Estimation of Primaries and Multiples by Sparse Inversion
Estimation of Primaries and Multiples by Sparse Inversion

PROEFSCHRIFT

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The chapters in this thesis are closely related to each other, but in order to study one particular theme not all of chapters need to be read. Below is an overview of the sections that address the different themes.

SRME: SRME (Chapter 2)
EPSI and near-offset reconstruction:
  SRME (2.1 notation, 2.2 forward model)
  EPSI (Chapter 3)
  Field data (Chapter 6)
Passive data: SRME (2.1 notation, 2.2 forward model)
  EPSI (3.1 algorithm)
  Passive data (Chapter 4)
Blended data: SRME (2.1 notation, 2.2 forward model)
  EPSI (3.1 algorithm)
  Passive data (4.2 missing angles, 4.3 results)
  Blended data (Chapter 5)
Field data: SRME (2.1 notation, 2.2 forward model)
  EPSI (3.1 algorithm, 3.3 near offset reconstruction)
  Field data (Chapter 6)

For the reader new to geophysics: if you know how to do a matrix multiplication and if you can keep in mind the three principal matrices: \( P \) (the data matrix), \( X \) (the earth transfer matrix), and \( S \) (the source matrix), you will be able to understand the essentials in this thesis.
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"A message that is repeated multiple times is easier to understand. A message that is repeated easier to understand. multiple times is that is repeated that is repeated easier to understand. multiple times is easier to understand. multiple times is easier to understand."

- A message from the subsurface
Introduction

1.1 An introduction to primaries and multiples

A message that is repeated multiple times is easier to understand. However, if the repetitions are started before the original message is finished, a method is needed to estimate the original message. In seismic recordings multiples are the repetitions that start before the original message, the primaries, is finished. In the past multiples were considered noise that had to be removed to obtain the primaries. This thesis describes a method that uses the multiples to obtain a better estimate of the primaries.

1.1.1 The seismic method

In order to locate or monitor reservoirs in the subsurface that contain oil and gas or can store CO$_2$, an image of the subsurface is needed. Seismic exploration is the most common way to obtain these images. The seismic method can be divided into several stages. During the seismic acquisition stage (Figure 1.1) a source at or close to the surface generates a wavefield. This wavefield propagates through the subsurface where it is partly reflected at places in the earth where two layers with different elastic properties meet. These reflections are measured by receivers, usually located at the surface. This experiment is repeated with different source positions.
Introduction

Figure 1.1: Seismic acquisition at sea. A boat pulls a source (air gun) and a cable with receivers (hydrophones) through the water. When fired the source will send a wavefield through the subsurface. The reflections that reach the surface are measured by the receivers.

In the seismic processing stage an image of the subsurface is constructed from the measured reflections.

1.1.2 Primaries and multiples

Figure 1.2 shows a synthetic subsurface model that is used to generate a seismic dataset. The dark colored layer with the curved top represents a high-velocity salt layer. From each shot in this dataset the recording of the receiver at zero offset is selected, and plotted in Figure 1.3a. Looking at Figure 1.3a, one might erroneously

Figure 1.2: Synthetic subsurface model containing a high velocity salt layer, with water as top layer.
1.1 An introduction to primaries and multiples

Figure 1.3: (a) Zero offset section with primaries and multiples. (b) Zero offset section with primaries only.

conclude that there are several salt domes on top of each other. However, the 'extra' salt domes are multiples; events that have traveled up and down at least twice (see Figure 1.1). Note that the multiples in Figure 1.3a not only give an erroneous image of the subsurface, but they also obscure the primaries; events that have traveled up and down only once.

Figure 1.4 shows the ray paths of possible events in a seismic measurement done at the surface.

- Direct and surface waves (Figure 1.4a): waves that have not propagated downward, but travel laterally, just below the surface. In the theory and all the examples in this thesis it is assumed that in the data that are given as input to the primary estimation methods the direct wave is removed in preprocessing. For marine data this preprocessing step usually is rather simple, via a muting operation.

The strict definition of a primary is an event that has traveled up and down only once, but in practice it is more convenient to make the distinction between primaries and multiples with respect to a boundary or depth level. If the surface is taken as
reference boundary then a distinction between surface-related primaries and surface-related multiples can be made.

- Surface-related primaries (Figure 1.4b): waves that have propagated through the subsurface and have not bounced at the surface. If a surface-related primary has reflected more than once in the subsurface (like ray path II in Figure 1.4b) it is called an internal multiple, but still belongs to the surface-related primaries. Note that the raypath I in Figure 1.4b is a primary according to the strict definition.

- Surface-related multiples (Figure 1.4c): waves that have propagated through the subsurface and have bounced at the surface at least once. The number of bounces at the surface determines the multiple order. Ray path I in Figure 1.4c describes a second-order multiple and ray path II a first-order multiple. Surface-related multiples are strong due to the high surface reflection coefficient (being close to -1 for the marine case).

For the sake of convenience this thesis will refer to surface-related primaries as primaries and surface-related multiples as multiples. From the synthetic dataset a shot gather is shown in Figure 1.5a. The events in this data are divided into primaries (Figure 1.5b) and multiples. The primary zero offset section shows that there is only one salt dome (Figure 1.3b) and that the other 'salt domes' in Figure 1.3a are multiples.
1.2 EPSI and its applications

Although the primaries are the desired output of seismic processing methods that separate primaries and multiples, these methods are seldom referred to as primary estimation methods. The most common names are multiple elimination, multiple suppression and multiple attenuation methods. This name was given to them due to the fact that most methods predict the location of the multiples and then subtract them from the data. The residual that is left is considered to be the primaries. The primary estimation method described in this thesis is the Estimation of Primaries by Sparse Inversion (EPSI) method, which was introduced by van Groenestijn and Verschuur [2009a]. EPSI estimates the primaries from information in both the primaries and the multiples. With the estimated primaries EPSI tries to explain the total data, primaries and multiples. The unexplained data, the residual, are considered to be noise. In this thesis EPSI is explained for the marine case. However, the findings are also useful for- and with a few modifications applicable to- land data. This thesis also describes various applications of EPSI beyond the conventional towed streamer situation. These applications are related to different acquisition strategies with which the data are collected. In the following subsections these applications

---

**Figure 1.5:** (a) Shot gather simulated in the model of Figure 1.2, containing primaries and multiples. (b) The same shot gather but now only containing primaries.
Introduction

1.2.1 Traditional acquisition and near offset reconstruction

Traditional marine seismic acquisition is depicted in Figure 1.6a for the 2D situation. A boat pulls a source and a cable with receivers. It is not possible to have receivers close to the source, because the pressure pulse of the source would damage them. Therefore, the near offsets primary ray path depicted in Figure 1.6a is not measured. However, if the boat sails further the same ray path shows up several times as a multiple that is measured (Figure 1.6b). It will be shown that EPSI is able to reconstruct the missing near offset data from the measured multiples.

Nowadays marine acquisition is done in a 3D fashion by boats pulling parallel cables to also capture the structure of the subsurface in the direction perpendicular to the sailing direction. However, in this thesis acquisitions are described that are done with only one sail line, i.e. assuming the 2D case.

1.2.2 Passive data

In passive seismics no controlled sources, like airguns, explosives or vibrator trucks, are used. The receivers record just noise and responses from passive sources, like mini earthquakes or heavy traffic. Figure 1.7 shows a ray path of a passive subsurface source. When the signal reaches the surface it will cause a series of multiples. EPSI can turn these multiples into primaries as if they came from a traditional acquisition, with sources and receivers at the surface.
1.2 EPSI and its applications

1.2.3 Blended data

In traditional acquisition the time interval between different shots is taken large enough to ensure that subsurface responses of different shots do not overlap. Recently an interest is grown in acquisition methods in which time intervals between
shots are deliberately taken too small. This allows to speed up the acquisition, or to improve illumination within the same acquisition time. The price that is paid is that the measured data consist of blended subsurface responses of different sources. Hence the name blended data. The only difference between the standard acquisition (Figure 1.5a) and the blended acquisition (Figure 1.8) is that in the first situation the primaries of one shot overlap with the multiples, and in the second situation these primaries also overlap with the primaries of another shot. EPSI can turn the blended primaries and multiples into primaries as if they came from a traditional, unblended acquisition.

1.3 Literature

1.3.1 Historical papers

In their collection of papers dealing with multiples, Weglein and Dragošet [2006] start with the historical papers from around 1948. These papers do not deal with how to eliminate multiples but with the question whether multiples exist or not. The authors that argued in favor of the existence of multiples also had a prophetic eye; Ellsworth [1948] already realizes that a multiple is a combination of two or more primaries (a foundation of surface-related multiple elimination): "each echo from the base of the weathering acts as another shot". Dix [1948] already describes how to make use of multiples by shifting primaries in time and then matching them to multiples (a foundation of EPSI): "This matching is often possible and serves to help make the accuracy of determination of the beginning of the primary reflection very much better". Ellsworth [1948] was not so prophetic when it came to the importance of multiples: ".o. the problem is worthy of close and continued study even though the multiple-reflection question as a whole does not seem to present a serious limitation to seismograph interpretation except in isolated cases". In defense of Ellsworth's last remark it should be noted that he studied land data and not marine data, where the air-water boundary acts as a perfect acoustical mirror and multiples are a major issue. Because physically moving the air-water boundary is not an option as noted by Riley and Claerbout [1976] (Figure 1.9) multiple removal methods had to be designed to obtain multiple-free data. Berkhout [1982] (Figure 1.10) was the first author who showed that for surface-related multiple removal no information of the subsurface is required. However, he also stated that knowledge of the source signature is essential for practical applications.
1.3 Literature

Figure 1.9: Riley and Claerbout [1976] noted that moving the air-water boundary is not an option.

1.3.2 Classification of multiple removal methods

In his book on seismic multiple removal techniques Verschuur [2006] classifies multiple removal methods into two main categories:


Verschuur [2006] further divides the second category into predictive deconvolution methods, and prediction and subtraction methods. This thesis adds inversion methods as a third group to the periodicity and predictability category.
1.3.3 Methods based on a difference in spatial behavior of primaries and multiples

Figure 1.11c shows a common midpoint (CMP) gather with primaries and multiples after normal move-out (NMO) correction with the primary velocities. Note that summing the traces together will conserve the aligned primary energy, but not the far-offset part energy of the multiples. Therefore, one might say that NMO stacking is a multiple attenuation method. Figure 1.11b shows only the primaries in the NMO corrected shot gather. In the gather of Figure 1.11c primaries are flat and multiples are curved downward. The idea is to transform the corrected CMP gather to a domain where the flat (primaries) and downward curved (multiples) events...
map into different areas. One can then select the primaries and transform these back to the spatial-temporal domain resulting in a primary estimation. It is also possible to select the multiples, and, after back transformation, subtract these from the data. Different transforms have been used; Ryu [1982] uses a transformation to the wavenumber-frequency domain, Hampson [1986] the parabolic Radon domain, and Thorson and Claerbout [1985] the high resolution hyperbolic Radon domain.

Figures 1.11b and c show that the assumption that primaries and their multiples have different move out holds for datasets obtained from areas where the subsurface has an increasing velocity profile (1.11a). Figure 1.12a-c shows that this assumption is not valid for areas where the subsurface does not only have an increasing velocity profile. Several multiples are flat in Figure 1.12c and cannot be distinguished from primaries. Also note the strong internal multiple (that is a surface-related primary) that is not flat in Figure 1.12b. Other issues with methods based on a difference in

Figure 1.11: (a) The velocity profile of a synthetic subsurface model. (b) An NMO corrected CMP gather obtained from the subsurface model of Figure a) with primaries only. (c) The same NMO corrected CMP gather, but now with primaries and multiples. [from Verschuur, 2006].
Figure 1.12: (a) A non-increasing velocity profile of a synthetic subsurface model. (b) An NMO corrected CMP gather obtained from the subsurface model of Figure a) with primaries only. (c) The same NMO corrected CMP gather, but now with primaries and multiples. [from Verschuur, 2006].

spatial behavior of primaries and multiples are events in the data that do not have their apex at zero offset in the CMP gather, like diffractions, syncline structures and dipping reflectors.

1.3.4 Predictive deconvolution

In the marine case each event that is measured at the surface, will reflect downward back into the water column where it is partly reflected upward by the water bottom and measured again as a multiple. For shallow water this effect is known as ‘ringing’. The periodicity of the water bottom can be used to remove the water bottom reflections by a filter. This is done by time shifting the data (Figure 1.13a) over the period of the water layer propagation (Figure 1.13b) and subtract this shifted data from the original data with the use of a matching filter (Figure 1.13c). This version of the
predictive deconvolution method [Robinson, 1954] tends to overestimate multiples that have only traveled in the water column compared to multiples that have reflections from below the water bottom. Backus [1959] and Peacock and Treitel [1969] have designed filters that repair this effect. The predictive deconvolution method has only removed the surface-related multiples associated with the water bottom, as can be seen in Figure 1.13c. Figure 1.13c also shows that a certain amount of lateral variation in the water bottom can be incorporated, but that the method will break down with strong lateral variations in the water bottom. Furthermore, the filters are designed such that they minimize the energy in the primary estimate. This can cause the elimination of primary energy as can be seen when Figure 1.13c is compared with the true primary zero-offset section in Figure 1.3b.

1.3.5 Multiple prediction and subtraction methods

In his book on seismic multiple removal techniques Verschuur [2006] describes the several approaches for wave-equation based multiple prediction and subtraction that have been proposed the last two decades; model-based methods [Berryhill and Kim, 1986; Wiggins, 1988; Lokshtanov, 1999] and data-driven methods, like surface-related multiple elimination (SRME) [Verschuur et al., 1992; Berkhout and Verschuur, 1997] and inverse scattering based algorithms [Weglein et al., 1997].

In Berkhout [1982] the theoretical framework behind SRME (see Chapter 2) is intro-
duced that shows the mathematics (integral equation of the second kind) and physics (feedback loop) of the complex multiple system in the subsurface. This framework makes clear that no information of the subsurface is required to remove surface-related multiples. It also shows the important role of the source signature in the subtraction process. Berkhout [1982] (page 211-215) proposes a multiple removal algorithm that consists of matrix convolutions. It is interesting to note that this matrix algorithm is also suitable to remove internal multiples if redatuming is used as a preprocessing step. Berkhout [1982] demonstrates the capability of the matrix convolution algorithm by giving an example for both surface-related multiples (see Figure 1.10) and internal multiples.

In his Ph.D. research Verschuur [1991] shows that by making the subtraction process data adaptive, assuming that primaries contain minimum energy, the unknown source signature can be estimated. This was an important step in making the matrix convolution algorithm (referred to as SRME) an attractive proposition for the industry [Verschuur et al., 1992]. In 1997, a new step forward was introduced by making SRME iterative [Berkhout and Verschuur, 1997]. We will treat the theory behind SRME in Chapter 2.

Both, model-based and data-driven, approaches involve an adaptive subtraction process in which unknowns in the method (water bottom reflectivity or the source wavelet) are estimated. The subtraction process is not obvious, as primaries can easily be distorted if multiples and primaries overlap [Guitton and Verschuur, 2004]. The most common way to adaptively subtract predicted multiples is by least-squares matching the predicted multiples to the data. By doing this one assumes that the primaries have minimum energy (the L2 norm). Guitton and Verschuur [2004] and van Groenestijn and Verschuur [2008] have demonstrated that using a different norm (respectively L1 and sparseness) can lead to different (and better) results. Herrmann et al. [2007] and Neelamani et al. [2008] have used curvelets to subtract the predicted multiples from the data.

1.3.6 Inversion methods

Inspired by the ideas of Berkhout and Verschuur [2006] and van Groenestijn and Verschuur [2006] to turn multiples into primaries, van Groenestijn and Verschuur [2009a] have introduced the estimation of primaries by sparse inversion (EPSI) and its application to near offset reconstruction. The main difference with the traditional
prediction and subtraction approaches is that EPSI considers the primary impulse responses as the unknowns in a large-scale inversion process. The primary impulse responses are parameterized by band limited spikes. They are estimated in such a way that they, together with their corresponding multiples, explain the input data (i.e. primaries and multiples). The residual, the difference between input data and estimated primaries plus multiples, is driven to zero during the optimization process. Recasting multiple removal as a large-scale inversion method, in which the primaries are unknowns, is not new. van Borselen et al. [1996] describe a method that estimates primaries as unknowns in an inversion process, provided that the input wavelet is known. Amundsen [2001] estimates primaries after up/down decomposition by a multi-dimensional division of the upgoing by the downgoing wavefields, provided that the direct wave, especially at the near offsets, is measured in the down going wavefield. EPSI operates on data where the direct wave is removed and does not need to know the source wavelet, because it will estimate it. Other inversion methods that operate on data where the direct wave is removed and do not need to know the source wavelet are described by Biersteker [2001] and Berkhout [2006]. The 2D-decon method as described by Biersteker [2001] estimates the (missing) shallow primaries under a minimum energy constraint of the final primary output. Berkhout [2006] obtains the inverse primaries and the inverse source wavelet by separating them in the inverse data space, which requires a multi-dimensional inversion. In contrast to 2D-decon, EPSI estimates all the primaries simultaneously, and not only the shallow part, and it does not assume that the primaries have minimum energy. Finally, unlike the inverse data space method [Berkhout, 2006], EPSI does not require a transformation to the inverse data space. It is interesting to note that SRME, the inverse data space method, and EPSI use the same feedback model as introduced by Berkhout [1982].

### 1.3.7 Missing near offsets

Especially in shallow water situations wave-equation based prediction suffers from the fact that the near offsets are not measured and, thus, need to be interpolated before the multiple prediction process is applied. This means that wrongly interpolated near offsets will produce errors in the predicted multiples and, therefore, limit the quality of the primary output. Current methods to reconstruct missing near offsets use the assumption that reflections have a certain parabolic or hyperbolic shape [see e.g. Kabir and Verschuur, 1995]. Quite often the data suit these
assumptions, especially after sorting the data to the CMP-offset domain. But for a subsurface with strong lateral variations these assumptions are no longer valid. Moreover, these methods also introduce NMO stretch errors, especially for the shallow events. EPSI can use the multiples to reconstruct the missing near offsets as demonstrated on synthetic data [van Groenestijn and Verschuur, 2009a] and on field data [van Groenestijn and Verschuur, 2009b]. Therefore, EPSI performs well on estimating primaries on shallow water data and does not introduce NMO stretch effects in the reconstructed data. Berkhout and Verschuur [2006] already demonstrated that it is possible to reconstruct the missing near offsets in the data if the primaries and multiples are known outside the near offsets. However, they did not mentioned how to obtain the primaries and multiples independently from the near offset data, a requirement for their method to work. In section 1.3.6 it is already mentioned that the 2D-decon method can reconstruct shallow missing near offsets. This section also mentioned the limitations of the 2D-decon method. Another data-driven reconstruction method is the pseudo primary method [Shan and Guitton, 2004] where a multidimensional auto correlation of the data is used to fill the near offset gap. Curry and Shan [2008] improved the pseudo primary method by extending it with prediction error filters. However, this improvement does not exclude cross correlation artifacts from the missing near offsets completely.

