GEOPHYSICAL RESEARCH LETTERS, VOL. 34, L23711, doi:10.1029/2007GL032098, 2007

Improved accuracy of GRACE gravity solutions through empirical orthogonal function filtering of spherical harmonics

B. Wouters[1] and E. J. O. Schrama[1]

Received 20 September 2007; revised 19 October 2007; accepted 5 November 2007; published 13 December 2007.

[1] One of the major problems one has to deal with when working with Gravity Recovery and Climate Experiment (GRACE) data is the increasing error spectrum at higher degrees in the provided Stokes coefficients, appearing as unphysical North-South striping patterns in the maps of equivalent water height (EWH). This phenomenon is commonly suppressed by application of a Gaussian smoothing filter, which unfortunately causes loss of signal and leakage between basins. In this paper we show how a significant amount of the striping can be removed by making use of the temporal characteristics of the error spectrum. The Stokes coefficients are decomposed using empirical orthogonal function analysis and the individual modes are tested for temporal noisiness. After filtering, maps of EWH are largely free of striping. Tests on simulated EWH estimates show that our filtering technique has a marginal effect on the predicted geophysical signal. **Citation:** Wouters, B., and E. J. O. Schrama (2007), Improved accuracy of GRACE gravity solutions through empirical orthogonal function filtering of spherical harmonics, *Geophys. Res. Lett.*, *34*, L23711, doi:10.1029/2007GL032098.

1. Introduction

[2] The Gravity Recovery and Climate Experiment (GRACE) mission, a joint project of the US National Aeronautics and Space Administration (NASA) and German Aerospace Center (DLR), has been providing the scientific community with global gravity observations since early 2002 [*Tapley et al.*, 2004]. One of the major problems one has to deal with when working with GRACE data is the increasing error spectrum at higher degrees in the provided Stokes coefficients C_{lm} and S_{lm}. As demonstrated by *Swenson and Wahr* [2006], these errors are not purely random, but also exhibit a strong correlation between even and odd degree coefficients respectively. When no post-priori filtering is applied, the errors show up as unphysical longitudinal striping patterns in the maps of equivalent water height (EWH). Commonly, these are suppressed by weighting the Stokes coefficients by means of a Gaussian smoothing function W_l which decreases in value with increasing degree l and thus attenuates the contribution of the ill-determined higher degree coefficients [*Wahr et al.*, 1998]. The value of the Gaussian smoothing function W_l depends on the degree l only and thus is isotropic [*Jekeli*, 1981]. However, as indicated by the correlation between the even and odd degree coefficients, the error has a non-isotropic character and consequently a large smoothing radius is required to remove all stripes in the GRACE maps, which implies a non-negligible loss of the information contained in the GRACE solutions [*Chen et al.*, 2007]. Moreover, with an increasing smoothing radius leakage between basins (e.g. land/ocean) will increase, making the separation of signals more cumbersome.

[3] This is illustrated in Figure 1, where we plotted a map of equivalent water height anomaly for March 2007 for several Gaussian smoothing radii. Without any smoothing (Figure 1a) the map is dominated by the meridional stripes and little geophysical signal can be discerned. At a smoothing radius of 350 km (Figure 1b), the larger hydrological basins (e.g. Amazon, Zambezi) start to stand out, but are still corrupted by the stripes. Increasing the smoothing radius to 500 km removes most of the striping over land, only at smoothing radii of 750 km and larger the oceans appear mostly stripe free.

[4] Various procedures have been proposed to remove the correlated errors in the Stokes coefficients and increase the resolution of EWH maps. Most of these make use of the calibrated error spectrum, the error covariance matrix or an a-priori signal covariance matrix, [see, e.g., *Swenson and Wahr*, 2002; *Chen et al.*, 2006; *Kusche*, 2007]. *Swenson and Wahr* [2006] propose to fit a quadratic polynomial in a moving window to the Stokes coefficients of even and odd degrees for a particular order m and remove this from the original Stokes coefficients. Using a slightly modified version, *Chambers* [2006b] showed that this reduces the uncertainty of the GRACE solutions by more than 51% over the oceans.

