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Drift reduction on sailing boats

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Abstract

The research explained in this paper was carried out to investigate the efficiency of different steering systems on sailing yachts. The steering system of a sail yacht mostly includes a simple steering system and a hydrodynamic shaped single rudder or multiple rudders, depending on boat characteristics. One of the basic design guidelines for fast sailing yachts is to reduce wetted surface to minimum allowed by the dynamic stability and maintaining the sailing performances. Deficiencies of different steering systems are discussed and their influences on total drag and yacht manoeuvrability in different sailing directions is analysed. The discussion is focused on steering systems applicable in practice and accepted by the yacht-building industry, although several innovations could be found that remained on their development stage because of their complexity in construction, maintenance, use itself and reliability.

All measurements have been conducted at sea applying on board sensors for position and accelerations acquirements. The purpose of the research was to demonstrate that the use of the bended rudder can reduce the leeway angle, the upwind sailing angle and increase the velocity made good to windward.

Introduction

The primary objective of the research is to provide design information as to the effect of sailing yacht hull and appendage characteristics and their interactions on the resistance and lift of the yacht. There is a wide field in yacht research and applications with and without the influence of the free surface where the drag and lift on the appendages have since long been an area of extensive research.

In their earlier publications "Course keeping qualities and motions in waves of a sailing yacht" (Gerritsma [1]) and later "Balance of helm of sailing yachts, a ship hydromechanics approach to the problem" (Nomoto and Tatano [2]) the authors have presented assessment methods for determining the force distribution and position of CLR (Center of Lateral Resistance) in yaw and sway over the hull and appendages in calm water and in waves. In this method use was made of a so called: Extended Keel Method (EKM) as introduced by Gerritsma in 1971, Ref [1] for calculating the side force on the keel and rudder (and hull) of a sailing yacht. EKM gave good results for the total side force of the hull, keel and rudder together in the upright condition, indicating that the major part of the side force is produced by the appendages, in particular for boats with average to high aspect ratio keels and rudders. The analyses of the vaw moment provided the results that the hull of the canoe body has a significant contribution that is not accounted for with the EKM. A modified approach to the correction method as introduced by Nomoto, Ref [2] yields good results for the yaw moment as well. With the development of Delft Systematic Yacht Hull Series (DSYHS) a large number of towing tank tests have been conducted on sailing yachts hulls and the results were used to develop improved methods for the calculation of lift and drag. Results were presented by Keuning in several publications for bare hull resistance [3, 4] and appendages [5] with particular attention paid to the interaction between hull and appendages and appendages themselves, as well as keel-rudder interactions. Rodriguez [6] has conducted his research on a series of experiments in the ETSIN towing tank focusing more on the influence of rudder, evaluating the distribution of forces in different conditions of navigation, as well as for the interactions between hull and appendages. In recent publications by Keuning he has been focusing on determining the force distribution in yaw and sway over the hull, keel and rudder. In [7] his method was used to deal with the yaw balance of a sailing yacht. In later publications a similar approach was used to determine forces and moments during the manoeuvring of sailing yachts [8].

The step forward

Changing the underwater geometry of a sailing yacht could change the stability and the hydrodynamic characteristics of the boat. A useful and competitive system is the canting keel that provided several advantages in sailing but also disadvantages. One of the main disadvantages is the need of additional foils or front rudder to reduce sideslip dagger boards. If they are extractible, has no influence on additional drag in downwind sailing, but if fixed it does. The second system is a twin foil manoeuvring system that is usually combined with a canting keel. In this system, the "working" rudder prevents sideslip but the upwind rudder produces additional resistance. In any case the best manoeuvring efficiency of the rudder is in its upright position when the maximum lift is produced. The idea is the introduction of the bending rudder, implying a system that maintains the rudder in an upright position independently of the heel of the boat.



Fig. 1. Device and system of bending rudder for sailing yachts

In the following, the effect of the bending rudder on force distribution in upright and heeled condition is analysed. Further the downwash effects are observed where the hypothesis is that its effects are reduced.

