Verification of a PC model for wave diffraction

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In the name of God
the Compassionate, the most Merciful
Summary

BELUGA is a mathematical model which has been developed by Booij, Holthuijsen and Tolman in Delft University of Technology, to compute wave penetration into a harbour taking diffraction and reflection into consideration. The aim of the model is to provide an efficient numerical method for calculation of a wave field in a relatively large harbour and executable on a personal computer.

The developed mathematical model is based on linear harmonic wave theory. The numerical technique employed in BELUGA programme can be classified as a boundary element method. The wave field in each boundary point and arbitrary other ones is composed of waves in a number of pre-defined directions; the waves in each direction are computed using the ray method. Diffraction around corner points in harbour contours is taken into account by applying linear superposition of the well-known Sommerfeld solutions.

In the present study, a verification of this mathematical model has been carried out thoroughly on academic as well as on realistic test cases. The numerical model outcomes have been compared with analytical solutions and with laboratory measurements which were performed by the Delft Hydraulics Laboratory. By considering the results of the computational method which have been evaluated critically, the model was modified. Re-testing of the model was implemented after the improvement.

It has been found that despite using an approximate solution the results of the model BELUGA are in good agreement with analytical exact solutions in simple cases such as a single breakwater and a breakwater gap. In the case of a complicated harbour the results are not in complete agreement with the laboratory measurements and the model needs still improvement.

Finally, recommendations are given for further model development and improvement.
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1. INTRODUCTION

1.1 General

Propagation of waves into a harbour is central to problems of harbour design. Knowledge of the wave response of harbours is not only of intrinsic importance in the design stages but it is also necessary for further studies, such as sedimentation, ship motions, mooring forces, and harbour wall design.

Scaled physical models have typically been used for detailed investigation of wave action in harbours, but a variety of mathematical models are now available which can provide an alternative to physical models or could be used in conjunction with them.

The irregularity of the waves in natural field conditions, the wave breaking phenomenon, and other nonlinear factors such as energy dissipation due to friction or turbulence, partial reflection, shoaling and refraction, makes wave propagation a very complex phenomenon. Due to such a complication a full mathematical description of the problem hardly can be achieved. Therefore, in modelling the wave propagation phenomenon some simplifications are usually involved depending on the aspects which are of interest.

Studies of wave propagation into harbours and coastal areas usually concentrate on individual physical effects, such as diffraction, refraction, shoaling and reflection. They can occur simultaneously with various degrees of interaction. Therefore it is desirable to develop mathematical models which simulate the combined effects in analogy to a physical model of a complete harbour.

Wave diffraction phenomena play the main role in the process of wave propagation into a harbour and is important for several reasons. Wave height distribution in a harbour is determined to some degree by the diffraction characteristics of both the natural and manmade structures affording protection from incident waves. Therefore, knowledge of the diffraction process is essential in planning such facilities. The proper design and
location of harbour entrances to reduce such problems as silting and
harbour resonance also require a knowledge of the effects of wave
diffraction.

BELUGA is a model developed for computation of wave propagation into a
relatively large harbour, with constant water depth, considering diffraction
and multi-reflection from the harbour contours.

The purpose of the present study is to verify this model through the
comparison with analytical as well as realistic test cases.

1.2 Literature Survey

Analytical and numerical treatment of wave diffraction was originated by
Sommerfeld (1886) through his solution for the diffraction of waves by a
semi-infinite screen or 'half-plane problem'. The Sommerfeld theory of
diffraction has been derived based on a certain set of assumptions: the
thickness of the screen, or the width of the breakwater, is theoretically
zero; the breakwater is theoretically rigid and impermeable; the screen or
breakwater is semi-infinite and straight line in plan.
Penney & Price (1944) used the Sommerfeld solution to construct the
diffracted field behind a semi-infinite breakwater. Blue & Johnson (1949)
used an approximate solution for the breakwater gap for the construction of
amplitude diagrams. Separate diagrams for different ratio of gap width to
wave length are required. A set of generalized wave diffraction diagrams for
the breakwater gap situation is given by Johnson (1952). These are included

The reflection of waves inside the harbour was not included in their model.
Carr (1952) showed a graphical way of including the reflection of waves
inside the harbour. The method was limited to harbours with only a few
straight harbour sides and a narrow harbour entrance.

Although the solutions of Penney and Price (1952) and Carr (1952) have
been the most widely used to date, other solutions are also available. Lamb
(1932) derived a theory for breakwater gap wave diffraction for the case
when the gap width B is small compared to the wavelength, L. Lacombe
(1952) derived an approximate solution for breakwater diffraction based on
a generalization of Huygens' principle under certain assumptions.
(1980) and Smallman and Porter (1985) have developed an exact solution for the diffraction of waves passing between two breakwater arms forming an angle. These solutions are more general than the classic solutions, which assume that the two breakwater arms lie on the same straight line.

The Penney-Price solution (1952) is only valid for the case of normal incidence, while the rest are valid for both normal and oblique incidence.

The resulting solution of the Penney and Price approach is reasonably accurate, only for gap widths large compared to the wave length. Carr and Stelzriede (1952) used the solution of Morse and Rubenstein for diffraction of sound and electromagnetic waves by a slit in an infinite plane. It is an exact solution for small gaps and bridges the gap between the method of Lamb for very small slits and the approximation based on Sommerfeld solution, and is useful for any angle of wave approach.

Several analytical methods and numerical models have been developed for computation of wave propagation in coastal areas, and wave penetration into a harbour with variable depth, over the past two decades.

A significant advance in the field of wave modelling was made by Berkhoff (1972, 1976), who derived the "combined refraction-diffraction" equation (mild-slope equation). This equation describes the complete wave transformation process for simple harmonic linear wave from deep water through shallow water in terms of the velocity potential. It can be used for a wide range of ocean wave frequencies. By including the effects of diffraction, this equation eliminates the usual problem encountered in refraction studies (viz. caustics). The usefulness of Berkhoff’s equation in yielding good simulations of wave behaviour in a wide variety of different situation has been demonstrated by many investigators.

Berkhoff (1976) and Rottmann-Sode, Schaper and Zielke (1985) have used it to investigate wave propagation in harbours; Pos and Kilner (1987) have used it to develop breakwater diffraction diagrams.

The refraction-diffraction equation is an inseparable elliptic partial differential equation with complex variables. This equation, with no additional physical assumptions, has been solved by means of finite element methods, Berkhoff (1976), Rottmann-Sode and Zielke (1983), Rottmann-Sode et al. (1985). In this case the requirements regarding computer speed and storage are high. A complete solution of the equation by the direct method is limited to
a domain about 10 times the wavelengths, Berkhoff et al. (1982). In view of these difficulties, it has become necessary to use approximation methods. One of these is the parabolic approximation (Radder, 1979) to the combined refraction-diffraction equation, but two limitations can be observed in this method:

(1) The waves must have a principal propagation direction, since diffraction effects are restricted to the lateral direction only, and (2) the reflected component of the wave potential in the negative direction should be negligibly small. The parabolic approximation is therefore inappropriate when the bathymetry or structures such as harbour walls, breakwaters, etc, reflect the energy in the negative direction. Booy (1981) has extended the elliptic model as well as parabolic model by introducing the influence of a current.

Ebersole (1985) has suggested a finite-difference solution to the elliptic combined refraction-diffraction equation, but this method solves the elliptic equation essentially as an initial value problem, so that reflections are again required to be negligible. Panchang et al. (1988) have suggested a finite difference solution of the combined refraction-diffraction equation. In their model the complete elliptic boundary value problem is solved following a marching or "Error Vector Propagation" method equation. This method is also restricted to the first assumption of the parabolic equation, but it overcomes the other limitation of the parabolic approximation, in that it allows back-scattering and propagation in the negative direction. Therefore it is possible to take into account the reflection phenomena.

More recently the mild-slope equation has been solved by means of an iterative procedure, Panchang (1990). The equations obtained by finite difference or finite element methods are modified and then they are solved by the conjugate algorithm.

For constant depth the mild-slope equation reduces to the Helmholtz equation. Lee (1971) developed a theory for wave penetration into harbours of arbitrary geometry by applying Weber's solution of the Helmholtz equation in both the regions inside and outside the harbour with the final solution obtained by matching the wave amplitudes and their normal derivatives at the entrance.

Berkhoff's equation has been derived for simple harmonic linear waves;
in more shallow water this assumption can not be applied because of non-linearity. Abbott et al. (1978), Rottmann-Sode et al. (1985) and Ross Warren et al. (1985) applied a mildly nonlinear formulation based on the Boussinesq equations for computing wave propagation. In more shallow water the method of Abbott et al. (1978) is the most general approach because the mathematical model on which the computational algorithm is based encompasses both linear and nonlinear effects including diffraction, refraction, shoaling, frequency and amplitude dispersion and partial reflection.

Another approach to the short wave penetration into harbours utilizes ray methods. These methods have been used for many years by coastal engineers to evaluate the effects of refraction and shoaling of waves as they approach to the coastline. If these methods are to be used in harbours, wave diffraction has additionally to be incorporated. For diffraction by breakwaters this is done by constructing extra sets of rays known as diffracted rays. Larsen (1978) and Southgate (1985a) used this method for computing wave penetration into harbours. Southgate (1985b) has developed a harbour ray model which describes the combined wave effects of diffraction around breakwaters and depth refraction. Diffraction caused by two types of breakwater lay-out have been considered: (1) A small gap between two straight breakwater; and (2) a single, straight, semi-infinite breakwater.

A new approach has been suggested by Booy et al. (1990) based on Sommerfeld’s theory and the ray method, in which the Helmholtz equation is solved by means of the boundary element method. This approach will be investigated in this study.

1.3 Outline

It is important in the verification of mathematical models to know the restrictions and simplification which have been made to obtain these models. Therefore chapter 2 starts with a description of the general equation of wave motion. The chapter continues with the description of existing wave propagation models and then analytical solutions of diffraction for some simple cases are given. These solutions are used to verify the mathematical model.
The mathematical BELUGA model is described in the chapter 3. Chapter 4 gives the test plan for verification of the model. In this chapter test criteria are formulated and for each test case the purpose of the test is given. The results of the tests and their evaluation are presented in chapter 5. Finally, discussions and conclusions about the model and some recommendations for extensions and improvements are given in chapter 6.
2 THEORETICAL APPROACHES

2.1 Introduction

In this study about mathematical models for wave propagation into harbours it will be useful to have an overview of the basic equations and mathematical properties of existing models.

The applicability of a mathematical description often depends on the number of space dimensions involved in the problem. In the case of two and three dimensional models the mathematical formulation is often restricted to the linear simple harmonic wave theory. The water surface waves are assumed to be time-harmonic and to have small amplitudes, so that linear water wave theory is applicable. The irregularity of the field is described as a linear stochastic phenomenon, characterised by an energy density function.

In this chapter first a short description of basic equations will be given, then governing equations and analytical solutions in this field will be briefly described. These analytical solutions will be used for verification of the model.

