"Frequency and Ventilation"
A survey of theoretical and experimental ventilation modelling

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Nomenclature

a  tube radius
B,C  correction factors for turbulent flow and entry effects
d  tube diameter
D(x)  energy dissipation rate in a tube per unit of length
E  energy dissipation rate in a tube per unit volume
F  total energy dissipation rate
FR  flow rate
G,H,J  coefficients used for calculating the tube cross-sectional surface area
l  tube length
l_0 lg  coefficient for calculating the tube length
n  generation number
p_k  kinetic pressure drop
p_s  static pressure drop
P_v  viscous pressure drop
Δp  pressure difference
Q  volume flow
Q_0, Q_3, Q_9  correction factors for pipe elasticity
r  radius
Re  Reynolds number according to the tube diameter
S  tube cross-sectional area
S_0, S_l  coefficient for calculating the tube cross-sectional area
TT(n)  number of tubes in the nth generation
u  velocity component in x direction
u̅  averaged axial velocity
U_0  undisturbed axial velocity
U_tr  axial velocity in the trachea
v  velocity component in y direction
V  lung volume
V_d  anatomical dead space
V_t  tidal volume
w  velocity component in z direction
W_p  work done by the pressure forces
x,y,z  cartesian coordinates
x,r,θ  cylindrical coordinates
\( Y \)  
ratio of the actual energy dissipation and the energy dissipation in a Poiseuille flow per unit length

\( Z \)  
ratio of the length integrated energy dissipation of the actual flow and a Poiseuille flow

\( \delta \)  
boundary layer thickness

\( \rho \)  
density of air

\( \mu \)  
dynamic viscosity

\( \nu \)  
kineamic viscosity

\( \theta \)  
angle in cylindrical coordinates

\( \omega \)  
angular frequency

subscripts

\( \text{p} \)  
Poiseuille flow.
1. Introduction

The final goal of this investigation is the establishment of a relation between ventilation efficiency and frequency. A first step in this process is the construction of a lung model. The lung model contains many material, geometrical and flow parameters. Obviously a simple model is preferred. Therefore some insight in the influence of the various material, geometrical and flow parameters in the lung model on the output quantities has to be obtained.

In order to get this insight a computer-program is constructed to analyze the influence of the various parameters in a human lung. The output quantities are for instance: concentration of the different gases, pressure-drop across the bronchii, flow velocities and diffusion rates. The conditions to be analyzed are normal ventilation, maximum expiration and high frequency ventilation occurring in human beings in the range from early born infants to adults.

In the literature various lung models can be found and also different aspects of respiration are considered. In this report a survey is given of existing models for the different aspects to be studied. The applicability in the computer-program is emphasized.
Explanation of terms used

Because of the use of some terms which may not be familiar to the reader, a short explanation will be given of the ones used. See the following figure.

![Diagram of lung structure](image)

Figure 1.1

The air we inhale enters via the mouth and/or nose. It passes a restriction (the larynx) before it enters the first tube (the trachea). In the model used each tube splits up in two identical tubes. So the number of tubes will increase following the simple formula $2^n$ with $n=0,1,2 \ldots$. For $n=0$ this gives one tube (the trachea) for $n=1$ two tubes, for $n=2$ four tubes etc. The number $n$ is called the generation number. A normal lung of an adult human exists of 23 generations. So at the lower side of the lung the tube system ends in $2^{23}=8,388,608$ tubes. These tubes end in small sacs (the alveoli). In a lung the alveoli start to appear at generation number 16 and their number will increase till generation 23 where the tube system ends. The alveoli are lined with a fluid to release their surface tension, and which is called the surfactant.
Besides terms used in lung geometry, also terms used in breathing have to be clarified. The following figure gives a typical view of the lung volume against the time.

![Graph](image)

Figure 1.2 An explanation of the abbreviations used.

(One must keep in mind that the lung considered, is the lung of an adult).

TLC = Total Lung Capacity.
   The amount of air that a lung contains after the deepest inhalation.

RV = Residual Volume.
   The amount of air which is still in the lung after the deepest expiration.

FRV = Functional Residual Volume.
   The amount of air you normally have in your lung after a normal expiration.

TV = Tidal Volume.
   The amount of air of a normal inhalation.

During inhalation the freshly inhaled air has first to fill the tube system before it reaches the alveoli. This amount of air has to be subtracted from TV if one wants to know the amount of air which effectively refreshes the alveoli where gas exchange with the blood can occur. It is called the anatomical dead space.
2. Parameters

In order to construct a flexible program first a list is made of input parameters. The input parameters are divided into three categories, geometrical quantities, material quantities and flow quantities.

For the lung model variation of the following parameters is desired.

geometrical parameters:
1 - number of generations of bronchii
2 - number of tubes per generation
3 - length and diameter of the tubes
4 - total lung volume
6 - (a-)symmetry of the lung
7 - anatomical dead space
8 - surface area

material parameters:
1 - tube elasticity (may differ per generation)
2 - surfactant

flow parameters:
1 - tidal volume
2 - flow rate
3 - ratio of inspiration and expiration time
4 - frequency

In this study especially the influence of the geometrical and material parameters on the flow parameters mentioned above will be emphasized, the final goal being the determination of an individual optimum ratio of inspiration and expiration time and frequency.

Of course the list of parameters mentioned above is not exhaustive. The more detailed the model is made, the more parameters can be distinguished. But the chosen ones are seen as a compromise between validity of the model and the computational effort.
3. Model boundaries

The model boundaries are formed by the trachea on the upperside and the alveoli on the lower side. The influence of the first part of the ventilation system formed by mouth, nose, throat and larynx is neglected. On the lower side the gas exchange between air and blood over the alveolar membrane is modelled by means of data obtained from the literature. Also energy consumption during ventilation in this model is not considered as a limiting constraint. During ventilation energy is consumed and exchanged between the following mechanisms:

1. Elasticity of the lungtissue, compliance work = (Δp * ΔV)/2
2. Lungtissue viscosity, energy dissipation by viscosity work
3. Airway resistance, energy dissipated in the airflow
4. Kinetic energy of the flow and the thorax wall, inertial work.

- Terms 2 and 3 form the so-called viscous pulmonary resistance.

These mechanisms needs some explanation.

1) Elasticity of the lungtissue is caused by the tension in the elastic fibres of the lung parenchyma, and the surface tension of the alveoli. These elastic forces tend to reduce the lung volume. Normal breathing therefore requires energy to inhale, while expiration is passive, and will be done by the elasticity of the lungtissue.

![Figure 3.1](image)

In the figure above the p-V curve for only elastic effects is the line AB. A straight line, because without viscous effects the lung volume will directly
change according to the pressure change. The area of the triangle ABC is defined as the compliance work.

2) Lungtissue viscosity is associated with the friction in the tissues and the non-elastic deformation of the thorax and abdomen. Normally however 80-90% of the viscous pulmonary resistance is caused by airway resistance (mechanism 3) and only 10-20% by lungtissue viscosity resistance.