Effect of heel on sailing balance

The lift that a foil generates is perpendicular to its surface – if our boat is upright any lift generated

by the keel or rudder acts horizontally. When we're sailing, it's unusual for the boat to be absolutely upright, as the boat heels the lift forces from the foils move away from the horizontal. We're interested in generating a horizontal force from the keel and rudder, to examine how these changes with heel angle we use the fact that a force at an angle can be represented as the combined effect of a horizontal force and a vertical force.

The lift from the rudder is used to turn the boat, and also to stop the boat from turning. This second point is important to remember when we're sailing to windward with some weather helm. The person on the helm will be steering to leeward to keep the boat running straight. The more weather helm the boat has the more force is required from the rudder to keep it on track, and the force from the rudder depends on the boat's speed and the angle of the tiller.

As the boat heels over the horizontal component of the rudder's lift is reduced. If the weather helm and boat speed are constant then we need to increase the rudder angle to generate more lift so that the horizontal component stays the same. At 25 degrees of heel the rudder has to generate about 10% more lift than it did when vertical to produce the same tuning force; if we push the boat to 40 degrees we're asking the rudder for 30% more lift.

The approach

As stated by Lin [9] the earlier used manoeuvring prediction methods are based almost entirely on empirical equations [10]. Such methods yield satisfying results for boats that are geometrically similar to models tested in towing tanks and the measurement results of which were used to derive coefficients of empirical equations. For new and unconventional ships and boats these empirical data are usually not available. The use of computers and advanced computational flow prediction numerical methods to predict the ship motion and steering capabilities allows the analysis of different hulls with different appendages configurations. El Moctar [11] has used viscous flow methods to predict the rudder flow, and Gaggero et al. [12], like several other authors have used the panel method in a potential flow to compute forces in 2D and 3D profiles. Although several improvements have been introduced in a panel method, the potential flow methods did not take into account the viscosity, turbulence, and flow separation. On the other side viscous flow methods applied in time dependent calculations like ship movement and steering still remain technically difficult and computationally expensive. Therefore, considering limitations of

Moro di Venezia	Lwl / Bwl	Bwl / Tc	Lwl / VOLc ^{1/3}	LCB%	LCF%	Cb	Ср	Cw	Cm	Aw / VOLc ^{1/3}
	6.15	4.7	7.433	-6.55	-8.73	0.41	0.54	0.65	0.631	7.417

Table 2. Geometry particulars	s for keel	is and ru	ader					
			Keel 1	Keel 3	Keel 5	Rudder	Keel Moro	Rudder Moro
Lateral Area	A_{lat}	[m ²]	0.086	0.086	0.086	0.066	0.0651	0.0188
Wetted Area	S	[m ²]	0.176	0.177	0.177	0.321	0.1432	0.0413
Aspect Ratio	AR	[-]	1.623	0.696	3.769	0.115	10.33	7.733
Span	b	[m]	0.374	0.245	0.57	0.321	0.620	0.290
Mean chord	c _{mean}	[m]	0.231	0.352	0.15125	0.115	0.12	0.075
Sweepback angle	Α	[°]	9.85	14.42	3	18	5	5
Volume	V_k	[m ³]	0.00155	0.0016	0.000853			
Thickness/chord ratio	t/c	[-]	0.1	0.066	0.1			

a)

Tabele 1. Hull form parameter of Moro model



Fig. 2. Line plan and longitudinal profile; a) Model 366, b) Moro di Venezia

potential flow methods, many practical flow problems are still solved by obtaining experimental data or computed by empirical methods, or by potential flow calculations [9].

The first presented analysis is based on the hull model which was used for extensive measurements at Delft Ship hydromechanics Laboratory of the Delft University of Technology in the late nineties. The DSYHS series model 3, named 366, is found to be well documented in several Keuning publications, like [6, 7] and [13], and is therefore used for the empirical model validation and the analysis of performance changes due to a bending rudder. The second is the model tested by the authors. The complete oversight of the hull shape parameters for the model tested is presented in table 2.

