MODEL EVALUATION AND DYNAMICS OF A VISCOELASTIC FLUID IN A COMPLEX FLOW
Model evaluation and dynamics of a viscoelastic fluid in a complex flow

PROEFSCHRIFT

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aan de Technische Universiteit Delft,
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## Contents

**Summary** vii  
**Samenvatting** xi  
**List of symbols** xv  

### 1 Introduction 1  
1.1 Viscoelastic flows around objects .......................... 2  
1.2 Objective ....................................... 3  
1.3 Approach & tools .................................. 3  
1.4 Outline ........................................ 6  

### 2 Fluid theory & models 7  
2.1 Basics of fluid mechanics ...................................... 7  
2.2 Dimensionless numbers .................................... 8  
2.3 Newtonian fluids ........................................ 8  
2.4 Material functions ....................................... 9  
2.4.1 Material functions in steady shear flow .................. 9  
2.4.2 Material functions in dynamic shear flow ............... 10  
2.4.3 Material functions in elongational flow .................. 10  
2.5 Generalised Newtonian models ............................... 11  
2.6 Continuum mechanics ...................................... 11  
2.6.1 Criteria for constitutive equations ....................... 13  
2.7 Elastic dumbbell models ................................... 16  
2.7.1 Outline of kinetic theory ................................ 16  
2.7.2 The Hookean dumbbell .................................. 19  
2.7.3 The non-Hookean dumbbell ............................... 21  
2.8 Fluid flow around objects: theory & results ................. 29  
2.8.1 Newtonian theory .................................... 29  
2.8.2 Non-Newtonian theory .................................. 31  

### 3 Experimental and numerical approach 39  
3.1 Experimental approach ...................................... 39  
3.1.1 Set-up .......................................... 39  
3.1.2 Measurement equipment .................................. 47  
3.2 Validation of the experiments ................................ 56  
3.2.1 Accuracy of the drag force measurements .................. 58  
3.2.2 Accuracy of the LDV measurements ........................ 64  
3.2.3 Calibration of the drag force device ....................... 64  
3.3 Numerical approach for 2D simulations ......................... 64
| 3.3.1 | Finite element formulation | 66 |
| 3.3.2 | Discretization and solvers | 66 |
| 3.3.3 | Boundary conditions | 66 |
| 3.3.4 | Finite element mesh | 66 |
| 3.3.5 | Convergence of the simulations | 66 |
| 3.4 | Numerical approach for 3D simulations | 68 |

### 4 Rheometry

| 4.1 | Choice of model fluid | 73 |
| 4.2 | Measurement of viscoelastic properties | 74 |
| 4.3 | Results | 75 |
| 4.3.1 | Dynamic shear experiments | 76 |
| 4.3.2 | Steady shear experiments | 76 |
| 4.3.3 | Stress growth and relaxation experiments | 79 |
| 4.3.4 | Elongational flow experiments | 79 |
| 4.4 | Characterisation of the fluid | 81 |
| 4.4.1 | Fit procedure of the dynamic data | 81 |
| 4.4.2 | Fit procedure of the steady shear data | 84 |
| 4.4.3 | Summary results fit parameters | 85 |

### 5 Results and analysis of experiments & computations

| 5.1 | Introduction | 89 |
| 5.2 | Results for the Newtonian fluid | 92 |
| 5.2.1 | Experimental results for the velocity | 92 |
| 5.2.2 | Experimental results for the drag on the cylinders | 96 |
| 5.2.3 | Comparison of experimental and numerical results for the velocity field | 99 |
| 5.2.4 | Comparison of experimental and numerical drag results | 105 |
| 5.2.5 | Discussion of differences between experiments and simulations | 106 |
| 5.3 | Results for the viscoelastic fluid | 107 |
| 5.3.1 | Viscoelastic fluid, 150 wppm: experiments | 109 |
| 5.3.2 | Comparison with Newtonian results | 117 |
| 5.3.3 | Comparison between experiments and simulations in 2D | 123 |
| 5.3.4 | First normal stress results from numerical computation | 128 |
| 5.3.5 | Numerical computation of the drag | 131 |
| 5.3.6 | Discussion of experiments and computations | 131 |

### 6 Summary conclusions

| 6.1 | Newtonian experiments and computations | 135 |
| 6.2 | Viscoelastic experiments and computations | 135 |

Bibliography | 137 |

Curriculum vitae | 144 |
Summary

Model evaluation and dynamics of a viscoelastic fluid in a complex flow

Jan M. Verhelst

In broad terms, rheology is the part of the science of fluid dynamics that focuses on the deformation behavior and flow of viscoelastic materials. These are materials that flow like a liquid but in combination with elastic properties or a flow dependent viscosity. An intuitive picture is that of a flowing rubber band, which has elastic properties, but does not exhibit complete recovery to the old shape after a deformation. For the description of the flow behavior of a certain material one uses balance equations of mass and momentum, which are generally valid for any type of material. In addition a so-called constitutive equation is needed, which describes the stress (the force per unit area) as a function of the deformation for that particular material history.

Most of the existing constitutive equations are based on experimental results obtained under very simple flow conditions, like simple shear or extensional flow. Only a few of them is also founded on rather simplified molecular models. As a result of this approach, the precise form of the constitutive equation depends not only on the material itself, but often it also depends on the type of flow used in those experiments. Therefore it is no surprise that most of these phenomenological constitutive descriptions fail as soon as they are used for flow conditions that differ from those for which they were originally devised.

Considering the above, there is a clear need for an extensive set of experimental data from complex flow experiments, e.g. flows with combined shear and extensional characteristics. Then, by means of these data, existing and future constitutive equations can be tested on these data. A result of these tests would be a clear understanding of the applicability of the theoretical expression under consideration to the experimental conditions of interest. On the other hand, it would become more clear under which circumstances the various constitutive equations are not of use anymore.

In this thesis both these needs are met. First, experimental results are described for the complex flow behavior of a series of different viscoelastic polymer fluids. The experimental conditions are chosen to be of practical interest. As model system a rectangular channel containing one or two cylinders was chosen. One reason for this choice is that the flow field in this system contains both shear and extensional properties. In the next step, the experimental results for the flow (=velocity) field are compared to the predictions of the most important constitutive theories for polymer solutions. Full 3D numerical simulations of complex flow in a system with submerged objects are not yet feasible. This is another reason for the choice of a channel geometry, as its dimensions could be chosen in such a way that a nominally 2D flow is obtained. This offered a reasonable way to compare the experimental results for the velocity
field with 2D numerical simulations.

Another important topic in rheology is the particle-particle interaction in viscoelastic fluids. This is a third reason for the presence of cylinders in the flow channel. To gain insight in this matter, in addition to the velocity field also the drag force on one of the two cylinders could be measured, under various flow conditions. Using different constitutive equations, also drag forces on the cylinders were calculated numerically for the 2D case, to enable comparison between the experiment and the existing theories.

As model fluid a Newtonian background fluid (a solution of 93% glucose syrup and 7% distilled water) was used, in which polyacrylamide was dissolved in different concentrations. By varying the concentration of the polymer, one obtains fluids with different viscoelastic properties. In this study three different fluids were used. A Newtonian fluid (the glucose syrup solution), an elastic fluid (glucose syrup solution with 150 wppm polyacrylamide) and an elastic fluid with shear thinning behavior (glucose syrup solution, but now with 400 wppm polyacrylamide).

The velocity field was measured using Laser Doppler Velocimetry. Drag force measurements could be performed on one cylinder submerged in the flow. To study the hydrodynamic interaction between particles, more than one cylinder were placed on the center line of the channel, and drag force measurements were performed on one.

The 2D model flow was simulated by numerical methods based on the discontinuous Galerkin method. Two different used constitutive equations were used for comparison with experiment: the FENE-P model and the Verhoeef model. The latter model has been developed by Verhoeef (Verhoeef et al. 1998) to apply in particular to extensional flow. In order to capture a wide spectrum of relaxation times of the fluid and to still allow for an objective comparison, an equal number of relaxation modes (three) was used for each model. As usual, the material parameters of the fluids, to be used in the numerical calculations, were obtained from measurements in pure shear flow. The shear viscosity and first normal stress difference were determined in steady shear flow and the complex parameters in dynamic shear flow measurements. Only the results of the Verhoeef model are presented as the results of the FENE-P model showed instabilities at the highest flow rate.

We found that velocity field measured in a Newtonian solution shows good agreement with the 2D calculations, although the experiments clearly show 3D flow effects near the cylinder. As a result the Newtonian force measurements show a qualitative representation of the drag behaviour compared to 2D simulations, the quantitative agreement is not good. Like in 2D simulations, and in accordance with theory, the measured drag force increases linearly with the velocity. However, the calculated drag force is about 20% higher than the measured drag force. Drag force calculations on the cylinder in a 3D geometry show that this difference can be well explained by the presence of the three dimensional flow patterns near the cylinder.

Next, the experimental data from the Newtonian fluid were compared with those from the viscoelastic fluids. The viscoelastic effects become more significant when the free stream velocity is increased. Eventually, at some critical velocity elastic instabilities occur. The experimental conditions were always chosen such that the flow patterns remained stable. The differences between the experimental flow patterns of the Newtonian and the viscoelastic fluid are most apparent in the wake of the cylinder, where the fluid is accelerated from zero to the final, undisturbed velocity. The 2D viscoelastic simulations qualitatively show the same effect. Again
the flow pattern of the Newtonian fluid in the wake of the cylinder differs from the flow pattern in the viscoelastic fluid. Quantitatively, however, the effect of fluid elasticity in the simulations is considerably smaller than in the experiments. More precisely the deviations in the flow pattern predicted by the simulations between the Newtonian fluid and the viscoelastic fluid are less significant as in the experiments. Finally we can note that, while in a Newtonian fluid the drag varies linearly with the flow rate, this is no longer the case in a viscoelastic fluid, where the drag increases with the square of the flow rate.

From experiments and calculations we conclude that the effects of viscoelastic behavior are most apparent in the wake of the cylinder. In this region large elongational flows occur and as a result all viscoelastic simulations fail quantitatively. The reason for this failure is mostly due to the fact that the material functions of the model fluids were determined in shear flow only. Obviously the predictions of the simulations would improve if the material functions are also determined for elongational flow. The viscoelastic effects become more significant when the free stream velocity is increased. Eventually, at some critical velocity elastic instabilities occur.
Samenvatting

Model evaluatie en dynamica van een viscoelastische vloeistof in een complexe stroming

Jan M. Verhelst

In algemene zin is reologie de wetenschap die zich op het gebied van vloeistof dynamica bezig houdt met de vervorming en de stroming van viscoelastische materialen. Dit zijn materialen die zich wel als een vloeistof gedragen maar in combinatie daarmee ook rekeigenschappen of een viscositeit die afhangt van de stroming. Een intuitieve voorstelling van zo’n vloeistof kan gemaakt worden door deze als een stromend elastiek voor te stellen die uitlekbaar is maar niet die niet volledig terugkeert naar zijn beginschatting na uitgerekt te zijn. Het stromingsgedrag van een materiaal wordt beschreven met behulp van voor elke vloeistof geldende evenwichtsvergelijkingen voor massa en momentum. Ook is er nog een zogenaamde constitutie-vergelijking nodig om de spanning als functie van de vervormingsgeschiedenis van een materiaal te beschrijven.

De constitutie-vergelijking beschrijft de spanning in het materiaal als functie van de deformatiesgeschiedenis. De meeste constitutie-vergelijkingen zijn gebaseerd op experimentele resultaten die gedaan zijn onder eenvoudige stromingscondities zoals een afschuifstroming of een rekstroming. Enkele constitutie-vergelijkingen zijn gebaseerd op betrekkelijk eenvoudige moluculaire modellen. Als gevolg hiervan hebben de meeste constitutie-vergelijkingen een vorm die niet alleen gebaseerd is op het materiaal zelf maar vooral ook afhangen van de stromingscondities waaronder de experimenten zijn gedaan. Het is dan ook geen verrassing dat de meeste fermenologische constitutie-vergelijkingen niet in staat zijn het gedrag juist te voorspellen als ze onder andere meer complexe stromingscondities gebruikt worden.

Het bovenstaande maakt duidelijk dat er een behoefte is aan een uitgebreide hoeveelheid meetgegevens in een complexe geometrie waarin zowel afschuiving als rek voorkomt. Met behulp van deze gegevens kunnen constitutie-vergelijkingen getest worden. Als resultaat hiervan kan er een uitspraak gedaan worden over de toepasbaarheid van deze constitutie-vergelijkingen.

In dit proefschift worden de experimentele resultaten beschreven van de stroming van een aantal vloeistoffen. Als stromingsgeometrie is gekozen voor een rechthoekig kanaal met 1 of 2 cilinders. Op deze manier wordt een stroming gecreeerd met zowel afschuiving als rek. Vervolgens zijn de experimentele resultaten vergeleken met numerieke berekeningen. Omdat het nog niet mogelijk is om 3D numerieke simulaties te doen in een complexe geometrie met viscoelastische modellen, is er gekozen voor een rechthoekig kanaal met dusdanige afmetingen dat in het midden van het kanaal een 2D stroming zou moeten ontstaan. Op deze manier is het mogelijk de experimentele resultaten met 2D numerieke berekeningen te vergelijken.

Een ander belangrijk onderwerp in de reologie is de interactie van deeltjes in een viscoelastische vloeistof. Om meer inzicht in dit onderwerp te verkrijgen is niet alleen de snelheid rondom de cilinders gemeten maar is ook de kracht gemeten op de cilinders onder verschillende stro-
Samenvatting

Mengscondities. Het snelheidsveld en de weerstand op de cilinders zijn berekend in een 2D geometrie en vergeleken met experimentele resultaten.

Voor de vloeistoffen in de experimenten is gekozen voor een basis-oplossing van 93% glucose en 7% gedestilleerd water waarin verschillende concentraties polyacrylamide zijn opgelost waardoor vloeistoffen met verschillende viscoelastische eigenschappen zijn verkregen. In totaal zijn drie verschillende vloeistoffen gebruikt. Een Newtonse vloeistof bestaande uit de genoemde basis-oplossing. Een elastische vloeistof, die bestaat uit de basis-oplossing met 150 wppm polyacrilamide. En een afschuifverdunnende elastische vloeistof, bestaande uit de basis-oplossing met daaraan toegevoegd 400 wppm polyacrilamide.

Het snelheidsveld is gemeten met Laser Doppler Velocimetry (LDV). De kracht op de cilinder is gemeten met een kracht-opnemer die speciaal voor deze doeleinden is ontwikkeld. De hydrodynamische interactie tussen twee deeltjes is bestudeerd door twee cilinders achter elkaar in het kanaal te plaatsen en op 1 van de 2 cilinders de kracht te meten.

De stroming is gesimuleerd in een 2D geometrie met een methode gebaseerd op de discontinue Galerkin methode. Voor de vergelijking met de experimenten zijn twee verschillende constitutie-vergelijkingen gebruikt: Het FENE-P model en het Verhoef model. Het laatste model is ontwikkeld door Verhoef (Verhoef et al. 1998) speciaal voor rekstromingen. Voor ieder model zijn drie relaxatiemodes gebruikt om een zo breed mogelijk spectrum aan relaxatie-tijden te beschrijven en een zo objectief mogelijke vergelijking te kunnen maken. De materiaaleigenschappen voor vloeistoffen gebruikt in de numerieke berekeningen zijn bepaald in een zuivere afschuifstroming. De afschuifviscositeit en het eerste normaalspanningsverschil zijn bepaald in een statische afschuifstroming en de complexe parameters in een dynamisch afschuifexperiment. Alleen de resultaten van het Verhoef model worden gepresenteerd omdat het FENE-P model instabiliteiten opleverde bij hogere snelheden.

Het gemeten snelheidsveld in de Newtonse oplossing komt goed overeen met de 2D numerieke berekeningen, hoewel dichtbij de cilinder 3D stromingseffecten merkbaar zijn. Als gevolg hiervan komt kwalitatief het gemeten gedrag van de weerstand van de cilinder wel overeen met de 2D berekeningen maar wijken de kwantitatieve waarden van de weerstand af van de 2D berekeningen. De gemeten weerstand van de cilinder neemt lineair toe met de snelheid. Maar de berekende 2D weerstand ligt 20% hoger dan de gemeten weerstand. De weerstand berekent met een 3D simulatie toont aan dat een groot deel van dit verschil verklaard kan worden met de aanwezigheid van de drie-dimensionale stroming dicht bij de cilinder.

Vervolgens zijn de experimenteel verkregen resultaten in de Newtonse vloeistof vergeleken met de resultaten in de viscoelastische vloeistoffen. De viscoelastische effecten worden beter zichtbaar met toenemende snelheid. Bij een bepaalde kritische snelheid treden instabiliteiten op. Tijdens de experimenten is erop gelet dat de snelheid altijd beneden deze kritische snelheid bleef en de stroming stabiel bleef. De grootste verschillen in het snelheidsveld tussen de Newtonse en de viscoelastische vloeistoffen vinden we in het zog van de cilinder waar de stroming wordt versneld van nul tot de ongestoorde stroomsnelheid. De 2D simulaties laten kwalitatief hetzelfde zien, ook hier zien we dat het stromingsgedrag in het zog van de cilinder van de viscoelastische vloeistoffen afwijkt van de Newtonse vloeistof. Hoewel kwantitatief de verschillen in het snelheidsveld tussen de viscoelastische vloeistoffen veel kleiner zijn dan in de Newtonse vloeistof. Tenslotte kunnen we nog opmerken dat de weerstand van de cilinder in de viscoelastische vloeistof bijna kwadratisch verandert met het debiet, terwijl in een Newtonse
vloeistof de weerstand lineair verandert met het debiet.

Uit zowel experimenten als numerieke berekeningen kunnen we concluderen dat het viscoelastische gedrag van de vloeistof het beste merkbaar is in het zog van de cilinder. In dit gebied treedt de grootste rek op waardoor de simulaties van de viscoelastische modellen hier de grootste afwijkingen vertonen met experimenten. De oorzaak hiervan ligt voor het grootste gedeelte aan het feit dat de eigenschappen van de vloeistoffen alleen in afschuifstromingen zijn bepaald. De resultaten van de numeriek berekeningen zouden verbeterd kunnen worden door ook de eigenschappen in rekstromingen te bepalen. De effecten van het viscoelastisch gedrag van de vloeistof worden beter zichtbaar als de stroomsnelheid toeneemt totdat bij een kritische snelheid instabiliteiten optreden.
Samenvatting
## List of symbols

<table>
<thead>
<tr>
<th>symbol</th>
<th>description</th>
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### List of symbols

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#### Greek symbols

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<td>unit tensor</td>
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</tr>
<tr>
<td>$\epsilon$</td>
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<td>(2.19)</td>
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<td>(2.4)</td>
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<tr>
<td>$\eta$</td>
<td>elongational viscosity</td>
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<tr>
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</tr>
<tr>
<td>$\eta_\infty$</td>
<td>infinite shear rate viscosity</td>
<td>(2.26)</td>
</tr>
<tr>
<td>$\kappa$</td>
<td>characteristic time scale based on the strain rate</td>
<td>(2.6)</td>
</tr>
<tr>
<td>$\kappa$</td>
<td>constant in the eq. of motion of the connector vector</td>
<td>(2.51)</td>
</tr>
<tr>
<td>$\lambda$</td>
<td>characteristic time scale of the fluid</td>
<td>(2.5)</td>
</tr>
<tr>
<td>$\rho$</td>
<td>density</td>
<td>(2.1)</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>total stress tensor</td>
<td>(2.2)</td>
</tr>
<tr>
<td>$\tilde{\sigma}$</td>
<td>deviatoric stress tensor</td>
<td>(2.3)</td>
</tr>
<tr>
<td>$\Psi_1$</td>
<td>first normal stress coefficient</td>
<td>(2.11)</td>
</tr>
<tr>
<td>$\Psi_2$</td>
<td>second normal stress coefficient</td>
<td>(2.12)</td>
</tr>
<tr>
<td>$\psi(Q,t)$</td>
<td>distribution function</td>
<td>(2.51)</td>
</tr>
<tr>
<td>$\Psi(r_1,r_2,t)$</td>
<td>configuration space distribution function</td>
<td>(2.51)</td>
</tr>
<tr>
<td>$\omega$</td>
<td>oscillating frequency</td>
<td>(2.5)</td>
</tr>
<tr>
<td>$\zeta$</td>
<td>friction coefficient</td>
<td>(2.51)</td>
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#### Miscellaneous symbols

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<thead>
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<th>Symbol</th>
<th>Description</th>
<th>Equation/Reference</th>
</tr>
</thead>
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<tr>
<td>$\nabla$</td>
<td>nabla-operator</td>
<td>(2.1)</td>
</tr>
<tr>
<td>$(\ )^T$</td>
<td>transpose</td>
<td>(2.39)</td>
</tr>
<tr>
<td>$(\ )_v$</td>
<td>upper-convective derivative</td>
<td>(2.46)</td>
</tr>
</tbody>
</table>

#### Abbreviations

<table>
<thead>
<tr>
<th>Abbreviation</th>
<th>Description</th>
<th>Page/Reference</th>
</tr>
</thead>
<tbody>
<tr>
<td>2D</td>
<td>two-dimensional</td>
<td></td>
</tr>
<tr>
<td>3D</td>
<td>three-dimensional</td>
<td></td>
</tr>
<tr>
<td>ARES</td>
<td>advanced rheometric expansion system</td>
<td>p. 70</td>
</tr>
<tr>
<td>LDV</td>
<td>laser Doppler velocimetry</td>
<td>p. 40, figure 3.1</td>
</tr>
<tr>
<td>PAMH</td>
<td>partially hydrolyzed polyacrylamide</td>
<td>p. 70, figure 4.1</td>
</tr>
<tr>
<td>PAA</td>
<td>polyacrylamide</td>
<td>p. 42, figure 3.3</td>
</tr>
<tr>
<td>PEO</td>
<td>polyethylene oxide</td>
<td>p. 42, figure 3.3</td>
</tr>
<tr>
<td>UCM</td>
<td>Upper Convected Maxwell fluid</td>
<td>p. 15</td>
</tr>
<tr>
<td>wppm</td>
<td>weight parts per million</td>
<td>p. 48, figure 3.14</td>
</tr>
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Chapter 1
Introduction

In the first subsection of this chapter we briefly discuss the background and behaviour of non-Newtonian fluids in general and polymeric fluids in particular. Next a brief overview is given of numerical and experimental results regarding viscoelastic fluid flow around objects. This is followed by a review of remaining problems in the description of the flow of non-Newtonian fluids. The approach to a part of this problem is discussed in the fourth subsection. Finally the outline of this thesis is given.

In everyday life we encounter many different kinds of fluids. Simple fluids, like water, honey and syrup consist of small molecules only and have no large internal structures and thus have a simple viscous behaviour. This means that their viscosity (and also the other material functions) depends only on the temperature and pressure, but not on the chosen flow. These fluids are called Newtonian. However, many fluids do not show such a simple behaviour. This is generally due to the presence of large internal structures, which are often caused by large molecules. The viscosity of these fluids does depend on the deformation rate, and some fluids also show elastic behaviour. Those types of fluid are therefore said to have a non-Newtonian behaviour. Examples of such fluids are paints, egg-white, dough, clay, salad dressing, yogurt, etc. Obviously, also polymer solutions belong to this category.

Polynomial solutions
Polymeric solutions are very interesting because of their unexpected behaviour, that differs from the behaviour of Newtonian fluids to which we are used to. In some cases this leads to surprising phenomena which are not always fully understood. A typical example is the "rod climbing" (or "Weissenberg") effect: when rotating a rod in a viscoelastic solution, the fluid moves inward and climbs the rod. One typically observes this effect when mixing dough with a mixer. It can be understood by realizing that, because of the stirring, the long polymer chains are extended along the circular streamlines around the mixer. As the polymer chains prefer to relax to more coiled (curled up) states, this results in an inward force directed to the rod. If the elastic stresses are large enough the fluid climbs the rod.

Another example is the "die swell effect": a polymer solution flowing out of a tube shows an increase in diameter. Again the explanation lies in the behaviour of the long chain polymer molecules. The flow in the narrow tube stretches them in stream-wise direction. Once the polymer chains have left the constraining walls of the tube they are free to relax to a more coiled conformation again. This effect is important in many extrusion processes in industry, as the polymer jet diameter may increase by a factor of up to 4 times the outflow value, see Goubilomme & Crochet (1992) and Goubilomme, Draily & Crochet (1993). Another problem in extrusion is the occurrence of flow instabilities, called "melt fracture", which is typical for polymer melts. After the polymer extrudate has passed through a die, the surface of the polymer melt shows an irregular distortion.
A well known application in industry of viscoelastic fluids is the drag reducing behaviour of long chain polymers in turbulent pipe flow. Addition of a very small amount of polymer (say 20 wppm) may lead to a reduction of the drag, even up to 80%. Industries, but also fire brigades use this phenomenon to their benefit. Another example is found in the recovery process of oil from porous rocks. When methods like pumping of water through the rock fails, due to instabilities between the water and oil interface, one uses "polymer flooding". This means that the water-oil interface is stabilized by using aqueous polymer solutions as the displacement fluid. Also in consumer products like paints and tooth pastes, polymers are used as thickeners. By adding types of polymers that form shear thinning solutions with only little elastic properties, a product is obtained which has a high viscosity at low shear (deformation) rates and a low viscosity at high shear rates. This property simplifies manipulation of products like paint and toothpaste. The same is true for margarine, that must spread easily from your knife when sheared, but should keep its shape without the applied stress. Fluids with this particular property are said to have a "yield stress". More examples are given by Bird et al. (1987) and Barnes et al. (1989).

In everyday life we also have to deal with various non-Newtonian fluids. For instance, non-Newtonian fluid behaviour can be found in many food products, emulsions like milk and mayonnaise and suspensions like margarine, which is, because of its yield stress, much easier to handle than, for instance, honey. Also the rod-climbing effect of dough, already mentioned before, is quite fascinating.

**Importance in industry**

A complete understanding of viscoelastic fluid flow in complex geometries is of importance in many industrial processes. Examples of such processes are extrusion, injection moulding and fibre spinning. Many processes are characterized by a flow geometry where the shear viscosity can be considered to be the dominant factor. If the polymer melt behaves like an inelastic non-Newtonian liquid, a simple power-law model is sufficient to describe the behaviour of the melt in the process modeling. In other processing situations, like film blowing, fibre spinning and flow through a contraction, elasticity is the dominant factor. In that case a more complex model is needed to predict the flow.

Further, the transport of particles moving in viscoelastic fluids is not well understood, which is of high importance in the oil industry. A model system for the study of this problem is that of a sphere settling in a viscoelastic fluid. This model system is also widely accepted as a benchmark problem for the validation of numerical simulation studies of viscoelastic flow against experiments.

### 1.1 Viscoelastic flows around objects

Much work has been done on the flow of viscoelastic fluids around various bluff bodies. In particular, flows around cylinders confined by walls, free cylinders, and configurations with packed cylinders have been studied. Furthermore, viscoelastic flows around a sphere have been examined. More recently the interaction between two or more spheres has been studied in more detail. We will present the results in this part so that we can use this as a frame of reference as where to place the results presented in this thesis.

Studies of the viscoelastic flow around a free cylinder (i.e without wall influence) have led to the following conclusions. Compared to the Newtonian case, a combination of strong elastic
1.2. Objective

and inertia effects \((Re >> 1, De > 1)\) has a considerable influence (about a factor of 10 higher) on the drag (James & Gupta 1971), on the mass and heat transfer (Piau 1980) and on the region disturbed by the cylinder (Koniuta et al. 1980). Simulations using constitutive models for viscoelastic fluids qualitatively predict those features (Hu & Joseph 1990). The effect of elasticity in creeping flow \((Re < 1, \text{i.e. no inertia effects})\) is much less drastic (Broadbent & Mena 1974). In experiments a small drag reduction is seen, but the flow region disturbed by the cylinder is similar to that in a Newtonian fluid.

