A STUDY OF THE BOW SHOCK INDUCED BY SECONDARY INJECTION INTO SUPersonic AND HYPERSONIC FLOWS

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ABSTRACT

The flow pattern resulting from perpendicular injection of a gas from a flat plate into free stream flows at Mach numbers ranging from 2.2 to 7 has been experimentally studied, with a primary interest in determining the validity of the assumption that the blast wave theory predicts the shape of the resulting bow shock wave. It has been found that by considering the effect of origin shift, the shock shape can be predicted with considerable accuracy by the second order blast wave solution.
LIST OF SYMBOLS

C \quad \text{velocity of sound}

C_D \quad \text{drag coefficient}

D \quad \text{drag force}

E \quad \text{energy per unit length}

J_o \quad \text{constant (see equation 1)}

\dot{m} \quad \text{mass flow rate}

n \quad \text{molecular weight}

P \quad \text{static pressure}

P_o \quad \text{stagnation pressure}

q \quad \text{dynamic head}

R \quad \text{radius of shock wave}

R_o \quad \text{characteristics length related to the energy of the explosion (see equation 2)}

R^* \quad \text{dimensionless blast wave radius, } R^* = \frac{R}{R_o}

R_e \quad \text{equivalent obstruction radius (see equation 21)}

t \quad \text{time}

U \quad \text{velocity of propagation of the line charge}

u \quad \text{flow velocity}

x \quad \text{horizontal distance from shock origin}

x^* \quad \text{dimensionless } x\text{-coordinate } x^* = \frac{x}{R}

X' \quad \text{horizontal origin shift}

Y \quad \text{vertical origin shift}
\( \gamma \) specific heat ratio
\( \delta \) boundary layer thickness at jet port with jet off
\( \lambda_1 \) constant (see equation 1)
\( \lambda_2 \) constant (see equation 1)
\( \omega \) see equation 14

Subscripts

i injectant
p primary flow
\( \infty \) free stream
l jet off conditions
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I. INTRODUCTION

In recent years the attitude control of rocket propelled vehicles has received considerable attention due to the increasing requirements for more precise trajectories. Aerodynamic surfaces are known to be inadequate as primary control devices since they are effective only during the portion of the flight when the vehicle is moving at very high speeds. Control techniques that accomplish a change in vehicle direction by altering the direction of the rocket exhaust have been used extensively in the form of jetavators, jet vanes and pivoting nozzles. However, these devices are all mechanical components that must operate in the very high temperature region of the rocket exhaust and consequently the reliability of such techniques has been marred by such failures as thermal deformation, melting and blockage due to propellant particles.

The United Aircraft Corporation (1) introduced in 1952 a method of changing the direction of the thrust vector of a rocket which required no moving mechanical components in the rocket exhaust region. The direction of the thrust vector was altered by the injection of a secondary gas into the expanding portion of the rocket nozzle. The side thrust resulting from this technique, which has been termed secondary injection, was found to be larger than that force which would result when injecting the same mass flow through a rocket vernier motor exhausting perpendicular to the rocket axis. The ratio of the force produced by secondary injection to that produced by the vernier motor is called the magnification factor, and this factor has been used as an index for measuring the effectiveness of various secondary injection configurations.
The absence of moving mechanical parts and a magnification factor of approximately two (1) stimulated sufficient interest to promote further study, until at present the practicality of changing the thrust direction of a rocket engine by injecting a gas or liquid into the nozzle is well established (2)(3).

The side force produced by secondary injection has been found to be a combination of the reaction force of the jet resulting from the momentum force imparted to the nozzle wall as the injectant flows from the port in the wall, and the interaction force which is a result of the shock envelope, induced by the jet obstruction. For instance, the higher pressure resulting from the flow being decelerated through the shock, acting over the projected area of the shock envelope will yield a force in the direction of the injection stream and will add to the reaction force of the jet.

Analytical attempts to mathematically predict the magnitude of the side force resulting from secondary injection have been made by several authors (4) (5) (6). However, in each of the theoretical studies listed here, an assumption was made concerning the shape of the shock envelope resulting from the jet issuing into the supersonic flow of the rocket exhaust gases from the rocket wall. And in each case, the various assumptions had to be made in the absence of experimental evidence. Therefore, it would be of considerable interest to a study of this kind to know the shape of the shock induced by secondary injection and to determine the validity and the range of validity for the various assumptions which have been proposed.
This study was made for the purpose of studying the flow pattern produced by a sonic jet issuing from a flat plate into supersonic and hypersonic streams, with a primary interest in determining the ability of the blast wave theory to predict the shape of the bow shock under various secondary and primary flow conditions.

