds, evalFN, evalOps, startPop, opts, ...
    termFN, termOps, selectFN, selectOps, xOverFNs, xOverOps, mutFNs, mutOps)

% GA run a genetic algorithm function
[x, endPop, bPop, traceInfo] = ga(bounds, evalFN, evalOps, startPop, opts, ...
    termFN, termOps, selectFN, selectOps, xOverFNs, xOverOps, mutFNs, mutOps)

% Output Arguments:
% x          - the best solution found during the course of the run
% endPop     - the final population
% bPop       - a trace of the best population
% traceInfo  - a matrix of best and means of the ga for each generation

% Input Arguments:
% bounds     - a matrix of upper and lower bounds on the variables
% evalFN     - the name of the evaluation .m function
% evalOps    - options to pass to the evaluation function ([NULL])
% startPop   - a matrix of solutions that can be initialized from initialize.m
% opts       - [epsilon prob_ops display] change required to consider two solutions different, prob_ops 0 if you want to apply the genetic operators probabilistically to each solution, 1 if you are supplying a deterministic number of applications and display is 1 to output progress 0 for quiet. ([1e-6 1 0])
% termFN       - name of the .m termination function
([maxGenTerm])
% termOps      - options string to be passed to the termination
function
% ([100]).
% selectFN     - name of the .m selection function
([normGeomSelect])
% selectOpts   - options string to be passed to select after
%                 select(pop,#,opts) ([0.08])
% xOverFNS     - a string containing blank seperated names of
%                 Xover.m
%                 files (['arithXover heuristicXover
%                 simpleXover'])
% xOverOps     - A matrix of options to pass to Xover.m files
%                 with the
%                 first column being the number of that xOver to
%                 perform
%                 similiarly for mutation ([2 0; 2 3; 2 0])
% mutFNs       - a string containing blank seperated names of
%                 mutation.m
%                 files (['boundaryMutation
%                 multiNonUnifMutation ...
%                 nonUnifMutation unifMutation'])
% mutOps       - A matrix of options to pass to Xover.m files
%                 with the
%                 first column being the number of that xOver to
%                 perform
%                 similiarly for mutation ([4 0 0; 6 100 3; 4 100
%                 3; 4 0 0])

n=argin; 
if n<2 | n==6 | n==10 | n==12
  disp('Insufficient arguments')
end
if n<3 %Default evaluation opts.
  evalOps=[];
end
if n<5
  opts = [le-6 1 0];
end
if isempty(opts)
  opts = [le-6 1 0];
end
if any(evalFN<48) %Not using a .m file
  if opts(2)==1 %Float ga
    elstr=['x=c1; c1(xZomeLength)=' evalFN ';'];
    e2str=['x=c2; c2(xZomeLength)=' evalFN ';'];
  else %Binary ga
    elstr='x=c1; c1(xZomeLength)=';
    e2str='x=c2; c2(xZomeLength)=';
  end
elstr=['x=b2f(endPop(j,:),bounds,bits);
endPop(j,xZomeLength)=',...
    evalFN ';']
end
else %Are using a .m file
    if opts(2)==1 %Float ga
        elstr=['c1 c1(xZomeLength)=' evalFN '(c1,[gen evalOps]);'
        e2str=['c2 c2(xZomeLength)=' evalFN '(c2,[gen evalOps]);'
    else %Binary ga
        elstr=['x=b2f(endPop(j,:),bounds,bits);[x v]=', evalFN ...
            '(x,[gen evalOps]); endPop(j,:)=f2b(x,bounds,bits)
            v];'
    end
end

if n<6 %Default termination information
    termOps=[100];
    termFN='maxGenTerm';
end
if n<12 %Default mutation information
    if opts(2)==1 %Float GA
        mutFNs=['boundaryMutation multiNonUnifMutation nonUnifMutation unifMutation'];
        mutOps=[4 0 0;6 termOps(1) 3;4 termOps(1) 3;4 0 0];
    else %Binary GA
        mutFNs=['binaryMutation'];
        mutOps=[0.05];
    end
end
if n<10 %Default crossover information
    if opts(2)==1 %Float GA
        xOverFNs=['arithXover heuristicXover simpleXover'];
        xOverOps=[2 0;2 3;2 0];
    else %Binary GA
        xOverFNs=['simpleXover'];
        xOverOps=[0.6];
    end
end
if n<9 %Default select opts only i.e. roulette wheel.
    selectOps=[];
end
if n<8 %Default select info
    selectFN=['normGeomSelect'];
    selectOps=[0.08];
end
if n<6 %Default termination information
termOps=[100];
termFN='maxGenTerm';
end
if n<4 %No starting population passed given
    startPop=[];
end
if isempty(startPop) %Generate a population at random
    startPop=zeros(80,size(bounds,1)+1);
end
startPop=initializega(80,bounds,evalFN,evalOps,opts(1:2));
end
if opts(2)==0 %binary
    bits=calcbits(bounds,opts(1));
end
xOverFNs=parse(xOverFNs);
mutFNs=parse(mutFNs);

xZomeLength = size(startPop,2); %Length of the xzome=numVars+fittness
numVar = xZomeLength-1; %Number of variables
popSize = size(startPop,1); %Number of individuals in the pop
endPop = zeros(popSize,xZomeLength); %A secondary population matrix
c1 = zeros(1,xZomeLength); %An individual
c2 = zeros(1,xZomeLength); %An individual
numXOvers = size(xOverFNs,1); %Number of Crossover operators
numMuts = size(mutFNs,1); %Number of Mutation operators
epsilon = opts(1); %Threshold for two fitness to differ
oval = max(startPop(:,xZomeLength)); %Best value in start pop
bFoundIn = 1; %Number of times best has changed
done = 0; %Done with simulated evolution
gen = 1; %Current Generation Number
collectTrace = (nargout>3); %Should we collect info every gen
floatGA = opts(2)==1; %Probabilistic application of ops
display = opts(3); %Display progress

while(~done)
    %Elitist Model
[bval, bindx] = max(startPop(:, xZomeLength)); %Best of current pop
best = startPop(bindx,:);

if collectTrace
    traceInfo(gen,1)=gen; %current generation
    traceInfo(gen,2)=startPop(bindx,xZomeLength); %Best fittness
    traceInfo(gen,3)=mean(startPop(:,xZomeLength)); %Avg fittness
    traceInfo(gen,4)=std(startPop(:,xZomeLength));
end

if ( (abs(bval - oval)>epsilon) | (gen==1)) %If we have a new best sol
    if display
        fprintf(1, '
%d %f
', gen, bval); %Update the display
    end
    if floatGA
        bPop(bFoundIn,:)=[gen startPop(bindx,:)]; %Update bPop Matrix
    else
        bPop(bFoundIn,:)=[gen b2f(startPop(bindx,1:numVar),bounds,bits)...  
            startPop(bindx,xZomeLength)];
    end

    bFoundIn=bFoundIn+1; %Update number of changes
    oval=bval; %Update the best val
else
    if display
        fprintf(1, '
%d ', gen); %Otherwise just update num gen
    end
end

endPop = feval(selectFN, startPop, [gen selectOps]); %Select

if floatGA %Running with the model where the parameters are numbers of ops
    for i=1:numXOvers,
        for j=1:xOverOps(i,1),
            a = round(rand*(popSize-1)+1); %Pick a parent
            b = round(rand*(popSize-1)+1); %Pick another parent
            xN=deblank(xOverFNs(i,:)); %Get the name of crossover function
        end
    end
end
[c1 c2] = feval(xN,endPop(a,:),endPop(b,:),bounds,[gen xOverOps(i,:)]);

if c1(1:numVar)==endPop(a,(1:numVar)) %Make sure we created a new
c1(xZomeLength)=endPop(a,xZomeLength); %solution before evaluating
elseif c1(1:numVar)==endPop(b,(1:numVar))
c1(xZomeLength)=endPop(b,xZomeLength);
else
%[c1(xZomeLength) c1] = feval(evalFN,c1,[gen evalOps]);
eval(e1str);
end
if c2(1:numVar)==endPop(a,(1:numVar))
c2(xZomeLength)=endPop(a,xZomeLength);
elseif c2(1:numVar)==endPop(b,(1:numVar))
c2(xZomeLength)=endPop(b,xZomeLength);
else
%[c2(xZomeLength) c2] = feval(evalFN,c2,[gen evalOps]);
eval(e2str);
end
endPop(a,:)=c1;
endPop(b,:)=c2;
end

for i=1:numMuts,
for j=1:mutOps(i,1),
a = round(rand*(popSize-1)+1);
c1 = feval(deblank(mutFNs(i,:)),endPop(a,:),bounds,[gen mutOps(i,:)]);
if c1(1:numVar)==endPop(a,(1:numVar))
c1(xZomeLength)=endPop(a,xZomeLength);
else
%[c1(xZomeLength) c1] = feval(evalFN,c1,[gen evalOps]);
eval(e1str);
end
endPop(a,:)=c1;
end
end

else %We are running a probabilistic model of genetic operators
for i=1:numXOvers,
xN=deblank(xOverFNs(i,:)); %Get the name of crossover function
cp=find(rand(popSize,1)<xOverOps(i,1)==1)
if rem(size(cp,1),2) cp=cp(1:(size(cp,1)-1)); end
cp=reshape(cp,size(cp,1)/2,2);
endPop(a,:)=c1;
endPop(b,:)=c2;
end
end

for i=1:numMuts,
for j=1:mutOps(i,1),
a = round(rand*(popSize-1)+1);
c1 = feval(deblank(mutFNs(i,:)),endPop(a,:),bounds,[gen mutOps(i,:)]);
if c1(1:numVar)==endPop(a,(1:numVar))
c1(xZomeLength)=endPop(a,xZomeLength);
else
%[c1(xZomeLength) c1] = feval(evalFN,c1,[gen evalOps]);
eval(e1str);
end
endPop(a,:)=c1;
end
end

else %We are running a probabilistic model of genetic operators
for i=1:numXOvers,
xN=deblank(xOverFNs(i,:)); %Get the name of crossover function
cp=find(rand(popSize,1)<xOverOps(i,1)==1)
if rem(size(cp,1),2) cp=cp(1:(size(cp,1)-1)); end
cp=reshape(cp,size(cp,1)/2,2);
endPop(a,:)=c1;
endPop(b,:)=c2;
end
end
end

for j=1:size(cp,1)
    a=cp(j,1); b=cp(j,2);
    [endPop(a,:) endPop(b,:)] = feval(xN,endPop(a,:),endPop(b,:),
        bounds,[gen xOverOps(i,:)]);
end
end
for i=1:numMuts
    mN=deblank(mutFNs(i,:));
    for j=1:popSize
        endPop(j,:) = feval(mN,endPop(j,:),bounds,[gen
        mutOps(i,:)])
        eval(e1str);
    end
end
end

for j=1:size(cp,1)
    a=cp(j,1); b=cp(j,2);
    [endPop(a,:) endPop(b,:)] = feval(xN,endPop(a,:),endPop(b,:),
        bounds,[gen xOverOps(i,:)]);
end
end
for i=1:numMuts
    mN=deblank(mutFNs(i,:));
    for j=1:popSize
        endPop(j,:) = feval(mN,endPop(j,:),bounds,[gen
        mutOps(i,:)])
        eval(e1str);
    end
end
end

gen=gen+1
done=feval(termFN,[gen termOps],bPop,endPop); %See if the ga
is done
startPop=endPop; %Swap the populations
[bval,bindx] = min(startPop(:,xZomeLength)); %Keep the best
    solution
    startPop(bindx,:) = best; %replace it with the worst
end
[bval,bindx] = max(startPop(:,xZomeLength));
if display
    fprintf(1,'%d %f
',gen,bval);
end
x=startPop(bindx,:);
if opts(2)==0 %binary
    x=b2f(x,bounds,bits);
    bPop(bFoundIn,:)=[gen
        b2f(startPop(bindx,1:numVar),bounds,bits)...
        startPop(bindx,xZomeLength)];
else
    bPop(bFoundIn,:)=[gen startPop(bindx,:)];
end
if collectTrace
    traceInfo(gen,1)=gen; %current generation
    traceInfo(gen,2)=startPop(bindx,xZomeLength); %Best
    fitness
    traceInfo(gen,3)=mean(startPop(:,xZomeLength)); %Avg
    fitness