Experiments on the confined flow around a cylinder show that the effect of elasticity is to decrease the drag by approximately 20% as compared to the Newtonian case (Dhahir & Walters 1989). In viscoelastic fluids the drag can be decreased up to a factor of 55%. Simulations with constant viscosity elastic models (Huang & Feng 1995), however, reveal that by the presence of a wall the drag is increased (up to 100% at a We number of 1 and a blockage ratio \(\beta = 0.5\), where \(\beta = d/h\), \(h\) is the channel height and \(d\) the diameter of the cylinder). Furthermore, the region disturbed by the cylinder is decreased. Other simulations (Baaijens et al. 1997), show good agreement between measured and calculated velocities and stresses. However, it should be mentioned that also a generalised Newtonian model describes the velocity field accurately.

Contradictory results were found in experiments on creeping flow around a sphere. In comparison to Newtonian fluids a large drag increase was found in an organic-based Boger fluid (Tirtaatmadja et al. 1990), while drag reduction was observed in a water-based Boger fluid (Chhabra et al. 1980). Experiments on two falling spheres were used to examine particle interaction in viscoelastic fluids. Also in these experiments contradictory results were found. Some experimentalists concluded that there exists a critical distance: if the initial distance between the spheres was smaller than this critical distance the spheres move towards each other, otherwise they separate (Riddle et al. 1977). However, in other experiments the two spheres always approach each other (Gheissary & van den Brule 1996), or a stable distance was found (Bot et al. 1998).

In chapter 2 an overview is given of references to viscoelastic flows around objects.

1.2 Objective

The objective of the research described in this thesis is to test and evaluate existing constitutive equations for polymer solutions in complex flows, by making a comparison of the results of flow experiments with numerical simulations. Furthermore, the hydrodynamic interaction between two bodies in viscoelastic fluids is studied and the predictive value of the FENE-P and Verhoef model regarding this interaction is examined.

1.3 Approach & tools

In order to address these objectives, the following requirements have to be met.

- Both shear and elongation flow aspects must be present in the flow that we consider, as the combination of both is not yet understood. Most constitutive models have been tested in simple shear flows only because of experimental difficulties regarding the creation of elongational flow. Even nowadays there is still little reliable data on elongational flows available, see (Tirtaatmadja & Sridhar 1993), (van Nieuwkoop & Muller von Czernicki 1996) and (Verhoef et al. 1998).
Due to limitations in computer power, one can only perform accurate 2D simulations on viscoelastic models. The flow studied should therefore be 2D. For the simulations the flow must be well defined, i.e. the grid may not contain any geometrical singularities like sharp corners.

We want to examine only a limited range of characteristic numbers ($Re$ and $De$). The Reynolds numbers of the employed flows will be kept low ($Re << 1$) to prevent vortices and other inertial flow phenomena. The Deborah number of the flow must be around one. At lower values elastic effects will diminish while at higher values elastic instabilities may occur.

![Diagram showing presentation of some published experimental results on viscoelastic flow past a cylinder. Experiments and simulations presented in this thesis are indicated by the dashed ellipse. Two relevant characteristic dimensionless numbers determine the axes. The Reynolds number ($Re = \frac{\rho Ud}{\eta}$) gives the ratio of inertial versus viscous effects. The importance of elasticity effects with respect to viscous effects is characterized by the Deborah number ($De = \frac{\lambda U}{d}$).](image)

To fulfill the requirements of a complex, nominally 2D flow with combined shear and elongational flow properties and without geometrical singularities, we have chosen for a geometry consisting of 1 or 2 cylinders placed in a rectangular channel, see figure 1.2. In the wake of the cylinders, where the fluid is accelerated from zero to the free stream velocity, strong elongational effects occur. Between the cylinder(s), which have a diameter of 10 mm, and the channel walls, which are 20 mm apart, aspects of both shear and elongational flow can be found. To ensure a nominally two dimensional flow, a channel with a cross section of 0.16 m in height and 0.02 m in width (see figure 1.2) was designed similar to the channel used in experiments presented in several papers ((McKinley et al. 1992), (McKinley et al. 1993), (Baaijens et al. 1994), (Baaijens et al. 1995) and (Baaijens et al. 1997)). This also enables us to compare our results with those of others. The total length of the channel is 1100 mm, which ensures a fully
1.4. Outline

developed velocity profile upstream of the cylinders and a complete relaxation behind them. A more elaborate explanation for the choice of this geometry is given in chapter 3.

![Figure 1.2](image)

Figure 1.2: Schematic representation of the used flow geometry of two cylinders in a rectangular channel. The distance $L$ between the cylinders can be varied between 3.5 and 7 times the cylinder diameter. The cylinder diameter is 0.01 m.

The measurements of the flow velocity and of the drag on the cylinder(s) were performed, as follows. For the determination of the flow velocity we used Laser Doppler velocimetry. For the measurement of the drag force on (one of) the cylinder(s) a special force transducer was designed and built. The distance $L$ between the two cylinder axes could be varied between 3.5 and 7 times the cylinder diameter ($d = 0.01 \text{ m}$).

The flow was calculated using an existing numerical method and existing theory (constitutive models). The flow was simulated with a finite element numerical method based on the discontinuous Galerkin method. With this technique the Navier Stokes equations of motion (which essentially is Newton’s law, $F = ma$, for a liquid) were solved. Many different constitutive equations (CE) exist, mostly phenomenological. For the description of the fluid we used the FENE-P model and a multi-mode model derived by Verhoef (Verhoef et al. 1998). However the results of the FENE-P model could not be used, as it showed instabilities at the highest flow rate. The material parameters of both models were determined by fitting the multi-mode FENE-P and Verhoef model to the results of experiments obtained in a simple shear flow (see chapter 4).

The effect of (visco)elasticity is studied by comparing the results of experiments in a polymer solution to the results measured in a Newtonian solution. The experiments are conducted at low Reynolds numbers and at different Deborah numbers so that we can study elastic effects without the results being influenced by inertia effects. The complexity of the flow is introduced by placing one or two cylinders in a channel.

Drag force measurements on two cylinders with varying distance may help to understand the mechanisms involved in particle interaction in non-Newtonian liquids and to clarify the existing confusion regarding this matter.
1.4 Outline

The outline of this thesis is as follows. In chapter 2 we discuss the theory on Newtonian and non-Newtonian fluid flow. The equations of motion and the various constitutive equations relevant for this study are presented. Next, the more specific theory on flows around objects is discussed, also both for Newtonian and non-Newtonian fluids. The flow facility, the Laser Doppler apparatus and the force transducer are described in detail in chapter 3. The used numerical method described in this chapter is quite common and therefore it is discussed in more general terms providing the proper references that contain all details. In chapter 4 we pay attention to the choice and characteristic behaviour of the chosen model fluid. All results are presented in chapter 5. First, the experimental Newtonian results are presented and compared to 2D and 3D simulations. Next we discuss the experimental non-Newtonian results and compare them to the simulations conducted with various constitutive equations. Finally, in chapter 6, conclusions are presented and recommendations are given regarding Newtonian and non-Newtonian experiments and simulations.
Chapter 2

Fluid theory & models

In this chapter the theory of viscoelastic fluid modeling is discussed. Starting from simple Newtonian behaviour we describe the behaviour of viscoelastic models in simple flows and end with the present theories for viscoelastic fluid flow in complex geometries. In particular we discuss different types of constitutive equations which relate the stress in the fluid to its deformation history. The discussion is based on the distinction between models based on continuum theory and models based on microscopic kinetic theory.

2.1 Basics of fluid mechanics

The flow of liquids and gases, generally called fluids, is described by the following system of equations,

- The conservation of mass equation, which alternatively is called the continuity equation

\[ \frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{u}) = 0 \]  

where \( \mathbf{u} \) is the velocity vector and \( \rho \) the density. One often assumes \( \rho \) to be constant (incompressible flow), so that (2.1) reduces to \( \nabla \cdot \mathbf{u} = 0 \).

- The conservation of momentum equation, or Newton’s second law \( \mathbf{F} = m \mathbf{a} \), adopted to the case of fluid flow situations, which is also denoted as the Navier Stokes equation. It reads

\[ \rho \frac{\partial \mathbf{u}}{\partial t} + \rho \mathbf{u} \cdot \nabla \mathbf{u} = \nabla \cdot \mathbf{\sigma} \]  

where \( \mathbf{\sigma} \) is the total stress tensor. The total stress tensor \( \mathbf{\sigma} \) is usually written as the sum of an isotropic part containing the hydrodynamic pressure \( p \) and a part determined by the deformation history, called the deviatoric stress tensor \( \mathbf{\tau} \)

\[ \mathbf{\sigma} = -p\mathbf{\delta} + \mathbf{\tau} \]  

where \( \mathbf{\delta} \) is the unit tensor.

- To close the set of equations (2.1) to (2.3) an explicit expression is needed that relates the stress \( \mathbf{\sigma} \) in the fluid to its deformation history. Such a relation is called a constitutive equation (CE). All differences between Newtonian and non-Newtonian behaviour are contained in this equation.
2.2 Dimensionless numbers

In fluid mechanics dimensionless numbers are introduced for scaling arguments and for defining the flow regime. In Newtonian fluid mechanics the Reynolds number is a well-known dimensionless number. The Reynolds number appears as a result of writing the equations of motion in dimensionless form, and it shows up as the ratio of inertia forces versus viscous forces,

\[ Re = \frac{\rho Ud}{\eta} \]  

(2.4)

where \( \rho \) is the density and \( \eta \) the viscosity of the fluid, \( U \) is the free stream velocity and \( d \) is a characteristic length scale of the flow. At high Reynolds numbers (e.g. \( Re > 1 \)) inertia plays a non-negligible role.

For viscoelastic fluids another dimensionless number is useful. The Deborah number gives the ratio of a characteristic time of the fluid (\( \lambda \)) to the characteristic time of the flow,

\[ De = \frac{\lambda}{d/U} \]  

(2.5)

where \( U \) and \( d \) are described above. For polymer fluids the characteristic time depends on the time of an extended molecule to relax to its coiled up equilibrium state. At high Deborah numbers (e.g. \( De > 1 \)) viscoelastic effects become important.

In some viscoelastic flows it is possible to discern another characteristic time scale, based on the strain rate \( \kappa \). In that case a second dimensionless group appears, the Weissenberg number, defined as,

\[ We = \lambda \kappa \]  

(2.6)

Again, at high Weissenberg numbers elastic effects become important. At high Weissenberg numbers, so at high strain rates, the fluid is still able to relax and shows elastic behaviour. At low Weissenberg numbers this is no longer the case.

2.3 Newtonian fluids

Newton has proposed a linear relationship between the deformation and the stress, based on the assumption that the friction between sliding fluid layers increases linearly with their velocity difference. This leads for these so-called "Newtonian fluids" to a very simple constitutive equation, as the deviatoric stress tensor \( \tau \) is then proportional to the deformation given by

\[ \tau = \eta \dot{\gamma} \]  

(2.7)

where \( \dot{\gamma} = \nabla u + (\nabla u)^T \) is the rate of strain tensor and \( \eta \) the viscosity. This equation describes standard fluids as air and water with great accuracy.

All other cases when (2.7) is not satisfied we call "non-Newtonian". We need the more complex constitutive equations, in which the viscosity itself does depend on the shear rate, or on the deformation history in general. Many propositions for all kinds of constitutive equations have been made, leading to very different expressions for material functions like the viscosity and the normal stress differences which will be considered in the next section. Some of the most important theories that have been developed to derive these constitutive equations are the subject of following sections.
2.4 Material functions

In experiments it is often required to characterize a Non-Newtonian fluid. There are two standard flow types that are used for this; a simple shear flow and an elongational flow.

2.4.1 Material functions in steady shear flow

In a simple shear flow with one velocity component in the x-direction which is a function of the y-coordinate only, the shear stress is nonzero. For a simple shear flow the deviatoric stress tensor $\sigma$ is given by

$$\sigma = -pI + \tau = \begin{pmatrix}
  p + \tau_{xx} & \tau_{xy} & 0 \\
  \tau_{xy} & -p + \tau_{yy} & 0 \\
  0 & 0 & -p + \tau_{zz}
\end{pmatrix}$$

(2.8)

For non-Newtonian fluids the relevant stresses for this case are defined as follows,

- Shear stress: $\tau_{xy}$
- First normal stress difference $N_1 = \tau_{xx} - \tau_{yy}$
- Second normal stress difference $N_2 = \tau_{yy} - \tau_{zz}$

(2.9)

The first normal stress difference $N_1$ is the normal stress in the flow direction minus that in the gradient direction. The second normal stress difference $N_2$ is the difference between the normal stress in the gradient direction and that in the neutral direction. By means of these stresses, one can define material functions. The shear viscosity $\eta$ is defined analogously to the viscosity for Newtonian fluids,

$$\eta(\dot{\gamma}) = \frac{\tau_{xy}}{\dot{\gamma}_{xy}}$$

(2.10)

where $\dot{\gamma}_{xy} = \frac{\partial u}{\partial y}$. For polymer melts and solutions the viscosity at low shear rates is independent of the shear rate. It approaches a constant value $\eta_0$ known as the zero shear rate viscosity or the Newtonian plateau value. At higher shear rates the viscosity usually decreases with increasing shear rate, which is called shear thinning behaviour. At very high shear rates the viscosity again becomes independent of the shear rate and approaches the so-called infinite shear rate viscosity $\eta_\infty$.

Likewise, we can define the first and second normal stress coefficients. The first normal stress coefficient is defined as

$$\Psi_1(\dot{\gamma}) = \frac{\tau_{xx} - \tau_{yy}}{\dot{\gamma}_{xy}^2}$$

(2.11)

And the second normal stress difference as

$$\Psi_2(\dot{\gamma}) = \frac{\tau_{yy} - \tau_{zz}}{\dot{\gamma}_{xy}^2}$$

(2.12)

For Newtonian fluids all normal stresses are zero for a simple shear flow, so also the first and second normal stress differences are zero.
2.4.2 Material functions in dynamic shear flow

In dynamic shear experiments the sample between two circular plates is subjected to a sinusoidal, oscillating shear flow with a small amplitude. One plate is rotated in an oscillating way, with a frequency \( \omega \) and a small amplitude \( \gamma_0 \), the other plate is held fixed. The shear strain and the shear rate are given by:

\[
\begin{align*}
\gamma_{yx} &= \gamma_0 \sin \omega t \\
\dot{\gamma}_{yx} &= \gamma_0 \omega \cos \omega t
\end{align*}
\] (2.13) (2.14)

The stress is written as the sum of a component in phase with the shear and a component out of phase with the shear:

\[
\tau_{yx} = G'(\omega) \gamma_0 \sin(\omega t) + G''(\omega) \gamma_0 \cos(\omega t)
\] (2.15)

where \( G' \) is the storage modulus, which is a measure of the elastic energy stored in the fluid, and \( G'' \) is the loss modulus, which is a measure of the dissipated energy. An alternative notation is in terms of the dynamic viscosities \( \eta' \) and \( \eta'' \)

\[
\tau_{yx} = \eta''(\omega) \gamma_0 \sin(\omega t) + \eta'(\omega) \gamma_0 \cos(\omega t)
\] (2.16)

2.4.3 Material functions in elongational flow

The second standard flow is the elongational flow. For uniaxial elongation we can write

\[
\begin{align*}
    u &= \dot{\epsilon} x \\
    v &= -\frac{1}{2} \dot{\epsilon} y \\
    w &= -\frac{1}{2} \dot{\epsilon} z
\end{align*}
\] (2.17) (2.18) (2.19)

where \( u, v \) and \( w \) are the velocity components in the \( x, y \) and \( z \) direction respectively. The material is extended in the \( x \) direction, while new material is supplied from the \( y \) and \( z \) directions. Consider in this elongational flow a cylindrical filament with length \( L \) and radius \( R \). We can then express the elongation rate \( \dot{\epsilon} \) as

\[
\dot{\epsilon} = \frac{L}{\partial t} \quad -\epsilon = \frac{2}{\partial R}
\] (2.20) (2.21)

The total strain is defined as

\[
\epsilon = \int_{t_0}^{t} \dot{\epsilon} dt = \ln\left(\frac{L}{L_0}\right) = -2 \ln\left(\frac{R}{R_0}\right)
\] (2.22)

The elongational viscosity \( \eta \) is derived from the equations of motion applied on a cylindrical filament, see (Bird et al. 1987) and (Macosko 1994), and defined in cylindrical coordinates as

\[
\eta(\dot{\epsilon}, t) = \frac{\tau_{xx}(t) - \tau_{rr}(t)}{\dot{\epsilon}(t)}
\] (2.23)
2.5 Generalised Newtonian models

The Trouton ratio is defined as the ratio of the transient extensional viscosity and the zero shear rate viscosity,

\[ T_r = \frac{\eta(\dot{\epsilon}, t)}{\eta_0} \]  

(2.24)

2.5 Generalised Newtonian models

The most simple extensions of the Newtonian constitutive equation are the generalised Newtonian models, also called viscous models. Elasticity is not incorporated. These models exhibit a shear dependent viscosity. An example is the powerlaw model which is only valid in the shear thinning region usually for high shear rates. For simple shear the powerlaw model is defined as

\[ \eta = k\dot{\gamma}^{n-1} \]  

(2.25)

This model has two parameters, \( k \) which determines the viscosity level at a certain shear rate and \( n \) which determines the level of shear thinning, usually the value of \( n \) lies in the range between 0.5 and 1.0 if \( n < 1 \). The powerlaw model is easy to use and is often applied when rough estimates of flow properties are needed. For very small values of the shear rate the power law model prediction for the viscosity diverges, while in real systems the viscosity always reaches its "Newtonian plateau value" \( \eta_0 \). In the Carreau model this plateau viscosity is included for small shear rates,

\[ \frac{\eta - \eta_\infty}{\eta_0 - \eta_\infty} = \left(1 + (\lambda \dot{\gamma})^2\right)^{\frac{n-1}{2}} \]  

(2.26)

This makes the Carreau model more appropriate for flows where regions of low shear rates are present. The Carreau model has three extra parameters: \( \eta_0 \) and \( \eta_\infty \) are the asymptotic values of the viscosity at very low and at very high shear rates, respectively, \( \lambda \) is a characteristic time of the fluid, which determines the shear rate where the crossover to shear thinning behaviour sets in.

Although a big disadvantage of viscous models is that they do not show any relaxation phenomena, they are still useful for flow calculations in systems where shear effects are dominant, for instance in polymer processing. In the following, we consider constitutive models where relaxation effects are taken into account.

2.6 Continuum mechanics

Just like the generalised Newtonian models the fluid behaviour of continuum models is represented as a continuum. An easy way to include relaxation effects is to describe its mechanical behaviour in terms of macroscopic notions like springs and dashpots. Where the spring represents the elastic behaviour of the continuum and the dashpot the viscous behaviour. An example is the Maxwell model, which consists of a spring and a dashpot in series (see figure 2.1). Applying the expressions for the stress in the spring and the dashpot and adding the strains, we obtain the Maxwell equation, which contains a linear relation between \( \tau \) and \( \dot{\gamma} \)

\[ \tau + \lambda \dot{\tau} = \eta_0 \dot{\gamma} \]  

(2.27)
Figure 2.1: Example of a continuum model represented by a mechanical model of a spring (with spring constant $G$) and a dash pot (with a resistance constant $\eta$) in series.

Here we have replaced $\eta/G$ by $\lambda$, the characteristic relaxation time of the system. For times short compared to $\lambda$, the Maxwell model has an elastic behaviour, while for large time scales the behaviour is predominantly viscous.

The Maxwell model can be generalised by a superposition of Maxwell models. In this way we obtain a so-called multi-mode Maxwell model. Equation (2.27) is then used to describe each mode, identified by the subscript ($k$)

$$\tau_k + \lambda_k \dot{\tau}_k = \eta_k \dot{\gamma}$$

(2.28)

The total stress is obtained by summing the stresses of all the different modes.

The solution of the first order differential equation 2.27 for $\tau$ as a function of time is (see (Bird et al. 1987))

$$\tau(t) = \int_{-\infty}^{t} \frac{\eta_0}{\lambda_1} e^{-\frac{t-t'}{\lambda_1}} \dot{\gamma}(t') dt'$$

(2.29)

Likewise the solution of the generalised Maxwell model 2.28 is

$$\tau(t) = \int_{-\infty}^{t} \sum_{i=1}^{N} \frac{\eta_k}{\lambda_k} e^{-\frac{t-t'}{\lambda_k}} \dot{\gamma}(t') dt'$$

(2.30)

where $N$ is the number of modes. The Maxwell model (2.29) and the generalised Maxwell model (2.30) can be written in a general form, the so called general linear viscoelastic model,

$$\tau(t) = \int_{-\infty}^{t} G(t-t') \dot{\gamma}(t') dt'$$

(2.31)

where $G(t)$ is called the relaxation modulus. The stress described by (2.31) can be considered to be a product of two functions. The first one ($G(t-t')$) depending on the material functions of the fluid and the second one ($\dot{\gamma}(t')$) depends on the flow characteristics. In chapter 4 we will use equation (2.31) for the description of a flow with small deformations.
2.6. Continuum mechanics

The Jeffrey’s model, also linear in stresses, deformation and their derivatives, is describes a system of a dashpot, in parallel with a series of a spring and dashpot (see figure 2.2),

\[ \tau + \lambda_1 \dot{\tau} = \eta_0 (\dot{\gamma} + \lambda_2 \ddot{\gamma}) \] (2.32)

where \( \eta_0 = \eta + \eta_s \) and \( \lambda_1 \) and \( \lambda_2 \) are two different time constants. For short and large time scales the Jeffrey’s model shows viscous behaviour (see figure 2.2). Another important feature of this model is given by the time derivative of the rate of strain \( \dot{\gamma} \). This means that the model not only shows stress relaxation after a deformation (\( \lambda_1 \) is the relaxation time) but also retardation behaviour (\( \lambda_2 \) is the retardation time). Although the Maxwell and Jeffrey’s model have certain merits regarding the description of simple “one-dimensional” viscoelastic flow problems, they do not obey the “objectivity rule”, i.e. the condition of frame indifference with respect to steady rotating frames. We return to this topic in the next section.

2.6.1 Criteria for constitutive equations

The simplest way to introduce relaxation phenomena is by using mechanical models like the Maxwell model (see equation (2.27)). A generalization to three dimensions would be,

\[ \tau + \lambda_1 \dot{\tau} = 2\eta D \] (2.33)

However, this equation does not obey the “objectivity rule”. This rule expresses the fact that material properties should be independent of the chosen frame. In particular, they should be invariant under a simple rotation of the system. To be more precise, when placing the system on a turntable and imposing the same deformations as before, one should measure the same stress, as long as inertial effects on the stress caused by the rotation itself can be neglected.

To treat this topic we must first introduce some important definitions. If a material is deformed, points in that material are displaced with respect to each other. Vectors that connect arbitrarily chosen pairs of points in the material are called material vectors. In other words,
due to the deformation, the directions and lengths of chosen material vectors may change. For small material vectors, deformation is a linear process, so that it can be described by means of a tensor; the deformation gradient tensor \(F\),

\[
\frac{dx}{dt} = F \cdot dX
\]  

(2.34)

where \(dX\) is the reference material vector at time \(t = 0\) (so before the deformation) and \(dx\) the same vector at time \(t\) after the deformation. Assuming the deformation is a sufficiently smooth process, also the velocity of the deformation can be described,

\[
\frac{dV}{dt} = \frac{d(dX)}{dt} = \dot{F} \cdot dX + F \cdot d\dot{X} = \dot{F} \cdot dX = \dot{F} \cdot F^{-1} \cdot dx = \dot{L} \cdot dx
\]  

(2.35)

where \(L \equiv \dot{F} \cdot F^{-1}\) is called the velocity gradient tensor \(\nabla V\); note that the time derivative of the reference vector \(X\) is zero. Now we have an expression for the time derivative of \(F\),

\[
\dot{F} = L \cdot F
\]  

(2.36)

The deformation \(F(x, t)\) causes a stress \(\tau(F)\) in the material. The precise form of the stress tensor depends on \(F(x, t)\), and is given by the CE. The stress \(t\) (traction) on a plane in the material can be calculated using the normal vector \(n\) of that plane,

\[
t = \tau \cdot n
\]  

(2.37)

Now we can proceed with the “objectivity condition”. Consider a material under a certain deformation \(F(x, t)\). Due to this deformation there will be a certain stress \(\tau(F)\) in the material. The deformation \(F\) could have been followed by a simple rotation \(Q\), so that the total ‘deformation’ of the material would be given by \(Q \cdot F\). If we neglect inertial effects caused by the rotation itself, the stress field would of course only be rotated compared to the unrotated case, and the traction with respect to any normal vector \(n\) would also be identical, up to this rotation,

\[
\tau(Q \cdot F) \cdot Q \cdot n = Q \cdot \tau(F) \cdot n
\]  

(2.38)

Note that in the rotated case the normal vector has to be rotated before \(\tau\) can act on it, in order to have the same orientation towards the material as in the unrotated case. We thus find that any expression for the stress in the material must obey

\[
\tau(Q \cdot F) = Q \cdot \tau(F) \cdot Q^T
\]  

(2.39)

where \(Q\) is a simple rotation, so that \(Q^{-1} = Q^T\). For the same reason as explained above, an appropriate time derivative of the stress tensor should obey the objectivity condition. Taking the time derivative of

\[
\tau_{QF} = Q \cdot \tau_{F} \cdot Q^T
\]  

(2.40)
where the subscripts \(QF\) and \(F\) refer to the rotated and unrotated case, respectively, and we obtain
\[
\dot{\tau}_F = \dot{Q}^T \cdot \tau_{QF} \cdot Q + Q^T \cdot \dot{\tau}_{QF} \cdot \dot{Q} + Q^T \cdot \tau_{QF} \cdot \dot{Q}
\] (2.41)

Using that for every deformation gradient tensor (and therefore also for the rotation \(Q\))
\[
\dot{\tau} = L_{QF} \cdot \dot{\tau}
\] (2.42)
\[
\dot{Q} = L_{QF} \cdot \dot{Q}
\] (2.43)
and that the time derivative of \(Q \cdot F\) can be written as
\[
(\dot{Q} \cdot F) = \dot{Q} \cdot F + Q \cdot \dot{F} = L_{QF} \cdot Q \cdot F
\] (2.44)

we express \(L_Q\) in terms of \(L_{QF}\) and \(L_{QF}\) by substituting (2.42) and (2.43) into (2.44). Next we can express equation (2.40) in terms of \(L_{QF}\) and \(L_{QF}\) by eliminating \(L_{QF}\). We finally find,
\[
\dot{\tau}_F - L_{QF} \cdot \tau_{QF} - \tau_{QF} \cdot L_{QF}^T = Q^T \cdot (\dot{Q}_{QF} - L_{QF} \cdot \tau_{QF} - \tau_{QF} \cdot L_{QF}^T) \cdot \dot{Q}
\] (2.45)

This means that not \(\dot{\tau}_F\) itself transforms in the desired way, but its combination with terms containing velocity gradients. This quantity, which emerges in a fairly natural way, has the units of a time derivative,
\[
\dot{\tau}_{QF} \equiv \dot{\tau}_F - L_{QF} \cdot \tau_{QF} - \tau_{QF} \cdot L_{QF}^T
\] (2.46)

The result is called the “upper convected time derivative” of \(\tau\), denoted as \(\ddot{\tau}\) or as \(\tau_{(1)}\). This means that we found an expression for a time derivative of \(\tau\) that also transforms in the way demanded by the objectivity rule,
\[
\dot{\tau}_{QF} \equiv \dot{Q} \cdot \ddot{\tau}_F \cdot Q^T
\] (2.47)

Although the stress tensor was used for this calculation, we would have obtained an analogous result by using any other physical quantity which depends on the real deformation of the material, excluding rigid body rotations. The time derivative(s) of the rate of strain tensor are an example. In more general terms, one can show that the upper convected time derivative appears naturally when the flow of the material is described in terms of convected coordinates. These are coordinates that follow the trajectories in the material. A rigorous treatment would lead somewhat too far in this thesis and can be found in appropriate textbooks, see Bird et al. (1987). Its natural emergence in the simple derivation given above, makes the use of the upper convected derivative to replace a simple time derivative at least plausible.