This investigation was carried out in partial fulfillment of the requirements for the diploma of the von Karman Institute. It was done under the supervision of Professor Jean J. Ginoux. Part of the financial support for the work was provided by the European Office of Aerospace Research, Contract Number 65-96.
II. THEORY

A. Blast wave analogy (first approximation)

Taylor's (7) analysis of the intense spherical explosion has been extended to the plane and cylindrical cases by Sakurai (8). It has been found that the radius $R$ of a strong cylindrical shock wave produced by a sudden release of energy per unit length $E$ can be constructed in the form of a power series of the squares of the inverse of the shock Mach number $\left( \frac{C}{U} \right)^2$ where $C$ is the sound velocity of the undisturbed fluid and $U$ is the velocity of propagation of the line charge. The distance from the line charge to the shock front is represented by

\[
\left( \frac{C}{U} \right)^2 \left( \frac{R}{R_0} \right)^2 = J_0 \left[ 1 + \lambda_1 \left( \frac{C}{U} \right)^2 + \lambda_2 \left( \frac{C}{U} \right)^4 + \ldots \right]
\]  

(1)

where $R_0$ is the characteristic length related to the energy of the explosion and is expressed as

\[
R_0 = \left( \frac{E}{2 \pi P_\infty} \right)^{\frac{1}{2}}
\]  

(2)

$J_0$ and $\lambda_1$ are constants which have been computed numerically and are presented in the study by Sakurai (9).

The first approximation of this theory is made by assuming that the product of $\lambda_1$ times the square of the inverse of the shock Mach number $\left( \frac{C}{U} \right)^2$ is small compared to one which will eliminate all terms in the expansion except the first, yielding

...
Realizing that the shock velocity $U$ is also the rate of change of the shock radius with respect to time $\frac{dR}{dt}$, this quantity can be substituted into equation 3, the variables can be separated and the expression can be integrated to yield

\[
\frac{R}{R_0} = \sqrt{\frac{2Ct}{R_0 J_0 J_{\frac{3}{2}}}}
\]  

(4)

However, when the above expression is applied to a condition where the disturbance remains at rest and the flow moves at a velocity of $u_\infty$ the time term $t$ can be replaced by $\frac{x}{u_\infty}$ while the velocity of sound $C$ can be replaced by $M_\infty u_\infty$ resulting in an expression for $R$ as follows

\[
\frac{R}{R_0} = \sqrt{\frac{2}{R_0 J_0 J_{\frac{3}{2}}} \frac{x}{M_\infty}}
\]  

(5)

Defining $\frac{x}{R_0}$ as $x^*$ and $\frac{R}{R_0}$ as $R^*$ we get the non-dimensional form of the blast wave radius $R^*$ as a function of the non-dimensional $x$ coordinate, $\frac{x^*}{M_\infty}$

\[
R^* = \sqrt{\frac{2}{J_0 J_{\frac{3}{2}}} \frac{x^*}{M_\infty}}
\]  

(6)
B. Blast wave analogy (second approximation)

In the second approximation the power series expansion of equation (1) is again considered and in this case, one more term is retained in the expansion yielding

\[ \left( \frac{C}{U} \right)^2 \left( \frac{R_0}{R} \right)^2 = J_0 \left[ 1 + \lambda_1 \left( \frac{C}{U} \right)^2 \right] \]  

(7)

then by letting

\[ X = \frac{R}{R_0}, \quad dX = \frac{dR}{R_0} \quad \text{and} \quad dR = R_0 dX \]

and substituting these quantities into equation (7) we get

\[ \int_0^{R/R_0} \left( \frac{1}{X} \right)^2 J_0 \lambda_1 - \int_0^t C dt \]

\[ \left[ \left( \frac{1}{X} \right)^2 J_0 \lambda_1 \right]^{\frac{1}{2}} = \int_0^t C dt \]

(8)

which can now be integrated to yield

\[ \frac{R}{R_0} = \left[ \frac{2Ct}{R_0 \lambda_1 J_0} \left( 1 - \frac{J_0 \lambda_1}{2R_0} \right) \right]^{\frac{1}{2}} \]  

(9)

Using the same reasoning in applying this theory to flow conditions where the disturbance is stationary and the flow moves at a velocity \( u_\infty \) as was outlined in the first order solution, we can again replace \( Ct \) by \( \frac{X}{M_\infty} \), and equation (9) can be arranged in the following non-dimensional form.
\[ R^* = \left[ \frac{2x^*}{J_0^{\frac{1}{2}} M_\infty^2} \left( 1 - \frac{\lambda}{2} \frac{J_0^{\frac{1}{2}}}{M_\infty} \right) \right]^{\frac{1}{2}} \] (10)

C. Energy term in blast-wave analogy

The energy term \( E \) in the expression for \( R^* \) (see equation 2) has been defined in the blast-wave analogy as the energy per unit length that is suddenly released to cause the resulting shock front. When this theory was applied to flow around blunt bodies in hypersonic flow by Lees and Kubota (10) the rate of energy imparted to the moving stream per unit time was assumed to be the product of the drag and the free stream velocity,

\[ \frac{dE}{dt} = D u \] (11)

while the energy per unit length \( E \) could be obtained by dividing the rate of energy input \( \frac{dE}{dt} \) by the free stream velocity \( u_\infty \), which finally shows that the energy imparted by a blunt body per unit length is simply the drag on the body

\[ E = D \] (12)