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8. Calculate the inner sound pressure: \texttt{FEM\_BEM\_cylinderSUCCESS.m}

```matlab
clc
clear all
close all

%----------------Sandwich cylinder FEM/BEM Coupling------------------%

% 5dof =five freedom [w,rx,ry,u,v]
% L: side length
L = 1;

tf=5.3e-4;
tc=0.015;

% kapa: shear correction factor
kapa=5/6;

% glass/epoxy material
E1=3.66e10;E2=5.4e9; G12=4.085e9;G13=4.085e9;G23=4.085e9;
v12=0.3;
v21 = E2*v12/E1;
Delta = 1-v12*v21;
Q11 = E1/Delta; Q22 = E2/Delta; Q66 = G12; Q12 = E2*v12/Delta;
Q21 = E2*v12/Delta;
Qf = [ Q11,Q12,0;Q21, Q22, 0; 0, 0, Q66];
rhof=1800;

% foam core material
Ec=1.51e8;Gc=6.87e7;
vC=0.3;
Deltac=1-vC^2;
Qc=[Ec/Deltac, Ec*vC/Delta, 0;Ec*vC/Delta, Ec/Deltac, 0;0,0,Gc]
;% stiffness maxtrix of core
rhoc=70;

% wooden materials
Ewood=6e9;
Gwood=2.4e9;
rho_w=700;
v_wood=0.25;

%stiffness and mass matrix generation for the cylinder part
Qt=[Qf,Qf,Qf,Qc,Qf,Qf,Qf];
layup=[90,90,90,0,90,90,90];
```
tlayer=[-3*tf-tc/2,-2*tf-tc/2,-tf-tc/2,-tc/2,tc/2,tc/2+tf,tc/2+2*tf,tc/2+3*tf]; 
[ Ae, Be, De] = ABD(layup, tlayer, Qt);

Q_sf=[G13, 0, 0, G23];
Q_sc=[Gc, 0, 0, Gc];
Q_shear=[Q_sf, Q_sf, Q_sf, Q_sc, Q_sf, Q_sf, Q_sf];
Se(2, 2)=0;

for i=1:length(layup)
    Qstem=Q_shear(:,(i-1)*2+1:2*i)*(tlayer(i+1)-tlayer(i));
    Se=Se+Qstem;
end

% ****************** read the coordinate of FEM and BEM

titleB=('D:\chongxinyuan\Dropbox\patran\full_cylinder_1000Hz103.bdf');
[quadE, triaE, coord]=read_node_element_cylinder(titleB);

temp=coord(:,1);
coord(:,1)=coord(:,3);
coord(:,3)=temp;

Ny=31; % element in hoop direction
Nz=20; % element in the axial direction

cylElem=quadE(1:Ny*Nz,:);
cylnode=coord(1:Ny*(Nz+1),:); % coord=[x,y,z] for the cylinder part
triaE(1:length(cylElem),:)=[]; % delete the 0 from the first lines

% **********************the cylinder part************************

% elementNodes=[[1:length(cylElem)'],cylElem]; % the dof of the bem element [id, elem1, elem2, elem3, elem4][
  elementNodes=cylElem;
% coord----nodes coordinates
% fex, fey, fez----element coordinates for the four element in x, y, z direction
Cylnode_n=length(cylnode); % the total number of grid of cylindrical part
FEele_n=Ny*Nz; % total number of FE element
GDof=5*Cylnode_n;
% nodeCoordinates= cylnode(:,1:2); % only x and y = axial and y;
%***************transform the Cylnode from castain to the
cylindedrical coordinates
for i=1:length(cylnode)
    cylnodeCy(i,:)=[cylnode(i,1),atan(cylnode(i,3)/cylnode(i,2))
                    ,sqrt(cylnode(i,2)^2+cylnode(i,3)^2)];
end
% keep the angle ascend
for j=1:21
    for i=(j-1)*Ny+9:(j-1)*Ny+24
        cylnodeCy(i,2)=cylnodeCy(i,2)+pi;
    end
    for i=(j-1)*Ny+25:j*Ny
        cylnodeCy(i,2)=cylnodeCy(i,2)+2*pi;
    end
end
nodeCoordinates= cylnodeCy(:,1:2);
% plot(cylnodeCy(1:31,2)) check the angel is increasing or not

% only theta and z HERE THE CYLINDRICAL COORDINATE IS USED
% form the stiffness matrix and mass matrix
% [K,M]=yuanStiffnessMass5doff(GDof,...
% elementNodes,nodeCoordinates,Ae,Be,De,Se,Ny,rhof,rhoc,3,3,tf,tc,cylnodeCy);

K=yuanStiffnessMatrixMindlinQ45laminated5dofB(GDof,...
        elementNodes,nodeCoordinates,Ae,Be,De,Se);

M=yuanMassMatrixMindlinQ4laminated5dof(GDof,elementNodes,no
deCoordinates,rhof,rhoc,3,3,tf,tc);

FEMnodecoord=[[1:length(cylnode)]',cylnode];

con=0;
%***********FEM displacement boundary condition******************
for i=1:length(FEMnodecoord)
    if FEMnodecoord(i,2)==0 | FEMnodecoord(i,2)==1
        con=con+1; % node number which has the boundary condition defined
        bc(con,:)=FEMnodecoord(i,1) 0];
    end
end

for i=1:length(FEMnodecoord)
    if FEMnodecoord(i,2)==0 | FEMnodecoord(i,2)==1

con=con+1; % node number which has the boundary condition defined
bc(con,:)=[FEMnodecoord(i,1)+Cylnode_n 0];
end
end

for i=1:length(FEMnodecoord)
    if FEMnodecoord(i,2)==0 | FEMnodecoord(i,2)==1
        con=con+1; % node number which has the boundary condition defined
        bc(con,:)=[FEMnodecoord(i,1)+2*Cylnode_n 0];
    end
end

%*******************the force vector*******************
f0=-1;
f=zeros(GDof,1);
f(311)=f0; % on the normal direction

% ************Raleigh damping matrix
a0=0;
a1=0;
C=a0*K+a1*M; % damping matrix

%*****BEM velocity and FEM translation coupling*****

%########################generate the coorinates for quad element and triangle element
couple_node=cylnode;
for i=1:length(quadE)
    bex(i,:)= coord(quadE(i,:),1)';
    bey(i,:)= coord(quadE(i,:),2)';
    bez(i,:)= coord(quadE(i,:),3)';

    femedof(i,:)=[i,quadE(i,1),quadE(i,1)+Cylnode_n,quadE(i,1)+2*Cylnode_n,quadE(i,1)+3*Cylnode_n,quadE(i,1)+4*Cylnode_n,...
                 quadE(i,2),quadE(i,2)+Cylnode_n,quadE(i,2)+2*Cylnode_n,quadE(i,2)+3*Cylnode_n,quadE(i,2)+4*Cylnode_n,...
                 quadE(i,3),quadE(i,3)+Cylnode_n,quadE(i,3)+2*Cylnode_n,quadE(i,3)+3*Cylnode_n,quadE(i,3)+4*Cylnode_n,...]
quadE(i,4),quadE(i,4)+Cylnode_n,quadE(i,4)+2*Cylnode_n,quadE(i,4)+3*Cylnode_n,quadE(i,4)+4*Cylnode_n;
end
eleQuad=[[1:length(quadE)]',quadE];
eleTri=[[1:length(triaE)]',triaE];
for i=1:length(triaE)
    bexTri(i,:)= coord(triaE(i,:),1)';
    beyTri(i,:)= coord(triaE(i,:),2)';
    bezTri(i,:)= coord(triaE(i,:),3)';
end
%******BEM pressure and FEM force coupling******
% for quad element
bemedof=[[1:length(quadE)]',quadE];
L=zeros(5*Cylnode_n,Cylnode_n);  %for quad element;
con=[1:Cylnode_n]';
for i=1:Nz*Ny  %element on the cylinder surface
    Le=bem_pressure_force(bex(i,:),bey(i,:),bez(i,:),1);
    t1=abs(con-bemedof(i,2)); t2=abs(con-bemedof(i,3));
    t3=abs(con-bemedof(i,4));
    t4=abs(con-bemedof(i,5));
    [val,po(1)]=min(t1); [val,po(2)]=min(t2);
    [val,po(3)]=min(t3); [val,po(4)]=min(t4);
    L(femedof(i,2:21),po)=L(femedof(i,2:21),po)+Le;
end
%*****Force vector*****
% f=zeros(3*Cylnode_n,1); p=1; A=disx*disy; f0=1;
f(1:Cylnode_n)=f0; % step=3
%******BEM boundary condition matrices*****
bcnv(:,1)=[Cylnode_n+1:length(coord)]'; % the BEM nodes except
those coupled with FEM nodes
bcnv(:,2)=0;  % the boundary velocity is zero for the uncoupled
nodes.
node_n=length(coord);
for loop=1:rev  % 99999999999999999999changed
    loop=1;
%*****Properties for the acoustic medium*****
%[angular frequency, sound velocity, density]
bep=[loop*step 340 1.21];
w=500*2*pi;  % frequency
%*****generate the tranform matrix T from w to v **********
Cylnode_n=length(couple_node);
T=zeros(Cylnode_n,5*Cylnode_n);
Te=1; % denotes that the velocity of the FE node is equal to
that of BE nodes
for i=1:Cylnode_n
    t=abs(FEMnodecoord(:,1)-i);
    [val,p]=min(t);
    % fn=[i,i+2*Cylnode_n];
    fn=i+2*Cylnode_n;%the normal direction of the cylinder
    T(p,fn)=T(p,fn)+Te;
    % T=bem_assemvel(T,Te,i,i*3-2,cylnode');
    % T=bem_assemvel(T,Te,bn,fn,con)
end

%*****Assemble the influence matrices for quad element*****ONE
G=zeros(node_n); % Quad+Tri
H=G;
for i=1:node_n %total bem node number
    for j=1:FEele_n %FE element number, they are quad.
        [He,Ge]=bem_infl4q(coord(i,:),bex(j,:),bey(j,:),bez(j,:),bep);
        t=abs(eleQuad(:,1)-j);
        [val,p]=min(t);
        H(i,eleQuad(j,2:5))=H(i,eleQuad(j,2:5))+He; %He, [1,4],
        eleQuad(2:5), different positions based on the 4 nodes
        G(i,eleQuad(j,2:5))=G(i,eleQuad(j,2:5))+Ge;
    end
end

%*****Assemble the influence matrices for tria element*****TWO
for i=1:node_n
    for j=1:length(triaE)
        [HeTri,GeTri]=bem_HG_3triangle_yuan(coord(i,:),bexTri(j,:),
        beyTri(j,:),bezTri(j,:),bep);
        t=abs(eleTri(:,1)-j);
        [val,p]=min(t);
        H(i,eleTri(j,2:4))=H(i,eleTri(j,2:4))+HeTri; %He, [1,3],
        eleTri(2:4), different positions based on the 3 nodes
        G(i,eleTri(j,2:4))=G(i,eleTri(j,2:4))+GeTri;
    end
end
H=H+0.5*diag(ones(node_n,1));

%*****Assemble and solve the coupled model*****
[Couple,f1,f2]=yuan_coupassem(K,C,M,L,T,H,G,w,bcnv);
forcetotal=[f;f1;f2]; %FEM force+ bem velocity
fd=[1:length(Couple)]';
% d=zeros(size(fd));
pd=bc(:,1);
dp=bc(:,2);
fd(pd)=[];
answ_dis_p=inv(Couple(fd,fd))*forcetotal(fd);
anwer=zeros(length(forcetotal),1);
anwer(fd)=answ_dis_p;
dis_radial=anwer(Cylnode_n*2+1:Cylnode_n*3);
pressure=anwer(Cylnode_n*5+1:length(anwer));
CvFEM=-i*w*dis_radial;
CvBEM=zeros(250,1);
% CvFEM=ones(651,1);
% CvBEM=ones(250,1);
Cv=[CvFEM',CvBEM']';
pressureBEM=inv(H)*G*Cv;

