1. Variable-frequency pulsed NMR (VFP-NMR) is niet geschikt voor het meten van kernspin-polarisatie.


4. Veel grote natuurkundige experimenten zouden duidelijk efficiënter verlopen, als meer fysici het belang zouden inzien van het opdoen van vaardigheden en kennis op het gebied van informatica en bedrijfswetenschap.

5. Kennisintensivering als reactie op lage-lonen landen kan niet eindeloos doorgaan. Door o.a. de bezuinigingen op de grote onderzoeksinstituten als TNO en ECN zal Nederland echter deze grens van kennisintensivering nooit bereiken.

6. Het principe dat studenten onafhankelijk dienen te zijn van al dan niet rijke ouders, een van de beginselen van de basisbeurs, is een luxe principe, en wordt, gezien de hoogte van de huidige beurs, ook niet verwezenlijkt.

7. De efficiëntie van Nederland als distributie-land wordt vooral in de weg gestaan door een niet-efficiënte distributie van de Nederlanders over hun land.

8. De jarenlange passieve houding van de overheid inzake de waterwering in het rivierengebied getuigt van een groot gebrek aan respect voor het leven en have en goed van de bewoners van dit gebied.

The spin dependent structure function $g_1$ of the deuteron and the proton

PROEFSCHRIJT

TER VERKRIJGING VAN DE GRAAD VAN DOCTOR AAN DE TECHNISCHE UNIVERSITEIT DELFT, OP GEZAG VAN DE RECTOR MAGNIFICUS PROF. IR. K.F. WAKKER, IN HET OPENBAAR TE VERDEDIGEN TEN OVERSTAAN VAN EEN COMMISSIE AANGEWERZEN DOOR HET COLLEGE VAN DEKANEN OP MAANDAG 27 FEBRUARI 1995 TE 16:00 UUR

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Chapter 1

Introduction

Scattering of charged particles has been used as a tool to study the structure of matter since the beginning of this century. The energy of these particles determines the scale at which the structure is examined; the more energetic the particles, the deeper the structure is probed. In the famous experiment by Geiger and Marsden in 1911, a gold foil was bombarded with α particles (from radio-active decay). Rutherford showed [1], that the angular distribution of the scattered α-particles was evidence for sub-structure of the atom; a very small nucleus with a cloud of electrons around it. After the invention of the accelerator, scattering of a.o. electrons was used to study the structure of the nucleus, which consists of protons and neutrons, together called nucleons. Neutrons were discovered in 1931 via (α-n)-reactions on light nuclei. The force that keeps the nucleons together in the nucleus is called the “strong force”. It turned out that apart from the proton and the neutron there were many more strongly interacting particles, fermions (half-integer spin), so-called baryons, and bosons (integer spin), called mesons because of their smaller mass. Baryons and mesons together are called hadrons.

More than 50 years after Rutherford’s discovery, the angular distribution of 20 GeV electrons scattered on protons [2] showed a similar behaviour, providing evidence for sub-structure of the proton. The electromagnetic force between the electron and proton is mediated via a photon, as described in quantum electrodynamics (QED). In scattering high energy electrons, a large four-momentum is exchanged between an electron and a proton, and the wavelength of the exchanged virtual photon is small enough to probe constituents of the proton. Because of the large four-momentum transferred to the probed constituent, the proton breaks up. Scattering in this regime is called deep inelastic scattering (DIS). The experimental data were interpreted in the quark parton model (QPM), a model in which DIS on the proton is the sum of elastic scattering on point-like spin $\frac{1}{2}$ particles within the proton, so-called partons, later identified with the quarks postulated by Gell-Mann and Zweig [3]. Since then, baryons and mesons were no longer “fundamental particles”, but consisted of quarks (three for baryons, two for mesons) in the QPM. Elastic scattering of electrons on point-like quarks can in principle be calculated exactly, provided the quark momentum and spin distributions are known. The scattering cross section can be expressed in terms of kinematic factors and so-called structure functions, which contain the momentum and spin distribution functions. Determination of the structure functions from the cross sections gives information on these distribution functions.
and thus on the interaction between the quarks inside the nucleon. For the unpolarised case the most important structure function is $F_2$. It was found, as predicted for protons composed of point-like constituents, that in the DIS kinematic range the structure functions are essentially independent of the four-momentum $Q^2$ exchanged between the electron and the proton. This property of the DIS structure functions is denoted as "scaling". In 1990 Friedman, Kendall and Taylor were awarded for this work the Nobel prize.

In the beginning of the eighties a muon beam line was installed at the scientific centre of the Conseil Européen de Recherche Nucléaire (CERN) in Geneva, providing muons with an energy from 90 GeV up to 280 GeV for DIS experiments. The beam line was first used by the European Muon Collaboration (EMC) and the BCDMS collaboration. The EMC a.o. compared the DIS cross sections of various nuclei, and found a dependence of the cross section per nucleon on the atomic number $A$, from then on known as the "EMC-effect". An important goal of the EMC and the BCDMS collaboration was to test perturbative quantum chromodynamics (P-QCD), describing the strong interaction between the quarks. In QCD the forces between quarks are mediated via gluons, which couple to colour charge, just like the photon, which mediates the electromagnetic force in QED, couples to electric charge. In QCD the structure functions, which to a zero-order approximation (QPM) show scaling behaviour, in fact have a logarithmic dependence on $Q^2$. Between 1986 and 1989 the New Muon Collaboration (NMC) has continued unpolarised DIS experiments with this muon beam, measuring $F_2$ for the proton, the deuteron and "heavy nuclei", with high precision and in an extended kinematic range.

The muon beam at CERN is spin-polarised naturally. If the target is spin-polarised as well, the cross section also depends on the relative orientation of the muon spin and proton spin, and therefore one can study the spin structure of the proton. On the basis of certain nuclear quark models one can expect that about 65% of the spin of the proton is carried by the spins of the quarks, and the rest by their orbital momentum. In polarised DIS the cross section is sensitive to the relative orientation of the quark spins with respect to the proton spin, and thus to the net polarisation of the quarks. For this reason the EMC also studied DIS on a polarised proton target, and determined the spin dependent structure function $g_1^p$ of the proton [4]. Earlier, $g_1^p$ had already been measured with polarised electrons at Stanford Linear Accelerator Centre (SLAC), but in a more restricted kinematic range. The results from the SLAC experiments [5] seemed consistent with quark model predictions. An important aspect was the extrapolation of the SLAC results to the kinematic domain where, later, the EMC measured $g_1^p$ for the first time. This extrapolation turned out to be inconsistent with the EMC data, although the EMC and SLAC data were in good agreement in the range of overlap. The interpretation of the combined EMC and SLAC data was that the quark spins are only responsible for a small fraction of the proton spin, compatible with zero. This surprising, counter-intuitive result was referred to in literature as the "EMC spin crisis", and was the incentive for a large theoretical and continuing experimental effort.

The Spin Muon Collaboration (SMC) was set up in 1988 by V.W. Hughes, who also initiated the pioneering spin-experiment at SLAC, to continue with polarised DIS measurements after the experiments of the NMC would be finished. Its goals were to verify and to improve the $g_1^p$ measurement of the EMC, and to measure $g_1^d$ for the deuteron as well, to obtain information on the neutron. For the combination of the proton and neutron results, there exists a rigorous
theoretical prediction, the Bjorken sum rule, which can be tested in this way.

This thesis is set up as follows. In Chapter 2 the formalism of polarised deep inelastic scattering is given, and the interpretation of the data both in the frame of the QPM as well as in QCD is prepared. Chapter 3 describes the set-up of the SMC experiment; the muon beam line, the spectrometer, the polarimeter and the data processing. The entire Chapter 4 is devoted to the polarised target, since a large fraction of the author's experimental work was related to this complex apparatus. The way the polarisation is achieved, and how it is measured once the target is polarised, are outlined in this chapter, together with several aspects of the apparatus. The data analysis and interpretation, another major subject of the author's work, are described and discussed in Chapter 5. The method to extract the photon-nucleon asymmetry $A_1$ from the observed count rate asymmetries is presented, and results for $A_1^p$ and $A_1^n$ are given. From these asymmetries the structure functions $g_1^d$ and $g_1^n$ are derived as discussed in Chapter 6, and the first moments of $g_1$ are determined and compared with theoretical predictions. The data are then compared and combined with those of other experiments, and Chapter 6 concludes with a discussion of the results.
1. Introduction
Chapter 2

Deep inelastic scattering and nucleon spin physics

This chapter summarises the formalism of deep inelastic scattering and the interpretation of the results of polarised deep inelastic scattering in the framework of the quark parton model and quantum chromodynamics. The experimental data and their interpretation before the start of the SMC experiment are also reviewed.

2.1 Deep inelastic scattering

Deep inelastic scattering provides a powerful tool to study the internal structure of the proton and neutron (nucleon). It implies the exchange of a virtual vector boson between a nucleon and a lepton, with such a high four-momentum exchange that the nucleon breaks up. In the case of lepton-nucleon scattering a neutral vector boson, $\gamma$ or $Z^0$, is exchanged, while a charged vector boson $W^\pm$ is exchanged in a neutrino-nucleon reaction.

2.1.1 Cross sections and structure functions

In the SMC experiment muon-nucleon deep inelastic scattering (DIS) is studied, and single virtual photon exchange is the dominant process in its kinematic domain. The Feynman diagram for this process is shown in Fig. 2.1. The four-momenta of the incident and scattered muon are $k = (k, E)$ and $k' = (k', E')$. The four-momenta of the virtual photon and the initial nucleon are $q = k - k'$ and $p$ respectively. We consider here only inclusive data, where the hadronic final state $X$ is not observed.

Kinematic invariants often used in the description of the scattering process are

$$\nu = \frac{p \cdot q}{M} = E - E'$$
2. Deep inelastic scattering and nucleon spin physics

\[ Q^2 = -q^2 \frac{ab}{2E'E'(1 - \cos \theta)} \]
\[ x = \frac{-q^2}{2(p \cdot q)} = \frac{Q^2}{2M^\nu} \]
\[ y = \frac{(p \cdot q)}{(p \cdot k)} \equiv \nu/E, \quad (2.1) \]

where natural units, \( h = c = 1 \), are used. The quantity \( Q^2 \) is minus the exchanged four momentum of the virtual photon and \( \nu \) the transferred energy in the laboratory frame. The mass of the proton is given by \( M \). The DIS regime is characterised by \( Q^2 >> \Lambda_{QCD}^2 \), the typical hadronic mass scale (\( \Lambda_{QCD} \approx 300 \text{ MeV} \)). The invariant \( x \) is known as the Bjorken scaling variable; it is dimensionless, just like \( y \). The physics can be described in terms of a combination of two of the above invariants, except \((\nu, y)\), but most commonly \( x \) and \( Q^2 \) are used as variables. Note that the kinematics of the scattering is completely defined by the measurement of the lepton energies and the scattering angle in the laboratory frame. The hadronic final state \( X \) has an invariant mass \( W \), with \( W^2 = (p + q)^2 \). The inelastic nature of DIS lies in the fact that in its regime \( W^2 \) is much larger than \( M^2 \).

In general, the cross section for a scattering process is given by the matrix element \( \mathcal{M} \) squared, divided by the incoming flux \( \Phi \) and summed over the phase space \( d\mathcal{P} \) of the hadronic final state \( X \).

\[ d\sigma = \frac{d\mathcal{P} |\mathcal{M}|^2}{\Phi}, \quad (2.2) \]

where in the specific case of inclusive DIS the components are given by [6, 7],

\[ d\mathcal{P} = \sum_X \frac{d^3k'd^3p_X}{(2\pi)^32E'} \delta^4(k + p - k' - p_X) \]
\[ |\mathcal{M}|^2 = \frac{e^4}{q^4} \langle k'|j_\mu^e(0)|k, s_i \rangle \langle k, s_i |j^\mu_i(0)|k' \rangle \langle p, s_n |j_{\nu n}(0) |X \rangle \langle X |j_{\nu n}(0) |p, s_n \rangle \]
\[ \Phi = 4\sqrt{(p \cdot k)^2 - m^2 M^2} \approx 4(p \cdot k)^{ab} 4M^E. \]

In \( |\mathcal{M}|^2 \) \( j_\mu^e \) and \( j_{\nu n}^\mu \) are the leptonic and hadronic electromagnetic currents. The spins of the incoming lepton and the nucleon are labeled \( s_i \) and \( s_n \). The spins of the scattered lepton and
of the final hadronic state are not measured, and have to be summed over. The mass of the lepton is denoted as $m_l$.

It is conventional to define the leptonic tensor $L^{\mu\nu}$ as

$$L^{\mu\nu} = \sum_{\text{final spin}} \langle k' | j_{\mu}^i(0) | k, s_i \rangle \langle k, s_i | j_{\nu}^j(0) | k' \rangle$$

(2.3)

and the hadronic tensor $W_{\mu\nu}$ as

$$W_{\mu\nu} = \frac{1}{4\pi} \sum_X d^3 p_X (2\pi)^4 \delta^4(q + p - p_X) \langle p, s_n | j_{\mu\nu}(0) | X \rangle \langle X | j_{\mu\nu}(0) | p, s_n \rangle.$$  

(2.4)

Inserting these definitions into eq.(2.2) leads to

$$d\sigma = \frac{e^4}{Q^4} \frac{d^3 k'}{(2\pi)^3 2E'} \frac{4\pi}{4EM} \cdot L^{\mu\nu} W_{\mu\nu},$$

(2.5)

which can be transformed into the commonly used double differential cross section

$$\frac{d^2\sigma}{dx dQ^2} = \frac{e^4}{8\pi Q^4} \frac{y^2}{Q^2} \cdot L^{\mu\nu} W_{\mu\nu}.$$  

(2.6)

Since leptons are point-like fermions, the leptonic tensor is equal to

$$L^{\mu\nu} = \bar{u}(k') \gamma^\mu u(k, s_i) \bar{u}(k, s_i) \gamma^\nu u(k')$$

$$= \bar{u}(k') \gamma^\mu u(k') \delta \bar{u}(k, s_i) \gamma_i u(k, s_i) \gamma^\nu \gamma_5,$$

(2.7)

where $\gamma^\mu$ are Dirac matrices and $u, \bar{u}$ spinors. In the normalisation of $2E$ particles per unit volume, leading to eq.(2.2), the external fermions are normalised to

$$\bar{u}(k) u(k) = (\not{p} + m_l) \quad \bar{u}(k, s) u(k, s) = (\not{p} + m_l) \frac{1}{2} (1 + \gamma_5 \frac{\not{r}}{m_l}),$$

(2.8)

where in general $\not{r} \equiv \gamma^\mu a_\mu$ and $\gamma_5 \equiv \gamma^0 \gamma^1 \gamma^2 \gamma^3$. The term $\frac{1}{2} (1 + \gamma_5 \frac{\not{r}}{m_l})$ is the spin projection operator. The sum over the lepton spins in the equation on the right leads to the first equation. Inserting both expressions into eq.(2.7), reordering and trace algebra leads to

$$L^{\mu\nu} = 2(k^\mu k'^\nu + k^\nu k'^\mu - g^{\mu\nu}(k \cdot k' - m_l^2)) - i\epsilon^{\mu\nu\alpha\beta} q_\alpha s_\beta$$

$$\approx 2(k^\mu k'^\nu + k^\nu k'^\mu - g^{\mu\nu}(k \cdot k')) - i\epsilon^{\mu\nu\alpha\beta} q_\alpha s_\beta,$$

(2.9)

where $g^{\mu\nu}$ is the metric tensor and $\epsilon^{\mu\nu\alpha\beta}$ is the anti-symmetric Levi-Civita tensor. In these expressions the spin vector $s$ has the dimension of mass, because of the chosen normalisation, eq.(2.8). The spin independent terms of the leptonic tensor are symmetric, the spin dependent term is anti-symmetric in $\mu\nu$.

Using translational invariance, and summing over the final states, the hadronic tensor (2.4) can be written as

$$W_{\mu\nu}(p, q) = \frac{1}{4\pi} \int d^4 x e^{iq \cdot x} \langle p, s_n | j_{\mu\nu}(x) j_{\mu\nu}(0) | p, s_n \rangle.$$  

(2.10)
2. Deep inelastic scattering and nucleon spin physics

Since the nucleon is not a point-like particle, theoretically one can not say much more about \( W_{\mu\nu} \). But our ignorance can be parametrised by splitting \( W_{\mu\nu} \) into terms containing the four-momenta occurring in the process, \( p, q, s_n \), and the invariant tensors \( g_{\mu\nu} \) and \( \epsilon_{\mu\nu\gamma\delta} \). Neglecting lepton masses and imposing parity conservation, translation and time-reversal invariance, hermiticity and current conservation leads to

\[
W_{\mu\nu} = F_1 \left( -g_{\mu\nu} + \frac{q_\mu q_\nu}{q^2} \right) + \frac{F_2}{p \cdot q} \left( p_\mu - \frac{(p \cdot q)q_\mu}{q^2} \right) \left( p_\nu - \frac{(p \cdot q)q_\nu}{q^2} \right) + \frac{g_1}{p \cdot q} \epsilon_{\mu\nu\gamma\delta} q^\gamma s_n^\delta + \frac{g_2}{(p \cdot q)^2} \epsilon_{\mu\nu\gamma\delta} q^\gamma ((p \cdot q) s_n^\delta - (s_n \cdot q) p^\delta),
\]

where \( F_1, F_2, g_1 \) and \( g_2 \) are the parity conserving \(^1\) structure functions of the nucleon. The above decomposition is the most general one for the case of spin \( \frac{1}{2} \). The spin 1 case is discussed below in this section. The structure functions are dimensionless and conventionally written as a function of \( x \) and \( Q^2 \) (actually \( x \) and \( Q^2/M^2 \)).

Contracting the obtained expressions for the hadronic and leptonic tensor in eq.(2.6) results in the differential cross section. In both the leptonic and hadronic tensor the spin independent part is symmetric and the spin dependent part antisymmetric in \( \mu, \nu \), which means that spin polarisation only influences the cross section if both the incoming leptons and the target nucleons are polarised. The spin independent part of the cross section is

\[
\frac{d^2 \sigma}{dx dQ^2} = \frac{e^4}{4\pi Q^4} \left[ y^2 F_1(x, Q^2) + \left( 1 - y - \frac{x^2 g_1^2 M^2}{Q^2} \right) \frac{F_2(x, Q^2)}{x} \right].
\]

(2.12)

In the specific case of a longitudinally polarised beam and target the spin dependent part of the cross section is

\[
\frac{d^2 \Delta \sigma}{dx dQ^2} = \frac{e^4}{4\pi Q^4} \left[ 2y^2 g_1(x, Q^2) \left( \frac{s_l \cdot s_n}{p \cdot q} + \frac{1}{2x} \frac{q \cdot s_l}{p \cdot q} \frac{q \cdot s_n}{p \cdot q} \right) + 2y^2 g_2(x, Q^2) \left( \frac{s_n \cdot s_l}{p \cdot q} - \frac{p \cdot s_l}{p \cdot q} \frac{s_n}{p \cdot q} \right) \right].
\]

(2.13)

with \( \Delta \sigma \equiv \sigma^{11} - \sigma^{1\dagger} \). The relation \( \epsilon^{\mu\nu\alpha\beta} \epsilon_{\mu\nu\gamma\delta} = -2(g^\alpha g^\beta - g^\alpha g^\beta g^\gamma) \) is used in the derivation. As already mentioned the spin vectors have the dimension of mass. The leptonic spin vector \( s_l \) in the relativistic limit equal to \( H_l k \), where \( H_l \) is the lepton helicity. Similarly the nucleon spin vector \( s_n \) is in the limit \( M^2/Q^2 \rightarrow 0 \) equal to \( -H_n p \). In this limit the effect of \( g_2 \) on the cross section is zero. Including also terms of order \( M^2/Q^2 \) leads to

\[
\frac{d^2 \Delta \sigma}{dx dQ^2} = \frac{e^4}{4\pi Q^4} \left( -H_l H_n \cdot \left[ y g_1(x, Q^2) \left( 2 - y - \frac{2x^2 y^2 M^2}{Q^2} \right) + g_2(x, Q^2) \frac{2x^2 y^2 M^2}{Q^2} \right] \right).
\]

(2.14)

The primary aim of the SMC experiment is to extract the structure function \( g_1 \). Since \( g_2 \) occurs in eq. (2.14) with a small kinematic weight factor, any uncertainty of \( g_2 \) tends to contribute little to the uncertainty of \( g_1 \).

\(^1\)In case of the parity-non-conserving neutrino scattering there is also a third spin independent structure function, \( F_3 \).
2.1.2 Physical meaning of structure functions; helicity amplitudes

The decomposition of the hadronic tensor into structure functions describes the various symmetry aspects of the hadronic "black box". The physical meaning of the structure functions can be understood by introducing helicity amplitudes.

The hadronic tensor $T_{\mu\nu}$ for Compton scattering of a virtual photon on a nucleon has the same symmetry properties as $W_{\mu\nu}$, and can be decomposed into similar structure functions. The optical theorem relates the imaginary ("absorptive") part of the forward Compton scattering structure function $\tilde{F}_1$ with the DIS $F_1$ as $\text{Im}\tilde{F}_1 = 2\pi F_1$. For the other structure functions similar relations exist. Forward Compton scattering can be represented as $\gamma_h + n_H \rightarrow \gamma_{h'} + n_{H'}$, where $h(h')$ and $H(H')$ indicate the helicities of the photons and nucleons of the initial(final) state. The Compton scattering cross section can be decomposed into so-called helicity amplitudes, denoted as $M_{hHh'H'}$. Using angular momentum conservation and symmetry arguments it can be shown that for a spin $\frac{1}{2}$ only four independent helicity amplitudes exist, $M_{+1+\frac{1}{2};+1+\frac{1}{2}}, M_{+1-\frac{1}{2};+1-\frac{1}{2}}, M_{0+\frac{1}{2};0+\frac{1}{2}}$ and $M_{+1-\frac{1}{2};0+\frac{1}{2}}$, consistent with the derivation of four independent structure functions. The helicity amplitudes are equal to $\epsilon^\mu_{h'} \epsilon^\nu_h T_{\mu\nu}$, where $\epsilon$ is the polarisation vector of the virtual photon. Expressing the helicity amplitudes in structure functions by using the equivalent of eq.(2.11) for $T_{\mu\nu}$, and applying the optical theorem, one can distinguish the following virtual photon cross sections [22],[7]:

\[
\sigma_T^{T} \propto \frac{1}{2\pi} \text{Im} \left( M_{+1+\frac{1}{2};+1+\frac{1}{2}} \right) = \left( F_1 + g_1 - \frac{4M^2x^2}{Q^2}g_2 \right)
\]

\[
\sigma_T^{L} \propto \frac{1}{2\pi} \text{Im} \left( M_{+1-\frac{1}{2};+1-\frac{1}{2}} \right) = \left( F_1 - g_1 + \frac{4M^2x^2}{Q^2}g_2 \right)
\]

\[
\sigma_L^{L} \propto \frac{1}{2\pi} \text{Im} \left( M_{0+\frac{1}{2};0+\frac{1}{2}} \right) = \left( -F_1 + \frac{F_2}{2x} \left( 1 - \frac{4x^2M^2}{Q^2} \right) \right)
\]

\[
\sigma_T^{L} \propto \frac{1}{2\pi} \text{Im} \left( M_{+1-\frac{1}{2};0+\frac{1}{2}} \right) = \frac{2Mx}{Q}(g_1 + g_2).
\]

(2.15)

The first two cross sections are absorption cross sections of transversely(T) polarised virtual photons, the third one is the absorption cross section of longitudinally(L) polarised photons. The interpretation of the fourth, "helicity flip"(TL), cross section in DIS is somewhat obscure. The subscripts $\frac{1}{2}$ and $\frac{3}{2}$ indicate the sum of the projection of the photon and nucleon spin on the virtual photon axis. The average of the two transversely polarised cross sections gives the absorption cross section of transversely polarised virtual photons:

\[
\sigma_T = \frac{1}{2}(\sigma_T^{T} + \sigma_T^{L}) \propto F_1.
\]

(2.16)

The frequently used ratio $R$ is defined as the ratio of transversely and longitudinally polarised virtual photon absorption cross section;

\[
R \equiv \frac{\sigma_L}{\sigma_T} = (1 + \gamma^2) \frac{F_2}{2xF_1} - 1,
\]

(2.17)

where $\gamma^2$ is the small kinematic factor $(4x^2M^2)/Q^2$. The quantity $R$ is rather small, which is related to the fact that the virtual photon is absorbed by charged spin-$\frac{1}{2}$ particles, as is discussed in the next section.
The asymmetry of the transversely polarised photon absorption cross sections $A_1$ and the so-called second asymmetry $A_2$ are defined as

$$ A_1 = \frac{\sigma_T^T - \sigma_T^L}{\sigma_T^T + \sigma_T^L} = \frac{g_1 - \gamma^2 g_2}{F_1} $$

$$ A_2 = \frac{\sigma_T^L}{\sigma_T^T} = \frac{\gamma (g_1 + g_2)}{F_1}. $$

(2.18)

The above definitions can be combined into the following expressions for $g_1$ and $g_2$

$$ g_1 = \frac{F_1}{1 + \gamma^2} (A_1 + \gamma A_2) = \frac{F_2}{2x(1 + R)} (A_1 + \gamma A_2) $$

$$ g_2 = \frac{F_1}{1 + \gamma^2} \left( \frac{A_2}{\gamma} - A_1 \right) = \frac{F_2}{2x(1 + R)} \left( \frac{A_2}{\gamma} - A_1 \right). $$

(2.19)

The first aim of the SMC experiment is to extract $g_1$ for the proton and the deuteron, from the measurement of the virtual photon absorption asymmetries $A_1$ and $A_2$ in muon scattering, using the earlier measured spin independent structure function $F_2$ and ratio $R$.

2.1.3 Extraction of photon asymmetries from scattering asymmetries

Not the virtual photon absorption cross section, but the muon-nucleon DIS cross section is experimentally observed in first instance. The muon-nucleon longitudinal scattering asymmetry $A$ is defined as

$$ A \equiv \frac{\sigma^{1\uparrow} - \sigma^{1\uparrow}}{\sigma^{1\uparrow} + \sigma^{1\uparrow}}, $$

where $\sigma^{1\uparrow}(\bar{1}\uparrow)$ is the scattering cross section with the muon and nucleon spin (anti-)parallel. Using (2.14), (2.12) and expressing $F_2$ in $F_1$ and $R$ via (2.17), leads to

$$ A = \frac{y}{F_1} \cdot \frac{g_1(2 - y - \frac{1}{2}y^2\gamma^2) + y^2\gamma^2 g_2}{2\gamma(1 - y - \frac{1}{4}y^2\gamma^2)((1 + R)/(1 + \gamma^2)) + y^2}. $$

(2.21)

Using eq.'s (2.19), the muon-nucleon cross section asymmetry can be expressed in terms of the virtual photon absorption asymmetries as

$$ A = D(A_1 + \eta A_2), \text{ and thus } $$

$$ A_1 = A/D - \eta A_2, $$

(2.22)

where $D$ is the depolarisation factor depending on kinematics and $R$, and $\eta$ a small purely kinematical factor:

$$ D = \frac{y(2 - y)(1 + \frac{1}{2}y^2\gamma^2)}{(1 + \gamma^2)\left[2(1 - y - \frac{1}{4}y^2\gamma^2)((1 + R)/(1 + \gamma^2)) + y^2\right]}, $$

$$ \eta = \frac{2\gamma(1 - y - \frac{1}{4}y^2\gamma^2)}{2 - y(1 + \gamma^2) - \frac{1}{2}y^2\gamma^2}. $$

(2.23)
2.1. Deep inelastic scattering

For $A_2$ a positivity limit $|A_2| < \sqrt{R}$ exists [10]:

$$A_2 \equiv \frac{\sigma^{TL}}{\sigma^{T}} < \frac{\sqrt{\sigma^{T}} \sqrt{\sigma^{L}}}{\sqrt{\sigma^{T}} \sqrt{\sigma^{T}}} = \sqrt{R}.$$  \hfill (2.24)

The small values of $R$ and $\eta$ lead to the approximation $A_1 \sim A/D$, where the uncertainty of $A_2$ can be accounted for by a systematic error.

2.1.4 Extensions for spin 1

In the case of a spin 1 nuclear target like the deuteron, the number of independent helicity amplitudes is eight instead of four, and consequently four more independent structure functions, $b_1$ to $b_4$, can be defined. In the DIS limit only the kinematical factors in front of $b_1$ in the relevant helicity amplitudes stay non-zero [11]. For the deuteron case $b_1$ is expected to be approximately zero, assuming non-relativistic nucleons in a potential. In fact for two non-interacting spin $\frac{1}{2}$ nucleons at rest all four $b_{1-4}$ structure functions are identical to zero [11].

Neglecting the $b_{1-4}$ structure functions implies

$$M_{1,0;1,0} = \frac{1}{2}(M_{+1+1/2;+1+1/2} + M_{+1-1/2;+1-1/2}).$$ \hfill (2.25)

To measure $A$, the cross sections of targets with opposite longitudinal polarisation are compared. In practice the target polarisation is not 100%, so that for the case of a spin 1 target also the spin state $I_z = 0$ can play a role. The polarisation $P_t$ is in case of spin 1/2 and spin 1 defined as $^2$

$$P_t^{I_z = 1/2} = \frac{n_+ - n_-}{n_+ + n_-}, \quad P_t^{I_z = 1} = \frac{n_+ - n_-}{n_+ + n_0 + n_-},$$ \hfill (2.26)

where $n$ indicates the number of nuclei in a certain spin state. The polarisation defined this way is the vector polarisation, while for spin 1 also a tensor polarisation, a measure for alignment, is defined. This is irrelevant in the present context. The measured asymmetry in both cases, assuming 100% polarised muons and a pure target, is

$$A_{meas}^{I_z = 1/2} = \frac{(n_+ \sigma^{11} + n_- \sigma^{11}) - (n_- \sigma^{11} + n_+ \sigma^{11})}{(n_+ \sigma^{11} + n_- \sigma^{11}) + (n_- \sigma^{11} + n_+ \sigma^{11})},$$

$$A_{meas}^{I_z = 1} = \frac{(n_+ \sigma^{11} + n_- \sigma^{11} + n_0 \sigma^{10}) - (n_- \sigma^{11} + n_+ \sigma^{11} + n_0 \sigma^{10})}{(n_+ \sigma^{11} + n_- \sigma^{11} + n_0 \sigma^{10}) + (n_- \sigma^{11} + n_+ \sigma^{11} + n_0 \sigma^{10})},$$ \hfill (2.27)

which, when including the definition of the polarisation, becomes

$$A_{meas}^{I_z = 1/2} = P_t \cdot \frac{\sigma^{11} - \sigma^{11}}{\sigma^{11} + \sigma^{11}},$$

$$A_{meas}^{I_z = 1} = P_t \cdot \frac{(n_+ + n_0 + n_-)(\sigma^{11} - \sigma^{11})}{(n_+ + n_-)(\sigma^{11} + \sigma^{11}) + 2n_0 \sigma^{10}},$$ \hfill (2.28)

$^2$The polarisation defined this way is the vector polarisation, while for spin 1 also a tensor polarisation, a measure for spin alignment, is defined. This is irrelevant in the present context.
If one assumes $b_{1-4} = 0$, by including relation (2.25) between the helicity amplitudes (and thus cross sections) the expressions for $I = 1$ and $I = \frac{1}{2}$ in eq. (2.28) become identical. Then, $A_{\text{meas}} = P_1 \cdot A$, so that in both cases $A_1$ and $g_1$ can be extracted from the measured scattering cross section asymmetry in the same way.

The deuteron bound state

A deuteron is a bound state of a proton and a neutron, with a wave function dominated by the spatially symmetric S-state (orbital angular momentum $L=0$), but with a small admixture of the D-state ($L=2$). In the S-state the proton and neutron spin are coupled to spin 1 of the deuteron. Without admixture of the D-state $2g_1^d$ would be equal to the sum of $g_1^p$ and $g_1^n$, neglecting nuclear modification of the internal nucleon structure. In the D-state the orbital angular momentum $L=2$ is coupled to the nuclear spin $S=1$, giving $J=1$, conventionally denoted as the $^3D_1$ state. A correction is needed for the probabilities of quantum states in which the proton and neutron spins are not parallel to the deuteron spin, in particular the D-state. This can be done using Clebsch-Gordan coefficients, which leads to the following relation between $g_1(x)$ for the proton, neutron and deuteron,

$$g_1^p + g_1^n = \frac{2g_1^d}{1 - \frac{3}{2} \omega_D}, \quad (2.29)$$

where $\omega_D$ is the probability of the D-state admixture (the structure functions are normalised per nucleon). The value for $\omega_D$, about 6%, has been determined in ref.[12], fitting a parametrisation of the so-called Paris nucleon-nucleon potential to experimental data.

2.2 Quarks and gluons

2.2.1 Scaling and the quark parton model (QPM)

In principle the decomposition of the hadronic tensor into structure functions is not specific for DIS, but it is valid for scattering in general. In elastic scattering one had observed a strong dependence of the "elastic structure functions" or "form factors" on $Q^2/m^2$, where the scale $m^2$ is related to the size of the nucleon. Bjorken [13] predicted that if the constituents of the nucleon would be pointlike and behave essentially as free particles, the hadronic scale $m^2$ would be irrelevant in DIS, leading to independence on $Q^2$. The so-called scaling of the DIS structure functions was indeed observed by the pioneering experiments at SLAC, studying deep inelastic scattering of electrons on protons with $2 < Q^2 < 7.5$ GeV$^2$ [2]. The scaling behaviour was the first indication of the existence of the spin $\frac{1}{2}$ quarks postulated by Gell-Mann and Zweig [3] to explain observed properties of hadrons. In Table 2.1 the charges and quantum numbers of the six quark flavours are given, where should be that the top quark is still not unambiguously discovered.

Hadrons are thought to be built up by a quark and antiquark (mesons) or three quarks (baryons). The nucleons are composed of three valence quarks ($uud$ for the proton and $ddu$ for the neutron) that determine the quantum numbers and a "sea" of quark-antiquark pairs. The forces between quarks are small at short ($\ll 1$ fm) distances ("asymptotic freedom")
2.2. Quarks and gluons

<table>
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<th>flavour</th>
<th>u(up)</th>
<th>d(down)</th>
<th>c(charm)</th>
<th>s(strange)</th>
<th>t(top)</th>
<th>b(bottom)</th>
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</tr>
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<td>1/2</td>
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<tr>
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<td>+1/2</td>
<td>-1/2</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>baryon number</td>
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<td>1/3</td>
<td>1/3</td>
<td>1/3</td>
<td>1/3</td>
<td>1/3</td>
</tr>
</tbody>
</table>

Table 2.1: The quantum numbers of the quarks.

and they become very large at long ($\approx 1$ fm) distances ("confinement"). This should explain why a quark has never been observed isolated, and hadronises, i.e. forms hadrons (mesons and baryons). The forces between quarks are discussed in the Section 2.2.2 about quantum chromodynamics.