Three different keel geometries were used for this study, varying in aspect ratio and thickness / chord ratio, the fourth is the keel of the tested model. These keels are labelled as #1, #3 and #5. The final test was conducted for the measured keel of the tested model. The principal dimensions are presented in table 2. Furthermore one rudder, and the rudder of the tested boat, of which the principal dimensions are also presented in table 2, was used in the calculation.

The line plane and the longitudinal profile of the model measured are presented on figure 2. The model has a low beam / draft ratio and represents a typical racing sailboat from the late nineties. The sailboat Moro di Venezia was an America's Cup class boat from 1992 to 1995.

Side force computation methods for the hull and appendages

The side force production of the hull and appendages is the key element in the sail yacht dynamic motion, because it allows for upwind sailing. Different models and empirical equations have been developed over the years, mainly based on towing tank testing results. DSYHS hull series tests conducted by Keuning at Delft University have yielded new answers to open questions left by L.F. Whicker and late by J. Gerritsma and K. Nomoto on lift production in different conditions. Tests or in practice sailing conditions demonstrates that yaw and heel angle have an influence on hull lift. Depending on hull form and type of appendages the magnitude of lift and the resistance/lift ratio varies from ship to ship - often on sister ships, too. The total side force of the hull and appendages and the separate contributions of hull, keel and rudder, are assessed differently in the upright and the heeled conditions. In the upright condition the so called Extended Keel Method, as derived by Gerritsma [14], is used to calculate the side force on the keel and rudder. The side force generated by the hull is accounted for by the virtually extended keel inside the canoe body to the waterline. The downwash angle on the rudder is approximated as 50% of the leeway angle and the water velocity over the rudder reduced by 10% to account for the wake of the keel.

The total side force is calculated as the sum of the force on extended keel and rudder.

$$Y_{\rm tot} = Y_{\rm ek} + Y_r \tag{1}$$

$$Y_{\rm ek} = \frac{1}{2} \rho V_S^2 A_{\rm ek} \left(\left(\frac{\mathrm{d} C_L}{\mathrm{d} \alpha} \right)_{\rm ek} \beta \right)$$
(2)

$$Y_r = \frac{1}{2} \rho \left(0.9 V_s^2 \right) A_r \left(\left(\frac{\mathrm{d} C_L}{\mathrm{d} \alpha} \right)_r 0.4 \beta \right)$$
(3)

where:

- Y_{tot} total side force in the horizontal plane [N];
- $Y_{\rm ek}$ side force generated by the extended keel [N];
- Y_r side force generated by the rudder [N];
- A lateral area of the foil [m²];
- $(dC_L/d\alpha)$ lift curve slope of the foil [deg⁻¹].

The extended keel method is often applied by yacht designers to make first approximations about forces and size of appendages. However, this procedure does not work under heel. Therefore, in these conditions the results of the side force polynomial as derived from the results of the DSYHS by Keuning and Sonnenberg [7] are used. This polynomial accounts for effects of heel angle and forward speed on the total side force production.

$$Fh\cos(\varphi) = \left(b_1 \frac{T^2}{Sc} + b_2 \left(\frac{T^2}{Sc}\right)^2 + b_3 \frac{Tc}{T} + b_4 \frac{Tc}{T} \frac{T^2}{Sc}\right) \cdot \frac{1}{2} \rho V_S^2 Sc(\beta - \beta_{Fh:0})$$
(4)

where:

$$\beta_{Fh:0} = B_3 \varphi^2 Fn$$
 and $B_3 = 0.0092 \cdot \frac{Bwl}{Tc} \cdot \frac{Tc}{T}$;

 $Fh\cos(\phi)$ – side force in horizontal plane [N];

T – total draft of hull with keel [m];

Tc - draft of the canoe body [m];

Sc – wetted surface of the canoe body [m²];

 $\beta_{Fh:0}$ – zero lift drift angle [deg];

Fn – Froude Number.

The coefficients b_1 to b_4 used in the presented function are obtained from DSYHS tests for the heeling angle between 0 and 30 degrees of heel and presented by Keuning [7] in table 3.