%**************
predB=20*log10(abs(pressureBEM)./2e-5);
%**************

%%%%%%transform elemnt id in the triaE for the patch plot
for i=1:length(triaE)
    for j=1:3
        if triaE(i,j)>=621
            triaE(i,j)=triaE(i,j)-621+32;
        end
    end
end
%************patch plot begin*****************************************
p1=patch('vertices',coord(1:651,:), 'Faces',quadE, 'FaceVertexCData',predB(1:651), 'FaceColor','interp', 'EdgeAlpha',0)
rot=[1:31, 621:901]; % the position of mesh grid of the endcape
p2=patch('vertices',coord(rot,:), 'Faces',triaE, 'FaceVertexCData',predB(rot), 'FaceColor', 'interp', 'EdgeAlpha',0)
view(3)
gird on
colorbar( 'EastOutside')
xlabel('x')
ylabel('y')
zlabel('z')
% %
%---------------------------------end---------------------------------
Appendixes F-I: Mathematica programs

Appendix F: Buckling calculations of sandwich cylinders
Appendix G: TL prediction of infinite sandwich cylinders
Appendix H: TL prediction of sandwich structures with absorption foams
Appendix I: Generate CNC codes for the filament winding of sandwich cylinders
Appendix F: Buckling computation of sandwich cylinders

**Geometry and material properties input**

\[ \begin{align*} 
Ef &= 0.49024 \times 10^{11} \text{ ;} \\
Gc &= 0.15 \times 10^9 \text{ ;} \\
Ec &= 0.35 \times 10^9 \text{ ;} \\
\nu_f &= 0.311 \text{ ;} \\
G_f &= 1.87 \times 10^9 \text{ ;} \\
t_f &= 0.0026 \text{ ;} \\
R &= 2 \text{ ;} \\
L &= 10 \text{ ;} \\
f_1 &= t_f \text{ ;} \\
f_2 &= t_f \text{ ;} \\
Tc &= 0.03 \text{ ;} \\
h &= \frac{f_1}{2} + Tc + \frac{f_2}{2} \text{ ;} \\
\lambda &= 1 - \nu_f^2 \text{ ;} 
\end{align*} \]

**Bending Buckling [Manuel Stein]**

\[ D_s = \frac{E_f t_f h^2}{2 (1 - \nu_f^2)} \text{ ; } (* \text{flexural stiffness of isotropic sandwich plate*}) \]

\[ D_s = \frac{E_f}{1 - \nu_f^2} \left( \frac{1}{6} t_f^3 + \frac{1}{2} h^2 \right) \text{ ;} \]

\[ D_Q = \frac{G_c h^2}{h - t_f} \text{ ; } (* \text{transverse shear stiffness of isotropic sandwich plate*} \text{ (here } h = f_1/2 + Tc + f_2/2 \text{)}) \]

\[ D_Q = G_c t_c + 2 G_f t_f \text{ ;} \]

\[ I_s = \frac{t_f h^2}{2} \text{ ;} \]

\[ za = \sqrt{\frac{2 t_f L^4 (1 - \nu_f^2)}{R^2 I_s}} \text{ ;} \]

\[ (* \text{curvature parameters for isotropic sandwich plate*} \text{;}) \]

\[ D_s \pi^2 \]

\[ \frac{r_a}{\pi L^2} \text{ ; } (* \text{transverse shear stiffness parameters for isotropic sandwich plate*}) \]

\[ k_{xa} = \left\{ \begin{array}{ll} 
\frac{1}{1 + r_a} + \frac{z_a^2}{\pi^2} & \frac{z_a}{\pi} \leq \frac{1}{1 + r_a} \\
\frac{z_a}{\pi} \left( 2 - \frac{z_a r_a}{\pi^2} \right) & \frac{1}{1 + r_a} < \frac{z_a}{\pi} < \frac{1}{r_a} \\
\frac{1}{r_a} & \frac{z_a}{\pi} \geq \frac{1}{r_a} 
\end{array} \right. \]

\[ (* \text{compressive load coefficients for isotropic sandwich plate*}) \]

\[ t_e q = \sqrt{6 t_f h} \text{ ;} \]

\[ \eta_b = 1 - 0.731 \left( 1 - e^{-\frac{1}{16 \eta_b / t_e q}} \right) \text{ ;} \]

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Appendix F: Buckling computation of sandwich cylinders

Torsion Buckling

\( (*Ef=90 \times 10^9; *\) \\
\( G_c = 180 \times 10^6; \)
\( Ef = 0.49024 \times 10^1; (G_c=0.15 \times 10^9); Ec = 0.35 \times 10^9; v_f = .311; \)
\( G_f = 1.87 \times 10^{-10}; \)
\( f_1 = 0.0026; R = 2; L = 10; f_2 = tf; \)
\( t_c = 0.03; h = f_1 / 2 + t_c + f_2 / 2; \lambda = 1 - v_f^2; \)
\( Ex = Ef; Ey = Ef; v_{xy} = v_f; v_{yx} = v_f; \)
\( C_x = G_c; C_y = G_c; \)
\( n = 4; (*the number of buckles in the circumferential direction*) \)
\( J = \frac{L^2}{h R}; \)
\( \mu_{xy} = \frac{1}{\frac{1 + v_{xy}}{Ex} + \frac{1 + v_{yx}}{Ey}}; \)
\( (*transverse modulus G_{xy} of the facings*) \)
\( \rho = \frac{\pi R}{n L}; \)
\( \psi_1 = \frac{J \rho^2}{4 \pi^2 x} \left( \frac{(x + \rho)^4}{Ex} + \frac{(x - \rho)^4}{By} - 2 v_{xy} \left( \frac{(x + \rho)^2 + 1}{\mu_{xy}} \right) \right); \)
\( \psi_{11} = \frac{J \rho^2}{4 \pi^2 x} \left( \frac{(x + \rho)^4}{(x + \rho)^2 + 1} + \frac{(x - \rho)^4}{(x - \rho)^2 + 1} \right); \)
\( \lambda = 1 - v_{xy} v_{yx}; \)
\( S_x = \frac{\phi Ex}{\lambda C_x R h}; \)
\( S_y = \frac{\phi Ex}{\lambda C_y R h}; \)
\( A_1 = \frac{2}{\lambda} \left( x^4 + 6 x^2 \rho^2 + \rho^4 \right) + 2 \frac{\mu_{xy}}{Ex} \left( x^2 + \rho^2 \right); \)
\( A_2 = 2 \left( \frac{\mu_{xy}}{\lambda} + \frac{\mu_{xy}}{Ex} \right) \left( x^2 + \rho^2 \right); \)
Torsion Buckling

\[
\begin{align*}
\text{(*)Ef}=90 \times 10^9; & \\
G_c = 180 \times 10^5; & \\
\text{Ef} = 0.49024 \times 10^4; & \text{(*)Gc}=0.15 \times 10^9; \\
\text{Ec} = 0.35 \times 10^{-9}; & \text{vf} = .311; \\
G_f = 1.87 \times 10^{10}; & \\
t_f = 0.0262; & R = 2; L = 10; f_1 = t_f; f_2 = t_f; \\
t_c = 0.03; h = f_1/2 + t_c + f_2/2; & \lambda = 1 - \text{vf}^2; \\
\text{Ex} = \text{Ef}; & \text{Ey} = \text{Ef}; \text{vxy} = \text{vf}; \text{vyx} = \text{vf}; \\
C_x = G_c; & C_y = G_c; \\
n = 4; \text{(*)the number of buckles in the circumferential direction} & \\
\end{align*}
\]

\[
J = \frac{L^2}{h R}; \\
\mu_{xy} = \frac{1}{\left(\frac{1 + \text{vxy}}{\text{Ex}} + \frac{1 + \text{vyx}}{\text{Ey}}\right)}; \\
(\text{*)transverse modulus Gxy of the facings} & \\
\rho = \frac{\pi R}{n L}; \\
\phi \text{Ex} & \lambda C_x R h \\
\phi \text{Ex} & \lambda C_y R h \\
\frac{2}{\lambda} \left(\text{x}^4 + 6 \text{x}^2 \rho^2 + \rho^4\right) + 2 \frac{\mu_{xy}}{\text{Ex}} \left(\text{x}^2 + \rho^2\right); \\
A_1 & \frac{\text{vyx}}{\lambda} + \frac{\mu_{xy}}{\text{Ex}} \left(\text{x}^2 + \rho^2\right); \\
A_2 & \frac{\text{vyx}}{\lambda} + \frac{\mu_{xy}}{\text{Ex}} \left(\text{x}^2 + \rho^2\right); \\
\end{align*}
\]
A3 = \frac{E_y}{E_x} + 2 \left( \frac{\mu_{xy}}{E_x} \right) (x^2 + \rho^2); \\
A4 = x^2 + \rho^2; \\
\phi = \frac{tc f_1 f_2}{f_1 + f_2}; \\
I_f = \frac{f_1 f_2}{12}; \\
I_t = \frac{f_1 f_2}{4 (f_1 + f_2)} (h + tc)^2; \\
\psi_2 = \frac{\pi^2}{(1 - \frac{tc}{R}) 4 J \rho^2 x} \left( \frac{I_t}{h^3} \left( A1 + 2 A2 + A3 + (A1 A3 - A2^2) \frac{\pi^2 \lambda}{J \rho^2} \frac{S_x A4}{A4} \right) \right) / \left( 1 + \frac{\pi^2 \lambda}{J \rho^2} \frac{S_x A1}{A4} + \frac{S_y A3}{A4} + 2 \frac{\pi^2 \lambda}{J \rho^2} \frac{S_x S_y A1 A3 - A2^2}{A4} \right) + \frac{I_t}{h^3} (A1 + 2 A2 + A3); \\
K_f = \psi_1 + \psi_2; \\
Plot[K_f, \{x, 0.01, 2\}, PlotLabel -> "Kf vs x"]; \\
K_f = \text{FindMinimum}[K_f, \{x, 0.01, 2\}][[1]]; \\
tcr = K_f * E_f * 0.8; \\
T = 2 \pi R^2 * (f_1 + f_2) * tcr \\
K_f vs x

torsion = Table[[E_f, T[E_f]], \{E_f, 20 \times 10^6, 200 \times 10^6, 20 \times 10^6\}]; \\
Export["bending_mement_Ef.xls", torsion]
Appendix G: Prediction of TL of Sandwich cylinders using FSDT

Geometry and flight definition

\[ E_1 = 1.25 \times 10^{11}; E_2 = 1 \times 10^{10}; E_3 = 1 \times 10^{10}; \]
\[ G_{12} = 5.9 \times 10^9; G_{23} = 5.9 \times 10^9; \]
\[ G_{13} = 3 \times 10^9; v_{12} = .4; v_{23} = 0.2; v_{13} = 0.2; \rho s = 1600; \]
\[ R = 1.975; \]
\[ h = 0.159 \times 10^{-2}; (*shell thickness*) \]
\[ M = h \cdot \rho s; \]
\[ ( * flight condition and incident angle* ) \]
\[ M_1 = 0; \]
\[ c_1 = 343; c_3 = 343; \rho_1 = 1.29; \rho_3 = 1.29; \]
\[ (*f=200;*) \]
\[ (*rohacell foam*) \]
\[ E_c = 350 \times 10^5; \]
\[ G_c = 150 \times 10^5; \]
\[ \rho_c = 205; \]
\[ v_c = E_c / G_c / 2 - 1; \]

the material property of the orthotropic

\[ tf = (1/10) * h; (*ply thickness*) \]
\[ tc = 0.02; (*core thickness*) \]
\[ (*plies sequence [0,90,45,-45,0]s*) \]
\[ Q_{11} = E_1 / Delta; Q_{22} = E_2 / Delta; Q_{66} = G_{12}; Q_{12} = E_2 \cdot v_{12} / Delta; \]
\[ Q_{21} = E_2 \cdot v_{12} / Delta; v_{21} = E_2 \cdot v_{12} / E_1; Delta = 1 - v_{12} \cdot v_{21}; \]
\[ Q = \begin{pmatrix} Q_{11} & Q_{12} & 0 \\
Q_{21} & Q_{22} & 0 \\
0 & 0 & Q_{66} \end{pmatrix}; \]
\[ \text{Deltac} = 1 - v_c^2; \]
\[ Q_c = \begin{pmatrix} E_c / \text{Deltac} & E_c \cdot v_c / \text{Deltac} & 0 \\
E_c \cdot v_c / \text{Deltac} & E_c / \text{Deltac} & 0 \\
0 & 0 & G_c \end{pmatrix}; \]
\[ \omega = 2 \cdot \pi \cdot f; p_0 = 200; V = 0; \]