1.4 Thesis outline

The objective of this thesis is to make use of the information that is present in the multiples to estimate primaries, even if the primaries are not directly measured. The thesis contains the following chapters:

- Chapter 2: *The forward model of primaries and multiples and SRME*. In this chapter the detail hiding operator notation is reviewed. This notation makes it possible to describe the forward model of primaries and multiples in terms of matrices for traditional acquisition, blended acquisition and passive acquisition. The forward model for traditional acquisition is used to review surface-related multiple elimination (SRME), and we will discuss the possibilities of applying SRME to data acquired with non traditional strategies. Some characteristics in datasets that can limit the success of SRME are discussed and illustrated.

- Chapter 3: *Estimation of Primaries by Sparse Inversion and its application to*
near offset reconstruction. This chapter introduces the estimation of primaries by sparse inversion (EPSI) and its application to near offset reconstruction. EPSI uses the same forward primary-multiple model as SRME and, like SRME, EPSI does not need subsurface information. EPSI tries to explain the total data, both primaries and multiples, in terms of primary impulse responses, by doing so an adaptive subtraction of multiples is avoided. EPSI does not need to know the complete downgoing wavefield. It estimates the source wavelet and it can reconstruct the missing near offsets. In this chapter the algorithm behind the method is explained. The method is tested on several synthetic datasets. Some of these datasets were used in the preceding chapter to illustrate data characteristics that limit the success of SRME.

- **Chapter 4: EPSI applied to passive data.** In this chapter EPSI is modified such that it can be applied to passive data. The EPSI results are compared with the cross-correlation results. From this comparison it will become clear that EPSI can obtain a true amplitude subsurface response without the uniform surface illumination assumption.

- **Chapter 5: EPSI applied to blended data.** For data acquired with conventional acquisition techniques, surface multiples are usually considered as noise events that obscure the primaries. However, in this chapter we demonstrate that for the situation of blended acquisition, meaning that different sources are shooting in a time-overlapping fashion, multiples can be used to ‘deblend’ the seismic measurements. With some modifications the EPSI method can be used for blended seismic data. As output this process gives unblended primary impulse responses with point sources and receivers at the surface, which can be used directly in traditional imaging schemes. It turns out that extra information is needed to improve on the deblending of events that do not have much associated multiple energy in the data, such as steep events at large offsets. We demonstrate that this information can be brought in during acquisition and during processing. The methodology is illustrated on 2D synthetic data.

- **Chapter 6: Field data examples.** This chapter shows the result of applying EPSI to two different field datasets. The first dataset is a marine dataset with moderate water depth. For this dataset the primary estimation result of EPSI is comparable with SRME. This dataset is used to illustrate the residual. The second field dataset is a marine dataset with shallow water depth. For
this dataset it is shown that EPSI gives a better result than the standard SRME result due to EPSI’s capability to accurately reconstruct the missing near offsets, such that they are consistent with the multiples.

- Chapter 7: Conclusions and Discussion. In this chapter we summarize the key elements of the EPSI method and its applications. We will discuss the relations between different geophysics methods that use multiples. Furthermore, we share some philosophical thoughts and we give a view on the future development of the EPSI method.
The forward model of primaries and multiples and SRME

In this chapter the detail hiding operator notation is reviewed. This notation makes it possible to describe the forward model of primaries and multiples in terms of matrices for traditional acquisition, blended acquisition and passive acquisition. The forward model for traditional acquisition is used to review surface-related multiple elimination (SRME), and we will discuss the possibilities of applying SRME to data acquired with non traditional strategies. Some characteristics in datasets that can limit the success of SRME are discussed and illustrated.

2.1 The detail hiding operator notation

2.1.1 The data matrix

The wavefield is recorded with respect to three coordinates; the source location, \( x_s \), the receiver location, \( x_r \), and time, \( t \). The recordings are discreet in all three coordinates. In this thesis it is assumed that the receivers and sources can only occupy positions on equidistant points on a line. Section 6.3 describes how to preprocess data such that it meets this requirement. The source is fired a distance \( \Delta x \) from the previous shot, such that \( x_s = j\Delta x \). The receivers are a distance \( \Delta x \) apart from
The forward model of primaries and multiples and SRME

Figure 2.1: The measured data are stored in a cube $p(t, x_r, x_s)$. The cube is transformed to the frequency domain. A frequency slice of this cube is the so called data matrix, $P$.

each other, such that $x_r = k \Delta x$. The recordings are sampled in time with $\Delta t$, and, therefore, $t = l \Delta t$. The values of $\Delta t$ and $\Delta x$ should be chosen such that aliasing is avoided. To order the data in the matrices of the detail hiding operator notation [Berkhout, 1982], we first store the measured data in a cube; $p(t, x_r, x_s)$ as is depicted in Figure 2.1. The coordinates $x_r$ and $x_s$ point at a trace that was measured at receiver location $x_r$ during an experiment where the source was fired at location $x_s$. Note that the receiver location $x_r$ is determined with respect to the coordinates of the earth and is not the receiver offset. The coordinate $l \Delta t$ points to the $l^{th}$ time sample of that trace. A Fourier transform along the $t$ axis will bring the cube into the frequency domain. For each frequency component we can now select the so called data matrix, $P$. In this matrix one column represents a monochromatic shot gather and one row a monochromatic receiver gather.

### 2.1.2 Conventions in the detail hiding operator notation

Some conventions in the detail hiding operator notation [Berkhout, 1982].

- Dipole sources. $P$ and all other matrices are assumed to contain responses...
2.1 The detail hiding operator notation

with a dipole source character.

- Depth levels. The subsurface is divided into depth levels \( z_0, z_1, z_2, \ldots, z_0 \) is the surface. \( P(z_m) \) indicates that the data are measured at depth level \( z_m \). \( P(z_m, z_n) \) indicates that the wavefields coming from a source at depth level \( z_n \) are measured at depth level \( z_m \). Omitting \( (z_m) \) or \( (z_m, z_n) \) means that the data are generated by surface sources and is measured at the surface: \( P = P(z_0, z_0) \).

- Upgoing and downgoing wavefields. To make a distinction between upgoing and downgoing wavefields the symbols "-" and "+" are used. \( P^- \) is the upgoing wavefield and \( P^+ \) is the downgoing wavefield. \( P \) without "-" or "+" stands for the sum of the up and downgoing wavefields, \( P = P^- + P^+ \). Note that \( P^- \) and \( P^+ \) both no longer have receiver ghosts, but still have a source ghost (= dipole source).

- Primaries. To indicate the surface related primaries in the data, \( P_0 \) is used. The subscript "0" indicates that no multiples related to the surface \( (z = z_0) \) are present in the matrix. \( P_n \) indicates that no multiples related to all layers up to and including depth level \( z_n \) are present in the matrix. To avoid confusion it is good to note that subscripts are also used to refer to a column or element of a matrix, an iteration, or the type of acquisition with which the data are acquired. The context in which the subscripts are used will clarify their role.

- Matrices, columns, and elements. Matrices are written in bold. The \( k^{th} \) column of \( P \) is a vector and is written as \( \vec{P}_k \). The element of \( P \) in the \( j^{th} \) row and the \( k^{th} \) column is a scalar and is written as \( P_{j,k} \).

- Iterations and estimates. In this thesis the subscript \( i \) is used to indicate that a property is a result of \( i \) iterations. The "\(^{\text{\textdagger}}\)" sign indicates that the quantity beneath it is an estimate. For example \( P_{0,i=3} \) is the primary estimate after three iterations.

- Non traditional acquisition types. The subscripts "\( b_l \)" and "\( p_a s \)" are used to indicate that \( P_{b_l} \) are data acquired with a blended acquisition and \( P_{p_a s} \) are data acquired with a passive acquisition.
2.1.3 Properties

Besides the data matrix $P$, we would like to introduce some more building blocks of the detail hiding operator notation that will allow us to describe the forward model of different acquisition strategies:

- The idea behind the source matrix $S$ is that each source wavefield can be represented by a number of dipole sources. Each element $S_{j,k}$ is one of the representing dipole sources. $j$ refers to the position of the representing dipole source, $j \Delta x$, and $k$ is the experiment number. If the source array consists of only one stable dipole source then $S$ is a diagonal matrix, $S = S(\omega)I$. Bringing all the frequency components of $S_{j,k}$ back to the time domain will result in the source wavelet of the dipole source at position $j$ during experiment $k$.

A source at the surface can only radiate downward, $S^+(z_0)$. A source in the subsurface can radiate both downward, $S^+(z_n)$, and upward, $S^-(z_n)$. In the marine case the physical source is never at the surface, but always a bit below it. Note that the source $S^+(z_0)$ describes the physical sources and their ghosts, which together form dipole sources at $z_0$.

- The subsurface impulse response, $X$. Matrix $X$ represents the impulse response due to a dipole source. It contains the propagation properties downward into the subsurface and, after reflections, upward to monopole receivers. A matrix multiplication of $X$ and $S^+$ will result in the upgoing data:

$$P^- = XS^+. \quad (2.1.1)$$

- In this thesis the primary impulse response, $X_0$, plays an important role. Note that the subscript "0" indicates that $X_0$ does not contain surface-related multiples. A matrix multiplication of $X_0$ and $S^+$ will result in the upgoing primaries:

$$P_0^- = X_0S^+. \quad (2.1.2)$$

- The detector array matrix $D$ is a detector array that transforms the upgoing wavefield $P^-$ into the measured wavefield $P$:

$$P = DP^-.$$  \hspace{1cm} (2.1.3)

Note that receiver deghosting algorithms try to approximate $D^{-1}$ and then apply this approximation to $P$, such that $D^{-1}P = P^-$. 
2.2 The forward primary multiple model

2.2.1 Traditional acquisition

In traditional acquisition a source at the surface is fired, \( S^+ \), and as a consequence a wavefield is sent into the subsurface. Parts of the wavefield are reflected in the subsurface and propagate back up to the surface, \( X_0 \), where they are detected, via

---

**Figure 2.2:** A schematic illustration of the relations between primaries and multiples. (a) A shot gather taken from a dataset with one single reflector. (b) The primary event in the shot gather is the consequence of firing the source. The star is placed at the location where (offset=0), and the time \( t=0 \) when the source is fired. (c) The up-going primaries will reflect at the surface and generate multiples. The same multiples are obtained when in each receiver location a secondary source is present, which is fired at the time the primary event reaches the receiver. These secondary sources of the primary event are depicted as stars. These stars are placed at the locations where, and the time when the secondary sources are fired. (d) The multiples are the result of adding all the delayed primaries.

- The surface reflector matrix \( R^\cap \) transforms each upgoing wavefield at the surface into a downgoing wavefield. For the air-water boundary it can be well approximated by \( R^\cap = -I \).
- The propagation operator \( W(z_m, z_n) \) describes the propagation of wavefields from depth level \( z_n \) to depth level \( z_m \). Between depth levels \( z_n \) and \( z_m \) internal scattering can occur.

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2.2 The forward primary multiple model
The forward model of primaries and multiples and SRME

\[ S(z_0) + X_0(z_0, z_0) P(z_0, z_0) + R(z_0, z_0) \]

\[ \text{Figure 2.3: Feedback loop (after Berkhout [1982]). This loop describes the surface multiple generating process.} \]

\[ \text{D, as primaries:} \]
\[ P_0 = DX_0 S^+. \quad (2.2.4) \]

The upgoing wavefield, \( P^- \), turns into a downgoing wavefield after it is reflected at the surface, \( R^\cap P^- \). Each element of matrix \( R^\cap P^- \) can be seen as a secondary or Huygens’ source [Huygens, 1690; Verschuur, 1992] and its consequences, delayed primaries, can be calculated through a matrix multiplication with \( X_0 \), resulting in the surface multiples, \( M \):

\[ M = DX_0 R^\cap P^- . \quad (2.2.5) \]

The combination of the primaries and multiples will result in the forward model for traditional acquisition:

\[ P = DX_0 S^+ + DX_0 R^\cap P^- , \quad (2.2.6) \]

which after receiver deghosting will give:

\[ P^- = X_0 S^+ + X_0 R^\cap P^- . \quad (2.2.7) \]

Figure 2.2a-d illustrate this forward model of primaries and multiples in terms of Huygens’ sources for the case of a subsurface with one reflector. The feedback loop in Figure 2.3 explains the multiple generating process as a flow diagram. Note that in equation 2.2.6 the data \( P \) are a function of the subsurface properties \( X_0 \) and \( R^\cap \), and the ‘man-made’ properties \( D \) and \( S^+ \).
2.2 The forward primary multiple model

2.2.2 Forward model in terms of up- and downgoing wavefields

If we reorder equation 2.2.7 into:

\[ P^- = X_0 (S^+ + R^- P^-) \]

(2.2.8)

we no longer look at the data in terms of a primary part, \( X_0 S^+ \), and a multiple part, \( X_0 R^- P^- \), but we can recognize an upgoing part, \( P^- \), a downgoing part, \( (S^+ + R^- P^-) \), and a primary impulse response, \( X_0 \), that connects the two. In equation 2.2.8 the receiver level is at the surface but with a receiver line that can measure the up and down going wavefields, as is the case in dual or multicomponent receiver lines, or OBC data, we can also choose the receiver line anywhere in the subsurface as long as the source is above the receiver level. This is depicted in Figure 2.4. We can express this in an equation as follows:

\[ P^-(z_r, z_0) = X_r(z_r, z_r) P^+(z_r, z_0) \]

(2.2.9)

where \( X_r(z_r, z_r) \) is the primary impulse response that transforms down going wavefields from receiver level \( z_r \) downward into the subsurface and upward to monopole receivers at level \( z_r \). Note that \( X_r \) is not a function of the subsurface above depth level \( z_r \), and that the source should be above depth level \( z_r \).

2.2.3 Blended acquisition

In Figure 2.4b and equation 2.2.9, the source is no longer explicitly present. Clearly there should be a source present to generate the downgoing wavefield, but the downgoing wavefield can also be generated by two or more sources. Figure 2.5 illustrates
The forward model of primaries and multiples and SRME

Figure 2.5: The downgoing wavefield allows more than one source wavefield.

this. Shooting with two or more source arrays during one experiment is done in blended acquisition. We can reformulate the forward primary-multiple model for blended acquisition. In Berkhout [2008] and Berkhout and Verschuur [2009] it is explained that the same forward primary-multiple model (equation 2.2.7) can be used for the case of blended acquisition, provided that a blended source matrix $S_{bl}$ is introduced:

$$P_{bl}^{-} = X_0 (S_{bl}^{+} + R^{\top} \bar{P}_{bl})$$

(2.2.10)

where the blended source matrix $S_{bl}^{+}$ carries the information of all the sources that are fired in each experiment. In Berkhout [2008] the structure of this blended source matrix is further explained. There, a blending operator $\Gamma_{bl}$ is introduced (see Figure 2.6), that describes this blending process: $S_{bl}^{+} = S^{+} \Gamma_{bl}$. If the blended experiment contains several sources that shoot with different time delays $t_i$, one column of blending operator $\Gamma_{bl}$ consists of time shift operators $e^{-j \omega \Delta t_i}$ at the spatial locations of the sources that are involved in one blended experiment. Looking at Figure 2.6 one can conclude that blending on the source side is the same as summing columns together, and, therefore, equation 2.2.10 is obtained by multiplying equation 2.2.7 on the left and right hand side with $\Gamma_{bl}$, yielding:

$$P^{-} \Gamma_{bl} = X_0 S^{+} \Gamma_{bl} + X_0 R^{\top} P^{-} \Gamma_{bl}.$$  

(2.2.11)

A special case is when all sources are fired within one blending experiment (i.e. a continuous recording during the firing of all sources in a time-overlapping fashion). In that case the blending operator has only one column vector, $\tilde{\Gamma}_{bl}$, and the resulting blended shot record is described by one column vector as well (i.e. one shot record, but with a very long time duration):

$$\tilde{P}_{bl}^{-} = X_0 S_{bl}^{-} + X_0 R^{\top} \tilde{P}_{bl}.$$  

(2.2.12)

Note that the subsurface properties $X_0$ and $R^{\top}$ are still described by full, unblended, matrices [see also Berkhout et al., 2008].
2.2 The forward primary multiple model

\[ S = \begin{pmatrix} S_{11}, & 0, & 0, & 0 \\ 0, & S_{22}, & 0, & 0 \\ 0, & 0, & S_{33}, & 0 \\ 0, & 0, & 0, & S_{44} \end{pmatrix}, \quad \Gamma_{bl} = \begin{pmatrix} 1, & 0 \\ 0, & 1 \\ \exp(i\omega t_1), & 0 \\ 0, & \exp(i\omega t_2) \end{pmatrix}, \quad S_{pl} = \begin{pmatrix} S_{11}, & 0 \\ 0, & S_{22} \\ \exp(i\omega t_1), & S_{33}, & 0 \\ 0, & \exp(i\omega t_2), & S_{44} \end{pmatrix} \]

\[ P = \begin{pmatrix} P_{11}, & P_{21}, & P_{31}, & P_{41} \\ P_{12}, & P_{22}, & P_{32}, & P_{42} \\ P_{13}, & P_{23}, & P_{33}, & P_{43} \\ P_{14}, & P_{24}, & P_{34}, & P_{44} \end{pmatrix}, \quad \Gamma_{bl} = \begin{pmatrix} 1, & 0 \\ 0, & 1 \\ \exp(i\omega t_1), & 0 \\ 0, & \exp(i\omega t_2) \end{pmatrix}, \quad P_{bl} = \begin{pmatrix} P_{11} + \exp(i\omega t_1), & P_{21}, & P_{31}, & P_{41} \\ P_{12} + \exp(i\omega t_1), & P_{22}, & P_{32}, & P_{42} \\ P_{13} + \exp(i\omega t_1), & P_{23}, & P_{33}, & P_{43} \\ P_{14} + \exp(i\omega t_1), & P_{24}, & P_{34}, & P_{44} \end{pmatrix} \]

**Figure 2.6:** The blending matrix blends columns together.

In Berkhout and Verschuur [2009] the interesting observation is made that when comparing the role of \( S^+_{bl} \) in equation 2.2.12 and \( R^\top P^- \) in equation 2.2.7 one might say that the multiples in equation 2.2.7 are a result of *natural blending*.

