Semi-analytical long-arc satellite orbit computation and the estimation of time-varying gravity parameters
Semi-analytical long-arc satellite orbit computation and the estimation of time-varying gravity parameters

Proefschrift

Ter verkrijging van de graad van doctor
aan de Technische Universiteit Delft,
op gezag van de Rector Magnificus prof. dr. ir. J.T. Fokkema
voorzitter van het College voor Promoties,
in het openbaar te verdedigen

op dinsdag 21 januari, 2002 te 10:30 uur

door

Sean Lanfrey BRUINSMA
geodetisch ingenieur
geboren te Leeuwarden
Dit proefschrift is goedgekeurd door de promotor:
Prof. ir. K.F. Wakker

Samenstelling promotiecommissie:

Rector Magnificus voorzitter
Prof. ir. K. Wakker Technische Universiteit Delft, promotor
Prof. ir. B.A.C Ambrosius Technische Universiteit Delft
Prof. dr. G. Balmino Centre National d'Etudes Spatiales
Prof. dr. ir. A.W. Heemink Technische Universiteit Delft
Prof. dr.-ing. R. Rummel Technische Universität München
Dr. P. Exertier Centre National de la Recherche Scientifique
Ir. R. Noomen Technische Universiteit Delft

Published and Distributed by: DUP Science
DUP Science is an imprint of
Delft University Press
P.O. Box 98
2600 MG Delft
The Netherlands
Telephone: +31 15 2785678
Telefax: +31 15 2785706
E-mail: Info@Library.TUDelft.NL


Keywords: orbit computation, perturbation modeling, parameter estimation

Copyright © 2003 by S.L. Bruinsma

All rights reserved. No part of the material protected by copyright notice may be reproduced or utilized in any form or by any means, electronic or mechanical, including photocopying, recording or by any information storage and retrieval system, without written permission from the publisher: Delft University Press.

Printed in The Netherlands
Contents

Summary

1 Introduction 1
  1.1 Historical Perspective 2
  1.2 Approach of the Problem 8
  1.3 Organization 10

2 Principles of Averaging 11
  2.1 General Averaging Principles 11
  2.2 Lie Transformation 13
    2.2.1 Example: Application to the \( J_2 \) Term of the Geopotential 18
  2.3 Averaged Kaula Potential 20
  2.4 Analytically Averaged Nongravitational Forces 22
  2.5 Numerically Averaged Nongravitational Forces 24
  2.6 Application of the Averaging Techniques 25

3 The Validation Procedure 27
  3.1 Numerical Integration in Osculating Elements 27
  3.2 High Frequency Filtering of the Orbits 29
  3.3 Numerical Integration in Mean Elements 34
  3.4 Resonance 34
  3.5 Example: Validation of the \( J_2 \) Modeling 35

4 Gravitational Force Modeling 37
  4.1 Extensions to the Gravitational Model 37
  4.2 Validation of the Gravitational Force Modeling 39
    4.2.1 Static Gravity Field Modeling 39
    4.2.2 Third-Body Perturbations 42
    4.2.3 Solid Earth Tide Modeling 43
    4.2.4 Ocean Tide Modeling 45
    4.2.5 The Perturbation due to the Atmosphere 46
  4.3 Discussion 51

5 Surface Force Modeling 53
  5.1 Mean Solar Radiation Pressure Modeling 54
  5.2 Mean Earth Albedo Modeling 56
  5.3 Mean Atmospheric Drag Modeling 56
  5.4 The Gauss Equations and Their Partial Derivatives 59
  5.5 Satellite-Specific Small Nongravitational Forces 61
  5.6 Simulation Results 64
    5.6.1 Validation of the Mean Solar Radiation Pressure Force Model 64
    5.6.2 Validation of the Mean Drag Force Model 65
    5.6.3 Validation of the Complete Mean Force Model 67
  5.7 Discussion 67
<table>
<thead>
<tr>
<th>Appendix</th>
<th>Title</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>Appendix A</td>
<td>Reference Systems</td>
<td>167</td>
</tr>
<tr>
<td>Appendix B</td>
<td>Numerical Integration Errors</td>
<td>171</td>
</tr>
<tr>
<td>Appendix C</td>
<td>Gravitational Perturbations</td>
<td>173</td>
</tr>
<tr>
<td>Appendix D</td>
<td>Perturbations due to Atmospheric Pressure</td>
<td>185</td>
</tr>
<tr>
<td>Appendix E</td>
<td>Nongravitational Perturbations</td>
<td>191</td>
</tr>
<tr>
<td>Appendix F</td>
<td>Typical SLR data distribution for LAGEOS-2 and GFZ-1 orbits</td>
<td>197</td>
</tr>
<tr>
<td></td>
<td>Samenvatting</td>
<td>199</td>
</tr>
<tr>
<td></td>
<td>Acknowledgements</td>
<td>205</td>
</tr>
<tr>
<td></td>
<td>Curriculum Vitae</td>
<td>207</td>
</tr>
</tbody>
</table>
Semi-analytical long-arc satellite orbit computation and the estimation of time-varying gravity parameters

Summary

This study deals with the long-period behavior of satellite orbits and the estimation of geophysical quantities. The three objectives of this thesis are the following:

1. the implementation, improvement and testing of force models for long-arc orbit computation;
2. improvement of atmospheric drag modeling, since it causes the largest nongravitational errors in the computation of low-Earth orbit satellite trajectories; and
3. estimation of secular and long-period geophysical parameter variations from single and multi-satellite solutions, using appropriate geodetic spacecraft.

The first objective aims at developing a theory and software program for the computation of an accurate long-arc orbit. The second objective aims at improving the atmospheric density model accuracy by employing a solar activity indicator that is more representative of solar Extreme Ultra-Violet (EUV) emissions than the nowadays-used solar radio flux at 10.7 cm wavelength. Finally, the long-arc computation program CODIOR is used to attain the third and main objective, namely the estimation of specific time-varying components of the Earth's gravity field.

The program CODIOR, presently used by CNES/Space Mathematics, and the corresponding satellite mean motion theory, developed by Exertier [1988] and Métris [1991], have been used as a starting point to attain the first objective. This theory models the mean motion of a satellite (free from short-period orbit perturbations) by numerically integrating a system of averaged equations of motion, which allows an orbit integration step-size of 3 to 12 hours and a fast orbit computation over periods of typically tens of thousands of orbital revolutions. It was found that the mean motion model was not adequate for low-altitude spacecraft, mainly because of the lack of an atmospheric drag force model. In the work described in this thesis, a mean drag force model has been implemented and validated, while the other force models have been verified and extended if this was required in view of precise long-arc computation.

The gravity field modeling in CODIOR has been extended using Lie-transforms up to degree 40 [Métris, 1991], and from 41 to 70 using the mean Kaula formulation [1966]. The luni-solar and Earth tide modeling did not require modification, but the ocean tide modeling has been extended. The semidiurnal \( N_2 \) and \( M_2 \) and the long-period \( M_n \) and \( M_f \) tides have been added, and
Summary

The model coefficients are presently taken into account up to degree 10 instead of 6. The mean nongravitational force model has been extended by implementing atmospheric drag and albedo force models. Each mean force model has been verified separately by comparison to the result of a numerical integration using an ultra-stable algorithm [Balmino and Barriot, 1990] and a filtering procedure [Exertier, 1990]. In this way, the precision of the mean motion model, which is mainly limited by the atmospheric drag force modeling, was established: 3 cm rms on the semimajor axis and a maximum drift of 1.3 m (metric values are obtained by multiplying the residuals by the semimajor axis) on the eccentricity over a period of 10 years. The precision of the gravitational mean motion model results in orbit errors that are an order of magnitude smaller. A specific nongravitational mean force model, representing thermal drag and anisotropic reflectivity of the spacecraft hemispheres, has been implemented for the LAGEOS satellites. Here, the validation procedure by simulation was not necessary, because the model is directly expressed in mean orbital elements. At this point in the study, the first objective of this thesis has been achieved.

Erroneous drag modeling is the largest error source in the trajectory determination of low-altitude satellites (altitude < 1000 km). The DTM (Drag Temperature Model) series of atmospheric density models ([Barlier et al., 1978]; [Berger et al., 1998]) have been developed for atmospheric constituent representation and precise orbit computation. These, and other, models employ the solar radio flux at 10.7 cm wavelength (\(F_{10.7}\)) because of its correlation with EUV radiation and its long and complete observational record. However, its representativity of chromospheric activity over periods of time less than several solar rotations of 27 days is inadequate [Avrett, 1992]. Since the last decade, the chromospheric emissions are thought to be governing upper atmosphere heating processes, for which the Magnesium index (Mg II) is a proxy [Heath and Schlesinger, 1986]. The correlation between \(F_{10.7}\) and the Mg II index over a period of 19 years is nearly 100%, but comparison on the scale of weeks to months reveals large differences.

In order to attain the second objective of this thesis, two test models have been constructed under identical conditions with 1½ years of Dynamics Explorer-2 (DE-2) density data, using either the Mg II index or \(F_{10.7}\) as solar activity proxy indicator. The comparison to DE-2 partial density data showed a 3-8% higher precision of the Mg II model. The external validation, by means of precise orbit computation, resulted in an up to 25% smaller bias and 25-50% smaller scatter of the estimated drag coefficients, and an up to 20% smaller rms-of-fit of the orbit to the tracking data. The drag modeling has significantly improved employing the Mg II index, so the second objective of this thesis has been achieved. Unfortunately, the Mg II index is not available on a daily, continuous basis. This makes it nowadays unusable for long-arc computations, and also for operational orbit computations.
Using recent models, except for the ocean tide model described in [Schwiderski, 1980], long-arc orbits have been adjusted to mean observations (called 'observed mean elements'). This allows the estimation of certain geophysical parameters, which is the third objective of this thesis. The observed mean elements have been obtained by filtering ephemerides (of 20-24 days in length) expressed in osculating elements which, in turn, have been derived from Satellite Laser Ranging (SLR) observations. The geodetic satellites Starlette, LAGEOS-1 and LAGEOS-2 have been used in this study because the available precise SLR tracking data allow analysis over long periods of time, and because their surface force modeling is relatively easy due to their cannonball shape. The Starlette (perigee height approximately 800 km) long-arc orbit, spanning 14 years, has an accuracy of 6.7 cm rms (about mean) for the semimajor axis and 3.7 m rms about mean for the ascending node. The eccentricity residuals are large (3.3 m rms) due to an unresolved coupling effect between the atmospheric drag and the gravity field. The long-arc orbit for LAGEOS-2 (perigee height approximately 5800 km) spans 4½ years, and shows an accuracy of 0.9 cm rms for the semimajor axis and 0.8 m rms for the ascending node. The eccentricity residuals are at the 0.3 m rms level. The length of this arc is not sufficient to estimate geophysical parameters representing other than seasonal variations. The 17-year LAGEOS-1 long-arc orbit has an accuracy of 4 cm rms for the semimajor axis and 0.6 m for the ascending node. Due to its length and accuracy, its contribution to the estimation of the secular variation in the even zonal terms of the gravity field and the 18.6 year tide is most important. The eccentricity residuals, 1.2 m rms, are too large to accurately estimate the variation in the odd zonal gravity harmonics. The thermal drag, the accuracy of which is highly dependent on the satellite spin-axis model, and albedo modeling require a higher accuracy than presently available to overcome this. A second LAGEOS-1 long arc, with a length of 5½ years, has been computed overlapping the period of the LAGEOS-2 arc. This allows comparison of different estimations of the same seasonal variations with different satellites, as well as the computation of a dual-satellite solution.

A combined solution derived from the above mentioned three satellites has been obtained for the secular variation in the degree 2 to 4 zonal (order zero) harmonic coefficients of the gravity field and the 18.6 year tide. The secular variation in the degree 5 zonal harmonic could not be estimated within reasonable error bounds: the uncertainty was larger than the estimated value and the value itself proved sensitive to small changes in orbit parameterization. The rates of change of the second- and fourth-degree zonal coefficients have been estimated accurately ((-2.7±0.2)·10^{-11}/yr and (-1.1±0.3)·10^{-11}/yr, respectively), due to the quality of the ascending node modeling. The rate of change in the degree 3 zonal harmonic coefficient of the gravity field is sensitive to the eccentricity and perigee modeling, and so a function of the accuracy of the nongravitational force modeling. Despite the fact that its estimated value of (-1.1±0.4)·10^{-11}/yr is in good agreement with
Summary

most published estimations, it should be used and interpreted with care. The amplitude of the 18.6
year tide has been estimated at 1.2 cm, when corrected for the anelasticity of the Earth. The phase
has been estimated at 90º, which corresponds to the equilibrium phase [Trupin and Wahr, 1990].
A large signal with a period of 9.3 years has been detected, but no explanation is proposed. A
similarly large amplitude has been found by Cheng et al. [1997].

The dual LAGEOS solution for the seasonal effects at the annual and semi-annual periods in \( J_2 \) and \( J_3 \) is in good agreement with earlier published values. The estimates are close to the
predicted values when the atmospheric mass redistribution, hydrological and snow cover cycle
[Chao and O’Connor, 1988] and ocean tide [Schwiderski, 1980] are accounted for. In addition, it
shows that the response of the ocean to atmospheric variations is not a simple mechanism.
However, over long periods of time it seems closer to the inverse barometer than to the non-
inverse barometer model.

The mean gravitational orbit perturbation caused by the atmosphere [Gegout, 1995], both
direct and indirect (deformation of the solid Earth and the ocean floor), has also been modeled in
the long-arc computation. This eliminates the largest part of the atmospheric signal from the total
seasonal variation estimations, leaving the hydrological and snow cover cycles, and the semi-
annual ocean tide as the main contributors. The estimates of the annual variation in \( J_2 \) and \( J_3 \) of the
gravity field are as expected closer to the hydrological signal amplitude. However, the differences
between the LAGEOS-1 and LAGEOS-2 solutions are significant, while that of Starlette has the
largest amplitudes (but which is within error bar limits of the LAGEOS-2 estimate). The annual
variation in \( J_3 \) of the gravity field displays a large discrepancy when compared to the surface water
and snow cover value: they have opposite phases. The amplitudes, however, are in agreement,
unlike the unrealistically large amplitudes of earlier determinations ([Nerem et al., 1993]; [Gegout
and Cazenave, 1993]). The signal at the semi-annual period is large and appears to be ocean tide
mainly (the predicted hydrological signal being small). Again, a considerable phase difference
(50º) exists between LAGEOS-1 and LAGEOS-2 estimates. The results listed above show that the
third objective has been achieved for the largest part; only the estimation of variation in \( J_3 \) still is
too inaccurate for geophysical purposes.

Author: S.L. Bruinsma
1 Introduction

This study deals with satellite long-arc orbit computation, yielding accurate predicted or adjusted satellite positions, in order to estimate secular and long-period variations in the gravity field and tides. It includes the following elements: orbit computation, orbit perturbation modeling, (geophysical) parameter estimation and modeling, and model evaluation. These elements have to be addressed as an ensemble, since the computation of an accurate orbit without mainly relying on empirical techniques (‘reduced dynamics’ technique: the estimation of empirical accelerations to obtain the best rms-of-fit to tracking data) requires accurate perturbation modeling. This, in turn, requires accurate models, of which those for the terrestrial gravity field and the atmospheric density are the most important ones for low-altitude satellites with altitudes up to approximately 1000 km. However, some inter-dependencies may have been introduced into some of these models (gravity field and drag in particular) because they are or have been obtained using the orbit perturbation technique. This technique basically links differences of a nominal orbit with respect to observations to deficiencies in the nominal force modeling and consequently to one or several nominal (geophysical) models. It can be applied to improve a specific geophysical model, for example the gravity field model, on the condition that the orbit perturbation is due mainly to errors of that particular model.

The above mentioned condition is particularly stringent when geophysical effects causing very small orbit perturbations are concerned, such as those due to the continually changing distribution of ice, snow, groundwater and atmospheric air mass, rebound effects from glacial loading of the last ice age, and very long-period tides for example. To assure the monitoring of strong and unambiguous orbit perturbations due to mass redistribution within, on and above the Earth, the data period under investigation must typically be longer than 10 years (more than half a period) in order to estimate the 18.6 year tide (amplitude and phase) and the secular variation in the $J_2$ term of the gravity field simultaneously. If shorter periods are analyzed, the correlation between these parameters becomes very large. One condition that must be met in order to directly estimate these signals is an orbit accuracy of a few cm rms on the semimajor axis over the entire length of the arc. If this is not the case, the accuracies of the orbital elements that are used to determine the geophysical parameters are insufficient. This condition is difficult to fulfil, so many of these small geophysical effects can only be estimated by very detailed and rigorous analysis methods and techniques. However, large discrepancies between the published values exist because of modeling inaccuracies and the use of different observations. A theory based on averaged equations of satellite motion, representing the motion in mean rather than osculating orbital
1 Introduction

Elements, allows accurate and fast integration over long periods of time and is therefore appropriate for this type of analysis. The fast computation also makes the method suitable for mission design, orbit analysis purposes, and particularly so for satellite lifetime estimation. The drawback of such a theory is that it requires much more work to implement and validate its (mean) force models than that of a purely numerical method. It is also much more specific than a purely numerical method, which may be used to estimate long and short-period signals.

1.1 Historical Perspective

During recent years there has been a significant increase in the number of Earth observation, navigation and telecommunication satellites. Furthermore, the launch of several scientific satellites (e.g. for astronomy, oceanography and aeronomy), as well as large constellations of communication satellites is planned in the immediate future. These satellites are, in varying degrees, all subject to gravitational and nongravitational forces. The amplitude of these forces is mainly a function of altitude and inclination, but also of the satellite's area-to-mass ratio. The dynamic modeling of satellite motion has benefitted from the greatly increased accuracy of most force models, the gravity field in particular. On the other hand, the models of the upper atmosphere, providing the atmospheric density values required in drag force modeling, have improved only modestly over the last decades. Due to new satellite tracking systems, such as the French DORIS (Détermination d'Orbite et Radiopositionnement Intégrés par Satellite; up-link Doppler measurements) and GPS (Global Positioning System; down-link pseudo-range and phase measurements) from the United States, which provide extensive tracking coverage, dynamic satellite orbit determination has become less sensitive to model errors, because reduced-dynamic schemes have become possible [Yunck et al., 1994]. The orbit errors caused by inaccurate geophysical models can be corrected for by estimating empirical parameters, although high correlations between these parameters exist sometimes. Obviously, this will not be possible in absence of tracking data, for instance in case of mission planning, nor will it illuminate the nature of the model errors. The large amount of high-quality satellite tracking data, SLR (Satellite Laser Ranging), DORIS and GPS, presently available, has largely contributed to the aforementioned accuracy of recent force models, of which the EGM96 gravity field [Lemoine et al., 1998] and the CSR 3.0 ocean tide solution [Eanes, personal communication; cf. CSR 2.0 by Eanes and Bettadpur, 1994] are typical examples. However, many small geophysical perturbations are still not well determined. As an example, the reader is referred to the 18.6 year lunar tide, which is difficult to estimate because of its long period, as well as the main terms of the secular variations in the gravity field, since both signals are detectable only after processing data over long periods of time.
The perturbations relative to a nominal satellite orbit due to the terrestrial gravity field have been studied since the beginning of the space age. Near-Earth orbiting satellites have become the basic tool of space geodesy, providing global data for gravity field modeling purposes. This field, normally represented as a series of spherical harmonic functions, may be represented by a static (or permanent) and a variable part. The static field characterizes the gravity field of the solid Earth for a specific epoch or the average field over a limited period of time. Examples of recent models representing the static gravity field are GRIM5 [Biancale et al., 2000] and EGM96 [Lemoine et al., 1998]. The zonal harmonic coefficients of degree $n$ (and order $m=0$) are designated $J_n$, of which $J_2$ represents the oblateness of the Earth. They are only a function of latitude, and only these coefficients cause secular orbit perturbations [Kaula, 1966]. Since these coefficients are not constant but also varying with time, the resulting orbit perturbations become quadratic with respect to the angle variables of the nominal orbit (the ascending node in particular), and therefore they are more easily recovered using long arcs. The secular change in the low-degree zonal harmonics ($\dot{J}_n$) is due to postglacial rebound since the last ice-age [Peltier and Jiang, 1996], but also due to the melting of ice caps, contributing to sea-level change [James and Ivins, 1997]. The first determination of $2J_2/G_26$ was done using 5½ years of SLR data by Yoder et al. [1983]. In spite of the relatively poor quality of the SLR observations available at the time and the limited time-span covered by the analysis, their estimate is only 10-15% larger than the outcome of recent calculations.

An accurate estimate of the secular variations is necessary because EGM96, for example, is referenced to 1 January 1986. When it is employed for a different date, a correction due to the secular rate must be applied. The periodic variations (mainly at the annual and semi-annual periods) in the Earth’s atmospheric pressure, ocean, and precipitation and snow cover represent a periodic mass redistribution on a planetary scale, and a varying potential over the satellite orbit. By estimating the most important term of the gravity field due to the non-sphericity of the Earth ($J_2$) and using the perturbations of a satellite orbit, the sum of the aforementioned parts can be determined. This may be expressed as follows:

$$J_2^{\text{observed}} = J_2^{\text{static}} + \dot{J}_2 + J_2^{\text{hydrology}} + J_2^{\text{atmosphere}} + J_2^{\text{ocean tide}}$$

(1.1)

where $J_2^{\text{observed}}$ is the recovered coefficient (also called 'effective' coefficient), $J_2^{\text{static}}$ the stationary part, $\dot{J}_2$ the secular rate of change, $J_2^{\text{hydrology}}$ the time-varying part due to hydrology and snow cover, $J_2^{\text{atmosphere}}$ the time-varying part due to the redistribution of atmospheric mass, and $J_2^{\text{ocean tide}}$ the time-varying part due to the ocean tide (or ocean tide model error).

This complicates the interpretation of the observed signal significantly, because the individual contributions are not accurately known except for the atmospheric signal, which is
1 Introduction

relatively well-observed. In theory, when estimating long-period perturbations, decorrelation of
these signals from other perturbations, secular ones in particular, is the only way to
unambiguously represent them. Small secular geophysical perturbations may probably be
determined more accurately using very long arcs than by cumulating information from many short
(in the order of days) or medium (in the order of weeks) length orbits, since their signal increases
quadratically in time and will thus be better distinguishable from other signals [Cheng et al., 1997].

The easier of the two approaches is the analysis of a time series of monthly estimates of the
parameter under investigation, provided that the individual solutions are sufficiently accurate.
Taking monthly estimates of $J_2$, for example, over a period of several years yields the static,
secular and sum of the periodic parts (see eq. (1.1)) by applying a regression with a mean, linear
and sinusoidal term to the data, respectively. This method has been used by Nerem et al. [1993]
and Cazenave et al. [1996] in particular.

The duration of such one-month arcs is too short to allow the perturbations to build up to
significant effects and estimate the parameters directly using the approach sketched above.
Therefore, semi-analytical and numerical long-arc techniques have been developed. These
techniques enable the direct estimation of the time-varying components, secular and (accumulated)
periodic, using a dynamically consistent long-arc orbit. The numerical long-arc approach basically
uses the same orbit propagation software as the one used in ‘short-arc’ computations, but employs
adapted integration algorithms with respect to robustness and precision [Lundberg et al., 1991].
This technique, successfully applied by the Center for Space Research (of the University of Texas
at Austin), enables the computation of arcs over periods of time ranging from one year ([Cheng et
al., 1989]; [Cheng et al., 1997]) to 20 years ([Eanes, 1995]; [Eanes and Bettadpur, 1996]), but at
high computational expense. The semi-analytical techniques, a combination of analytically
transformed equations of motion integrated numerically, have been developed since the beginning
of the seventies ([Wagner et al., 1974]; [Borderies, 1976]; [Salama and Tapley, 1985]; [Métris and
Exertier, 1995]). These techniques were originally developed for the determination of the (static)
zonal gravity field coefficients, which were not accurately known, as well as for mission analysis
purposes. When compared to direct numerical integration of the equations of motions they are
significantly faster (CPU-time) because the equations of motions have been averaged analytically,
which implies that the mean instead of the instantaneous motion of a satellite is represented. This
allows large integration steps and, in most cases, less complex force models. This characteristic of
semi-analytical techniques makes them appropriate tools for the study of slowly-evolving
perturbations as well as for mission analysis and design. On the other hand, semi-analytical
methods are much more difficult to develop, and are specifically adapted to study the slow
evolution of satellite orbital elements. The recovery of parameters producing fast variations, such
as those induced by tesseral harmonic coefficients of the terrestrial gravity field and most diurnal
and semi-diurnal ocean tide coefficients, is no longer possible.

The observations constitute a second difficulty: the motion of the satellite is expressed in
mean orbital elements, while the tracking data refer to the instantaneous elements. The elimination
(or filtering) of only the short-period perturbations (to be defined in Chapter 2) from satellite
orbits in osculating elements, adjusted to tracking data, produces the mean elements required for a
mean motion model. This has been done analytically [Kozai, 1969], numerically ([Uphoff, 1973];
[Velez and Fuchs, 1974]), or semi-analytically ([Douglas et al., 1972]; [Goad, 1977]) in the past.
The analytical filtering is too inaccurate, because it may only filter out the known short-period
perturbations, which leaves a large amount of ‘noise’ in the mean elements. A purely numerical
filter, on the other hand, never has an ideal frequency response, resulting in not-completely filtered
signals, while it may introduce artifacts in the mean elements due to edge-effects [Oppenheim et
al., 1983]. The semi-analytical filtering method first applies an analytical filter, which removes
most of the short-period signals, and subsequently a numerical filter for the removal of the
remaining ‘noise’. At present, this method yields mean elements with cm precision [Exertier,
1990]. It is this high precision of the mean observations, which had not been attained before 1990,
that presently allows the estimation of small geophysical signals employing a mean motion model.

Because of the scientific and operational advantages of an accurate long-arc orbit theory, a
semi-analytical theory of satellite mean orbital motion has been under development at the
Observatoire de la Côte d’Azur (Grasse) since the end of the 1980s [Métris and Exertier, 1995],
and implemented in the program CODIOR (COrrection Différentielle d’ORBite). This theory is
based on the numerical integration of the Lagrange or Gauss equations of motion, in which the
disturbing potentials have been averaged analytically over the variable representing the motion
during one orbital revolution: the mean anomaly $M$ (the orbital elements are introduced in
Appendix A). The two remaining angle variables (as opposed to the so-called action variables $a$, $e$
and $i$), the argument of perigee $\omega$ and the right ascension of the ascending node $\Omega$, typically have
angular velocities two orders of magnitude smaller, with periods of months to years, and do not
need to be averaged. This numerical integration of an analytically averaged differential system
free of short-period terms benefits from the speed of an analytical calculation as well as the
flexibility of a numerical integration. With respect to a purely analytical method, the numerical
integration covers long-period perturbations that are extremely difficult to model, such as
atmospheric drag. Compared to a numerical integration of the actual motion, the increase of
computational speed constitutes a significant advantage. This is also true for the numerical
stability, due to the smoother behavior of the variables and therefore faster convergence per
numerical integration step. This point is of importance for mission design, because the numerical
1 Introduction

Integration of the actual motion is still very time-consuming (especially when long periods of time are involved), and even more so in case of a constellation of satellites.

Low-altitude satellites (altitude approximately below 1000 km) in particular are decelerated by the residual atmosphere, which causes a drag force. The atmospheric neutral density modeling has become an important issue since the dawn of the space era, because it causes the satellite orbit (altitude) to decay. The first known analysis of observations of the semimajor axis decay of artificial satellites was performed by Jacchia in 1959 [Jacchia, 1959], and a model based on this kind of observations was constructed next [Jacchia and Slowey, 1962]. Up to 1974, atmospheric density modeling was primarily based on satellite drag total density data, which changed with the MSIS model by Hedin et al. [1974] based on in situ compositional data from the OGO-6 satellite mass spectrometer. OGO-6 also carried a Fabry-Pérot interferometer, which provided the absolute temperature observations used in the model of Thuillier et al. [1977], which has been combined with satellite drag total density data in the elaboration of the DTM78 model [Barlier et al., 1978]. These empirical density models predict the atmospheric density as a function of the solar and geomagnetic activity, as well as the position (altitude, latitude, longitude and local time) and the day of the year. This kind of models is well adapted for orbit computation purposes because of their relatively simple and rapidly-computed predictions, but they are highly dependent on the assimilated data since their predictions are not directly based on physical processes in the upper atmosphere. The accuracy of the empirical models has improved over the last decades, but the progress has been made mainly by assimilating more data, with a higher spatial resolution, as well as under low, medium and high solar activity conditions. The most recent models are DTM94 [Berger et al., 1998] and MSIS86 [Hedin, 1987], with 1σ accuracies of 15-25%. The main limitations of the empirical models may be summarized as follows: the use of the radio flux $F_{10.7}$ as a solar activity proxy (it is not generated in the correct region of the Sun, but correlated with emissions emanating from it, which is why it is called a ‘proxy’ indicator), a coarse spatial resolution, and a limited accuracy and temporal and spatial resolution of the available data base.

General Circulation Models (GCM) integrate the equations for the conservation of mass, momentum and energy for the atmosphere as a whole, as well as for the individual species [Marcos et al., 1993]. These models have a solid physical basis, but they require appropriate boundary conditions and accurate values for a number of physical parameters [Roble et al., 1988], and the uncertainty of some of these is considerable. Their advantages are a higher spatial resolution, while the variability of the atmospheric density is predicted respecting the physical principles (dynamical, chemical and energetic processes) to which the atmosphere is subjected, rather than by empirical coefficients based on observations. When observations are available, the GCM results are almost as good as those of an empirical model in which these have been
assimilated; for other conditions, the extrapolation by a GCM is based on physical processes fully accounted for, and the results may be significantly different. In that case, the GCM results are probably more reliable. However, this kind of models is not adapted for orbit computation purposes, because their output consists of densities (amongst other things) on some grid in latitude and longitude, given on constant pressure surfaces. This makes the computation of an atmospheric density prediction several orders of magnitude slower than that by an empirical model, which is the reason why they are not employed in precise orbit computation.

The (magnitude and variation of) atmospheric density, and thus air drag, is governed by solar EUV emissions (wavelength < 200 nm), whose intensity follows a phenomenon called the solar cycle. While nearly all solar energy emerges in the visible light and infrared part of the spectrum, with a nearly constant intensity, for wavelengths smaller than approximately 400 nm this intensity is highly variable, and quasi-periodic with a period of approximately 11 years [Lantos, 1994]. Because the quality of the atmospheric drag modeling is proportional to the accuracy of the atmospheric density prediction, most studies concentrate on improving the thermosphere model. As already mentioned, this mainly boiled down to (partial) density data assimilation, while the solar EUV activity proxy-indicator has rarely been at the center of effort. Hedin [1984] has correlated several rays from the solar spectrum with partial density observations, but due to the noisy data he could not recommend a new proxy. However, there are indications that using $F_{10.7}$ probably is the most important source of error in most existing density models ([Neupert, 1998]; [Berger et al., 1998]). Therefore, in order to improve satellite drag modeling, a proxy that is more representative of EUV radiation is required.

After this historical perspective, in which the full scope of long-arc orbit computation has been outlined, the main objectives of this study may be formulated. These are:

4. the implementation, improvement and testing of force models for long-arc orbit computation;
5. improvement of atmospheric drag modeling, since it causes the largest nongravitational errors in the computation of low-Earth orbiting satellite trajectories; and
6. estimation of secular and long-period geophysical parameters in single and multi-satellite solutions, using appropriate geodetic spacecraft.

These three objectives aim at obtaining a theory and software program able to compute an accurate long-arc orbit, from which specific time-varying components of gravity can be derived, and a more accurate atmospheric drag modeling by employing a more representative solar EUV proxy than the nowadays used solar radio flux. The next section describes how these goals may be achieved, by solving specific modeling problems.
1 Introduction

1.2 Approach of the Problem

The first task consists of analyzing the initial version of CODIOR, in order to decide which models should be improved or modified for obtaining an accurate long-arc satellite orbit representation. This is necessary because of the small amplitude of the variations of the geophysical parameters to be estimated. Subsequently, the initial model in Keplerian elements has been extended and upgraded. The initial version of CODIOR has been developed for the modeling of the mean motion in Keplerian elements of the geodetic satellite LAGEOS-1 (LAser GEOdetic Satellite) and for the estimation of certain geophysical parameters [Exertier et al., 1995]. This first version, however, lacked an accurate and complete nongravitational force model so it could not be applied to low-altitude spacecraft (altitude lower than 1000 km). These types of orbits are of primary interest for future missions, and secondly because several geodetic satellites (Starlette, as well as Stella, Ajisai and GFZ-1) fit in this category of orbits as well. Therefore, the existing mean motion theory has to be refined on the following points:

- augmentation of the maximum degree of the gravity field, from 17 to 50 or higher;
- augmentation of the number of ocean tide constituents and their maximum degree;
- implementation of a mean terrestrial (visible and infrared) radiation model;
- implementation of a mean atmospheric drag force model; and
- implementation of a LAGEOS-specific nongravitational force model.

The filtering of the high frequencies (the \( m \)-daily induced periods and smaller, \( m>0 \)) from the accurate orbits (by adjustment to SLR data) results in the "observed mean elements" [Exertier, 1990]. These observations contain the long-period signals only, and can be used in CODIOR. The mean orbit is fitted to these mean elements in a least-squares procedure, and so the mean state vector at epoch and geophysical parameters can be estimated. This was the procedure in case of the geodetic satellites Starlette, LAGEOS-1 and LAGEOS-2. The orbit configurations are shown in Table 1.1. Figure 1.1 shows the satellite Starlette, which is a smaller version of the identical LAGEOS satellites.

These satellites are well suited for the estimation of secular variations in the low-degree zonal gravity harmonics \( J_n \) and of the closely related (since it causes a similar perturbation over periods of a few years) 18.6 and 9.3 year ocean tide coefficients [Eanes, 1995]. Also, the annual and semiannual variations in the low-degree zonal gravity harmonics have
been modeled and estimated, and an interpretation of the observed signals is presented. The geodetic satellites Starlette and LAGEOS-1 have been chosen in this study because precise SLR tracking data are available over a long period of time, and because their surface force modeling is relatively easy due to their spherical shape. Their observation period exceeds 20 years (although with a much poorer data quality before 1983), which is theoretically long enough to decorrelate the $J_2$ effects from the 18.6 and 9.3 year ocean tide signals.

The second objective of this study is the improvement of atmospheric drag modeling by increasing the accuracy of the atmospheric density model prediction. The DTM94 model (Drag Temperature Model; [Berger et al., 1998]) has been employed in the orbit computations throughout this study. It was also used as the initial model for density model adjustments. Since upper atmosphere modeling is too vast a subject to cover entirely, this study addresses the topic that probably results in the most significant gain in modeling accuracy.

This topic concerns the modeling of the heating of the upper atmosphere by solar extreme ultraviolet and ultraviolet (EUV and UV, respectively) radiation. These originate mainly in the region of the Sun which is called the chromosphere, but also in the transition region and the corona. This energy flux is parameterized in upper atmosphere models by a single solar parameter, the solar radio flux at 10.7 cm wavelength, which originates in the lower corona at a specific altitude of the solar atmosphere. So, while this observed solar emission is not in the EUV wavelength range (20-200 nm), their long-term variations are found to be highly correlated [Hedin, 1984] because they originate in approximately the same region of the Sun. This is why it is a so-called ‘proxy-indicator’. However, an EUV or UV observation by satellite-borne spectrometer is probably more representative of the solar chromospheric EUV emission than the 10.7 cm radio flux is. The use of the most promising observation in terms of ray intensity (and thus observability), with an already demonstrated correlation with chromospheric EUV radiation, has been tested: the Mg II index [Heath and Schlesinger, 1986].

| Table 1.1: Satellite and orbit characteristics for Starlette, LAGEOS-1 and LAGEOS-2. |
|---------------------------------|-------------|-------------|-------------|
| Semimajor axis (km)            | 7335        | 12269       | 12162       |
| Eccentricity                   | 0.02        | 0.004       | 0.013       |
| Inclination (degrees)          | 49.2        | 109.9       | 52.6        |
| Diameter (m)                   | 0.24        | 0.60        | 0.60        |
| Area-to-mass ratio (m²/kg)     | 0.00096     | 0.00069     | 0.00069     |
| Launch date                    | February 1975 | May 1976    | October 1992 |
Introduction

The Mg II index is, however, not yet available for (semi) real-time computations, but with a delay of at least several weeks. Secondly, due to missing observations (gaps up to several months in some occasions), it could not be used in the long-arc orbit computations.

1.3 Organization

Chapters 2-5 of this dissertation have a preparatory and theoretical character, in which the reader will be familiarized with the particular equations of motion and relevant mean force models that are included in CODIOR. These chapters present simulation results only, whereas the consecutive Chapters 6 to 8 present results based on observations.

Chapter 2 reviews the (analytical and numerical) averaging procedures and the choice of variables, as well as the corresponding differential equations of motion. Chapter 3 describes the test procedure adopted to validate the mean motion model. CODIOR is a semi-analytical theory and therefore each new mean force model must be verified and validated with respect to a purely numerical integration, in order to establish the precision (mathematical, actual error) with which it may reconstruct the motion. The results of the tests with respect to gravitational forces, as well as the orders of magnitude of the various forces, are presented in Chapter 4. Chapter 5 is dedicated to the nongravitational force modeling. It describes the radiation pressure forces, as well as the atmospheric and thermal drag forces, that act on a satellite.

In Chapter 6 atmospheric density modeling is discussed and it can be read independently from the previous chapters. The physical model hypotheses and equations are reviewed briefly. A new proxy, the Mg II index, describing the intensity of the solar chromospheric EUV emissions, has been proposed for atmospheric density modeling purposes and tested successfully.

Precise orbit computation in true-of-date rectangular elements (x,y,z) is a major subject in this study because these orbits are required for computing mean observed elements. They are also used in the evaluation of atmospheric density models in Chapter 6. The SLR tracking system, force models and adjustment results are presented in Chapter 7 for the satellites Starlette, LAGEOS-2, LAGEOS-1 and GFZ-1.

The long-arc adjustments for the satellites Starlette, LAGEOS-2 and LAGEOS-1 are presented in Chapter 8. The estimated secular ($J_2 - J_4$) and periodic (annual, semiannual and 18.6 year variations) time-varying components of the gravity field and tides are presented also in that chapter. Interpretations of the estimated parameters are discussed.

Chapter 9 gives the general conclusions of this study, as well as an outlook.
2 Principles of Averaging

For studies in which the short-period orbit perturbations (in this study, periods of up to slightly over 1 day) are of no interest, for example in case of satellite lifetime studies or the estimation of certain geophysical parameters representing secular or long-period variations, one may consider removing this source of ‘noise’. This chapter describes the averaging techniques, which filter the short-period perturbations from a signal without affecting the long-period and secular perturbations, employed in the CODIOR software.

The choice of variables is very important in analytical averaging methods such as the Lie transformation [Deprit, 1969], as will be shown in Section 2.2. Section 2.3 presents a simple way to eliminate short-period perturbations, using Kaula's [1966] first-order theory, which is, however, less accurate. The fourth and fifth sections present methods to average nongravitational forces. Finally, Section 6 gives an overview of where the respective averaging methods are used.

2.1 General Averaging Principles

Averaging has been applied in celestial mechanics since secular perturbations have for a long time been the prominent area of interest, and by filtering the less interesting short-period terms the solution accuracy improved. The oldest averaging theory used in celestial mechanics is that from Poincaré-von Zeipel [von Zeipel, 1916], which aims at eliminating the ‘fast’ variable. In this particular case, this variable has to be removed from the Hamiltonian function, or Hamiltonian. The Hamiltonian is equivalent to the total amount of energy (kinetic plus potential), and thus must be constant for bodies evolving under the influence of gravitational (i.e. conservative) forces, as is the case in celestial mechanics. Its value is independent of the variables in which it is expressed. It has been used by Brouwer [Brouwer, 1959] in his second-order theory of motion perturbed by $J_2$. A more complete theory of mean motion, modeling all gravitational perturbations averaged analytically and nongravitational perturbations (atmospheric drag and solar radiation pressure) averaged numerically, has been incorporated in the program ROAD [Wagner et al., 1974]. There, the second-order terms of Brouwer's theory have been added to the terms described by Kaula's first-order theory [Kaula, 1966]. Kaula's theory, however, is not accurate enough because it is only a first-order theory and this penalizes the result. This motivated Borderies to develop a similar theory in France [Borderies, 1976], based on a more solid mathematical formulation and a more accurate definition of 'mean' observations. This work has been continued and improved again by Exertier [Exertier, 1988], who added higher-order developments to the theory, and who finally established a filtering method [Exertier, 1990] in
accordance with current tracking data precision. However, the theory was still not precise enough
with respect to the now very precise observables, i.e. the observed mean state-vector elements, and
so Métris established a new, homogeneous mean differential system obtained by Lie
transformation [Métris, 1991]. The initial version of CODIOR is based on this system, which will
be discussed in the following section. Section 2.2 gives a short overview of the developments done
by Métris [1991], to which work the reader is referred to for a more detailed description.

For a given gravitational perturbation expressed in Hamiltonian form, the aim of averaging
is to obtain a new Hamiltonian \( H' \) that is no longer a function of the mean anomaly \( l \). This is
achieved by a canonical transformation of variables [Métris, 1991]:

\[
\begin{align*}
\text{variables:} & \quad (L, G, H, l, g, h) \to (L', G', H', l', g', h') \\
\text{Hamiltonian:} & \quad H(L, G, H, l, g, h) \to H'(L', G', H', l', g', h')
\end{align*}
\] (2.1)

where the Hamiltonian has been expressed in Delaunay variables, themselves expressed in orbital
elements (Figure a4, Appendix A):

\[
L = \sqrt{\mu a}, \quad G = L\sqrt{(1-e^2)}, \quad H = G \cos i, \text{ and the angle variables } l = M, \quad g = \omega, \quad h = \Omega
\]

\[
\mu = GM = \text{product of universal gravitational constant and mass of the central body}
\]

for reasons which will be explained in the next section. In practice this means that the motion will
be free of the otherwise typical \( n \) per revolution (an integer number \( n \) of sinusoidal cycles per
revolution) perturbations, while the \( m \)-daily (\( m \) sinusoidal cycles per day) perturbations are
eliminated by taking \( m=0 \) (since perturbations with periods of one day or less are of no interest to
this study). The perigee period is still present, which is a perturbation due to the (odd) zonal
geopotential coefficients (\( m=0 \)) only. The tesseral harmonics induce only perturbations with
periods of slightly more than a day, except in case of resonance [Sansò and Rummel, 1989]. This
means that, after the averaged Hamiltonian has been obtained, the motion may be represented
using only the zonal harmonics. For other, smaller, perturbations, it is sometimes easier to average
a Kaula type potential [Kaula, 1966], which is described in Section 2.3. The classical gravity
potential \( R \) developed in spherical harmonics with coefficients \( C_{nm} \) and \( S_{nm} \) (of degree \( n \) and order \( m \)) is given below in spherical coordinates \( r, \phi, \) and \( \lambda \) [Kaula, 1966]:

\[
R(r, \phi, \lambda) = \sum_{n=2}^{\infty} \sum_{m=0}^{n} \left( \frac{a_e}{a} \right)^n \left( C_{nm} \cos m\lambda + S_{nm} \sin m\lambda \right) P_{nm}(\sin \phi)
\] (2.2)

where \( a_e \) is the mean equatorial radius, \( r \) the radius, \( \lambda \) the longitude, and \( \phi \) the geocentric latitude
of the satellite, respectively, and \( P_{nm} \) the associated Legendre function of degree \( n \) and order \( m \).

Kaula was the first to express this potential as a function of the orbital elements \((a, e, i, \Omega, \omega, M)\),
which is more practical in satellite geodesy [Kaula, 1966].
2.2 Lie Transformation

Finally, for most forces that may not be represented in the form of a potential, such as surface forces, the averaging has to be performed numerically. This implies the use of numerical quadrature methods [Bulirsch and Stoer, 1980], such as Simpson’s or Gauss’, which average the forces over the mean anomaly, that is, over one orbital revolution. This is easy to implement, but less precise and less computationally efficient. The averaging is done during the long-arc orbit computation, so at each integration step instead of once for all, as is the case for the conservative forces. This fundamental difference will be explained in the next sections.