\[ T_{1[\theta]} = \begin{pmatrix} \cos[\theta]^2 & \sin[\theta]^2 & 2 \cos[\theta] \sin[\theta] \\
\sin[\theta]^2 & \cos[\theta]^2 & -2 \cos[\theta] \sin[\theta] \\
\cos[\theta] \sin[\theta] & -\cos[\theta] \sin[\theta] & \cos[\theta]^2 - \sin[\theta]^2 \end{pmatrix}; \]
\[ T_{2[\theta]} = \begin{pmatrix} \cos[\theta]^2 & \sin[\theta]^2 & \cos[\theta] \sin[\theta] \\
\sin[\theta]^2 & \cos[\theta]^2 & -\cos[\theta] \sin[\theta] \\
2 * \cos[\theta] \sin[\theta] & -2 * \cos[\theta] \sin[\theta] & \cos[\theta]^2 - \sin[\theta]^2 \end{pmatrix}; \]
\[ Q_0 = Q; \]
\[ Q_{90} = T_1[\pi/2] \cdot Q \text{Inverse}[T_2[\pi/2]]; \]
\[ Q_{45} = T_1[\pi/4] \cdot Q \text{Inverse}[T_2[\pi/4]]; \]
\[ Q_{54} = T_1[-\pi/4] \cdot Q \text{Inverse}[T_2[-\pi/4]]; \]
\[ Q = \{ Q_0, Q_{90}, Q_{45}, Q_{54}, Q_0, Q_c, Q_0, Q_{54}, Q_{45}, Q_{90}, Q_0 \}; \]
\[ x_i = \{-5 \cdot tf - tc/2, -4 \cdot tf - tc/2, -3 \cdot tf - tc/2, \]
\[ -2 \cdot tf - tc/2, -tf - tc/2, -tc/2, tc/2, tc/2 + tf, \]
\[ tc/2 + 2 \cdot tf, tc/2 + 3 \cdot tf, tc/2 + 4 \cdot tf, tc/2 + 5 \cdot tf\}; \]
\[ np = 11; \] (* total layer number *)
\[ A = \sum_{k=1}^{np} Q[[k]] \cdot (x[[k+1]] - x[[k]]); \]
\[ B = \frac{1}{2} \sum_{k=1}^{np} (Q[[k]] \cdot (x[[k+1]]^2 - x[[k]]^2)); \]
\[ DD = \frac{1}{3} \sum_{k=1}^{np} (Q[[k]] \cdot (x[[k+1]]^3 - x[[k]]^3)); \]
\[ EE = \frac{1}{4} \sum_{k=1}^{np} (Q[[k]] \cdot (x[[k+1]]^4 - x[[k]]^4)); \]
\[ A_1 = A + B/R; \]
\[ B_1 = B + DD/R; \]
\[ DD_1 = DD + EE/R; \]
\[ A_2 = A - B/R; \]
\[ B_2 = B - DD/R; \]
\[ DD_2 = DD - EE/R; \]
\[ K_1 = \sqrt{\frac{5}{6}}; \]
\[ K_2 = \sqrt{\frac{5}{6}}; \]
\[ G_{23Q} = \{ G_{23}, G_{23}, G_{23}, G_{23}, G_{23}, G_{23}, G_{23}, G_{23}, G_{23}, G_{23} \}; \]
\[ G_{13Q} = \{ G_{13}, G_{13}, G_{13}, G_{13}, G_{13}, G_{13}, G_{13}, G_{13}, G_{13}, G_{13} \}; \]
\[ A_{44} = \sum_{k=1}^{np} K_1 \cdot K_2 \cdot G_{23Q}[[k]] \cdot (x[[k+1]] - x[[k]]); \]
\[ B_{44} = (1/2) \sum_{k=1}^{np} K_1 \cdot K_2 \cdot G_{23Q}[[k]] \cdot (x[[k+1]]^2 - x[[k]]^2); \]
\[ A_{55} = \sum_{k=1}^{np} K_1 \cdot K_2 \cdot G_{13Q}[[k]] \cdot (x[[k+1]] - x[[k]]); \]
\[ B_{55} = (1/2) \sum_{k=1}^{np} K_1 \cdot K_2 \cdot G_{13Q}[[k]] \cdot (x[[k+1]]^2 - x[[k]]^2); \]
\[ A_{244} = A_{44} - B_{44}/R; A_{155} = A_{55} + B_{55}/R; \]

the strain and stress differential equations
Appendix G: Prediction of TL of Sandwich cylinders using FSDT

\[
\text{strain} = \left( \begin{array}{c} I \ast U_n \\ I \ast V_n \\ W_n \end{array} \right) \ast e^{i(\omega z + k z z n - \phi)}
\]

\[
p = p_0 \ast e^{i(\omega z + k z z n - \phi)};
\]

\[
u_0 = \text{strain}[1];
\]

\[
v_0 = \text{strain}[2];
\]

\[
w_0 = \text{strain}[3];
\]

\[
\Psi_{11} = I \ast \Psi_{1n} \ast e^{i(\omega z + k z z n - \phi)};
\]

\[
\Psi_{12} = I \ast \Psi_{2n} \ast e^{i(\omega z + k z z n - \phi)};
\]

\[
x_0 z = D[u_0, z];
\]

\[
x_0 \phi = (\partial_\phi v_0 + w_0) / R;
\]

\[
x_0 z \phi = \partial_\phi v_0;
\]

\[
x_0 z \phi = \partial_\phi u_0 / R;
\]

\[
k_z = D[\Psi_{11}, z];
\]

\[
k_\phi = D[\Psi_{12}, \phi] / R;
\]

\[
k_z \phi = D[\Psi_{11}, \phi] / R;
\]

\[
k_z \phi = D[\Psi_{12}, z];
\]

\[
gamma_0 z = D[w_0, z] + \Psi_{51};
\]

\[
gamma_0 \phi = D[w_0, \phi] / R - v_0 / R + \Psi_{22};
\]

\[
(*z_1 \text{ is the distance from the midplane } u(z, \phi, z_1)*)
\]

\[
v = v_0 + z_1 \ast \Psi_{22}; w = w_0; (*)
\]

\[
\{ \{ i e^{i (2 \pi n - k z z n - \phi)} U_n \}, \{ i e^{i (2 \pi n - k z z n - \phi)} V_n \}, \{ e^{i (2 \pi n - k z z n - \phi)} W_n \} \}
\]

\text{combined stiffness matrix}

\[
x_{ik} = \left( \begin{array}{c} x_{i0z}[1] \\ x_{i0\phi}[1] \\ x_{i0z}[3] \\ k_z \\ k_\phi \\ k_z \phi \\ k_\phi z \end{array} \right)
\]

\[
(*x_{ik} = \text{MatrixForm}[\text{Flatten}[x_{ik}]];\ast) (*\text{flatten is to remove the } \{\} \ast)
\]

\[
\text{ABD} =
\]

\[
\left( \begin{array}{cccccccc}
A_1[1, 1] & A_1[1, 2] & A_1[1, 3] & A_1[3, 1] & B_1[1, 1] & B_1[1, 2] \\
B_1[1, 1] & B_1[1, 2] & B_1[1, 3] & B_1[1, 3] & D_1[1, 1] & D_1[1, 2] \\
\end{array} \right)
\]

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\[
\begin{align*}
\text{NM} &= ABD.x[k]; \\
\text{Nz} &= \text{Simplify}[[\text{NM}[1]]]; \\
\text{N}\phi &= \text{Simplify}[[\text{NM}[2]]]; \\
\text{Nz}\phi &= \text{Simplify}[[\text{NM}[3]]]; \\
\text{N}\phi z &= \text{Simplify}[[\text{NM}[4]]]; \\
\text{Mz} &= \text{NM}[[5]]; \\
\text{M}\phi &= \text{NM}[[6]]; \\
\text{Mz}\phi &= \text{NM}[[7]]; \\
\text{M}\phi z &= \text{NM}[[8]]; \\
\text{Qshear} &= \begin{pmatrix} A155 & 0 \\ 0 & A244 \end{pmatrix} \cdot \begin{pmatrix} \text{gamma}0z \\ \text{gamma}0\phi \end{pmatrix}; \\
\text{Qz} &= \text{Qshear}[[1]]; \\
\text{Q}\phi &= \text{Qshear}[[2]]; \\
\rho_{\text{total}} &= \{\rho_s, \rho_s, \rho_s, \rho_s, \rho_s, \rho_s, \rho_s, \rho_s, \rho_s, \rho_s\}; \\
\text{m1} &= \sum_{k=1}^{np} \int_{x[k]}^{x[k+1]} \rho_{\text{total}}[k] \, dx; \\
\text{m2} &= \sum_{k=1}^{np} \int_{x[k]}^{x[k+1]} \rho_{\text{total}}[k] \, x \, dx; \\
\text{m3} &= \sum_{k=1}^{np} \int_{x[k]}^{x[k+1]} \rho_{\text{total}}[k] \, x^2 \, dx; \\
\text{m4} &= \sum_{k=1}^{np} \int_{x[k]}^{x[k+1]} \rho_{\text{total}}[k] \, x^3 \, dx; \\
\text{mm1} &= \text{m1} + \text{m2} / R; \\
\text{mm2} &= \text{m2} + \text{m3} / R; \\
\text{mm3} &= \text{m3} + \text{m4} / R; \\
\text{xin} &= \text{Piecewise}[[1, n == 0], [2, n > 1]]; \\
\end{align*}
\]
build the differential equation of \((Un, Vn, Wn, P1nR, P3nT)\)

\[
\begin{align*}
C_1 &= \sqrt{A[1, 1]} m \ast \lambda; \\
\text{eqn1} &= \left(\partial_z N_z + 1/R \ast \partial_\phi N_\phi - mm1 \ast \partial_\phi \phi U_0 - mm2 \ast \partial_\phi \phi P3nT\right)/e^{i(\omega z - klz)} \left[1\right]; \\
\text{eqn2} &= \left(\text{Flatten}\left[D[N_\phi, \phi]/R + D[N_\phi, z] + Q_\phi/R - mm1 \ast (D[V_0, t, t]) - mm2 \ast D[Psi12, t, t]\right)/e^{i(\omega z - klz)} \left[1\right]; \\
\text{eqn3} &= \left(\text{Flatten}\left[-(N_\phi/R + D[Qz, z]) + D[Q_\phi, \phi]/R - mm1 \ast D[w0, t, t]\right)/e^{i(\omega z - klz)} - \text{HankelH2}[n, klr \ast R] \ast P1nR + \text{HankelH1}[n, k3r \ast R] \ast P3nT\right]\left[1\right]; \\
\text{eqn4} &= \left(\text{Flatten}\left[D[Mz, z] + D[N_\phi z, \phi]/R - Q_\phi - mm2 \ast (D[u0, t, t]) - mm3 \ast (D[Psi1, t, t])\right)/e^{i(\omega z - klz)} \left[1\right]; \\
\text{eqn5} &= \left(\text{Flatten}\left[(D[M_\phi, \phi]) / R + D[N_\phi, z] - Q_\phi - mm2 \ast (D[v0, t, t]) - mm3 \ast (D[Psi2, t, t])\right)/e^{i(\omega z - klz)} \left[1\right]; \\
\text{eqn6} &= P1nR \ast (\text{HankelH2}[n + 1, klr \ast R] + n \ast \text{HankelH2}[n, klr \ast R] / (klr \ast R)) \ast klr - \text{rho1} \ast (\omega^2 + \omega z - klz - klz^2 - 2 \ast \omega \ast \omega k1z) \ast Wn; \\
\text{eqn7} &= \text{P3nT} \ast (\text{HankelH1}[n + 1, k3r \ast R] + n \ast \text{HankelH1}[n, k3r \ast R] / (k3r \ast R)) \ast k3r - \text{rho3} \ast \omega^2 \ast Wn; \\
\text{eqns} &= \{\text{eqn1} = 0, \text{eqn2} = 0, \text{eqn3} = 0, \text{eqn4} = 0, \text{eqn5} = 0, \text{eqn6} = 0\}; \\
\text{CoefficientArrays}[\text{eqns}, \{\text{Un}, Vn, Wn, P1nR, P3nT, Psi1n, Psi2n\}] \\
(*\text{find coefficient matrix}*) \\
\text{AAFlag} = \text{Normal}[] (*\text{include the coefficient matrix and vector}*) \\
b = \text{AAFlag}[1]; \\
\text{AA} = \text{AAFlag}[2]; \\
\end{align*}
\]