[5] In this paper we use an alternative approach to remove the North-South striping in the GRACE solutions and use empirical orthogonal functions (EOFs) to isolate significant geophysical signal. The effectiveness of this procedure was demonstrated by *Schrama et al.* [2007], in which EOF analysis of the Gaussian smoothed EWH maps was used to suppress noise. We apply the EOF procedure directly to the Stokes coefficients and use the temporal (un)correlatedness of the degree dependent correlation in the coefficients, which allows a significantly better separation of noise and 'real' signal. Moreover, we will show that this method retains most of expected geophysical signals, by applying the filter to modelled maps of EWH variations.

2. EOF Filtering of GRACE Spherical Harmonics

2.1. Empirical Orthogonal Functions

[6] EOF analysis is a technique based on the fact that a general real-valued, scalar, homogeneously dimensioned

[1]Delft Institute of Earth Observation and Space Systems, Delft University of Technology, Delft, Netherlands.

0094-8276/07/2007GL032098$05.00

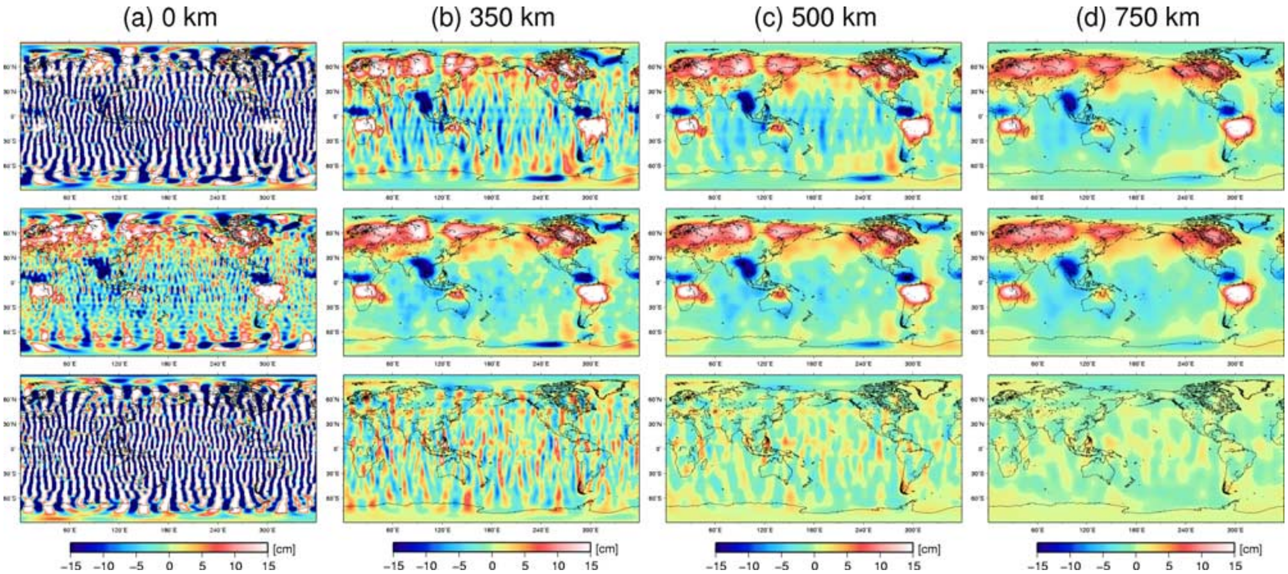

Figure 1. GRACE based maps of EWH anomaly for March 2007 smoothed with various radii. (a) 0 km, (b) 350 km, (c) 500 km, and (d) 750 km. The upper figures show the unfiltered fields, the middle figures are filtered with the EOF method. The lower figures represent the difference between the figures in the upper and middle row. Units are in cm.

data set Z with elements $\{z(t, x): x = 1, \ldots, p; t = 1, \ldots, n\}$ consisting of n observations of p variables, has a set of dominant directions of variability in the Euclidean space E_p, [see *Preisendorfer*, 1988]. These directions may be obtained by finding the eigenvectors e_j of the covariance matrix of the data set Z^TZ and form an orthogonal basis for the data space, such that:

$$z(t,x) = \sum_{j=1}^{\rho} a_j(t)e_j(x) \qquad (1)$$

where $a_j(t)$ is a temporal weight to the contribution of the eigenvector e_j. These weights are uncorrelated and their variance is equal to the variance of the data set along the direction of the associated eigenvector e_j. The variables a_j and e_j are commonly referred to as the jth principal component respectively EOF mode. The number of modes is at most min[$n - 1$, p], denoted as ρ in equation (1).