2.2 Basic Equations

The mathematical formulation, used in the conventional linear models, is based on the following assumptions:
- the water behaves like an ideal fluid, which means no viscosity and the water motion is irrotational,
- the fluid is homogeneous and incompressible,
- the deviations of the free surface from the still water level are small, resulting in a linear approach of the problem,
- the water motion is harmonic in time.
Assuming an irrotational flow, one can define a velocity potential \( \Phi(x,y,z,t) \) such that the fluid particle velocity vector can be expressed as \( u = \nabla \Phi \), where
(x,y,z) are cartesian coordinates. For time harmonic motion with frequency ω,

\[ \Phi(x,y,z,t) = \Phi(x,y,z) e^{-i\omega t} \]

Thus, from the continuity equation for an incompressible fluid, Laplace's equation is obtained:

\[ \nabla U - \nabla^2 \phi = 0 \quad (2.1) \]

or:

\[ \frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} + \frac{\partial^2 \phi}{\partial z^2} = 0 \quad (2.1) \]

φ is a three-dimensional wave potential function \( \phi(x,y,z) \), which must satisfy the domain equation (2.1) with the boundary conditions in the vertical z-direction (using above assumptions):

\[ \frac{\partial \phi}{\partial x} \frac{\partial h}{\partial x} + \frac{\partial \phi}{\partial y} \frac{\partial h}{\partial y} + \frac{\partial \phi}{\partial z} = 0 \quad \text{at} \quad z = -h(x,y) \quad (2.2) \]

and

\[ \frac{\partial \phi}{\partial z} - \frac{\omega^2 \phi}{g} = 0, \quad \text{at} \quad z = 0 \quad (2.3) \]

in which:

- \( x, y \) = horizontal coordinates
- \( z \) = vertical coordinate
- \( h \) = water depth
- \( g \) = acceleration due to gravity
- \( \omega \) = angular frequency
- \( \phi \) = three dimensional wave potential

These equations (2.1),(2.2),(2.3), supplemented with the boundary condition in \( x,y \) plane (e.g. radiation condition of Sommerfeld) and a given expression
of the incident wave field and reflection conditions, give a full definition of the wave potential function $\phi$.

However, due to the three-dimensional formulation, a method of solution will not be suitable in general cases because of computation time. A simplification of the formulation to a two-dimensional $(x,y)$ model can be obtained, by assuming moderate slopes of the bottom in the solution domain ("mild slope" approximation). In this way the problem can be described by a two-dimensional field equation due to an integration in the vertical direction from the bottom to the surface.

The relationship between the two-dimensional potential $\Phi$ and the three-dimensional potential $\phi$ is given by:

$$\Phi(x,y,z) = \int_0^z \frac{g}{2\omega} \frac{i}{\cosh(k(h+z))} \frac{\cosh(kh)}{\cosh(kh)} \varphi(x,y) dz$$

where

$i = \text{imaginary unit (} i^2 = -1\text{)}$.

2.3 Computational Methods

2.3.1 Combined refraction-diffraction method

Using the mild slope approximation of the three-dimensional formulation (2.1) - (2.3), water wave propagation for simple harmonic linear waves in the horizontal plane can be described by the combined refraction-diffraction, derived by Berkhoff (1972, 1976):

$$\frac{\partial}{\partial x} (cc_x \frac{\partial \varphi}{\partial x}) + \frac{\partial}{\partial y} (cc_y \frac{\partial \varphi}{\partial y}) + k^2 cc_x \varphi = 0$$

where:

$\varphi = \text{two-dimensional complex wave potential function}$

$c = \text{local phase velocity}$

$c_g = \text{local group velocity}$
k = wave number, defined by the relation:

\[ \omega^2 - gk \tanh(kh) \]

The phase velocity \( c \) is defined by:

\[ c = \frac{\omega}{k} \]

and the group velocity \( c_g \) by:

\[ c_g = \frac{1}{2} c \left[ 1 + \frac{2 kh}{\sinh 2kh} \right] \]

The field equation (2.5) is of elliptic type. For solving the equation a boundary condition along the whole contour of the solution domain is necessary. Possible boundary conditions are:

- partial or full reflection at fixed boundaries,
- known incident wave field and radiation condition at open boundaries for the radiating waves,
- prescribed potential, or prescribed fluid velocity.

For more detailed information about the mild slope approximation and the numerical solution of combined refraction–diffraction model (2.4) reference is made to Berkhoff (1976).

Major advantages in solving the complete equation is the ability to treat incident as well as reflected waves and easy to apply boundary conditions. Solving wave propagation problems using Eq. (2.4) is practical when the dimensions of the spatial area of interest do not exceed approximately ten times the length scale of the wavelengths being considered, Berkhoff et al. (1982). In fact the models which solve the eq. (2.4) are well suited to model multiple reflections and harbour resonances but the computing effort increases rapidly with the size per wavelength of the domain. Because of this difficulty simplifications of the method can be of more practical use.

For constant depth the mild-slope equation reduces to the well known Helmholtz equation:
It is possible to simplify the combined refraction-diffraction model significantly by assuming that the non-uniformity of the bottom slope and the boundaries cause negligible reflection. In that case the domain equation (2.4) can be reduced to the equation:

$$\frac{\partial^2 \Phi}{\partial x^2} + \frac{\partial^2 \Phi}{\partial y^2} + k^2 \Phi = 0$$  \hspace{1cm} (2.6)

2.3.2 **Parabolic refraction-diffraction approximation**

Equation (2.7) is of parabolic type and needs the following boundary conditions:
- at x = 0 the potential function is given as a function of the coordinate y (incident wave function),
- in y-direction the model must have two boundaries with appropriate conditions (for instance reflecting or absorbing conditions).

In this model there is no possibility for waves travelling in the negative x-direction, so reflection of waves in that direction is impossible. As a consequence the parabolic refraction-diffraction model can be used for the computation of the wave propagation and deformation due to bottom lay-out, assuming the main direction of propagation will be maintained. In the parabolic model only diffraction effects along the wave fronts are taken into account and not in the direction of propagation.

The parabolic equation can be solved by a marching method which uses considerably less computing time and storage than the elliptic models. For more information about the derivation of the equation and the numerical method of solution reference is made to Radder (1978).
2.3.3 Ray method

Ray methods have been used for many years by coastal engineers to evaluate the effects of refraction and shoaling of waves as they approach the coastline. Ray plotting or refraction models are based on the assumption of geometric optics that the wave field behaves locally as it were uniform. The solution is an asymptotic approximation to the free surface linear wave theory ignoring diffraction effects. These models solve an initial value problem along each wave ray or wave-front orthogonal. The main drawback of these models is their inability to model diffraction.

To obtain the refraction approximation, the substitution:

\[ \varphi = a e^{iS} \]  

is made in the governing elliptic differential equation (mild slope equation), in which

- \( a = \) wave amplitude,
- \( S = \) wave phase function.

This substitution gives the two equations:

\[
\frac{1}{a c_s} \left( \frac{\partial}{\partial x} \left( c_s \frac{\partial a}{\partial x} \right) + \frac{\partial}{\partial y} \left( c_s \frac{\partial a}{\partial y} \right) \right) + \left( \frac{\partial S}{\partial x} \right)^2 + \left( \frac{\partial S}{\partial y} \right)^2 - k^2 = 0
\]  

(2.9)

\[
\frac{\partial}{\partial x} (c_s a^2 \frac{\partial S}{\partial x}) + \frac{\partial}{\partial y} (c_s a^2 \frac{\partial S}{\partial y}) = 0
\]  

(2.10)

Assuming that:

\[
\frac{\nabla^2 a}{k^2 a} < 1
\]

\[
\frac{|\nabla a|}{k a} < 1
\]

and neglecting the corresponding terms in the equation (2.9), the following
equation is obtained

\[
(\frac{\partial S}{\partial x})^2 + (\frac{\partial S}{\partial y})^2 - k^2 = 0
\]  

The replacement and assumptions yield differential equations (2.12 and 2.10) for determination of the rays and the field on the rays. In this way the elliptic boundary-value problem is turned into a initial value problem along each ray.

The so-called "eikonal" equation (2.12) for the phase function $S$ can be solved with the aid of the method of characteristics, resulting in the classical wave ray method, Berkhoff (1976), and giving the well-known refraction wave ray diagrams.

With the aid of energy balance equation (2.10) it is possible to compute the wave amplitude function along a wave ray. In the refraction model all diffraction effects are neglected and reflection phenomena due to bottom changes are not taken into account. Each ray can be computed independently from all others and this reduces the computation effort strongly, giving the opportunity to apply the method for large areas.

If ray methods are to be used in harbours, wave diffraction has additionally to be incorporated. For diffraction breakwaters this is done by constructing extra sets of rays known as diffracted rays, Larsen (1978). When a ray strikes an edge point in the harbour (e.g. the tip of a breakwater) diffracted rays are produced and emitted in all direction into the harbour in accordance with the law of edge diffraction. In the harbour the reflection phenomenon also plays an important role. When a ray strikes the surface of an obstacle a reflected ray is produced in accordance with the law of reflection. The field on the reflected ray is determined from the field on the incident ray by multiplication with a reflection coefficient.
2.4 Reflection

Water waves may be either partially or totally reflected from both natural and manmade structures. Wave reflection is important in the design of coastal structures, particularly for structures associated with harbour development. Reflection of waves implies a reflection of wave energy as opposed to energy dissipation. Consequently, multiple reflections and absence of sufficient energy dissipation within a harbour complex can result in build up of energy which appears as wave agitation and surging in the harbour. These surface fluctuations may cause excessive motion of moored ships and other floating facilities, and result in the development of great strains on mooring lines. Therefore seawalls, bulkheads, and revetments inside of harbours should dissipate rather than reflect incident wave energy whenever possible.

A measure of how much a barrier reflects waves is given by the ratio of the reflected wave height $H_r$ to the incident wave height $H_i$ which is termed the reflection coefficient $K_r$; hence $K_r = H_r / H_i$. The magnitude of $K_r$ varies from 1 for total reflection to 0 for no reflection. For partial reflection, the reflection coefficient $K_r$ must be given by the complex value including phase. In general, the reflection coefficient depends on the geometry and composition of structure and the incident wave characteristics such as wave steepness and relative depth $d/L$ at the structure site.

2.5 Diffraction

Diffraction of water waves is a phenomenon in which energy is transferred laterally along a wave crest. Diffraction occurs when there is a sharp variation in wave energy along a wave crest. When a wave train is passing an obstacle there are, in the first instance, no waves in the lee of the obstacle. There will therefore be a gradient in the wave energy along the wave crest. The water away from the obstacle has more energy (all the
initial wave energy) than the water behind the obstacle (in first instance zero, since there are no waves). Energy is now transported along the wave crest to the part behind the obstacle and bending waves develop in the lee of the obstacle.

The degree of diffraction depends on the ratio of a characteristic lateral dimension of the obstacle, e.g., the length of detached breakwater, D, to the wavelength. When a thin pile is standing in waves with large wave length, D/L << 1, clearly the diffraction will be nearly 100% implying that the wave field is approximately the same as if there is no pile. In the case of a detached breakwater, D/L >> 1, diffraction occurs around each breakwater head. There is a large zone in which diffracted waves develop. Both the undisturbed waves passing the breakwater and also the reflected waves are diffracted.

There are mathematical solutions of some specific problems which have been taken from the theory of sound and light waves and applied to water waves for the case of constant depth, and for impermeable rigid structures. Several such solutions of considerable practical importance which have been used for the test of BELUGA are presented in this chapter.

2.5.1 Diffraction around a semi-infinite breakwater:

The wave height around a thin, semi-infinite obstacle can be estimated by means of the well known Sommerfeld solution. Penney and Price (1952) showed that the solution presented by Sommerfeld for diffraction of light polarized in a plane parallel to the edge of a half-plane is also a solution of the water wave diffraction around a semi-infinite breakwater.