In lepton-nucleon scattering at high energy the virtual photon "sees" the nucleon as a swarm of quarks with almost parallel momenta. The component of the quark momentum perpendicular to the nucleon momentum can often be neglected. In the quark parton model (QPM) [14], DIS is described as an incoherent superposition of quark-virtual photon absorption interactions, where the struck quark is considered a free massless, pointlike particle carrying a fraction $\xi$ of the nucleon momentum $p$. With free is meant that within the time scale of the interaction with the virtual photon, the struck quark is not influenced by its fellow quarks, the spectators. The momentum fraction $\xi$ is equal to the scaling variable $x$ in the limit $x^2 M^2 / Q^2 \to 0$. This can be shown by momentum conservation:

$$(p')^2 = (\xi p + q)^2 \quad \to \quad m_q^2 = \xi^2 M^2 + q^2 + 2\xi p \cdot q \quad \to \quad \xi = \frac{-q^2}{2(p \cdot q)}, \quad (2.30)$$

where $p'$ is the four-momentum of the struck quark, on-shell after the photon absorption and before the hadronisation. In the scaling region the scaling variable (Bjorken-) $x$ can thus be interpreted as the fraction of the nucleon momentum carried by the struck quark.

The hadronic tensor for elastic scattering of a lepton on a free quark is not a "black box" like the DIS nucleon hadronic tensor, since the quark is a pointlike particle. One has to integrate over the phase space of the non-observed final quark states. Calculating the quark hadronic tensor, and comparing it to eq.(2.11) leads to the following contribution of a massless quark with momentum $\xi p$ and helicity $h$ to the structure functions:

$$F_1^q(x) = \frac{e_i^2}{2} \delta(\xi - x), \quad F_2^q(x) = e_i^2 \xi \delta(\xi - x), \quad g_1^q(x) = \frac{e_i^2}{2} \delta(\xi - x), \quad g_2^q(x) = 0, \quad (2.31)$$

where $e_i$ indicates the charge of a certain quark flavour. In the QPM approximation the nucleon structure function is obtained by summing over all possibilities of momentum fractions and helicities, weighted with their probability. The quark distribution functions like $q_{i+(-)}(\xi)$ stand for the probability to find a quark or anti-quark $u, \bar{u}, d, \bar{d}, s, \bar{s}, ..$ in the nucleon with helicity equal(+) or opposite(-) to the nucleon helicity $H$ and carrying momentum fraction $\xi$. The structure functions then become:

$$F_1(x) = \sum_i \frac{e_i^2}{2} (q_i(x) + \bar{q}_i(x) + q_i(x) + \bar{q}_i(x))$$

$$F_2(x) = 2x F_1(x) \quad \to R(x) = 0$$
\[ g_1(x) = \sum_i \frac{2}{\Delta q} (q_+(x) + \bar{q}_+(x) - q_-(x) - \bar{q}_-(x)) \equiv \sum_i \frac{e_i^2}{\Delta q} \]
\[ g_2(x) = 0, \quad (2.32) \]

where the summation is over the active quark flavours. In combination with eq.(2.15) one can see that \( \sigma_2 = \Sigma (q_+ + \bar{q}_+) \), while \( \sigma_2 = \Sigma (q_- + \bar{q}_-) \). This is because a quark (spin-\( \frac{1}{2} \)) can only absorb a photon (spin-1) if the spins are anti-parallel. Also related to the spin-\( \frac{1}{2} \) nature of quarks is the small value of \( R \). In the limit where one can neglect the transverse momenta of the quarks, they can only absorb transversely polarised virtual photons, so that \( \sigma_L \rightarrow 0 \), and thus \( R \rightarrow 0 \). In the QPM context the structure functions are \( Q^2 \)-independent, as observed in the first DIS experiments. In later more precise experiments over a large \( Q^2 \) range scaling violations were found however, indicating that the Quark Parton Model gives only a simplified description.

### 2.2.2 Quantum chromodynamics (QCD)

Quantum chromodynamics (QCD) is a non-abelian gauge field theory of the strong interaction, where the force is mediated by a vector boson called "gluon". The gluon couples to colour charge, like the vector boson of the electromagnetic force, the photon, couples to electric charge. Hadrons are colour neutral, but quarks carry the quantum label "colour". There are three colour charges, commonly denoted as red, green and blue (r, b, g), where anti-quarks have the complementary colours \( \bar{r}, \bar{b} \) and \( \bar{g} \). Gluons are bi-coloured objects, e.g. \( r\bar{b} \), and there are eight independent combinations that actually carry colour. A fundamental difference between QED and QCD is that unlike the charge neutral photon, a gluon carries colour and can therefore couple to other gluons. The strong interaction coupling strength is \( \alpha_s \).

An electron is surrounded by virtual electron-positron pairs, which screen the electric charge with increasing distance. In QCD, due to the gluon-gluon coupling, the observed colour coupling to a quark decreases when one penetrates the colour cloud around it. The variation of the coupling constant with varying resolution, i.e. varying \( Q^2 \), is normally referred to as "the running coupling constant \( \alpha_s \)" and the fact that \( \alpha_s \) decreases with decreasing distance to "asymptotic freedom". The quarks behave as if they are free at very small distances, thus at very high \( Q^2 \). On the other hand, the coupling constant increases rapidly with increasing distance, leading to confinement. The variation of \( \alpha_s \) with \( Q^2 \) can be calculated in QCD, but only relative to a given scale, not in absolute value. It is common practice nowadays to use as a reference the measured value of \( \alpha_s \) at the \( Z_0 \)-mass, and use the QCD \( Q^2 \)-dependence of \( \alpha_s \),

\[
\frac{1}{\alpha_s^2(Q^2)} = \frac{1}{\alpha_s(M_Z^2)} + b_0 \ln \left( \frac{Q^2}{M_Z^2} \right) \\
\frac{1}{\alpha_s(Q^2)} = \frac{1}{\alpha_s^2(Q^2)} - b_1 \ln \left( \frac{\alpha_s^2(Q^2)}{\alpha_s(M_Z^2)} \right), \quad (2.33)
\]

with \( b_0 = (11 - \frac{2}{3} N_f)/(2\pi) \) and \( b_1 = (51 - \frac{10}{3} N_f)/(4\pi^2) \), to calculate the coupling strength at other values of \( Q^2 \). If the number of active flavours \( N_f \) is different at \( Q^2 \) and \( M_Z^2 \), the continuity of \( \alpha_s \) at the flavour thresholds is used. The first expression, leading to \( \alpha_s^2 \), includes
2.2. Quarks and gluons

corrections to first order in \( \alpha_s \) to the quark-gluon interaction, the second expression is up to second order. Also the third order expression has already been calculated [8].

For the quark distribution functions there is a similar situation as for \( \alpha_s \). One is still restricted to calculations in the perturbative regime of QCD, where \( \alpha_s \) is relatively small and consequently the number of significant Feynmann diagrams not too large. With perturbative QCD the quark distribution functions can not be calculated, but their evolution can be, given a starting value for the functions. The quantitative procedure introduced by Altarelli and Parisi [15] describes two ways in which the momentum distribution of quarks can change: Quarks can lose momentum by radiation of a gluon, or quarks can be generated by pair production from a gluon. Furthermore also gluons can decompose into two gluons. These processes are described by so-called splitting kernels \( P_{a\rightarrow b}(x) \). They represent the probability to find a parton \( a \) carrying momentum fraction \( x \) of the parent parton \( b \). The partons can either be gluons or quarks, where quark masses and thus flavour dependence are neglected. The coupled equations describing the \( Q^2 \)-evolution of the spin averaged quark and gluon distribution functions are

\[
\frac{d\bar{q}_i(x,Q^2)}{dQ^2} = \frac{\alpha_s(Q^2)}{2\pi Q^2} \int_x^1 \frac{dx'}{x'} \left( \bar{q}_i(x',Q^2)P_{q\bar{q}}(x/x') + g(x',Q^2)P_{qG}(x/x') \right)
\]

\[
\frac{dG(x,Q^2)}{dQ^2} = \frac{\alpha_s(Q^2)}{2\pi Q^2} \int_x^1 \frac{dx'}{x'} \left( \sum_{i=1}^{2N_f} \bar{q}_i(x',Q^2)P_{G\bar{q}}(x/x') + g(x',Q^2)P_{GG}(x/x') \right).
\]

For the spin dependent case the separate quark and gluon distribution functions for partons with helicity parallel(+) or antiparallel(-) to the nucleon helicity are taken, together with helicity dependent splitting kernels. Parity conservation implies \( P_{a-b\pm} = P_{a+b\mp} \), which leads to the Altarelli-Parisi (AP) evolution equations:

\[
\frac{d\Delta\bar{q}_i(x,Q^2)}{dQ^2} = \frac{\alpha_s(Q^2)}{2\pi Q^2} \int_x^1 \frac{dx'}{x'} \left( \Delta\bar{q}_i(x',Q^2)\Delta P_{q\bar{q}}(x/x') + \Delta G(x',Q^2)\Delta P_{qG}(x/x') \right)
\]

\[
\frac{d\Delta G(x,Q^2)}{dQ^2} = \frac{\alpha_s(Q^2)}{2\pi Q^2} \int_x^1 \frac{dx'}{x'} \left( \sum_{i=1}^{2N_f} \Delta\bar{q}_i(x',Q^2)\Delta P_{G\bar{q}}(x/x') + \Delta G(x',Q^2)\Delta P_{GG}(x/x') \right),
\]

with the definition \( \Delta P_{ab}(x) \equiv P_{a+b}(x) - P_{a-b}(x) \).

The set of equations can be solved numerically, given the distribution functions at a starting value of \( Q^2 \). On the other hand one can fit the distribution functions to the observed scaling violations of structure functions. A beautiful example of this is shown in Fig. 2.2, where the scaling violations of \( F_2^d \) in different \( z \)-bins are shown, together with the QCD-fit. The precision data and the fit are from the New Muon Collaboration (NMC) [16]. Probing the quarks with decreasing resolution hides the fact that the probed momentum, due to gluon radiation, is distributed over more partons (quarks and gluons), and thus leads to an increase in the high-\( x \) region and a decrease in the low-\( x \) region of the quark distribution functions and consequently of \( F_2 \). Another effect of gluon radiation is an increase of the transverse momentum carried by quarks, and thus an increase of \( R \), at small \( x \).

In the spin dependent case the data \( (g_1) \) are much less accurate than in the spin independent case \( (F_2^d) \). With some assumptions one can however estimate the \( Q^2 \) dependence of \( g_1 \) without
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Figure 2.2: The structure function $F_2^N$ as a function of $Q^2$ for different values of $x$. Plotted are the data points of NMC, together with a QCD fit. The dotted curve represents $F_2$ without higher twist contributions.

explicit quark polarization functions $\Delta q(x)$ [17]. One has to assume an Ansatz functional form for the gluon polarization $\Delta G(x, Q^2)$. Combining eq.'s (2.35) and (2.32) leads to the following differential equation for $g_1$:

$$\frac{dg_1(x, Q^2)}{dQ^2} = \frac{\alpha_s(Q^2)}{2\pi Q^2} \int_x^1 \frac{dz'}{z'} \left( g_1(x', Q^2) \Delta P_{qq}(x/z') + \frac{1}{3} \Delta G(x', Q^2) \Delta P_{gq}(x/z') \right).$$

(2.36)

Neglecting the $Q^2$-dependence of $g_1$ and $\Delta G$ with respect to the variation of $\alpha_s(Q^2)/Q^2$ leads to

$$g_1(x, Q^2) - g_1(x, Q_0^2) = \frac{2}{9} \ln \frac{Q_0^2}{Q^2} \int_x^1 \frac{dz'}{z'} \left( g_1(x') \Delta P_{qq}(x/z') + \frac{1}{3} \Delta G \Delta P_{gq}(x/z') \right).$$

(2.37)

The splitting kernels in these expressions are equal to

$$\Delta P_{qq}(z) = \frac{4}{3} \frac{1 + z^2}{(1 - z)_+}, \quad \Delta P_{gq}(z) = \frac{z^2 - (1 - z^2)}{2},$$

(2.38)

where the + prescription, which regularises the singularity at $z = 1$, is defined such that $\int_0^1 f(z)/(1 - z)_+ dz = \int_0^1 (f(z) - f(1))/(1 - z) dz$.

Above the mass of the quark has been neglected. From eq.(2.30) one can see that the equality of the momentum fraction and the Bjorken-$x$ deviates more, the lower $Q^2$ becomes. The
correction for this is called the target mass correction. The structure functions as a function of $\xi$ rather than $x$, obey the Altarelli-Parisi evolution equations.

Also important at low $Q^2$ are non-perturbative effects. In the Operator Product Expansion (OPE, see below) these contributions are called "higher twist effects". The operators are classified in the OPE according to their "twist", which is dimension minus spin. The lower the twist, the more important the contribution. The lowest possible twist in DIS is two. The higher twist terms include a.o. quark-quark correlations, and give contributions to the power $(M/Q)^{t-2}$ for twist $t$.

### 2.3 Sum rules and quark spin contributions

Within perturbative QCD one cannot deduce the quark distributions and thus structure functions, but solely predict their evolution given the $x$ dependence at an initial value of $Q^2$. However, QCD does have predictive power over some of the moments of structure functions. These predictions are the so-called sum rules. The sum rules of importance for the present work can be derived in the QPM, and, more importantly, also directly within QCD.

Because the experiments only cover a limited range in $x$, extrapolations at large and small $x$ are needed in order to determine moments of structure functions. Normally a Regge type behaviour [18] is used to extrapolate structure functions to $x = 0$ at fixed $Q^2$. This leads to [19]

$$ g_1(x) \propto x^{-\alpha_t} $$

$$ -0.3 < \alpha_t < 0.0, \quad (2.39) $$

where $\alpha_t$ is the intercept of the dominant Regge trajectory, which is not well known, but is expected to be within the given range. It is not clear what is the starting point in $x$ of the Regge domain.

### 2.3.1 Derivation and interpretation of sum rules in the QPM

From the QPM expression for $g_1$ of eq.(2.32) one can express the integrals of $g_1^p$ and $g_1^n$ in terms of quark polarisation distributions:

$$ \Gamma_1^p = \int_0^1 g_1^p(x)dx = \frac{1}{2} \left( \frac{4}{9} \Delta u + \frac{1}{9} \Delta d + \frac{1}{9} \Delta s \right) $$

$$ \Gamma_1^n = \int_0^1 g_1^n(x)dx = \frac{1}{2} \left( \frac{4}{9} \Delta d + \frac{1}{9} \Delta u + \frac{1}{9} \Delta s \right), \quad (2.40) $$

where isospin symmetry is used to refer to distributions in the proton only. The net contribution $\Delta q$ of the quarks of a certain flavour($q=u,d,s$) to the nucleon spin is equal to $\int_0^1 \Delta q(x)dx$.

It is interesting to express the moments in $\Delta q$'s, since some combinations can be linked to nucleon and hyperon $\beta$-decay. The octet axial currents which mediate the Gamov-Teller
transitions between octet baryons are defined by

\[ A_k^\mu \equiv \bar{\psi} \gamma^\mu \gamma^5 \lambda_k \frac{\psi}{2}, \quad k = 1..8, \]  

(2.41)

where \( \psi \) is the quark field triplet \((u,d,s)\) and \( \lambda_k \) are the Gell-Mann SU(3) matrices. Some matrix elements can also be expressed as functions of \( F \) and \( D \), the symmetric and antisymmetric reduced SU(3) matrix elements, conventionally used to interpret the hyperon \( \beta \)-decay results in the framework of flavour SU(3). The proton matrix element \( \langle p| A^3_\mu | p \rangle \) is via the Wigner-Eckart theorem equal to the isospin raising operator \( \langle p| A^+ | n \rangle \), measured in neutron \( \beta \)-decay, and determines \( \Delta u - \Delta d \):

\[ (\Delta u - \Delta d) s_\mu = 2 \langle p, s| A^3_\mu | p, s \rangle = s_\mu \left( \frac{g_a}{g_v} \right) _{n \rightarrow p} = (F + D) s_\mu. \]  

(2.42)

Here \( g_a/g_v \) is the ratio of the axial vector over the vector weak coupling constant. The matrix element \( A^3_\mu \) can be measured in hyperon \( \beta \)-decay only, but it is equal to the proton matrix element under the assumption of SU(3) flavour symmetry, i.e.

\[ (\Delta u + \Delta d - 2\Delta s) s_\mu = 2\sqrt{3} \langle p, s| A^8_\mu | p, s \rangle = (3F - D) s_\mu. \]  

(2.43)

The above two matrix elements are of non-singlet axial-vector currents. The flavour singlet axial-vector current matrix element \( A^0_\mu \) is proportional to the sum of quark contributions of different flavour to the proton spin:

\[ (\Delta u + \Delta d + \Delta s) s_\mu = \langle p, s| A^0_\mu | p, s \rangle. \]  

(2.44)

The difference between the first moments of \( g_1 \) for the proton and the neutron can be predicted using only isospin symmetry and information from \( \beta \)-decay of the neutron,

\[ \Gamma^p_1 - \Gamma^n_1 = \frac{1}{6} (\Delta u - \Delta d) = \frac{1}{6} \left( \frac{g_a}{g_v} \right) _{n \rightarrow p}. \]  

(2.45)

This is the fundamental Bjorken sum rule, first derived from current algebra [20], and later shown to be anchored in QCD. Its verification is considered a fundamental test for QCD.

The Ellis-Jaffe sum rule [21] predicts the value of \( \Gamma^p_1 \) and \( \Gamma^n_1 \) separately, using stronger assumptions. After some trivial algebra the first moments can be written as

\[ \Gamma^{p(n)}_1 = (-) \frac{1}{12} (\Delta u - \Delta d) + \frac{5}{36} (\Delta u + \Delta d - 2\Delta s) + \frac{1}{3} \Delta s = (-) \frac{1}{12} (F + D) + \frac{5}{36} (3F - D) + \frac{1}{3} \Delta s. \]  

(2.46)

Assuming the net contribution of the strange sea to be zero \( (\Delta s = 0) \), the value of \( \Gamma^{p(n)}_1 \) can be predicted using measurements of \( F \) and \( D \). The stronger assumptions, SU(3) flavour symmetry and \( \Delta s = 0 \), make the Ellis-Jaffe sum rule less fundamental than the Bjorken sum rule, which uses SU(2) flavour (isospin) symmetry only.
2.3. Sum rules and quark spin contributions

The total contribution of quark spins to the proton spin can be derived from the first moments of $g_1$, using slightly different decompositions:

$$
\Gamma_1^{(n)} = \left(-\frac{1}{12}(\Delta u - \Delta d) + \frac{1}{36}(\Delta u + \Delta d - s\Delta s) + \frac{1}{9}(\Delta u + \Delta d + \Delta s) + \frac{1}{12}(F + D) + \frac{1}{36}(3F - D) + \frac{1}{9}\Delta \Sigma\right).
$$

(2.47)

The total contribution $\Delta \Sigma \equiv (\Delta u + \Delta d + \Delta s)$ of the quark spins to the proton spin can thus be extracted from the measurement of the first moment of $g_1^{(n)}$, in combination with the experimental values for $F$ and $D$.

2.3.2 Sum rules in QCD

One can derive the sum rules also directly from QCD using the Operator Product Expansion (OPE). In the OPE the time-ordered product of currents in $W_{\mu\nu}$ (see eq.(2.10)) can, at sufficiently high $Q^2$, which means small separation, be expressed as a sum of local operators $\mathcal{O}_i(0)$ multiplied by Wilson coefficients $C_i'(z)$ (see e.g. [7]), where $z$ is a space four-vector,

$$
\lim_{z^2 \to 0} j_h(z)j_h(0) = \sum_i C_i'(z)\mathcal{O}_i(0).
$$

(2.48)

Inserting this in the expression of the hadronic tensor leads (schematically) to

$$
W(p, q, s_n) = \frac{1}{4\pi} \int d^4 z e^{i q \cdot z} \langle p, s_n | \sum_i C_i'(z)\mathcal{O}_i(0) | p, s_n \rangle
$$

$$
= \frac{1}{4\pi} \sum_i C_i'(Q^2)\langle p, s_n | \mathcal{O}_i(0) | p, s_n \rangle.
$$

(2.49)

The Wilson coefficients, where $C_i(Q^2)$ is the Fourier transform of $C_i'(z)$, are calculable in QCD. The operators are chosen to be traceless and symmetric in their tensor indices, where the number of indices $n$ is called their “spin”. The “twist” $t$ of an operator is defined as the mass dimension of an operator minus its spin. The lowest twist operators contribute dominantly to the amplitude, higher twist contributions are suppressed by $(M/Q)^{t-2}$. It can be shown that from this set of lowest twist operators only the ones with spin $n$ contribute to the $n$th moment of the structure function, either polarised or unpolarised. For an arbitrary structure function $S$ of a proton this means

$$
\int_0^1 x^{(n-1)} S(x) dx = \sum_i C_i^n(Q^2)\langle p, s_n | \mathcal{O}_i^n | p, s_n \rangle.
$$

(2.50)

In the case of the first moment of $g_1$ the spin-1 operator with twist is the quark operator $\bar{\psi} \gamma_{\mu} \gamma_5 \psi$ with twist-2 (dimension-3). There is no gauge invariant twist-2, spin-1 gluon operator, so the first moment of $g_1$ can be expressed in terms of quark operators only.

The first moment of $g_1$ for the proton and neutron can be decomposed into two non-singlet(ns) terms with the axial current operators $A^3$ and $A^8$, and one singlet(s) term with the $A^0$
operator, as
\[
\sigma_\mu \Gamma_1^{(n)}(Q^2) = \frac{1}{6} \left( (-) C_\mu^1(Q^2) \langle p, s_n | A_\mu^0 \rangle | p, s_n \rangle + \frac{1}{\sqrt{3}} C_\mu^1(Q^2) \langle p, s_n | A_\mu^8 \rangle | p, s_n \rangle + \frac{2}{3} C_s^1(Q^2) \langle p, s_n | A_\mu^0 \rangle | p, s_n \rangle \right). \tag{2.51}
\]
Using SU(3) flavour symmetry through eq.'s (2.42)-(2.44) this can be written as
\[
\Gamma_1^{(n)}(Q^2) = \frac{1}{6} \left( C_\mu^1(Q^2) \left(-\frac{1}{2} (F + D) + \frac{1}{6} (3F - D) \right) + \frac{2}{3} C_s^1(Q^2) \Delta \Sigma(Q^2) \right). \tag{2.52}
\]
The Wilson coefficients for the non-singlet and singlet case have been calculated for three active flavours as [9]
\[
C_\mu(Q^2) = 1 - \frac{\alpha_s(Q^2)}{\pi} - 3.5833 \left( \frac{\alpha_s(Q^2)}{\pi} \right)^2 - 20.215 \left( \frac{\alpha_s(Q^2)}{\pi} \right)^3
\]
\[
C_s(Q^2) = 1 - \frac{\alpha_s(Q^2)}{\pi} - 1.0959 \left( \frac{\alpha_s(Q^2)}{\pi} \right)^2 + ...	ag{2.53}
\]
For the singlet piece the difficulty appears that \( \langle p, s_n | A_\mu^0 \rangle | p, s_n \rangle = s_\mu \Delta \Sigma(Q^2) \) is renormalised, and scale dependent. This is due to the axial anomaly, which will be treated in more detail in the next subsection.

The scale dependence of \( \Delta \Sigma \) can be isolated by extracting \( \Delta \Sigma(Q^2 \to \infty) \):
\[
\Delta \Sigma(Q^2) = \left( 1 + \frac{2\alpha_s}{3\pi} + 1.2130 \left( \frac{\alpha_s}{\pi} \right)^2 \right) \Delta \Sigma(Q^2 = \infty), \tag{2.54}
\]
This identity, which enables one to do an evolution of \( \Delta \Sigma \) to any value of \( Q^2 \) (within the domain of perturbative QCD), is derived in a paper by Larin [9].

Since the coefficients in front of \( A_\mu^8 \) and \( A_\mu^0 \) are not the same, one can not isolate the value for \( \Delta s \) as was done in the last subsection for the Ellis-Jaffe sum rule. But assuming \( \Delta s \) to be zero, both matrix elements are equal, and the integral becomes
\[
\Gamma_1^{(n)} = \frac{1}{12} \left( (-) C_\mu(F + D) + \left( \frac{1}{3} C_\mu + \frac{4}{3} C_s \right) (3F - D) \right), \tag{2.55}
\]
which can be calculated knowing \( F, D \) and \( \alpha_s \). Since \( A_\mu^0 \) is scale independent and \( A_\mu^8 \) is not, their matrix element can in principle only be equal at a certain value of \( Q^2 \). In the Ellis-Jaffe sum rule one implicitly assumes \( A_\mu^0 \) and \( A_\mu^8 \) to be equal at the average \( Q^2 \) value of the experiment. The QCD corrections on the Ellis-Jaffe sum rule can only be made up to next-to-next-to-leading order (NNLO), since \( C_s \) has not yet been calculated to higher order.

The Bjorken sum rule expressed in Wilson coefficients is equal to
\[
\sigma_\mu (\Gamma_1 - \Gamma_7) = \frac{1}{6} C_\mu \langle p, s_n | A_\mu^0 \rangle | p, s_n \rangle
\]
\[
(\Gamma_1 - \Gamma_7) = \frac{1}{12} C_\mu (F + D). \tag{2.56}
\]
Contrary to $A^0$, the non-singlet axial current $A^3$ (like $A^6$) is conserved and thus not renormalised. This in combination with the fact that flavour singlet contributions like $\Delta s$ and gluon contributions cancel, only assuming the accurately established SU(2) flavour (isospin) symmetry, makes that the Bjorken sum rule can be considered as a rigorous prediction of QCD.

2.4 Some models

![Graph with data points and lines.](image)

**Figure 2.3:** The EMC result for $\Gamma^0$. Plotted is the build-up of the integral from large $x$ to small $x$, and the theoretical prediction.

The incentive to do the SMC experiment came from the surprising measurement of $g_1^p$ by the European Muon Collaboration (EMC) in 1989 [4]. The value for $\Gamma^p$ was found to be more than 2.5 standard deviations away from the Ellis-Jaffe prediction (see fig. 2.3), and the fraction from the proton spin coming from the quark spins was derived in the frame of the QPM to be $0.12 \pm 0.17$, with the strange quarks being negatively polarised. Along with the experimental efforts like that of the SMC, the EMC result also gave rise to a lot of theoretical work. Below some of the models and explanations are summarised.

**SU(6) quark model**

In the SU(6) symmetric quark model hadrons are considered to consist of massive $U, D$ and $S$ constituent quarks. The masses of the constituent quarks can be regarded as the effective masses of the quarks bound within the hadron. They are much different from the current quarks $(u, d, s)$ appearing in the QCD Lagrangian [23, 24]. The model is quite effective in describing the hadronic mass spectrum and the ratio of the proton and neutron magnetic moments. The QCD currents can be rewritten in terms of the constituent quark fields, where renormalisation constants are needed, which can not be calculated. The values for the non-singlet matrix elements $\Delta u - \Delta d$ and $\Delta u + \Delta d - 2\Delta s$ can be evaluated in terms of the
non-singlet current normalisation constant $g_A^{n_s}$ as

$$\Delta u - \Delta d = \frac{5}{3} g_A^{n_s}, \quad \Delta u + \Delta d - 2\Delta s = g_A^{n_s}. \quad (2.57)$$

Using the experimental values of these matrix elements from hyperon decay (using SU(3) flavour symmetry), one obtains a value of $g_A^{n_s}$ of about 0.75. Also in the expression for the singlet current there is a singlet current renormalisation constant $g_A^s$, and the total (current) quark spin contribution $\Delta \Sigma$ is equal to this $g_A^s$. In the case where non-direct coupling, like the axial anomaly, is suppressed, it does not matter for the renormalisation what combination of $\Delta q_i$ is under study:

$$\frac{\Delta \Sigma}{1} \approx \frac{\Delta u + \Delta d - 2\Delta s}{1} \approx \frac{\Delta u - \Delta d}{5/3}, \quad (2.58)$$

where the denominator values are the SU(6) current quark values with $\Delta u = 4/3$, $\Delta d = -1/3$ and $\Delta s = 0$. The renormalisation is the same, $g_A^s = g_A^{n_s}$ and thus $\Delta \Sigma \approx 0.75$. In this model the nucleon spin is coming completely from the spin of the constituent quarks, which are extended objects which derive some of their spin from gluons or orbital angular momentum. The value for $\Delta \Sigma$ is somewhat different from the Ellis-Jaffe prediction $\Delta \Sigma \approx 0.6$, since there $\Delta \Sigma$ is taken equal to $\Delta u + \Delta d - 2\Delta s$, instead of an average of the two expressions in eq. (2.58).

**Polarised gluons**

In Fig. 2.4 the Feynman diagram of the so-called Adler-Bell-Jackiv anomaly [25, 26], is shown. As already indicated, the axial anomaly links the polarised gluon distribution to the quark spin contribution to the nucleon spin as measured in DIS. At first glance it seems contradictory that the polarised gluons significantly contribute, since in the OPE theory the first moment of $g_1$ is dominated by the quark operators only, since there is no spin-1, twist-2 gluon operator. But the axial anomaly is included in the OPE, since its operators are gauge-invariant and the quark operator includes the local photon-gluon interaction, as described in a review article by Bass and Thomas [27]. This means that the definition of $\Delta q_i$ in eq.’s (2.42)-(2.44) is different.

![Figure 2.4: The triangle graph of the axial anomaly.](image)
for the OPE compared to the Quark Parton Model. The difference between the two definitions is equal to

\[
\left( \Delta q_i(Q^2) \right)_{OPE} = \left( \Delta q_i - \frac{\alpha_s(Q^2)}{2\pi} \Delta G(Q^2) \right)_{QPM}
\]

\[
\left( \Delta \Sigma(Q^2) \right)_{OPE} = \left( \Delta \Sigma - \frac{3\alpha_s(Q^2)}{2\pi} \Delta G(Q^2) \right)_{QPM}.
\]

(2.59)

The term with \( \Delta G \) does not vanish with increasing \( Q^2 \), since the logarithmic increase of \( \Delta G \) compensates the decreasing value of \( \alpha_s \). What is extracted in the EMC experiment is \( \left( \Delta \Sigma \right)_{OPE} \). This was realised first by Efremov-Teryaev and then by Altarelli-Ross [28, 29], who suggested that a large polarised gluon component in the nucleon could retain the Ellis-Jaffe hypothesis of \( \Delta s = 0 \). Somewhat disturbingly, the large value (\( \approx 4 \)) of \( \Delta G(10\text{GeV}^2) \) needed to obtain the EJ value of \( \Delta \Sigma \approx 0.6 \) has to be compensated in this context by a similarly large orbital angular momentum contribution, since

\[
\frac{1}{2} = \frac{1}{2} \Delta \Sigma + \Delta G^+ < L_z >.
\]

(2.60)

**Models describing \( A_1(x) \)**

One of the first models describing the \( x \)-dependence of \( A_1 \) is due to Carlitz and Kaur [31], who start with the SU(6) wave function and introduce a phenomenological “spin dilution function” to fulfill the Bjorken sum rule. Using experimentally derived unpolarised valence quark distribution functions, and introducing separate “spin dilution functions” for the \( u \) and \( d \) quark, Schäfer [30] describes the EMC data quite well with this model, and makes a prediction for \( A_1^p \). The spin dilution functions account for quark-gluon coupling.

In the Schäfer model the sea is assumed to be unpolarised, but Close and Shivers [32] suggested this may not be the case. Starting from the valence quarks, sea-quarks are generated through gluons emitted from the valence quarks. From the AP-splitting kernels it can be seen that the higher the fractional momentum carried by the emitted gluon, the higher the probability to have its helicity parallel to the parent quark. From the produced quark pair the one with the highest fractional momentum will have the highest probability to have the same helicity as the gluon. This results in a sea polarised parallel to the nucleon spin at relatively large \( x \), and anti-parallel at very small \( x \). However, this standard P-QCD mechanism does not generate a net polarisation of the sea; it was introduced in the Schäfer model by Woloshin [33].

The EMC result gave rise to much more discussion, models and interpretations. An overview of the theoretical activity can e.g. be found in [34] and references therein.
2. Deep inelastic scattering and nucleon spin physics
Chapter 3

The Experiment

The primary aim of the SMC experiment is to measure $A_1$ and $g_1$ for the proton and the deuteron. As explained this requires a longitudinally polarised lepton beam and a longitudinally polarised proton or deuteron target. The SMC uses a beam of naturally polarised positive muons, the polarisation of which is measured downstream of the main experiment using a muon polarimeter. The target consists of two target cells with opposite longitudinal polarisations, placed behind each other in the beam. A spectrometer is built up around the target to select and register potentially useful scattering events. Hits in the detectors of the spectrometer are used to reconstruct the tracks of the incoming and scattered muons, after which the tracks are combined to determine interaction vertices. Events with the interaction vertex in one of the two target cells are used in the asymmetry analysis, described in Chapter 4 and 5. Below the main properties of the muon beam and the spectrometer are summarised. Finally the software which is used to reconstruct the muon trajectories and kinematics is discussed.

3.1 The muon beam and its polarisation

The SMC experiment makes use of the high energy muon beam from the CERN M2 beam line. This beam line was first implemented in 1974 for the EMC and BCDMS experiments, and upgraded [35] to meet the requirements for the SMC experiment. It has been redesigned for a muon intensity of $4 \cdot 10^7$ muons per spill, a muon energy between 100 and 225 GeV and an average muon polarisation of about 80%.

3.1.1 Muon production

The muons of the beam originate from the decay of pions (and kaons), which are produced by bombarding a target with high energy protons. A schematic view of the various accelerators involved is shown in Fig. 3.1. The protons are first accelerated in the linear accelerator (LINAC) to an energy of 0.8 GeV. After a temporary storage in the Booster, they are injected
in the Central Proton Synchrotron (CPS), where they obtain an energy of about 14 GeV before they are injected in the Super Proton Synchrotron (SPS). In the SPS the protons are accelerated to an energy of 450 GeV. The protons are extracted in pulses of 2.6 seconds long, with a cycle time of 14.4 seconds. Each pulse counts roughly $10^{12} - 10^{13}$ protons.

![Diagram of proton accelerator facility and M2 beam line at CERN.](image)

**Figure 3.1:** The proton accelerator facility and the M2 beam line at CERN.

The extracted protons are directed onto a beryllium production target of 30-50 cm thickness, where a.o. pions are produced. Pions within a momentum bite $\Delta p/p = \pm 4\%$ are selected and led to a decay channel of about 600 metres length. The parity violating decay process $\pi \to \mu \nu_\mu$ causes the muon beam to be naturally polarised. The helicity $h$ of the muon is related to the centre-of-mass decay angle $\theta^*$ of the muon with respect to the pion momentum vector in the laboratory frame by

$$h = \mp \frac{E^* \cos \theta^* + p^*}{E^* + p^* \cos \theta^*},$$

(3.1)

where $E^*$ and $p^*$ refer to the muon energy and momentum in the pion rest frame. The $-(+)$ sign holds for positive (negative) muons. The angle $\theta^*$ is related to the fraction of the parent pion momentum carried by the muon, and the pion-muon mass ratio. The longitudinal polarisation expressed in laboratory frame variables is in the relativistic limit

$$h = \mp \frac{m_\pi^2 + (1 - 2 \frac{E_\pi}{E_\mu}) m_\mu^2}{m_\pi^2 - m_\mu^2}.$$  

(3.2)

The decay muon energy is constrained by $m_\pi^2/m_\mu^2 < E_\mu/E_\pi < 1$. The maximum polarisation is obtained at $\cos \theta^* = \pm 1$, or equivalently with $E_\mu/E_\pi$ close to one of the two limits mentioned.
3.1. The muon beam and its polarisation

above, but there the intensity goes to zero. The optimum between beam polarisation and beam intensity is chosen by selecting $E_\mu/E_\pi \approx 0.9$, which leads to a polarisation of about $-80\%$. The hadron beam does not only consist of pions, but also contains a few percent kaons, which decay to muons with a slightly higher polarisation. At the end of the decay channel about 10% of the pions (for a 100 GeV beam) have decayed, and the remaining hadrons are absorbed in a 7.7 metres long beryllium block, through which the muons easily pass. Beryllium is a.o. chosen because of its low proton number and thus low multiple scattering of muons. The design of the absorber is such that the pion contamination behind it is of the order of $10^{-6}$.