Tabele 3. Coefficients for the lift force Polynom

φ	0°	10°	20°	30°
b_1	2.025	1.989	1.980	1.762
b_2	9.551	6.729	0.633	-4.957
b_3	0.631	0.494	0.194	-0.087
b_4	-6.575	-4.745	-0.792	2.766

The use of this expression, however, yields no information on the contribution of the three different components - i.e. hull, keel and rudder - and therefore no result for the yaw moment can be found. Verwert and Keuning [13] have developed a new formulation for keel and rudder lift calculation that takes into account the interaction effect of the hull on the keel and the rudder. To overcome this problem, the distribution over keel and rudder as found in the upright condition is used in the heeled condition. The Munk moment on the hull is calculated taking the geometry of the heeled hull into account. This procedure is also described in [4]. Keuning, Katgert and Vermeulen [7] improved the prediction of the side force production for higher aspect ratio keels and the yaw moment under heel by taking the newly derived formulation for the influence of the downwash of the keel on the rudder into the calculations.

This situation of using two different approaches was considered undesirable and inconsistent. So, in the framework of the present study a new method has been developed.

In this new method the side force generated by keel and rudder is calculated using the expression derived by Whicker and Fehlner (W&F) for thin airfoils [6]:

$$\frac{\mathrm{d}C_L}{\mathrm{d}\alpha} = \frac{a_0 \mathrm{AR}_e}{\cos\Lambda\sqrt{\frac{\mathrm{AR}_e^2}{\cos^4\Lambda} + 4 + \frac{57.3a_0}{\pi}}} \tag{5}$$

where:

 AR_e – effective aspect ratio [m];

- Λ the sweepback of quarter-chord line [rad];
- α angle of attack [deg];
- α_0 corrected section lift curve slope [–];

 $a_0 = 0.9(2\pi/57.3)$ per degree.

The aspect ratio is obtained from the expression:

$$AR_e = 2\frac{b}{c_{mean}}$$

where: b is the span of the foil and c_{mean} the mean geometric chord in meters.

In this calculation, the keel is not extended to the free surface, but taken as it is. The effect of the hull is therefore calculated separately.

Another effect is the lift carry over from the keel to the hull that is expressed over the ratio between the entire lift of the appended hull and the lift generated by the keel and rudder computed from equation (5). This ratio is represented as the hull influence coefficient c_{hull} . The formulation for the extended range of keels in upright conditions states:

$$c_{\rm hull} = 1.8 \frac{Tc}{bk} + 1 \tag{6}$$

where *bk* is the span of the keel.

The influence of the heel angle on the lift production is represented by the lift reduction of appendages and expressed by the heel influence coefficient c_{hell} . The second is the zero lift drift angle β_0 that originates from the asymmetry of the hull when heeled. This reduces the angle of attack on appendages and the effect increases with heel angle. As presented by Verwerft [13] a linear relation between lift reduction and heel angle is applied:

$$c_{\text{heel}} = 1 - b_0 \varphi \tag{7}$$

with $b_0 = 0.382$ and φ in radians.

The influence of hull asymmetry when heeled is represented by the zero lift drift angle obtained from the measurements:

$$\beta_0 = \left(c_0 \frac{B_{wl}}{Tc}\varphi\right)^2 \tag{8}$$

with $c_0 = 0.405$ and φ in radians.

The downwash angle of the keel on the rudder is calculated from the formulation of Keuning [15]:

$$\boldsymbol{\Phi} = a_0 \sqrt{\frac{C_{Lk}}{ARe_k}} \tag{9}$$



Fig. 3. Side force for hull model 366; a) $F_n = 0.3$, $\alpha = 4^\circ$, $f = 0^\circ$, $\delta_{ru} = 0$; b) $F_n = 0.3$, $\alpha = 4^\circ$, $f = 10^\circ$, $\delta_{ru} = 0$; c) $F_n = 0.3$, $\alpha = 4^\circ$, $f = 20^\circ$, $\delta_{ru} = 0$; d) $F_n = 0.3$, $\alpha = 4^\circ$, $f = 30^\circ$, $\delta_{ru} = 0$

where:

$$\phi$$
 _ downwash angle at the rudder [rad];

$$ARE_k$$
 – effective aspect ratio of the keel;

$$C_{Lk}$$
 – lift coefficient of the keel;

 $a_0 = 0.137$ for 15° heel angle.