Computation for diffused acoustic field

\[
\tau_d = \int_0^{\pi/2} \frac{\tau(\phi) \sin \phi \cos \phi d\phi}{\int_0^{\pi/2} \sin \phi \cos \phi d\phi} = \int_0^{\pi/2} \tau(\phi) \sin 2\phi d\phi
\]

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\[
\lambda = 0.001; \\
Z = 11; \\
Z2 = 32; (*\text{number of frequency}*) \\
f0 = 20; \\
aa = Table[\{0, 0\}, \{Z2 - 1\}]; \\
For[i = 1, i < Z2, i++, \\
\tauT = 0; \\
f = 2^i f0; \\
For[j = 1, j < Z, j++, \\
Upsilon = \frac{\pi}{Z}; \\
k1 = \omega / (c1*(1 + M1*Cos[Upsilon])); \\
k3 = \omega / c3; \\
k1z = k1*Cos[Upsilon]; \\
k1r = k1*Sin[Upsilon]; \\
k3z = k1z; \\
k3r = \sqrt{k3^2 - k1z^2}; \\
\]

an = Inverse[AA].b; \\
Wn = an[[3]]; \\
P3nT = an[[5]]; \\
\tau = \sum_{n=0}^{30} (-Re[P3nT*HankelH1[n, k3r*R]*Conjugate[I*\omega*Wn]) * \\
rho1*c1*\pi) / (xin*Cos[Upsilon]*p0^2); \\
\tauT = \tau + \tau*Sin[2*Upsilon]*\pi/2/Z; \\
TL = -10*Log10[\tauT]; \\
]; (*\text{don't omit the fenhao}*) \\
aa[[\{i, All\}]] = {{N[\{f\}}, N[TL]} \\
]

For[i = 1, i < Z2, i++, \\
aa[[i]] = Flatten[aa[[i]]] \\
](*\text{delete the excessive bracket}*) \\
ListLogLinearPlot[aa, Joined \rightarrow True, GridLines \rightarrow Automatic]
Output to excel or notebook

(*SetDirectory[$UserDocumentsDirectory];*)
fname = FileNameJoin[
   {$UserDocumentsDirectory, "thick_diffusedSandwichGC150e5"}];

strm = OpenWrite[fname, FormatType -> OutputForm];
For[i = 1, i ≤ Length[aa], i++,
   Write[strm,
      aa[[i, 1]], " ",
      aa[[i, 2]]
   ];
Close[strm];
Appendix H: Transmission through a plate-porous-plate structure

This program is used to compute the sound transmission of sandwich panels with porous foam under a diffused acoustic field or plane acoustic waves. It is created by Chongxin Yuan. The theory is based on the book of Allard (1993).

generate the transform matrix for the foam layer

\[ \varphi_{1s} = A_1 \exp \left( I \left( \omega t - k_{13} x_3 - k t x_1 \right) \right) + A_{1b} \exp \left( I \left( \omega t + k_{13} x_3 - k t x_1 \right) \right); \]
\[ \varphi_{2s} = A_2 \exp \left( I \left( \omega t - k_{23} x_3 - k t x_1 \right) \right) + A_{2b} \exp \left( I \left( \omega t + k_{23} x_3 - k t x_1 \right) \right); \]
\[ \varphi_{2f} = A_3 \exp \left( I \left( \omega t - k_{33} x_3 - k t x_1 \right) \right) + A_{3b} \exp \left( I \left( \omega t + k_{33} x_3 - k t x_1 \right) \right); \]
\[ \varphi_{1f} = \mu_1 \varphi_{1s}; \]
\[ \varphi_{2f} = \mu_2 \varphi_{2s}; \]
\[ \varphi_{3f} = \mu_3 \varphi_{3s}; \]
\[ \varphi_{1s} = \partial_{x_1} \left( \varphi_{1s} + \partial_{x_3} \varphi_{2s} - \partial_{x_3} \varphi_{2s} \right); \]
\[ \varphi_{2s} = \partial_{x_3} \left( \varphi_{2s} + \partial_{x_3} \varphi_{2s} + \partial_{x_1} \varphi_{2s} \right); \]
\[ \varphi_{3f} = \partial_{x_1} \left( \varphi_{1f} + \partial_{x_1} \varphi_{2f} + \partial_{x_1} \varphi_{2f} \right); \]

\[ (\text{stress}) \]
\[ \nabla \psi_s = \partial_{x_1, x_1} \left( \varphi_{1s} + \varphi_{2s} \right) + \partial_{x_3, x_3} \left( \varphi_{1s} + \varphi_{2s} \right); \]
\[ \nabla \psi_f = \partial_{x_1, x_1} \left( \mu_1 \varphi_{1s} + \mu_2 \varphi_{2s} \right) + \partial_{x_3, x_3} \left( \mu_1 \varphi_{1s} + \mu_2 \varphi_{2s} \right); \]
\[ \psi_{1s} = \partial_{x_1} \left( \varphi_{1s} + \varphi_{2s} \right) + \partial_{x_1} \psi_{2s}; \]
\[ \psi_{2s} = \partial_{x_1} \left( \varphi_{1s} + \varphi_{2s} \right) - \partial_{x_3} \psi_{2s}; \]
\[ \sigma_{33s} = \left( P - 2 \, N_s \right) \nabla \psi_s + Q \nabla \psi_f + 2 \, N_s \partial_{x_3} u_{3s}; \]
\[ \sigma_{33f} = \left( P - 2 \, N_s \right) \nabla \psi_f + Q \nabla \psi_s; \]
\[ \psi_{1s} = \text{Simplify}[\text{ExpToTrig}[\psi_{1s}]] / (\cos[k t x_1 - t \omega] - i \sin[k t x_1 - t \omega]); \]
\[ \psi_{3s} = \text{Simplify}[\text{ExpToTrig}[\psi_{3s}]] / (\cos[k t x_1 - t \omega] - i \sin[k t x_1 - t \omega]); \]
\[ \psi_{3f} = \text{Simplify}[\text{ExpToTrig}[\psi_{3f}]] / (\cos[k t x_1 - t \omega] - i \sin[k t x_1 - t \omega]); \]
\[ \sigma_{33s} = \text{Simplify}[\text{ExpToTrig}[\sigma_{33s}]] / (\cos[k t x_1 - t \omega] - i \sin[k t x_1 - t \omega]); \]
\[ \sigma_{33f} = \text{Simplify}[\text{ExpToTrig}[\sigma_{33f}]] / (\cos[k t x_1 - t \omega] - i \sin[k t x_1 - t \omega]); \]

\[ 1 \]
\[ 2 \]
\[ A_1 = \frac{1}{2} (\text{var1} + \text{var2}); \]
\[ A_{1b} = \frac{1}{2} (\text{var1} - \text{var2}); \]
\[ A_2 = \frac{1}{2} (\text{var3} + \text{var4}); \]
\[ A_{2b} = \frac{1}{2} (\text{var3} - \text{var4}); \]
\[ A_3 = \frac{1}{2} (\text{var5} + \text{var6}); \]
\[ A_{3b} = \frac{1}{2} (\text{var5} - \text{var6}); \]
generate the transform matrix for the solid layer

\[\varphi = \text{Exp}[I \omega t - I k l x_1] (A_{1s} \text{Exp}[-I k l x_3] + A_{2s} \text{Exp}[I k l x_3]);\]
\[\psi = \text{Exp}[I \omega t - I k l x_1] (A_{3s} \text{Exp}[-I k l x_3] + A_{4s} \text{Exp}[I k l x_3]);\]

\[v_{1sD1} = I \omega (\partial_{x_1} \varphi - \partial_{x_3} \psi);\]
\[v_{3sD1} = I \omega (\partial_{x_3} \varphi + \partial_{x_1} \psi);\]
\[\sigma_{33sD1} = \lambda (\partial_{x_1} x_1 \varphi + \partial_{x_1,x_1} \psi) + 2 G_1 (\partial_{x_3,x_3} \varphi + \partial_{x_1,x_3} \psi);\]
\[\sigma_{13sD1} = G_1 (2 \partial_{x_1,x_3} \varphi + \partial_{x_1,x_1} \psi - \partial_{x_3,x_3} \psi);\]

\[T = \text{CoefficientArrays}([v_{1s}, v_{3s}, v_{3f}, \sigma_{33s}, \sigma_{13s}, \sigma_{33f}],\]
\[\{\text{var1, var2, var3, var4, var5, var6}\};\]
\[\Gamma[x_3] = \text{Normal}[T[[2]]];\]
\[\text{MatrixForm}[\Gamma[x_3]];\]
\[\text{In} = \text{Simplify}[\text{Inverse}[T[0]]];\]

\(\varphi = \text{Exp}[I \omega t - I k l x_1] (A_{1s} \text{Exp}[-I k l x_3] + A_{2s} \text{Exp}[I k l x_3]);\)
\(\psi = \text{Exp}[I \omega t - I k l x_1] (A_{3s} \text{Exp}[-I k l x_3] + A_{4s} \text{Exp}[I k l x_3]);\)

\[v_{1sD1} = I \omega (\partial_{x_1} \varphi - \partial_{x_3} \psi) / (\cos[k l x_1 - t \omega] - i \sin[k l x_1 - t \omega]);\]
\[v_{3sD1} = \text{Simplify}[\text{ExpToTrig}[v_{3sD1}]] / (\cos[k l x_1 - t \omega] - i \sin[k l x_1 - t \omega]);\]
\[\sigma_{13sD1} = \text{Simplify}[\text{ExpToTrig}[\sigma_{13sD1}]] / (\cos[k l x_1 - t \omega] - i \sin[k l x_1 - t \omega]);\]
\[\sigma_{33sD1} = \text{Simplify}[\text{ExpToTrig}[\sigma_{33sD1}]] / (\cos[k l x_1 - t \omega] - i \sin[k l x_1 - t \omega]);\]

(*distill the matrix*)

\[1 \quad A_{1s} = - (\text{Bvar1} + \text{Bvar2});\]
\[2 \quad A_{2s} = - (\text{Bvar1} - \text{Bvar2});\]
\[2 \quad A_{3s} = - (\text{Bvar3} + \text{Bvar4});\]
\[2 \quad A_{4s} = - (\text{Bvar3} - \text{Bvar4});\]

\[\sigma_{33sD1} = \text{Simplify}[\sigma_{33sD1}];\]
\[\sigma_{13sD1} = \text{Simplify}[\sigma_{13sD1}];\]
\[T_{\text{solid}} = \text{CoefficientArrays}([v_{1sD1}, v_{3sD1}, \sigma_{33sD1}, \sigma_{13sD1}], [\text{Bvar1}, \text{Bvar2}, \text{Bvar3}, \text{Bvar4}]);\]
\( h_1 = 0.00127; \) (*the thickness of the solid panel*)
\( E_1 = 7.1 \times 10^{10}; \)
\[ G_1 = \frac{E_1}{2 (1 + v_{12})} \]
\( v_{12} = 0.3; \)
(*D1=16.3;*)
\[ D_1 = \frac{E_1 h_1^3}{12 (1 - v_{12}^2)}; \) (*bending stiffness for the panel*)
\( \rho_1 = 2700; \)
\( h_2 = 0.000762; \)
\( E_2 = 7.1 \times 10^{10}; \)
(*D2=16.3;*)
\[ D_2 = \frac{E_2 h_1^3}{12 (1 - v_{12}^2)} \]
\( \rho_2 = 2700; \)

(*porous materials, cited from an example materials of Actran*)
\( \phi = 0.9; \) (*porosity*)
\( d = 0.027; \) (*the thickness of the porous materials*)
\( \sigma = 5000; \) (*flow resistivity*)
\( \alpha = 7.8; \) (*tortuosity*)
\( \rho = 30; \) (*solid density*)
\( N_s = 8 \times 10^5 + 1.265 \times 8 \times 10^5; \) (*skeleton young modulus*)
(*c=1.5;*)
\( (\Lambda_2 = \frac{1}{c_1} + \frac{8 \eta}{\phi \sigma})^{1/2}; \) (*viscous dimensions*)
\( \Lambda_1 = \frac{1}{c_1} + \frac{8 \eta}{\phi \sigma})^{1/2}; \) (*thermal dimensions*)
\( \Lambda = 226 \times 10^{-6}; \) (*viscous dimensions*)
\( \Lambda_1 = 226 \times 10^{-6}; \) (*thermal dimensions*)
(*c1=1/c;) (*)
\( v_{f, \text{foam}} = 0.4; \) (*skeleton poisson Ratio*)

