Fatigue crack growth in ARALL
A hybrid aluminium-aramid composite material

Crack growth mechanisms and quantitative predictions of the crack growth rates

June 1988

R. Marissen
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Keywords:

fatigue, aluminium alloy sheets, ARALL, aramid fibres, structural adhesive, crack bridging, residual stresses, stress intensity factor, crack growth, delamination, crack opening displacement, adhesive shear deformation.

Abstract

ARALL (Aramid Reinforced ALuminium Laminates) is a fatigue resistant hybrid material consisting of thin high strength aluminium alloy sheets which are laminated, using an aramid fibre reinforced adhesive. The good fatigue properties of ARALL are caused by intact aramid fibres which are bridging fatigue cracks, thus reducing the crack opening displacement and the stress intensity factor at the crack tip. The efficiency of the crack bridging can be enhanced by the introduction of a favourable residual stress system. As a consequence, the crack growth rate of ARALL can be reduced with orders of magnitude as compared to monolithic high strength aluminium alloy sheets.

The fatigue behaviour of ARALL is investigated under different types of fatigue loading. The main fatigue mechanisms are identified and investigated separately. A model for the calculation of crack growth rates in ARALL is developed, based on analytic solutions for the stress-strain system in cracked ARALL and on the results of the experiments on separate mechanisms. A computer programme has been prepared. The results of the programme agree with experimental fatigue crack growth rates with a satisfactory accuracy.
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<tr>
<td>$a$</td>
<td>half crack length</td>
<td>mm</td>
</tr>
<tr>
<td>$a_1$</td>
<td>half axis of a quarter elliptical crack (in z-direction)</td>
<td>mm</td>
</tr>
<tr>
<td>$a_2$</td>
<td>half axis of a quarter elliptical crack (in x-direction)</td>
<td>mm</td>
</tr>
<tr>
<td>$a_{eff}$</td>
<td>'effective' length of a crack in a notch stress field</td>
<td>mm</td>
</tr>
<tr>
<td>$b$</td>
<td>1. distance between the artificial crack and the delamination</td>
<td>mm</td>
</tr>
<tr>
<td></td>
<td>front in a delamination specimen</td>
<td></td>
</tr>
<tr>
<td></td>
<td>2. half length of the short axis of a delamination ellipse</td>
<td></td>
</tr>
<tr>
<td>$b_s$</td>
<td>half distance between the delamination boundaries at $x = s$.</td>
<td>mm</td>
</tr>
<tr>
<td>$c$</td>
<td>constant in the Paris equation for crack growth</td>
<td>-</td>
</tr>
<tr>
<td>$c_1$</td>
<td>stiffness ratio</td>
<td>-</td>
</tr>
<tr>
<td>$c_2$</td>
<td>stiffness ratio</td>
<td>-</td>
</tr>
<tr>
<td>$C$</td>
<td>geometrical correction factor</td>
<td>-</td>
</tr>
<tr>
<td>$C_{ad,d}$</td>
<td>correction factor on the stress intensity factor due to adhesive</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>deformation, for the influence of delamination</td>
<td></td>
</tr>
<tr>
<td>$C_{wa}$</td>
<td>correction factor on the efficiency of the crack bridging stresses,</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>related to the axis ratio of the delamination ellipse</td>
<td></td>
</tr>
<tr>
<td>$C_{w_1}$</td>
<td>finite width correction factor on the stress intensity factor</td>
<td>-</td>
</tr>
<tr>
<td>$C_{w_2}$</td>
<td>finite width correction factor on the crack opening displacement</td>
<td>-</td>
</tr>
<tr>
<td>$C_s$</td>