With the result we can return to the Maxwell model and introduce the convective derivative. If we replace the partial time derivative in (2.27) by the upper convected time derivative, we obtain for the 3D case the so-called upper convected Maxwell model (UCM),
\[
\dot{\tau} + \lambda \tau_{(1)} = \eta \gamma_{(1)}
\] (2.48)
where the subscript \(^{(1)}\) denotes the upper convected derivative which replaces the 1D time derivative. Although the upper convected Maxwell model still does not predict a shear dependent viscosity, it does show stress relaxation in time dependent flows.

In the same way we can introduce the convective derivative into the Jeffreys model (given by (2.32)). In this way the Oldroyd-B model is obtained,

\[
\tau + \lambda_1 \tau^{(1)} = \eta_0 (\gamma^{(1)} + \lambda_2 \gamma^{(2)})
\]

where \(\gamma^{(2)}\) is the upper convected time derivative of \(\gamma^{(1)}\). Note that \(\gamma^{(1)}\) is equal to \(\dot{\gamma}\). The Oldroyd-B model has three parameters, a zero shear-rate viscosity \(\eta_0\), a relaxation time \(\lambda_1\) and a retardation time \(\lambda_2\). The UCM is recovered for \(\lambda_2 = 0\). The UCM model can also be derived from Hookean dumbbell theory, as we will see in the next sections.

2.7 Elastic dumbbell models
2.7.1 Outline of kinetic theory
In this section we give a brief outline of the kinetic theory. For a more elaborate description of the kinetic theory see chapter 13 of (Bird et al. 1987).

Modeling a flowing polymer: assumptions made in kinetic theory
The kinetic theory of polymers is based on a very simple model for a polymer molecule: a chain of beads and springs. The simplest case is an elastic dumbbell, containing only two beads and a connecting spring, as shown in figure 2.3. In the kinetic theory the following assumptions have been made.

- The polymer molecule is modeled by an elastic dumbbell, i.e. two beads joined by a non-separable spring.
- The overall orientation and the internal configuration of the polymer molecule are described by the so-called connector vector \(\vec{Q}\)

\[
\vec{Q} = \vec{r}_2 - \vec{r}_1
\]

where \(\vec{r}_1\) and \(\vec{r}_2\) are the position vectors of bead 1 and 2, respectively.
- The flow field of the polymer solution is assumed to be homogeneous on the length scale of the dumbbells.
- The phase-space distribution function of the ensemble of connection vectors can be written as the product of a configuration space distribution function and a velocity-space distribution function. The velocity-space distribution function is assumed to be a Maxwellian, as for the solvent velocity, at the center of mass of the dumbbell. The configuration-space distribution function is independent of the location of the center of mass of the polymer molecule.

Time evolution equation for the configurational distribution function
The (diffusion) equation for the time evolution of the configurational distribution function is
2.7. Elastic dumbbell models

derived by combining the equations of motion of the beads with an equation of continuity that accounts for the normalization of the distribution function. The equation of motion for the beads of the dumbbells is derived for each bead separately. The net force on the bead, e.g. the sum of all forces acting on it, is equal to the mass of the bead times its acceleration. In addition, some other assumptions are made:

- Inertial terms, containing the bead masses, are neglected, e.g. the net force on the beads is always equal to zero.
- Hydrodynamic interaction between beads, e.g. the perturbation of the solvent velocity near a bead due to the motion of the other beads is neglected.
- External forces (e.g. gravity) are independent of the position of the dumbbell. The external forces on the beads of the same dumbbell need not be equal.

Denoting velocity averaged quantities by $\| \|$, the equation of motion of the connector vector $Q$ becomes

$$\| \dot{Q} \| = [\kappa \cdot Q] - \frac{2kT}{\zeta} \frac{\partial}{\partial Q} \ln \psi - \frac{2}{\zeta} \zeta F^{(c)} - \frac{1}{\zeta} [F_{e}^{(c)} - F_{r}^{(c)}]$$

(2.51)

where $\zeta$ is the friction coefficient of the beads in the solvent and $\kappa = (\nabla u)T$ is constant over all space, $\psi(Q, t)$ is the distribution function of the configuration space distribution function $\Psi(\Gamma_1, \Gamma_2, t)$, $F^{(c)} = F_{e}^{(c)} = -F_{r}^{(c)}$ is the connector force on the two beads and $F^{(c)}_{e}$ is the external force on bead $\nu$. The first term on the right-hand side of (2.51) represents the affine motion in the flow field (e.g. the contribution due to hydrodynamic interaction with the surrounding fluid). The second term takes the Brownian motion into account, the third term is due to the spring in the dumbbell. The last contribution stems from the influence of external forces.
Equation of continuity for the configuration-space distribution function $\psi$

The equation of continuity for $\psi(Q, t)$ is given by

$$\frac{\partial \psi}{\partial t} = -\left( \frac{\partial}{\partial Q} \cdot \|Q\| \psi \right)$$

The equation of continuity for the configuration space distribution function is similar to the equation of continuity in hydrodynamics. It essentially states that the distribution function remains normalised to one, while its shape may change under the influence of the different time-dependent forces that act on the dumbbell ensemble.

Diffusion equation for the configuration-space distribution

The diffusion equation for $\psi(Q, t)$ is now obtained by substituting the equation of motion (2.51) for the dumbbell connector vector $Q$ into the equation of continuity (2.52) for $\psi$

$$\frac{\partial \psi}{\partial t} = -\left( \frac{\partial}{\partial Q} \cdot \left\{ [\kappa \cdot \dot{Q}]\psi - \frac{2kT}{\zeta} \frac{\partial \psi}{\partial Q} - \frac{2}{\zeta} F^{(c)}\psi + \frac{1}{\zeta} (F_2^{(e)} - F_1^{(e)})\psi \right\} \right)$$

This equation describes the way in which the distribution of configurations changes with time. The time-dependent homogeneous velocity field is specified by $\kappa(t)$. Finally, if we multiply equation (2.53) by a function of the connector vector $B(Q)$ and integrate over all configuration space, we obtain the equation of change for $\langle B \rangle$. For the special case $\dot{B} = Q$, to which we will return later, we obtain

$$\langle Q, Q \rangle (1) = \frac{4kT}{\zeta} \delta - \frac{4}{\zeta} \langle Q F^{(c)} \rangle + \frac{1}{\zeta} (F_2^{(e)} - F_1^{(e)}) Q + Q (F_2^{(e)} - F_1^{(e)})$$

where subscript 1 indicates the convective derivative.

Stress tensor

The stress tensor is derived from three contributions of the force on the dumbbells:

- the connector force $\tau^{(c)}$;
- the external forces $\tau^{(e)}$;
- bead contributions to the stress $\tau^{(b)}$ due their momentum.

The contribution of the connector force to the stress tensor is

$$\tau^{(c)} = -n \langle Q F^{(c)} \rangle$$

where $n$ is the number of dumbbells per unit volume. The contribution of the external forces $F_1^{(e)}$ and $F_2^{(e)}$ is given by,

$$\tau^{(e)} = \frac{1}{2} n \langle Q (F_2^{(e)} - F_1^{(e)}) \rangle$$
2.7. Elastic dumbbell models

Assuming that the velocity distribution is Maxwellian, the contribution to the stress tensor due
to the bead motion will be isotropic,

$$\tau^{(b)} = 2nkT\delta$$  \hspace{1cm} (2.57)

Thus, assuming a Maxwellian velocity distribution, the expression for the total stress tensor of
the solution becomes

$$\sigma = \tau_{s} - n\langle QF^{(c)} \rangle + \frac{1}{2}n\langle Q[F^{(c)} - F^{(e)}] \rangle + 2nkT\delta$$  \hspace{1cm} (2.58)

where $\tau_{s}$ is the contribution from the Newtonian solvent. Subtracting the corresponding ex-
pression at equilibrium and substituting (2.7) we get the equation for the extra stress tensor,

$$\tau_{p} = -\eta_{s}\dot{\gamma} - n\langle QF^{(c)} \rangle + \frac{1}{2}n\langle Q[F^{(c)} - F^{(e)}] \rangle $$

$$+ nkT\delta$$  \hspace{1cm} (2.59)

where $\eta_{s}$ is the viscosity of the Newtonian solvent. This way of presenting the result is called the
“Kramers form” for the stress tensor. In the absence of external forces, the polymer contribution
to the extra stress tensor is given by

$$\tau_{p} = -n\langle QF^{(c)} \rangle + nkT\delta$$  \hspace{1cm} (2.60)

The Kramers equation for the stress tensor (2.60) and the equation of change for $\langle QQ \rangle$
(2.54) are used to derive different constitutive equations by substituting different descriptions
for the connector force in these general expressions. In the next sections we will present different
choices for the connector force. This leads to different CEs.

2.7.2 The Hookean dumbbell

The simplest choice one can make for the connector force is the Hookean spring, for which the
retractive force depends linearly on the connector vector,

$$F^{(c)} = HQ$$  \hspace{1cm} (2.61)

where $H$ is the spring constant. For this case the Kramers expression for the polymer part of
the stress tensor becomes

$$\tau_{p} = -nH\langle QQ \rangle + nkT\delta$$  \hspace{1cm} (2.62)

With (2.54) we obtain

$$\tau_{p} + \lambda\tau_{p^{(i)}} = -\eta_{p}\dot{\gamma}$$

where $\lambda = \frac{\zeta}{4H}$ and $\eta_{p} = nkT\lambda$. Thus we have recovered the upper convected Maxwell model.

**Giesekus model**

Giesekus (1982) postulated a non-isotropic friction,

$$\zeta^{-1} = \frac{1}{\zeta} \left( \delta - \frac{\alpha}{nkT} \tau_{p} \right)$$  \hspace{1cm} (2.64)
Chapter 2: Fluid theory & models

The connector force in this theory is still Hookean. This expression contains the assumption that the friction for dumbbells having the same orientation as most others is less than for dumbbells not aligned with the stress field. Using the Kramers expression for the stress tensor (2.60) we take the convected derivative (see (2.46), indicated by the subscript \( (1) \)),

\[
\tau_p^{(1)} = -nH\langle QQ \rangle^{(1)} + n\kappa \dot{\gamma} \tag{2.65}
\]

where we used that \( \delta^{(1)} = \gamma \). We obtain an equation for the stress tensor \( \tau_p \) by substituting the result into (2.54), the equation of change for \( \langle QQ \rangle \), which gives,

\[
\lambda_H \tau_p^{(1)} + \tau_p + \alpha \frac{\lambda_H}{\eta_p} \tau_p \cdot \tau_p = \eta_p \dot{\gamma} \tag{2.66}
\]

where \( \lambda_H = \frac{\zeta}{4H} \) is the relaxation time of a dumbbell and \( n\kappa = \eta_p/\lambda_H \) is the shear modulus of a solution of \( n \) dumbbells per unit volume.

The Giesekus model contains three parameters: a relaxation time \( \lambda_H \), the polymer viscosity \( \eta_p \) and the mobility parameter \( \alpha \). Except for the quadratic stress term the Giesekus equation is similar to the UCM model, although both models have been derived from different starting points. This expression (2.66) containing the quadratic stress term, gives more realistic material functions than the UCM model. For instance the Giesekus model predicts that the viscosity and first normal stress difference coefficient decrease with increasing shear rate, whereas in the UCM model they are constant. The behaviour of the one mode Giesekus model in shear for three values of the \( \alpha \) parameter is shown in figure 2.4 where we see the dimensionless viscosity in steady shear, the first normal stress coefficient in steady shear and the dimensionless stress growth, \( \eta'/\eta_0 \) and \( \Psi_1'/\Psi_{1,0} \), for different values of \( \alpha \). In figure 2.5 we see the behaviour in strain flow.

Some typical qualities and disadvantages of the Giesekus model are summarized below:

- For \( \alpha = 0 \) the term \( \tau_p \cdot \tau_p \) disappears and a simple Oldroyd-B model is obtained (see (2.49)). The term \( \tau_p \cdot \tau_p \) gives a decrease in the viscosity and the normal stress coefficients with increasing shear rate (shear thinning behaviour). For all \( \alpha \neq 0 \) the viscosity is proportional to \( \dot{\gamma}^{-1} \).

- For \( \alpha = 1/2 \) the elongational viscosity (defined as \( \eta_e \) for a steady state strain flow) is bounded and reaches a constant value at large strain rates.

- Material functions for the Giesekus model:
  - Small amplitude oscillatory shear flow
    \[
    \eta' = \frac{G\lambda}{1 + (\omega\lambda)^2} \tag{2.67}
    \]
    \[
    \eta'' = \frac{G\lambda^2}{1 + (\omega\lambda)^2} \tag{2.68}
    \]
2.7. Elastic dumbbell models

- Simple shear flow, normalised shear viscosity

\[
\frac{\eta}{\eta_0} = \frac{(1 - n_2)^2}{1 + (1 - \alpha)n_2}
\]  

(2.69)

where

\[n_2 = \frac{1 - \Lambda}{1 + (1 - 2\alpha)\Lambda}
\]  

(2.70)

and

\[\Lambda^2 = \frac{1}{8\alpha(1-\alpha)}\left(\frac{1}{\lambda^\gamma} \sqrt{1 + 16\alpha(1 - \alpha)(\lambda^\gamma)^2} - 1\right)
\]  

(2.71)

The first normal stress difference coefficient is given by

\[\Psi_1 = \frac{n_2^2(1 - \alpha n_2)}{(\lambda^\gamma)^2\alpha(1 - n_2)}
\]  

(2.72)

and the second normal stress difference coefficient,

\[\Psi_2 = \frac{n_2}{(\lambda^\gamma)^2}
\]  

(2.73)

- At high strain values the Giesekus model is not able to predict the observed extensional viscosity (see Tirtaatmadja & Sridhar (1995)).

2.7.3 The non-Hookean dumbbell

For small extensions the Hookean spring seems appropriate. However, the Hookean dumbbell can be extended infinitely, while a real dumbbell cannot be extended beyond its maximum extension, \(Q_0\). One expects the retractive force to strongly increase for extensions that approach this maximum extension, \(Q_0\). An expression for the connector force that contains this behaviour is given by,

\[F(c) = HQ - \frac{(Q/Q_0)^2}{1 - (Q/Q_0)^2}
\]  

(2.74)

where \(Q < Q_0\) (see also figure 2.6). This is the so-called finitely extensible nonlinear elastic (FENE) spring. For the FENE dumbbell model the Kramers expression for the stress tensor reads, analogous to (2.60),

\[\tau_{pq} = -nH\left(\frac{QQ}{1 - (Q^2/Q_0^2)}\right) + nkT\delta_{pq}
\]  

(2.75)

Neglecting external forces, the equation of change for \(\langle QQ \rangle\) (2.54) becomes

\[\langle Q Q \rangle (1) = -\frac{4H}{\zeta}\left(\frac{QQ}{1 - (Q^2/Q_0^2)}\right) + \frac{4kT}{\zeta}\delta_{pq}
\]  

(2.76)
Chapter 2: Fluid theory & models

Figure 2.4: Behaviour of the one mode Giesekus model for three values of the $\alpha$ parameter in simple shear flow. In the top graphs we see the steady state behaviour of the viscosity (left) and the first normal stress difference coefficient as a function of shear rate. In the bottom graphs we see the time dependent behaviour of the viscosity and first normal stress difference coefficient at a fixed shear rate value of 1.

Figure 2.5: Behaviour of the one mode Giesekus model for different values of the $\alpha$ parameter in simple extension flow at a fixed extension rate value of 1. The Trouton ratio is defined as the ratio of the extensional viscosity and the shear viscosity.
2.7. Elastic dumbbell models

It is not possible to derive a constitutive equation from (2.75) and (2.76) by simple substitution. A solution to this problem is obtained by introducing a pre-averaging approximation. The result is called the FENE-P model.

The FENE-P model

Peterlin approximated the average value in equation (2.76) in the following way (see Bird et al. (1980)),

$$\langle \frac{Q Q}{1 - (Q^2/Q_0^2)} \rangle = \frac{\langle Q^2 \rangle}{1 - \langle Q^2/Q_0^2 \rangle}$$

(2.77)

Substituting equation (2.77) into the Kramers equation for the stress tensor (2.75) we obtain

$$\tau_p = -nH \frac{\langle Q Q \rangle}{1 - \langle Q^2/Q_0^2 \rangle} + nkT\delta$$

(2.78)

Moreover, the Giesekus equation for the stress tensor is applicable:

$$\tau_p = \frac{n\zeta}{4} \langle \frac{Q Q}{(1)} \rangle + nkT\delta$$

(2.79)

If we look at the equilibrium situation, where $\tau_p = 0$, equation (2.78) becomes,

$$nH \frac{\langle Q Q \rangle_{eq}}{1 - \langle Q_{eq}^2/Q_0^2 \rangle} = nkT\delta$$

(2.80)

Taking the trace of equation (2.80),

$$nH \frac{Q_{eq}^2}{1 - Q_{eq}^2/Q_0^2} = 3nkT$$

(2.81)
equation (2.81) can also be written as,

\[
\frac{Q_{eq}^2}{1 - Q_{eq}^2/Q_0^2} = 3 \frac{kT}{HQ_0^2} \equiv \frac{3}{b} \tag{2.82}
\]

where we have introduced the dimensionless ratio \( b \) defined as,

\[
b = HQ_0^2/(kT) \tag{2.83}
\]

Using the dimensionless ratio \( b \), we can write equation (2.82), as follows,

\[
\frac{Q_{eq}^2}{Q_0^2} = \frac{3}{b + 3} \tag{2.84}
\]

Or the equilibrium length of the dumbbell is,

\[
Q_{eq}^2 = \frac{kT}{H} \frac{3b}{b + 3} \tag{2.85}
\]

One now introduces a dimensionless tensor \( \mathbb{b} \), by

\[
\mathbb{b} = \frac{3 \langle Q Q \rangle}{Q_{eq}^2} \tag{2.86}
\]

Substituting the previous result into \( \mathbb{b} \),

\[
\mathbb{b} = \frac{H}{kT} \frac{b + 3}{3b} \langle Q Q \rangle \tag{2.87}
\]

or

\[
\langle Q Q \rangle = \frac{kT}{H} \frac{3b}{b + 3} \mathbb{b} \tag{2.88}
\]

Again we take the trace of equation (2.88),

\[
\langle Q^2 \rangle = \frac{kT}{H} \frac{3b}{b + 3} tr(\mathbb{b}) \tag{2.89}
\]

Using the Peterlin approximation for the stress tensor (2.78) and the Giesekus equation for the stress tensor (2.79),

\[
\zeta \frac{4H}{\langle Q Q \rangle} (1) = \frac{\langle Q Q \rangle}{1 - \langle Q^2/Q_0^2 \rangle} + \frac{kT}{H} \delta \tag{2.90}
\]

Substituting equation (2.87) and (2.89) into equation (2.90),

\[
\frac{\zeta}{4H} \frac{b}{b + 3} (1) = -\frac{kT}{H} \frac{b}{b + 3} b(1 - \frac{kT}{HQ_0^2} \frac{b}{b + 3} tr(\mathbb{b}))^{-1} + \frac{kT}{H} \delta \tag{2.91}
\]
2.7. Elastic dumbbell models

Or in short,

\[ \lambda_{b(1)} = -bf + \delta \]  \hspace{1cm} (2.92)

where \( \lambda \) is a time constant defined as \( \lambda = \frac{\zeta H}{n b + 3} \), \( nkT = \frac{\eta}{\lambda} \) and \( f \) is given by

\[ f = \frac{b}{b + 3} (1 - \frac{1}{b + 3} \text{tr} b)^{-1} \]  \hspace{1cm} (2.93)

The stress tensor becomes,

\[ \tau_p = \frac{n\zeta}{4} (Q Q)_{(1)} = \frac{n\zeta kT}{4} \frac{b}{b + 3} b_{(1)} = nkT \zeta \frac{b}{4H b + 3} b_{(1)} = nkT \lambda b_{(1)} \]  \hspace{1cm} (2.94)

Substituting in equation (2.92) and multiplying by \( nkT = \eta/\lambda \) we find a constitutive equation for the Peterlin approximation, the FENE-P model,

\[ \tau_p = -nkTbf + nkT \delta = -\frac{\eta}{\lambda} (bf - \delta) \]  \hspace{1cm} (2.95)

The FENE-P model contains three parameters: a relaxation time \( \lambda_H \), the dimensionless energy ratio \( b = HQ_0^2/kT \) and the Brownian motion energy \( nkT \), where \( n \) is the number of dumbbells per unit volume. The FENE-P model has non-linear terms in \( \tau_p \). The behaviour of the FENE-P model in shear flow is shown in figure 2.7 and in figure 2.8 we see the behaviour in strain flow. Furthermore the FENE-P model has the following properties:

- For small dumbbell deformations \( b \ll \text{tr}(b) \) a multi-mode Oldroyd-B model is obtained.
- In steady-state shear flow the viscosity behaves like,

\[ \eta - \eta_s = \begin{cases} 
    nkT\lambda_H \left( \frac{b}{b + 3} \right) & \text{for } \dot{\gamma} = 0 \\
    \sim nkT\lambda_H \left( \frac{\sqrt{b/6}}{\lambda_H(6+2\zeta)} \right)^{2/3} & \text{for } \dot{\gamma} \to \infty
  \end{cases} \]

- At high strain the FENE-P model is not able to predict the extensional viscosity (see (Tirtaatmadja & Sridhar 1995)).

The Verhoef model

A special case of the FENE dumbbell is the Verhoef model, which was introduced by Verhoef et al. (1998). In the FENE dumbbell model an additional internal spring force is introduced. Verhoef assumed that the contribution of the viscous connector force (denoted with subscript \( \nu \)) to the total connector force \( F^{(c)} \) is directed along the line connecting two beads and is proportional to the deformation rate tensor \( Q Q \), i.e.

\[ F^{(c)} = \mu (Q Q : \dot{\gamma}) Q \]  \hspace{1cm} (2.96)
Figure 2.7: Behaviour of the one mode FENE-P model for four values of the $b$ parameter in simple shear flow. In the top graphs we see the steady behaviour of the viscosity (left) and the first normal stress difference coefficient as a function of the shear rate. In the bottom graphs we see the time dependent behaviour of the viscosity and first normal stress difference coefficient at a fixed shear rate value of 1.

Figure 2.8: Behaviour of the one mode FENE-P model for different values of the $b$ parameter in simple extensional flow at a fixed extension rate value of 1.
where $\mu$ is a viscous parameter. The spring force (denoted with subscript $e$) is still assumed to be directed along the line connecting two beads. The magnitude of the force is some function of the connector length of the dumbbell. The spring force might for instance have FENE behaviour. Generally it is written as

$$F_e^{(c)} = H(Q^2)Q$$

(2.97)

The dumbbell connector force then becomes

$$F^{(c)} = [H(Q^2) + \mu(Q Q : \dot{\gamma})]Q$$

(2.98)

The Kramers expression for the stress tensor (2.60) reads

$$\tau_p = -n\left[H(Q^2) + \mu(Q Q : \dot{\gamma})\right]Q + nkT\delta$$

(2.99)

If we again neglect external forces, the equation of change (2.54) reads

$$\langle Q Q \rangle^{(1)} = -4\zeta\left[H(Q^2) + \mu(Q Q : \dot{\gamma})\right]Q + 4kT\zeta \delta$$

(2.100)

In order to derive a constitutive equation Verhoef introduced the following approximation for the average value:

$$\left[H(Q^2) + \mu(Q Q : \dot{\gamma})\right]Q = \left[H(Q_0^2) + \mu(Q Q : \dot{\gamma})\right]Q$$

(2.101)

Substituting (2.101) into the Kramers equation (2.99) we obtain

$$\tau_p = -n\left[H(Q_0^2) + \mu(Q Q : \dot{\gamma})\right]Q + nkT\delta$$

(2.102)

And the equation of change (2.100) becomes

$$\langle Q Q \rangle^{(1)} = -4\zeta\left[H(Q_0^2) + \mu(Q Q : \dot{\gamma})\right]Q + 4kT\zeta \delta$$

(2.103)

Verhoef then made a first order approximation for the non-linear elastic connector force

$$F_e^{(c)} = H(Q^2)Q = \left[H_0^{(c)} + H_1^{(c)}(\langle Q^2 \rangle - \langle Q_{eq}^2 \rangle)\right]Q$$

(2.104)

where $\langle Q_{eq}^2 \rangle$ is the mean square FENE dumbbell length in the equilibrium state of the fluid. And used this to introduce the tensor $b$

$$b = \frac{H_0}{kT}Q$$

(2.105)
with trace,
\[ \text{tr} \mathbf{b} = \frac{H_0}{kT} \langle Q^2 \rangle \] (2.106)

We can rewrite (2.102) in terms of tensor \( \mathbf{b} \), which gives
\[ \tau_p = -n \left[ g(\text{tr} \mathbf{b}) + \mu \frac{kT}{H_0} (\mathbf{b} : \dot{\gamma}) \right] \frac{kT}{H_0} \mathbf{b} + nkT \delta \] (2.107)

and for (2.103), the equation of change, we obtain
\[ \lambda \mu \mathbf{b}^{(1)} = - \left[ g(\text{tr} \mathbf{b}) + \mu \frac{kT}{H_0} (\mathbf{b} : \dot{\gamma}) \right] \mathbf{b} + \delta \] (2.108)

where we have used the time constant \( \lambda = \frac{\zeta}{4H_0} \) and the elastic function \( g(\text{tr} \mathbf{b}) \) is determined by the function \( H(\langle Q^2 \rangle) \)
\[ H(\langle Q^2 \rangle) = g(\text{tr} \mathbf{b}) = H \left( \frac{kT}{H_0} \text{tr} \mathbf{b} \right) \] (2.109)

The Verhoef model has a finite upper-bound for \( \text{tr} \mathbf{b} \) (see (Verhoef et al. 1998)) and Verhoef assumes that the deviations from the Hookean spring are small and that the nonlinear spring force can be described by a linear function for \( H(\langle Q^2 \rangle) \)
\[ g(\text{tr} \mathbf{b}) = H_0 + H_1 \left( \frac{\text{tr} \mathbf{b}}{3} - 1 \right) \] (2.110)

See for more details (Verhoef et al. 1998).

The Verhoef model is similar in behaviour to the FENE-P model in shear flows. However, in extensional flow the prediction of the new model is better in relaxation after stretching. In figure 2.9 we see the behaviour of the Verhoef model in strain flow. Other typical disadvantages and qualities of the Verhoef model are:

- For small deformations and deformation rates the Verhoef model reduces to the Oldroyd-B model.
- In steady shear flow FENE-P model is identical to the Verhoef model if we take \( H_1 = 0 \).
- Finite extensibility is not explicitly included. However for \( \dot{\epsilon} \to \infty \) the extensional viscosity reaches the asymptotic value \( \eta_e = 2\eta \mu / \mu^\ast \).
- Predictions in transient extensional flow are good and comparable to those of the FENE model. The Verhoef model shows good agreement with experiments over a large range of strains and strain rates in start-up and relaxation.
- If we choose the parameter \( H_1 = 0 \), in shear flow we have \( \mu = \lambda / L^2 \) where \( L^2 = b + 3 \) from FENE-P.
2.8 Fluid flow around objects: theory & results

We briefly describe the theory of the Newtonian laminar flow around a cylinder. Subsequently we discuss the non-Newtonian theory, which we have divided into five subcategories: i) The viscoelastic flow around a free cylinder, ii) the viscoelastic flow around a confined cylinder, iii) viscoelastic flow around an array of cylinders, iv) the falling sphere in a viscoelastic liquid and v) the problem of two or more spheres falling in a viscoelastic liquid.