The assumptions underlying the Penney and Price solution in addition to the main assumption in section 2.2 are:

1. The water depth is constant.
2. At the breakwater boundary, the normal component of the orbital velocity is zero.

The solution derived by Sommerfeld consists of the sum of two terms,
which can be considered as the diffracted, incident wave field and the diffracted, reflected wave field. The solution can be written in the following form:

$$\Phi = \Phi_i + \Phi_r$$  \hspace{1cm} (2.13)

The following is a brief summary of the solution by Penney and Price. The water surface elevation based on linear theory can be expressed as

$$\eta = \frac{ai \omega t}{g} e^{i \omega t} \cosh kh \cdot F(x,y)$$  \hspace{1cm} (2.14)

The wave potential function $\phi$ can be written as

$$\phi = ae^{-iky} \cosh k(h+z) \cdot F(x,y)$$  \hspace{1cm} (2.15)

Thus, the Helmholtz equation (2.6) can be represented as

$$\frac{\partial^2 F}{\partial x^2} + \frac{\partial^2 F}{\partial y^2} + k^2 F = 0$$  \hspace{1cm} (2.16)
For the general case of waves approaching the breakwater at any angle $\theta_0$ with respect to the breakwater (Fig 2.1) Penney and Price (1952) show

$$F(r, \theta) = f(\sigma) e^{-i kr \cos(\theta - \theta_0)} + f(\sigma') e^{-i kr \cos(\theta + \theta_0)}$$  \hspace{1cm} (2.18)

where

$$f(\sigma) = \frac{1 + i}{2} \int_{-\infty}^{1} e^{-i \pi r^2} dt$$  \hspace{1cm} (2.19)

$$f(\sigma') = \frac{1 + i}{2} \int_{-\infty}^{1} e^{-i \pi r^2} dt$$  \hspace{1cm} (2.20)

$$\sigma = 2 \sqrt{\frac{kr}{\pi}} \sin\left(\frac{\theta - \theta_0}{2}\right)$$

$$\sigma' = -2 \sqrt{\frac{kr}{\pi}} \sin\left(\frac{\theta + \theta_0}{2}\right)$$

Eqs. 2.19 and 2.19 can be written as follows

$$f(\sigma) = \frac{1}{\sqrt{2}} e^{i \pi/4} \int_{-\infty}^{1} e^{i \pi r^2/2} dt - \frac{1}{2} \left[ 1 + C(\sigma) + S(\sigma) \right] + i \left[ C(\sigma) - S(\sigma) \right]$$  \hspace{1cm} (2.21)

$$f(-\sigma) = \frac{1}{\sqrt{2}} e^{i \pi/4} \int_{-\infty}^{1} e^{i \pi r^2/2} dt - \frac{1}{2} \left[ 1 - C(\sigma) - S(\sigma) \right] + i \left[ C(\sigma) + S(\sigma) \right]$$  \hspace{1cm} (2.22)

where $C(\sigma)$ and $S(\sigma)$ are Fresnel integrals

$$C(\sigma) = \int_{0}^{1} \cos\left(\frac{\pi}{2} r^2\right) dt \quad , \quad S(\sigma) = \int_{0}^{1} \sin\left(\frac{\pi}{2} r^2\right) dt$$  \hspace{1cm} (2.23)
The diffraction coefficient, $k_d$, is defined as the ratio of wave height in the area affected by diffraction to the wave height in the area unaffected by diffraction, and given by the modulus of $F(x,y)$ for the diffraction wave. That is,

$$k_d = |F(x,y)| = \sqrt{R^2 + I^2} \quad (2.24)$$

Where $R$ and $I$ are the real component and the imaginary component of $F(x,y)$. The phase value is given by the argument of $F(x,y)$.

To discuss the physical problem, line $OS'$ and $OR'$ are marked in Fig. 2.1 as shadow lines which are generated through the incoming and reflected wave. One gets three regions $S$, $Q$, $R$ and $\sigma$ and $\sigma'$ possess a different sign in each of these parts. The wave solution in the three regions is obtained as follows:

a) in the region of the geometrical shadow $\sigma < 0$, $\sigma' < 0$

$$F(r,\theta) = f(-\sigma) e^{-ikr \cos(\theta-\theta_0)} + f(-\sigma') e^{-ikr \cos(\theta+\theta_0)} \quad (2.25)$$

b) in the exposed region $\sigma > 0$, $\sigma' < 0$

$$F(r,\theta) = e^{-ikr \cos(\theta-\theta_0)} - \{f(\sigma) e^{-ikr \cos(\theta-\theta_0)} + f(-\sigma') e^{-ikr \cos(\theta+\theta_0)} \} \quad (2.26)$$

c) in the reflecting region $\sigma > 0$, $\sigma' > 0$

$$F(r,\theta) = e^{-ikr \cos(\theta-\theta_0)} + e^{-ikr \cos(\theta+\theta_0)} - \{f(\sigma) e^{-ikr \cos(\theta-\theta_0)} + f(-\sigma') e^{-ikr \cos(\theta+\theta_0)} \}$$

The graphical representation of the Fresnel integrals is known as the Cornu spiral and is used for diffraction calculation.

$$X = \frac{1}{\sqrt{2}} \int_0^\sigma \cos \frac{\pi}{2} \tau^2 d\tau \quad , \quad Y = \frac{1}{\sqrt{2}} \int_0^\sigma \sin \frac{\pi}{2} \tau^2 d\tau$$
2.5.2 Diffraction through a breakwater gap

There are three theoretical methods for the general problem of water-wave diffraction through a breakwater gap. The first one can be expressed the method due to Rayleigh. It is for diffraction through a very small gap compared with the wave length (Lamb, 1932).

The second method of approach may be attributed to Penney and Price (1952), in which the Sommerfeld solution for a semi-infinite screen is extended by superposition to two breakwaters with a gap. The resulting solution is reasonably accurate for a gap width large compared to the wave length.

The third method of approach may be credited to Carr and Stelzriede (1952), involving the solution of Morse and Rubenstein for the diffraction of waves by ribbons and by slits with the two boundary condition of zero wave function and zero normal gradient. This analysis, based on elliptic-cylinder coordinates and the associated Mathieu functions, was originally developed for the diffraction of sound and electromagnetic waves. Carr and Stelzriede (1952) showed that it can be used with a high degree of success in water-wave diffraction studies, especially for gap widths of the order up to three wave lengths.

2.5.2.1 Rayleigh Method

The solution of wave diffraction through a single gap small compared with the wave length has been given by Lamb (1932). Assume the origin of the coordinate is the gap middle and incident wave direction is normal to the gap. Assuming the incident wave propagation direction to be in the negative direction of y-coordinate, the solution by Lamb reads

\[ \phi(x,y) = e^{iky} + e^{-iky} + \psi(x,y) \]  

(2.28)

and

\[ \phi(x,y) = \psi'(x,y) \]

for two sides of the breakwaters respectively.
\[
\psi(x,y) = \frac{\sqrt{\frac{L}{4r}} e^{-i\left(\frac{\pi}{4} + \phi\right)}}{(\log \frac{kb}{4} + \gamma + \frac{\pi}{2})}
\]

(2.29)

for \(y > 0\) and

\[
\psi'(x,y) = -\psi(x,-y)
\]

for \(y < 0\)

where

- \(r\) = distance from origin,
- \(b\) = gap width,
- \(L\) = wave length,
- \(\gamma\) = Euler constant.

The value of \(\psi'(x,y)\) at any point \(P(x,y)\) on the negative side of the plane \(y=0\) is equal and opposite to the value of \(\psi(x,y)\) at the image of \(P(x,y)\) with respect to the plane.

However, this solution is not widely used because the gap width usually is not very small compared to the wave length.

2.5.2.2 Penney-Price Method

The theory for large gaps was developed by Penney and Price (1944, 1952) for waves approaching at 90 degrees. Blue and Johnson (1949) extended this work to include waves incident from any direction. In this method wave heights at any point are affected by both of the breakwaters, Fig 2.3.

The solutions for \(F(r,\theta)\) are as shown in Fig. 2.3 (Eq 2.30), where

\[
\sigma_1 = -\sqrt{\frac{4(r_1 - y)}{L}}, \quad \sigma_2 = -\sqrt{\frac{4(r_2 - y)}{L}}
\]

\[
f_1 = e^{-iky}f(-\sigma_1), \quad f_2 = e^{-iky}f(-\sigma_2), \quad g_1 = e^{-iky}f(-\sigma'_1), \quad g_2 = e^{-iky}f(-\sigma'_2),
\]
Fig. 2.2 Definition sketch for breakwater gap

\[ \sigma_1' = -\sqrt{\frac{4(r_1 + y)}{L}} , \quad \sigma_2' = -\sqrt{\frac{4(r_2 + y)}{L}} \]

and \( f(\sigma) \) and \( f(\sigma') \) are defined by Eq. (2.29).

\[ \begin{array}{ccc}
\begin{pmatrix}
 f_1 + g_1 \\
 f_2 + g_2
\end{pmatrix} &=& \begin{pmatrix}
 f_1 + g_1 \\
 f_2 + g_2
\end{pmatrix} \\
\begin{pmatrix}
 e^{iky} \\
 -f_1 + g_1
\end{pmatrix} &=& \begin{pmatrix}
 -f_2 + g_2 \\
 e^{iky}
\end{pmatrix} \\
\begin{pmatrix}
 e^{-iky} + e^{iky} \\
 -f_1 + g_1
\end{pmatrix} &=& \begin{pmatrix}
 1 \\
 f_1 - g_1
\end{pmatrix} \\
\begin{pmatrix}
 -f_2 - g_2 \\
 -f_2 - g_2
\end{pmatrix} &=& \begin{pmatrix}
 wave \ direction \\
 -f_2 - g_2
\end{pmatrix}
\end{array} \]

Penney and Price concluded from a mathematical analysis that the solution as given above is a good approximation as long as the width of the breakwater gap is greater than 1 wave length.

2.5.2.3 Morse-Rubenstein Solution

The deficient features of the Penney-Price method are largely avoided by
the approach outlined by Morse and Rubenstein. It is an exact solution for small gaps. Application of the exact boundary conditions of zero potential gradient to the breakwater with a gap is expedited by the use of elliptic-cylinder coordinates. Morse and Rubenstein separate the wave equation in elliptic cylinder coordinates using a transformation defined by \((\xi, \phi)\)

\[
x - \frac{d}{2} \cosh \xi \cos \phi, \quad y - \frac{d}{2} \sinh \xi \sin \phi, \quad z - z.
\]

They obtained the total transmission and the angular distribution of the scattered or diffracted waves in terms of Mathieu functions. Following Carr and Stelzriede (1952), the diffracted wave beyond the rigid breakwater with a gap is expressed by the equation:

\[
\phi = \sqrt{8\pi} \sum_{m}^{i} \frac{\gamma_{m} e^{i\alpha}}{N_{m}} - [J_{e_{m}}(s,\xi) + iY_{e_{m}}(s,\xi)]
\]

where:

- \(\alpha\) = the angle of incidence wave with the breakwater,
- \(s = (wh/L)^2\),
- \(N_{m}\) = normalization factor,
- \(\gamma_{m}\) = the phase angle of the partial wave, and \(\gamma_{m} = [(Y_{e_{m}}(s,0))/[J_{e_{m}}(s,0)]\)
- \(J_{e_{m}}(s,\xi)\) = Bessel’s function of the first kind and \(m\)-th order
- \(Y_{e_{m}}(s,\xi)\) = Bessel’s function of the second kind and \(m\)-th order

\[
J_{e_{m}}(s,\xi) = \sum_{k=0}^{\infty} D_{e_{k}} \cos k\xi - Y_{e_{m}}(s,\xi)
\]

\(D_{e_{k}}\) = the Mathieu coefficients

In these relations subscript \(e\) indicates the even solution.

The modulus and argument of equation 2.33 represent the amplitude and phase of the diffracted wave respectively.

2.5.3 Diffraction around a circular pile

The solution of diffraction around a circular pile (see Fig. 2.4) has been
given by many authors e.g. Lamb (1932), McCamy and Fuchs (1954) and Mei (1984) with the aid of Bessel series development.

If \((r, \theta, z)\) are the cylindrical coordinates of the point \(P\), then the solution in this point is given by:

\[
\phi(r, \theta, z) = - \frac{1}{2} \frac{\rho g}{\omega} \frac{\cosh(k(z + h))}{\cosh(kh)} \varphi(r, \theta)
\]

with

\[
\varphi(r, \theta) = \sum_{m=0}^{\infty} e_m i^m A_m(kr) \cos(m\theta)
\]

in which \(e_0 = 1; \ e_m = 2\) for \(m = 1, 2, 3, \ldots\)

\[
A_m(kr) = - \frac{J_m(kr) Y_m'(ka) - J'_m(ka) Y_m(kr)}{J'_m(ka) + iY'_m(ka)}
\]

\(J_m(\cdot)\) = Bessel function of the first kind and \(m\)-th order,

\(Y_m(\cdot)\) = Bessel function of the second kind and \(m\)-th order,

\(a\) = radius of the pile,

and a prime indicating a differentiation with respect to the argument.
3 MODEL DESCRIPTION

3.1 General

The BELUGA computer programme is developed to compute wave diffraction and reflection in a confined area. The aim of the model is to provide an efficient numerical method for calculation of wave penetration into a harbour, executable by a personal computer.