3) Airway resistance is the resistance caused by the viscosity of the flow passing through the conducting airways from the mouth to the alveoli. It is due to the resistance of the laminar or turbulent boundary layer in the pipes. The viscous terms cause the line AB to deform in a loop AEBFA. For instance inspiration is achieved by lowering the pressure in the lung by increasing its volume. The lung volume achieved, by a certain pressure drop, will be less than the lung volume achieved if only elastic forces were working, due to the viscous resistance.

4) The inertial resistances are assumed to be so small compared with the elastic and viscous resistances that they may be neglected.

During natural ventilation an energy balance exists between the mechanisms mentioned above and the work done by the muscles. The participation of the various mechanisms in the balance depends on the ventilation frequency. Inversely the natural ventilation frequency range and also the optimum ventilation frequency is influenced by maintenance of the energy balance.

During artificial ventilation, in which a person is ventilated by a ventilatory machine outside his natural respiration, an extra work term is induced in the balance. This source term is contributed by the external ventilation apparatus. Therefore during artificial ventilation the frequency range is no longer limited by the maintenance of the natural energy balance and hence an optimum artificial ventilation frequency is no longer influenced by the maintenance of this balance.
4. Ventilation mechanisms

During both natural and artificial ventilation in general five mechanisms may be distinguished. In the five mechanisms two physical principles are involved:

a. Convection
b. Diffusion of one gas with respect to other gases.

Depending on the part of the lung considered one or more of the five ventilation mechanisms play a role. The five mechanisms, which will be explained more thoroughly later, are (for explanatory figures see page 13):

1. Direct (convective) ventilation. This kind of ventilation for instance applies for the alveoli close to the mouth. It should be noted that this mechanism is not effective for a tidal volume (Vt) smaller than the anatomical dead space (Vd).

2. "Pendelluft"-mechanism. The "pendelluft"-mechanism is a convective mechanism and it corresponds to the relaxation phenomenon occurring in the transition phase between inhalation and expiration when internal pressure differences are equalized. The pressure differences developed during inhalation because of the asymmetry in the lung geometry.

3. Effective gas transport caused by differences between the velocity profiles occurring during inhalation (inspiration) and expiration.

4. Taylor dispersion (=augmented diffusion) (ref 11). In the flow of a viscous medium with an axial concentration gradient through a (branched) tubesystem axial dispersion of species originates from convection and diffusion. If the flow is unsteady an extra contribution is added to the axial dispersion. This contribution originates from an interaction between axial convection and radial diffusion.

5. Molecular diffusion. In this process the molecules move from a place where their concentration is high to a place where it is lower, the work of displacement being supported by the kinetic energy of the molecules. If
convective velocities are small diffusion becomes the dominating transport mechanism.

In the remaining part of this paragraph various aspects of the five mechanisms will be elaborated.

During artificial ventilation at higher frequencies usually the direct (convective) ventilation mechanism is assumed to be of negligible influence since in that case the tidal volume is smaller than the anatomical dead space. In the past it was assumed as a rule of thumb that overall ventilation efficiency depends only on the volume of air ventilated per unit time. Hence with a constant ventilation efficiency, an increase of the ventilation frequency corresponds to a decrease of the tidal volume.
However as soon as $V_t < V_d$, further increase in ventilation frequency should affect the efficiency since the alveoli no longer are reached directly by the small tidal volume. Hence for high frequency ventilation the direct convection mechanism is only important in the higher airways.

To clarify the process of the "pendelluft" mechanism one is referred to figure 4.2. It shows a pipe splitting up into two pipes ending into two sacks. The pipes leading to the sacks may have different resistances and the sacks may have different compliances. The product of compliance times resistance gives a time constant. This time constant governs the rate of charge and discharge of a sack in the presence of a given pressure gradient. So if unit A has the smaller time constant it is observed that at the end of an inspiration unit A is ready to empty while unit B is still filling. Air thus flows from unit A to unit B. This is called "pendelluft".

The importance of the "pendelluft" mechanism depends on the lungmodel adopted. In the literature often the lungmodel of Weibel (ref 12) is used. This is a symmetrically branched model and therefore the "pendelluft" mechanism plays no role. If however an asymmetrical model is used as suggested by Horsfield and Cummings (ref 13) the "pendelluft" indeed is of importance.

Differences in velocity profiles between inhalation and expiration result in a redistribution of the time average transport over the cross-section of the lungtubes. Although the transport averaged over both, the cross-section and the
time is zero because of continuity, the time averaged transport in the neighbourhood of the centerline of the tube may be non-zero and is balanced by a time averaged counterflow near the wall. During high frequency ventilation this mechanism is enhanced since the generated pressure oscillations act differently on the viscous flow in the boundary layer and on the inertial flow near the centerline.

The same enhancement applies for Taylor dispersion, which is supposed to be the main transport mechanism during high frequency ventilation. Because of the generated velocity profile a radial concentration gradient develops resulting in radial diffusion of species, which again results in an enhanced axial transport of species.

Summarizing it should be noted that during high frequency ventilation descending down the airways the species transport mechanism shift from unsteady pure convection in the upper airways via a time averaged interaction between convection and diffusion to almost steady diffusion in the neighbourhood of the alveolair membranes, where convection velocities are negligible.
Figure 4.1  pure convection

Figure 4.2  "pendelluft" - mechanism

Figure 4.3  respiration with different velocity profiles for in- and expiration

Figure 4.4  Taylor dispersion

Figure 4.5  pure diffusion
5. Strategy

During the construction of the computer model the various ventilation mechanisms will be incorporated one by one and the influence of the different variables on the output parameters will be analyzed step by step.

In a first step a quasi-steady convection model is made for the inhalation and the expiration cycle. This step will mainly be based on existing literature (see for instance Pedley et al.). In a next step the effect of Taylor-dispersion in an oscillating flow is considered and added to the previous model. Simultaneously the normal diffusion is added at the deep end (the alveoli).
In a separate step the influence of asymmetry of the velocity-profiles is analyzed and if necessary added to the model. Next elasticity of the tubes and the alveoli (compliance) is added.
In a final step geometrical asymmetry is introduced and the "pendelluft" mechanism is analyzed.

Once the lung model and ventilation mechanisms have been established in a satisfactory way the influence of external disturbances like for instance cardiogenic oscillations will be analyzed and the results are used for a refinement of the model. The validity of the modelled mechanisms will be tested by comparing the calculated results with results of measurements given in literature.

If all steps are successfully performed optimum values of the various artificial ventilation parameters like frequency and time resolved tidal profile may be calculated and if possible experimentally verified.
6. Quasi-steady convection

The lungmodel used to study convection is Weibels model (ref 12). The lungmodel of Weibel consists of 23 generations of branched tubes. In one generation the number of tubes is doubled. Hence the number of tubes in the n'th generation is $2^n$, the zero'th generation being the trachea. According to Weibel tube lengths and diameters are given by the following formulas:

$$0 < n < 3 : \quad l = l_0 e^{-n} \quad S = S_0 e^{-n} G$$

$$4 < n < 23 : \quad l = l_0' e^{-n} u \quad S = S_0' e^{(H+n)J}$$

For a normal adult man, his lungs inflated till 75% TLC, this becomes (ref 12):

$$l = 1.2e^{-0.92n} \quad S = 2.54e^{-0.083n}$$

$$l = 2.5e^{-0.17n} \quad S = 1.32e^{(0.1074+0.0125n)}$$

For reasons of flexibility instead of these formulas a table with results has been incorporated in the program. This form provides the possibility to bypass one or more generations.