2.8.1 Newtonian theory

Lamb determined the drag force \( F \) on a cylinder using Oseen’s equation, which takes inertia terms partially into account (Lamb 1932). The dimensionless drag coefficient per unit length of the cylinder is

\[
C_d = \frac{F}{\eta l_{cyl}} = \frac{4\pi}{\frac{1}{2} - \gamma - \ln \frac{l_{cyl}}{4\eta}}
\]

(2.111)

where \( \gamma \) is Euler’s constant (\( \gamma = 0.5772 \)), \( \rho \) is the density and \( \eta \) is the viscosity of the fluid, \( d \) is the diameter of the cylinder, \( l_{cyl} \) its length and \( U \) is the fluid velocity, at infinity also called the free stream velocity. This equation is consistent with observations of Tritton (1959) as seen in figure 2.10. Tritton measured the drag on circular cylinders by observing the bending of quartz fibres in a flow with Reynolds numbers in the range 0.5-100.

Faxen (1946) considered the 2D problem of a circular cylinder between two parallel walls in great detail (see Happel & Brenner (1965) and Tritton (1959)). He looked at the problem of a moving cylinder and a motionless cylinder where the fluid is forced between two plates. Using complex function theory and the concept of potential flow (see (Batchelor 1967)), Faxen derived a series expansion for the resistance of a cylinder per unit length, up to \((d/h)^8\), where
Figure 2.10: Comparison of the drag coefficient of a free cylinder measured by Tritton (1959) with the theory by Lamb using the Oseen equation. One can see that the Oseen equation is valid up to a Reynolds number of approximately 0.1.

Figure 2.11: Drag coefficient determined from the series expansion of Faxen in equation (2.112) as a function of $d/h$ the cylinder-diameter to channel-height ratio compared to simulations of Liu et al. (1998) and simulations presented in this thesis.
2.8. Fluid flow around objects: theory & results

\( d \) is the diameter of the cylinder and \( h \) the distance between the walls,

\[
C_d = \frac{F/l}{\eta U} = \frac{4\pi}{U_0 - (1 + 0.5(d/h)^2 + W_4(d/l)^4 + \ldots) \ln(d/h) + V_2(d/h)^2 + \ldots}
\] (2.112)

where \( U_0, W_4 \) and \( V_2 \) are constants. In figure 2.11 the result of this series expansion is shown, compared with simulations of Liu et al. (1998). In table 2.1 we have collected the results of various drag coefficients \( C_d \) for a cylinder in a confined flow.

<table>
<thead>
<tr>
<th>( d/h )</th>
<th>author</th>
<th>( C_d )</th>
<th>remarks</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.5</td>
<td>Faxen</td>
<td>92.32</td>
<td>approximate result</td>
</tr>
<tr>
<td>0.5</td>
<td>Liu</td>
<td>88.23</td>
<td>in/out-flow boundary conditions</td>
</tr>
<tr>
<td>0.5</td>
<td>Liu</td>
<td>83.63</td>
<td>periodic boundary conditions</td>
</tr>
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<td>0.5</td>
<td>This thesis</td>
<td>88.25</td>
<td>One cylinder</td>
</tr>
<tr>
<td>0.5</td>
<td>This thesis</td>
<td>85.70</td>
<td>Two cylinders, ( L=3r )</td>
</tr>
<tr>
<td>0.5</td>
<td>This thesis</td>
<td>87.84</td>
<td>Two cylinders, ( L=4r )</td>
</tr>
<tr>
<td>0.5</td>
<td>This thesis</td>
<td>88.25</td>
<td>Two cylinders, ( L=6r )</td>
</tr>
</tbody>
</table>

Table 2.1: Drag coefficients of a confined cylinder (pair) in a Newtonian solution. The ratio of the cylinder diameter to the channel height is \( d/h = 0.5 \). \( L \) is the distance between the cylinder axes. The results of the approximate series expansion of the drag formula of Faxen (1946) and the results of the simulations of Liu et al. (1998) are compared with simulations results presented in this thesis. The latter were obtained with 2D Dynaflow simulations using periodic boundary conditions.

2.8.2 Non-Newtonian theory

In non-Newtonian theory one discerns roughly three types of fluids which are classified by their behaviour in a simple shear experiment:

1. A viscoelastic fluid, that exhibits shear thinning properties as well as elastic properties. The shear viscosity decreases with increasing shear rate. The elastic effects are expressed by the normal stress differences \( N_1 \) and \( N_2 \).

2. A shear thinning fluid, which exhibits shear thinning properties but hardly any normal stress effects.

3. A Boger fluid shows an almost constant shear viscosity (with increasing shear rate) in combination with elastic properties. \(^1\)

Together with Newtonian fluids this makes four fundamentally different fluid types.

**Free cylinder**

The effect of elasticity in combination with inertia was examined by James & Acosta (1971). Drag experiments in dilute aqueous polyethyleneoxide solutions (PEO) showed that above a certain critical velocity the drag became independent of the free stream velocity. Compared to

\(^1\)It was found by Prilutski et al. (1983) that generally such a fluid can be obtained by adding a small amount of a high molecular weight polymer to a highly viscous Newtonian fluid. A solution of 0.31% polyisobutylene tetradecane (C14) added to 94.86% polybutene (PB) is an example of this type of fluids.
pure Newtonian flow, the drag was about a factor of 10 higher. Also heat loss experiments by James & Acosta (1970) and Piau (1980) revealed the existence of this critical velocity. Above the critical velocity the Nusselt number is also independent of the velocity. Where the Nusselt number is defined as

$$\text{Nu} = \frac{h d}{k} = \frac{\text{total heat transfer}}{\text{conductive heat transfer}}$$

(2.113)

where $h$ is the heat transfer coefficient and $k$ the thermal conductivity and $d$ is the diameter of the cylinder. Above this critical velocity the heat loss, compared to Newtonian flow, is reduced by about 70%.

An explanation for the phenomenon that above a critical velocity the drag or heat loss becomes independent of the free stream velocity, was given by Koniuta et al. (1980) and Manero & Mena (1981). They showed with LDV measurements that the wake of the cylinder becomes broader and longer than in a Newtonian fluid. The flow region disturbed by the cylinder in a polyethylene oxide solution in water is roughly an order of magnitude larger than in water, which accounts for the factor of 10. Flow visualisation experiments of Manero & Mena (1981) revealed that the observed critical phenomena occurred at Deborah numbers above 1.

The effect of elasticity without inertia (at very low flow velocities; creeping flow regime) was examined by Broadbent and Mena (1974). They found a small drag reduction in aqueous solutions of polyacrylamide and noticed that the region disturbed by the cylinder is similar to that region in a Newtonian fluid.

Viscoelastic flow with inertia effects was simulated by Hu & Joseph (1990). Using UCM and Oldroyd-B models (both CEs incorporate elastic properties but give a constant viscosity) they tried to describe the experimental results obtained by James & Acosta (1970). The results are shown in figures 2.12 and 2.13. The simulations show a qualitative representation of the features measured by James & Acosta, although the predicted increase in the drag for high Reynolds numbers is not measured as we see in figure 2.12. The quantitative agreement is also unsatisfactory.

Cylinder confined by walls

Extensive drag measurements were performed by Dhahir & Walters (1989) on a cylinder with considerable wall effects. The ratio cylinder diameter to channel width was only 0.5 in their experiments. They used the following four fluids:

1. a Newtonian fluid of water in maltose syrup;
2. a Boger fluid consisting of polyacrylamide in a mixture of water and maltose syrup;
3. a shear thinning solution with negligible elastic properties, of Xanthan gum (2% and 3%) in water;
4. a viscoelastic fluid, with elasticity and shear thinning effects, of polyacrylamide (1.5% and 2%) in water.

The Boger fluid shows a decrease in drag on the cylinder by approximately 20% as compared to Newtonian fluid. The drag is decreased about 50% in the shear thinning solution. If elasticity is introduced, slightly larger drag reduction is seen: roughly 55%.
Numerical simulations were used by Huang & Feng (1995) to study the viscoelastic fluid flow around a cylinder confined by two parallel walls. They applied three types of fluid models: a Newtonian fluid, a Carreau liquid (shear thinning) and an Oldroyd-B fluid (Boger). First they studied the influence of the wall proximity. The results indicated that the influence of the wall is to shorten the region disturbed by the cylinder and to increase the drag on the cylinder. The drag is increased up to 100% at a We number of 1 and at a blockage ratio $\beta = d/h$ where $h$ is the channel width and $d$ the diameter of the cylinder of. Next the results of the different fluid models were compared in the presence of walls. It appeared that the drag in the shear thinning Carreau fluid and the elastic Oldroyd-B fluid is smaller than in the Newtonian fluid.

Recently quite some experiments and simulations have been done using a shear thinning polyisobutylene (PIB) in tetradecane (C14) solution. The experimental results of the velocity fields and the stress fields (see (Baaijens et al. 1994), (Baaijens 1994), (Baaijens et al. 1995), (Baaijens et al. 1997) and (Schoonen 1998)) show good agreement with numerical predictions. A generalised Newtonian model could also describe this velocity field accurately, but of course fails to predict the normal stresses. The elongational viscosity of the model fluid used depends only weakly on elongation rate. Hence the predictions in the wake of the cylinder, where the strongest elongational flow occurs are good. The simulations (see also Barakos & Mitsoulis (1995)) were done with a four mode PTT and Giesekus model. The drag was not measured, but the predictions show that the drag decreases with increasing Weissenberg number.
Other experiments using a constant viscosity Boger fluid of 0.31% polyisobutylene (PIB) in a PB/C14 solvent (by McKinley et al. (1992) and (1993)) showed elastic instabilities at Weissenberg numbers above approximately one. Because the instabilities already occur at low Reynolds numbers, they cannot be due to inertial effects. It is plausible to suggest that the instabilities are therefore caused by elastic effects only.

**Packed cylinders in a channel**
Similar to the geometry of a confined cylinder is the problem of an array of cylinders, arranged in a rectangular channel. This type of geometry closely resembles flow through porous media. Experiments on Boger fluids show a large pressure drop compared to the Newtonian case (see Skartsis et al. (1992), Chmielewski & Jayaraman (1993) and Vossoughi & Seyer (1974)). As the friction factor is proportional to the pressure drop, this means that the resistance of the flow should increase substantially above some critical Weissenberg number. However, simulations with UCM and Oldroyd-B models do not show a pressure drop at all (see Talwar & Khomami (1992) and (1995)).

Recent experiments by Khomami & Moreno (1997) using Boger fluids show elastic instabilities, as in the case of a single confined cylinder. They concluded that these elastic flow instabilities are responsible for the increase in flow resistance. Moreover, the same authors found that different cylinder packing geometries led to different types of instabilities. In a densely packed channel (low porosity) the flow changes from a steady two-dimensional flow to a three-dimensional, unsteady flow. In a less densely packed channel (higher porosity) the flow changes to a steady three-dimensional flow. In the latter case, one sees the development of periodic cellular flow structures along the cylinder axis.

**Creeping flow around a sphere**
The flow around a sphere is, from a certain point of view, the three-dimensional equivalent of a flow past an infinitely long cylinder. The problem of a sphere in a cylindrical tube with a ratio of the sphere diameter to the cylindrical tube diameter of 1:2 is generally accepted as the benchmark system for numerical viscoelastic flow simulations. The first simulations on this system were performed using elastic, constant viscosity models, like UCM and Oldroyd-B. To compare the simulations with experiments, David Boger (1977) composed the well-known solution of polyacrilamide in corn syrup.

A Boger fluid, known as fluid M1, was used in experiments by Tirtaatmadja et al. (1990). At low Weissenberg numbers a small drag reduction was seen, followed by a large drag increase at higher Weissenberg numbers (see figure 2.14). Similar experiments were done by Bisgaard and Hassager (1982) although they used very concentrated shear thinning solutions. The results of their experiments showed that the velocity in the wake was directed in the opposite direction of the flow. They called this phenomenon a "negative wake". This so-called negative wake was not seen in the experiments of Tirtaatmadja.

Simulations with the UCM and the Oldroyd-B model were compared to experiments using a polyisobutylene Boger fluid (Becker et al. 1994). At low Weissenberg numbers the Oldroyd-B simulations do not even qualitatively agree with the experiments. The experiments of a

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2The first Boger fluid consisted of 500 ppm PIB and 97/3 % PB/kerosene mixture, the second of 1000 ppm PIB and 98.2/1.8 wt% PB/kerosene mixture. PB with a MW of 950 and PIB with a MW of 4.7 million.

3Fluid M1 consists of 0.244% PIB, 6.98% kerosene and 93% Hyvis 3
falling sphere show an overshoot in the falling velocity, which then slowly approaches a steady state. This qualitative behaviour is predicted by the Oldroyd-B model, but for a quantitative prediction of transient phenomena in viscoelastic liquids a multi-mode formulation is necessary.

Experiments and simulation results for a falling sphere were presented by Arigo et al. (1996) and Rajagopalan et al. (1996). The simulations using the Oldroyd-B model, the Chilcott-Rallison and the 4-mode PTT model provide a good representation of the drag. The drag curves only deviate from experiments at the point where numerical convergence could no longer be achieved (see figure 2.15). For low Weissenberg numbers the axial velocity predicted by the Chilcott-Rallison model shows good agreement with LDV measurements. However, for We > 1 the quantitative agreement is lost. The agreement between experiments and simulations is encouraging, although close quantitative agreement requires computations with a multi-mode nonlinear model. Unfortunately, multi-mode simulations can be performed for relatively small Weissenberg numbers only.

Rajagopalan et al. (1996) compared the results of simulations using a quasi-linear continuum model (Oldroyd-B), a nonlinear dumbbell model (Chilcott-Rallison) and a nonlinear network model (PTT) with experiments. The dumbbell model and the network model give improved predictions (compared to UCM and Oldroyd-B models) if a spectrum of time constants is used.

Experiments using another Boger fluid, polyacrylamide (Separan AP30 in water) dissolved in corn syrup (Staley 200), show a different drag behaviour on the falling spheres as the PIB in PB Boger solution. Drag reduction is obtained for We > 0.1 (Chhabra et al. 1980) (see figure 2.14). A comparison of these experiments with an analysis of Chilcott and Rallison (1988) suggests that the PIB molecules in PB are stretched more by flow than are the PAA molecules in corn syrup. Measurements of the intrinsic viscosity $[\eta]$ indicate that the aqueous corn syrup solution is a better solvent for PAA than the PB is for PIB. Hence in equilibrium PAA will be more extended than PIB so that by a small additional stretching caused by the flow, the PAA molecules already enter the nonlinear regime where drag reduction takes place.

4The intrinsic viscosity is defined as $[\eta] = (\mu - \mu_s)/(c\mu_s)$ where $\mu$ is the total viscosity, $\mu_s$ is the viscosity of the solvent and $c$ the concentration of the polymer.

5High extensibility means that up to higher extension rates the polymers are stretched further, and therefore show extension hardening effects, until at full extension of the polymer chains a constant extension viscosity is reached. In the Chilcott-Rallison model the extensibility is defined by the parameter $L$, which is the ratio of the fully extended chain length to the equilibrium coil length. In the CR model this the single parameter controlling the extensibility of the chain. For $L \to \infty$ the model reduces to the Oldroyd-B model.
Figure 2.14: The drag ratio $X_e$ as a function of the Weissenberg number for spheres moving through the PAA-CS and the PIB-PB solutions. The difference in drag might be explained by the elongational flow characteristics (see figure 2.16). The drag ratio is defined as $X_e = \frac{C_D}{C_{Ds}}$, where $C_D$ is the drag coefficient of a sphere translating in Boger fluid and $C_{Ds}$ is the drag on a sphere translating in a Newtonian fluid. (Obtained from Solomon & Muller (1996)).

Figure 2.15: Viscoelastic drag correction factor $K$ for a 0.31% PIB in PB Boger fluid. Numerical simulations are shown for the Oldroyd-B correction factor is defined as $K = \frac{U_{Stokes}}{U_s}$, (Old-B) model, the Chilcott-Rallison model (CR) and the 4-mode PTT model. Obtained from Arigo et al. (1996). The drag where $U_{Stokes}$ is the Stokes settling velocity based on the zero shear rate value for the viscosity ($\eta_0$) and $U_s$ is the measured steady-state velocity.
shear thinning with hardly any elasticity.

Also Bot et al. (1998) did not find a critical distance. Experiments in a Boger fluid, consisting of 9 g partially hydrolysed polyacrylamide in 3 kg water and 57 kg glucose syrup, showed that if two spheres are far apart, the wake of the lower sphere induces a higher velocity for the upper sphere. For small distances the spheres separate because the velocity of the lower sphere increases considerably. As a result the spheres attract for large distances but separate for small distances. Eventually these two effects result in a stable distance.

Experiments of Mariappan et al. (1998) in a 0.3 wt% aqueous Carbopol-941 dispersion show that settling results of two particles are consistent with the hypothesis of network damage caused by shear with subsequent healing. The network damage can be ascribed to the shear experienced by the fluid as the spheres moves by. Evidence is found by the observation that the steady terminal settling velocity of the sphere was seen to decrease with increasing time intervals between the moments that the spheres were dropped. Moreover, it was noticed that denser spheres, causing a higher shear rate, have a higher settling velocity than the less denser spheres.
Chapter 2: Fluid theory & models
Experimental and numerical approach

In this chapter we describe the experimental arrangements, the equipment and the procedures used in the experiments. Also the choice of the dimensions of the channel and the cylinder geometry is discussed. The design of the force transducer is explained in more detail as it was specially designed and constructed for these experiments. Finally, the numerical approach and simulation tools are described briefly. The numerical methods used are quite common, so that we restrict ourselves here to a brief description with proper references.

3.1 Experimental approach

A schematic overview of the flow circuit is shown in figure 3.1. The fluid is stored in the reservoir (1) and transported with the pump (2) via a flow meter (3) into the test section (4) and back into the reservoir. The fluid is kept at a constant temperature by means of an external thermostatic device (5). The flow velocity is measured with the Laser Doppler Velocimetry (LDV) technique (6). To determine the drag force on the cylinder, a special force transducer (7) was designed and built. The signals from the LDV apparatus and force transducer are directed to data acquisition (8) system consisting of an A/D converter, a personal computer and a line plotter.

3.1.1 Set-up

In the following we describe these parts in more detail. Specifications of the LDV set-up and the design of the force transducer are explained in more detail in appendix A and B, respectively.

Fluid reservoir (1). Since we want to use a re-circulatory test facility it is possible that mechanical degradation of the polymer solution occurs. Due to the action of the pump the long polymer molecules break up in smaller molecules and hence the properties of the solution change. This can be checked by for instance measuring the pressure drop as a function of time, as was done by Toonder et al. (1995), or by taking samples of the solution from time to time and measure the properties. The fluid reservoir has been given a capacity of 80 l. In the reservoir two glass spiral heat exchangers control the temperature of the fluid.

Pump (2). The pump was selected according to the following criteria. First the pump had to meet obvious requirements as the possibility of a variable flow rate and a flow capacity up to 1 l/s. But the most important requirement the pump has to meet, is minimal degradation of the polymer solution. This has led to the choice of a so-called rotary pump from Nakamura (type RM-40-VT). The operation of the pump is based on the use of a pair of two vanes that in turn displace a compartment of fluid. Therefore this pump is able to pump liquids containing large solid material, such as porridge with strawberries, without crushing them.

The rotary pump from Nakamura has a maximum capacity of 2 l/s. The pump is driven by a planetroll variator (type 2.2 D4-ARG-3), 0-550 revolutions per minute. The motor is controlled
Figure 3.1: Schematic representation of the flow circuit. The test section with the force transducer and the two cylinders can be closed by two valves so that it is possible to empty the channel and replace the cylinders without having to empty the entire flow circuit.

by a Hitachi frequency regulator (type J100-022-SFE2) which is shielded from electromagnetic interference. The frequency control unit, the variator and the pump are mounted on a separate platform to minimise transmission of pump vibrations to the experimental set-up.

**Flow meter (3).** To record the flow rate a magnetic inductive flow meter was used (Altimeter SC 100 AS). As this type of flow meter is actually designed for turbulent flow measurements, we merely used it to roughly estimate the flow rate. The actual flow rate was determined by integration of the velocity profile, obtained from LDV measurements.

**Test section (4).** The test section consists of a rectangular channel in which one or two cylinders can be placed half way the channel (see figure 3.2). The dimensions of the channel are based on two requirements: We want to create a two dimensional flow in order to validate the two-dimensional calculations. We also want to achieve a fully developed flow near the cylinder as well as a long exit length to enable the viscoelastic fluid to relax in the wake of the cylinder. The walls of the test section have to be transparent to allow optical experiments. The cylinder
3.1. Experimental approach

Figure 3.2: Dimensions of the rectangular channel. Also some measurement results of the stream-wise velocity component $u$ are shown. The measurements have been performed in an empty channel. In the empty channel the flow near $x = 0$ is fully developed, and is, for $z$ values around the mid-depth $z = 0$, independent of the $z$-coordinate.
axis has to be placed in the vertical direction, in this way gravity has no influence on horizontal drag force measurements.

The aspect ratio of the channel cross-section is based on other experiments with a similar geometry as presented by McKinley et al. (1992), (1993) and Baaijens et al. (1994), (1994) and (1995). In those papers a channel with a cross-section of 0.16 m in height and 0.02 m in width was used as we also did in our experiments.

The choice of the length of the channel is based on literature concerning entry lengths in channels and tubes see Damsteeg (1984), Collins & Schowalter (1963) and Brocklebank & Smith (1968), and on the basis of the influence of the cylinder in upstream and downstream direction (Huang & Feng 1995). Numerical simulations by Damsteegt for a Newtonian fluid in a 2D channel flow show that to reach 99% of the free stream velocity is obtained at entry lengths (in terms of the Reynolds number) of

$$l_e = \frac{1.24}{0.06 Re + 1} + 0.088 Re$$

Thus we find that for Reynolds numbers lower than 1 the entry length necessary to reach 99% of the free stream velocity is of the same order of magnitude as the width of the channel. There are not many experimental data on entry lengths of viscoelastic fluids. The experimental data of Brocklebank & Smith (1968) for a tube flow are plotted in figure 3.3. They used three test fluids: a Newtonian fluid of glycerol ($\eta = 0.095 \text{ Pa.s}$), a polyacrylamide (Cyanamer P250) solution of 1.6% in water ($\eta_0 = 0.6 \text{ Pa.s}$) and a polyethylene oxide solution (Polyox WSR 301)
3.1. Experimental approach

of 0.6% in water (\(\eta_0 = 0.606\text{ Pa.s}\)). The results in figure 3.3 indicate that the entry length in a polymer fluid does not differ very much from that of a Newtonian fluid. We see that also in a polymer fluid the entry length increases with the \(Re\) number.

To see if a fully developed velocity profile is obtained, velocity profiles were measured in an empty channel using a Newtonian solution of glycerol. In figure 3.2 we have indicated the directions of the coordinates system axis. The \(x\) axis is in the direction of the free stream velocity, the \(y\) axis is perpendicular to the free stream direction and the \(z\) axis is parallel to the cylinder axis. In figure 3.2 we also plotted the results of the velocity component in stream-wise direction. Three velocity profiles have been plotted at three different stream-wise positions. In the \(y\) direction a fully developed parabolic velocity profile is found. In the neutral \(z\) direction the velocity profile is flat in the middle, so that a two dimensional flow is obtained.

Next we performed some experiments using a viscoelastic fluid in order to test the equipment and to determine the criteria for the model fluid for our measurements. The fluid consists of 0.1% A-110 polyacrylamide dissolved in distilled water. It has a zero shear viscosity of \(\eta_0 = 11.23\text{ Pa.s}\) and a shear thinning parameter of \(n = 0.378\) (see eq. 2.25), hence it has extremely shear thinning properties. Results of velocity measurements in an empty channel are plotted in figures 3.4 and 3.5 and compared to velocity profiles of the Newtonian fluid glycerol. The differences in velocity profiles along both the \(y\) axis and the \(z\) axis are striking. While in the \(y\) direction the velocity profile of the polymer solution is flattened, in the \(z\) direction the opposite happens. To check whether the effect seen in figure 3.4 is caused by shear thinning rather than elastic effects we performed numerical calculations using a Carreau model.

![Figure 3.4: LDV measurements of the stream-wise velocity component (u) as a function of y position for a fully developed two-dimensional channel flow. The fluids are a Newtonian fluid (glycerol) and a viscoelastic fluid (0.1% A-110 in tap water). Both the measurements in glycerol and polyacrylamide (A-110) have been performed in an empty channel at approximately the same flow rate.](image-url)
Chapter 3: Experimental and numerical approach

The simulations were performed in an empty channel and the results in figures 3.6 and 3.7 are for a fully developed flow. The shear thinning parameter $n$ has been varied from $n = 1$ (Newtonian viscosity) to $n = 0.4$. The results in figure 3.6 reveal that the Newtonian velocity profile in the $y$ direction has a parabolic shape which flattens at the channel axis and steepens near the walls with increasing shear thinning parameter. In the $z$ direction the velocity profile becomes less steep near the channel wall and more curved in the middle of the channel which is consistent with the experiments. Moreover, these simulations indicate that the shear thinning parameter $n$ of the model fluid must be between 1 and 0.8. A shear thinning parameter smaller than 0.8 would lead to a boundary layer thickness in the $z$ direction that would be too large to maintain the assumption of a nominally 2D flow.

Next we performed velocity measurements in the presence of one cylinder. In particular we looked at the magnitude of the stream-wise velocity component ($u$) in the centre of the channel ($y = 0$), as a function of the height $z$. The results in a Newtonian fluid are presented in figures 3.8 and 3.9. As the Newtonian liquid reaches the cylinder, not only does its velocity decrease, but what is even more important, also the boundary layer thickness decreases. If we look at the velocity component in the $z$ direction ($w$), parallel to the cylinder axis, we see that far in front of the cylinder this velocity component is practically zero as we expect. However, to our surprise near the cylinder the velocity in the $z$ direction is no longer negligible. Hence it appears that fluid is flowing in the direction of the walls perpendicular to the $z$ axis which explains the thinning of the boundary layers at both walls. The peak value of the $w$ velocity component is approximately 10% of the free stream velocity and almost 30% of the maximum $u$ velocity component near the cylinder. This means that the assumption of a 2D flow in the
3.1. Experimental approach

Figure 3.6: Calculated stream-wise velocity component ($u$) for a fully developed two-dimensional channel flow at different shear thinning factors $n$. The model used for these simulations is a Carreau model.

Figure 3.7: Calculated stream-wise velocity component ($u$) for a fully developed two-dimensional channel flow at different shear thinning factors $n$. The model used for these simulations is a Carreau model.
middle of the channel is not fully met.