The developed mathematical model is based on linear harmonic wave theory. The numerical method employed in BELUGA programme can be classified as a boundary element method. Hence, the contour of the harbour or obstacle is split up into a number of finite segments; in each segment the unknown wave characteristics, in terms of height, period are approximated.

With respect to the spectral method, it is assumed that the wave field in each boundary point is coming up via various harmonic directional components, therefore a ray method can be employed to evaluate the wave characteristics. Both the incident and reflected waves are considered in this computation.

Herein, the diffraction is imagined to take place only at the corners; according to the well-known Sommerfeld solution, it is assumed that the wave direction is almost in a straight line from each corner point, where the distance from the corner points is more than a few wave-lengths.

The superimposition of the reflected and diffracted waves coming from other points along the boundary for each point, gives an initial value of the wave characteristics in that point. Evidently the points affect each other, then an iterative procedure can determine the final result of wave field inside the harbour.

3.2 Model Schematisation

3.2.1 Geometry

A boundary element method is involved, therefore it is necessary that a harbour boundary can reasonably be schematized to straight lines, named
segments. The geometry data are the coordinates of the ends of these lines. The model accepts arbitrary geometry with straight lines and constant depth.

The segments are divided into several smaller lines, called the line elements. Data (coordinates and reflection coefficients) are given at ending point of as well as along the segments whereas the line elements are calculated using linear interpolation. All the mentioned points on which the calculations are carried out are labelled as the boundary points. Diffraction takes place at the corner points which are called diffraction points and/or corner points. All the other points act as reflection points. Fig. (3.1) shows an imaginary harbour lay-out marked by corner and intermediate boundary points.

The incident wave is schematizes to a straight line and is considered the same as harbour contours. The direction of this incident wave contour is the crest direction of undisturbed wave. The location and length of this wave incident contour should be selected as far as it indicates the wave field at infinity. The incident contour is also divided into several smaller lines (see Fig. 3.1). In the incident contour, it is assumed that reflected waves are completely absorbed.

In the model Cartesian convention is employed, therefore the coordinate of all points are given in this coordinate system. The direction of incident waves is the direction to which the waves propagate, the direction is measured from the x-axis positive in counterclockwise direction. For more information about the input data reference is made to Booy and Tolman (1990). The main input data are:

- wave length,
- wave height,
- incident wave direction,
- geometry of the harbour,
- reflection coefficient of the harbour contours,
- maximum segment length in the reflecting part of the harbour contours,
- maximum segment length in the incident part of the contours,
- size of the directional increment in the spectral sector of each boundary contour,
- the maximum number of iterations,
- desired accuracy of wave height.
3.2.2 Boundary condition

The full reflection condition at a boundary contour is

\[ \frac{\partial \phi}{\partial n} = 0 \]
where \( \varphi \) is the total wave potential \( \varphi = \varphi_0 + \varphi_d \),

\( \varphi_0 \) = incident wave potential

\( \varphi_d \) = diffracted wave potential.

Thus, the boundary condition for the two-dimensional scattered wave potential (Helmholtz equation) are:

\[
\frac{\partial \varphi_d}{\partial n} - \frac{\partial \varphi}{\partial n} = 0
\]  

(3.1)

at full reflecting contours and a radiation condition

\[
\lim_{r \to \infty} \sqrt{r} \left( \frac{\partial \varphi_d}{\partial r} - i k \varphi_d \right) = 0
\]

(3.2)

at infinity. With these conditions the diffracted wave potential will be uniquely defined where the derivation is carried out assuming the water depth is constant and the obstacles are cylindrical extending from the bottom to the free surface, and fully reflecting.

In practical problem the walls of the harbour contours and obstacle are not always vertical and very often not fully reflecting as well. To treat these boundaries in diffraction models the walls are schematized as vertical, but partial reflection is considered by using a mixed boundary condition for the total wave potential. The reflection coefficient \( K_r \) must be given as a complex value with information of phase except for the cases of \( k_r = 1 \) and 0, but the reflection coefficient cannot generally be given as the complex value. Therefore, as in most existing models in BELUGA, the phase shift is assumed to be zero and the real values from 0 to 1 have been taken as the \( k_r \) values.

The following reflection boundary conditions can be distinguished for line elements (reflection points) considering the reflection law:

a. the angle of incidence is equal to the angle of reflection. \( \theta_i = \theta_r \)

b. the amplitude of reflected wave is the product of wave incidence amplitude and the reflection coefficient, i.e. \( A_r = K_r * A_i \)

c. the waves are reflected with the same phase as the incident wave phase.
3.3 **Computational procedure**

The boundary element technique has been employed for this calculation. In this method each boundary element receives wave energy from other elements, and sends wave energy to the other elements, depending on the boundary condition of the element. By means of an iterative solution, the incident and reflected waves are considered for each boundary point as separate components applying the boundary conditions. The convergence of the iteration process has to be checked in this process as well.

It is assumed that the wave field at each boundary point is composed of waves in a number of pre-defined directions, i.e. the wave amplitude and phase are the functions of direction.

In the computational procedure the contribution of diffracted and reflected waves are calculated separately. Then the reflected and diffracted components in each target point are superimposed in order to get the wave field in that point.

The computational procedure can be distinguished into preprocessing and processing stages.

In the preprocessing stage the geometric problems are solved, and it can be described as follows:

- The geometric characteristics of line elements are determined by interpolation of input data for segments.
- For each boundary point a spectral sector is determined. The first and the last spectral direction are parallel to the contour in opposite direction; the sector in between is divided into evenly spaced directions.
- Straight line elements (rays) are extended from the boundary points in each of the spectral directions. The location of intersections of these rays with the harbour contours and incident contour are determined. The intersection point is assumed to be a source point (sending point) of wave energy in opposite direction (see Fig. 3.2).
- In each intersection point (source point), the angle of the ray with harbour contour or incident contour is determined.
- The number and location of visible corner points are determined for every boundary point. These corner points contribute to the wave field in the boundary point.
In the processing stage the calculations of reflection and diffraction are performed separately. Following, the steps of the computation procedure are given briefly. The detailed computation of reflection and diffraction is provided in the next sections. According to the applied technique in the model, an iteration procedure is used to calculate the wave field in the boundary contours. The following computations are carried out in each iteration:

I- Reflection computation

In each boundary point (receiving point) the approaching waves from corresponding source points (in the harbour or incident contour) are calculated. The contribution of the wave field (amplitude and phase) in each direction is calculated directly from the related parameters at the source point in that direction. Then, the wave field in the boundary point is determined by superposition of all calculated wave components, e.g. the wave field in point R in Fig. 3.2 is calculated by superposition of the wave components coming from the source points B1 up to B14. The wave field (amplitude and phase) in each source point is calculated using linear interpolation from the nearest two line element points (boundary points), e.g. the wave field in the source point B5 in Figure 3.2. is determined from the point boundary points P1 and P2.

II- Diffraction computation:

The contributions of the visible corner points to the wave field in each boundary point are calculated using an approximated Sommerfeld solution. The total incoming wave field is calculated by superposition of incident, reflected and diffracted waves reaching the boundary point.

III- Applying boundary conditions

In each boundary point the boundary conditions are applied in order to determine the outgoing wave field for next iteration process.

This procedure is repeated for every boundary point. The calculated wave fields in each iteration are used for calculation of wave field in next iteration process. In next iteration the above steps are repeated and new wave characteristics
(amplitude and phase) are computed for all boundary points. In each iteration the computed wave height of each boundary point is compared with the computed wave height in previous iteration. The difference should be less than the desired accuracy; otherwise, the iteration process should be continued until an appropriate accuracy is achieved for all boundary points. Figure 3.3 provides a schematic sketch presentation of the procedure.

The procedure is the same for target point other than the boundary points except that the iteration is not needed.

Figure 3.2 Schematic presentation of a receiving point and corresponding source points
Input data

Preprocessing
(geometric problems)

Processing

Reflection computation
Diffraction computation
Applying boundary conditions

Accuracy check

Generating output

Fig. 3.3 Schematic presentation of the procedure
3.3.1 Reflection computation

It was assumed that the harbour boundary is composed of straight walls so that the canonical problem of a straight boundary can be used. Waves incident from a direction forming an angle $\theta_1$ with the normal to the boundary and with amplitude $a_1$ will excite reflected waves in the direction forming an angle $\theta_2$ with the normal to the boundary and with amplitude $a_2$, according to the law of reflection. In the reflection procedure the amplitude emitted is equal to the amplitude received multiplied with the reflection coefficient; the phase of the emitted wave is equal to the phase of the received wave.

The emitted wave field is characterized by its amplitude and phase. For every receiving point and direction both amplitude and phase are calculated. The contributions of reflection points to the wave motion in a target point are determined directly from the corresponding parameters at the sending point.

The amplitude at the receiving point $R$ is assumed to be equal to the amplitude at the sending point $B$:

$$a_R = a_B$$

The amplitude in the sending point $B$ is calculated by means of linear interpolation from the amplitudes at the nearest two line element points $P_1$ and $P_2$ (see Fig. 3.2).

$$a_B = RD * a_{P_1} + (1 - RD) * a_{P_2} \quad (3.3)$$

where $RD$ is the linear interpolation coefficient which depends on position of the intersection point $B$ between $P_1$ and $P_2$:

$$RD = \frac{BP_2}{P_1P_2}$$

where $BP_2$ is distance between $B$ and $P_2$, and $P_1P_2$ is also distance between $P_1$ and $P_2$.

The phase at point $B$ is calculated from the phases at $P_1$ and $P_2$ ($\phi_{P_1}$ and
$S_{P2}$) applying the theoretical phase difference ($\delta S$) between the two points, which results in estimates for $S_B$ as:

\[
S_{B1} = S_{P1} - (1 - RD) \ast \delta S_{PIP2},
\]
\[
S_{B2} = S_{P2} + RD \ast \delta S_{PIP2}
\]  

(3.4)

where $S_{B1}$ and $S_{B2}$ are phases at point B calculated from the point P1 and P2 respectively, and $\delta S_{PIP2}$ is theoretical phase difference between points P1 and P2:

\[
\delta S_{PIP2} = \overrightarrow{P2P1} \cdot \hat{F}
\]

(3.5)

where $\overrightarrow{P1P2}$ is the displacement vector from P1 to P2.

For purely reflecting long crested waves the difference between $S_{B1}$ and $S_{B1}$ should be zero (or $2\pi n$ with $n = \ldots -1, 0, 1, \ldots$). However, due to numerical errors and corner solutions a difference $\Delta S$ exists between $S_{B1}$ and $S_{B2}$:

\[
\Delta S = S_{B1} - S_{B2} + 2\pi n
\]

(3.6)

or

\[
\Delta S = S_{P1} - S_{P2} + \delta S_{PIP2} + 2\pi n
\]

To define $\Delta S$ uniquely, $n$ is chosen so that $-\pi < \Delta S < \pi$. To obtain a consistent correction of $S_{B1}$ and $S_{B2}$ the phase at B is defined as:

\[
S_B = S_{P1} + (1 - RD) \ast \Delta S + 2\pi n
\]

(3.7)

or

\[
S_B = S_{P2} + RD \ast \Delta S
\]

By these solutions wave characteristics are defined in each sending point. The wave components coming from each sending point are calculated in the receiving point R whereas the amplitude at the receiving point R is equal to the amplitude at the sending point B, and the phase at the receiving point
is calculated from the phase at point B as:

$$S_R = S_B + \delta S_{BR}$$

where $\delta S_{BR}$ is theoretical phase difference between B and R:

$$\delta S_{BR} = \vec{BR} \cdot \vec{k}$$

$\vec{BR}$ = displacement vector from B to R.