6.1. Theory and experiments

Following Pedley and based on both theoretical and experimental results an estimate for the viscous pressure drop ($\equiv$ the pressure drop due to the working of viscous forces in the fluid) over a branched tube system in a steady flow will be given. This branched tube system models the first five generations of lungtubes.

The energy dissipation rate per unit volume in a steady viscous flow is expressed in cylindrical coordinates by:
\[ E = \mu \left( 2 \left( \frac{\partial u}{\partial x} \right)^2 + 2 \left( \frac{\partial v}{\partial r} \right)^2 + 2 \left( \frac{\partial v}{\partial r} + \frac{1}{r} \frac{\partial w}{\partial \theta} \right)^2 + \left( \frac{\partial u}{\partial r} + \frac{\partial v}{\partial x} \right)^2 + \left( \frac{1}{r} \frac{\partial u}{\partial \theta} + \frac{\partial w}{\partial x} \right)^2 + \right) \]

\[ + \left( \frac{1}{r} \frac{\partial v}{\partial \theta} + \frac{\partial w}{\partial r} - \frac{w}{r} \right)^2 \]  \hspace{1cm} (6.5)

In slender tubes the radial and swirl velocity components are small compared to the axial velocity. Hence from the continuity equation,

\[ \frac{\partial u}{\partial x} + \frac{1}{r} \frac{\partial (rv)}{\partial r} + \frac{1}{r} \frac{\partial w}{\partial \theta} = 0 \]  \hspace{1cm} (6.6)

it follows that \( \frac{\partial u}{\partial x} \) will be negligible also.

The expression for the dissipation rate then simplifies to \( v = w = \frac{\partial u}{\partial x} = 0 \):  

\[ E = \mu \left( \left( \frac{\partial u}{\partial r} \right)^2 + \frac{1}{r^2} \left( \frac{\partial u}{\partial \theta} \right)^2 \right) \]  \hspace{1cm} (6.7)

Integration of the dissipation rate per unit of volume over the tube cross-section yields the energy dissipation rate per unit of length \( D(x) \):

\[ D(x) = \int_{0}^{2\pi} \int_{0}^{r} \frac{\partial u}{\partial r} E \, r \, dr \, d\theta \]  \hspace{1cm} (6.8)

The total energy dissipation rate in the tube between \( x_1 \) and \( x_2 \) is:

\[ F = \int_{x_1}^{x_2} D(x) \, dx \]  \hspace{1cm} (6.9)

The dissipated energy is balanced by the work done by the pressure forces. (See appendix E) If the pressure is supposed to be constant over a cross-section (slender tubes) the work done by the pressure force is

\[ W_p = \Delta p(x_1, x_2) Q \]  \hspace{1cm} (6.10)

So  \[ W_p = F \]  \hspace{1cm} (6.11)
where $\Delta p(x_1,x_2)$ is equal to the pressure difference between a cross-section at axial station $x_1$ and a cross-section at $x_2$ and $Q$ is the volume flow. Equating the viscous energy dissipation rate and the work done by the pressure forces yields an expression for the effective viscous pressure drop $\Delta p(x_1,x_2)$.

If the flow in the tube is a fully developed Poiseuille flow the energy dissipation rate per unit length is constant and given by:

$$
\text{with: } u = 2\pi \left( \frac{r^2}{a^2} \right), \text{ so } \frac{\partial u}{\partial \theta} = 0 \text{ and } \frac{\partial u}{\partial r} = \frac{-4ru}{a^2} \text{ one obtains (6.12)}
$$

$$
D_p = \int_0^a \int_0^{2\pi} \mu \left( \frac{-4ru}{a^2} \right)^2 + 0 \text{ rdrd$\theta$} = 8\pi \mu u^2. \quad (6.13)
$$

The amount of energy dissipated per unit time in a section between $x_1$ and $x_2$ is:

$$
F_p = 8\pi \mu u^2 (x_2-x_1). \quad (6.13)
$$

And hence

$$
\Delta p = \frac{F_p}{Q} = \frac{F_p}{\pi a^2 u} = 8\mu u \frac{x_2-x_1}{a^2}. \quad (6.14)
$$

With these results a factor $Y(x)$ may be defined being the ratio of the actual energy dissipation rate per unit length and the energy dissipation rate in a Poiseuille flow:

$$
Y = \frac{D(x)}{D_p}. \quad (6.15)
$$

In the same way a factor $Z$ may be defined being the ratio of the integrated energy dissipation rate in the actual flow and in a Poiseuille flow:

$$
Z = \frac{F_p}{F} = \frac{1}{(x_2-x_1)} \int_{x_1}^{x_2} Y(x)dx. \quad (6.16)
$$

In order to calculate $Y(x)$ or $Z$ first $D(x)$ has to be determined. Therefore the axial velocity $u$ is measured as a function of radial distance $r$ and the circumferential angle $\theta$ at a few stations in axial direction. However since in a branched tube system like the lung never a completely developed Poiseuille flow exists the factors $Y(x)$ and $Z$ will probably differ considerably from 1.
Therefore the results of the measurements will also be compared with some simple theoretical results obtained by means of a simplified boundary layer model for an axially symmetrical configuration.

In a developing tube flow one can distinguish two domains. Near the wall a small boundary layer in which the no slip conditions must be fulfilled, and at the center a layer in which the flow is still undisturbed. The axial velocity is supposed to vary linearly between the value \( U \) at the outer edge of the boundary layer and 0 at the wall. From boundary layer theory it is known that the boundary layer thickness on a flat plate increases proportional to the square root out of the axial distance. It is assumed that this result may also be applied to the boundary layer on the inner wall of a tube (see figure 1).

\[
\alpha \delta = \sqrt{\frac{2 \mu x}{U_0}}. 
\]

(6.17)

\[ u=U_0 \quad 0 \leq \delta \leq a(1-\delta) \]

\[ u=U_0 \left( \frac{a-R}{a \delta} \right) \quad a(1-\delta) \leq \delta \leq a \]

Figure 6.1. Boundary layer model.