The lift of the keel and the rudder is than calculated from:

$$Lc_{\text{keel}} = C_{L \text{ keel}_{WF}} \cdot \alpha_k \cdot \frac{\rho \cdot v_{e \text{ keel}}^2 \cdot A_{lat \text{ keel}}}{2} \cdot c_{\text{hull}} \cdot c_{\text{heel}}$$
(10)

where $v_{e \text{ keel}}$ is assumed to be equal to the velocity of the boat v_B .

$$Lc_r = C_{L_{r_{WF}}} \cdot \alpha_r \cdot \frac{\rho \cdot v_{er}^2 \cdot A_{latr}}{2} \cdot c_{hull} \cdot c_{hell} \quad (11)$$

where v_{er} is assumed to be $0.9v_B$.

The equilibrium obtained is in practice very tenuous and is controlled by the helmsman acting on rudder.

Simulations conducted on a hull model 366 and presented on figure 3 shows the side force distribution on appendages, keel and rudder. Increasing a heel angle influences the side force on appendages. The side force on keel and rudder, produced by the heel at 30° is balanced when the angle of

attack β is about 5° for the keel and 7.5° for the rudder (Fig. 4d). Another $\delta_{ru} = 2^{\circ}$ of rudder deviation is required to correct this additional side force on rudder. This increases the drag in slowdown the boat.

Applying on the same model the vertical rudder assumptions the equilibrium of side forces on appendages because of heel is reached at about 5° with 30° of heel.

The balance of underwater side forces controls the yaw and the drift of the boat. The reduction of the side forces on the appendages caused by heel angle is presented in figure 4 as calculated by the above method. Applying the bending rudder that is kept vertical independently of the heel angle neglects some parameters in the calculation of rudder side force. This is the downwash angle Φ that represents the influence of the keel leaving flow to the rudder and the coefficient c_{hell} of the rudder. The result is more side force on the rudder and the ability to reduce the leeway angle by pushing the boat upwind with a reduced drift.

Way to reduce leeway angle

Modern racing boats like open 60 s have wide, flat sterns. This style is beginning to appear in some



Fig. 4. Side forces assuming a vertical position of the rudder or hull model 366; a) $F_n = 0.3$, $\alpha = 4^\circ$, $f = 0^\circ$, $\delta_{ru} = 0$; b) $F_n = 0.3$, $\alpha = 4^\circ$, $f = 10^\circ$, $\delta_{ru} = 0$; c) $F_n = 0.3$, $\alpha = 4^\circ$, $f = 20^\circ$, $\delta_{ru} = 0$; d) $F_n = 0.3$, $\alpha = 4^\circ$, $f = 30^\circ$, $\delta_{ru} = 0$

cruiser-racer designs, particularly in smaller boats. With a single rudder this means that at large heeling angles some of the rudder is out of the water where it's not doing any good at all, so the force available from the rudder is reduced even more. Many boats of this design get around this problem by having twin rudders canted outwards a little. As the boat heels the windward rudder lifts out of the water but the leeward rudder is submerged perpendicular to the water. The disadvantage of two rudders is in having more wetted surface in all other sailing conditions than windward rudder is not completely out of water.

As opposed to twin rudders, the single bending rudder (Fig. 6) does not influence the original underwater geometry of the boat and can be maintained perpendicular to the water surface at any time.



Fig. 5. Leeway angle measured for booth rudder positions

Measurements conducted in water, as presented in the next paragraph, demonstrate the positive influence of the bended rudder in reducing the leeway angle (see Fig. 5).

Measurement

The analysis of boat sailing and steering improvement was conducted on sailboat prototype of 2 m LOA. The boat is a remote controlled, applying a standard rig of mainsail and jib. Main dimensions of the sailboat model are presented in figure 6. Measurements are conducted at sea without, of course, controlled conditions up to a certain point because boat movements in all three directions are measured with an accelerometer with 25 Hz and positions were measured with GPS at 4 Hz. At the same time the wind speed and direction was measured with 2D anemometer.