Broadwell (4) applied this drag analogy to blast-wave theory for the case of secondary injection by assuming that the drag imparted to the free stream was equal to the momentum change that would occur by allowing the secondary flow to accelerate to free stream velocity. Fluid which is injected perpendicular to the main flow enters the stream with zero
axial momentum and is accelerated to the free stream velocity thus imparting an effective force on the primary stream equal to \( \dot{m}_1 u_\infty \) where \( \dot{m}_1 \) is the mass flow rate of the secondary injectant. However, it must be recalled that \( E \) represents the energy per unit length in the case of axisymmetrical flow, whereas, in secondary injection flow the disturbance is confined to the space above the flat plate. Therefore, Broadwell reasoned that the energy \( E \) in secondary injection is in fact producing half the effect and therefore should be equal to twice the drag force.

\[
\frac{E}{2} = D_1 = \dot{m}_1 u_\infty
\]  

(13)

Broadwell next generalized the preceding analysis to allow for volume addition to the flow by assuming "that heat is added to a portion of the primary flow at such a rate that the resulting change in volume is equal to the added volume". This analysis alters the energy term in the following way

\[
E = 2 \omega \dot{m}_1 u_\infty
\]  

(14)

where

\[
\omega = \frac{1}{2} + \frac{(\gamma - 1)}{2} \left[ \frac{M_\infty^2}{n_\infty \frac{T_\infty}{T_{Oi}}} \right]
\]  

\[
\frac{1}{2} + \frac{1}{2} \left[ \frac{M_\infty^2}{n_i \frac{T_i}{T_\infty}} \right]
\]  

(15)

An attempt has also been made by Dahm (11) to derive the energy behind the blast-wave directly from thermodynamic considerations in order to avoid making any assumptions concerning the magnitude of the drag resulting from the stream
of injectant. The work that is done on the primary fluid by the injectant is found by considering the secondary fluid to be injected into an arbitrary volume, composed initially of primary fluid, then an arbitrary amount of adiabatic mixing of the two streams is assumed to occur and finally the mixture is assumed to expand from this initial state to free stream conditions. This analysis results in an energy expression of the following form

$$E = 2 \omega \dot{m}_1 u_\infty$$  \hspace{1cm} (16)

where

$$\omega = \left[ \frac{1}{2} + \frac{1}{\gamma_\infty (\gamma_\infty - 1) M_\infty} + \frac{\gamma_1}{\gamma_1 - 1} \left( \frac{1 + \frac{\gamma_\infty - 1}{2} \frac{M_\infty^2}{\gamma_\infty M_\infty^2} \frac{n_\infty}{n_1} \frac{T_0}{T_1} }{\gamma_1 - 1} \right) \right]$$  \hspace{1cm} (17)

The similarity between the results of Broadwell and Dahm is quite remarkable especially in view of the two totally different approaches to the problem.

Using the above results of Dahm and substituting them into equation 2 we can now write the full expression for the characteristic length related to the strength of the explosion $R_o$, in terms of the mass flow rate of injectant $\dot{m}_1$, the Mach number of the free stream $M_\infty$, and the molecular weights and the stagnation temperatures of the primary and secondary streams.

$$R_o = \left( \frac{\omega \dot{m}_1 u_\infty}{\pi P_\infty} \right)^{\frac{1}{2}}$$  \hspace{1cm} (18)
III. EXPERIMENTS

A. Apparatus

1. Facilities

The tests designed to study secondary injection into hypersonic flows were conducted in the VKI intermittent hypersonic wind tunnel H-1. This facility which is described in detail in reference 12, has a 12 cm x 12 cm test section and is capable of operating at Mach numbers ranging from 4 to 8. This series of tests employed hypersonic flow at two different Mach numbers, 7 and 5.4.

All of the experimental data used in this report for flow at a free stream Mach number of 2.2 was obtained from tests made by B. Gilman at the von Karman Institute in 1962 and reported in reference 13. Gilman conducted these tests in VKI supersonic tunnel S-1 at a stagnation pressure of $\frac{1}{4}$ atmosphere and at a total temperature of 29.4°C.

2. Models

The model tested at Mach numbers of 5.4 and 7 is shown in fig. 1. It is a flat steel plate model with a sharp leading edge of approximately .02 millimeter in thickness and equipped with an injection port 3.25 millimeters in diameter which operates as a sonic nozzle. Along the centerline of the model 18 static pressure taps were located at positions shown in fig. 2. A total pressure probe was fitted into the wall of
the injection stagnation chamber to allow a determination of the mass flow from the sonic injection port. The injection stagnation chamber was supplied with air from a 14 atmosphere supply through copper tubing which passed through the tunnel side wall and through the floor of the diffuser section. The pressure in the injection stagnation chamber was maintained by a manually operated globe valve.

The models used by Gilman are described in detail in reference 13. It can be observed that the flat plate model is very similar in design to the one tested at hypersonic velocities. The conical model, on the other hand, which gave a Mach number of 1.86 along its surface, is not expected to yield flow patterns identical to those obtained on the flat plate models. However, it is still felt that an order of magnitude check could be made using this model in the origin shift study which will be described under a future heading.

3. Instrumentation

The total temperature and total pressure of the tunnel were measured by probes placed in the tunnel settling chamber and referenced to ambient conditions.