Integrating the continuity equation over the cross-section yields a relation between the mean velocity and the boundary layer thickness:

at the center:
\[
\pi [a(1-\delta)]^2 U_0 = \pi a^2 (1-2\delta+\delta^2) U_0. 
\]

(6.18)

and in the boundary layer:
\[
\frac{2\pi}{a} \int_0^a \int_{a(1-\delta)}^a u \, r \, dr \, d\theta = 2\pi \int_0^{a(1-\delta)} U_0 \left( \frac{a-R}{a \delta} \right) r \, dr = \pi a^2 (\delta-\delta^2) U_0. 
\]

(6.19)

\[
\pi a^2 \bar{u} = \pi a^2 (1-2\delta+\delta^2) U_0 + \pi a^2 (\delta-\frac{2}{3} \delta^2) U_0, \quad U_0 = \frac{\bar{u}}{1-\delta+\frac{\delta}{3}}. 
\]

(6.20)

Also the dissipation rate \( D(x) \) may be calculated in this model flow:
at the center \( \Omega r^2 a(1-\delta) \):
\[
  u = u_0, \quad \frac{\partial u}{\partial r} = \frac{\partial u}{\partial \theta} = 0, \quad E = 0.
\] (6.21)

in the boundary layer:
\[
  \frac{\partial u}{\partial \theta} = 0, \quad \frac{\partial u}{\partial r} = -\frac{U_0}{a}, \quad E = \frac{U_0^2}{a^2},
\] (6.22)

\[
  D = \int \int E r dr d\theta + \int \int E r dr d\theta =
  \begin{bmatrix}
  2\pi a(1-\delta) & 2\pi a \\
  0 & 0 \\
  0 & a(1-\delta)
  \end{bmatrix} = \begin{bmatrix}
  \frac{U_0}{a^2} & \frac{a}{2a^2} \\
  0 & a(1-\delta)
  \end{bmatrix}
\]

\[
  = 0 + 2\pi \mu \frac{U_0}{a^2} \int r dr = \frac{\pi \mu U_0^2}{\delta}.
\] (6.23)

and for the factor \( Y(x) \) is found:
\[
  Y(x) = \frac{D(x)}{D_0} = \frac{\frac{\pi \mu U_0^2}{\delta}}{\frac{8\pi \mu}{2a}} = \frac{1-\delta/2}{4\delta(1-\delta+rac{1}{2}\delta^2)^2}.
\] (6.24)

If the boundary layer thickness is small this result simplifies to:
\[
  Y(x) = \frac{1}{4\delta}.
\] (6.25)

Introducing the Reynolds number with respect to the diameter \( d \):
\[
  Re = \frac{U_0 d}{v} = \frac{U_0 \rho d}{\mu},
\] (6.26)

the boundary layer thickness \( \delta \) can be determined as a function of Reynolds number:
\[
  \delta = \frac{1}{a} \sqrt{\frac{2\mu x}{\rho U_0}} = \frac{\sqrt{\frac{8\pi}{2a}} \sqrt{\frac{\mu}{2a\rho U_0}}}{d Re} = \frac{\sqrt{\frac{8\pi}{d Re}}}{d Re} = k \sqrt{x}.
\] (6.27)

The expressions for \( Y(x) \) and \( Z \) then are approximated as:
\[
  Y(x) = \frac{1}{4 \sqrt{\frac{8\pi}{d Re}}} = \frac{1}{8\sqrt{2}} \sqrt{\frac{d}{Re}} \frac{1}{x}.
\] (6.28)
and: \( Z = \int_{x_2}^{x_1} \frac{1}{x^2-x_1} \int_{x_1}^{x_2} \frac{1}{4\delta} \ dx = \int_{x_2}^{x_1} \frac{1}{4k\sqrt{x}} \ dx = \)

\[
= \frac{1}{2k(\sqrt{x_1}+\sqrt{x_2})} = \frac{1}{2(\delta_2+\delta_1)} = \frac{\sqrt{\text{Re}}}{4\sqrt{2}} \left( \frac{\sqrt{x_2}}{d} + \frac{\sqrt{x_1}}{d} \right)
\]

(6.29)

The reliability of these results is greatest if the boundary layer thickness is small. The measurements of Pedley were performed at Reynolds numbers of 177 and above. For the tubes in the lung a typical length/diameter ratio is 3.5. For the boundary layer thickness at the end of a typical tube applies:

\[
\delta = \sqrt{\frac{8\pi}{d \text{ Re}}} \leq \sqrt{\frac{8(3.5)}{177}} = 0.38
\]

(6.30)

which is a moderate number. Hence the quantities \( Y(x) \) and \( Z \) follow as:

\[
Y = \frac{C}{8\sqrt{2}} \sqrt{\text{Re} \frac{d}{x}}
\]

(6.31)

\[
Z = \frac{C}{4\sqrt{2}} \frac{\sqrt{\text{Re}}}{\left( \frac{\sqrt{x_1}}{d} + \frac{\sqrt{x_2}}{d} \right)}
\]

(6.32)

The proportionality constant \( C \) depends on the velocity profile at the entrance of the tube. If the flow starts with a block profile \( C=1 \). In a branched tube system each new generation starts with a velocity distribution inherited from the former generation. From the measurement (ref 4) results a mean value for \( C \) is obtained of 1.85 with a standard deviation of less than 15%. This result appeared to be independent of the Reynolds number and independent of the position in axial direction of the measuring point. During the measurements the bifurcation angle was fixed at a typical value for lungs and so was the cross-section ratio between generations of tubes.
6.2. Implementation

In the program first an effective Poiseuille viscosity is calculated by means of the formula:

\[ R_{p(n)} = \frac{1}{TR(n)} \frac{128\mu L}{\pi d^4} \]  \hspace{1cm} (6.33)

The factor \( Z \) correcting this viscosity for entrance effects and secondary flows is given in (6.32) and takes the form:

\[ Z = \frac{1.85}{4\sqrt{2}} \sqrt{\frac{d}{Re}} \]  \hspace{1cm} (6.34)

The correction factor is never allowed to become smaller than one. Because if this would be the case, the viscous pressure drop would be less than the Poiseuille pressure drop which is impossible. This correction is used to calculate the viscous pressure drop. In addition to the viscous pressure drop a pressure increase occurs since the flow decelerates down the lung because of the increasing cross-sectional area. An estimate for this compensating effect can be obtained by means of Bernoulli's equation, where viscous effects are neglected. Theoretically one finds for the inviscid pressure difference between trachea where \( u = U_{tr} \) and the alveoli where \( u = 0 \):

\[ \Delta p_s = -(p_{k1} - p_{k2}) = -B \rho U_{tr}^2 \]  \hspace{1cm} (6.35)

Experimentally viscosity being included is found from:

\[ p_{s1} + p_{k1} = p_{s2} + p_{k2} + \Delta p_v \rightarrow p_{s1} - p_{s2} = \Delta p_v - (p_{k1} - p_{k2}) \]

leading to \( \Delta p_s = \Delta p_v - B \rho (U_1^2 - U_2^2) \) (with \( B = 0.85 \)),

\[ \text{in which } p_k \text{ is defined as } p_k = \frac{1}{2} \rho u^3 dA / \rho u dA = B p u^2 \]  \hspace{1cm} (6.36)

Measurements of a complete tube system (ref. 6) showed that the value of \( B \) should be 0.85. For Poiseuille flow the value of \( B \) may also be predicted by theory yielding \( B = 1 \). For fully turbulent flow \( B = 0.5 \). In this way the pressure
drop per generation can be calculated and summation over all 23 generation yields the pressure drop between trachea and alveoli.

So far the considerations applied to steady flow. During normal ventilation however the flow is unsteady.

Since normal ventilation frequencies are rather low (.2 Hz) a quasisteady approximation seems reasonable. This implies that the pressure drop is calculated in correspondence with the instantaneous flow rate. An other complication is formed by the elasticity of the tube walls. The wall compliance introduces a change in cross-section. The wall stretch depends on the tidal volume (see figure 6.2)

![Figure 6.2. Ventilation volume](image)

The lung model of Weibel is based on ventilation at 75% of the Total Lung Capacity. Hence with respect to this unsteady aspect, again, a quasisteady approximation is chosen and an instantaneous stretch factor is introduced proportional to the amplitude of the tidal volume. Because the inhaled amount of air is directly responsible for the expansion of the lung. Based on the corrected cross-section a pressure drop is calculated.