Downstream of the absorber a vertical bend is used for momentum selection, after which the muon beam is transported to the experimental area. On the way several magnetic collimators (scrapers) deflect the halo muons. Another vertical bend is used to measure the momentum of the incoming muons. This Beam Momentum Station (BMS) is described below. Behind the BMS the muon beam enters the experimental hall and passed through the polarised target. At the target the beam diameter is about 3 cm FWHM, the halo is 2-3% and the momentum spread is $\Delta p/p \approx \pm 3\%$. Roughly 3% of the muons originate from kaon-decay. The incident muon energy was 100 GeV for the deuteron measurement and 190 GeV for the proton measurement.

In principle the beam polarisation can be calculated by simulation of the beam transport system. Such a Monte Carlo (MC) simulation [36] has been used to define the beam set-up, predicting a muon polarisation at the target of about $-80\%$. The uncertainty in the simulation is difficult to estimate, but stems mainly from the uncertainty of the momentum spectrum of the hadrons. Also the ratio of pions over kaons, the precise shape of the magnetic fields, the amount of material in the beam line and the radiative energy loss in the absorber are sources of uncertainty. For the EMC experiment, which had to rely on the MC simulation, the uncertainty in the muon polarisation was estimated to be 7%. For the SMC experiment the beam polarisation is measured, since its error would be dominant otherwise.

Figure 3.2: The SMC muon decay polarimeter.
3.1.2 Measurement of the muon polarisation

The polarisation measurement is based on the feature that the momentum spectrum of positrons from the decay $\mu^+ \to e^+ \nu_\mu \bar{\nu}_\mu$ is polarisation dependent, since the decay positron is preferentially emitted in the direction of the muon spin [37]. The momentum spectrum of the decay positron (or electron for negative muons) is known as the Michel spectrum, and is given by

$$\frac{dN}{dr} = N \left[ \frac{5}{3} - 3r^2 + \frac{4}{3}r^3 - P_\mu \left( \frac{1}{3} - 3r^2 + \frac{8}{3}r^3 \right) \right],$$  

(3.3)

where $r = E_e/E_\mu$ is the ratio between positron and muon energy, and $N$ the number of muon-decays. Fig. 3.3 gives an indication of the sensitivity of the spectrum to the polarisation.

![Figure 3.3: The Michel spectrum. The graph on the left gives an indication for the sensitivity of the spectrum versus polarisation. On the right a typical (corrected) spectrum obtained with the SMC polarimeter. Indicated is the range of $r$ which is used in the fit, the range where the acceptance function $a(r)$ is quite flat.](image)

The measurement of the Michel spectrum and thus of the muon polarisation requires the determination of both the muon and positron energies. The set-up designed and built for this purpose is shown in Fig. 3.2. The central part is a 33 m long decay channel. It is evacuated in order to reduce Bremsstrahlung of the decay positron. Upstream of the decay channel the incident particles are identified as muons or as decay positrons by a shower veto, which consists of an 8 mm thick layer of lead followed by scintillators. Positrons cause showers in the lead and are vetoed on recognition. Downstream of the decay channel the momentum of a decay positron is determined by the deflection in the analysing magnet MNP26. The upstream arm consists of three sets of multiwire proportional chambers (MWPC's) in the decay channel, the track downstream of the magnet is also measured by three sets of proportional chambers.
Muons are deflected so little in the magnetic field that they are not detected in the chambers downstream of MNP26. The energy of the positrons is measured in lead glass calorimeters, and compared with the value determined by the deflection in MNP26 in order to remove background. The momentum of the incoming muon is determined with the beam momentum station (BMS), taking into account energy loss in the target and absorber. The BMS and MNP26 were intercalibrated\[38],[39]. Background of positrons from other mechanisms than muon decay are charge symmetric, and are subtracted by looking at the response of a negative muon beam, where only background positrons are observed.

The muon polarisation depends on the absolute shape of the positron momentum spectrum. The acceptance function \( a(r) \) of the detector was simulated with a Monte Carlo (MC) simulation. Radiative corrections to the decay process turned out to be quite important; their effect was included in \( a(r) \). For the simulation the efficiencies and positions of the chambers and an input description of the muon beam polarisation and phase space are needed. The polarisation was taken to be -80%, but the dependence of \( a(r) \) on \( P_\mu \) is negligible with muon polarisations between -70 and -90%. The simulation was also used to define cuts to eliminate the background from muon decay in the MNP26. The measured spectrum was corrected for \( a(r) \) in the region \( 0.34 < r < 0.6 \), the region where \( a(r) \) is quite flat. The polarisation is obtained by fitting the corrected spectrum, of which Fig. 3.3b shows a typical example, to eq. (3.3), keeping \( N \) and \( P_\mu \) as free parameters.

<table>
<thead>
<tr>
<th>( E_\mu ) (GeV)</th>
<th>( P_\mu )</th>
<th>( \Delta P_\mu ) stat.</th>
<th>( \Delta P_\mu ) sys.</th>
<th>( P_\mu ) MC</th>
</tr>
</thead>
<tbody>
<tr>
<td>100</td>
<td>-0.840</td>
<td>0.050</td>
<td>0.020</td>
<td>-0.83</td>
</tr>
<tr>
<td>190</td>
<td>-0.803</td>
<td>0.029</td>
<td>0.020</td>
<td>-</td>
</tr>
</tbody>
</table>

**Table 3.1: The average muon polarisation in the 1992 and 1993 SMC experiments.**

In Table 3.1 the results [40] for the muon polarisation are collected. The systematic error is mainly caused by the uncertainties in \( r \), the radiative corrections, the assumed energy loss between the BMS and MNP26, positron background and the geometrical acceptance. The contributions of these errors are comparable. The measured polarisation values are in quite good agreement with the results from the above mentioned simulation of the beam transport [35].

### 3.2 The target system

The polarised target consists of two target cells with opposite longitudinal polarisation, placed one behind the other in the beam line. This makes an asymmetry extraction possible without knowledge of the beam flux. The latter cancels when determining the event yield asymmetry for the two cells, since an asymmetry is in principle derived from a ratio: \( \frac{(a - b)}{(a + b)} = \frac{((a/b) - 1)}{((a/b) + 1)} \). The ratio of the number of scattering events in the two target cells is dominated though, by the ratio of their spectrometer acceptances. This ratio cancels by combining data before and after a reversal of both target cell polarisations, provided the acceptance ratio is constant. Because of the sensitivity to possible acceptance ratio drifts, the polarisation is reversed frequently. The targets are not pure hydrogen or deuterium, but the material used by the SMC is normal or deuterated butanol, with a typical deuteron and
proton polarisation of 35 and 80% respectively. The polarisation mechanism, the polarisation measurement, and technical details of the polarised target are treated in the next chapter.

3.3 The spectrometer

In order to determine the kinematics of the scattering, and the interaction point in the target, knowledge of the trajectories of incident and scattered muon is necessary, where the trajectories in a magnetic field are used to derive both momenta. The spectrometer has a large redundancy in the number of detector planes, which makes the experiment insensitive to drifts in individual detector behaviour. This is important in view of the fact that in the experiment an asymmetry is measured at the level of a fraction of a percent. The acceptance ratio between the upstream and the downstream target cell has to be stable over reversal periods (see Chapter 4).

The SMC took over the spectrometer from the New Muon Collaboration (NMC), which in turn inherited it from the European Muon Collaboration (EMC). Both the NMC and the SMC have made significant upgrades to the spectrometer system. The general set-up of the spectrometer is shown in Fig. 3.4. The muon beam enters the experimental hall from the left hand side. Several veto counters define the size of the beam. Upstream of the polarised target the momentum and trajectory of the incident muon of each recorded event is determined. The momenta of the scattered muon and of produced hadrons are determined by their deflection in the Forward Spectrometer Magnet (FSM). Scattered muons are identified by the fact that they traverse a 150 cm thick iron absorber and are detected downstream of it, a requirement
for any physics trigger. Originally the center of the FSM was taken as the center of the coordinate system in the hall. The x-axis is almost parallel to the beam, the z-axis is vertical and the y-axis is perpendicular to both, positive in the Jura direction. In the SMC experiment the FSM magnet is located at y = -7.6 cm.

3.3.1 Detection of the incoming muon

The momentum of incoming muons is determined in the beam momentum station (BMS). The bending of the particles in its dipole magnet, “Bend 6”, is determined by the hit coordinates in four hodoscope planes, two in front of and two behind the magnet. The planes consist of 64 elements of 5 mm width and various lengths according to the beam profile, to obtain roughly equal count rates. The elements are 2 cm thick, causing large signals which makes precise timing possible. This is necessary to distinguish between hits from different beam tracks. The stability of the field in Bend 6 is monitored by Hall probes. The BMS was calibrated with respect to the precisely mapped MNP26, by moving the downstream chambers of the polarimeter set-up into the beam during a dedicated low-intensity run. The precision of the momentum determination in the BMS is \(3 \cdot 10^{-3}\).

The trajectory of an incoming muon is determined using the beam hodoscopes and a MWPC. The hodoscopes consist of two sets of four planes, BHA and BHB, roughly 10 metres apart. Each set consists of pairs of \(Y, Z, T^+\) and \(T^-\) planes, where the names of the planes indicate the coordinate they measure. The \(T\) planes are at angles of \(\pm 45^\circ\) in the \(Y-Z\) plane. Each pair consists of two staggered planes of 20, 4 mm wide strips, leading to a 2 mm resolution per pair. The spectrometer counts five \(P_0\) detectors, 14 cm diameter MWPC’s with 2Y, 2Z and 4T staggered planes, and a wire spacing of 1 mm. They were designed to be able to withstand the high muon flux in the beam. The precision of the reconstruction of the incoming track is improved considerably by \(P_0B\), one of these precision chambers. It is positioned close to the target, which makes the position of the reconstructed interaction vertex less sensitive to the angular resolution. The precision of the beam track position at the target, perpendicular to the beam, is 0.2 mm, with an angular resolution of 0.2 mrad. The beam track is basically determined by BHA and \(P_0B\). The hodoscopes BHB are still necessary, since only the timing information from the hodoscopes makes it possible to distinguish hits from different incident muons, and to associate the BMS momentum measurement with a trajectory.

3.3.2 Detection of the scattered muon

The momentum of the scattered muon is derived from the curvature of the track in the FSM, which has a magnetic field in the vertical direction. The magnetic field at the two settings used for the 100 and 190 GeV incident energy has been mapped carefully [41]. The integrated magnetic field \(\int B \cdot dl\) is equal to 2.328(4.414) Tm for the 100(190) GeV muon beam, with a relative uncertainty of less than \(2 \cdot 10^{-3}\). The overall field was calibrated using the measurement of the \(J/\Psi\) and \(K^0\) masses, and monitored with an NMR probe during the experiment. Wire chambers of various designs, depending on the demand of resolution, speed and covered area, are placed in front of, inside and behind the FSM. In a multiwire proportional chamber a wire
gives a signal if a particle passes next to it, which leads to a plane resolution equal to the wire spacing. Drift chambers give also information on the time it takes the charge, induced by the particle, to reach the wire. The time information from the so-called Drift Time Recorder (DTR) can be translated into the distance from the wire closest to the passing particle, thus increasing the resolution considerably. Because of this, the wire spacing can be much larger for the same resolution, making it possible to cover far larger areas with a similar amount of electronics and wires, as compared to MWPC’s. The chambers are not suited for high count rates, however, because of the long drift times and dead times of the DTR’s. Information about the detectors in the spectrometer is collected in Table 3.2.

<table>
<thead>
<tr>
<th>Chamber name</th>
<th>Size (cm)</th>
<th>Wire orientations Y(vert), Z(hor), T±</th>
<th>Wire spacing (mm)</th>
<th>Position x (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>P0B</td>
<td>Φ 14.4</td>
<td>2T⁺ 2Y 2Z 2T⁻</td>
<td>1</td>
<td>-7.1</td>
</tr>
<tr>
<td>P0C</td>
<td>Φ 14.4</td>
<td>2T⁺ 2Y 2Z 2T⁻</td>
<td>1</td>
<td>-3.2</td>
</tr>
<tr>
<td>P0D</td>
<td>Φ 14.4</td>
<td>2Z T⁻ 2Y T⁺ 2Z</td>
<td>1</td>
<td>-1.9</td>
</tr>
<tr>
<td>P0E</td>
<td>Φ 14.4</td>
<td>2T⁺ 2Y 2Z 2T⁻</td>
<td>1</td>
<td>2.3</td>
</tr>
<tr>
<td>P0A</td>
<td>Φ 14.4</td>
<td>2T⁺ 2Y 2Z 2T⁻</td>
<td>1</td>
<td>8.3</td>
</tr>
<tr>
<td>PV1</td>
<td>150 × 94</td>
<td>T⁺ YT⁻ Y</td>
<td>2</td>
<td>-2.6</td>
</tr>
<tr>
<td></td>
<td></td>
<td>T⁺ = ±80°</td>
<td></td>
<td></td>
</tr>
<tr>
<td>PV2</td>
<td>154 × 100</td>
<td>YT₁ T₂ T₁⁺ T₂⁺ T₁⁻ T₂⁻ Y</td>
<td>2</td>
<td>-2.2</td>
</tr>
<tr>
<td></td>
<td></td>
<td>T⁺₂ = ±45,72°</td>
<td></td>
<td></td>
</tr>
<tr>
<td>P1A</td>
<td>180 × 80</td>
<td>Z Y T⁻</td>
<td>2</td>
<td>-1.6</td>
</tr>
<tr>
<td>P1B</td>
<td>180 × 80</td>
<td>Z Y T⁻</td>
<td>2</td>
<td>-1.0</td>
</tr>
<tr>
<td>P2</td>
<td>180 × 80</td>
<td>T⁺ Y Z</td>
<td>2</td>
<td>-0.3</td>
</tr>
<tr>
<td>P3</td>
<td>180 × 80</td>
<td>T⁺ Y Z</td>
<td>2</td>
<td>0.3</td>
</tr>
<tr>
<td>W1AB</td>
<td>220 × 120</td>
<td>T⁻ YT⁺ ZT⁻ YT⁻ Z</td>
<td>20</td>
<td>2.6</td>
</tr>
<tr>
<td>W2AB</td>
<td>220 × 120</td>
<td>T⁻ YT⁺ ZT⁻ YT⁺ Z</td>
<td>20</td>
<td>3.0</td>
</tr>
<tr>
<td>W45C</td>
<td>530 × 260</td>
<td>2T 2Y 2Z 2T</td>
<td>40</td>
<td>6.3 - 6.7</td>
</tr>
<tr>
<td>W4AB</td>
<td>530 × 260</td>
<td>2Y 2Z 2T 2Y</td>
<td>40</td>
<td>6.9 - 7.4</td>
</tr>
<tr>
<td>W5AB</td>
<td>530 × 260</td>
<td>2Z 2T 2Y 2Z</td>
<td>40</td>
<td>7.6 - 8.0</td>
</tr>
<tr>
<td>P45</td>
<td>Φ 90</td>
<td>2(T⁻ Y T₄) Y T⁻</td>
<td>2</td>
<td>6.8 - 8.1</td>
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<tr>
<td>P67</td>
<td>Φ 90</td>
<td>2(T⁻ Y T₄) Y T⁺</td>
<td>2</td>
<td>14. - 17.</td>
</tr>
<tr>
<td>T⁺ = ±30°</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>ST6</td>
<td>410 × 410</td>
<td>2(T⁺ Y) 2(Z T⁻)</td>
<td>10</td>
<td>13.8 - 14.9</td>
</tr>
<tr>
<td>ST7</td>
<td>410 × 410</td>
<td>2(Z T⁻) 2(T⁺ Y)</td>
<td>10</td>
<td>15.9 - 17.1</td>
</tr>
<tr>
<td>T⁺ = ±10°</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>DT57</td>
<td>500 × 420</td>
<td>3 (2Y 2Z)</td>
<td>52</td>
<td>13.5 - 17.3</td>
</tr>
</tbody>
</table>

Table 3.2: Wire chamber parameters

The first chamber downstream of the target is the MWPC named P0C. This chamber sees the beam muons as well as the scattered muons. Downstream of P0C, but still upstream of
the FSM is a cluster of proportional chambers, PV1, PV2 and P0D. The PV1 and PV2 are large chambers covering an area of 1.5x1.0 m², and have a wire spacing of 2 mm. PV1 has four planes, PV2 has six planes. The central area of 6.5 cm diameter of PV2 is permanently inactivated. The central area of PV1 is inactivated by a potential, but can be made sensitive again, which is of importance for the alignment procedure described below. The small chamber P0D (8 planes, wire spacing 1 mm) covers the dead area of PV1 and PV2. The chamber P0D is physically connected to PV2, but is not in a field free area anymore. Three chambers are positioned in the FSM, P1, P2 and P3, consisting of three planes (YZT) each. The chambers cover the whole air gap of the FSM, 1.8x0.8 m², and have a wire spacing of 2 mm. The central area of 6.5 cm diameter is inactivated. Downstream of the FSM there are two groups of drift chambers, W12, two modules of eight planes directly downstream of the FSM, and W45, six modules of four planes 5 m from the magnet. Those chambers are large, 2.2x1.2 m² for W12 and 5.0x2.5 m² for W45. They allow reconstruction of low momentum hadrons and muons, which get a large deviation in the FSM. The wire spacing is 2 cm for W12, leading to a resolution of 250 – 300μm per plane, and 4 cm for W45 with a plane resolution of 350μm. The dead area of W12 is covered by P0E, and the inner region of W45 is covered by a set of 1 m diameter proportional chambers, P45. Those chambers, five modules of two planes (YT) and a wire spacing of 2 mm, give extra redundancy in the central region of W45, where the background count rate is high.

The next piece of apparatus the particles pass through is the calorimeter H2, which is not used in this analysis, and thus not described here. Downstream of it is a 2 m thick layer of steel, the hadron absorber. All the particles which pass this absorber are identified as muons, and are detected by a set of streamer tubes, ST67. This detector of size 4.10x4.10 m² counts 32 planes of 10 mm wide streamer cells. Copper read out strips with a spacing of 10 mm are mounted separately. The inner area of ST67 is covered by four P67 modules, similar to the P45 modules. The ST67-P67 detector group is used to construct a muon track behind the absorber, the extrapolation of which is then used to distinguish the muon track from all the hadron tracks in the W45-P45 group. Apart from ST67 and P67 also a set of drift tubes, DT67, was installed, in order to have extra redundancy in case of a low reconstruction efficiency in ST67.

3.3.3 On-line event selection

The read-out of information from the various detectors is triggered by a pulse of a fast electronic system based on a predefined hodoscope coincidence pattern. Between the spills the data are written to magnetic tape for off-line analysis. The data are used for on-line data quality checking as well.

The trigger system consists of a number of so-called Veto hodoscopes to define the beam, and a combination of trigger hodoscopes to select a scattered muon. In Fig. 3.4 the Veto counters are indicated as V1.5, V3, V2.1, V4 and V2. An event can only be accepted by the trigger if none of the veto counters have fired within a given time interval, thus diminishing the chance that halo muons cause fake triggers. The trigger hodoscopes for the main physics trigger, trigger 1 or T1, are H1H and H1V just behind the FSM, H3V and H3H just downstream of the absorber and H4H behind a second 40 cm thick steel wall behind the ST67 equipment.
3. The Experiment

<table>
<thead>
<tr>
<th>plane</th>
<th>number of elements</th>
<th>element width(cm)</th>
<th>sensitive area $\times 2 (cm^2)$</th>
<th>hole size $\times 2 (cm^2)$</th>
<th>position in x (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>H1V</td>
<td>36</td>
<td>7</td>
<td>252×130</td>
<td>14×14</td>
<td>3.37</td>
</tr>
<tr>
<td>H1H</td>
<td>18</td>
<td>7</td>
<td>250×128</td>
<td>14×14</td>
<td>3.36</td>
</tr>
<tr>
<td>H3V</td>
<td>50</td>
<td>15</td>
<td>750×340</td>
<td>45×55</td>
<td>12.49</td>
</tr>
<tr>
<td>H3H</td>
<td>2×23</td>
<td>15</td>
<td>752×345</td>
<td>55×45</td>
<td>12.80</td>
</tr>
<tr>
<td>H4H</td>
<td>2×29</td>
<td>15</td>
<td>972×435</td>
<td>55×45</td>
<td>18.30</td>
</tr>
<tr>
<td>H1'</td>
<td>24</td>
<td>1.4</td>
<td>46×50</td>
<td>17</td>
<td>8.63</td>
</tr>
<tr>
<td>H3'</td>
<td>24</td>
<td>1.4</td>
<td>54×50</td>
<td>25</td>
<td>13.17</td>
</tr>
<tr>
<td>H4'</td>
<td>32</td>
<td>1.4</td>
<td>87×50</td>
<td>31</td>
<td>18.72</td>
</tr>
</tbody>
</table>

Table 3.3: The hodoscope planes for T1 and T2.

The “H” or “V” indicate that a hodoscope has horizontal or vertical strips. A hit in H3V/H can ideally only be caused by a muon, but a beam muon scraping the sides of the hole in the absorber could cause electromagnetic showers and thus cause a trigger. The required coincidence with H4H avoids such events. The H1V and H1H hodoscopes are used for rough target pointing selection. Downstream of H4H is H5, used as a beam trigger.

The hodoscopes have a large number of strips, from 18 for H1H to 50 for H3V, where the strip widths were chosen such that the angle cut can be varied in steps of 0.5°. The patterns of the programmable coincidence matrices [42] define which combinations of hit hodoscope elements are allowed, and thus define the angle cut and target pointing criteria. The hodoscope elements are read out by photomultipliers at both ends of the scintillators. The average time of the two pulses is used for the coincidence, thus being insensitive to the position of the muon along the strip. H3H and H4H are split into two parts in order not to have too long strips and thus larger chance for accidental coincidences. Before data taking the veto and trigger hodoscope elements have to be timed with an accuracy of about 10 ns, since the pulse widths used in the coincidence are of the order of 20 ns.

The hodoscope H3V is used for the timing of the event, and has two TDC’s connected to both ends of each element as well. This time information is used in the linking of the scattered muon that caused the trigger to its beam track, via a comparison with the BMS and beam hodoscope times. The H3V time is also used as a reference time for the drift chambers. The TDC values are used for the off-line analysis only.

The logic of the main physics or large angle physics trigger T1 is shown in Fig. 3.5. The coincidence patterns loaded into the matrices are obtained from a Monte Carlo simulation of hits on the hodoscopes. The matrix patterns define allowed combinations of firing hodoscope elements, e.g. of H3V and H3H in case of M7. Most fake triggers would be unmasked in the reconstruction process, but since the dead time (and processing time) of the experiment is critical, pixels with a small ratio of interesting events to fake triggers or uninteresting events have been turned off. The matrices M0 to M3 impose the target pointing. Matrix 6 requires a minimum scattering angle with respect to the beam axis, and thus rejects a large fraction of low $Q^2$ events. Matrix 7 selects events with a minimal angle with respect to the beam axis displaced in the bending direction of the magnet. In this way events with high $\nu$ and small $Q^2$, which have a large fraction of radiative events, are rejected to a large
extent. In the 1992 data taking on the deuteron the matrices were programmed the same way the NMC had used them. The data with $y > 0.85$ were to a large extent removed by the trigger logic. It became clear, however, that the requirements are different for the SMC and NMC experiments. The disadvantage of high $y$ events, a large fraction of radiative events, is compensated in the SMC experiment by the lower depolarisation of those events, since that leads to a decrease of the dominant error for the SMC, the statistical error. After this was realised, a study was done to define new T1 matrix settings. Data were taken with completely opened matrices, recording all hits on the hodoscope strips. The matrix logic was simulated in the reconstruction software, thus doing a realistic study including all incidental hits on the trigger hodoscopes. By applying kinematic cuts the population of the various matrix pixels selecting highly useful events was compared to that of selecting useless events. The result of the study was that opening M7 much more, indeed leads to an unacceptable increase of trigger rate and thus dead time, but that a large fraction of the useless acquired extra events ($y > 0.9$ and $Q^2 < 1.0$) can be removed by using a more stringent horizontal tagging with M0 and M1. Fig. 3.6 shows the increase in reconstructed event rate $N$ between the matrix pattern used in 1992 and the one used in 1993 as a result of this study. Plotted is $D^2 \cdot N$, taking
into account the improved statistical error on the extracted $A_1$ due to higher depolarisation factor $D$. The effect of decreasing dilution factor $f$ as a function of $y$ (see Chapter 4) is also taken into account, as well as the increased dead time (from 20 to 25%). For 100 GeV the improvement is much smaller, because at that energy the pixels causing increased efficiency for the 190 GeV beam, collect to a large extent events with $Q^2 < 1.0$. Such events are not used in the analysis (see Section 5.1.6). The SMC decided to run in 1993 with a 190 GeV beam, despite of the lower cross section compared to 100 GeV. The main motivation was that at higher energy the kinematic range is extended towards small $x$, the domain where the spin structure measurements of the SMC are unique.

![Figure 3.6: The increase in efficiency of the physics trigger 1. Plotted are the effective number of events (arbitrary units) per $x$-bin for the original matrices used in 1992 (shaded) and the ones used in 1993, at 100 GeV (left) and 190 GeV (right) incident muon energy.](image-url)

To be more selective in the region close to the beam and to cover the region excluded by the holes in the trigger 1 hodoscopes, the NMC implemented a similar hardware setup for a small angle physics trigger, called T2. The hodoscopes are shown in Fig. 3.4 and are called H1', H3' and H4'. For the NMC the trigger was set-up to collect mainly data at lower $Q^2$ than T1. For the SMC that is not useful; therefore the hodoscopes were moved more out of the beam line. The kinematic domain covered by the two triggers of the experiment, after kinematic cuts, is shown in Fig. 5.2. The experiment also has a “small-$x$” trigger, trigger 14, which is not used in the present analysis, since its kinematic domain is essentially at $Q^2 < 1.0$ GeV$^2$.

Apart from by the physics triggers, the data acquisition can be triggered by T5, T7 and T8, triggers used for the alignment and calibration of the spectrometer. Trigger 5 is a beam trigger using H5. Trigger 7 is used for the near halo and trigger 8 for far halo. Dedicated runs with the FSM switched off were taken with these triggers, resulting in a sample with straight tracks over a large area in $y$ and $z$ for alignment purposes. Triggers 3 and 4 are for beam normalisation, which was not used in this analysis. Finally trigger 10 is a random trigger used to obtain a sample of beam tracks as input for Monte Carlo simulations.
3.4 Data processing

The raw data written to magnetic tape consist of element number information for all detector hits, and timing information for hodoscopes and drift chambers. Before the physics data are reconstructed, alignment data are used to determine the position of the detectors in an overall coordinate system, and to perform time calibrations.

3.4.1 Survey, alignment and calibration

Hit information from the detectors is only of use if the position of the elements is known in an overall coordinate system. The first step in achieving this is the positioning of the detectors with help of optical measurements by CERN surveyors. The aim of this surveying is to measure the x-position of the chambers, to position the chamber in y and z on the theoretical beam line, and to make sure that the detectors are not tilted in any direction. Measured are the positions of reference points on the frame of each chamber. The positions of the planes and wires are deduced from these measurements, using construction drawings of the chambers. The precision of the surveyor measurements is a few mm in x, and a fraction of a mm in y and z.

To achieve a higher precision in the alignment, special alignment data were taken with the FSM switched off, that is with straight tracks. The intensity of the beam is lowered for these alignment runs, in order to reduce background from other tracks than the ones involved in the triggers. The software alignment of the chambers is done in a few steps, where each step requires iteration:

- Time calibrations. The trigger hodoscopes H3V or H4' define the time of an event. The software induced TDC offsets of the different strips are adjusted such that the software time of all strips with respect to the trigger time is zero. For H3V, which has long strips with TDC's on each end, the calibration is done by centering the individual TDC's at zero for tracks at the centre of the strip, i.e. at z=0. Furthermore the beam hodoscopes and BMS hodoscopes are internally calibrated, again by applying appropriate TDC offsets. Then the relative timing between BMS, BHOD and H3V(H4') is adjusted, in order to be able to link the beam and scattered muon tracks in the time domain during the vertex reconstruction process.

- The chambers ST67 and P67 are aligned with respect to each other by reconstructing close halo T7 and far halo T8 tracks with the available plane position information. For each plane the average deviation from the track reconstructed by ST67/P67 is calculated and used as a correction to its previous position. All position information is collected in the so-called alignment file, which also contains information on the angles with respect to the coordinate system, wire spacing and x-coordinates of the planes.

- The next step is to align the large chambers used for the reconstruction of the scattered muons and hadrons with respect to each other, and to align the muon recognition system behind the absorber with respect to it as well. This is done with T7 tracks, and therefore called trigger 7 alignment. The positions of all planes of W45, P45, W12, H1, H1', P123 and PV12 are corrected by the average distance between the muon tracks reconstructed in W45/W12, and corresponding hits in the planes. The muon tracks are found by extrapo-
lation of ST67/P67 tracks. The trigger hodoscopes H3, H4, H3' and H4' are aligned with respect to ST67/P67, after which the misalignment of the system behind the absorber with respect to the detectors in front of it is corrected.

- After the first few iterations of the trigger 7 alignment, the drift chambers W12 and W45 are calibrated. The dependence of position versus drift time, and the offset of the DTR's are determined. To first approximation the drift velocity is constant, and the distance versus drift time thus linear. The non-uniformity of the drift field and thus velocity is accounted for by fitting a higher order polynomial to the observed relation between distance and drift time. The position information stems from tracks reconstructed by W12 and W45 themselves, which makes iteration necessary. After the drift time calibration the T7 alignment is iterated a few times.

- The small P0 chambers and the beam hodoscopes are aligned with respect to each other, after which they are aligned with respect to the large chambers by including PV1 and P45 in the P0/BHOD track. For this purpose the center regions of these two large chambers are made alive while taking the T5 data used for this alignment. The precision of the alignment procedure is of the order of 0.1-0.2 mm.

- All detectors are aligned with respect to each other after this procedure, but might be completely wrongly aligned with respect to the FSM. This is of importance since the field map of the FSM is used for the track reconstruction. The absolute position of two detectors, as measured by the surveyors, is used to rotate and/or shift the system of detectors in such a way that its position in the alignment file corresponds to the actual position in the hall.

### 3.4.2 Event reconstruction

A chain of programs is used to reject fake triggers and to determine the position and kinematics of the muon-nucleon interaction vertices. In the first step of the analysis chain, taken care of by the program "Phenix", the raw data are translated into position information, making use of the alignment file. The coordinates are then used to reconstruct the beam track and the trajectory of the scattered muon and produced hadrons.

The muon track is also reconstructed in the BMS, where timing information is used to associate hits from different planes a track. The beam hodoscopes are used to define the straight beam track upstream of the target, where again timing information is used to distinguish between different beam muons where necessary. The P0B hits are then used to increase the precision of the track(s). Only if a BHOD and BMS track are found within a certain time window, the event has an identified beam track, and reconstruction of the scattered muon is attempted.

The reconstruction of the scattered muon starts behind the absorber, by tagging hits in ST67 and P67. The track found is traced backward through the spectrometer to the target, as follows. The hits in the trigger hodoscopes H3H and H4H are used for tagging in the (x,z)-plane, while the hits in H3V in combination with the knowledge that the muon should come from the target area is used for the definition of the (x,y)-plane to look for ST67/P67 hits. The tracks formed by these hits are extrapolated through the absorber to the W45/P45 region, where they are used to tag hits to construct W45/P45 tracks. These tracks are extrapolated to W12/P0E, and the hits found in these detectors are included in the track. The ST67/P67 information is not included in the scattered muon tracks, the system behind the absorber is
3.4. Data processing

just used to find tracks in W45/P45 that continue behind the absorber and thus are muon tracks. For the reconstruction in the FSM the track in its field is first approximated by a circular trajectory. For the incoming tangent target pointing is used, the W45/W12 track is the outgoing tangent. Hits possibly belonging to the track are first looked for in P0D, after which hits in the other magnet chambers are included in the track step by step. A spline fit, starting with the W45/W12 track and using the found hits in the magnet, is used to extrapolate to PV12/P0C. Only now a choice is made between different magnet tracks in case of an ambiguity. For the reconstruction procedure several parameters have to be carefully tuned. The so-called road widths determine how far away from an extrapolation, hits should be included in the track. Plane requirements give the minimum number of planes within a detector or group of detectors that should have a hit in order to speak of a track. For the magnet reconstruction the inhomogeneity of the field causes the tracks to slightly deviate from the circular trajectory. Using the field map of the FSM this deviation is parametrised for each plane as a function of the \((y,z)\) position behind the magnet and the momentum, using results from a Monte Carlo simulation.

<table>
<thead>
<tr>
<th>reconstruction stage</th>
<th>efficiency (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>100 GeV T1</td>
</tr>
<tr>
<td>beam track</td>
<td>97</td>
</tr>
<tr>
<td>ST67/P67 track</td>
<td>90</td>
</tr>
<tr>
<td>full muon line</td>
<td>81</td>
</tr>
<tr>
<td>total phenix</td>
<td>71</td>
</tr>
<tr>
<td>beam cuts</td>
<td>97</td>
</tr>
<tr>
<td>same sign muon</td>
<td>96</td>
</tr>
<tr>
<td>trigger test</td>
<td>81</td>
</tr>
<tr>
<td>vertex</td>
<td>84</td>
</tr>
<tr>
<td>total geometry</td>
<td>64</td>
</tr>
</tbody>
</table>

Table 3.4: Efficiencies of the various reconstruction steps from the raw data to the micro-DST, for the 100 GeV deuteron and 190 GeV proton data set.

The output of "Phenix" is fed to the program "Geometry", where the preliminary tracks found by "Phenix" are combined into vertices, after some checks, fittings and calculations are done. Events which have two beam tracks too close in time or space are thrown away, since the kinematics of the scattering can not be determined unambiguously. The momentum of the incoming muon is determined from the BMS track. The scattered muon trajectory is refitted; per detector group a point \((x,y,z)\) or line \((x,y,z,\partial y/\partial x,\partial z/\partial x)\) is constructed, which is used as input for a fifth order spline fit. Tracks are forced to be straight in field-free areas. A trigger check is done by requiring that the combination of trigger hodoscope elements hit by the resulting track satisfies the trigger. Then the interaction vertex is reconstructed as the point of closest approach of the two tracks. The kinematic variables of the scattering are calculated, taking into account the energy loss of the muon in the target material. The multiple scattering is included in the error calculation. After a muon vertex is found, hadron tracks if present are used to improve the fit of the interaction point. In this analysis the hadron information itself is not used. Only vertices with incoming and scattered muons of the same charge are used, but other interaction vertices occur as well, e.g. the \(J/\Psi\) and \(K^0\).
decay vertices. The track and vertex reconstruction is well described in [43].