### 2.2.4 Passive acquisition

Next, the situation of passive seismic data is considered. In passive seismics the sources, that can be located at the surface or in the subsurface, cannot be controlled. In the passive cases that we will describe the sources are also on the same side of the receiver line as the zone of interest. Following Berkhout and Verschuur [2009], we will separately describe the upward and downward radiating part of the passive sources. Figure 2.7a describes the forward model of the downward radiating part of a passive source:

\[
\tilde{P}^-_{pas}(z_0, z_n) = [X_0(z_0, z_n)W(z_n, z_0)]R \cap \tilde{P}^-_{pas}(z_0, z_n) + X_0(z_0, z_n)S^+(z_n) = X_0(z_0, z_n)R \cap \tilde{P}^-_{pas}(z_0, z_n) + X_0(z_0, z_n)S^+(z_n). \quad (2.2.13)
\]
The forward model of primaries and multiples and SRME

\[
S(z_n) X_0(z_0, z_n) + W(z_0, z_n) - P(z_0, z_n) - R(z_0, z_0) X_0(z_0, z_n) + S(z_0) X_0(z_0, z_0) + W(z_0, z_0)
\]

Figure 2.7: (a) Feedback loop for the downward radiating part of a passive seismic source. (b) Feedback loop for the upward radiating part of a passive seismic source.

If we sum for all depth levels:

\[
\tilde{P}_{\text{pas}}^{-}(z_0) = \sum_n \tilde{P}_{\text{pas}}^{-}(z_0, z_n)
\]

\[
= \sum_n X_0(z_0, z_0) R \cap \tilde{P}_{\text{pas}}^{-}(z_0, z_n) + X_0(z_0, z_n) S^+(z_n)
\]

\[
= X_0(z_0, z_0) R \cap \tilde{P}_{\text{pas}}^{-}(z_0) + \sum_n X_0(z_0, z_n) S^+(z_n). \quad (2.2.14)
\]

Figure 2.7b describes the forward model of the upward radiating part of a passive source:

\[
\tilde{P}_{\text{pas}}^{-}(z_0, z_n) = [W(z_0, z_n) X_0(z_n, z_0)] R \cap \tilde{P}_{\text{pas}}^{-}(z_0, z_n) + W(z_0, z_n) S^-(z_n)
\]

\[
= X_0(z_0, z_0) R \cap \tilde{P}_{\text{pas}}^{-}(z_0, z_n) + W(z_0, z_n) S^-(z_n). \quad (2.2.15)
\]

Again we sum for all depth levels:

\[
\tilde{P}_{\text{pas}}^{-}(z_0) = \sum_n \tilde{P}_{\text{pas}}^{-}(z_0, z_n)
\]

\[
= \sum_n X_0(z_0, z_0) R \cap \tilde{P}_{\text{pas}}^{-}(z_0, z_n) + W(z_0, z_n) S^-(z_n)
\]

\[
= X_0(z_0, z_0) R \cap \tilde{P}_{\text{pas}}^{-}(z_0) + \sum_n W(z_0, z_n) S^-(z_n). \quad (2.2.16)
\]

Adding both the summations of the downward and upward radiating sources together results in:

\[
\tilde{P}_{\text{pas}}^{-}(z_0) = X_0(z_0, z_0) R \cap \tilde{P}_{\text{pas}}^{-}(z_0) + \sum_n W(z_0, z_n) S^-(z_n) + X_0(z_0, z_n) S^-(z_n)
\]

\[
= X_0(z_0, z_0) R \cap \tilde{P}_{\text{pas}}^{-}(z_0) + \tilde{P}_{\text{dir}}^{-}(z_0). \quad (2.2.17)
\]
\(P_{dir}(z_0)\) is called the direct arrival, because its events have not yet reflected at the surface. Note that \(X_0(z_0)R^\cap P_{pas}(z_0)\) describes the multiples in the same way as done in other acquisition strategies.

2.3 SRME

2.3.1 Iterative SRME

From equation 2.2.7 it can be derived that surface multiples, \(M\), for traditional acquisition can be predicted by a multidimensional convolution of the primaries with the total data and with the surface operator, \(A\):

\[M^-=P^+_0AP^-\]

(2.3.18)

where \(A = S^{-1}R^\cap\). Because the primaries are not known beforehand, the relationship is written as an iteration process [Berkhout and Verschuur, 1997]:

\[\hat{P}^-_{0,i+1} = P^- - \hat{A}_{i+1}\hat{P}^-_{0,i}P^-\]

and

\[\hat{P}^-_{0,i=0} = P^-\]

(2.3.19)

where \(i\) represents the iteration number, \(A\) is replaced by \(A(\omega)I\). We will refer to \(\hat{P}^-_{0,i}P^-\) as the predicted multiples and to \(\hat{A}_{i+1}\hat{P}^-_{0,i}P^-\) as the estimated multiples.

Since there are more unknowns (\(\hat{P}^-_{0,i+1}\) and \(\hat{A}_{i+1}\)) then knowns (\(P^-\)) in equation 2.3.19 an extra constraint is needed. Typically it is assumed that the primaries have minimum energy (the L2 norm). This constraint is used when \(\hat{A}_{i+1}\) is estimated as a filter that matches the predicted multiples, \(\hat{P}^-_{0,i}P^-\), to the data in the time domain after which the estimated multiples are subtracted from the data, resulting in the new primary estimation, \(\hat{P}^-_{0,i+1}\), [Verschuur and Berkhout, 1997]. Figure 2.8a shows a shot gather with primaries and multiples. Figures 2.8b,c,e show the primary estimate, \(\hat{P}^-_{0,i}\), for three different iterations. The true primaries are depicted in Figure 2.8d.

2.3.2 SRME for different acquisition strategies

SRME is applied to the upgoing data measured at the surface, \(P^-(z_0, z_0)\). Therefore, data gathered by non-traditional acquisition strategies has to be transformed to \(P^-(z_0, z_0)\) first.
Figure 2.8: Iterative SRME example for synthetic data belonging to the subsurface model of Figure 1.2. (a) Shot gather with all multiples. (b) The SRME primary estimation after the 1st iteration. (c) The SRME primary estimation after the 2nd iteration. (d) The true primaries. (e) The SRME primary estimation after the 3rd iteration.
2.4 Data characteristics limiting the success of SRME

Blended data need to be deblended (obtain $P^-$ from $P^- \Gamma_{mid}$) before SRME can be applied to it.

An estimated subsurface response from passive data can be used as input data for SRME. If this subsurface response is obtained with the correlation method (see Chapter 4), then one should keep in mind that the estimated subsurface response will contain erroneous events coming from cross terms and amplitude inconsistencies (also see Chapter 4), that will have an influence on the SRME outcome.

2.4 Data characteristics limiting the success of SRME

SRME has become a widely accepted multiple removal method in the seismic processing industry and for many datasets it has proven to be of great value. There are some datasets that have characteristics that can limit the success of SRME. We will discuss and demonstrate these characteristics on synthetic datasets. In the next chapter it will become clear whether the EPSI method [van Groenestijn and Verschuur, 2009a] has an answer to these characteristics.

2.4.1 Overlapping primaries and multiples

Figure 2.9a shows a shot gather from a dataset obtained from a synthetic model with two horizontal reflectors. By comparing the data with the true primaries (Figure 2.9b) it becomes clear that the primary reflection from the second reflector overlaps with the first-order multiple from the first reflector. The multiples are predicted (Figure 2.9c) and subtracted from the data, resulting in the primary estimate of Figure 2.9d. It can be seen that the subtraction has attenuated the primary event from the second reflector in its apex in order to obtain a minimum energy solution [see also Guitton and Verschuur, 2004]. Predicting multiples with primaries that are partly attenuated results in predicted multiples that are not strong enough to eliminate the multiples. Note that a subtraction in smaller windows will remove the residual multiples at $t = 1.2s$ and $t = 1.6s$, but will also further attenuate the primary event at $t = 0.8s$. 
Figure 2.9: (a) Shot gather of a dataset obtained from a synthetic model with two horizontal reflectors. (b) The true primaries. (c) The predicted multiples with SRME after three iterations. (d) The estimated primaries with SRME after three iterations.

2.4.2 Noise

If noise is added to the data, $P^- + N$, then this noise will enter the multiple prediction in SRME:

$$P^-_{0,i+1} = P^- + N - A^-_{1+i+1}P^-_{0,i}(P^- + N), \quad (2.4.20)$$
2.4 Data characteristics limiting the success of SRME

![Figure 2.10](image)

Figure 2.10: To one shot gather of the dataset obtained from the complex salt model in Figure 1.2 a horizontal noise event is added. (a) Shot gather with noise event. (b) The true primaries. (c) The predicted multiples for the shot in a). (d) The estimated primaries for the shot in a). (e) The predicted multiples for a shot next to the shot in a). (f) The estimated primaries for a shot next to the shot in a).
The forward model of primaries and multiples and SRME directly as $N$ and indirectly through subtraction of the predicted multiples. Note that in the predicted multiples the noise is not only present in $(P + N)$ but also in $\hat{P}_{0,i}$. To demonstrate the above a horizontal noise event is added to one shot (Figure 2.10a) in a dataset obtained from the synthetic model described in Figure 1.2. The direct effect of the noise can be clearly seen in the primary estimate in Figure 2.10d as the horizontal event at $t = 0.55s$, but also the indirect effect of the subtraction of the predicted multiples (Figure 2.10c) that contain noise, for example the erroneous multiples around $t = 0.8s$. The effect of the horizontal noise event on the primary estimation of the neighboring shot (Figure 2.10f) is still present through the predicted multiples (Figure 2.10e).

### 2.4.3 Sources

In equation 2.3.19 $\hat{A}$ is replaced by $\hat{A}(\omega)I$. This is based on the assumption that the source matrix $S$ can be represented by a matrix with only constant values on the main diagonal, such that $S = S(\omega)I$ and $A = -S^{-1}(\omega)I$. Physically this would equal a source "array" with only one source, and this source would fire the same signal into the subsurface during each experiment. In reality, however, a source array contains more sources. Often the array behaves in such a way that it can be seen as a single source with respect to its influence on the SRME outcome, but if this is not the case equation 2.3.19 can no longer be used, and the surface operator $\hat{A}$ has to be estimated. The estimation of $\hat{A}$ will involve a lot of degrees of freedom, something that is not desired during the adaptive subtraction of the predicted multiples from the data [Prein, 1996].

### Arrays

The effect of estimating primaries with equation 2.3.19 on data shot with a big source array is demonstrated in Figure 2.11. The data are created from the synthetic model in Figure 1.2 and with an array that consists of five dipole sources placed 15 meter (=receiver distance) from each other. All sources have the same source wavelet, such that one can say that each shot in the new dataset is created by summing five successive shots from the dataset used in Figure 2.8. In the summation of the five successive shots the diffractions, visible in the shot gather of Figure 2.8a, have destructively interfered and are no longer visible in the shot gather of Figure 2.11a. The primary estimate is shown in Figure 2.11d. The multiple removal has done
2.4 Data characteristics limiting the success of SRME

Figure 2.11: (a) A shot gather coming from a synthetic dataset, obtained from the complex salt model in Figure 1.2, that is created with a source array of five sources. (b) The true primaries coming from an "array" with only one source. (c) The predicted multiples, after three iterations. (d) The estimated primaries, after three iterations.

...a good job on the multiple events that are not steep. Steep multiples have not been removed properly. The reason for this is that the flat events in the summed primaries appear at the more or less same position as in the true non-summed...
primaries (Figure 2.11b). The steep events are shifted outwards in offset in the summed case, a convolution of the data and the summed primaries will therefore, position the steep parts of the predicted multiples (Figure 2.11c) outward in offset. However, note that (shallow) steep events (both primary and multiple) are usually removed in further processing steps.

**Varying sources**

Another case in which equation 2.3.19 cannot be used is when the source is not firing the same signal into the subsurface during each experiment. To demonstrate this we take the example of a source that is firing a signal into the subsurface which amplitude varies according to a sine wave. The effect of this can be seen in the zero offset section (Figure 2.12e). Figure 2.12c shows the predicted multiples belonging to the shot gather of Figure 2.12a. As can be seen the primaries, that have different amplitudes, convolved with the data have predicted multiples in which the amplitudes appears to have a sine wave. The primary estimation for the same shot can be seen in Figure 2.12d and for a zero offset section in Figure 2.12f. The best way to improve the multiple elimination in this varying source case is to remove the variation in the source wavelet in preprocessing.

**2.4.4 Missing near offsets**

In SRME near offsets are important offsets. This can be quickly seen by applying SRME to data with a near offset gap. Figure 2.13b shows a shot gather from the dataset of Figure 1.2, but this time the near offsets have been removed for each shot. The gap in the predicted multiples (Figure 2.13f) appears twice as large as in the shot gather. This is due to the fact that the gap is also a product of the spatial convolution in equation 2.3.19. The results of subtracting these predicted multiples from the data can be seen in Figure 2.13d. From this result it becomes clear that the missing near offsets should be interpolated. However, in this interpolation process errors will be made in the near offset reconstruction. The causes of these errors are; NMO corrections that cannot satisfy the velocities of both primaries and multiples, events that do not obey to the hyperbolic or parabolic assumption made in Radon interpolation and the unavoidable wavelet stretch. Also note that the predicted multiples have to be matched to the interpolated near offset data. Figures 2.13a,c, and e show the results for data without missing near offsets.
2.4 Data characteristics limiting the success of SRME

Figure 2.12: In a synthetic dataset, obtained from the complex salt model in Figure 1.2, the amplitude of each source signal varies. (a) A shot gather. (b) The true primaries. (c) The predicted multiples, after three iterations. (d) The estimated primaries, after three iterations. (e) The zero offset section. (f) The estimated primaries in a zero offset section.
Figure 2.13: (a) Shot gather. (b) Shot gather with missing near offsets (≤ 165m). (c) The true primaries for the shot in a). (d) The estimated primaries for the shot in b). (e) The true predicted multiples for the shot in a). (f) The predicted multiples for the shot in b).
2.4.5 3D effects

In practice 3D effects in the data are a very important characteristic in a dataset that can limit the success of 2D SRME [see e.g. Dragoset and Jeričević, 1998]. A 3D effect can manifest itself in two ways into the predicted multiples; it can change both the amplitude and the timing of an event. Figure 2.14a is a shot gather from a synthetic dataset. The 2D dataset is created from a 3D model that consists of two plane reflectors that are dipping perpendicular to the sail line. Comparing the position of the predicted multiples (Figure 2.14b) with the multiples in the data shows that the timing is not correct. The estimated least squares filter chooses to lower the energy of all multiples, but does not remove any of the multiples completely (Figure 2.14c).
Figure 2.14: (a) A shot gather coming from a synthetic dataset, obtained from a 3D model with two reflectors dipping perpendicular to the sail line direction. (b) The predicted multiples, after three iterations. (c) The estimated primaries, after three iterations. (d) The true primaries.
Estimation of Primaries by Sparse Inversion and its application to near offset reconstruction

This chapter introduces the estimation of primaries by sparse inversion (EPSI) and its application to near offset reconstruction. EPSI uses the same forward primary-multiple model as SRME and, like SRME, EPSI does not need subsurface information. EPSI tries to explain the total data, both primaries and multiples, in terms of primary impulse responses, by doing so an adaptive subtraction of multiples is avoided. EPSI does not need to know the complete downgoing wavefield. It estimates the source wavelet and it can reconstruct the missing near offsets. In this chapter the algorithm behind the method is explained. The method is tested on several synthetic datasets. Some of these datasets were used in the previous chapter to illustrate data characteristics that limit the success of SRME.
3.1 Estimation of primaries by sparse inversion

3.1.1 The algorithm

In this chapter we propose to estimate primaries through full waveform inversion, such that they are consistent with the primaries and their multiples in the input data. In this way an explicit multiple subtraction step is avoided. The algorithm is based on the same primary-multiple model as discussed in Chapter 2. We repeat equation 2.2.7:

\[ P^- = X_0S^+ + X_0R^0P^- . \]  

We will assume that \( R^0 = I \), but then still this equation has more unknowns, \( X_0 \) and \( S^+ \), than knowns, \( P^- \), and, therefore, an extra constraint is needed to solve it. We will use the constraint that \( X_0 \) is sparse in the time domain. We assume that \( X_0 \) can be represented by a limited number of spikes with large amplitudes (from the major reflecting boundaries) and many small amplitude spikes (from all other events).

A synthetic dataset based on a 2D subsurface model (Figure 1.2) is used to illustrate all the steps involved in this method. Figure 3.1a shows a shot gather from this dataset.

To ensure that the multiples have enough energy to play a role in this process, an exponential gain \( e^{\gamma t} \) (\( t \) is the time and \( \gamma \) is a positive constant) might be applied to each trace. Note that with such an exponential gain the convolutional relationship between primaries and multiples remains intact. The main idea of the proposed method is to estimate \( X_0 \) and \( S^+ \) such that equation 3.1.1 becomes valid. Therefore, the objective function \( J \) is introduced as:

\[ J_i = \sum_\omega \sum_{j,k} |P^- - \hat{X}_{0,i}\hat{S}^+_i - \hat{X}_{0,i}R^0P^- |_{j,k}^2 , \]

where \( i \) denotes the iteration number, \( \sum_{j,k} \) indicates a summation over all the squared elements of the matrix (i.e. a summation over all sources and receivers), and \( \sum_\omega \) indicates a summation over all the frequencies. Note that the objective function will go to zero if the correct \( \hat{X}_0 \) and \( \hat{S}^+ \) are found. This in contrast to SRME, where usually minimum energy on the primaries is assumed.

The objective function is minimized iteratively by estimating \( \hat{X}_0 \) and \( \hat{S}^+ \) in alternate steps. The initial values of \( \hat{X}_0 \) and \( \hat{S}^+ \) are set to zero. The update, \( \Delta \hat{X}_0 \), is a steepest
3.1 Estimation of primaries by sparse inversion

\[
\Delta \mathbf{X}_0 = (\mathbf{P}^\perp - \hat{\mathbf{X}}_{0,i} \hat{\mathbf{S}}_i^+ + \hat{\mathbf{X}}_{0,i} \mathbf{R}^\perp \mathbf{P}^\perp) (\hat{\mathbf{S}}_i^+ + \mathbf{R}^\perp \mathbf{P}^\perp)^H,
\]

where \((\hat{\mathbf{S}}_i^+ + \mathbf{R}^\perp \mathbf{P}^\perp)^H\) is the complex adjoint of \((\hat{\mathbf{S}}_i^+ + \mathbf{R}^\perp \mathbf{P}^\perp)\). The term \((\mathbf{P}^\perp - \hat{\mathbf{X}}_{0,i} \hat{\mathbf{S}}_i^+ - \hat{\mathbf{X}}_{0,i} \mathbf{R}^\perp \mathbf{P}^\perp)\) represents the unexplained data or the residual, which is minimized during the iteration process. The matrix multiplication \((\mathbf{P}^\perp - \hat{\mathbf{X}}_{0,i} \hat{\mathbf{S}}_i^+ - \hat{\mathbf{X}}_{0,i} \mathbf{R}^\perp \mathbf{P}^\perp)(\hat{\mathbf{S}}_i^+)^H\) can be interpreted as information of primaries in the residual that has not yet been explained by the algorithm. The matrix multiplication \((\mathbf{P}^\perp - \hat{\mathbf{X}}_{0,i} \hat{\mathbf{S}}_i^+ - \hat{\mathbf{X}}_{0,i} \mathbf{R}^\perp \mathbf{P}^\perp)(\mathbf{R}^\perp \mathbf{P}^\perp)^H\) can be interpreted as multiples in the residual that are mapped to the primary locations. This mechanism demonstrates that it is possible to transform multiples into primaries, an idea proposed by Berkhout and Verschuur [2006]. Thus, both multiples and primaries are simultaneously used to update the primary impulse responses. The ratio between the contributions of the primaries and multiples can be changed by adjusting the exponential gain, \(e^{\gamma t}\). A higher gain

\[a)\] Shot gather for a source at \(x=2685\) m with all multiples. \(b)\) The corresponding update of the primary impulse response before imposing sparseness, \(\Delta \mathbf{X}_0\). \(c)\) The corresponding update of the primary impulse response after imposing sparseness, \(\Delta \mathbf{X}_0\).
will emphasize the contribution of the multiples, a low gain will emphasize the contribution of the primaries. There are examples in this chapter where a high gain is necessary to come to a correct answer, but note that setting the gain too high will make the system instable.