[7] Each EOF mode represents a certain percentage of the total variability in the data set, with the first few modes having the most power and other modes representing superfluous noise in the original data set. Thus, by summing over only a selected set of eigenvectors in equation (1), noise can be greatly reduced and only physically significant signal should remain in the compressed data set.

2.2. Kolmogorov-Smirnov Test

[8] To determine which EOF modes should be retained or rejected in the summation, we test the principal components $a_j(t)$ for temporal noisy behavior using the Kolmogorov-Smirnov (KS2) test [e.g., *Preisendorfer*, 1988]. Through a Fourier transform the power spectrum density (PSD) of each principal component $a_j(t)$ is obtained, which is then tested for spectral whiteness. This is done by comparing the cumulative distribution function (CDF) of the PSD of the principal components a_j with the CDF of the spectrum of a random normal distributed population. If the maximum value of the absolute difference between the two CDFs is below a critical value D_{KS2}, the test hypothesis that a_j is a random sample from a white noise process is accepted. The value of D_{KS2} depends on the significance level [e.g., *Press et al.*, 1992], which we set at 5%.

2.3. Application to GRACE Spherical Harmonics

[9] In our case, the data set to be decomposed comprises the GRACE solutions of Stokes coefficients. For this study we used the release 04 (RL04) processed by the Center for Space Research (CSR). Currently, 52 monthly solutions are available, spanning January 2003 to April 2007. Since the construction of the PSD requires continues data sets, we interpolated the missing 2 months in the time series (June 2003 and January 2004), yielding a set of 54 solutions. Since the GRACE project does not supply degree 1 coefficients, we use the model of *Chen et al.* [1999] for seasonal variations of the geocenter, as suggested by *Chambers et al.* [2004].

[10] The CSR RL04 solutions consists of Stokes coefficients estimated up to a degree and order $l_{max}/m_{max} = 60$. Since we want to remove the correlation between the spherical harmonics as a function of degree, we follow the findings of *Swenson and Wahr* [2006] and order the coefficients keeping the order m fixed. Consequently, we apply the EOF decomposition to each of these series $\{C_m(t, l): l = m, \ldots, l_{max}; t = 1, \ldots, 54\}$ for $m = 0, \ldots, l_{max}$ (and equivalent for the S_{lm} coefficients). For example, the data matrix Z for the C_{l0} coefficients will be:

$$Z_{C_{m=0}} = \begin{pmatrix} C_{00}(t=1) & \ldots & C_{00}(t=54) \\ \vdots & \ddots & \vdots \\ C_{l_{max}0}(t=1) & \ldots & C_{l_{max}0}(t=54) \end{pmatrix} \qquad (2)$$