After "Geometry" the data are processed by "Snomux" and "Micro", which mainly perform data reduction and process some information not yet used. Only events that satisfy the trigger and have a valid vertex are kept, and raw data are discarded at this moment. Some rough kinematic cuts are applied, as well as a beam phase space cut, making sure that the incoming muon would have passed both target halves. The target polarisation records are separated from the spectrometer information here and stored in a separate file. Information of the calorimeter H2, the beam triggers T3, T4 and T10, as well as scaler information from the end-of-run and start-of-run records are processed, but are not used in the present analysis. The remaining information is compressed in Zebra-banks format, and stored in so-called "Micro DST (Data Summary Tape)"-files, which are used for the subsequent physics analysis of the data, as discussed in Chapter 4 and 5. Table 3.4 shows typical reconstruction efficiencies of the various stages of the event reconstruction. The last row shows the number of events acquired on Micro DST in 1992 and 1993 with the above described apparatus.
Chapter 4

The Polarised Target

In the SMC experiment a fixed target with spin polarised nuclei (protons or deuterons) is used. The polarisation is achieved by a microwave pumping technique in a suitable solid state material. This method of dynamic nuclear polarisation is explained in terms of simplified models, after which the hardware to achieve this nuclear polarisation is described in some detail. The way the obtained polarisation is measured is outlined in the last section.

4.1 Method of nuclear polarisation

Materials suitable for polarising nuclei by microwave pumping must contain a small amount of unpaired electrons, which magnetic properties resemble those of free electrons. Placed in a magnetic field of several tesla with the material at a temperature of about 1 K, these electrons spins are highly polarised by "brute force". With the microwave pumping technique the electron spin polarisation can be transferred to the nuclei, protons or deuterons in the case of SMC.

The target material used by the SMC is hydrogenic or deuterated butanol, because of the reasonable fraction of protons or deuterons, the required solid state properties and the (practically total) absence of other polarisable nuclei. Since a non-magnetic material like butanol has its electron spins coupled to zero, some paramagnetic centres have to be introduced. One way to do this is to add a compound with (partially) unpaired electrons. The compound chosen by the SMC is a Cr(V)-complex [44], typically in a concentration of $5 \cdot 10^{19}$ spins/cm$^3$. The target material has to be amorphous, since crystallisation would destroy the homogeneous distribution of the paramagnetic molecules. This is achieved by rapid cooling of droplets of liquid butanol (with the complex dissolved in it) in liquid nitrogen, resulting in "glassy", amorphous beads of target material[45] with a size of about 1 mm.

The mechanism of dynamic nuclear polarisation (DNP) by microwave pumping, can be described in a simplified way by considering isolated electron-proton pairs, in combination with bulk proton-spin diffusion. This mechanism, which is known as the solid effect[46], is introduced below. The properties of the SMC target material result in the fact that the actual
polarisation mechanism is more complicated, and can be described with thermal reservoirs, as is done in Section 4.1.2. The description of the polarisation mechanisms focuses on the proton material. The deuteron material is polarised essentially the same way, but the situation is slightly more complicated due to the spin 1 of the deuteron.

### 4.1.1 Solid effect

Consider a system of a free proton and electron spin in a magnetic field. The energy splitting of an electron-proton pair together with the first and second order transitions are as sketched in Fig. 4.1. The sublevel splitting of the proton is much smaller than the electron splitting.

![Diagram](image)

**Figure 4.1:** a) The level splittings due to a magnetic field of an electron-spin/proton-spin pair with the first and second order transitions. b) With these energy levels where the first and second order transitions are well separated, the solid effect is the dominant mechanism for polarisation.

In order to obtain proton polarisation the target material is irradiated with microwaves of the frequency of one of the two second order transitions, depending on which sign of polarisation is wanted. In these transitions an electron spin and a proton spin are simultaneously reversed ("flip-flop"). Because the spin-lattice relaxation time of the electrons is much shorter than that of the protons, the electron falls back to the state of its preferred spin direction (first-order transition), whereas the proton spin does not return to its original state. The electron spin is available again to interact with a proton, i.e. to flip another proton spin. The concentration of electron spins (paramagnetic centres) is small. After some time all proton spins in the direct vicinity of the electron spin will be polarised, and the process would stop, with only a small fraction of the protons polarised. However, due to proton spin-spin diffusion the polarisation will finally be distributed throughout the target material. In this state the total proton dipole-dipole energy is minimal. The spin diffusion process in the solid state consists of many two proton spin reversals ("flip-flops").

The proton polarisation is defined as \( P = \langle I_z \rangle / I = (n_+ - n_-)(n_+ + n_-) \), where \( n_+(-) \) are the number of proton spins parallel or anti-parallel to the magnetic field. The proton polarisation can never become higher than the electron polarisation. For this reason a strong magnetic
4.1. Method of nuclear polarisation

field and a relatively low temperature are essential. The proton polarisation can be reversed by changing the microwave frequency from $\omega_e - \omega_n$ to $\omega_e + \omega_n$ and vice versa.

![Diagram](image)

**Figure 4.2:** The thermal reservoirs relevant for dynamic nuclear polarisation

4.1.2 Cooling of the spin-spin interaction reservoir

The solid effect is the dominant nuclear polarisation mechanism if the first order electron spin transition and the second order electron-proton spin transitions are well separated (as shown in Fig. 4.1b), or if the electron spin density is so low that the electrons hardly interact. However, if the electron spin resonance (ESR) spectrum ($\omega_e$ in Fig. 4.1b) is broad, first-order electron transitions of the same frequency occur on selecting one of the second-order electron-proton transitions. In case of a sufficiently high electron spin density a mechanism using these first order transitions becomes dominant.

In the target material used by the SMC the ESR-line is broadened by the dipole-dipole interaction between the paramagnetic centers, the induced internal magnetic field, the hyperfine interactions and the anisotropic spin-orbit interaction. The total resonance width is about 350 MHz, to be compared to the deuteron and proton sublevel transition frequencies of 16.35 MHz and 106 MHz.

The polarisation mechanism using the first-order transitions can be described with a thermal reservoir model; "dynamic cooling of the electron spin-spin interaction reservoir"[47]. Fig. 4.2 shows the relevant thermal reservoirs. The nuclear Zeeman reservoir with temperature $T^n_z$ is the reservoir to be cooled, since a low temperature in this reservoir means high proton polarisation. Similar to this is the electron Zeeman reservoir at $T^e_z$. The electron spin-spin reservoir in between depends on the energy related to the electron dipole-dipole interactions and has a temperature $T^{es}_s$. There is finally the lattice reservoir of phonons at the temperature $T_L$.

Fig. 4.3a shows the initial occupation of the energy levels of the electron spin system (Zeeman and dipole-dipole together) in thermal equilibrium. The proton spins are not taken into account. All the temperatures in the system are the same and equal to the lattice temperature.
Figure 4.3: The electron spin system in thermal equilibrium a), and with microwaves saturating an electron Zeeman transition b) and c). The grey bands indicate the population of the broadened energy levels. Saturation makes the population of the two levels of the transition chosen by $h\nu$ equal, as shown in b) and c). The electron Zeeman temperature of the system is roughly the same in a), b) and c), since the total occupation of the high and low Zeeman level are the same for the three cases. However, the system has a very low positive electron spin temperature in state b), a very small negative electron spin temperature in state c).

During DNP either the transition with roughly the lowest excitation frequency, Fig. 4.3b, or the one with roughly the highest excitation frequency, Fig. 4.3c, is saturated by microwave irradiation. Saturation means in this case that the two energy levels involved in the transition become equally populated. The electron spin-spin system relaxes to its own temperature now; the electron spin temperature $T_{ss}^e$. The electron spin system in Fig. 4.3b has to be described by two different temperatures; the electron Zeeman temperature $T_{Z}^e$, which is hardly changed by the reordering of the electron spins, and the electron spin temperature, $T_{ss}^e$, which is orders of magnitude lower than the lattice temperature. In Fig. 4.3c again the electron Zeeman temperature is hardly different from the lattice temperature, but within the spin-spin system the higher energy levels are more densely populated than the lower levels, which corresponds to a negative electron spin temperature. The absolute value of this temperature is again orders of magnitude lower than the lattice temperature.

Although the electron spin-spin reservoir is cooled by this mechanism, this does not yet result in proton polarisation. Since these two reservoirs are in good thermal contact however, the nuclear Zeeman reservoir will reach roughly the same temperature as the electron spin-spin reservoir. This thermal contact is established by a threefold spin exchange, where two electron spins and a proton spin flip simultaneously. Since an electron spin flip changes its dipole-dipole energy with respect to its neighbours, a twofold electron spin exchange has an energy effect, which in this spin exchange process is equal to the energy to flip the proton spin. Another mechanism to establish thermal contact between the reservoirs goes via the already mentioned microwave stimulated second-order transition, where a proton spin and an electron spin flip at the same time. In this transition energy is transferred from the electron spin-spin reservoir to the nuclear Zeeman reservoir, while the electron Zeeman reservoir is influenced as well. But since the electron flip can either be up → down or down → up, the net effect of many
transitions on the electron Zeeman temperature is zero. So, having microwave power of the right frequency not only cools the electron spin-spin reservoir, but also improves the thermal contact between this reservoir and the nuclear Zeeman reservoir. Typical temperature values are 0.5 K for the lattice, and 2 mK for the spin temperature.

The nuclear polarisation is given by the Brillouin function $B_I(x)$ for spin $I$:

$$P = B_I\left(\frac{\mu B}{kT_n}\right),$$  \hspace{1cm} (4.1)

where $\mu$ is the nuclear magnetic moment, $B$ is the magnetic field, and $k$ is the Boltzmann factor.

4.1.3 Frozen-spin mode

DNP by microwave pumping is realised at temperatures of about 0.5-1.0 K. Switching off the microwaves once the target is polarised, and leaving it at these temperatures leads to a fast decay of the proton polarisation. An interesting use of the behaviour of the nuclear-spin relaxation time, which depends on the lattice temperature and the magnetic field, can be made. At temperatures in the tens of mK range and magnetic field of a few tesla this relaxation time becomes of the order of thousand hours. This enables us to operate the target in the so-called frozen-spin mode, by rapidly cooling the target to $\leq 50$ mK after having switched off the microwaves. This can be done using a "dilution refrigerator" of special design, as described in the next section. The operation in the frozen-spin mode is stable and economic, and the microwave high-voltage tubes, which have a limited lifetime and are quite expensive, are not worn down.

4.2 Dilution refrigerator

In this and the following two sections the different sub-systems of the polarised target are described. For the 1991 and 1992 runs on deuterium the polarised target system of the EMC experiment [48] was used. For this purpose several improvements were made to the target. Since the 1993 run a new polarised target of similar design but larger and with still better performance characteristics. The description of the target system in this thesis is based upon the EMC target plus some changes and improvements on which I worked. Where relevant, features of the new polarised target system are discussed.

The heart of the target is the dilution refrigerator. The refrigerator has a low base temperature (50 mK), enabling the use of the frozen-spin mode during a considerable fraction of the experiment. The design cooling power at 0.5 K is 2 Watt, which is very large for a dilution refrigerator, but necessary to polarise this large target. During the polarisation procedure a high microwave power is needed at a low lattice temperature, to saturate the electron transition and to have sufficiently long relaxation times and a high electron polarisation.
4.2.1 The principle of a $^3$He/$^4$He dilution refrigerator

If one pumps on a normal cryogenic liquid like $^3$He or $^4$He, the cooling power is $\dot{Q} = L \dot{n}$, where $L$ is the heat of evaporation per mole and $\dot{n}$ the number of moles removed per second. The lowest temperature obtained in this way depends, apart from the heat input, on the lowest pressure reached above the liquid, and the relation between the vapour pressure and the temperature. At low temperatures the vapour pressure is a steep function of temperature, such that the cooling power decreases rapidly while cooling down. Equilibrium of this cooling power with the heat load to the pumped liquid determines the lowest temperature. With pumped $^4$He or $^3$He about 1.0 K and 0.3 K can be reached, but then the cooling power is very limited.

The principle of a dilution refrigerator is shown in Fig. 4.4. A mixture of $^3$He and $^4$He is used. If this mixture is cooled below 0.7 K a separation occurs between a liquid phase of concentrated $^3$He, and a phase of low $^3$He concentration dissolved in superfluid $^4$He. The concentrated $^3$He phase floats on top of the dilute $^3$He phase. This separation occurs in the mixing chamber. The dilute phase is being pumped on at the “still”. The evacuated gas contains mostly $^3$He due to the higher vapour pressure compared to $^4$He. By removing $^3$He from the still a $^3$He gradient occurs between the mixing chamber and the still, and $^3$He “evaporates” from the concentrated phase into the dilute phase. The heat needed for this transition is extracted from the environment, which is thus cooled. The concentration of $^3$He in the dilute phase can be considered as the “vapour pressure” of the concentrated $^3$He phase. At absolute zero there is still a finite amount of $^3$He in the dilute phase (6%). Consequently cooling continues down to very low temperatures. A heater is installed at the still which is kept at a higher temperature to increase the vapour pressure. A higher vapour pressure means a higher $^3$He flow and hence a higher cooling power. On the other hand the higher temperature of the still means a higher heat load to the mixing chamber, and a higher $^4$He content of the gas being pumped. The optimal temperature of the still, depending on the cooling power needed, is about 0.7 K. The helium gas pumped from the still is circulated and recondensed in the condenser of the refrigerator at about 1-2 K. It is brought back to the mixing chamber through heat exchangers, which use the $^3$He in the dilute phase in counter flow to cool the incoming concentrated $^3$He phase. The design of the heat exchangers is very critical, since the incoming helium represents a heat load to the mixing chamber. They must have large contact areas especially at the lowest temperatures since the heat transport between the two helium phases via the walls of the heat exchangers is poor and decreases very rapidly with decreasing temperature.

A dilution refrigerator also has a separate $^4$He circuit. A pumped $^4$He bath is used to cool the condenser for the $^3$He. In addition liquid and evaporated $^4$He is used to cool radiation shields inside the cryogenic system. A more rigorous treatment of the principles of cryogenics can be found in the book by Lounasmaa[50].

4.2.2 The EMC dilution refrigerator

A more realistic drawing of the dilution refrigerator [49] used for the 1991 and 1992 runs of this experiment is shown in Fig. 4.5. The target material is positioned in the horizontal mixing
4.2. Dilution refrigerator

Figure 4.4: A principle sketch of a dilution refrigerator. Mainly $^3$He is pumped from the still, and the equilibrium between the two phases of the liquid helium causes $^3$He to migrate from the concentrated phase to the dilute phase. During this process cooling occurs at the phase boundary. The $^3$He is circulated, and brought back to the mixing chamber. The incoming helium is cooled by the helium extracted from the mixing chamber.

chamber, in two 40 cm long target cells of 5 cm diameter, with a 20 cm separation. The two cells are polarised in opposite sense. An important feature is that the pre-cooled target holder has to be inserted into the refrigerator when it is already cold, since the temperature of the target material should not exceed 100 K. At temperatures slightly higher than 100 K devitrification occurs, which is fatal for the polarisability of the material. The target material, held in perforated Mylar cartridges, is supported by a light glass epoxy structure, which is connected to the end of a vacuum chamber. Since this vacuum chamber, once inserted into the refrigerator, has one side at room temperature and one side at mixing chamber temperature, there is in principle a heat path directly to the mixing chamber. The minimisation of the heat load to the mixing chamber is a critical design aspect of the refrigerator.

The refrigerator has an additional $^4$He circuit for cooling screens and precooing the incoming $^3$He. Helium, transferred from a dewar to the refrigerator, is separated into a liquid phase and a gaseous phase in a separator. This cold gas is used to cool the radiation shields, the liquid helium is transferred to the evaporator, where it is being pumped and thus cooled to about 2 K. The cold gas coming from the evaporator is used in a heat exchanger to cool the incoming $^3$He gas, which thereafter is condensed at the evaporator itself. The mixing chamber is surrounded by a copper microwave cavity, which serves as a radiation shield as well, cooled by helium. Operation of the refrigerator involves the regulation of the $^3$He flow with mainly the still heater, and the operation of several needle valves to control the $^4$He flows and the level in the evaporator.
Figure 4.5: The polarised target used for the 1991 and 1992 runs of the SMC experiment.

4.2.3 Improvements to the EMC-refrigerator

Thermal anchoring of the NMR cables

In order to measure the polarisation of the target material at various positions by means of NMR, ten coaxial lines were led from the mixing chamber through the vacuum chamber of the target holder to room temperature. The low-loss coaxial lines are made of materials with good electrical and thus good thermal conductivity. It is clear that the thermal anchoring of these lines is important for reaching a low base temperature in the mixing chamber. The inner conductor, which is separated from the outer tube of the cable by teflon poses a special problem.

Figure 4.6: Detailed view of one the thermal anchors of the semi-rigid coaxial lines.
The cryogenic performance of the refrigerator allows one to determine the heat load to the mixing chamber. During the EMC runs the load was determined to be about 10 mW at a base temperature of 80 mK [49]. These values were surprisingly large, and remained unexplained until the thermal conduction characteristics of the coaxial line were determined, enabling a study of the problem.

The lines traversing the evacuated target holder chamber were thermally anchored at five positions. A detailed sketch of a heat sinking point is shown in Fig. 4.6. The thermalising rings were made of a titanium alloy having a thermal dilatation lower than that of the stainless steel wall of the vacuum chamber of the target holder.

In order to find an explanation for the large heat input in the mixing chamber various models were set up and examined. All the models are schematically described by Fig. 4.7, which displays a network of heat resistances, showing also the temperatures of the anchoring points. In equilibrium the different heat flows are related to each other by a set of equations:

$$\dot{Q}_{Li} = \dot{Q}_{Li+1} + \dot{Q}_{Ti},$$

where $i$ goes from 1 to 5.

The longitudinal and transverse heat flows depend on the five unknown temperatures $T_i$ to $T_5$. Once these are determined, also the heat load to the mixing chamber, $\dot{Q}_{L6}$, is known.

![Figure 4.7: Simplified model of the heat flow through one coaxial line to the 5 thermal anchors and to the mixing chamber.](image)

The transverse heat flow from the thermal anchors on the cable to the helium coolant is limited by a number of thermal resistances in series. The various models differ from each other by the conductances included in the expression for the transverse heat flow. For the calculation the models were simplified by only including the dominant transverse thermal resistance.

The inner and outer conductor of the cryogenic cable UT-T-85-B-B are made of beryllium copper, and the insulation material is teflon. One hypothesis was that, because of the difference in thermal expansion coefficients, the teflon could become partly detached from the outer conductor at low temperatures. This could impair the transverse heat conduction, and thus cause a large heat input through the inner conductor to the mixing chamber. For this reason both the transverse and the longitudinal thermal conduction properties of the cable had been measured at low temperatures[51]. To study this hypothesis it was assumed that the
intrinsic transverse conduction in the coaxial line itself is the only limiting factor. The outer conductor was assumed to be perfectly thermally anchored. Furthermore the assumption was made that the inner and outer conductor are only in contact at the anchoring points. The result of the model calculation is that, using the measured cable properties, the temperatures $T_1$ to $T_5$ are not significantly higher than those of the anchoring points. This means that the transverse heat conduction of the cable is sufficient, and for the remaining models the cable was approximated as a conductor with the measured longitudinal resistance only.

Next the following situations were modeled: 1) Thermal contact was assumed between the ring and the steel wall through a layer of Apiezon grease around the entire circumference of the titanium ring. 2) The grease layer is effectively absent and only spot contact exists between the titanium ring and the steel at positions on the ring at some distance from the coaxial lines. 3) The contact spots of situation 2) are the thermal bottle neck. This last situation was modeled by making the spot size very small. The heat will first spread into the steel, a poor heat conductor, and then pass through the helium gas. If the thermal contact spot is of the same order or larger then the thickness of the steel, the problem is two dimensional. A spot diameter of 0.25 mm, half of the thickness of the steel, was assumed.

From the model calculations heat inputs into the mixing chamber of the order of milliwatts were only found in the case of spot contacts between the titanium and the stainless steel wall far away from the cables. In this case the poor heat conductivity of titanium becomes apparent. The spot contacts themselves were found not to be critical. As a result of this study, which is described in more detail in [52], the thermallyising rings inside the target holder were exchanged by copper rings. The thermal contact between the ring and the wall was improved by providing the rings with an expansion mechanism. After these improvements a mixing chamber base temperature of 50 mK or lower could be obtained. The heat load to the mixing chamber has not been determined due to lack of time.

**Cavity cooling**

The microwave cavity was originally cooled to about 4 K by a flow of cold helium gas from the separator through a cooling tube soldered to the cavity. For the SMC target a superconducting dipole coil was wound onto the cavity to be able to do a magnetic rotation of the polarisation (see next section). In order to make this possible the cooling of the cavity had to be improved. Therefore liquid helium is now extracted from the bottom of the separator and pumped through a needle valve acting as a Joule-Thomson expansion valve. In this way a cavity temperature of about 2 K could be obtained.

**$^3$He thermometer**

For the thermal equilibrium measurement of the polarisation, treated in detail in Section 4.5, a precise determination of the mixing chamber temperature at about 1K is essential. For this purpose a $^3$He vapour pressure thermometer was installed. $^3$He vapour pressure thermometry is used in the new International Temperature Scale (ITS90)[53]; it has the advantage of a large sensitivity due to the steep temperature dependence in the temperature range covered. A 3 cm$^3$ bulb in the mixing chamber was connected by a capillary pipe through the target holder vacuum chamber to a baratron pressure gauge outside the refrigerator. The absolute mixing chamber temperature could be determined with a precision of 10 mK.
4.2.4 The new SMC dilution refrigerator

The new dilution refrigerator[55] built for the runs in 1993 and later, is larger but of a design very similar to that of the EMC target. The target cells, which are 50% longer than those of the EMC target, increase the luminosity with the same factor. The larger separation between the cells, also made 50% longer, increases the possibility to separate the two target cells in the vertex reconstruction. Apart from the size, a major difference between the old and new refrigerator is the ease and reliability with which the new one can be operated. The most critical needle valve, the one regulating the flow of liquid helium to the evaporator, is automated with the evaporator level as input parameter. The target was operated smoothly in 1993 and no blockages of the needle valves did occur.

4.3 The magnet

Another major part of the target is the superconducting solenoid magnet, producing a 2.5 T field along the beam axis. The magnet consists of a main coil and a number of trim coils. In order to obtain a homogeneous field over the whole volume of the target cells the trim coil currents have to be carefully tuned. This is discussed below.

4.3.1 Trim coil adjustment

The magnet of the EMC target has twelve trim coils. The trim coil currents have been determined by calculating the field along the axis produced by each coil, summing them and minimising the variation of the field using the trim coil currents as parameters. The inner and outer radii of the coils, the numbers of windings and the lengths are needed as input to be able to calculate the field produced by the coils.

The total field of the trim coils as a function of the position can be expressed as a vector with \( n \) elements, for \( n \) positions along the axis. If one also expresses the trim coil currents as a 12-element vector, the following set of linear equations can be set up:

\[
\begin{pmatrix}
B_1 \\
\vdots \\
B_n
\end{pmatrix} =
\begin{pmatrix}
A_{1,1} & \cdots & A_{1,12} \\
\vdots & \ddots & \vdots \\
\vdots & \ddots & \vdots \\
\vdots & \ddots & \vdots \\
A_{n,1} & \cdots & A_{n,12}
\end{pmatrix}
\begin{pmatrix}
I_1 \\
\vdots \\
I_{12}
\end{pmatrix}
\]

(4.3)

The matrix \( A \) contains the proportionality constants of current versus field at a given position for the trim coils. The field \( B_n \) in these equations is the field measured at the position minus the field to be obtained, in other words the field to be generated by the trim coils. With a field measurement at more than 12 positions the set of equations of (4.3) is over-determined. A computer program was written to solve the system of equations, using the trim coil currents
Figure 4.8: Fieldmap of the EMC magnet after tuning the trim coils.

and the field to be obtained as free parameters. The field along the axis was measured using an NMR probe. A few iterations of calculating trim coil currents and measuring the field were done, giving corrections to the trim coil currents, since the measured field already included trim coil fields.

The final field maps on the axis and 2.5 cm off-axis are shown in Fig. 4.8. The limiting factor for the homogeneity is a winding error in coil 5. The homogeneity obtained is $0.75 \cdot 10^{-4}$ over the whole target cell volume. Despite coil 5 this is not far from the theoretically best obtainable homogeneity with the chosen design of the trim coil configuration. The values of all the trim coil currents were lower during the SMC runs compared to the earlier EMC experiment, and the volume of sufficiently homogeneous magnetic field extended a few more centimeter along the axis than previously.

For comparison, the new magnet of the SMC target[54] has a main coil and sixteen trim coils. The obtained homogeneity is $2 \cdot 10^{-5}$ over a length of 1.5 meter and a radius of 2.5 cm.

4.3.2 Magnetic rotation of polarisation

During the EMC experiment the polarisation was reversed about once a week by DNP, by interchanging the microwave frequencies for the two target cells. The most important systematic error on the asymmetry measurements was due to the uncertainty of the assumed constant acceptance ratio for the two cells. The higher the rate of polarisation reversals, the smaller this uncertainty. The change of polarisation by interchanging the microwave frequen-
cies has the disadvantage that it takes a long time to reach a high polarisation, since each time one goes through zero polarisation. For example in the case of the deuteron target material it takes 3(6) hours to reach a polarisation of 60(90) % of the maximum value.

A much faster way to reverse the orientation of the nuclear spins with respect to the muon beam direction is to rotate the magnetic field over 180°. This requires a magnetic field perpendicular to the solenoid field in order to avoid a transition through zero field. For the SMC experiment a dipole coil was wound on the cavity for this purpose. The nuclear polarisation is defined with respect to the magnetic field. Changing the orientation of the field does not change the sign of the polarisation, but it does change the orientation of the nucleon spins with respect to the muon spins.

The dipole wound on the microwave cavity has a maximum field of 0.2 T transverse to the main field. It can only carry current with the main field at 0.5 T, because otherwise the forces on the dipole coil would become to large. During field rotation the dilution refrigerator is regulated to its base temperature. The main field is lowered to 0.5 T, after which the dipole magnetic field is slowly increased to 0.2 T. Once the dipole field has reached 0.2 T, the main field is ramped through zero field to -0.5 T, after which the dipole field is turned off slowly and the main field is increased in strength to -2.5 T.

In this procedure the nuclear spin lattice relaxation time is an important property of the target material. Since the energy splitting is proportional to the magnetic field, this relaxation time is a steep function of the field in the low field region. It also depends strongly on temperature, therefore a low base temperature is very important during magnetic rotation of the polarisation. Relaxation time measurements at several fields and temperatures were made during dedicated "technical runs". Fig. 4.9 shows some of the results. Because of the relatively short relaxation time at 0.2 T and 50 mK the ramping speed of the main field should be as high as possible without risking a quench or mechanical damage to the dipole coil.

A field rotation performed this way takes 30 minutes and has an efficiency of 99%. During the data taking in 1992 the polarisation was rotated every eight hours, whereas once a week the spin orientation was reversed with respect to the magnetic field by DNP.

The new longer magnet for the SMC target used for the 1993 proton run has been equipped with a dipole coil with which a maximum transverse field of 0.5 T can be obtained. The magnet rotation procedure became computer controlled and turned out to be so reliable that field rotations were carried out up to five times a day. In order to reduce time losses due to the field rotations data taking is continued while ramping down the main field, until the transverse field is switched on. During this procedure the magnet current is written to tape continuously, thus enabling the track reconstruction to take the different target field into account. In this way the down time during a field rotation is reduced from about 40 minutes to 10 minutes.

The SMC target also enables the first measurement of $g_2(x)$, the transverse spin structure function. The necessary transverse polarisation is obtained by polarising in the normal way, and then using the 0.5 T transverse dipole field as holding field. Because of the rapid polarisation decay in a 0.2 T field this was not possible with the EMC target.
Figure 4.9: Polarisation decay rate of the deuteron material at various temperatures and magnetic field strengths.

4.4 The Microwave System

4.4.1 General description

The microwave set-up consists of an independent system for each target cell, since the two cells have to be polarised in opposite sense. The microwave sources are two Extended Interaction Oscillators (EIO) tubes. The tubes have a frequency of about 70 GHz, an emission width of 0.1 MHz and 20 W of power. The frequency can be varied by adjusting the cavity size in the tube mechanically, or by varying the cathode voltage. The latter method is used to fine tune the frequency, the first one to change the frequency for the DNP polarisation reversal.

In order to be able to adjust the power arriving in the mixing chamber without changing the frequency, two high power attenuators were designed at Delft University of Technology, based on destructive interference rather than absorption [56]. The power is first split in a Hybrid Tee into two arms, one of which is variable in length, and thereafter recombined in another Tee. The outgoing power now depends on the relative phase of the waves in the two arms, which can be regulated by changing the length of one of the arms with a step motor. The power arriving in the mixing chamber can be varied from zero to about one watt in each cell.

The multi-mode microwave cavity is divided into two separate sections, since the microwaves that polarise one cell in positive sense are destructive for the negative polarisation in the other cell. To prevent this, a microwave insulator using various absorptive and reflective screens
is positioned in the 20 cm separation between the two target cells. A difficulty is that the insulator should not obstruct the flow of helium in the mixing chamber. In between the mixing chamber wall and the microwave cavity wall the leakage of microwaves is prevented as well.

### 4.4.2 Microwave modulation

In the course of the experiment it turned out that frequency modulation of the microwaves is essential to reach high deuteron polarisation [57]. In several microwave DNP experiments modulation is used to achieve a moderate increase in polarisation. In the case of our large double target microwave modulation increased the deuteron polarisation by about a factor of two, which is a quite remarkable feature. It was found that a 1 kHz modulation over a 30 MHz range at the resonance frequency was the most efficient. For the proton target material microwave modulation also turned out to be effective, primarily because the speed of polarisation increased by a factor of 1.5-2.

### 4.5 Polarisation measurement

One of the important parameters in the analysis of the experiment is the target polarisation. This is measured by means of nuclear magnetic resonance (NMR). The polarisation of the material is monitored by ten NMR coils, five in each target cell. The deuterons coils are made out of thin-walled cupro-nickel tubing of 2 mm diameter, to have as little material as possible in the beam, and a 3.5 mm teflon sleeve around these tubes to avoid saturation of the material next to the coil. The proton coils are 4 mm in diameter. In order to study a possible radial dependence of the polarisation two of the coils are small sampling coils. The polarisation measurement with the method of Continuous Wave NMR (CW-NMR) is used with series Q-meters [58].

#### 4.5.1 Principle

The absorption of radio frequency (RF) power in an NMR coil is a measure for the polarisation of the material around the coil. An RF-field of the resonance frequency stimulates both spin up → spin down and spin down → spin up transitions. A sample without polarisation has no net absorption. A sample with a positive polarisation absorbs power, whereas a negative polarisation gives a (stimulated) net emission of RF-power.

An RF-coil embedded in a material with magnetic susceptibility \( \chi(\omega) \) has an inductance

\[
L(\omega) = L_0(1 + 4\pi \eta \left( \chi'(\omega) + i\chi''(\omega) \right)).
\]

In this expression \( L_0 \) is the inductance without the material and \( \eta \) the filling factor of the material around the coil, whereas \( \chi'(\omega) \) is the dispersive and \( \chi''(\omega) \) the absorptive part of the complex susceptibility.
Usually there is not just one sharp spectral line, but a rather broad NMR spectrum. In this case the polarisation is proportional to the integral of the absorptive part of the spectrum, the so-called integrated spectrum\cite{59}:

$$P \sim \int_0^\infty \chi''(\omega) \cdot d\omega.$$  \hspace{1cm} (4.5)

![Graphs showing signal from Q-meter and Q-curve subtracted](image)

**Figure 4.10:** A typical spectrum produced by the Q-meter, before(left) and after(right) parabola subtraction. The parabola is the response of the electronic circuit. Superimposed on it is the absorption spectrum of the target material. The parabola, the so-called Q-curve or baseline, is measured by shifting the magnetic field by 5%.

The RF-coil is connected to the Q-meter with a $\lambda/2$ transmission line. The specific length makes the line transparent for the signals. With an RF-transmitter a frequency scan (16.35 MHz ± 250 kHz for deuterons, 106.0 MHz ± 200 or 300 kHz for protons) of the NMR spectrum is made. This frequency scan, which takes about 20 seconds, consists of 400 measurements at a discrete frequency. At each frequency the response of the circuit is measured, resulting in a signal of 400 channels. The absorptive part of the signal is filtered out by using a phase-sensitive receiver. A typical proton spectrum produced by the Q-meter is shown on the left in Fig. 4.10. The parabola is the response of the circuit alone. Superimposed on it is the absorption spectrum of the material, here given for negatively polarised protons. After subtraction of the parabola, the so-called baseline or Q-curve, this results in the spectrum on the right hand side of Fig. 4.10.

Due to quadrupole broadening, the NMR spectrum of the deuteron with spin 1 has a different shape compared to the NMR spectrum of the proton with spin $\frac{1}{2}$. Fig. 4.11 shows the nuclear levels of the deuteron in butanol. On the left only the effect of Zeeman splitting is plotted, leading to only one spectral line. In deuterated butanol the O-D and C-D bonds each result in a quadrupole interaction (Q1), given by

$$E_{Q1} = h\nu Q \left( 3\cos^2(\theta) - 1 \right) \cdot \left( 3m^2 - I(I + 1) \right),$$  \hspace{1cm} (4.6)

where $\theta$ is the angle between the axis of the bond and the magnetic field. The strength of the interaction $\nu Q$ depends on the local electric field gradient and the quadrupole moments of the
4.5. Polarisation measurement

\[ \text{Zeeman} + \text{quadrupole interaction} \]

\[
\begin{array}{c}
\text{m = -1} \\
\text{0} \\
\text{1} \\
\end{array}
\]

\[\theta = 0 \quad \theta = \pi/2\]

**Figure 4.11:** Energy levels of the deuteron spin in deuterated butanol. The energies depend on the angle between the C-D and O-D bonds in the butanol with respect to the external magnetic field. The material is isotropic, so the two transitions are broadened and partly overlap, resulting in the spectrum on the right.

nucleus. The right hand side of Fig. 4.11 shows the energy levels when the QI is included, for the two extreme angles \( \theta \) equal to zero and \( \pi \). The hyperfine splitting results in two spectral lines, but since the electric field gradient of the material has no preferred directions, the two peaks are broadened and overlap partly. A typical deuterium NMR spectrum is shown at the right hand side of Fig. 4.11.

### 4.5.2 The thermal equilibrium calibration

**Figure 4.12:** A typical NMR spectrum of the deuteron(l) and proton(r) target material in thermal equilibrium at 1.0 K.

Since the integrated spectrum is proportional to the polarisation, the proportionality constant must be determined by taking an integrated spectrum for the same material in thermal equilibrium at a well-known temperature and magnetic field, hence with a known polarisation.
This is the thermal equilibrium (TE) method. To achieve sufficient temperature stability and homogeneity, and to have sufficiently short relaxation times to arrive at equilibrium within a reasonable time, the calibration is performed at 1 K, having only (superfluid) $^4$He in the dilution refrigerator. This relatively high temperature results in a very low nuclear polarisation, especially for deuterons ($\sim 0.05\%$) because of the small hyperfine splitting. The polarisation as a function of temperature is given by eq. (4.1).