In figures 3, 4 and 5 some calculated results are presented. It is noticed that the pressure drop is low if the flow rate is low. And the flow rate is low in the higher generations because of the large cross-sectional area. Therefore the larger part of the pressure drop occurs in the first generations. If the flow rate is high the deviation from Poiseuille flow is large and the corresponding pressure drop is also large.

In this model different from reality expiration is treated in the same way as inhalation.
References


12) E.R. Weibel. Morphometry of the human lung (1963)

    J. Appl. Physiol. (24): 373-383
APPENDIX A

Captions and remarks for the figures

Figure 1: The total lung cross-section area for each generation.

One observes that the total cross-sectional area of a lung becomes very high in the last generations. Therefore by the continuity equation the airspeed in the last generations is very low, so near the alveoli diffusion is in general the main process.

Figure 2: The Poiseuille pressure drop in each generation.

At generation 4 there is a jump in the pressure drop. This is due to the fact that the total cross-sectional area decreases from generation 0 till generation 3. At generation 4 the cross-sectional area jumps back to the value at generation 0 and for higher generations it increases very rapidly. These details can not be seen in figure 1 because the value of the total cross-sectional area is too small.

The top of this curve lies at generation 6 because $(1/2^*n)*(L(n)/D(n)^*4)$ is the largest for $n=6$.

Figure 3: The viscous pressure drop and the Poiseuille pressure drop at three flow rates.

Comparing the Poiseuille pressure flow with the real pressure drop we note that for the first generations the real pressure drop is up to three times larger. This is due to the fact that the dissipation factor $Z$ is proportional to the Reynolds number. The Reynolds number is high in the first generations due to the high flow speed there.

The real pressure drop for two higher flow rates are given (FR=50 l/min and FR=100 l/min) as an extra.
Figure 4: The sum of real- and Poiseuille-pressure drop over the generations at a flow rate of 10 liter/minute.

The Poiseuille pressure fall and the real pressure drop at a flow rate of 10 l/min is now superposed over the generations starting with the last one (= generation 23).

One can see more clearly the difference between Poiseuille pressure drop and real pressure drop for the first few generations.

Figure 5: The superposed real- and Poiseuille pressure drop over the generations at a flow rate of 100 liter/minute.

We see here the same as in figure 5 but for a higher flow rate. The pressure drop is now considerably higher (almost up to 10-20 times) and the number of generations where the real pressure drop differs from the Poiseuille pressure drop is increased. Because at a larger amount of generations there is no Poiseuille flow.

Figure 6: The superposed static real- and Poiseuille pressure drop over the generations at a flow rate of 10 liter/minute.

One should compare this figure with figure 4 and observes that the static pressure drop is only slightly less than the not static corrected pressure drop.

Figure 7: The superposed static real- and Poiseuille pressure drop over the generations at a flow rate of 100 liter/minute.

The same as figure 6 but now it must be compared with figure 5. We note that for this high flow rate the Poiseuille pressure even becomes negative.
Figure 8: Lungvolume against time.

The relation is \( V = 3.5 - 2.3 \cos(2\pi t/5) \). So from \( t = 0 \) \((V=1.2 \text{ (=FRC)})\) till \( t = 2.5 \) \((V=5.8 \text{ (=TLC)})\) there is inspiration. The observed curve is a negative cosine curve.

Figure 9: Flow-rate against time.

The flowrate is the derivative of the lungvolume so here the derivative of figure 8 is obtained. We see that the flow rate varies from 0 up to almost 3 liter/second. This 3 liter/second flow rate yields 180 liter/minute. Comparing this with figure 2, 4, 6 and with reality we note that this is a very high flow rate. This is due to the fact that the respiration is chosen as extreme respiration which indicates normal frequency but maximal lung-volume variation.

Figure 10: Pressure drop for each specific time against time.

The pressure drop has a maximum near \( t = 1.1 \text{ sec.} \) instead of 1.25 sec. This is due to the fact that, as seen in figure 8, the lungvolume grows larger when time proceeds. So for the same flow rate (which is symmetric around \( t = 1.25 \text{ see figure 9} \)) the pressure drop for small times will be larger than for large times (for \( t < 2.5 \text{ sec} \)).

Figure 11: The time-integrated pressure drop against time.

This is figure 10 but now time integrated. Although it looks like an ordinary cosine wave it's not since fig 10 is not symmetric.
Figure 12: The time integrated pressure drop against lung volume.

This is the final result of computing the pressure drop in an elastic lung model. The form of this curve is dependent upon the respiration technique which is chosen here cosine-like. The maximum pressure drop of 4 cm water is a normal value for a lung. The line shown is in fact only valid for inspiration (one must start at the bottom left for t=0 and end at upper right for t=2.5). For expiration I obtain here exactly the same line because:

- the elasticity of the tubes is taken as instantaneously
- the flow profiles for expiration are different from those for inspiration so the constants used in the computer-program should be different. However till now I have not seen useful proposals.

The expiration line starts at t=2.5 at the upper right side and ends at t=5 at the lower left side following exactly the inspiration line. In fact one should not see it as a line but a closed loop from which the two sides coincide since the model is not sufficiently accurate.
APPENDIX B