![Figure 3.8: Velocity profiles of the $u$ component along the $z$ axis at two different stream-wise positions in a Newtonian fluid. 105 mm upstream of the cylinder axis ($y = 0$) and 7.5 mm upstream of the cylinder axis at a flow rate of 0.2 L/s.](image1.png)

![Figure 3.9: Velocity profiles of the $w$ component along the $z$ axis at two different stream-wise positions in a Newtonian fluid. 105 mm upstream of the cylinder axis ($y = 0$) and 7.5 mm upstream of the cylinder axis at a flow rate of 0.2 L/s.](image2.png)

We did the same experiments in a water based viscoelastic solution. The polymer that we use is a partially hydrolysed polyacrylamide (PAMH) called Superfloc A-110 from Cytec Industries. In section 4.1 more details are given of the model fluid we used for our experiments.
3.1. Experimental approach

In figure 3.10 we have plotted the viscosity and the first normal stress difference as a function of the shear rate of this fluid. If we look at the $u$ and $w$ velocity components in the shear thinning polymer solution of 0.1% A-110 in distilled water (see figure 3.11 and figures 3.12 and 3.13), where we see that the flow shows even larger 3D effects than in the Newtonian fluid (figures 3.8 and 3.9). Upstream of the cylinder we still see a relatively flat velocity profile along the $z$ axis. But close to the cylinder we see that two peaks appear in the axial velocity. Once again we can conclude from the $w$ velocity profiles that near the cylinder the fluid is flowing in the direction of the walls perpendicular to the cylinder axis. In a shear thinning fluid this phenomenon is enhanced. Near the channel walls the largest shear rate occurs, which means that the viscosity of the fluid in the boundary layers is lower than in the middle of the channel.

![Figure 3.10: The viscosity and first normal stress difference of a solution of 0.1% A-110 polyacrylamide as a function of the shear rate (left figure). The measurements were done on the ARES (Advanced Rheometric Expansion System) of Rheometric Scientific.](image)

**Thermostat** (5). As the viscosity of the polyacrylamide solution is very sensitive to temperature changes (0.1°C temperature difference already causes a 1% change in viscosity see figure 3.14), the system must be well thermostated. For this purpose two spiral glass tubes have been made that act as a heat exchanger. The spiral tubes are placed in the reservoir containing the solution. The water can be cooled or heated with a thermostatic bath. The temperature is monitored at three different positions. A thermometer in the reservoir and two PT100 elements in the channel, one at the entrance and one at the exit of the channel. The temperature could be maintained constant within 0.2 degrees Celsius on the ARES (Advanced Rheometric Expansion System) of Rheometric Scientific.

3.1.2 Measurement equipment

**The LDV system** (6). The principle of the LDV system is as follows. Laserlight is focused in a small measuring volume in the flow. Particles present in the flow, scatter the light of the
Chapter 3: Experimental and numerical approach

Figure 3.11: The $u$ velocity component measured along the $x$ axis ($y = 0, z = 0$), in a Newtonian glycerol solution compared to the same polyacrylamide solution. Due to 3D effects, as explained in the text, the velocity profile in the polyacrylamide solution deviates considerably from that in the Newtonian glycerol solution.

Figure 3.12: LDV measurements of the $u$ velocity component for a viscoelastic fluid (0.1% A-110 in tap water). The velocity profiles have been measured at two stream-wise positions: 100 mm and 40 mm upstream of the cylinder in the middle of the channel.
3.1. Experimental approach

Figure 3.13: LDV measurements of the $w$ velocity component for a viscoelastic fluid (0.1% A-110 in tap water). The velocity profiles have been measured at two stream-wise positions: 100 mm and 40 mm upstream of the cylinder in the middle of the channel.

Figure 3.14: Shear viscosity as a function of temperature for a 400 wppm polyacrylamide (A-110) solution dissolved in a mixture of 7% distilled water and 93% glucose syrup. The viscosity measurements were performed at a constant shear rate of 0.5 s$^{-1}$ in a Couette geometry.
Chapter 3: Experimental and numerical approach

laser beams. The scattered light has a Doppler shift due to the fact that the particles have a velocity. This Doppler shift is directly proportional to the velocity of the particle. Assuming that the particles have the same velocity as the fluid, the fluid velocity is then also known. For more details, see Goldstein (1983) and Drain (1980).

The velocity fields presented in this thesis were measured with a 5 mW Helium-Neon Spectra Physics laser, model 120. The wavelength of the laser beam is 632.8 nm and the beam has a diameter of 1 mm. The optical system in figure 3.15 was developed at the TPD of TNO (type 400). In our experiments the optical system is used with a two component reference beam mode in forward scatter mode. The optics (see figure 3.15) consists of:

1. Lens L1, focuses the laser beam on the grating.
2. Rotating grating, the laser beam is split in an illuminating beam (26% intensity) and several reference beams (2% intensity).
3. Lens L2, makes all laser beams parallel. The beam separation is determined by the focal length of lens L2.
4. Mask, used to select the illuminating beam and two reference beams from several outcoming beams of the rotating grating.

Figure 3.15: Details of the LDV set-up. The Helium Neon laser is mounted on an optical table which also contains the optical system as described in the text. The scattered light is collected with two photodiodes and sent to the filters. After the raw signals have been filtered the tracker converts the frequency difference into a voltage. The signals are then read into a computer via an A/D converter or sent to a line plotter to view the signal online.

---

**Diagram:**

- **Laser**
- **Lens L1**
- **Grating**
- **Lens L2**
- **Lens L3**
- **Mask**
- **Dropout 1**
- **Dropout 2**
- **Tracker**
- **Signal 1**
- **Signal 2**
- **Filter 1**
- **Filter 2**
- **Oscilloscope**
- **A/D converter**
- **Computer**
- **Line plotter**
- **Measuring volume**
- **Illuminating beam**
- **Reference beams**
- **Photo diodes**
3.1. Experimental approach

5. Lens L3, focuses the beams in the measuring volume. The focal length of lens L3 determines the beam angle and the size of the measuring volume.

The output device consists of two photo-diode detectors, type 731. A filter, Krohn-Hite model 3940, used in bandpass mode from 200-500 MHz to filter the raw signal from the photo diodes. A two-channel frequency tracker, type 8102, used in low frequency mode, 1V/40kHz. The output signal of the tracker is amplified and directed to an oscilloscope. The analog voltage signal is send to an AD converter and read into a PC.

According to their specifications, the trackers convert a frequency shift of 40 MHz into a 1 volt signal. This was checked by offering a sine function with known frequency to the trackers. The frequency of this signal was varied in the range of 0 to 200 MHz. It appeared that the outcoming signals needed a small correction, the first tracker converts a frequency shift of 40229 Hz into a 1 volt signal and the second tracker converts a frequency of 40363 Hz into a 1 volt signal.

The two measured Doppler frequency shifts have to be converted into the two velocity components $u$ and $w$. The conversion factor is determined by the following relation

$$\Delta f_{Doppler} = \frac{u}{\lambda} \sin \beta$$

The wave length is determined from the specification of the laser ($\lambda = 632.8$ nm) and $\beta = 7.62^\circ$. This angle is determined by projecting all three beams on the wall and measuring the angles between the centres of the three beams.

The mixture of glucose syrup and distilled water hardly contain any particles that are able to scatter the light of the laser beams. Hence we added particles that would take care of light scattering. The signal was optimised by adding seeding (TSI particles model 10087), density 2.6 g/cm$^3$, mean diameter 4 $\mu$m, 2% mass in distilled water.

The velocity measured with the HeNe laser was compared with the velocity measurements of an ArI laser at the same flow rate. The two velocity profiles appeared to be identical within 2%.

The force transducer (7). The hydrodynamic interaction between two cylinders in a viscoelastic fluid was studied by measuring the force on both cylinders. For this reason a force transducer was designed and built. The design and manufacturing of the force transducer was done in close collaboration with the section of Micro-techniques of the group of Design and Production of the Faculty of Mechanical Engineering of the Delft University of Technology. The actual design was done by Biloen (Biloen 1996) and based on the following list of requirements.

1. We wanted to be able to measure a relatively large range of possible forces and moreover, we required an accuracy of within 1%.

2. The force transducer may not disturb the flow by unwanted deflections.

3. It should be possible to determine the force on the middle part of the cylinder where the influence of the boundary layer is minimal.

For the measurement of the force on a cylinder we used a spring leaf construction, with two parallel springs. Such a construction shows a linear spring behaviour, has a high reproducibility.
and a well defined displacement along an almost straight line. See figure 3.16 for an example of a spring leaf construction. The vertical movement and rotation of the cylinder are negligible for small horizontal displacements (a displacement in the direction of the flow). Moreover a range of forces (0.01 to 1 N) can be measured by simply changing the leaf springs and at the same time an accuracy within approximately 1% can be maintained.

A drawback of the two-leaf spring construction is the modular assembly of the leaf spring construction which causes a small hysteresis effect. Although a one-piece construction of the parallel leaf platform would not show hysteresis behaviour, we could not use this option because of problems in producing a one-piece construction of this size.

The displacement of the two-leaf spring construction is a measure of the force on the cylinder. To measure the displacement we required a sensor which is contactless and has an accuracy of order 1% over a range of 1 mm displacement. A displacement detector made by Philips using the eddy-current principle met these requirements. The sensor of type PR6422 comes with a converter, and a power supply of 24 V. Extra advantages are that the eddy-current sensor is easy in use and the output signal has a relative error of 1% in the working range. Further, the eddy-current sensor is relatively cheap and water resistant.

An indirect method, using a weight balance, was used to calibrate the force transducer (see figure 3.17). In this way the force transducer could be calibrated using a horizontal force, in the same way as it is used in the experimental set-up. Therefore, the friction was kept as small as possible by using a weight balance with a hinge using a knife-edge bearing. The disadvantage of a direct calibration is that the force transducer is deflected because of the weight of the cylinder and hence the calibration does not start from zero deflection. The disadvantage is that due to friction the actual applied load $F_c$ is lower.

To determine whether the point of application of the force is of influence on the measured displacement, the force is applied at two different positions. One position at 30 mm from
3.1. Experimental approach

Figure 3.17: Calibration set-up of the leaf spring construction. Two types of calibrations were used. The first type is a direct one where the load \( F_w \) is directly applied to the force transducer; a disadvantage is that the leaf spring construction is already bent under the weight of the cylinder. The second method is by using a weight balance which makes it possible to calibrate under the same conditions as in flow starting from zero load. The disadvantage is that due to friction the actual applied load \( F_c \) is lower.

Figure 3.18: Calibration force transducer at two different points along the cylinder axis. The same output signal is obtained for two different points of action of the calibration force.
the bottom and one at 150 mm. As one can conclude from figure 3.18 there is no significant difference between both measurements.

![Figure 3.19: Correction force measurements for boundary layers to a force which acts only on the middle part of the cylinder where the flow is approximately two-dimensional. The influence of the boundary layers on the force is eliminated by subtracting the force of the 1/4 cylinder (which covers only the boundary layer) from the force of the 3/4 cylinder (which covers the boundary layer and the middle of the channel where the flow is nominally two-dimensional).](image-url)

As we want to compare the experimental results with two-dimensional simulations, we have to correct for the effect of the varying velocity on the drag force. This contribution of the boundary layer to the force is eliminated in the following way. First the force is measured on 3/4 part of the cylinder $F_{3/4}$ (cylinder 1 in figure 3.21), which includes the boundary layer on the top part of the channel, the results of the drag force measurements have been plotted in figures 3.19 and 3.20. Next the experiment is repeated with a cylinder which only covers the top 1/4 part of the channel, this force $F_{1/4}$ practically only contains the contribution of the boundary layer on the top part of the channel. By subtracting $F_{1/4}$ from $F_{3/4}$ we obtain the total drag force on the cylinder ($F_{1/2}$) between 1/4 and 3/4 of its height. Clearly, to do so, both measurements were performed on different cylinder/dummy-cylinder combinations (1/4 dummy-cylinder, 3/4 cylinder and 3/4 dummy-cylinder, 1/4 cylinder). The forces $F_{1/4}$ and $F_{3/4}$ were measured for different velocities $U_{\text{max}}$ and the desired values were obtained by linear interpolation. We can write the non-dimensional drag force $K$ as follows

$$K = \frac{F_{1/2}}{\tau d} \quad (3.3)$$

and assuming that

$$\tau \sim \eta \frac{U}{d} \quad (3.4)$$
3.1. Experimental approach

Figure 3.20: As the force depends linearly on the velocity (in a Newtonian fluid) a constant value is obtained if we divide by the velocity. That is why we have introduced a non-dimensional drag force coefficient \( K \) which is obtained from the drag force by dividing by the free stream velocity (see equation 3.6).

we can write for the non-dimensional drag force

\[
K = \frac{F_{1/2}}{\eta U}
\] (3.5)

For the experiments we divide \( F_{1/2} \) by the maximum free stream velocity velocity, the half-length of the cylinder and the viscosity of the fluid and we obtain a non-dimensional force

\[
K = \frac{F_{1/2}}{\eta U l_{cyl1/2}}
\] (3.6)

where \( l_{cyl1/2} \) is the half-length of the cylinder, \( U \) is the maximum value of the free stream velocity and \( \eta \) the viscosity of the fluid.

The distance \( L \) between the two cylinders can be varied by replacing the interchangeable wall section by another one. The distance between the two cylinder axes, \( L \), is given in terms of the cylinder radius \( r \). If the distance between the axes of the two cylinders is 6 times the radius then we denote this by \( L = 6r \) (see figure 3.21). Results for the following separation distances have been taken: \( L = 3r \), \( L = 3.5r \), \( L = 4r \) and \( L = 6r \).

Data acquisition (8). The output signals of the two photo detectors are sampled with a predefined frequency and number of samples and sent to the two trackers. The output signals of the trackers are amplified and directed to an oscilloscope. Thus the Doppler signals can be viewed and analysed on-line. The mean values and the variance of the two laser Doppler signals are calculated. The analog voltage signals of the trackers, force transducer, flow meter and PT-100 elements are sent to an AD converter (DAS 40, Keithley Metrabyte) and read into
a PC. The data is sampled and stored on the PC harddisk using the data acquisition program Viewdac (version 2.1, Keithley Instruments, Inc.).

The signals of the trackers are also directly sent to a line plotter. The output of the line plotter is used to check if the cylinders are properly aligned in the middle of the channel. A velocity profile along the $y$ axis is obtained by plotting the axial velocity signals just upstream or downstream the cylinders. If the cylinder is not positioned in the middle of the channel then the resulting velocity profile is not symmetric (see $#1$ in figure 3.23). The cylinder fixed to the force transducer can be translated and the fixed cylinder can be rotated as indicated in figure 3.22. If the measured velocity profile is symmetric around $y=0$ (velocity profile $#2$) we assume that the cylinder is properly positioned in the middle of the channel and the system is ready for measurements.

3.2 Validation of the experiments

To validate the velocity (LDV) and drag force measurements we have performed multiple reproduction measurements. The measurement error is determined explicitly. The results of those measurements are presented by means of error bars. We start by discussing the accuracy of the force measurements in detail, as the force measurements were employed with a new technique. In a following section we briefly discuss the accuracy of the LDV measurements. The latter one is a more commonly used technique and therefore we only briefly discuss this measurement method and its accuracy.
3.2. Validation of the experiments

Figure 3.22: Schematic representation of the procedure used to place the cylinder in the middle of the channel. Cylinder 1 is fixed to the force transducer (cylinder 1) and cylinder 2 is fixed to the channel wall. Cylinder 1 can be shifted in the middle of the channel by translating the transducer housing as indicated in this figure. Cylinder 2 is mounted eccentric on a cylinder with a larger diameter. By rotating this larger cylinder, cylinder 2 can be positioned in the middle of the channel.

Figure 3.23: Velocity profiles in the $x$ direction, 2.5 mm upstream of cylinder 1. Velocity profile #1 is measured when the cylinder is not in the middle of the channel. Cylinder 1 and 2 are shifted until velocity profile #2 is obtained and we can assume that both of them are now placed in the middle of the channel.
3.2.1 Accuracy of the drag force measurements.
The drag force measurements on the cylinder were done at different flow velocities. In figure 3.24 an example is shown of such a measurement. Starting at zero velocity the flow rate was

![Figure 3.24: Two different drag force measurements as a function of velocity. The cylinder has been removed in between the two measurements. The error bars are also plotted in this graph.](image1)

![Figure 3.25: Two different drag force measurements as a function of velocity, we have subtracted a linear fit from the drag force measurements which enables us to see the size of the error bars.](image2)
3.2. Validation of the experiments

gradually increased. Each force measurement was done for a steady state velocity. The steady state situation was detected by plotting the force signal on-line and ensuring the signal had reached a steady state value after each change of the flow rate. After reading the maximum flow rate, we decreased the velocity again, so that for each velocity two independent drag force measurements were done. The maximum signal of the force measurements is 7 N, as can be seen in figure 3.24. The error bars in the right graph are approximately 0.05 N large. Therefore the error of the force measurements is approximately 1%.

To check the reproducibility of the force measurements, the drag force measurements were repeated on the same cylinder but in between removed it from the channel and then put it back again. The reason for doing this experiment is that two force measurements had to be performed for each experimental situation, to determine the influence of the boundary layer. This influence was determined by measuring the force on a cylinder with a length of 3/4 of the channel width (see figure 3.32) and then replace it by a cylinder with a length of 1/4 of the channel width, and repeat the measurements. This procedure is illustrated in figure 3.32. In figures 3.26 and 3.27 we show the results of two measurements, whereby in between the cylinder has been removed from the channel and put back again. The agreement between the two measurements is very good and the differences found for the different cylinders are not larger than differences between different measurements (e.g. while increasing or decreasing the flow velocity in steps).

![Figure 3.26: Drag force on the cylinder as a function of velocity. First the drag force of the full cylinder was measured, next on the 3/4 cylinder and finally on the full cylinder again.](image)

Also the influence of the narrow slit (see figure 3.32) between the measurement cylinder and the fixed cylinder was investigated. The results of the drag measurements using two different slit widths are shown in figures 3.28 and 3.29. The drag measurements were done using a 400 wppm solution of A-110 in glucose syrup. The drag becomes larger when the slit between the fixed and measurement cylinder is increased from 0.7 to 1.7 mm.
Chapter 3: Experimental and numerical approach

Figure 3.27: Drag force on the cylinder as a function of velocity. A linear fit has been subtracted from the drag force measurements.

Figure 3.28: Drag force measurement on a cylinder as a function of velocity. The drag force of the cylinder was measured using two different slit widths.
3.2. Validation of the experiments

Figure 3.29: Drag force measurement on a cylinder as a function of velocity. A linear fit has been subtracted from the drag force measurements.

Also the flow velocity was measured behind the slit, for these two different slit widths, at the same overall flow rate (see figures 3.30 and 3.31). Surprisingly, it is observed that for the large slit the u velocity component shows a peak value while for the small slit we see a minimum value just behind the slit. This peculiar behaviour might be caused by the viscoelastic properties of the 400 wppm solution. Those experiments were not done on a Newtonian solution.

First it was decided to measure the drag force on a 3/4 cylinder and on a full cylinder because for these cases the measured force on the cylinder is large and therefore the relative measurement error smaller. However in the case of a 3/4 cylinder, a slit is present, which is absent in the full cylinder. So it is not possible to correct for the extra drag caused by the slit. To eliminate the influence of the slit, the final measurements were done using both a 3/4 and a 1/4 cylinder. In figure 3.28 the results have been plotted of the drag force measurements varying the slit between the measuring cylinder and the fixed cylinder. These results show that the larger the slit the larger the drag force becomes. The effects of the slit on the drag force measurements were minimized by always using a configuration in which a slit is present. As discussed, we calculate the drag on a full cylinder, corrected for boundary layer effects, as follows

\[ F_{\text{full}} = 2(F_{3/4} - F_{1/4}) \]  

(3.7)

Assuming that the extra drag due to the presence of the slit is the same for the 3/4 cylinder and the 1/4 cylinder, a correction is made not only for boundary layer effects, but also for the presence of the slit at the same time. As different slit sizes cause different drag contributions, the necessarily constant slit size must be carefully controlled. We did this by measuring the slit width after cylinders had been installed and if necessary some corrections were made. The width of the slit between both cylinder parts was kept as constant as possible. The slit was
Figure 3.30: The \( u \) velocity component behind the cylinder slit in 400 wppm solution of A-110 in glucose syrup. The velocities were measured using two different slit widths.

Figure 3.31: The \( w \) velocity component behind the cylinder slit in 400 wppm solution of A-110 in glucose syrup. The velocities were measured using two different slit widths.
3.2. Validation of the experiments

Figure 3.32: Overview situation of the measurement method of the drag force on 3/4 cylinder and on a 1/4 cylinder. The first option we considered was measuring first the drag on the full cylinder and next the drag on the 3/4. The influence of the boundary layer was calculated by subtracting the drag force of the 3/4 cylinder from the drag of the full cylinder. The second option is by subtracting the force of the 1/4 cylinder from the force of the 3/4 cylinder which gives us the drag force on the middle part of the cylinder.

Figure 3.33: Measuring the slit width using a movable binocular. The width is measured by moving the horizontal line on the lens of the binocular from the bottom of the top cylinder to the top of the bottom cylinder and then determining the distance the binocular has moved.
measured using a binocular with a scale on its lens. The binocular is mounted on a tripod and can be moved up and down. The slit between the two cylinders is measured by moving the scale on the lens from the bottom of the upper cylinder to the top of the lower cylinder as presented in figure 3.33. The slit width could be adjusted by extending the fixed cylinder, using filling rings of different sizes.

3.2.2 Accuracy of the LDV measurements

To determine the accuracy of the LDV measurements, the signal was measured as a function of time. We also compared the velocity profile measured with the Ar-I laser to the velocity profile measured with the He-Ne laser at the same flow rate, see figure 3.34.

![Figure 3.34: Comparison of velocity profiles measured with the Ar-I laser and the He-Ne laser, both at a flow rate of 0.2 L/s. The fluid is a solution of 400 wppm A110, dissolved in glucose syrup.](image)

3.2.3 Calibration of the drag force device.

The calibration of the force transducer was done with help of weights. First all the weights were calibrated on an accurate weight-balance. The calibrated weights were used to determine the signal of the force transducer as a function of the applied weight. This was done before and, as a check, after each measuring session. The result of a force transducer calibration is a straight line, as one sees in figure 3.35. A fit program is used to determine the calibration factor to be used during drag force measurements, to convert the transducer signal into a force. In figure 3.36 we subtracted the line which we have fitted through the measured values to see the variation around the fitted straight line. From this figure (left graph) we can clearly see that the signal shows some hysteresis effects. The hysteresis effect is about 0.5% of the full scale of a measurement.

3.3 Numerical approach for 2D simulations

The flow is simulated by numerical methods based on the discontinuous Galerkin method, see Hulsen (1992), (1996). To capture a wide spectrum of relaxation times of the fluid we used a multi-mode model for the CE.
3.3. Numerical approach for 2D simulations

Figure 3.35: Calibration of the force transducer using 0.5 mm leaf springs. The signal is plotted as a function of weight.

Figure 3.36: Calibration of the force transducer using 0.5 mm leaf springs. We have subtracted a linear line fit from the signal, in order to show the deviations.
3.3.1 Finite element formulation

For the simulations a finite element program is used called DYNAFLOW (Hulsen 1997). This program allows simulation of unsteady inviscid, viscous or viscoelastic, incompressible fluid flow. The program is based on the finite element package SEPRAN. The mesh generation and postprocessing part in DYNAFLOW is the same as in SEPRAN.

The system solution vector consists of velocities, gradients and pressures. They are solved simultaneously. By default a penalty method is used. The system matrix is solved by a direct matrix method.

3.3.2 Discretization and solvers

For the discretization of the equations of motion (momentum balance and continuity equation) we use the Discrete Elastic Viscous Stress Split (DEVSS) formulation (Guénette & Fortin 1995). We use quadrilateral elements with continuous biquadratic polynomials for the velocity space.

The viscoelastic stresses are discretised using the discontinuous Galerkin (DG) method (Fortin & Fortin 1989). The DG method was introduced to handle numerical difficulties in viscoelastic fluid flow at high Weissenberg numbers. A decoupled approach is used between the computation of the velocity and the extra stress. The extra stress is computed element by element.

3.3.3 Boundary conditions

Periodic boundary conditions are considered. To generate the flow a constant flow rate is defined. The pressure gradient is computed at each timestep. No-slip boundary conditions are assumed on the channel and the cylinder walls. Only half of the geometry (c.q. \( y > 1 \)) is considered for the simulations, as we assume the flow to be symmetric; hence symmetry conditions are imposed on the centre line (\( y = 0 \)).

3.3.4 Finite element mesh

The meshes that were used for the various simulations are shown in figure 3.37. In this figure only the coarsest meshes have been plotted. The entry length is 15 times the cylinder diameter (not shown). Because periodic boundary conditions are used we actually simulate a flow geometry where the cylinders are 30 times the diameter apart. Since the assumption is made that the flow problem is symmetric, only half of the domain has to be considered. The mesh is refined near the cylinder and channel walls. In the table 3.1 the grid sizes for the different 2D numerical simulations are presented.

<table>
<thead>
<tr>
<th>Configuration</th>
<th>number of elements</th>
<th>number of nodal points</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 cylinder</td>
<td>2097</td>
<td>8441</td>
</tr>
<tr>
<td>2 cylinders, L=3r</td>
<td>2801</td>
<td>11345</td>
</tr>
<tr>
<td>2 cylinders, L=4r</td>
<td>3089</td>
<td>12521</td>
</tr>
</tbody>
</table>

Table 3.1: Mesh sizes for the 2D numerical simulations.

3.3.5 Convergence of the simulations

The convergence of the 2D numerical simulations was checked by plotting the drag coefficient as a function of time. In figure 3.38 we show the results at a \( De \) number of 1.25, in figure 3.39 the
Figure 3.37: Central part of three different meshes used for the 2D simulations of one cylinder and two cylinders with different distances.
Chapter 3: Experimental and numerical approach

Figure 3.38: Convergence of the simulations for different mesh sizes. Results for numerical simulations at $De = 1.25$. I.e. the drag is plotted as a function of time.

Figure 3.39: Convergence of the simulations for different mesh sizes. Result for numerical simulations at $De = 3.14$. I.e. the drag is plotted as a function of time.

results have been plotted at a $De$ number of 3.14. In both graphs it can be seen that the drag converges to a steady state value when an increasing fine mesh is used for the computations.

3.4 Numerical approach for 3D simulations

The experiments on a Newtonian fluid already showed that the flow becomes three-dimensional in the neighbourhood of the cylinder. Comparison of the velocity field close to the cylinder of both experiments and 2D simulations reveal a difference in the axial velocity along the symmetry axis in stream-wise direction (see figure 3.40). However, also the measured non-dimensional
3.4. Numerical approach for 3D simulations

The drag force on one cylinder in a Newtonian fluid differs considerably from the 2D simulations. This could be caused by differences in pressure and stress on the cylinder surface between the 2D simulations and the experiments, which are necessarily three dimensional. Therefore, we also performed 3D numerical simulations of the flow of a Newtonian fluid past a cylinder in a rectangular channel.

![Figure 3.40: Results of experimental data compared to 2D and 3D simulations of a confined cylinder in a Newtonian fluid. Shown are the results of the stream-wise velocity component (v) along the symmetry line of the rectangular channel.](image)

The three-dimensional calculations were performed with the finite element package SEPRAN. The 2D mesh was used to create a 3D mesh. For the two-dimensional simulations only half of the geometry was used for the calculations because of symmetry reasons. In the three-dimensional channel geometry there are two planes of symmetry which makes it possible to use only a quarter of the channel cross-section. Since the 3D problem could only be solved on a very coarse mesh by a direct solver, an iterative method was used to perform calculations on a fine mesh. The iteration process is carried out by starting with the Stokes solution followed by Picard iterations, to improve the convergence behaviour.