The rather poor signal to noise ratio of the measurement makes signal averaging necessary. Because of the drift in the electronics there is an optimum averaging time. For deuterons 10000 frequency scans were taken, which takes 20 minutes. For protons, where the signal to noise ratio is much less of a problem, 2000 scans were averaged to one measurement. For each measurement a new baseline is taken with the same number of scans.

Thanks to the excellent NMR system with very low noise a relatively good TE signal is obtained even for deuterium, despite the very low signal. Typical TE signals for deuterons and protons are shown in Fig. 4.12. The temperature measurement is an important aspect in the calibration. A $^3$He vapour pressure thermometer was used to determine the temperature during the calibration.

### 4.5.3 TE signal analysis

The Q-meter baselines are obtained by shifting the magnetic field by 5%. After subtraction of this baseline from the signals a small residual baseline remains, due to small drifts in the circuitry. The residual base-line of the TE spectra for deuterons is subtracted by fitting a parabola through the wings of the spectrum, typically the first and last seventy channels out of the four hundred channels for the whole spectrum. Because of the wide spectrum of the proton signal the residual baseline is subtracted by fitting a straight line through the first and last forty points of the spectrum. After this the integral of the signals is taken by summing the contents of all the channels of the spectrum.

The temperature at which the calibrations were taken is about 1.1 K for the deuterons and between 1.0 K and 1.6 K for the protons. The average temperature was determined per NMR measurement and used to calculate a calibration coefficients per coil for each measurement. The averages over the measurements were taken to obtain the calibration coefficients of the coils. The uncertainty in the absolute value of the temperature corresponds to a global shift of all the calibration coefficients, and is taken into account in the systematic error (see below)[60].

### 4.5.4 Treatment of the enhanced signals

The spectra from polarised material, the so-called enhanced signals, are treated on-line. Again the baseline is subtracted, after which the residual baseline is fitted and subtracted. This is done using the same fitting routine as used for the analysis of the TE signals. The integral of the signal is taken by adding the contents of the four hundred channels, and is written to the data tape in a separate record together with other target information. This is done every two minutes. While processing the data-tape the run and event number during which the
measurement is recorded are added to the target information. This is necessary in order to use only polarisation information of times when data were actually taken. A separate file with this information is written, enabling an analysis with updated calibration constants without scanning through all the data tapes.

For the '93-run data have also been taken during ramping of the target magnet during the field rotation. During this time the polarisation of the target can not be measured. The polarisation is assumed to be constant during ramping, and is extrapolated forward (before actual rotation) and backward (after rotation) from the measurements done. The difference in polarisation before and after rotation was monitored to check for reversals with polarisation loss. In that case data taken during the reversal were not used.

For each run the average polarisation per coil is calculated by averaging over the different measurements done during the run, using the calibration coefficients determined as described is the section above. The polarisation of each target cell is taken as the average of the measurements per longitudinal section of the cells. For the deuteron target the small coils were not used in the averaging. In the proton target one section of the upstream target cell was monitored by two small coils. The average polarisation of these two coils was taken as the polarisation of that section. The average polarisations per run were used for the asymmetry calculation.

4.5.5 Systematic errors and corrections

The magnetic moment of the deuteron is about six times as small as that of the proton. Apart from that the deuteron spectrum is broadened by the quadrupole interaction. This leads to a much smaller NMR signal for the deuteron than for the proton, which makes the deuteron TE calibration much more sensitive to drifts or shifts in the Q-curve during the calibration. For the deuteron measurement the error on the enhanced signals is negligible compared to the uncertainties in the calibration. For the proton the errors on the enhanced signals are significant. This is on one hand due to the better determination of calibration coefficients, on the other hand due to effects of the large magnetic moment. Various uncertainties and corrections on the target polarisations are mentioned here. A more detailed treatment on the deuteron and proton error analysis is given in [62] and [63].

Calibration signal uncertainties
An important source of a systematic error on the TE calibration is the effect of a shift in the magnetic field on the baseline. The baseline is taken with a 5% offset of the magnetic field, and subtracted from the signal assuming the baseline is field independent. The possible effect is largely cancelled by fitting and subtracting a residual baseline. For the deuteron measurement the residual base-line was subtracted using a parabolic fit, so only third and higher order effects are of importance. The relative error on the deuteron calibration constants due to this is 4%. For protons the residual baseline was subtracted with a straight line fit. The residual error is only 0.5% due to the larger TE signal relative to the change in the baseline, compared to the deuteron signals. Furthermore a drift in the Q-meter circuitry between taking the baseline and taking the signal resulted in an uncertainty in the extracted calibration constants of 1.5% for the deuteron and 0.5% for the proton.
For the proton measurement a 2% effect of the direction of the magnetic field on the calibration coefficients was observed. It was largely taken into account by using two sets of calibration coefficients. The residual uncertainty arising from the polarity effect was estimated as 0.6%. A satisfactory explanation for this effect has not yet been found.

For the proton measurement a correction also was made for the presence of non-polarisable protons, present mainly in the Kevlar cartridges. Unpolarisable protons do contribute to the TE signal, and their contribution has to be subtracted. The background was determined in an empty target run, and results in a 1% correction to the TE signal with an uncertainty of 0.4%.

Adding the mentioned and other smaller errors quadratically leads to a total error on the TE signal of 1.15% for the proton. For the deuteron the total systematic relative uncertainty is 4.4%.

**Calibration temperature uncertainty**

The effect of an uncertainty in temperature at which the calibration was done follows from eq. (4.1). The effect is the same for all the coils, and amounts to 1.0% for the deuteron measurement, using $^3$He vapour pressure thermometry. Because of a problem with the $^3$He bulb during the proton run, the temperature of the first proton calibration was determined using a calibrated Speer resistor, leading to an uncertainty of 2.1%. For the following calibrations the $^4$He vapour pressure in the still was used for temperature determination, which lead to an uncertainty in the temperature between 0.4 and 1.1%, depending on the temperature at which the calibration was done. At higher temperature its uncertainty is smaller, but the smaller TE polarisation results in a lower signal as well. The lower error on the $^4$He-thermometry compared to $^3$He is due to a more careful error analysis.

**Enhanced signal corrections and errors**

An effect of the large magnetic moment of the proton is that the signals are so large that non-linearity effects in the Q-meters become important. An additional problem is that because of the large signal width of the proton signal the residual baseline subtraction results in a possible “missing area”. This was the reason for changing the width of the NMR sweep from 400 to 600 kHz during the proton run. The two effects have been dealt with simultaneously by a circuit simulation, as described in [64]. A correction of order 3-5% was applied, with an estimated error of 0.9(0.6)% for the 400(600) kHz sweep.

Due to the large proton magnetic moment the polarised protons produce a non-negligible net magnetic field, causing the NMR line-centre to be approximately 10 channels off-centre. This effects the measurement since the electronics gain is slightly frequency dependent, and the changing wings also result in a shift dependent residual baseline subtraction. The effect was quantified by looking at the effect of shifting the signal at constant polarisation by slightly changing the magnetic field. A correction of the order of one percent was applied, with an estimated uncertainty of 0.5(0.4)% for the 400(600) kHz sweep.

In combination with some other errors this leads to a total systematic error on the proton enhanced signals of 1.3(1.1)%.
### 4.5. Polarisation measurement

<table>
<thead>
<tr>
<th>error source</th>
<th>rel.error deuteron</th>
<th>rel.error proton cal1,400kHz</th>
<th>cal2,400kHz</th>
<th>cal2,600kHz</th>
<th>cal2,600kHz</th>
</tr>
</thead>
<tbody>
<tr>
<td>B-shift</td>
<td>4.0%</td>
<td>0.5%</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>drift</td>
<td>1.5%</td>
<td>0.5%</td>
<td>-</td>
<td>0.5%</td>
<td>0.6%</td>
</tr>
<tr>
<td>background</td>
<td>-</td>
<td>0.5%</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>B-polarity</td>
<td>-</td>
<td>0.6%</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>TE temp.</td>
<td>1.0%</td>
<td>2.1%</td>
<td>0.8%</td>
<td>1.4%</td>
<td>1.1%</td>
</tr>
<tr>
<td>total TE</td>
<td>4.5%</td>
<td>2.4%</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>non-linearity</td>
<td>-</td>
<td>0.9%</td>
<td>-</td>
<td>0.5%</td>
<td>-</td>
</tr>
<tr>
<td>signal shift</td>
<td>-</td>
<td>0.5%</td>
<td>-</td>
<td>-</td>
<td>0.4%</td>
</tr>
<tr>
<td>total enhanced</td>
<td>-</td>
<td>1.3%</td>
<td>-</td>
<td>-</td>
<td>1.1%</td>
</tr>
<tr>
<td>total error per coil</td>
<td>4.6%</td>
<td>2.7%</td>
<td>1.9%</td>
<td>1.8%</td>
<td></td>
</tr>
<tr>
<td>sampling</td>
<td>2.0%</td>
<td>0.75%</td>
<td>1.2%</td>
<td>0.75%</td>
<td></td>
</tr>
<tr>
<td>error on $\Delta P$</td>
<td>5.0%</td>
<td>2.7%</td>
<td>1.9%</td>
<td>2.1%</td>
<td>1.9%</td>
</tr>
</tbody>
</table>

**Table 4.1:** The various contributions to the error on the target polarisation. The proton data are split in periods, by a combination of two TE calibrations and two sweep widths. The difference between the last two columns is the number of coils that could be used. The total errors include some not-mentioned smaller contributions as well. The "−" means that the contribution is negligible compared to the other errors, and thus not studied in detail.

---

**Polarisation sampling and averaging**

The polarisation values from the NMR coils were combined into polarisation values per target cell. In doing so the uncorrelated errors on the individual measurements cancel to some extent, while the error has to be increased due to the fact that the NMR coils do not sample all the target material.

The polarisation of the target material around the NMR coils is sampled according to the profile of the RF field produced by the coils. The 8 large deuteron coils are 8 cm long, and have three windings at a radius of 2.0 cm. The 8 large proton coils are 7 cm long and have one winding at radius 1 cm. The deuteron coils are made larger than the proton coils to compensate to some extent the weaker deuteron signal. In the proton and deuteron target holders small coils were added to study a possible radial dependence of the polarisation. The deuteron target cells of 40 cm length have four large coils, while one small coil is added in the centre of one of them. The proton target cells in the new SMC dilution refrigerator are 60 cm long. The downstream cell has five and the upstream cell three large coils, while one quarter of the upstream cell is sampled by two 7 cm long coils with 3.5 mm radius, one at the centre and one off-centre.

Radial dependence of the polarisation is important if the radial sampling of the material by the beam is different from that of the NMR coils. For the deuterium target there was a slight difference between the beam and coil sampling, and the difference in polarisation values of the small coil and large coil around it gives an upper limit to the systematic error due to imperfect radial sampling of 2%[61]. The proton coils were designed such that the radial sampling was practically identical to the sampling of the material by the muon beam. Apart from that no difference was observed between the polarisations of the small proton coils.
The variation of the deuteron polarisation values between the different NMR-coils of up to 10% indicates a variation of the polarisation along the beam axis. The deuteron coils cover practically the whole length of the target cells, and averaging of the measurements takes the longitudinal polarisation variation into account properly. The largest difference in the proton polarisation measurements between the coils is only 2-3%, but since the proton target has smaller coils in larger target cells a possible longitudinal polarisation inhomogeneity should be taken into account. The errors not correlated between the coils could roughly account for the observed difference already, but an average polarisation inhomogeneity of ±1.0% absolute over the target cell was assumed, leading to an extra relative uncertainty of 0.75-1.2% on the difference between the upstream and downstream polarisation, depending on the number of coils used for the polarisation measurement.

The mentioned contributions to the error on the polarisation measurements per coil are collected in Table 4.1. The total error on the difference between upstream and downstream polarisation is given as well, where the data taking is split in some subperiods for the proton. The effect of the uncorrelated errors is diminished by the averaging over the coils.

**Polarisation values**
The average polarisations of the 1992 deuteron and 1993 proton target are collected in Table 4.2.

<table>
<thead>
<tr>
<th>target</th>
<th>upstream</th>
<th>downstream</th>
</tr>
</thead>
<tbody>
<tr>
<td>deuteron 1992</td>
<td>35.0</td>
<td>-39.8</td>
</tr>
<tr>
<td>proton 1993</td>
<td>85.4</td>
<td>-89.7</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>target</th>
<th>upstream</th>
<th>downstream</th>
</tr>
</thead>
<tbody>
<tr>
<td>deuteron 1992</td>
<td>33.0</td>
<td>-39.0</td>
</tr>
<tr>
<td>proton 1993</td>
<td>83.2</td>
<td>-84.3</td>
</tr>
</tbody>
</table>

*Table 4.2: The average positive and negative target polarisation values per target cell, for the deuteron and proton targets.*
Chapter 5

The extraction of $A_1$

In this chapter the first part of the physics analysis of the data, the extraction of $A_1$ and the determination of its error, is discussed. After a general discussion about the methods of asymmetry extraction several aspects are treated in detail: the dilution factor, the radiative corrections and possible false asymmetries. Finally the results of $A_1$ for the deuteron and the proton are given. The numbers presented in this chapter include some additional corrections that were applied after the respective SMC publications, and are the results of a private analysis. Final values will appear in forthcoming SMC publications.

5.1 General method

In the SMC, data are taken simultaneously with two target cells with opposite longitudinal polarisation, placed one behind the other in a beam of longitudinally polarised muons. The polarisation direction of the two target cells is periodically reversed (two to five times per day). The two polarisation configurations are depicted in Fig. 5.1. The event yield asymmetry from either configuration is dominated by the difference in spectrometer acceptance for the two cells. In combining data from the two polarisation configurations, this acceptance difference and the muon fluxes cancel, which makes it possible to extract the spin dependent cross section asymmetry.

The method of extracting the asymmetry $A_1$ in our analysis is introduced in steps. We start with a refinement of the first order method used in the EMC analysis.

5.1.1 Extraction of the asymmetry

The event yield $N_i$ from the two target cells, upstream(u) and downstream(d), in the two configurations $I$ and $II$ is given by
5. The extraction of $A_1$

\[ P_\mu \longleftrightarrow \Phi_{i,j} \]

\[ P_{i\mu} \longleftrightarrow P_{i\mu} \]

\[ N_1 \]

\[ N_2 \]

\[ P_{i\mu} \longleftrightarrow \Phi_{3,4} \]

\[ P_{i\mu} \longleftrightarrow P_{i\mu} \]

\[ N_3 \]

\[ N_4 \]

\textbf{Figure 5.1:} The two polarisation configurations, I and II, in which data were taken. The beam direction is from left to right. Arrows represent polarisation directions.

\begin{align*}
N_1 &= \Phi_{1,2} \sigma_0 \left( 1 - f P_{\mu} P_{i\mu} A \right) \\
N_2 &= \Phi_{1,2} \sigma_0 \left( 1 - f P_{\mu} P_{i\mu} A \right) \\
N_3 &= \Phi_{3,4} \sigma_0 \left( 1 - f P_{\mu} P_{i\mu} A \right) \\
N_4 &= \Phi_{3,4} \sigma_0 \left( 1 - f P_{\mu} P_{i\mu} A \right)
\end{align*}

(5.1)

where $\Phi$ is the integrated beam flux, $a$ the acceptance of the spectrometer and $n$ the number of target nucleons for the target cell under consideration, $P_{\mu}$ the beam polarisation and $P_{i\mu}$ the target polarisation. The dilution factor $f$ (see eq. (5.21)) accounts for the presence of non-polarisable material in the beam. The spin independent cross section per nucleon is denoted by $\sigma_0$. The prime on some of the variables indicates the possibility of a slightly different value before and after polarisation reversal.

The quantity to be extracted is the muon-nucleon scattering asymmetry $A$. As described in Chapter 2, $A$ is related to the virtual photon-nucleon absorption asymmetry $A_1$ by

\[ A = D(A_1 + \eta A_2), \]

(5.2)

where $D$ is the depolarisation factor from the polarised muon to the polarised virtual photon, $A_2$ the asymmetry arising from longitudinal/transverse polarisation interference, and $\eta$ a (small) kinematic factor. In the $A_1$ analysis the possible contribution of $A_2$ is included in the systematic error.

\textbf{First order method}

The measured asymmetries in the two configurations can be written as:
\[ A_{\text{meas}(1,2)} = \frac{N_1 - N_2}{N_1 + N_2} = \frac{(n_u a_u - n_d a_d) - f P_\mu (n_u a_u P_{\mu 1} - n_d a_d P_{\mu 2}) A}{(n_u a_u + n_d a_d) - f P_\mu (n_u a_u P_{\mu 1} + n_d a_d P_{\mu 2}) A} \]

\[ A_{\text{meas}(3,4)} = \frac{N_4 - N_3}{N_4 + N_3} = \frac{(n'_u a'_u - n'_d a'_d) - f P_\mu (n'_d a'_d P_{\mu 4} - n'_u a'_u P_{\mu 3}) A}{(n'_d a'_d + n'_u a'_u) - f P_\mu (n'_d a'_d P_{\mu 4} + n'_u a'_u P_{\mu 3}) A} \] (5.3)

Neglecting the small term on the right hand side of the denominator with respect to the first term, the average of the two measured asymmetries of eq. (5.3) is equal to

\[ \overline{A_{\text{meas}}} = \frac{1}{2} \Delta P_1 \cdot P_{\mu 1} \cdot f \cdot A, \] (5.4)

provided that the ratio of acceptance times target thickness for upstream and downstream target cells is constant, and the difference of the two target polarisations is the same before and after reversal; \( \Delta P_1 = P_{\mu 1} - P_{\mu 3} = P_{\mu 4} - P_{\mu 2} \). Note that the extracted asymmetry is independent of beam flux. The beam polarisation is assumed to be constant. The assumption of constant acceptance and target thickness ratios is discussed in Section 5.1.2.

**Second order method**

The above explained first order method has the disadvantage that it is not exact and that assumptions are needed for the target polarisation. Using a second order equation avoids this. Keeping the four target polarisation values as parameters and eliminating \( \Phi \sigma_0 \), eqs. (5.1) can be combined per configuration into

\[ N_1 (1 - P_\mu P_{\mu 2} f A) - N_2 (1 - P_\mu P_{\mu 1} f A) \left( \frac{a_u n_u}{a_d n_d} \right) = 0 \]

\[ N_3 (1 - P_\mu P_{\mu 4} f A) - N_4 (1 - P_\mu P_{\mu 3} f A) \left( \frac{a'_u n'_u}{a'_d n'_d} \right) = 0. \] (5.5)

A second order equation in \( A \) can now be constructed by eliminating the ratio \( (a_u n_u)/(a_d n_d) \), assuming it to be the same before and after polarisation reversal. Solving this equation yields:

\[ A = \frac{1}{f P_\mu} \cdot \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}, \text{ with} \]

\[ a = N_1 P_{\mu 2} P_{\mu 3} - N_2 P_{\mu 1} N_3 P_{\mu 4} \]

\[ -b = N_1 P_{\mu 2} N_4 + N_1 N_4 P_{\mu 3} - N_2 P_{\mu 1} N_3 + N_2 N_3 P_{\mu 4} \]

\[ c = N_1 N_4 - N_2 N_3. \] (5.6)

One of the two solutions is the asymmetry, the other solution has \(|A| >> 1\) and thus is unphysical. The numerical stability of the solution, obtained from the difference of two large numbers, has to be monitored. In the case that eq. (5.6) does not give a stable solution the first order method of eq. (5.4) is used.

**Refined second order method**

The above mentioned expression accommodates polarisation changes at reversal, but assumes the target polarisation to be constant during data taking in one configuration. Often however
the polarisation slowly increases during data taking, especially right after repolarisation by microwaves. Considering the data runs of a polarisation configuration separately, one can write eqs. (5.5) for each run. Combining the runs and proceeding in the same way as before, the coefficients of eq. (5.6) have instead of for example \( N_1 P_{22} \) the sum of such products per run. In using that expression to extract the asymmetry, the variation of polarisation over time is properly taken into account.

5.1.2 Acceptance effects

Regarding the influence of acceptance variations, one notices that not the acceptances, but the ratio of the upstream and downstream acceptances, or more precisely the ratio of the acceptance times target thickness, has to be the same before and after reversal in order not to cause a falsely extracted asymmetry. This requirement is not necessarily fulfilled. By introducing \( K \) to describe a change in this ratio:

\[
\frac{n_u a_u}{n_d a_d} = K \cdot \frac{n_u a_u}{n_d a_d},
\]

(5.7)

the resulting asymmetry is given by

\[
A = \frac{1}{|\frac{1}{2} \Delta P_t \cdot P_\mu| \cdot f} \left( A_m + \frac{K - 1}{4} \right).
\]

(5.8)

instead of by eq. (5.4):

In the analysis the acceptance ratio is assumed (and checked) to be constant \((K=1)\). The upper limit of the false asymmetry \( A_f \),

\[
A_f = \frac{1}{|\frac{1}{2} \Delta P_t \cdot P_\mu| \cdot f} \left( \frac{K - 1}{4} \right),
\]

(5.9)

is estimated by performing spectrometer studies, described elsewhere in this chapter. It is included in the systematic error.

The target thicknesses may vary since the ratio of \(^3\text{He}/^4\text{He}\) and thus the density of the liquid helium between the target material beads can change. This is the case if the target is in frozen spin mode and the microwaves are switched on after reversal to increase the polarisation. As long as the variation of the helium density is the same for upstream and downstream target cells this does not lead to a false asymmetry. Assuming a reasonable upper limit for the difference in the variation of the helium density for the upstream and downstream target cell \((e.g. \ 0.1)\), the maximum possible false asymmetry caused by this effect is an order of magnitude smaller than the estimated effect of the variation of detection efficiencies.

5.1.3 Statistical error and weighting of the data

For each pair of polarisation configurations the asymmetry \( A_1 \) is extracted for sufficiently populated \((x,Q^2)\) bins, using eq. (5.2). Assuming Gaussian distributions, the asymmetries
from a number of configuration pairs, \( i \), can be combined by weighting them with their statistical error:

\[
A_1(x, Q^2) = \frac{\sum_i (A_i / \sigma_i^2)}{\sum_i (1/ \sigma_i^2)}.
\]

The combined statistical error is given by

\[
\Delta A_1(x, Q^2) = \sqrt{\frac{1}{\sum_i (1/ \sigma_i^2)}}.
\]

If \( A_1 \) is assumed to scale, that is if \( A_1 \) is \( Q^2 \) independent, the summation can be extended over the \( Q^2 \) bins within an \( x \) bin.

In the first order method the statistical error of a measurement of \( A_1 \) from a pair of polarisation configurations is equal to

\[
\sigma_{1st} = \frac{1}{|\Delta P_i \cdot P_\mu| Df} \sqrt{\left( \frac{1}{(N_1 + N_2)} + \frac{1}{(N_3 + N_4)} \right)}.
\]

The values of the statistical error in the asymmetry extracted by the second order method, and likewise for the refined second order method, are not substantially different.

### 5.1.4 The \( Df \) method of asymmetry extraction

A problem with the methods described above is that the data have to be split in \((x, Q^2)\) bins, since eq. (5.1), in combination with eq. (5.2), is only valid at fixed \( f \) and \( D \). The dilution factor \( f \) is treated in detail in Section 5.3. The depolarisation factor \( D \) (eq. (2.23)) is proportional to \( y \) in first order. The \((x, Q^2)\) bins have to be chosen in such a way that the variation of \( f \) and \( D \) within a bin remains small. This may lead to a problem at large \( x \), where the scattering cross section is low, and the number of events per configuration pair in a bin becomes small, causing the rejection of data if one out of four count rates in eq. (5.1) becomes less than a certain cut-off value.

If one assumes \( Q^2 \) independence (scaling) of \( A_1 \), a variation of the first order method, the so-called \( Df \) method, makes an asymmetry calculation possible without splitting in \( Q^2 \) bins. This method uses an expression that does not assume \( f \) and \( D \) to be constant; the \( Q^2 \) variation of \( f \) and \( D \) within the bin is properly treated with this method.

Assuming for the moment equal acceptances for the upstream and downstream target cells one can write, starting from eq. (5.10) and using eq. (5.4) for \( A_1 \) at bin centre \( x \), the following expression

\[
A_1(x) = \frac{\sum_i \left[\left(\frac{N_i - N_2}{N_i + N_2}\right)\frac{1}{|\frac{1}{2} \Delta P_i \cdot P_\mu| Df}\right] \frac{1}{\sigma_i^2}}{\sum_i (1/ \sigma_i^2)},
\]

where the summation is over the \( Q^2 \) bins. Using for \( \sigma_i \) eq. (5.12) with only \( N_1 \) and \( N_2 \), yields

\[
A_1(x) = \frac{|\frac{1}{2} \Delta P_i \cdot P_\mu|}{\left(\frac{1}{2} \Delta P_i \cdot P_\mu\right)^2} \sum_i \left[\left(\frac{N_1 - N_2}{N_1 + N_2}\right) Df \right] \frac{1}{\sigma_i^2}.
\]
For infinitesimally small $Q^2$ bins the summation over $i$ becomes a summation over events weighted with $Df$. Taking now into account the difference in acceptance between the two target halves, the equation changes into

$$A_1(z) = \frac{1}{\Delta P_t \cdot P_{\mu}} \left[ \frac{\sum_i (Df)_{1i} - \sum_i (Df)_{2i}}{\sum_i (Df)^2_{1i}} - \frac{\sum_i \left[ \frac{\Delta x_{\mu} - \Delta x_d}{\Delta x_{\mu} + \Delta x_d} Df \right]_{1i}}{\sum_i (Df)^2_{1i}} \right]. \quad (5.15)$$

Combining data from the polarisation configurations $I$ and $II$ results in

$$A_1(z) = \frac{1}{\Delta P_t \cdot P_{\mu}} \left[ \frac{\sum_i (Df)_{1i} - \sum_i (Df)_{2i}}{\sum_i (Df)^2_{12i}} + \frac{\sum_i (Df)_{3i} - \sum_i (Df)_{3i}}{\sum_i (Df)^2_{34i}} \right], \quad (5.16)$$

since the second term on the right hand side of eq. (5.15) is opposite in sign for the data before and after reversal. The average $(x, y)$ and thus $Df$ in a bin are the same before and after reversal, apart from statistical fluctuations. This expression does not require $f$ and $D$ to be constant within the bin under study. This implies that the division of data in $Q^2$ bins is no longer necessary if one assumes scaling of $A_1$.

The statistical error of the asymmetry extracted in this way is given by

$$\sigma_{df} = \frac{1}{\Delta P_t \cdot P_{\mu}} \sqrt{\frac{1}{\sum_i (Df)^2_{12i}} + \frac{1}{\sum_i (Df)^2_{34i}}}. \quad (5.17)$$

### 5.1.5 Discussion

The advantage of the $Df$ method is the proper treatment of the variation of $f$ and $D$ with $Q^2$ and hence its insensitivity to the $Q^2$ binning. A disadvantage is the fact that apart from the possible acceptance changes ($K \neq 1$), statistical fluctuations also influence the second term on the right hand side of eq. (5.15), which complicates the estimate of a possible false asymmetry. These fluctuations are caused by the fact that neither $Df$ nor the acceptances in eq. (5.15) are quantities averaged over the whole data taking, but are averaged only over the configuration under study. One should also notice that since the $Df$ method is based on the first order approach, it is inexact and needs assumptions on target polarisation variations.

The second order methods require binning in $Q^2$. This implies that at large $x$, where the count rates are low and the range in $Q^2$ and hence the effective number of $Q^2$ bins is large, the extracted asymmetry could be too low: The chance of rejection, because one of the four count rates is below the cut-off value, is higher if a fluctuation has resulted in a large asymmetry, than if it has lead to small asymmetry.

For the refined second order method and the $Df$ method Monte Carlo studies were done [65] in order to estimate the accuracy of both methods in representative cases. Given an input asymmetry, events were generated in a much finer $(x, Q^2)$ grid than the binning used in the analysis, this in order to study the effect of the $Q^2$ dependence within the bins. The actual number of runs between polarisation reversals and the actual variation of target polarisation were used in this simulation. The average $Df$ values were also taken from the data. The
generated event rates per mini-bin and run were analysed using different asymmetry extraction methods. It turned out that the input asymmetry was recovered most accurately using the \( D_f \) method. Furthermore the study showed that by just demanding none of the four count rates to be zero results in the lowest bias. Since the statistical error used for weighting the different measurement is based on Gaussian statistics, this was not obvious from the beginning.

Therefore, we have used the \( D_f \) method for the analysis. The difference between the first order and the refined second order method was used to monitor the effect of the target polarisation variations. The event cut was put to one event.

### 5.1.6 The data samples

In the off-line analysis several kinematic cuts were applied to the data, although the hardware trigger already eliminates the majority of useless events. The kinematic cuts for the proton and deuteron data are collected in Table 5.1, and are discussed below.

<table>
<thead>
<tr>
<th></th>
<th>deuteron</th>
<th>proton</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \mu ) energy</td>
<td>100 GeV</td>
<td>190 GeV</td>
</tr>
<tr>
<td>( Q^2 )</td>
<td>&gt; 1 GeV(^2)</td>
<td>&gt; 1 GeV(^2)</td>
</tr>
<tr>
<td>( \nu )</td>
<td>&gt; 10 GeV</td>
<td>&gt; 15 GeV</td>
</tr>
<tr>
<td>( \theta )</td>
<td>&gt; 13 mrad</td>
<td>&gt; 9 mrad</td>
</tr>
<tr>
<td>( y )</td>
<td>&lt; 0.85</td>
<td>&lt; 0.90</td>
</tr>
<tr>
<td>( E' )</td>
<td>&gt; 10 GeV</td>
<td>&gt; 19 GeV</td>
</tr>
<tr>
<td># events after cuts</td>
<td>(3.2 \cdot 10^8)</td>
<td>(4.4 \cdot 10^8)</td>
</tr>
</tbody>
</table>

**Table 5.1**: The data selection criteria for the deuteron and proton data sample.

The reason for the cut in \( \nu \) is that because of the resolution of the spectrometer \( \nu \) is poorly defined below the cut-off value. A cut in the scattering angle \( \theta \) is necessary since the vertex resolution decreases rapidly below the values listed in Table 5.1. The contamination with muons from the decay of pions and kaons produced in the target requires a cut on the momentum of the scattered muon. The cut in \( y \) is motivated by the uncertainty in the radiative corrections. For the deuteron data the cut was placed at \( y = 0.85 \). Before taking the proton data in 1993 it had become clear that the cut could even be placed at \( y = 0.90 \), and the acceptance of the spectrometer for large \( y \) events was increased, as described in Section 3.3.3. The increase in the systematic error due to larger radiative corrections is compensated by a decrease in the statistical error, since events with high \( y \) have low depolarisation (high value for \( D \)). Events with \( Q^2 < 1 \text{ GeV}^2 \) were not used, since the validity of perturbative QCD is questionable below this value. Geometrical cuts, which are discussed in Section 5.3.4, were applied to select scattering events from the target cells only.

The resulting deuteron data sample, taken with a muon energy of 100 GeV, is \(3.2 \cdot 10^8\) events. The proton data sample of \(4.4 \cdot 10^8\) events was taken at incident energy of 190 GeV. The kinematic domain of the proton trigger 1 and trigger 2 data is shown in Fig. 5.2. For the deuteron measurement only trigger 1 data were used. The shape of the kinematic domain for 100 GeV data is essentially the same as for data with 190 GeV incident muon energy, but shifted to the right in Fig. 5.2, the lowest value of \( x \) being equal to 0.006.
5.2 Radiative corrections

To obtain the one photon exchange cross sections, which are directly related to the structure functions, from the measured "full" scattering cross sections, one has to correct for the contributions from higher order QED processes. These are the so-called radiative corrections.

The event yields \( N_i \) from the target cells are expressed in eq. (5.1). Both \( A \) and \( f \) are quantities which involve cross sections. The definitions of \( N_i \), \( f \) and \( A \) in eq. (5.1) have to be consistent, which means that if \( N_i \) is related to the measured cross section, the same holds also for \( f \) and \( A \). Alternatively, \( N_i \), \( f \) and \( A \) can be related to one photon exchange cross sections.

In the SMC analysis the asymmetry of full scattering cross sections is determined by starting from the measured count rates and using full cross sections to calculate the dilution factor. The calculation of full cross sections from the published structure functions requires spin independent radiative corrections for the unpolarised nuclei in the target cells. Spin dependent radiative corrections are applied to the extracted proton or deuteron asymmetry \( A \) to obtain \( DA_1 \), the asymmetry of one photon exchange cross sections. In applying corrections to the asymmetry, radiative corrections to the cross sections cancel to a large extent.

Radiative corrections can in principle also be applied on an event by event basis. After that, the proton or deuteron asymmetry of one photon exchange or Born cross sections is obtained directly, using the dilution factor of Born cross sections. The radiative corrections depend on the target (and beam) polarisation, which complicates the procedure. Since apart from that, the cancellation of systematic errors is more transparent in the first procedure, as is discussed
below, we do not apply the radiative corrections on an event by event basis.

5.2.1 Spin-independent radiative corrections

For the dilution factor, the ratios of the cross sections for each of the non-polarisable nucleons, mainly from carbon, oxygen, helium and copper, and the target nucleus, hydrogen or deuterium, are of importance. In order to determine the ratios of full cross sections for these nuclei, the NMC results for the $F_2$ ratios, equal to the one photon exchange cross section ratios, were used. The radiative corrections applied by the NMC to obtain these ratios were reintroduced to retrieve the full cross section ratios. For the latest NMC results on the $F_2$ ratios, the radiative corrections were calculated using the program TERAD [66] based on a method by Akhundov, Bardin and Shumeiko. In order to minimise the systematic error in the present analysis, the same program was used to retrieve the ratios of full cross sections from the $F_2$ ratios.

The set of Feynman diagrams taken into account by TERAD is shown in Fig. 5.3[66]. The first order QED corrections can be split in contributions from vacuum polarisation, corrections to the lepton current and to the hadron current.

The vacuum polarisation contribution from lepton loops (diagram 2) can be calculated because of the well-known lepton masses. The quark loop contributions (diagram 6) have been determined by making use of the hadronic cross section in $e^+e^-$ annihilation.

One of the contributions to the lepton current correction is the so-called inelastic radiative tail, due to the processes of the form $\mu + N \rightarrow \mu + \gamma + \text{Hadrons}$ (diagram 4 and 5). To calculate this contribution one has to integrate over all possible final energies of the hadronic system and all transferred four-momenta. This requires knowledge of the DIS structure functions over the kinematic domain as plotted in Fig. 5.4. Although the integral is divergent, the result becomes finite when combined with the also divergent vertex correction (diagram 3).

For the lepton current diagrams not only scattering on quarks has to be considered, but also elastic scattering on nucleons (quasi-elastic scattering) and, except for hydrogen, scattering on the whole nucleus, referred to as coherent or elastic scattering. These processes contribute if real photon radiation occurs, which brings the kinematics of the observed incoming and outgoing lepton into the DIS region. In order to calculate the contributions of this “radiative tail of the elastic peak” the DIS structure functions in the expressions for the various cross sections have to be replaced by combinations of the electro-magnetic form factors of the proton and the neutron. A nuclear suppression factor accounts for the fact that the nucleons are bound in a nucleus (not for hydrogen). For the elastic scattering on nuclei the form factors for the proton have to be replaced by those of the nucleus. At small $x$ and large $y$ the contribution of the elastic tail for heavier nuclei such as copper and carbon is up to several times as large as the Born cross section. Important in this respect is that the higher order corrections to the elastic tail (diagrams 16 to 26) are only 2-3% of the first order correction in the kinematic domain where the contribution of the tail is large.