The centerline static pressure survey was accomplished by the 18 pressure taps shown in fig. 2 and connected to variable reluctance differential transducers.

The injection chamber stagnation pressure was measured with Bourdon type gages.
The starting of the tunnel during each test was confirmed by flow visualization which was provided by the schlieren optical system. The shock shapes in the vertical plane were obtained by shadowgraphs which were made by placing photographic film in front of the optical glass viewing ports. The film was allowed to intercept light which was generated by a spark from the discharge of a condenser at 15,000 volts. This light originating at the spark, passed first to a flat mirror where the direction is changed by reflection, then it was reflected from a parabolic mirror to yield parallel beams of light and finally another flat mirror was used to direct these parallel rays through the tunnel test section to the photographic plate.

B. Tests

1. Preliminary tests

At the outset of this investigation it was necessary to determine the feasibility of such a test to be conducted from the point of view of tunnel blockage, maximum allowable injection rates, and the strength of the model support.

The shadowgraph method of shock determination was also tested and found to give adequate detail for the determination of the various shock patterns studied under the flow conditions.

Repeatability was also studied in this series of preliminary tests and it was found that the shock shape produced by secondary injection, although very unsteady in certain areas
especially at high injection rates, is quite repeatable from test to test. The unsteady action is usually seen superimposed on the basic shock envelope (see fig. 4) and does not therefore interfere with the determination of the dominant bow shock wave. The repeatability of the static pressure survey however, was found to exhibit large variations particularly in the region of the main bow shock (see fig. 5). These variations were no doubt caused by the large static pressure gradients in this region which are moved to different locations by a change in injectant flow. Therefore, a small change in injectant flow could move the high pressure region a very small distance still resulting in a rather large change in static pressure. This lack of repeatability, however, was not considered to be fatal to the static pressure study since this information was after all intended to be of a qualitative nature which might reveal certain characteristics about the flow in the region of the injected stream.

2. **Secondary injection study**

The principal part of this investigation was designed to study the flow pattern resulting from the interaction of a sonic jet and primary flow at both supersonic and hypersonic velocities with the secondary mass flow as the primary parameter. With this goal in mind the model was first tested in a stream flowing at a Mach number of 7 and the secondary mass flow rate was varied by altering the injection stagnation chamber pressure in five steps from 1.5 kg/cm² to 6 kg/cm². During each of these five runs the static pressure data was obtained while a shadowgraph picture was being made. The same procedure was repeated
at a free stream Mach number of 5.4, except that in this case it was possible to vary the injectant stagnation pressure from 2 kg/cm² to 13 kg/cm² in eight steps, the possible range of this quantity being determined by requiring that the shock envelope remains entirely on the plate.

In an attempt to gather information that would aid in constructing a flow model for the interference region between the two streams, sublimation tests were conducted at both hypersonic Mach numbers and for various injection rates (see figs. 6A, 6B and 6C) by first painting the model with marking blue and then coating it with a thin uniform coating of acenaphtene. The model was then placed into the tunnel and subjected to primary flow at one of the Mach numbers listed above with the desired secondary flow issuing from the injection port, until a pattern of dark and white areas appeared on the surface of the plate. Assuming constant plate temperature, the white spots represent areas of low surface friction such as separated regions, while black areas are known to be caused by high friction such as turbulent regions of flow, reattachment areas or any very high velocity region.

C. Discussion of results

1. Centerline static pressure study

The static pressure measured along the centerline of the model is displayed graphically in the form of nondimensional pressure coefficients in figs. 7, 8, 9 and 10. The first two curves were plotted from data acquired at a free stream Mach
number of 7 with the secondary injection stagnation chamber pressure as a variable. It can be seen that the pressure at the leading edge is higher than the static pressure of the free stream. This can be explained by realizing that unless the leading edge of the model is infinitely sharp the leading edge shock is slightly detached allowing the higher pressure from the stagnation point to be felt along the leading edge of the plate. In addition to this leading edge effect, in the hypersonic flow regime the boundary layer which has a high rate of growth interacts with the inviscid flow field causing an additional increase in pressure at the leading edge. Both of these effects become less pronounced as the flow travels over more plate length and therefore the pressure approaches that of the free stream. The pressure coefficient can thus be seen to initially decrease as the distance is increased from the leading edge. At some distance upstream of the injection port the flow separates from the plate not being able to support the high pressure being fed back from the shock system and the static pressure experiences a rise followed by a plateau which exists through most of the separated region. After the plateau the pressure decreases, and then sharply increases to a maximum value under the edge of the main bow shock and then decreases again under the jet plume (fig. 5). From the number of experimental points exhibited on these curves it is quite clear that the spacing of the static taps on the present model was too large to provide an accurate picture of the static pressure in the region from the end of the separation plateau to the injection port. However, an extensive study of surface pressure distribution with a sonic jet issuing from a flat plate into flow ranging in Mach number from 2.92 to 6.4 has been made by
Cubbison, Anderson and Ward (14). The flat plate model tested in this study was equipped with a sufficient number of static pressure taps to indicate a consistent pattern of pressure distribution along the centerline of model similar to that shown in fig. 5. The dotted lines in figs. 6 and 7 indicate the lack of experimental points, but the strong suspicion that a pressure pattern of this kind actually exists.