![Table 3.2: The drag coefficient obtained from the 3D computations for three different mesh sizes. K_mid is the drag coefficient calculated near the middle of the cylinder. K_out is the drag coefficient calculated at the outer part of the cylinder.](image)

For the three-dimensional simulations no periodic boundary conditions were imposed. However the inflow velocity depends on space, so in the y direction a quadratic inflow profile was chosen. As the fully developed velocity profile in the neutral z direction is flat, we also choose a flat inflow condition in this direction. An entry length of 0.5 m ensured the development of
Figure 3.41: The drag coefficient as a function of mesh size obtained from 3D computations at different mesh sizes for three parts of the cylinder (the middle part of the cylinder, $K_{mid}$, the outer part of the cylinder, $K_{out}$ and the full cylinder). From right to left the mesh size is increased. A second order fit curve was used to determine the extrapolated drag coefficient.

Figure 3.42: Definition and dimensions of the three-dimensional mesh.
a boundary layer near the wall in the $z$ direction. The same inflow conditions were used for simulations in an empty channel to be able to choose the right entry and exit length to guarantee a fully developed flow when the cylinder is reached. The entry length of the 3D simulation is 60 times the cylinder radius and the exit length 40 times.

The Newtonian flow was computed for different mesh sizes. The value of the drag on the middle part of the cylinder and the outer part of the cylinder are given in table 3.2. In figure 3.41 the results of different mesh sizes on the drag have been plotted. Also a second order fit curve to the data points was calculated to determine the extrapolated drag coefficient. The convergence of the drag coefficient as a function of mesh size is evident. All three-dimensional calculations were performed with an iterative method. The results of the most coarse mesh show that the drag on the outer part of the cylinder, close to the channel wall, is smaller than the drag in the middle of the channel. However, the more fine mesh we use, the smaller this difference. From this we can conclude that there is hardly any difference between the drag on the inner part of the cylinder, and on the outer part of the cylinder, close to the wall where the boundary layer is present.
Chapter 4

Rheometry

In this chapter the model fluid is described. Our criteria for a model fluid and its choice are discussed. Further the properties in shear flow are presented and also some results of extensional measurements are shown. Next, the numerical fit procedure and the results are discussed. Finally a summary is given of the properties used for the numerical simulations presented in chapter 5.

4.1 Choice of model fluid
The model fluid that we want to use for experiments has to meet some criteria: first of all it should have elastic properties. We want to be able to separate shear thinning and elastic behaviour. As a reference we start with a Newtonian fluid. Further we want to consider a Newtonian fluid, an elastic fluid without shear thinning and an elastic fluid with shear thinning behaviour. However, some restriction must be imposed on the shear thinning fluid. The shear thinning parameter must be \( n \geq 0.8 \) because of 3D effects mentioned earlier (see section 3.1.1.4).

In order to create an elastic fluid with little or no shear thinning effects, a model fluid was composed, after doing several rheological experiments. It is based on a Newtonian fluid with a high viscosity combined with a small amount of polymer with long molecular chains. Experiments showed that when using a low viscous Newtonian solvent a high percentage of polymer has to be added to the solvent to reach noticeable elastic effects. However, this also led to a solution with a strong shear thinning behaviour. A plausible explanation between the difference of dissolving a polymer in a low viscous solvent or a high viscous solvent is that the polymer chains deform and relax more easy in a low viscous solvent than in a high viscous solvent. As a result, all fluid solutions that we used are based on a high viscous Newtonian glucose syrup mixture with water.

The glucose syrup (Cerestar MS 01610) is a purified and concentrated aqueous solution of natural sugars. It is derived from starch by enzymatic hydrolysis. The usual applications for this glucose are confectionery products, such as gums and jellies, food products such as canned fruit and vegetables, and bakery products such as fat fillings.

The polymer we use is a partially hydrolysed polyacrylamide (PAMH) (Superfloc A-110 from Cytec Industries). Superfloc A-110 has a molecular weight of \( 4-6 \cdot 10^6 \) g/mol. The chemical structure of PAMH is shown in figure 4.1. The reason for choosing this polymer is that it shows relatively little mechanical degradation (Toonder et al. 1995). The high molecular weight implies that the polymer has long chains. As already explained a high viscous solution combined with a long chain polymer leads to a highly elastic solution without shear thinning.

All the solutions are based on a proportion of 93% glucose syrup and 7% distilled water. First a master solution of PAMH in distilled water is made, because PAMH is very hard to

73
Partially hydrolyzed polyacrylamide (PAMH)

\[
\begin{aligned}
\text{CH}_2 & \quad \text{CH} & \quad \text{CH}_2 & \quad \text{CH} \\
\text{C} & \equiv & \text{O} & \quad & \text{C} & \equiv & \text{O} \\
\text{OH} & & \text{NH}_2
\end{aligned}
\]

Figure 4.1: Chemical structure of PAMH Superfloc A-110 used in the experiments.

dissolve in syrup. In early rheological experiments (not presented in this thesis) Delft tap water was used. However, the quality of the Delft tap water varies in time and we decided to use distilled water instead. After the master solution was prepared it was mixed with the glucose syrup. The three different types of solutions that were prepared for our experiments are:

1. Newtonian fluid (93% glucose syrup, 7% distilled water);
2. Boger fluid (93% syrup, 7% water, 150 wppm PAMH);
3. shear thinning viscoelastic fluid (93% syrup, 7% water, 400 wppm PAMH).

4.2 Measurement of viscoelastic properties

For a Newtonian fluid it suffices to characterise the fluid by its viscosity. For a non-Newtonian fluid a range of material functions should be obtained, as these are needed to determine the constants in the constitutive equations used for numerical simulations.

Usually two kinds of flows are used to determine characteristic parameters of the fluid, namely shear flows and extensional flows (also called shear-free flows). Because the motion of the fluid is very different in both flows, the material information derived from them is also very different. Therefore it is interesting to determine the material parameters for both flow types as they also occur simultaneously in complex flows like a rectangular channel with two cylinders. Unfortunately it is very difficult to perform reliable extensional measurements, especially for water based solutions which evaporate easily and which have little consistency.

4.2.1 Measurement techniques

All rheological experiments presented in this chapter were done on an ARES (Advanced Rheometric Expansion System) rotational rheometer from Rheometrics. Both a Couette and a cone-plate geometry were used to obtain the shear data (see figure 4.2). The Couette geometry has several advantages over the cone-plate geometry. The Couette geometry has a larger surface which, in combination with a larger fluid content leads to a larger torque at small shear rates and thus more accurate measurements. Moreover, it is easier to protect the fluid sample from evaporation of water because of a smaller free surface. To prevent evaporation, a thin layer of silicon oil is used as a protective skin. In case of the cone-plate geometry the presence of a thin layer of silicon oil can significantly influence the value of the first normal stress difference. The main disadvantage of the Couette geometry is that it is not possible to measure the first
4.3 Results

Small amplitude oscillatory flow experiments, steady flow experiments and transient shear flow experiments were done with three fluids mentioned above. The results of those measurements will be discussed in the next subsections.

Figure 4.2: Illustration of a cone-plate and Couette geometry. The cone-plate geometry can be used for both viscosity and first normal stress measurements, the Couette geometry only for viscosity measurements.
Chapter 4: Rheometry

4.3.1 Dynamic shear experiments

The results of the measurements of the storage modulus $G'$ and the loss modulus $G''$ for the Boger fluid (150 wppm) and the shear thinning viscoelastic solution (400 wppm) are shown in figure 4.4 for two temperatures, 23.1 °C and 24.1 °C. The storage modulus $G'$ is a measure of the elastic energy stored in the fluid, whereas the loss modulus $G''$ is a measure of the dissipated energy.

In figure 4.4 we see that the storage modulus (also called the elastic modulus) is smaller than the loss modulus (viscous modulus) indicating the small elasticity of the solutions in shear. Of course the storage modulus of the 400 wppm solution is higher than the storage modulus of the 150 wppm solution due to the higher concentration of polyacrylamide.

4.3.2 Steady shear experiments

In steady shear flow we find that for Newtonian fluids the only nonzero stress is the shear stress ($\tau_{xy}$). In viscoelastic fluids we will find nonzero values for the first normal stress difference ($N_1 = \tau_{xx} - \tau_{yy}$) and the second normal stress ($N_2 = \tau_{yy} - \tau_{zz}$). Usually, the second normal stress difference has the opposite sign and its magnitude is much smaller than the first normal stress difference. Therefore $N_2$ cannot be measured accurately, so that we present only data of the first normal stress difference.

The results of steady shear experiments ($\eta(\dot{\gamma})$ and $\Psi_1(\dot{\gamma})$) at two different temperatures (23.1 °C respectively 24.1 °C) are shown in figure 4.5. The 150 wppm A-110 solution shows only minor shear thinning effects with a shear thinning parameter of $n=0.97$ ($n=1$ for a Newtonian fluid) and a zero-shear viscosity of $\eta_0=2.38$ Pa·s. The higher concentrated 400 wppm solution
4.3. Results

Figure 4.4: Dynamic shear data, $G'$ and $G''$ versus $\omega$ for the Newtonian, 150 wppm and 400 wppm A-110 solution. The measurements were done in a Couette geometry at different temperatures. Above for a temperature of 23.1 °C. Below for a temperature of 24.1 °C.
Figure 4.5: Steady shear data, $\eta$ and $\Psi_1$ of the Newtonian, 150 wppm and 400 wppm A-110 solution. The viscosity was measured in a Couette geometry, the first normal stress difference in a plate-cone geometry. The measurements were done at a temperature of 23.1 °C (above) and at a temperature of 24.1 °C (below).
has a higher zero-shear viscosity \((\eta_0=3.62 \text{ Pa-s})\) and a steeper slope \((n=0.90)\). The first normal stress coefficient has a greater rate of decline with \(\dot{\gamma}\) than \(\eta\) with \(\dot{\gamma}\). Theoretically, at low shear rates the first normal stress difference is proportional to \(\dot{\gamma}^2\), so that \(\Psi_1\) tends to a constant value. This plateau value is not reached in the ARES measurements, the first normal stress difference is too small at low shear rates for accurate measurements.

4.3.3 Stress growth and relaxation experiments

The aim of stress growth experiments is to see how a fluid behaves upon the inception of a steady shear flow. At a time \(t = 0\) a steady shear rate is applied to a fluid in rest. Shear stress growth function \(\eta(t, \dot{\gamma})\) of three different solutions are shown in the left graphs of figure 4.6. All data have been reduced with the steady shear viscosity \(\eta(\dot{\gamma})\) so that the curves approach the asymptote \(\eta(t, \dot{\gamma})/\eta(\dot{\gamma})=1\) for large \(t\) values. For the Newtonian solution we see that the shear stress approaches its steady value monotonically. The non-Newtonian solutions (150 wppm A-110 and 400 wppm A-110 solutions) also show that the shear stress approaches its steady value monotonically, but only for small shear rates. For larger shear rates \(\eta(t, \dot{\gamma})\) goes through a maximum and then approaches the steady state value. Furthermore we see that at larger shear rates the time needed to reach a steady state decreases.

In a stress relaxation experiment (right graphs of figure 4.3.3) the stress on a fluid undergoing steady shear is suddenly stopped at \(t=0\) \((\dot{\gamma}=0\) for \(t \geq 0)\). The decay of the shear stress is observed until it reaches zero. For all liquids the shear stress relaxes monotonically to zero. For the Newtonian liquid the stress relaxation process is independent of the applied shear rate \(\dot{\gamma}\), the relaxation time is the same for all \(\dot{\gamma}\). The polymer solutions relax more rapidly at higher shear rates.

4.3.4 Elongational flow experiments

For the elongational flow measurements a filament stretching device was used at the SHELL company (in the laboratory SIEP-RTS). The operational principle of this device is: A droplet of the sample fluid is held between two disks. The upper disk is fixed in space, while the lower disk is moved downward with an exponential velocity. The force on the upper disk is then measured. At the same time the diameter of the stretched sample is measured by means of a laser beam, moving at half the lower disk, thus following the decrease of the diameter in the middle of the sample. More details of the stretching device are given by Van Nieuwkoop & Muller von Czernicki (1996) and (Verhoef et al. 1998).

In chapter 2 we introduced the time dependent Trouton ratio as the ratio of the transient extensional viscosity and the zero shear rate viscosity

\[
Tr = \frac{\eta(\dot{\epsilon},t)}{\eta_0}
\] (4.1)

Time dependent elongational stress growth experiments have been performed for the Newtonian, the 150 wppm and the 400 wppm solutions. Due to problems with dehydration at the surface of the fluid in combination with lack of elasticity made it impossible to perform reliable experiments for the Newtonian and the 150 wppm A-110 solution. Moreover, in all experiments the fibre broke, so it was not possible to do relaxation experiments.

In figure 4.7 we see the time dependent Trouton ratio for different values of the extension rate \(\dot{\epsilon}\). The Trouton ratio curves, measured at different strain rates, all show the same qualitative
Chapter 4: Rheometry

Figure 4.6: Stress growth (left graphs) and relaxation behaviour (right graphs) of three solutions. Top graphs: Newtonian solution, middle graphs: Boger solution (150 wppm A-110) and bottom graphs: viscoelastic solution (400 wppm A-110). The Boger solution and the viscoelastic solution show overshoot effects in start-up flow, due to elastic behaviour. The little hump in start-up flow at shear rate 0.3 [s\(^{-1}\)] is due to start-up effects of the rheometer at low shear rates.
behaviour. The solution shows a rapid increase of the elastic stresses and the Trouton ratio reaches high values. At strains between 4.5 and 5 Tirtaatmadja and Sridhar (1993) found a steady state viscosity. We could not confirm this result, as in all our experiments all fibres broke between strains of 4 and 5.

\[ \dot{\epsilon}_L = 3 \quad \text{s}^{-1} \]
\[ \dot{\epsilon}_L = 2 \quad \text{s}^{-1} \]
\[ \dot{\epsilon}_L = 1 \quad \text{s}^{-1} \]

![Figure 4.7: The transient Trouton ratio ($T_r$) as a function of actual elongational strain $\epsilon$ for different strain rates. The solution used in these experiments is a 400 wppm A-110 polyacrylamide in 93% glucose syrup and 7% distilled water. The other two solutions were not consistent enough to perform elongational measurements.]

### 4.4 Characterisation of the fluid

We determine the fit parameters for our model fluid in two steps. In the limit of small deformations the polymer stress for each model reduces to the linear Maxwell model. The dynamic measurements at small strain are therefore used to fit the viscoelastic parameters $\lambda$ and $\eta$ to the linear Maxwell model (see equation 2.27). Then the steady-shear data are used to fit the non-linear viscoelastic parameters by simultaneously fitting the viscosity and first normal stress difference.

#### 4.4.1 Fit procedure of the dynamic data

The material functions $G'$ and $G''$, defined in section 2.4.2, are important for characterising the fluid behaviour at small deformations. For a small amplitude oscillatory flow between two parallel plates, where one plate is oscillating with a frequency $\omega$ and a small amplitude $\dot{\gamma}_0$, the velocity gradient is changing with time

\[ \dot{\gamma}_{yx}(t) = \dot{\gamma}_0 \cos(\omega t) = \omega \gamma_0 \cos(\omega t) \]

(4.2)
If we substitute 4.2 into 2.31, we obtain
\[ \tau_{yx} = \omega \int_{-\infty}^{t} G(t - t') \dot{\gamma}_0 \cos(\omega t) dt' \] (4.3)
\[ = \omega \gamma_0 \cos(\omega t) \int_{0}^{\infty} G(s) \cos(\omega s) ds + \omega \gamma_0 \sin(\omega t) \int_{0}^{\infty} G(s) \sin(\omega s) ds \] (4.4)
where we have changed variables; \( s = t - t' \). Using definition 2.15 we find for the dynamic properties \( G' \) and \( G'' \)
\[ G'(\omega) = \omega \int_{0}^{\infty} G(s) \sin(\omega s) ds G''(\omega) = \omega \int_{0}^{\infty} G(s) \cos(\omega s) ds \] (4.5)
(4.6)
By using a generalised Maxwell model to describe the relaxation modulus (2.30), we arrive at
\[ G''(\omega) = \sum_{i=1}^{k} \frac{G_i \lambda_i \omega}{1 + (\lambda_i \omega)^2} \] (4.7)
\[ G'(\omega) = \sum_{i=1}^{k} \frac{G_i (\lambda_i \omega)^2}{1 + (\lambda_i \omega)^2} \] (4.8)
where the subscript \( i \) counts the \( k \) different relaxation modes. For a perfect elastic solid \( G' \) is equal to the shear modulus \( G \). For a Newtonian fluid \( \eta' = G'' / \omega \) is equal to the viscosity \( \eta \). At very low frequencies the loss modulus \( G'' \) for viscoelastic fluids must approach the zero-shear rate viscosity at low frequency,
\[ \lim_{\omega \to 0} \frac{G''(\omega)}{\omega} = \eta_0 \] (4.9)
Likewise, the storage modulus \( G' \) and \( \Psi_1 \) have the same values at low shear rates and frequencies,
\[ \Psi_{1,0} = \lim_{\omega \to 0} \frac{2G'(\omega)}{\omega^2} \] (4.10)
The linear parameters were determined by fitting the linear viscoelastic Maxwell model to the two components of the complex viscosity
\[ G''(\omega) = \sum_{i=1}^{k} \frac{G_i \lambda_i \omega}{1 + \omega^2 \lambda_i^2} \] (4.11)
and
\[ G'(\omega) = \sum_{i=1}^{k} \frac{G_i \lambda_i^2 \omega}{1 + (\lambda_i \omega)^2} \] (4.12)
The index \( i \) counts the different relaxation modes of the fluid. Once the parameters \( G_i \) and \( \lambda_i \) for each mode have been determined the parameters of the non-linear model can be determined.
4.4. Characterisation of the fluid

Newtonian solution: $\eta = 1.675 \, \text{Pa.s}$
$\rho = 1370 \, \text{kg/m}^{-3}$

150 wppm viscoelastic solution: $\eta = 1.764 \, \text{Pa.s}$
$\eta_0 = 2.416 \, \text{Pa.s}$
$\bar{\lambda} = 1.254 \, \text{s}$

<table>
<thead>
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<th>$\eta_k$ (Pa.s)</th>
<th>$\lambda_k$ (s)</th>
</tr>
</thead>
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<tr>
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<td>0.2379</td>
<td>3.165</td>
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<td>2</td>
<td>0.1940</td>
<td>0.3196</td>
</tr>
<tr>
<td>3</td>
<td>0.2200</td>
<td>0.01115</td>
</tr>
</tbody>
</table>

400 wppm viscoelastic solution: $\eta = 1.617 \, \text{Pa.s}$
$\eta_0 = 3.596 \, \text{Pa.s}$
$\bar{\lambda} = 1.574 \, \text{s}$

<table>
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<td>3</td>
<td>0.3426</td>
<td>0.009078</td>
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Table 4.1: Properties of three solutions and results of the fit of a three-mode Maxwell model to the experimental storage and loss modulus of the 150 wppm solution and the 400 wppm. All measurements have been performed for a temperature of 23.1 °C.

This was done by a least-squares method (see also Quinzani et al. (1990)), e.g. minimisation of

$$
\sum_{j=1}^{N} \left[ \log G'_j - \log G'(\omega_j) \right]^2 + \left[ \log G''_j - \log G''(\omega_j) \right]^2
$$

for all parameters $G_i$ and $\lambda_i$ simultaneously, with the data points being given by $(\omega_j, G'_j)$ and $(\omega_j, G''_j)$. $G'(\omega_j)$ and $G''(\omega_j)$ are the calculated values determined from the fit and $N$ the number of experimental data points. Further we can determine the total viscosity, which is equal to the zero-shear rate viscosity $\eta_0$, by the fact that $\eta'(\omega \to 0) = \eta_0$

$$
\eta_0 = \eta_s + \sum_{i=1}^{N} \eta_i
$$

where $\eta_s$ is the viscosity of the Newtonian solvent. The mean relaxation time is given by

$$
\bar{\lambda} = \frac{\sum_{i=1}^{N} \eta_i \lambda_i}{\sum_{i=1}^{N} \eta_i} = \lim_{\dot{\gamma} \to 0} \frac{\sum_{i=1}^{N} N_i(\dot{\gamma})}{2\dot{\gamma} \sum_{i=1}^{N} \tau_i}
$$

In table 4.1 the fit results are listed for the dynamic measurements for 150 and 400 wppm A-110 measured for a temperature of 23.1 °C and in table 4.2 at a temperature of 24.1 °C.
Chapter 4: Rheometry

Newtonian solution: \( \eta = 1.48 \text{ Pa.s} \)
\( \rho = 1370 \text{ kg/m}^3 \)

150 wppm viscoelastic solution: \( \eta_s = 1.472 \text{ Pa.s} \)
\( \eta_l = 2.0411 \text{ Pa.s} \)
\( \overline{\lambda} = 1.089 \text{ s} \)

<table>
<thead>
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<th>( \lambda_k ) (s)</th>
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<td>0.1196</td>
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<tr>
<td>3</td>
<td>0.1968</td>
<td>0.00778</td>
</tr>
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400 wppm viscoelastic solution: \( \eta_s = 1.412 \text{ Pa.s} \)
\( \eta_0 = 3.041 \text{ Pa.s} \)
\( \overline{\lambda} = 1.780 \text{ s} \)

<table>
<thead>
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<th>( \lambda_k ) (s)</th>
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</thead>
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<td>0.4309</td>
</tr>
<tr>
<td>3</td>
<td>0.3297</td>
<td>0.01599</td>
</tr>
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</table>

Table 4.2: Results of the fit of a three-mode Maxwell model to the experimental storage and loss modulus of the 400 wppm solution for a temperature of 24.1 °C.

4.4.2 Fit procedure of the steady shear data

The non-linear viscoelastic parameters were determined by simultaneously fitting the shear viscosity and the first normal stress difference to the Verhoef model. This is done by minimising the expression

\[
\sum_j \left[ \log \eta_j - \log \eta(\gamma_j) \right]^2 + \left[ \log N_{1,j} - \log N_1(\gamma_j) \right]^2
\]

4.16

where subscript \( j \) again counts over the \( N \) experimental data points. The results of this fit procedure for 3 modes are presented in table 4.3 and shown in figures 4.8, 4.9, 4.10 and 4.11.

<table>
<thead>
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<th>( \mu(s) ) (24.1 °C)</th>
<th>( \mu(s) ) (23.1 °C)</th>
<th>( \mu(s) ) (24.1 °C)</th>
</tr>
</thead>
<tbody>
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<td>2</td>
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<td>1.861·10^{-3}</td>
<td>7.690·10^{-3}</td>
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<tr>
<td>3</td>
<td>1.397·10^{-6}</td>
<td>6.278·10^{-5}</td>
<td>1.120·10^{-6}</td>
<td>1.392·10^{-6}</td>
</tr>
</tbody>
</table>

Table 4.3: Results of a three-mode Verhoef model fit on the viscosity and the first normal stress difference of the 150 wppm solution and the 400 wppm solution for two different temperatures.

The properties \( G' \) and \( G'' \) are important in characterising the fluid behaviour in small deformations. For a perfect elastic solid \( G' \) is equal to the constant shear modulus \( G \). For a Newtonian fluid \( \eta' = G''/\omega \) is equal to the viscosity \( \eta \). As mentioned before the dynamic
viscosity $\eta'$ for viscoelastic fluids is found to approach the zero shear rate viscosity at low frequencies. Likewise the storage modulus $2G'$ and $\Psi_1$ have the same values at low shear rates and frequencies (see equation 4.10).

Figure 4.8: Rheological data of 150 wppm A-110 in 93% glucose syrup and 7% distilled water compared to 3 mode fit using the Verhoef model. The measurements were done for a temperature of 23.1°C. The results of this fit were used in the 2D computations presented in chapter 5.

In figures 4.8, 4.9, 4.10 and 4.11 we have compared data of dynamic shear experiments with steady shear results. We have also plotted the results of the various fits to check if the fit results also comply to the above mentioned empirical rules. The dynamic experiments of both the 150 and 400 wppm fluid do not show an approach to a plateau value because no reliable measurements could be done at frequencies below 0.1 s$^{-1}$.

4.4.3 Summary results fit parameters.
We have shown several results of rheometrical measurements. Only the dynamic and steady shear data could be used to determine the parameters for the numerical calculations. The data measured at a temperature of 23.1 °C were used for this purpose. These data were first thought to be measured at a temperature of 24.1 °C. However, due to an error in the temperature device of the ARES rheometer this was a false assumption. The measurements at a temperature of 24.1 °C were performed after the numerical simulations were finished.

In table 4.4 we summarize the rheometrical parameters used in the numerical simulations presented in chapter 5.
Figure 4.9: Rheological data of 400 wppm A-110 in 93% glucose syrup and 7% distilled water compared to 3 mode fit using the Verhoef model. The measurements were done for a temperature of 23.1°C. The results of this fit were used for the 2D computations presented in chapter 5.

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<th>( \lambda_k , \text{s} )</th>
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<table>
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<tr>
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<th>( \lambda_k , \text{s} )</th>
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</table>

Table 4.4: Summary of the fit parameters used for numerical computations. Three-mode Verhoef model fit for the 150 wppm solution and the 400 wppm solution.
4.4. Characterisation of the fluid

Figure 4.10: Rheological data of 150 wppm A-110 in 93% glucose syrup and 7% distilled water compared to 3 mode fit using the Verhoef model. The measurements were done for a temperature of 24.1°C which is the same temperature used in the flow experiments around the cylinders.

Figure 4.11: Rheological data of 400 wppm A-110 in 93% glucose syrup and 7% distilled water compared to 3 mode fit using the Verhoef model. The measurements were done for a temperature of 24.1°C which is the same temperature used in the flow experiments around the cylinders.
Chapter 5

Results and analysis of experiments & computations

In this chapter we present the results of experiments and simulations in the set-up described in chapter 3, using a Newtonian and two viscoelastic fluids. First a brief overview is given of the experiments and numerical simulations that we have performed for different positions in the channel. Next the results are discussed. First we compare, for the Newtonian case, the observed drag and velocity field to numerical computations for a Newtonian fluid in a 2D and a 3D geometry.

Next we describe the results for the viscoelastic fluid. The experimental data obtained for different geometries, at different Deborah numbers, are compared with each other. Then the results computed for the viscoelastic fluid are compared to those obtained for the Newtonian solution. Finally the experimental data are compared with the results of a 2D numerical computation performed with the Verhoef model for a viscoelastic fluid (discussed in chapter 3).

5.1 Introduction

As discussed in chapter 3 we have performed measurements and numerical computations in a rectangular channel with one and two cylinders, as illustrated in figure 5.1. The radius of the cylinders used in experiments and numerical computations is 5 mm. The distance between the two cylinder axes was varied between 1.5 to 2 mm. This distance, denoted \( L \), is expressed in terms of the cylinder radius. For instance, a two-cylinder geometry in which the axes of the two cylinders are three cylinder radii apart, is indicated by \( L = 3r \).

The LDV measurements were done along the \( x \), \( y \) and \( z \) direction, at different positions in the channel. In the following a measurement of a flow velocity component, as a function of the position along a certain chosen line is called a “traverse”. The different traverses for which measurements were carried out are defined in figure 5.1. The \( x \)-traverses are in the free stream direction. All \( x \)-traverses were done in the symmetry plane of the channel along the \( x \)-axis (i.e. \( y = 0 \)). The \( y \)-traverses were also done in the symmetry plane of the channel, perpendicular to the cylinder axis (i.e. \( z = 0 \)). Most \( z \)-traverses were done in the symmetry plane of the channel parallel to the cylinder axis (i.e. \( y = 0 \)).

The \( y \)-traverses were taken at different stream-wise (\( x \)) positions as sketched in figure 5.2 and are indicated with Roman numbers. Traverse I was done 105 mm upstream of the cylinder axis where the flow is still undisturbed. The traverses II and III are also upstream of the cylinder but at 15 mm and 7.5 mm, respectively, from the cylinder axis. The traverse indicated by IV is always between the two cylinders in case of a two-cylinder geometry. The traverses V and VI are always in the wake of the cylinder or in the wake of the second cylinder, in case of the two-cylinder geometry. Traverse V is taken at 7.5 mm downstream of the cylinder axis and traverse VI is at 15 mm downstream of the cylinder axis. Those positions were chosen by
plotting the LDV signal on-line, as a function of time. It turned out that at those particular positions the most interesting flow features occur.