Corrections to the hadron current, diagrams 7 to 9, are model dependent. In TERAD they are calculated in the framework of the quark parton model, where the quarks are assumed not
to interact with each other and leptons interact directly with one of the quarks or antiquarks. Calculating the Bremsstrahlung contribution leads to quark-mass singularities, which can be factorised out and have to be absorbed in the quark distribution functions, which means in the structure functions[67]. The remaining corrections are small.

In Fig. 5.5 the contributions of several processes are given as a function of $y$ for two values
5.2. Radiative corrections

Figure 5.4: The kinematic domain, plotted as \((x, Q^2)\) and \((x, y)\), in which the one photon exchange process with Bremsstrahlung contributes to the cross section at the kinematic point \(P\).

Figure 5.5: The various contributions to the radiative corrections for carbon as a function of \(y\) for two \(x\) values. The corrections are expressed as a percentage of the one photon exchange cross section.

The corrections are given in \((x, y)\) instead of \((x, Q^2)\) because they are to a large extent independent of the incident energy.

5.2.2 Spin-dependent radiative corrections

In the case of polarised lepton-nucleus scattering the spin dependence of the various cross sections has to be taken into account. This is done in the radiative correction program POLRAD[68], developed by the Minsk group of Shumeiko. The spin dependent contributions to the cross section are all linear with respect to lepton and nucleon polarisations. This means
that the Born and full scattering cross sections can be calculated with the two spins parallel and anti-parallel (and "perpendicular" in the case of spin 1), after which the cross sections for an arbitrary set of polarisation values can be obtained by taking the correctly weighted average of the two (or three in the case of the deuteron).

In POLRAD, like in TERAD, the divergence in the deep inelastic tail calculation is circumvented by combining it with the vertex correction contribution. An exact expression is used for the calculation of the first order radiative corrections. The multi soft-photon contribution to the elastic and inelastic tail is taken into account by an exponentiation procedure[69]. Corrections to the hadron current are not applied; they are assumed to be small (see above).

In the SMC analysis an additive correction $\Delta A$ to the asymmetry $A$ of full cross sections is used,

$$A_1D = A_{\text{Born}} = A - \Delta A$$  \hspace{1cm} (5.18)

since a multiplicative correction $(1 - \Delta A/A)$ would amplify statistical fluctuations. This is especially the case in regions where the asymmetry is close to zero, since $\Delta A$ remains small but a multiplicative correction would become large. In this expression $D$ is the depolarisation factor that relates the lepton-nucleon asymmetry $A_{\text{Born}}$ to the photon-nucleon asymmetry $A_1$.

The full cross section for an arbitrary polarisation (e.g. polarised(pol) or unpolarised(unp)) can be expressed as

$$\sigma_{\text{full}}^{unp,pol} = f_{\nu,t} \cdot \sigma_o^{unp,pol} + \sigma_{\text{tails}}^{unp,pol},$$  \hspace{1cm} (5.19)

where $\sigma_o$ is the Born cross section and the polarisation independent $f_{\nu,t}$ accounts for the so-called factorising part of the vertex correction and radiative tail contributions. The remaining contribution of the inelastic(inel), quasi-elastic(qe, i.e. scattering on nucleons) and elastic(el) radiative tail cross sections is expressed as $\sigma_{\text{tails}}$.

One of the inputs necessary for the calculation of the radiative corrections is the spin dependent structure function $g_1(x)$ itself. The correction $\Delta A$ and its dependence on $g_1$ can be expressed as

$$\Delta A = \left( \frac{-\sigma_{\text{tails}}^{pol}}{\sigma_{\text{full}}^{unp}} \right) DA_1(g_1) + \frac{\sigma_{\text{tail}}^{pol}(g_1)}{\sigma_{\text{full}}^{unp}},$$

with

$$\sigma_{\text{tail}}^{pol}(g_1) = \sigma_{\text{inel tail}}^{pol}(g_1) + \sigma_{\text{qe tail}}^{pol} + \sigma_{\text{el tail}}^{pol}.$$  \hspace{1cm} (5.20)

In $\sigma_{\text{tails}}^{pol}$ only the inelastic radiative tail depends on $g_1$. Equations (5.19) and (5.20) show that only the non-factorising part of the full cross section ($\sigma_{\text{tails}}$) leads to a correction on the asymmetry. In Fig. 5.6 the two terms of $\Delta A$ are plotted as a function of $x$ for the proton and the deuteron. For the proton the $A_1^p$ measured by the EMC has been used as input asymmetry and for the deuteron $A_1^d$ has been combined with the Schäfer[70] parametrisation of $A_1^d$.

The dependence of the radiative corrections on the initial assumption on $g_1(x)$ and hence on $A_1(x)$ can be removed by an iteration procedure, where one uses the uncorrected $g_1$ result from the data as input to calculate the radiative corrections. These are used to determine a corrected $g_1$, which is the starting point for the next iteration, and so on, until convergence. This procedure removes the dependence on the initial assumption of $A_1$ within the measured
Figure 5.6: The two terms, both with inverted sign, of $\Delta A^p$ and $\Delta A^d$ as given by eq.(5.20), in the kinematics of the SMC experiment. The closed triangles correspond to the term linear with $A_1$, the open triangles to the term with the spin dependent part of the radiative tails. At small $x$ the elastic and quasi-elastic tails dominate, for large $x$ the inelastic radiative tail dominates.

kinematic region (the first term of the right-hand side of eq. (5.20) ), while the contribution of the spin dependent part of the inelastic tail also depends on the value of $g_1$ outside the measured region.

For the deuteron measurement we have applied the radiative corrections calculated using $A_1^d$ from the Schäfer model and $A_1^p$ from the EMC proton measurement. For the proton measurement, because of the good statistics, the iteration procedure could have been of use. But the radiative corrections to the asymmetry are so small in the first step, that further iteration was not done. The systematic error was for both nuclei determined by shifting the $A_1$ input for POLRAD by half the SMC statistical error.

5.3 The dilution factor

The dilution factor takes into account the presence of other nuclei than the (polarisable) hydrogen(p) or deuterium(d); it is defined as

$$ f = \frac{n_{p,d} \sigma_{p,d}}{n_{p,d} \sigma_{p,d} + \sum_c n_c \sigma_c}. $$

(5.21)

In this expression $n$ stands for the number of nuclei within the beam diameter and $\sigma$ for the (spin independent) nuclear cross section. The index $c$ indicates contaminant elements (mainly carbon, oxygen, helium and copper) present in the target cells.

As discussed in Section 5.2 the dilution factor used in the present analysis is determined with full scattering cross sections. In this section the steps to obtain this dilution factor
are discussed. First the target composition was determined. The one-photon exchange cross sections as measured by the NMC were used to calculate the dilution factor of one photon exchange cross sections, after which the radiative corrections were reintroduced to obtain the dilution factor for full cross sections.

5.3.1 Target composition

First the relative abundance of all the elements present in the target cells was determined. Of importance are the composition of the target material and the masses of the liquid helium around the target beads and of the cupronickel NMR coils relative to the target material mass.

The target material beads for the deuterated material have been produced by G. Baum and S. Bültmann at Bielefeld University. The deuterated butanol, C\textsubscript{4}D\textsubscript{9}OD, with a deuterisation of 99.4 %, was mixed with 5.085 ± 0.005 mass per cent D\textsubscript{2}O. This was to improve the glass formation. To the butanol-water mixture 4.131 ± 0.007 mass per cent of Cr(V)-complex, the paramagnetic centers, was added. The chemical formula for the complex is C\textsubscript{12}D\textsubscript{20}O\textsubscript{7}CrNa\textsubscript{2}D\textsubscript{2}O. The density of the target material was determined at liquid nitrogen temperature to be 1.106 ± 0.012 g/cm\textsuperscript{3}. After the 1992 run the mass of the target material in both target cells together was measured to be 1031.5 ± 4.0 g. All these numbers and the way they were obtained can be found in [71].

The proton material has been produced at GKSS Geesthacht. The mass ratio between butanol, water and the Cr(V)-complex is 90.5 : 5.0 : 4.5. The density of the material is 0.985 ± 0.010 g/cm\textsuperscript{3}. The total amount of target material in the two cells during the 1993 proton run was determined to be 1424 ± 12 g.

The relative amount of liquid helium around the target material beads is determined by the packing factor. Due to the slightly irregular shape of the beads and the restrictions during loading (the beads should not exceed 100 K, otherwise devitrification occurs, fatal for the polarising performance of the material) the closest packing (74%), can not be obtained. The packing factor was not measured directly, but the relative amount of helium was obtained from the ratio of the total mass of the target material and the mass of the helium, determined by the density of the helium, discussed below, and its volume. The volume \(V_{\text{He}}\) occupied by helium is equal to

\[
V_{\text{He}} = V_{\text{tc}} - \frac{M}{\rho},
\]  

(5.22)

where \(V_{\text{tc}}\) is the volume of the cells, \(\rho\) the density of the target material and \(M\) its mass. The dimensions of target cells used for the deuterium measurement are (including a 0.5% shrinkage) 399.5 ± 1.0 mm for the length and 49.5 ± 0.2 mm for the diameter. The proton target cartridges are made of Kevlar which has a very small thermal shrinkage coefficient. The length of the target cells is 600.0 ± 0.5 mm and the diameter 49.0 ± 0.2 mm.

To obtain the density of the helium mixture one must know the ratio between \(^3\text{He}\) and \(^4\text{He}\) in the mixing chamber of the dilution refrigerator, where the target material is situated. The ratio changes with cryogenic parameters like the condensation pressure or the temperature in the mixing chamber. The numbers for the total amount of \(^3\text{He}\), the volumes of the relevant parts of the \(^3\text{He}\) circuit and the limits on the \(^3\text{He}/^4\text{He}\) ratio in these parts result in a ratio in
the mixing chamber of $0.35 \pm 0.10$. Under the assumption of perfect turbulence, which means no clear horizontal phase boundary in the mixing chamber but $^3$He-rich bubbles surrounded by the diluted phase, the helium ratio in the target cells is the same as in the mixing chamber. Assuming however a perfect horizontal phase separation boundary, the limits of the separation in the mixing chamber (diameter 72 mm) correspond to a ratio $^3$He/$^4$He between 0.10 and 0.45 in the target cells (diameter 50 mm). Combining the limits given by these extreme assumptions results in a value for the $^3$He/$^4$He ratio of $0.30 \pm 0.20$.

The deuterium NMR coils are made of CuNi(70/30) tubing (outer diameter 2 mm), surrounded by 3.5 mm diameter teflon sleeve, $C_2F_4$. This prevents the beads from being too close to the coil, which would saturate the NMR transition. The mass of the cupro-nickel is 40.5 g, the mass of the teflon 34.4 g and the volume of the coils is 33.5 cm$^3$. Since a large fraction of the coil material is at a radius of 20 mm, and the beam profile is approximately Gaussian with $\sigma = 15$ mm, the effective masses and volume are smaller. The numbers after weighting with the beam profile are 31 g for the copper, 26 g for the teflon and 25 cm$^3$ for the volume. Relative uncertainties of 5 per cent are taken for the error calculation. Since the NMR signal for protons is much larger than that for the deuterons, the proton NMR coils are smaller than those for the deuterium measurement. The upstream cell has three large coils of 3.6 g and two small coils of 1.15 g with in total 1.2 g of teflon coating. In the downstream cell five coils of 3.6 g and 0.2 g teflon coating are mounted. The total volume of the coils is 16.6 cm$^3$.

The choice of vertex cuts has an influence on the dilution factor. For the deuterium target, used in 1992, two cases were examined, the so-called narrow and wide cuts as shown in Fig. 5.7. For the wide cuts the amount of helium between the target cells, upstream and downstream of the cells, as well as the copper screens (a.o. the microwave isolator) in these volumes has to be taken into account when calculating the dilution factor.

![Diagram](Figure 5.7: The narrow and wide vertex cuts visualised via a schematic view of the target cells in the mixing chamber (see also Fig. 5.10).)

For the two target materials the relative abundances of all elements is shown in Table 5.2. For the deuterated material three columns are given, one for the narrow cuts, one for the upstream cell with the wide cuts and one for the downstream cell with wide cuts. The nucleon ratios with errors are given, but not those for the individual elements, since many are correlated. The errors are discussed in detail in the section treating the systematic errors on $A_1$. 
Table 5.2: The relative abundance of nucleons from the various nuclei in the target cells. For the wide cuts only the changed values for copper and the helium isotopes are given.

<table>
<thead>
<tr>
<th>nucleus</th>
<th>nucleons per proton or deuteron nucleon</th>
<th>deuterium target</th>
<th>proton target</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>narrow cuts</td>
<td>wide cuts</td>
</tr>
<tr>
<td></td>
<td></td>
<td>upstream</td>
<td>downstream</td>
</tr>
<tr>
<td>proton</td>
<td>0.003</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>deuteron</td>
<td>1</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>carbon</td>
<td>2.360</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>oxygen</td>
<td>0.975</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>copper</td>
<td>0.131</td>
<td>0.181</td>
<td>-</td>
</tr>
<tr>
<td>helium-4</td>
<td>0.255</td>
<td>0.527</td>
<td>0.467</td>
</tr>
<tr>
<td>helium-3</td>
<td>0.061</td>
<td>0.127</td>
<td>0.112</td>
</tr>
<tr>
<td>sodium</td>
<td>0.010</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>chromium</td>
<td>0.023</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>fluor</td>
<td>0.080</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>nucleon ratio</td>
<td>0.204±0.003</td>
<td>0.189±0.004</td>
<td>0.193±0.004</td>
</tr>
</tbody>
</table>

5.3.2 One photon exchange cross sections

The expression for the dilution factor can be rewritten by dividing the numerator and the denominator of the right hand side of eq. (5.21) by $\sigma_{p,d}$:

$$f = \frac{n_{p,d}}{n_{p,d} + \sum_c n_c \cdot (\sigma_c / \sigma_{p,d})}.$$  (5.23)

From this it is clear that only cross section ratios are needed to calculate the dilution factor. First it is calculated with Born cross sections. In the case of isoscalar nuclei, the ratio of Born cross sections per nucleon is equal to the ratio of the $F_2$ structure functions. If the number of protons and neutrons in the nucleus is not equal however, a correction term with the ratio of the $F_2$ for the neutron and the proton has to be applied. The fact that the $F_2$ ratios for nuclei differ from unity, which means that the cross section per nucleon depends on the nucleus, is known as the EMC effect.

The New Muon Collaboration measured the $F_2$ ratios for carbon, helium and calcium with respect to deuterium [72] and that for the neutron to the proton [73]. The results show that within the small errors there is no $Q^2$ dependence, that is, the $F_2$ ratios scale. The NMC has provided a fit to the $F_2$ ratios including also the SLAC data at large $x$. Fig. 5.8 shows the EMC effect for carbon together with the fit used.

The $F_2$ ratios for other nuclei to deuterium were extrapolated from the measured ones, using the parametrisation of the $F_2$ ratios in the $A - x$ plane ($A$ is the atomic number):

$$\frac{F_2^A}{F_2^d} = C(x) \cdot A^{\alpha(x)}.$$  (5.24)

The parameters $C(x)$ and $\alpha(x)$ were determined by the EMC [74], from EMC, NMC and SLAC data. Only the parameter $\alpha$ is important for the calculation of the EMC effect from
5.3. The dilution factor

![Graph showing the ratio of F_2^c/F_2^d]

Figure 5.8: World data on the $F_2$ ratio of carbon over deuterium, and the fit provided by the NMC.

that of a nearby mass nucleus, e.g. for O from that of C:

$$\frac{F_2^O}{F_2^d} = \left(\frac{16}{12}\right)^{\rho(x)} \cdot \frac{F_2^C}{F_2^d}. \quad (5.25)$$

Similarly, the $^3$He effect is linked to that of $^4$He. The EMC effect for copper is extrapolated from the measured ratio for calcium to deuterium.

The dashed line in Fig. 5.9 shows the dilution factor with one photon exchange cross sections for the proton and deuteron target as a function of $x$.

5.3.3 Full scattering cross sections

The deuterium data were taken by the SMC at 100 GeV incident muon energy, while the proton data were obtained at 190 GeV. The ratio of the full cross section of nucleus $A$ to that of deuterium, both at 100 GeV, is equal to

$$\frac{\sigma_A^{100}(x, y)}{\sigma_d^{100}(x, y)} = \frac{F_2^A/RC(A, x, y, 100)}{F_2^d/RC(d, x, y, 100)}. \quad (5.26)$$

where $RC(A, x, y, 100)$ stands for the radiative corrections for nucleus $A$ as a function of $x$ and $y$ at incident energy of 100 GeV. Since the latest results[75] for the $F_2$ ratios from the NMC
5. The extraction of $A_1$

Figure 5.9: The dilution factor for both target materials. Plotted is $f$ with full cross sections and its error band as a function of $z$, where for each $z$ its average $y$ was taken. The dilution factor with one photon exchange cross sections is plotted with the dashed line.

have been obtained with the radiative corrections program TERAD[66], the same program was used to retrieve the full cross sections. The corrections were calculated on a grid of $(x,y)$ and an interpolation was done to calculate the dilution factor at other $(x,y)$ values. Fig. 5.9 shows the dilution factor for the SMC proton and deuteron target as a function of $x$ at the average $y; <y>(x)$. The error bands are plotted as well, and are discussed below. The dashed line correspond to the dilution factor with one photon exchange cross sections.
For the discussion about the systematic error of the full cross section ratios it is important to realise that the NMC results of the structure function ratios have mostly been obtained from measurements of the full cross section ratios at 200 GeV. Therefore the full cross section ratios at 100 (and 190) GeV are sensitive only to the energy dependence of the radiative corrections, i.e.

\[
\frac{\sigma_A^{100}(x,y)}{\sigma_d^{100}(x,y)} = \frac{RC(A,x,y,200) \cdot RC(d,x,y,200)}{RC(d,x,y,200) \cdot RC(A,x,y,100)} \cdot \frac{\sigma_A^{200}(x,y)}{\sigma_d^{200}(x,y)}.
\] (5.27)

The dependence of the radiative corrections in a given \((x,y)\) point on the muon energy is small, at most a few per cent between 100 and 200 GeV. Since the RC ratios have correlated errors, the systematic error introduced by translating the full cross section ratios at 200 GeV to the corresponding ones at 100 (or 190) GeV is at most the same as the error introduced in deriving the \(F_2\) ratios from the full cross section ratios. Hence the errors on the full cross section ratios at 100 (and 190) GeV were taken to be the errors on the \(F_2\) ratios as quoted in the NMC papers. Since these errors already include an uncertainty due to the radiative corrections, we do not introduce an extra uncertainty due to the RC as used in eq. (5.26).

For the proton measurement special attention was paid to the small \(x\) region. The average \(y\), and therefore the magnitude of the RC, is larger than that for the NMC measurement. For the run with the proton target in 1993 the SMC has increased the acceptance for events with high \(y\), for which the depolarisation is smaller (\(D\) larger). The contribution of the radiative elastic tail (elastic scattering on the nucleus with radiation of a real photon) for heavier nuclei such as copper and carbon is up to several times as large as that of the Born cross section in this kinematic region. The theoretical uncertainty in the radiative elastic tail, however, is less than one per cent [76]. Also the dependence on the electromagnetic form factors is small since the contribution goes as \(1/Q^4\) and at small \(Q^2\) different parametrisations give similar values.

### 5.3.4 Vertex smearing and target cuts

The vertex cuts influence the value of the dilution factor. In order to find the best cut, the uncertainty due to vertex smearing was examined for the two cases already introduced, wide and narrow cuts. This study was done for the deuterated target used in 1992.

**Corrections and uncertainties**

With the help of fits to the vertex distribution an estimate was made of the size of the corrections to the dilution factor. For that purpose, a box convoluted with a Lorentzian and a Gaussian was fitted to each of the helium volumes and the target cells, where the relative positions and widths of the boxes were taken from target holder drawings. An example of a vertex fit for the deuterium target is shown in Fig. 5.10.

In the case of wide cuts, in which the helium between the target cells, upstream and downstream of the cells is taken within the cuts, the only smearing effect of significance is the "cross talk" between the two cells. Since one of the cuts is taken in the middle of the helium between the two cells, the net leak-in/out of events from helium is zero. At first glance one would assume the same to be true for events from the target material. The exchanged events however have opposite polarisation. This means that the asymmetry will be reduced by a
factor of \((1 - 2\delta)\), where \(\delta\) is the leak fraction. From the vertex fit results the correction to be applied to the asymmetry, which can be put as a correction to the dilution factor, is calculated to be \(-0.8\%\) with an estimated error of \(0.3\%\). In fact the two target halves have slightly different \(\delta\). This has been accounted for properly.

If the vertex cuts are placed at the edges of the two target cells (narrow cuts), the interaction vertex of some events originating from helium just outside the cells will be smeared into the region within the vertex cuts, and some of the events from the target material will be smeared out. The correction that has to be applied to the dilution factor to account for these effects, requires the knowledge of the fraction of events form helium leaked-in with respect to events from the target cell within the vertex cuts. It is not necessary to know the fraction of events from the target material leaked out, since it is similar to a decrease in target thickness or acceptance. The dilution factor \(f'\), taking into account the vertex smearing, is then equal to

\[
f' = \frac{N_{tc}}{N_{tc} + N_{He-li}} \cdot (1 - 2\delta) \cdot f, \tag{5.28}
\]

where \(N\) represents numbers of events within the vertex cuts, “tc” stands for all the material.
within the target cuts, and "He-li" for the leak-in events from helium outside the cuts. The term \((1 - 2\delta)\) accounts for the fact that in the case of the narrow cuts still some cross talk occurs. In this equation \(f\) is the dilution factor calculated without taking vertex smearing into account. Again using the vertex fit one finds a correction due to helium events being smeared into the target cells of \(-1.3 \pm 0.6\%\). The correction for the cross talk in the case of narrow cuts is \(-0.3 \pm 0.15\%\). The total correction to the dilution factor due to vertex smearing for the narrow cuts is then \(-1.6 \pm 0.7\%\).

**Target position**

From the fits to the vertex distribution the center of the target was determined to be \(x = -4.859\) m, which is 4 mm from the design value. The effect of a shift of the center of 4 mm was examined; in both choices of vertex cuts the correction to the dilution factor is less than 0.1%.

**Statistics**

On one hand the wide cuts have the advantage that all the events from the polarised material are used. On the other hand the dilution factor is on average 0.93 of that for the narrow cuts (see Table 5.2). The relative statistical error in the measured asymmetry \(A\), equal to the relative statistical error in \(A_1\), depends on both aspects:

\[
\frac{\delta A}{A} = \sqrt{1 \over A \cdot |N_1 - N_2|},
\]  

(5.29)

where \(N_1\) and \(N_2\) are the measured numbers of events for the two spin orientations (averaged to cancel acceptance differences). The measured asymmetry depends on the dilution factor, while the difference \(|N_1 - N_2|\) depends on the number of events from the polarised material. The estimated fraction of events from the target material that is cut away by the narrow cuts is 6.5%. Combined with the value of the dilution factor ratio one finds that the statistical uncertainty on the asymmetry \(A_1\) is essentially the same for the two vertex cuts.

**Conclusion**

On statistical grounds there is no preference for either of the cuts. The error due to smearing is larger for the narrow cuts, but since this is compensated by the fact that the relative amount of helium and copper is better known in that case, as can be seen in Table 5.2, the narrow vertex cuts were used in the present analysis.

The vertex smearing correction to the dilution factor for the proton data taken with the new polarised target was determined in a similar way to be between \(-1.8 \pm 0.6\%\) for large angles, and \(-2.5 \pm 0.9\%\) for small scattering angles.

### 5.3.5 Polarised background

Due to imperfect deuterisation, the deuteron target material still contains a small fraction of polarisable protons. This leads to a correction on the extracted deuteron asymmetry. To evaluate this correction, eq. 5.1 is modified to:

\[
N = \Phi \cdot a \cdot n \cdot \sigma_0 \left[ 1 - f P_T P_n (A^d + \frac{P_T}{P_d} \frac{\sigma_P}{\sigma_d} \frac{n_P}{n_d} A^P) \right].
\]  

(5.30)
The polarisation of the proton is taken twice as high as that of the deuteron, with a relatively large error of 20%, which reflects that the equal spin temperature assumption is not necessarily fulfilled in this case. For $A_T^p = A_T^d/D$ and its error the EMC result [4] is taken, and for the cross section ratio that of the NMC [73]. The nucleon ratio follows from the deuteronisation figure of $99.4 \pm 0.3\%$. In Table 5.3 the resulting corrections to the deuteron asymmetry of up to a few per cent are given.

<table>
<thead>
<tr>
<th>$x$</th>
<th>$A_T^p$</th>
<th>$A_{pb}$</th>
<th>$\Delta A_{pb}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.009</td>
<td>-0.003</td>
<td>0.00008</td>
<td>0.00024</td>
</tr>
<tr>
<td>0.015</td>
<td>0.005</td>
<td>0.00014</td>
<td>0.00027</td>
</tr>
<tr>
<td>0.025</td>
<td>0.007</td>
<td>0.00025</td>
<td>0.00031</td>
</tr>
<tr>
<td>0.035</td>
<td>-0.010</td>
<td>0.00037</td>
<td>0.00036</td>
</tr>
<tr>
<td>0.050</td>
<td>-0.013</td>
<td>0.00056</td>
<td>0.00046</td>
</tr>
<tr>
<td>0.079</td>
<td>-0.014</td>
<td>0.00092</td>
<td>0.00065</td>
</tr>
<tr>
<td>0.123</td>
<td>0.022</td>
<td>0.00149</td>
<td>0.00097</td>
</tr>
<tr>
<td>0.173</td>
<td>0.071</td>
<td>0.00213</td>
<td>0.00133</td>
</tr>
<tr>
<td>0.241</td>
<td>0.163</td>
<td>0.00296</td>
<td>0.00182</td>
</tr>
<tr>
<td>0.343</td>
<td>0.302</td>
<td>0.00412</td>
<td>0.00253</td>
</tr>
<tr>
<td>0.470</td>
<td>0.504</td>
<td>0.00540</td>
<td>0.00334</td>
</tr>
</tbody>
</table>

Table 5.3: The polarised background correction $A_{pb}$ to $A_T^p$ due to the contaminant hydrogen in the target, and the error $\Delta A_{pb}$ on the correction.

Other sources of polarised background may arise from the non-zero spin nuclei $^{13}$C, $^{23}$Na and $^{53}$Cr. The quantities of these Na and Cr isotopes are however so small, that their effects are negligible. For $^{13}$C, with an abundance of 1.1%, the effect is at most 0.1%.

### 5.4 Possible false asymmetry

As indicated in Section 5.1.2, a false asymmetry could be introduced by drifts of the detector acceptance, and could overshadow the small asymmetry due to the spin dependence of the cross section, especially for the deuteronium measurement. The ratio of acceptances for the upstream and downstream target cells has to be constant before and after polarisation reversal, in order to avoid such an effect.

#### 5.4.1 Avoiding significant false asymmetries

During data taking the performance of the spectrometer was kept as constant as possible. The target polarisation was reversed roughly every eight hours in the deuteron and every five hours in the proton experiment, so that only drifts in the spectrometer performance over such time scales would contribute.

In the off-line analysis the data of runs with a certain polarisation configuration are combined with those taken with the opposite one. The way the runs are combined in sets of configurations is governed by the concern for false asymmetries. Data before and after a jump observed
in the behaviour of one or more detectors were not combined. Therefore, an important part of the analysis consists of searching for such discontinuities in spectrometer performance. The parameters in this evaluation are the following.

- Information from the Phenix reconstruction phase: track reconstruction efficiencies for the detector groups, timing information and $\chi^2$ probabilities for various classes of reconstructed tracks. These quantities are available in so-called Phenix run summary files, and discontinuities were traced by plotting them as a function of run number.

- Plane efficiencies. They are determined by investigating tracks in a chamber and calculating the ratio of numbers of tracks that actually had a hit in the plane under investigation, and tracks that should have had a hit in that plane. For every plane the average efficiencies per run were calculated and for most detectors the planes were split into an inner and outer region as well. Although a small jump in one of these efficiencies does not necessarily have an effect on the reconstruction, combining data before and after an observed efficiency change was avoided.

Runs before and after a data taking interruption of more than several hours were not combined either.

![Diagram](image)

**Figure 5.11:** Two ways to combine the data, the full and half configurations.

These precautions take care of jumps in the detector performance, provided they are observed. The effect of gradual changes, however, is not ruled out, but depends on the way data are combined. Two ways, see Fig. 5.11, are considered. One is to construct "full configurations", i.e. to combine all runs of one polarisation configuration with all runs of the next configuration. The alternative is to use "half configurations", in which each time the first half of the data of a configuration is combined with the last part of the previous configuration, and its second half with the first part of the next configuration. In this way a gradual change in the ratio of acceptances is largely cancelled, since the order of the A(I) and B(II) configurations is now AB-BA-AB-BA, instead of the original time order AB-AB-AB-AB for the full configuration pairs. The effect of a linear change of the acceptance ratio is opposite for an AB and a BA configuration set. In the deuteron analysis the "half configuration pairs" were used in the asymmetry extraction. For the proton analysis the "full configuration pairs" were used, since it already leads, in combination with the restrictions due to observed discontinuities in
spectrometer behaviour, to short data periods to extract an asymmetry.

5.4.2 Estimate of remnant false asymmetries

The above-mentioned procedure is only a safeguard against false asymmetries to first order. Therefore we now evaluate the upper limit for the remnant false asymmetry. The procedures are first outlined, after which the estimates are given.

One may trace false asymmetries by using the data such as to calculate an asymmetry that should be zero, for example by combining only data with the same polarisation configuration. Another check would be to compare the asymmetry for data from, for example, the middle of the target cells to that of the outer target regions. The main problem with these studies is however that the error in the estimate is always at best the same as that in the physics asymmetry extracted from the same data.

An estimate of the remaining false asymmetry that is statistically much more significant can be obtained from studies using the same data twice. If an asymmetry were to be calculated by using the data of a certain configuration, and combining it with the same data assuming the polarisation to be inverted, the resulting asymmetry would be zero. Now, if one modifies the data for one of the "polarisation configurations" to simulate an efficiency change in a chamber, this will lead to a non-zero asymmetry, which equals the false asymmetry due to this change. Because of the correlated input, the error is zero for the zero asymmetry case, and increases with increasing asymmetry, since the correlation weakens. The (still) correlated statistical error on the false asymmetry can be determined by a slight modification of the error calculus. Efficiency changes of importance are those between configurations that are combined in the physics analysis. If the treated configuration is combined with a later one in the physics analysis, a decrease in efficiency of a plane over time is simulated by killing hits in the second processing of this data, i.e. after the "reversal". For the same data an increase in plane efficiency is simulated by killing hits for that plane in the first processing, leading to a higher efficient plane in the second processing, after the "reversal". The assumption is that the effect of a real increase from $\epsilon$ to $\epsilon + \delta$ is equivalent to that of the simulated increase from $\epsilon - \delta$ to $\epsilon$, which seems reasonable for small $\delta$'s and high $\epsilon$'s. Data of configurations combined with an earlier one in the physics analysis are treated the opposite way, since an efficiency increase over time implies that the data under study is combined with a configuration with a lower efficiency. In Fig. 5.12 the treatment of data for the false asymmetry studies is visualised.

The estimated maximum of the false asymmetry was extracted with a slightly modified program, and by using the same configuration and polarisation files as for the extraction of $A_1$. The real variation over time of the various plane efficiencies, obtained from the global (or split in inner and outer region) plane efficiencies versus run number (see above), was used as input to modify the data. Hits were discarded at the micro level according to the efficiency drop to be simulated. The criterion for an event not to survive is that after having killed the hits the plane requirements are no longer fulfilled. Since the weighting of the physics asymmetries is done as in eq. (5.10), the false asymmetries from the various sets of configurations enter in
Figure 5.12: The treatment of the data for the false asymmetry studies. The data are processed twice, where, depending on the combination of data in the physics analysis, hits are killed in the first and second processing, according to observed efficiency variations. An asymmetry is calculated from the combination of the first and second processing of the same data.

$A_1$ with the same weight, i.e.

$$A_f(x, Q^2) = \frac{\sum_i (A_{fi}/\sigma_i^2)}{\sum_i (1/\sigma_i^2)},$$

(5.31)

where $\sigma_i^2$ are the uncorrelated errors used in the extraction of $A_1$. The (correlated) errors on the false asymmetries per configuration are combined in the same way, weighted with the uncorrelated errors.

For the proton data such systematic studies resulted in an upper limit of the false asymmetry contribution to $A_1^P$ of 0.004. A more conservative estimate is used however, which is determined by imposing the largest observed efficiency changes between polarisation reversals on the data. This results in a false asymmetry of $A_f < 0.007$, which translated in terms of $K$ yields the estimate $|K - 1| < 0.0007$. From the studies of the deuteron data, taken with a less stable spectrometer, and with less frequent polarisation reversals as for the proton data, we concluded that $|K - 1| < 0.002$. Because of the lower polarisation the effect of $K \neq 1$ on $A_1$ is larger.

5.5 Systematic errors on $A_1$

A dangerous source of systematic error, the possible false asymmetry, is treated separately above. The experiment is largely ruled by the concern of avoiding any significant and estimating any remnant false asymmetries. Some other errors were already treated elsewhere, such as that on the target polarisation. The other systematic errors are discussed in this section.
5.5.1 Error due to neglecting $A_2$

The effect of neglecting $A_2$ in the analysis follows directly from eq. (5.2); it is equal to $\eta A_2$, where $\eta$ is a kinematical factor of the order of 0.01. Part of the programme of the 1993 run was to measure $A_2$ for the proton; $A_2^p$ was found to be compatible with zero with an uncertainty of $0.2$ [77]. As is shown in Fig. 5.13, this leads to an improvement of the error due to $A_2$ compared to the only information available before, i.e. the positivity limit $|A_2| \leq \sqrt{R}$. For the $A_2^d$ analysis the highest value for $R$ allowed by the SLAC parametrisation for $R$ [78] was used as an estimate for the error introduced by neglecting $A_2$.

![Figure 5.13: The measurement of $A_2^p$ by the SMC, compared to the positivity limit $A_2 < \sqrt{R}$ (solid line).](image)

5.5.2 Error due to the uncertainty in the dilution factor $f$

From eq. (5.4) it follows that the contribution to the relative error in $A_1$ equals the relative error in $f$. The main uncertainty in the dilution factor stems from that in the cross section ratios (Section 5.3.3): For the deuteron measurement the relative error on the cross section ratios was taken to be that on the published $F_2$ ratios. For the proton measurement the error at small $x$ was increased, since the average $y$, and thus the size of the radiative corrections, is larger than that for the NMC measurement of the $F_2$ ratios. Since the dominant correction at small $x$ is the coherent tail, the theoretical uncertainty in this contribution was included in the error if the radiative correction was larger than 1.6, the value used as a cut in the NMC analysis. Some cross section ratios were derived from (partly) correlated data. In such cases the errors were combined linearly.