(*air in the porous materials************* *)
\( \rho_0 = 1.205; \) (*density of air at 20[kg/m3]*)
(*in case of the air at 20 C*)
\( c_p = 1005; \) (*Heat capacity at constant pressure[J/(Kg.K)]*)

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\[ cv = 718; \] *(Heat capacity at constant volume\([J/(Kg.K)]\]*)

\[ k_{\text{cond}} = 0.0257; \] *(\([W/m.K]\]*)

\[ \gamma = cp/cv; \]

\[ P_0 = 1.10132 \times 10^5; \] *(ambient pressure\([\text{pa}]\]*)

\[ c_0 = 340; \] *(wave speed of air\([\text{m/s}]\]*)

\[ \eta = 1.84 \times 10^{-5}; \] *(the viscosity of the air\([\text{poiseuille}]\))*

\[ \eta cp \]

\[ \text{prandt} = \frac{\eta}{k_{\text{cond}}}; \]

\[ (*\text{the square root of the Prandtl number defined in} \text{*}) \]

\[ k_t = \omega/c_0 \sin[\theta]; \]

\[ k_{13} = \text{Re}\left[\sqrt{\delta_{1}^{2} - k_t^{2}}\right]; \]

\[ k_{23} = \text{Re}\left[\sqrt{\delta_{2}^{2} - k_t^{2}}\right]; \]

\[ k_{33} = \text{Re}\left[\sqrt{\delta_{3}^{2} - k_t^{2}}\right]; \]

\[ k_{13b} = -k_{13}; \]

\[ k_{23b} = -k_{23}; \]

\[ k_{33b} = -k_{33}; \]

\[ k = \omega/c_0; \]

\[ k_1 = k \sin[\theta]; \]

\[ \mu_1 = \frac{P \delta_{1}^{2} - \omega^2 \rho_{11b}}{\omega^2 \rho_{12b} - Q \delta_{1}^{2}}; \] *(the ratio of the velocity of the frame over the velocity of the air of the first compressional wave)\)*

\[ \mu_2 = \frac{P \delta_{2}^{2} - \omega^2 \rho_{11b}}{\omega^2 \rho_{12b} - Q \delta_{2}^{2}}; \]

\[ \mu_3 = \frac{-\rho_{12b}}{\rho_{22b}}; \]

\[ \rho_{11b} = \rho + \rho_{a} - I \sigma \phi^2 \frac{G_{\omega}}{\omega}; \]

\[ \rho_{12b} = -\rho_{a} + I \sigma \phi^2 \frac{G_{\omega}}{\omega}; \]

\[ \rho_{22b} = \phi \rho_{0} + \rho_{a} - I \sigma \phi^2 \frac{G_{\omega}}{\omega}; \]

\[ \rho_{a} = \rho_{0} \phi (\alpha - 1); \]

\[ R = \phi Kf; \]

\[ Q = Kf (1 - \phi); \]

\[ P = \frac{4}{3} Ns + K_b + \frac{(1 - \phi)^2}{\phi} K_f; \]

\[ K_b = \frac{2 Ns (vfoam + 1)}{3 (1 - 2 vfoam)}; \] *(the bulk modulus of frame)*

\[ (*\text{there are two methods to compute the} G_{\omega} \text{*}) \]

\[ (*\text{first*}) \]

\[ G_{\omega} = \left(1 + \frac{4 I \alpha^2 \eta \rho_{0} \omega}{\sigma^2 \Lambda^2 \phi^2}\right)^{1/2}; \]
Appendix H: Transmission through a plate-porous-plate structure

\[
\begin{align*}
\Delta \frac{1}{c} \left( \frac{8 \eta}{\rho \sigma} \right)^{1/2} \ast
\end{align*}
\]

\[
\begin{align*}
Kf &= (\gamma P0) \left( 1 + (\gamma - 1) \left[ 1 + (8 \eta) / \left( I \Lambda 1^2 \text{ prandt}^2 \omega \rho 0 \right) \left( 1 + I \rho 0 \frac{\omega \text{ prandt}^2 \Lambda 1^2}{16 \eta} \right)^{1/2} \right]^{-2} \right) ;
\end{align*}
\]

\[
\begin{align*}
\delta_1 &= \left( \frac{\omega^2}{2 (P \gamma - Q^2)} \left( P \rho 2 \rho 22b + \rho 11b - 2 Q \rho 12b - \sqrt{\Delta}\right) \right)^{1/2} ; (*the squared complex wave number of the first compressational wave*)
\end{align*}
\]

\[
\begin{align*}
\delta_2 &= \left( \frac{\omega^2}{2 (P \gamma - Q^2)} \left( P \rho 2 \rho 22b + \rho 11b - 2 Q \rho 12b + \sqrt{\Delta}\right) \right)^{1/2} ;
\end{align*}
\]

\[
\begin{align*}
\delta_3 &= \frac{\omega^2}{Ns} \left( \rho 11b \rho 22b - \rho 12b^2 \right) ;
\end{align*}
\]

\[
\begin{align*}
\Delta &= (P \rho 2 \rho 22b + R \rho 11b - 2 Q \rho 12b)^2 - 4 \left( P \gamma - Q^2 \right) \left( \rho 11b \rho 22b - \rho 12b^2 \right) ;
\end{align*}
\]

\[
\begin{align*}
k_{13s} &= \sqrt{\delta_1^2 - \kappa t^2} ;
\end{align*}
\]

\[
\begin{align*}
k_{33s} &= \sqrt{\delta_3^2 - \kappa t^2} ;
\end{align*}
\]

\[
\begin{align*}
\delta_1s^2 &= \frac{\omega^2}{\lambda + 2 G_1} ;
\end{align*}
\]

\[
\begin{align*}
\lambda &= \frac{G_1 (E_1 - 2 G_1)}{3 G_1 - E_1} ; (* \text{Lamé’s first parameter}*)
\end{align*}
\]

\[
\begin{align*}
\delta_3s^2 &= \frac{\omega^2}{G_1} ;
\end{align*}
\]

\[V_m = \begin{align*}
\begin{cases}
p A, & v 3 f A, \ v 1 s M 2, \ v 3 s M 2, \ \sigma 33 s M 2, \ \sigma 13 s M 2, \ v 1 s M 4, \ v 3 s M 4, \ \sigma 33 s M 4, \ \sigma 33 f M 4, \ \sigma 13 s M 4, \ v 1 s M 6, \ v 3 s M 6, \ \sigma 33 s M 6, \ \sigma 13 s M 6, \ p
\end{cases}
\end{align*}\]
calculate the TL at different frequencies

\[ N_t = 16; \]
\[ Z_B = 415; (* impedance of the fluid 2*) \]
\[ \omega = 2 \pi f; \]
\[ N_{mm} = 20; (* number of incidence angle \theta*) \]
\[ N_{nn} = 20; (* number of the frequency*) \]
\[ \frac{89 \pi}{90} \frac{\pi}{2N_{mm}}; \]
\[ \sum W = \text{Table}[x, \{x, 1, N_{mm}\}]; \]
\[ \text{TL}_\omega = \text{Table}[[0, 0], \{x, 1, N_{nn}\}]; (* frequency of TL*) \]
\[ f_0 = 100; \]
\[ \text{For} [\text{nn} = 1, \text{nn} <= \text{Nnn}, \text{nn}++, (* frequency*)] \]
\[ f = 2^n f_0; \]
\[ \text{For} [\text{mm} = 1, \text{mm} <= \text{Nmm}, \text{mm}++, (* incidence angle*)] \]
\[ \theta = \text{tt} \cdot \text{mm}; \]
\[ \text{Ttrans} = \Gamma[-d].\text{In}\Gamma; (* transform matrix for porous layer*) \]
\[ \text{Ttranssolid1} = \Gamma_{\text{solid}}[-h_1].\text{In}\Gamma_{\text{solid}}; (* transform matrix for the solid*) \]
\[ \text{Ttranssolid2} = \Gamma_{\text{solid}}[-h_2].\text{In}\Gamma_{\text{solid}}; (* transform matrix for the solid*) \]
\[ \text{T02} = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}. \text{Ttranssolid1}; \]
\[ \begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{pmatrix}; \]
\[ \text{Jsp} = -\begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 \end{pmatrix}; \]
\[ \text{T24} = \text{Jsp}.\text{Ttrans}; (* the transform matrix*) \]
\[ \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix}; (* the right is the porous*) \]
\[ \text{Isp} = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}; \]
\[ \text{T46} = \text{Isp}.\text{Ttranssolid2}; \]
Appendix H: Transmission through a plate-porous-plate structure

\[
\text{Dmatrix} = \begin{pmatrix}
0 & -1 & 0 & 0 & 0 & T02[[1, 1]] & T02[[1, 2]] & T02[[1, 3]] & T02[[1, 4]] \\
1 & 0 & 0 & 0 & 0 & T02[[2, 1]] & T02[[2, 2]] & T02[[2, 3]] & T02[[2, 4]] \\
0 & 0 & 0 & 0 & 0 & T02[[3, 1]] & T02[[3, 2]] & T02[[3, 3]] & T02[[3, 4]] \\
0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\end{pmatrix}
\]

\[
\text{T24}[[1, 1]] \quad \text{T24}[[1, 2]] \quad \text{T24}[[1, 3]] \quad \text{T24}[[1, 4]] \quad \text{T24}[[1, 5]] \\
\text{T24}[[2, 1]] \quad \text{T24}[[2, 2]] \quad \text{T24}[[2, 3]] \quad \text{T24}[[2, 4]] \quad \text{T24}[[2, 5]] \\
\text{T24}[[3, 1]] \quad \text{T24}[[3, 2]] \quad \text{T24}[[3, 3]] \quad \text{T24}[[3, 4]] \quad \text{T24}[[3, 5]] \\
\text{T24}[[4, 1]] \quad \text{T24}[[4, 2]] \quad \text{T24}[[4, 3]] \quad \text{T24}[[4, 4]] \quad \text{T24}[[4, 5]] \\
\text{T24}[[5, 1]] \quad \text{T24}[[5, 2]] \quad \text{T24}[[5, 3]] \quad \text{T24}[[5, 4]] \quad \text{T24}[[5, 5]] \\
\text{T24}[-1, 1] \quad 0 \quad 0 \quad 0 \quad 0 \\
0 \quad -1 \quad 0 \quad 0 \quad 0 \\
0 \quad 0 \quad -1 \quad 0 \quad 0 \\
0 \quad 0 \quad 0 \quad -1 \quad 0 \\
0 \quad 0 \quad 0 \quad 0 \quad 0 \\
0 \quad 0 \quad 0 \quad 0 \quad 0 \\
0 \quad 0 \quad 0 \quad 0 \quad 0 \\
0 \quad 0 \quad 0 \quad 0 \quad 0 \\
0 \quad 0 \quad 0 \quad 0 \quad 0 \\
0 \quad 0 \quad 0 \quad 0 \quad 0 \\
0 \quad 0 \quad 0 \quad 0 \quad 0 \\
0 \quad 0 \quad 0 \quad 0 \quad 0 \\
0 \quad 0 \quad 0 \quad 0 \quad 0 \\
\end{pmatrix}
\]

\[
\text{T46}[[1, 1]] \quad \text{T46}[[1, 2]] \quad \text{T46}[[1, 3]] \quad \text{T46}[[1, 4]] \\
\text{T46}[[2, 1]] \quad \text{T46}[[2, 2]] \quad \text{T46}[[2, 3]] \quad \text{T46}[[2, 4]] \\
\text{T46}[[3, 1]] \quad \text{T46}[[3, 2]] \quad \text{T46}[[3, 3]] \quad \text{T46}[[3, 4]] \\
\text{T46}[[4, 1]] \quad \text{T46}[[4, 2]] \quad \text{T46}[[4, 3]] \quad \text{T46}[[4, 4]] \\
\text{T46}[[5, 1]] \quad \text{T46}[[5, 2]] \quad \text{T46}[[5, 3]] \quad \text{T46}[[5, 4]] \\
\text{T46}[-1, 1] \quad 0 \quad 0 \quad 0 \\
0 \quad 0 \quad 0 \quad 0 \\
0 \quad 0 \quad 0 \quad 0 \\
0 \quad 0 \quad 0 \quad 0 \\
0 \quad 0 \quad 0 \quad 0 \\
0 \quad 0 \quad 0 \quad 0 \\
0 \quad 0 \quad 0 \quad 0 \\
0 \quad 0 \quad 0 \quad 0 \\
0 \quad 0 \quad 0 \quad 0 \\
0 \quad 0 \quad 0 \quad 0 \\
0 \quad 0 \quad 0 \quad 0 \\
0 \quad 0 \quad 0 \quad 0 \\
\end{pmatrix}
\]
Appendix H: Transmission through a plate-porous-plate structure