2.2 Lie Transformation

The basic theory as described in [Métris, 1991] will be reviewed in this section. The algorithms developed by Deprit [1969] and Deprit and Rom [1970], which are specific applications in celestial mechanics of the Lie transformation (given in eq. (2.6)), have been used in that study. An advantage of this analytical transformation is that the accuracy of the approximation as a function of order of development is known. Secondly, when the equations of motion are expressed in canonical Delaunay variables, they are much more compact and skew symmetric; this form is typical of a differential equation system when it is expressed in canonical variables. The equations of motion are now given as:

\[
\begin{align*}
\frac{dL}{dt} &= \frac{\partial H}{\partial l}, & \frac{dl}{dt} &= -\frac{\partial H}{\partial L}, \\
\frac{dG}{dt} &= \frac{\partial H}{\partial g}, & \frac{dg}{dt} &= -\frac{\partial H}{\partial G}, \\
\frac{dH}{dt} &= \frac{\partial H}{\partial h}, & \frac{dh}{dt} &= -\frac{\partial H}{\partial H}
\end{align*}
\]  

(2.3)

where the Hamiltonian in general depends on the six variables \(L, G, H, l, g, h\) (cf. eq. (2.1)), (some of) which become singular for small eccentricities and/or small inclinations. The Poisson brackets, necessary in the averaging procedure, benefit from the use of canonical variables. Poisson brackets have the following general form, where the notation \(\{\phi, \psi\}\) represents Poisson brackets of the functions \(\phi\) and \(\psi\):

\[
\{\phi, \psi\} = \sum_{j=1}^{3} \left( \frac{\partial \phi}{\partial \psi_j} \frac{\partial \psi}{\partial \phi_j} - \frac{\partial \phi}{\partial \phi_j} \frac{\partial \psi}{\partial \psi_j} \right)
\]  

(2.4)

The Poisson brackets are invariant under a canonical transformation of variables, and this characteristic simplifies the calculations of Lie transforms significantly. The following relationship between the Delaunay variables and the Poisson brackets illustrates this, where \(V_i\) are the metric variables \((L, G, H)\), \(v_i\) the angles \((l, g, h)\) and \(W\) a generating function (described in eq. (2.7)): 
2 Principles of Averaging

\[ \dot{V}_i = \frac{\partial H}{\partial v_i}, \text{ and } \dot{v}_i = -\frac{\partial H}{\partial V_i} \]  \hspace{1cm} (2.5)

\[ \{H, W\}_V = \{H; W\}_{V, V} \]

Within the framework of canonical transformation of variables from osculating \((V_i, v_i)\) to mean variables \((V'_i, v'_i)\), the initial Hamiltonian \(H\) (lower index: order of the development) and the averaged Hamiltonian \(H'\) (upper index: order of the development) are formally developed in a Taylor series of the small parameter \(\epsilon\) (\(\epsilon = J_2\), the first-order perturbation):

\[ H(L, G, H, l, g, h) = H_0^0(L) + \sum_{k=2}^\infty \frac{\epsilon^k}{k!} H_k^0(L, G, H, l, g, h) \]

\[ H'(L', G', H', l', g', h') = H_0^0(L') + \sum_{k=2}^\infty \frac{\epsilon^k}{k!} H_k^0(L', G', H', l', g', h') = E_w H \]  \hspace{1cm} (2.6)

and \(H_0^0(L) = H_0^0(L') = -\mu / 2a\)

where \(E_w\) represents the Lie transformation. The transformation is generated by a generating function \(W(L, G, H, l, g, h)\) likewise developed in powers of \(\epsilon\):

\[ W = \sum_{k=0}^\infty \frac{\epsilon^k}{k!} W_{k+1} \]  \hspace{1cm} (2.7)

For example, the transformation of variables up to second order is expressed as follows:

\[
\begin{align*}
V_i - V'_i &= \epsilon \frac{\partial W_1}{\partial v_i} + \frac{\epsilon^2}{2} \left( \frac{\partial W_2}{\partial v'_i} + \left( \frac{\partial W_1}{\partial v'_i} \right) \right) + \ldots \hspace{1cm} (i = 1, 3) \\
\dot{v}_i - \dot{v}'_i &= -\epsilon \frac{\partial W_1}{\partial V'_i} + \frac{\epsilon^2}{2} \left( \frac{\partial W_2}{\partial V'_i} + \left( \frac{\partial W_1}{\partial V'_i} \right) \right) + \ldots \hspace{1cm} (i = 1, 3)
\end{align*}
\]  \hspace{1cm} (2.8)

The algorithm of Deprit ([Deprit, 1969]; [Deprit and Rom, 1970]) using Lie transformation as defined in eq. (2.6) by \(E_w\) produces the expression of the functions \(H_k^0\) as functions of the initial Hamiltonians \(H^0\) and \(W\). For example, the expressions up to second order are the following:

\[
\begin{align*}
H_0^0 &= H_0 \\
H_0^1 &= H_0^1 + \left\{ H_0^0; W_1 \right\} \\
H_0^2 &= H_0^2 + \left\{ H_0^1; W_1 \right\} + \left\{ H_0^0; W_2 \right\} \\
H_0^k &= H_0^k + \left\{ H_0^k; W_i \right\}
\end{align*}
\]  \hspace{1cm} (2.9)

\(H_0^0\) is the Hamiltonian of the unperturbed 2-body problem (Keplerian motion). The equations (2.9) are solved for order after order in a recursive procedure with each time one equation with two unknowns: the new Hamiltonian \(H_0^k\) and the generating function \(W_i\). It is this
indetermination that allows choosing $H_0^k$ free of short-period signals, that is to say, averaged over
the mean anomaly:

$$H_0^k = \left\{ H_0^k + \left\{ H_0^0; W_1 \right\} \right\}_p = \left\{ H_0^k \right\}_p = \frac{1}{2\pi} \int_0^{2\pi} H_0^0 dl^\prime \quad \text{(since } H_0^0 \text{ is only a function of } a)$$

(2.10)

$$\frac{\partial W_1}{\partial l} = \frac{L^3}{\mu^2} \left( H_0^0 - H_0^1 \right) \quad \text{(order 1)}$$

$$H_0^2 = \left\{ H_1^1 + \left\{ H_0^2; W_1 \right\} \right\}_p = \left\{ H_1^1 + \left\{ H_0^0; W_1 \right\} + \left\{ H_0^1; W_1 \right\} \right\}_p$$

$$\frac{\partial W_2}{\partial l} = \frac{L^3}{\mu^2} \left( H_0^2 - H_0^3 + \left\{ H_2^0; W_1 \right\} + \left\{ H_0^1; W_1 \right\} \right) \quad \text{(order 2)}$$

The generating function ($W$) is always the result of an integration with respect to the mean anomaly. Therefore, it is defined except for a constant of integration $w$ independent of $l^\prime$:

$$W_k = W_k^* + w_K \left( L^*, G^*, H^*, \ldots, g^*, h^* \right) \quad (2.11)$$

This means there is an infinite number of transformations available for the elimination of the mean anomaly from the initial Hamiltonian. The mean elements are consequently defined up to the long-period variations. Indeed, taking into account the expression for the transformation of the variables at the second-order (eq. (2.10)), it can be shown that if the generating function ($W_k$) is not chosen with a zero mean at the time of the integration that defines it, long-period terms will be removed by the transformation of the variables and will no longer be present in the Hamiltonian $H_0^1$. The long-period terms are conserved as often as possible in the expressions for the Hamiltonians $H^j$ in this theory, creating generating functions of zero mean. However, at the second-order expression in eq. (2.10), the Poisson brackets themselves contain long-period terms that can no longer be transferred to any Hamiltonian. Those terms form a true transformation of variables by themselves, which unfortunately is not canonical. This means that no "centered" Hamiltonian exists that corresponds to the centering problem in space geodesy, or an averaged Hamiltonian that contains all long-period terms contained in osculating elements.

The numerical procedure of the ephemeris computation is accomplished in two steps. The first step of the computation consists in numerically integrating the Hamiltonian equations with the "best possibly centered" Hamiltonian to obtain the mean orbit in ($V_i', v_i'$):

$$\left\{ \begin{array}{l}
\dot{V}_i' = \frac{\partial H'}{\partial V_i'} \\
\dot{v}_i' = -\frac{\partial H'}{\partial V_i'}
\end{array} \right. \quad (i = 1,3)$$

(2.12)
The second step consists of a non-canonical transformation of variables of the mean orbit to obtain this time the rigorously-centered variables \( (\tilde{V}_i', \tilde{v}_i') \):

\[
\begin{align*}
\tilde{V}_i' &= \langle v_i' \rangle_r = V_i' + \langle v_i' - V_i' \rangle_r \\
\tilde{v}_i' &= \langle v_i' \rangle_r = v_i' + \langle v_i' \rangle_r \\
\end{align*}
\]  \hspace{1cm} (2.13)

The complete computation, for all gravitational perturbations treated in this way, produces hundreds of thousands of terms that could not have been obtained without the use of specific software for the manipulation of Poisson series. The programs MS \[ Claes et al., 1988 \] and MINIMS \[ Moons, 1991 \], developed at the University of Namur for this purpose especially, have been used for this specific calculation.

The algorithm uses a development of the disturbing function with respect to a small parameter. This does not require the small parameter to appear explicitly in all perturbations, but rather implies a classification of the perturbations by order of magnitude. It is not possible to define a hierarchy valid for all satellites: for low-orbiting spacecraft (altitude < 500 km) the terrestrial potential and drag are dominant, whereas for high-orbiting platforms (altitude > 2000 km) those perturbations diminish while the third-body (Moon-Sun) perturbations increase. The inclination and the eccentricity also modulate the different effects. However, the following classification by order (of magnitude) seems to be a good compromise, by choosing \( J_2 \) as small parameter (order \( i=(J_2)^i \)):

- **Order 1**: spherical harmonic \( J_2 \) of the geopotential.
- **Order 2**: spherical harmonics \( J_2^2, J_1 \) to \( J_6 \) of the geopotential, luni-solar perturbations.
- **Order 3**: spherical harmonics \( J_2^3 \), and \( J_6 \) to \( J_{40} \) and coupling terms of the geopotential (\( J_2 \cdot J_3 \) to \( J_2 \cdot J_5 \)) and \( J_2 \) with luni-solar perturbations, resonant tesseral harmonics, tidal effects, atmospheric drag, solar radiation pressure.
- **Order 4**: \( J_2^4 \), all relevant coupling terms (\( J_2^2 \) to \( J_2^2 \cdot J_6 \) etc.) and all remaining harmonics of the geopotential (> \( J_{40} \)), other nongravitational forces, relativistic effects.

Referring to Deprit's algorithm \[ Deprit, 1969 \], a function of a given order (generating function or Hamiltonian) contains all combinations of harmonics that can produce the perturbations of that particular order. Specifically:

\[
\begin{align*}
W_1 &= W_1(J_2) \\
W_2 &= W_2(J_2^2) + W_2(J_3, J_4, J_5) \\
W_3 &= W_3(J_2^3) + W_3(J_2 \cdot J_3) + W_3(J_2 \cdot J_4) + \ldots + W_3(J_6, J_7, \ldots, J_{20}) + \ldots \\
\end{align*}
\]  \hspace{1cm} (2.14)
2.2 Lie Transformation

The perturbations cannot all be accounted for independently; an expression like $W_3 (J_2 \cdot J_3)$ is the mathematical representation of coupling terms. At each order $O$ (in this study, order 1 is $J_3$), the following set of combinations has to be accounted for:

Order 1: $O(1)$

Order 2: $O(2) + O(1) O(1)$

Order 3: $O(3) + O(2) O(1) + O(1) O(1) O(1)$

Order 4: $O(4) + O(3) O(1) + O(2) O(2) + O(2) O(1) O(1) + O(1) O(1) O(1) O(1)$

This is realized automatically if the algorithm of canonical transformation is applied rigorously, and the set of perturbations is inserted in the initial Hamiltonian, which is then globally accounted for. For reasons of convenience, transparency and computational technique, the perturbations have been successively averaged up to third order. At the fourth order, certain perturbations have been rearranged, since the coupling effects are many and tedious to calculate one at a time. Fifth-order secular perturbations have been computed for $J_2$ and the coupling of $J_2$ with the fourth-order zonal harmonics only, because they may still represent an effect of several decimeters per year.

The list shown below summarizes the produced results, old (up to degree 17: [Métris, 1991]) and new (degrees 18 and up have been implemented and validated in the present study), and the number of terms obtained in the resulting series for the geopotential. It is given as an indication only, since the calculations are not restrictive: no theoretical limit prevents the calculation of the series for higher-degree harmonics or to calculate up to a higher order. Practical limits undoubtedly exist, but were not attained by the completely automated algorithm. However, for degrees exceeding 40, it is much easier and faster to use the averaged Kaula potential (Section 2.3), which provides sufficient precision for these small perturbations (for altitudes > 600 km).

Order 1: $J_2$ (Closed Form (CF))

Order 2: $J_2^2, J_3, J_4, J_5$ (CF)

Order 3: $J_2^2, J_2 \cdot J_3, J_2 \cdot J_4, J_2 \cdot J_5, J_2 \cdot J_6$ to $J_10$ (CF)

Order 4: $J_2^4, J_2^3 \cdot J_3, J_2^2 \cdot J_4, J_2^2 \cdot J_5, J_2 \cdot J_3 \cdot J_4, J_3 \cdot J_5$,

As an illustration of the importance of the various terms, the orders of magnitude of the perturbations on the Starlette orbit that correspond to these relative orders, especially the effect of
2 Principles of Averaging

the coupling terms, are presented in Table 2.1. As may be seen in the list, coupling terms have no longer been computed for the degrees 36 to 40. The use of the Kaula formulation without coupling for degrees 41 and up (approximately 1% of the perturbation) is indeed justified.

Table 2.1: the amplitude of the gravity perturbations and the coupling effect of degrees 31-35 with \( J_2 \) in particular, on the Starlette orbit. The values are given in meters (\( a, e, i \)) or m/year (\( \Omega, \omega, \omega+M \)).

<table>
<thead>
<tr>
<th>degrees</th>
<th>( a )</th>
<th>( e )</th>
<th>( i )</th>
<th>( \Omega )</th>
<th>( \omega )</th>
<th>( \omega+M )</th>
</tr>
</thead>
<tbody>
<tr>
<td>3-17</td>
<td>0.002</td>
<td>5500</td>
<td>100</td>
<td>85000</td>
<td>580000</td>
<td>400</td>
</tr>
<tr>
<td>17-30</td>
<td>0.000</td>
<td>28</td>
<td>0.5</td>
<td>2000</td>
<td>5000</td>
<td>250</td>
</tr>
<tr>
<td>31-35</td>
<td>0.000</td>
<td>0.62</td>
<td>0.01</td>
<td>150</td>
<td>260</td>
<td>50</td>
</tr>
<tr>
<td>coupling 31-35</td>
<td>0.000</td>
<td>0.03</td>
<td>0.000</td>
<td>1.6</td>
<td>17</td>
<td>3</td>
</tr>
<tr>
<td>36-40</td>
<td>0.000</td>
<td>0.5</td>
<td>0.01</td>
<td>50</td>
<td>250</td>
<td>11</td>
</tr>
<tr>
<td>41-50</td>
<td>0.000</td>
<td>0.36</td>
<td>0.006</td>
<td>10</td>
<td>105</td>
<td>11</td>
</tr>
</tbody>
</table>

2.2.1 Example: Application to the J2 Term of the Geopotential

An example of the principle of averaging will be given for the case of flattening of the Earth, called the \( J_2 \) problem in celestial mechanics. This perturbation is treated separately in celestial mechanics, since it is a first-order perturbation, much larger than all other higher-order (gravitational) perturbations. In the case of flattening only, represented by the \( C_{2,0} = -J_2 \) term, the disturbing potential in spherical coordinates, as given in eq. (2.2), reduces to \( n=2, m=0 \):

\[
R_{2,0} = \frac{\mu^2}{L^5} a^2 C_{2,0} (\sin \phi)
\]  \hspace{1cm} (2.15)

If this classical expression is developed in orbital elements and after that in Delaunay variables (eq. (2.1)), the Hamiltonian of the problem has the following form, where \( f \) is the true anomaly:

\[
H_1^0(J_2) = \frac{\mu^2}{L^5} a^2 C_{2,0} \left[ -\frac{1}{4} \frac{3 H^2}{G^2} + \frac{3}{4} \frac{\cos(2f + 2g)}{G^2} - \frac{3}{4} \frac{H^2 \cos(2f + 2g)}{G^2} \right] \frac{a^3}{r^3}
\]

where \( \frac{a^3}{r^3} = \frac{a^2 L^2}{G^2} (1 + e \cos f) \), and \( L = \sqrt{\mu a} \), \( G = L \sqrt{(1 - e^2)} \). \hspace{1cm} (2.16)

The averaging of the expression of this perturbation in \( J_2 \), the first order, gives:

\[
H_0^0 = \left\langle H_1^0(J_2) \right\rangle_f = \frac{\mu^2}{L^5 G^3} a^2 C_{2,0} \left[ -\frac{1}{4} + \frac{3 H^2}{4 G^2} \right]
\]  \hspace{1cm} (2.17)
and next for the second order:

\[
H_0 = \mu \left( a_e \right)^4 \frac{1}{\eta} \frac{C_2^2 a}{16} \left( \frac{15}{8} \left( 1 - 2\theta^2 - 7\theta^4 \right) + \frac{3}{2} \eta \left( 1 - 6\theta^2 + 9\theta^4 \right) + \frac{15}{8} \eta^2 \left( 1 - \frac{15}{8} \theta^2 + \theta^4 \right) + \frac{3}{4} e^2 \left( 1 - 16\theta^2 + 15\theta^4 \right) \cos 2g + 3e^2 \left( 1 - 6\theta^2 + 5\theta^4 \right) \frac{1 + 2\eta}{(1 + \eta)^2} \cos 2g \right)
\]

where \( \eta = \frac{G'}{L} = \sqrt{1 - e^2} \), and \( \theta = \frac{H'}{G'} = \cos i' \) \hspace{1cm} (2.18)

Note the large similarity between these two expressions (first and second-order) of the transformed Hamiltonian as obtained by Métris [1991] and the results obtained by Brouwer [1959] after its first transformation (elimination of the mean anomaly). The last term in the multiplier of \( \cos 2g \) in the expression of \( H_0 \) does not appear in Brouwer's theory. It originates from a constant term that was added to \( W_1 \) in order to obtain a generating function with zero mean. The sum \( \left( H_0^3 + H_1^3 + H_2^3 \right) \) represents the second-order development of the new Hamiltonian \( H'(J_2) \) free of the mean anomaly. Only secular and long-period terms appear in these expressions. The terms in the multiplier of \( \cos 2g \) in \( H_0^3 \) represent the long-period part to the first-order in the variables \( (h', i') \). Starting with a relatively simple and above all finite expression of the perturbation, eq. (2.17), we are now led to introduce the partial derivatives of the development of \( H' \) in the new equations of motion. Also, for reasons of precision, the necessity to develop the calculations by algebraical manipulation up to fourth-order was shown [Métris, 1991], which implies calculating the analytical expression of \( H_0^3 \), followed by that for \( H_0^4 \), in \( J_2 \) to the power four.

Originally, and as demonstrated in this section, the averaged Hamiltonians have been obtained expressed in Delaunay elements. These elements are well-known in celestial mechanics, but not usual in space geodesy, where orbital elements, rectangular elements, or elements projected in a local satellite frame are most widely used. The Kaula type potential in Keplerian orbital elements, presented in the next section, and many of the other perturbing forces, are more conveniently expressed in orbital elements than in Delaunay elements. A second important reason to switch to orbital elements is that it reduces the number of terms in the averaged Hamiltonians by up to 30%. It was not necessary to recommence all computations, since it is possible to project the averaged Hamiltonians in orbital elements.

There is a large number of different analytical terms, representing the derivatives of the disturbing function with respect to the orbital elements, in these expressions. Therefore, they have been encoded and subsequently saved in a permanent file. They can subsequently be read by a computer program that evaluates them during each step of the numerical integration procedure of the mean orbit. This is the general principle to access the theory that has been retained in the entire
program for each perturbation averaged using Lie transforms. The series are organized per perturbation type (geopotential, luni-solar attraction, Earth tides) and order of development, giving for example for the averaged second-order Hamiltonian in $J_2$ the following expression:

$$H_0^G = \mu^4 a^4 J_2^2 \sum_{i=1}^{\text{ind}} c^4(i) \sum_{l=1}^{7} e^4(i) e^6(i) e^7(i) \left(\frac{1}{1+G}\right)^{e^7(i)} \cos(2i \Omega)$$

where $c$ to $c^7$ indicate the column and $i$ the line number where the values may be found in the file, which begins by giving the total number of lines, $\text{ind}$, which for this particular example of a perturbation equals 13.

For a satellite like Starlette, this Hamiltonian formulation up to degree 40 is not sufficient, which has been shown in Table 2.1. The number of terms becomes increasingly large for higher degrees, which has a delaying effect on the computational speed. For smaller perturbations, like those due to the gravity field coefficients of degree 40 and higher, it is easier to average a Kaula type potential [Kaula, 1966], which was presented in eq. (2.2). This technique is discussed in the next section.

**2.3 Averaged Kaula Potential**

The equations that describe the perturbations of the orbital elements are known as the Lagrange planetary equations, which are no longer canonical as in eq. (2.3). They are presented below for the sake of completeness, $n$ being the mean Keplerian motion ($n=\left(\frac{\mu}{a^3}\right)^{1/2}$), where the disturbing potentials $R$ are either obtained using a simple relation linking them to the averaged Hamiltonians, or by taking Kaula type potentials as presented in eq. (2.2) [Kaula, 1966]:

$$\begin{align*}
\frac{da}{dt} &= \frac{2}{na} \frac{\partial R}{\partial a} \\
\frac{de}{dt} &= -\sqrt{1-e^2} \frac{\partial R}{\partial e} + \frac{1-e^2}{na} \frac{\partial R}{\partial e} \\
\frac{di}{dt} &= \frac{-1}{na^2 \sqrt{1-e^2} \sin i} \frac{\partial R}{\partial \Omega} + \frac{\cos i}{na^2 \sqrt{1-e^2} \sin i} \frac{\partial R}{\partial \Omega} \\
\frac{d\Omega}{dt} &= \frac{1}{na^2 \sqrt{1-e^2} \sin i} \\
\frac{d\omega}{dt} &= \frac{\sqrt{1-e^2}}{na^2 e} \frac{\partial R}{\partial \omega} - \frac{\cos i}{na^2 \sqrt{1-e^2} \sin i} \frac{\partial R}{\partial \omega} \\
\frac{dM}{dt} &= n \left(\frac{2}{na} \frac{\partial R}{\partial a} - \frac{1-e^2}{na^2 e} \frac{\partial R}{\partial e}\right)
\end{align*}$$

(2.19)
2.3 Averaged Kaula Potential

These equations represent true motion when osculating elements are used on the right-hand side of the differential equations, and not-averaged potentials. When averaged potentials are used (and, consequently, mean orbital elements), the equations represent the mean motion desired in this study. The relation linking the averaged disturbing potential $R'$ to the averaged Hamiltonian $H'$ is the following: $R' = n - H'$. Thus, one only needs to compute the partial derivatives of $R'$, so basically $H'$, with respect to the mean orbital elements to obtain the system of mean motion equations. The Lie transformation is a very powerful averaging technique, the precision of the averaging being a function of the highest developed order. Because this transformation involves a large amount of careful computing, which becomes more and more difficult and ponderous with increasing order, it is more advantageous from a practical point of view to employ an averaged Kaula potential from some point on. This transition point is not arbitrary, but a function of the magnitude of the perturbation. For the gravity field, for example, the perturbation decreases with increasing degree $n$, as may be seen in eq. (2.2) and Table 2.1, as well as in the equation below, which equally represents the geopotential, but expressed in orbital elements:

$$R = \frac{\mu}{r} \sum_{n=2}^{\infty} \left( \frac{a_n}{r} \right) \sum_{m=0}^{n} \sum_{p=0}^{n} F_{nmn}(i) \sum_{q=-\infty}^{\infty} G_{nmp}(e) S_{nmpq}(\mathcal{M}, \omega, \Omega, \theta) \quad (2.20)$$

where $F_{nmn}(i)$ is the inclination function, and $G_{nmp}(e)$ the eccentricity function. These result from the transformation of eq. (2.2), which is in spherical coordinates, to eq. (2.20) in orbital elements. The functions $S_{nmpq}$ are defined as follows:

$$S_{nmpq} = \begin{cases} C_{nm}^{(n-m) \text{even}} \cos \psi_{nmpq} + S_{nm}^{(n-m) \text{odd}} \sin \psi_{nmpq} & \text{if } n-m \text{ is even} \\ C_{nm}^{(n-m) \text{odd}} \cos \psi_{nmpq} + S_{nm}^{(n-m) \text{even}} \sin \psi_{nmpq} & \text{if } n-m \text{ is odd} \end{cases} \quad (2.21)$$

with the argument $\psi_{nmpq} = (n-2p)\omega + (n-2p+q)M + m(\Omega - \theta)$.

The representation of the gravity field as a Kaula potential, eq. (2.20), is a sum of perturbations at all frequencies $\Psi_{nmpq}$, which in itself is a linear combination of three basic frequencies (cf. eq. (2.21)). These frequencies are:

1. the orbital frequency $\Omega + \dot{\mathcal{M}}$, for degree $n$ commonly known as $n$ cycles/revolution ($n$/rev);
2. the perigee frequency $\dot{\omega}$; and
3. the nodal day frequency $\dot{\Omega} - \dot{\Theta}$, expressing a complete rotation of the Earth with respect to the precessing orbit plane, which for order $m$ is commonly known as $m$-daily.

Since only the secular and long-period perturbations are of interest for the mean model, averaging of the potential is required. Because the frequencies of the contributing terms are explicitly known using the Kaula formulation, this operation is fairly simple, and basically a selection process instead of an averaging algorithm. First of all, the tesseral harmonics only
Principles of Averaging

engender short-period perturbations, the largest period being slightly over a day. This means that the last part in eq. (2.20) will not be a part of the argument in the averaged potential. The even zonal harmonics engender secular perturbations \((n\cdot 2p\cdot n\cdot 2p\cdot q=0)\), long-period perturbations (for \(n\cdot 2p\cdot q=0\) and \(n\cdot 2p>0\)) at the perigee period and its submultiples, and short-period perturbations (for \(n\cdot 2p\cdot q\neq 0\)). Selecting only \(npq\) combinations that satisfy the first or second condition \((m\) is always 0), the short-period perturbations will have been eliminated.

The potential of eq. (2.21) may now be expressed in the following specific form:

\[
\langle R_{n,0} \rangle = \frac{\mu}{r} \left( \frac{d_x}{r} \right)^n C_n \sum_{p=0}^{n} G_{n,p,2p-n}(e) F_{n,0,p}(t) \begin{cases} \cos[(n-2p)\omega] & \text{when } n = \text{even} \\ \sin[(n-2p)\omega] & \text{when } n = \text{odd} \end{cases}
\]

(2.22)

where the same notation as in eq. (2.20) has been used. This potential is no longer a function of the mean anomaly \(M\), nor of the node \(\Omega\). Partial derivatives with respect to the semimajor axis \(a\) and perigee \(\omega\) are easily computed, while partial derivatives with respect to the eccentricity \(e\) or inclination \(i\) are obtained by differentiating the eccentricity and inclination function, respectively.

The eccentricity function has the following specific form in the case of a potential as eq. (2.22) (and more complex otherwise) [Kaula, 1966]:

\[
G_{n,p,2p-n}(e) = \frac{1}{(1-e^2)^{n-0.5}} \sum_{d=0}^{n-1} \frac{2d+n-2p}{d} \left( \frac{e}{2} \right)^{2d+n-2p'}
\]

with \(p' = p \) for \(p \leq \frac{n}{2}\), and \(p' = n - p \) for \(p > \frac{n}{2}\). (2.23)

The notation \(\binom{x}{y}\) is short for \(\frac{x!}{y!(x-y)!}\).

The Kaula inclination function has the following expression, given here in a form easier to implement and faster than the original function as developed by Kaula [1966]:

\[
F_{n,m,p}(i) = C \sum_{u} (-1)^u \binom{2p}{u} \left( \frac{2n-2p}{n-m-u} \right) \cos \frac{\pi}{2} u + m + 2u \sin \frac{\pi}{2} u + m + 2u
\]

\[
C = \frac{(-1)^{(n-m)/2} \left( \frac{2^{n-m}}{(n-m)!} \right)^{n}}{\left( \frac{2^{n-m}}{(2^{n-m})!} \right)^{p}}
\]

(2.24)

where the summation over \(u\) is from the larger of \((0,2p\cdot n\cdot m)\) to the smaller of \((n-m,2p)\).

Analytically Averaged Nongravitational Forces

If the perturbing accelerations can no longer be expressed as a gradient of some potential, Lagrange equations cannot be derived. Instead, the Gauss equations have to be used, which also give the (first) time derivatives of the orbital elements, but with the accelerations explicitly
2.5 Numerically Averaged Nongravitational Forces

expressed in a local orbital reference frame (referred to as r,t,n; see Figure a5, Appendix A). When the expression of the force is straightforward, such as is the case for Yarkovsky thermal drag [Barlier et al., 1986], or just the Yarkovsky effect, the force may still be averaged analytically. This force is due to the photon thrust along the satellite spin axis, caused by heating of the satellite by Earth infrared (IR) radiation. The components of the acceleration in the \( PWQ \) frame are \( a_p \), the acceleration in the direction of the perigee, \( a_h \), the acceleration perpendicular to the orbital plane in the direction of the angular momentum, and \( a_Q \) being the acceleration in the direction \( Q = \hat{W} \wedge \hat{P} \) completing the orthonormal frame. Without going further into the nature of the force (which will be discussed in Section 5.5), since this chapter is meant only to explain different ways of averaging, the Yarkovsky acceleration may be expressed as follows (Métris, personal communication), with \( \hat{s} \) being the spin-axis orientation vector of the satellite:

\[
\ddot{\hat{s}} = 4\gamma_0\Delta T_0 \cos \delta \left[ \cos(v - \delta)\hat{P} \cdot \hat{s} + \sin(v - \delta)\hat{Q} \cdot \hat{s} \right] \hat{s}
\] (2.25)

where the variable of interest is \( v \), the true anomaly, and \( \delta \) represents a thermal delay angle. Subsequently, this force is inserted into the Gauss equation for the semimajor axis, for example, expressed in the \( PWQ \) frame:

\[
\frac{da}{dt} = -\frac{2}{\sqrt{1-e^2}} \left[ -\sin v \hat{P} \cdot \hat{P} + (\cos v + e)\hat{Q} \cdot \hat{Q} \right]
\] (2.26)

yielding a complicated equation

\[
\frac{da}{dt} = -\frac{2}{\sqrt{1-e^2}} \left[ 4\gamma_0\Delta T_0 \cos \delta \left[ -\sin v \cos(v - \delta)(\hat{P} \cdot \hat{s})^2 + \cos v \sin(v - \delta)(\hat{Q} \cdot \hat{s})^2 + \cos(2v - \delta)(\hat{Q} \cdot \hat{s})(\hat{P} \cdot \hat{s}) \right] \right]
\] (2.27)

which, after averaging over the mean anomaly, yields the mean partial derivative, indicated by the bar (Métris, personal communication):

\[
\frac{d\bar{a}}{dt} = -\frac{2\gamma_0\Delta T_0 \sin 2\delta}{\sqrt{1-e^2}} \left( (\hat{P} \cdot \hat{s})^2 + (\hat{Q} \cdot \hat{s})^2 \right)
\] (2.28)

The same procedure has to be repeated for the remaining 5 Gauss equations. Due to the dependence of the force on the true anomaly, eq. (2.25), the result is specific for this force. Each force averaged in this way thus requires independent averaging, each yielding a specific set of mean Gauss equations.
2 Principles of Averaging

2.5 Numerically Averaged Nongravitational Forces

When a force may not be expressed as a gradient of a potential, nor as a relatively simple explicit acceleration as was discussed in Section 2.4, the Gauss equations have to be used in combination with a numerical quadrature to obtain the mean temporal derivatives of the orbital elements. The discrete, numerical quadrature to obtain the averaged time derivatives of the orbital elements, \( \dot{E} \), is an approximation of the following, continuous, integral:

\[
\dot{E} = \frac{1}{T} \int_{T-0.5T}^{T+0.5T} (G a_{RSW}) \, dt
\]  
(2.29)

where \( T \) is the orbital period of revolution of the satellite, \( G \) the Gauss matrix and \( a_{RSW} \) the accelerations in the directions \( R \), \( S \) and \( W \), which are best suited for these types of perturbation. The components of this acceleration are \( a_R \), the radial acceleration in the direction from the Earth's Center of Mass (CM) to the satellite's, \( a_S \) is the tangential acceleration component in the direction of the motion and \( a_W \) is the acceleration perpendicular to the orbital plane in the direction of the angular momentum. The right-hand members of the Gauss equations in approximate osculating elements have to be averaged over one orbital revolution (instantaneous perigee ± 180°) by a numerical quadrature. The perigee position is not constant, \( \omega \neq 0 \), but this motion is negligible when only one orbital revolution is considered (for example, the period of the argument of perigee of Starlette is 109 days). Expressed in this particular orbital frame, the Gauss equations (\( \dot{E} = G a_{RSW} \)) have the following form [Zarrouati, 1987]:

\[
\begin{align*}
\frac{da}{dt} &= \frac{2}{n\sqrt{1-e^2}} \left[a_R \sin \nu + a_S (1 + e \cos \nu)\right] \\
\frac{de}{dt} &= \frac{\sqrt{1-e^2}}{na} \left[a_R \sin \nu + a_S (\cos E + \cos \nu)\right] \\
\frac{di}{dt} &= \frac{a_W a (1 - e \cos E) \cos(\omega + \nu)}{n a^2 \sqrt{1-e^2}} \\
\frac{d\Omega}{dt} &= \frac{a_W a (1 - e \cos E) \sin(\omega + \nu)}{n a^2 \sqrt{1-e^2} \sin i} \\
\frac{d\omega}{dt} &= \frac{\sqrt{1-e^2}}{n a e} \left[-a_R \cos \nu + a_S \left(1 + \frac{1}{1+e \cos \nu}\right) \sin \nu\right] \\
& \quad - \frac{a_W a (1 - e \cos E) \cos(\omega + \nu)}{n a^2 \sqrt{1-e^2} \sin i} \\
\frac{dM}{dt} &= n + \frac{1-e^2}{n a e} \left[a_R \left(-\frac{2e}{1+e \cos \nu} + \cos \nu\right) - a_S \left(1 + \frac{1}{1+e \cos \nu}\right) \sin \nu\right]
\end{align*}
\]  
(2.30)
2.6 Application of the Averaging Techniques

where \( \nu \) is the true anomaly and \( E \) the eccentric anomaly. These equations will be used in Chapter 5, when the surface force modeling will be introduced. The averaging over the mean anomaly, which corresponds to one orbital revolution, of these forces is performed by a numerical quadrature of eq. (2.30), which means short-period perturbations will be required. This procedure cannot be replaced by averaging of the Gauss equation over one revolution (so averaging over the true and eccentric anomaly of eq. (2.30)), since the perturbations are functions of both position and time explicitly.

2.6 Application of the Averaging Techniques

The averaging techniques described in the Sections 2.2-2.5 have been applied in CODIOR. The Lie transforms are used in the terrestrial gravity field modeling (up to degree 40), solid Earth tides and third-body force models. The direct solar radiation pressure force (i.e. in absence of shadowing effects, or a full-Sun orbit) can be derived from a potential, which allows analytical averaging through Lie transformations. Averaged Kaula potentials are used in the ocean tide force model and in order to compute the gravitational effect of the redistribution of the atmosphere, as well as for the terrestrial gravity field (for degrees 41-70). These gravitational force models will be described in Chapter 4. The expressions for the nongravitational forces that are described in Section 5.5 are analytically averaged, of which an example was given in Section 2.4. Finally, the numerical averaging technique explained in Section 2.5 is applied to the solar radiation pressure (with shadowing effects), Earth albedo and atmospheric drag forces, which are described in Sections 5.1, 5.2 and 5.3, respectively.

This concludes the chapter concerning the four averaging principles applied in this study. The next chapter describes the validation procedure that is employed to test the consistency and precision of the mean force models.
2 Principles of Averaging
3 The Validation Procedure

The implementation of a mean force model requires a validation procedure to verify the modeling precision, and this is performed by adjustment to simulated observed mean elements. The simulated observations tie the averaged force models to a reference: a precise numerically integrated orbit. The simulated observed mean elements are obtained by analytical and numerical filtering of extrapolated orbits in osculating elements, computed with a small integration step-size. This small step-size assures a very precise ephemeris (all perturbations are well modeled) when a robust integrator is employed. This ephemeris contains long-period as well as short-period fluctuations (the $n/\text{rev}$ and the $m$-daily, for example), but the latter signals are filtered out of the orbit. Only the long-period and secular signals are contained in the observed mean elements [Exertier, 1990]. This procedure yields six mean orbital elements per filtered orbit, or more in general, for a given epoch. These simulated observed mean elements are then used in the adjustment of the initial conditions of a CODIOR long arc, which uses a large integration step-size thanks to its averaged differential system. The difference between the CODIOR ephemeris and the simulated observed mean elements gives a measure of the mean model's precision. Ideally, the difference would be zero: any deviation from zero is due to mismodeling, imperfect filtering of the short-period perturbations and/or model approximations. The chain of activities necessary to test a mean force model is presented in the following three sections. Resonance is discussed in the fourth section separately, since this kind of perturbation has to be treated in a special way where mean motion is concerned. An example of this procedure for the $J_2$ perturbation only, providing error magnitudes per order of (analytical) development, concludes this chapter.

3.1 Numerical Integration in Osculating Elements

All numerical integrations have been performed using multi-step integrators (i.e. prediction and correction steps), in particular the Adams-Bashforth-Moulton algorithm (ABM) [Bulirsch and Stoer, 1980], which integrates first-order differential equations, and the Adams-Moulton-Cowell algorithm (AMC) [Bulirsch and Stoer, 1980], which integrates second and first-order differential equations, as well as the ultra-stable single-step Bulirsch-and-Stoer extrapolator (BS) [Bulirsch and Stoer, 1980]. The integrators have to be reliable (accurate and robust) in order to be useful in this validation procedure, since the fully numerically integrated reference orbit has the same arc length as the CODIOR long arc. Due to the inevitably small step-size (typically 60 s or less) of the reference arc, since all high-frequency perturbations have to be well represented, this had to be verified. The computations have been performed on a DEC Alpha 500, in double
precision or 16 significant digits, while the integrators themselves have been coded in quadruple precision, or 32 digits. To assess the round-off errors of the integrators, the difference between a forward and backward (negative step-size) integrated orbit is computed. In this scheme, the orbital elements obtained at the end of the forward integration are subsequently used as initial state vector for the backward integration. This testing scheme is also relevant in case of dissipation of energy, which is the case when modeling the atmospheric drag force for example. However, the orbit variations caused by that perturbation are smooth due to the limited resolution of atmospheric density models. The round-off errors will be well represented by a simulation without employing the atmospheric drag force, since the gravity perturbation causes the most rapid orbit osculations. Although a complete description of the errors of the three integration algorithms used here has been published some time ago [Balmino and Barriot, 1990], they have been re-evaluated since the computers and the number of significant digits are not identical.

The programs LAGRAN2 and MZ are existing orbit extrapolators at CNES, capable of taking into account all major perturbations, which they integrate in orbital elements and rectangular elements, respectively. They are solely employed in simulation procedures. The eccentricity should be larger than $3 \times 10^{-3}$ for LAGRAN2, since the motion equations in Lagrange/Gauss form (equations 2.19 and 2.30) have not been expressed in non-singular variables. Since MZ is developed in rectangular variables (the coordinates $x, y, z$, and the velocities $\dot{x}, \dot{y}, \dot{z}$), it is capable of integrating any orbit configuration, with both very small eccentricities (where the Lagrange/Gauss equations expressed in orbital elements become singular) and large ones. However, they cannot process observations and their force models are not entirely complete. GINS is the orbit computation program of the GRGS/CNES (Toulouse), which has been employed in the construction of the GRIM5 gravity field [Biancale et al., 2000]. It has been used to compute the observed mean elements used in this study by fitting the orbits to Satellite Laser Ranging measurements. The orbits extrapolated with LAGRAN2 and MZ software are comparable in precision to those computed by GINS when the same (simplified) force model is selected.

The results of forward and backward integration tests, performed over a period of 12 years, are presented in Table 3.1, using Starlette as a test case with a simplified force model, which includes $J_2 - J_{10}$ only. The difference at the initial position is small, meaning that the integrator step-size has been well-chosen, and the error is mainly due to round-off and not truncation. The largest differences in this case are in the middle (as may be seen in Appendix B) of the integration interval [Balmino and Barriot, 1990]. The global step-size in case of BS was 1200 s and 600 s, with a convergence criterion ($\varepsilon$) of $1 \times 10^{-12}$ (the maximum difference between prediction-correction steps), while the step-size of ABM was 60 s, with a convergence criterion of $1 \times 10^{-11}$. 

3 The Validation Procedure

...
3.2 High Frequency Filtering of the Orbits

The BS algorithm uses a global step-size, which is divided into substeps, which was parameterized at 10 maximum [Bulirsch and Stoer, 1980].

**Table 3.1**: results of the maximum difference of the forward minus backward integration, in meters, evaluated for Starlette (12-year arc length and a simplified force model of $J_2$ up to $J_{10}$).

<table>
<thead>
<tr>
<th></th>
<th>$a$</th>
<th>$e$</th>
<th>$i$</th>
<th>$\Omega$</th>
<th>$\omega$</th>
<th>$M$</th>
</tr>
</thead>
<tbody>
<tr>
<td>BS (1200 s)</td>
<td>0.005</td>
<td>-0.006</td>
<td>0.002</td>
<td>-0.011</td>
<td>0.27</td>
<td>-3.7</td>
</tr>
<tr>
<td>BS (600 s)</td>
<td>0.023</td>
<td>-0.045</td>
<td>0.011</td>
<td>-0.042</td>
<td>0.65</td>
<td>-29.7</td>
</tr>
<tr>
<td>ABM (60 s)</td>
<td>11.05</td>
<td>10.99</td>
<td>4.68</td>
<td>19.4</td>
<td>-531</td>
<td>-7377</td>
</tr>
</tbody>
</table>

The AMC results, not shown in the table, diverged by 61 m on the semimajor axis. This is due to the less-stable integration in rectangular elements, one of the results of Balmino and Barriot [1990], which require adapted algorithms, such as Encke’s algorithm.

These tests show the importance of using a very stable integrator, such as BS, when long periods of time are concerned. Otherwise purely numerical errors may leak into the overall validation procedure. Appendix B presents the orbit differences of the forward minus backward tests over the entire interval in case of BS (1200 s step-size) and ABM (60 s step-size). Tests over a period of two years with ABM (60 s step-size) showed a difference of only a few cm for the metric variables. Therefore, when the simulations span less than two years, the orbits are extrapolated employing the ABM integrator and subsequently filtered (Section 3.2). When a force model has to be validated over periods of time longer than two years, the BS integrator (1200 s step-size) is employed to compute an arc over the entire period. This arc cannot be filtered directly due to the large step-size, but it provides the initial state vectors of the shorter, to be filtered orbits, which are computed with the ABM integrator with a step-size of 60 s.

3.2 High Frequency Filtering of the Orbits

The program CANEL2 (Calcul Analytique et Numérique d'ELéments; [Exertier, 1990]) is used to calculate 'observed' mean orbital elements by filtering all signals with a frequency above a certain threshold from an input ephemeris (extrapolated by LAGRAN2 or MZ, or adjusted by GINS), analytically as well as numerically. The cut-off frequencies to be selected depend on the chosen model and the satellite-specific resonance periods, because all perturbations due to resonance have to be removed from the mean elements, if possible. CODIOR, which integrates averaged equations of motion (Chapter 2) expressed in mean orbital elements, cannot model resonance very accurately since its effect depends on the orbit configuration. This would necessitate specific Hamiltonians per satellite, which is neither practical nor general. However, when the chosen gravitational force model only depends on zonal harmonics (degree>1, order=0),
certain resonance effects are not present since these are due to particular combinations of degree
and order (degree>1, order>0) as will be explained in Section 3.4. This is only possible in case of
simulations, in order to test other mean force models. In precise orbit computation or the
adjustment of geophysical parameters a complete force model has to be employed. Since
resonance is an important (the perturbations can be large and their periods may vary from days to
months) and inevitable (it is always present) aspect of orbit computation, it is discussed separately
in Section 3.4.

The analytical theories of Berger [Berger, 1975] for the zonal harmonics up to degree 7
and Kaula [Kaula, 1966] for the zonal harmonics of degree 8 and up as well as the tesseral
harmonics, are used to filter most gravitational signals with a frequency larger than the analytical
cut-off frequency from the input ephemeris. To remove the (high frequency) noise due to non-
modeled forces and the imperfect analytical theories employed in the filter, a numerical filter is
applied to the ephemeris subsequently. This numerical filter requires a time-interval that is 8 times
as large as the cut-off period in order to function with high precision [Exertier, 1990]. For
example, in order to completely filter (i.e. analytically and numerically) the perturbation due to the
order 14 resonance with a period of 2.8 days from the Starlette ephemerides, these need to be at
least 8-times longer, resulting in 24-day ephemerides. The filtering of the input ephemeris yields
the observed mean elements, one for each orbital element (6 observations per ephemeris,
calculated in the middle of the arc). The high accuracy (better than 1 cm) of the observed mean
metric elements obtained in this way is due to the relatively small dependence on the numerical
filtering, because the analytical filter already removes more than 90% of the perturbation. A purely
numerical approach suffers from edge-effects, since filters with an ideal frequency response are not
available for finite arc lengths. When the interval is 8-times longer than the resonance periods
present in the orbit the error due to edge-effects becomes small, but not negligible in view of the
accuracy required in this study. The use of the second-order theory of Berger, complemented with
Kaula's first-order theory, removes already most of the short-period gravitational perturbations.
For a thorough review of this semi-analytical filtering method, the reader is referred to [Exertier,
1990]. Table 3.2 summarizes the programs that are employed in the simulation and/or adjustment
procedure.

Diagrams 3.1 and 3.2 are schematized representations of the validation (simulation) or
estimation (using real mean observed elements) procedure, in which the programs described in
Table 3.2 are employed. The first step (Diagram 3.1) concerns the extrapolation (simulation) or
adjustment (to tracking data of some kind) of medium-length arcs, which span 24 days in case of
Starlette. When a simulation is performed to test a particular mean force model of CODIOR, first a
continuous long-arc orbit is extrapolated with a large step-size (1200 s in case of Starlette)
employing the BS integrator. This long arc subsequently provides the initial state vectors of the 24-day arcs that are computed with a small integration step-size (60 s in case of Starlette). Once all ephemerides are computed, they are filtered in the second step, first analytically and then numerically, using the program CANEL2 (Diagram 3.2). The filtering of each arc produces 6 mean observed elements (i.e. \( L, G, H, I, g, h \)) in the middle of the time-interval (24-days in case of Starlette) spanned by it. Finally, in step 3, the mean observed elements are used in CODIOR to adjust a long-arc orbit. When tracking data such as SLR underlie these mean observations, geophysical parameters may be estimated.

**Table 3.2:** description of the programs used in the simulation and/or adjustment chain.

<table>
<thead>
<tr>
<th>Program</th>
<th>Function</th>
<th>Input</th>
<th>Output</th>
<th>Specific limit(s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>LAGRAN2</td>
<td>Orbit extrapolation</td>
<td>State vector(^1)</td>
<td>Ephemeris</td>
<td>( e &gt; 0.005 )</td>
</tr>
<tr>
<td>MZ</td>
<td>Orbit extrapolation</td>
<td>State vector(^2)</td>
<td>Ephemeris</td>
<td>few force models</td>
</tr>
<tr>
<td>GINS</td>
<td>Orbit adjustment</td>
<td>Tracking data</td>
<td>Ephemeris (and estimated parameters)</td>
<td>All purpose (terrestrial and planetary)</td>
</tr>
<tr>
<td>CANEL2</td>
<td>Orbit filter</td>
<td>Ephemeris</td>
<td>(Observed) mean elements</td>
<td>( e &gt; 0.001 )</td>
</tr>
<tr>
<td>CODIOR</td>
<td>Orbit extrapolation and adjustment</td>
<td>(Observed) mean elements</td>
<td>Mean long arc (and estimated geophysical parameters)</td>
<td>( 0.001 &lt; e &lt; 0.75 ) only secular/long-period signals estimated</td>
</tr>
</tbody>
</table>

\(^1\) in osculating Keplerian elements

\(^2\) in osculating rectangular variables
Diagram 3.1: the computation of the orbits, which must be filtered subsequently. Typical values for Starlette are printed in italic.

**Simulated observations**

Selection of mean model to test (gravity, drag, tides etc.)