The abundances of different materials in the beam have also uncertainties that contribute to
the error in \( f \). The only significant contributions arise from the uncertainties in the relative amounts of copper, teflon and helium with respect to the amount of target material, and in the \(^3\text{He}/^{4}\text{He}\) ratio. The relative amount of copper (and teflon) depends on the amount of target material and copper seen by the beam. The estimated uncertainty in the target material mass is increased to 4\%, to account for a possible empty space in the upper part of the target cells. Since such an empty space at the top would hardly be seen by the beam, it would effectively lead to a higher dilution factor. The attributed uncertainty corresponds to the effect of a 3 mm high empty space. The uncertainty in the relative amount of helium depends on the uncertainty in the filling factor, which in turn depends on the uncertainty in the target cell volume and the mass and density of the target material. The error in the composition of the target beads is negligible.

The errors due to vertex smearing are taken to be half of the correction.

Fig. 5.9 shows the dilution factor for the deuterium and proton targets together with their errors. In Table 5.4 the input errors and their effect on the dilution factor are presented for the relevant variables.

<table>
<thead>
<tr>
<th>source</th>
<th>contribution to ( \Delta f/f(%) )</th>
<th>deuteron</th>
<th>proton</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( x=0.006 )</td>
<td>( x=0.02 )</td>
<td>( x=0.5 )</td>
</tr>
<tr>
<td>cross sections</td>
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<td>1.2</td>
<td>0.9</td>
</tr>
<tr>
<td>target mat. mass</td>
<td></td>
<td>1.0</td>
<td></td>
</tr>
<tr>
<td>(^3\text{He}/^{4}\text{He})-ratio</td>
<td></td>
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<td></td>
</tr>
<tr>
<td>vertex smearing</td>
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<td>0.7</td>
<td></td>
</tr>
<tr>
<td>cell volume</td>
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<td></td>
<td></td>
</tr>
<tr>
<td>copper mass</td>
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<td>0.20</td>
<td>0.15</td>
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<tr>
<td>target mat. density</td>
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<td>0.1</td>
<td></td>
</tr>
<tr>
<td>teflon mass</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>total error</td>
<td></td>
<td>1.9</td>
<td>1.8</td>
</tr>
</tbody>
</table>

Table 5.4: The various contributions to the uncertainty in the dilution factor.

5.5.3 Error due the target and muon polarisation

The error in the target polarisation is treated in Chapter 4. The total relative uncertainty on the target polarisation was found to be 5\% for the deuteron target and 2.5\% for the proton target.

The muon polarimeter is described in Chapter 3. The relative statistical error in the measurement of the polarisation of the 100 GeV muon beam used for the deuteron measurement was found to be 5\%; the error for the 190 GeV beam used for the proton measurement 3\%. The overall systematic error was determined to be 2\% for both the 100 GeV and the 190 GeV measurement. Since the extraction of the asymmetry is insensitive to the cause of the uncertainty in the polarisation, the systematic and statistical error in the muon polarisation measurement were added in quadrature and propagated as a systematic error on \( A_1 \).
Deuteron

Figure 5.14: The deuteron asymmetry as a function of $Q^2$ for different values of $x$. The squares are values obtained from E142 neutron data combined with E80+E130 proton data. The numbers in parentheses correspond to the vertical scale offset of the values for each value of $x$.

5.5.4 Other systematic errors

The other sources of systematic errors are listed below. Some of them are discussed above. The effects of the individual uncertainties are listed in Tables 5.5 and 5.6.
Figure 5.15: The proton asymmetry as a function of $Q^2$ for different values of $x$. The data plotted include SMC (circles), EMC (squares) and SLAC E80 and E190 (diamonds) data. The numbers in parenthesis correspond to the vertical scale offset of the values for each value of $x$.

Radiative corrections to the asymmetry
Although the list of diagrams taken into account in the program POLRAD is not as complete as in the program TERAD, the main uncertainty in the radiative corrections on the asymmetry is due to the uncertainty in the asymmetry itself, since it is needed as an input. The input asymmetries were therefore varied in accordance with the statistical error of the experimental data to determine the error in the radiative corrections.

Polarised background correction
Uncertainties in the polarised background correction for the deuterium measurement arise
mainly from the error in the polarisation of the contaminant hydrogen and from the error in $A_1^q$.

**Kinematic smearing**

The finite resolution of the spectrometer does not only lead to smearing of the position of the interaction vertices, but also to smearing of the kinematic variables. The effect of this smearing was studied by a Monte Carlo (MC) simulation. The main effect on the extracted $A_1$ is due to the dependence of $f$ and $D$ on the kinematic variables, which leads to a multiplicative correction

$$\frac{(D \cdot f)_{\text{true}}}{(D \cdot f)_{\text{MC}}}, \quad (5.32)$$

the ratio of the "true" and reconstructed quantities in the MC study. For every $x$ bin the weighted sum was taken over the $Q^2$ bins. The resulting correction factor is significantly different from unity only for the highest three $x$ bins, the largest effect being a 3% deviation. This result was included in the systematic error.

**Momentum determination**

Uncertainties in the calibrations of the incoming muon momentum, measured in the BMS, and the scattered muon momentum, determined from the deviation from a straight track in the FSM field, lead to an uncertainty in the determination of $x$ and $Q^2$. The effect of an uncertainty of 0.2 % in the FSM calibration, combined with and 0.5% uncertainty in the incoming muon momentum, was determined and included in the systematic error in $A_1$.

**Error due to $R$**

The error in $R$ propagates to $A_1$ via its influence on the depolarisation factor:

$$\Delta A_{1,R} = \left( \frac{\partial A}{\partial D} \right) \left( \frac{\partial D}{\partial R} \right) \Delta R = \frac{2(1-y)}{y(2-y)} A_1 D \cdot \Delta R \quad (5.33)$$

The error in $R$ was taken from the SLAC R1990 parametrisation[78].

### 5.6 Results for $A_1$ of the deuteron and proton

The final results for $A_1^d$ and $A_1^p$ are given below with their errors, and some consistency checks are presented.
5.6. Results for $A_1$ of the deuteron and proton

Figure 5.16: SMC results for $A_1^d$. The error bars are the statistical error. The systematic error is indicated by the band.

Figure 5.17: SMC results for $A_1^p$. The error bars are the statistical error. The systematic error is indicated by the band. Included as well are the measurements available before the SMC experiment.

In Fig. 5.14 and 5.15 the deuteron and proton asymmetry and their statistical errors are plotted per $x$ bin as a function of $Q^2$. Each $x$ bin has a different offset. Since within the statistical error there is no significant $Q^2$ dependence, data of various $Q^2$ were combined to
determine the asymmetry as a function of \( x \) only. The results for \( A_1^d \) and \( A_1^p \) are shown in fig. 5.16 and 5.17. The error bars are statistical errors and the systematic error is visualised by the grey bar along the \( x \) axis. The same results are also presented in Tables 5.5 and 5.6, where the systematic errors are split into the various contributions as well. All results were obtained using the \( Df \) method. The average values for \( x, y \) and \( Q^2 \) were obtained by weighting events with their statistical significance, i.e. \( (Df)^2 \).

The deuteron result published by the SMC [85] is slightly different from the result presented here, mainly due to the fact that for the publication the refined second order method, which requires \( Q^2 \) binning, was used, with an event cut of 10. The MC studies that made clear that this is not the best way to extract the asymmetry were done after the publication. The proton asymmetries published by the SMC [86] are slightly lower than the ones presented here, due to the fact that for the publication the shape of the NMR coils was not weighted with the beam profile, and that the vertex smearing was not taken into account.

<table>
<thead>
<tr>
<th>( x ) range</th>
<th>( &lt; x &gt; )</th>
<th>( &lt; Q^2 &gt; )</th>
<th>( &lt; y &gt; )</th>
<th>( A_1^d )</th>
<th>#events</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.006-0.010</td>
<td>0.0085</td>
<td>1.2</td>
<td>0.745</td>
<td>(-0.042 \pm 0.070 \pm 0.013)</td>
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</tr>
<tr>
<td>0.010-0.020</td>
<td>0.0145</td>
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<td>0.665</td>
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</tr>
<tr>
<td>0.020-0.030</td>
<td>0.0246</td>
<td>2.8</td>
<td>0.608</td>
<td>(-0.040 \pm 0.056 \pm 0.016)</td>
<td>313784</td>
</tr>
<tr>
<td>0.030-0.040</td>
<td>0.0347</td>
<td>3.6</td>
<td>0.564</td>
<td>(-0.079 \pm 0.069 \pm 0.020)</td>
<td>262602</td>
</tr>
<tr>
<td>0.040-0.060</td>
<td>0.0490</td>
<td>4.8</td>
<td>0.524</td>
<td>(0.088 \pm 0.064 \pm 0.023)</td>
<td>419389</td>
</tr>
<tr>
<td>0.060-0.080</td>
<td>0.0770</td>
<td>6.7</td>
<td>0.471</td>
<td>(0.005 \pm 0.066 \pm 0.024)</td>
<td>594975</td>
</tr>
<tr>
<td>0.060-0.100</td>
<td>0.1216</td>
<td>9.3</td>
<td>0.413</td>
<td>(0.118 \pm 0.091 \pm 0.030)</td>
<td>475202</td>
</tr>
<tr>
<td>0.100-0.150</td>
<td>0.1721</td>
<td>12</td>
<td>0.372</td>
<td>(0.170 \pm 0.133 \pm 0.034)</td>
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<tr>
<td>0.150-0.200</td>
<td>0.2402</td>
<td>15</td>
<td>0.335</td>
<td>(0.232 \pm 0.150 \pm 0.039)</td>
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</tr>
<tr>
<td>0.300-0.400</td>
<td>0.3410</td>
<td>19</td>
<td>0.296</td>
<td>(0.211 \pm 0.266 \pm 0.042)</td>
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</tr>
<tr>
<td>0.400-0.600</td>
<td>0.4651</td>
<td>23</td>
<td>0.263</td>
<td>(0.60 \pm 0.403 \pm 0.042)</td>
<td>39335</td>
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</tbody>
</table>

Table 5.5: Results for \( A_1^d \) per \( x \) bin. In the lower section the various contributions to the systematic error are given, where the total error is obtained by adding in quadrature.\(^1\)

In fig. 5.18 the results for \( A_1 \) obtained with the various methods are compared. The fact that

\( ^1 \)These numbers include some additional corrections that were applied after the respective SMC publications, and result from a private analysis. Final values will appear in forthcoming SMC publications.
Table 5.6: Results for $A_1^p$ per $x$ bin. In the lower section the various contributions to the systematic error are given, where the total error is obtained by adding in quadrature.\(^1\)

\[\begin{array}{|c|c|c|c|c|c|c|}
\hline
x range & < x > & < Q^2 > & < y > & A_1^p & \# events \\
\hline
0.003-0.006 & 0.0047 & 1.3 & 0.812 & 0.056 ± 0.026 ± 0.007 & 518045 \\
0.006-0.010 & 0.0078 & 2.1 & 0.776 & 0.044 ± 0.025 ± 0.006 & 510236 \\
0.010-0.020 & 0.0143 & 3.7 & 0.737 & 0.050 ± 0.023 ± 0.005 & 653037 \\
0.020-0.030 & 0.0246 & 6.0 & 0.692 & 0.052 ± 0.033 ± 0.005 & 379467 \\
0.030-0.040 & 0.0347 & 8.1 & 0.663 & 0.072 ± 0.040 ± 0.006 & 281747 \\
0.040-0.060 & 0.0490 & 10.8 & 0.629 & 0.129 ± 0.036 ± 0.009 & 425064 \\
0.060-0.080 & 0.0771 & 15.5 & 0.574 & 0.167 ± 0.036 ± 0.013 & 554018 \\
0.080-0.100 & 0.1218 & 22.1 & 0.517 & 0.286 ± 0.049 ± 0.020 & 426126 \\
0.100-0.150 & 0.1725 & 28.5 & 0.471 & 0.283 ± 0.069 ± 0.021 & 272769 \\
0.150-0.200 & 0.2415 & 36.3 & 0.429 & 0.277 ± 0.073 ± 0.023 & 300967 \\
0.300-0.400 & 0.3416 & 46.4 & 0.387 & 0.549 ± 0.120 ± 0.044 & 125947 \\
0.400-0.700 & 0.4808 & 57.9 & 0.345 & 0.540 ± 0.162 ± 0.051 & 71332 \\
\hline
\end{array}\]

the first order method has results very similar to those of the refined second order method indicates that the effect of the actual polarisation variations is small. This is an important observation since the polarisation treatment of the $Df$ method is the same as of the first order method. The $Df$ method is very similar to the refined second order method. Only in the high $x$ region of the proton measurement there is a significant difference. Monte Carlo studies\(^65\), as mentioned in Sec. 5.1.5, show that this difference would become smaller if the time between reversals, and thus the count rates, would increase, and that the $Df$ method recovers the input asymmetry best in all cases. Although for the “full configurations” exactly the same runs and “combination inhibits” were used as for the “half configurations”, the effective use of the data is slightly different. Combining ten runs of one configuration with four of the following configuration brings the impact of the ten runs down compared to combining the same ten runs with ten other runs. It was shown via the MC study that the difference seen between the half and full configurations can be explained by this effect.
Figure 5.18: The results for $A_1$ obtained with the different methods of asymmetry extraction. The values for these methods are slightly shifted in $x$. 
Chapter 6

The spin dependent structure function $g_1$ of the deuteron and the proton

In this chapter the extraction of the structure functions $g_1$ from the measured $A_1$ for the proton and the deuteron in combination with the unpolarised structure functions $R$ and $F_2$ is discussed. The SMC data are compared and combined with results from other experiments. The first moments $\Gamma_1$ of $g_1(x)$ determined from these results, are compared with the Ellis-Jaffe predictions. The Bjorken sum rule is tested by combining the values of $\Gamma_1$ obtained in the various experiments. In addition, values for the contributions of quark spins of different flavours to the nucleon spin are derived. The chapter concludes with a discussion of the results and an outlook. The numbers presented in this chapter include some additional corrections that were applied after the respective SMC publications, and are the result of a private analysis. Final values will appear in forthcoming SMC publications.

6.1 From $A_1$ to $g_1$

To begin with, the structure functions $g_1^d$ and $g_1^p$ as a function of $x$ were determined at the average $Q^2$ of the corresponding $x$-bin. In order to obtain $\Gamma_1$, they were also evaluated at a fixed value of $Q^2$.

6.1.1 Determination of $g_1(x, < Q^2 >)$

The spin dependent structure function $g_1$ is related to the virtual photon absorption asymmetry $A_1$ presented in the previous chapter, through

$$g_1(x, Q^2) = \frac{A_1(x, Q^2)F_2(x, Q^2)}{2x(1 + R(x, Q^2))}, \quad (6.1)$$
where the influence of $A_2$, see eq. 2.19, is included in the systematic error on $A_1$. Using this relation one can determine $g_1$ from $A_1$ obtained in a number of $(x, Q^2)$ bins, provided that $F_2$ and $R$ are known over the measured range.

![Figure 6.1: The NMC, SLAC and BCDMS data on $F_2^d$. The solid line through the data points is the QCD-inspired parametrisation used in the SMC analysis.](image)

The spin independent structure functions $F_2^p(x, Q^2)$ and $F_2^d(x, Q^2)$ have been measured recently with high precision by the NMC [79]. This collaboration published for both nuclei a QCD inspired parametrisation of their own data combined with earlier results from SLAC[80] and BCDMS[81]. Fig.'s 6.1 and 6.2 show these structure function data and their parametrisations, which were used for the SMC analysis. For the lowest $x$ point of the SMC proton data, $x = 0.003$, $F_2^p$ is not known experimentally. The parametrisation of $F_2^p(x, Q^2)$ is consistent however with more recent HERA $F_2^p$ data[82] at even much smaller $x$, as shown in Fig. 6.3. For the structure function $R$, the SLAC parametrisation from 1990[78] was used. Fig. 6.4 shows the parametrisation at the $x$-bin averages $<Q^2>$ and at fixed $Q^2 = 10$ GeV$^2$.

The statistical significance of $A_1$ (and hence of $g_1$) determined for individual $(x, Q^2)$ bins is relatively low (see fig. 5.15). Since, as shown in section 5.6, there is no experimental evidence for $Q^2$ dependence of $A_1$ within each $x$ bin, the data at different $Q^2$ within an $x$ bin were combined.

The statistical and systematic uncertainties in $A_1$ propagate to $g_1$. The uncertainty in $g_1$ due to that in $R$ requires special care, since $R$ influences $g_1$ in three ways. It affects $A_1$ via the depolarisation factor, eq. (2.23), $R$ appears explicitly in eq. (6.1), and the NMC used the same parametrisation of $R$ to extract $F_2$ from measured cross sections. The relative uncertainty in $g_1$ associated with $R$ is roughly half of that in $A_1$, because the effects of the uncertainties in $R$ partially cancel each other. For the uncertainty in $F_2^p$ and $F_2^d$, the errors as
Figure 6.2: The NMC, SLAC and BCDMS data on $F_2^p$. The solid line through the data points is the QCD-inspired parametrisation used in the SMC analysis.

quoted by the NMC, were taken. A relative error of 15% was estimated for $F_2^p$ at $x = 0.003$.

The results for $g_1(x)$ at $<Q^2>$ per $x$ bin are presented in Tables 6.1 and 6.2. The value of $<Q^2>$ per $x$ bin was obtained by weighting the events with their statistical significance in the $A_1$ analysis (see Section 5.1.1).

6.1.2 Evaluation of $g_1(x)$ at fixed $Q^2$

The physics interpretation of the first moment of the spin dependent structure function $g_1(x)$ at a given value of $Q^2$ requires relatively few assumptions, as is discussed in Chapter 2. In order to calculate this first moment, $g_1$ was evaluated at a fixed value of $Q^2$, central to the data, by using eq. (2.19) with $F_2$ and $R$ at this central $Q^2$-value for all $x$ bins. For the deuteron measurement the central value of $Q^2$ is 4.6 GeV$^2$, for the proton it is equal to 10.0 GeV$^2$.

The systematic error in $g_1$ was calculated as mentioned in subsection 6.1.1. Additional attention was paid to $F_2^D$ in the lowest four $x$ bins. At those small $x$ values, $F_2^D$ has not been measured at the central value of $Q^2 = 10$ GeV$^2$, as can be seen in figure 6.2. In investigating the gluon distributions[16], the NMC determined the logarithmic derivative $\partial \ln(F_2(x, Q^2))/\partial \ln(Q^2)$ and its uncertainty for the proton and the deuteron, consistent with the parametrisation of $F_2^p$. The error in this derivative was used to estimate the error in $F_2^D$ due to the extrapolation uncertainty.

The results for $g_1^p(x)$ and $g_1^d(x)$ at $Q^2 = 4.6$ GeV$^2$ and $Q^2 = 10.0$ GeV$^2$ for the deuteron and
6. The spin dependent structure function $g_1$ of the deuteron and the proton

![Graph showing $F_2^p$ data from HERA at small $x$, and the parametrisation by the NMC using only data with $x > 0.005$, all at $Q^2 = 15 \text{ GeV}^2$. Although this value of $Q^2$ is higher than the ones used in the SMC analysis, it makes a comparison possible. The points on the parametrisation correspond to the SMC bin centers. The lowest point in $x$ was given an error of 15%.](image)

**Figure 6.3:** $F_2^p$ data from HERA at small $x$, and the parametrisation by the NMC using only data with $x > 0.005$, all at $Q^2 = 15 \text{ GeV}^2$. Although this value of $Q^2$ is higher than the ones used in the SMC analysis, it makes a comparison possible. The points on the parametrisation correspond to the SMC bin centers. The lowest point in $x$ was given an error of 15%.

The proton are given in Tables 6.1 and 6.2, respectively, together with the statistical and total systematic errors. The results for $g_1$ at fixed $Q^2$ are also shown in figure 6.5.

6.2 The first moment of $g_1$

The determination of the first moment of $g_1(x)$ requires the knowledge of $g_1$ over the full $x$ range. In practice the integral of $g_1(x)$ is determined over the measured $x$ range, and extrapolations are made to $x = 0$ and $1$.

6.2.1 Integral over the measured $x$ range

The integral of $g_1$ over the range covered by the experiment ($x_{\text{min}} < x < x_{\text{max}}$) is calculated while multiplying the extracted value of $A_1$ in each $x$ bin with the integral containing $F_2$ and
6.2. The first moment of $g_1$

![SLAC parametrisation of $R$](image)

**Figure 6.4:** The SLAC-R1990 parametrisation of $R$, at the average value of $Q^2$ per $x$ bin, and at fixed value of $Q^2$.

$R$, over the $x$ range of the bin:

$$
\int_{x_{\text{min}}}^{x_{\text{max}}} g_1(x)dx = \sum_{i=1}^{n} \left[ A_i(x_i) \int \frac{F_2(x)dx}{2x(1 + R(x))} \right],
$$

where the summation is over all measured $x$ bins $i$, and the integral is taken over the width of each bin.

For the error in the integral a distinction is made between errors correlated (c) between $x$-bins and uncorrelated (u) errors, the latter including the statistical error. The uncertainty $\Delta I$ in the integral is then equal to

$$
\Delta I = \sqrt{\sum_{c} \left( \sum_{i} \Delta I_{c,i} \right)^2 + \sum_{i,u} (\Delta I_{i,u})^2}.
$$

The errors for the individual data points in the deuteron measurement were assumed to be correlated. For the proton measurement the statistical errors and the extrapolation errors at small $x$ were treated as uncorrelated, while the systematic errors on $F_2$ were taken to be correlated over the $x$-bins.
The spin dependent structure function $g_1$ of the deuteron and the proton

<table>
<thead>
<tr>
<th>$x$</th>
<th>$&lt;Q^2&gt;$</th>
<th>$g_1^d(x, &lt;Q^2&gt;)$</th>
<th>$g_1^d(x, Q_c^2)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.008</td>
<td>1.2</td>
<td>$-0.54 \pm 0.91 \pm 0.11$</td>
<td>$-0.79 \pm 1.33 \pm 0.27$</td>
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<td>$-0.48 \pm 0.38 \pm 0.07$</td>
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<td>0.025</td>
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<td>$-0.22 \pm 0.31 \pm 0.04$</td>
<td>$-0.25 \pm 0.34 \pm 0.09$</td>
</tr>
<tr>
<td>0.035</td>
<td>3.6</td>
<td>$-0.32 \pm 0.29 \pm 0.04$</td>
<td>$-0.34 \pm 0.30 \pm 0.08$</td>
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<td>0.049</td>
<td>4.8</td>
<td>$0.26 \pm 0.19 \pm 0.03$</td>
<td>$0.26 \pm 0.19 \pm 0.06$</td>
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<tr>
<td>0.077</td>
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<td>$0.01 \pm 0.13 \pm 0.02$</td>
<td>$0.01 \pm 0.12 \pm 0.03$</td>
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<tr>
<td>0.122</td>
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<td>$0.06 \pm 0.07 \pm 0.01$</td>
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<tr>
<td>0.465</td>
<td>23</td>
<td>$0.01 \pm 0.04 \pm 0.00$</td>
<td>$0.01 \pm 0.05 \pm 0.00$</td>
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</table>

Table 6.1: Results on the spin structure function $g_1^d$ of the deuteron, for every $x$ bin given at its average $Q^2$, and at the central value of the experiment, $Q_c^2 = 4.6 \text{GeV}^2$. The first error is statistical, the second one systematic. For the evaluation of $g_1^d$ at $Q^2 = Q_c^2$, it has been assumed that $A_1^d$ does not depend on $Q^2$.

<table>
<thead>
<tr>
<th>$x$</th>
<th>$&lt;Q^2&gt;$</th>
<th>$g_1^p(x, &lt;Q^2&gt;)$</th>
<th>$g_1^p(x, Q_c^2)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.005</td>
<td>1.3</td>
<td>$1.41 \pm 0.65 \pm 0.28$</td>
<td>$2.60 \pm 1.21 \pm 0.51$</td>
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<td>0.008</td>
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<td>$0.77 \pm 0.44 \pm 0.11$</td>
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<tr>
<td>0.014</td>
<td>3.7</td>
<td>$0.54 \pm 0.25 \pm 0.06$</td>
<td>$0.69 \pm 0.32 \pm 0.08$</td>
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<td>0.025</td>
<td>6.0</td>
<td>$0.35 \pm 0.22 \pm 0.03$</td>
<td>$0.40 \pm 0.25 \pm 0.04$</td>
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<td>0.035</td>
<td>8.1</td>
<td>$0.37 \pm 0.20 \pm 0.03$</td>
<td>$0.38 \pm 0.21 \pm 0.03$</td>
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<td>11</td>
<td>$0.48 \pm 0.13 \pm 0.03$</td>
<td>$0.47 \pm 0.13 \pm 0.03$</td>
</tr>
<tr>
<td>0.077</td>
<td>16</td>
<td>$0.40 \pm 0.09 \pm 0.03$</td>
<td>$0.38 \pm 0.08 \pm 0.03$</td>
</tr>
<tr>
<td>0.122</td>
<td>22</td>
<td>$0.42 \pm 0.07 \pm 0.03$</td>
<td>$0.39 \pm 0.07 \pm 0.03$</td>
</tr>
<tr>
<td>0.172</td>
<td>29</td>
<td>$0.27 \pm 0.07 \pm 0.02$</td>
<td>$0.26 \pm 0.06 \pm 0.02$</td>
</tr>
<tr>
<td>0.241</td>
<td>36</td>
<td>$0.17 \pm 0.04 \pm 0.01$</td>
<td>$0.17 \pm 0.04 \pm 0.01$</td>
</tr>
<tr>
<td>0.342</td>
<td>46</td>
<td>$0.17 \pm 0.04 \pm 0.01$</td>
<td>$0.19 \pm 0.04 \pm 0.01$</td>
</tr>
<tr>
<td>0.481</td>
<td>58</td>
<td>$0.06 \pm 0.02 \pm 0.01$</td>
<td>$0.08 \pm 0.02 \pm 0.01$</td>
</tr>
</tbody>
</table>

Table 6.2: Results on the spin structure function $g_1^p$ of the proton, for every $x$ bin given at its average $Q^2$, and at the central value of the experiment, $Q_c^2 = 10 \text{GeV}^2$. The first error is statistical, the second one systematic. For the evaluation of $g_1^p$ at $Q_c^2$, it has been assumed that $A_1^p$ does not depend on $Q^2$.

The resulting integrals over the measured region for the deuteron and proton then are

\[
\int_{0.006}^{0.6} g_1^d(x) dx = 0.021 \pm 0.019 \pm 0.012, \quad (Q^2 = 4.6 \text{ GeV}^2)
\]

\[
\int_{0.003}^{0.7} g_1^p(x) dx = 0.136 \pm 0.012 \pm 0.011, \quad (Q_c^2 = 10 \text{ GeV}^2)
\]

(6.4)

where the first error is statistical, the second systematic.


6.2. The first moment of $g_1$

6.2.2 Extrapolations

Figure 6.5: The extrapolations of $g_1^d$ and $g_1^p$ to the unmeasured $x$-range, and their errors. Plotted is $xg_1(x)$ versus $\log x$, which implies that the area below the $g_1$-values represents the contribution to the integral. The error bars on the measurements represent their statistical errors.

The extrapolation of $g_1$ from $x = 0.6(0.7)$ to $x = 1$ was done separately for $A_1^d$ and $F_2$ as follows. For the deuteron, $A_1^d$ was extrapolated using a parametrisation of the data, and the error on the measured $A_1^d$ in the highest $x$ bin was taken as a conservative estimate for the uncertainty of the extrapolation. The proton data extend to the somewhat larger $x = 0.7$. For their extrapolation to $x = 1$ the asymmetry was taken constant, $A_1^p = 0.7 \pm 0.3$. For $F_2^d$ and $F_2^p$ ($R \approx 0$ for $x > 0.6$) the mentioned parametrisations were used. The large $x$ extrapolations of $A_1$ and of the unpolarised structure functions lead to contributions to the first moment of $g_1$ for the deuteron and the proton of

$$
\int_{0.6}^{1.0} g_1^d(x) dx = 0.002 \pm 0.004,
$$

$$
\int_{0.7}^{1.0} g_1^p(x) dx = 0.0015 \pm 0.0007. \quad (6.5)
$$

For the extrapolation to $x = 0$ a Regge type behaviour, see Section 2.3, was assumed. For both nuclei $g_1(x)$ was fitted to a constant in the two lowest $x$ points. The error on the fitted constant was increased to include the values obtained when fitting to the lowest, or to the three lowest $x$ points. The thus estimated contributions of the not measured small-$x$ region to the deuteron and proton integral are

$$
\int_{0}^{0.006} g_1^d(x) dx = -0.004 \pm 0.003,
$$

$$
\int_{0}^{0.003} g_1^p(x) dx = 0.005 \pm 0.003. \quad (6.6)
$$
The extrapolations (with their uncertainties) are shown in figure 6.5, where \( xg_1 \) is plotted versus \( \log x \).

### 6.2.3 Results on \( \Gamma_1 \)

Combining the above results, one obtains the following values for the first moments \( \Gamma_1 \) of \( g_1 \) for the deuteron and proton

\[
\Gamma_1^d = 0.020 \pm 0.019 \pm 0.013 \quad (Q^2 = 4.6 \text{ GeV}^2)
\]

\[
\Gamma_1^p = 0.142 \pm 0.012 \pm 0.011 \quad (Q^2 = 10 \text{ GeV}^2).
\] (6.7)

In fig. 6.6 the values of \( xg_1(x) \) and that of the integral as a function of the lower integration limit are shown. The contributions to the systematic error on \( \Gamma_1 \) are given in Table 6.3.

<table>
<thead>
<tr>
<th>Source of the error</th>
<th>( \Delta \Gamma_1^d )</th>
<th>( \Delta \Gamma_1^p )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Acceptance variation ( \Delta r )</td>
<td>0.0100</td>
<td>0.0030</td>
</tr>
<tr>
<td>Neglect of ( A_2 )</td>
<td>0.0054</td>
<td>0.0022</td>
</tr>
<tr>
<td>Extrapolation at high ( x )</td>
<td>0.0040</td>
<td>0.0007</td>
</tr>
<tr>
<td>Extrapolation at low ( x )</td>
<td>0.0030</td>
<td>0.0030</td>
</tr>
<tr>
<td>Beam polarisation</td>
<td>0.0015</td>
<td>0.0059</td>
</tr>
<tr>
<td>( F_2 )</td>
<td>0.0012</td>
<td>0.0054</td>
</tr>
<tr>
<td>Target polarisation</td>
<td>0.0006</td>
<td>0.0041</td>
</tr>
<tr>
<td>Dilution factor</td>
<td>0.0004</td>
<td>0.0036</td>
</tr>
<tr>
<td>Radiative corrections</td>
<td>0.0009</td>
<td>0.0022</td>
</tr>
<tr>
<td>Kinematic resolution</td>
<td>0.0008</td>
<td>0.0010</td>
</tr>
<tr>
<td>Momentum measurement</td>
<td>0.0005</td>
<td>0.0020</td>
</tr>
<tr>
<td>( R )</td>
<td>0.0005</td>
<td>0.0020</td>
</tr>
<tr>
<td>Polarised proton background</td>
<td>0.0005</td>
<td>—</td>
</tr>
<tr>
<td>Total systematic error</td>
<td>0.0127</td>
<td>0.0114</td>
</tr>
<tr>
<td>Statistical error</td>
<td>0.0188</td>
<td>0.0119</td>
</tr>
</tbody>
</table>

**Table 6.3: Contributions to the error on \( \Gamma_1^d \) and \( \Gamma_1^p \)**

From these first moments one can obtain \( \Gamma_1^p \) provided that \( \Gamma_1^d \) and \( \Gamma_1^p \) are given at the same \( Q^2 \) value. This can be achieved by applying eq. (2.19) at \( Q^2 = 5 \text{ GeV}^2 \) to the measured values of \( A_1^d \) and \( A_1^p \). This yields

\[
\Gamma_1^d = 0.020 \pm 0.019 \pm 0.013 \quad (Q^2 = 5 \text{ GeV}^2)
\]

\[
\Gamma_1^p = 0.133 \pm 0.011 \pm 0.011 \quad (Q^2 = 5 \text{ GeV}^2).
\] (6.8)

In principle \( \Gamma_1^p = 2\Gamma_1^d - \Gamma_1^p \), if nuclear effects in the deuteron are neglected. The D-state adv-

\(^1\)The value for the proton published in [86] is \( \Gamma_1^p = 0.136 \pm 0.011 \pm 0.011 \). This is smaller due to the fact that the corrections mentioned in the previous chapter (Section 5.6) have an accumulative effect on the integral. The published value for the deuteron in [85] is \( \Gamma_1^d = 0.023 \pm 0.020 \pm 0.15 \).
6.3. Comparison and combination with other experiments

\[ \Gamma_1^p + \Gamma_1^n = \frac{2\Gamma_1^d}{1 - \frac{3}{2}\omega_d}. \] (6.9)

This leads to

\[ \Gamma_1^p + \Gamma_1^n = 0.044 \pm 0.041 \pm 0.028 \quad (Q^2 = 5 \text{ GeV}^2), \] (6.10)

and subtracting \( \Gamma_1^p \) from eq. (6.8) gives

\[ \Gamma_1^n = -0.09 \pm 0.05 \pm 0.04 \quad (Q^2 = 5 \text{ GeV}^2). \] (6.11)

6.3 Comparison and combination with other experiments

In this section the results of the experiments published before the completion of this thesis are compared to each other and a combined analysis of the world data is presented.

6.3.1 World data on \( A_1^p \), \( A_1^n \) and \( A_1^d \)

At the start of the SMC experiment the available data on the proton were from the measurements at SLAC from the E80(1976/78) and E130(1983)[5] SLAC/Yale collaborations and at CERN by the EMC(1989)[4]. Although the SMC covers a somewhat broader range in \( z \), the \((x,Q^2)\) range of the EMC and the SMC data is quite similar. The experiments carried out at SLAC used a beam of polarised electrons with an energy up to 20 GeV and therefore at
given $x$ have lower $Q^2$ than the muon experiments at CERN. Moreover the SLAC/Yale data do not cover $x$ values below 0.1. Very recently a new measurement of $g_1^p$, with high statistical accuracy, was carried out at SLAC by the E143 collaboration [83]. The experiment covers the range $0.029 < x < 0.8$ at $1.3 < Q^2 < 10$, and extracts the ratio $g_1/F_1$ from a combination of the asymmetries with electron and proton spins parallel and perpendicular. In this way no assumptions on $A_2$ are needed. The world data on $A_1^p(x)$ are collected in figure 6.7. A statistical analysis, which is described below, shows that the world proton asymmetry data are consistent within errors. This consistency of the SLAC and CERN data, in combination with their different $(x,Q^2)$ coverage, lends also support to our assumption of scaling of $A_1$.

In another recent experiment at SLAC, by the E142 collaboration [84], a measurement was done of the deep inelastic scattering asymmetry of a polarised electron beam on a polarised $^3$He gas target. The incident energy was between 19 and 26 GeV. Since $^3$He consists of a neutron and two protons, the latter mostly coupled to spin zero, the E142 target nuclei can to first approximation be considered as polarised neutrons with an intrinsic dilution factor of $1/3$. The neutron asymmetry $A_1^n$ has been determined in the range $0.03 < x < 0.6$, with a central value for $Q^2$ of 2 GeV$^2$. The results, which have good statistical accuracy, are presented in figure 6.8.