<td>correction factor for the effect of the starter notch on the stress</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>intensity factor caused by adhesive shear deformation</td>
<td></td>
</tr>
<tr>
<td>COD</td>
<td>crack opening displacement</td>
<td>mm</td>
</tr>
<tr>
<td>$d$</td>
<td>function of $a$ and $h$</td>
<td>-</td>
</tr>
<tr>
<td>$da/dN$</td>
<td>crack growth rate</td>
<td>mm/cycle</td>
</tr>
<tr>
<td>$db/dN$</td>
<td>delamination growth rate</td>
<td>mm/cycle</td>
</tr>
<tr>
<td>$D$</td>
<td>bore hole diameter</td>
<td>mm</td>
</tr>
<tr>
<td>$e'$</td>
<td>width of an &quot;artificial crack&quot;</td>
<td>mm</td>
</tr>
<tr>
<td>$E$</td>
<td>Young's modulus</td>
<td>N/mm²</td>
</tr>
<tr>
<td>$E_{Al}$</td>
<td>Young's modulus of the aluminium sheets</td>
<td>N/mm²</td>
</tr>
<tr>
<td>$E_w$</td>
<td>Young's modulus of the fibre-adhesive layers in the fibre direction</td>
<td>N/mm²</td>
</tr>
<tr>
<td>$E_s$</td>
<td>Young's modulus of the aramid fibres</td>
<td>N/mm²</td>
</tr>
<tr>
<td>$E_m$</td>
<td>Young's modulus of the ARALL laminate in the fibre direction</td>
<td>N/mm²</td>
</tr>
<tr>
<td>$f$</td>
<td>ligament length</td>
<td>mm</td>
</tr>
<tr>
<td>$F$</td>
<td>Work per mm width, done by an external load</td>
<td>Nmm</td>
</tr>
<tr>
<td>$F_{sd}$</td>
<td>shear stiffness of an adhesive interface between the fibres and</td>
<td>N/mm³</td>
</tr>
<tr>
<td></td>
<td>the aluminium sheets (section 5.3)</td>
<td></td>
</tr>
</tbody>
</table>
\( F_{AI} \) stiffness of the aluminium sheets (section 5.2) N/mm
\( F_{av} \) stiffness of the fibre-adhesive layers in the fibre direction (section 5.2) N/mm
\( F_a \) stiffness of the ARALL laminate in the fibre direction (section 5.2) N/mm
\( G \) energy release rate N/mm
\( G_{ad} \) shear modulus of the adhesive N/mm²
\( G_{Al} \) shear modulus of the aluminium sheets N/mm²
\( G_r \) energy release rate for delamination, not corrected for the number of fibre aluminium interfaces N/mm
\( G_a \) shear modulus of ARALL N/mm²
\( G_i \) energy release rate for delamination, after correction for the number of fibre aluminium interfaces N/mm
\( G_{is} \) shear modulus in an anisotropic material N/mm²
\( h \) half distance between parallel cracks mm
\( j \) number of interfaces between the fibre and the aluminium layers -
\( K \) stress intensity factor N/mm², or, MPa√m
\( K_{ad} \) stress intensity factor in the aluminium sheets as caused by the adhesive shear deformation N/mm²
\( K_{Al} \) stress intensity factor in the aluminium sheets of ARALL N/mm²
\( K_a \) stress intensity factor in ARALL N/mm²
\( K_{fia} \) final stress intensity factor in ARALL, considering all relevant effects N/mm²
\( K_{I0} \) asymptotic stress intensity factor N/mm²
\( K_{mr} \) stress intensity factor at the mean stress in flight of the TWIST spectrum N/mm², or, MPa√m N/mm²
\( K_{IA} \) stress intensity factor in the aluminium sheets considering delamination, external load and residual stress N/mm²
\( K_{Ia} \) stress intensity factor in the total ARALL laminate considering delamination, external load and residual stress N/mm²
\( K_i \) stress concentration factor (based on the nett section stress) -
\( K_{i,gross} \) stress concentration factor (based on the gross section stress) -
\( \Delta K \) cyclic stress intensity factor N/mm², or, MPa√m
\( \Delta K_{eff} \) effective cyclic stress intensity factor N/mm², or, MPa√m
\( l \) gauge length mm
\( L \) length mm
\( \Delta l \) elongation mm