Superposition of all components gives the wave field in the receiving point. This wave field is used to define wave fields in other points after applying the boundary conditions. When the receiving point R in the iteration procedure contributes as a sending point for other points, its amplitude is reduced in proportion with the reflection coefficient of its line element.

In each iteration, the calculated wave amplitude and phase in the previous iteration (in the boundary points or receiving points) are used to determine the new wave field in the boundary points.

### 3.3.2 Diffraction computation

The contribution of diffraction points (corner points) are computed in each boundary point using the Sommerfeld solution. The diffraction point receives wave energy from various directions and emits wave energy in all its spectral directions. It is assumed that the incoming wave fields are uniform in the neighbourhood of the diffraction point.

With respect to the Sommerfeld solution of the contributions of the diffracted, incident wave and the diffracted, reflected wave should be considered. Theoretically, the Sommerfeld solution is for a thin, semi-infinite and straight obstacle. However, in the approximate solution which has been used in BELUGA, it is assumed that this solution can be used for corner points with adjacent lines. The contribution of the corner point is considered for both adjacent lines separately. In fact in the model, it is assumed that each adjacent line of a corner point acts as a straight and semi-infinite obstacle. Therefore, at the corner point the contribution of four wave fields
is considered as follows:
- The incoming wave field at both adjacent lines.
- The reflected wave field at both adjacent lines.

For calculation of incoming wave diffraction in each corner point a local coordinate system is defined. The corner point is the origin of this coordinate system. The orientation of \( x_i \)-coordinate is the direction of incoming wave. The orientation of \( y_i \)-coordinate is defined by convention that the shadow zone is specified by negative value of \( y_i \)-coordinate. The local coordinate system for reflecting wave is similar to the incoming wave; the orientation of \( x_r \)-coordinate is the direction of reflecting wave and again the negative values of \( y_r \)-coordinate are related to the shadow zone (see Fig. 3.4).

For a given field and direction for which the amplitude is non-zero, the \( x \) and \( y \) coordinates of the target relative to the corner point and the propagation direction of the undisturbed wave are calculated (see Fig. 3.4).

![Fig. 3.4 coordinates system for diffraction computation](image)
In this part, the methods for considering four mentioned wave fields are explained. The approximated Sommerfeld solution for calculation of these wave fields is given in next part.

In a target point T (Fig. 3.4), the contribution of four wave fields are considered as follows:

- The contribution of incident, diffracted wave around line 1 (first line) is calculated directly using the first term of Sommerfeld solution.
- The effect of line 2 (second line) in the target point is considered using a mirror solution. In this mirror solution the effect of second line of the corner point is calculated using the image mirror point of the target point on line 2, e.g. point T' in Fig. 3.5. The procedure for the projected target point T' is similar to the first solution (solution for point T), i.e. the wave field in the mirror image of the target point T' is calculated. The resultant component is multiplied by the corresponding reflection coefficient, and is added to the first solution for the target point T.

The second component is calculated only if the angle between the direction of the incoming waves and the direction of the second line is less than \( \pi/2 \).

Fig. 3.5 definition sketch for mirror solution
- The contribution of reflected wave on line 1 is also calculated directly using second term of Sommerfeld solution.
- The effect of line 2 on reflected wave is considered using the mirror solution. The procedure is similar to the incident wave; the contribution of reflected wave in project target point T' is calculated and is multiplied to the reflection coefficient. The resultant component is added to the wave field in target point T.

According to the wedge angle and wave attack if the second element is seen by the undisturbed wave component, a diffraction solution occurs and mirror solution are not taken into account. For instance in Fig. 3.6 the incident wave is seen by line 1 and line 2, therefore, diffraction around two lines are calculated and the mirror solution is not taken into account in this case.

![Incident wave on two adjacent lines](image_url)

**Fig. 3.6 Incident wave on two adjacent lines**
Approximation of the diffraction computation:

For diffraction calculations the Sommerfeld solution is applied. In this solution Fresnel integrals appear which have been tabulated in mathematic handbooks. The approach of Penney and Price (1952) for the Sommerfeld solution was described in chapter 2, so here only the approximate solution which has been used in the model is described.

The wave field in terms of wave height and amplitude is calculated by means of Eq. 2.16, which is repeated here for convenience:

\[
F(x,y) = \frac{1}{\sqrt{2}} e^{i k \cos(\theta_0 - \theta)} \int_{-\infty}^{\infty} e^{i \pi r^2} dt + \frac{1}{\sqrt{2}} e^{i k \cos(\theta_0 + \theta)} \int_{-\infty}^{\infty} e^{-i \pi r^2} dt
\]  

(3.8)

The integrals in (3.8) can be expressed (Abramowitz and Stegun, 1968) as

\[
\int_{-\infty}^{\infty} e^{-i \frac{z^2}{2}} dt = 1 - i e^{-i \pi z^2} [g(z) - if(z)]
\]

(3.9)

for \(z\) positive, and otherwise

\[
\int_{-\infty}^{\infty} e^{-i \pi r^2} dt = e^{i \pi z^2} [g(-z) + if(-z)]
\]

(3.10)

The functions \(f\) and \(g\) have fairly accurate rational approximation (Abramowitz and Stegun, 1964) which read:

\[
f(z) = \frac{1 + 0.926z}{2 + 1.792z + 3.104z^2} + \epsilon
\]

\[
g(z) = \frac{1}{2 + 4.142z + 3.492z^2 + 6.670z^3} + \epsilon
\]  

\(|z| \leq 2 \times 10^{-3}
\]

(3.11)

The solution for the target amplitude and phase \((a_t\) and \(S_t)\) caused by diffraction depends on the parameter (see Fig.3.4):
\[ W = \frac{R-X}{L} \]  

(3.12)

and

\[ z = 2 \sqrt{W} \]

R : Distance between target and corner.
X : X coordinate of the target point defined in the local coordinate system.

The value of Z is always computed positive, so the effect of the sign of Z should be considered by another method in the calculation procedure. Since cartesian coordinates are used in the programme, instead of considering the effect of sign Z in various regions, the sign of Y (Y coordinate of the target point) has been evaluated in the programme. This is considered in such a way that the phase of target point varies by an increment of \( \pi \) for different signs of Y. In fact, the shadow zone corresponds to negative value of Y, thus the effect of various signs for Z in the shadow zone and the exposed zone is considered by means of adding \( \pi \) to the phase.

For calculation of diffraction contributions in a target point, an approximate solution had been used before the modification of the programme. Because of incorrect results and discontinuity in some points, the model was modified to have better results. In the following, formulation of the diffraction contribution before and after this modification are given. The comparisons between the model results before and after modification are provided in chapter 5. It should be mentioned that in the previous version of the programme there were also some other errors which in the course of the present study have been corrected before the modification for the diffraction calculation.
In the previous version of the programme the amplitude in a target point is calculated as:

\[ a_i = Kz \cdot a_u \]  

(3.13)

where

\[ Kz = F(Z) \frac{1 + 6.5 \cdot Z \sqrt{2}}{1 + 6.5 \cdot Z} \]  

(3.14)

\[ F(Z) = \frac{1 + 0.926 Z}{2 + 1.792 Z + 6.208 W} \]

\( a_u \) and \( S_u \) are amplitude and phase of undisturbed wave.

The diffraction phase difference was estimated as:

\[ \delta S = \pi \left( 0.75 + \frac{0.25}{1 + 10 \cdot W} \right) \]  

(3.15)

\[ \delta S = \pi \left( 0.75 + \frac{0.25}{1 + 10 \cdot W} \right) + \pi \]  

(3.16)

for positive and negative y-direction respectively. The total phase at the target point became:

\[ S_i = S_u + 2 \pi R + \delta S \]  

(3.17)

Test results showed that the approximate solution which had been used for diffraction calculation was not proper. Here, two points can be considered. First, the derivation for the approximate formulation based on the Sommerfeld solution was given. Secondly, as can be observed in Eq. 3.17 for the calculation of the phase difference in the target point, the theoretical phase difference between the corner and target point has been added to the phase of undisturbed wave. However, this is not correct because the contribution of the reflected, diffracted waves, and of the undisturbed waves
are calculated separately. These components are superimposed to compute the total wave. Therefore the phase difference between the corner point and the target one should not be added to the calculated phase difference in the diffraction computation.

**Modification of the model**

The modification of the programme is based on the Sommerfeld exact solution except for calculation of the Fresnel integrals, for which the approximate expressions (3.9-3.11) have been used.

In a target point, the contributions of diffracted, incident waves and diffracted, reflected waves are calculated by the same formulae. But for calculation of \( W \), a distinction between incident and reflected waves is made by selecting appropriate coordinates.

In equation 2.19 and 2.20 the following expressions can be used for the Fresnel integrals (Abramowitz and Stegun, 1968):

\[
C(z) = \frac{1}{2} + f(z) \cdot \sin(\frac{\pi z^2}{2}) - g(z) \cdot \cos(\frac{\pi z^2}{2})
\]

\[
S(z) = \frac{1}{2} - f(z) \cdot \cos(\frac{\pi z^2}{2}) - g(z) \cdot \sin(\frac{\pi z^2}{2})
\]

where \( f(z) \) and \( g(z) \) are given by (3.11). After calculation of the Fresnel integrals the values of \( U \) and \( V \) are determined:

\[
U = \frac{1}{2} \left( 1 - S - C \right)
\]

\[
V = \frac{1}{2} \left( S - C \right)
\]

Then the diffraction coefficient and phase difference are calculated as follows:
\[ A = U \cos kR + V \sin kR \]
\[ B = V \cos kR - U \sin kR \]

where \( R \) is the distance between target point and corner point.

\[ Kd = \sqrt{A^2 + B^2} \]

\[ \delta S = \arctan \left( \frac{B}{A} \right) \]

The wave component is computed as:

\[ a_2 = Kd \cdot a_1 \]
\[ S_2 = S_1 + \delta S \]

where \( a_1 \) and \( S_1 \) are wave amplitude and phase at the corner point.

The diffracted, incident and diffracted, reflected wave fields are computed by the same procedure. The total wave field in each boundary point or in each arbitrary target point are computed by superposition of all wave components in that point.

Comparisons between model results, before and after modification, and analytical solutions are given in chapter 5. In the comparisons, the first version of the model (the version with which the present study was started) is labelled model version 1 or B1. As already mentioned there were some errors in the programme which were corrected during this study. The model version 2 (B2) and version 3 (B3) are corresponded to the improvement of the model before the main modification due to diffraction calculation. The results of the modified model (due to diffraction calculation) are shown under the label of model version 3.1 (B3.1).

### 3.3.3 Accuracy of the model

The amplitudes at all boundary points are estimated and then changes in amplitude over the last iteration step are compared to the desired accuracy. If the changes of the amplitude at all points are smaller than the specified accuracy, the iteration is stopped, otherwise the iteration is continued until
the changes reach the specified accuracy.
The number of iterations and desired accuracy are input values. The computation stops as soon as one of these criteria is fulfilled. The rate of convergence of the process depends on the average reflection coefficient; if there are strong reflections, a large number of iterations is needed.

3.4 Computer programme description

The computer programme has been built up in separate routines and in each routine a special activity is considered. The activities that have to be done by the programme are: initialization, input, preprocessing, calculation, and output. The preprocessing routine handles geometric problems, such as the problem to determine basic elements of contours and the problem to find the intersection point of a ray in given direction from a given target point with the harbour contour. Another problem is to determine the number of visible corner points and to consider whether or not a ray from a given target point to the diffraction points is interrupted by other harbour contours.

The reflection and diffraction computations are carried out in the processing routine. The calculation of the incoming waves, the application of the boundary conditions, and inspection on accuracy are performed in the check routine.