Program listing

10 DIM OP(23)
20 DIM D(23)
30 DIM L(23)
40 DIM TT(23)
50 DIM TOP(23)
60 DIM U(24)
70 DIM RE(23)
80 DIM RP(23)
90 DIM Z(23)
100 DIM RR(23)
110 DIM INH(23)
120 DIM PVP(23)
130 DIM PVR(23)
140 DIM PSP(23)
150 DIM PSR(23)
160 DIM DPSR(25)
170 FOR T=0 TO 2.5 STEP .1
180 MID=0
190 FR=ABS(2890.265*SIN(1.2566371#*T))
200 VOL=3500-2300*COS(1.2566371#*T)
210 PRINT FR; VOL; T
220 RNT = 0
230 QO=.98838-.0314365*COS(1.2566371#*T)
240 Q3=.96791-.08683*COS(1.2566371#*T)
250 Q9=.950432-.134118*COS(1.2566371#*T)
260 PRINT QO;Q3;Q9
270 D(0) = 1.8:D(1) = 1.22:D(2) = .83:D(3) = .56:D(4) = .45:D(5) = .35:D(6) = .2813
290 D(14) = .074:D(15) = .066:D(16) = .06:D(17) = .054:D(18) = .05:D(19) = .047:D(20) = .045
300 D(21) = .043:D(22) = .041:D(23) = .041
310 L(0)=12:L(1)=4.76:L(2)=1.9:L(3)=.76:L(4)=1.27:L(5)=1.07:L(6)=.9
320 L(7) = .76:L(8) = .64:L(9) = .54:L(10) = .46:L(11) = .39:L(12) = .33:L(13) = .27
330 L(14)=.23:L(15)=.2:L(16)=.165:L(17)=.141:L(18)=.117:L(19)=.099:L(20)=.083
340 L(21)=.07:L(22)=.059:L(23)=.05
350 FOR X=0 TO 2
360 D(X)=Q0*D(X):L(X)=Q0*L(X)
370 NEXT
380 FOR X=3 TO 8
390 D(X)=Q3*D(X):L(X)=Q3*L(X)
400 NEXT
410 FOR X=9 TO 23
420 D(X)=Q9*D(X):L(X)=Q9*L(X)
430 NEXT
440 TT(0)=1:TT(1)=2:TT(2)=4:TT(3)=8:TT(4)=16:TT(5)=32:TT(6)=64:TT(7)=128
450 TT(8)=256:TT(9)=512:TT(10)=1024:TT(11)=2048:TT(12)=4096:TT(13)=8192
460 TT(14)=16384:TT(15)=32768:TT(16)=65536:TT(17)=131072:TT(18)=262144
470 TT(19)=524288:TT(20)=1048576:TT(21)=2097152:TT(22)=4194304:TT(23)=
=8388608!
480 FOR X=0 TO 23
490 REM LPRINT X;D(X);L(X)
500 NEXT
510 PRINT "gen. opp tot.opp Ugem Rey"
520 FOR N = 0 TO 23
530 OP(N)=.78539816#*D(N)^2
540 TOP(N)=OP(N)*TT(N)
550 INH(N)=TOP(N)*L(N)
560 RNT=INH(N)+RNT
570 U(N)=FR/TOP(N)
580 RE(N)=D(N)*U(N)*6.55
590 RP(N)=((1/(2^N))*L(N)*7.4560908#)/(D(N)^4)
600 Z(N)=((RE(N)*D(N)/L(N))^-.5)*.32703689#
610 IF Z(N) < 1 THEN Z(N)=1
620 RR(N)=Z(N)*RP(N)
630 PRINT N;OP(N);TOP(N);U(N);RE(N)
640 NEXT N
650 PRINT "int=",RNT
660 PRINT:PRINT:
670 PRINT"gen. Rpois. Z=RR/Rp Rreal"
680 FOR I = 0 TO 23
690 PRINT I;RP(I);Z(I);RR(I)
700 NEXT I
710 XN=0
720 XO=0
730 PRINT:PRINT:
740 PRINT"gen.Prss.visc.pois.Prss.visc.real Prss.stat.pois Prss.stat.real"
750 FOR K = 0 TO 22
760 J=22-K
770 PVP(J)=FR*.000001*RP(J+1)+XN
780 PVR(J)=FR*.000001*RR(J+1)+XO
790 XN=PVP(J)
800 XO=PVR(J)
810 NEXT K
820 FOR L = 0 TO 23
830 PSP(L)=PVP(L)-.0000012*(U(L)^2)
840 PSR(L)=PVR(L)-1.02E-06*(U(L)^2)
850 PRINT L;PVP(L);PVR(L);PSP(L);PSR(L)
860 NEXT L
870 IF RE(O)>2000 THEN B=1 ELSE B=.5
880 B=.5
890 DPSP=PVP(0)-.0000012*B*U(0)^2
900 U=INT(10*T+.1)
910 DPSR(U)=PVR(0)-.0000012*B*U(0)^2
920 FOR H=1 TO U-1
930 MID=MID+DPSR(H)
940 NEXT H
950 PRINT"mid=",MID
960 RINT=.1*(.5*DPSR(0)+MID+.5*DPSR(U))
970 PRINT:
980 PRINT "pvp=",PVP(0)
990 PRINT "dpsp=", DPSP
1000 PRINT "dpsr=",DPSR(U);T
1010 PRINT "rint=",RINT
1020 PRINT:
1030 NEXT T
APPENDIX C

Description of the program listing

lines:
10-160 Dimension declaration of the arrays.
A lot of the arrays are made to store and to retrieve the calculated
results in each stage. The array DSPR is dimensioned on 25 because there
is calculated with 25 time steps (see line 170). The other arrays have
23 units because the model has 23 generations of tubes.
170 The time loop, 25 steps from t=0 till 2.5 sec.
the loop is closed in line 1030.
180 Resetting to zero from MID because otherwise line 930 would go wrong
because it would go further with his old value.
190 The flow rate is a derivative of the volume where \( \omega = 1/5 \) Hz that is one
period in 5 sec.; so from t=0 till t=2.5 is exactly one complete
inspiration.
The ABS sign is necessary because otherwise for t=2.5 sec. the computer
calculated a very small negative flow rate like -0.00001 and this would
go wrong.
200 This line shows how the breathing is chosen.
We see that the frequency is 0.2 Hz so twelve breathings (inspiration +
expiration each minute). This is the normal breathing frequency in an
adult.
V varies from 1.2 till 5.8 liter so from RV till TLC this are the
borders for a normal man. I call this in future extreme breathing. The
reason that here I choose for extreme breathing is that I want to see
the maximal effect from the airway changes which are incorporated in
this model. If you want to change the type of breathing you should at
least change the lines 170,190,200 and 230-250.
210 Control.
220 Resetting to zero because otherwise line 560 would go wrong.
230-250 Expanding factors for the diameter and the length of the tubes. These factors are valid for extreme breathing. There are three factors: Q0 for generation 0-2 with a maximum expanding of 5% Q3 for generation 3-8 with a maximum expanding of 15% Q5 for generation 9-23 with a maximum expanding of 25%.

260 control.

270-340 Tube diameter and tube length inputs according to Weibel

350-430 Multiplying the tube diameter and the tube lengths with the expanding factors. If these lines stood before line 170 they would each time step again be multiplicated by the expanding factor. Because they stand behind line 340 they are now automatically resetted.

450-470 Read in from the number of tubes for each generation. For Weibel this would be simple 2^-n. Here it is written explicitly to make it easier adjustable for other cases.

480-500 control

510 Headline for the calculated results which are printed in line 630

520 For each generation do (closing of the loop in line 640)

530 Calculate the cross-section area for each tube (and store it in an array)

540 Calculate the total cross-section area for one generation by multiplicating the cross-section area from one tube times the number of tubes in each generation

550 Calculate the volume from that generation by multiplicating with the length of the tubes in that generation

560 Summate to get the total volume of all the generations first time (n=0) only the volume of generation 0 is calculated (the trachea); second time (n=1) the volume of generation 1 is added by that of generation 0..etc

570 Calculation of the averaged speed U (the flow rate divided by the total cross-sectional area of a generation)

580 Because Reynolds=UD/v and v=μ/ρ and in this program I took v=1.53 cm²/sec so 1/v=6.55 (ρ = 1.20 kg/m³3 and μ=0.183) we get for the Reynolds number (related on the tube diameter) Rey=6.55*UD

590 Calculating the Poiseuille resistance for a generation

600 Calculating the factor Z for effective pressure fall instead of Poiseuille pressure fall
if $Z<1$ then the effective pressure fall would be less than the Poiseuille pressure fall which is impossible. So that it is replaced by 1 (the value of a fully developed Poiseuille stream). Remark: $1/d$ is never much bigger than 3 so that $Z$ becomes 1 if Reynolds is less than 3.

The effective resistance is the Poiseuille resistance times the factor $Z$.