The SMC has measured $A_1^d$ at an incident muon energy of 100 GeV. Apart from relatively small corrections, $A_1^d$ represents the average of $A_1^p$ and $A_1^n$. One way to investigate the internal

\[ A_1^p \]

\[ \bullet \text{E80-E130} \]
\[ \bullet \text{EMC} \]
\[ \bullet \text{SMC} \]
\[ \bullet \text{E143} \]
consistency of the world data is to use $A_1^p$ and $A_1^n$ to construct $A_1^d$, and compare the results with the direct measurement of $A_1^d$ by the SMC. The result of such an investigation is shown in figure 6.9. Qualitatively, it is clear that the world data on $A_1$ are consistent, assuming $Q^2$ independence.

6.3.2 Combination of proton data

The proton data of the SMC, EMC, E80, E130 and E143 experiments were combined at the level of the measured asymmetries. For $x < 0.03$ the SMC binning, which coincides with that of the EMC in the region of overlap, was taken. In the range $0.03 < x < 0.8$ the E143 binning was used, while the SMC, EMC, E80 and E130 data points were moved to the $x$ value of the nearest E143 data point. This was done using a fit to the world data on $A_1^p$, by applying the expression

$$A_1^p(x_{\text{bin}}) = A_1(x_{\text{data}}) + \left( A_1^{fit}(x_{\text{bin}}) - A_1^{fit}(x_{\text{data}}) \right),$$

(6.12)

where $x_{\text{data}}$ is the mean $x$ of the original data point, and where the fit is given by

$$A_1^{fit}(x) = x^\alpha \left( 1 - e^{\beta x} \right),$$

with $\alpha = 1.54 \cdot 10^{-4}$ and $\beta = -2.26$.

(6.13)

The systematic errors on $A_1$, except the effect of the uncertainty in $A_2$ and $R$, are assumed to be uncorrelated among the different experiments. The asymmetries of the experiments were combined while weighting them with their statistical error.
Figure 6.9: Consistency of data on $A_1^p$, $A_1^n$ and $A_1^d$: Plotted are the $A_1^d$ measured by the SMC, and $A_1^d$ constructed from the $A_1^n$ measurement of E142 and a fit to the $A_1^p$ data of E80/E130/EMC/SMC.

A $\chi^2$-probability consistency test was done, in which the asymmetries from the various experiments ($j = 1, 5$) were compared with the weighted average:

$$\chi^2 = \sum_j \sum_i \left( \frac{(A_{j,i} - A_{\text{world},i})^2}{\Delta A_{j,i}} \right),$$

where the summation over $i$ involves the bins in which the experiment under study has data. The result is $\chi^2 = 28$ for 30 degrees of freedom. From this test no sign of inconsistency in the world proton data appears.

The integral of $g_1^p(x)$ over the measured $x$-range was found to be $0.135 \pm 0.005 \pm 0.009$ at $Q^2 = 5$ GeV$^2$. The small-$x$ extrapolation, $g_1^p = 1.19 \pm 0.81$, was made using the world data, giving a contribution to the first moment of $g_1^p(x)$ of $0.0037 \pm 0.0026$. For the large-$x$ extrapolation, $A_1^p = 0.7 \pm 0.3$ results in a contribution of $0.0006 \pm 0.0003$ to $\Gamma_1$. The result for the full $x$ range becomes therefore:

$$\Gamma_1^p = 0.140 \pm 0.005 \pm 0.009 \ (\text{world data, } Q^2 = 5 \ \text{GeV}^2),$$

where the systematic error includes the uncertainties in the extrapolations. In Table 6.4 a specification of the systematic error on $\Gamma_1^p(\text{world data})$ is given.
6.3. Comparison and combination with other experiments

| $P_t$ (E143) | 0.0023 |
| $P_c$ (E143) | 0.0019 |
| $f$ (E143) | 0.0023 |
| RC (E143) | 0.0019 |
| $P_t$ (SMC) | 0.0007 |
| $P_\mu$ (SMC) | 0.0009 |
| $K$ (SMC) | 0.0003 |
| $A_2$ (SMC/EMC) | 0.0019 |
| $f$ (SMC) | 0.0005 |
| RC (SMC) | 0.0013 |
| $P_t$ (EMC) | 0.0005 |
| $P_\mu$ (EMC) | 0.0004 |
| $f$ (EMC) | 0.0003 |
| $K$ (EMC) | 0.0008 |
| RC (EMC) | 0.0001 |
| $F_2^p$ | 0.0053 |
| $R$ | 0.0046 |
| Small $x$ extrap. | 0.0026 |
| Large $x$ extrap. | 0.0003 |
| Total sys. error | 0.0090 |

Table 6.4: The different contributions to the systematic error on the combined result for $\Gamma_1^p$. For the E80 and E130 data points only total errors were given, which are treated here as statistical errors.

As a check the data were also combined in another way. The integral of $g_1^p(x)$ over the measured $x$ range was divided in four parts; the $x$ range where all experiments have data, the range $0.03 < x < 0.1$ covered by SMC, EMC and E143 data, the range in $x$ where only the SMC and EMC have data and a fourth range $0.03 < x < 0.01$ only covered by the SMC. For each sub-range the results on the integral of $g_1(x)$ in that range were combined while weighting them with their total errors. The E80 data were merged with those of E130. The extrapolations were made as described above. The result for the first moment is $\Gamma_1^p = 0.139 \pm 0.005 \pm 0.009$ ($Q^2 = 5 \text{ GeV}^2$). The results from the two ways of combining the data agree well with each other. The physics results, collected below for the p,d and n(3He) targets, were extracted from $\Gamma_1^p$ combined at the asymmetry level, (6.15).

6.3.3 Combination of neutron data

The neutron asymmetries from the E142 experiment[84] have a high statistical accuracy. The muon data on $A_1^p$[85] and $A_1^n$[86] were combined to determine $A_1^n$ as well, however with much lower statistical accuracy. Although in the $x$ range covered by the E142 data the extra information coming from the muon data is negligible, the fact that the SMC data extend to $x$ values smaller than the lowest $x$ bin of E143 (0.03) makes a combination of all neutron data significant. The two experiments were combined at a $Q^2$ value of 5 GeV$^2$, which represents a reasonable intermediate value for the three data sets.
Figure 6.10: The combination of E142 neutron data with the muon data at small $x$. Plotted is $xg_1^n(x)$ as a function of log $x$. The small-$x$ extrapolations made in the E142 analysis and the one after including the muon data at small $x$ are indicated by the solid lines.

The combination is made at the asymmetry level. The SMC-binning, with which the E142 data points practically coincide, is used. In the range $0.006 < x < 0.6$ the muon data on $A_1^n$ and $A_1^d$ were combined to obtain $A_1^n$, using

$$
\sigma_n^{1\gamma} A_1^n = \frac{\left(\sigma_p^{1\gamma} + \sigma_n^{1\gamma}\right) A_1^d}{1 - \frac{1}{2} \omega_d} - \sigma_p^{1\gamma} A_1^p.
$$

(6.16)

The neutron asymmetry $A_1^n$ is determined most accurately if eq. (6.16) is rewritten in terms of $\sigma_n^{1\gamma}/\sigma_p^{1\gamma}$, which has been measured with high precision by the NMC[73]. In the range $0.006 < x < 0.03$ data on $A_1^n$ are solely available from the muon experiments. In the range of overlap, $0.03 < x < 0.6$, the data of E142 and the muon data were combined while weighting them with their statistical errors. The systematic errors on the asymmetry were assumed to be independent for the three experiments, except for the influence of the uncertainty in $A_2$ and in $R$.

From the combined asymmetries $g_1^n(x)$ was calculated. The structure function $F_2^n$ was derived from the $F_2^p$ parametrisation also used for $\Gamma_p^n$, and the measured ratio $F_2^n/F_2^p$ [73]. From this ratio, one also obtains $\sigma_n^{1\gamma}/\sigma_p^{1\gamma}$, since the two ratios are equal under the assumption that $R$ is the same for proton and neutron. The integral of $g_1^n$ in the measured range is equal to

$$
\int_{0.006}^{0.6} g_1^n(x) dx = -0.062 \pm 0.020 \pm 0.009.
$$

For the extrapolation at large $x$ the approach of
E142 was used, leading to a contribution of $0.003 \pm 0.003$. The small $x$ extrapolation was taken as the combination of the extrapolation of proton world data and SMC deuteron data, leading to a contribution of $-0.017 \pm 0.007$. The resulting value for $\Gamma_1^n$ from the world data on the neutron then is
\[ \Gamma_1^n = -0.076 \pm 0.020 \pm 0.012 \quad \text{(world data } Q^2 = 5 \text{ GeV}^2). \] (6.17)

The small-$x$ extrapolation used in the E142 analysis gives a less negative contribution to $\Gamma_1^n$ than the muon data and their extrapolation. This is visualised in figure 6.10, where the values obtained for $xg_1^n$ are shown with the two extrapolations. The extrapolation made by the E142 collaboration has a smaller error attributed to it compared to the combination of the small-$x$ muon data and their extrapolation. Ignoring $Q^2$ dependence effects, this may indicate that the systematic error assigned to the E142 extrapolation is slightly underestimated.

6.4 Interpretation

The interpretation of the polarised deep inelastic scattering data in this thesis is based on the first moments of $g_1$. The values obtained from the various experiments and (combined) analyses are compared to the quark parton model predictions by Ellis and Jaffe. Moreover, the Bjorken sum rule is checked. The contribution of the quark spin from different flavours to the nucleon spin is also extracted, including information from other sources. Each item is first treated using the SMC data only, after which all available data are interpreted in the same way. The values are all evaluated at $Q^2 = 5 \text{ GeV}^2$.

6.4.1 The Ellis-Jaffe sum rules

The theoretical basis of the Ellis-Jaffe sum rules, SU(3) flavour symmetry and an unpolarised strange sea, is discussed in Chapter 2. The resulting expressions, including second order QCD corrections[9], are
\[ \Gamma_1^{(n)}(EJ) = \left( -\frac{1}{12} (F + D) + \frac{1}{36} (3F - D) \right) \left[ 1 - \frac{\alpha_s}{\pi} - 3.5833 \left( \frac{\alpha_s}{\pi} \right)^2 \right] + \right] + \frac{1}{9} (3F - D) \left( 1 - \frac{\alpha_s}{\pi} - 1.0959 \left( \frac{\alpha_s}{\pi} \right)^2 \right) \] (6.18)

for three active quark flavours, where $F$ and $D$ are the symmetric and anti-symmetric SU(3) coupling constants for $\beta$-decay in the baryon octet. The bracketed minus sign in the first term of this equation applies to the neutron.

The value for $F + D$ can be extracted from the neutron $\beta$-decay, assuming isospin symmetry only, whereas other combinations require data from other baryon decays under the assumption of SU(3) flavour symmetry. Canonical values at present are $F + D = 1.257 \pm 0.003$ and $F/D = 0.575 \pm 0.016[87]$, from which the value for $3F - D$ is determined.
For the QCD corrections the value of the strong coupling constant $\alpha_s$ at the $Q^2$ of the experiment was determined starting from $\alpha_s(M_W^2) = 0.113 \pm 0.004$ [88]. The value of $\alpha_s$ at the central value of $Q^2$ at which the interpretation is done is $\alpha_s(Q^2 = 5 \text{ GeV}^2) = 0.258 \pm 0.020$.

The resulting values for the Ellis-Jaffe sum rule are given in the last column of Table 6.5. For the deuteron value the D-state admixture was accounted for using eq. (6.9). The table allows a comparison with the experimental results for $\Gamma_1$ of the proton, neutron and deuteron. Both for the SMC deuteron and proton measurement about two standard deviation differences between the Ellis-Jaffe prediction and the data are observed. The same is true for the world proton data. The result using the E142 neutron data, and extrapolations based on their data only, is consistent with the prediction. For the world neutron data, however, one observes an about two standard deviation difference.

<table>
<thead>
<tr>
<th>$Q^2$</th>
<th>$\Gamma_1$</th>
<th>$\Gamma_{EJ}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Proton SMC</td>
<td>10</td>
<td>$0.142 \pm 0.012 \pm 0.011$</td>
</tr>
<tr>
<td>5</td>
<td>$0.133 \pm 0.011 \pm 0.011$</td>
<td>$0.168 \pm 0.006$</td>
</tr>
<tr>
<td>Proton World</td>
<td>5</td>
<td>$0.140 \pm 0.005 \pm 0.009$</td>
</tr>
<tr>
<td>Neutron E142</td>
<td>2</td>
<td>$-0.022 \pm 0.006 \pm 0.009$</td>
</tr>
<tr>
<td>Neutron World</td>
<td>5</td>
<td>$-0.076 \pm 0.020 \pm 0.012$</td>
</tr>
<tr>
<td>Deuteron SMC</td>
<td>5</td>
<td>$0.020 \pm 0.019 \pm 0.013$</td>
</tr>
</tbody>
</table>

Table 6.5: Experimental values for $\Gamma_1$, together with the Ellis-Jaffe predictions, which include second order QCD corrections.

6.4.2 The Bjorken sum rule

The Bjorken sum, $\Gamma_1^p - \Gamma_1^n$, for the combination of SMC deuteron and proton data was obtained through $2\Gamma_1^p - (\Gamma_1^p + \Gamma_1^n)$. This leads to the following experimental value for the Bjorken sum:

$$\Gamma_1^p - \Gamma_1^n = 0.223 \pm 0.059 \quad (SMC'93, \quad Q^2 = 5 \text{ GeV}^2),$$

which is within its still large error in agreement with the theoretical prediction at the same value of $Q^2$,

$$\Gamma_1^p - \Gamma_1^n = 0.186 \pm 0.003 \quad (theory, \quad Q^2 = 5 \text{ GeV}^2).$$

The QCD corrections up to third order in $\alpha_s$, see eq. (2.53), were taken into account. The effect of the third order QCD corrections are also included in the error estimate.

The most accurate value for the Bjorken sum is obtained when extracting it from all available data. Since the three experimental results, $\Gamma_1^p$-world, $\Gamma_1^n$-world and $\Gamma_1^n$-SMC are not independent, the value of $\int g_1^p(x)dx - \int g_1^n(x)dx$ was determined in three intervals of $x$: The range $0 < x < 0.03$ where only the two muon experiments have data, the range $0.03 < x < 0.6$ where data on all three nuclei is available, and the interval $0.6 < x < 1.0$ where the extrapolations on the world proton and E142 data on the neutron were used. The intervals at large $x$ and small $x$ are straightforward, since only two independent experiments deliver data. In the range where proton, neutron as well as deuteron data are available, the three independently measured integrals were fit to free parameters representing $\int g_1^p(x)dx$ and $\int g_1^n(x)dx$ over this
x-range. Combining the results from the intervals one obtains

\[ \Gamma^p_1 - \Gamma^n_1 = 0.216 \pm 0.026 \quad (\text{world data, } Q^2 = 5 \text{ GeV}^2), \]  

which does not differ from the value where only the proton and neutron world result are combined (see Table 6.5), since the influence of the deuteron result at low \( x \) is already included in the value of \( \Gamma^p_1 \). The experimental value of the Bjorken sum is compatible with the sum rule within about one standard deviation.

### 6.4.3 Contribution of the quark spins to the nucleon spin

If one allows for a polarised strange sea, and assumes flavour SU(3) symmetry, then the following expressions hold, as is discussed in detail in Chapter 2,

\[
\begin{align*}
F + D &= \Delta u - \Delta d \\
3F - D &= \Delta u + \Delta d - 2\Delta s \\
\Gamma^{\text{eff}}_1 &= f(F, D, \alpha_s, \Delta \Sigma).
\end{align*}
\]

Here the third equality indicates that the experimentally determined value of \( \Gamma_1 \) can be expressed as a function of the coupling constants \( F, D, \alpha_s \) and the quark spin polarisation \( \Delta \Sigma \equiv \Delta u + \Delta d + \Delta s \). The value of \( \Delta \Sigma \) was independently derived from the SMC proton and deuteron first moments, using the values for \( F, D \) and \( \alpha_s \) as quoted earlier. The results are

\[
\begin{align*}
\Delta \Sigma(\text{deuteron}) &= 0.08 \pm 0.21 \pm 0.13 \quad (Q^2 = 5 \text{ GeV}^2) \\
\Delta \Sigma(\text{proton}) &= 0.27 \pm 0.11 \pm 0.12 \quad (Q^2 = 5 \text{ GeV}^2).
\end{align*}
\]

As discussed in Chapter 2 the first two equations of (6.22) represent non-singlet contributions to \( \Gamma_1 \) and are scale independent. The singlet piece \( \Delta \Sigma \) on the contrary, is scale dependent. Here the QCD corrections are taken such that the extracted \( \Delta \Sigma \) is at a \( Q^2 \) equal to the central value of the experiment.

Solving the set of equations (6.22) results in the following net contributions of quark spins for the different flavours

\[
\begin{align*}
\text{proton}(Q^2 = 5 \text{ GeV}^2) & \quad \text{deuteron}(Q^2 = 5 \text{ GeV}^2) \\
\Delta u &= 0.81 \pm 0.03 \pm 0.03 & \Delta u &= 0.75 \pm 0.07 \pm 0.04 \\
\Delta d &= -0.44 \pm 0.03 \pm 0.03 & \Delta d &= -0.51 \pm 0.07 \pm 0.04 \\
\Delta s &= -0.10 \pm 0.03 \pm 0.03 & \Delta s &= -0.17 \pm 0.07 \pm 0.05.
\end{align*}
\]

Only non-singlet combinations like \( \Delta u - \Delta d \) are scale independent, thus all the quark contributions \( \Delta q \) themselves are scale dependent.

The SMC values for \( \Delta s \) and \( \Delta \Sigma \) are also given in Table 6.6, where the values for \( \Delta \Sigma \), the total contribution of the quark spins to the nucleon spin, and \( \Delta s \), the contribution of the strange quark spin, are presented for the different experiments and analyses. For each analysis the values are given at the fixed value \( Q^2 = 5 \text{ GeV}^2 \).
6. The spin dependent structure function $g_1$ of the deuteron and the proton

<table>
<thead>
<tr>
<th></th>
<th>$Q^2$</th>
<th>$\Delta s$</th>
<th>$\Delta \Sigma$</th>
<th>$\Delta s$ ($Q^2 = 5 \text{ GeV}^2$)</th>
<th>$\Delta \Sigma$ ($Q^2 = 5 \text{ GeV}^2$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Proton SMC</td>
<td>10.0</td>
<td>-0.09</td>
<td>0.32</td>
<td>$-0.10 \pm 0.04 \pm 0.04$</td>
<td>$0.27 \pm 0.11 \pm 0.12$</td>
</tr>
<tr>
<td>Proton World</td>
<td>5.0</td>
<td>-</td>
<td>-</td>
<td>$-0.08 \pm 0.02 \pm 0.03$</td>
<td>$0.33 \pm 0.05 \pm 0.09$</td>
</tr>
<tr>
<td>Neutron World</td>
<td>5.0</td>
<td>-</td>
<td>-</td>
<td>$-0.19 \pm 0.07 \pm 0.04$</td>
<td>$0.02 \pm 0.20 \pm 0.12$</td>
</tr>
<tr>
<td>Deuteron SMC</td>
<td>4.6</td>
<td>-0.17</td>
<td>0.08</td>
<td>$-0.17 \pm 0.07 \pm 0.05$</td>
<td>$0.08 \pm 0.21 \pm 0.13$</td>
</tr>
</tbody>
</table>

Table 6.6: Quarks spin contributions extracted from the first moments of the different $g_1(x)$'s, including second order QCD corrections.

6.5 Discussion

The world data on $\Gamma_0$, $\Gamma_1$ and $\Gamma_1$ deviate from the Ellis-Jaffe predictions by about two standard deviations. Interpreted in the QPM with QCD corrections, the total contribution $\Delta \Sigma$ of the quark spins to the nucleon spin is consistent for the proton, neutron and deuteron world data, and amounts to $26 \pm 9\%$, while the strange sea polarisation $\Delta s$ is $-11 \pm 4\%$. These values are mainly determined by the world result on $\Gamma_1$, because of its high precision. Compared to the EMC result, which largely triggered the recent experimental efforts, the value of $\Delta \Sigma$ has come closer to the naive expectation, and is no longer compatible with zero, as it was just after the EMC experiment. For example, a value $\Delta G \approx 2$ is now sufficient to bring $(\Delta s)_{QPM}$ and $(\Delta \Sigma)_{QPM}$ in agreement with the naive expectations (see eq. (2.59)). The gluon polarisation has to be partly compensated by orbital angular momentum.

The Bjorken sum rule is independent of these considerations; it is a fundamental test for QCD. It is experimentally verified now, at the one standard deviation level to within 16% of the theoretical value.

Mulders and Pollock [89] investigated the $Q^2$ evolution of $\Delta \Sigma$ from low energy to the scale of the DIS experiments. They determined the value $Q^2_0$ at which the non-polarised gluon distribution is zero, by a downward $Q^2$ evolution of the gluon distribution fitted at $Q^2 = 4 \text{ GeV}^2$, since $\Delta \Sigma(Q^2)$ must also be zero then. From low energy quark bag models, they take $\Delta \Sigma(Q^2_0) = 0.65$, from which $\Delta \Sigma$ can be evolved to any higher value of $Q^2$, using eq. (2.54). In the "normal" proton, i.e. a proton with no anomalous strangeness, $\Delta s$ would be zero at the strangeness threshold $Q^2_s$, and the strange sea polarisation at higher values of $Q^2$ would be accounted for by the Altarelli-Parisi $Q^2$ evolution. This would imply $\Delta \Sigma(Q^2_s) = 0.58$. Mulders and Pollock find $Q^2_s \approx 0.3 \text{ GeV}^2$, a reasonable value. Evolution to the central $Q^2$ value of the experiment results in $\Delta \Sigma(Q^2 = 5 \text{ GeV}^2) \approx 0.4$. Thus, they argue that a value of $\Delta \Sigma(Q^2 = 5 \text{ GeV}^2) \approx 0.4$ would be reasonable for the "normal" proton. This value of $\Delta \Sigma$ is fairly close to the experimental value, at least much closer than the value 0.58 used in the Ellis-Jaffe predictions for $\Gamma_1$. The theoretical uncertainty of this approach comes from: the bag model value, and from using perturbative QCD with values of the strong coupling constant up to $\alpha_s \approx 1.8$. However, using first or second order QCD evolution doesn't change the results dramatically.

The use of SU(3) symmetry in extracting the matrix elements $\langle p, s | A^3_n | p, s \rangle$ (the values $F$ and $D$) from hyperon decay, is contested in a paper by Ehrnsperger and Schäfer [90]. They use the mass difference between the hyperons as a measure of SU(3) symmetry breaking, and come to a value of $F/D = 0.492 \pm 0.083$. With this value the Ellis-Jaffe prediction for the
first moment of the proton is $\Gamma_1^p = 0.141 \pm 0.023$, consistent with the experimental result, while the quark sum extracted from $\Gamma_1^B$ is $\Delta \Sigma(Q^2 = 5 \text{ GeV}^2) = 0.38 \pm 0.11$, consistent with the prediction of Mulders and Pollock.

After the publication of the experimental results of $g_1^p$, $g_1^n$ and $g_1^d$, several theoretical groups published parametrisations of the polarised quark distribution functions. De Florian et al. [91] performed a fit to the E80/E130-EMC-SMC proton, E142 neutron and SMC deuteron asymmetries. They continue with the Schäfer/Woloshin approach, including either a polarised gluon distribution or a net polarised strange sea contribution. At this stage the experimental data can not distinguish between the two proposed descriptions.

Bartelski and Tatur [92] started with a set of unpolarised quark distribution functions from Martin, Roberts and Stirling [93]. They include eight new parameters to fit the $\Delta q$'s, allowing the strange sea to be polarised, but including no gluon polarisation. From a fit on the E80/E130-EMC-SMC proton and E142 neutron asymmetries, and by adapting Regge behaviour for $x < 0.003$, they find $\Gamma_1^p = 0.168$, $\Gamma_1^n = -0.038$ and $\Delta \Sigma = 0.43$. The (non-fitted) deuteron asymmetries are predicted to stay positive at small $x$.

Another approach is presented by Gehrmann and Stirling [94], who assume no significant polarisation of the strange sea, but allow a modest amount of the nucleon spin to be carried by gluons, namely $\Delta \Sigma \approx 2$. The data show no preference for one of their proposed shapes of the polarised gluon distribution. They start with a parametrisation of polarised valence quarks at $Q^2 = 4 \text{ GeV}^2$, and dynamically generate a polarised sea for $Q^2 > 4 \text{ GeV}^2$. For their fit only data with $Q^2 > 4 \text{ GeV}^2$ are used, which means that practically all data points of E142 are not taken into account. Also here $g_1^d(Q^2 = 4 \text{ GeV}^2)$ stays positive at small $x$.

Another parametrisation was proposed by Bourrely and Soffer [95]. They relate unpolarised quark distribution functions based on Fermi-Dirac distributions with polarised quark distribution functions without any additional free parameters, and find good agreement with data on unpolarised structure functions as well as with the recent SMC $g_1^p$ and E142 $g_1^n$ results.

The experimental groups all published their results on $\Gamma_1^1$ under the assumption of scaling of $A_1$. Although, within the present accuracy, there is no experimental evidence for scaling violations of $A_1$, this assumption has to be further investigated. The difference in the value between $\Gamma_1^B(Q^2 = 10 \text{ GeV}^2)$ and $\Gamma_1^B(Q^2 = 5 \text{ GeV}^2)$ derived from the SMC $A_1$ data, is larger than expected from the QCD evolution of $\Gamma_1$. Especially at small $x$ the dependence of $F_1$ on $Q^2$ is large, which, under the scaling assumption for $A_1$, results in a large $Q^2$ dependence of $g_1$ as well. However, the parton distribution functions derived in ref. [94], and their evolutions, result for $x < 0.02$ in a decrease of $g_1^n(x)$ with increasing $Q^2$, while $F_1$ increases. This would imply that assuming $A_1$ to scale results in too high a value for $g_1^n(x)$ at small $x$. The approach of a simplified evolution of $g_1$ discussed in section 2.2.2 [17] also has the same tendency, an approximately constant $g_1$ at small $x$.

The observation that $g_1^p$ shows a tendency to increase at small $x$, in combination with the fact that the small-$x$ HERA data on $F_2$ show such an increase, gave rise to discussions [96, 97] on the validity of the Regge type extrapolations used by the experimental groups.

A point of consideration is that small-$x$ data will always have a relatively large error on $g_1$, since the small asymmetries are multiplied with $F_1$, which becomes large at small-$x$. If more stringent limits can be derived, from a.o. the world data on $A_1$, for the small-$x$ behaviour of $g_1$ than those given by the errors on the small-$x$ data points, the precision of $\Gamma_1$ can be increased.
Another uncertainty aspect of the small-$x$ data, because of the inherent low $Q^2$, arises from higher twist effects. In a recent paper from Ehrnsperger et al. [98] it is pointed out that there is no inconsistency anymore between the results of different groups for higher twist corrections to the moments of structure functions. The values for the correction to $\Gamma_1^p$ are given by

$$(0.018 \pm 0.007)/Q^2,$$

which is a small correction at the $Q^2$ values where $\Gamma_1^p$ is determined. The fact remains however, that both SLAC and CERN experiments measure the asymmetries in their smallest $x$ bins at $Q^2 \approx 1$ GeV$^2$. Higher twist corrections to $A_1$ or $g_1$ have not yet been published. From the consistency between the data at small-$x$ and $Q^2 \approx 1$ GeV$^2$ from the E143, and the SMC/EMC data in the same $x$ range but at higher values of $Q^2$, one can conclude that at $x > 0.03$ the higher twist effects are not important in view of the experimental precision. For the small-$x$ data of the SMC there are no higher $Q^2$ data available to compare with.

The theoretical uncertainties mentioned above do not drastically affect the values of $\Gamma_1$ derived from the present data, since the experimental uncertainties still dominate. More and higher precision experiments are thus desirable. Precise data on $A_1$ for different nuclei and at different values of $Q^2$ will increase the possibilities to attack the above issues. The SMC has continued its measurement on $g_1^d$ in 1994, using a beam of muons with an energy of 190 GeV, which will extend the data down to $x = 0.003$ (like for the proton data), and diminish the error on the small-$x$ points by more than a factor of two. Apart from the theoretical uncertainties in the interpretation, the slightly negative deuteron asymmetries at small $x$ need verification.

Analysis of already collected SMC proton data with $Q^2 < 1.0$ GeV$^2$ will give information on the scaling behaviour at small $x$, and on $g_1^p(x < 0.003)$. The E143 experiment has already taken data on a ND$_3$ target, and is expected to publish its results of $A_1^d(x > 0.03)$ shortly. In 1995 the SMC will collect still more data on deuterium. At SLAC a continuation of E142/E143 at an energy of 50 GeV, E154/E155, has been approved. These experiments might run in 1996, measuring both $g_1^p$ and $g_1^d$ at $x > 0.015$. The HERMES experiment at DESY will start data taking in 1995, and aims at high statistics measurements of $g_1^p$, $g_1^d$ and $g_1^{3He}$ in the range $x > 0.02$, as well as of $g_2$ for all three nuclei. Apart from the inclusive data, semi-inclusive data may for example give information on the valence and non-strange sea polarisation. For the SMC experiment a semi-inclusive analysis has been carried out [99]. The HERMES experiment is designed to be well suited for semi-inclusive measurements. Concluding, one can say that nuclear spin physics is a very exciting field, in which several experimental groups will continue to provide interesting new, partly complementary, data in the coming years.
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De spin-afhankelijke structuurfunctie $g_1$ van het deuteron en het proton

Dit proefschrift beschrijft een studie naar de spinstructuur van het nucleon, via diep-inelastische verstrooiing van gepolariseerde muonen aan gepolariseerde protonen en deuteronen. Dit werk is gedaan in de Spin Muon Collaboration (SMC), op het CERN in Genève.

Uit de asymmetrie in de werkzame doorsnede van de muon-kern verstrooiing voor kernspin en muonspin parallel en anti-parallel, kan de spin-afhankelijke structuurfunctie $g_1$ worden bepaald; deze bevat informatie over de spin-verdelingsfuncties van de quarks en gluonen. De interpretatie in het kader van het quark parton model (QPM) van eerdere resultaten voor $g_2^p$, gemeten door de European Muon Collaboration (EMC), gaf een indicatie dat de spins van de quarks slechts voor een klein gedeelte verantwoordelijk zijn voor de kernspin. De SMC is opgezet met het doel dit onverwachte resultaat met grotere nauwkeurigheid te verifiëren, en om uit een combinatie van metingen van $g_1^d$ en $g_1^p$ een fundamentele somregel in de quantum chromodynamica (QCD), de Bjorken somregel, te toetsen.

De SMC resultaten die in dit proefschrift worden gepresenteerd zijn gebaseerd op meetgegevens uit 1992, waarbij gebruik is gemaakt van een gepolariseerd deuterium-target en gepolariseerde muonen met een energie van 100 GeV, en op gegevens uit 1993 van de verstrooiing van muonen van 190 GeV aan een proton-target. De waarden voor $A_1$, de asymmetrie van de werkzame doorsnede van de absorptie van een virtueel foton door een nucleon, worden gegeven als een functie van de Bjorken schalingsvariabele $x$. Het breken van de schaling van $A_1$, i.e. een afhankelijkheid van $A_1(x)$ van $Q^2$, is niet waargenomen. Onze waarden van $A_1^p$ en $g_1^p$ zijn consistent met de eerdere resultaten (E80/E130) van het Stanford Linear Accelerator Center (SLAC) en van de EMC.

De SMC meetgegevens zijn vergeleken met recente resultaten van de E142 en E143 experimenten op SLAC. De in de verschillende experimenten gevonden $A_1$-waarden zijn consistent, en zijn derhalve gecombineerd om tot de op dit moment meest precieze resultaten te komen. De interpretatie van de gecombineerde waarden in het QPM met QCD correcties leidt tot de conclusie dat de bijdrage van de quarkspins tot de kernspin $26 \pm 9\%$ bedraagt bij $Q^2 = 5 \text{ GeV}^2$. De voorspelling van Ellis en Jaffe voor het eerste moment van $g_1^p$, $g_1^d$ en $g_1^\nu$ zijn niet in overeenstemming met de metingen. In het QPM kan dit worden verklaard door een $-11 \pm 4\%$ polarisatie van de vreemde(strange) quarks bij $Q^2 = 5 \text{ GeV}^2$ te veronderstellen. Als de zogenaamde axiale anomalie en de $Q^2$-evolutie van verstorings-QCD in aanmerking wordt genomen, zijn de verkregen waarden redelijk in overeenstemming met een model-afhankelijke verwachting dat de quark spins bij lage energie verantwoordelijk zijn voor ongeveer 65% van de kernspin.

De fundamentele Bjorken somregel is nu, gebruik makend van alle beschikbare meetgegevens, op het niveau van een standaard-afwijking bevestigd tot op 16% van zijn theoretische waarde.
Summary

This thesis presents a study on the spin structure of the nucleon, via deep inelastic scattering (DIS) of polarised muons on polarised proton and deuterium targets. The work was done in the Spin Muon Collaboration (SMC) at CERN in Geneva.

From the asymmetry in the scattering cross section for nucleon and lepton spins parallel and anti-parallel, one can determine the spin dependent structure function $g_1$, which contains information on the quark and gluon spin distribution functions. The interpretation in the frame work of the quark parton model (QPM) of earlier results on $g_1^p$ by the European Muon Collaboration (EMC), gave an indication that only a small fraction of the proton spin, compatible with zero, is carried by the spins of the constituent quarks. The SMC was set up to check this unexpected result with improved accuracy, and to combine measurements of $g_1^p$ and $g_1^n$ to test a fundamental sum rule in quantum chromodynamics (QCD), the Bjorken sum rule.

The SMC results presented in this thesis are based on data taken in 1992 using a polarised deuterium target and polarised muons with an incident energy of 100 GeV, and 1993 data with a proton target and an incident muon energy of 190 GeV. The virtual photon-nucleon asymmetries $A_1^p$ and $A_1^n$ are given as a function of the Bjorken scaling variable $x$ only, since scaling violation in $A_1$, i.e. $Q^2$ dependence of $A_1(x)$, was not observed. From the $A_1$ data the structure functions $g_1^p$ and $g_1^n$ and their first moments were determined. Our values of $A_1^p$ and $g_1^p$ are consistent with earlier results (E80/E130) from the Stanford Linear Accelerator Center (SLAC) and from the EMC.

The SMC data are compared to recent data of the E142 and E143 collaborations at SLAC. The values on $A_1$ from the different experiments are consistent. They were combined to give the most accurate results possible at present. The interpretation of the world data in the QPM with QCD corrections, leads to the conclusion that the quark spin content of the proton spin is $26 \pm 9\%$ at $5 \text{ GeV}^2$. The Ellis-Jaffe predictions for the first moments of $g_1^p$, $g_1^n$ and $g_1^n$ are violated, which in the QPM context can be interpreted in terms of a strange quark polarisation of $-11 \pm 4\%$ at $Q^2 = 5 \text{ GeV}^2$. By taking into account the so-called axial anomaly and the $Q^2$ evolution of perturbative QCD, the obtained values can to a large extend be reconciled with a model dependent expectation that the quark spins carry about 65% of the nucleon spin at low energy.

Using all available data, the fundamental Bjorken sum rule has now been verified at the one standard deviation level to within 16% of its theoretical value.
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