\[
\begin{align*}
DN_1 &= \text{Drop}[\text{Dmatrix}, [], \{\text{Nt} + 1\}] ; \\
D_{\text{first}} &= \text{Drop}[\text{Dmatrix}, [], \{1\}] ; \\
D_{\text{second}} &= \text{Drop}[\text{Dmatrix}, [], \{2\}] ; \\
Z_s &= -\text{Det}[D_{\text{first}}] / \text{Det}[D_{\text{second}}] ; \\
\text{Reflect} &= \frac{Z_s \cos[\theta] - Z_0}{Z_s \cos[\theta] + Z_0} ; \\
(*\text{Reflect}=0.3;*) \\
Z_0 &= 415 ; (*\text{impedence of air pamm}^-1s*) \\
\text{Transmission} &= -(1 + \text{Reflect}) \frac{\text{Det}[DN_1]}{\text{Det}[D_{\text{first}}]} ; \\
\text{sumW}[[\text{mm}]] &= 2 \text{Abs}[\text{Transmission}^2] \cos[\theta] \sin[\theta] tt ; \\
\] ; \\
T_{\text{Lw}}[[\text{nn}, \text{All}]] &= \left\{ N[f] , -10 \log10 \left[ \sum_{k=1}^{\text{Num}} \text{sumW}[k] \right] \right\} ; \\
\] ; \\
T_{\text{Lw}}[[\text{All}, 2]] \\
\text{ListLogLinearPlot}[T_{\text{Lw}}, \text{Joined} \to \text{True}, \text{GridLines} \to \text{Automatic}] \\
\text{Export}[^{"diffuse_sandwich_porousTMM.csv", \text{T}_{\text{Lw}}}]
Appendix I: Generation of CNC codes for the filament winding

(Mathematica program for Appendix C)

Basic parameters

\[
R_0 = 50; (* radius of polar end*)
Lc = 80; (* length of cone*)
Lcy = 1200; (* length of cylinder*)
R = 250; (* radius of cylinder*)
Ang = \pi / 6; (* the fiber angle at cylinder*)
\]

\[
\phi = \text{ArcTan}[(R - R0) / Lc];
(* the incline angle for the cone/end cap*)
d0 = 270;
L0 = 300; (* distance from reference point to cylinder end*)
X0 = 735; (* the x coordinate of mandrel axis*)
\]

Cone1

\[
r = R0 + (R - R0) \frac{z - L0 + Lc}{Lc};
\]

\[
\text{ConeSurface1} = \{r \cdot \text{Cos}[\theta], r \cdot \text{Sin}[\theta], z\};
\]

\[
\text{Cone1Plot} = \text{ParametricPlot3D}[\text{ConeSurface1}, \{z, L0 - Lc, L0\}, \{\theta, 0, 2 \pi\}]
\]

Cylinder part

\[
Lcy = 1200; (* length of cylinder*)
\]

\[
\text{CylinderSurface} = \{R \cdot \text{Cos}[\theta], R \cdot \text{Sin}[\theta], zz\};
\]

\[
\text{CylinderPlot} = \text{ParametricPlot3D}[
\text{CylinderSurface}, \{\theta, 0, 2 \pi\}, \{zz, L0, L0 + Lcy\}];
\]

Create fiber trajectory on cylinder part

\[
z1 = \text{Cos}[\text{Ang}] \cdot l + L0;
\]

\[
\text{Sin}[\text{Ang}] \cdot l
\]

\[
R \quad \text{sol2} = \text{Solve}[z1 = \text{Lcy} + L0, l];
(* find the l end point at cylinder*)
L1 = 1 ./ \text{sol2[[1]]};
\]

\[
\text{FiberTrajectoryCylinder} = \{R \cdot \text{Cos}[\theta1], R \cdot \text{Sin}[\theta1], z1\};
\]

\[
\text{CylinderTrajectoryPlot} = \text{ParametricPlot3D}[\text{FiberTrajectoryCylinder}, \{l, 0, L1\},
\text{PlotStyle} \rightarrow \{\text{Thickness}[0.01]\}, \text{PlotRange} \rightarrow \text{All}];
\]

\[
\text{Show}[
\text{CylinderPlot}, \text{CylinderTrajectoryPlot}]
\]

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Cone part 2

\[
rrr = R0 + (R - R0) \cdot \frac{Lcy + L0 + Lc - zzz}{Lc};
\]

\[
\text{ConeSurface2} = \{rrr \cdot \cos[\theta \theta \theta], \ rrr \cdot \sin[\theta \theta \theta], \ zzz\};
\]

\[
\text{Cone2Plot} = \text{ParametricPlot3D}[\text{ConeSurface2}, \{\{\theta \theta \theta, 0, 2 \pi\}\}]
\]

create fiber trajectory in cone2

dl*Cos[\alpha] = -dr/Sin[\phi];
R Sin[\alpha] = r Sin[\alpha];

\[
\theta f1 = \theta 1 /. l \rightarrow L1; (*\text{determine the fiber angle } \theta f*)
\]

\[
dsol1 = \text{DSolve}\left[\left\{-\sin[\phi] \cdot \sqrt{1 - \frac{R^2 \cdot (\sin[\alpha])^2}{r2[1]^2}}, r2'[1], r2[1] = R\right\}, r2[1], 1\right]; (*\text{cone radius}*)
\]

\[
r2 = N[r2[1] /. dsol1[[1]]];
\]

\[
z2 = 1200 + L0 + 250 \cot[\phi] - r2 \cot[\phi];
\]

\[
(*\text{DSolve}\left\{-\cot[\phi] = z4'[rc3], z4[R] = Lc + Lcy, z4[rc3], rc3\right\}(*\text{when } l = 12, \theta \theta \theta = \theta 2 *)
\]

\[
a1 = \text{ArcSin}\left[\frac{R \cdot \sin[\alpha]}{r2}\right];
\]

\[
\text{Clear}[\theta 2]
\]

\[
dsol2 = \text{DSolve}\left[\left\{\frac{R \cdot \sin[\alpha]}{r2^2} = \theta 2'[1], \theta 2[L1] \rightarrow \theta f1, \theta 2[1], 1\right\}, \theta 2[1], 1\right];
\]

\[
\theta 2 = \theta 2[1] /. dsol2[[1]];
\]

\[
\text{sol3} = \text{Solve}[z2 = L0 + Lcy, 1];
\]

(*\text{find the fiber length at the combination of cone and cylinder}*)

\[
L2 = 1 /. \text{sol3}[[2]]; (*\text{starting length of fiber at cylinder}*)
\]

\[
\text{DSolve::bvnul: \text{For some branches of the general solution, the given boundary conditions lead to an empty solution.} \gg}
\]

\[
\text{FiberTrajectoryCone2} = \{r2 \cdot \cos[\theta 2], \ r2 \cdot \sin[\theta 2], \ z2\};
\]

\[
\text{Cone2TrajectoryPlot} = \text{ParametricPlot3D}[\text{FiberTrajectoryCone2}, \{l, L1, L2\}, \text{PlotStyle} \rightarrow \{\text{Thickness}[0.01]\}, \text{PlotRange} \rightarrow \text{All}];
\]

\[
\text{Show}[\text{Cone2TrajectoryPlot}, \text{Cone2Plot}]
\]
Round way in cylinder part

\[ \theta f2 = \theta f2 / . \ l \rightarrow L2; \]
\[ z3 = -\cos[\text{Ang}] \ast (1 - L2) + L0 + Lcy; \]
\[ \theta 3 = \frac{\sin[\text{Ang}] \ast (1 - L2)}{R} + \theta f2; \]
\[ \text{sol4} = \text{Solve}[z3 = L0, l]; \]
\[ (\ast \text{find the fiber length at the combination of cone and cylinder}) \]
\[ L3 = l / . \text{sol4[[1]]}; (\ast \text{starting length of fiber at cylinder}) \]
\[ \text{FiberTrajectoryCylinder1} = \{R \ast \cos[\theta 3], R \ast \sin[\theta 3], z3\}; \]
\[ \text{CylinderTrajectoryPlot1} = \]
\[ \text{ParametricPlot3D}[	ext{FiberTrajectoryCylinder1}, \{l, L2, L3\}, \]
\[ \text{PlotStyle} \rightarrow \{\text{Thickness}[0.01]\}, \text{PlotRange} \rightarrow \text{All}\];
\[ \text{Show}[\text{CylinderTrajectoryPlot1}, \text{CylinderPlot}] \]

Round way in Cone1

\[ \theta f3 = \theta f3 / . \ l \rightarrow L3; \]
\[ \text{dsol3} = \text{DSolve}\left[ \right] \]
\[ \left\{ \frac{\sin[\phi] \ast \sqrt{1 - \frac{R^2 \ast (\sin[\text{Ang}])^2}{r4[1]^2}}}{r4[1]^2} = r4'[1], r4[L3] = R\right\}, r4[1], 1]; \]
\[ r4 = r4[1] / . \text{dsol3[[1]]}; \]
\[ z4 = L0 - 250 \cot[\phi] + r4 \cot[\phi]; \]
\[ (\ast \text{DSolve}[\{\cot[\phi] = z4'[rc3], z4[R] = L0\}, z4'[rc3], rc3]*) \]
\[ a2 = \arcsin\left[ \frac{R \ast \sin[\text{Ang}]}{r4} \right]; \]
\[ \text{dsol4} = \text{DSolve}\left[ \right] \]
\[ \left\{ \frac{R \ast \sin[\text{Ang}]}{r4^2} = \theta 4'[1], \theta 4[L3] = \theta f3\right\}, \theta 4[1], 1]; \]
\[ \theta 4 = \theta 4[1] / . \text{dsol4[[1]]}; \]
\[ \text{sol5} = \text{Solve}[z4 = L0, l]; \]
\[ (\ast \text{find the fiber length at the combination of cone and cylinder}) \]
\[ L4 = l / . \text{sol5[[2]]}; (\ast \text{starting length of fiber at cylinder}) \]
\[ \text{DSolve::bvnul:} \]
\[ \text{For some branches of the general solution, the given boundary conditions} \]
\[ \text{lead to an empty solution.} \]
Fiber geometry test

cone and cylinder radius $\rho$

$$\rho = \text{Piecewise}[
\{(R, 0 < l < L1), (r2, L1 < l < L2), (R, L2 < l < L3), (r4, L3 < l < L4)\}];$$

Plot[$\rho$, {1, 0, L4}]

fiber orientation $\alpha$

$$\alpha = \text{ArcSin} \left( \frac{R \cdot \text{Sin}[\text{Ang}]}{\rho} \right) \cdot \frac{180}{\pi};$$

Plot[$\alpha$, {1, 0, 5000}, PlotRange -> All, PlotRangeClipping -> True, Frame -> True, GridLines -> Automatic]

fiber z position

$$z_t = \text{Piecewise}[
\{(z1, 0 < l < L1),
\{z2, L1 < l < L2), (z3, L2 < l < L3), (z4, L3 < l < L4)\}];$$

Plot[
$z_t$, 
{1, 0, L4}]

Create machine movement

1st stage: $l \in (0, L_1)$

$$L = \sqrt{d_0^2 - R^2} ;$$

$$\beta_1 = \text{ArcTan} \left( \frac{L \cdot \text{Sin}[\text{Ang}]}{R} \right);$$

$$X_{b1} = L \cdot \text{Sin}[\text{Ang}] \cdot \text{Sin}[\beta_1] + R \cdot \text{Cos}[\beta_1];$$

$$Z_{b1} = z_1 + L \cdot \text{Cos}[\text{Ang}];$$

$$\gamma_1 = \beta_1 + \theta_1;$$

$$X_{bb} = R / \text{Cos}[\beta_1];$$

$$10 \sqrt{651}$$

Plot11 = Plot[{z1, Zb1}, {1, 0, L1}, PlotLabel -> "Zb1"];