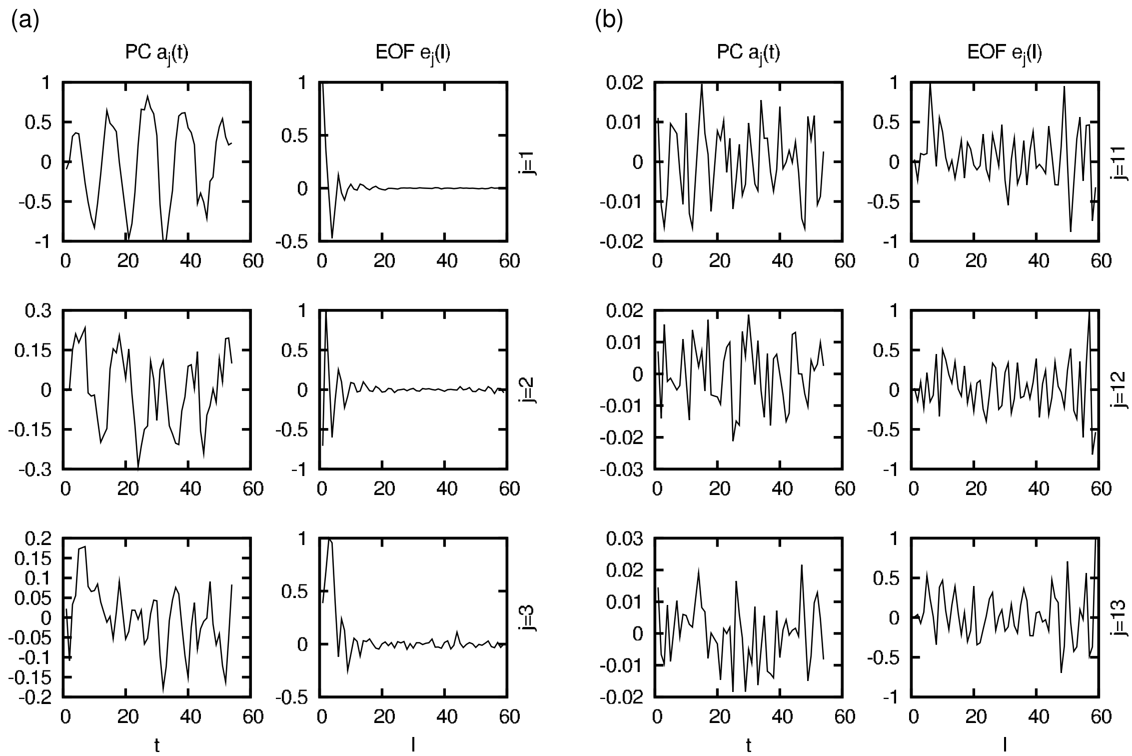

Figure 2. Principal Components (PC) and Empirical Orthogonal Functions (EOF) of the $C_{m=0}$ Stokes coefficients. (a) EOF mode 1–3, passing the KS2 test. (b) EOF mode 11–13, failing the KS2 test. The PC's have been scaled by 2.9e+10 to enhance presentation. On the x-axes, 't' stands for observation epoch and 'l' for coefficient degree.

[11] After obtaining the EOF modes e_j and associated PCs a_j, j = 1, …, ρ, we perform the KS2 test on each component. For m = 60, there is only one observation and we apply the KS2 test directly to coefficients series. If the test hypothesis is accepted, we assume that the EOF mode represent noise signal and the principal component and the corresponding EOF mode will not be used in the summation. The remaining EOF modes and PCs are used to rebuild the series $C_m(t, l)$ and $S_m(t, l)$ using equation 1.

[12] We illustrated the procedure for m = 0 in Figure 2, in which we plotted the jth EOF modes $e_j(l)$ for j = 1–3 in Figure 2a and j = 11–13 in Figure 2b (where j = 11–13 correspond to the first order 0 modes to fail the KS2 test), together with their associated PCs $a_j(t)$. The PCs of the first