\( \Delta L \) relative displacement of the two delamination boundaries, at \( x=0 \) mm
<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
<th>Unit</th>
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<tr>
<td>m</td>
<td>Paris exponent for delamination growth</td>
<td></td>
</tr>
<tr>
<td>n</td>
<td>Paris exponent for crack growth</td>
<td></td>
</tr>
<tr>
<td>N</td>
<td>cycle number</td>
<td></td>
</tr>
<tr>
<td>P</td>
<td>external load</td>
<td>N</td>
</tr>
<tr>
<td>q</td>
<td>constant in the Paris equation for delamination growth</td>
<td></td>
</tr>
<tr>
<td>Q</td>
<td>ratio of the mean adhesive shear stress, to the cyclic shear stress</td>
<td></td>
</tr>
<tr>
<td>R</td>
<td>stress ratio</td>
<td></td>
</tr>
<tr>
<td>R_sp</td>
<td>apparent stress ratio for the adhesive shear stresses at the delamination front</td>
<td></td>
</tr>
<tr>
<td>S</td>
<td>stress</td>
<td>MPa</td>
</tr>
<tr>
<td>S_{A,1}</td>
<td>remote stress in the aluminium sheets</td>
<td>MPa</td>
</tr>
<tr>
<td>S_{A,1,0}</td>
<td>remote stress in the aluminium sheets at which the stress intensity factors caused by the residual stress and the external load compensate each other</td>
<td>MPa</td>
</tr>
<tr>
<td>S_e</td>
<td>remote stress in the fibre-adhesive layers</td>
<td>MPa</td>
</tr>
<tr>
<td>S_{cr}</td>
<td>crack bridging stress calculated over the thickness of ARALL</td>
<td>MPa</td>
</tr>
<tr>
<td>S_{e}</td>
<td>stress difference in the fibre-adhesive layers between the delaminated and the non delaminated area</td>
<td>MPa</td>
</tr>
<tr>
<td>S_n</td>
<td>stress in the delaminated fibre-adhesive layers</td>
<td>MPa</td>
</tr>
<tr>
<td>S_{a}</td>
<td>stress in the aramid fibres</td>
<td>MPa</td>
</tr>
<tr>
<td>S_{gint}</td>
<td>gross section stress</td>
<td>MPa</td>
</tr>
<tr>
<td>S_a</td>
<td>remote stress in the total ARALL laminate</td>
<td>MPa</td>
</tr>
<tr>
<td>S_{a,0}</td>
<td>remote stress on ARALL at which the stress intensity factors caused by the residual stress and the external load compensate each other</td>
<td>MPa</td>
</tr>
<tr>
<td>S_{m}</td>
<td>mean stress in flight for the TWIST flight simulation sequence</td>
<td>MPa</td>
</tr>
<tr>
<td>S_{peak}</td>
<td>peak stress at the notch root</td>
<td>MPa</td>
</tr>
<tr>
<td>S_{r,1}</td>
<td>residual stress in the aluminium sheets</td>
<td>MPa</td>
</tr>
<tr>
<td>S_{r,2}</td>
<td>residual stress in the aramid-adhesive layers.</td>
<td>MPa</td>
</tr>
<tr>
<td>S_{r,3}</td>
<td>stress in the aluminium sheets as caused by external loading and residual stresses</td>
<td>MPa</td>
</tr>
<tr>
<td>S_{r,4}</td>
<td>stress in the aramid-adhesive layers as caused by external loading and residual stresses</td>
<td>MPa</td>
</tr>
<tr>
<td>t_{a,1}</td>
<td>thickness of one interfacial adhesive layer</td>
<td>mm</td>
</tr>
<tr>
<td>t_{A}</td>
<td>total thickness of the aluminium layers</td>
<td>mm</td>
</tr>
<tr>
<td>t_{2}</td>
<td>total thickness of the fibre adhesive layers</td>
<td>mm</td>
</tr>
<tr>
<td>t_{5}</td>
<td>thickness of a fictitious pure fibre layer within the fibre-adhesive layer</td>
<td>mm</td>
</tr>
<tr>
<td>t_{m}</td>
<td>thickness of the ARALL laminate</td>
<td>mm</td>
</tr>
<tr>
<td>U</td>
<td>stored elastic energy per unit of thickness</td>
<td>N</td>
</tr>
<tr>
<td>v_r</td>
<td>fibre volume content of the fibre adhesive layer</td>
<td></td>
</tr>
<tr>
<td>v</td>
<td>displacement in the y direction</td>
<td>mm</td>
</tr>
</tbody>
</table>
\( v_a \) adhesive volume content of the fibre adhesive layer