The output result of the programme is provided in files such as table form and plot file which can be demonstrated by means of a service programme. It is possible to observe the wave condition in the boundary points after every iteration. It is also possible to observe wave components in each target point.

3.5 Efficiency of the model compared with existing methods

In recent decades much effort has been devoted to modelling wave propagation in coastal areas specially wave action in a confined area and harbour resonance using finite-difference and finite-element techniques. These methods have a computational limitation in that they require a certain
minimum number of grid points per wavelength. This limitation makes these methods unsuitable for modelling short waves in large areas because the computational effort is considerable, and also the computer storage must be large.

With respect to the applicability of the models it can be stated that these models can be used best when the study area is restricted to a relatively small area. Thus, they are suitable to modelling wave activity in small harbour areas. They would be best applied to modelling resonant wave periods and long waves.

Some types of model are also restricted in the range of harbour layouts and incident wave conditions by the computing size and cost of the model. BELUGA is a model using a ray method which is a far-field method and not suffering from the above mentioned computational limitation. Therefore it is well-suited for modelling short waves in large harbour areas.

Nowadays using the personal computer is common for engineering purpose, and preparing a computer program running on a PC seems to be very useful. The greatest advantages of BELUGA are its speed and capability to run on a personal computer, while the above mentioned models can be used only on large main-frame computers.

With the BELUGA model, it is a simple procedure to consider wave penetration into any harbour layout with constant water depth and any wave condition. On the other hand harbour models based on finite-difference or finite-element techniques are able to incorporate the combined primary wave effects of refraction, diffraction, and reflection. And depending on the governing equations, some non-linear effects as well. However, in BELUGA only the effects of diffraction and reflection in constant water depth have been taken into account. This is a limitation to the BELUGA programme but in numerous cases the harbours depth are constant and the model can then be used with appropriate accuracy.
4 Test Plan:

4.1 Introduction

For verification of a mathematical model, two aspects are important. First, it must be verified whether the method of solution is correct in the sense that the numerical solution is in agreement with the exact solution of the mathematical description of the problem. Secondly, it must be verified whether the mathematical description of the problem and the corresponding solution agree in an appropriate way with the desired physical process. The second verification gives the most problems, because the physical process always has some effects which do not satisfy the restrictions of the mathematical formulation.

A real comparison is difficult since many different independent parameters are involved. In this study results of numerical computation are compared with analytical solutions and laboratory measurements.

The tests are carried out for the calculation of wave diffraction and reflection phenomena. In some test cases only the effect of diffraction is considered, but in most cases the combined effect of both diffraction and reflection are taken into account.

4.2 Criteria

To test the model its results are compared with those of analytical solutions, laboratory measurements, and existing reliable mathematical models.

4.2.1 Analytical solutions:

The numerical solution of the model is verified with the aid of analytical solutions given in section 2.5. The outcomes of the model are compared with the results of these analytical solutions. The analytical tests are carried out for the following cases:

a. a single, straight, semi-infinite breakwater
b. a breakwater gap
c. diffraction around a circular pile.
The Sommerfeld graphical solution (Cornu spiral) is involved for the cases of a single breakwater and a gap between two breakwaters. A set of generalized wave diffraction diagrams for the breakwater gap situation, based on the Penny-Price (1944, 1952) method and Carr-Stelzriede (1952), is given by Johnson (1952), and these are included in CERC (1984). The outcomes of the model are compared with these graphs in some cases. These graphs are for vertical, rigid and impermeable breakwater, i.e. full reflection.

The variable parameters, reflection coefficient and wave direction have to be regarded in the case of a single breakwater. In the case of a gap the variation of gap width is taken into consideration as well.

In the case of a circular pile, the results of the exact solution are compared with the result of the model for the schematized pile.

4.2.2 Laboratory measurements

The numerical model results are compared with the laboratory experiments reported by Berkhoff (1981) concerning regards wave penetration into a harbour. The hydraulic model was built in a wave basin of the Delft Hydraulics Laboratory. More information about the measurements can be found in Report W 154-VI of the Delft Hydraulics Laboratory by Berkhoff (1981).

4.2.3 Existing reliable computer programmes

Solving the Helmholtz equation with full boundary condition for wave penetration into a harbour with arbitrary shape is possible only numerically. A number of computer programmes are available which can be used for this purpose. As discussed in the previous chapter, these models solve the equation by finite element or finite difference techniques and are useful for long waves or relatively small harbours.

The results of the model are also compared with the numerical results of model GOLDHA reported by Berkhoff (1981) for the same hydraulic model harbour as mentioned in part 4.2.2.
4.3 Variable parameters

The following parameters should be varied for the verification of the model:
- wave length
- reflection coefficient
- wave direction
- gap width

Since the depth is constant, variation of wave length indicates the variation of wave period. In the case of analytical approaches, i.e. the Sommerfeld solution, the breakwater length is assumed to be infinite but in the program this length has to be finite. Breakwater length is involved as a variable to consider the effect of finite length. The model is based on linear wave theory, consequently, the obtained result for various wave height and wave length should be linear. Since dimensionless parameters are used for comparison of the results, it is not necessary to consider wave height as a variable parameter. But in one test case the wave height is changed to verify whether the model is fully linear.

For each category of tests the relevant parameters will be changed and computed results will be compared to the available results.

4.4 Tests description

4.4.1 Analytical tests

Table 4.1 shows a summary of the various tests carried out in the case of a single breakwater. Test TS1a is carried out to check the linearity of the model by varying the wave height. Test TS1b demonstrates the effect of breakwater length. Tests TS2 and TS2a are performed to consider the effect of reflection coefficient. Variation of wave direction is considered by means of tests TS3 and TS3a. Test TS4 is carried out to consider the effect of variation of wave length.

The configuration for a single breakwater and the lines for which the results are shown in Fig. 4.1. Diffraction coefficients are computed along the lines A and B on the lee-side of the breakwater, with y/L= 5 and y/L=2 respectively, and along line C in front of the breakwater with y/L=-3.

Table 4.2 displays the summary of tests for the case of a breakwater gap. In all tests except TG2 full reflection is considered in order to compare the results.
with the CERC graphs.

Test TG3 is carried out to consider a gap problem with oblique wave direction. Tests TG4, TG5 are performed to observe the effect of the gap width.

Fig. 4.2 shows the breakwater configuration which is considered in the breakwater gap tests. Here again diffraction coefficients along lines A and B are calculated for comparison of the results.

According to the results obtained from the first version of the model, a further set of tests were carried out to recognize the error points in the model. Table 4.3 shows a summary of these tests. Here, the reflection coefficient at both sides of a single breakwater has been changed for two cases of wave direction. In test TN1 the wave incident direction is perpendicular to the breakwater and the reflection coefficient in front of the breakwater is zero, but there is full reflection at the back side. Test TN2 is similar to test TN1 except for the reflection coefficient. In test TN3, incident wave direction to the single breakwater is 45 degrees and there is full reflection at both sides of the breakwater. In test TN4 reflection coefficients are the same but the wave direction is 135 degrees.

In the case of diffraction around a circular pile, results of a boundary integral method and laboratory measurements are available for a cylinder with 96 meter diameter, Van Oortmerssen (1972). The results of BELUGA are compared with these results.

4.4.2. Hydraulic Model Tests

4.4.2.1 Description of hydraulic model

Fig. 4.3 shows the wave penetration model, located in a wave basin of the Delft Hydraulic Laboratory. The model geometry was selected so as to have a shadow section in the harbour, depending on reflection against the back walls of the harbour. The required partial reflection was obtained by constructing gravel beaches under a specific slope. A gravel beach was also constructed outside of the harbour, under a 1:5 slope, in order to minimize the reflection of wave
energy toward the wave generator.

Inside the harbour the wave height was measured in a square grid of 0.5 m. The wave direction was perpendicular to the harbour entrance. The water depth of the harbour was 0.3 m.

The deflection of the water surface was recorded on paper tape during approximately ten waves during the wave tests, simultaneously at six points in a line inside the harbour, and at one single reference point in front of the harbour entrance (see Fig. 4.3).

4.4.2.2 Measurements and computational tests

Table 4.4 shows a summary of the tests carried out with regular waves. Water depth was the same for all tests (d=0.30 m). Parameters that varied were: the slope angle of the back walls (BC in Fig. 4.4), the wave period, and the wave height.

For calculating the wave climate in the model harbour by means of the mathematical BELUGA model, all data on geometry and wave parameters are identical with those given for scale model. The model harbour lay-out, as applied in the mathematical model, is shown in Fig 4.5. The boundaries correspond with the intersections of the undisturbed water level and the slopes.

Throughout the area the water depth is kept constant (d= 0.30 m). The direction of incidence of the entering wave is, in all cases, perpendicular to the harbour entrance. Parameters that have been varied are: the reflection coefficient of boundary BC, and the wave period T. In the model the wave length should be given as an input data instead of wave period, so the wave length is computed from the given water depth and wave period.

Table 4 lists the various computed cases referred to in this study. The table indicates the applied reflection factor R for boundary BC. For boundary FB a factor R= 0.23 applies, and for the vertical boundaries CD and EF the factor R=1.00 applies.

50
<table>
<thead>
<tr>
<th>Code</th>
<th>Wave Length</th>
<th>Wave Direction</th>
<th>Reflection Coefficient</th>
<th>Br.Length /L</th>
<th>Incident Wave Height</th>
</tr>
</thead>
<tbody>
<tr>
<td>TS1</td>
<td>50</td>
<td>90</td>
<td>0</td>
<td>10</td>
<td>5</td>
</tr>
<tr>
<td>TS1a</td>
<td>50</td>
<td>90</td>
<td>0</td>
<td>10</td>
<td>2.5</td>
</tr>
<tr>
<td>TS1b</td>
<td>50</td>
<td>90</td>
<td>0</td>
<td>20</td>
<td>5</td>
</tr>
<tr>
<td>TS2</td>
<td>50</td>
<td>90</td>
<td>100</td>
<td>10</td>
<td>5</td>
</tr>
<tr>
<td>TS2a</td>
<td>50</td>
<td>90</td>
<td>50</td>
<td>10</td>
<td>5</td>
</tr>
<tr>
<td>TS3</td>
<td>50</td>
<td>60</td>
<td>0</td>
<td>10</td>
<td>5</td>
</tr>
<tr>
<td>TS3a</td>
<td>50</td>
<td>45</td>
<td>0</td>
<td>10</td>
<td>5</td>
</tr>
<tr>
<td>TS4</td>
<td>25</td>
<td>90</td>
<td>0</td>
<td>10</td>
<td>5</td>
</tr>
</tbody>
</table>

Table 4.1 Summary of tests for single breakwater

<table>
<thead>
<tr>
<th>Code</th>
<th>Wave Length (m)</th>
<th>Gap width / L</th>
<th>Incident wave Direction(degree)</th>
<th>Reflection Coefficient(%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>TG1</td>
<td>50</td>
<td>2</td>
<td>90</td>
<td>100</td>
</tr>
<tr>
<td>TG2</td>
<td>50</td>
<td>2</td>
<td>90</td>
<td>0</td>
</tr>
<tr>
<td>TG3</td>
<td>50</td>
<td>2</td>
<td>90</td>
<td>100</td>
</tr>
<tr>
<td>TG4</td>
<td>50</td>
<td>1</td>
<td>90</td>
<td>100</td>
</tr>
<tr>
<td>TG5</td>
<td>50</td>
<td>.5</td>
<td>90</td>
<td>100</td>
</tr>
</tbody>
</table>

Table 4.2 Summary of tests for the case of a gap between two breakwater
<table>
<thead>
<tr>
<th>Code</th>
<th>Wave Direction</th>
<th>Ref. Coe. BW In front</th>
<th>Ref. Coe. BW Backside</th>
</tr>
</thead>
<tbody>
<tr>
<td>TN1</td>
<td>90</td>
<td>100</td>
<td>0</td>
</tr>
<tr>
<td>TN2</td>
<td>90</td>
<td>0</td>
<td>100</td>
</tr>
<tr>
<td>TN3</td>
<td>45</td>
<td>100</td>
<td>100</td>
</tr>
<tr>
<td>TN4</td>
<td>135</td>
<td>100</td>
<td>100</td>
</tr>
</tbody>
</table>