- LAGRAN2 ($e<0.005$)
- Bulirsch and Stoer, or Adams-Bashforth-Moulton
  first-order integrator
  integration in Keplerian elements
- or MZ
  Adams-Moulton-Cowell second-order integrator
  integration of rectangular accelerations

Initial state vectors

- (osculating) continuous long arc
  (720-4600 days, Bulirsch and Stoer, step=1200 s)
- (osculating) medium-length ephemerides
  (24 days, ABM, step=60 s)

**Real observations**

Complete force model, adjustment of:
- * state-vector at epoch
- * drag coefficients (24)
- * solar radiation pressure coefficient (1)

- GINS
  Adams-Moulton-Cowell second-order integrator
  integration of rectangular accelerations
  (step 60s, 8th order progression, eps.=10^{-11})

Observations

- (SLR, 2-lines,)

- (osculating) ephemerides, depend on data availability
  (24 days, typical rms SLR fit: 15 cm)
Diagram 3.2: the complete 3-step simulation or estimation procedure. Typical values for Starlette are printed in italic.

**STEP 1** Input orbit (LAGRAN2, MZ, or GINS)
(24 days, tabulated per 6 minutes)

1. **Analytical filter**
   - $J_2-J_7$: 2nd order theory of Berger [1975]
   - All other harmonics: 1st order theory of Kaula [1966], which is also used to filter/reduce the effects of resonance with periods $> 3$ days.
   - (Cut-off period: 30 days)
   - Iterates until fit is good, requires mean values of $a$, $e$, and $i$ to start.
   - ($a = 7335$ km, $e = 0.02$, $i = 49.2^\circ$)

2. **Numerical filter**
   - Convolution with sinc function in the time domain to filter all residual noise (high-degree harmonics, luni-solar perturbations, etc.).
   - Requires a window that is 8x as large as the cut-off period to reduce edge effects and at least 3500 points per window.
   - (Cut-off period: 3 days, window: 24 days)
   - The mean observation is calculated in the middle of the window.
   - (Start date of the arc + 12 days)

**STEP 2** CANEL2

**STEP 3** CODIOR

Adjustment of initial state vector and:
- verification of mean model precision;
or (with real mean observations):
- estimation of geophysical parameters.

Simulated or real mean observations in Delaunay variables (6 mean observations in the middle of each filtered arc).
3.3 Numerical Integration in Mean Elements

The program CODIOR is capable of orbit extrapolation, for mission analysis, and orbit adjustment, for the estimation of orbital and geophysical parameters. The former is accomplished by integrating a system of averaged first-order differential equations of motion (the Lagrange and Gauss equations), which no longer contain short-period perturbations. The estimation of parameters requires at least one iteration, so residuals (observed - calculated) can be computed, followed by a least-squares adjustment. When the adjustment is performed with simulated observed mean elements, the (mathematical) capacity of the program to model certain perturbations may be evaluated. This does not exclude erroneous physical modeling, however, which may only be verified using real observations. The complete 3-step simulation procedure is schematized in Diagram 3.2.

The integration step-size may be taken between three and twelve hours in CODIOR, since short-period perturbations are not present in its equations of motion. The ABM algorithm is used, as in the LAGRAN2 program. The round-off error is a function of the number of integration steps, as was shown in Section 3.1. The large step-size in CODIOR divides the number of steps over twelve years by a factor of ten to several hundreds when compared to LAGRAN2. The numerical error is negligible in that case, comparable to the round-off error of an ABM LAGRAN2 orbit integration over a period of the order of a month.

3.4 Resonance

Resonance is an effect that relates the satellite's orbital period to the Earth's rotation. The simplest resonance to compute ($q = 0$ in eq. (3.1)) takes place when after exactly $\alpha$ nodal periods of the satellite, the Earth will have rotated $\beta$ times, where $\alpha$ and $\beta$ are integers. This condition is approximately fulfilled when the number of revolutions per day is close to an integer value. The satellite reiterates its tracks relative to the Earth after a repeat period, which engenders dynamical resonances. The frequencies at which these occur can be calculated by determining those particular $lmpq$ combinations (indices used in Kaula's [1966] first-order theory) for which the following argument becomes small:

$$\Psi_{lmpq} = (l - 2p)\dot{\omega} + (l - 2p + q)\dot{M} + (\Omega - \theta)m \equiv 0$$

(3.1)

which is the denominator in Kaula's theory for satellite motion [Kaula, 1966]. When the condition mentioned above is not met exactly, we speak of shallow resonance, which is a periodic perturbation with a period significantly larger than one day. These perturbations can be considerable, so in one way or another they have to be taken into account when the CODIOR long
3.5 Example: Validation of the J2 Modeling

The orbit perturbation due to the flattening of the Earth, so due to $J_2$, is by far the largest for Earth-orbiting satellites. The concept of order, as discussed in Section 2.2, is based on the magnitude of this perturbation. All other perturbations are referenced to the effect due to $J_2$. This implies that its modeling must be very precise if all kinds of smaller perturbations have to be taken into account to obtain a homogeneous precision ultimately. As was mentioned in Section 2.2, the $J_2$ force model is developed up to fifth order. To validate the precision of its modeling in CODIOR, and thus to verify whether no mistakes were made during the analytical averaging and to check whether fifth order is precise enough to achieve the objectives of this study, a two-year numerical LAGRAN2 $J_2$ orbit has been extrapolated (ABM integrator, step-size 60 s). The Starlette orbit parameters (Table 1.1) have been used, which means that the modeling of lower-orbiting satellites will be slightly less precise due to the height attenuation factor (eq. (2.15)). This arc is computed. As has already been mentioned in Section 3.2, CODIOR does not model resonance through averaged differential equations, since this perturbation differs per satellite.

The resonance perturbations can either be added to the mean elements of the CODIOR long arc, or they can be removed from the observed mean elements. When the main resonance combinations $lmpq$ have been determined, these terms can be introduced in CODIOR, and they are then added to the mean perturbations. The accuracy of this modeling is limited by Kaula's theory, which is first order. When the satellite enters deep resonance (in this study: when the magnitude in the modeled resonance becomes larger than the required orbit accuracy), this will no longer work well, and the long arc will be less and less accurate. In case of simulated observations, only the resonance up to some maximum order is present (which is a function of the maximum degree and order selected for the gravity field). Real observed mean elements obviously contain all possible resonances and all other perturbations. When the resonance periods are not too large, say shorter than 3 days, they can be filtered from the (simulated) observed mean elements. In case of extrapolation (for mission analysis for example) the resonance terms always have to be added, if they are significantly large, as well as when the periods are too large to filter in case of an adjustment.

For Starlette, the order 14 resonance has periods of approximately 1.5 and 2.8 days. The higher-order resonance, at order 28, has a period of 1.4 days, but the order 41 resonance has a period of approximately 16.6 days. So while the order 14 and 28 resonance can be filtered from the ephemerides (assuming these span 24 days), this is no longer completely the case for the order 41 effects. Only analytical filtering is available for that perturbation; the numerical filter would require ephemerides of 8x16.6=133 days in order to function accurately, which is too long.
orbit was processed with CANEL2, and this filtering procedure yielded 32 simulated observed mean elements. These simulated observations have subsequently been used in a least-squares adjustment of the initial state vector of a CODIOR mean long arc, where only the $J_2$ mean force model has been selected. The adjustments have been performed at several precision levels: the complete force model, no fourth and fifth-order effects, no third and higher-order effects, and no second and higher-order effects. This allows to show the magnitude of the perturbations due to a certain order. The results of these tests are presented in Table 3.2.

Table 3.2: the validation by simulation of the $J_2$ force modeling in CODIOR with respect to a fully numerically integrated $J_2$ orbit (ABM, step-size 60 s) for Starlette, and the residual errors per order of development. The complete model developed up to fifth order is compared to truncated models, where the fifth down to second-order effects have been neglected. The rms (about the mean) are given in meters or the drift in meters or kilometers per year (m/yr or km/yr, respectively), by multiplying dimensionless and angular elements by the semimajor axis.

<table>
<thead>
<tr>
<th></th>
<th>$a$</th>
<th>$e$</th>
<th>$i$</th>
<th>$\Omega$</th>
<th>$\omega$</th>
<th>$M$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$J_2$ complete</td>
<td>0.0006</td>
<td>0.0033</td>
<td>0.0002</td>
<td>0.042</td>
<td>0.034</td>
<td>0.055</td>
</tr>
<tr>
<td>no 4$^{th}$/5$^{th}$ order</td>
<td>0.0006</td>
<td>0.0033</td>
<td>0.0002</td>
<td>0.078</td>
<td>0.065</td>
<td>0.142</td>
</tr>
<tr>
<td>no 3$^{rd}$/5$^{th}$ order</td>
<td>0.0007</td>
<td>0.073</td>
<td>0.0011</td>
<td>126 m/yr</td>
<td>700 m/yr</td>
<td>50 km/yr</td>
</tr>
<tr>
<td>no 2$^{nd}$/5$^{th}$ order</td>
<td>0.0007</td>
<td>26.2</td>
<td>0.47</td>
<td>120 km/yr</td>
<td>165 km/yr</td>
<td>9000 km/yr</td>
</tr>
</tbody>
</table>

The effect of the third and second order on the semimajor axis is small in terms of rms about mean, but is large in terms of bias, which is not given in Table 3.2. The semimajor axis is systematically smaller when the order of development is too low: 1.19 m for when the development is up to and including second order, and 201 m when the development is up to first order, which causes the secular variation of the angle variables, expressed in m or km per year in Table 3.2. The most important variables (for this study), the semimajor axis and the ascending node, are restituted precisely (rms=0.06 cm and 4.2 cm, respectively, for the complete model), as well as unbiased (no systematic offset between CODIOR and a filtered numerically integrated orbit in osculating elements). This precision is obtained employing the complete model, containing developments of up to fifth order. This is adequate for this study, which aims at estimating signals that are at least an order of magnitude larger on the ascending node (Chapter 8).
4 Gravitational Force Modeling

Since the first analytical theories of satellite motion around a non-spherical central body developed by Brouwer [1959], Kozai [1962] and others ([Kaula, 1966]; [Berger, 1975]; [Kinoshita, 1977]), many studies have been undertaken to efficiently model the perturbation due to the zonal harmonics of the terrestrial potential. Only these particular harmonics induce long-period (resonance excepted) and linear orbit perturbations (with periods of a full revolution of the ascending node and the perigee) for reasons explained in Section 2.3. The low-degree zonal coefficients are the largest of the gravity field. It is remarkable that the progress in analytical modeling has essentially been made on the precision in the computation of effects due to the low-degree harmonics, neglecting the higher-degree terms. In the case of degree 2, for instance, the evolution has been from Brouwer's second-order theory in 1959 to Coffey and Deprit's fourth-order theory in 1981. One of Deprit's co-workers has even attained order 6. This results in theories close to perfection for the harmonics of degree 2 to 7, verified by an equivalent numerical computation. However, the effects of $J_2$ of fourth or higher order are smaller than the orbit perturbation due to $J_{3n}$, for example, which is not included in those theories.

For a theory to be really useful in practice, a homogeneous precision for all perturbations (if possible) acting on the satellite is required. An effort to improve the low-degree zonal harmonics modeling therefore has to be accompanied by an improvement of the modeling of the other perturbations to obtain a truly accurate model. In the case of high-orbiting satellites, this essentially concerns the external third-body perturbations, while in case of low-orbiting satellites this concerns atmospheric drag and high-degree harmonics of the geopotential. This chapter concerns the gravitational force modeling, both the part already existing in the software and the extensions for low-orbiting satellites, at a homogeneous precision level. Chapter 5 presents the nongravitational force modeling.

4.1 Extensions to the Gravitational Model

The basic gravitational force model had already been implemented and tested, both by simulation and orbit restitution, using the relatively high-orbiting satellite LAGEOS-1 (altitude approximately 5900 km). Due to the height attenuation factor $(a_e/r)^6$, see eq. (2.2), the terrestrial gravity perturbations could be adequately modeled using the coefficients $J_2$ to $J_7$ only [Exertier et al., 1995]. This is no longer the case for satellites orbiting at lower altitudes, like Starlette. Based upon an analysis of the amplitude of the perturbations due to higher-degree zonal terms in the
4 Gravitational Force Modeling

gravity field, it was decided to extend the gravity model effects up to $J_{40}$ using the Hamiltonian formulation (Section 2.2). The number of terms in the averaged Hamiltonians being large for high-degree gravity field coefficients, it was not efficient to implement even higher degrees. The power in the degrees 41 to 50 is, however, still high for the Starlette orbit, as may be seen in Table 2.1: approximately 0.4 m periodically on the eccentricity, and a drift of the ascending node of approximately 10 m/year. For a satellite at 500 km altitude in an otherwise identical orbit, these values are 10 m and 350 m/year, respectively. So, these effects need to be modeled also. This is done using the averaged Kaula potential (Section 2.3) for degrees 41 and higher. Although this is only a first-order formulation of the perturbation, the relatively small amplitude of these perturbations limits the absolute error and justifies its use.

The external third-body gravitational perturbation modeling was already of high accuracy for LAGEOS-1. This perturbation becomes smaller for low-orbiting satellites (see eq. (4.2)), since it is a differential effect on the Earth and the satellite through the third-body attraction, and so modifications were not required. An orbit with an apogee height of 36000 km has been used in Section 4.2.2 to demonstrate the precision of the luni-solar perturbation modeling. The effect of solid Earth tides, described in Section 4.2.3, has been developed sufficiently accurately using the Kaula formulation, including the $J_2$ coupling terms [Métris, 1991], and did not need modification either. The ocean tide modeling will be explained in Section 4.2.4, using the Kaula formulation without coupling (since the perturbation is small). The force model did not yet include the long-period tidal constituents (solar annual $S_a$ tide, solar semi-annual $S_{sa}$ tide, lunar monthly $M_m$ tide and the lunar fortnightly $M_f$ tide), nor the semidiurnal tidal constituents $N_2$ and $M_2$. The maximum degree was limited to 6. The Starlette orbit perturbations due to the aforecited tidal constituents are quite small, a few meters on the ascending node or some 10 cm on the eccentricity, except for the $M_2$ tidal constituent. This particular constituent, using Schwiderski’s coefficients [1980], causes a variation with an amplitude of approximately 1 m on the inclination of Starlette. The amplitude of the contributions of the degrees 7 to 10 with respect to those of the degrees 2 to 6 due to the diurnal constituents is approximately one meter on the eccentricity. The missing tidal constituents, as well as a spherical harmonic development up to degree 10, have been implemented in the mean ocean tide force model. The gravitational effect due to air mass redistribution on a satellite orbit is described in Section 4.2.5 and has been implemented as well, to the author’s knowledge for the first time directly in a long-arc orbit computation. It primarily causes annual and semi-annual variations in gravity.
4.2 Validation of the Gravitational Force Modeling

The mean gravitational force modeling was verified for LAGEOS-1 and Starlette. These satellites, presented in Section 1.1, have different orbit configurations, as can be seen in Table 1.1, but both are passive cannonball type satellites. The most important difference between the two satellites is their altitude. The lower-orbiting Starlette is more sensitive to (high degree and order) gravity perturbations, and consequently requires a more complete gravity model than LAGEOS-1. All earlier results obtained with LAGEOS-1 [Exertier et al., 1995] were computed with a gravity model up to $J_{17}$ only, and so were the simulations. The following subsections describe the gravitational forces acting on a satellite (the gravity field perturbation has already been introduced in Chapter 2, and so only simulation results are given), and the simulation results following the validation procedure described in Chapter 3.

4.2.1 Static Gravity Field Modeling

The static part of the terrestrial gravity field corresponds to its mean value, which is referenced to a specific epoch given in the model. It reflects an instantaneous, non-unique image of the (very) slowly evolving mass distribution inside the Earth, and it is also (weakly) correlated with topography. For a different epoch, the gravity field coefficients must be corrected for, for example because of the secular change due to postglacial rebound, which will be discussed in Section 8.3. The seasonal gravity perturbations equally presented in that section are variations with respect to the static gravity field corrected for secular effects. The equipotential surface best fitting at mean sea level of a hypothetical ocean at rest is called the geoid, which is defined by the Earth's gravity field. Figure 4.1 shows the geoid heights, in meters above the GRS80 ellipsoid, computed with the GRIM5 model [Biancale et al., 2000]. These equipotential surface variations become smoother with increasing altitude due to height attenuation, as may be seen in eq. (2.2).

To verify and validate the gravity modeling in orbital elements of the Hamiltonian series from $J_2$ up to $J_{30}$ and $J_2$ up to $J_{40}$, two simulations called F230 and F240 (30x30 and 40x40 gravity field used in the LAGRAN2 orbits; see Diagram 3.1), respectively, have been done. They have been compared with the simulation of the initial gravity model (called ‘Starlette F217’, $J_2$ up to $J_{17}$) so that a possible degradation of the mean model precision with increasing gravity field harmonic degree may be detected. The validation of the averaged Kaula formulation has been performed by a simulation called F270, a simulation up to degree 70 in CODIOR (70x70 gravity field used in the LAGRAN2 orbits). The results of the gravity field modeling simulations are presented in Table 4.1. These simulations have been performed following the validation procedure as presented in Chapter 3, in which it has been described how a mean orbit (CODIOR software)
4 Gravitational Force Modeling

may be compared to a purely numerically integrated precise orbit (LAGRAN2 software). The LAGEOS-1 simulation is most precise, as expected due to its altitude, with a submillimetric rms about mean for the metric variables and the ascending node as shown on the first line of Table 4.1. The Starlette simulations show a small decrease in precision with increasing degree of the gravity field: the rms (about the mean) of the mean ascending node residuals, for example, decreases from 1.4 cm with the F217 simulation (degrees 2 to 17) to 7.1 cm with the F270 simulation (degrees 2 to 70). This precision is still an order of magnitude better than the amplitude of the smallest geophysical signal to be estimated, as will be shown in Chapter 8. The most realistic simulation, F270, shows that the mean semimajor axis and the eccentricity may be modeled with a precision of 1.4 and 7.0 cm, respectively.

The angle variables $\omega$ and $M$ are least well modeled with the mean gravity model, which holds true for all mean force models as will be seen in the next sections and Chapter 5. The drift presented in Table 4.1 ranges from 0.5 to 92 m per year for $\omega+M$ in these simulations, which is due to truncation (the Hamiltonians are developed in powers of eccentricity) and the use of the first-order Kaula theory, decreasing filter performance with increasing gravity field degree and order, and coupling between $\omega$ and $M$ and the semimajor axis (a few millimeters error in the semimajor axis modeling results in an error of the order of meters in those variables). Since $\omega$ and $M$ will intervene only slightly in the estimation of the geophysical parameters to be discussed in Chapter 8 (for which the ascending node is the most important variable), their simulation results will not be commented in the following subsections.

The time-varying gravity field perturbations will be discussed in Chapter 8, since they are not modeled, but estimated (one of the thesis goals). Therefore, a simulation is not required.

<table>
<thead>
<tr>
<th></th>
<th>$a$</th>
<th>$e$</th>
<th>$i$</th>
<th>$\Omega$</th>
<th>$\omega$</th>
<th>$\omega+M$</th>
</tr>
</thead>
<tbody>
<tr>
<td>LAGEOS-1 F217</td>
<td>0.000</td>
<td>0.006</td>
<td>0.000</td>
<td>0.000</td>
<td>1</td>
<td>0.5</td>
</tr>
<tr>
<td>Starlette F217</td>
<td>0.000</td>
<td>0.007</td>
<td>0.000</td>
<td>0.014</td>
<td>3</td>
<td>4</td>
</tr>
<tr>
<td>F230</td>
<td>0.004</td>
<td>0.023</td>
<td>0.007</td>
<td>0.042</td>
<td>16</td>
<td>21</td>
</tr>
<tr>
<td>F240</td>
<td>0.004</td>
<td>0.045</td>
<td>0.008</td>
<td>0.058</td>
<td>20</td>
<td>30</td>
</tr>
<tr>
<td>F270</td>
<td>0.014</td>
<td>0.070</td>
<td>0.010</td>
<td>0.071</td>
<td>40</td>
<td>92</td>
</tr>
</tbody>
</table>

Table 4.1: results of the simulations of geopotential modeling for Starlette, over a period of 2 years. The rms-of-fit (about the mean) is given in meter ($a$, $e$, $i$, $\Omega$) and the drift in meter per year ($\omega$, $\omega+M$). ‘LAGEOS-1 F217’ is an equivalent simulation performed for the LAGEOS-1 orbit, added for reference purposes.
Figure 4.1: GRIM5 geoid heights (m) above the GRS80 ellipsoid.
4 Gravitational Force Modeling

4.2.2 Third-Body Perturbations

The effect of perturbing forces due to the gravitational attraction of external bodies on an orbit, in particular the Moon and the Sun (several orders of magnitude more important than those of the planets in the Solar System), can be computed with millimeter accuracy considering the third-bodies in a point-mass approximation [Sansò and Rummel, 1989]. The position of the Earth, Moon, Sun and the planets can be obtained using planetary ephemerides, for example those of the Bureau de Longitude, called ELP-2000 and VSOP82 (BDL, Connaissance des Temps, 1984, Paris), or the Jet Propulsion Laboratory Development Ephemeris-403 (JPL DE-403; recommended as IERS Convention [McCarthy, 1996]). The acceleration can be expressed in rectangular coordinates in a geocentric coordinate system as follows [Sansò and Rummel, 1989]:

\[
\mathbf{\ddot{a}} = -GM_3 \frac{\mathbf{r} - \mathbf{r}_b}{r^3} + \frac{\mathbf{r}_b}{r_b^3}
\]

(4.1)

where \( GM_3 \) is the product of the universal gravitational constant and the third-body mass, \( \mathbf{r} \) is the satellite (geocentric) radius vector, and \( \mathbf{r}_b \) the third-body radius vector. The equation shows the differential effect of this perturbation, attracting both the Earth (the origin of the adopted reference system) and the satellite.

This perturbation has also been averaged using the Hamiltonian formalism, which resulted in the following expression for the second-order Hamiltonian [Métris, 1991]:

\[
H_0^2(E_n) = \frac{GM_3}{r^2} \frac{L^2}{a}\left(\frac{r_{sat}}{a_{c}}\right) \sum_{j=1}^{n} T_j (\nu, \omega, \Omega) P_j (\cos i, \sin i, A_A, B_A, C_A, C) \\
\cos \psi = A_A \frac{\cos \Omega \cos (\omega + \nu) - \sin \Omega \sin (\omega + \nu) \cos i}{\sin \omega + \nu} \\
+ B_A \frac{\sin \Omega \cos (\omega + \nu) + \cos \Omega \sin (\omega + \nu) \cos i}{\sin \omega + \nu} \\
+ C \frac{\sin (\omega + \nu) \sin i}{\sin \omega + \nu}
\]

(4.2)

where \( T_j \) is a periodic function, \( P_j \) a polynomial which is a function of the inclination \( i \) and the direction cosines \( (A_A, B_A, C_A) \), and \( \psi \) the geocentric angle between the satellite and the perturbing third-body. Since the third-body orbit is ‘outside’ the satellite orbit \( (r_{sat} > r_{sat}) \), the equation shows that the perturbation increases with \( r_{sat} \) (the numerator), contrary to the geopotential perturbation (where \( r_{sat} \) is the denominator; \( r \) in eq. (2.2)). This expression requires averaging over the mean anomaly of the satellite only, since the contribution with shortest periods coming from the Moon still amounts to days (due to her orbital period around the Earth, approximately 28 days, and the harmonic frequencies, of which the 14-day period is most important). The precision of the luni-solar perturbation modeling has been verified by comparison with precise numerical integration, in which eq. (4.1) has been used. The results are presented in Table 4.2 for the satellites Starlette,
4.2 Validation of the Gravitational Force Modeling

LAGEOS-1 and Hipparcos (geostationary transfer orbit: \(e=0.72\), \(i=7^\circ\), apogee height=36000 km), which is only used in this particular simulation because of its high apogee altitude. The length of the simulations was 2 years, using only a (15x15) gravity field to include the strongest resonance effects and the luni-solar perturbations as force model. Table 4.2 shows the decreasing modeling performance with increasing altitude for Starlette, LAGEOS-1 and Hipparcos, as well as the effect of the large eccentricity of the Hipparcos orbit. The eccentricity of 0.72 is too large to compute a precise orbit using the Lagrange planetary equations in orbital elements. The precision of the ascending node modeling of Starlette and LAGEOS-1 is at the millimeter level, more than an order of magnitude better than the terrestrial gravity modeling as presented in Table 4.1.

Table 4.2: the precision of the third-body perturbation modeling. The rms-of-fit (about the mean) is given in meter (\(a, e, i, \Omega, \omega\)) and the drift in meter per year (\(\omega+M\)).

<table>
<thead>
<tr>
<th></th>
<th>(a)</th>
<th>(e)</th>
<th>(i)</th>
<th>(\Omega)</th>
<th>(\omega)</th>
<th>(\omega+M)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Starlette</td>
<td>0.000</td>
<td>0.008</td>
<td>0.000</td>
<td>0.003</td>
<td>1.8</td>
<td>4.2</td>
</tr>
<tr>
<td>LAGEOS-1</td>
<td>0.000</td>
<td>0.010</td>
<td>0.002</td>
<td>0.001</td>
<td>2.0</td>
<td>0.9</td>
</tr>
<tr>
<td>Hipparcos</td>
<td>4.7</td>
<td>5.3</td>
<td>10.3</td>
<td>121</td>
<td>141</td>
<td>300</td>
</tr>
</tbody>
</table>

4.2.3 Solid Earth Tide Modeling

The gravitational attractions of the Sun and the Moon deform the Earth, which is not a perfectly rigid body. These deformations are considered to be elastic, and can be described using an induced free space potential with the Love numbers \(k_2\) and \(k_3\) ([Melchior, 1978]; [McCarthy, 1996]). The vertical deformations at the Earth surface are modeled as being proportional to the perturbing potential (approximately 22 cm amplitude for the lunar tide, and 10 cm for the solar tide), and the Love numbers are the proportionality coefficients. In addition to the geometric effect, the changes in geopotential caused by these luni-solar tides are typically expressed in terms of time-dependent geopotential coefficients (\(\Delta C_{nm}, \Delta S_{nm}\)). These are added to the expression of the geopotential, eq. (2.2), which constitutes the usual way of computing this perturbation in purely numerical orbit computation programs.

The changes in the geopotential \(\Delta U\) can also be computed using a similar approach as was done for the third-body perturbations ([Melchior, 1978]; [Métris, 1991]):

\[
\Delta U = \frac{GM_{ab}}{r_{ab}} \sum_{n>1} k_n \left(\frac{d}{r_{ab}}\right)^n \left(\frac{d}{r}\right)^{n+1} P_n(\cos{\psi})
\]

(4.3)
where the notation is the same as in eq. (4.2), and \( a_e \) represents the Earth’s equatorial radius and \( P_n \) are the Legendre functions of degree \( n \).

The Wahr solid Earth model [Wahr, 1981] takes into account the frequency-dependent response of the Earth, in the form of frequency-dependent Love numbers \( k_n \). It is most efficiently computed in two steps according to the IERS Conventions [McCarthy, 1996], the first being the computation of the perturbation due to a frequency-independent Love number \( k_2 \) (third-degree terms are neglected). This step has just been described in eq. (4.3), using the Hamiltonian form. The second step consists in correcting those arguments of a harmonic expansion of the tide generating potential for which the error of using the \( k_2 \) of step 1 is above some cut-off (IERS amplitude cut-off: \( 9 \times 10^{-12} \)). The changes in normalized geopotential coefficients from step 2 are:

\[
\Delta \tilde{C}_{nm} = \sum_{s(n,m)} A_m \delta k_s H_s (1 \ n + m = \text{even}, e^{i \theta_s} \ n + m = \text{odd})
\]

where \( A_m = \frac{(-1)^m}{a_e \sqrt{4\pi(2 - \delta_m)}}\), \( \delta_m = \begin{cases} 1 & (m = 0) \\ 0 & (m \neq 0) \end{cases} \)

\( \delta k_s \) is the difference between the Wahr model for \( k \) at the frequency \( s \) and the nominal value \( k_2 \) in the sense \( k_s - k_2 \)

\( H_s \) is amplitude (m) of term at frequency \( s \) from a harmonic expansion of the tide generating potential using the Doodson variables \( \beta \), which relates this frequency to a linear combination of the fundamental arguments of the nutation series

\( \theta_s = \sum_{j=1}^{6} n_j \beta_j \) (\( n_j \) multipliers of the Doodson variables)

The resulting change in potential can now be obtained in a Kaula development, which is justified by the much smaller perturbation represented by the Wahr terms in step 2 with respect to step 1. The simulation results, using filtered LAGRAN2 orbits with a force model due to a 15x15 gravity field and the solid Earth tide perturbations spanning a period of 2½ years as reference, are given in Table 4.3 ('SET') at the end of Section 4.2 together with the simulation results of the ocean tide and atmospheric gravity perturbation models. The precision of 16 cm rms on the ascending node is the worst modeling performance so far, but sufficient to estimate signals with amplitudes of more than 1 m, examples of which are given in Appendix C. The modeling precision of the metric variables is sufficient, at the 1-2 cm level, but not at the luni-solar perturbation (which is much larger) modeling level. Figure c2 (Appendix C) presents the amplitude of solid Earth tides in mean orbital elements, with the aim of showing the relative importance of this perturbation and to appreciate the obtained modeling precision.
4.2 Validation of the Gravitational Force Modeling

4.2.4 Ocean Tide Modeling

The dynamical effect of the ocean tides due to the gravitational attraction of the Sun and the Moon may be represented by periodic variations in the geopotential coefficients [McCarthy, 1996]. The variations can be expressed as follows:

$$\Delta \tilde{C}_{nm} - i \Delta \tilde{S}_{nm} = \xi_n \sum_{r(n,m)} \sum_{s} (C_{nm}^{+} + iS_{nm}^{+}) \exp(z^m \cdot \theta),$$

where

$$\xi_n = \frac{4\pi G \rho_w}{g} \frac{1 + k'_n}{(n-m)! (2n+1)(2-\delta_{mn}) 2n+1},$$

$$g = 9.798261 \text{ m}^2 \text{ s}^{-2} \text{ (surface gravity)}$$

$$G = \text{ universal gravitational constant}$$

$$\rho_w = \text{ density of sea water (1025 kg/m}^3)$$

$$k'_n = \text{ load deformation coefficients}$$

$$C_{nm}^{+}, S_{nm}^{+} = \text{ ocean tide amplitude coefficients (m) for constituent s}$$

$$\theta = \text{ argument of the tide constituent s (see Section 4.2.3)}$$

The load deformation coefficients model the effect of the deformation of the terrestrial crust, similar to the solid Earth tide modeling using Love numbers, due to the time-varying ocean loading. By substituting eq. (4.5) in eq. (2.20) the resulting change in the gravity potential \(R\) due to the ocean tide coefficients \(C_{nm}\) and \(S_{nm}\) can be expressed as follows:

$$\Delta R = \frac{G M}{r} \sum_{n=2}^{n} \sum_{m=0}^{m} \sum_{s} \left( a_e \right) r^{n} \sum_{r(n,m)} \sum_{s} \left( F_{nmp}(i) \xi_n \sum_{s(n,m)} \left( C_{nm}^{+} - iS_{nm}^{+} \exp(i \psi_{nmp} + \Theta_s) \right) \right)$$

$$+ \left( C_{nm}^{-} + iS_{nm}^{-} \exp(i \psi_{nmp} - \Theta_s) \right) \right)$$

$$\psi_{nmp} = (n-2)p(\alpha + \nu) + m(\Omega - \theta)$$

$$\Theta_s = \sum_{j=1}^{6} n_{ij} \beta_j = n_{i4}(\theta + \pi - s) + \sum_{j=2}^{6} n_{ij} \beta_j$$

where \(s\) and \(\beta\) are slowly varying angles related to the positions of the Moon and the Sun, and \(\theta\) is the sidereal time. The only rapidly-varying angle is the true anomaly \(\nu\), and long-period perturbations are obtained by averaging functions of the form \((a / r)^{m+1} \exp(i(n-2)\nu\) [Métris, 1991], since \(m\) is zero. A second source of long-period perturbations is due to resonance effects, which are computed in a similar way as was done for the gravity field induced resonances (Section 3.4): by computing the combination of indices, and retaining only those that produce perturbations
4 Gravitational Force Modeling

of the order of days or more with amplitudes of at least 1 cm on the metric and 10 cm on the angular orbital elements, respectively.

The result of this averaging showed that only long-period tide coefficients of order \( m=0 \) (annual \( S_a \), semiannual \( S_{sa} \), monthly \( M_m \), and fortnightly \( M_f \)), the order \( m=1 \) diurnal tide coefficients \( (K_1, O_1, Q_1 \text{ and } P_1) \), and the order \( m=2 \) semidiurnal tide coefficients \( (K_2, S_2, M_2 \text{ and } N_2) \) produce long-period perturbations.

The mean ocean tide simulation results over a testing period of 2½ years are given in Table 4.3. The diurnal tidal constituents ('DOT') and the semidiurnal constituents ('SDOT') were validated separately, with a simplified force model complemented by a (15x15) gravity field. The semidiurnal tide is less well restituted in the simulation than the diurnal tide (5.9 cm precision on \( \Omega \)), but the 8.5 cm rms of the ascending node residuals is sufficiently precise for the estimation of geophysical signals and mission design. The tide modeling precision on the inclination is at the few millimeter level. Appendix C presents the perturbation amplitude on the mean orbital elements, for specific tidal constituents. This allows the identification of a specific tidal constituent (diurnal, semidiurnal or long-period), because of its orbit-specific period: Figure c3 in Appendix C shows the long-period orbit perturbation induced by the diurnal tides. It also illustrates the different effect on specific mean orbital elements of the long-period (ascending node) and diurnal and semidiurnal (inclination and eccentricity) tidal constituents.

4.2.5 The Perturbation due to the Atmosphere

Atmospheric pressure is not constant over time at a specific location. It is measured by meteorological stations globally distributed over the Earth. The data are processed by the ECMWF (European Centre for Medium-Range Weather Forecasts) in the United Kingdom, amongst others, which makes the data available in the form of 6-hourly pressure maps. These maps provide pressure fields over the continents and over the oceans, which may be converted into spherical harmonics, complete to degree and order 50. Pressure fields represent varying, moving air masses, resulting in a varying gravitational potential as sensed by a satellite, since its orbit is outside the atmosphere. A second, indirect, effect sensed by a satellite is due to the induced deformation of the continental crust, the ocean floor and the ocean surface (the water column height). Several authors have correlated the atmospheric pressure successfully with satellite orbit perturbations (for example [Gegout and Cazenave, 1993], [Nerem et al., 1993], [Dong et al., 1996]). Since this perturbation is not well-known, even amongst precise orbit computation specialists, it will be explained in more detail than the preceding gravitational perturbations.

Gegout [1995] has studied the deformation of the solid Earth and the oceans due to air mass redistribution. In that study, a non-global (since it does not cover the entire Earth),
incompressible ocean has been assumed, as well as an elastically deformable solid Earth. The responses of the oceans and the solid Earth to a higher than average atmospheric pressure have been modeled by simultaneously solving the equations of hydrostatic equilibrium of the non-global oceans and the gravity-elasticity equations (hypothesis of Love) for the solid Earth. The basics of Gegout's model [1995] will be described next.

Local atmospheric overloading (highs and lows with respect to a mean surface pressure) over the oceans causes a variation of the sea surface height that is proportional to the local pressure (provided by the ECMWF pressure maps), but does not induce deformation of the ocean floor under the ‘inverse barometer’ hypothesis. The mean pressure, in Pascal, is for example computed as the average over the years 1998-1999, which is shown in Figure 4.2. This figure shows the expected behavior of pressure, namely that it is highly dependent of altitude. Figure 4.3 displays the situation for 6 April 1999. It is very similar to Figure 4.2, because of the altitude dependence, but significant differences can still be detected. Modeling the gravitational effect of these variations, despite their apparent smallness, in the long-arc computation significantly improves the orbit fit, as will be shown in Chapter 8.

The deflection of the sea surface and the corresponding water mass redistribution without affecting the solid Earth is known under the name of ‘inverse barometer’ effect; the hypothesis that the ocean floor will be deformed is simply called ‘non-inverse barometer’ effect. If the entire Earth would be covered by water, the inverse barometer would be the correct and only effect. Atmospheric overloading over the continents, however, does induce deformation of the solid Earth. This deformation does not stop at continent-ocean boundaries, but also causes a deformation of the sea floor (the tectonic plates are continuous at these boundaries or are in contact with each other). The redistribution of ocean water mass counterbalances the atmospheric pressure over it and the deformation of the sea floor. This effect is known under the name of ‘continental loading’, which thus exists besides the earlier mentioned inverse barometer effect. The potential \( U \) representing this kind of mass redistribution is a function of atmospheric overloading over the continents only. Gegout [1995] derived the following equation, based on earlier studies ([Wahr, 1981]; [Farrell, 1972]):

\[
U = 4\pi G a_0 \sum_{l} \frac{1}{\beta_l (2l+1)} \left( a_e \right)^{l+1} \sum_{m} \frac{\Delta P(\phi, \lambda, t)}{g} \\
\text{where } \beta_l = 1 - \frac{3}{2l+1} \frac{\rho_w}{\rho} (1 + k'_l - h'_l) \tag{4.8}
\]

where the same notation has been used as in eq. (4.5). The continental pressure \( \Delta P \) is computed using the ECMWF spherical harmonic coefficients. The coefficient \( \beta_l \) models the effect of the pressure over the oceans on the solid Earth, using the load deformation coefficients \( k'_l \) and \( h'_l \).
Figure 4.2: mean atmospheric pressure (1998-1999), in Pa, derived from ECMWF 6-hourly pressure grids.
4.2 Validation of the Gravitational Force Modeling

Figure 4.3: ECMWF atmospheric pressure grid, in Pa, for 6 April 1999 (000 UT).
and the mean density of the Earth $\rho$ (5500 kg/m$^3$). For infinite degree $l$ it becomes 1, which represents the inverse barometer case, but up to degree 10 this coefficient ranges from 0.7 to 0.9. This indicates significant deviation of the inverse barometer case for the low degrees and orders. The long-period perturbation on the mean orbit can only be caused by the zonal coefficients of the spherical harmonic expansion of the pressure fields (except for a resonance effect). The long periods in question are at the monthly, semi-annual, annual and interannual frequencies, since atmosphere dynamics are driven by the Sun. By computing the power spectra of certain individual atmospheric pressure harmonic coefficients, using 20 years of ECMWF data, the dominance of annual, semi-annual and daily frequencies has been established. Figures d1-d7 (Appendix D) show the power spectra of the degrees 2 to 8 of the zonal pressure coefficients, as well as a degree-variance spectrum, which represents the power of a signal for a given degree. The former plots show that the degrees 2, 3, 7 and 8 (and order 0) only have power at the annual period, while the degrees 4 and 6 have power at both annual and semi-annual periods. The strongest signal of order 0 is due to the terms of degree 3 (Figure d2 of Appendix D), which mainly perturbs the eccentricity and perigee. The degree-variance spectrum (Figure d8; shown are degrees 1-50) shows that the power is strong up to degree 10, and by far strongest for terms of degree 2. The temporal evolution of the atmospheric $C_{2,0}$ term is displayed in Figure d9 of Appendix D to show that it is indeed not a sinusoidal, stationary signal, but varying from year to year.

Based upon these results, the gravitational effect due to atmospheric pressure (degrees 2 to 8) has been added to the mean gravitational model, by using pressure coefficients averaged over 3 days. The result of the mean atmospheric pressure model validation is given in Table 4.3 (‘PRES’) for LAGEOS-1. In this test, the simulated observed mean elements issued from filtering a LAGRAN2 orbit spanning 2½ years using the degrees 2 to 6 of the atmospheric pressure coefficients and a (15x15) gravity field as simplified force model, have been compared to the CODIOR long-arc orbit. The precision for the metric variables and the ascending node is (sub) centimetric. This simulation has been done with LAGEOS-1 rather than Starlette (which is more sensitive to this perturbation due to its lower altitude, but the signal-to-noise ratio is smaller due to inaccurate atmospheric drag modeling) for practical reasons, since small geophysical signals may be more accurately determined with the LAGEOS satellites, as will be shown in Chapter 8. The total amplitudes of the nominal orbit perturbations are displayed in Figures c10 and c11 of Appendix C.
4.3 Discussion

Table 4.3: results of the simulations for the solid Earth tide model (SET), diurnal (DOT) and semidiurnal (SDOT) ocean tide model for Starlette, and the atmospheric pressure (PRES) model for LAGEOS-1. The rms-of-fit (about the mean) is given in meter ($a$, $e$, $i$, $\Omega$, $\omega$) and the drift in meter per year ($\omega + M$). All simulations have been performed with a force model consisting of a (15x15) gravity field in addition to the to be tested force model (MT, DOT, SDOT and PRES) over a period of 2½ years.

<table>
<thead>
<tr>
<th>Variable</th>
<th>SET</th>
<th>DOT</th>
<th>SDOT</th>
<th>PRES</th>
</tr>
</thead>
<tbody>
<tr>
<td>$a$</td>
<td>0.010</td>
<td>0.000</td>
<td>0.003</td>
<td>0.000</td>
</tr>
<tr>
<td>$e$</td>
<td>0.020</td>
<td>0.014</td>
<td>0.010</td>
<td>0.01</td>
</tr>
<tr>
<td>$i$</td>
<td>0.010</td>
<td>0.002</td>
<td>0.001</td>
<td>0.003</td>
</tr>
<tr>
<td>$\Omega$</td>
<td>0.160</td>
<td>0.059</td>
<td>0.085</td>
<td>0.008</td>
</tr>
<tr>
<td>$\omega$</td>
<td>11</td>
<td>1.7</td>
<td>1.9</td>
<td>3.8</td>
</tr>
<tr>
<td>$\omega + M$</td>
<td>50</td>
<td>5</td>
<td>10</td>
<td>5</td>
</tr>
</tbody>
</table>

4.3 Discussion

The precision of the complete gravitational force model has been validated by separate testing of the gravity field, third-body perturbations, solid Earth tides, ocean tides and the atmospheric gravity effect force models. The precision of the restitution of the metric variables is at the few cm level, as was shown in Tables 4.1-4.3. The most important angle variable for geophysical parameter estimation purposes, the ascending node, is restituted with a precision of 16 cm rms-of-fit at worst, as was shown in Table 4.3.
4 Gravitational Force Modeling
5 Surface Force Modeling

The most important nongravitational forces, in terms of satellite orbit perturbation, are atmospheric drag, solar radiation pressure and Earth radiation pressure. The drag force dominates more and more with decreasing altitude, due to increasing atmospheric density. Both forces depend explicitly on the satellite shape, material characteristics, mass and attitude, which makes it convenient to express them in a local, orbital frame, the origin being the satellite's Center of Mass (CM). This frame is composed of a radial component $R$, in the direction of the Earth's CM to the satellite, the normal component $W$ in the direction of the angular momentum, and $S (=RxW)$ in the direction of motion. When the forces are expressed in this frame, the time derivatives of the orbital elements (or another set of variables) are given by the Gauss form of the Lagrange planetary equations. The Lagrange planetary equations are valid only for forces that can be derived from a potential function (and are consequently only a function of position). The averaging of these forces can no longer be performed analytically due to their dependence on satellite speed (the force cannot be expressed as the derivative of a potential function), as well as observed environmental parameters (solar flux, for example), or discontinuities (shadowing effects). Their averaging over the mean anomaly is assured by a numerical quadrature of the right-hand members of the Gauss equations, an operation that has to be performed at each integration step (of 3-12 hours). This type of averaging has been described in Section 2.5. The realistic evaluation of the nongravitational forces, however, requires coupling with the short-period perturbations, with $J_2$ in particular, the effect of which may be quite large. The integration being in mean elements, the short-period perturbations due to $J_2$ are added to the mean elements using Brouwer's analytical theory [Brouwer, 1959], thus obtaining approximate osculating elements.

The actual modeling of these forces is described in the following sections. The solar radiation pressure modeling is described in Section 5.1, followed by a section concerning the albedo modeling. The third section describes the averaged drag force modeling. The partial derivatives of the Gauss equations (eq. (2.30)) with respect to the orbital parameters, necessary for the estimation of drag scaling coefficients, are presented in Section 5.4. Very small (less than 1% of the total of the nongravitational perturbations), satellite-specific nongravitational forces, in particular for the LAGEOS satellites, are discussed in Section 5.5. Finally, the results of the mean surface force modeling validation are presented in Section 5.6.
5.1 Mean Solar Radiation Pressure Modeling

The direct solar radiation pressure, acting on satellites not intersecting Earth's shadow cone, is a nearly constant force in the direction Sun-satellite for spherical spacecraft. It may be modeled as derived from a potential, or Hamiltonian, and as such it can be averaged analytically. In this expression, the distance and direction Sun-satellite have been approximated by the distance and direction Sun-Earth, which induces very small errors. The (instantaneous or averaged) acceleration due to direct solar radiation pressure is given by:

\[ \ddot{a}_{srp} = C \left( \frac{\text{AU}}{R_{\text{Sun-sat}}} \right) \frac{S}{m} k_R \hat{\mathbf{s}} \]  

(5.1)

where

\[ C = \text{the solar constant at 1AU}, \quad R_{\text{Sun-sat}} = \text{the actual Sun-satellite distance}, \quad S = \text{the surface of the satellite which is exposed to the sunlight}, \quad m = \text{its mass}, \quad k_R = \text{its global reflectivity}, \quad \hat{\mathbf{s}} = \text{the unit vector Sun-Earth}. \]

The global reflectivity coefficient is itself a function of the specular (\( K_S \); absent in case of a spherical satellite) and diffuse (\( K_D \)) reflectivity coefficients. For a sphere, it is as follows:

\[ k_R = \left( 1 + \frac{4}{3} K_D \right) \]  

(5.2)

Once the satellite orbit plane intersects the Earth's shadow cone, the solar radiation pressure is no longer nearly constant over a revolution. The force decreases to zero very rapidly in the light-shadow transition zone, the penumbra, in a matter of up to a few minutes as a function of increasing altitude. Figure 5.1 gives a 2-dimensional representation of the Earth's shadow cone and the solar radiation pressure unit force vector for an orbit that intersects the shadow cone. In case of shadow, the analytical formulation eq. (5.1) is no longer valid, since it assumed solar radiation over the complete revolution in the averaging procedure. In this case, the averaged force is obtained numerically, approximating eq. (2.29) by a 200-point quadrature. This large number of points is necessary to have at least two points in the transition zone, which is absolutely indispensable for a precise evaluation of this force. In case of Starlette, a complete penumbra crossing is approximately 1 minute, while the orbital period is approximately 104 minutes, so that a 200-point quadrature is sufficient. A shadow function is applied to the computed solar radiation pressure, which leaves this computed value unchanged in case of full sunlight, and reduces it to zero in case of full shadow. The satellite crosses the penumbra in 1 to 3 minutes, depending on the satellite's altitude, and during these shadow entrance or exit occurrences the shadow function regularizes the resulting rapid change in force. In summary, where \( f_{\text{reg}} \) is the shadow function and \( Rr \) a function of Earth and satellite radius vector and the Sun-satellite angle:
5.1 Mean Solar Radiation Pressure Modeling

\[ f_{\text{reg}} \mathbf{a}_{\text{up}} = f_{\text{reg}} (\mathbf{a}_{\text{up}}) \]

\[ f_{\text{reg}} = 1, \text{light} \]

\[ f_{\text{reg}} = \frac{1}{1 + e^{-\frac{d}{a}}}, \text{penumbra} \]

\[ f_{\text{reg}} = 0, \text{shadow} \]

A numerical quadrature is not efficient in terms of computational speed, which is why both a numerical and an analytical solar radiation pressure model are maintained in the software. Secondly, the numerical quadrature is a precise approximation, but suffers from small but non-negligible numerical errors. That is why in case of shadow intersection the solar radiation pressure is always computed as a sum of an analytically computed mean force (no shadow) plus a numerically computed mean force with shadow minus a numerically computed mean force without shadow. This distinction between intervals with and without shadow and the accessory fast analytical and numerical mean force computation is most efficient for high-orbiting satellites.