\( V \) programme control parameter

\( \Delta v_{ai} \) elongation of aluminium sheets near the edges of an "artificial crack" between the gauge pins

\( w \) specimen width

\( W \) energy which is available per unit of thickness or width for an increment of delamination or crack growth.

\( x \) distance from the crack centre in the crack direction

\( x_0 \) x-coordinate

\( y \) distance from the crack in the load direction

\( y_g \) y-coordinate of a gauge pin

\( y_p \) plastic zone size in the adhesive

\( y_0 \) y-coordinate

\( z \) distance in the thickness direction

\( z \) complex variable \( z = x + iy \)

\( \bar{z} \) complex variable \( z = x - iy \)

\( z_0 \) complex parameter \( z_0 = x_0 + iy_0 \)

\( Z(z) \) Westergaard stress function

\( \bar{Z}(z) \) Westergaard stress function

\( \alpha \) long axis of an elliptical notch

\( \beta \) short axis of an elliptical notch

\( \gamma \) shear strain

\( \epsilon \) strain

\( \eta \) empirical correction factor for the shear compliance of an adhesive interface

\( \nu \) Poisson's ratio

\( \rho \) notch root radius

\( \tau_{ad} \) adhesive shear stress at the delamination front

\( \tau_{ad,m} \) mean adhesive shear stress at the delamination front

\( \tau_p \) adhesive shear yield stress (one percent plastic deformation)

\( \tau_v \) adhesive shear stress at distance \( y \) from the delamination front

\( \Delta \tau_{ad} \) cyclic adhesive shear stress at the delamination front

- N/mm²

- mm

- -
### Terminology

<table>
<thead>
<tr>
<th>Term</th>
<th>Explanation</th>
</tr>
</thead>
<tbody>
<tr>
<td>alclad</td>
<td>standard aircraft aluminium alloy sheet, covered with a thin layer of (nearly) pure aluminium as a corrosion protection</td>
</tr>
<tr>
<td>bare sheet</td>
<td>aircraft aluminium alloy sheet without surface layer.</td>
</tr>
<tr>
<td>crack bridging</td>
<td>aramid fibres remain intact behind the tip of a fatigue crack in the aluminium sheets of ARALL, thus reducing crack opening and crack growth.</td>
</tr>
<tr>
<td>fabric-adhesive layer</td>
<td>fibre-adhesive layer with 90% of the fibres in the load direction.</td>
</tr>
<tr>
<td>fibre-adhesive layer</td>
<td>aramid reinforced adhesive layer between the aluminium sheets in ARALL. The layer contains about 50% fibres which are mainly or completely orientated in the direction of the fatigue load.</td>
</tr>
<tr>
<td>laminates</td>
<td>nonreinforced laminates of aluminium alloy sheets.</td>
</tr>
<tr>
<td>prepreg</td>
<td>Fibre-adhesive layer, consisting of AF-163-2 adhesive (3M company) and 50% unidirectional aramid fibres. All fibres are in the load direction.</td>
</tr>
</tbody>
</table>
1. Introduction

The structural weight of aircraft has a significant effect on flight performance, transport capacity and fuel costs. In order to achieve weight savings for aircraft structures, high design stress levels have to be adopted. That means, that materials with a high specific strength are required. The aircraft's structure in service will meet with an extensive spectrum of fatigue loads. The cyclic stresses may cause initiation and propagation of fatigue cracks. For several important structural components fatigue is the limiting design parameter and fatigue resistant materials are then required. The fatigue resistance of a material is characterized by its resistance against the initiation of fatigue cracks and the resistance against fatigue crack growth. In general there is a large number of geometrical notches present in an aircraft structure (e.g. joints). Moreover incidental damage and damage due to fretting may occur. 'Natural' scatter in the fatigue crack initiation behaviour is another significant aspect. Consequently it appears to be impossible to prevent crack initiation in a reliable and economically acceptable manner.

After cracks have been initiated the operation of aircraft can still be safe, provided that cracks can be found before they have reached a critical size. In this case inspections are essential to ensure that the probability of the occurrence of a catastrophic failure remains extremely low. The probability that cracks are not detected early enough decreases if the cracks grow slowly, or if the inspection intervals become small. The latter approach implies an expensive maintenance which is not a favourable proposition.