The centerline pressure acquired at a free stream Mach number of 5.4 (figs. 10 and 11) show that at this reduced Mach number the extent of the separated region is considerably reduced. Therefore, the wide spacing of the static pressure taps in this case did not detect the separated plateau region, or the pressure decrease before the sharp rise to a maximum underneath the main shock. Therefore, this static pressure data could only be used to show the consistent action of the quantities that are detectable from a study of the curves. For instance, the point of flow separation which is known to occur at the first pressure rise, can be observed to move upstream as the injectant mass flow rate is increased. The maximum static pressure rise under the main shock system also increases with an increase in this mass flow rate. And finally the overexpansion downstream of the injection port can also be seen to increase as the mass flow rate from the injectant port is increased.

2. Sublimation tests

A surface flow study using the sublimation technique was conducted for both hypersonic Mach numbers and for various secondary flow rates. The full scale photographs shown in
figs. 6A and 6B were made for the same injectant rate at Mach numbers of 5.4 and 7. Although the separated region which was originally of interest in this study is not clearly marked in these photographs due to the high sublimation of theacenaphthene at the leading edge which threatened to erase the entire pattern, before the separated region had time to appear, another interesting area just in front of the injection port and immediately beneath the main shock system is clearly shown. Recalling that the dark areas represent regions of high friction, it was surprising to find these areas in a region which was originally believed to be entirely separated flow. By studying shadowgraph and sublimation tests made under the same flow conditions, it has been determined that the outer centerline extremity of the white region surrounding the jet port corresponds to the position of the intersection of the jet boundary and the jet shock. The next white region proceeding toward the leading edge has been found to be located slightly ahead of the intersection of the main bow shock and the separated flow region. It is also believed that the position of this region of low surface friction corresponds to that of the high static pressure ridge observed in the centerline static pressure studies (figs. 7 and 8) and also noted by other investigators (15)(14)(13). However, due to the limited number of sublimation tests made and the fact that the centerline static pressure curves do not contain a sufficient number of experimental points, there is no conclusive evidence to this fact and it is therefore listed here as a point to be checked in further studies of this subject. The upstream extremity of the outermost black area corresponds to the position of the low pressure region between the separation plateau and the injection port.
3. Flow model

The main features of the flow pattern resulting from the interaction of the injection stream, the boundary layer and the hypersonic or supersonic free stream is illustrated schematically in fig. 12. This centerline view of the three dimensional flow reveals that the jet expands from a sonic flow in the nozzle to a high Mach number and then experiences a shock that decelerates the flow to a subsonic level. The obstruction of this jet plume to the main stream causes a strong bow shock wave ahead of the jet which decelerates a portion of the primary stream to a subsonic Mach number. The high pressure behind the bow shock is fed upstream through the subsonic region of the boundary layer causing extensive separation of the boundary layer from the model surface. This separation which is accompanied by a swelling of the boundary layer results in a separation shock. All of the characteristics of the flow in this separated region have not been conclusively determined but the experimental evidence accumulated in this study suggests at least three possible explanations of this phenomena which have been observed by the sublimation tests, the centerline static pressure survey and the shadowgraph pictures.

The first explanation to be presented is one suggested by Cubbison, Anderson and Ward (14), who have proposed that the mass from both the jet and the free stream create a pair of counterrotating vortices with a stagnation region between the two, which would cause the region of high local static pressure ahead of the jet and the trailing off to lower values of the static pressure in front and behind the stagnation point.
Also this explanation seems to be consistent with the findings in the sublimation tests, that is, two regions of high velocity flow separated by a stagnation region.

Amick and Hayes (15) have presented a model that contains an expansion of the air from the relatively high pressure of the stagnation point between the main bow shock and the jet shock to the separated region. This air is suggested to flow from this high pressure region downward onto the plate and radially outward along the surface of the plate increasing in velocity and losing static pressure. Then when this flow meets the air in the separated plateau region it is decelerated to the lower velocity and higher static pressure of this region. As this reverse flow proceeds outward it turns toward the downstream direction thus leaving the separated region and making room for the mass of air which is continually being supplied from the high pressure region described above.

The final flow model to be presented follows the classical ideas of boundary layer separation and reattachment that have been shown to exist in two dimensional shock wave, boundary layer interaction. That is, the flow is separated by the high pressure existing behind the shock which is fed back through the subsonic portion of the boundary layer and then reattaches just ahead of the jet plume. This action would account for the additional pressure rise and the high skin friction in this area. However, it does not explain in obvious ways the low pressure trough on each side of the so called stagnation point (see fig. 5).
Finally the flow from the jets meets that of the primary flow along some contact surface, and both are reaccelerated around expansion corners to supersonic speeds. The secondary flow is overexpanded and must change its direction through a compression shock system downstream of the jet plume.