Starlette's eclipse season lasts nearly six months, alternated by only six days of fully sunlit orbits. LAGEOS-1, on the other hand, has an eclipse season of three to six months, alternated by five to six months of no shadow-intersecting orbits. This should lead to a more precise mean solar radiation pressure modeling for LAGEOS-1, which will be verified in Section 5.6.1. The perturbation on the orbital elements is shown in appendix E.

![Figure 5.1: direction and relative magnitude of the solar radiation pressure force.](image-url)
5 Surface Force Modeling

5.2 Mean Earth Albedo Modeling

The albedo of a surface is defined as the ratio of the total flux emanating from it, in all
directions, to the total flux incident on it, in the visible spectrum (200–400 nm) [Knocke et al.,
1988]. The incident flux is sunlight, which is reflected and re-emitted from all illuminated surfaces
of the Earth. The albedo of the Earth is not constant due to differences in reflective properties of
ocean, continent and ice, for example, as well as cloud cover. Its force is mainly radially directed.
It becomes zero when the surface element is not illuminated (surface elements in the shadow emit
infrared radiation, producing a very small force on the satellite; it is not accounted for by the
albedo, but it is modeled as a radial force with only a seasonal variation). The acceleration due to a
single Earth surface element \(i\) may be expressed as:

\[
\vec{a}_{\text{alb}(i)} = C \alpha \frac{\rho S}{m \pi D^2} \hat{r}_i
\]

where \(\alpha\) is the albedo coefficient (0≤\(\alpha\)≤1), \(dA_i\) the area element \(i\) of the Earth reflecting the
incoming sunlight, and \(D\) the distance Earth element-satellite [Knocke et al., 1988]. The remaining
variables and constants are as in equation (5.1). The total acceleration on a spherical satellite of
invariant reflective properties due to the albedo may be approximated by a summation of all
visible surface elements \(i\). The surface elements may be arranged in consecutive concentric circles
and meridians around the sub-satellite point, in such a way that the flux per surface element is
constant. When all surface elements of the visible cap are illuminated, the force is radial for the
largest part. However, when shadow intersects the visible cap, some surface elements do not
contribute to the albedo force, which then deviates significantly from a radial force. The height
attenuation, embodied by the term \(1/D^2\), is in part compensated for. This is due to the surface of
the visible cap, which increases with height, and so the albedo perturbation remains relatively
strong even for high-orbiting spacecraft [Rozanes, 1993].

5.3 Mean Atmospheric Drag Modeling

The atmospheric drag force is caused by collisions of the spacecraft with gas particles. Its main
effect on the orbit is a secular decrease of the semimajor axis, and ultimately the loss of the
satellite. The atmospheric density decreases with altitude approximately exponentially, becoming
negligible for most applications above 2000 km altitude. It is not constant at altitudes below 2000
km, but a function of latitude, longitude and local solar time, the day of the year, and the solar
activity. These input parameters are used in density models to estimate the atmospheric density \(\rho\)
at a given satellite position. The drag acceleration for spherical satellites is modeled as follows
[Zarouatti, 1987]:

\[
\vec{a}_{\text{drag}} = -\frac{6\pi \rho}{m} \vec{v} \times \vec{v} = -\frac{6\pi \rho}{m} \vec{v} \times \frac{\vec{v}}{v^2}
\]
5.3 Mean Atmospheric Drag Modeling

\[ \bar{a}_D = -\frac{1}{2}C_D \frac{A}{m} \rho \bar{v}^2 \]  \hspace{1cm} (5.5)

where \( A \) is the effective satellite cross-sectional area, \( m \) its mass, \( \rho \) the atmospheric density and \( \bar{v} \) its speed with respect to the atmosphere. Lift will not be discussed here since spherical satellites only are used in this study. The accurate computation of the drag force requires, in case of spherical satellites, an atmospheric density model and a model for the energy transfer between the satellite surface and the atmosphere. The prediction of density is presently the limiting factor in drag modeling because it contributes the largest uncertainty (20-25%) to eq. (5.5). Drag model improvement can thus best be achieved by more accurately modeling the atmospheric density, which is the subject of the next chapter.

Equation (5.5) shows that a small area-to-mass ratio minimizes the drag effect. \( C_D \), the aerodynamic or drag coefficient, is a function of satellite geometry, the mode of reflection of particles off its surface and the temperature of the incident and re-emitted particles. It is a dimensionless parameter, the value of which is known to better than 5% for the geodetic satellites [Gaposhkin and Coster, 1994]. When Cook’s [1966] theory for spherical satellites is used, the drag coefficient is the sum of a specular and diffuse reflection part:

\[
C_D(\text{spec.}) = \left( 2 + 2 \frac{s^2}{s^2} - \frac{1}{s^2} \right) \text{erf}(s) + 2s \frac{s^2}{s^2} + 1 - e^{-s} s \frac{1}{s^2}, \quad \text{where} \quad \alpha = \frac{v_{sat}}{v_{molecule}} = \frac{v_{sat}}{\sqrt{2RT/mm}}
\]

\[
C_D(\text{diff.}) = \frac{2\sqrt{\pi}}{3s} \left[ 2 - \frac{T_{sat}}{T} + \sqrt{1 - \alpha} \left( s + 1 - 2 \frac{T_{sat}}{T} \right) \right]
\]

\[ \text{where} \quad \alpha = \frac{3.6 mmr}{(1 + mmr)^2}, \quad \text{and} \quad \text{erf}(s) = \frac{2}{\pi} \int_0^\infty e^{-u^2} du \]  \hspace{1cm} (5.6)

where \( \text{erf} \) is the error function, \( T \) is the kinematic gas temperature, \( R \) the universal gas constant, and \( mmr \) the ratio of the mass of the incident gas atom (which is mainly a function of altitude) to the mass of the (satellite) surface atom. The speed ratio, \( s \), is low (< 5) for Starlette due to its altitude (>> 800 km), which is why the random thermal motion of the atmospheric atoms must be considered, leading to eq. (5.6) instead of much simpler equations for lower-orbiting spacecraft [Cook, 1966]. Several quantities are not well-known, such as the actual mode of reflection and energy transfer, the most probable molecular speed, and the satellite surface temperature, causing the uncertainty (<5%) stated above. It is not possible to create a realistic space environment under laboratory conditions, and this prohibits improvement of drag coefficient modeling presently. Figure 5.2 shows the drag coefficients over a period of 13 years, including the solar cycle maximum in the middle, for Cook's model and a model that does not take the temperature of the atmosphere into account [Afonso et al., 1989], resulting in a nearly constant value for \( C_D \) of 2.74.
5 Surface Force Modeling

Figure 5.2: the Starlette drag coefficients over a period of 13 years using Cook's [1966] model (dots) and that of Afonso et al. [1989] (line at nearly constant value of 2.74).

The effective cross-sectional area of a satellite may be difficult to model, except in the case of a spherical satellite, when complex attitude modeling is required. The mass of an active satellite changes during the mission due to maneuvering, and needs to be monitored accurately. Only for geodetic satellites, like Starlette or GFZ-1, is exact calculation possible, due to their spherical form and constant mass. The velocity of the satellite relative to the atmosphere, which is assumed to be corotating with the Earth, is given by the following equation:

\[ \vec{v} = \vec{v}_{\text{sat}} - \vec{v}_{\text{atm}}, \quad \text{where} \quad \vec{v}_{\text{atm}} = \omega \times \vec{r} \]

(5.7)

where \( \vec{r} \) is the geocentric radius vector of the spacecraft. For satellites at altitudes below 300 km, the velocity of the atmosphere may be significantly different from a corotating atmosphere due to high-latitude winds [Hedin, 1991], which are strongest at latitudes exceeding plus or minus 60°.
(the auroral zones). This effect is negligible at Starlette’s altitude (perigee height 800 km), where
the atmosphere is well modeled as corotating with the Earth. There is no super-rotation of the
atmosphere, but there are both zonal and meridional winds, which are functions of season ([Hedin,
1991]; [Fauliot et al., 1993]). The maximum wind speeds, which occur at high latitudes during
geomagnetic storms, are of the order of 500 m/s. Not modeling these maximum wind speeds
causes errors of less than 10\% of the satellite velocity (approximately 7 km/s) relative to the
atmosphere. Fortunately, geomagnetic storms are rare events. Moreover, Starlette’s orbit
inclination of approximately 50° limits the perturbations due to high-latitude winds.

Unlike the solar radiation pressure modeling, the mean drag force can only be computed
numerically. This is not computationally efficient, eq. (5.5) being approximated by a 24-point
quadrature at each 3-hour integration step, since the very high computational speed gain achieved
by analytical filtering of the gravitational forces is partly lost. Since no accurate analytical model is
available, the numerical error cannot be corrected for in a similar way as was done for the solar
radiation pressure. A second problem is the high variability of the environmental input parameters,
\textit{i.e.} solar flux and the geomagnetic activity in particular. The latter may cause changes in density
up to an order of magnitude over a period of a few hours only. Since the integration step-size is 3
hours, in order to coincide with the geomagnetic activity observations, the mean orbit will
inherently be less precise than under quiet geomagnetic conditions. The quality assessment
presented in Section 5.6 will confirm this. Figure e2 (Appendix E) shows the modeled orbit
perturbation due to atmospheric drag for the satellite Starlette.

\section*{5.4 The Gauss Equations and Their Partial Derivatives}

The predicted atmospheric density from a model is subject to several error sources. An
erroneous density and/or aerodynamic coefficient will directly affect the calculated drag
acceleration, as may be seen in eq. (5.5). When tracking data are available, a linear multiplicative
drag scale factor ($c_d$) may be estimated to compensate for errors in the predicted density and the
modeled aerodynamic coefficient (although its 5\% error is much smaller than the approximately
20-25\% uncertainty of density models; see Chapter 7). This scale factor is essential in precise orbit
computation and is adjusted daily or more frequently when the available tracking data allow this. It
has unit value in the ideal case, but otherwise compensates for a too low or too high density as
predicted by the atmospheric density model, and also for a too large or small aerodynamic
coefficient ($C_D$).

To likewise adjust a linear multiplicative drag scale factor $c_d$ in CODIOR in an iterative,
least-squares way, the partial derivatives with respect to this coefficient have to be calculated. The
actual variables in CODIOR are Keplerian elements, which makes the adjustment of a drag scale coefficient much more ponderous than would be the case for rectangular variables. The derivation, using eqs. (2.29-2.30), is as follows:

\[
\frac{\partial E}{\partial d} = \frac{\partial}{\partial E} \left[ \int_{t-0.5T}^{t+0.5T} \left( \frac{\partial G}{\partial E} \vec{a}_{RSW} + G \frac{\partial \vec{a}_{RSW}}{\partial \vec{X}} \frac{\partial \vec{X}}{\partial E} \right) dt \right] \frac{\partial E}{\partial d} + \frac{\partial (G \cdot \vec{a}_{RSW})}{\partial d} =
\]

(5.8)

where \( \vec{X} \) represents the Cartesian state vector \((X,Y,Z,\dot{X},\dot{Y},\dot{Z})\), and the second term on the right-hand side is simply the mean acceleration divided by the coefficient. The acceleration \( \vec{a}_{RSW} \) is computed by rotation of the acceleration into the Cartesian true of date frame XYZ:

\[
\begin{bmatrix}
F_X \\
F_Y \\
F_Z
\end{bmatrix} = \sigma \begin{bmatrix}
\dot{X} + \omega Y \\
\dot{Y} - \omega X \\
\dot{Z}
\end{bmatrix} \quad \sigma = \frac{-1}{m} \frac{4}{C_d} \rho \quad \omega = \text{Earth rotation rate}
\]

(5.9)

The rotation from the acceleration in XYZ coordinates to RSW coordinates is performed as follows, where only the equation for \( a_R \) is given explicitly:

\[
a_R = \frac{\vec{r}}{\| \vec{r} \|} \cdot \vec{a}_{XYZ} = a_X \frac{X}{\| \vec{r} \|} + a_Y \frac{Y}{\| \vec{r} \|} + a_Z \frac{Z}{\| \vec{r} \|}
\]

\[
a_W = \frac{\vec{r} \times \vec{r}}{\| \vec{r} \|^2} \cdot \vec{a}_{XYZ}
\]

\[
a_S = \frac{(\vec{r} \times \vec{r}) \times \vec{r}}{\| \vec{r} \|^3} \cdot \vec{a}_{XYZ}
\]

(5.10)

The tangential acceleration, \( a_S \), is by far the largest. The derivatives with respect to \( X, Y, Z \) and the velocities \( V_X, V_Y, V_Z \) of these equations have to be derived, as can be seen in eq. (5.8). This leads to a (3x6) matrix, which subsequently has to be multiplied by the (6x6) matrix of \( X \) derived with respect to the orbital elements \( E \), which produces the derivatives of the drag force with respect to Keplerian elements. The derivatives of the (6x3) Gauss matrix \( G \) with respect to the Keplerian elements are independent of the ascending node (see the Gauss equations (2.29)), so 5 matrices have to be evaluated. Since the eccentric anomaly \( E \) and the true anomaly \( v \) are functions of the eccentricity \( e \) and the mean anomaly \( M \), their partial derivatives have to be taken as well. It is the description in eccentric and true anomaly of the Gauss equations that makes the derivation of \( G \) with respect to the orbital elements \( E \) relatively complicated.
A drag scale factor determined in this manner has a linear effect on the metric variables (\(a\), \(e\), \(i\)), and consequently a parabolic effect on the angle variables. To verify its correct implementation, its precision and the effect of the filtering procedure on its determination, a simulation with deliberately erroneous observations was performed. The drag force in the ephemerides that were at the base of these pseudo-observations was multiplied by 1.05, and these ephemerides were filtered subsequently. After adjustment of the initial state vector and a drag scale coefficient, the recovered value in CODIOR of the coefficient was 1.05003. The precision of its determination is high, and the filter does not alter its determination significantly (0.003%).

5.5 Satellite-Specific Small Nongravitational Forces

The dominant nongravitational forces are solar radiation pressure, atmospheric drag and Earth albedo radiation pressure. These forces are independent of satellite spin in case of spherical satellites, which is the common factor of the small surface forces that will be described in this section. A sophisticated mean force model has been developed for the LAGEOS satellites, the orbit and tracking accuracy of which allow their estimation. Two of the three forces are thermal drag forces, or photon thrust, due to a temperature imbalance between the satellite’s southern and northern hemispheres. This imbalance is nonexistent longitudinally because of the (still) rapid spinning motion of the satellites, the rotation period of which is at least a factor 200 smaller than the thermal re-emission time lag. The heat absorbed by the satellite, either from the Sun or from the Earth, is reradiated along its spin-axis away from the heated hemisphere, but with a certain time lag due to thermal inertia of the retroreflector material.

**Yarkovsky effect** - The Yarkovsky effect, which is illustrated in Figure 5.3a, is that part of the thermal drag that is due to the heating of the satellite by infrared radiation of the Earth ([Rubincam, 1988]; [Métris et al., 1997]). The temperature distribution is symmetric about the spin-axis, along which it produces a net along-track acceleration when averaged over an orbital revolution, which is just the average required in the mean motion theory. Due to the thermal inertia of the satellite, there is a phase delay between the hottest point of the satellite and the point closest to Earth. Expressed as a lag angle, in recent studies it varies between 53.85° [Métris et al., 1997] and 55.4° [Scharroo et al., 1991] of an orbital revolution. The amplitude is modulated by the precession of the ascending node, which represents the rotation of the orbital plane about the equator. The Yarkovsky along-track effect is largest when the spin-axis is in the plane of the orbit and has an along-track component, and this component is zero when the spin-axis and the radiation vector are parallel.
5 Surface Force Modeling

**Yarkovsky-Schach effect** - The Yarkovsky-Schach effect, which is shown in Figure 5.3b, is that part of the thermal drag that is due to the heating of the satellite by solar radiation. The satellite hemisphere on the Sun-side will always be warmer than the opposite hemisphere, which creates a constant temperature difference when we assume a fixed spin-axis orientation in space. In absence of shadow this will not cause a significant effect on the orbit; it can be modeled by a slightly modified solar radiation pressure force. However, when the orbit intersects the Earth's shadow cone, the solar heating drops off to zero in a matter of a few minutes. The Yarkovsky-Schach effect is thus only important during the eclipse season, at which times it engenders spike-like along-track accelerations, compared to the slowly-varying Yarkovsky effect. The thermal response time of a retroreflector has been estimated at nearly a quarter of an orbital period (or 83.1°) by Scharroo and his co-workers [1991], but in a recent study [Métris et al., 1997] it has been estimated at more than half the orbital period (183.3°). In orbit adjustment, the phase lag is a function of the amplitude: large phase lags are required for large amplitudes.

**Anisotropic reflectivity** - The third small force is due to an assumed anisotropic reflectivity of the satellite hemispheres. This may be due to different reflective characteristics of the retroreflectors, or by means of a different protective coating [Scharroo et al., 1991]. It is shown in Figure 5.3a. An asymmetric reflectivity has consequences for the solar radiation pressure modeling, and will cause a force directed along the spin-axis away from the hemisphere with better reflecting properties. This will cause a small constant shift in the solar radiation pressure, which is easily absorbed by the adjusted solar radiation pressure scaling coefficient. However, when the satellite intersects Earth's shadow cone, there is a net along-track acceleration due to this asymmetry, similar to the Yarkovsky-Schach effect. It equally produces the spike-like accelerations. Figures 5.3 represent the effects of Yarkovsky and the Yarkovsky-Schach thermal drag and anisotropic reflection effects (not to scale). This particular orbit configuration has the spin-axis in the plane of the orbit and a phase lag of 90° for visual purposes. The terrestrial IR radiation is assumed to be radially outward, and constant over the globe with only an annual variation. Orbit perturbations due to these forces are shown in Figures e3-e5 (Appendix E).

These three surface forces may only be modeled for high-orbiting, geodetic spacecraft. They are small, and require a very accurate tracking and a particle drag that is of the same order of magnitude or smaller. These conditions exclude all satellites except the LAGEOS pair. At their altitude of nearly 6000 km, the charged and neutral particle drag is very small, and may be modeled as a constant along-track acceleration. Semi-empirical atmospheric density models such as DTM94 are not valid at altitudes above approximately 1500 km due to the fact that adequate data have not been assimilated because they are not available. This empirical constant along-track acceleration completes the satellite-specific nongravitational force model.
5.6 Simulation Results

**Figure 5.3a:** Direction and relative magnitudes of the forces acting on the satellite due to Yarkovsky thermal drag, and anisotropic reflectivity in case of a better reflecting northern hemisphere of the spacecraft (solid black arrow).

**Figure 5.3b:** Direction and relative magnitudes of the forces acting on the satellite due to the Yarkovsky-Schach effect and solar radiation pressure.
5.6 Simulation Results

5.6.1 Validation of the Mean Solar Radiation Pressure Force Model

The solar radiation pressure force modeling has been verified for two separate cases, namely LAGEOS-1 and Starlette, according to the procedure described in Chapter 3. The difference between these cases is the long periods of time that the LAGEOS-1 orbit stays permanently sunlit, 5-6 months, compared to 6 days without shadowing in case of Starlette. This simulation is meant to evaluate the precision of the mean solar radiation pressure force modeling by means of a numerical quadrature (Section 2.5, eq. (2.29)), required in case of shadowing effects on the orbit, instead of by means of a purely analytically averaged force model in case of a permanently sunlit orbit. The numerical quadrature is less precise than an analytically averaged force model, and thus the LAGEOS-1 mean solar radiation pressure modeling is on the average expected to be more precise, resulting in a more precise mean orbit and thus smaller residuals. The results of simulations (simulated observed mean elements issued from filtering LAGRAN2 orbits compared to a CODIOR long arc; see Diagrams 3.1 and 3.2) spanning 3 years with a (10x10) and (15x15) gravity field, and luni-solar perturbations in addition to solar radiation pressure with shadowing effects to obtain a realistic force model, for LAGEOS-1 and Starlette, respectively, are presented in Table 5.1. From this table, it is evident that the numerical quadrature, which is much more often used in case of Starlette than LAGEOS-1, introduces numerical errors into the modeling: the precision of the metric variables and the ascending node is lower. However, this result is still precise, with the error on the semimajor axis being less than 1 cm. This is more than adequate to model the mean solar radiation pressure for low orbits, since the atmospheric drag force induces errors that will be more and more important with decreasing altitude. The precision is excellent for LAGEOS-1, and an accurate mean solar radiation pressure modeling at the cm-level will be possible for the elements $a$, $e$, $i$ and $\Omega$.

<table>
<thead>
<tr>
<th></th>
<th>LAGEOS-1</th>
<th>Starlette</th>
</tr>
</thead>
<tbody>
<tr>
<td>Semimajor axis</td>
<td>0.002</td>
<td>0.009</td>
</tr>
<tr>
<td>Eccentricity</td>
<td>0.011</td>
<td>0.08</td>
</tr>
<tr>
<td>Inclination</td>
<td>0.007</td>
<td>0.014</td>
</tr>
<tr>
<td>Ascending node</td>
<td>0.019</td>
<td>0.32</td>
</tr>
<tr>
<td>Perigee</td>
<td>12.81</td>
<td>7.33</td>
</tr>
</tbody>
</table>
5.6.2 Validation of the Mean Drag Force Model

The simulations were performed during high and low solar activity, with Starlette, so at a relatively high altitude (perigee height 800 km). The simulation with Starlette serves as a starting point, since at its altitude the drag force is still relatively weak, a third-order perturbation (with respect to $J_2$; see Section 2.2). It also serves as a precision indicator for the adjustment of real Starlette observations.

The first test employed a simplified dynamic force model (with only zonal gravity field coefficients), so there will be no resonance to complicate the interpretation of the modeling error due to drag. The represented perturbations in both LAGRAN2 and CODIOR consisted of the gravity perturbations due to the zonal coefficients $J_2$ to $J_{15}$, luni-solar attraction and the atmospheric drag force, which simulation is called "J215LSD". This verification using the Starlette orbit parameters (Table 1.1) spanned a period of 10 years. It started during the solar cycle minimum in 1986 and passed the entire solar cycle maximum of 1989-1991. The observations for this test were filtered with a cut-off period of 3 hours (which is longer than the orbital period of 109 minutes, the longest short-period perturbation induced by zonal gravity coefficients), in a 1-day window. To be able to assess the modeling error due to drag, the same simulation without the drag force was also done. Table 5.2 presents the maximum values of the residuals for both tests for the mean metric elements. The angle elements were restituted worse, but this is likely due to weighting. The maximum deviations are given because the eccentricity residuals showed a trend, for which an rms-of-fit value is not representative. These maximum values are indicative, higher values are possible, since not many mean observations were generated for this simulation (6 per year for each orbital element) due to CPU-time considerations at that time. However, the observations were adjusted in both low and high solar activity epochs, so their values are representative of the modeling error.

The large value of 1.28 m for the eccentricity is due to an unmodeled $J_2$-drag coupling effect. This unmodeled coupling effect gives rise to a trend, where high gradient values correspond to high solar activity. This coupling is thus not totally accounted for, but thanks to this simulation its effect may now be corrected for (which is important when real observations will be used) by applying an empirical force. The total loss of energy due to atmospheric drag over 10 years

<table>
<thead>
<tr>
<th></th>
<th>without drag</th>
<th>with drag</th>
</tr>
</thead>
<tbody>
<tr>
<td>semimajor axis</td>
<td>0.004</td>
<td>0.2</td>
</tr>
<tr>
<td>eccentricity</td>
<td>0.04</td>
<td>1.28</td>
</tr>
<tr>
<td>inclination</td>
<td>0.002</td>
<td>0.005</td>
</tr>
</tbody>
</table>
is represented by the decreasing value of the semimajor axis, which in this case decays approximately 320 m. The relative precision of the mean drag model is thus high, since the drag force is modeled up to 0.06% (0.2 m/320 m) for the semimajor axis at worst for this first test. The angle variables are restituted with the same relative precision, which however gives rise to much larger absolute values when these are expressed in meters.

The second series of partial model tests were more realistic, and included the tesseral harmonics, employing a (20x20) gravity field in addition to the luni-solar and drag perturbations in the LAGRAN2 orbits. These were calculated under low and high solar activity conditions over a period of 2 years. These forces cause the major orbit perturbations at Starlette altitude. These tests, called "J2020LSD", allowed the assessment of the effect of a weak and much stronger drag force (up to 30 times stronger) on the mean drag model precision, taking into account that the gravitational modeling is precise (see Section 4.3). The order 14 resonance (1.4 and 2.8 days) was filtered from the observations using a cut-off frequency of three days in a 24-day window. Table 5.3 presents the rms-of-fit (about mean) values of the residuals for these tests, where the corresponding simulation without drag ("J2020LS") has been added for completeness. The large difference between low and high activity shows mostly on the semimajor axis (4.4 compared to 2.0 cm rms about the mean) and eccentricity (13.5 compared to 3.0 cm rms), which was to be expected of a force that acts principally along-track. The $J_2$-drag coupling effect is only visible in the residuals under high solar activity. The semimajor axis decreases by 169 m during the two-year simulation under high solar activity, which sets the relative precision for this test at 0.03%. For low solar activity, the semimajor axis decreases by only 14.9 m, which sets the relative precision at 0.13%. The ascending node precision of 69 cm under high solar activity conditions is adequate because this error is an order of magnitude smaller than the amplitude of the smallest periodical geophysical signal to be estimated in Chapter 8.

**Table 5.3:** the results for the second series of Starlette drag modeling tests (J2020LSD), for low and high solar activity, and without drag (J2020LS), over 2 years. Rms-of-fit about the mean of the residuals is given, in cm. Values in brackets indicate the total secular effect due to drag, in m.

<table>
<thead>
<tr>
<th></th>
<th>J2020LSD</th>
<th>J2020LS</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>High activity</td>
<td>Low activity</td>
</tr>
<tr>
<td>Semimajor axis</td>
<td>4.4 (169)</td>
<td>2.0 (14.9)</td>
</tr>
<tr>
<td>Eccentricity</td>
<td>13.5 (94)</td>
<td>3.0 (4.7)</td>
</tr>
<tr>
<td>Inclination</td>
<td>0.9 (3)</td>
<td>0.9 (0.3)</td>
</tr>
<tr>
<td>Ascending node</td>
<td>69.0 (13900)</td>
<td>11.3 (1378)</td>
</tr>
<tr>
<td>Argument of perigee</td>
<td>1300 (15300)</td>
<td>1400 (1174)</td>
</tr>
<tr>
<td>Mean anomaly</td>
<td>45700</td>
<td>5700</td>
</tr>
</tbody>
</table>
5.6.3 Validation of the Complete Mean Force Model

Finally, a simulation spanning thirteen years was performed with a complete force model: a (60x60) gravity field, luni-solar perturbations, solid Earth and ocean tides, solar radiation pressure, albedo and atmospheric drag. This simulation covers the same period as the Starlette long-arc adjustment with real observations to be presented in Section 8.2.1, and uses almost the same dynamical model. The simulation starts in November 1983, under moderate solar activity conditions, and runs until January 1997, during low solar activity conditions. The results for this simulation, presented in Table 5.4, are 8 cm, 33 cm and 4 cm rms-of-fit (about the mean) for the semimajor axis (with 15 cm secular drift during solar maximum), eccentricity and inclination residuals, respectively. The ascending node residuals present a quadratic signature. The perigee has a secular variation of 15 m/year, while the mean anomaly residuals also present a quadratic signature due to the drifting offset of 15 cm of the semimajor axis. When the drift is not present, which will be the case using real observations and estimating drag scale factors to absorb such errors (which was not done in this simulation), the ascending node and perigee residuals will only present unmodeled perturbations, and not this artificial curvature. The precision of the mean model for the ascending node is better than the here obtained 12 m, being at the 70 cm level during high solar activity, as was shown in Table 5.3.

Table 5.4: the results for the Starlette complete model test over a period of 13 years. Rms-of-fit about the mean of the residuals is given, in cm ($a$, $e$, $i$, $\Omega$) and the drift in meter per year ($\omega$).

<p>| | | | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>$a$</td>
<td>$e$</td>
<td>$i$</td>
<td>$\Omega$</td>
<td>$\omega$</td>
<td></td>
</tr>
<tr>
<td>8</td>
<td>33</td>
<td>4</td>
<td>1200</td>
<td>15</td>
<td></td>
</tr>
</tbody>
</table>

5.7 Discussion

The precision of the gravitational (Chapter 4) and nongravitational force models has been established by simulation. The obtained precisions, which are functions of the order of the analytical developments, approximations and numerical techniques (described in Chapter 2), are adequate for the estimation of certain secular and time-varying components of the Earth's gravity field (Chapter 8) and very long-period tides, the amplitudes of which are shown in Appendix C. The results of the complete model validation, given in Table 5.4, demonstrate this (taking the remarks concerning the ascending node and the perigee of the former paragraph into account).
5 Surface Force Modeling
6 Upper Atmosphere Density Modeling

Thermosphere models, representing temperature and (partial) density as a function of environmental parameters, are used in satellite orbit determination to compute the atmospheric drag force, as may be seen in eq. (5.5). Since the atmosphere engenders a major perturbation of the (mean) motion of low-orbiting spacecraft like Starlette, it needs to be modeled accurately. However, precise satellite orbit computation requires the estimation of many density scale factors to absorb the errors in orbit predictions induced by the density model. This is only possible when a tracking system is available, with stringent requirements concerning the accuracy and temporal and spatial continuity of its observations. In spite of a large modeling effort in terms of data processing, the typical uncertainty is at the 20% level [Berger et al., 1998]. The reasons for the slow progress in density modelling since models as early as J71 [Jacchia, 1971] are the following, though not necessarily in order of importance:

1. simplified description of atmospheric physics;
2. the use of proxy indicators for solar and geomagnetically induced atmospheric heating processes; and
3. the inaccurate or sparse (or both) datasets available for partial or total density and temperature data available.

The first item of the list is the most difficult subject to tackle, which is why recent empirical density models such as DTM94 [Berger et al., 1998] or MSIS86 [Hedin et al., 1987] are still based on the same algorithms as their earliest versions. The DTM model algorithm will be discussed in Section 6.1. The basic difference between the first DTM model, DTM78 [Barlier et al., 1978], and DTM94, is the quantity and quality of density data incorporated in the model, allowing the modeling of smaller density variations under low and high solar activity conditions.

This chapter concerns the heating of the atmosphere by solar extreme ultraviolet (EUV; \( \lambda = 20-200 \) nm) radiation, which is its most important energy source, and thus addresses item 2 of the list. Since the EUV is completely absorbed by the upper atmosphere, it is not observable on the ground, whereas the radio emission at 10.7 cm wavelength (\( F_{10.7} \)) is. The National Research Council of Canada has been measuring it on a daily basis since 1947. The EUV radiation originates in the solar high chromosphere, transition region and corona, including emission lines at different temperatures, while the radio flux at 10.7 cm originates at a specific altitude of the solar atmosphere. This is why looking for an EUV variability indicator, or proxy, is not a straightforward task. Calculating averaged values over 3 solar rotations, the mean radio flux is correlated with the mean EUV radiation (up to 0.99), but the correlation is weaker between the
daily values (approximately 0.88). Correlations between EUV and density are 0.97 and 0.85 for mean and daily values, respectively [Hedin, 1984]. However, $F_{10.7}$ suffers from several defects as EUV proxy due to the temperature dependence of the various emissions composing this domain, due to their specific process of emission, and due to the optical depth of the solar atmosphere at the wavelength of emission [Neupert, 1998]. Furthermore, the EUV emission exhibits a 13-day periodicity which is not present in the $F_{10.7}$ time series [Donnelly and Puga, 1990]. Consequently, $F_{10.7}$ cannot represent all the EUV emissions accurately, especially not on a short-term basis.

The Mg II index [Heath and Schlesinger, 1986], derived from satellite observations of the Mg II emission lines (doublet) at 280 nm, has been considered in this study. It will be described in more detail in Section 6.2.1. First, because it allows the description of the spectral solar irradiance as a function of Mg II index change with a better accuracy than $F_{10.7}$, as shown by Lean et al. [1992], down to the Lyman $\alpha$ emission at 121.6 nm. Secondly, the Mg II index exhibits a 13-day periodicity [Cebula and DeLand, 1998], as do the EUV emissions. A correlation has also recently been shown between the helium line at 30.4 nm (He II) measured on board the satellite SOHO (Solar and Heliospheric Observatory), and the Mg II variation observed at the same time (Viereck and Judge, personal communication, 1999). The above stated reasons suggest to use the Mg II index to describe the heating of the thermosphere by EUV radiation.

The third item of the list is the consequence of several reasons. The satellite drag total density data used in DTM78 is not accurate due to the inadequate orbit determination resulting mainly from the inaccuracy of the terrestrial gravity field models at the time (1970-1975). Accelerometer-derived total density data cover only a small altitude or latitude range. Partial density data are accurate, but only give information on the relative variations, since the instrumental bias of the mass spectrometers has not been determined under laboratory conditions. Secondly, no single-instrument data set has been obtained under both high and low solar activity conditions for the same altitude interval.

The next section describes the DTM upper atmosphere model algorithm. Section 6.2 introduces the Mg II index and the results that have been obtained using this index instead of $F_{10.7}$. The chapter finishes with an overall conclusion and some future prospects concerning atmospheric density modeling.

### 6.1 Density Model Description

The upper atmosphere in DTM is modeled using the law of hydrostatic equilibrium, which has proven to be a realistic hypothesis. This law describes the decrease of pressure $P$ and density $\rho$ with increasing altitude $z$: 

$$ P(z) = P_0 e^{-z/h}, 
\rho(z) = \rho_0 e^{-z/h} $$

where $P_0$ and $\rho_0$ are the pressures and densities at $z = 0$, and $h$ is the scale height.
6.1 Density Model Description

\[
\frac{\text{d}P}{\text{d}z} = -nmg \Rightarrow \frac{1}{P}\frac{\text{d}P}{\text{d}z} = - \frac{mg}{RT} = \frac{1}{H}
\]

(6.1)

where \( n \) is the number density (particles/volume), \( m \) is the molecular mass (gram/mole), \( g \) is gravity, \( R \) is the Boltzmann constant \((1.3803 \times 10^{-23} \text{JK}^{-1})\), \( T \) is the temperature, and \( H \) is called the density scale height, obtained by substitution of the gas law. The scale height is the vertical distance in which \( P \) changes by a factor \( e \) (2.718). Integration of eq. (6.1) under the assumption that \( H \) is constant gives:

\[
\frac{P}{P_0} = \frac{\rho}{\rho_0} = \exp \left[ -\frac{z-z_0}{H} \right] = e^{-Z}
\]

(6.2)

where \( P=P_0 \) at \( z=z_0 \) and \( Z \) is the reduced height [Hargreaves, 1979]. The reference height \( z_0 \) at which the partial densities \( \rho_0 \) of each constituent are given in the density models is approximately 120 km (turbopause), above which altitude diffusive separation has taken place. Equation (6.2) is the result of the integration of a simplified hydrostatic equation and is meant as an example. The actual equation that is used in DTM will be presented in the following.

As a consequence of this law (cf. eq. (6.1)), the heavier atmospheric constituents are in abundance at low altitudes while the concentration of the lighter constituents becomes more and more important with height. Above the turbopause altitude, each constituent has its own independent height distribution governed by its own scale height, which is related to the mass of the constituents, see eq. (6.1); below this height the modeling is much more complicated due to turbulent mixing and chemical processes. The major atmospheric constituents with height are molecular nitrogen \( \text{N}_2 \), atomic oxygen \( \text{O} \), helium \( \text{He} \) and atomic hydrogen \( \text{H} \). The partial densities of the minor atmospheric constituents (primarily argon, atomic nitrogen, and molecular oxygen) are small compared to those of the major constituents. Molecular oxygen is the only modeled minor constituent.

In the DTM models the temperature and partial densities are given at 120 km altitude. The concentration \( c \) (or number density \( n \)) of constituent \( i \) is modeled as [Berger et al., 1998]:

\[
c_i(z) = c_i(120 \text{ km}) \exp(G_i(L)) f_i(z)
\]

(6.3)

where the function \( f \) is the result of the integration of a diffusive equilibrium distribution, using the temperature profile of Bates [1959], which has the following form for each constituent:
6 Upper Atmosphere Density Modeling

\[
f(z) = \left( \frac{T_{120}}{T(z)} \right)^{1+\alpha+\gamma} \exp(-\sigma \xi)
\]

\[
\alpha = \frac{T_m - T_{120}}{T_m}
\]

\[
\gamma = \frac{m g_{120}}{\sigma R T_m}
\]

(6.4)

where \( \alpha \) is the thermal diffusion factor, \( \sigma \) the relative vertical temperature gradient, \( T_{120} \) and \( T_m \) are the temperature at 120 km and at the thermosphere, respectively, \( g_{120} \) is the acceleration of gravity at 120 km altitude (approximately 9.44 m/s\(^2\), compared to 9.80 m/s\(^2\) on average at sea level) and.

When eq. (6.3) is multiplied by \( m_i / N_A \) (molecular mass of constituent \( i \) divided by Avogadro's number) and summed, the density is obtained:

\[
\rho(z) = \sum_{i} \frac{m_i}{N_A} c_i (120 \text{ km}) \exp(G_i(L)) f_i(z)
\]

(6.5)

The function \( G(L) \) models the variation of density and temperature with the environmental parameters, represented by \( L \), which are assumed to be at the origin of the observed variations: geocentric latitude and longitude, local solar time, day of year, mean (\( F_{10.7} \)) and daily (\( F_{10.7} \)) solar radio flux and the 3-hourly geomagnetic index \( k_p \). DTM94 models the variations in atmospheric density with 38 coefficients for the exospheric temperature and each atmospheric constituent, but the temperature and its gradient at 120 km are represented by constant values. The function \( G \) contains non-periodic terms (NP) which depend on solar and geomagnetic activity and latitude. The modeled periodic terms have annual (PA), semi-annual (PSA), diurnal (PD), semidiurnal (PSD) and terdiurnal (PTD) periods for each constituent. This can be expressed as follows:

\[
G = \sum_i a_{iNP} + \sum_j a_{jPA} + \sum_k a_{kPSA} + \sum_l a_{lPD} + \sum_m a_{mPSD} + \sum_n a_{nPTD}
\]

(6.6)

where the coefficients \( a \) are estimated model coefficients. The effects due to phenomena with periods smaller than 8 hours, such as gravity waves or solar flares, are not represented in the model, but are considered as sources of noise in the data when the terdiurnal variations and signals with larger periods are estimated.

The periodicities in atmospheric density are due to the motion of the Earth about its rotational axis and about the Sun. The longest ‘period’ coincides with the solar cycle of approximately 11 years [Lantos, 1994], but this is not modeled explicitly in eq. (6.6): the periodicity results from the proxy indicator (\( F_{10.7} \)). The solar cycle causes order-of-magnitude differences at satellite altitudes between atmospheric density taken at solar minimum and
maximum, the ratio of which is highly altitude-dependent. The day-to-night temperature difference causes density differences of up to a factor 5. The daily period with respect to the Earth is characterized by a diurnal bulge, approximately at the Sun's latitude but lagging it by two to three hours due to delayed atmospheric response to energy input and Earth rotation.

The main thermospheric heat source is solar EUV radiation. The physical reasons underlying this atmospheric heating are ionization, absorption/emission and recombination of the atmospheric components. The result of this radiation is that the upper atmosphere is for an important part a plasma: a (kinetically) very hot, ionized medium. The energy is mainly deposited at low latitudes (due to the ecliptic), and it controls the circulation within the thermosphere from low to middle latitudes. The second energy source is the interaction of the solar wind (a hot plasma of mainly electrons, protons and helium nuclei traveling at speeds exceeding 170 km/s, spiraling radially outward from the Sun) with the terrestrial magnetosphere at high latitudes, which results in kinetic and Joule heating. The proxy indicator for these processes is the planetary geomagnetic index $k_p$. The energy balance of the upper atmosphere is difficult to model, because of several not very well understood cooling and heating mechanisms (NO cooling for example [Wells et al., 1997]), and secondly because it has to be calculated using proxy-indicators like $F_{10.7}$ and $k_p$. The radio flux will be discussed in more detail in the next section.

A magnetic storm takes place when the value of $k_p$ attains a value of 5 or higher. These magnetic storms have an enormous impact on the atmospheric density, which can change by nearly an order of magnitude within a few hours. The ratio of densities under high and low geomagnetic conditions behaves similarly as the ratio for solar activity with respect to altitude, but is much smaller, having a maximum of about 10. The largest impact on the density is at high latitudes close to the (magnetic) poles, but during major storms ($k_p > 6$) the atmospheric circulation becomes equatorward, and the densities at all latitudes become affected. Besides an approximately 27-day recurrence tendency (1 solar rotation), there is no periodicity in geomagnetism [Mugellesi and Kerridge, 1991].

Unfortunately, the global distribution of the dedicated stations that observe (variations in) geomagnetism is not very good: the network is unevenly distributed per hemisphere (11 in the northern and 2 in the southern), and the same holds for the distribution by longitude. This affects the representativity of $k_p$ for high latitudes and local time effects in particular. Partly for that reason, the response of the atmosphere to geomagnetic storms is not modeled very well in the density models, the correlation of model error and $k_p$ being large. The highest correlation of $k_p$ with density is 0.6 for midlatitudes and a time lag of about 4 hours [Marcos et al., 1993], which demonstrates that this index does not contain all necessary information to model auroral heating. However, as is the case for $F_{10.7}$, it is the only readily available and continuous index of
geomagnetic activity. Tests performed with alternative indices ([Marcos et al., 1993]; [Gaposhkin and Coster, 1994]) have not been encouraging, and suggest the reformulation of auroral heating processes. Fortunately, on the other hand, energy input through auroral heating is much smaller than the EUV energy input due to the limited duration and low frequency of major and severe storms.

6.2 The Mg II Index

This section concerns the use of the Mg II index in upper atmosphere modeling instead of the usual and widely known $F_{10.7}$ radio flux. The relatively small correlation of daily $F_{10.7}$ radio flux data with density, as mentioned in the introduction of this chapter, suggests taking an indicator originating in the chromosphere, with high intensity of the emission lines for good observability. Figure 6.1 shows the evolution of $F_{10.7}$, the Fe XV emission at 28.4 nm, the He II emission at 30.4 nm (extracted from Figure 1 of Avrett [1992]) as well as that of the Mg II index.

Figure 6.1: the 10.7 cm (top curve), 28.4 nm (Fe XV), the 30.4 nm (He II) solar emissions and the Mg II index (bottom curve) over a period of 5 months (top plot extracted from Avrett [1992]).
from 1 May, 1980, to 1 October, 1980. During the first 2 weeks in May $F_{10.7}$ is out of phase with the EUV emissions (Fe XV and He II), as well as at the end of September (it is declining instead of rising). Furthermore, the two maxima in August and the first half of September are a high one followed by a smaller one, except for $F_{10.7}$, which exhibits the opposite evolution. Consequently, the correlation of Mg II with Fe XV and He II is higher than that of $F_{10.7}$. The complexity of the matter is revealed by Figure 6.1 as well, which shows important differences between the four observables. The density modeling should become better by using the Mg II index rather than $F_{10.7}$, although some EUV emissions are already decorrelated over timespans of the order of months.

6.2.1 The Mg II Data

The Mg II index, proposed by Heath and Schlesinger [1986], is computed as the irradiance ratio of the Mg II emission at 280 nm to the line wings continuum. This is shown in Figure 6.2. It is quasi-independent of the calibration and degradation due to aging of the instrument in the harsh space environment.

![Figure 6.2: the definition of the Mg II core-to-wing index.](image)

\[ \text{Mg II core-to-wing index} = \frac{a}{b} \]
The Mg II emissions originate in the chromosphere, where the solar EUV radiation is generated that governs upper atmosphere heating [Donnelly, 1992]. The $F_{10.7}$ index that is commonly used to represent solar EUV variability is not generated at the same altitude, but likely in the high chromosphere/lower corona. This is probably the cause for its smaller correlation with short-term UV radiation variation [Donnelly and Puga, 1990]. The Ca II plage at 393 nm also originates in the chromosphere, and has been tested in the past [Hedin, 1984] for similar reasons as Mg II in this study. The result of that study was negative: $F_{10.7}$ was better correlated with EUV radiation and atmospheric density than Ca II was. The Mg II emission lines are twice as intense as those are from Ca II, and thus more easily observed accurately [Donnelly et al., 1994].

The index used here has been derived from instruments aboard satellites from the National Oceanic and Atmospheric Administration (NOAA), and has been corrected for instrumental errors [Cebula et al., 1998]. The Mg II data from Nimbus-7, NOAA-9 and NOAA-11, derived from the SBUV (Solar Backscatter Ultraviolet) aboard Nimbus-7 and SBUV/2 instruments aboard the latter two satellites [DeLand and Cebula, 1993], have been made available. The first observations by Nimbus-7 have been taken in November 1978. Since that date the index is available: 4 out of 5 days in general, but extensive data gaps exceeding a month exist due to instrumental or orbit control errors.