The main purpose of taking measurements is to find the difference between sail characteristics applying a classic rudder and a bending rudder.



Fig. 6. Sailboat model applying bending rudder device

The steering system mounted on a sailboat model allows the rudder to bend 35 degrees in each direction, without the influence of underwater geometry of the hull or on rudder profile. Figure 7 shows the path of the model test and the true wind directions and strengths. The point indicated by a single star is the point where the rudder position is changed from normal, perpendicular to the boat, to perpendicular to the water. At that point the wind has not changed its direction and strength.

When sailing, the course of the sailboat was kept as much as possible upwind related to the jib wool tickers mounted on the luff. The obvious conclusion is that the boat is going more upwind when the rudder is in the vertical position. However, this is still not an overall indication of more efficient sailing.



Fig. 7. Sailing path and true wind characteristics

Further review of wind conditions and boat speed are necessary. Figure 8 shows the true and apparent wind speed and the speed of the boat. The line at time 14:40:18 indicates when the rudder changes from the normal to the vertical position. At that time, the true wind is slightly slowing down, reducing the speed of the boat. There is a contradiction between figures 8 and 9 because at lower boat speed the drift increases, which did not happen in this case where the boat is sailing more upwind. Just before the wind has changed direction, around 14:41:10, the boat speed touched the velocity of the true wind speed, which is quite good for a small sailboat model.



Fig. 8. Apparent and true wind speed compared with sail boat speed

Next is the review of wind angle dynamics during measurements. Figure 9 shows the angles of the true and apparent wind that on average do not change particularly up to the 14:41:10, when the true wind angle is from about 250 degrees. The apparent wind angle is going to reduce at 14:40:18, because the wind is decreasing and consequently so is the speed of the boat.



True wind direction bendet rudder
Apparent winnd angle bendet rudder

Fig. 9. Wind direction and apparent wind angle for booth rudder positions

The benefit of the vertically positioned rudder is found by calculating the (V_{mg}) velocity made good for windward sailing. Applying the model described in [16] the V_{mg} is increasing after the change of the rudder in the vertical position. This is the result the authors were looking for to confirm the benefit of the bending rudder in windward sailing (Fig. 10).



Fig. 10. Apparent and true wind speed compared with velocity made good for windward sailing

The second test regarded the resistance of the bending system holding the rudder, its stiffness and water sealing. The survey after several measurements and several hours in the water in different weather and sea conditions shows that not one drop of water entered through the mechanism.

Conclusions

The bending rudder system and its holding mechanism was installed in a 2 meter sailboat model and tested at sea. The initial hypothesis was that the boat applying a bending rudder could achieve better sailing performances in windward sailing. During tests, the boat position, accelerations in three directions, wind speed and direction were measured. The analysis of the results demonstrates that the sailboat reduces the drift and sails at a lower upwind angle. The result is a better V_{mg} for windward sailing. The bending rudder system was also tested for mechanical resistance and water sealing. Results were positive; the mechanism maintained its stiffness and water tightness throughout the time of testing.

From the author's point of view the analyses conducted and presented in this paper gives enough information and proves that the system could be applicable to larger racing and cruising sail yachts.

References

- 1. GERRITSMA J.: Course keeping qualities and motions in waves of a sailing yacht. Technical Report, Delft University of Technology, May 1971.
- NOMOTO K., TATANO H.: Balance of helm of sailing yachts, a shiphydromechanics approach on the problem. HISWA. 1979.