4. Shock shape

A visual study of the flow by schlieren and shadowgraph pictures has revealed that as the secondary mass flow rate is increased, the jet plume (jet shock and jet boundaries) increases in size and therefore moves the bow shock origin upstream. In addition, the extensive amount of flow separation resulting from the injectant is accompanied by a large amount of swelling of the boundary layer which in effect changes the geometry of the flat plate, since the bow shock is now sitting above the plate on the separated flow. This action of the flow again moves the origin of the bow shock, this time in the vertical direction in amounts that vary with the injectant flow rate.

The analytical studies of secondary injection which were discussed in the introduction have not dealt with the problem of an origin shift. Each has placed the origin of the bow shock at the leading edge of the injection port and has left it there for all injection flow rates. The present investigation has shown that this assumption is not valid at least for the flow conditions dealt with in this study. In fact, by ignoring the phenomena of origin shift, the percent variation from theory increases from 5% to 26% for the flow at a Mach number of 1.
No theoretical attempts at predicting the magnitude of either component of origin shift were made in this study, but experimental information was gathered that will show the magnitude and the direction of variation in this quantity as various parameters are changed.

In experimentally studying the shift of the origin in the horizontal direction (X') (see fig. 11) it was believed that this quantity would be in some way proportional to the size of the jet plume, so the horizontal direction origin shift (X') was non dimensionalized by dividing it by a quantity which will be called the radius of the equivalent obstruction, \( R_e \).

It was assumed that the jet obstruction acts as a blunt nosed body which has a constant drag coefficient of 0.90. Then as the mainstream flow expands around the obstruction, it will assume a size that will produce a drag equal to the drag \( D \), that the injectant imparts to the primary flow.

\[
D = C_D q_\infty \left( \frac{\pi R_e^2}{2} \right)
\]  

(19)

The drag \( D \) is equal to the energy per unit length shown in equation 16

\[
D = \omega \dot{m}_1 u_\infty = C_D q_\infty \left( \frac{\pi R_e^2}{2} \right)
\]

(20)

Solving for \( R_e \), we get

\[
R_e = \sqrt{\frac{2 \omega \dot{m}_1 u_\infty}{C_D q_\infty \pi}}
\]
The x-direction origin shift (X') was divided by this equivalent obstruction radius and plotted as a function of the jet stagnation pressure ratio ($P_{oi}/P_1$) (fig. 13). For constant free stream flow conditions it was found that the ratio $X'/R_e$ was a constant value. An increase in Mach number, however, resulted in a decrease of the quantity $X'/R_e$ (fig. 14).

Vertical origin shifts (Y') were also recorded and plotted as a nondimensional quantity by dividing the origin displacement by the boundary layer thickness that would exist at the injection port location without injection ($\delta$). This swelling of the boundary layer ($Y'/\delta$) is displayed graphically as a function of the injectant stagnation pressure ratio ($P_{oi}/P_1$) in figure 15.

Typical bow shocks are presented in full scale dimensional form in figs. 16 through 19, along with the shock shape predicted by the second order blast wave theory. One can observe that for a given Mach number the percent variation from the theoretically predicted shape increases as the injectant mass rate is increased. For example, at Mach 7 the variation from theory was only 2.5% when the injectant stagnation pressure ratio was 236, however, as the injectant rate was increased by raising the jet stagnation pressure ratio to 711, the variation from theory was found to be 9%. For the reduced Mach number of 5.4 the variation from theory followed the same pattern. For instance, at an injection stagnation pressure ratio of 57.6 the variation from theory was less than 1.5% and when the mass flow rate of the injectant was increased by a stagnation pressure ratio of 374, the variation from theory of the shock shape increased to 9%.
At a free stream Mach number of 2.2 the variation from theory of the bow shock wave is quite pronounced at high injection rates. For instance, the variation at an injectant stagnation pressure ratio of 189.5 was of the order of 25% while at the lower injectant pressure ratio of 18.1 the variation was reduced to 7%. However, it must be noted that the size of the shocks at hypersonic tunnel speed had to be limited to approximately 50 mm at a distance behind the origin of 100 mm due to the limiting size of the flat plate and tunnel test section. On the other hand, all of the shock shape obtained by Gilman at a Mach number of 2.2 were much larger in size than those obtained during the hypersonic tests. For example, the smallest bow shock had a vertical dimension of 80 mm at a distance of 100 mm behind the origin. The point of the shock size is raised here because as the injectant flow rate is increased beyond a certain limit, it is expected that the shock will no longer approach an axisymmetrical shape but will assume a shape that becomes more and more elliptical. Therefore, it is believed that a true comparison of the ability of the blast wave theory to predict shock shapes at various Mach number should be made between shock shapes of the same size.

The bow shock shapes obtained over a range of Mach numbers from 7 to 2.2 and injectant pressure ratios from 18.1 to 711 are presented, along with the first and second order blast wave solution, in nondimensional form, in figs. 21 through 25. It should be noted that the dimensionless form of the theoretical curves are independent of free stream conditions and injectant mass flow rate, therefore, all of the curves would be identical in shape if the same graphical scales were used on each. However, the scales were varied for the range of
flow conditions in this investigation since the experimental
points at the lower Mach number would have otherwise been
cluttered into the region near the origin of the curves.