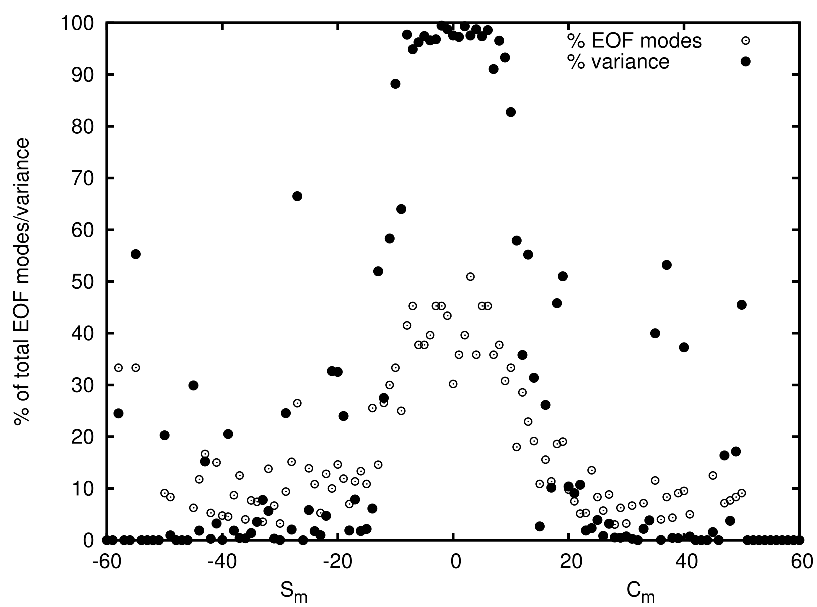

Figure 3. Percentage of available EOF modes accepted by the KS2 test (open circles) and percentage of the original variance contained in these accepted modes (black circles) for each order m series of Stokes coefficients.

three modes exhibit a clear annual, semiannual, inter- and intrannual signal and pass the KS2 test. Most power is contained in the Stokes coefficients of lower degrees up to degree ~20. In contrast, the power in the EOF modes for j = 11 – 13 is spread out over the entire range of coefficients with a bias towards the higher degrees. The temporal evolution of the PCs a much more random pattern and does not pass the KS2 test. Consequently, these modes are not used to rebuild the filtered Stokes coefficients for order $m = 0$.

[13] The number of accepted EOF modes depends on the order m. In Figure 3 we plotted the amount of original variance of the GRACE spherical harmonics retained after applying the KS2 test, i.e. the sum of the variances of the PCs a_j passing the KS2 test normalized by the sum of variances of all PCs a_j. Up to an order of ~10, almost all variance is retained, which agrees with the finding of *Swenson and Wahr* [2006] that the correlation between the even and odd coefficients starts at approximately this order.

3. Results

[14] Figure 1 illustrates the effect of our filtering method on the GRACE monthly EWH anomaly maps. Figures in the top row display the original fields, the middle row figures the fields after applying the EOF filtering to the spherical harmonics. Due to the fact that almost all original variance is retained in the lower order spherical harmonics, the effect of the filter is relatively modest at a larger smoothing radii (Figure 1d). Going down to lower smoothing radii (Figures 1b and 1c), the impact of the filter becomes more evident. Little to none of the striping remains in the filtered fields, whereas both meridional and longitudinal geophysical structures are clearly distinguishable. Without any Gaussian smoothing (Figure 1a), the striping is still significantly reduced, although some noisy patterns remain at low and mid-latitudes. Additional tests show that this is caused by the Stokes coefficients for $m > 50$, where the EOF decomposition is based on fewer sample points and thus will be less effective.

[15] As a second tool to analyse the effectiveness of our filter, we use the method suggested by *Wahr et al.* [2004]. We fit an annual and semi-annual sinusoid to the series of EWH maps and take the spatial rms of the residual signal as a measure for the errors in the GRACE solutions. We find a decrease of the estimated error from 23.7 mm to 16.7 mm at 500 km Gaussian smoothing. The effect is even more pronounced at lower smoothing radii, at 350 km a reduction from 43.5 mm to 21 mm is obtained. Using an updated version of the filter of *Swenson and Wahr* [2006], where we removed the even/odd degree correlation by fitting an 6th order polynomial to the harmonics for $m = 11–60$ (as given by *Chambers* [2006a]), we find comparable values, i.e. 17.1 mm and 21.1 mm respectively. These numbers should be regarded as an upper bound for the true accuracy, since the residuals will still contain true intra- and interannual geophysical signals that are not accounted for.

[16] We have shown that EOF filtering of the Stokes coefficients successfully removes the North-South striping in the GRACE EWH maps. Next, we will evaluate to which extent the filter might remove true geophysical signal. The difference between the filtered and unfiltered fields, i.e. the signal removed by filtering, is displayed in the lower row of Figure 1. Inspection of these figures reveal only little correlation between the removed signal and expected geophysical signal.