Printing of the calculated results for each generation:
- number of generation
- tube cross-section area
- total cross-section area
- averaged airspeed
- Reynolds number based on tube diameter and averaged airspeed

next generation (closes the loop opened in 520)

Print the total volume of the tube system (this line must of course be outside of the loop because the volume first must be summed in the loop)

print the new headline for the results yet to calculate

for each generation do

print
- number of the generation
- Poiseuille resistance
- factor $Z$
- effective resistance remark: if the factor $Z = 1$ the effective resistance is equal to the Poiseuille resistance

close the loop opened in 680 remark this extra loop is necessary because not all of the results could be printed on one line (see line 630)

resetting of the starting values of the summated Poiseuille and effective resistance. Otherwise it would by each new time step go wrong (line 170)

headline for the calculated results which are printed in line 850

for each generation (except number zero) do

Change to start with the last one

summation of the viscous pressure fall over the generations remark: the pressure fall over the trachea (=generation zero) is not taken into account the factor $10^{-6}$ comes from the conversion of the units. Take care of the boundaries: example k=0 : j=22 : PVP(22)=$FR*10^{-6}*RP(23)$
$0 \text{ k=1 : j=21 : } PVP(21)=FR*10^{-6}*RP(22) + PVP(22)$ etc.

the same for the effective resistance
810 closing of the loop opened at line 750
820 for each generation do
830-840 "correction" for the static pressure fall line 830 deals with Poiseille pressure so B=1 so we get $1.2^{-6}$. Line 840 deals with effective pressure so b=.85 so we get $1.02^{-6}$
850 print the results
860 close the loop opened at line 820
870 correction factor if there is Poiseille stream (alike Pedley)
890 Calculation of the total Poiseille pressure fall from trachea till alveoli corrected for static pressure fall (the factor B=1 because it deals with Poiseille stream)
900 translation from the time steps into integers the adding of 0.1 is necessary to avoid rounding to a lower integer
910 calculation of the total effective pressure fall and storing in an array the first round only DSPR(0) exists, the second round DSPR(0) and DSPR(1) etc.
920 integration of the mid part of the total time integral by skipping the first one (H=0) and skipping the last one (H=U)
930 summation from the mid part:
  first time U=0  so H does not exist
  second time U=1 so H does not exist
  third time U=2  H=1 to 1 so MID=DSPR(1)
  fourth time U=3  H=1 to 2 so MID=DSPR(1)+DSPR(2) etc
One must take in mind that this is an integration over time. so normally would by each time step the values of the previous be summed (as example: for U=3 would also the values of U=0,1 and 2 be added so that we get for the midpart of the integral: MID=2*DPSR(1)+DPSR(2).
Therefore this part is resetted in line 180 (just inside of the timeloop)
940 Close the loop opened in line 920
950 control
960 The total time integrated effective pressure fall is according to the trapezium rule: timespace*{(1/2*F(begin) + midpart + 1/2*F(end))}
980 print the Poiseille pressure fall
990 Poiseille pressure fall corrected for static pressure fall
1000 effective pressure fall corrected with static pressure fall
1010  Total time integrated pressure fall
1030  Closing of the loop opened in line 170

note: sometimes I have added some "print:" statements which give an empty line while printing to see the results more clearly separated
APPENDIX D

Implementation of elastic tubes

Explanation how to calculate time integrated pressure fall with and without elastic tubes.

I define normal ventilation as: \( V = 2550 - 250 \cos(2\pi \omega t) \) with \( \omega = 0.2 \) (twelve breaths each minute).

So inspiration is from \( t = 0 \) till \( t = 2.5 \) sec, expiration from \( t = 2.5 \) till \( t = 5 \) sec.

The flow rate is: \( FR = \dot{V} = 250 \omega 2\pi \sin(2\pi \omega t) \). \hspace{1cm} (D.1)

Analog we get for extreme breathing: (\( \omega = 0.2 \) but the lungvolume \( V \) varies from 1.2 till 5.8 liter):

\( V = 3500 - 2300 \cos \left( \frac{2\pi}{5} t \right) \hspace{1cm} (\text{cm}^3) \hspace{1cm} (D.2) \)

\( FR = 920 \pi \sin \left( \frac{2\pi}{5} t \right) \hspace{1cm} (\text{cm}^3/\text{sec}) \hspace{1cm} (D.3) \)

An example of a calculation procedure:

\( t = 0 \quad V = 1.2 \text{ liter} \quad FR = 0 \) so no viscous pressure drop,

\( t = 0.1 \quad V = 1.22 \text{ liter} \quad FR = 362.2 \). Calculate the pressure drop according to this flowrate. Do this for each time step \( t \) and integrate the pressure drop from \( t = 0 \) till \( t = 2.5 \) (for instance with the trapezium rule).

Adjustments for elastic tubes

Because especially for extreme breathing the diameter changes will affect the pressure drop seriously, I have made a small adjustment for the computer program. To bring elasticity in the model I worked on a simple way which was based on an article which stated some measured results (ref 6). Elasticity was taken lineair with the lung volume. In reality there may be a relation with the
1/3 power of the lung volume but this adjustment has not been taken into account. They stated an elasticity of 5% for generation 0.1 and 2 for the tube length and the tube diameter if the lung varies from FRC till TLC. For generation 3 till 9 it was 15% and for generation 9 and higher it was 25%. One must take into mind that the formerly used values for the pipe length and the pipe diameters were based on Weibels lung model which is a lung of an adult male inflated till 75% of TLC.

With this the elasticity factors can be calculated which one has to multiply with the original pipe diameters and pipe lengths to achieve the right values for a certain lungvolume.

For extreme breathing the results are:

- generation 0.1 and 2: \[ Q_0 = 0.98838 - 0.0314365 \cos \left( \frac{2 \pi}{5} t \right) \] (D.4)
- generation 3-8: \[ Q_3 = 0.96791 - 0.08683 \cos \left( \frac{2 \pi}{5} t \right) \] (D.5)
- generation 9-23: \[ Q_9 = 0.950432 - 0.134118 \cos \left( \frac{2 \pi}{5} t \right) \] (D.6)

A quick control for Q0 gives:

From above we can see that Q0 varies from 0.95694 (at V=1.2 liter =RV) till 1.0198165 (at V=5.8 liter =TLC).

Weibels lung was a lung at 75% of TLC so it contains \((0.75)\times(5.8)=4.35\) liter.

So Q0 for 75% TLC is:

\[ Q_{0,75\%TLC} = \frac{(4.35-1.2)}{5.8-12} (1.0198165-0.95694) + 0.95694 = 1.0 \] (D.7)

A calculation for extreme breathing then works like this: For a certain time t: Calculate the lungvolume V (one must keep in mind the difference between the lung volume V and the volume which is occupied by the tube system. The last one is just a part of the lung volume). Calculate the flow-rate FR. Calculate the elasticity factors and multiply the tube diameters and the tube lengths with...
the elasticity factors. You now can calculate the pressure drop on the known way which fits at that lung-geometry. If ready start with time step t+1.