Figure 26 displays graphically the ability of the
blast wave analogy to predict shock shapes under three different
free stream flow conditions. An attempt was made here to use
shock shapes as near the same size as possible.
IV. CONCLUSIONS

The following conclusions have been drawn from the information gathered during this investigation.

1. Within the range of free stream parameters investigated, it can be concluded that the second order solution of the blast wave theory predicts with considerable accuracy the shape of the shock produced by a sonic jet issuing into a hypersonic or supersonic stream.

2. The first order solution does not predict with sufficient accuracy the bow shock produced by secondary injection.

3. As the injectant rate increases, the ability of the second order blast wave theory to predict the shock shape decreases due to the break-up of the axisymmetrical shape.

4. The essential features of the interaction flow model have been established with the exception of the flow in the separated region.

5. The effect of origin shift on the shock shape cannot be ignored in applying the blast wave theory.

6. The origin of the blast wave is a function of the secondary mass flow rate and the free stream conditions.
REFERENCES


7. Gilman, B.: An investigation of the interference effect of a high speed jet issuing normally into a supersonic stream.


10. Hayman, L.O. Jr & McDearman, R.W.: Jet effects on cylindrical afterbodies housing sonic and supersonic nozzles which exhaust against a supersonic stream at angles of attack from 90° to 180°.


15. Liepman, H.P.: Interaction effects of side jets with supersonic streams, experimental results and wind tunnel techniques.


NASA TR R-6, 1959.

18. Love, E.S.: A reexamination of the use of simple concepts for predicting the shape and location of detached shock waves.


NACA TN 1921, June 1949.

ARS Jnl, Aug. 1962, pp. 1203-1211.


FIGURE 1
FLAT PLATE MODEL
DIMENSIONS IN MM
FIGURE 4: SHADOWGRAPH - M = 5.4, p_i/P_1 = 374, p_0i = 13.02 kg/cm^2
FIGURE #5
TYPICAL CENTERLINE STATIC PRESSURE DISTRIBUTION
FULL SCALE DRAWING
M_∞ = 7  P_{oi}/P = 236

LEADING EDGE SHOCK
SEP.ATION SHOCK
SEP.ATED REGION
INTERFACE
JET SHOCK
FLAT PLATE MODEL

P.RESSURE COEFFICIENT, C_p

0.08
0.06
0.04
0.02
0
0.02
0
A) $M = 7, \frac{P_{oi}}{P_1} = 358, P_{oi} = 3.01 \text{ kg/cm}^2$

B) $M = 5.9, \frac{P_{oi}}{P_1} = 86.2, P_{oi} = 3.01 \text{ kg/cm}^2$

C) $M = 5.4, \frac{P_{oi}}{P_1} = 201, P_{oi} = 7.01 \text{ kg/cm}^2$

**FIGURE 6: SUBLIMATION TESTS**
CENTERLINE STATIC PRESSURE SURVEY
ON SURFACE OF MODEL

\[ \frac{P_{i}}{P} = 594 \]

\[ \frac{P_{i}}{P} = 236 \]

FIGURE 7

PRESSURE COEFFICIENT, \( C_P \)

DISTANCE FROM JET, mm
FIGURE #8
CENTERLINE STATIC PRESSURE SURVEY ON SURFACE OF MODEL $M_{\infty} = 7$

$P_{0i}/P_1 = 711$

$P_{0i}/P_1 = 358$

PRESSURE COEFFICIENT, $C_p$

DISTANCE FROM JET, mm
Figure #9

Centerline Static Pressure Survey

on surface of model

\[ \Delta \quad \frac{Po_{i}}{P_{i}} = 14 \]

\[ \quad \circ \quad \frac{Po_{i}}{P_{i}} = 57.6 \]

Distance from jet, mm

Pressure Coefficient, \( C_{p} \)
Figure # 10
Centerline Static Pressure Survey
On Surface of Model
M∞ = 5.4

--- Δ --- P_o / P_i = 316
--- O --- P_o / P_i = 201
FIGURE 12
CENTERLINE FLOW MODEL

MAIN SHOCK
INTERFACE
TYPICAL STREAMLINE
JET SHOCK
STREAMLINE
SEPARATION SHOCK

M_p > 1
M_z > 1

COMPRESSION SHOCK
JET BOUNDARY
SEPARATED REGION
EDGE OF JET SHOCK
FIGURE 14
MACH NUMBER EFFECT
ON HORIZONTAL ORIGIN SHIFT