In EPSI the sparseness is imposed on the update of $\hat{X}_0$. First, the update $\Delta X_0$ is calculated according to equation 3.1.3. Next, $\Delta X_0$ is divided by $\sqrt{\omega_i t}$, after which it is transformed to the time domain. Figure 3.1b shows this result for the same shot gather. The term $\sqrt{\omega_i t}$ represents the frequency dependent behavior of a two dimensional dipole source [Berkhout, 1982]. By dividing out this term, the events in $\Delta X_0$ will have a zero phase appearance in the time domain. Note that since both $\hat{X}_0$ and $\hat{S}$ are zero in the first iteration step, $\Delta X_0$ equals a multi dimensional correlation of the data with itself, $-P(R^\top P^-)^H$. Next, a suitable time window is placed over $\Delta X_0$ in the time domain and the event(s) per trace with the largest amplitudes are selected. The result of this procedure is displayed in Figure 3.1c. By increasing the size of the window in each iteration convergence is improved. In each iteration the window should exclude strong events in the update that are not associated with primaries (e.g. the event at zero offset and $t = 0.45$ s in Figure 3.1b) as much as possible. The primaries precede the multiples that they create. A window will allow us to select mainly primaries. When the window is increased so far that it will include the position of the multiples, then the multiples will be largely removed from the residual. The starting size of the window and the speed with which the window increases are therefore a function of the position of the dominant primaries and dominant multiples. The window will also exclude events arriving before the first arrivals ($t < 0.15$ s).

After bringing $\Delta X_0$ back to the frequency domain it is multiplied by the factor $\sqrt{\omega_i t}$ in order to restore the two dimensional dipole behavior. The sparse update, $\Delta X_0$, is added to the primary impulse response:

$$\hat{X}_{0,i+1} = \hat{X}_{0,i} + \alpha \Delta X_0,$$

where $\alpha$ is a positive frequency independent factor that scales the update step in such a way that the objective function value decreases. For the examples in this chapter we have used the method presented in van Groenestijn and Verschuur [2009a] to determine $\alpha$; the value of $\alpha$ is on purpose chosen too high, such that value of the objective function in equation 3.1.2 is increased after replacing $\hat{X}_{0,i}$ with $\hat{X}_{0,i+1}$. Then the value of $\alpha$ is repeatedly halved until the objective value decreases. A more
advanced way to estimate \( \alpha \) might be by least squares matching 
\[-\alpha \Delta \tilde{X_0}(\hat{S}_i^+ + R^\top P^-) \]
to 
\[(P^- - \tilde{X}_0,i \hat{S}_i^+ - \tilde{X}_0,i R^\top P^-) \]
in the time domain. Throughout the process the value of \( \alpha \) should not vary too much. Large variations in the beginning of the process are a sign of multiples being too weak compared to the primaries (in this case a stronger exponential gain should be applied to the data). Large variations at the end may be an indication that the iteration process is better stopped.

The update of \( \hat{S}_i^+ \) is executed in the same way as the update of \( \tilde{X}_0 \):

\[
\Delta S = \tilde{X}_{0,i+1}^H (P^- - \tilde{X}_{0,i+1} \hat{S}_i^+ - \tilde{X}_{0,i+1} R^\top P^-),
\]

(3.1.5)

At this point prior knowledge on the source array should be used. We know that the dataset of Figure 3.1a is created with a ‘source array’ of one source. This source fired the same signal into the subsurface for each experiment. Therefore, from the full matrix \( \Delta S \) all elements on the main diagonal are replaced by the average of the elements on the main diagonal and other elements are set to zero. \( \Delta S \) is brought to the time domain where the wavelet length is limited. After transforming \( \Delta S \) back to
Estimation of Primaries by Sparse Inversion and its application to near offset reconstruction

Figure 3.3: Direct (a) and conservative (b) estimation of the primaries of a source at \( x=2685 \) m. c) The true primaries. These figures are plotted with the same gain and clipping values as Figure 3.1a.

The frequency domain the update is scaled to ensure that the next objective function of equation 3.1.2 is lower than the previous one. Section 3.2.3 will treat the case of a source array consisting of more sources, and the case of a source signal that is not constant. Figure 3.2 shows how the primary estimation, \( \hat{X}_0, \hat{S}_i \), is being built up during the iterations.

Finally, the estimates of \( \hat{X}_0 \) and \( \hat{S}^+ \) can be used in two ways to obtain a primary estimation: 1) directly, by convolving the estimated (spiky) impulse responses with the estimated wavelets; \( \hat{P}_0^- = \hat{X}_0 \hat{S}^+ \), and 2) in a conservative way, by creating the multiples and subtracting them from the total data; \( \hat{P}_0^- = \hat{P}^- - \hat{M}^- = \hat{P}^- - \hat{X}_0 R^\top \hat{P}^- \). Note that the estimated multiples are simply subtracted from the data without any matching filter. The result of the direct approach is shown in Figure 3.3a and its conservative counterpart in Figure 3.3b. Figure 3.3c shows the true primaries. Note that for this ideal, noise-free situation both primary results are
3.1 Estimation of primaries by sparse inversion

virtually identical. The weak events below 1.0s are internal multiples. These are very well visible in the zero-offset section of the direct primary estimation (Figure 3.4. Internal multiples are surface-related primaries and, therefore, they should not be removed by this process. Residual surface multiples are hardly visible.

3.1.2 Stability and convergence

Note that imposing the sparseness on the update is required to give the primary impulse response $X_0$ its spiky behavior in the time domain. Furthermore, it ensures that strong primaries - and their corresponding multiples - are resolved before the weaker reflections are recovered. In that respect EPSI resembles a matching pursuit
Figure 3.5: (a) Logarithmic contour plot of the objective function value as function of the final estimated primary impulse responses, $\hat{X}_0$, and the final estimated source wavelet, $\hat{S}$; $J(\lambda, \mu) = \sum_\omega \sum_{j,k} |P - \lambda \mu \hat{X}_0 \hat{S} + \lambda \hat{X}_0 R^c P|_{j,k}^2$. The parameter $\lambda$ varies $\hat{X}_0$, and the parameter $\mu$ varies $\hat{S}$. The contour plot has a minimum in point $\mu = 1, \lambda = 1$. Away from this point each contour line represents an increase of 0.6dB. (b) The same experiment as in a, but this time an exponential gain $\gamma = 1s^{-1}$ is applied to the data and the final estimated primary impulse responses in the time domain before calculating the objective function. Away from the minimum each contour line represents an increase of 0.3dB. (c) The same experiment as in a, but this time an exponential gain $\gamma = 2s^{-1}$ is applied to the data and the final estimated primary impulse responses in the time domain before calculating the objective function. Away from the minimum each contour line represents an increase of 0.2dB.

approach. However, it should be pointed out that EPSI is robust in the sense that during iterations it can repair errors made in earlier iterations. As an example, iteration artifacts in Figures 3.2a, b and c (e.g. around the apex of the first event) are not visible anymore in Figure 3.3a. To get some insights in the convergence of the method we calculate the effect of varying the final estimated $\hat{S}^+$ and $\hat{X}_0$ in the objective function:

$$J(\lambda, \mu) = \sum_\omega \sum_{j,k} |P - \lambda \mu \hat{X}_0 \hat{S}^+ + \lambda \hat{X}_0 R^c P|_{j,k}^2,$$

where $\lambda$ is a scalar that scales the final estimated $\hat{X}_0$, and $\mu$ is a scalar that scales the final estimated $\hat{S}^+$. In Figure 3.5a the effects of varying both $\lambda$ and $\mu$ between 0.1 and 10 can be seen. There appears to be a clear minimum in $\lambda = 1$ and $\mu = 1$. The dominant direction of the contour lines is the line $\lambda + \mu = \text{constant}$. This line
3.2 EPSI dealing with specific datasets

In this section we will apply EPSI to the same datasets that were treated in section 2.4 to see if EPSI can overcome some of the data characteristics that limit the success of SRME.

3.2.1 Overlapping primaries and multiples

In Figure 3.6 the experiment with the synthetic model with two horizontal reflectors of Figure 2.9 is repeated but now using the EPSI method. By comparing the data
with the true primaries (Figure 2.9b) it becomes clear that the primaries reflection from the second reflector overlaps with the first-order multiple from the first reflector. As can be seen EPSI has succeeded in estimating both primary events (Figure 3.6b). In order to achieve this result an exponential gain is applied with $\gamma = 2s^{-1}$ to put more emphasis in the contribution from the multiples. Note that in practice more information is present in the multiples than in the data of Figure 3.6a due to the fact that there will be more reflections in the subsurface, and there will be a longer recording time. Figure 3.6c shows the estimated multiples.

### 3.2.2 Noise and data inconsistencies

If we add noise to the data then $P^-$ will be replaced in all formulas with $P^- + N$ for the objective function this results in:

$$J_i = \sum_{\omega} \sum_{j,k} |(P^- + N - \hat{X}_{0j}S^+_i + \hat{X}_{0j}R^\cap(P^- + N))|^2_{j,k}. \quad (3.2.7)$$

To demonstrate the effect this will have on the primary estimation a noise event is added to one shot (Figure 3.7a) in a dataset obtained from the synthetic model described in Figure 1.2. Figure 3.7b shows the primary estimation and Figure 3.7c the residual. Compared to the SRME result on the same data in Figure 2.10 there are some interesting differences. Parts of the noise event have been left out of the primary estimation, and are still in the residual. The indirect influence of the erroneous multiples (the noise event convolved with the primary impulse responses, $X_0R^\cap N$) has also been partly left out the primary estimate. The result of Figure 3.7b is far from perfect, and more research is needed to fully understand it, but it shows the potential of using the relationship between primaries and multiples to attenuate noise. In section 6.1, dealing with the EPSI results on field data, more remarks are made on the role of the residual with respect to more realistic forms of noise and data inconsistencies, such as 3D effects, incorrect 3D to 2D conversions and interpolation errors.

### 3.2.3 Sources

Below equation 3.1.5 it is discussed how to treat $\Delta S$ in case of a 'source array' of one laterally constant dipole source. Two more cases are described here; array effects and source variations.
3.2 EPSI dealing with specific datasets

Figure 3.7: In a synthetic dataset a noise event is added to one shot. (a) The shot gather with the noise event. (b) The estimated primaries. (c) The residual.

Source array

Figure 3.8a shows a shot gather coming from a synthetic dataset that is created with a source array of five sources placed 15m (=receiver distance) from each other and the synthetic subsurface model of Figure 1.2. All sources have the same wavelet such that one can say that each shot in this dataset is created by summing five successive shots from the dataset used in Figures 3.1 till 3.4. For this case the five main diagonals of matrix $\Delta S$ (equation 3.1.5) are selected. We do not assume that the five sources have the same wavelet, but we assume that the wavelet of each source does not change during acquisition. Therefore, we have to estimate five unknown source wavelets. To do this we average the main diagonal of $\Delta S$ and each of its four closest neighboring diagonals. We replace the elements on these diagonals with their averages. The primary estimation shown in Figure 3.8b is the estimated primary impulse response convolved with the estimated wavelet of the middle source in the source array. The shot gather of Figure 3.8a, which is a result of summing five shots,
Figure 3.8: (a) A shot gather coming from a synthetic dataset that is created with a source array of five sources. (b) The estimated primary impulse response convolved with the estimated source wavelet of the middle source. The gain of this figure is three times as high as Figures a) and c). (c) The estimated primary impulse responses convolved with the entire source array.

is wider than this primary estimate. Comparing Figure 3.8b with the true primaries in Figure 3.3c reveals that the diffractions are not visible in the primary estimate.
3.2 EPSI dealing with specific datasets

Note that they are also not visible in the shot gather (Figure 3.8a). Also note that the events are slightly jittered. However, if we look at the primary impulse responses convolved with the entire source array (Figure 3.8c), this jitter is eliminated in the summation. Figure 3.9 shows the estimated source wavelets for the five sources in the source array. The EPSI result can be compared with the SRME result in Figure 2.11.

Varying sources

EPSI can also estimate the wavelet of each shot separately. We demonstrate this on a synthetic dataset where the amplitude of the source wavelet is a function of the source position. This function is a sine wave as can be seen in the zero-offset section of Figure 3.10d. Figure 3.10b shows the primary estimate and Figure 3.10c the estimated multiples. For this example there is a significant difference with the
Figure 3.10: (a) A shot gather coming from a dataset that is created by one source that does not fire the same wavelet into the subsurface during each experiment. (b) The estimated multiples. (c) The estimated direct primaries. (d) Zero-offset section of the data. (e) Zero-offset section of the estimated primary impulse responses convolved with the same wavelet for each shot.
SRME result in Figure 2.12. Figure 3.10e shows the estimated primary impulse responses convolved with the same wavelet for each shot in a zero offset section. As can be seen the variation in the source strength does not have an influence on the primary impulse responses that are only a function of the subsurface. In Figure 3.11a the estimated source wavelets are plotted on top of each other, they only differ in amplitude. In Figure 3.11b the maximum amplitude of each estimated source wavelets is plotted as a function of the source position.

Note that in Chapter 5 we will deal with another variation of source arrays, being blended data. For the blended situation sources are fired (semi) simultaneously, but
with large distances between them.

3.2.4 3D effects

Figure 3.12a shows a shot gather from a synthetic dataset. The 2D dataset is created from a 3D model that consists of two plane reflectors that are dipping perpendicular to the sail line. The 2D forward primary-multiple model does not incorporate the 3D effects. As a consequence of that EPSI does not succeed in explaining the data after 120 iterations, which can be seen in the residual (Figure 3.12b). During the iterations the energy in the residual has been reduced but the estimation of the primary impulse responses has become a compromise between explaining part of the primary energy and part of the multiple energy, as can be seen in the estimated direct primaries (Figure 3.12c). The conservative estimated primaries (Figure 3.12d) are not much better off. The dipping in the synthetic model has been chosen very extreme to show the possible 3D effect on a 2D algorithm. In Chapter 6 EPSI will be applied to field datasets with 3D effects. These 3D effects are not so extreme and do not hinder EPSI to come to a good result. In Chapter 6 the role of the residual will be further discussed.

3.3 Reconstruction of missing near offsets

In this section we will review the reconstruction of missing near offsets in EPSI, as proposed by van Groenestijn and Verschuur [2009a]. The reconstruction is completely data driven, and, therefore, it does not create wavelet stretch. The basic EPSI method is extended to the situation of missing near offsets. Equation 2.2.7 shows how the upgoing wave field, $P^-$, is built from sources, $S^+$, reflected data (which can be seen as secondary sources), $R^\gamma P^-$, and primary impulse responses, $X_0$:

$$P^- = X_0 S^+ + X_0 R^\gamma P^-.$$  \hspace{1cm} (3.3.8)

Although the missing near offset data are not measured, the consequences of "firing" the secondary sources in the near offset region are measured in the multiples. Therefore, the near offset data can be reconstructed from the multiples. Figure 3.13a-f illustrates this. We will discuss the points in the EPSI algorithm for data with missing near offsets that differ from the algorithm for the data without missing near offsets.
3.3 Reconstruction of missing near offsets

Figure 3.12: (a) A shot gather coming from a synthetic dataset, obtained from a 3D model with two reflectors dipping perpendicular to the sail line direction. (b) The residual. (c) The estimated direct primaries. (d) The estimated conservative primaries.
The total data obtained in iteration $i$ consist of two subsets: $P_i^- = \hat{P}_i^\prime + P''$, where $P''$ is the part of the upgoing data that does not need to be reconstructed and $\hat{P}_i^\prime$ is the missing near offset part that has to be reconstructed. In the objective function $P^\prime$, $\hat{X}_0$, and $\hat{S}^+$ are the unknowns:

$$J_i = \sum_\omega \sum_{j,k} |P_i^- - \hat{X}_{0,j}\hat{S}_j^+ - \hat{X}_{0,i}\hat{R}^\prime \hat{P}_i^-|^2_{j,k}. \quad (3.3.9)$$

For the first iteration, the values for $\hat{X}_0$, $\hat{S}^+$ and $\hat{P}^\prime$ are set to zero. It is interesting to point out that the difference between the 2D-decon method described by Biersteker [2001] and EPSI is in essence the term $\hat{X}_0\hat{S}^+$ in equation 3.3.9 that is not present in 2D-decon. This means that 2D-decon relies on the minimum energy assumption of the primaries. Furthermore, note that 2D-decon typically aims at reconstructing only the shallow part of the full $X_0$ (say the first few hundred milliseconds).

The iteration process consists of three consecutive steps. First, the update of $X_0$ is calculated. It is the same as in equation 3.1.3, except that we use the data $P_i^-$.\[\text{Figure 3.13: A schematic illustration of the relations between primaries and multiples. (a) A shot gather taken from a dataset with one single reflector. (b) The primary event in the shot gather is the consequence of firing the source. The star is placed at the location where (offset=0), and the time (t=0) when the source is fired. (c) The up-going primaries will reflect at the surface and generate multiples. The same multiples are obtained when in each receiver location a secondary source is present, which is fired at the time the primary event reaches the receiver. These secondary sources of the primary event are depicted as stars. These stars are placed at the locations where, and the time when the secondary sources are fired. (d) The multiples are the result of adding all the delayed primaries. (e) The same shot gather as in a). The shaded area indicates the offset gap in the data. (f) The first order multiple is built from delayed primaries caused by secondary sources. The secondary source inside the missing data gap has not been measured but its consequences have an effect outside the gap.}\]
(with reconstructed near offset data from the previous iteration) instead of $P^-\cdots$:

$$\Delta X_0 = (P_i^- - X_{0,i}^+ - X_{0,i} R^\circ P_i^-)(S_i + R^\circ P_i^-)^H. \quad (3.3.10)$$

The same substitution, $P_i$ for $P$, is also the only thing that changes for the update of $S^+$:

$$\Delta S = X_{0,i+1}^H (P_i^- - X_{0,i+1}^+ - \tilde{X}_{0,i+1} R^\circ P_i^-). \quad (3.3.11)$$

Finally, the update of $\hat{P}'$ is applied, which is given by the following steepest descent step:

$$\Delta \hat{P}' = -(I - \tilde{X}_{0,i+1} R^\circ)^H (P_i^- - \tilde{X}_{0,i+1}^+ - \tilde{X}_{0,i+1} R^\circ P_i^-). \quad (3.3.12)$$

Events that arrive before the primary water bottom reconnection are removed from $\Delta P'$ in the time domain, together with parts of the data we do not want to reconstruct (because they are already in $P^\alpha$). The update of $\hat{P}'$ is scaled to ensure that the next objective function value of equation 3.3.9 is lower than the previous one.