Table 4.3

<table>
<thead>
<tr>
<th>code</th>
<th>slope BC</th>
<th>Hi</th>
<th>T</th>
</tr>
</thead>
<tbody>
<tr>
<td>T1</td>
<td>1:5</td>
<td>0.04 m</td>
<td>1.4 s</td>
</tr>
<tr>
<td>T2</td>
<td>1:1½</td>
<td>0.04 m</td>
<td>1.4 s</td>
</tr>
<tr>
<td>T3</td>
<td>vert.</td>
<td>0.04 m</td>
<td>1.4 s</td>
</tr>
<tr>
<td>T4</td>
<td>1:5</td>
<td>0.08 m</td>
<td>1.4 s</td>
</tr>
<tr>
<td>T5</td>
<td>1:1½</td>
<td>0.06 m</td>
<td>1.4 s</td>
</tr>
<tr>
<td>T6</td>
<td>1:5</td>
<td>0.04 m</td>
<td>0.7 s</td>
</tr>
<tr>
<td>T6</td>
<td>1:1½</td>
<td>0.04 m</td>
<td>0.7 s</td>
</tr>
</tbody>
</table>

Table 4.4 measurement cases

<table>
<thead>
<tr>
<th>code</th>
<th>$R_{bc}$</th>
<th>T</th>
</tr>
</thead>
<tbody>
<tr>
<td>TC1</td>
<td>0.05</td>
<td>1.4 s</td>
</tr>
<tr>
<td>TC2</td>
<td>0.50</td>
<td>1.4 s</td>
</tr>
<tr>
<td>TC3</td>
<td>1.00</td>
<td>1.4 s</td>
</tr>
<tr>
<td>TC4</td>
<td>0.00</td>
<td>0.7 s</td>
</tr>
</tbody>
</table>

Table 4.5 computed cases for scale model harbour
5 Results and Comparison

5.1 Analytical tests

In the previous chapters, it was mentioned that the results of the first version of the model were not adequate for most cases. During this study some modifications were made to the model. The model results after these modifications are shown in this chapter. In the figures which are provided for the comparisons, the results of the first version of the model are shown by legend "model version 1" or "B1". The model results after the modification due to diffraction calculation (last modification in this study) are shown by legend "B3.1" or "model version 3.1". The obtained results for two stages of model modification are also shown by the legend "model version 2" or B2 and "model version 3" or B3.

5.1.1 Single breakwater

Fig. 5.1 shows the computed wave diffraction coefficient compared with the Cornu spiral results in the test case TS1. The effects of modification to the model are clearly visible in the relative wave height trend along the lines A, B and C (see Fig. 4.1).

The wave height is changed in the test TS1a. The obtained diffraction coefficients are the same as for test TS1. This confirms the linear nature of the solution.

Relative wave heights ($K_d$) are computed in a number of points to draw isolines of $K_d$ using a standard package. The diffraction pattern in the test case TS2, normal incidence and full reflection, is shown in Fig. 5.2. A comparison between Cornu spiral and model outcome is shown for this test case in Fig 5.3 along the lines A, B, and C. As this figure indicates, the result of the modified model are very close to the analytical solutions on both sides of the breakwater. It can be observed that along the line C, in front of the breakwater, a discontinuity had occurred before the modification of the model which has been overcome by the model improvement.

In Fig. 5.4, the computed wave heights have been compared with the Cornu
spiral solution for partial reflection (Test TS2A) along the Lines A, B, and C. As can be clearly observed, the results are in good agreement with the graphical Cornu spiral solution.

The results of test cases TS3 and TS3a (60 and 45 degrees incident wave direction) have been compared with the analytical solution in Fig. 5.5 and Fig. 5.6 respectively. These figures indicate that the results of the modified model are also in agreement with the analytical solution in the case of oblique incidence.

Fig 5.7 shows the model results compared with the analytical solution for the test TS4. In this test case the wave length has been changed. A comparison was been made between the results of test case TS2 and TS4 to observe the effect of wave length in the computation procedure. For the purpose, the diffraction coefficients along a line $y/L=2$ with two wave lengths were computed. A distance interval $\Delta x/L=.4$ was selected for both cases. The computed results are exactly the same for both wave length and are presented in Fig. 5.8. This obtained results indicate that there is no error in the programme with due to variation of wave length.

5.1.2 Breakwater gap

Fig 5.9 shows the computed wave heights for test case TG1, $B/L=2$, along the lines A and B compared with the Cornu spiral approximate solution (see Fig. 4.2). In this figure, model improvement steps during this study are shown. As can be observed, the result was not in agreement with the analytical solution before the modification. Specially in the exposed zone, the result was completely inaccurate. After modification of the model, the result is very close to the theoretical solution in this zone.

In Fig. 5.10 comparisons have been made between the outcome of the model and the graph given in CERC (1984) based on Penney and Price solution for the test cases TG1 ($B/L=2$). The comparison indicates that the model result is in agreement with the solution of Penney & Price except that in the shadow region the BELUGA gives higher results.

Fig 5.11 shows a comparison between model outcome and the result of the Cornu spiral solution for test case TG2 along the lines A, B, and C. In this test case, a fully absorbing breakwater has been adopted (reflection
coefficient is zero). A very good agreement of the model result with the analytical approach can be observed in the figure.

In Fig 5.12 a comparison has been made between test case TG1, full reflection, and TG2, full absorption, along line A \((y/L=6)\) and line B \((y/L=2)\). The figure indicates that the contribution of the reflected wave is negligible along lines A and B except far in the shadow zone. One can conclude that the Cornu spiral solution can be used for breakwater gap problems without taking reflection into account (except for the points very near to the breakwater).

In Fig 5.13 a comparison has been made between model results and the Cornu spiral solution for oblique incidence (Test TG3). The figure shows that the model outcome is reasonably close to the Cornu spiral solution in the shadow region. It is also in agreement with the analytical solution in the exposed zone.

Figures 5.14 and 5.15 show comparisons between model results and the solution of Carr & Stelzriede (1952) for the test cases TG4 and TG5 \((B/L=1\) and \(B/L=0.5)\). In Fig 5.16, comparison between model results and analytical solutions can be observed for the test case TG4, \(B/L=1\), along the lines A and B. The result of the model in this case is almost in agreement with the result of the Cornu spiral solution (Penney and Price method). As discussed in chapter 2 the Penney & Price method is not valid for \(B/L<1\). According to the result which is shown in Fig. 5.16, it can be concluded that the calculations are performed accurately based on Sommerfeld approximate solution. According to the Carr & Stelzriede solution, the model results are underpredict in the exposed zone while they are overpredict in the shadow zone.

5.2.3 Circular pile

The circular pile has been schematized to 12 and 24 segments. The outcomes of the model for a single circular pile have been compared with the computed and measured wave height given by Van Oortmerssen (1972) in Fig 5.18 and 5.19 for the lines A and B in front of and behind the pile respectively.

The results of the model along line A, in front of the breakwater, do not
have acceptable correlation with the results of the boundary integral method and of the measurements. In this case the results for 12 segments and those for 24 segments are very close together.

The results for line B, at the lee side of the pile, are completely different. In this case the computed amplitudes are zero for the points near to the pile and they differ widely for different number of segments. The reason for this inappropriate result is that in the computation procedure only diffraction around the corner points that receive wave energy from the wave incident contour are taken into account. It means that the corner points which do not receive any incident or reflected waves are not considered as a diffraction point. e.g. in Fig. 4.5 the corner points C8 up to C12 are not taken into account as a diffraction point. The points near the pile (such as point A and B in the figure) can only receive energy from the corner points behind the pile (C8 to C12) and these corner points have no energy; therefore, zero amplitudes are obtained for such points close to the pile. This is also explain the sensitivity of the result to the number of segments.

5.2 Laboratory tests of wave penetration into a harbour

The result of laboratory measurements in a harbour on a (0.5m*0.5 m) grid are given in Figs. 5.20 to 5.23. These figures correspond to test cases that have been selected for verification of the model (TC1-TC4). At these grid points the wave heights inside the harbour have been computed by BELUGA. Using these computed values, line of equal wave heights have been drawn manually. The computed wave height patterns inside the harbour are shown in Figs. 5.24 to 5.27 for the computed test cases TC1, TC2, TC3, and TC4 respectively. Also the computed values of wave height for different points in the harbour in each test case are presented in Figs. 5.28 to 5.31.

Using Fig. 5.24 and 5.28, computed results of the model TC1 can be compared with the laboratory measurements T1. In this test there was very little reflection against the back wall; for computation TC1 a 5 percent reflection was taken into account. Comparison indicates that computation TC1 corresponds fairly well with measurements, except that in the exposed area the results computed by BELUGA are too low.

In Figs 5.32 to 5.35 comparisons have been made between model results,
laboratory measurements, and the results computed by the GOLDHA model (Berkhoff, 1981) along the lines A, B and C. By comparing all the results it appears that the model gives fairly appropriate result for this complicated harbour.

The wave height patterns show clearly the effect of the reflection coefficient. It is obvious that, with higher reflection, the wave heights within the shadow zone increase.

In Fig 5.32 the model results (TC1), the measurements (T1) and the results computed by the GOLDHA model (B1) have been plotted along the lines A, B, and C (see Fig 4.4). The results of the model are in fair agreement with the measurements. Only along the exposed part of the line A, the computed wave height is relatively low and the peak wave height is not predicted well.

Computation TC2, for which 50 percent back wall reflection was taken into account, shows the same correspondence when compared with the measurement results of test T2 (see Fig. 5.33). Comparison of the results along the lines A, B, and C shows that the results are qualitatively in agreement with the measurements. Only in line B the computed result is slightly too low in the shadow zone.

In test T3 there is full back wall reflection. Consequently, a reflection coefficient of one was adopted in computation TC3. The computation results TC3 are plotted in Fig 5.26. The comparison between computation and measurement along the lines A, B, and C is shown in Fig. 5.34. As the figure indicates, the result is qualitatively the same as in the test case TC2. Along the exposed part of line A the computed wave height is low. Also the computed result is relatively low in the shadow area.

The hydraulic model test T6 corresponds with test TC4. In this test case the wave period is less, which causes shorter waves. In principle, the reflection from the slopes will be less for shorter waves. The reflection coefficient of the back wall is zero in this test. Comparison of TC4 and T6 along the lines A, B and C (see Fig. 5.35) shows that the results correspond accordingly with the measurement, except that in line C at the point near to reflection back wall, the computed wave heights are too high.
6 Discussion, conclusions and recommendations

6.1 Conclusion

The comparison of the analytical and computation results, as outlined in the present report, gives rise to some conclusions. The comparisons indicate that for the cases of a simple breakwater configuration such as single breakwater and a single breakwater gap, the model functions very accurately. The result of the model is in very good agreement with the analytical solutions for these cases.

The correlation between the numerical and the analytical results in the case of a single breakwater is very good. The result of the modified model is very close to the analytical solution both in front of and behind the breakwater.

In the case of a breakwater gap problem, the comparisons indicate that the model outcomes are in very good agreement with the analytical solution, if the gap width is equal to two wavelength.

In the numerical model the Sommerfeld solution has been used and it is assumed that the gap width is relatively large as compared to the wave length. The comparisons between the model result and the solution of Carr and Stelzriede (1952) indicate that the numerical results is overpredict the amplitudes for the case of a relative narrow breakwater gap.

The model can be applied for the problem of wave diffraction around any obstacle by schematizing the obstacle into a number of segments. Observing the result for the test case of a circular pile, it is concluded that the model results are in agreement with the exact analytical solution and with the laboratory measurements in the area in front of the pile, as far as the location of maximum and minimum amplitudes are concerned; however, the model overpredicts the amplitude variations significantly.