DIMENSIONLESS ORIGIN SHIFT, $x'/R_l$

FREE STREAM MACH NO. - $M_\infty$
Figure 15
VERTICAL ORIGIN SHIFT

M* = 5.4
M* = 2.2
M* = 7.0

INJECTION STAGNATION PRESSURE RATIO, P*/P
FIGURE 16
DIMENSIONAL BOW SHOCK SHAPE

--- SECOND ORDER BLAST-WAVE THEORY

- EXPERIMENTAL

---

A. $M_\infty = 7$, $P_{0i}/P_1 = 594$

---

B. $M_\infty = 7$, $P_{0i}/P_1 = 236$
FIGURE 17
DIMENSIONAL BOW SHOCK SHAPE

---

SECOND ORDER BLAST-WAVE THEORY

EXPERIMENTAL

---

**A.** $M_\infty = 7 \quad \frac{P_{oi}}{P_i} = 7.11$

---

**B.** $M_\infty = 5.4 \quad \frac{P_{oi}}{P_i} = 3.74$
FIGURE 18

DIMENSIONAL BOW SHOCK SHAPE

--- SECOND ORDER BLAST-WAVE THEORY

O EXPERIMENTAL

A. $M_\infty = 5.4$, $P_{0i}/P_1 = 144$

B. $M_\infty = 5.4$, $P_{0i}/P_1 = 57.6$
FIGURE 19
DIMENSIONAL BOW SHOCK SHAPE
--- SECOND ORDER THEORY
--- EXPERIMENTAL

\[ R \text{ - mm} \]
\[ x \text{ - mm} \]

A. \( M_\infty = 2.2, \frac{P_{out}}{P_1} = 18.5 \)

B. \( M_\infty = 2.2, \frac{P_{out}}{P_1} = 18.1 \)
FIGURE 21
SHOCK SHAPE AT $M_{\infty} = 7$

--- --- FIRST ORDER SOLUTION

--- --- SECOND ORDER SOLUTION

$\Delta$ - $P_0 v_0 / P_i = 594 \quad M_{\infty} = 7$

$\bigcirc$ - $P_0 v_0 / P_i = 711$

DIMENSIONLESS BLOW-WAVE RADIUS, $R^*$

DIMENSIONLESS $x$ COORDINATE, $x^*/M_{\infty}$
FIGURE 22
SHOCK SHAPE AT M∞ = 7

--- --- --- FIRST ORDER SOLUTION
--- --- --- SECOND ORDER SOLUTION

\( \Delta \) - \( P_{\infty} = 177 \)
\( \bigcirc \) - \( P_{\infty} = 286 \quad M_{\infty} = 7 \)
\( \bigodot \) - \( P_{\infty} = 358 \)

DIMENSIONLESS BLAST-WAVE RADIUS, \( R^* \)

DIMENSIONLESS X COORDINATE, \( x^*/M_{\infty} \)
FIGURE 23
SHOCK SHAPE AT $M_{\infty} = 5.4$

--- --- --- FIRST ORDER SOLUTION
--- --- --- SECOND ORDER SOLUTION

$\Delta$ - $P_{0i}/P_1 = 374 \quad M_{\infty} = 5.4$

$\diamond$ - $P_{0i}/P_1 = 201$

$\bigcirc$ - $P_{0i}/P_1 = 316$

$\bigotimes$ - $P_{0i}/P_1 = 143$

DIMENSIONLESS BLAST-WAVE RADIUS, $R^*$

DIMENSIONLESS X COORDINATE, $x^*/M_{\infty}$
**Figure 24**

Shock Shape at $M_\infty = 5.4$

- ---- First Order Solution
- --- Second Order Solution

- $P_0/P_1 = 115$
- $P_0/P_1 = 86.2$
- $P_0/P_1 = 57.6$

Dimensionless Blast-Wave Radius, $R^*_r$

Dimensionless X Coordinate, $x^*/M_\infty$
FIGURE 25
SHOCK SHAPE AT Mᵦ = 2.2

--- FIRST ORDER SOLUTION

----- SECOND ORDER SOLUTION

△ - P₀/\(P₁\) = 4.72
◆ - P₀/\(P₁\) = 18.1
○ - P₀/\(P₁\) = 185

DIMENSIONLESS BLAST-WAVE RADIUS, \(R^*\)
DIMENSIONLESS X COORDINATE, \(X^*/Mᵦ\)
**Figure 26**

**Shock Shape at 3 Mach Nos**

- **First Order Solution**
- **Second Order Solution**

- $M_\infty = 7$, $P_0/R = 711$
- $M_\infty = 5.4$, $P_0/R = 374$
- $M_\infty = 2.2$, $P_0/R = 18.1$
The flow pattern resulting from perpendicular injection of a gas from a flat plate into free stream flows at Mach numbers ranging from 2.2 to 7 has been experimentally studied, with a primary interest in determining the validity of the assumption that the blast wave theory predicts the shape of the resulting bow shock wave. It has been found that by considering the effects of origin shift, the shock shape can be predicted with considerable accuracy by the second order blast wave solution.
A STUDY OF THE BOW SHOCK INDUCED BY SECONDARY INJECTION INTO SUPERSONIC AND HYPERSOONIC FLOWS, by J.L. Evers.

The flow pattern resulting from perpendicular injection of a gas from a flat plate into free stream flows at Mach numbers ranging from 2.2 to 7 has been experimentally studied, with a primary interest in determining the validity of the assumption that the blast wave theory predicts the shape of the resulting bow shock wave. It has been found that by considering the effects of origin shift, the shock shape can be predicted with considerable accuracy by the second order blast wave solution.