To demonstrate the above, we again use the data related to the model in Figure 1.2 and remove near offsets. Figure 3.14a shows one input shot record for the same source location as in Figures 3.1 and 3.3. The direct and conservative primary estimation results are shown in Figures 3.14b and c. Figure 3.14c has missing near offsets since it is obtained by subtracting the multiples from the data with missing near offsets. Note the excellent resemblance with the results in Figure 3.3a and b. Also note the quality of the reconstructed near offsets in Figure 3.14b. Figures 3.15a and b show the zero offset section of the true data and the reconstructed total data. Except from some minor artifacts, the reconstruction is very good. Note that the inversion method also yields the primary estimate at the near offsets (Figure 3.15c), which appears very satisfactory as well. Especially note the accurately reconstructed diffraction events. Also note that no NMO corrections were applied to the data during the near offset reconstruction process and, therefore, a wavelet stretch has been avoided.

### 3.4 Discussion

Although the number of iterations in EPSI is higher (for the synthetic examples typical 30 to 60 iterations and for the marine data with moderate water depth 180 iterations and for the shallow water marine data 240 iterations in Chapter 6)
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than the number of iterations in SRME (for the synthetic and field datasets three iterations), we see reasons to assume that the calculation time does not have to be a factor 10 to 80 bigger: First, because $\tilde{X}_0$ is spiky in the time domain, its convolution with the total data can be carried out in the time domain saving on the costs of going through a Fourier transform. Secondly, the correlation $P(R^T P)^H$ has to be calculated only once. Furthermore, the number of iterations can be brought down by making the number of events that are selected from $\Delta X_0$ a function the iteration number. Also note that it takes a lot of iterations to build the direct primary estimation, however, the conservative primary estimation takes its form much faster since most of the multiples are the result of a few dominant primaries that are estimated in the beginning of the process.

We believe that there is large scope to apply this new combined primary and near offset estimation method for shallow water situations, as this is where SRME is
3.5 Conclusion

In this chapter we have presented a new primary estimation method; estimation of primaries by sparse inversion (EPSI). The primaries are considered unknowns in a full waveform inversion process. Compared to SRME there are two interesting differences: there is no adaptive subtraction step that matches multiples to data and EPSI does not need interpolated near offsets to estimate primaries. Results for synthetic 2D datasets show its feasibility.

known to have difficulties [see e.g. Verschuur, 2006].

EPSI is a very open method in the sense that it is easy to build in other applications, like the reconstruction of missing near offsets, or to apply it to other forms of data, like blended data (Chapter 5) or passive data (Chapter 4). Therefore, we think that EPSI can also be reformulated for the full 3D marine case, where the missing data between the streamer lines might be treated in a similar way as the missing near offsets in the 2D method.

3.5 Conclusion

Figure 3.15: Zero offset section of the true data (a), the reconstructed data (b) and the estimated primaries, $X_0 \hat{S}$ (c). These figures have the same gain and clipping values as Figure 3.1a.
Estimation of Primaries by Sparse Inversion and its application to near offset reconstruction
EPSI applied to passive data

In this chapter EPSI is modified such that it can be applied to passive data. The EPSI results are compared with the cross-correlation results. From this comparison it will become clear that EPSI can obtain a true amplitude subsurface response without the uniform surface illumination assumption.

4.1 Introduction

For passive seismic data surface multiples are used to obtain an estimate of the subsurface responses, usually by a cross-correlation process. This cross-correlation process relies on the assumption that the surface has been illuminated uniformly by subsurface sources in terms of incident angles and strength. If this is not the case the cross-correlation process cannot give a true amplitude estimation of the subsurface response. Furthermore, there are cross terms in the cross-correlation result that are not related to actual subsurface inhomogeneities. In this chapter we will demonstrate that, with some modifications to the algorithm, EPSI can obtain true amplitude subsurface responses without the uniform surface illumination assumption. The EPSI method will go beyond the cross-correlation step and will estimate surface-related primaries only from the surface-related multiples in the available signal. The estimated primary impulse responses, with point sources and receivers at the surface, can be used directly in traditional imaging schemes. The methodology is illustrated
for 2D synthetic data.

In passive seismics no controlled sources, like airguns, explosives or vibrator trucks, are used but passive sources, like mini earthquakes within the subsurface of the earth, reservoir rocks cracking due to fluid pressure during production, or heavy traffic. We will compare the passive data model with the primary-multiple model and demonstrate that cross-correlating data, as usually done in seismic interferometry [see e.g. Claerbout, 1968; Schuster, 2001; Shapiro and Campillo, 2004; Wapenaar, 2004; Wapenaar et al., 2004; Snieder et al., 2006; Dellinger and Yu, 2009; Draganov et al., 2009], is only the first step in a modified version of the recently introduced estimation of primaries by sparse inversion (EPSI) method [van Groenestijn and Verschuur, 2009]. In Berkhout and Verschuur [2009] a framework for describing active and passive seismic data was given. There, it was proposed to find the primary impulse responses via an inversion method. It turns out that the modified EPSI algorithm does that, as is demonstrated in this chapter (the contents of this chapter are also published in van Groenestijn and Verschuur [2010]).

An advantage of estimating primary impulse responses compared to cross-correlating data is that the surface no longer has to be illuminated uniformly by the passive sources in terms incident angles and strengths. Wapenaar et al. [2008] also describes a method that has the same favorable characteristic. However, they assume that the direct signal arriving from all passive sources is known, which makes it possible to obtain the total impulse responses (including multiples) through multi-dimensional deconvolution. In this chapter we have assumed that this direct arrival is unknown.

### 4.2 Passive seismic data

The main difference with the surface seismic data is that primary reflection responses from controlled sources are not present. Instead, there is a direct arrival from passive sources towards the surface. This direct arrival is the cause of a series of surface multiples (Figure 4.1). In section 2.2.4 the forward model for passive data is derived:

$$\vec{P}_\text{pas} = \vec{P}_\text{dir} + X_0 R^\dagger \vec{P}_\text{pas},$$  \hspace{1cm} (4.2.1)

where $\vec{P}_\text{pas}$ is the measured upgoing wavefield and $\vec{P}_\text{dir}$ is the direct signal arriving from all passive sources in the subsurface [see also Berkhout and Verschuur, 2009]. $\vec{P}_\text{pas}$ and $\vec{P}_\text{dir}$ are both described by one column vector (i.e. one shot record, but with a very long time duration). The first term in equation 4.2.1 cannot be expressed
in terms of primary impulse responses, but the second term $X_0 R \cap \tilde{P}_{\text{pas}}$ equals the multiple term in the primary-multiple model (equation 2.2.7). Rearranging terms leads to $\tilde{P}_{\text{dir}} = \tilde{P}_{\text{pas}} - X_0 R \cap \tilde{P}_{\text{pas}}$. The sparseness constraint on $X_0$ in the time domain is no longer enough to solve the two unknowns $\tilde{P}_{\text{dir}}$ and $X_0$. Therefore, an extra constraint is required and we assume that $\tilde{P}_{\text{dir}}$ has minimum energy. Thus the objective function to minimize is now:

$$J_i = \sum \sum |\tilde{P}_{\text{pas}} - X_0, i R \cap \tilde{P}_{\text{pas}}|^2.$$  

(4.2.2)

We assume that $R \cap = -I$. The algorithm starts with setting $X_0$ to zero. The update of $X_0$ is given by:

$$\Delta X_0 = (\tilde{P}_{\text{pas}} - X_0, i R \cap \tilde{P}_{\text{pas}})(R \cap \tilde{P}_{\text{pas}})^H.$$  

(4.2.3)

Note that $\Delta X_0$ is again a full matrix. For the first iteration this update is equal to the multidimensional correlation of the data with itself, $\tilde{P}_{\text{pas}} (R \cap \tilde{P}_{\text{pas}})^H$, as used in seismic interferometry [see e.g. Wapenaar et al., 2004]. Figure 4.2 illustrates how the correlation $(\tilde{P}_{\text{pas}} - X_0, i R \cap \tilde{P}_{\text{pas}})(R \cap \tilde{P}_{\text{pas}})^H$ removes the first part of the raypath, resulting in the path of the primary impulse response. Note that the correlation will also create artifacts and gives incorrect amplitudes for the primary impulse responses. These artifacts and incorrect amplitudes will influence the residual and, therefore, are dealt with in the later iterations. Next, a window is placed over the update $\Delta X_0$ in the time domain and sparseness is imposed on $\Delta X_0$. Note that the choice of the window for the passive data case is less trivial than for the active data case. For active data, the observed shallow events, such as the water bottom
reflection, will help to optimally design a window that in the first iterations does not pick up the first water bottom multiple. For passive data this prior knowledge is not present and needs to be extracted from the cross-correlation result. The strongest primaries need to be identified and the window must be based on that interpretation. The update of $X_0$ is made sparse, scaled and added to $X_0$ in the same was as described in section 3.1.1.

In each iteration $X_0$ is updated. The iterations are stopped when no more (visible) changes are observed in the residual. What is left in the residual is an estimate of $\tilde{F}_{dir}$. Note that EPSI applied to passive data does not estimate the source signals emitted by the passive sources, and does not make an assumption on their properties.

For EPSI applied to actively and passively acquired data only the upgoing wavefields are used. For land/ocean bottom cable (OBC) data we assume that surface/interface waves are removed by filtering and multicomponent measurements are used to obtain an up/down separation. For marine data it means that deghosting is applied.

### 4.3 Results

#### 4.3.1 two reflector model

The proposed inversion method is tested on a synthetic dataset. The 2D two-reflector model used to create this data can be seen in Figure 4.3. 81 subsurface sources,
that are emitting small bursts, were placed randomly in an area below the second reflector.

The first update of $X_0$, before application of the sparseness constraint, is shown in Figure 4.4a. Note again that this is the traditional interferometry result, obtained by cross-correlating the traces of the passive data. Figure 4.4b shows the first update of $X_0$ after applying sparseness. How the estimation of the primary impulse responses develops during the iterations can be seen in Figures 4.4c-e. For display purposes the spiky primary impulse response estimates have been convolved with an arbitrary wavelet.

Figures 4.5d-f show the obtained primary impulse responses after 30 iterations. Note the reduction of cross-terms, visible in the traditional interferometric result (Figures 4.6d-f) after 30 iterations of our algorithm (Figures 4.5d-f). For comparison, the modeled primaries from a standard reflection survey at the surface have been displayed in Figures 4.5a-c. Note that the use of a sparseness constraint yields small discontinuities at various locations. Modeled shot gathers are displayed in Figure 4.6a-c for comparison of the interferometric result with the total (primaries and multiples) subsurface response. Also note that not all angles are present in the estimated primary impulse responses. This can be understood because the surface is not illuminated with all angles, and, therefore, the surface reflection cannot illuminate the

Figure 4.3: The subsurface model used to generate the passive seismic data. It shows the location of the passive sources below the second reflector. The top layer is water.
Figure 4.4: (a) The first update of the primary impulse responses before windowing and making it sparse for a 'shot' at receiver position 1000m. Note that this represents the traditional interferometry result. (b) The result of a) after it is windowed and made sparse. (c-e) The estimated primary impulse response for different iterations with an arbitrary wavelet for display purpose.

subsurface under all angles. The illumination angles per offset can be estimated from the 'crosses' at and around offset=0 and t=0 in the correlation, \( \tilde{P}_{\text{pass}}^{-1}(R \tilde{P}_{\text{pass}})^H \).

In Figures 4.6g-i the interferometry results are plotted in the \( \tau - p \) domain. The cross in Figure 4.6e is the sum of the correlations of each local plane wave arriving at receiver position \( x = 1000m \). Correlating the plane wave event in the trace \( x = 1000m \) with itself will result in a peak at \( t = 0 \). Correlating the plane wave event in trace \( x = 1000m \) with the same plane wave event in trace \( x = 1025m \) will show a peak with a time shift inversely proportional to the apparent velocity of the plane wave.
wave event. Since the lowest apparent velocity in our model is the water velocity ($1500 \text{ms}^{-1}$) the steepest angle in the crosses can be $1/(1500 \text{ms}^{-1}) = 66 \cdot 10^{-5} \text{sm}^{-1}$. As can be seen from the event at $\tau = 0$ in Figure 4.6h the ray parameter associated with the steepest angle in the water layer is missing. Figure 4.7a displays the zero offset section of the interferometric result and Figure 4.7b displays the zero offset section of the estimated primary impulse responses convolved with the same arbitrary wavelet as in Figures 4.5d-f.
Figure 4.6: Modeled shot gathers obtained from standard acquisition belonging to a shot at receiver position: (a) 500m, (b) 1000m, (c) 1500m. (d-f) The interferometric result obtained from the same passive data as used for Figures 4.5. (g-i) $\tau-p$ transforms of the interferometric results. Note that these displays are centered around zero time.
4.3 Results

Figure 4.7: Zero offset section of (a) the interferometric result and (b) the EPSI result for data from subsurface sources that are equal in strength. Zero offset section of the (c) interferometric result and (d) EPSI result for data from subsurface sources that vary in source strengths from left to right with a factor 4. Note that the EPSI result is insensitive to the source strength variations.
### 4.3.2 Different source strengths

As stated earlier, one interesting aspect of our approach is that it can obtain $X_0$ also in the case that the passive sources have different strengths and non-uniform illumination angles, this in contrast to the traditional cross-correlation method. Wapenaar et al. [2008] also described a method that has the same favorable characteristic. However, they assume that $\vec{P}_{\text{dir}}$ is known. In this chapter we have assumed that $\vec{P}_{\text{dir}}$ is unknown. However, if (parts of) $\vec{P}_{\text{dir}}$ are known they could be subtracted from $(\vec{P}_{\text{pas}} - X_0R\cap \vec{P}_{\text{pas}})$ in equation 4.2.2. In that case the objective function will get (closer to) zero. Furthermore, note that our method provides the primary impulse responses instead of the total impulse responses that include the surface multiples.

We test the case of passive sources with different strengths for the subsurface model of Figure 4.3. We use the same subsurface source locations except this time the source strengths are no longer equal, but increase linearly as function of the horizontal source position, such that a subsurface source at $x = 2000m$ is four times as strong as a source at $x = 0m$. Figure 4.7c shows the zero offset section of the interferometric result, $\vec{P}_{\text{pas}}(R\cap \vec{P}_{\text{pas}})^H$. Clearly the influence of the different source strengths can be seen. Figure 4.7d shows the zero offset section of the EPSI result. As expected, no influence of the different source strengths can be observed.

### 4.3.3 Subsurface source area that are limited horizontally

Next, we test the case where the area of subsurface sources is limited horizontally. For this we use the same subsurface model of Figure 4.3, but this time we only take the subsurface sources between $x = 800m$ and $x = 1200m$. Figures 4.8a-c show the interferometric result of this data. Figures 4.8d-f show the EPSI result. Apart from a small leakage in the EPSI result, we can see that the events that are visible are in the right location. This in contrast to the cross-correlation result that also shows events in the wrong locations. Compared to Figures 4.5d-f fewer angles are present in the primary impulse response estimates, but this can be understood by the fact that the illumination angles of the surface have been reduced. Thus, this is not a limitation of our method, but an intrinsic limitation of the data. The limitation becomes visible in the $\tau-p$ plots in Figures 4.8g-i.
4.3 Results

Figure 4.8: The EPSI result for data with a limited horizontal range of subsurface sources for a 'shot' at receiver position: (a) 500m, (b) 1000m, (c) 1500m. (d-f) The interferometric result on the same data. (g-i) $\tau - p$ transforms of the interferometric result.
4.3.4 Complex subsurface model

Finally, we test the EPSI method with a more complex subsurface model (Figure 4.9a). 361 subsurface sources (equal in source strengths) are randomly distributed in the area between depth 980m and 1045m and between lateral distance 0m and 5400m (Figure 4.9b). Figure 4.10a shows a part of the input data. The events in the figure are generated by one subsurface source. EPSI has explained the multiples, $\mathbf{X}_0 \mathbf{R} \hat{\mathbf{P}}_{pas}$, (Figure 4.10b) in this data. The unexplained data (Figure 4.10c) are considered to be the direct arrival, $\hat{\mathbf{P}}_{dir}$.

Figures 4.11d-f show the obtained primary impulse responses. For comparison the modeled primaries from a standard acquisition (Figures 4.11a-c) and the interferometric result, being the first step of our inversion algorithm, (Figures 4.11g-i) are shown. It is clearly visible that not all angles are present in the estimates. This can be understood by the fact that the surface points are not illuminated by all angles, as can be seen in Figures 4.12a-c. Apart from the discontinuity due to missing illumination angles, it is clearly visible that the primary impulse responses are discontinuous in some other parts. The EPSI method works with placing spikes (see Figure 4.4b). For a simple subsurface model as in Figure 4.3, this happened in a continuous fashion, but for a more complex model some parts in the end result are discontinuous. However, the multiples that are created through a multidimensional convolution and summation, $\mathbf{X}_0 \mathbf{R} \hat{\mathbf{P}}_{pas}$, with these primary impulse responses are
4.4 Discussion

4.4.1 Algorithm

An extra constraint may be added to the objective function in equation 4.2.2 to enforce events to be laterally more consistent. Taking into account that this constraint will also force true discontinuous events to be erroneously continuous, plus the fact that a lot of processes that will follow primary estimation in the seismic processing chain are not hindered by discontinuity, one might decide to keep the discontinuous parts. Stacking is an example of a process that is not hindered by discontinuity as is demonstrated in the stacked estimated primary impulse responses in Figure 4.10.
Figure 4.11: A shot gather obtained from standard acquisition belonging to a shot at receiver position: (a) 1800m, (b) 2700m, (c) 3600m. (d-f) The estimated primary impulse responses via EPSI obtained from passive data with an arbitrary wavelet for the same positions. (g-i) The interferometric result for the same passive data.
4.4 Discussion

4.13b. With respect to discontinuous events it is interesting to note that in Lin and Herrmann [2009] the curvelet transform is combined with the EPSI algorithm for simultaneous source data. By minimizing the L1 norm of the estimated primary impulse responses in the curvelet domain, lateral continuity is improved.