- KEUNING J.A., ONNINK R., VERSLUIS A., VAN GULIK A.A.: The Bare Hull Resistance of the Delft Systematic Yacht Hull Series. 14th International HISWA Symposium on Yacht Design and Yacht Construction, Amsterdam 1996.
- KEUNING J.A., SONNENBERG U.B.: Approximation of the Calm Water Resistance on a Sailing Yacht Based on the Delft Systematic Yacht Hull Series. The 13th Chesapeake Sailing Yacht Symposium, Annapolis, Maryland, USA, 1997.
- 5. KEUNING J.A., BINKHORST B.J.: Appendage Resistance of a Sailing Yacht Hull. The 13th Chesapeake Sailing Yacht Symposium, Annapolis, Maryland, USA, 1997.
- KEUNING J.A., VERMEULEN K.J.: The yaw balance of sailing yachts upright and heeled. Chesapeake Sailing Yacht Symposium, 2003.
- 7. KEUNING J.A., SONNENBERG U.B.: Approximation of the Hydrodynamic forces on a sailing yacht based on the Delft Systematic Yacht Hull Series. HISWA Symposium on Yacht Design and Construction, 1998.
- KEUNING J.A., KATGERT M., VERMEULEN K.J.: Keel-Rudder Interaction on a Sailing Yacht. 19th International HISWA Symposium on Yacht Design and Yacht Construction, Amsterdam, The Netherlands, November 2006.
- LIN R.Q., HUGHES M.L., SMITH T.: Prediction of ship steering capabilities with a fully nonlinear ship motion model. Part 1: Maneuvering in calm water, Journal of Marine Science and Technology, 2010, vol. 15, 2, 131–142.
- WHICKER L.F., FEHLNER L.F.: Free-stream characteristics of a family of low-aspect-ratio, all-movable control surfaces for application to ship design. Technical report 933, David Taylor Model Basin, 1958.
- 11. El Moctar OM. Numerical determination of rudder forces. Euromech 374, Futurescope, Poitiers, 27–29 April 1998.
- GAGGERO S., BRIZZOLARA S.: A potential based panel method for the analysis of marine propellers in steady flow. 6th International conference on high-performance marine vehicles, Naples (Italy) 2008.

- 13. VERWERFT B., KEUNING J.A: The Modification and Application of a Time Dependent Performance Prediction Model on the Dynamic Behaviour of a Sailing Yacht. 20th International HISWA Symposium on Yacht Design and Yacht Construction, Amsterdam, The Netherlands, 2008.
- GERRITSMA J., KEUNING J.A.: Sailing yacht performance in calm water and waves. 12th International HISWA Symposium on Yacht Design and Yacht Construction, The Netherlands, 1992.
- 15. KEUNING J.A., KATGERT M., VERMEULEN K.J.: Further Analysis of the Forces on Keel and Rudder of a Sailing Yacht. The 18th Chesapeake Sailing Yacht Symposium, Annapolis, Maryland, USA, March 2007.
- 16. GARRET R.: The Symetry of sailing-the physics of sailing for yachtsman. Adlard Coles Ltd., London 1987.

Others

- RODRÍGUEZ R.Z., YERÓN J.I., VERA E.B.: Sailing Yacht Rudder Behaviour. 47º Congreso de Ingeniería Naval e Industria Marítima, España 2008.
- KEUNING J.A., KAPSENBERG G.K.: Wing Body Interaction on a Sailing Yacht. The 18th Chesapeake Sailing Yacht Symposium, Annapolis, Maryland, USA, 2005.
- 19. FOSSATI F.: Aero-hydrodinamics and the performance of sailing yachts. Politecnico di Milano, Milano 2007.
- BINNS J.R., KLAKA K., DOVELL A.: Hull-Appendage Interaction of a Sailing Yacht, Investigated with Wave Cut Techniques. The 13th Chesapeake Sailing Yacht Symposium, Annapolis, Maryland, USA, 1997.
- HOERNER S.F.: Fluid Dinamic Drag. Hoerner Fluid Dynamics, Bricktown, N.J., 1965–1992.
- 22. MESEGUER RUIZ J., SANZ ANDRÉS A.: Aerodinámica Básica. ETSI Aeronáuticos, Universidad Politécnica de Madrid, 2005.
- 23. VAN OOSSANEN P.: Predicting the Speed of Siling Yacht. SNAME transactions, 1993, vol. 101, 333–397.
- 24. http://www.oceansail.co.uk