It would of course be more accurate if we would calculate the pressure drop continuously with the time. For simplicity here is calculated for 25 time steps of 0.1 second each. If we integrate over time we obtain the pressure drop from alveoli till trachea for one inspiration. We can easily see that this pressure drop varies with the used breathing technique. If a person has to inhale 4 liter and he does it on a very slow way (so it takes a long time) the flow rate would be low. So he would achieve a Poiseuille flow in almost the whole lung which gives a relatively low resistance. If in the contrary one has to inhale the same amount of air in a quick time the flow rate would be high. This would give a turbulent flow in a big part of the lung or a correction factor Z which is not one. So the pressure drop will be relatively high.

The program is for this second case less accurate because the build in of turbulence is by the factors B and C. This can have a high influence on the solution because the highest flow speeds are achieved in the first generations so they will be less accurate then the rest. But the first generations take the largest part of the pressure drop into account.

Remarks: Expiration in this model is the same as inspiration, which is of course not true in reality. Expiration will certainly affect the factor's B and C but there are no accurate measurements which give a good quantitative prediction of the right values.

There is also no time delay factor for the elasticity factor. Therefore the P-V loop will be a closed loop (so it forms a line) see figure 12.

To choose a sinus wave as breathing pattern is perhaps a bit rough. We have seen that the way of breathing can influence the pressure drop. If necessary the breathing pattern can be adjusted.
Appendix E

The proof of formula (6.11)

The dissipation rate for a stationary tube flow with no radial or swirl velocity components is

\[ D = \mu \left[ \left( \frac{\partial u}{\partial r} \right)^2 + \frac{1}{r^2} \left( \frac{\partial u}{\partial \theta} \right)^2 \right]. \quad \text{(E.1)} \]

See also (5.1).

Rewriting from polar coordinates to cartesian coordinates gives

\[ x = x, \]
\[ y = r\cos \theta, \]
\[ z = r\sin \theta, \quad \text{(E.2)} \]
\[ r^2 = x^2 + y^2. \]

The derivative of \( u(r, \theta) \) yields:

\[ \frac{\partial u}{\partial r} = \frac{\partial u}{\partial y} \frac{\partial y}{\partial r} + \frac{\partial u}{\partial z} \frac{\partial z}{\partial r} = \frac{\partial u}{\partial y} \cos \theta + \frac{\partial u}{\partial z} \sin \theta, \quad \text{(E.3)} \]

\[ \frac{\partial u}{\partial \theta} = \frac{\partial u}{\partial y} \frac{\partial y}{\partial \theta} + \frac{\partial u}{\partial z} \frac{\partial z}{\partial \theta} = \frac{\partial u}{\partial y} (-r\sin \theta) + \frac{\partial u}{\partial z} (r\cos \theta). \quad \text{(E.4)} \]

Substitution of (E.3) and (E.4) in (E.1) yields:

\[ D = \mu \left[ \left( \frac{\partial u}{\partial y} \right)^2 + \left( \frac{\partial u}{\partial z} \right)^2 \right]. \quad \text{(E.5)} \]

With

\[ \left( \frac{\partial u}{\partial y} \right)^2 = \frac{\partial}{\partial y} \left( u \frac{\partial u}{\partial y} \right) - u \frac{\partial^2 u}{\partial y^2}, \quad \text{(E.6)} \]

\[ \left( \frac{\partial u}{\partial z} \right)^2 = \frac{\partial}{\partial z} \left( u \frac{\partial u}{\partial z} \right) - u \frac{\partial^2 u}{\partial z^2}. \quad \text{(E.7)} \]
(E.5) can be written as:

\[ D = \mu \left( \frac{\partial}{\partial y} \left( u \frac{\partial u}{\partial y} \right) + \frac{\partial}{\partial z} \left( u \frac{\partial u}{\partial z} \right) \right) - \mu u \left( \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} \right) = D_1 + D_2. \quad (E.8) \]

Taking the volume integral of the first term of (E.8) over a pipe with constant circular cross-section we obtain:

\[ \int \left( \mu \frac{\partial}{\partial x} \left( u \frac{\partial u}{\partial y} \right) \right) \, dV = \mu \int \left( \frac{\partial}{\partial x} (0) + \frac{\partial}{\partial y} \left( u \frac{\partial u}{\partial y} \right) + \frac{\partial}{\partial z} \left( u \frac{\partial u}{\partial z} \right) \right) \, dV. \quad (E.9) \]

With the theorem of Gauss (E.9) becomes:

\[ \int D_1 \, dV = \mu \int \left( \frac{\partial}{\partial x} (0) + \frac{\partial}{\partial y} \left( u \frac{\partial u}{\partial y} \right) + \frac{\partial}{\partial z} \left( u \frac{\partial u}{\partial z} \right) \right) \, dV \]

\[ \quad + \mu \int \left( o \cdot n_x + \left( u \frac{\partial u}{\partial y} \right) n_y + \left( u \frac{\partial u}{\partial z} \right) n_z \right) \, dS_V \]

\[ \quad + \mu \int \left( o \cdot n_x + \left( u \frac{\partial u}{\partial y} \right) n_y + \left( u \frac{\partial u}{\partial z} \right) n_z \right) \, dS_A \quad (E.10) \]

For \( S_V \) and \( S_A \), \( n_x = 1 \) and \( n_y = n_z = 0 \) (see figure (E.1)) so:

\[ \int D_1 \, dV = \mu \int \left( u \frac{\partial u}{\partial y} n_y + \frac{\partial u}{\partial z} n_z \right) \, dS = \mu \int \frac{\partial u}{\partial n} \, dS = 0. \quad (E.11) \]

Because \( u \) is zero on \( S \) (the no-slip condition).
Therefore

\[ \int \nabla \cdot \mathbf{D} \, dV = \int \nabla \cdot \mathbf{D} \, dV = - \int \mu \left( \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} \right) \, dV \quad (E.12) \]

The stationary Navier-Stokes equation in x direction states

\[ \rho u \frac{\partial u}{\partial x} + \frac{\partial p}{\partial x} = \mu \left( \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} \right) \quad (E.13) \]

Because radial and swirl velocities (v and w) are omitted, and therefore by the continuity law \( \frac{\partial u}{\partial x} = 0 \), (E.13) can be written as

\[ \frac{\partial p}{\partial x} = \mu \left( \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} \right) \quad (E.14) \]

So (E.12) and (E.14) gives

\[ \int \nabla \cdot \mathbf{D} \, dV = - \int u \frac{\partial p}{\partial x} \, dV = - \int \int \int u \frac{\partial p}{\partial x} \, dx \, dy \, dz \quad (E.15) \]

With \( \frac{\partial u}{\partial x} = 0 \), so u is constant in x direction one obtains

\[ \int \nabla \cdot \mathbf{D} \, dV = - \left[ \int \int p \, dy \, dz - \int \int p \, dy \, dz \right]_{S_A}^{S_V} \quad (E.16) \]

and if p is constant over the cross sectional area

\[ \int \nabla \cdot \mathbf{D} \, dV = - \left[ p_A Q - p_V Q \right] = \left[ p_V - p_A \right] Q \quad . \]

So the volume integral of the dissipation over a tube with constant circular cross-section and equal pressure over that cross-section, is equal to the pressure difference over the tube times the volume flow if radial and swirl velocities in the tube are omitted.