We have no physical justification for the assumption that the direct arrivals have minimum energy, but we would like to point out that more or less the same assumption is made in multiple elimination methods applied to actively acquired data, like SRME (section 2.3). In these multiple elimination methods minimum energy of the primaries is assumed when the predicted multiples are adaptively subtracted from the data. For most cases this assumption results in the correct primary estimation. However, in datasets where primaries and multiples overlap in the same way everywhere in the dataset, the assumption turns out to be invalid. The data from passive subsurface sources, however, might find itself in a more favorable position with respect to the overlap between the direct arrival and the multiples. This is due to the fact that the variation in direct arrivals is bigger than the variation in primaries from actively acquired data, and therefore the direct arrivals and multiples overlap less in the same way everywhere in the dataset.

It might give some insights to reorder equation 4.2.1 into:

\[ \hat{p}_{\text{pas}} - \hat{p}_{\text{dir}} = X_0 R \hat{p}_{\text{pas}}', \]  

\( (4.4.4) \)
Figure 4.13: Stacks of (a) the true primaries obtained from standard acquisition, (b) the EPSI result on passive data, (c) the total data obtained from standard acquisition, (d) the interferometric result on passive data.
In this way we have a downgoing wavefield \( (\mathbf{R}^{\text{down}}) \), the consequences of this downgoing wavefield \( (\hat{\mathbf{P}}_{\text{pas}} - \hat{\mathbf{P}}_{\text{dir}}) \), and the primary impulse responses that connect both. The inversion approach is in essence making the division of the upgoing wavefield by the downgoing. By looking at EPSI as a method that divides the upgoing by the downgoing wavefield it becomes clear that the receivers do not have to be positioned at the surface.

EPSI can be of great value for reservoir monitoring through passive acquisition. The primary impulse responses are a function of the (changing) subsurface and not of the subsurface source strengths. Moreover, the primary impulse responses estimated by a passive acquisition can be combined with the primary impulse responses obtained by EPSI from a standard acquisition. EPSI is a very open method in the sense that it is easy to build in other applications, like the reconstruction of missing near offsets [see van Groenestijn and Verschuur, 2009a]. Therefore, we think that EPSI can also be reformulated to incorporate the estimation of the direct arrivals (shown in Figure 4.10c) in equation 4.2.2.

### 4.4.2 Application to field data

The question is how well our method will behave on field data. The synthetic models that we have chosen both have their random sources located in a small layer, thus mimicking reservoir rocks that crack during production. These microseismic events can be monitored [see e.g. Maxwell and Urbancic, 2001]. Current studies on field data, however, show that in practice applying the interferometric method is not trivial. Dellinger and Yu [2009] only managed to reconstruct Scholte waves from passive OBC data. This means that our method, that uses the cross-correlation process as a first step, cannot be used in such case. Draganov et al. [2009] managed to reconstruct reflection energy only after pre-processing the passive data, amongst others by applying dip filters to remove the surface waves. For such data our proposed method can be used. Using densely sampled passive receiver arrays will increase the chance of success of our method, because this allows better pre-processing, such as aliasing-free dip filters.

It is known that attenuation is a problem for interferometry [Ruigrok et al., 2008]. EPSI, however, can handle attenuation and will estimate it as part of the impulse response. This means that for example dispersion effects become visible at later arrival times, meaning that EPSI needs a few spikes to describe each event at later
times, requiring more iterations in the process. In Chapter 6 EPSI is applied to two marine field datasets and there it it is demonstrated that the attenuation of higher frequencies will slow the convergence of EPSI down since it will create an $X_0$ that is less spiky. However, the end result is not affected by it.

4.5 Conclusions

In this chapter we presented the extension of EPSI to the situation of passive seismic data. EPSI uses the result of the usually applied cross-correlation process for passive data as input of an inversion process that will provide the impulse responses of the subsurface. Compared to the cross-correlation method, the EPSI method will remove the spurious correlation events and end up with primaries only. Furthermore, the obtained primary impulse responses are true amplitude, without the sensitivity to the distribution and strengths of the various noise sources as observed in cross-correlation results.
EPSI applied to blended data

For data acquired with conventional acquisition techniques, surface multiples are usually considered as noise events that obscure the primaries. However, in this chapter we demonstrate that for the situation of blended acquisition, meaning that different sources are shooting in a time-overlapping fashion, multiples can be used to 'deblend' the seismic measurements. With some modifications the EPSI method can be used for blended seismic data. As output this process gives unblended primary impulse responses with point sources and receivers at the surface, which can be used directly in traditional imaging schemes. It turns out that extra information is needed to improve on the deblending of events that do not have much associated multiple energy in the data, such as steep events at large offsets. We demonstrate that this information can be brought in during acquisition and during processing. The methodology is illustrated on 2D synthetic data.

5.1 Introduction

In traditional acquisition methods the time interval between different shots is taken large enough to ensure that subsurface responses of different shots do not overlap. Beasley et al. [1998], de Kok and Gillespie [2002], Stefani et al. [2007], Hampson et al. [2008], Berkhout [2008], Akerberg et al. [2008], Fromyr et al. [2008], Moore et al. [2008], and Spitz et al. [2008] have described acquisition methods where time
intervals between shots are deliberately taken too small. This allows speeding up the acquisition, or improving illumination within the same acquisition time. The price that is paid is that the measured data consist of blended subsurface responses of different sources. Hence the name blended data, as proposed by Berkhout [2008]. The latter also suggests two routes for processing of blended data: 1) try to 'deblend' the data into individual shot records without overlap or 2) redesign processing steps such that they can take the blended effect into account. With data deblending we mean the reconstruction of the original separate source responses from the blended measurements. Beasley et al. [1998] and Stefani et al. [2007] show that stacking already reduces a lot of the noise from the overlapping signals.

In this chapter (which contents are also published in van Groenestijn and Verschuur [2010b]) we extend the application of EPSI to blended data. We will modify the EPSI algorithm such that it can deblend blended seismic data. Furthermore, it will provide primary impulse responses as the output. Thus, multiple removal and separation into the individual single source responses is obtained in one integrated process. It is important to realize that the information used to deblend the data comes from the surface multiples.

First, we will discuss the forward primary-multiple model for standard seismic acquisition and briefly review the EPSI method (for more details see Chapter 3). Then, we will show that the forward primary-multiple model can be easily extended to the situation of blended seismic data. By making some modifications to the algorithm of EPSI it can reconstruct primaries from blended data. The reconstruction can be improved by adding extra information to the data. This can be done during acquisition or during processing. In this chapter we will demonstrate both aspects by simulating acquisition with a marine vibrator and by bringing in arrival time information.

### 5.2 EPSI for unblended data

We will (quickly) go through the EPSI algorithm for unblended data in order to set the scene for the blended data. Again, we can express the upgoing data at the surface, $P^-$, as:

$$P^- = X^+ S^+ + X^+ R^\circ P^-.$$  \hspace{1cm} (5.2.1)
If we reorder the above equation into:

$$P^+ = X_0 (S^+ + R^\top P^-),$$  \hspace{1cm} (5.2.2)

we no longer look at the data in terms of a primary part, $X_0 S^+$, and a multiple part, $X_0 R^\top P^-$, but we can recognize an upgoing part, $P^-$, a downgoing part, $(S^+ + R^\top P^-)$, and the primary impulse responses, $X_0$, that act as a transfer function to connect the down- and upgoing wavefields. Figure 5.1a illustrates the up- and downgoing wavefields. In case that the entire up- and downgoing wavefields are known, $X_0$ might be obtained by a division of the upgoing wavefield by the downgoing. This is, for example, possible in the case of OBC data [Amundsen, 1999]. In the case of surface data this is not possible because $S^+$ is an unknown part in the downgoing wavefield.

If we take $S^+ = S(\omega)I$, meaning a constant source wavelet for all shots and neglecting the directivity of the source array, equation 5.2.1 becomes:

$$P^+ = X_0 S + X_0 R^\top P^-.$$  \hspace{1cm} (5.2.3)

We aim at an inversion process that estimates $\hat{X}_0$ and $\hat{S}$ such that the total data, $P^-$, are explained according to equation 5.2.3. Thus, the objective function $J$ is introduced as:

$$J_i = \sum_\omega \sum_{j,k} |P^- - \hat{X}_{0,i} \hat{S}_i - \hat{X}_{0,i} R^\top P^-|^2_{j,k}. \hspace{1cm} (5.2.4)$$

First, $\hat{X}_0$ is updated. The update, $\Delta \hat{X}_0$, is a steepest descent step:

$$\Delta \hat{X}_0 = (P^- - \hat{X}_{0,i} \hat{S}_i - \hat{X}_{0,i} R^\top P^-)(\hat{S}_i I + R^\top P^-)^H,$$  \hspace{1cm} (5.2.5)
where \((\hat{S}_i I + R^\odot P^-)^H\) is the complex adjoint of \((\hat{S}_i I + R^\odot P^-)\). The term \((P^- - \hat{X}_{0,i} \hat{S}_i - \hat{X}_{0,i} R^\odot P^-)\) can be interpreted as the unexplained data or the residual. A synthetic dataset based on a 2D subsurface model (Figure 5.2) is used to illustrate the EPSI method. Figure 5.3a shows one shot gather from this dataset. Figure 5.3b shows the first update step, \(\Delta X_0\), for the same shot position. Because both \(\hat{X}_0\) and \(\hat{S}\) are zero in the first iteration step, the first update step equals a multidimensional correlation of the data with itself, \(P^- (R^\odot P^-)^H\). In equation 5.2.5 the residual contributes in two ways to the update, \(\Delta X_0\). The contribution, \((P^- - \hat{X}_{0,i} \hat{S}_i - \hat{X}_{0,i} R^\odot P^-)(\hat{S}_i)^H\), aims at transforming residual primaries to the update and the other contribution, \((P^- - \hat{X}_{0,i} \hat{S}_i - \hat{X}_{0,i} R^\odot P^-)(R^\odot P^-)^H\), aims at transforming residual multiples to the update. Figures 5.4a and b demonstrate this last feature. For the first iteration, the information in Figure 5.3b comes entirely from the second contribution. The far offset part of the primaries is clearly not present in this contribution. Figure 5.4c illustrates that this is due to the fact that the multiples that contain the necessary information for the far offset primaries are not in the measurement aperture.

In order to constrain the inversion process, it is proposed in section 3.1.1 to enforce sparseness on the update of \(\hat{X}_0\), which is achieved in a separate step. A window is placed over the update of \(\hat{X}_0\) in the time domain and the strongest event(s) per

Figure 5.2: The subsurface model with two horizontal interfaces used to generate (blended) seismic data. The top layer is water.
Figure 5.3: Results for the unblended data simulated in the model of Figure 5.2: (a) Shot record. (b) The corresponding update of the primary impulse response before imposing sparseness, $\Delta X_0$. (c) The corresponding update of the primary impulse response after imposing sparseness, $\Delta X_0$. (d-f) Corresponding updates of the estimated primaries, $\hat{X}_{0,i}, \hat{S}_i$, for various iterations. (g) The direct and (h) the conservative primary estimation.
The result of this process is shown in Figure 5.3c. Next, the sparse update, \( \Delta \tilde{X}_0 \), is added to the primary impulse response:

\[
\tilde{X}_{0,i+1} = \tilde{X}_{0,i} + \alpha \Delta \tilde{X}_0,
\]

where \( \alpha \) is a positive frequency independent factor that scales the update step. Scale factor \( \alpha \) is chosen such that the objective function value decreases.

The update of \( \tilde{S} \) is executed in a similar way as the update of \( \tilde{X}_0 \):

\[
\Delta \tilde{S} = \tilde{X}^H_{0,i+1}(P - \tilde{X}_{0,i+1}\tilde{S}_i - \tilde{X}_{0,i+1}R^P).
\]

From the full matrix \( \Delta \tilde{S} \) the diagonal elements are selected and summed to get the scalar \( \Delta \tilde{S} \). \( \Delta \tilde{S} \) is brought to the time domain where the wavelet length is limited. After that the update is scaled to ensure that the next objective function of equation 5.2.4 is lower than the previous one.

These two update steps are repeatedly applied until the residual is small enough. Figures 5.3d-f show how the primary estimation, \( \tilde{X}_{0,i} \tilde{S}_i \), is built up during the iterations. Finally, the estimates of \( \tilde{X}_0 \) and \( \tilde{S} \) can be used in two ways to obtain a primary estimation: 1) directly, by convolving the estimated (spiky) impulse responses with the estimated wavelets; \( \tilde{P}_0^\tau = \tilde{X}_0\tilde{S}_i \), and 2) in a conservative way, by creating the multiples and subtracting them from the total data; \( \tilde{P}_0^\tau = \tilde{P}^- - \tilde{M}^- = \tilde{P}^- - \tilde{X}_0R^P \).

Note that the estimated multiples are simply subtracted from the data without any matching filter. The result of the direct approach is shown in Figure 5.3g and its
conservative counterpart in Figure 5.3h. For this simple model and for data without noise they are virtually identical.

5.3 Epsi for blended seismic data

In this section we will reformulate the forward primary-multiple model for blended seismic acquisition and demonstrate that EPSI can be modified for this situation.

In the case of blended acquisition the downgoing wavefield is generated by two or more sources. Figure 5.1b illustrates this situation. In Berkhout [2008] and Berkhout and Verschuur [2009] it is explained that the same forward primary-multiple model (equation 5.2.2) can be used for the case of blended acquisition, provided that a blended source matrix $S_{bl}$ is introduced:

$$P_{bl} = X_0 (S_{bl}^+ + R^\Gamma P_{bl}^-), \quad (5.3.8)$$

where the blended source matrix $S_{bl}^+$ carries the information of all the sources that are fired in each experiment. In Berkhout [2008] the structure of this blended source matrix is further explained. There, a blending operator $\Gamma_{bl}$ is introduced (see Figure 2.6), that describes this blending process: $S_{bl}^+ = S^+ \Gamma_{bl}$. If the blended experiment contains several sources that shoot with different time delays $t_i$, one column of blending operator $\Gamma_{bl}$ consists of time shift operators $e^{-j\omega \Delta t_i}$ at the spatial locations of the sources that are involved in one blended experiment. Looking at Figure 2.6 one can conclude that blending on the source side is the same as summing columns together in a weighted fashion, and, therefore, equation 5.3.8 is obtained by multiplying equation 5.2.2 on the left and right hand side with $\Gamma_{bl}$, yielding:

$$P^- \Gamma_{bl} = X_0 (S^+ \Gamma_{bl} + R^\Gamma P^- \Gamma_{bl}). \quad (5.3.9)$$

Note that an upgoing and a downgoing part can be recognized again both in equation 5.3.9 and Figure 5.1b. Also note that the subsurface properties $X_0$ and $R^\Gamma$ are still described by full, unblended, matrices [see also Berkhout et al., 2008]. A deblending result can be obtained by estimating the unblended $X_0$.

$X_0$ and $S_{bl}^+$ are estimated via EPSI. We take $S_{bl}^+ = S(\omega) \Gamma_{bl}(\omega)$, assuming that the blended acquisition is done with several identical sources. Thus, the new objective function to be minimized becomes:

$$J_i = \sum_{j,k} \sum_{\omega} |P_{bl}^- - \hat{X}_{0,i,j,k} \hat{\Gamma}_{bl} - \hat{X}_{0,i} R^\Gamma P_{bl}^- |^2_{j,k}. \quad (5.3.10)$$
Initially, $\hat{X}_0$ and $\hat{S}_i$ are set to zero. For the blended situation the update of $\hat{X}_0$ is given by:

$$\Delta \hat{X}_0 = (P_{bl}^- \hat{X}_{0,i+1} - \hat{X}_{0,i} \hat{S}_i \Gamma_{bl} - \hat{X}_{0,i} R \cap P_{bl}^-) (\hat{S}_i \Gamma_{bl} + R \cap P_{bl}^-)^H.$$  \hfill (5.3.11)

Next, both the increasing window and sparseness are applied to the update as is done in the EPSI method for traditional data. The update of $\hat{S}$ is executed in the same way as the update of $\hat{X}_0$:

$$\Delta \hat{S} = \hat{X}_{0,i+1}^H (P_{bl}^- \hat{X}_{0,i+1} - \hat{X}_{0,i+1} \hat{S}_i \Gamma_{bl} - \hat{X}_{0,i+1} R \cap P_{bl}^-) \Gamma_{bl}^H.$$  \hfill (5.3.12)

From the full matrix $\Delta \hat{S}$ the diagonal elements are selected and summed to get the scalar $\Delta S$. $\Delta S$ is brought to the time domain where its length is limited. After that the update is scaled to ensure that the next objective function of equation 5.3.10 is lower than the previous one.

### 5.4 Results for a two reflector subsurface model

We will illustrate the algorithm with a blended dataset obtained from the two reflector subsurface model in Figure 5.2. Each blended shot record contains a shot coming from a boat sailing from the first receiver position to the middle receiver position and an other boat sailing from the middle receiver position to the last receiver position. In Figure 5.5a a shot record from the blended dataset is selected. The reflections caused by the right shot have arrived a little earlier than the reflections from the left shot. This is due to a random delay with which the left source is fired during each blended experiment. The random delays are current practice in blended acquisition [see e.g. Stefani et al., 2007].

Figure 5.5b shows the first update step, $\Delta \hat{X}_0$, for the left shot in Figure 5.5a. The noise is coming from events that have correlated with events from the right shot. It is clear that due to the extra blending noise in the update, a proper definition of the selection window becomes more crucial. Note that the far offset part of the water bottom primary (indicated by the arrow in Figure 5.5b) is not completely present in the correlation. The reason for this is the same as in the unblended situation (Figure 5.3b); the multiples associated with the steep, far offset part of the water bottom primary are not present within the measurement aperture. Therefore, the multiples cannot provide any information on the deblending of this part of the primaries. However, the primary update in equation 5.3.11 consists of two terms: one
5.4 Results for a two reflector subsurface model

Figure 5.5: Results for the blended data simulated in the model of Figure 5.2: (a) Blended shot record. (b) The corresponding update of the primary impulse response before imposing sparseness, \( \Delta \mathbf{X}_0 \). (c) The corresponding update of the primary impulse response after imposing sparseness, \( \Delta \hat{\mathbf{X}}_0 \). (d-f) Corresponding updates of the estimated primaries, \( \hat{\mathbf{X}}_{0,i} \hat{\mathbf{S}}_i \), for various iterations. The arrow points at the location where reconstruction of primaries was insufficient due to limited aperture in the multiples.

that maps multiples in the blended data to unblended primaries, \( (\mathbf{P}_{bl} - \hat{\mathbf{X}}_{0,i} \hat{\mathbf{S}}_i \Gamma_{bl} - \hat{\mathbf{X}}_{0,i} \mathbf{R}_i \mathbf{P}_{bl}) (\mathbf{R}_i \mathbf{-P})^H \), and one that maps primaries in the blended data directly into unblended primaries \( (\mathbf{P}_{bl} - \hat{\mathbf{X}}_{0,i} \mathbf{R}_i \mathbf{P}_{bl}) (\hat{\mathbf{S}}_i \Gamma_{bl})^H \). So with the second term the primaries can still be found, however, this route is more sensitive to
cross-talk. Part of this cross-talk can be reduced in following iterations if wrongly positioned primary energy will create multiples that do not match with multiples in the blended data.