Plot12 = Plot[$\gamma_1 \cdot 180 / \pi$, {1, 0, L1}, PlotLabel -> "\gamma_1"];
Appendix I: Generation of CNC codes for the filament winding

\[ \beta_2 = \text{ArcTan} \left( \frac{L \sin[\alpha_1]}{L \cos[\alpha_1] \sin[\phi] + r_2} \right); \]

(* \(\alpha_1\) is the fiber angle at the cone *)

\[ X_{b2} = L (\sin[\alpha_1] \sin[\beta_2] + \cos[\alpha_1] \sin[\phi] \cos[\beta_2]) + r_2 \cos[\beta_2]; \]

\[ Z_{b2} = z_2 + L \cos[\alpha_1] \cos[\phi]; \]

(* the feedeye movement at 1st stage *)

\[ \gamma_2 = \beta_2 + \theta_2; \]

(* the rotational angle of mandrel *)

\[ \text{Plot21} = \text{Plot}[Z_{b2}, \{1, L_1, L_2\}, \text{PlotLabel} \rightarrow \text{"Zb2"}]; \]

\[ \text{Plot22} = \text{Plot}[\gamma_2 \ast 180 / \pi, \{1, L_1, L_2\}, \text{PlotLabel} \rightarrow \text{"\gamma2"}]; \]

\[ \text{Plot23} = \text{Plot}[X_{b2}, \{1, L_1, L_2\}, \text{PlotLabel} \rightarrow \text{"Xb2"}]; \]

\[ \text{Plot}[(z_2, Z_{b2}), \{1, L_1, L_2\}] \]

3rd stage: \( l \in (L_2, L_3) \)

\[ \beta_3 = \text{ArcTan} \left( \frac{L \sin[\text{Ang}]}{R} \right); \]

\[ X_{b3} = L \sin[\text{Ang}] \sin[\beta_3] + R \cos[\beta_3]; \]

\[ Z_{b3} = z_3 - L \ast \cos[\text{Ang}]; \]

\[ \gamma_3 = \beta_3 + \theta_3; \]

\[ \text{Plot31} = \text{Plot}[Z_{b3}, \{1, L_2, L_3\}, \text{PlotLabel} \rightarrow \text{"Zb3"}]; \]

\[ \text{Plot32} = \text{Plot}[\gamma_3 \ast 180 / \pi, \{1, L_2, L_3\}, \text{PlotLabel} \rightarrow \text{"\gamma3"}]; \]

4th stage: \( l \in (L_3, L_4) \)

\[ \beta_4 = \text{ArcTan} \left( \frac{L \sin[\alpha_2]}{L \cos[\alpha_2] \sin[\phi] + r_4} \right); \]

(* \(\alpha_2\) is the fiber angle at the second cone *)

\[ X_{b4} = L (\sin[\alpha_2] \sin[\beta_4] + \cos[\alpha_2] \sin[\phi] \cos[\beta_4]) + r_4 \cos[\beta_4]; \]

\[ Z_{b4} = z_4 - L \cos[\alpha_2] \cos[\phi]; \]

\[ \gamma_4 = \beta_4 + \theta_4; \]

(* the rotational angle of mandrel *)

\[ \text{Plot41} = \text{Plot}[Z_{b4}, \{1, L_3, L_4\}, \text{PlotLabel} \rightarrow \text{"Zb4"}]; \]

\[ \text{Plot42} = \text{Plot}[\gamma_4 \ast 180 / \pi, \{1, L_3, L_4\}, \text{PlotLabel} \rightarrow \text{"\gamma4"}]; \]

Show \( Z_{bt} \) and \( \\gamma t \) in one graph

\[ Z_{bt} = \text{Piecewise}[\{(Z_{b1}, 0 \leq l < L_1), \{Z_{b2}, L_1 \leq l < L_2\}, \{Z_{b3}, L_2 \leq l < L_3\}, \{Z_{b4}, L_3 \leq l < L_4\}\}]; \]

\[ X_{bt} = \text{Piecewise}[\{(X_{b1}, 0 \leq l < L_1), \{X_{b2}, L_1 \leq l < L_2\}, \{X_{b3}, L_2 \leq l < L_3\}, \{X_{b4}, L_3 \leq l < L_4\}\}]; \]

\[ \Theta t = \text{Piecewise}[\{\{\Theta_1, 0 \leq l < L_1\}, \{\Theta_2, L_1 \leq l < L_2\}, \{\Theta_3, L_2 \leq l < L_3\}, \{\Theta_4, L_3 \leq l < L_4\}\}]; \]

\[ \gamma t = \text{Piecewise}[\{\{\gamma_1, 0 \leq l < L_1\}, \{\gamma_2, L_1 \leq l < L_2\}, \{\gamma_3, L_2 \leq l < L_3\}, \{\gamma_4, L_3 \leq l < L_4\}\}]; \]

\[ \beta t = \text{Piecewise}[\{\{\beta_1, 0 \leq l < L_1\}, \{\beta_2, L_1 \leq l < L_2\}, \{\beta_3, L_2 \leq l < L_3\}, \{\beta_4, L_3 \leq l < L_4\}\}]; \]

\[ \text{Plot}[Z_{bt}, \{0, 0, L_4\}, \text{PlotLabel} \rightarrow \text{"Zbt"}] (* z position of feedeye *) \]

\[ \text{Plot}[X_{bt}, \{0, 0, L_4\}, \text{PlotLabel} \rightarrow \text{"Xbt"}] \]

\[ \text{Plot}[(\gamma t, 0, L_4), \text{PlotLabel} \rightarrow \text{"\gamma t"}] (* \text{rotational angle of mandrel}*) \]

\[ N[X_{bt} /. L \rightarrow 20]; \]
compute the cycles

\[ \text{wid} = 2; (*\text{suppose the fiber width is 2mm}*) \]
\[ \text{Ncyc} = \text{Round}\left(\frac{2\pi R}{2\text{wid}/\sin\text{Ang}}\right); \]

create table

\[ \Delta l = 10; (*\text{step of fiber}*) \]
\[ \text{ZbTa} = \text{Table}[N[\text{Zb4}], \{1, L3, L4, \Delta l\}]; \]
\[ \gamma \text{Ta} = \text{Table}[N[\gamma 4]*180/\pi, \{1, L3, L4, \Delta l\}]; \]
\[ \text{XbTa} = \text{Table}[N[X0 - \text{Xb4}], \{1, L3, L4, \Delta l\}]; \]
(*change to the reference coordinates*)
\[ \text{AbsCombi} = \]
\[ \text{Table}[[j, \gamma \text{Ta}[[j]], \text{ZbTa}[[j]], \text{XbTa}[[j]]], \{j, 1, \text{Length}[\text{ZbTa}]\}]; \]
\[ \text{AbsTableCombi} = \text{TableForm}[\text{AbsCombi}, \text{TableHeadings} \rightarrow \]
\[ \{\text{Automatic}, \{"Point", "C[deg]", "pz[mm]", "px[mm]"}\}]; \]
\[ \text{IncrCombi} = \text{Table}[[10 j, \gamma \text{Ta}[[j + 1]] - \gamma \text{Ta}[[j]], \text{ZbTa}[[j + 1]] - \]
\[ \text{ZbTa}[[j]], \text{XbTa}[[j + 1]] - \text{XbTa}[[j]]], \{j, 1, \text{Length}[\text{ZbTa} - 1]\}]; \]
\[ \text{IncrTableCombi} = \text{TableForm}[\text{IncrCombi}, \text{TableHeadings} \rightarrow \]
\[ \{\text{Automatic}, \{"Block#", "\Delta C[deg]", "\Delta pz[mm]", "\Delta px[mm]"\}\}]; \]

compute Speed limit

\[ \Delta C = \text{Table}[[\gamma \text{Ta}[[j + 1]] - \gamma \text{Ta}[[j]], \{j, 1, \text{Length}[\gamma \text{Ta}] - 1\}]; \]
(*angle increase*)
\[ \Delta Z = \text{Table}[[\text{ZbTa}[[j + 1]] - \text{ZbTa}[[j]], \{j, 1, \text{Length}[\text{ZbTa}] - 1\}]; \]
(* z increase*)
\[ \Delta X = \text{Table}[[\text{XbTa}[[j + 1]] - \text{XbTa}[[j]], \{j, 1, \text{Length}[\text{XbTa}] - 1\}]; \]
(* x increase*)
\[ \Delta CC = \text{Prepend}[\text{Table}[\text{Sqrt}[\text{Abs}[\gamma \text{Ta}[[j + 1]] - 2*\gamma \text{Ta}[[j]] + \gamma \text{Ta}[[j - 1]]]], \]
\[ \{j, 2, \text{Length}[\gamma \text{Ta}] - 1\}], 0]; \]
(*speed difference*)
\[ \Delta ZZ = \]
\[ \text{Prepend}[\text{Table}[\text{Sqrt}[\text{Abs}[\text{ZbTa}[[j + 1]] - 2*\text{ZbTa}[[j]] + \text{ZbTa}[[j - 1]]]], \]
\[ \{j, 2, \text{Length}[\text{ZbTa} - 1]\}], 0]; \]
\[ \Delta XX = \text{Prepend}[\text{Table}[\text{Sqrt}[\text{Abs}[\text{XbTa}[[j + 1]] - 2*\text{XbTa}[[j]] + \text{XbTa}[[j - 1]]]], \{j, 2, \text{Length}[\text{XbTa} - 1]\}], 0]; \]
Cvlimit = 600; (*rotational speed deg/s*)
Zvlimit = 1000; (*z movement speed deg/s*)
Xvlimit = 500; (*x movement speed deg/s*)
Calimit = 6/4*180 (*rotational acceleration deg/s^2*)
Zalimit = 3500; (*z acceleration m/s^2*)
Xalimit = 3500; (*x acceleration m/s^2*)

ΔTv = Table[Max[{Abs[ΔC[[i]]]/Cvlimit, Abs[ΔZ[[i]]]/Zvlimit, Abs[ΔX[[i]]]/Xvlimit}], {i, 1, Length[ΔC]}];
ΔTa = Table[Max[{Abs[ΔCC[[i]]]/Calimit, Abs[ΔZZ[[i]]]/Zalimit, Abs[ΔXX[[i]]]/Xalimit}], {i, 1, Length[ΔCC]}];

ΔT = Table[Max[{ΔTv[[i]], ΔTa[[i]]}], {i, 1, Length[ΔT]}]; (*the time at each step*)
Feed = Table[Abs[ΔC[[i]]]/(ΔT[[i]]/60), {i, 1, Length[ΔT]}]; (*the time unit is changed to deg/min*)
ListLinePlot[Round[Feed]]
T = Total[ΔT] (*unit is second*)

Appendix I: Generation of CNC codes for the filament winding

CNC codes Export

SetDirectory[$UserDocumentsDirectory];

SPprog[IncrCombi_, name_] := Module[{strm, i},
  strm = OpenWrite[ToString[name], FormatType → OutputForm];
  For[i = 1, i ≤ Length[IncrCombi], i++,
    Write[strm,
      "N", IncrCombi[[i, 1]], " ",
      "C", IncrCombi[[i, 2]], " ",
      "F", Feed[[i]], " ",
      "Z", IncrCombi[[i, 3]], " ",
      "X", IncrCombi[[i, 4]], " ",
      "A", 0];
    Write[strm, "RET"];
  ]
  Close[strm];

SPprog[IncrCombi, SPYUANFiber30c]

Show all the parts

YbTa = Table[0, {i, 0, Length[ZbTa]}];
TrajectoryPoint = Table[{N[Xbt], 0, N[Zbt]}, {i, 0, L4, Δl}];
TrajacPlot = ListPointPlot3D[TrajectoryPoint,
  PlotRange → All, PlotStyle → {Thickness[0.1]}];
Show[CylinderTrajactroyPlot, CylinderPlot, Cone2TrajactroyPlot,
Cone2Plot, CylinderTrajactroyPlot1,
Cone1TrajactroyPlot2, Cone1Plot, TrajacPlot]