Figure 6.3 presents the Mg II index derived from the SBUV and SBUV/2 instruments aboard the three NOAA satellites. The offsets that exist between the three data sets are due to small instrumental differences (minor differences in wavelength sampling and slit function), since no two instruments are perfectly identical. The difference between the two SBUV/2 instruments (NOAA-9 and NOAA-11) is small, but the SBUV data (Nimbus-7) are significantly different and approximately 4% larger than the NOAA-9 data. Thanks to overlapping observation intervals, the Nimbus-7 and NOAA-11 data have been fitted to the NOAA-9 scale by linear regression, as described by DeLand and Cebula [1993], resulting in a composite Mg II data set (Figure 6.4). NOAA-9 provides the longest data set, and since the Mg II observations are of comparable quality, it has been used to scale the other data sets.

The composite data span 19 years on a single scale, from November 1978 to October 1997, but there are still too many days missing to be useful in density modeling. Atmospheric density models like DTM94 [Berger et al., 1998] require a mean solar radio flux ($F_{10.7}$) averaged over the three last solar rotations. Since the DTM94 algorithms will be used, a similar requirement exists for the Mg II index, requiring continuous data. Since data gaps are existing, a cubic spline in case of one or two consecutive missing days has been used to assure continuity. When the data gap is longer than two days, a realistic interpolation is not possible and the existing gaps are left. The
composite data set after interpolation, shown in Figure 6.4, exhibits a variability as expected from the Schwabe (the discoverer of the solar cycle) cycle.

![Figure 6.3: the original Mg II data sets corrected for instrumental drift.](image)

![Figure 6.4: the composite Mg II dataset scaled to NOAA-9.](image)
The short-time variations of the $F_{10.7}$ and Mg II indices are not identical. The Mg II index has been scaled to $F_{10.7}$ by means of a linear regression. This allows the differences between these data to be detected and analyzed. The correlation between Mg II and $F_{10.7}$ is 0.99 over the entire data set, but an rms about mean of 25 solar flux units (sfu) exists for short periods of time. The usual averaging over three solar rotations has been shortened to a single one due to the still remaining data gaps. Applying the same averaging to the $F_{10.7}$ and Mg II index, the correlation remains 0.99, but the rms (about mean) reduces to 10 sfu. The following equation has been obtained by applying a linear regression law:

$$\text{Mg II} = 0.000128 \times F_{10.7} + 0.25068 \quad (6.7)$$

Figure 6.5 presents the rescaled composite Mg II index in sfu and the difference with respect to $F_{10.7}$. The largest differences occur during solar cycle maxima, associated with the solar rotation. It is known that the solar rotation generates significant variations in UV and EUV due to the appearance of sunspots and associated phenomena. This result may be interpreted as possibly supporting the use of the Mg II index in providing a better description of solar irradiance change associated with the solar activity. However, despite the data correction for instrumental drift made

![Figure 6.5: the rescaled Mg II index (top) and the difference with $F_{10.7}$ (bottom).](image-url)
6.1 Density Model Description

by Cebula et al. [1998], they still seem to increase linearly. Since the trend is smaller than 1 sfu/year, and data will be used over a two-year period only, it has been ignored for the time being.

6.2.2 Mg II Model Results

The comparison between using $F_{10.7}$ and Mg II as solar activity indicators is achieved through comparison of two test models named DTM-F107 and DTM-Mg II, respectively. These models have been constructed under identical conditions using the same one-year of density data, employing either the Mg II index or the $F_{10.7}$ cm radio flux as solar activity proxies. The problem encountered is that there is only one large density data set concurrent with Mg II, namely that of the Dynamics Explorer 2 (DE-2) mass spectrometers ([Carignan et al., 1981]; [Spencer et al., 1981]), while all other data sets, in particular the CACTUS total density [Villain, 1980] and the Atmosphere Explorer-C and -E partial density data sets ([Nier et al., 1973]; [Spencer et al., 1973]), have been obtained before the observation of the Mg II emission lines.

The residuals, in aeronomy usually defined as Observed/Calculated (O/C instead of the usual O-C), reflect the relative precision of the models. The DE-2 mass spectrometer does not provide absolute densities, and its observations require scaling by calibration factors. These have not been derived in this study, but values from DTM94 were taken. The mean value of the residuals corresponds to the instrumental calibration factor, which is different for each constituent, and to the model capability to represent the observations. An erroneous value of a calibration factor, which in this case has an uncertainty of approximately 30%, thus causes a model bias. However, they could not be estimated in this study because only the DE-2 data set is used. The rms of the residuals about the mean value reflects the short-term modeling performance.

The DE-2 residuals have been divided over altitude, latitude, local time, month, $F_{10.7}$, $F_{10.7}$, and $k_p$ bins. The mean and rms (always about mean in this study) of the residuals have then been computed per bin, of which specific results per constituent only will be shown in Figures 6.6 through 6.9. The modeling results for the temperature and the constituents are given in the following subsections.
6 Upper Atmosphere Density Modeling

Table 6.1: the modeling performance with respect to the DE-2 dataset. The number of residuals (after 3σ editing) is given on the first line, followed by their mean and rms about mean on the second line.

<table>
<thead>
<tr>
<th></th>
<th>He</th>
<th>O</th>
<th>N₂</th>
<th>T</th>
</tr>
</thead>
<tbody>
<tr>
<td>DTM94</td>
<td>513879</td>
<td>360191</td>
<td>226868</td>
<td>262008</td>
</tr>
<tr>
<td></td>
<td>0.808 / 0.182</td>
<td>0.888 / 0.181</td>
<td>0.665 / 0.198</td>
<td>0.988 / 0.066</td>
</tr>
<tr>
<td>DTM_F107</td>
<td>513795</td>
<td>359729</td>
<td>226774</td>
<td>261976</td>
</tr>
<tr>
<td></td>
<td>0.893 / 0.189</td>
<td>0.930 / 0.193</td>
<td>0.640 / 0.194</td>
<td>1.009 / 0.067</td>
</tr>
<tr>
<td>DTM_Mg II</td>
<td>513635</td>
<td>359873</td>
<td>227022</td>
<td>261988</td>
</tr>
<tr>
<td></td>
<td>0.933 / 0.195</td>
<td>0.937 / 0.182</td>
<td>0.627 / 0.188</td>
<td>1.005 / 0.062</td>
</tr>
</tbody>
</table>

6.2.2.1 The Temperature Modeling

The temperature modeling in terms of both bias and precision is the best with DTM-Mg II, as may be seen in the last column of Table 6.1. Its global mean of the residuals is 0.5% biased, and the rms of 0.062 is 6% smaller than that of the two other models. Table 6.1 also shows that the temperature is the best represented variable of the thermosphere models, with an rms of 6 to 7%, compared to 18 to 20% rms of the residuals of the constituents.

Figure 6.6 presents the temperature residuals binned as a function of local time, which shows the stable mean per bin with DTM-Mg II (left plot; varying between 0.99 and 1.015). Both test models present a semidiurnal signature, implying imperfect modeling of this variation.

Verification of the residuals after binning as a function of the model parameters did not reveal other systematic modeling errors.

Figure 6.6: the mean and rms of the DE-2 T residuals presented in local time bins.
6.1 Density Model Description

6.2.2.2 The Atomic Oxygen Modeling

The atomic oxygen (O) modeling is better with DTM-Mg II than with DTM-F107, demonstrated by their respective values of 0.182 and 0.193 for the global rms (about mean) of the residuals in Table 6.1. It is at the level of DTM94, but the residual mean is higher with the test models: 3% higher with respect to the 0.91 calibration factor of DE-2 oxygen estimated by Berger et al. [1998] than with DTM94, which has a 2% smaller mean of 0.888. Figure 6.7a presents the mean (left) and rms (right) of the residuals per \((F_{10.7} - \overline{F}_{10.7})\) bin for the test models and DTM94. The histogram of the \((F_{10.7} - \overline{F}_{10.7})\) is shifted towards negative values for the period July 1981-January 1983, with most residuals in the bin \(-30 < F_{10.7} - \overline{F}_{10.7} < -10\), which is shown in Figure 6.7b. This is not the case of the Mg II index, whose distribution of differences is centered around 0. The mean of the residuals per bin, on the other hand, drifts linearly, which is not the case for the DTM-F107 or DTM94 residuals (Figure 6.7a).

Further analysis of the DTM-Mg II residuals showed that their smallest rms is in the 200-400 km altitude range, approximately 0.13, steadily increasing to 0.30 in the 600-700 km altitude range (not shown). A similar behavior is seen in the DTM94 and DTM-F107 residuals, whose rms is approximately 5% larger in the 200-400 km altitude range. The latitude modeling is the best with DTM94, which has the most stable mean per 10° bin. The DTM-Mg II residuals display a typical latitudinal signature: higher values at the poles than at the equator. This distribution is due to the auroral activity, which perturbation of the neutral atmosphere increases with latitude. The local time modeling is the best with DTM94. Both test models have some large steps between bins, particularly between the 20-22 (mean=0.98) and 22-24 (mean=0.87) hr local time bins.

It is important to note that: (i) the rms of the residuals per \(F_{10.7}\) bin is smallest for the

![Figure 6.7a](image-url): the mean and rms (right) of the DE-2 oxygen residuals presented in \((F_{10.7} - \overline{F}_{10.7})\) bins, where \(\overline{F}_{10.7}\) represents the 27-day mean of the solar radio flux or the rescaled Mg II index.
highest solar activity bin (220-250 sfu) for all models, and (ii) the rms of the 130-160 sfu bin is 18% smaller with DTM-Mg II than with DTM-F107. The DTM-Mg II residuals binned as a function of $k_p$ presented the most stable mean up to $k_p=6$, and the smallest rms of the 3 models for $k_p$ values equal to or less than 3. DTM94 predicts density best for higher values of $k_p$.

6.2.2.3 The Molecular Nitrogen Modeling

As presented in Table 6.1, the N$_2$ modeling precision (rms about mean: 0.188) has improved 3% with respect to DTM-F107 (0.194) and 5% compared to DTM94 (0.198). The mean of the residuals per latitude bin is stable with DTM-Mg II (between 0.60-0.65), and does not have the latitudinal signature as described in section 6.2.2.2. The corresponding intervals with DTM-F10.7 and DTM94 are (0.61-0.67) and (0.63-0.68), respectively. Figure 6.8 shows that this signature is present, however, in the rms of the binned residuals of the three models.

At altitudes below approximately 240 km, where N$_2$ becomes the major constituent, the gain in precision is 10% and 6% with respect to DTM-F107 and DTM94, respectively (not shown). Analysis of the residuals binned per hour local time revealed systematic modeling errors in both test models, namely a semidiurnal signal. The residuals binned per $F_{10.7}$ showed the smallest rms with DTM-Mg II for the 220-250 sfu bin, 9% better than obtained with DTM-F107, which is however 16% more precise for a flux from 130 to 160 sfu.

Figure 6.7b: the number of DE-2 oxygen residuals presented in ‘($F_{10.7} - \bar{F}_{10.7}$)’ bins.
6.2.2.4 The Helium Modeling

The helium modeling has deteriorated using the Mg II index, as shown by the increased rms in Table 6.1. The rms (about mean) of 0.195 is the largest, as is the mean of the residuals (0.933). Helium is a light element, and therefore becomes the major constituent at altitudes exceeding 700 to 1000 km, depending on the level of solar activity. Figure 6.9 presents the residuals per altitude bin, which shows that the rms obtained with DTM-Mg II is 3 to 6% smaller than that obtained with DTM-F107 in the 800 to 1000 km altitude range.

Binning of the residuals as a function of latitude presents means ranging from approximately 0.97 at the poles to 0.87 at the equator for DTM-Mg II (not shown). This variation is twice as large as with DTM-F107. The latter model’s local time binned residuals on the other hand show a semidiurnal signal which is twice as large (mean per bin ranging from 0.83 to 0.97) as those of DTM-Mg II. The most stable mean per $F_{10.7}$ bin is with DTM-Mg II, whose values range from 0.92-0.96. DTM-F107 does not represent changes in density as a function of mean flux correctly: the means of its residual per 30 sfu bin lie in the interval 0.86-0.97.

Figure 6.8: the rms (about mean) of the N$_2$ residuals binned as a function of latitude.
6 Upper Atmosphere Density Modeling

6.2.2.5 The Total Density Modeling

An external validation of the model accuracies may be obtained by precise orbit computation [Marcos et al., 1993]. Drag coefficients, estimated on a daily or more frequent basis, scale the predicted atmospheric density to the atmospheric drag experienced by the satellite. This is only possible in case of accurate and sufficiently dense tracking of the satellite. In this study, the SLR system was used. A second and equally important condition is the employment of the most accurate force models in the orbit computation, so that the estimated drag coefficients ($c_d$; this coefficient scales the density $\rho$ in eq. (5.5)) indeed represent the density model scaling error. Otherwise, modeling errors not due to the atmospheric drag, due to gravity and solar radiation pressure in particular, may be partially absorbed by the drag coefficients.

The geodetic satellites GFZ-1 and Starlette, presented in Section 6.2.4 and Table 1.1 respectively, have been chosen for this evaluation. GFZ-1 was (end-of-life in June 1999) the only target in very-low Earth orbit. Sufficient SLR data for orbits of 4 days long are available from June 1995 until October 1997, and that is to say, during low solar activity only (the solar cycle 23 minimum was in 1996). Starlette has been launched in 1975, allowing orbit computation under both high and low solar activity conditions. The major atmospheric constituents at 380 km under

Figure 6.9: the rms about mean of the He residuals binned as a function of altitude.
low solar activity are O and N₂, while at 800 km the major constituents are He, and O and He for low and high solar activity, respectively.

The results of the GFZ-1 precise orbit tests are given in Table 6.2, which displays the average SLR rms-of-fit of 10 orbits of 4 days arc length, as well as the mean and rms (about mean) of the 160 estimated drag coefficients. The GFZ-1 orbit computation benefits from the use of the Mg II index, resulting in the best orbit fit to the SLR observations (24 cm rms on average) and the smallest bias (7%) and rms (0.18) of the estimated drag coefficients. The DTM-F107 model is the most biased one, predicting a density at 380 km altitude that is 26% too small. These results corroborate the results displayed in Table 6.1, which showed a more precise O, N₂ and temperature modeling using Mg II rather than using $F_{10.7}$.

Table 6.2: the mean SLR rms-of-fit and drag coefficients ($c_d$) resulting from the GFZ-1 precise orbit computations.

<table>
<thead>
<tr>
<th>Model</th>
<th>Mean SLR rms (m)</th>
<th>Mean / rms $c_d$</th>
</tr>
</thead>
<tbody>
<tr>
<td>DTM94</td>
<td>0.27</td>
<td>1.08 / 0.21</td>
</tr>
<tr>
<td>DTM-F107</td>
<td>0.31</td>
<td>1.26 / 0.34</td>
</tr>
<tr>
<td>DTM-Mg II</td>
<td>0.24</td>
<td>0.93 / 0.18</td>
</tr>
</tbody>
</table>

The results obtained for Starlette are given in Table 6.3, based on 7 orbits of 8 days arc length computed during low solar activity (LF), and 7 orbits during high solar activity (HF), estimating 1 or 2 drag coefficients per day. The Starlette orbits have the smallest rms-of-fit of the SLR residuals employing DTM-Mg II during high solar activity (6.5 cm), as was to be expected of a model incorporating only high solar activity density observations. DTM94 has a 13% smaller rms in the drag coefficients and has a 5% smaller bias during low activity than DTM-Mg II has. The DTM-F107 model has a bias of 30%, and with 0.29 the largest rms of the drag coefficients.

Table 6.3: the mean SLR rms-of-fit and drag coefficients ($c_d$) resulting from the Starlette precise orbit computations, during low (LF) and high (HF) solar activity.

<table>
<thead>
<tr>
<th>Model</th>
<th>Mean SLR rms (m)</th>
<th>Mean / rms $c_d$</th>
</tr>
</thead>
<tbody>
<tr>
<td>DTM94</td>
<td>0.076</td>
<td>1.04 / 0.19</td>
</tr>
<tr>
<td>DTM-F107</td>
<td>0.080</td>
<td>1.34 / 0.29</td>
</tr>
<tr>
<td>DTM-Mg II</td>
<td>0.077</td>
<td>1.09 / 0.22</td>
</tr>
</tbody>
</table>
6 Upper Atmosphere Density Modeling

6.2.3 Discussion

The Mg II index and the solar radio flux present a high correlation over an entire solar cycle (0.99), but not on shorter time scales, in particular the one linked to solar rotation. The application of the Mg II index has been evaluated using only DE-2 data, spanning 1½ year during high to moderate solar activity. This period of time is sufficient to produce the test models, but at least an entire solar cycle of data is required to construct a representative new DTM model with homogeneous accuracy. However, the results obtained with DTM-Mg II are better than those obtained with DTM-F107, and only the DTM94 He modeling (and thus Starlette during low activity) is more accurate. DTM-F107 suffers more from the short time span of the data than DTM-Mg II, which reflects the average character of $F_{10.7}$: the correlation with chromospheric activity is 0.99 over an entire solar cycle, but not so on shorter time scales. The modeling results are better with Mg II under high solar activity than with $F_{10.7}$, as was shown in Table 6.3. Not surprisingly, it is just under those conditions that the difference between these proxies becomes large, as was shown in Figure 6.5.

It may seem contradictory that the global rms of the DE-2 residuals with DTM-F107 is larger than with DTM94, since more data sets have been used in the latter model. The DTM model algorithm has been developed for low and high solar activity, which was not the case here. Secondly, the difference $F_{10.7} - F_{10.7}$ for the DE-2 period is mainly negative instead of centered around 0. This, together with the poorer representation of chromospheric activity on short time scales by $F_{10.7}$, introduces systematic errors and noise.

6.3 Conclusions and Future Prospects

Comparison of the test models DTM-Mg II and DTM-F107 in Section 6.2 demonstrated the superior representativity of the Mg II index than $F_{10.7}$ with respect to upper atmosphere heating processes, in particular under high solar activity conditions. The smaller standard deviations of the DE-2 residuals at altitudes where they are the major constituents show a 3 to 8% higher precision. The orbit tests with DTM-Mg II present an up to 25% smaller bias and an rms that is typically 25-50% smaller than with DTM-F107. The fact that the density model based on Mg II is more accurate than that based on $F_{10.7}$ under high solar activity, where these proxies differ most, demonstrates the higher representativity of the Mg II index of upper atmosphere heating processes. The lack of density data during low solar activity introduced a bias in the test models, and a larger and more representative density data set is required to construct a more homogeneous DTM model based on the Mg II index.

The DTM_Mg II test model has clearly identified one of the points responsible for the slow progress in upper atmosphere model accuracy, namely the use of a proxy indicator that is
only statistically representative on the average. This is the probable cause for the inaccuracy of the test model DTM_F107, which is derived from only 1.5 years of DE-2 data, and thus not benefiting from the fact that the solar radio flux is representative when taken over a long period (more than 99% correlated with the Mg II index over a period of 19 years), at least a solar cycle. A suitable index to substitute \( k_p \), which is a large source of error, has not been found, but it is possible that indices, already tested but rejected in the past, such as the Hemispheric Power index [Marcos et al., 1993] or the Auroral Electrojet index [Berger et al., 1988], have to be reconsidered now that the EUV indicator proved to be only moderately representative.

Unfortunately, the Mg II index cannot be used in the Starlette long-arc adjustment presented in Chapter 8 (Section 8.2.1) due to data gaps, some of which are visible in Figure 6.5. While atmospheric density model errors will be corrected for by adjusting drag scale coefficients, the orbit fit results shown in Tables 6.2 and 6.3 prove that residual errors remain when employing the radio flux instead of the Mg II index. In fact, a mean long-arc computation is more affected by the lesser representativity of \( F_{10.7} \), because the drag scale coefficients are not adjusted frequently. It is a function of the number of observed mean element state vectors, which is one per month at best due to the filtering technique (window length). However, density predicted using \( F_{10.7} \) instead of Mg II sometimes presents erroneous offsets and/or drifts over periods of weeks [Bruinsma and Biancale, 2001], and these cannot be accurately corrected for with less than one drag scale factor per week. Therefore, a significantly better Starlette long-arc fit would be obtained if the Mg II index were available. The Starlette orbits presented in Chapter 6, which will be used to compute the mean observations for CODIOR, have also not been computed with Mg II. Because of the filtering procedure (Section 3.2), however, this does not affect the accuracy of the observed mean elements.

It is not possible to use \( F_{10.7} \) (to fill the gaps) and Mg II simultaneously because of the discontinuities at the transitions, which are incompatible with the dynamically consistent long-arc approach. A solution to this problem may be the use of an index, for example based on a combination of \( F_{10.7} \), Ca plage, Lyman \( \alpha \), Mg II or other observations, which assures its availability on a daily basis and guarantees continuity. This type of index is constructed by Lean et al. [2001], but it has not been validated yet by actual density modeling.
6 Upper Atmosphere Density Modeling
7 Orbit Determination

Orbit determination is an important topic in space geodesy, which uses the orbit perturbation technique to estimate, for example, gravity field and ocean tide coefficients, station coordinates and velocities, and geocenter motion. It concerns this study also, because the observed mean elements that are used in the long-arc computations (Chapter 8) are obtained by filtering of orbits fitted to tracking data. Secondly, atmospheric density models may be evaluated via precise orbit computation [Marcos et al., 1993], and this technique has been used in Chapter 6. The Mg II index, however, is not employed in the orbit computations presented in this chapter because of the data gaps in the observations. The orbit error caused by employing the solar radio flux instead of the Mg II index is filtered and does not significantly affect the observed mean elements. The state-of-the-art dynamical and geometrical models have to be employed in order to obtain the accuracy required to estimate geophysical parameters. These models and observations may be based upon several tracking data types, such as SLR (Satellite Laser Ranging), GPS (Global Positioning System), DORIS (Doppler Orbitography and Radio positioning Integrated by Satellite), VLBI (Very Long Baseline Interferometry) and LLR (Lunar Laser Ranging) for example. Other data types contribute to the present orbit accuracy also, since they are used in the elaboration of models important to orbit determination. Examples are atmospheric density and temperature data (for upper atmosphere density modeling), Earth radiation data (for albedo and IR radiation pressure modeling), or meteorological data (both for troposphere modeling and the gravitational orbit perturbation provoked by moving air mass). In the framework of this thesis, only the SLR tracking system has been used and will therefore be presented briefly in the next section. Section 7.2 presents the results of the orbit fits in terms of rms-of-fit of Starlette, LAGEOS-2, LAGEOS-1, and GFZ-1 precise orbit determinations. The orbits presented in these subsections are used in the long-arc adjustments of Chapter 8 (except for GFZ-1, which was used in the previous chapter), providing the required mean observed elements after filtering.

7.1 The SLR Tracking System

The SLR technique has been developed in the 1960s and has nowadays a single-shot precision of better than 1 cm for the most modern third-generation YAG lasers, i.e. comparable with the orbit precision level attainable for LAGEOS. The second-generation ruby lasers, a few of which are still operational, have a single-shot precision of 10-20 cm, while the first operational stations were ranging at the meter level. The technique is based upon the measurement of the round-trip travel time of a short laser pulse, to which purpose some satellites carry a retroreflector.
array while the geodetic satellites are covered with cube corner reflectors. These geodetic satellites, such as Starlette (Figure 1.1) and LAGEOS-1/-2, have been specifically designed for this tracking system. The theoretical equation for the range measurement $\rho_{\text{station-satellite}}$ is simply:

$$\rho_{\text{station-satellite}} = \frac{1}{2} c T_{\text{roundtrip}} + \Delta \rho (\text{corrections})$$  \hspace{1cm} (7.1)

where $c$ is the speed of light in vacuum, $T_{\text{roundtrip}}$ the round-trip travel time, and $\Delta \rho (\text{corrections})$ represent instrumental, tropospheric and relativistic corrections [McCarthy, 1992] to the range, respectively. Any error in the round-trip travel time results in erroneous distances, which is why an SLR system has to be calibrated before and after every tracking event, to determine a possible range bias. Many SLR stations have also been collocated with mobile systems to verify the accuracy of the bias determinations.

The advantages of the SLR system, besides that they provide absolute measurements, are its accuracy, the simplicity of its measurement and data processing, and the reliability of the space segment: there are no electronics involved. This last point makes it the ideal back-up tracking system of present and future satellite missions. The ground segment, on the other hand, is more time-consuming to operate and technically complicated (and hence expensive) than DORIS or GPS. The SLR stations have to aim at the satellite, to which purpose a target acquisition system utilizes accurate orbit predictions, while fine-tuning often has to be performed manually. A second disadvantage is its weather dependency; optical measurements require a clear sky. Finally, the tracking station network is not evenly distributed over the globe, as may be seen in Figure 7.1, which shows the network in the nineties. Most stations are in the Northern Hemisphere, either in Europe or the United States. The resulting orbit coverage is typically less than 20\% for high-
7.2 Orbit Computation Results

The next 4 subsections present the employed models, orbit parameterizations, and the obtained orbit fit results of Starlette, LAGEOS-1/-2, and GFZ-1, respectively. The orbit computations have been performed using the GINS software of the GRGS/CNES Toulouse, which has been used both in the framework of the GRIM4 [Schwintzer et al., 1997] and the GRIM5 gravity field projects [Biancale et al., 2000].

7.2.1 Starlette

The length of the Starlette orbits is 24 days, which is imposed by the CANEL2 filter (Section 3.2), which is long with respect to the model accuracies. For GRIM5 or for atmospheric density model evaluation purposes, for example, the Starlette orbits have a length of 8 days to limit the effect of error accumulation. The dynamical and geometrical models employed in the Starlette orbit adjustments are presented in Table 7.1. The IERS Conventions [McCarthy, 1996] have been respected in most instances.

The solved-for parameters per orbit were the 6 initial state-vector elements at epoch (x,y,z,x\dot{y},y\dot{z}), 1 solar radiation pressure scaling coefficient and 24 linear drag scale coefficients (1/day). A total of 64 orbits has been adjusted using this particular computational model. The first orbit is in November 1983, while the last orbit has been adjusted in December 1996. The results, in terms of rms-of-fit of the SLR residuals, are given in Figure 7.2, which shows that on average an orbit fit of 17 cm to 1800 observations has been achieved. This is a good result taking into account the arc length of 24 days, engendering model error accumulation (to limit this effect, orbits used for GRIM5 gravity field modeling purposes have a length of 8 days resulting in an orbit fit of 5 cm on average). The weighting of the observations of the individual SLR stations is a function of the station precision in terms of the instrumental precision, accuracy of its coordinates and the standard deviation of its residuals with respect to a LAGEOS-1 reference orbit [Biancale et al., 1998]. Old stations systematically have a small weight due to their lower instrumental precision, and modern stations occasionally in case of operational problems or inaccurate coordinates (in case of a new site or a station modification). The orbit computation in this study has been done using weighting of the SLR observations, while stations that provided dubious observations have been eliminated (no participation in the adjustment).

The estimated empirical drag and solar radiation pressure scaling coefficients have been verified for each orbit. Due to the too large number of drag scale coefficients to properly visualize, only the solar radiation pressure scaling coefficients (Cr; scales equation 5.1) and the formal error of their estimation are presented in Figure 7.3. The estimated values are well within 10% bounds in 55 cases, which is a realistic limit when taking the accuracy of the albedo modeling into account.
(the albedo force modeling error may be partly absorbed when estimating Cr). Where this is not the case, important leakage of other model errors (most probably drag) into the solar radiation pressure scaling coefficient has occurred. The average value is slightly over 1, which means that the used reflectivity coefficient was slightly too small. The standard deviations of the estimates presented in Figure 7.3 have \(1\sigma\) values between 0.001 to 0.008, even in case of unrealistically high estimations of Cr (>1.2, taking into account that this coefficient also absorbs albedo modeling errors). This formal (computed) error is in most instances not representative of the accuracy of the estimated value, which is highly dependent on the number and quality of the observations, their distribution along the arc or the arc length.

**Table 7.1:** dynamical and geometrical model for the Starlette SLR orbit adjustment.

<table>
<thead>
<tr>
<th>Model</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Dynamical model:</strong></td>
<td></td>
</tr>
<tr>
<td>Earth gravity</td>
<td>EGM96 70x70 [Lemoine et al., 1998]</td>
</tr>
<tr>
<td>third bodies (Moon, Sun and 5 planets)</td>
<td>JPL DE403 ephemerides [McCarthy, 1996]</td>
</tr>
<tr>
<td>Earth tides</td>
<td>due to Moon and Sun, Wahr terms</td>
</tr>
<tr>
<td>ocean tides</td>
<td>FES95.2 (degree 20, order 4) [LeProvost et al., 1994]</td>
</tr>
<tr>
<td>atmospheric drag</td>
<td>DTM97 [Bruinsma et al., 1997]</td>
</tr>
<tr>
<td>solar radiation</td>
<td>solar constant (4.5605 \times 10^{-6} \text{ Nm}^{-2}), shadow function</td>
</tr>
<tr>
<td>albedo (visible+IR)</td>
<td>analytical function using mean (observed) albedo tables [Knocke et al., 1988]</td>
</tr>
<tr>
<td>relativity</td>
<td>Schwarzschild correction [McCarthy, 1992]</td>
</tr>
<tr>
<td>area-to-mass ratio</td>
<td>0.00094166 \text{ m}^2/\text{kg}</td>
</tr>
<tr>
<td>reflectivity visible/IR</td>
<td>1.1688 / 1.10</td>
</tr>
<tr>
<td>aerodynamic coefficient</td>
<td>Cook [1966]</td>
</tr>
<tr>
<td>precession/nutation</td>
<td>w.r.t. J2000 inertial reference frame, using DE403</td>
</tr>
<tr>
<td>integration, step-size</td>
<td>8th order Cowell multi-step, 60 s.</td>
</tr>
<tr>
<td><strong>Geometrical model:</strong></td>
<td></td>
</tr>
<tr>
<td>Earth rotation parameters</td>
<td>EOP97C04 (IERS)</td>
</tr>
<tr>
<td>SLR station positions and velocities</td>
<td>ITRF94 [Boucher et al., 1996]</td>
</tr>
<tr>
<td>site displacement</td>
<td>Earth tides, ocean loading based on FES95.2 model</td>
</tr>
<tr>
<td>tropospheric refraction</td>
<td>Marini and Murray [1973]</td>
</tr>
<tr>
<td>relativity</td>
<td>range effect [McCarthy, 1992]</td>
</tr>
</tbody>
</table>
The drag scale factors fluctuate much more about 1, due to the less-accurate density modeling. The uncertainty of density models is typically 20-25% (1σ), which was shown in Chapter 6, and this is also the average scatter of the estimated drag coefficients (0.23). For 3 orbits, unrealistically small drag coefficients were obtained (0.1), most probably related to the small number of observations for those orbits. Since the empirical parameters will not be used at a later stage (for example for density modeling), their values are relatively unimportant, as long as the resulting orbit solution is accurate. The rms of the orbit fit is an internal precision indicator, but often is a good accuracy indicator as well. For this to be true, the data volume (at least 6 passes tracked per day) and temporal spacing need to be homogeneous (not all passes tracked during a
small part of the day), and a large number of accurate SLR stations have to participate in the adjustment (in the Northern and Southern Hemisphere, with a good longitude distribution).

Unfortunately, it is the heterogeneity of the SLR data quality that poses an obstacle. This is the reason why the SLR residuals are weighted, since the data quality of some stations is an order of magnitude less than that of the best ones. As an example, Figure 7.4 shows this difference for three stations, from below average (Orroral Valley, Australia) to average (Simosato, Japan) to excellent (Graz, Austria) performance. The offset presented in Figure 7.4 is not the instrumental bias (range or timing bias), but the mean of the residuals with respect to the adjusted orbit. However, the mean value over a long period of time of these biases is an indicator of the station’s instrumental bias and/or inaccurate coordinates.

Although Orroral Valley presented an average rms-of-fit of 27 cm, which is worse than average (17 cm), it is a valuable station in the network, since it is one of the few in the Southern Hemisphere. Its observations were retained in the adjustments, which was not the case for the obsolete stations (ruby lasers). They were nearly always eliminated (3σ criterium) during the iterative adjustment process due to their too high dispersions. Simosato has an average rms-of-fit of its observations of 18 cm, while Graz is one of the best stations in the network with an average rms-of-fit of 13 cm. Graz, Grasse and Herstmonceux in Europe, Yarragadee in Australia and Greenbelt, Quincy and Monument Peak in the United States form the backbone of the SLR network by their continuous operation and precise observations. However, even these stations may suffer from instrumental upsets or calibration errors, and a file containing most of the station calibration errors has been used to obtain the best fit.

The mean observed elements to be used in Chapter 8 are referenced to the epoch in the middle of each orbit. In order to evaluate the orbit quality in the middle of the 24-day orbits, orbit overlaps have been computed there. 4-day orbits have been adjusted which have subsequently been compared with each 24-day orbit over the common interval. The 4-day orbits are more accurate than the 24-day orbits (but too short to filter), since the cumulative errors due to nongravitational model errors and the state vector at epoch are much smaller despite the smaller volume of tracking data. The accuracy of the determination of the state vector and the empirical parameters is higher, and the rms of the SLR residuals is much smaller: it is at the 4 cm level. They may thus be used as references to assess the accuracy of the 24-day arcs.

The overlaps revealed a relatively high sensitivity of the inclination to the orbit length: overlap differences of up to 35 cm (mainly bias, and some drift) have been observed. The semimajor axis overlap differences were always smaller than 2 mm, the eccentricity overlap differences exceeded 10 cm on 5 occasions only. No orbits were found where the overlaps were
unsatisfactory, which would be the case for semimajor axis overlap differences exceeding 5 mm for example, since this would cause important trends (meters) in the angle variables.

Figure 7.4: the rms and (mean) bias of the residuals in meters of 3 SLR stations used in the Starlette orbit adjustment, performing below average (top) to excellent (bottom).
7 Orbit Determination

7.2.2 LAGEOS-2

The dynamical and geometrical models used in the LAGEOS-2 and LAGEOS-1 orbit adjustments are presented in Table 7.2. The solved-for parameters per 20-day orbit were the 6 initial state-vector elements at epoch, 1 solar radiation pressure scaling coefficient, and 2 empirical constant along-track accelerations (1 per 10 days) to accommodate the unmodeled Yarkovsky and Yarkovsky-Schach effects, as well as neutral and charged particle drag. Note the following modeling differences compared to Starlette, which orbits at a lower altitude: maximum degree 30 of the gravity field development due to height attenuation (although the perturbation due to degree 20-30 is at cm-level), no specific drag modeling but empirical along-track acceleration estimation, and the integration step-size has increased from 60 to 120 seconds.

Table 7.2: dynamical and geometrical model for the LAGEOS-1/-2 SLR orbit adjustment.

<table>
<thead>
<tr>
<th>Dynamical model:</th>
</tr>
</thead>
<tbody>
<tr>
<td>Earth gravity</td>
</tr>
<tr>
<td>third bodies (Moon, Sun and 5 planets)</td>
</tr>
<tr>
<td>Earth tides</td>
</tr>
<tr>
<td>ocean tides</td>
</tr>
<tr>
<td>gravitational effect due to atmospheric pressure</td>
</tr>
<tr>
<td>solar radiation</td>
</tr>
<tr>
<td>albedo (visible+IR)</td>
</tr>
<tr>
<td>relativity</td>
</tr>
<tr>
<td>area-to-mass ratio</td>
</tr>
<tr>
<td>reflectivity visible/IR</td>
</tr>
<tr>
<td>precession/nutation</td>
</tr>
<tr>
<td>integration, step-size</td>
</tr>
<tr>
<td>Geometrical model:</td>
</tr>
<tr>
<td>Earth rotation parameters</td>
</tr>
<tr>
<td>SLR station positions and velocities</td>
</tr>
<tr>
<td>site displacement</td>
</tr>
<tr>
<td>tropospheric refraction</td>
</tr>
<tr>
<td>relativity</td>
</tr>
</tbody>
</table>

EGM96 30x30 [Lemoine et al., 1998]
JPL DE403 ephemerides [McCarthy, 1996]
due to Moon and Sun, degree 2 and 3, Wahr terms
FES95.2 (up to degree 20, order 4) [LeProvost et al., 1994]
as a field in spherical harmonics, daily tables from 6/86-10/97 (ECMWF)
solar constant \(4.5605 \cdot 10^{-6} \text{Nm}^{-2}\), shadow function
analytical function using mean (observed) albedo tables [Knocke et al., 1988]
Schwarzschild correction [McCarthy, 1992]
0.00069287 \(\text{m}^2/\text{kg}\)
1.1377 / 1.11
w.r.t. J2000 inertial reference frame, using DE403
8th order Cowell multi-step, 120 s.

EOP97C04 (IERS)
ITRF94 [Boucher et al., 1996]
Earth tides, ocean loading based on FES95.2 model
Marini and Murray [1973]
range effect [McCarthy, 1992]
The amplitudes of the forces acting on LAGEOS-2 are given in Figure 7.5. The third-body attraction is large at 6000 km altitude, while the effect of terrestrial gravity is much attenuated.

**Figure 7.5**: the amplitudes of the main accelerations, in m/s², acting on LAGEOS-2.

A total of 83 orbits has been adjusted, from November 1992 through June 1997, of which 74 used the computational model given in Table 7.2. The last 9 orbits, from November 1996 to June 1997, have been computed without the gravitational effect of observed atmospheric pressure, because these data were not available. The orbit fit, in terms of the weighted rms of the SLR residuals, is given in Figure 7.6. The average orbit fit was 3.8 cm, to an average of 2600 observations per arc. This result is not state-of-the-art, which is better than 2 cm. However, thanks to the filtering, this has no effect on the computed observed mean elements (Chapter 3). The distribution of the used observations over the globe for a typical 20-day arc is plotted in Figure f1 of Appendix F.

**Figure 7.6**: the rms-of-fit of the SLR residuals for the 20-day LAGEOS-2 orbits.
The solar radiation pressure scaling coefficients and their uncertainties are presented in Figure 7.7. The estimated values are within 3% of the initial value in all cases. Leakage of other model errors into the solar radiation pressure coefficients has not occurred or has been very small. The accurate determination of these coefficients was to be expected, since all other surface forces are (much) smaller, and cause different perturbing signals in the orbital elements. The decay of the semimajor axis depends mainly on the along-track acceleration, while the solar radiation and albedo (the latter force is small for LAGEOS-2) mostly affect the eccentricity and the semimajor axis, but on a periodic basis. A characteristic of the LAGEOS-2 solar radiation pressure scaling coefficients is that they seem to decay, approximately exponentially, indicating a decreasing reflectivity (also observed by Ries et al. [1997]). This is supposedly due to surface degradation caused by the space radiation environment. This effect has not been observed on LAGEOS-1, although it must be stressed that the parameter estimation for this satellite was limited by the SLR accuracy at the beginning of its mission from 1976 to 1983. SLR accuracy improved significantly after 1983, at which date the third-generation YAG laser technology became available, improving single-shot precision from 30 to 1 cm or even better.

The along-track accelerations, depicted in Figure 7.8, fluctuate considerably at the $10^{-12}$ m/s$^2$ level, with a formal precision of the estimations of better than $10^{-13}$ m/s$^2$ in most cases. The overall features of this figure correspond to earlier determinations by Ries [Ries et al., 1997] of the Center for Space Research (CSR) at the University of Texas at Austin, who smoothed the accelerations using a Kalman filter. There is no correlation between the along-track accelerations and the magnitude of the solar and geomagnetic activity. This suggests that the along-track accelerations are primarily due to the solar radiation pressure.
acceleration component due to atmospheric drag is constant, or at least that the fluctuations are too small to be observable, even under high solar activity conditions.

The estimates of the empirical parameters have been used in the LAGEOS-2 long arc as a file of observed values. This is justified by their precision and temporal validity of 10 days for the along-track accelerations and 20 days for the solar radiation pressure coefficients. The use of the along-track accelerations should ensure an empirical yet precise semimajor axis modeling, in the absence of a nongravitational force model. Secondly, they may be used as a reference for the nongravitational force model thanks to their high precision.

The heterogeneity of the SLR data quality is large, which was also noted in the case of Starlette. Because LAGEOS-2 is an easier target to track than Starlette, much more stations participate in the orbit adjustments. This leads to more observations, which are also better distributed geographically. The accuracy of the orbit benefits from both factors. As an example: Riga, a station which performs worse than average during the time-interval under consideration here, hardly has any weight in the adjustments since the data weighting is a function of data quality. The average rms-of-fit of 9.5 cm of this station is high, and the average of its residuals of -3.9 cm is significant (in this case probably caused by a range bias). Its average number of normal points per 20-day arc is 63. Grasse (France) is an example of an average station, with an average rms-of-fit of 4.7 cm rms, but only 76 normal points per arc on average. In addition, the bias of 3.3 cm hints at calibration errors (a position error is unlikely taking the long occupation and several collocations into account). Monument Peak (California) is one of the best stations, with a constant high precision, 3.5 cm rms-of-fit on average, and no bias (-0.03 cm). The high quality of the tracking data is accompanied by a large average number of normal points per arc for this station: 332. The most accurate station over the 4½ years of LAGEOS-2 orbits was Graz, with an average precision of 3 cm rms-of-fit, a -0.3 cm bias, and an average of 157 normal points per arc.

**Figure 7.8:** the estimated along-track accelerations for LAGEOS-2.
7 Orbit Determination

7.2.3 LAGEOS-1

The 24-day orbits for LAGEOS-1 have been computed employing the same dynamical and geometrical models used in the LAGEOS-2 orbit adjustments, for which the reader is referred to Table 7.2. The solved-for parameters per orbit were the 6 state-vector elements at epoch, 1 solar radiation pressure scaling coefficient, and this time 4 empirical along-track accelerations (1 per 6 days) to absorb the unmodeled Yarkovsky and Yarkovsky-Schach effects, as well as neutral and charged particle drag. The frequency of the along-track acceleration estimation has been increased here, because analysis of the LAGEOS-1 orbits and long arcs revealed that once per 10 days provided insufficient temporal resolution. The GINS and CODIOR orbit fit for LAGEOS-1 improved significantly by doubling the frequency of the along-track estimations, especially where they presented the large spike-like amplitudes, similar to those shown in Figure 7.8.

A total of 82 orbits has been adjusted, from January 1992 through June 1997. 75 used this particular computational model. The last 7 orbits, from November 1996 to June 1997, have been computed without the gravitational effect of observed atmospheric pressure, because these data were not available. The orbit fit, in terms of the rms of the SLR residuals, is given in Figure 7.9. The average orbit fit was 6.8 cm, to an average of 3140 observations. This fit is not at the LAGEOS-2 level, the orbit of which is less perturbed by the thermal drag forces. Orbit computations (6/1998) in the framework of the GRIM5 project [Biancale et al., 2000] by J.M. Lemoine (GRGS/CNES, Toulouse) provided insight into the nature of this lesser accuracy. By estimating two empirical accelerations, one in the plane of the orbit in the direction Center of mass of the Earth-ascending node and the second one perpendicular to that (Bx and By, respectively), as well as an along-track acceleration, the unmodeled thermal drag was much better accommodated. This is simply due to the right choice of parameterization, since thermal drag and radiation pressure forces can only in part be decomposed into a tangential component. The effect on the eccentricity and perigee in particular may be modeled using this specific moving satellite frame, bearing in mind that they are less well-defined than in case of LAGEOS-2, the eccentricity of which is three times larger. The empirical accelerations have been made available by J.M. Lemoine, and they have been used in a similar way as was done with LAGEOS-2 (Section 8.2.3a).

The estimates of the empirical along-track accelerations and solar radiation pressure scaling coefficients have been reviewed for each orbit. The solar radiation pressure scaling coefficients and their uncertainties are presented in Figure 7.10. The estimated values are not within 3% of the initial value (1.0) in all cases (as was the case for LAGEOS-2) due to leakage of other nongravitational forces. The LAGEOS-1 solar radiation pressure scaling coefficients do not present a decreasing signal. If degradation of its surface has taken place it was presumably before 1992 and it no longer changed afterwards.
The along-track acceleration estimates fluctuate considerably at the level of $10^{-12}$ m/s$^2$, with a formal precision of the estimations of $10^{-13}$ m/s$^2$ in most cases. Considerable differences exist between the along-track accelerations determined by CSR (courtesy of J. Ries), estimated per 3 days, and the GINS along-track accelerations estimated per 6 days. This is probably due to error transfer of the inaccurately estimated solar radiation pressure scaling coefficients into the along-track acceleration data set. The empirical along-track accelerations have been used in the long arc (Chapter 8) as ‘observed’ values, as was done for LAGEOS-2. It turned out that the semimajor axis modeling is less accurate using the GINS observed along-track accelerations, but the CSR data set provided observations ensuring an accurate reference semimajor axis modeling. The empirical accelerations provided by J.M. Lemoine are most complete, and should improve the eccentricity and perigee modeling as well, besides the semimajor axis. This has been verified in Section 8.2.3a, where the smallest residuals have been obtained with this particular data set.

Figure 7.9: the rms-of-fit of the SLR residuals for the 24-day LAGEOS-1 orbits.

Figure 7.10: the estimated LAGEOS-1 solar radiation pressure scaling coefficients (connected dots; left axis) and their formal error (diamonds; right axis).

The along-track acceleration estimates fluctuate considerably at the level of $10^{-12}$ m/s$^2$, with a formal precision of the estimations of $10^{-13}$ m/s$^2$ in most cases. Considerable differences exist between the along-track accelerations determined by CSR (courtesy of J. Ries), estimated per 3 days, and the GINS along-track accelerations estimated per 6 days. This is probably due to error transfer of the inaccurately estimated solar radiation pressure scaling coefficients into the along-track acceleration data set. The empirical along-track accelerations have been used in the long arc (Chapter 8) as ‘observed’ values, as was done for LAGEOS-2. It turned out that the semimajor axis modeling is less accurate using the GINS observed along-track accelerations, but the CSR data set provided observations ensuring an accurate reference semimajor axis modeling. The empirical accelerations provided by J.M. Lemoine are most complete, and should improve the eccentricity and perigee modeling as well, besides the semimajor axis. This has been verified in Section 8.2.3a, where the smallest residuals have been obtained with this particular data set.
GFZ-1 is a geodetic satellite, which was launched in a nearly circular, 51.6° inclined orbit at approximately 380 km altitude on April 19, 1995. It is basically a smaller version of Starlette. The 4-day orbits have been computed employing the same dynamical and geometrical models used in the Starlette orbit adjustments (Table 7.1), but with gravity field modeling up to degree and order 90. The arc length of 4 days is a trade-off imposed by the sparseness of the observations and the dynamic modeling errors. The area-to-mass ratio of GFZ-1 is 0.0017598 m²/kg. The estimated parameters per orbit were the usual 6 initial state-vector elements, 2 to 4 piecewise linear drag coefficients per day and a 1/rev drag coefficient for the entire arc. The first of the 67 orbits starts on June 19, 1995, and the last orbit ends on October 30, 1997. The rms of the SLR orbit fit is displayed in Figure 7.11. The average rms-of-fit is 24 cm, and is based on approximately 500 SLR observations per orbit on average. Some GFZ-1 orbits have been used in density model evaluation, a topic which was addressed in Chapter 6. Since they are not filtered in such an application, the orbit accuracy is an important issue, requiring the smallest residuals possible and realistic values of the empirical parameters.