Figure 5.2: Sketch of the channel geometries with one and two cylinders. The arrow indicates the flow direction. The different stream-wise positions at which the y-traverses took place are indicated by dotted lines and Roman numbers. All y-traverses took place in the symmetry plane of the channel \((z = 0)\). I: 105 mm upstream of cylinder 1, II: 15 mm upstream of cylinder 1, III: 7.5 mm upstream of cylinder axis 1, IV: between cylinder 1 and 2, V: 7.5 mm downstream of cylinder axis 2, VI: 15 mm downstream of cylinder axis 2.

The different z-traverses were done for a single cylinder and are illustrated in figure 5.3. The z-traverses that were performed in the symmetry plane of the channel (i.e. \(y = 0\)) are indicated
by Roman numbers. Position I is 100 mm upstream of the cylinder axis, position II is 20 mm upstream, position III is 10 mm upstream and position IV is 7.5 mm upstream of the cylinder axis. The $z$-traverses that were done at $y = 7.5$ mm (i.e. close to one of the channel walls) are denoted by Roman numbers and subscript “o” (i.e. off-centre). Those $z$-traverses were done at the following $x$ positions (again for a single cylinder and upstream of the cylinder): 100 mm upstream of the cylinder (position $I_o$), 20 mm upstream ($II_o$), 10 mm upstream ($III_o$) and 7.5 mm upstream of the cylinder ($IV_o$).

The drag experiments were done using the same cylinder geometries as for the LDV measurements. The drag was measured both for one cylinder and for the following two-cylinder geometries: $L = 3r$, $L = 3.5r$ and $L = 4r$. All drag measurements were done for different flow rates. The drag was measured while step-wise increasing the flow rate, followed by step-wise decreasing the flow rate. Thus for each flow rate the drag was measured twice, following two different flow histories.

We have used three different fluids (see also chapter 4 where their properties are described in detail):

1. a Newtonian fluid consisting of 93% glucose syrup mixed with 7% distilled water;
2. a viscoelastic fluid consisting of 150 wppm A-110 dissolved in a mixture of 93% glucose syrup and 7% distilled water. This viscoelastic fluid exhibits almost no shear thinning and can be considered to be a so-called Boger fluid;
3. a viscoelastic fluid consisting of 400 wppm A-110 dissolved in a mixture of 93% glucose syrup and 7% distilled water. This fluid behaves as a viscoelastic shear thinning fluid.
Numerical computations were done for a 2D and a 3D geometry. For the 3D-computations the finite element package SEPRAN was used. The numerical computations in 3D were only performed for the Newtonian fluid. It was not possible to perform 3D numerical computations with a viscoelastic fluid. The 2D numerical computations were performed with the finite element package DYNAFLOW (Hulsen 1997). For the 2D numerical computations we used a Newtonian fluid and two viscoelastic fluids. The numerical computations for the viscoelastic fluids were performed with help of the Verhoef model (Verhoef et al. 1998). This model is used because it guarantees a stable solution at the same two flow rates ($Q = 0.20 \, l/s$ and $Q = 0.52 \, l/s$) at which the experiments took place. The FENE-P model could not be used for the simulations, as it showed instabilities at the highest flow rate.

In most figures presented in this chapter the experimental results are indicated by symbols and the results of numerical computations by curves. Each symbol in the experimental results represents a data point.

In the following section the results of the experiments and numerical computations are presented. The first section contains a discussion of the results for the Newtonian fluid. In the next section the results for the viscoelastic fluids are presented. We start each section with a presentation of the experimental results, followed by a comparison between the experimental and numerical computation results. Each section ends with an interpretation and an explanation of the relevant differences between experiments and numerical computations.

5.2 Results for the Newtonian fluid

The $Re$ number (see equation 2.4) presented in this chapter is based on the diameter $d$ of the cylinder and the maximum flow velocity $U$ in the channel. The viscosity of the Newtonian fluid in experiments was determined at a temperature of 24.1°C as the experiments were done at this particular temperature. However, the numerical computations were done using a viscosity for the Newtonian fluid determined at a temperature of 23.1°C for the reason we explained in chapter 4. Most of the experiments in the Newtonian solution, of which the results are presented in this chapter, were performed at a Reynolds number of 0.232.

5.2.1 Experimental results for the velocity

In figure 5.4 we present the results for the velocity profile for the case of one cylinder. These results show that the velocity profile is almost symmetrical with respect to the $y$ axis. In figures 5.5 and 5.6 we present the results of the two-cylinder geometry for $L = 3r$ and $L = 4r$. For the case $L = 3r$ we see that the velocity becomes almost zero at $y = 0$. For the case $L = 4r$ we observe that the flow velocity at position IV (between the cylinders) is not zero anymore.

The $x$-traverses for the single cylinder and for the two-cylinder geometry with $L = 3r$ and $L = 4r$ are shown in figures 5.7, 5.8 and 5.9, respectively. In figure 5.7 we see that it takes a long distance compared with the 2D computations for the flow to return to the undisturbed velocity again. In the 2D computations it takes approximately 2 cylinder diameters for the flow to return to 99% of the undisturbed velocity. The 3D computations show that it takes approximately 6.5 cylinder diameters to return to 99% of the undisturbed velocity (see figure 5.25). So we can conclude that it is mainly due to the three-dimensional flow effects that it takes such a long distance compared with 2D computations for the flow to again reach the undisturbed velocity field, after passing the cylinder. This is explained later in more detail.
5.2. Results for the Newtonian fluid

Figure 5.4: Results of experiments for one cylinder. Velocity component $u$ versus $y$ coordinate at different stream-wise positions at $z = 0$. The fluid is Newtonian, consisting of 93% glucose syrup mixed with 7% distilled water. The flow rate is $Q = 0.052$ l/s, corresponding to a Reynolds number of $Re = 0.23$.

Figure 5.5: As in figure 5.4, now for two cylinders where $L = 3r$. 
Chapter 5: Results experiments and computations

For the case of $L = 3r$ shown in figure 5.8 the zero velocity for $y = 0$, between the cylinders, indicates that there is a stagnation region present. When the distance between the cylinders increases to $L = 4r$ (figure 5.9), we see that this stagnation region disappears. Unlike the results obtained by Taneda (1979), presented in the book of Van Dyke (1988), we did not find separation between the two cylinders. This is probably due to the large influence of the channel walls which are absent in the experiments of Taneda.

Next we consider the traverses in the $z$ direction shown in figures 5.10 and 5.11 at different stream-wise positions. Both the velocity component $u$ (in stream-wise direction, figure 5.10) and the velocity component $w$ (in the direction parallel to the cylinder, figure 5.11) have been

Figure 5.6: As in figure 5.4, now for two cylinders where $L = 4r$.

Figure 5.7: Results of experiments for one cylinder. Velocity component $u$ versus $x$ coordinate, the flow is from left to right. The fluid is Newtonian, consisting of 93% glucose syrup mixed with 7% distilled water. The flow rate is $Q = 0.052$ l/s, corresponding to a Reynolds number of $Re = 0.23$. 

Next we consider the traverses in the $z$ direction shown in figures 5.10 and 5.11 at different stream-wise positions. Both the velocity component $u$ (in stream-wise direction, figure 5.10) and the velocity component $w$ (in the direction parallel to the cylinder, figure 5.11) have been
5.2. Results for the Newtonian fluid

Figure 5.8: As in figure 5.10, now for two cylinders where $L = 3r$.

Figure 5.9: As in figure 5.10, now for two cylinders where $L = 4r$. 
Chapter 5: Results experiments and computations

plotted. Close to the cylinder the velocity component $u$ becomes smaller and is for a large part more or less independent of $z$. We also see that closer to the cylinder the boundary layer at the channel wall becomes smaller. As one would expect for a purely two-dimensional flow, the $w$ velocity component is almost zero far away from the cylinder. However, closer to the cylinder a peak appears near the channel wall, which means a velocity in the direction towards the channel wall. The $w$ velocity component remains zero in the middle of the channel (at $z = 0$) due to symmetry. Although the large peak values in the $w$ velocity component indicate a three-dimensional flow field, we can consider the flow near $z = 0$, i.e. the symmetry plane of the channel to be two-dimensional. Our velocity measurements are taken in this symmetry plane.

![Figure 5.10](image_url)

Figure 5.10: Results of experiments for a single cylinder. Velocity component $u$ versus $z$ coordinate at different stream-wise positions at $y = 0$. The fluid is Newtonian, consisting of 93% glucose syrup mixed with 7% distilled water. The flow rate is $Q = 0.052$ l/s, corresponding to a Reynolds number of $Re = 0.23$.

In figures 5.12 and 5.13 we present some other $z$-traverses, but now at $y = 5.25$ as indicated in figure 5.3 (b). Here the velocity component $u$ increases as the cylinder is approached. At the same time the boundary layer at the channel wall becomes smaller. The $w$ velocity component again shows a peak when approaching the cylinder.

5.2.2 Experimental results for the drag on the cylinders

Finally we consider the results of the drag force measurements. These have been plotted in figure 5.14. The results show that the drag on each of the cylinders separately in the two-cylinder geometry with $L = 3r$ is smaller than the drag in the case of a single cylinder. Due to the presence of the second cylinder the growth of the wake behind the first cylinder is disturbed. Furthermore, the second cylinder does not experience the same upstream flow as the first cylinder. The combination of two cylinders in the case $L = 3r$ therefore turns out to be a more streamlined body than a single cylinder. If the distance between the cylinders is increased, the drag on the first cylinder ($K_1$) in the two-cylinder geometry approaches the value of the drag on the single cylinder (see figure 5.15). However, for $L = 4r$ the difference in
5.2. Results for the Newtonian fluid

Figure 5.11: As in figure 5.10, now for velocity component $w$.

Figure 5.12: Results of experiments for a single cylinder. Velocity component $u$ versus $z$ coordinate at different stream-wise positions in the plane $y = 5.25$. The fluid is Newtonian, consisting of 93% glucose syrup mixed with 7% distilled water. The flow rate is $Q = 0.052$ l/s, corresponding to a Reynolds number of $Re = 0.23$. 
Chapter 5: Results experiments and computations

Figure 5.13: As in figure 5.12, now for velocity component $w$.

Figure 5.14: Results of experiments for a single cylinder and a two cylinder geometry with $L = 3r$. Drag coefficient versus $Re$ number. The fluid is Newtonian, consisting of 93% glucose syrup mixed with 7% distilled water. $K_1$ indicates the drag on the first cylinder and $K_2$ indicates the drag on the second cylinder. The drag coefficient has been corrected for the boundary layer and slit influence as explained in chapter 3 (subsection 3.2.1).
5.2. Results for the Newtonian fluid

The drag between the first and second cylinder in the two-cylinder geometry is very small.

![Graph showing drag coefficient versus Re number](image)

Figure 5.15: Results of experiments for one cylinder and two cylinders where \( L = 4r \). Drag coefficient versus Re number. The fluid is Newtonian, consisting of 93% glucose syrup mixed with 7% distilled water. We have plotted the results \( K_1 \) indicates the drag on the first cylinder and \( K_2 \) indicates the drag on the second cylinder. In this case the drag coefficient has not been corrected for the boundary layer and slit influence. So what we see is the drag coefficient on a 3/4 cylinder.

5.2.3 Comparison of experimental and numerical results for the velocity field

Ptasinski (1997) performed 2D flow calculations on a Newtonian fluid for the same three geometries as the experiments (i.e. single-cylinder and two-cylinder geometries with \( L = 3r \) and \( L = 4r \)). However, the results of experiments showed that the flow is not quite 2D near the cylinder. Therefore, also 3D numerical calculations have been done to determine the effect of the three-dimensional flow on the drag on the cylinder. For the 3D computations only a single cylinder in a rectangular channel was considered. First consider the velocity profiles in stream-wise direction, in the plane \( z = 0 \). In figure 5.16 the \( u/U \) profiles are shown, where \( U \) is the velocity far upstream of the cylinder, at three different stream-wise (\( x \)) positions (see figure 5.2) for both experiments and 3D computations. The agreement between the experiments and the three-dimensional calculations is good. Far upstream of the cylinder (position I), the undisturbed parabolic velocity profile is found. Approaching the cylinder the parabolic profile changes (position II). Very close to the cylinder (position III), the velocity profile is dominated by the presence of the cylinder and the fluid is flowing away from the centre line, in the direction of the channel wall.

Further, 2D computations (Ptasinski 1997) were carried out and the results were compared with the experimental data in figure 5.17. The influence of introducing a third flow dimension can thus be seen by comparing figures 5.16 and 5.17. Far upstream of the cylinder (position I), where the parabolic profile is formed, the velocity profiles of the 2D and 3D computations are identical. When the fluid reaches the cylinder surface (positions II and III) the velocity profiles predicted by the 2D and the 3D computations are different, in particular near the centreline. In general we find that the velocity component \( u \) predicted by the 3D computations is somewhat
Chapter 5: Results, experiments and computations

Figure 5.16: Results of experiments and 3D numerical computations for a single cylinder. Velocity component $u/U$ versus the $y$-coordinate at different stream-wise positions in the plane $z = 0$. The fluid is Newtonian, consisting of 93% glucose syrup mixed with 7% distilled water. The flow rate is $Q = 0.052$ l/s, corresponding to a Reynolds number of $Re = 0.23$.

lower than the prediction by the 2D computations. In view of the velocity profiles found in figures 5.12 and 5.13 this means that part of the flow in the centre region near $z = 0$ is diverted towards the upper and lower wall in the channel. One of the consequences is that although at $z = 0$ the $w$-component is near zero, a purely 2D flow is not obtained in the $x$-$y$ plane, but rather a straining flow with the strain in the $z$ direction.

In figure 5.18 the 2D results are presented for the two-cylinder configuration ($L = 3r$, for this case no 3D computations were done). Just like in the experiments, the simulations show that in the region between the cylinders the velocity is almost zero. Close to the cylinder, at positions III and V, the velocity is larger for the simulations which we attribute to the three dimensional flow in the experiments.

The results for the two-cylinder geometry, where $L = 4r$, are shown in figure 5.19. Again near the cylinders, the flow velocities for the experiments are somewhat smaller than for the numerical computations. Apart from that, the qualitative agreement between experiments and computations is quite good.

The experimental and computational results of the $x$-traverses are compared in figures 5.20, 5.21 and 5.22. Again the predictions of the computations are quite reasonable, provided we disregard the greater experimentally observed region of influence, both in front and behind the cylinders, in comparison with the 2D computations. In figure 5.21 we see that for the two cylinder geometry ($L = 3r$) the numerical computations predict a stagnation region between the cylinders, while the experiments show a small but finite flow velocity.

In figure 5.23 the results are shown of the 3D computations, for the dimensionless $u/U$
5.2. Results for the Newtonian fluid

Figure 5.17: Comparison of experiments (symbols) and 2D numerical computations (curves) for one cylinder. Velocity component $u$ versus the $y$ coordinate at different stream-wise positions in the plane $z = 0$. The fluid is Newtonian, consisting of 93% glucose syrup mixed with 7% distilled water. The flow rate is $Q = 0.052 \text{ l/s}$, corresponding to a Reynolds number of $Re = 0.23$.

Figure 5.18: As in figure 5.17, now for two cylinders where $L = 3r$. 
Chapter 5: Results experiments and computations

Figure 5.19: As in figure 5.17, now for two cylinders where $L = 4r$.

Figure 5.20: Comparison of experiments (symbols) and 2D numerical computations (curves) for one cylinder. Velocity component $u$ versus the $x$-coordinate, the flow is from left to right. The fluid is Newtonian, consisting of 93% glucose syrup mixed with 7% distilled water. We have plotted the results for one cylinder. The flow rate is $Q = 0.052 \text{ l/s}$, corresponding to a Reynolds number of $Re = 0.23$. 
5.2. Results for the Newtonian fluid

Figure 5.21: As in figure 5.20, now for two cylinders where $L = 3r$.

Figure 5.22: As in figure 5.20, now for two cylinders where $L = 4r$. 
velocity component at different positions in front of the cylinder. The boundary layers at the top and bottom wall of the channel are well predicted by the computations. Far upstream of the cylinder (at position I) the boundary layer thickness amounts to about one quarter of the channel width and the velocity profile is flat in the middle of the channel. Closer to the cylinder (at position III) the velocity component $u/U$ has not only decreased, but the boundary layer is much smaller than far away from the cylinder in the undisturbed flow. Both the experiments and computations agree on this phenomenon. We expect this is caused by the negative pressure gradient resulting from the acceleration of the flow.

![Figure 5.23: Results of experiments and 3D numerical computations for a single cylinder. Velocity component $u/U$ versus the z-coordinate at different stream-wise positions (indicated by I and III) in the plane $y = 0$. The fluid is Newtonian, consisting of 93% glucose syrup mixed with 7% distilled water.](image)

In the figure 5.24 the corresponding data for the $w/U$ velocity component are presented. Here too, the computations confirm the behaviour already observed in the experiments, i.e. an increase of $w$ near the channel walls, when the cylinder is approached. The peak value of $w/U$ is approximately 30% of the value of the $u/U$. Note the scale difference between figures 5.23 and 5.24. The shape and value of the $w/U$ velocity implies that the flow is in the direction of the wall, i.e. away from the centre of the channel.

The same three-dimensional flow pattern near the cylinder can also be deduced from figure 5.25, where the dimensionless velocity $u/U$ along the $x$-axis for $y = z = 0$ have been plotted. The 2D computations predict that the cylinder influences the flow upstream as well as downstream of the cylinder up to a distance of approximately 4 times the cylinder radius (based on 99% of the fully developed velocity profile). The experiments and the 3D numerical computations lead to a quite different result. Both the experiments and the 3D numerical computations suggest that upstream as well as downstream of the cylinder, the $u/U$ velocity component is influenced for a distance of more than 10 times the cylinder radius (see table 5.2.3. Therefore, the wake in the experiments and three-dimensional simulations is much longer than the 2D computations predict, and we must conclude that even in the plane $z = 0$, where the $w$ component is close to zero, the flow can still not be considered 2D. This must be considered
5.2. Results for the Newtonian fluid

Figure 5.24: As in figure 5.23, now for velocity component $w/U$.

an artefact of the measurement method, leading to significantly different values for the drag coefficient at the low Reynolds numbers for which the experiments were done.

Figure 5.25: Results of experiments and numerical simulations, 2D and 3D, for a single cylinder, $u/U$ velocity component versus $x$, the flow is from left to right. The fluid is Newtonian, consisting of 93% glucose syrup mixed with 7% distilled water. The flow rate is $Q = 0.052$ l/s corresponding to a $Re$ number of 0.23.

5.2.4 Comparison of experimental and numerical drag results

The results for the 2D and 3D drag computations, together with those from the experiments are shown in figures 5.26 and 5.27. With the definition given for the drag coefficient (see chapter 3) it should be independent of the Reynolds number, which is confirmed by the computations. Also the experiments show a drag coefficient independent of the Reynolds number, at least for the higher Reynolds numbers. The drag measurements at low Reynolds numbers are less accurate. In figure 5.26 we see that there is a large difference in the drag coefficient derived
from the computations and the drag coefficient found from experiments. The drag coefficient calculated from two-dimensional simulations is 88.25 on one cylinder in a Newtonian fluid. Experiments show a drag coefficient of 76.61. A large part of this difference can be explained by the 3D flow phenomenon, as the 3D computations resulted in a drag coefficient of 81.14 (see chapter 3). For the latter case there is only a relative error of about 6% with experiments.

![Diagram showing results of experiments and numerical computations for the drag coefficient on a single cylinder versus Re number. The fluid is Newtonian, consisting of 93% glucose syrup mixed with 7% distilled water. The drag coefficient has been corrected for the boundary layer and slit influence as described in chapter 3.](image)

In figure 5.27 we see that the results of the 2D computations in a two-cylinder geometry where \( L = 3r \) show that the drag coefficient on the first and second cylinder is the same and that the drag coefficient on each of the cylinders separately is slightly lower than the drag coefficient on the single cylinder. Both phenomena are confirmed by the experiments which, however, predict a somewhat larger difference.

### 5.2.5 Discussion of differences between experiments and simulations

As we have seen above, a comparison of the velocity field between experiments and 3D numerical simulations shows that the agreement is quite good. Also the agreement between experiments

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<tr>
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<td>2D Computations</td>
</tr>
</tbody>
</table>

Table 5.1: Dimensionless wake length (based on the 99% of the fully developed velocity profile at \( y = z = 0 \)) as a function of \( Re \) derived from experiments and computations in a Newtonian fluid. The wake length is made dimensionless by the cylinder radius.
5.3 Results for the viscoelastic fluid

The $De$ number (definition see chapter 2) presented in this chapter is based on the diameter $d$ of the cylinder and the maximum flow velocity $U$ in the channel. For the characteristic time scale in experiments we choose the mean relaxation time of the fluid. For the experiments this is the mean relaxation time at a temperature of 24.1°C (see chapter 4) as the experiments were done at this particular temperature. Because the numerical simulations were performed using the parameters determined at a temperature of 23.1°C, we also choose the mean relaxation time at a temperature of 23.1°C as the characteristic time scale.

In chapter 3 we have discussed some of the problems we encountered when using viscoelastic solutions of a polymer, dissolved in a low viscosity solvent like water. Using this type of fluids,
it seems we need relatively highly concentrated solutions to find viscoelastic effects. However, in these fluids a 3D flow is already observed at positions which are 20 times the cylinder radius upstream of the cylinder, see figures 3.12 and 3.13.

By using higher viscosity solvents, like glucose syrup, it was possible to use a much smaller concentration of polymer and still obtain viscoelastic effects. The drawback, however, is that in our case the solutions with a higher viscous base solution (glucose syrup) show flow instabilities in the wake of the cylinder, above a certain flow rate. These instabilities were detected by online plotting the LDV signal as a function of time. A typical result of the output is shown in figures 5.28 and 5.29, where we have plotted some time-traces of the \( u \) velocity component upstream and downstream of the cylinder. Upstream of the cylinder the flow is stable at a flow rate of \( Q = 0.2 \) l/s. Behind the cylinder the flow becomes unstable for \( Q \geq 0.1 \) l/s, as we see in figure 5.29. If the flow rate is decreased the flow behind the cylinder becomes stable again. These instabilities are probably due to the elastic behaviour of the fluid in combination with a straining deformation flow behind the cylinder, where the fluid is accelerated from zero to the free stream velocity. If the amount of polymer in the solution is decreased the instabilities still occur, but at a higher flow rate. For the Newtonian solution this kind of instabilities were not found at all.

\[
\begin{align*}
\text{Figure 5.28: Time series of the velocity component } u. & \quad \text{The fluid is a viscoelastic solution, consisting of 400 wppm A-110 dissolved in a mixture of 93\% glucose syrup and 7\% distilled water. The velocity component } u \text{ is plotted, upstream of the cylinder, at a distance of 100 mm from the cylinder and at a flow rate of 0.2 l/s.}
\end{align*}
\]

In the following we therefore restrict ourselves to the lowest of both polymer concentrations and to flow rates below \( Q = 0.1 \) l/s to remain in the stable flow regime. All experiments in the viscoelastic solution were performed at two different flow rates; at 0.020 l/s and at 0.052 l/s. For the 150 wppm solution these flow rates correspond to Deborah numbers of 1.42 and 2.72, respectively, based on the diameter of the cylinder and the maximum velocity at the centre line \( y = z = 0 \) for the undisturbed velocity profile upstream of the cylinder. The numerical
5.3. Results for the viscoelastic fluid

computations were done at roughly the same Deborah numbers as the experiments, at 1.63 and 3.14, respectively. The Deborah numbers of the numerical computations correspond to the Deborah numbers of the experiments based on properties of the viscoelastic fluid at a temperature of 23.1 °C instead of the temperature value of 24.1 °C, due to an error in the thermometer of the ARES rheometer.

5.3.1 Viscoelastic fluid, 150 wppm: experiments

Let us first look to the results of the $y$-traverses for a single cylinder in figures 5.30 and 5.31, for the two flow rates. Both figures show the $u$ profiles at 5 different positions with respect to the cylinder. Upstream of the cylinder the same effects as found in the Newtonian solution are observed, i.e. the velocity becomes smaller when approaching the cylinder and the velocity profile of course changes shape due to the presence of the cylinder. The results become more interesting when we turn to the wake of the cylinder, where the velocity increases from about zero at the centre line to the free stream velocity. Here large differences are observed as compared to the Newtonian solution (see figure 5.4). In figure 5.30 at position number V, just behind the cylinder, the velocity at the centre line behind the cylinder ($y = 0$) shows a local maximum, which becomes more pronounced when the flow rate is increased (see figure 5.31). This suggests that the acceleration of the fluid at the centre line behind the cylinder is larger than in the Newtonian case where the flow velocity was symmetrical with respect to the position of the cylinder axis ($y = 0$). However, at position VI, further behind the cylinder, where the Newtonian flow has relaxed almost to a parabolic profile, we see that the opposite occurs. In this case the velocity has reached a parabolic profile at the edges of the wake, but it still deviates strongly at the centre line, where a minimum value is observed.

The results for a geometry with two cylinders, at a distance of $L = 3r$, are shown in figures
Figure 5.30: Results of experiments for one cylinder. $u$ velocity component versus $y$ coordinate at different stream-wise positions for $z = 0$. The fluid is a viscoelastic solution, consisting of 150 wppm A-110 dissolved in a mixture of 93% glucose syrup and 7% distilled water. The flow rate is 0.020 l/s, corresponding to a Deborah number $De = 1.42$.

Figure 5.31: As in figure 5.30, now for $De = 2.72$. 
5.3. Results for the viscoelastic fluid

5.32 and 5.33. We see that between the cylinders, at position IV, the velocity is no longer zero, as observed in the Newtonian case (figure 5.5). This must be caused by the viscoelastic
Chapter 5: Results experiments and computations

behaviour of the fluid. Apparently the long molecules in the solution prevent the fluid from coming to rest or otherwise creating a stagnation zone in the region between the cylinders.

This effect is visible at both flow rates studied. The peculiar flow behaviour as observed in the wake, (figures 5.30 and 5.31) is also seen in the two-cylinder geometry, in the wake behind the second cylinder. Again this effect is stronger for higher flow rates. For the configuration where the distance between the two cylinders is at its largest \( L = 4r \), results are plotted in figures 5.34 and 5.35. More or less the same phenomena occur as in the case of the smaller cylinder distance \( L = 3r \).

![Figure 5.34: As in figure 5.30, now for two cylinders where \( L = 4r \) and \( De = 1.42 \).](image)

In figures 5.36 up to 5.41 the results of the traverses have been plotted along the \( x \)-axis in the plane \( y = 0 \) for the two flow cases that we have studied. First, the results for the single cylinder geometry, in figures 5.36 and 5.37 are considered. At the lowest \( De \) number we see that the wake is larger than in a Newtonian fluid (see figure 5.7). Later an overview is presented of the wavelength for different flow situations (table 5.3.3). If the \( De \) number is increased (figure 5.37) the wake again becomes larger. For the two-cylinder geometries shown in figures 5.38, 5.39, 5.40 and 5.41 we observe the same effect behind the second cylinder, i.e. the wake behind the second cylinder increases with increasing \( De \) number. For the flow between the two cylinders no essential difference is found for the two cases at different \( De \) numbers. Also upstream of the cylinder there is no relevant difference between the results for the two \( De \) numbers.