[17] We further address this problem by applying the filter to a set of independent EWH fields. We created 54 maps of EWH variability by combining the output of the Global Land Data Assimilation System (GLDAS) [*Rodell et al.*, 2004] with ocean bottom pressure variations from JASON-1 altimetry observations, corrected for steric expansion using the World Ocean Atlas 2005 (WOA05) [*Locarnini et al.*, 2006; *Antonov et al.*, 2006]. Although these fields are not error-free themselves, they do not exhibit the strong North-South error structures and therefore serve as a good basis for testing our filter. The synthetic EWH maps are converted to spherical harmonics and filtered as described above. The maps of the filtered fields compare closely to the original fields, although some small scale features are attenuated (Figure S1 in the auxiliary material).[1] Globally, the rms of the 54 fields reduces from 43.4 mm to 39.9 mm (no Gaussian smoothing) respectively from 30.3 to 29.0 mm (350 Gaussian smoothing).

4. Discussion

[18] In this paper, we have described a filter that effectively removes the North-South stripping in the GRACE EWH fields. Our method is based on the fact that these have a random temporal behavior and therefore can be separated from signals with a non-random character. The power of the method lies in the fact that we test the PCs of the GRACE spherical harmonics, rather than decomposing the Gaussian smoothed EWH fields. This way, we are able to retain a maximum amount of information contained in the original spherical harmonics solutions.

[19] In contrast with other methods, the EOF filter is based on the entire temporal history of the monthly GRACE solutions. This means that monthly solutions with a lower quality will deteriorate the other solutions in the filtered data set. Indeed, we find that elimination of such months (e.g. September 2004, when the GRACE satellites were in a near-repeat orbit) results in a slight decrease of striping in the EOF filtered fields. However, as the mission time of GRACE increases and more solutions become available, the influence of such months is expected to decrease. Alternatively, one could think of assigning a weight to the individual GRACE solutions before applying the EOF decomposition.

[20] We demonstrated that our filter has little effect on the true geophysical signal. However, there is a caveat concerning episodic events. Such signals are generally small in amplitude and dimension and thus may be misinterpreted as being of random nature and therefore be removed in the filtered fields. We do indeed find that the fingerprint of the Sumatra Earthquake [*Chen et al.*, 2007] is slightly attenuated in our solutions (not shown here). This remark does only concern episodic events, we found no attenuation of trends in our solution compared to other filtering methods.

[1]Auxiliary materials are available in the HTML. doi:10.1029/2007GL032098.

[21] Given that the EOF filter is based on temporal correlations, whereas the destriping filter of *Swenson and Wahr* [2006] is based on spectral correlations, we expect these techniques to be complementary. We find the combination of the two filters further reduces the rms of the non-seasonal signal in the GRACE maps to 17.9 mm at 350 km smoothing. The rms of the synthetic EWH map reduces to 28.4 mm at this scale and to 37.3 mm without any Gaussian smoothing. Note that a proper combination of the two filters would actually require a comparison of the GRACE data with some independent data set (e.g., GPS, ocean model or altimetry as given by *Chambers* [2006b]) to tune the parameters of the two filters, which will be subject to further research.

[22] **Acknowledgments.** We would like to thank the various institutes and researchers for providing their data: The GRACE data was made available by the Center for Space Research at the University of Texas at Austin; World Ocean Atlas 2005 data was obtained from NOAA/NODC at http://www.nodc.noaa.gov/OC5/WOA05/pr_woa05.html; Matthew Rodell (GSFC/NASA) supplied us with GLDAS data. The altimetry data was processed using the DEOS RADS V2.2 software. The filtered GRACE solutions are available upon request. We are grateful to Don Chambers and an anonymous reviewer for their comments which helped to improve the manuscript.

References

Antonov, J. I., R. A. Locarnini, T. P. Boyer, A. V. Mishonov, and H. E. Garcia (2006), *World Ocean Atlas 2005*, vol. 2, *Salinity*, 182 pp., U.S. Gov. Print. Off., Washington, D. C.

Chambers, D. P. (2006a), Observing seasonal steric sea level variations with GRACE and satellite altimetry, *J. Geophys. Res.*, *111*, C03010, doi:10.1029/2005JC002914.