Precise orbit computation for GFZ-1 is difficult and tedious when compared to the effort required in the case of LAGEOS or Starlette. The reasons for this will be explained in the remainder of this paragraph. The orbit perturbations acting on GFZ-1 are large because of its low altitude, in particular the perturbations due to the gravity field and the atmospheric drag. The amplitudes of the forces acting on the satellite are given in Figure 7.12, where they are presented in the same order as was done for LAGEOS-2 in Figure 7.5. The gravity acceleration due to degrees 3 and higher (and all orders) on GFZ-1 is nearly 2 orders of magnitude larger than that on LAGEOS-2. The atmospheric drag is even 5 orders of magnitude larger. The observations and model errors impose a maximum arc length of 4 days, although longer arcs are required to be able to estimate the high-degree and order resonances with periods exceeding 10 days. The much larger
model noise and error at 380 km are reflected by the slow convergence of the iterative least-squares adjustment algorithm and a large sensitivity of the orbit fit to the initial state vector. The LAGEOS orbit adjustments converge twice as fast, and the same fit is obtained if the initial state vector is changed by several hundred meters.

The empirical parameter estimation is delicate due to the absolute magnitude of the model errors and the sparseness of the SLR tracking data. The estimation of a realistic solar radiation pressure scaling coefficient (with solutions ranging between 0.8-1.2) is not possible without an a-priori constraint due to the atmospheric drag and gravity modeling errors, which are orders of magnitude larger. Without a constraint, in the form of an a-priori variance (σ=0.1) of the estimate, unrealistically large or even (physically impossible) negative values are obtained. The drag scaling coefficients have been estimated every 12, 8 or 6 hours, depending on the tracking data availability and distribution over the day. A 1/rev acceleration parameter, absorbing errors of both gravitational and nongravitational origin, has to be estimated if one wants to obtain a precise orbit with an average orbit fit of the SLR observations of better than 40 cm.

Unfortunately the GFZ-1 tracking is not dense: there are often days with less than 4 tracked passes, as well as days without any observation at all. The importance of the empirical parameters is revealed in such instances: extrapolation of even a single day to the next available tracking data epoch provides initial state-vector elements that result in a diverging orbit adjustment. The drag coefficients have to be estimated every 6 hours if possible in order to correct for modeling deficiencies (although they do not represent density model error only; they also contain gravitational signal). They have to be constrained as well using an a-priori variance,
7 Orbit Determination

although not for the same reason as the solar radiation pressure scaling coefficient. This time the constraint ($\sigma=0.01$) counters the effect of an estimation without or with few tracking data, allowing the estimation of more parameters than the data make possible. This does not mean that 4 drag coefficients per day may be estimated for every orbit; it only guarantees that if 1 or 2 days of a specific 4-day orbit have less observations than the other 3 or 2, it is still possible to estimate all coefficients based upon the better-tracked days.

The 67 4-day orbits over a period of more than 2 years have been obtained by selecting always 4 consecutive days with at least 5 passes tracked from a minimum of 2 continents. This has been done in view of the orbit accuracy, which otherwise may not be satisfactory (orbit overlap tests exhibiting along-track differences of hundreds of meters). The low orbit makes the satellite a difficult target to acquire for an SLR station, and typically yields passes of less than 5 minutes length. An example of the distribution of the observations for a typical arc is given in Appendix F, Figure f1, which illustrates the data density of a GFZ-1 orbit. This may be compared to a typical LAGEOS-2 orbit, shown in Figure f2. The data selection and best parameterization (with the right constraints) for each orbit required a large effort. Conversely, a LAGEOS orbit adjustment may be performed at any particular date using the parameterization given in Section 7.2.2.

7.3 Discussion

The orbit computation results (in instantaneous orbital or rectangular elements) described in the last 4 subsections demonstrate the high dependence of the quality of the orbit fit on altitude. The highest-orbiting satellites, LAGEOS-1 and –2, have the best orbit fit (6.8 and 3.8 cm rms-of-fit on average, respectively) thanks to the attenuation of the gravitational perturbations and the very small effect of atmospheric drag. GFZ-1, on the other hand, has an average rms-of-fit of 24 cm due to the error in the drag (atmospheric density) modeling, as well as due to inaccurately modeled terrestrial gravity perturbations.

The orbit fits obtained for the four geodetic satellites do not have the highest precision one can achieve. This is mainly due to the parameterization of the orbit, which is not high (1/rev terms are only estimated for GFZ-1, for example), but also due to the data editing procedure, which was not very stringent. The rms-of-fit of the LAGEOS orbits, for example, is presently at the 1 cm level. However, the filtering procedure (Chapter 3) that is employed in order to compute the observed mean elements, using the ephemerides presented in this chapter as input, is very robust with respect to orbit fit: the rms-of-fit must be of the order of meters before their accuracy decreases significantly. This has been verified by filtering increasingly inaccurate orbits, represented by an increasing rms of the SLR orbit fit. Even orbits at the 50 cm rms level (which is bad indeed for a LAGEOS-1 orbit) yielded the same mean observed elements at the 0.1 mm level.
8 Long Arcs and Geophysical Parameter Estimation

In Chapter 1 it has been argued that long-period and secular signals may be recovered best using dynamically-consistent long arcs. In the Chapters 2 up to 5 it has been demonstrated how the averaged equations of motion enable the computation of long arcs with negligible numerical error and small errors due to the averaging process. Chapter 7 discussed the arcs issued from precise orbit computation that were filtered to obtain the ‘mean observed elements’. Using these observations in long-arc adjustments allows the estimation of the aforecited parameters, which may next be used in geophysical studies, for example to constrain mantle viscosity [Ivins et al., 1993] or sea-level rise due to global warming [James and Ivins, 1997].

The results of the long-arc computations and geophysical parameter estimation will be presented for the satellites Starlette, LAGEOS-2 and LAGEOS-1. The observed mean elements (number and origin) are presented in Section 8.1. The results of the mean orbit fit in terms of the rms of the residuals (CODIOR long arc - observed mean elements), obtained with satellite-specific force models, are given in the second section. Certain geophysical parameters have been estimated, the choice of which depends mainly on the length of the orbit and the temporal density of the observed mean elements. Multi-satellite solutions enable the estimation of parameters that, due to their high correlation with other parameters for one satellite, cannot be estimated in single-satellite solutions. Section 8.3 starts with a review of the geophysical phenomena that give rise to the observed orbit perturbations and the difficulties in estimating their amplitudes, followed by their estimations. Conclusions end this chapter.

8.1 The ‘Observations’: Mean Observed Elements

The observations used in the long-arc computation are mean observed elements, and these are obtained following the procedure given in Diagrams 3.1 and 3.2. The choice of the arc-length in view of the perturbations to be filtered, resonance in particular, is explained for the satellites Starlette, LAGEOS-1 and LAGEOS-2. The time-coverage and number of observations of each resulting data set of mean observed elements is also given.

Starlette is the oldest passive spherical SLR target, covered with cube corner reflectors (see Figure 1.1), launched in February 1975. The orbit and satellite characteristics were given in Table 1.1. The orbit plane precesses through one cycle with respect to the true-of-date equinox in 91 days in the retrograde sense (i.e. clockwise as seen from the North Pole). The period of the
8 Long Arcs and Geophysical Parameter Estimation

Argument of perigee is 109 days, moving in a prograde sense. The satellite altitude (800-1100 km) attenuates the effect and hence errors of the very high-degree terms of the gravity field (>70), but the atmospheric drag force, always difficult to model, is considerable. The LAGEOS-1 spacecraft was launched in May 1976 and is basically a larger and heavier version of Starlette. LAGEOS-2, launched in October 1992, is a copy of LAGEOS-1. Its orbital inclination of 52.6º has been chosen to allow optimal SLR tracking from Europe. Their orbit and satellite characteristics were given in Table 1.1. Their orbit planes precess through one cycle with respect to the true-of-date equinox in +1051 days (prograde) and −570 days (retrograde) for LAGEOS-1 and LAGEOS-2, respectively. In addition, the period of the argument of perigee is -1695 and +823 days for LAGEOS-1 and LAGEOS-2, respectively. The high altitude (approximately 5900 km) attenuates the errors of the high-degree gravity field coefficients, and the drag due to the neutral atmosphere is so small that it may be modeled as a constant, empirical along-track force.

The observed mean elements are obtained by filtering orbits of medium length that have been adjusted to SLR observations. As was discussed in Chapter 7, the GINS software of the GRGS/Toulouse, used in the construction of the GRIM4-S4 gravity field model [Schwintzer et al., 1997] and presently for GRIM5 [Biancale et al., 2000], has been used to adjust these orbits. The filter configuration is a function of the satellite orbit, since the \( n/\text{rev} \) and \( m/\text{daily} \) effects (eq. (2.21)), and resonance in particular, have to be removed as completely as possible from the orbits expressed in osculating elements. The former two perturbations pose no problems to the filter employed in this study (which requires a filtering window at least 8 times larger than the cut-off period), since they are always shorter than 1.1 day. An arc length of 10 days with a numerical cut-off period of 1.25 days would suffice in this case. However, this is not realistic, since all orbits are resonant to some level. The resonant perturbations for Starlette and the LAGEOS satellites have been determined using Kaula’s theory [1966], by successive calculation of the amplitudes for a given degree and order. Resonance amplitudes for Starlette exceeding 1 cm for \( a, e \) and \( i \), 10 cm for \( \Omega \), and 1 m for \( \omega \) and \( M \) have been computed. For the LAGEOS pair, the minimum resonance amplitudes were 1 cm for \( a, e \), and \( i \), 5 cm for \( \Omega \), 0.3 m for \( \omega \) and 1 m for \( M \). These criteria are based on the expected amplitudes, which are at least an order of magnitude larger, of the geophysical signals (Appendix C) that will be estimated in Section 8.3 as well as the precision of the mean nongravitational force model (Section 5.6.3). The results are presented in Table 8.1 for a computation up to degree and order 70 using the EGM96 gravity field [Lemoine et al., 1998].
8.1 The 'Observations': Mean Observed Elements

Table 8.1: resonant orders and degrees with corresponding periods in days using EGM96.

<table>
<thead>
<tr>
<th>Starlette</th>
<th>LAGEOS-1</th>
<th>LAGEOS-2</th>
</tr>
</thead>
<tbody>
<tr>
<td>order $m$</td>
<td>degree $n$</td>
<td>period</td>
</tr>
<tr>
<td>13</td>
<td>13-40</td>
<td>1.5-1.6</td>
</tr>
<tr>
<td>14</td>
<td>14-44</td>
<td>2.6-3.0</td>
</tr>
<tr>
<td>27</td>
<td>28-51</td>
<td>3.2-3.7</td>
</tr>
<tr>
<td>28</td>
<td>30-49</td>
<td>1.3-1.4</td>
</tr>
<tr>
<td>40</td>
<td>47</td>
<td>1.1</td>
</tr>
<tr>
<td>41</td>
<td>46-61</td>
<td>11-23</td>
</tr>
</tbody>
</table>

The resonance cannot be entirely removed from the orbits because of practical considerations. The filter requirement of a window eight times larger than the period of the perturbation would impose an arc length of $8 \times 23$ days = 184 days for Starlette for example. The filtered Starlette orbits were 24 days in length, thus removing the strongest order 13, 14 and 28 resonances almost completely (up to 2.8 m on the eccentricity and inclination) and the order 27 perturbation for the major part, as was shown in Section 4.2.1. The order 27 resonance is still important, with maximum amplitudes of 10 cm on the semimajor axis, 30 cm on the eccentricity and 50 cm on the inclination. The high accuracy of the EGM96 gravity field limits the mismodeling to less than one percent of those amplitudes, taking into account that the largest part of the resonance is filtered analytically using Kaula’s theory. The order 41 resonance is small, causing perturbations on the perigee and mean anomaly of a few meters, and is well modeled by EGM96, in which the Starlette contribution is the most important (F. Lemoine, private communication).

For practical considerations the LAGEOS-1 arcs were selected to be 24 days in length also, assuring the complete filtering of the order 6, 7 and 12 perturbations (up to 1 m on the semimajor axis, 1.8 m on the eccentricity and 3.4 m on the inclination, and 440 m on the perigee). The order 13 resonance is weak, being 2.2 m on the argument of perigee, and is filtered analytically. Since the order 13 resonance effect with a period of 9 days on the LAGEOS-2 orbit of 90 cm on the perigee is removed by the analytical filter, an orbit length of 20 days suffices for the numerical filter to remove the resonances due to the orders 6 and 7.

The filtering of 64 Starlette orbits between November 1983 and December 1996 provided 384 observed mean elements (64 observations for each mean orbital element). The 486 computed LAGEOS-1 mean observed elements over a period of 5½ years ending in July 1997 have been obtained by filtering orbits with an average rms of the SLR fit of 6 cm. A second set of 822 observed mean elements, computed in 1994 by Y. Boudon of CERGA/Grasse, has been obtained.
by filtering less accurate orbits. These orbits, from July 1980 through December 1991, have an rms-of-fit at the 15 cm level. The filtering procedure is very robust with respect to orbit precision, as was argued in Section 7.3. This older set of observations spans 1 ¹/₂ years, from July 1980 until December 1991. The 498 observed mean elements computed for LAGEOS-2 cover the period January 1992 until June 1997.

8.2 Long-Arc Computation

The CODIOR continuous, dynamically-consistent long arcs (one for each satellite) have been adjusted to the observed mean elements. The accuracy, given as a bias and an rms about mean (which equals the rms when the bias is small) of the residuals, with which this is achieved validates the mean force modeling and allows the estimation of specific geophysical parameters. The results of the long-arc computations are presented here for each satellite separately.

8.2.1 Starlette

The long arc has been fitted to the observed mean elements by estimating the initial state vector, 2 to 6 drag scale coefficients per year, and a single solar radiation pressure scaling coefficient. The adjusted long arc represents the mean motion of Starlette over a period of thirteen years, including the solar cycle 22 maximum in 1990. The adopted dynamical model is presented in Table 8.2. A geometrical model is absent in CODIOR, since the observed mean elements are expressed in the same variables as those in which the equations of motion have been expressed.

By estimating or fixing certain parameters, its mean effect on the orbit may be evaluated over a very long time. Using a classical numerical integration, this effect may not be visualized, simply because the effect in question is too small over the adjustment period (24 days, for example), or because the short-period perturbations mask the long-period signal.

A first application of the long arc is to evaluate a density model's performance over a long period of time, more than a solar cycle. This has been done by adjusting only the initial state vector to the observed mean elements, and thus not correcting for density model errors by adjusting drag scale coefficients as well. Figure 8.1 shows the residuals of this adjustment employing the DTM94 atmospheric density model [Berger et al., 1998], amongst others. The atmospheric drag, which is proportional to atmospheric density, causes the semimajor axis to decrease. Thus, the quality of the drag modeling can be evaluated by analysis of the semimajor axis residuals. The absolute value of the last residual (2 m; see Figure 8.1), or the largest residual (5.5 m), divided by the total decay of the semimajor axis (377.7 m), reflects the density model accuracy. This establishes the DTM94 long-term accuracy at 0.5% to 1.5 % (2 m or 5.5 m / 377.7 m, respectively), for the last or the largest semimajor axis residual, respectively.
Table 8.2: mean dynamical model for the Starlette long-arc adjustment.

<table>
<thead>
<tr>
<th>Dynamical model:</th>
</tr>
</thead>
<tbody>
<tr>
<td>Earth gravity</td>
</tr>
<tr>
<td>third bodies (Moon, Sun and 5 planets)</td>
</tr>
<tr>
<td>Earth tides</td>
</tr>
<tr>
<td>ocean tides</td>
</tr>
<tr>
<td>atmospheric drag</td>
</tr>
<tr>
<td>solar radiation</td>
</tr>
<tr>
<td>albedo (visible+IR)</td>
</tr>
<tr>
<td>relativity</td>
</tr>
<tr>
<td>area-to-mass ratio</td>
</tr>
<tr>
<td>reflectivity visible/IR</td>
</tr>
<tr>
<td>aerodynamic coefficient</td>
</tr>
<tr>
<td>precession/nutation</td>
</tr>
<tr>
<td>integration, step-size</td>
</tr>
</tbody>
</table>

Figure 8.1: the semimajor axis residuals of a Starlette long arc without estimating drag scale coefficients, employing three different atmospheric density models.
This high accuracy using a mean force model is in sharp contrast with the short-term (less than an orbital revolution) accuracy, as the estimated drag coefficients using GINS have shown: their scatter is 23% as was stated in Section 6.2.1. However, it should be stressed that this (good) result is highly dependent on the type of orbit, and that the total decay was small. It is for example possible that DTM94 will not perform as well for a polar orbit, or a very low one. What is important, though, is that the evolution of the semimajor axis residuals reveals no drift, just an offset and a step during solar maximum. This is not the case using MSIS86 [Hedin, 1987], which is less accurate than DTM94 when the largest residual is taken into consideration, as may be seen in Figure 8.1. Because the residuals drift secularly (14.8 m), this is also the last residual. This is probably due to a more accurate helium modeling of DTM94 (helium is the principal constituent at Starlette altitude under low solar flux conditions) than that of MSIS86, since the drift is small during low solar flux. This may have consequences when the arc is longer, or when the experienced drag force is stronger in case of a larger satellite area-to-mass ratio. Lastly the DTM78 model [Barlier et al., 1978] has been tested. This predecessor of DTM94 has been constructed without high solar activity data, resulting in the secular drift of the semimajor axis residuals during the solar cycle maximum of approximately 66 m as shown in Figure 8.1.

The drag force modeling may also be evaluated by investigating the effect of the employed aerodynamic coefficient. Basically, only two models are available: Cook's theory (Section 5.3), which takes surface and environmental parameters into account, and theories adapted for spherical satellites only, where the aerodynamic coefficient is a function of altitude only (for example [Afonso et al., 1989]). Using this type of simple models, which provide values approximately averaged over the solar cycle, causes large errors. Starlette’s aerodynamic coefficient, shown in Figure 5.2, varies between 2.55 (high solar flux, and thus a very hot atmosphere) and 2.95 (for flux at the minimum of the cycle) using Cook’s formulation given in eq. (5.6), and this has a direct effect on the computed drag, as may be seen in eq. (5.5). Using the altitude-dependent only $C_D$ values (2.74-2.75) resulted in a secular trend in the semimajor axis residuals of 18 m during solar maximum, corresponding to an error of 4.8%. Of course, by adjusting drag scale coefficients these errors will be absorbed, but this will not be possible when tracking data are not available. Cook developed his model independently of Starlette observations, and the good results obtained with his model prove the validity of this formulation, at least for spherical satellites.

Satellite mean motion depends solely on the zonal gravity coefficients (and resonance), whose even terms induce secular variations on the ascending node and the argument of perigee. The residuals of these angle variables constitute a measure of accuracy of the zonal coefficients of the gravity field employed in the adjustment. A necessary condition for this type of evaluation to be valid, is the precise and unbiased modeling of the semimajor axis, which otherwise will
8.2 Long-Arc Computation

Introduce secular trends on the angle variables. This boils down to a precise adjustment of drag scale coefficients, of which 2 to 6 per year have been estimated. Their mean value is 1.01, with a scatter of 0.10 (minimum value 0.80, maximum value 1.28), mainly due to variations during high solar activity. This evaluation has been performed with EGM96, the residuals of which are presented in Figure 8.2, where, compared to Figure 8.1, drag coefficients have been adjusted. The rms about mean of the semimajor axis residuals is 6.7 cm over an arc with a length of 13 years. A prior adjustment using GRIM4-S4 [Schwintzer et al., 1997], with an equally precise semimajor axis modeling, had a total perigee drift error of 3200 m. No drift was detected in the present adjustment, owing to the higher accuracy of the EGM96 gravity field model, or at least of its zonal coefficients.

The eccentricity residuals in Figure 8.2 drift during the solar maximum (1989-1992) due to unaccounted-for coupling effects in the mean model, in accordance with the simulation results presented in Section 5.6.2. There is, however, also a quasi-periodic signal visible, which is mainly due to ocean tide modeling error, as well as unmodeled (in CODIOR) air mass redistribution and precipitation (gravitational) effects.

The noisy residuals on the inclination are due to ocean tide model errors. The residuals diminish by estimating the $K_1(2,1)$ coefficients from 45 cm to the 34 cm rms shown in Figure 8.2. Estimating other ocean tide coefficients did not improve the inclination modeling, and produced very unreliable estimates. This is conceivable after studying the third frame from the top of Figures c3 and c4 of Appendix C, which show the perturbations on the inclination due to the diurnal and semidiurnal tides; the period of most of these tides is too small to be estimable with only 4 observations per year, which is the case here. Since the geophysical signals that will be estimated in this chapter may be recovered primarily by means of an accurate ascending node modeling, the inclination accuracy achieved is satisfactory.

The ascending node and the argument of perigee residuals (the latter are not shown in Figure 8.2) also display quasi-periodic signals. These are also due to ocean tide modeling error and the unmodeled air mass redistribution and precipitation. Although not visible in the residuals, mismodeling due to errors in the very long-period tides as well as in the secular rate of the zonal gravity coefficients is present. The residuals representing the sum of the argument of perigee and the mean anomaly (not shown; these variables are not considered in the analysis) display large linear drifts, due to offsets in the semimajor axis modeling. These drifts of 24 km maximum (during solar maximum activity) over a period of 700 days correspond to an error of 2.4 m per revolution. Due to the significant value of this error, the mean anomaly observations are downweighted in the adjustment and play an insignificant role in the least-squares estimation algorithm.
Figure 8.2: the residuals of the final Starlette long arc (state vector, ocean tide coefficients, drag scale coefficients and a solar radiation pressure scaling coefficient estimated) [Bruinsma and Exertier, 1997].
8.2 Long-Arc Computation

8.2.2 LAGEOS-2

A first LAGEOS-2 long arc with a length of 4½ years has been fitted to the observed mean elements by estimating the initial state vector only, or a so-called nominal adjustment. The adopted mean dynamical model is presented in Table 8.3, in which only the satellite-specific surface forces are set to 0 in the nominal adjustment. In this case, the long-arc residuals essentially reflect the effects of photon thrust and neutral and charged particle drag (semimajor axis decay), mismodeled solar radiation and albedo (eccentricity and argument of perigee), as well as errors due to certain ocean tide constituents (inclination and ascending node).

Table 8.3: mean dynamical model for the LAGEOS-2 long-arc adjustment.

<table>
<thead>
<tr>
<th>Dynamical model</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>Earth gravity</td>
<td>EGM96 up to degree 30 [Lemoine et al., 1998]</td>
</tr>
<tr>
<td>third bodies (Moon, Sun and 5 planets)</td>
<td>Bureau de Longitude ephemerides</td>
</tr>
<tr>
<td>Earth tides</td>
<td>due to Moon and Sun, degree 2 and 3, Wahr terms</td>
</tr>
<tr>
<td>ocean tides</td>
<td>Schwiderski (up to degree 10, order 2) [Schwiderski, 1980]</td>
</tr>
<tr>
<td>charged and neutral particle drag</td>
<td>constant empirical along-track acceleration</td>
</tr>
<tr>
<td>solar radiation</td>
<td>solar constant $4.5605 \times 10^{-6} \text{ Nm}^{-2}$, shadow function</td>
</tr>
<tr>
<td>albedo (visible+IR)</td>
<td>analytical function</td>
</tr>
<tr>
<td>relativity</td>
<td>Schwarzschild correction [McCarthy, 1992]</td>
</tr>
<tr>
<td>area-to-mass ratio</td>
<td>0.00069287 $\text{m}^2/\text{kg}$</td>
</tr>
<tr>
<td>satellite-specific surface forces</td>
<td>Yarkovsky, Yarkovsky-Schach, anisotropic reflectivity</td>
</tr>
<tr>
<td>reflectivity visible/IR</td>
<td>1.14 / 1.11</td>
</tr>
<tr>
<td>precession/nutation</td>
<td>w.r.t. J2000 inertial reference frame, analytical model</td>
</tr>
<tr>
<td>integration, step-size</td>
<td>$10^{th}$ order Adams-Bashforth-Moulton multi-step, 12 hr.</td>
</tr>
</tbody>
</table>

The observed decay of the semimajor axis over a period of 4½ years is approximately 1.35 m. It is attributable to the along-track (AT) component of the ensemble of nongravitational forces acting on the satellite, the orbital signatures of which are shown in Figures e3-e5 of Appendix E. A large periodic perturbation on the eccentricity is due to mismodeled solar radiation and albedo, because a scaling coefficient was not adjusted. A large perturbation on the inclination with a period of approximately 560 days is due to mismodeling of the $K_1$ tidal constituent. This diurnal tide constituent has a total amplitude of 3.9 m (on the inclination) using Schwiderski’s coefficients,
8 Long Arcs and Geophysical Parameter Estimation

but recent models (i.e., FES95.2 [LeProvost et al., 1994]) indicate that these coefficients are in error of up to 20%. The ascending node residuals present the typical parabolic signature with time due to the not properly restituted semimajor axis, the decay of which was not modeled in this nominal adjustment. This nominal adjustment serves as a reference, since it enables to quantify the effect of the nongravitational force model and parameter estimations on the orbit fit when it is compared to a complete adjustment, which takes all perturbations into account.

As was stated before, an accurate semimajor axis modeling is crucial for the restitution of the angle variables. The estimated along-track accelerations issued from the same GINS precise orbit computation that provided the input orbits for the filtering procedure have been used first directly as ‘observed’ accelerations in CODIOR. These were estimated every 10 days as a constant value, as was described in Section 6.2.2. These ‘observed’ AT-accelerations empirically assure the quality of the semimajor axis modeling of the long arc, since they contain the yet unmodeled (thermal and atmospheric) drag perturbation. The post-fit residual bias and rms about mean were 0.3 cm and 1.9 cm, respectively, for the semimajor axis (not shown). The unmodeled accelerations causing the drift present in the semimajor axis residuals of the nominal adjustment have been completely reproduced by the AT-observations. However, some major peaks were still distinguishable in the residuals, corresponding to large unmodeled AT-accelerations. These peaks in the semimajor-axis residuals became larger after smoothing the AT-accelerations, using a 30-day averaging window, implying that they are not constant over 10 days. Based on this information, the AT-accelerations have been estimated at a higher frequency in the 20-day orbit adjustments with GINS, notably once per 5 days. When these accelerations were applied in the long arc, the residual bias and rms (about mean) became 0.02 cm and 1.5 cm, respectively, for the semimajor axis (not shown). This result may be interpreted as a confirmation of the high accuracy of the along-track estimations by GINS, because otherwise the semimajor-axis residuals of the LAGEOS-2 long arc would still drift secularly and the rms-of-fit would be much larger. Angle variable residuals have a parabolic signature when the modeled semimajor axis drifts secularly, and are secularly drifting when the modeled semimajor axis has a systematic offset (bias). When the AT-observations were applied, only signatures of yet unmodeled geophysical perturbations show on the residuals of the angle variables.

The empirical AT-accelerations have been used in CODIOR for two reasons: to verify the accuracy of the estimated empirical estimations, but mainly in order to evaluate the quality of the mean nongravitational force model. Since the LAGEOS-2 orbit fits, realized with GINS and presented in Section 6.2.2, are good (3.8 cm rms on the average), the estimated empirical accelerations are probably accurate. This has been confirmed by the CODIOR long arc as well as by comparison to estimates of [Ries et al., 1997] of CSR/University of Texas. The value of 1.5 cm
8.2 Long-Arc Computation

rms (about mean) of the semimajor-axis residuals using the CSR AT-accelerations may be used as a reference value for the long-arc adjustment using the complete model that is presented next.

Modeling subsequently a solar radiation pressure scaling coefficient, the Yarkovsky and Yarkovsky-Schach effects, anisotropic reflection, degree 2 $K_1$ and $K_2$ tide coefficients and annual and semiannual variations in $J_2$ (or equivalently for the adjustment, the annual and semiannual solar tide coefficients), ultimately led to the best-fitting long arc. The spin-axis of LAGEOS-2 is still stable and easy to model, since it has not been in space yet for a long time. Here an analytical model from D. Vokrouhlický was used (an improved version of the model of Farinella et al. [1996]), based on magnetic and gravitational torques acting on the satellite. The long-arc adjustment results, presented in Figure 8.3, and the ‘observed’ AT-accelerations (top and middle frames) and modeled AT-components of the mean nongravitational accelerations (bottom frame) described in Section 5.5 are presented in Figure 8.4. The observed AT-acceleration serving as reference, this figure shows that the large perturbations are well modeled, but that some smaller spikes are not present. These smaller spikes are probably noise for a large part, since the semimajor-axis modeling is at the level of 0.9 cm rms about mean for this long arc (top frame of Figure 8.3), compared to 1.5 cm using the observed AT-accelerations. Thus, for this LAGEOS-2 long arc, the nongravitational model performs better than using the observed AT-accelerations.

The modeling of a decreasing reflectivity, visible in the GINS estimations of the reflectivity coefficient (Figure 7.7) and observed by Ries et al. [1997] too, did not improve the fit on the eccentricity, nor perigee. The rms-of-fit of the eccentricity residuals is 26 cm, which still display small signatures. These are due to the combined effects of errors in the nongravitational model and unmodeled temporal variations in the odd-degree gravity field mainly. The adjustment of the $K_1$ and $K_2$ ocean tide constituents improved the inclination modeling considerably (32 cm rms-of-fit, compared to 72 cm without adjustment), but an unknown periodic signal remains. Fortunately, the inclination is not a key variable for this study since it does not contain significant information on the geophysical parameters that will be estimated.

The ascending node residuals (74 cm rms-of-fit, or 0.8 mas; Figure 8.3) still show signature, despite the estimation of the annual and semiannual gravity field coefficients of degree 2 as well as selected ocean tide coefficients. Half a period of the 9.3 year tide, which has not been estimated because of the insufficient arc length, seems to be present as well. The argument of perigee residuals are better than expected (not shown). A remaining quasi-periodic perturbation is not associated with shadowing events, and is probably due to tide modeling errors. The mean anomaly residuals are well restituted for LAGEOS-2 (rms-of-fit of 54 m; not shown); however, this variable will in the following no longer be discussed because it plays no role due to the very small weight it is assigned to in the adjustments.
8 Long Arcs and Geophysical Parameter Estimation

Figure 8.3: the residuals of the final LAGEOS-2 long arc (state vector, seasonal variations, ocean tide, constant drag and a solar radiation pressure scaling coefficient estimated) [Bruinsma and Exertier, 1997].
8.2 Long-Arc Computation

Figure 8.4: the estimated along-track acceleration (GINS) every 5 days (top), or 10 days (middle), and the modeled one ('Total': solid line) in CODIOR (bottom) in pm/s². The along-track acceleration components are the Yarkovsky (Y: dotted line) and Yarkovsky-Schach (YS: dash-dotted line) thermal drag effects, anisotropic reflection (An: large-dashed line) and atmospheric drag (D: dashed line).
8 Long Arcs and Geophysical Parameter Estimation

8.2.3a The LAGEOS-1 Long Arc over 5½ Years

Analogous to the procedure applied to LAGEOS-2 and using the same dynamical model (Table 8.2), a nominal long arc has been adjusted without employing the satellite-specific nongravitational force model, estimating only the initial state vector. The decay of the semimajor axis over a period of 5½ years (1992-1997.5) is approximately 2.20 m, attributable for the major part to (not modeling of) satellite thermal thrust (the Yarkovsky effect), and for a smaller part (not exceeding 35%) to neutral and charged particle drag. Judging by the semimajor axis decay, these forces are 33% stronger than in case of LAGEOS-2. A large periodic perturbation on the eccentricity is due to mismodeling of solar radiation and albedo effects (Appendix E). A perturbation on the inclination with a period of approximately 1000 days is due to mismodeling of the \( K_1, S_2 \) and \( K_2 \) tidal constituents. The \( K_1 \) diurnal tide and \( K_2 \) semidiurnal tide constituents required adjustment in the case of LAGEOS-2 as well. The ascending node residuals have a parabolic signature, since the semimajor axis is not properly restituted (unbiasedly in particular) in the nominal adjustment.

Applying the CSR/University of Texas (courtesy of J. Ries) AT-accelerations instead of a nongravitational model resulted in a bias of -1.3 cm and rms (about the mean) of 1.7 cm, a confirmation of their accuracy. A second set of empirical accelerations, provided by J.M. Lemoine of GRGS/Toulouse, spanning the period August 1985 to August 1996, does not entirely cover the 5½ year long arc. The empirical accelerations (constant plus 1-cpr) are decomposed in the along-track and Bx and By directions (in the plane of the orbit in the direction Center of mass of the Earth-ascending node and perpendicular to that, respectively). This judiciously chosen satellite reference frame allows the absorption of the thermal drag and radiation pressure forces, which do not only have a tangential component. This 8 months shorter long arc (January 1992-August 1996) has the smallest residuals indeed, and this time the eccentricity and perigee modeling benefitted as well. The bias and rms about mean of the semimajor axis residuals are 2.4 cm and 0.5 cm, those for the eccentricity being 2.1\( \times 10^{-9} \) and 3.5\( \times 10^{-8} \) (corresponding to 2.6 and 42.9 cm), and those of the perigee being 3.2\( \times 10^{-7} \) and 1.23\( \times 10^{-5} \) rad (corresponding to 3.93 and 151 m), respectively. Figure 8.5 displays the residuals for this adjustment, where a constant along-track acceleration has been adjusted as well to correct for an otherwise secularly-drifting semimajor axis. This hints at correlation between the Bx and By components and the AT-component (all three were estimated in GINS and applied in CODIOR), which projections are not orthogonal. However, the empirical Bx and By accelerations do not only absorb the unmodeled nongravitational forces, but the non-static part of the odd zonal gravity field harmonics as well (secular, annual and semiannual variations). Using these accelerations is thus only valid when the most accurate orbit is required, as is the case for reference system computation. The empirical accelerations have been estimated in the
When applied in the long-arc computation, they make an accurate estimation of odd-degree zonal gravity field coefficients impossible. The even coefficients can be estimated, since these depend on

**Figure 8.5:** the residuals of the LAGEOS-1 long arc in which the nongravitational accelerations in the AT, Bx and By directions were provided by J.M. Lemoine (GRGS/Toulouse).
the accuracy of the ascending node modeling. Unfortunately, the data set does not span a sufficient
time interval to accurately recover the very long-period tides and the secular variation in the even-
degree zonal gravity coefficients.

The complete nongravitational model does not perform as well as for LAGEOS-2, but this
is for a large part due to a less accurate spin-axis orientation model (which is not necessary when
applying empirical accelerations issued from precise orbit computation), in particular since the
beginning of 1997. An unexpected event took place at the beginning of 1997, which resulted in
very high along-track accelerations (represented by the large spike in Figure 8.9 at approximately
modified Julian date 50400). This event could be due to unstable behavior of the spin-axis, which
can no longer be modeled using the dynamical model from D. Vokrouhlický. Instead, an empirical
correction to the dynamical spin-axis motion [Métris et al., 1998], based on a fit to the nominal
excitations, had to be used in order for the nongravitational model to function more or less
properly in 1997 as well. The credibility of these (unexplained) empirical corrections is enhanced
by their good agreement with direct observations of the spin-axis orientation.

The determination of the amplitudes and phase lags of the nongravitational model proved
to be difficult and ambiguous. The 'best' nongravitational model appears to be different for the
semimajor axis and the eccentricity. Either the semimajor axis is biased by 1 to 2.5 cm, and has a
small rms, or the bias is less than 1 cm, but with a much higher rms. Nongravitational model
parameters from a previous long-arc adjustment (called 'PX'; [Exertier et al., 1995]) were used,
based on an older version of the mean motion software. Secondly, the model parameters published
by Métris et al. [1997] were used (GM), characterized by a large Yarkovsky-Schach amplitude
(three times larger than in Scharroo et al. [1991]) and phase lag. The third model parameterization
(SB; [Bruinsma and Exertier, 1997]) is the result of several iterations, mainly to find the optimal
phase lag of the Yarkovsky-Schach effect. The estimated values turned out to be close to those of
Slabinski [1997]. The three nongravitational force model parameterizations are given in Table 8.4.

<table>
<thead>
<tr>
<th>Table 8.4: Nongravitational model amplitudes and phase lags from three studies for LAGEOS-1.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Parameter</td>
</tr>
<tr>
<td>----------------------------------</td>
</tr>
<tr>
<td>Particle drag</td>
</tr>
<tr>
<td>Yarkovsky</td>
</tr>
<tr>
<td>Yarkovsky-Schach</td>
</tr>
<tr>
<td>Anisotropy</td>
</tr>
</tbody>
</table>
Figure 8.6: the residuals of the LAGEOS-1 long arc spanning 5½ years, employing three nongravitational model parameterizations, given in Table 8.4 (the statistical numbers, mean and rms about the mean, hold for the ‘GM’ parameterization).
For this particular 5½ year long-arc orbit, the model parameterization of Métris et al. [1997] ('GM' in Table 8.4) realizes the best orbit fit. Their predicted accelerations have the highest correlation (0.78) with the 'observed' AT-accelerations of CSR/University of Texas. The orbit adjustment residuals, using the amplitudes of the nongravitational forces as determined by Métris et al. [1997], are presented in Figure 8.6. Selected ocean tide coefficients, a solar radiation pressure force scaling coefficient, seasonal variations in the degree 2 gravity field coefficient, and a single along-track acceleration have also been estimated. The residuals obtained with the SB and PX parameterizations are shown only for those orbital elements that were restituted significantly different from those obtained employing the GM model parameters.

The mean and rms about mean of the semimajor axis residuals is 1.0 and 4.8 cm, respectively, with GM, as may be seen in the top frame of Figure 8.6. The corresponding numbers employing the SB model parameterization are 1.9 and 2.6 cm, and those with PX 2.0 and 2.9 cm. The rms (the bias is negligible) of the eccentricity residuals is 83 cm (with GM). The eccentricity residuals are relatively stable during the first 3 years and the last 2 years of this arc, but there is a drift of approximately 1.5 m starting in February 1995 over a period of 10 months. The results obtained with SB and PX are significantly less precise for the eccentricity, with rms' of fit of 128 and 143 cm, respectively. When these results are compared to Figure 8.5, the shortcomings of the nongravitational model (and/or the spin-axis model) become evident, in particular its AT-component, which is most important for the semimajor axis modeling. The semimajor axis residuals of the long-arc orbit computed with the empirical, observed accelerations have an rms about mean of 0.5 cm (Figure 8.5), while the best model result (with the SB parameterization) is 2.6 cm. The perigee residuals display a large ‘valley’ over the same interval of time as the drift in the eccentricity residuals. The origin of this modeling error has not been established. The best results are obtained with the GM model parameters, as may be seen in the bottom plot of Figure 8.6. The rms about mean is 101 m. The corresponding numbers obtained with SB and PX are 185 and 217 m, respectively.

The fit of the inclination and ascending node residuals is independent of the nongravitational model parameterization. The restitution of the inclination has improved (with respect to Figure 8.5) thanks to the adjustment of selected ocean tide coefficients ($K_1$, $S_2$ and $K_2$), from $5.7\cdot10^{-9}$ (70 cm) to $1.8\cdot10^{-8}$ rad (22 cm or 9 mas). The ascending node residuals have also benefited from the adjustment of the ocean tide coefficients, and their rms has nearly been divided by three to become $3.4\cdot10^{-8}$ (41 cm or 19 mas).
8.2 Long-Arc Computation

8.2.3b The LAGEOS-1 Long Arc over 17 Years

This long-arc orbit spans a period of 17 years, from July 1980 to July 1997, but with fewer mean observations per year prior to 1992. This smaller data density has no consequences for the estimation of very long-period signals, such as the 18.6 year tide, nor for the estimation of secular signals, such as the $J_n$. Annual, and semiannual signals even more so, will be more difficult to recover with this lower temporal resolution.

Again, a nominal long arc has been adjusted first, estimating only the initial state vector, mainly to display the impact of the nongravitational forces on the orbit. The semimajor axis decay over 17 years, shown in Figure 8.7, is nearly 8 m. The eccentricity (rms-of-fit: 1.71 m) is restituted more accurately before 1993 than after that date, indicating a change in the nongravitational forces acting on the spacecraft. The inclination residuals (rms-of-fit: 67 cm) present signature due to mismodeling of the $K_1$, $S_2$ and $K_2$ tidal constituents. The ascending node residuals have a parabolic signature, because the semimajor axis is drifting in the nominal adjustment.

To obtain an idea of how accurate the semimajor axis may be modeled using a complete nongravitational force model, the 'observed' along-track accelerations from CSR/University of Texas (courtesy of J. Ries) were applied. The resulting fit of the long arc is presented in Figure 8.8, where the 18.6 year signal on the ascending node is now visible in the residuals due to the accurate semimajor axis modeling. The rms-of-fit (about mean) of the semimajor axis residuals of this 17-year long arc is 1.8 cm. Analysis of the top frame of Figure 8.8 shows that significant errors are still present, in particular two 4 cm peaks and the drop of 4 cm at the end of the arc. The eccentricity residuals, with an rms-of-fit of 1.71 m, still present signatures due to unmodeled nongravitational and gravitational (odd-degree gravity field harmonics) forces. The AT-accelerations also do not improve the inclination modeling, which depends mainly on the tide model, as may be seen by comparing the third frames from the top of Figures 8.8 and 8.7.
Figure 8.7: the residuals of the nominal LAGEOS-1 long-arc adjustment (adjustment of the initial state vector only).
Figure 8.8: the residuals of the LAGEOS-1 17 year long arc when the along-track accelerations of CSR/University of Texas have been applied, instead of modeling small nongravitational forces.
The three nongravitational force model parameterizations (given in Table 8.4) that have been evaluated in the 5½ year long arc (Section 8.2.3a), have been evaluated over 17 years as well. The along-track component of the nongravitational accelerations of the three models is visualized in Figure 8.9, demonstrating the good agreement with the CSR 'observations'. The orbit residuals of this long arc are presented in Figure 8.10, where the same selected ocean tide coefficients, a solar radiation pressure (force) scaling coefficient, seasonal variations in the degree 2 gravity field coefficient, and a single along-track acceleration have been estimated, in accordance with the 5½ year arc. The results are not similar, since for this much longer time span the SB model parameters [Bruinsma and Exertier, 1997] perform best, while the GM [Métris et al., 1997] model parameters perform much worse. In the latter model, the semimajor axis suffers from the too high Yarkovsky-Schach amplitude, which is not sufficiently attenuated by the already large phase lag of 183.3º. A possible explanation of this less-accurate orbit reconstitution with the 'GM' model parameterization is its correlation with the employed spin-axis model; empirical corrections, based upon SLR observations from the middle of the eighties and nineties and that particular nongravitational model parameterization, were applied to it. This may have affected the spin-axis model for the beginning of the eighties, where the semimajor axis residuals are large with the GM parameterization, as may be seen in Figure 8.10. The correlation between the predicted and the 'observed' AT-accelerations, however, is still best with 0.78. The SB nongravitational model accelerations have a correlation coefficient of 0.75, those predicted with PX [Exertier et al., 1995] 0.74. Neither model is significantly biased with respect to the CSR/University of Texas along-track observations, the mean value of which over the 17 year arc is -3.50·10^{-12} m/s², compared to -3.54·10^{-12} m/s² for the three models.

The semimajor axis residuals have an rms-of-fit of 4.0 cm (top plot of Figure 8.10), which is better than that of the 5½ year arc. The better spin-axis modeling, and thus more accurate nongravitational force modeling, up to 1989 than after that date can have played a role here. It is also evident that the nongravitational force modeling is not at the performance-level as that of LAGEOS-2, since this time the long-arc fit realized with the CSR AT-accelerations (rms: 1.8 cm) is significantly more accurate than that obtained with the complete nongravitational model. The eccentricity residuals (second plot from the top of Figure 8.10) have an rms-of-fit of 1.19 m. They are relatively stable during the first 2000 days, but then they rise over a period of 2000 days, while the last 2000 days display a negative trend (of more than 5 m). The inclination residuals exhibit a larger scatter during the first 4000 days than they do during the last 2000 days, so where there are more observations per year. The mean of the residuals is 0.0 cm, the rms-of-fit is 37 cm.

The ascending node modeling and the estimation of the seasonal variation in J₂ result in an rms-of-fit of 58 cm. The residuals still display the signatures of unmodeled perturbations. One of
the reasons for this is that the estimation of a single seasonal signal for a period of 17 years is not realistic; the annual hydrology/snow cover and air mass redistribution perturbations (Figure d9 of Appendix D) are not constant. Secondly, the ocean tide modeling suffers from errors, and the adjustment of a single amplitude and phase for each of the 3 tidal constituents $K_1$ (estimation of the degree 2, order 1 coefficients only), $S_2$ (estimation of the degree 2, order 2 coefficients only) and $K_2$ (estimation of the degree 2, order 2 coefficients only) is not sufficient to absorb the total error due to that particular constituent (the sum of all degrees in the mean model). The argument of perigee, with an rms about mean of 175 m, equally suffers from the above-mentioned effects, as well as from inaccurate nongravitational force modeling.

Figure 8.9: the modeled along-track accelerations using the PX, GM (top) and SB (bottom) nongravitational model parameterizations (Table 8.4), and the 'observed' accelerations computed by CSR (bottom).
Figure 8.10: the residuals of the final 17 year LAGEOS-1 long arc (state vector, ocean tide coefficients, constant drag, a solar radiation pressure scaling coefficient and seasonal variation in $J_2$ adjusted) with the SB, PX and GM nongravitational model parameterizations (the statistical numbers, mean and rms, hold for the SB parameterization).
8.3 Temporal Gravity Variation and Very Long-Period Tides

The Starlette, LAGEOS-1, and LAGEOS-2 long arcs have been used to estimate several geophysical parameters, of which the secular variation in the low-degree zonal harmonics, their seasonal variation and the very long-period lunar tides are important for geophysical studies. These effects represent a global mass redistribution of the solid Earth-ocean-atmosphere system. The secular variation in the zonal harmonic coefficients is predominantly due to postglacial rebound, so due to mantle processes, as well as sea-level change, retreating glaciers, and snow mass changes in Greenland and at the poles [Vermeersen et al., 1998]. Geophysical models, such as ICE-4G [Peltier and Jiang, 1996], require an accurate determination of the secular variation in the zonals in order to constrain their solution. The seasonal variability with annual and semiannual periods is the result of meteorological and hydrological effects, as well as of the solar annual and semiannual ocean tides. This variability may be and is commonly expressed in terms of seasonal variation in $J_2$ and $J_3$, or as effective $S_a$ and $S_{sa}$ tide coefficients. Values from Schwiderski [1980] have been used in this study for comparison. The 18.6 year lunar tide estimate, assuming the ocean tide is in equilibrium or very near, yields information about the anelasticity of the mantle [Zhu et al., 1996]. It is not clear what signal is contained in the large estimates of the 9.3 year tide coefficients, which have been estimated with LAGEOS-1 and Starlette.