In figures 5.42 and 5.43 we have plotted the results of some \( z \)-traverses at positions defined in figure 5.3. Far upstream of the cylinder (position I) the velocity profile is equal to the undisturbed flow, as also measured in the Newtonian case. There the velocity component in the \( z \) direction is almost zero. Close to the cylinder, at positions II, III and IV, the velocity profile of the \( u \) component is flattened. Far away from the cylinder, the \( w \) velocity component
5.3. Results for the viscoelastic fluid

Figure 5.35: As in figure 5.30, now for two cylinders where \( L = 4r \) and \( De = 2.72 \).

Figure 5.36: Results for velocity component \( u \) versus \( x \) coordinate along the symmetry axis \( y = 0, z = 0 \). The flow is from left to right. The fluid is a viscoelastic solution, consisting of 150 wppm A-110 dissolved in a mixture of 93% glucose syrup and 7% distilled water. The results are for one cylinder and the flow rate is 0.020 l/s, corresponding to a Deborah number \( De = 1.42 \).
Chapter 5: Results experiments and computations

Figure 5.37: As in figure 5.36, now for $De = 2.72$.

Figure 5.38: As in figure 5.36, now for two cylinders where $L = 3r$ and $De = 1.42$.

Figure 5.39: As in figure 5.36, now for two cylinders where $L = 3r$ and $De = 2.72$. 
5.3. Results for the viscoelastic fluid

Figure 5.40: As in figure 5.36, now for two cylinders where $L = 4r$ and $De = 1.74$.

Figure 5.41: As in figure 5.36, now for two cylinders where $L = 4r$ and $De = 2.7$. 
is zero. Near the cylinder the $w$ velocity component has a maximum value near the channel wall (at $z = 70 \text{ [mm]}$) which we also saw for the Newtonian fluid. However, close to the cylinder the peak shifts into the direction of the channel wall, which is not seen for the Newtonian fluid (figure 5.11).

![Figure 5.42: Results of experiments for a single cylinder. Velocity component $u$ versus $z$ coordinate at different stream-wise positions in the plane $y = 0$. The fluid is a viscoelastic solution, consisting of 150 wppm A-110 dissolved in a mixture of 93% glucose syrup and 7% distilled water. The flow rate is $Q = 0.052 \text{l/s}$, corresponding to a Deborah number $De = 2.72$.](image)

The results for the drag coefficient in the 150 wppm A-110 solution are shown in figures 5.44, 5.45 and 5.46. While the drag coefficient, according to its definition in equation (3.3), is expected to be constant as a function of $Re$ in a Newtonian fluid and the experiments for
5.3. Results for the viscoelastic fluid

Re $\geq 0.2$ showed more or less this expected behaviour, these figures show that the drag in a viscoelastic fluid increases with increasing $De$ number. Moreover, for $L = 3r$ the drag on the second cylinder remains much smaller than the drag on the first cylinder. This in contrast to the Newtonian solution for $L = 3r$ (see figure 5.14), where the drag on each of the two cylinders separately is smaller than the drag on one cylinder, but on both cylinders the drag is approximately equal. This behaviour of the drag coefficient, which differs from that in the Newtonian case, may perhaps be connected to the behaviour of the entangled polymers, which as they arrive at the cylinder have to flow around the cylinder either above or below the cylinder. This means that the polymers have to be disentangled which causes the increase in drag on the (first) cylinder with increasing Deborah number. Once the polymers have travelled past the first cylinder they get the opportunity to return to their entangled conformation again, provided the distance between the two cylinders is large enough. In the case where $L = 3r$ the polymers most likely do not get the opportunity to get entangled again, hence the lower drag on the second cylinder. When the distance between the two cylinders is increased ($L = 4r$) the drag on the second cylinder increases and approaches the same value as the drag on the first cylinder.

![Figure 5.44: Drag coefficient of a single cylinder versus Deborah number. The fluid is a viscoelastic solution, consisting of 150 wppm A-110 dissolved in a mixture of 93% glucose syrup and 7% distilled water. The drag coefficient has been corrected for the boundary layer and slit influence.](image)

5.3.2 Comparison with Newtonian results

In this section we compare the results of the experiments with the viscoelastic fluid to those using the Newtonian fluid. In the following, the results of the Newtonian solution are indicated by $De=0$.

In figures 5.47 and 5.48 we present the velocity results in the wake of the cylinder at the three different $De$ numbers. In figure 5.47 which gives the position $V$ just behind the cylinder, we observe that the Newtonian solution behaves differently from the viscoelastic solution. While the velocity profile in the Newtonian solution ($De=0$) goes to a minimum value near $y = 0$, the
Chapter 5: Results experiments and computations

Figure 5.45: As in figure 5.44, now for two cylinders where $L = 3r$

Figure 5.46: As in figure 5.44, now for two cylinders where $L = 4r$
5.3. Results for the viscoelastic fluid

velocity profile in the viscoelastic solution ($De=1.42$ and 2.72) first goes to a minimum value, but shows a small but significant local maximum value at $y = 0$.

![Graph showing velocity profile](image)

Figure 5.47: Results of experiments for three different $De$ numbers at position V behind a single cylinder. Velocity component $u$ versus $y$ coordinate. For $De=0$ the fluid is a Newtonian solution consisting of a mixture of 93% glucose syrup and 7% distilled water. For $De=1.42$ and $De=2.72$ the fluid is a viscoelastic solution, consisting of 150 wppm A-110 dissolved in a mixture of 93% glucose syrup and 7% distilled water.

Further downstream of the cylinder, at position VI (figure 5.48) we see that the opposite happens. While the velocity profile in the Newtonian solution goes to a maximum value near $y = 0$, the velocity profile in the viscoelastic solution goes to a local minimum value at $y = 0$ and needs much more time to fully return to a parabolic profile. Probably the Newtonian fluid is continuously accelerated in the wake of the cylinder. This, in contrast to the viscoelastic solution which first seems to be accelerated behind the cylinder with respect to the Newtonian solution. Further downstream the viscoelastic solution is decelerated again with respect to the Newtonian solution. These effects are enhanced for larger $De$ numbers.

In figures 5.49, 5.50 and 5.51 we have plotted the velocity along the symmetry axis $y = 0$, $z = 0$. Again the velocity profile in the Newtonian solution is indicated by $De=0$. The results of the viscoelastic solution are indicated by $De=1.42$ and 2.72. The acceleration just behind the cylinder is not visible in these figures. The deceleration of the fluid further downstream with respect to the Newtonian solution is clearly visible. Only for $L = 3r$ we see a difference in behaviour between the three $De$ numbers. For $De=0$ (Newtonian solution) the velocity between the cylinders is zero and the velocity between the cylinders increases with increasing Deborah number. This can be understood by the elastic behaviour of the viscoelastic solutions, caused by the entanglement-like interactions of the polymer molecules, which prevent the surrounding fluid to come to a stand-still between the two cylinders. For $L = 4r$, we see no relevant difference between the three $De$ numbers.
Chapter 5: Results experiments and computations

Figure 5.48: As in figure 5.47, now at position VI.

Figure 5.49: Results of experiments for three different $De$ numbers for a single cylinder. Velocity component $u$ versus $x$ coordinate, the flow is from left to right. For $De=0$ the fluid is a Newtonian solution, consisting of a mixture of 93% glucose syrup and 7% distilled water. For $De=1.42$ and $De=2.72$ the fluid is a viscoelastic solution, consisting of 150 wppm A-110 dissolved in a mixture of 93% glucose syrup and 7% distilled water.
5.3. Results for the viscoelastic fluid

Figure 5.50: As in figure 5.49, now for two cylinders where $L = 3r$.

Figure 5.51: As in figure 5.49, now for two cylinders where $L = 4r$. 
The phenomenon of the acceleration followed by a deceleration of the viscoelastic fluid in the wake of the cylinder, can be explained by the presence of the long polymer molecules in this fluid. In the wake of the cylinder the fluid (containing long molecules in a more or less coiled equilibrium conformation) is accelerated from zero to the undisturbed velocity. Just behind the cylinder the acceleration in the $x$ direction is at its maximum, further downstream the acceleration becomes smaller. In other words, the coiled molecules experience an extension behind the cylinder, which is largest just behind the cylinder. Therefore, the molecules will be extended in this region. Further downstream, where the acceleration diminishes, the molecules relax to a more coiled conformation. In the polymer solution these processes will be connected, as the polymers on these different locations are expected to be at least somewhat connected to each other, via entanglement-like interactions. Just behind the cylinder, the fluid containing the coiled molecules is accelerated by the extended molecules further downstream, which are trying to relax. Vice versa, the fluid more downstream is decelerated by the same process.

![Combined plot of the contour lines of the velocity and of the streamlines for the flow of a Newtonian fluid behind a single cylinder in a rectangular channel.](image)

We illustrate this viewpoint by means of the flow of a Newtonian fluid past a single cylinder in a rectangular channel in figure 5.52. In this figure a contourplot connecting points of equal velocity (bold curves) is combined with a plot of the streamlines (thinner curves) in the fluid. In the regions where the streamlines are parallel to the contour lines of the velocity, the polymer molecules experience a shear flow. The shear is largests in those regions where the contour lines of the velocity are closest to each other. In the regions where the streamlines are perpendicular to the contour lines of the velocity the polymer molecules go through an elongational flow. The highest strain occurs in the regions where we find the highest concentration of contour lines. We now describe the flow of polymer molecules along three different streamlines (indicated by I, II and III in figure 5.52). Along each of these three streamlines the polymer molecule experiences a different deformation history.

Along streamline I the polymer molecule is decelerated by the cylinder and experiences a small strain in the $y$ direction. The shape of the molecule changes in this direction. More downstream, between the cylinder wall and the channel wall the molecule goes through an increasing shear flow. In this region the polymer molecule is rotated and also elongated under an angle of 45°. Behind the cylinder the shear flow decreases and an increasing strain flow in $x$ direction is experienced by the molecule, hence the polymer molecule is strained in $x$ direction. In chapter 4 we found that the average time scale of the polymer molecule to relax
5.3. Results for the viscoelastic fluid

is approximately 1 s (see table 4.2). This is confirmed by the stress growth and relaxation experiments also presented in chapter 4 (see figure 4.6). Because the fluid is moving very slowly near the cylinder it is likely that the polymer molecules have enough time to relax from the shear flow experienced between the channel wall and the cylinder.

If we follow a polymer molecule along streamline II, we see that far upstream it feels a shear flow which is gradually changed into a strain flow near the cylinder. Behind the cylinder the strain flow decreases and changes into a shear flow again. The shear flow is at its largest in the region between the channel wall and the cylinder. The residence time of the polymer molecule in this region is approximately 1 s at a flow rate of $Q = 0.021/s$ and approximately 0.4 s at a flow rate of $Q = 0.0521/s$. This is very close to the mean relaxation time of the polymer molecules. This means that these molecules, when arriving behind the cylinder, are not fully relaxed, and will try to transfer some of their stress to the more relaxed molecules closer to the cylinder. As a result, these are somewhat accelerated, while the molecules further downward are somewhat decelerated in the flow. This is precisely the kind of behaviour seen in, for instance, figures 5.47 and 5.48.

The molecules that follow streamline III only experience a shear flow which is maximum at $x = 0$. Along this streamline the fluid is again moving very slowly, which gives the polymer molecule enough time to relax to its equilibrium conformation. As a result, these molecules will not show much viscoelastic behaviour.

5.3.3 Comparison between experiments and simulations in 2D

Now we compare the results of the experiments with the 2D computations (see Ptasinski (1997)), using the Verhoef model for the viscoelastic fluid, as described in chapter 2. In all following figures, the symbols indicate experiments and the lines represent numerical computation results.

First we consider the $y$-traverses. In figures 5.53 and 5.54 we plotted the data for the one-cylinder geometry at three different stream-wise positions and for the various $De$ numbers. At position III (upstream of the cylinder) the simulations agree qualitatively with the experiments. Note that due to the error in the temperature measurement, we cannot expect quantitative agreement. Again we must stress, that the flow in experiments exhibits three-dimensional effects, while the numerical computations are 2D. In our opinion this explains most of the differences between the experimental and numerical data upstream of the cylinder. In the wake of the cylinder, at positions V and VI, the experiments differ even qualitatively from the computations. The peculiar behaviour of the flow near the centre line behind the cylinder as seen in experiments (see figures 5.30 up to 5.35) is not predicted at all by the numerical computations at low $De$ numbers (figure 5.53). At high $De$ numbers (figure 5.54) the peculiar behaviour in the wake of the cylinder (at VI) is also qualitatively predicted by the numerical computations, although the viscoelastic effects are much less manifest than in the experiments.

The large differences between experiments and 2D computations in the wake of the cylinder are to a large extent caused by the method we used for determining the material parameters of the model fluid. The properties of the fluid used for the 2D numerical calculations were determined by fitting the numerical model to shear flow data of the model fluid. However, in the wake of the cylinder and particularly near the centre line of the channel ($y = 0$), the flow is mainly elongational. So it is to be expected that numerical computations fail to predict the flow in this region.
Chapter 5: Results experiments and computations

Figure 5.53: Comparison of experiments (symbols) and 2D numerical computations (curves) using the Verhoef model for a single cylinder. Velocity component $u$ versus $y$-coordinate at different stream-wise positions in the plane $z = 0$. The fluid is a viscoelastic solution, consisting of 150 wppm A-110 dissolved in a mixture of 93% glucose syrup and 7% distilled water. For the simulations $D_e = 1.63$ and for the experiments $D_e = 1.42$.

Figure 5.54: As in figure 5.53, now for $D_e = 3.14$ for the simulations and $D_e = 2.72$ for the experiments.
In figure 5.55 we present the results for a two-cylinder geometry for the case $L = 3r$. Again we see that the results of the numerical computations differ from the experiments in the wake near $y = 0$. Also we notice that in the region between the cylinders, i.e. at position IV, the calculations predict a zero velocity at $y = 0$, while the experiments show a finite velocity. At $De$ number 1.63, the numerical results for the viscoelastic fluid are quite similar to those of the Newtonian fluid, while the experiments show substantial viscoelastic effects.

In figures 5.56 and 5.57 the results of a two-cylinder geometry have been plotted for the case $L = 4r$. In figure 5.56 we present the results at a low $De$ number, i.e. 1.63 for computations and 1.42 for experiments and in figure 5.57 the results at higher $De$ numbers are shown, i.e. 3.14 for computations and 2.72 for experiments. At low $De$ numbers we find more or less the same differences between experiments and computations that we observed in figure 5.55 for the case $L = 3r$: i.e. the computations show a kind of Newtonian behaviour, whereas the experiments show clear viscoelastic effects. At the higher $De$ number, however, computations show some, but still too small, viscoelastic effects. For instance when we look at the velocity profile of $u$ at position VI on the symmetry axis ($y = 0$) we observe a small local minimum in the velocity, much smaller than the minimum observed in the experimental data, but at least qualitatively in agreement with the experiments.

We have also compared the numerical and experimental results for the $x$-traverses. In figure 5.58, we show the results for two different $De$ numbers for the one-cylinder geometry. This figure clearly illustrates that the computations fail to predict the slow return of the viscoelastic fluid to the undisturbed velocity profile and thus the occurrence of an enlarged wake behind the cylinder, as compared to the Newtonian case. Although part of the enlargement of the wake seen in experiments may be caused by three-dimensional effects, it is believed that this contribution is
Chapter 5: Results experiments and computations

Figure 5.56: As in figure 5.53, now for two cylinders where $L = 4r$ and $De = 1.63$ for the simulations, $De = 1.42$ for the experiments.

Figure 5.57: As in figure 5.53, now for two cylinders where $L = 4r$ and $De=3.14$ for the simulations, $De=2.72$ for the experiments.
5.3. Results for the viscoelastic fluid

De = 4.38, exp
De = 4.39, sim
De = 1.76, exp
De = 1.88, sim

Figure 5.58: Comparison of the experimental and numerical results for velocity component $u$ versus $x$ coordinate for one cylinder, the flow is from left to right. The fluid is a viscoelastic solution, consisting of 150 wppm A-110 dissolved in a mixture of 93% glucose syrup and 7% distilled water. The Verhoef model was used for the numerical computations. The results are for different De numbers, $De=1.63$ and $De=3.14$ for simulations and $De=1.42$ and $De=2.72$ for experiments.

negligible compared to the viscoelastic effects themselves. The numerical computations predict that for increasing $De$ number, the wake becomes larger. This can be seen as at least a qualitative confirmation of the experimental data.

In figure 5.59 we present the results for two different $De$ numbers in the two-cylinder geometry, for $L=3r$. Also in this geometry the computations quantitatively fail to predict the large increase of the wake behind the second cylinder. In the region between the cylinders the computations predict a stagnation region for both $De$ values while the experiments clearly show that the velocities are not negligible here.

Figure 5.59: As in figure 5.58, now for two cylinders where $L = 3r$.

In figure 5.60 we present the results of the $x$-traverses for two different $De$ numbers, in the two-cylinder geometry for $L = 4r$. The results for the wake are similar as for the cases discussed above. In contrast, in this case the flow between the two cylinders is predicted rather well.

Finally, in table 5.3.3 we show the wake length as a function of $De$ number and $Re$ number. The wake length is defined as the position downstream of the cylinder where the velocity at $y = z = 0$ has reached 95% of the undisturbed velocity. The increase of the wake length with increasing $De$ number is clear for both experiments and 2D computations. Although the predicted increase in wake length is smaller in case of the 2D computations. Further we notice
that the 2D computations for the Newtonian fluid ($De = 0$) at two different $Re$ numbers (0.06 and 0.21) show that the wake length is almost constant as a function of $Re$ number. These results indicate that viscoelastic effects are mainly responsible for the variation in wake length seen in different situations.

<table>
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<th>Wavelength</th>
<th>$Re$</th>
<th>$De$</th>
<th>Exp/comp</th>
<th>Fluid</th>
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</thead>
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<td>0</td>
<td>Experiments</td>
<td>Newtonian</td>
</tr>
<tr>
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<td>0.068</td>
<td>1.42</td>
<td>Experiments</td>
<td>150 wppm</td>
</tr>
<tr>
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<td>0.171</td>
<td>2.72</td>
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<td>150 wppm</td>
</tr>
<tr>
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<td>0.207</td>
<td>0</td>
<td>3D Computations</td>
<td>Newtonian</td>
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<tr>
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<td>0</td>
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<td>Newtonian</td>
</tr>
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<td>0.145</td>
<td>3.14</td>
<td>2D Computations</td>
<td>150 wppm</td>
</tr>
</tbody>
</table>

Table 5.2: Dimensionless wake length (based on the 95% of the fully developed velocity profile at $y = z = 0$) as a function of $Re$ and $De$ derived from experiments and computations. The wake length is made dimensionless by the cylinder radius. The case with $De = 0$ corresponds to the results found for the Newtonian solution. The other two $De$-values are for the results found for the viscoelastic solution 150 wppm A-110 in glucose syrup.

### 5.3.4 First normal stress results from numerical computation

The numerical model described in chapter 2 allows us to compute the normal stresses, which play an important role in viscoelastic flow. The results for the first normal stress difference (defined in chapter 2) versus the $x$ coordinate are shown in figure 5.61 for the 150 wppm solution, and in figure 5.62 for the 400 wppm solution. Upstream of the cylinder the fluid is decelerated to zero, and a negative first normal stress occurs. In front of the cylinder the fluid is decelerated to zero in the stagnation area. In this area the polymers are stretched perpendicular to the flow direction, hence the negative peak value in the $N_1$ development along the $x$-axis.

In the wake of the cylinder(s) we find a positive first normal stress difference, which means that the polymers are stretched more in the $x$ direction than in the $y$ direction. This leads to a positive $N_1$ value in the wake of the cylinder.
5.3. Results for the viscoelastic fluid

Figure 5.61: Results of numerical simulations. First normal stress difference versus $x$ coordinate, the flow is from left to right. The fluid is a viscoelastic solution, consisting of 150 wppm A-110 dissolved in a mixture of 93% glucose syrup and 7% distilled water. The Verhoef model was used for the numerical computations. Top: single cylinder, middle: two-cylinder geometry where $L=3r$, bottom: two-cylinder geometry, $L=4r$. Data are presented for two Deborah numbers $De=1.63$ and $De=3.14$. 
Figure 5.62: As in figure 5.61, now for $De=2.31$ and $De=4.45$. 
5.3.5 Numerical computation of the drag
The drag obtained from the computations has been plotted in figures 5.63 and 5.64 for the 150 wppm and the 400 wppm solution, respectively. The drag coefficient is plotted versus the $De$ number. For higher $De$ numbers we see for the one-cylinder geometry in the 150 wppm solution the same behaviour as found in the experiments (see figures 5.44, 5.45 and 5.46), i.e. the drag coefficient increases with increasing $De$ number. For low $De$ numbers (between 1 and 4) we see a decrease in drag, this was not found in the experiments.

The two-cylinder geometries show the same behaviour as the one-cylinder geometry. For increasing $De$ numbers we see a decrease in drag followed by an increase of the drag at higher $De$ numbers. For the two-cylinder geometry, ($L = 3r$) we see that the drag of the first cylinder is larger than the drag of the second cylinder, which agrees with the experiments (see figure 5.45). The drag on both cylinders is smaller than the drag of the one-cylinder geometry, which is in agreement with the experimental results. For the case $L = 4r$ a totally different behaviour is observed. The drag on the second cylinder is larger than the drag of the first cylinder which equals the drag on the single cylinder. This behaviour of the drag coefficient is in disagreement with the experimental results (see figure 5.46), where the cylinders experience approximately equal drag forces.

If we compare, in figure 5.63, the $De$ numbers at which we have presented the velocity results of the numerical computations we see that the low $De$ number ($De=2.82$) corresponds to the region where a decrease of the (numerical) drag takes place. The higher $De$ number ($De=6.57$) corresponds to the region where an increase in drag takes place. The latter was also what we found in experiments and can be explained in the following way. The long molecules are extended in the wake of the cylinder and relax to their equilibrium conformation. This causes a larger wake by which the drag is increased.

The results of the drag coefficient in the 400 wppm solution (shown in figure 5.64) are quite different from the results of the 150 wppm solution. The drag now decreases with increasing $De$ number. Further the drag coefficient for all configurations becomes almost equal, except that for the first cylinder in the ($L = 3r$) two-cylinder geometry.

5.3.6 Discussion of experiments and computations
The experiments on the viscoelastic solution of 150 wppm have revealed that the largest differences with the Newtonian fluid for the one-cylinder geometry occur in the wake of the cylinder.

For the two-cylinder geometry we see the same results, as mentioned above, for the wake behind the second cylinder. Behind the first cylinder the development of the wake is disturbed by the presence of the second cylinder, especially when the distance between the two cylinders is small, $L = 3r$. Further we see that the drag of the first cylinder does not show the large drag enhancement with increasing $De$ number which we saw for the single cylinder. For the case where $L = 3r$ we saw that the numerical computations predicted a stagnation area for the viscoelastic solution between the cylinders, but this was not confirmed by the experiments. This stagnation area was also found for the experiments in a Newtonian solution. Apart from the differences in the flow field found between the experiments and the computations, this is another indication for the fact that the numerical model used for the computations behaves more like a Newtonian fluid than a viscoelastic one.

We saw that the numerical simulations qualitatively show the same tendency for the vis-
Chapter 5: Results experiments and computations

Figure 5.63: Numerical results for the drag coefficient versus $De$ number obtained from the numerical computations using the Verhoef model. The fluid is a viscoelastic solution, consisting of 150 wppm A-110 dissolved in a mixture of 93% glucose syrup and 7% distilled water.

Figure 5.64: As in figure 5.63, now for a viscoelastic solution, consisting of 400 wppm A-110 dissolved in a mixture of 93% glucose syrup and 7% distilled water.
5.3. Results for the viscoelastic fluid

A viscoelastic solution of 150 wppm, i.e. a larger wake with increasing $De$ number. Quantitative the numerical simulations deviate from experiments, the predicted wake is much smaller as seen in experiments. This could be due to the method we used for determining the properties of the numerical model used for computations. The properties have been found by fitting the numerical model to shear properties of the viscoelastic fluid as described in chapter 4. We did not determine the properties of the numerical model in an extensional flow. However, in the wake of the cylinder the flow can be considered to be much like an extensional flow, as the fluid is accelerated from zero to the undisturbed velocity. So it is very likely that the numerical simulations will go wrong in the wake of the cylinder.

The behaviour of the drag predicted by numerical computations with increasing $De$ number for the 150 wppm solution at higher $De$ numbers is confirmed by the experiments. At low $De$ numbers the behaviour of the drag predicted by computations differs from the drag found in experiments. Experiments show that the drag increases with increasing $De$ number, while numerical simulations first show a decrease of the drag coefficient with increasing $De$ number. Only above approximately $De=2.5$ the drag coefficient starts to increase. For the 400 wppm solution the numerical result is even worse. There the computations show only a decrease of the drag coefficient with increasing Deborah number.
Chapter 5: Results experiments and computations
Chapter 6

Summary conclusions

In this chapter we summarize the main results and present the conclusions derived from these results.

6.1 Newtonian experiments and computations

- The aspect ratio of the channel cross-section (height 0.16 m and width 0.02 m) is too small to create an acceptable 2D flow in the middle of the channel. This causes the differences between experiments and 2D computations in both the velocity field and the drag on the cylinder.

- Near the cylinder the 2D computations differ from the experiments because of the 3D effects seen in experiments. 3D computations reveal that a major part of these differences can indeed be attributed to 3D flow effects.

- For the two-cylinder geometry where $L = 3r$ there is a stagnation region present between the two cylinders. This stagnation region disappears after a small increase of the cylinder distance ($L = 4r$). Unlike the results obtained by Taneda (1979), presented in the book of Van Dyke (1988), we did not find separation between the two cylinders. This is probably due to the large influence of the channel walls, which is absent in the experiments of Taneda.

- For the two-cylinder geometry where $L = 3r$ the drag on the two cylinders is equal and the drag for each cylinder separately is smaller than the drag on a single cylinder. After a small increase of the cylinder distance ($L = 4r$), the drag on each of the two cylinders separately approaches the value of the drag on the single cylinder.

6.2 Viscoelastic experiments and computations

- If the shear thinning behaviour is increased, the 3D flow effects become more pronounced.

- The influence of the viscoelastic properties of the fluid is felt strongest in the wake of the cylinder. The wake length increases with increasing Deborah number.

- For the two-cylinder geometry where $L = 3r$ the stagnation region is absent, unlike in the Newtonian case which is a consequence of the viscoelastic behaviour of the fluid.

- The flow in the wake of a cylinder becomes unstable above a critical Deborah number.
• The drag coefficient, as defined in equations 3.6 in chapter 3, increases with increasing Deborah number for a single cylinder. For the two-cylinder geometry with $L = 3r$ the drag on the first cylinder is larger than the drag on the second cylinder at high Deborah numbers. A small increase of the distance ($L = 4r$) between the two cylinders makes the drag on the two cylinders become practically equal.

• The quantitative differences between the 2D numerical computations and the experiments in the wake are probably due to the fact that the fluid parameters of the viscoelastic fluid could only be determined in shear flow, while in the wake of the cylinder elongational aspects dominate. This makes it hard to draw unambiguous conclusions on the validity of the Verhoef model.
Bibliography


141

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Nawoord

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Nawoord

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Met veel plezier denk ik terug aan mijn werkzaamheden voor Panta Rhei en het contact met alle studenten. Ik was nog maar twee maanden in dienst bij de vakgroep toen Maarten mij al vroeg als secretaris van Panta Rhei. Ik heb altijd erg prettig met jou samengewerkt bij mijn werkzaamheden voor Panta Rhei. Ik ben erg benieuwd wanneer ik weer een keer bij jou en Conny langs mag komen en nog meer benieuwd waar dat zal zijn, want jullie wonen nu wel al heel erg lang in hetzelfde huis.

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