Chambers, D. P. (2006b), Evaluation of new GRACE time-variable gravity data over the ocean, *Geophys. Res. Lett.*, *33*, L17603, doi:10.1029/2006GL027296.

Chambers, D. P., J. Wahr, and R. S. Nerem (2004), Preliminary observations of global ocean mass variations with GRACE, *Geophys. Res. Lett.*, *31*, L13310, doi:10.1029/2004GL020461.

Chen, J. L., C. R. Wilson, R. J. Eanes, and R. S. Nerem (1999), Geophysical interpretation of observed geocenter variations, *J. Geophys. Res.*, *104*, 2683–2690, doi:10.1029/1998JB900019.

Chen, J. L., C. R. Wilson, and K.-W. Seo (2006), Optimized smoothing of Gravity Recovery and Climate Experiment (GRACE) time-variable gravity observations, *J. Geophys. Res.*, *111*, B06408, doi:10.1029/2005JB004064.

Chen, J. L., C. R. Wilson, J. S. Famiglietti, and M. Rodell (2007), Attenuation effect on seasonal basin-scale water storage changes from GRACE time-variable gravity, *J. Geod.*, *81*, 237–245, doi:10.1007/s00190-006-0104-2.

Jekeli, C. (1981), Alternative methods to smooth the Earth's gravity field, *Tech. Rep. 327*, Dep. of Geod. Sci. and Surv., Ohio State Univ., Columbus.

Kusche, J. (2007), Approximate decorrelation and non-isotropic smoothing of time-variable GRACE-type gravity field models, *J. Geod.*, *81*, 733–749, doi:10.1007/s00190-007-0143-3.

Locarnini, R. A., A. V. Mishonov, J. I. Antonov, T. P. Boyer, and H. E. Garcia (2006), *World Ocean Atlas 2005*, vol. 1, *Temperature*, 182 pp., U.S. Gov. Print. Off., Washington, D. C.

Preisendorfer, R. W. (1988), *Principal Component Analysis in Meteorology and Oceanography*, 425 pp., Elsevier, Seattle, Wash.

Press, W. H., S. A. Teukolsky, W. T. Vetterling, and B. P. Flannery (1992), *Numerical Recipes in FORTRAN: The Art of Scientific Computing*, 2nd ed., Cambridge Univ. Press, New York.

Rodell, M., et al. (2004), The Global Land Data Assimilation System, *Bull. Am. Meteorol. Soc.*, *85*(3), 381–394.

Schrama, E. J. O., B. Wouters, and D. A. Lavallée (2007), Signal and noise in Gravity Recovery and Climate Experiment (GRACE) observed surface mass variations, *J. Geophys. Res.*, *112*, B08407, doi:10.1029/2006JB004882.

Swenson, S., and J. Wahr (2002), Methods for inferring regional surface-mass anomalies from Gravity Recovery and Climate Experiment (GRACE) measurements of time-variable gravity, *J. Geophys. Res.*, *107*(B9), 2193, doi:10.1029/2001JB000576.

Swenson, S., and J. Wahr (2006), Post-processing removal of correlated errors in GRACE data, *Geophys. Res. Lett.*, *33*, L08402, doi:10.1029/2005GL025285.

Tapley, B. D., S. Bettadpur, M. Watkins, and C. Reigber (2004), The gravity recovery and climate experiment: Mission overview and early results, *Geophys. Res. Lett.*, *31*, L09607, doi:10.1029/2004GL019920.

Wahr, J., M. Molenaar, and F. Bryan (1998), Time variability of the Earth's gravity field: Hydrological and oceanic effects and their possible detection using GRACE, *J. Geophys. Res.*, *103*(B12), 30,305–30,229.

Wahr, J., S. Swenson, V. Zlotnicki, and I. Velicogna (2004), Time-variable gravity from GRACE: First results, *Geophys. Res. Lett.*, *31*, L11501, doi:10.1029/2004GL019779.

E. J. O. Schrama and B. Wouters, Delft Institute of Earth Observation and Space Systems, Delft University of Technology, Kluyverweg 1, NL-2629 HS Delft, Netherlands. (bert.wouters@tudelft.nl)