Due to high correlation, several of the parameters cannot be estimated accurately with the mean motion model using a single satellite. This was the case for $J_2$ and the 18.6 year tide coefficients (a correlation of -0.9), and between $J_2$ and $J_4$ or any higher even-degree coefficient (they all induce a quadratic effect on $\Omega$). The (lumped) seasonal variations in $J_2$ and $J_3$ could be estimated in single-satellite solutions, except for the semiannual signal with Starlette because of the small number of mean observed elements per year.

The multi-satellite solutions have been obtained by computing and saving the observation equations for each satellite before the first iteration in the usual orbit adjustment procedure, and subsequently accumulating these equations in a single observation matrix. This matrix contains satellite-dependent columns (the initial state vectors, constant drag terms and solar radiation pressure scaling coefficients), and satellite-independent columns (the geophysical parameters to be estimated). This is a dynamical solution method solving for consistent arc parameters and geophysical parameters simultaneously. As has already been explained, the solutions from CSR are also based on solving for the geophysical parameters from a single long-arc orbit ([Cheng et al., 1989], [Eanes and Bettadpur, 1994]), or from consecutive yearly arcs [Cheng et al., 1997]. However, these long arcs have been integrated in instantaneous orbit variables and fitted to SLR data directly, which demands much CPU time. Another method consists of estimating monthly values of $J_2$ and $J_3$, the solutions of which vary from month to month due to their time-varying...
components. The secular and periodic variations have subsequently been determined from time series of these monthly solutions ([Nerem et al., 1993]; [Gegout and Cazenave, 1993]; [Cazenave et al., 1996]; [Deviòt et al., 2001]). This method is not successful in accurately determining the rates of change in the odd zonal harmonics, since unmodeled nongravitational forces also contaminate the monthly solutions. It is the easiest method, since orbits of up to a month are used, which does not require special algorithms in the orbit integration software. However, for this method to yield accurate results, the contributing monthly solutions, and thus the orbits, must be sufficiently accurate to be able to separate the signals from the noise. Since this method does not benefit from the cumulative perturbing effects on the orbit as do consistent long arcs, the signal-to-noise ratio in the estimation is lower because each individual orbit solution may absorb and hence obscure part of the geophysical signal, for example in the initial state vector.

The origins and effects on the orbit of the time-varying low-degree zonal harmonics will be described in the subsections 8.3.1 and 8.3.3, while the very long-period lunar tides are discussed in subsection 8.3.2. Recently published values are presented to give an impression of the amplitude of these parameters, as well as their scatter. Sections 8.3.4 to 8.3.6 present the estimates obtained for these parameters in this study.

8.3.1 Secular Variations in Zonal Harmonics

The static gravitational potential is customarily represented as a spherical harmonic expansion in the constant $C_{nm} / S_{nm}$ Stokes coefficients, as in eq. (2.2). The zonal coefficients $J_{n0}$ ($= -C_{n0}$ for un-normalized coefficients) of degree $n$ are a function of the Earth's mass density distribution, $\rho = \rho (r, \theta, \lambda)$, its total mass $M$ and mean radius $R$ and the co-latitude $\theta$ in the following way [Rummel, 1991]:

$$J_n = \frac{1}{M} \int \int \int \rho P_n(\cos \theta) dV (dV = r^2 \sin \theta dr d\theta d\lambda) (8.1)$$

where the integration is over the entire Earth, which at satellite altitude includes its atmosphere as well. Consequently, any temporal variation in $\rho$ gives rise to temporal variation in $J_n$, of which $\dot{J}_n$ represents the secular part of the variation. The first successful determination of the secular variation in $J_2, \dot{J}_2$, was based on the analysis of the secular acceleration observed in the node residuals of a 5½ year LAGEOS-1 arc [Yoder et al., 1983]. The node is primarily sensitive to the even-degree zonal coefficients, which induce a secular variation (precessing orbit) due to their constant (static) part, and a quadratic variation in time due to their secular time-varying part ($\dot{J}_n$). It is the best modeled angle variable for LAGEOS-1, whose nearly circular orbit makes the argument of perigee $\omega$ difficult to observe accurately. It is also the best modeled
angle variable for Starlette, because the modeling of the node depends only weakly on atmospheric drag (via the semimajor axis, the precision of which is increased by means of estimated drag scale coefficients). The odd-degree zonal coefficients primarily induce a periodic variation of the eccentricity and perigee (with the perigee period), while \( \hat{J}_{\text{odd}} \) induces the same periodic variation, the amplitude of which increases quadratically in time. The corresponding orbit perturbations on LAGEOS-1 are displayed in Figures c6 and c7 of Appendix C for \( J_3 \) and \( J_5 \), respectively.

Unfortunately, the \( \hat{J}_{\text{even}} \) all have predominantly the same satellite-specific signature (quadratic on \( \Omega \)), all effectively contributing to the estimate of \( J_2 \) in a linear way. This gives for LAGEOS-1, for example, the following linear constraint equation [Eanes, 1995]:

\[
\hat{J}_{2\text{eff}} \text{ (LAGEOS-1)} = J_2 + 0.371 \hat{J}_4 + 0.079 \hat{J}_6 + 0.006 \hat{J}_8 - 0.003 \hat{J}_{10} + \ldots \tag{8.2}
\]

This implies that when \( J_2 \) is estimated with LAGEOS-1, one actually estimates the linear combination of even \( \hat{J}_n \), which may be far from the true value one wished to determine if the multipliers and/or the rates of change in the higher-degree zonals are large. Separating the individual \( \hat{J}_{\text{even}} \) requires at least a second satellite, for which the linear constraint equation must be sufficiently different (due to a different orbit configuration). The corresponding equations for Starlette and LAGEOS-2 are [Eanes and Bettadpur, 1996]:

\[
\begin{align*}
\hat{J}_{2\text{eff}} \text{ (Starlette)} &= J_2 + 0.040 \hat{J}_4 - 0.555 \hat{J}_6 - 0.150 \hat{J}_8 + 0.283 \hat{J}_{10} + \ldots \\
\hat{J}_{2\text{eff}} \text{ (LAGEOS-2)} &= J_2 + 0.073 \hat{J}_4 - 0.065 \hat{J}_6 - 0.014 \hat{J}_8 + 0.003 \hat{J}_{10} + \ldots \tag{8.3}
\end{align*}
\]

The equation for Starlette still has large multipliers for degree 10, rendering this satellite less suitable for estimation of the secular variation in the low-degree even zonal harmonics than LAGEOS-1 and LAGEOS-2. It also increases the uncertainty of a multi-satellite solution in which it participates. It has been used anyway in many studies, as well as in the present analysis, for three reasons: first of all, it is the oldest of the cannonball type geodetic satellites yielding a long and complete tracking data set; secondly, there are only few geodetic satellites; and thirdly, the magnitude of the time-varying part of the zonal coefficients is assumed to decrease with increasing harmonic degree. Satellite solutions for the rates of change of degrees 8 and higher are not available, and even \( \hat{J}_6 \) is badly determined ([Cheng et al., 1997]; [Devoti et al., 2001]). This is not a critical remark, but a consequence of the correlation and the insensitivity of most satellites, especially under the assumption of diminishing signal strength with increasing degree. A realistic error estimate requires bounds for the coefficients of degrees 6 and 8, and higher if possible, which must be provided by some other technique (postglacial rebound models, for example), or by using more sensitive, lower-orbiting satellites. The error in \( J_2 \) due to not taking \( \hat{J}_6 \) into account (or erroneously) may represent up to 20% (when using the value of Cheng et al. [1997] and taking its uncertainty into account).
The linear constraint equations can be constructed equally for the odd-degree zonal coefficients, which gives for LAGEOS-1, LAGEOS-2 and Starlette:

\[
\begin{align*}
\dot{J}^\text{eff}_3 (\text{LAGEOS} - 1) &= \dot{J}_3 - 0.27\dot{J}_5 - 0.22\dot{J}_7 - 0.06\dot{J}_9 + \ldots \\
\dot{J}^\text{eff}_3 (\text{LAGEOS} - 2) &= \dot{J}_3 - 0.53\dot{J}_5 + 0.03\dot{J}_7 + 0.05\dot{J}_9 + \ldots \\
\dot{J}^\text{eff}_3 (\text{Starlette}) &= \dot{J}_3 + 1.04\dot{J}_5 + 0.53\dot{J}_7 + 0.81\dot{J}_9 + \ldots
\end{align*}
\]

These equations imply that only \( \dot{J}^\text{eff}_3 \) may be estimated, so the 'lumped' value, which holds for Starlette in particular. Even the lumped coefficient will be victim to (significant) error due to its dependence on the quality of the nongravitational model. The equations for the rate of change in the odd zonals will be considered in this study, but must be used with care. The amplitude of this perturbation is small for LAGEOS-1, LAGEOS-2 and Starlette, compared to other perturbations and the achieved nongravitational modeling accuracy. Figure c7 (Appendix C) displays the orbit perturbation due to \( \dot{J}_3 \) on LAGEOS-1, over a period of 12 years, assuming a value of \(-1\cdot10^{-11}/\text{yr}\). This value is close to the estimate of \(-1.3\cdot10^{-11}/\text{yr}\) of Cheng et al. [1997], and so the magnitude of the perturbation can be considered to be a realistic one. This simulation demonstrates the delicateness of attempting its estimation:

- the amplitude of the perturbation is very small, only 30 cm and 60 m on the eccentricity and perigee, respectively, and that after 12 years. The LAGEOS-1 long arc, the residuals of which were shown in Figure 8.10, is used in a multi-satellite solution (Section 8.3.4), but is clearly not at the accuracy level required to provide an even moderately-accurate estimation;

- the perturbation affects the same orbital elements as that due to the Yarkovsky-Schach and anisotropic reflection effects (Section 5.5), and it is highly correlated with both forces. This means that any mismodeling of these nongravitational forces will corrupt the \( \dot{J}_3 \) estimate. This has been diagnosed by several authors ([Nerem et al., 1993]; [Gegout and Cazenave, 1993]; [Eanes and Bettadpur, 1996]; [Cheng et al., 1997]), but has not always withheld them from presenting quite dissimilar estimates;

- the annual variation in \( J_3 \), presented in detail in the next section, causes perturbations of the same order of magnitude as its secular variation \textit{after 12 years}. This adds to the already considerable noisy and inaccurate signal from which the secular trend has to be derived. This is especially true for methods that deduce the rate of change from analysis of a long time series of monthly \( J_3 \) estimations.
8.3 Temporal Gravity Variation and Very Long-Period Tides

The results obtained with one or several satellites, as well as with different techniques, are presented in Table 8.5. Several conclusions may be drawn from this table, the most important one being that only $J_2$ is relatively well estimated, but an uncertainty of nearly 10% level persists despite many studies. The estimations of $J_4$ seem to converge towards two values: $-1 \cdot 10^{-11}/\text{yr}$ or $0.3 \cdot 10^{-11}/\text{yr}$ (but both with a large uncertainty, see Table 8.5). This parameter is also an even-degree zonal term, so mainly dependent on the quality of the modeling of the ascending node. The value for $J_2$ from Stephenson and Morrison [1995] is the only estimate not based on the satellite perturbation technique, but on ancient lunar and solar eclipse data. The remarkable difference between the (lumped) estimates from Nerem and Klosko [1996] and those from Cazenave et al. [1996], while 3 out of 4 satellites participating in the estimation were identical, suggests very different modeling of the nongravitational forces (actually, a nongravitational model was not employed; empirical accelerations were estimated). The estimate from Exertier et al. [1995] is the smallest one, and is the only one that is not within the error-bar range of all other solutions. However, this estimate has been obtained with a (CODIOR) long arc of 12 years, which is not long enough to successfully separate $J_2$ from the 18.6 year tide. The differences between solutions, while using approximately the same observations, may also be caused by weighting of the contributions of each satellite, or the application of constraints (for example, using eq. (8.4), or by imposing a maximum on the solar radiation pressure scaling coefficients).

The satellite-derived observations of $J_n$ can be related to postglacial rebound processes in the lower mantle (depth between 670 - 2885 km), whose viscous profile may be constrained by them, only if it is certain within reasonable limits that the signal as sensed by the satellite has its origin in them [Ivins et al., 1993]. The present-day observed global sea-level rise, a phenomenon for which a wide consensus exists (although not on what is causing it), is estimated between 1 and

**Table 8.5:** recent estimates of the secular change of the zonal harmonics, in $10^{-11}/\text{yr}$.

<table>
<thead>
<tr>
<th># satellites</th>
<th>$J_2$</th>
<th>$J_3$</th>
<th>$J_4$</th>
<th>$J_5$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Devoti et al. [2001]</td>
<td>4</td>
<td>-2.9±0.2</td>
<td>0.5±0.2*</td>
<td>0.6±0.5</td>
</tr>
<tr>
<td>Cheng et al. [1997]</td>
<td>8</td>
<td>-2.7±0.4</td>
<td>-1.3±0.5</td>
<td>-1.4±1.0</td>
</tr>
<tr>
<td>Eanes and Bettadpur [1996]</td>
<td>2</td>
<td>-2.6±0.3</td>
<td></td>
<td>-0.9±1.3</td>
</tr>
<tr>
<td>Nerem and Klosko [1996]</td>
<td>4</td>
<td>-2.8±0.3</td>
<td>1.6±0.4*</td>
<td>0.2±1.5</td>
</tr>
<tr>
<td>Cazenave et al. [1996]</td>
<td>3</td>
<td>-3.0±0.5</td>
<td>-1.8±0.1*</td>
<td>-0.8±1.5</td>
</tr>
<tr>
<td>Exertier et al. [1995]</td>
<td>1</td>
<td>-2.3±0.2</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Nerem et al. [1993]</td>
<td>1</td>
<td>-2.6</td>
<td>-1.86</td>
<td></td>
</tr>
<tr>
<td>Gegout and Cazenave [1991]</td>
<td>1</td>
<td>-2.8±0.4</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Cheng et al. [1989]</td>
<td>1</td>
<td>-2.5±0.3</td>
<td>-0.1±0.3</td>
<td>0.3±0.6</td>
</tr>
<tr>
<td>ancient eclipse data **</td>
<td>**</td>
<td>-3.5±0.8</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

* Denotes the lumped result ** [Stephenson and Morrison, 1995]
It is caused by a combination of melting of the Greenland and Antarctic ice sheets, the melting of mountain glaciers and small ice caps, and thermal expansion effects in the oceans [James and Ivins, 1997]. This mass transfer (not the thermal expansion part) contributes to the satellite observed \( \dot{J}_n \), and so must be taken into account when solving for mantle viscosity. Using accurately-derived satellite estimates, in particular of \( \dot{J}_3 \) and \( \dot{J}_4/\dot{J}_3 \), may in the future help discriminate between the world's ice sheet mass balance (Antarctica primarily) and mantle structure, respectively [Vermeersen et al., 1998]. However, \( \dot{J}_{\text{odd}} \) is difficult to estimate at the present time, and will probably be estimated accurately for the first time using the combined observations of the CHAMP (launched July 2000; [Reigber et al., 1996]) and GRACE (launched March 2002; [GRACE, 1998]) missions. These satellites will benefit from their accelerometers, which will observe the nongravitational forces that otherwise are difficult to model accurately, as was shown in Section 8.2. The powerful constraint placed on the mass balance of Antarctica and Greenland by the odd-degree zonal harmonics is necessary, and justifies the effort of several groups to obtain a most accurate satellite long-arc orbit to be able to estimate them reliably.

The present-day postglacial \( \dot{J}_n \) can equally be reconstituted [Ivins et al., 1993], assuming a certain viscosity profile of the lower mantle (the upper mantle viscosity is usually taken as \( 10^{21} \) Pa s) and the ice-load history of the Laurentide, Fennoscandian (both in the northern hemisphere) and Antarctic ice sheets as estimated with ICE-3G [Tushingham and Peltier, 1993] or ICE-4G [Peltier and Jiang, 1996] models. The deglaciation history of Antarctica and its timing with respect to the northern hemispheric ice sheets (the time lag) are not well known, but using a phase-lagged Antarctic deglaciation changes the predicted value for \( \dot{J}_3 \) by a large, positive value compared to a situation with no southern hemispheric phase lag (Mitrovica and Peltier [1989] predicted a weakly negative \( \dot{J}_3 \)). Adding the sea-level rise contribution to the one due to postglacial rebound yields the predicted \( \dot{J}_n \), which may be compared to the satellite-derived estimates. It is again Antarctica's contribution, this time due to present-day global sea-level rise, which is most uncertain. While the predicted values due to postglacial rebound and melting of the Antarctic and Greenland ice sheets and mountain glaciers (a satellite senses the cumulated signal) of \( \dot{J}_2 \) are in the interval \((-2.8 \pm 0.5) \cdot 10^{-11}/\text{yr}\), and \( \dot{J}_3 \) is in the interval \((1.6 \pm 0.5) \cdot 10^{-11}/\text{yr}\), the prediction of \(-3.3 \cdot 10^{-11}/\text{yr} < \dot{J}_4 < 0.7 \cdot 10^{-11}/\text{yr}\) [James and Ivins, 1997] is useless as a starting value for a satellite-derived estimation. Predictions for higher degrees have not been computed, but a prior study indicated the possibly large amplitudes of \( \dot{J}_5 \) and \( \dot{J}_7 \) (\( 2 \cdot 10^{-11}/\text{yr} \)) in case of a high-viscosity layer in the lower mantle, and small amplitudes (\( 0.2 \cdot 10^{-11}/\text{yr} \)) in case of a low-viscosity layer [Ivins et al., 1993]. This indicates the importance of obtaining accurate estimates of these coefficients, which may contribute to the modeling of the lower-mantle stratification.
8.3 Temporal Gravity Variation and Very Long-Period Tides

8.3.2 The 18.6 and 9.3 Year Tides

These very long-period tides are difficult to determine because of the long time series required to make a reliable estimation, ideally several periods of 18.6 or 9.3 years. Satellite data cover slightly more than a complete 18.6 year period, whereas tide gauge data are available for over a century for a few stations only. The second-degree coefficients of these tides \((n=2, m=0\) and \(s=18.6\) year tide, which hereafter will be written as a superscript; eq. (4.5)), \(C^{18.6+}_{2,0}\) and \(S^{18.6+}_{2,0}\), cause a periodic (sinusoidal) perturbation on the ascending node of the satellite orbit. This resembles a parabolic signal if the arc length is too short (less than half a period), which explains its correlation with \(J_{\text{even}}\) in that case. The amplitude and phase of the theoretical equilibrium value of the 18.6 year tide, assuming an elastic Earth, are: 0.967 cm and 90º, or \(C^{18.6+}_{2,0}=0.967\) cm, \(S^{18.6+}_{2,0}=0\) when expressed in spherical harmonics [Zhu et al., 1996]. The theoretical amplitude of the 9.3 year tide does not exceed 0.01 cm, inducing a negligible orbit perturbation. Taking into account the anelasticity of the Earth (using 0.299 for the elastic Love number \(k_2\)), a larger amplitude for the satellite-observed 18.6 year tide has been predicted by Zhu et al. [1996]: 1.53 cm, with a lower bound of 1.12 cm and an upper bound of 1.95 cm. The phase remains close to 90º, but with a phase lag or lead due to the dissipation induced by the anelasticity of ~26 to 20º because of the large observational uncertainties. These numbers have been obtained adopting an amplitude of 1.09 cm and a phase of (90±30)º for the 18.6 year tide and adding a correction of 0.44 cm for anelasticity [Zhu et al., 1996]. The 18.6 year tide estimate of 1.09 cm is based on global tide gauge data, using 260 stations with at least 19 years of data [Trupin and Wahr, 1990]. That analysis yielded a multiplicative factor of 1.13 with respect to the equilibrium value of 0.967 cm, and the authors concluded that the tide is near equilibrium.

8.3.3 Periodic Variations in Zonal Harmonics

The temporal variations in the low-degree zonal and tesseral harmonic coefficients also have periodic components, primarily with annual and semiannual periods. These seasonal gravitational variations have their origin in the Sun, whose influence on the Earth's mass distribution is modulated by the Earth's orbit around it. The main perturbations are the solid Earth tides, which are well modeled, the ocean tides with annual (\(S_a\)) and semiannual (\(S_{a1}\)) periods, the redistribution of air mass, and the variation in surface water storage and snow cover. The solid Earth tides are separately modeled, as a well-distinguished perturbation, but this is not the case for the ocean tide components, the signal of which is correlated with the atmosphere. The not well-known steric effect (thermal expansion), which changes sea-level height, constitutes only an insignificant gravitational change. As a consequence, these long-period tides are not well known. The redistribution of air mass in the atmosphere changes the external gravitational field at satellite
altitude, and at the local scale of an SLR station causes a vertical load site displacement. Both effects have been measured, the former for the first time by Gutierrez and Wilson [1987], using the nodal perturbations of Starlette and LAGEOS-1. They also provided a first estimation of the gravitational signal due to the hydrological cycle. Estimations of the air mass signal in the effectively estimated annual and semiannual variation in the low-degree zonals, \( J_n^{\text{ann}} / J_n^{\text{semi}} \) respectively, have been computed using gridded surface pressure data, available every 6 hours.

The European Centre for Medium-Range Weather Forecasts (ECMWF), for example, provides this type of data. Spectral analysis of the zonal coefficients, after decomposition of the gridded data in spherical harmonics, showed that the annual signal is the strongest and that the semiannual signal is not present in all coefficients. This may be seen in Appendix D. The seasonal gravitational signal induced by surface water and snow cover, customarily expressed as an amplitude and a sine phase with respect to the first day of the year, has been estimated by Chao and O'Connor [1988], for \( J_2 \) through \( J_4 \). That study excluded Antarctica and Greenland from its computations, however, because of large uncertainties in their mass balances. Secondly, the geographical distribution of the contributing meteorological stations measuring precipitation was not homogeneous, leaving large areas of the world uncovered.

The satellite-derived seasonal gravity variations are the sum of the above mentioned effects (ocean tide, air mass and hydrology/snow cover), and may help solve the question on the type of response the oceans have to changes in atmospheric pressure: an inverse barometer (IB), non-inverse barometer (NIB), or mixed reaction. The former response means that the ocean reacts isostatically to local changes in pressure, redistributing water uniformly and instantaneously over all oceans and changing ocean bottom pressure only as an average of all atmospheric pressure changes over all oceans, without deforming the ocean floor. This response reduces the net gravitational effect, whereas it is enhanced by the other extreme, NIB, which assumes a pressure transfer through a ‘rigid’ ocean, thus causing a deformation of the ocean floor. Secondly, the satellite-derived phases may give information on which perturbation is predominant, or indicate which of the three perturbations requires remodeling. Table 8.6 in Section 8.3.6 will present the results of some recently published satellite estimates of the annual and semiannual variations in \( J_2 \) and \( J_3 \), as well as variations induced at these periods due to ocean tides, atmospheric mass redistribution, hydrology and snow cover.

**8.3.4 Estimated Secular Variations**

It is in practice not possible to accurately estimate the secular variation in the zonal harmonics in satellite solutions without equally estimating or accurately modeling (for example, by using the amplitude and phase estimated by Zhu et al. [1996]) the 18.6 year tide. The estimate
of $J_2^{\text{effective}}$ is highly correlated with the 18.6 year tide if only arcs spanning less than 12 years are used due to the similarity of the orbit perturbations they induce on the ascending node as may be seen in Figures c5 and c6 of Appendix C. The multi-satellite solution allowed a good decorrelation of the secular variations up to degree 4 and the 18.6 year tide (the correlation coefficients are smaller than 0.5). The determination of $J_3$ was too inaccurate due to high correlation (0.9) with $J_3$, with an uncertainty of $3.5 \times 10^{-11}/\text{yr}$, and has not been retained in the solution.

The results obtained for the rates of change in the zonals of degrees 2 and 4, using LAGEOS-1, Starlette and LAGEOS-2, are:

$$
\begin{align*}
J_2' &= -(2.7 \pm 0.2) \times 10^{-11}/\text{yr} \\
J_4' &= -(1.1 \pm 0.3) \times 10^{-11}/\text{yr}
\end{align*}
$$

Both estimates are consistent with estimates obtained with the CSR long-arc technique ([Eanes and Bettadpur, 1996]; [Cheng et al., 1997]). The $J_2'$ estimate agrees, within the error bar limits, with the values determined by other groups listed in Table 8.5. This is not the case for $J_4'$, for which both positive and negative values have been reported (cf. Table 8.5). However, Devoti et al. [2001] report a large correlation of this coefficient with their $J_2'$ estimate, causing the estimation of a partially lumped $J_4$, probably caused by their too short data processing period (1987-1998).

The estimate of $J_3'$ of $-(1.1 \pm 0.4) \times 10^{-11}/\text{yr}$ is relatively close to that of Cheng et al. [1997], but the comparison with other recent estimates shows that large differences remain. $J_3$ could not be estimated accurately, as the aforementioned formal uncertainty of the estimation indicates, and represents the limit that could be attained with the present 3 satellites. Tests without either LAGEOS-1 or Starlette have not been performed, because the LAGEOS-2 long-arc orbit is too short (4½ years) to decorrelate the secular variation in the zonals of degrees 2 and 4. Estimation of $J_2^{\text{effective}}$ and the 18.6 year tide coefficients with Starlette and LAGEOS-1 separately showed that estimates obtained with the former satellite are larger (-2.9 versus -2.6 $\times 10^{-11}/\text{yr}$, and 1.6 versus 1.9 cm, respectively) and with larger uncertainties. This may be explained by the less-accurate Starlette orbit, which moreover is nearly 4 years shorter than the LAGEOS-1 long arc. The weighting of the mean observed elements in the mixed solution corresponds to the weighting applied to each separate satellite in a usual orbit adjustment. This means that the argument of perigee and the mean anomaly are downweighted, and that the solutions consequently are based on least-squares fitting of the ascending node residuals. There is a form of natural weighting, by the sensitivity of the orbit to a given perturbation, and secondly by the volume of observed mean elements. The smaller LAGEOS-1 sensitivity compared to Starlette (due to altitude) is so partly compensated for by its larger amount of observations. LAGEOS-2 mainly aided in the
8 Long Arcs and Geophysical Parameter Estimation

decorrelation and in the reduction of the variance of the estimates, but that satellite contributed significantly to the estimation of seasonal parameters.

8.3.5 Estimated 18.6 and 9.3 Year Tides

The values estimated in this study, in cm, uncorrected for the Earth anelasticity effect of 0.44 cm [Zhu et al., 1996], are: $C_{2,0}^{18.6+}=1.64\pm0.04$, $S_{2,0}^{18.6+}=0.0\pm0.08$ This corresponds to an ocean tide amplitude of $1.20\pm0.05$, with a phase of $(90\pm4)^\circ$. These are only slightly larger than the values estimated by Cheng et al. [1997] (and within the error bar limits), who estimated $C_{2,0}^{18.6+}=1.57\pm0.15$, $S_{2,0}^{18.6+}=0.08\pm0.25$. Taking the anelasticity effect of 0.44 cm on the estimate into account, the estimated amplitude is also within the error bounds of the estimate by Trupin and Wahr [1990] of $1.09\pm0.22$ cm, and equilibrium phase $(90\pm30)^\circ)$. All results have in common that they yield a higher amplitude than the near-equilibrium value of $1.09$ cm plus the $0.44$ cm of the Earth anelasticity effect. It is most probable, assuming the 18.6 year tide at equilibrium [Eanes, 1995] or near-equilibrium, that the effect of anelasticity is slightly larger than $0.44$ cm (between 0.48 and 0.55 cm). The upper bound given by Zhu et al. [1996] is 0.65 cm, which would give an amplitude of $1.74$ cm $(1.09+0.65)$, which is larger than the estimate found here.

The estimated 9.3 year tide values are: $C_{2,0}^{9.3+}=0.37\pm0.03$, $S_{2,0}^{9.3+}=0.11\pm0.03$. The origin of this perturbation is currently unknown [Cheng et al., 1997], and further investigation is required.

8.3.6 Estimated Seasonal Variations

Seasonal variations with annual and semiannual periods have been estimated in $J_2$, and only with an annual period in $J_3$. The main effects of the signal with an annual period are assumed to be related to redistribution of atmospheric mass, and variations in surface water storage and snow cover ([Cheng et al., 1989]; [Chao and O'Connor, 1988]; [Gegout and Cazenave, 1993]; [Nerem et al., 1993]; [Schutz et al., 1993]; [Chao and Eanes, 1995]; [Dong et al., 1996]). The resulting change in the gravity field experienced by a satellite allows the estimation of these seasonal variations, on the condition that the satellite orbit is perturbed sufficiently to guarantee an accurate determination. Small perturbations require an accurate orbit description to be able to successfully isolate them, and accurate modeling of specific orbital elements in particular. Sensitivity to and an accurate modeling of the ascending node are required to determine the even-degree terms of the seasonal variations, while the eccentricity and perigee provide the information on the odd terms. Taking these conditions into account, the LAGEOS satellites provide the best possible orbit geometry and tracking accuracy to estimate seasonal variations of the existing satellites. The orbit perturbations are still sufficiently large at their altitudes (see Figures c8 and c9 of Appendix C), while they have the advantage of being only very slightly perturbed by (a
8.3 Temporal Gravity Variation and Very Long-Period Tides

However, their eccentricity modeling is not achieved with the same high accuracy as that of their ascending nodes, so special care must be taken to verify and interpret the annual variation in $J_3$.

A dual-satellite solution has been obtained by accumulating the observation equations of a 5½ year LAGEOS-1 (Section 8.2.3a) and a 4½ year LAGEOS-2 long arc (Section 8.2.2), using their nongravitational models as described in Section 8.2. Not all 17 years of LAGEOS-1 data have been included, because the older data have a much larger temporal spacing, and are slightly noisier. Consequently, the secular and very long period variations could not be estimated in this solution, and they have been fixed to the values presented in Sections 8.3.4 and 8.3.5. All $S_a$ (annual) and $S_s_a$ (semiannual) ocean tide coefficients have been set to zero to recover unbiased estimates in each of the presented solutions, which represent the sum of the geophysical signals at those two periods. The modeled annual and semiannual ocean tides are inaccurate due to nontidal effects absorbed into these coefficients [Dong et al., 1996]. Therefore, modeling these tides will cause those errors to leak into the solution. The results of this study are presented in Table 8.6, which lists results from other studies as well. The nongravitational model parameterization employed in the 17-year LAGEOS-1 arc has been used (SB in Table 8.4), as well as the model parameters (Yarkovsky-Schach in particular) from Métris et al. [1997] (GM in Table 8.4), which realize a better orbit fit over the 5½ years under consideration, to investigate the impact of the nongravitational model on the estimates.

The obtained and reported satellite results for $J_2$ show a good agreement of the phase solutions, while the differences in estimated amplitudes are large for the annual variation. This can partly be explained by the period under consideration, since corresponding atmospheric signals are equally different, as may be seen in Table 8.6. The hydrological signal also is non-stationary. However, reliable and accurate observations as for the atmospheric pressure are not available. The estimates of the semiannual amplitudes are less dispersed than the annual ones, which is consistent
8 Long Arcs and Geophysical Parameter Estimation

Table 8.6: total seasonal variations (no atmosphere, \( S_a \) and \( S_w \)) in \( J_2 \) obtained with a LAGEOS-1 and LAGEOS-2 mixed solution. The 'combined' signal is the sum of atmospheric, hydrologic, and tidal variations including the anelastic effect. Sine amplitude \( A \) (in \( 10^{-10} \)) and phase (degrees) relative to 1 January.

<table>
<thead>
<tr>
<th>solution</th>
<th>( A_{\text{annual}} )</th>
<th>phase</th>
<th>( A_{\text{semiannual}} )</th>
<th>phase</th>
</tr>
</thead>
<tbody>
<tr>
<td>this study ( \text{SB} )</td>
<td>2.3 ± 0.2</td>
<td>207 ± 8</td>
<td>2.4 ± 0.4</td>
<td>297 ± 11</td>
</tr>
<tr>
<td>this study ( \text{GM} )</td>
<td>1.8 ± 0.2</td>
<td>207 ± 8</td>
<td>1.9 ± 0.4</td>
<td>300 ± 10</td>
</tr>
<tr>
<td>Cheng et al. [1989]</td>
<td>3.9</td>
<td>172</td>
<td>1.0</td>
<td>234</td>
</tr>
<tr>
<td>Gegout &amp; Cazenave [1993]</td>
<td>3.2</td>
<td>197</td>
<td>1.7</td>
<td>291</td>
</tr>
<tr>
<td>Nerem et al. [1993]</td>
<td>2.7</td>
<td>205</td>
<td>2.5</td>
<td>288</td>
</tr>
<tr>
<td>Cheng et al. [1993]</td>
<td>3.2</td>
<td>209</td>
<td>2.0</td>
<td>286</td>
</tr>
<tr>
<td>Schutz et al. [1993]</td>
<td>3.2</td>
<td>212</td>
<td>2.0</td>
<td>280</td>
</tr>
<tr>
<td>Dong et al. [1996]</td>
<td>2.5 ± 0.2</td>
<td>208 ± 5</td>
<td>2.1 ± 0.2</td>
<td>294 ± 6</td>
</tr>
</tbody>
</table>

anelasticity effect\(^a\) | 0.1 | 72 | 0.3 | 273 |
snow+surface water\(^b\) | 1.4 | 244 | 0.5 | 92 |
tide (Schwiderski) | 0.2 | 50 | 1.5 | 242 |
atmosphere IB\(^2\) | 0.7 | 186 | 0.2 | 290 |
atmosphere IB\(^3\) | 1.6 ± 0.2 | 198 ± 7 | 0.7 ± 0.2 | 272 ± 17 |
atmosphere IB\(^4\) | 1.3 | 235 | 0.2 | 212 |
atmosphere NIB\(^2\) | 2.0 | 161 | 0.2 | 277 |
atmosphere NIB\(^3\) | 2.9 ± 0.3 | 187 ± 7 | 0.9 ± 0.3 | 286 ± 22 |
atmosphere NIB\(^4\) | 2.4 | 205 | 0.2 | 215 |
combined IB\(^5\) | 2.7 | 214 | 1.6 | 290 |
combined NIB\(^5\) | 3.9 | 201 | 1.9 | 294 |
combined IB\(^6\) | 2.5 | 235 | 1.1 | 283 |
combined NIB\(^6\) | 3.5 | 214 | 1.3 | 284 |
combined IB\(^7\) | 1.6 | 144 | 1.5 | 315 |
combined NIB\(^8\) | 3.8 | 217 | 1.5 | 279 |

\(^{a}\text{SB, GM}\) Solution obtained with non-gravitational model parameterization given in Table 8.4.
\(^{c}\text{The tide is the Schwiderski tide as estimated in Nerem et al. [1993].}\)
\(^{d}\text{Zhu et al. [1996] / Schutz et al. [1993]}\)

with the smaller non-stationary contributions (atmosphere and hydrology) with respect to the stationary one (ocean tide). Moreover, only the estimates from Cheng et al. [1989] and Gegout and Cazenave [1993] are larger than the sum of the ocean tide and the anelastic effect.

When the satellite solutions of variation in \( J_2 \) (top 8 lines of Table 8.6) are confronted with the total estimated seasonal amplitudes and phases ('combined' solutions in Table 8.6), it can only be concluded that the ocean response is somewhere between the inverted and non-inverted barometer hypothesis. The annual variation obtained with the 'SB' parameterization agrees best
8.3 Temporal Gravity Variation and Very Long-Period Tides

with the combined signal using the atmospheric effect under the inverse barometer hypothesis of Dong et al. [1996] (underlined in Table 8.6), although the latter still has a larger amplitude.

For the estimated semiannual variation the combined signal with an NIB response (underlined in Table 8.6) seems more probable. From the 'SB' (and also the 'GM') estimates it can thus be concluded that the response of the oceans at the annual and semiannual periods is not the same. Either the snow and surface water amplitude or the atmosphere amplitude seems too large in the combined annual signal, the ocean tide contribution already being small. Based upon the purely atmospheric variations in $J_2$ calculated in the two other studies, this suggests an overestimation by Dong et al. [1996], although these differences may be caused by differences in the period under study. In the study of Dong et al. [1996], data from the National Meteorological Center have been used, while in the other studies it concerns ECMWF data. Secondly, the atmospheric signal is an average computed over 1984-1992; in Nerem et al. [1993], the 1980-1989 period has been analyzed, and in Gegout and Cazenave [1993] it concerns the period 1985-1989. The non-stationarity of the atmospheric $J_2$ term, which probably causes the reported averages to be different, is shown in Figure d9 (Appendix D). The semiannual amplitude of the satellite solutions, on the other hand, is in most cases larger than the sum of the model values. It is not clear which estimate is erroneous; most probably they all are to a certain degree. However, at the semiannual period the ocean tide indeed represents the dominant perturbation (cf. Table 8.6).

The annual variation in $J_3$ is difficult to estimate accurately, and its determination requires careful and accurate nongravitational force modeling. The amplitude and phase of a satellite solution correspond to the combined effects of the atmospheric data and the snow and surface water model (negligible ocean tide contribution). Results are summarized in Table 8.7. The estimate by Nerem et al. [1993], is unrealistically large in view of the periodic perturbation this would produce on the eccentricity and perigee, with amplitudes of 1.5 and 220 m, respectively, and seems very inaccurate. The estimate by Gegout and Cazenave [1993] also is large. The estimated phases correspond to the model predicted values for the inverted barometer response in case of Nerem et al. [1993], and is close to the non-inverted barometer response for Gegout and Cazenave [1993]. The results reported above have been obtained with LAGEOS-1, while the estimation of Schutz et al. [1993] is based on a Starlette long-arc analysis. Their estimate is in relatively good agreement with the predicted geophysical (combined) signal, although the amplitude is too large. The amplitudes and phases estimated in this study of the (both 'SB' and 'GM') annual variation in $J_3$ lie in the interval or within the error bar limits defined by the combined signals of Gegout and Cazenave [1993], Nerem et al. [1993], and Schutz et al. [1993], respectively (last 6 lines of Table 8.7). For these estimates, the oceanic response agrees best with the NIB case. The estimation of the semiannual variation has also been attempted in this study, but
because the formal error was larger than the estimates, it has not been retained in the solution. The estimates using the Starlette long arc have for the same reason equally not been retained here.

The impact of using one or the other set of nongravitational model parameters did not change the formal precision of the estimations. Their recovered values, however, are quite different in case of the annual and semiannual amplitudes of \( J_2 \), their phases being relatively constant (Table 8.6). Based on the orbit fit, the results obtained with the nongravitational model parameters of Métris et al. [1997] (GM in Table 8.4) are probably more accurate, also for the variation in \( J_3 \), but the formal errors of its solution were only marginally smaller.

Using the atmospheric pressure directly in the orbit computation may decorrelate the contributions from the ocean tides and the hydrology better than by *a-posteriori* computing the combined contribution of the three perturbations and comparing the result to the estimated sinusoidal signal. The same LAGEOS-1 and LAGEOS-2 arcs have been computed using the force model described in Section 4.2.5, but this time only single-satellite solutions have been produced. This has also been done with Starlette (Section 8.2.1), but, due to the inadequate temporal

<table>
<thead>
<tr>
<th>solution</th>
<th>( A_{\text{annual}} ) (in ( 10^{-10} ))</th>
<th>phase (degrees)</th>
</tr>
</thead>
<tbody>
<tr>
<td>this study ( ^{\text{SB}} )</td>
<td>1.7 ± 0.3</td>
<td>79 ± 14</td>
</tr>
<tr>
<td>this study ( ^{\text{GM}} )</td>
<td>1.6 ± 0.3</td>
<td>78 ± 12</td>
</tr>
<tr>
<td>Nerem et al. [1993]</td>
<td>12.3</td>
<td>117</td>
</tr>
<tr>
<td>Gegout&amp;Cazenave [1993]</td>
<td>5.2</td>
<td>82</td>
</tr>
<tr>
<td>Schutz et al. [1993]</td>
<td>2.8</td>
<td>65</td>
</tr>
<tr>
<td>snow+surface water ( ^{1} )</td>
<td>1.4</td>
<td>234</td>
</tr>
<tr>
<td>atmosphere IB ( ^{2} )</td>
<td>2.42</td>
<td>87</td>
</tr>
<tr>
<td>atmosphere NIB ( ^{3} )</td>
<td>3.27</td>
<td>75</td>
</tr>
<tr>
<td>atm.BI+snow ( ^{1} )</td>
<td>1.0</td>
<td>114</td>
</tr>
<tr>
<td>atm.NIBI+snow ( ^{1} )</td>
<td>1.8</td>
<td>58</td>
</tr>
<tr>
<td>atm.BI+snow ( ^{1} )</td>
<td>1.5</td>
<td>118</td>
</tr>
<tr>
<td>atm.NIBI+snow ( ^{1} )</td>
<td>2.0</td>
<td>88</td>
</tr>
<tr>
<td>atm.BI+snow ( ^{1} )</td>
<td>1.3</td>
<td>120</td>
</tr>
<tr>
<td>atm.NIBI+snow ( ^{1} )</td>
<td>1.9</td>
<td>87</td>
</tr>
</tbody>
</table>

\( ^{\text{SB, GM}} \): Solution obtained with nongravitational model parameterization given in Table 8.4.

\( ^{1} \) Chao & O'Connor [1988] / \( ^{2} \) Gegout & Cazenave [1993] / \( ^{3} \) Schutz et al. [1993]

\( ^3 \) The tide is the Schwiderski tide as estimated in Nerem et al. [1993].
coverage of the data, only the annual variation could be estimated with acceptable accuracy. It is not necessary to re-estimate the secular and very long-period tides using the atmospheric pressure force modeling, because its perturbation is only seasonal.

The arcs have been processed using two adjustment schemes to recover the seasonal variations: by estimating a sinusoidal total analytical signal without modeling the effect of the atmospheric pressure ('T'), and an estimation using the atmospheric pressure effect in the orbit computation, thus recovering the tidal and hydrological signal ('W') only. This allows the determination of the atmospheric part in the estimates by differencing the results. The modeling of the atmospheric pressure effect directly during the orbit computation improved the ascending node modeling significantly, the rms-of-fit decreasing by 20% with respect to an orbit without this force model. However, a part of the atmospheric effect will not be present on the orbits, since the model of Gegout [1995] is not perfect, nor are the meteorological observations. The results are shown in Tables 8.8 and 8.9 for $J_2$ and $J_3$, respectively. For LAGEOS-1, the nongravitational model parameters of Métris et al. [1997] have been used in the orbit computation besides the parameters determined in this study (GM and SB, respectively; cf. Table 8.4).

### Table 8.8: seasonal variations in $J_2$ determined with a 5½ year LAGEOS-1 arc, a 4½ year LAGEOS-2 arc, and a 13 year Starlette arc, for the total (T) and hydrology and snow cover plus tides signal (W). Sine amplitude A (in $10^{-10}$) and phase (degrees) relative to January 1.

<table>
<thead>
<tr>
<th>Solution: LAGEOS-1</th>
<th>A\text{annual}</th>
<th>phase</th>
<th>A\text{semiannual}</th>
<th>phase</th>
</tr>
</thead>
<tbody>
<tr>
<td>*(T)\text{SB}</td>
<td>2.2 ± 0.3</td>
<td>187 ± 9</td>
<td>2.0 ± 0.5</td>
<td>316 ± 17</td>
</tr>
<tr>
<td>*(W)\text{SB}</td>
<td>1.4 ± 0.2</td>
<td>183 ± 7</td>
<td>2.3 ± 0.4</td>
<td>323 ± 14</td>
</tr>
<tr>
<td>*(T)\text{GM}</td>
<td>2.1 ± 0.3</td>
<td>186 ± 9</td>
<td>2.7 ± 0.5</td>
<td>284 ± 17</td>
</tr>
<tr>
<td>*(W)\text{GM}</td>
<td>1.4 ± 0.2</td>
<td>182 ± 7</td>
<td>2.3 ± 0.4</td>
<td>284 ± 14</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Solution: LAGEOS-2</th>
<th>A\text{annual}</th>
<th>phase</th>
<th>A\text{semiannual}</th>
<th>phase</th>
</tr>
</thead>
<tbody>
<tr>
<td>*(T)</td>
<td>2.6 ± 0.3</td>
<td>224 ± 10</td>
<td>2.1 ± 0.6</td>
<td>194 ± 18</td>
</tr>
<tr>
<td>*(W)</td>
<td>1.9 ± 0.3</td>
<td>223 ± 10</td>
<td>2.1 ± 0.6</td>
<td>230 ± 18</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Solution: Starlette</th>
<th>A\text{annual}</th>
<th>phase</th>
</tr>
</thead>
<tbody>
<tr>
<td>*(T)</td>
<td>2.9 ± 0.3</td>
<td>213 ± 10</td>
</tr>
<tr>
<td>*(W)</td>
<td>2.3 ± 0.3</td>
<td>220 ± 12</td>
</tr>
</tbody>
</table>

* nongravitational model parameterization according to Table 8.4

The annual variation in $J_2$ estimated with LAGEOS-2 is systematically larger than the results obtained for LAGEOS-1. The Starlette estimate is larger still, but the atmospheric effect is comparable, being at the level of 0.6-0.8 $10^{-10}$ for the three satellites. The estimated phases differ
significantly, and are smaller with LAGEOS-1 (187º) and larger with LAGEOS-2 (224º) than the results of earlier studies (197º-208º in Table 8.6). The phase recovered with Starlette (213º), however, is within the error bar limits of results of earlier studies. The amplitude estimation with LAGEOS-1 using the atmospheric pressure modeling ('W') is close to the combined signal predicted by the hydrology and tide model, as given in Table 8.6, but the Starlette estimation in particular is large. The phase estimated with LAGEOS-1 does not correspond (within error bar limits) with the combined model signal. The better agreement of the LAGEOS-2 and Starlette phases, both mutual and with the model of Chao and O'Connor [1988], than that of LAGEOS-1 (solutions 'W' in Table 8.8) with the Chao and O'Connor model may be due to their smaller inclinations (approximately 50º vs. 109.9º); the model has not assimilated data over Antarctica and Greenland and is thus less representative of the LAGEOS-1 estimate. Large-amplitude signals are obtained again with a semiannual period. The effect of the atmospheric mass redistribution is small (0.3-0.4 \times 10^{-10}), at least a factor 2 smaller than the annual effect, in accordance with the power spectra of the atmospheric pressure field coefficients displayed in Appendix D.