Figure 5.5c shows the update after it is made sparse. Figures 5.5d-f show the primary estimations for the left shot in Figure 5.5a during the iterations. The primary estimations are obtained by convolving the estimated primary impulse responses with the estimated source wavelet, $\hat{X}_{0,i}^j$.

The final results after 120 iterations are shown in Figures 5.6. Figure 5.6a shows the blended shot record. In Figure 5.6d we can see that the far offset part of the water bottom primary of the right shot has leaked into the estimated primaries of the left shot (indicated by the arrow). For this part of the primaries the multiples could not provide information. Also in Figure 5.6e we can see that the water bottom primary of the other side has leaked into the primary estimation of the right shot (indicated by the arrow). Apart from these small leakages the shots have been deblended well and they can be further processed in traditional imaging schemes (route number 1 in Berkhout [2008]).

The primary impulse responses can also be convolved with the blended data to obtain the multiples, $\hat{X}_{0}^\text{R}^j \cdot \hat{P}_{0}^j$. These multiples (Figure 5.6c) can be subtracted from the blended data to get a conservative estimate of the primaries (Figure 5.6b). These conservative estimates of the primaries can be one step in a processing scheme that takes the blended effect into account (route number 2 Berkhout [2008]). The residual is shown in Figure 5.6f. Note that, although the estimated unblended primaries (Figures 5.6d,e) exhibit a small leakage, the multiple removal quality is very good (Figure 5.6b).

### 5.5 Results for a complex subsurface model

Next, we will test the EPSI method with a more complex subsurface model (Figure 5.7). Again each blended shot record contains a shot coming from a boat sailing from the first receiver position to the middle receiver position and another boat sailing from the middle receiver position to the last receiver position. The boat that sails from the first receiver position to the middle receiver position fires with a random delay time.
5.5 Results for a complex subsurface model

Figure 5.6: Results for the blended data simulated in the model of Figure 5.2: (a) Blended shot record. (b) Multiple removal result. (c) Removed multiples. (d) The primary estimation belonging to the left shot. (e) The primary estimation belonging to the right shot. (f) The residual. The arrows point at remaining cross-talk in the deblended result.

5.5.1 EPSI

The final results of EPSI on the blended dataset after 120 iterations are shown in Figures 5.8 and 5.9. Note that all data results related to this model are plotted with a linear time gain. Figure 5.8a shows the blended shot record. Figure 5.8b shows the primary estimation for the left shot, and Figure 5.8c the primary estimation for the
right shot. The reconstruction of the two primary wavefields is quite good given the fact that they are completely mixed in Figure 5.8a. Comparing the estimated primaries in Figures 5.8b-c with the true primaries in Figures 5.8d-e shows that the reconstruction is not perfect. It appears that the steep water bottom events are more difficult to reconstruct than the flatter events. Again, the reason for this is that multiples associated with the steep dipping primaries are hardly present within the measurement aperture, whereas the flatter primaries are implicitly present in several multiples in the data. The leakage of primary events from one shot into the other shot and the small multiple leakage into the primaries can be seen as the price one pays for acquiring blended data. Note again, that the estimated primary impulse responses are reasonably well deblended and that they can be further processed in traditional imaging schemes.

The multiples, $\tilde{X}_d R^c P_{bl}$, (Figure 5.9c) can be subtracted from the blended data (Figure 5.9a) to get a conservative estimate of the primaries (Figure 5.9b). For comparison the true blended primaries are displayed in Figure 5.9e. Note again that the leakage in the estimated primaries (Figure 5.8b-c) does not produce noticeable discrepancies in the estimated blended primaries (Figure 5.9b). The residual is shown in Figure 5.9d.

To improve the deblending result for the steep dipping events extra information has to be added. This can be done during acquisition and during processing. We will give a demonstration of both.
Figure 5.8: Results from the blended complex subsurface data: (a) Blended shot record. (b) The primary estimation belonging to the left shot. (c) The primary estimation belonging to the right shot. (d) The true primaries belonging to the left shot. (e) The true primaries belonging to the right shot.
Figure 5.9: Results from the blended complex subsurface data: (a) Blended shot record. (b) Multiple removal result. (c) Removed multiples. (d) The residual. (e) The true blended primaries.
5.5 Results for a complex subsurface model

5.5.2 Adding information during processing

By knowing the depth of the water bottom and the water velocity it is easy to roughly predict where the steep water bottom events are located in the data. We can bring this information into the EPSI algorithm. We will add an extra window, referred to as 'steep dip window' on top of the window that already is applied to the update of the primary impulse responses, $\Delta X_0$. The steep dip window that we use for the update to the primary impulse response belonging to the left shot is drawn in Figure 5.10b and the one belonging to the right shot in Figure 5.10c. During the first series of iterations the steep dip window will not expand such that the steep water bottom cannot be attributed to the wrong source. After the first series of iterations is done, the size of the steep dip window is increased and a new series of iterations is started with the values of $\hat{X}_0$ and $\hat{S}$ from the previous series as starting values. The process of a series of iterations followed by an increase in the steep dip window is repeated until the steep dip window covers the whole shot gather.

Figure 5.10: (a) Blended shot record. (b) Steep dip window for the left shot. (c) Steep dip window for the right shot.
Figure 5.11: Results from the blended complex subsurface data estimated with the use of the steep dip window: (a) Blended shot record. (b) Multiple removal result. (c) Removed multiples. (d) The primary estimation belonging to the left shot. (e) The primary estimation belonging to the right shot. (f) The residual.
A comparison of the reconstruction results of Figures 5.8b-c with Figures 5.11d-e shows that now the leakage is much smaller and the primary water bottom reflection is largely attributed to the correct source position in Figure 5.11d-e.

### 5.5.3 Adding information during acquisition

We use the same blending acquisition strategy and subsurface model as the one with which the dataset of Figure 5.8 and 5.11 is created, except the airgun sources are replaced with marine vibrators that emit complex source signals, $ST_{bl}$. $S$ is unknown and $T_{bl}$ known, but much more complex than in the previous acquisition. Compared to the previous acquisition the non-zero elements in the blending matrix, are replaced with a number of random delays $(\gamma_1 e^{-j\omega \Delta t} + \gamma_2 e^{-j\omega 2\Delta t} + \gamma_3 e^{-j\omega 3\Delta t} + \ldots + \gamma_n e^{-j\omega n\Delta t})$, where $\gamma_1$ till $\gamma_n$ are set randomly and differ for each source and for each shot. Instead of a single time shift for each shot we now have created a convolution filter that will cause the marine vibrator to behave as if its source is fired $n$ times with source amplitudes that are multiplied by $\gamma_1$ till $\gamma_n$. The effect of selecting this blending matrix can be seen in Figure 5.12a. It is hard to judge how well the blended primaries and multiples are estimated (Figure 5.12b and c), but Figures 5.12d and e show that the deblended primaries have been obtained very well.

### 5.6 Discussion

In Berkhout and Verschuur [2009] the interesting observation is made that when comparing the role of $S^T T_{bl}$ in equation 5.3.9 and $R^\top P^-$ in equation 5.2.1, one might say that the multiples in equation 5.2.1 are a result of natural blending. A non-zero trace of $S^T T_{bl}$ in the time domain in the case of the marine vibrator and a trace of $R^\top P^-$ in the the time domain look very similar. The $S^T T_{bl}$ trace consists of a wavelet convolved with a series of spikes (the coding), the $R^\top P^-$ trace consists of a wavelet convolved with the subsurface response (a natural coding). Both traces force the algorithm to find an $X_0$, that convolved with the trace, explains the data. The difference between the two traces is that a trace of $R^\top P^-$ does not have zero traces as its neighbors, but has traces that are very similar to itself. This has as a consequence that steep dipping events in $X_0$ are difficult to reconstruct from the multiples as was demonstrated.

On passive seismic data (Chapter 4) we have reported similar issues with the retrieval
Figure 5.12: Results from the blended complex subsurface data with the marine vibrator using random delay codes: (a) Blended shot record. (b) Multiple removal result. (c) Removed multiples. (d) The primary estimation belonging to the left shot. (e) The primary estimation belonging to the right shot. (f) The residual.
of steep events. The fact that the steep dipping events are hard to deblend might in many cases be a non-issue because shallow, steep events are often removed in further processing steps.

In the last example where EPSI and source encoding are used together, both methods are helping each other. The source encoding improves the information on primaries that have no multiples in the measurement aperture, and EPSI reduces the number of events (from data with primaries and multiples to primary impulse responses without wavelets) that need to be separated. This was also concluded by Lin and Herrmann [2009]. Lin and Herrmann [2009] also noted the interesting fact that their deblending algorithm based on source encoding combined with EPSI gave a better primary estimation result than their deblending algorithm followed by EPSI. This is again a good example of the fact that EPSI takes the physics into account.

When it comes to processing of blended field data we think that EPSI should be used in combination with other methods (source encoding, incoherency of the data in an other domain) because all these methods are helping each other. In order to make use of the deblending possibilities of EPSI, multiples need to be present in the data. The blended acquisition strategy should sample the surface with sources and receivers densely, such that EPSI has the right information. The last condition will most likely be easily satisfied, because a dense sampling is one of the reasons to do a blended acquisition. We expect no practical issues with a receiver cable that moves with the sources, like in marine acquisition. Similar to the case of regular SRME, reciprocity can be used for the parts of the primary impulse responses that are not present in the measured data to get the estimation more stable. Similar to the case of the 2D SRME method, an application to 3D data will not be trivial. At first, we need to investigate the impact of 3D subsurface structures on the 2D implementation of the proposed method. When the earth has structural variations in the cross-line direction, the estimated primaries and their corresponding multiples will not be kinematically and dynamically precise. Therefore, we envision the need of extra adaptation filters to be included in the EPSI process to compensate for these effects. Of course, the introduction of these extra filters may also introduce unwanted interaction between primaries and multiples. This needs to be further investigated. However, the blended acquisition will develop itself into the direction of 3D as that is the area where the added value of blended data can be found. Therefore, a 3D implementation of EPSI is necessary.
5.7 Conclusions

In this chapter we presented the extension of the estimation of primaries by sparse inversion (EPSI) to the situation of blended data. For the blended data case EPSI was able to reconstruct the primaries as if they were obtained from a standard acquisition. The natural encoding of the measured data in the downgoing wavefield allows for the estimation of deblended primary impulse responses from the multiples. Bringing in extra information during acquisition or processing improves the deblending of steep events.
Field data examples

This chapter shows the result of applying EPSI to two different field datasets. The first dataset is a marine dataset with moderate water depth. For this dataset the primary estimation result of EPSI is comparable with SRME. This dataset is used to illustrate the residual. The second field dataset is a marine dataset with shallow water depth. For this dataset it is shown that EPSI gives a better result than the standard SRME result due to EPSI’s capability to accurately reconstruct the missing near offsets, such that they are consistent with the multiples.

6.1 Results on marine data with moderate water depth

In Figure 6.1a a shot gather from the marine data with moderate water depth can be seen. Figure 6.1b shows the corresponding update of the primary impulse response. In Figure 6.1c the same primary impulse update is displayed after windowing and making it sparse. The build-up of the primary estimation during the iterations can be seen in Figure 6.2.

As demonstrated in section 3.1.1 the estimates of $\hat{X}_0$ and $\hat{S}^+$ can be used in two ways to come to a primary estimation: a direct way, by convolving the estimated impulse responses with the estimated wavelets, $\hat{P}^- = \hat{X}_0 \hat{S}^+$, and a conservative way, by creating the multiples and subtracting them from the data; $\hat{P}^- = P^- - M^-$.
Figure 6.1: (a) Input shot gather with multiples from the moderately deep water marine data. (b) The corresponding update of the primary impulse response ($\Delta X_0$). (c) The corresponding update of the primary impulse response after windowing and applying sparseness ($\Delta X_0$).

$P^* - \hat{X}_0 R P^*$. Note that the estimated multiples are simply subtracted from the data without any matching filter. The difference between the two options is that in the second option, the residual data are added to the direct primary estimate. The direct primary estimation for the shot gather under consideration (Figure 6.3a) is displayed in Figure 6.3d. The corresponding estimated multiples are plotted in Figure 6.3e. Iterative SRME has been applied to the same dataset (Figures 6.3b and c). Both SRME and EPSI have removed the water bottom multiple. It is interesting to look at the strong reflection around 1.0 seconds in the primaries at the near offsets (indicated by the arrows). One can observe that SRME has weakened this primary.

The stacks in Figure 6.4 compare the direct primary estimate of EPSI with the SRME result. Figure 6.5 zooms in on the region between 2.2 and 3.0 seconds and shows that there is a diagonal multiple residual in the SRME output around 2.4 seconds (top arrow). Another multiple is visible around 2.7 seconds on the right side of the stack (bottom arrow). These multiples are (not so strongly) present in
6.1 Results on marine data with moderate water depth

The primary estimation, $\hat{X}_0$, during different iterations.

The stacked residual in Figure 6.6c shows the events in the data that are not yet explained by the model. A good example is the strong reflector just above 1.0 seconds. Note also the residual dipping multiples around 3 seconds and the low frequencies below 3.5 seconds. If we continue to iterate this residual will go to zero. Noise in the data will be explained as primaries. These fake primaries will cause multiples at later times. These fake multiples can then be compensated for by explaining them as primaries. By continuing the iterations, the errors are pushed out of the time window.

An answer to this unwanted effect can be to stop iterating before the residual is zero. Maybe equation 3.1.4 can play a role here. A sparse update, $\Delta \tilde{X}_0$, without noise will, when added to the primary impulse responses, have a bigger impact on the objective function (it is explaining primaries and multiples) then the impact of a sparse update with only noise on the objective function (it is explaining noise in the primaries and is creating noise in the multiples). Therefore, a sparse update without noise will have a bigger scaling factor, $\alpha$, then a sparse update with noise.
Figure 6.3: (a) A shot gather of the moderately deep water marine dataset with multiples. (b) Primary estimation with iterative SRME. (c) Subtracted multiples with iterative SRME. (d) The direct primary estimate obtained with the EPSI method. (e) The estimated multiples from the EPSI method.
Figure 6.4: (a) Stacked section of the moderately deep water marine data with multiples. (b) Stack of the primaries obtained with iterative SRME. (c) Stack of the direct primary estimate from the EPSI method.
A decrease in the scaling factor at the end of the process might, therefore, be a sign that the method is trying to explain noise. An other answer may be to disallow the creation of primaries in a certain area, for example below 3 seconds. The correct primary estimation will then leave a residual that would consist of the primaries below 3 seconds. Actually, the EPSI process can be used as a quality control to
check the consistency of the input data. If the residual goes to zero smoothly, the
data are ok. If there are clear events residing in the residual data, this may be an
indication that something is not consistent in the data, which may be resolved by
changing the preprocessing sequence. This is a topic for further investigation.

6.2 Results on shallow water marine data

Next, we demonstrate the application of EPSI to a shallow water field dataset. The
preprocessing steps can be found in section 6.3. Figure 6.7a shows one shot record
after this preprocessing sequence. Note that the near offsets have been filled by
interpolation, except for the first 0.4 seconds.

The modified EPSI algorithm, described in the section 3.3, is applied with \( P'' \) being
the measured data outside the near offset gap and the interpolated data in the lower
part of the missing near offset gap (such as shown in Figure 6.7a). Thus, \( \hat{P}' \) is the
top part of the near offset data that will be reconstructed. Figures 6.7a-d show the
result of the first iteration of the EPSI algorithm. Note the amount of multiples in
Figure 6.7c that is predicted by a small part of the first reflection only (Figure 6.7b).
From this result the importance of having the shallow near offset data for predicting
the multiples becomes clear.

Figure 6.8 show the final EPSI and SRME results for one shot gather. Since the
parabolic Radon interpolation underestimates the strengths of the water bottom
reflection, iterative SRME can not provide good results. For example the peg-leg
multiple around 0.7 seconds is not properly removed (lower arrows). Note that the
wavelet stretch that can be seen in the data (upper arrow) is not present in the
EPSI result. Also note that the near offsets in the primary estimation of EPSI
is discontinuous in some parts. When these discontinuous parts in the primary
impulse responses are convolved with the data they will sum up to create continuous
multiples, that will match the continuous multiples in the data. So the discontinuous
primary impulse responses are a solution of the equations. A smoothing constraint
may be used to force the primary impulse responses to be continuous. However,
the stack of the primary impulse responses (Figure 6.9c) shows that a process like
stacking is not hindered by discontinuous primaries.

The fact that SRME needs correct near offset data is known [see e.g. Verschuur,
2006]. To by-pass the influence of the shallow near offset reflections it is common
practice to remove multiples related to shallow primaries with a deconvolution first
Figure 6.6: (a) Stacked section of EPSI’s direct primary estimate on the moderately deep water marine data. (b) Stack of the conservative primary estimate of EPSI. (c) stack of the residual of the EPSI method, i.e. Figure b-a.
Figure 6.7: The EPSI result for one iteration. (a) Shot gather with the top part of the near offsets missing. (b) The primary estimate after one iteration. (c) The explained part of the data after one iteration. (d) The residual after one iteration. Note the significant reduction of multiples in d) solely based on the estimated first primary in b). This illustrates the importance of a correct near offset reconstruction.
Figure 6.8: (a) Input shot gather of a shallow water marine dataset with multiples. For the EPSI method the first 0.4 seconds of the interpolated near offsets are set to zero again. (b) Primary estimation with iterative SRME. (c) Subtracted multiples of iterative SRME. (d) The direct primary estimate of EPSI. (e) Estimated multiples by EPSI.
Figure 6.9: (a) Stacked section of the shallow water marine data with multiples. (b) The primaries obtained with iterative SRME. (c) The direct primary estimate of EPSI.
[Biersteker, 2001] and then apply SRME to this data without the shallow primaries. However, deconvolution relies on minimum energy, and the deconvolution and SRME processes are two separate steps, although their multiples interfere with each other. This can provide sub-optimum results. This in contrast to EPSI that does not rely on minimum energy in the primaries and estimates all primaries and multiples in one step.

Stacks were made for the complete line, as shown in Figure 6.9. The estimated near offsets have been included in the stacking process. Except for the shallow events, the result was similar as stacking without the near offsets. This was done to further emphasize the effect of the near offsets as reconstructed by the EPSI method. As can be observed the water bottom has had a tremendous impact on the multiple removal results. The area between 1.0 and 1.5 seconds is a lot cleaner in the EPSI result (Figure 6.9c) compared to the SRME result (Figure 6.9b). Furthermore, the structure around 1.7 seconds just right of the middle is much better visible for the EPSI result. A zoom on the water bottom area is made in Figure 6.10. A residual

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**Figure 6.10:** Zooms of the top parts of the stacks of Figure 6.9. (a) Stack of the SRME primaries. (b) Stack of the EPSI primaries.
of the first order water bottom multiple (between the arrows) is clearly visible in the SRME result (Figure 6.10a), while the EPSI result (Figure 6.10b) clearly reveals the reflector that was partly covered by the first order multiple in the data.