The results are fully incorrect for the back side of pile. The reason for this is that in the model, diffraction around the corner points at the lee-side is not taken into account. This requires correction of the model.

From the comparison of the measurement and computations for the scale
model harbour, it can be concluded that the computed results are encouraging and correspond qualitatively well to the measurements for the case of a complicated harbour. The comparison demonstrates the effect of various magnitudes of reflection coefficient for the harbour back wall.

The results computed by the model are not quantitatively close to the laboratory measurements. In this respect correspondence some reasons could be mentioned such as that the assumptions that are made for the reflection coefficients in the numerical calculation may not reproduce exactly the reflection occurring in the hydraulic model. The location where the reflection point is to be assumed is a point that should be considered as well. Also in the computation procedure it is assumed that the phase shift is zero.

An important problem is that the mathematical computation for all test cases is not correct for the corner with wedge angle of 90 degree. The results of a test case for a single corner with 90 degree wedge angle showed that zero wave amplitudes are obtained for the shadow region and the wave remained undisturbed in the exposed area. It means that diffraction around such a corner is not taken into account.

Another problem is that the mirror solutions, which have been used for the effect of a wedge angle, are not solved properly in the programme. Also here the assumptions which have been made in the model should be considered:

In the model it is assumed that the diffraction only takes place at corners in the harbour contours. Also it is assumed that wave direction is in a straight line from the corner point at a distance more than a few wave length from the corner point. This assumption restricts the model for wave penetration into a relatively large harbour compared with wave length.

The Sommerfeld solution is used for diffraction computation in the model. This solution is mathematically exact for a thin, straight, semi-infinite, and fully reflecting breakwater. For breakwaters with low reflection, and if guidewalls are used, a modified solution is commonly applied reducing the second term of the solution proportional to the degree of reflection. In the model the modified solution is used in terms of reducing the second expression of the Sommerfeld solution. This modified solution has been applied for the case of corners in the harbour contour. In the hydraulic model test guidewalls exist; this can be the reason for obtaining incorrect
result in the points near to the corner points. However, the obtained results are encouraging and they confirm that the relevant phenomena are basically correctly represented. But the geometric problems have not been solved completely and the model needs improvements in this respect.

The BELUGA model is computationally suited to determining short wave propagation into relatively large harbours. This model does not include diffraction effects in a general manner. However, in BELUGA only the effects of diffraction and reflection in constant water depth have been taken into account. This is a limitation to the programme, but where the harbour depth is constant the model can be used for estimation of wave propagation after some modifications and improvements.

6.2 Recommendations

The correct basis and the efficiency of the model make it suitable for future improvements and extensions. The model should be improved in two steps. First, it should be improved in terms of numerical solution in order to have accurate results under the predefined assumptions. Secondly, it will be very useful if it is improved to incorporate even more complicated wave phenomena, such as refraction, shoaling, interaction between current and waves, partially transmitting structures, etc.

6.2.1 Suggestions for model improvement

The model can be improved maintaining the same main assumptions. In this step the following recommendations are given:

a) For diffraction calculation, it is very important to know that the target point is either in the shadow zone or in the exposed zone because in each target point, the sign of diffraction components depends on the location (either shadow or exposed zone) of the point. In the model for all cases the distinction between shadow and exposed zones has not been completely and correctly incorporated yet. According to the incident wave direction and directions of adjacent lines, various cases can occur. All the
possible cases should be considered in the model.

b) Mirror solutions have been used in the model for the effect of wedge angle of corner points and angle of wave attack. This problem has not been solved properly and it is required to improve the model in this respect. A reason for the deficiency in the mirror solution is due to incorrect distinction between shadow zone and exposed one. After modification of the model in this respect, the mirror solutions should be checked in the model. In addition to the mirror solutions, one possibility would be to introduce a set of weighting factors for considering the effect of wedge angle in the model. Regarding the exact solution for wedges, it is possible to determine a special wedge factor for the second term of the Sommerfeld solution (see Daemrich and Kohlhase, 1978). This weighting factor has to be determined with regard to the wave direction and the reflection coefficient.

c) In the programme all the corner points are considered as a diffraction point. This is very time consuming for the computer, and in some cases it also produces incorrect result in diffraction computations. Diffraction around the corner points depends on the wedge angle of the corner and the angle of wave attack. With respect to the harbour layout, only a few corner points are important in the diffraction calculation procedure. The impact of the remaining corner points is not considerable; therefore, it is not necessary to add diffraction at such points. It is possible to distinguish between the corner points which are important for diffraction computation and those which are less important regrading to the corner wedge and wave incident angles. For instance, in Fig. 3.1 (page 27) only the corners C1, C4, C7 and C10 are important in the diffraction computation and diffraction around remaining points can be neglected because only reflected waves are diffracted around these points. Comparison between model results for a wedge angle less than 90 degree and more than 180 degree indicates that for the second case, diffraction component are negligible if compared with the reflection component.

d) In the model, only diffraction around the corner points that receive wave energy from the incident contour or from reflecting contours are taken
into account. It means that the corner points which receive incident or reflected waves are considered as a diffraction point. Therefore, diffraction around other corner points is not considered. Diffraction components around such corner points are negligible for most cases, but in some cases, specially when a large obstacle or an island exists, the effect will be important. This is a restriction in the present version of the model which should be improved in the future.

e) The convergence of the iteration process is another problem that should be considered carefully. The rate of convergence of the process depends on the average reflection coefficient. For relatively strong reflection a large number of iterations is needed.

With the present version of the model, it is not possible to reach a high desirable accuracy. It has been experienced that the accuracy can be upgraded nearly to the level of the desired one. When iteration continues a divergency is observed in the results (instead of convergency). The following suggestions are given in order to improve the model in this respect.

- A check test should be incorporated in the model, in order to be aware that a shift has been taken place from convergence to divergence in the process, and the last result of the convergence should be considered as the final result.
- It should be possible to continue the test with convergence by redistribution of the wave field in the harbour contours. In such redistribution process, the difference between the wave height (in a point where a desired accuracy has not been reached) of two successive iteration should be redistributed among other boundary points.

6.2.2 Suggestions for model extension

The model is restricted to constant depth, i.e. only the diffraction and reflection phenomena are considered. The efficiency of the model makes it suitable to improve it for more complicated conditions. First it can be extended to consider wave refraction and shoaling, i.e. non-uniform depth. In the model the Helmholtz equation has been solved numerically using boundary integral and ray method. For considering wave propagation in
variable water depth, the problem can be solved by taking into account curved rays. In this method the incident and/or reflected wave rays under the influence of variable water depth (refraction and shoaling) are extended until they meet the reflecting contours. Then the wave rays follow the reflection laws. It should be mentioned that the diffraction rays are considered separately.

The effect of the variation of the wave amplitude should also be taken into account, specially for cases of ray crossings (caustic). But for practical problems ray methods cannot be used to model the variation of the wave amplitude. The reason is that the location of the points in which the effect of variation wave amplitude is important, cannot be predicted in advance. Error arising from neglecting the variation of wave amplitude can be reduced by interpolation a procedure presented by Battjes and Bouws (1981) which is related to an averaging process in space. Also it can be reduced by averaging the effect of rays as given by Southgate (1984).

The extension of the model to consider some other physical effects which can be linearized such as interaction between current and waves, partially transmitting structures, bottom friction, etc is a matter that can be incorporated in the model in the future.
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Fig. 4.1 Single breakwater

Fig. 4.2 Breakwater gap configuration
GEOMETRY OF WAVE PENETRATION MODEL
IN THE WAVE BASIN
DELFT HYDRAULICS LABORATORY

SCALE 1:100

Fig. 4.3
MEASURING POINT LOCATIONS IN MODEL

SCALE 1:50

DELFt HYDRAULICS LABORATORY

W 154 VI Fig. 1.1
$K_a = 4$
Cylinder diameter = 96 meters

Fig. 4.5 Circular pile
Fig. 5.1 Comparison of analytical and computation results along the lines A, B and C for test TS1
Fig. 5.2 Calculated Kd-values for a semi-infinite breakwater (TS2).
Fig. 5.3 Comparison of analytical and computation results along the lines A, B and C for test TS2

Cs. = Cornu spiral solution
B1 = model version 1
B3 = model version 3
B3.1 = model version 3.1
Fig. 5.4 Comparison of analytical and computation results along the lines A, B and C for test TS2A.
Fig. 5.5 Comparison of analytical and numerical results along lines A, B and C for test TS3
Fig. 5.6 Comparison of analytical solutions and model results for test TS3a.
Fig. 5.7 Comparison of analytical and numerical results along lines A and B for test TS4
Fig 5.8 Comparison of tests TS1 and TS4

![Graph showing comparison between TS1 and TS4 with x/L and y/L=2 axes. The graph depicts fluctuations in Kd values for each test.]

- TS1
- TS4
Fig. 5.9 Comparison between analytical solution and model results for test TG1.
Fig. 5.10 Comparison of wave height pattern; Penney & Price solution ——— and model computation ———— for breakwater gap, B/L = 2.
Fig. 5.11 Comparison between analytical solution and model results for test TG2.
Fig. 5.12 Comparison Between test TG1 and TG2
Fig. 5.13 Comparison between analytical solution and model results for test TG3.

C.S. = Cornu spiral solution  B1 = BELUGA version 1  B3.1 = BELUGA version 3.1
Fig. 5.14 Comparison of wave height pattern; Carr & Stelzriede solution and model computation for breakwater gap, $B/L=1$.

Fig. 5.15 Comparison of wave height pattern; Carr & Stelzriede solution and model result for breakwater gap, $B/L=0.5$. 

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Fig. 5.16 Comparison between analytical solutions and model results for test TG5, B/L = 1
C&S = Cornu spiral solution.
B1 = BELUGA version 1.
B3.1 = BELUGA version 3.1. C&S = Carr & Steilrieder solution
Test TG5 (Line A)

Diffraction Coefficient Kd vs x/L for B/L = 0.5, y/L = 6

Test TG5 (Line B)

Diffraction Coefficient Kd vs x/L for B/L = 0.5, y/L = 2

C.S. = Cornu spiral solution,  B1 = BELUGA version 1,  B3.1 = BELUGA version 3.1,  C&S = Carr & Stelzriede solution

Fig. 5.17 Comparison between analytical solutions and model results for test TG5, B/L = 0.5
Figure 5.18 The wave height in front of a circular cylinder.

"Calculated" refers to boundary integral method.

Figure 5.19 The wave height behind a circular cylinder.
MEASURED WAVE HEIGHT PATTERN

DELFT HYDRAULICS LABORATORY
Fig. 5.24 Computed Wave Height Pattern
Fig. 5.25 Computed Wave Height Pattern
Fig. 5.26 Computed Wave Height Pattern
Fig. 5.27 Computed Wave Height Pattern
Fig. 5.28 Computed Wave Height Value Inside the Harbour
Fig. 5.29 Computed Wave Height Value Inside the Harbour.
Fig. 5.30 Computed Wave Height Value Inside the Harbour
Fig. 5.31 Computed Wave Height Value Inside the Harbour
Fig. 5.32 Computation of measurement and computation results along the lines A, B and C.

T1 = measurement results
B1 = numerical results, GOLDHA model
TC2 = numerical results, BELUGA model
Fig. 5.33 Computation of measurement and computation results along the lines A, B and C.

T2 = measurement results
B2 = numerical results, GOLDHA model
TC2 = numerical results, BELUGA model
T3 = measurement results
B3 = numerical results, GOLDHA model
TC3 = numerical results, BELUGA model

Fig. 5.34 Computation of measurement and computation results along the lines A, B and C.
T6 = measurement results
B6 = numerical results, GOLDHA model
TC4 = numerical results, BELUGA model

Fig. 5.35 Computation of measurement and computation results along the lines A, B and C.