The results obtained with the LAGEOS satellites presented in Tables 8.6 and 8.8, as well as independent model values, have been plotted in Figures 8.11 to 8.14 as sinusoidal signals. Figure 8.11 displays the total annual $J_2$ solutions (without atmospheric pressure modeling), as well as the 3 (summed) external values given in Table 8.6 (atmosphere+snow/water+ocean tide). The LAGEOS-2 estimate, given in Table 8.8 (T), is within 5% of the underlined value of Table 8.6. The satellite solutions and the external values are reasonably consistent, except for the atmospheric effect of Gegout and Cazenave [1993], which presents a significant phase difference.

The semiannual variations in $J_2$ are presented in a similar form in Figure 8.12. The satellite solutions have a large phase difference, and the LAGEOS-1 estimate is in better accordance with the models and meteorological data than the estimate obtained with LAGEOS-2. All satellite estimates have amplitudes that are larger than the combined model signals.

Figures 8.13 and 8.14 display the results of the estimated annual and semiannual variations in $J_2$, respectively, when the atmospheric pressure effect has been modeled in the orbit computations (the solutions ‘W’ in Table 8.7). The estimated total signals are given as well (solutions ‘T’ in Table 8.7), to show the effect of including the atmospheric pressure modeling on the estimates. The effect on the annual variation estimates is mainly on the amplitudes, which diminish $0.8 \times 10^{-10}$ and $0.7 \times 10^{-10}$ in case of LAGEOS-1 and LAGEOS-2, respectively (cf. Table 8.8). The situation is less clear for the semiannual variations (Figure 8.14), as has already been stated. The LAGEOS-2 phase estimate (230º in Table 8.7) is near the phase of the sum of the ocean tide and snow and surface water signal of 223º. The amplitude of that signal ($1.1 \times 10^{-10}$), however, is much smaller than the satellite-derived values.
Figure 8.11: graphical representation of the estimated total annual variation in $J_2$ as a function of day-of-year (relative to January 1). The satellite solutions obtained with the 'SB' nongravitational model parameters (Table 8.4) are displayed.
Figure 8.12: graphical representation of the estimated total semiannual variation in $J_2$ as a function of day-of-year (relative to January 1). The satellite solutions obtained with the 'SB' nongravitational model parameters (Table 8.4) are displayed.
8.3 Temporal Gravity Variation and Very Long-Period Tides

Figure 8.13: the satellite-derived annual variation in $J_2$ as a function of day-of-year (relative to January 1). (1) Chao and O’Connor [1988].

Figure 8.14: the satellite-derived semiannual variation in $J_2$ as a function of day-of-year (relative to January 1). (1) Chao and O’Connor [1988].
The annual variation estimated in $J_3$, shown in Table 8.9, is not reliable in case of LAGEOS-1, the formal error being at the level of 20-100%. The estimates obtained employing the 'SB' nongravitational model parameters are large ($7.1$ and $4.8 \cdot 10^{-10}$, respectively), and comparable to reported estimates shown in Table 8.6. The large formal error of the result, combined with the orbit perturbation that these large amplitudes would provoke, makes it clear that these estimates are highly corrupted by nongravitational force modeling errors. The large Yarkovsky-Schach amplitude determined by Métris et al. [1997] appears to be correct, judging by the result obtained when employing their nongravitational force model parameters (for the 5½ year long arc): the result using the atmospheric pressure modeling in the orbit computation (W) has the amplitude of the snow cover and surface water signal ($1.4 \cdot 10^{-10}$; Table 8.7) but nearly the opposite phase. The formal uncertainties of the estimates are too large, however, to be able to draw any other conclusion than that the nongravitational modeling errors are still at an unacceptably high level for this type of geophysical studies, at least when using LAGEOS-1.

Table 8.9: seasonal variations in $J_3$ determined with a 5½ year LAGEOS-1 arc and a 4½ year LAGEOS-2 arc, for the total (T) and hydrology and snow cover plus tides signal (W). Sine amplitude $A$ (in $10^{-10}$) and phase (degrees) relative to January 1.

<table>
<thead>
<tr>
<th>Solution: LAGEOS-1</th>
<th>$A_{\text{annual}}$</th>
<th>phase</th>
</tr>
</thead>
<tbody>
<tr>
<td>(T)$^\text{SH}$ *</td>
<td>$7.1 \pm 2.0$</td>
<td>$86 \pm 12$</td>
</tr>
<tr>
<td>(W)$^\text{SB}$</td>
<td>$4.8 \pm 1.6$</td>
<td>$95 \pm 15$</td>
</tr>
<tr>
<td>(T)$^\text{GM}$</td>
<td>$1.2 \pm 1.8$</td>
<td>$302 \pm 70$</td>
</tr>
<tr>
<td>(W)$^\text{GM}$</td>
<td>$1.4 \pm 1.1$</td>
<td>$70 \pm 35$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Solution: LAGEOS-2</th>
</tr>
</thead>
<tbody>
<tr>
<td>(T)</td>
</tr>
<tr>
<td>(W)</td>
</tr>
</tbody>
</table>

* nongravitational model parameterization according to Table 8.4

The LAGEOS-2 estimates have reasonable amplitudes and a formal error at the level of 10-30%. The effect of the atmospheric pressure force modeling is large for the degree 3 zonal coefficient, as was to be expected (see Appendix D), and is comparable for both satellites. One reason for the smaller estimates obtained with LAGEOS-2 is that the nongravitational forces acting on it are approximately one-third smaller than those acting on LAGEOS-1, which makes it less sensitive to errors in the nongravitational force model. The estimated amplitude when the atmospheric pressure has been taken into account ($0.7 \cdot 10^{-10}$) is smaller than the hydrology/snow cover signal, and with a phase that is $150^\circ$ smaller (Table 8.7), similar to the LAGEOS-1 results.
8.4 Discussion and Conclusion

However, as already mentioned before, the periodic variations in $J_3$ due to the hydrologic and snow cover cycle have not been accurately estimated [Chao and O’Connor, 1988].

The single-satellite solutions (Tables 8.8 and 8.9) logically have larger formal uncertainties than the dual-satellite solutions (Table 8.6 and 8.7), and two estimates are significantly different. The first difference is in the estimated semiannual variation in $J_2$ using LAGEOS-2, in particular the phase of 194° of the total signal. However, when the atmospheric pressure is used in the orbit computation, the phase becomes 230°, so closer to the LAGEOS-1 determined phase of 284°, and also closer to the tide model phase of 242°. The second difference concerns the annual variation in $J_3$, which becomes very large for LAGEOS-1 ($7.1 \times 10^{-10}$) employing the 'SB' nongravitational force model parameterization. This is not so in the dual-satellite solution, where the estimated amplitude is $1.7 \times 10^{-10}$. The LAGEOS-2 only estimate is also larger ($2.9 \times 10^{-10}$) than the dual-satellite result, which is in accordance with atmospheric pressure and snow and surface water models. Estimating the signal using both satellites simultaneously has thus largely aided in decorrelating the nongravitational modeling error from the geophysical signals.

8.4 Discussion and Conclusion

The ability of the mean-motion theory to accurately propagate orbital motion over long periods of time has been demonstrated. Thanks to its large integration step-size and its smaller force model (i.e., no tesseral harmonics) compared to purely numerical force modeling and integration, the gain in computational speed (i.e. CPU time) is at least an order of magnitude. These issues are important in mission analysis and satellite lifetime estimation, allowing the fast computation of several mission scenarios with negligible numerical truncation or round-off errors.

The mean force model of CODIOR is presently at the required high level of precision to be able to recover small geophysical signals, in particular the secular and periodic variation in the low-degree even zonal harmonics. The odd zonal harmonics, however, could not be estimated accurately due to their correlation with small nongravitational forces (notably thermal drag), which are still sources of brain-racking. The modeling of the atmospheric gravity effect in the long-arc computation has been performed for the first time to the author’s knowledge, and the higher accuracy of the orbit is demonstrated by the smaller residuals. Orbit errors due to this effect persist, however, because the atmospheric pressure over the oceans is not as well-known as that over the continents because there are much less meteorological stations. Secondly, the response of the ocean (IB, NIB or mixed) is not well-known. Thirdly, the perturbation model is not perfect.

The Starlette long arc of approximately 13 years validated the mean atmospheric drag modeling and allowed the testing of (some) atmospheric density models over slightly more than a solar cycle. The long-term accuracy of the DTM94 and MSIS86 models is at the few percent level;
their short-term accuracy is 20-25% (Chapter 6). DTM78 on the other hand, which is an older model, has an estimated long-term accuracy of approximately 17%.

It was not possible to recover the static part of the low-degree zonal harmonics with sufficient precision, the estimated corrections being too small. This implies a high accuracy of the a-priori zonal harmonics, which in this case were EGM96's, so actually not a surprising result. When GRIM4-S2 was used, a secular drift of several kilometers was detected on the perigee, a signal that would have been large enough to adjust static zonal coefficients.

The dual and multi-satellite solutions do not only have a high (formal) precision, as may be seen in Section 8.3.4 and Table 8.6 in particular, but also agree well with other reported solutions and geophysical model predictions and estimations. The secular variation in the low-degree zonal gravity field coefficients has only been estimated for the degrees 2 and 4 with good accuracy, in combination with the 18.6 year tide. Accurate recovery of the secular variation in the odd-degree zonals depends on the quality of the nongravitational force modeling. Only the (lumped) degree 3 coefficient has been estimated, although much less accurately than the even-degree terms. This is the case of several other studies as well, making nongravitational force modeling an important issue for the future. However, the value for $\dot{J}_3$ derived in this study is probably more accurate than most solutions, because of the extensive nongravitational force model employed in this study. This is confirmed by the obtained solution of the annual variation in $\dot{J}_3$, which agrees well with the sum of the geophysical signals (Table 8.6), contrary to the other satellite solutions. The future CHAMP and GRACE missions will not encounter this particular problem if the accelerometers that are in their respective payloads will function nominally. The good agreement of the results for $\dot{J}_2 - \dot{J}_4$ obtained with the CSR ([Cheng et al., 1997]; [Eanes and Bettadpur, 1996]) and the mean long-arc techniques, despite their different equations of motion and observations, demonstrates the consistency and validity of the 'long-arc' approach to recover secular signals.

To obtain a higher-degree solution of the rate of change in the zonal harmonic coefficients, more satellites are required, which preferably should not be much perturbed by atmospheric drag. Unfortunately, this only leaves Ajisai, a satellite with a large (for a geodetic satellite) area-to-mass ratio and an inhomogeneous surface, which makes its nongravitational force modeling even more difficult [Sengoku et al., 1996]. The tracking frequency of the Etalon satellites [Tatevian and Zakharov, 1989] is not sufficient and their orbits are too high ($a=19130$ km) to be sensitive to the small seasonal gravitational signals. Secondly, Etalon-1 has a perigee period of about 30 years, whereas for Etalon-2 this is even larger. This makes it, at least for the moment, impossible to represent even one perigee revolution, required in long-arc computations to accurately adjust the initial state vector.
8.4 Discussion and Conclusion

The solutions for seasonal variation obtained with LAGEOS-1, LAGEOS-2 and Starlette demonstrate the improved orbit accuracy by modeling the orbit perturbation due to the atmospheric mass redistributions directly in the long-arc computation. The effects on the even (degree 2) and odd (degree 3) degree terms were as expected, in accordance with the power spectra displayed in Appendix D. The annual variation observed in $J_2$, in the LAGEOS dual-satellite as well as the single-satellite total signal (‘T’ in Table 8.8) solutions, is in good agreement with the external estimate obtained by the sum of the IB atmospheric signal of Dong et al. [1996], the surface water and snow cover signal [Chao and O’Connor, 1988], and the ocean tide signal [Schwiderski, 1980]. It cannot be verified if the solutions are within error bar limits because the uncertainties of the reported satellite solutions are not given. In particular the LAGEOS solutions given in Table 8.8 obtained when modeling the effect of atmospheric pressure in the orbit computation also are consistent with the snow cover and surface water signal (the ocean tide signal is probably small), as was shown in Figure 8.13. The uncertainty of that signal (data for Antarctica and Greenland have not been assimilated) does not allow to identify the more accurate satellite solution. The solution for the semiannual variation obtained with LAGEOS-2 when modeling the effect of atmospheric pressure in the orbit computation appears to be more accurate than the one obtained with LAGEOS-1. The estimated phase (230°) is close to the phase of the sum of the snow cover and surface water and ocean tide signals (223°), but its amplitude is significantly larger. Thus, the semiannual variation is mainly due to the ocean tide; the snow cover and surface water signal is much weaker but not stationary, while the atmospheric contribution is the smallest. The atmospheric effect, calculated using meteorological pressure grids, seems to be larger using the NMC data than when using the ECMWF data. It is probably due to the data selection criteria and the processing method, but this difference should be investigated further.

The solutions of the annual variation in $J_3$ demonstrated their large dependence on nongravitational force modeling. The amplitude, estimated using LAGEOS-1, changed by a factor of approximately 5 just by changing the Yarkovsky-Schach model parameters (the amplitude and phase lag). The dual-satellite solution agrees well with the predicted geophysical total signal. The LAGEOS-1 solution, using the Métris et al. [1997] nongravitational model parameterization and taking the effect of atmospheric pressure in the orbit computation into account (‘W’ in Table 8.9), has the expected amplitude, based on the Chao and O’Connor [1988] model, but is approximately out of phase. The corresponding LAGEOS-2 solution is approximately out of phase also, but has an amplitude which is twice as small. The model uncertainty of the surface water and snow cover signal is largest for $J_3$ due to the geographical location of Antarctica and Greenland [Chao and O’Connor, 1988]. An error of 180° seems excessive, but can at present not be ruled out.
The LAGEOS-2 nongravitational force modeling appears to be more accurate than that of LAGEOS-1, indicated by the smaller rms-of-fit of the eccentricity residuals, but the force is smaller as well. As was stated before, a more accurate model for the hydrology and snow cover signal is required. However, it is not sufficient to have a single (i.e. an average estimated over the observation interval) estimate of the signal, since it is not sinusoidal with a constant amplitude, but it varies from year to year. The best solution may probably be obtained using daily precipitation data in a similar way as was done with atmospheric pressure data. However, for this scheme to function, the surface characteristics must be known too, which is not the case today. Secondly, snow-height data are nowadays available measured by satellite-borne altimeters [Wingham et al., 1998], allowing a more accurate snow cover model. The evolution of the second-degree zonal coefficient of the atmospheric pressure over 10 years (Appendix D) shows large interannual differences, which is the reason for modeling its perturbation directly during the orbit computation here.

It is typical of long-arc techniques that the gain in orbit accuracy, when parameters are estimated, can be quantified (smaller rms of the residuals). The signals are not recovered from the analysis of estimated parameters of a large number of ‘short’ arcs, but by computing the partial derivatives and residuals and solving them in a few least-squares iterations. The estimation of successive ‘effective’ monthly parameters and subsequent estimation of a secular and seasonal signal in such time series do not allow the direct computation of orbit residuals. This may cause a surprise when verification orbits are computed, since they may not be more accurate.

The information content of the observed mean elements is very well suited for this type of investigations, since they are very compact (10-25 Kb for over 10 years of data) and almost independent of the force models used in the filtered orbits. The smallest period that may be observed is limited by these same elements, which concentrate SLR measurements taken over one month (actually per 24 or 20 days) into one observation, so in a way similar to SLR normal points. Consequently, signals with periods shorter than 2 months (according to the Nyquist cut-off rule) may not be estimated with the mean-motion technique. However, in practice signals with periods shorter than 6 months should not be estimated with the long-arc technique and the observed mean elements presented in this study, because below this limit a myriad of periods is present in the residuals as may be deduced from the plots in Appendix D.
The first goal (Chapter 1) of this thesis has been achieved: the mean force models have been upgraded and additional force models have been implemented. These have all been validated by simulation and subsequently by long-arc adjustment.

Concerning the second objective of this thesis, the Mg II index has been validated as a solar activity proxy in the EUV/UV range for atmospheric drag modeling purposes. It is more representative of short-period solar EUV activity than F10.7 is, which was demonstrated by means of comparison of test model results. The smaller rms about mean of the DE-2 residuals at altitudes where they represent the major constituents show a 3 to 8% higher precision. The orbit tests with the Mg II-based test model show a bias and a scatter in the drag scale coefficients, which are up to 25% and 25-50% smaller, respectively, than those obtained with F10.7. Moreover, a better fit to the SLR tracking data is achieved. The density model based upon Mg II is more accurate than the one based upon F10.7 under high solar activity, where these proxies differ most, which demonstrates that the Mg II index is better suited to represent upper atmosphere heating processes. Due to data gaps, however, the index could not be used in the long-arc analysis. Since its representation of EUV radiation is more adequate than the solar radio flux data employed here, its use in the long arc would probably significantly improve the quality of the adjustment because of its positive contribution to the dynamical consistency.

The third objective of this thesis has for the most part been achieved. The simultaneously estimated geophysical parameters, the secular and seasonal variations in J2 and the 18.6 year tide in particular, confirm (the few) earlier published values, and their uncertainties have diminished. Moreover, they have been obtained by an independent method, which has been employed for the first time. The estimation of the geophysical parameters improves the LAGEOS-1 long-arc orbit fit for the ascending node to the level of 58 cm or 20 mas rms. This establishes the precision and stability over long periods of time of the mean motion model. The variations in J3, however, are still inaccurate due to remaining errors in the nongravitational forces despite the use of the most recent and extensive models.

Low-orbiting geodetic satellites, such as Starlette, are less suited for the estimation of small parameters, although their induced orbit perturbations are larger. This is mainly caused by the atmospheric density model, which at present (2002) is still too inaccurate. The atmospheric drag force decreases with increasing altitude. The LAGEOS altitude is the best compromise...
Conclusions and Outlook

between sensitivity to small perturbations and orbit accuracy, but a longer time series of LAGEOS-2 is required to estimate secular variations and the very long-period tides with the LAGEOS pair. The presently-orbiting satellites do not allow an accurate determination of the secular variation in zonal coefficients of the gravity field of degrees 6 and higher due to a combination of force model errors and too large sensitivities of certain satellites to high-degree coefficients that will contaminate the solution. The accurate estimation of odd harmonics requires a substantial and additional effort in nongravitational force modeling. The CHAMP (launched in July 2000) and GRACE (launched March 2002) missions will probably allow an accurate estimation of both the (lumped) secular and seasonal variations in the degrees 2 to 5 of the gravity field coefficients thanks to their accelerometers. Their orbit accuracies will in principle not depend on nongravitational force models, but instrument performance and data noise; these factors may be functions of time. However, they will depend on the accurate determination of the calibration parameters of their respective accelerometers, as well as on their (near)-nominal operation. Combining observations of these satellites with the long time series already existing today may enable the accurate estimation of the variations in the gravity field coefficients of degrees 6 and 8. They may also contribute to the improvement of the nongravitational force models employed in the computation of the LAGEOS long arcs by feedback of the CHAMP/GRACE solution of the variations in the zonal coefficients of degree 3 and 5.

The residual seasonal signal diminishes significantly by modeling the gravitational effect of atmospheric pressure directly in the orbit computation, and it improves the orbit fit. The dual-satellite solution for the annual and semiannual variation in $J_2$ and the annual variation in $J_3$ obtained with the LAGEOS pair is in good agreement with the sum of the atmospheric signal and the ocean tide and hydrology/snow cover models. The solutions of the annual and semiannual variations are always average values over the period under study. However, the atmospheric, surface water and snow cover signals are not stationary but may vary considerably from year to year. Therefore, the (sub)seasonal variations should be estimated on a yearly basis, which would allow for a better separation in contributing components. For that kind of analysis the long-arc technique can still be used if at least one set of mean observed elements per month is available, but it is not well-adapted to the problem. This analysis should be done by computing monthly gravity field solutions (i.e. in line with the GRACE mission), while the long-arc method is best adapted to retrieve secular- and very long-period signals.

A conclusion concerning (geophysical) observations in general is the following: their temporal and spatial continuity is in most cases inadequate. The SLR network is unevenly distributed over the globe, being mainly concentrated in Europe and the continental United States;
more stations in the Southern Hemisphere are required to increase orbit coverage. Fortunately, the observed mean elements are not very sensitive to the short-arc orbit accuracy due to the filtering procedure. The meteorological stations are also not evenly distributed, causing large uncertainties in global models, over Antarctica and Greenland in particular. Mainly for that reason the models for hydrology and snow cover/height in particular are inaccurate, or only valid locally. This makes the interpretation of the rate of change in $J_3$ difficult, since it contains the signal due to sea-level rise caused by melting of the ice sheets. The Mg II index was not available on a daily, continuous basis at the time of this study, which makes it inadequate for long-arc and also operational (short-arc) orbit computations. Moreover, the necessary density data for modeling purposes have not been collected since the DE-2 mission, which ended in 1983.

Geodesy, and gravity field modeling in particular, presently gain momentum thanks to the CHAMP and GRACE missions, and the future GOCE (Gravity field and steady-state Ocean Circulation Explorer; [GOCE, 1999]) mission. The impact of high-low Satellite-to-Satellite Tracking (SST) and accurately measured nongravitational accelerations with an accelerometer, which is the CHAMP configuration, on the gravity field is demonstrated by the EIGEN-1S model [Reigber et al., 2002] with only 3 months of data. The GRACE mission, which is based on the low-low SST concept (very precise inter-satellite distance measurements), will in case of nominal operation improve the accuracy of the current state-of-the-art gravity field models by 2 orders of magnitude. In addition, temporal gravity variations down to monthly periods will be determined by analysis of consecutive monthly solutions. The GOCE mission, representing the high-low SST combined with a gravity gradiometer concept, will be the sumum of static gravity field modeling: the geoid accuracy at degree 200 shall, mission configuration and instrument performance permitting, be at the 2 mm level.

Despite the extreme precision of the above-described missions, the long-arc analysis technique will contribute to the determination of the secular variations in the gravity field and to their separation from the very long-period tides. Moreover, the Starlette long-arc will soon benefit from both a more accurate atmospheric density model thanks to assimilation of CHAMP accelerometer-derived density data [Bruinsma and Biancale, 2002] as well as from the Mg II index. A continuous composite time series, which is constructed with data from several satellites, is presently available [Viereck et al., 2001]. The cumulative effect of the perturbations on the long arcs represents a second advantage of the technique: the accuracy of the estimation improves with the length of the arc, which in case of the geodetic satellites increases continuously. Therefore, it is expected that the contribution of the Starlette long arc to the determination of the secular variation in the low-degree zonal harmonics will significantly increase. Furthermore, the long-arc analysis
9 Conclusions and Outlook

will soon benefit from LAGEOS-2 also, which lifetime will attain 10 years in October 2002, so long enough to contribute to the estimation of the signal with 9.3 year period.

The long-arc technique is also used in other types of investigations: in lifetime estimation studies and the computation of the trajectories of large space debris, for example. These applications represent a large computational burden on the one hand, while on the other hand the short-period perturbations, which are most time-consuming to compute, are not required. These conditions are thus compatible with the concept of mean motion. CODIOR is employed by the division Orbital Mechanics of CNES for these studies because of its demonstrated capability of accurate and very fast extrapolation of the mean orbit over long periods of time.
References


References


References

Eanes, R.J., S. Bettadpur (1994) Ocean tides from two years of TOPEX/POSEIDON altimetry (abstract), Eos Trans. AGU, 75(44), Fall Meeting supplement, 61.


GOCE (1999) Gravity Field and Steady-State Ocean Circulation Mission; Reports for the four candidate Earth Explorer Core Missions, ESA SP-1233(1).


References


References


References


Appendix A

Reference Systems

A reference system consists of a reference frame, a time scale and constants and models to be used in the computations (for detailed reviews see [Sansò and Rummel, 1989], [McCarthy, 1996]). For reasons of convenience and tradition in geodesy, two Earth-centered reference systems are in use: a pseudo-inertial space-fixed system, and a terrestrial Earth-fixed system. The satellite motion is expressed in the pseudo-inertial space-fixed frame, while the tracking stations are on the Earth, and thus follow its daily rotation. The relation between these two frames can be expressed by three rotations, which are functions of time. A third reference frame often used in orbital mechanics is the orbital reference frame 'rtn', see figure a5.

The Conventional Inertial Reference System (CIRS) and the Conventional Terrestrial Reference System (CTRS) have been established by the International Earth Rotation Service (IERS) [McCarthy, 1996]. Figure a1 represents the pseudo-inertial reference frame, which has the Earth Center-of-Mass as origin. To transform the terrestrial reference frame, which has the same origin as the inertial one, and whose X-axis points to the Greenwich meridian and whose Z-axis points to the mean pole of rotation, to the inertial reference frame, three rotations have to be performed:
1. The polar motion x correction to the Z-axis (figure a2)
2. The polar motion y correction to the Z-axis (figure a2)
3. The angle corresponding to the daily rotation, GAST (figure a3)

Figure a1: visualization of the inertial reference frame (X-axis passes the CoM of the Sun)
Appendix A

**Figure a2**: the polar motion correction (x,y), and its evolution with respect to the mean pole CIO.

**Figure a3**: the Greenwich Apparent Sidereal Time (GAST), UT1, UTC and the atomic time TAI.
Appendix A

Figure a4: the orbital elements $a$, $e$, $\Omega$, $\omega$, in the orbit plane and the Kepler equation for computing $M$. The inclination $i$ is the angle between the inertial X-Y plane and the orbit plane.

Figure a5: the orbital reference frame $(r,t,n)$ with respect to the inertial reference frame at epoch.
Appendix A
Appendix B

Numerical Integration Errors

Figure b1: round-off error test result, in m, for the 10th order Bulirsch and Stoer algorithm (Starlette orbit spanning 4300 days; x-axis), with a 1200 s step-size.
Figure b2: round-off error test result, in m, for the 8th order Adams-Bashforth-Moulton integrator (Starlette orbit spanning 4300 days; x-axis), with a 60 s. step-size.
Appendix C

Gravitational Perturbations

Figure c1: the LAGEOS-1 orbit perturbation due to luni-solar attraction (x-axis: days).
Appendix C

Figure c2: the Starlette orbit perturbation due to solid Earth tides with $k_2=0.299$ (x-axis: days).
Appendix C

Figure c3: the Starlette orbit perturbation due to the diurnal tidal constituents, using Schwiderski’s model [1980] (x-axis: days).
Appendix C

Figure c4: the Starlette orbit perturbation due to the semidiurnal tidal constituents, using Schwiderski’s model [1980] (x-axis: days).
Figure c5: the LAGEOS-1 orbit perturbation due to the 18.6 year tide with an amplitude of 1.64 cm (x-axis: days)

<table>
<thead>
<tr>
<th>Figure</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Semimajor axis (μm)</td>
<td>1.0</td>
</tr>
<tr>
<td>Eccentricity (mm)</td>
<td>1.1</td>
</tr>
<tr>
<td>Inclination (cm)</td>
<td>0.7</td>
</tr>
<tr>
<td>Ascending node (m)</td>
<td>14.9</td>
</tr>
<tr>
<td>Argument of perigee (m)</td>
<td>2.4</td>
</tr>
<tr>
<td>Argument of perigee plus mean anomaly (m)</td>
<td>-7.2</td>
</tr>
</tbody>
</table>
Figure c6: the LAGEOS-1 orbit perturbation due to $\dot{J}_2 = -2.71 \cdot 10^{-11}/yr$ (x-axis: days).
Figure c7: the LAGEOS-1 orbit perturbation due to $J_3 = -1.0 \times 10^{-11}/\text{yr}$ (x-axis: days).
Figure c8: the LAGEOS-1 orbit perturbation due to an annual variation in $J_2 = 1.8 \times 10^{-11} / \text{yr}$ (x-axis: days).
Figure c9: the LAGEOS-1 orbit perturbation due to an annual variation in $J_5 = 2.9 \cdot 10^{-11} \, \text{yr}^{-1}$ (x-axis: days).
Figure c10: the LAGEOS-1 orbit perturbation due to the $J_2$ term of the atmospheric pressure field (x-axis: days).
Figure c11: the LAGEOS-1 orbit perturbation due to the $J_3$ term of the atmospheric pressure field (x-axis: days).
Appendix D

Perturbation due to Atmospherical Pressure

Figure d1: power spectrum of the $J_2$ coefficient of the atmospherical pressure fields (10 years of ECMWF data), x-axis in days (top plot) and years (bottom plot).

Figure d2: power spectrum of the $J_3$ coefficient of the atmospherical pressure fields (10 years of ECMWF data), x-axis in days (top plot) and years (bottom plot).
Figure d3: power spectrum of the $J_4$ coefficient of the atmospherical pressure fields (10 years of ECMWF data), x-axis in days (top plot) and years (bottom plot).

Figure d4: power spectrum of the $J_5$ coefficient of the atmospherical pressure fields (10 years of ECMWF data), x-axis in days (top plot) and years (bottom plot).
Figure d5: power spectrum of the $J_6$ coefficient of the atmospherical pressure fields (10 years of ECMWF data), x-axis in days (top plot) and years (bottom plot).

Figure d6: power spectrum of the $J_7$ coefficient of the atmospherical pressure fields (10 years of ECMWF data), x-axis in days (top plot) and years (bottom plot).
Appendix D

Figure d7: power spectrum of the $J_8$ coefficient of the atmospheric pressure fields (10 years of ECMWF data), x-axis in days (top plot) and years (bottom plot).

Figure d8: degree-variance (power spectrum where the orders are summed per degree) of the atmospheric pressure field for a specific date.
Figure d9: temporal evolution of the $J_i$ coefficient of the atmospherical pressure fields (10 years of ECMWF data). The data has been low-pass filtered, with a filter cut-off of 150 days.
Appendix E

Nongravitational Perturbations

Figure e1: the Starlette orbit perturbation due to direct solar radiation pressure with Earth shadowing effect (x-axis: days).
Figure e2: the Starlette orbit perturbation due to atmospheric drag employing the DTM94 model (x-axis: days).
Figure e3: the LAGEOS-1 orbit perturbation due to the Yarkovsky effect (phase lag = 53.85°, amplitude $0.3 \cdot 10^{-11} \text{ms}^{-2}$). The x-axis represents the arc-length in number of days.
Figure e4: the LAGEOS-1 orbit perturbation due to the Yarkovsky-Schach effect (phase lag = 2.5 rad, amplitude $-1.0 \cdot 10^{-10} ms^{-2}$). The x-axis represents the arc-length in number of days.
Figure e5: the LAGEOS-1 orbit perturbation due to the anisotropic reflectivity of the two satellite hemispheres (amplitude $0.1 \cdot 10^{-10} \, \text{ms}^{-2}$). The x-axis represents the arc-length in number of days.
Appendix E
Appendix F

Typical SLR data distribution for LAGEOS-2 and GFZ-1 orbits

Figure f1: the SLR tracking passes for a typical LAGEOS-2 10-day arc (top), and an average GFZ-1 4-day arc (bottom).
Semianalytische lange-arc satellietbaan berekening en het schatten van tijdsafhankelijke zwaartekrachtparameters

Samenvatting

Deze studie heeft betrekking op de lang-periodieke eigenschappen van satellietbanen en de geofysische grootheden die men hieruit kan afleiden. De drie doelstellingen van dit proefschrift zijn:

1. het implementeren, verbeteren en testen van krachtmodellen voor de berekening van langdurige satellietbanen;
2. het verbeteren van de modellering van atmosferische weerstand, aangezien dat de grootste niet-gravitationele fouten veroorzaakt in de berekening van lage satellietbanen;
3. het schatten van seculaire en (lang-)periodieke geophysische parameters in mono- en multi-satelliet oplossingen, gebruikmakend van geschikte, geodetische satellieten.

De eerste doelstelling moet resulteren in een theorie en een bijbehorend computerprogramma dat nauwkeurige langdurige satellietbanen kan berekenen. De tweede doelstelling heeft tot doel de nauwkeurigheid van het model voor atmosferische dichtheid te verbeteren door gebruik te maken van een alternative indicator voor zonne-activiteit die meer representatief is voor zonne- emissie in het (extreem) ultraviolette spectrum (EUV) dan de nu gebruikte radioflux. Het uit de eerste doelstelling resulterende computerprogramma wordt vervolgens gebruikt om de derde doelstelling te verwezenlijken, namelijk het schatten van tijdsafhankelijke componenten van het aardse zwaartekrachtveld.

Het reeds bestaande lange-baan berekeningsprogramma CODIOR, gebruikt door de divisie ‘Mathématiques Spatiales’ van het Franse ruimtevaartagentschap CNES, en de bijbehorende theorie voor gemiddelde bewegingsvergelijkingen van een satelliet (ontwikkeld door Exertier [1988] en Métris [1991]) zijn gebruikt als beginpunt om de eerste doelstelling te verwezenlijken. Deze theorie modelleert de satellietbeweging door gemiddelde bewegingsvergelijkingen numeriek te integreren; kort-periodieke baanverstoringen worden hierbij niet in rekening gebracht. Dit maakt integratie met een stapgrootte van 3-12 uur mogelijk, evenals snelle baanberekening over tijdvakken van enkele tienduizenden baanomwentelingen. Het initiële model voor de gemiddelde baanbeweging bleek niet geschikt voor lage satellietbanen, vanwege de afwezigheid van hoge-orde termen in het model voor de aardse zwaartekracht en, met name, van een luchtrestandsmodel. Tijdens deze studie is een gemiddeld atmosferisch weerstandsmodel aan de software toegevoegd.
Samenvatting

en getest, en zijn alle overige modellen gecontroleerd en uitgebreid, wanneer dit noodzakelijk bleek te zijn uit het oogpunt van precieze lange-satellietbaan berekening.

De modellering van het zwaartekrachtveld is uitgebreid tot en met graad 40 gebruikmakend van Lie transformaties [Métris, 1991], terwijl voor graad 41-70 de gemiddelde formulering van Kaula [1966] is toegepast. De modellering van de gravitatie van de maan, zon en de planeten, alsmede de vaste aardgetijden hoefden niet te worden verbeterd, maar het model voor de gravitatiekracht uitgevoedend door het oceaangetijde is uitgebreid. De \( N_2 \) en \( M_2 \) (12-uurs) getijden, en de lang-periodieke \( M_n \) en \( M_f \) golven zijn aan het gravitationele krachtmodel toegevoegd, en de ontwikkeling gaat nu tot en met graad 10 in plaats van 6. Het model voor de beschrijving van oppervlaktekrachten (d.w.z. niet-gravitationele krachten) is gecompleteerd met een model voor atmosferische weerstand en albedo. Alle gemiddelde krachtmodellen zijn gecontroleerd door vergelijking met numerieke integraties die gebruik maken van een zeer robuust algoritme [Balmino en Barriot, 1990] gevolgd door een filterprocedure [Exertier, 1990]. Op deze wijze is de precisie, hoofdzakelijk begrensd door de modellering van luchtweerstand, van het gemiddelde bewegingsmodel vastgesteld: 3 cm rms voor de halve lange as en een maximale lineaire drift van 1.3 m voor de eccentriciteit over een tijdvak van 10 jaar (waarden in meters worden hier verkregen door vermenigvuldigen met de halve lange as). De precisie van het gravitationele bewegingsmodel resulteert in baanfouten die een orde van grootte kleiner zijn. Een specifiek krachtmodel, dat de effecten van eigenstraling en anisotrope reflectie-eigenschappen van de (halfronden van de) LAGEOS satellieten bevat, is toegevoegd. De validatieprocedure door vergelijken met precieze numerieke integratie was niet nodig, omdat dit model direct in gemiddelde baanparameters is ontwikkeld. De eerste doelstelling is hiermee verwezenlijkt.

Onnauwkeurige modellering van luchtweerstand veroorzaakt de grootste fouten in de baanbepaling van satellieten in lage banen (tot 1000 km hoogte). De luchtdichtheidmodellen uit de serie DTM (Drag Temperature Model; [Barlier et al., 1978]; [Berger et al., 1998]) zijn gecreëerd om de atmosferische componenten goed weer te geven, met een directe toepassing voor precieze baanberekening. Om de inbreng van de zonne-energie te kwantificeren gebruiken deze (en andere) modellen de zonne-radioflux met een golflengte van 10.7 cm (\( F_{10.7} \)), vanwege zijn correlatie met de EUV straling en zijn ononderbroken observatie. Deze flux is echter niet representatief voor de activiteit van de chromosfeer over een tijdsbestek korter dan een zonnecyclus: de verhitting van de hoge atmosfeer wordt recentelijk aan de chromosferische straling toegeschreven, waarvoor de Mg II index [Heath and Schlesinger, 1986] als een goede proxy-indicator geldt. Over een tijdvak van 19 jaar is de correlatie tussen de Mg II index en \( F_{10.7} \) bijna 100%, maar vergelijking over periodes van enkele weken toont soms grote verschillen.
Om de tweede doelstelling van dit proefschrift te verwezenlijken zijn twee testmodellen gecreëerd onder identieke verevenningscondities (dichtheidsgegevens van de satelliet Dynamics Explorer 2, verkregen over een periode van 1½ jaar), waarbij in één geval de Mg II index en in het andere geval $F_{10,7}$ is gebruikt als indicator voor de zonne-activiteit. Vergelijking met partiële luchtdichtheidsgegevens gaf een 3-8% hogere precisie aan van het model dat de Mg II index gebruikt. Een externe test, door middel van precieze baanberekeningen, liet een 25% kleinere systematische fout en 25-50% kleinere dispersie van de ggemoduleerde luchtdichtheiden zien, alsmede een tot 20% nauwkeuriger beschrijving van de satellietbaan, getoetst aan onafhankelijke baanobservaties. De tweede doelstelling van dit proefschrift is hiermee vervuld, aangezien de modellering van de atmosferische weerstand aanzienlijk wordt verbeterd door gebruik te maken van de Mg II index. Helaas is de Mg II index niet dagelijks en continu beschikbaar, waardoor hij niet bruikbaar is voor lange-baan berekeningen of operationele doeleinden.

Oplossingen van het baangedrag van satellieten over lange termijn zijn berekend via verevenning met gemiddelde observaties, ('geobserveerde gemiddelde baanelementen' genoemd), waarbij gebruik is gemaakt van recente modellen, met uitzondering van een relatief oud model voor het oceaangetijdige van Schwiderski [1980]. Dit maakt het schatten van bepaalde geophysische parameters mogelijk, hetgeen de derde doelstelling van dit proefschrift is. Deze gemiddelde observaties zijn verkregen door het filteren van satellietbanen (van 20-24 dagen lang) die zijn verevenfend met Satellite Laser Ranging (SLR) afstandswaarnemingen, en herleid naar (gemiddelde) baanparameters op het tijdstip midden in het betreffende tijdsinterval. De geodetische satellieten Starlette, LAGEOS-1 en LAGEOS-2 zijn voor deze studie gebruikt, omdat de nauwkeurige SLR metingen die voor deze satellieten beschikbaar zijn een analyse over een lang tijdsbestek mogelijk maken, en omdat de modellering van oppervlaktekrachten vergemakkelijkt wordt door de sferische vorm van deze satellieten. De baan van Starlette (perigeumhoogte ongeveer 800 km), berekend over een tijdsbestek van 14 jaar, heeft een geschatte nauwkeurigheid van 6.7 cm rms voor de halve lange as en 3.7 m rms voor de klimmende knoop. De residuen van de eccentriciteit zijn aanzienlijk, omdat een koppelingseffect tussen de weerstandskracht en de zwaartekracht niet volledig is ggemodelleerd. De oplossing voor de baan van LAGEOS-2 (perigeumhoogte ongeveer 5800 km), met een lengte van 4½ jaar, heeft een nauwkeurigheid van 0.9 cm rms voor de halve lange as en 0.8 m voor de klimmende knoop. De eccentriciteit heeft een geschatte nauwkeurigheid van 0.3 m rms. De lengte van deze baan is slechts voldoende om seizoenseffecten te schatten. De opgeloste baan van LAGEOS-1 (perigeumhoogte ongeveer 5900 km; lengte 17 jaar) tenslotte, heeft een geschatte nauwkeurigheid van 4 cm rms voor de halve lange as en 0.6 m voor de klimmende knoop. Door zijn lengte en nauwkeurigheid levert deze baan de belangrijkste bijdrage aan de schatting van de seculaire variatie van de even zonale harmonische coëfficiënten van het
Samenvatting

aardse zwaartekrachttveld en het oceaan getijde met een periode van 18.6 jaar. De residuen van de
eccentriciteit, met een rms van 1.2 m, zijn te groot om de variatie van de oneven zonale
coefficiënten nauwkeurig te kunnen schatten. De thermische krachten, waarvan de nauwkeurigheid
in hoge mate afhanger van het model voor de rotatie van de satelliet, en de albedomodellen zijn
vooralsnog niet nauwkeurig genoeg. Een tweede LAGEOS-1 baanoplossing, met een lengte van
5½ jaar, is berekend voor een tijdvak dat de LAGEOS-2 baan overlap. De schattingen van
seizoenseffecten met verschillende satellieten zijn op deze wijze vergeleken. Een gecombineerde
oplossing (d.w.z gebaseerd op beide satellieten) is eveneens berekend.

Door de baaninformatie van de 3 bovengenoemde satellieten te combineren is een
oplossing van de seculaire variatie van de zonale coëfficiënten tot en met graad 4 en het 18.6 jaar
oceaan getijde verkregen. Variaties van de termen met graad 5 (en hoger) kunnen niet worden
geschat met een redelijke nauwkeurigheid: deze laatste is groter dan de geschatte parameter zelf en
het schattingsproces wordt numeriek instabiel. De lineaire tijdsvariaties in de tweede- en vierde-
graads zonale coëfficiënten zijn nauwkeurig bepaald (respectievelijk -2.7±0.2 en –1.1±0.3, in 10^{-11}/jaar), hetgeen te danken is aan de kwaliteit van de modellering van de klimmende knoop. De
seculaire variatie van de zonale term van de derde graad is afhankelijk van de modellering van de
eccentriciteit en het perigeum, en is dus een functie van de nauwkeurigheid van het niet-
gravitationele krachtmodel. Ondanks de goede vergelijkbaarheid van de in deze studie geschatte
waarde (-1.1±0.4)·10^{-11}/jaar met het merendeel van eerder gepubliceerde waarden, is bij gebruik
en interpretatie voorzichtigheid geboden. De amplitude van het oceaan getijde met een periode van
18.6 jaar is op 1.2 cm geschat (gecorrigeerd voor de anelasticiteit van de Aarde). De fase is
geschat op 90°, hetgeen overeenkomt met de evenwichtsfase [Trupin and Wahr, 1990]. Een groot
signaal met een periode van 9.3 jaar is aanwezig in de observaties, maar de bron is onbekend. Dit
signaal is eveneens waargenomen door Cheng et al. [1997].

De gecombineerde oplossing van de seizoenseffecten in J_2 en J_3 met jaarlijkse en
halfjaarlijkse perioden uit de SLR observaties van de beide LAGEOS satellieten komt goed
overeen met eerdere publicaties. De oplossing is een goede benadering van de som van de signalen
van verplaatsing van atmosferische massa, grondwater en sneeuwbedekking [Chao and O’Connor,
oceanen met de atmosfeer geen eenvoudig mechanisme is. Bekeken over een lange periode is het
evenwel waarschijnlijk dat deze interactie meer de modellering van de zogenaamde inverse
barometer benadert dan het tegengestelde model.

De gemiddelde gravitationele baanverstoring veroorzaakt door de atmosfeer [Gegout,
1995], direct en indirect (deformatie van de Aarde en de oceaanbodem), is toegepast in de lange-
baan berekening. Dit verwijdt het grootste gedeelte van het atmosferische signaal uit het totale
Samenvatting

geschatte signaal (de som van atmosfeer, grondwater en oceaan getijde) en derhalve heeft de schatting betrekking op de seizoensvariaties alleen veroorzaakt door grondwater en oceaan getijde. De schattingen van de jaarlijkse variatie van $J_2$ en $J_3$ hebben zoals verwacht ongeveer de amplitude van het grondwater signaal. De schattingen op basis van (afzonderlijk) LAGEOS-1 en LAGEOS-2 zijn echter significant verschillend, terwijl de grootste amplitude wordt verkregen met Starlette (maar binnen de onzekerheids grenzen van LAGEOS-2). Vooral de schatting van de jaarlijkse variatie van $J_3$ vertoont een groot verschil met betrekking tot het grondwater- en sneeuwbedekkingssignaal: de fase is tegengesteld. De amplitude, echter, komt overeen, hetgeen niet het geval is in eerder gepubliceerde oplossingen ([Nerem et al., 1993]; [Gegout and Cazenave, 1993]). Het geschatte signaal met een periode van 6 maanden is groot en lijkt voornamelijk veroorzaakt door het oceaan getijde (het gmodelleerde hydrologische signaal is klein). Een aanzienlijk faseverschil ($50^\circ$) bestaat tussen de afzonderlijke schattingen gebaseerd op respectievelijk LAGEOS-1 en LAGEOS-2 banen. De derde doelstelling is voor het grootste deel gehaald, zoals blijkt uit de bovenvermelde resultaten; alleen de schatting van de variatie van $J_3$ is nog te onnauwkeurig voor geofysische toepassingen.

Auteur: S.L. Bruinsma
Acknowledgements

I wish to thank Pierre Exertier for giving me the opportunity to do Ph.D research in his department, for his help in accomplishing this thesis, and for all kinds of support during my years spent in Grasse. I acknowledge the efforts of Gilles Métris, who taught me the discipline of Celestial Mechanics and computed the Hamiltonians. The enthusiasm and support of François Barlier have also contributed largely to this study. I thank all my former colleagues of OCA/CERGA for the good time I had during my stay at the observatory, Yves Boudon and Pascal Bonnefon in particular.

I acknowledge the contribution of Gérard Thuillier of the CNRS/Service d'Aéronomie to the atmospheric research presented in this thesis.

I thank my supervisor Ron Noomen for his help and the time spent over the years in improving this thesis. The careful reading of this thesis and the useful comments from the reading committee are also gratefully acknowledged. I wish to thank Prof. K.F. Wakker and all the members of the Ph.D committee for reading this thesis.

Finally, I thank Georges Balmino from CNES for his help in my making it to the finishing line.
Curriculum Vitae

Sean Lanfrey Bruinsma

Born 30 December 1968 in Leeuwarden, The Netherlands

1995 – 1998  Ph.D. Student at the Observatoire de la Côte d'Azur, department CERGA, Grasse, France.