To get some insights in the convergence of the method we calculate the effect of varying the final estimated $\hat{S}^+$ and $\hat{X}_0$ in the objective function:

$$J(\lambda, \mu) = \sum_\omega \sum_{j,k} |P^- - \lambda \mu \hat{X}_0 \hat{S} - \lambda \hat{X}_0 R^\top P^-|^2_{j,k}, \quad (6.2.1)$$

where $\lambda$ is a scalar that scales the final estimated $\hat{X}_0$, and $\mu$ is a scalar that scales the final estimated $\hat{S}^+$. To save computation time we will not calculate equation 6.2.1 for the total data ($P^-$), direct primaries ($\hat{X}_0 \hat{S}^+$) and multiples ($\hat{X}_0 R^\top P^-$) but for the stacked data and primaries in Figure 6.9 after 0.4 seconds and stacked multiples. In Figure 6.11 the effects of varying both $\lambda$ and $\mu$ between 0.1 and 10 can be seen. There appears to be a clear minimum in $\lambda = 1$ and $\mu = 1$. The dominant direction of the contour lines is the line $\lambda * \mu = constant$. This line will keep the scaled primaries, $\lambda \mu \hat{X}_0 \hat{S}^+$, constant. For values of $\lambda > \mu$ the scaled multiples, $\lambda \hat{X}_0 R^\top P^-$ will deviate the contours. Note the big resemblance between this figure and the plot in Figure 3.5 which is made with synthetic data.

## 6.3 Preprocessing and implementation details

In order to apply a 2D algorithm to a 3D dataset some preprocessing was done on both field datasets described in this chapter. Here follows a list of the preprocessing steps that were taken:

- Removal of random noise.
- Deconvolution for the source bubble effects.
- Removal of the direct wave by a muting operation.
- Differential NMO to bring the receivers and the sources on the same grid, because each line was actually selected from a 3D survey.
- $f-k$ interpolation of missing shots in the common offset domain. The marine dataset with moderate water depth has a shot spacing of 50 meter and a receiver spacing of 12.5 meter. Therefore, three extra shots per shot needed to
Figure 6.11: Logarithmic contour plot of the objective function as function of the final estimated primary impulse responses, $X_0$, and the final estimated source wavelet, $S^+$; $J(\lambda, \mu) = \sum_j \sum_{j,k} |p^+ - \lambda(X_0 S^+ - \lambda X_0 R^+ p^-)^\perp |^2_{j,k}$. The parameter $\lambda$ varies $X_0$, and the parameter $\mu$ varies $S^+$. The contour plot has a minimum in point $\mu = 1, \lambda = 1$. Away from this point each contour line represents an increase of 0.7dB.

be interpolated. The shallow water marine dataset has a shot spacing of 37.5 meter and a receiver spacing of 12.5 meter, such that two extra shots per shot were interpolated.

- Parabolic Radon domain-based interpolation of missing near offsets [Kabir and
6.4 Conclusions

Verschuur, 1995]. For the shallow marine dataset the first 0.4 seconds were set to zero again, because the interpolated near offsets are not reliable there; they were part of the data to be estimated by the EPSI process. The same dataset, but without the first 0.4 seconds set to zero, was used as input for iterative SRME.

- A factor \(\sqrt{t}\) was applied to the data as a 3D to 2D amplitude correction.
- Reciprocity was used to convert the data from a marine end-on geometry to a split spread geometry.

Since the current implementation of the EPSI algorithm is based on matrices, the data had to be fed to EPSI in blocks. The maximum offset that was present in these blocks \((148\times12.5\,m = 1850\,m\), for both field datasets\), was also used as the maximum offset that was fed into iterative SRME. The blocks overlapped in such a way that a primary estimate was present for all the offsets between \(0\,m\) and \(49\times12.5\,m = 612.5\,m\).

For a proper comparison of both the EPSI and iterative SRME results, only these offsets were stacked.

Iterative SRME [Verschuur and Berkhout, 1997] was used in three iterations using a wavelet of 31 samples for the global matching and a filter of 3 samples for the local matching in windows of 250 samples by 25 traces.

6.4 Conclusions

In this chapter we have demonstrated the EPSI method on two marine datasets. The results obtained with EPSI and iterative SRME for a marine dataset with moderate water depth were similar in quality. An interesting property of EPSI is the residual, which can be used as quality control to see if certain events are not properly explained, indicating an inconsistency in the data. For the shallow marine data, EPSI showed a much better result than iterative SRME. This is mainly due to the fact that the reconstruction of missing near offsets can be incorporated into EPSI, such that they explain the multiples observed in the data.
Conclusions and discussion

In this chapter we summarize the key elements of the EPSI method and its applications. We will discuss the relations between different geophysics methods that use multiples. Furthermore, we share some philosophical thoughts and we give a view on the future development of the EPSI method.

7.1 Conclusions

In this thesis we have investigated the estimation of primaries by sparse inversion (EPSI) method. The key elements of this method are:

- EPSI uses the same forward primary-multiple model as SRME (the 'feedback model'). This forward model is written in terms of data and does not rely on subsurface information. Therefore, EPSI is completely data-driven and does not need subsurface information.

- EPSI tries to explain the total data, both primaries and multiples. By doing so an adaptive subtraction of multiples is avoided.

- EPSI estimates the source wavelet and it can reconstruct the missing near offsets. Compared to SRME, this leads to better primary estimation results in shallow water and wavelet stretch in interpolated near offsets is avoided.
EPSI turns out to be a very flexible method in the sense that it can be easily modified to be applied to datasets acquired with different acquisition methods. The key factor in this is that these different acquisition methods can all be described with the same detail hiding operator notation, as was introduced by Berkhout [1982], and that a primary impulse response term is present in the forward model of each of these acquisition methods. Apart from traditional marine acquisition, in this thesis the EPSI method has also been applied to:

- **Passive data.** We have demonstrated that it is possible to separate the direct arrivals of the subsurface source signals from the surface-related multiples in the upgoing wavefield. In contrast to the cross-correlation method, EPSI gave true amplitude primary impulse response estimates. Certain angles of the primary impulse responses cannot be recovered from the multiples in the passive data, because they are not present in the recorded multiples.

- **Blended data.** In principle the EPSI algorithm for blended data is a division of the upgoing blended wavefield with the downgoing blended wavefield. Not all angles can be recovered from the surface-related multiples in the blended data, like in passive data. We have demonstrated how to use the multiples in the data to estimate the primary impulse responses and how to make changes in processing or acquisition in order to obtain information from the primaries that is not in the multiples.

### 7.2 Discussion

#### 7.2.1 Convolution, correlation, division, and inversion

In this thesis we have described two interpretations of the forward primary-multiple model; in terms of primaries and multiples:

$$ P^- = X_0 S^+ + X_0 R^\circ P^-, \quad (7.2.1) $$

and in terms of up- and downgoing wavefields:

$$ P^- = X_0 (S^+ + R^\circ P^-). \quad (7.2.2) $$

Multiple prediction and subtraction methods, like SRME, use the first view, and get their primary estimates through a convolution. The convolution approach is
not limited to surface data, but it also works for e.g. internal multiples [Berkhout, 1982; Berkhout and Verschuur, 2005; Verschuur and Berkhout, 2005] and OBC data [Verschuur and Neumann, 1999].

Wavefield deconvolution uses the second view, and, as its name implies, gets its primary estimate through a deconvolution or scaled correlation. Approaches that use a division approach to estimate primary impulse responses also use the second view. These methods are not only bound to one type of data, as they can be found in vertical seismic profile data [Ross and Shah, 1987], OBC data [Amundsen, 1999], or surface data [van Groenestijn and Verschuur, 2009a].

It is interesting to see that the correlation approach has been applied to similar types of data; virtual source data [Mehta et al., 2007], OBC data [Cao, 2009], and surface data [Claerbout, 1968]. Here the correlation of the upgoing wavefield by the downgoing is used to obtain an estimate of the total subsurface response instead of the primary impulse response (see Chapter 4). However, the information also comes from the multiples. The similarities between all approaches make it clear that they can benefit from each other’s findings. However, this notion is only slowly growing in the geophysics community.

### 7.2.2 Some philosophical thoughts

- An undesirable way of blending data is firing two or more sources located very close together and simultaneous. This is, however, exactly the way it is done in a source array. A blended use of source arrays should be implemented such that better angle sensitivity and a broader bandwidth can be obtained.

- There are some interesting similarities between full waveform inversion methods and the inversion method EPSI. Both describe how an input (the source signal at the surface) is transformed into an output (the measured upgoing wavefield at the surface). And although these transformations are calculated different a lot of things stay the same. For example the blending of shot gathers is possible in EPSI and in full waveform inversion [Krebs et al., 2009], and also the estimation of the source wavelet is done in EPSI and in full waveform inversion [Wang et al., 2009].

- Instead of writing the forward primary-multiple model as:

\[
P^- = X_0 S^+ + X_0 R^\dagger P^-,\tag{7.2.3}
\]
we can also write it as:

\[ P^- = (X_0 + X_0 R \cap X_0 + X_0 R \cap X_0 R \cap X_0 + X_0 R \cap X_0 R \cap X_0 R \cap X_0 + \ldots) S^+. \]

(7.2.4)

If we use an \( X_0 \) that contains a small error, e.g. a small time delay, then the primaries and multiples created with this \( X_0 \) on the right side of equation 7.2.3 are all shifted with this small time delay. If we use the same \( X_0 \) in equation 7.2.4 then we will also create primaries that are shifted with the small time delay, but the first order multiples will be shifted with twice this time delay, the second order multiple will be time shifted three times this time delay, and so on. Applying equation 7.2.4 in an inversion algorithm will result in larger residuals when the wrong \( X_0 \) is used, making it possible to estimate \( X_0 \) even more precise.

\section*{7.2.3 Future plans}

A lot of questions regarding the estimation of primaries through an inversion have been answered by this thesis. Especially the question of its practical feasibility has been given a lot of attention with synthetic and field data examples. The next feasibility question for EPSI is 3D field data. Furthermore, the EPSI applications on blended data and passive data still wait for their field data tests. This thesis has not spoken about land data at all. However, since EPSI uses the same forward model as SRME, it is expected that (at least in principle) it could be applied to land data, just like SRME.

Also the question of what information can we extract from the data in order to estimate primaries has been given a lot of attention. Especially the EPSI applications to passive (missing surface illumination angles), and blended data (multiples created outside the measurement aperture) have enforced a deeper understanding.

Questions regarding the mathematics of the EPSI method (like convergence, stability, choice of parameters) have mostly been answered with giving safe upper boundaries obtained from our experience. We need more insight in these mathematical issues and a numerical support on the choices made. It is to be expected that future versions of EPSI will differ from the current version due to a better insight in the mathematics.
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Summary

Estimation of Primaries and Multiples by Sparse Inversion

Surface-related multiple reflections are often considered noise in the seismic reflection measurements. By seismic processes, such as migration and inversion, they can be mistakenly seen as primary reflections and give an erroneous image of the earth. Therefore, a method is needed to separate the multiples from the primaries.

This thesis describes a primary estimation method named estimation of primaries by sparse inversion (EPsI). The interesting aspect of this method is that it does not see the multiples as noise, but uses the multiples to come to a better estimation of the primaries.

Other wave equation based primary estimation methods first predict the multiples and then adaptively subtract them from the data. During this adaptive subtraction primary energy may be removed. EPsI tries to explain the total data, both primaries and multiples, in terms of primary impulse responses. By doing so an adaptive subtraction of multiples is avoided. In fact EPsI is a large-scale inversion process that estimates primaries such that they and their corresponding multiples explain the total data. To constrain this process, a sparseness constraint is used, which assumes that our estimated primaries have a certain amplitude distribution (large and small ones).
A general characteristic of wave equation based primary estimation methods is that the near-offset data are very important for estimating water column reverberations, especially in shallow water. However, it is not possible to record the near-offset data and, therefore, most wave equation based primary estimation methods have great difficulties with shallow water marine data. A major advantage of EPSI is that it can reconstruct the missing near-offset data from information in the multiples and, therefore, show a good primary estimation result on shallow water marine data.

Furthermore, the EPSI method can be extended to other measurement configurations, exploiting a similar relation between primaries and surface multiples. In this thesis both passive and blended seismic data have been considered.

For passive seismic data multiples are used to obtain an estimate of the subsurface responses, usually by a cross-correlation process. This cross-correlation process relies on the assumption that the surface has been illuminated uniformly by subsurface sources in terms of incident angles and strength. If this is not the case the cross-correlation process cannot give a true amplitude estimation of the subsurface response. Furthermore, there are cross terms in the cross-correlation result that are not related to actual subsurface inhomogeneities. In this thesis it is demonstrated that, with some modifications to the algorithm, EPSI can obtain true amplitude subsurface responses without the uniform surface illumination assumption. The EPSI method will go beyond the cross-correlation process and will estimate primaries only from the multiples in the available signal. The estimated primary impulse responses, with point sources and receivers at the surface, can be used directly in traditional imaging schemes.

This thesis demonstrates that for the situation of blended acquisition, meaning that different sources are shooting in a time-overlapping fashion, multiples can be used to 'deblend' the seismic measurements. With some modifications the EPSI method can be used for blended seismic data. As output EPSI gives unblended primary impulse responses with point sources and receivers at the surface, which can be used directly in traditional imaging schemes.

The feasibility of the EPSI method is demonstrated in this thesis by a successful application of the method to two marine field datasets, one with a moderate water depth and one with shallow water. It demonstrates that for deeper water EPSI can compete with the standard surface-related multiple elimination (SRME) method, where for the shallow water EPSI clearly shows better results than SRME. The latter is mainly attributed to the fact that near offset reconstruction, which plays a
crucial role in shallow water data, is included in the EPSI method.

Gert-Jan van Groenestijn
Samenvatting

Estimation of Primaries and Multiples by Sparse Inversion

Oppervlakte gerelateerde meervoudige reflecties (in de geofysische wereld aangeduid als 'surface-related multiples' of 'multiples') worden vaak gezien als ongewenste signalen in seismische reflectiemetingen. Door seismische processen, zoals migratie en inversie, kunnen ze abusievelijk worden aangezien als enkelvoudige reflecties (in de geofysische wereld aangeduid als 'primaries') en een verkeerde afbeelding van de aarde opleveren. Daarom is er een methode nodig die de multiples van de primaries scheidt.

Dit proefschrift beschrijft een schattingsmethode voor primaries, genaamd 'schatting van primaries door schaarse inversie'. In het Engels heet deze methode 'estimation of primaries by sparse inversion' en wordt het afgekort tot 'EPSI'. Het interessante aspect van deze methode is dat het multiples niet ziet als ongewenst signaal, maar dat het multiples gebruikt om tot een betere schatting van de primaries te komen.

Andere op de golfvergelijking gebaseerde schattingsmethoden voor primaries voorstellen eerst de multiples en proberen deze daarna uit de data te verwijderen met behulp van een adaptieve aftrekking. Deze adaptieve aftrekking kan echter ook energie van de primaries verwijderen. EPSI probeert de totale data, zowel primaries als multiples, te verklaren in termen van enkelvoudige impuls responses en op deze wijze wordt het adaptieve aftrekkingsproces vermeden. In feite is EPSI een grootschalig
Samenvatting

inversieproces dat primaries schat, zodat deze met hun bijbehorende multiples de totale data verklaren. Om dit proces naar het goede antwoord toe te leiden wordt een schaarste randvoorwaarde gebruikt, die aanneemt dat de amplitude van de primaries in de data een bepaalde verdeling hebben (bestaande uit grote en kleine amplitudes).

Een algemeen kenmerk van op de golfvergelijking gebaseerde schattingsmethoden voor primaries is dat er dicht bij de bron metingen nodig zijn om reverberaties in de waterkolom te kunnen schatten. Deze nabije afstand metingen zijn vooral belangrijk voor de schatting van primaries in ondiep water. Het is echter niet mogelijk om deze nabije afstand data daadwerkelijk te meten en daarom hebben de meeste op de golfvergelijking gebaseerde schattingsmethoden voor primaries grote moeite met ondiep water data. Een groot voordeel van EPSI is dat het de nabije afstand data kan reconstrueren vanuit de informatie uit de multiples en daarom komt het tot goede schattingen van de primaries in ondiep water data.

Verder kan de EPSI methode worden uitgebreid naar andere meetconfiguraties door gebruik te maken van gelijksoortige relaties tussen primaries en multiples. In dit proefschrift worden passieve seismische data en 'blended' seismische data behandeld.

Voor passieve seismische data worden multiples gebruikt om een responsie te krijgen van de ondergrond, gewoonlijk door een kruiscorrelatieproces. Dit kruiscorrelatieproces berust op de aanname dat het oppervlakte uniform belicht is door ondergrondse bronnen in termen van invalshoeken en amplitudes. Als dit niet het geval is schat het kruiscorrelatieproces niet de correcte amplitudes van de responsie van de ondergrond. Verder bevinden zich kruistermen in het kruiscorrelatieresultaat die niet gerelateerd zijn aan echte inhomogeniteiten in de ondergrond. In dit proefschrift wordt gedemonstreerd dat, met enige aanpassingen aan het algoritme, EPSI de responsie van de ondergrond met correcte amplitudes kan schatten zonder de aanname van de uniforme beliching van het oppervlak. De EPSI methode zal verder gaan dan het kruiscorrelatieproces en kan primaries schatten vanuit de multiples in het aanwezige signaal. De geschatte primaries met puntbronnen en puntontvangers op het oppervlak kunnen direct gebruikt worden in traditionele processen voor het afbeelden van de ondergrond.

Dit proefschrift demonstreert dat in het geval van een 'blended' acquisitie, hetgeen betekent dat verschillende bronnen schieten met overlap in tijd, de multiples gebruikt kunnen worden om de data te 'deblenden'. Met enkele aanpassingen aan het algoritme kan de EPSI methode gebruikt worden voor deze blended data. Als uitvoer geeft EPSI 'unblended' primaries met punt bronnen en punt ontvangers op
het oppervlak. Deze kunnen direct worden gebruikt in traditionele processen voor het afbeelden van de ondergrond.

De haalbaarheid van de EPSI methode wordt in dit proefschrift aangetoond door een succesvolle toepassing van de methode op twee veld datasets. En dataset is afkomstig van een gebied op zee met een middelgrote waterdiepte en de andere dataset komt van een gebied met ondiep water. Dit proefschrift toont aan dat voor het diepere water EPSI zich kan meten met SRME (een standaard schattingsmethode voor primaries). Echter, voor het ondiepe water toont dit proefschrift aan dat EPSI duidelijk betere resultaten laat zien dan SRME. Dit laatste wordt voornamelijk veroorzaakt doordat de reconstructie van de nabije afstand metingen onderdeel is van de EPSI methode.

Gert-Jan van Groenestijn
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