An important goal of aircraft materials development is to improve the crack growth properties of structural materials. At the Delft University of Technology, it was found that the fatigue crack growth rates in sheet materials can be reduced, if they are built up by laminating thin sheets of the material which are connected by adhesive bonding, instead of using one thick monolithic sheet [1]. The advantage becomes highly evident if cracks start in one of the thin sheets of the laminate only. Under such circumstances the sheets which are still uncracked, reduce the crack growth rate in the cracked sheet. The reduction in the crack growth rates lasts until a crack is also initiated in the neighbouring sheets.

Another fundamental improvement of the fatigue behaviour was obtained by introducing high strength, fatigue resistant fibres into the adhesive [2]. The fibre orientation is chosen into the direction of the main load. The laminate is designed such that the fibres do not fail, when fatigue cracks develop. That means that they remain intact behind the tip of the propagating crack in the metallic layers. They hinder the opening of the crack, and consequently, the crack tip stress intensity in the aluminium sheets is reduced. This behaviour can lead to improvements in the crack growth rates by a factor of hundred and even more, as compared to monolithic aluminium sheets. This improvement allows for weight savings up to 30% in fatigue critical aircraft components. In this way a new hybrid material ARALL (Aramid
Reinforced ALuminium Laminates) has been obtained. In 1984 two international patents [3] were accepted and a pilot production of ARALL was started by ALCOA.

Aramid fibres were chosen for the reinforcement of ARALL. Aramid fibres combine a high specific tensile strength and a low fatigue sensitivity. Other properties, like the Young's modulus and the fracture strain, can be seen as an optimum choice for the present reinforcing purpose. Aramid fibres behave electrically neutral. That means that galvanic corrosion needs not to be expected for the combination with aluminium, in contrast to the selection of carbon fibres.

Optimal properties of the laminate are achieved if the fibre content is high enough to prevent fibre failure during the entire fatigue life. The efficiency of the crack bridging mechanism of the fibres can be increased if the thickness of the individual layers is decreased. The interaction between the different layers then becomes more intense. The laminated material optimized in this manner is ARALL. The production occurs by a hot curing process of the laminate.

Due to the low thermal expansion coefficient of the aramid fibres, residual stresses build up in the laminate during the cooling process from the curing temperature of 120°C to room temperature. These stresses are a tensile stress in the aluminium and a compressive stress in the aramid fibres. Tensile stresses in the aluminium sheets basically have an unfavourable influence on the crack growth behaviour. However, by prestraining or prestressing ARALL the residual curing stresses can be annihilated or even reversed, in such a way that the stress in the aluminium sheets become compressive and the stress in the aramid fibres become tensile. Consequently, the fatigue properties of prestrained ARALL become significantly better than those of as cured ARALL, whereas the crack growth resistance of the latter material is already significantly superior to the resistance of the aluminium alloy sheet material. Cracks remain closed as long as compressive stresses are present and the cracks do not propagate. The presence of compression in the aluminium sheets shifts the fatigue stresses to a less damaging level.

It was already pointed out that the excellent fatigue properties of ARALL are caused by the crack bridging mechanism of the aramid fibres. The actual crack growth behaviour is especially dependent on the efficiency of crack bridging and on the crack growth resistance of the aluminium sheets. The crack bridging efficiency is mainly related to the following basic mechanisms:

1. The crack bridging forces are transferred from the fibres into the aluminium sheets by shear stresses in the adhesive. The resulting shear deformations in the adhesive allow for some crack opening and, consequently, some crack growth can occur.
2. The cyclic adhesive shear stresses will cause some fatigue damage in the adhesive, and as a consequence a local area with debonding develops around the crack. This delamination between fibres and aluminium reduces the crack bridging efficiency.
A significant part of the present thesis is covered by the investigation of these two mechanisms. A fatigue crack growth model for ARALL is formulated on the basis of these mechanisms, which are investigated separately by experimental as well as analytical techniques.

A general survey of the engineering properties of ARALL is presented in chapter 2, where the production of the material is described, including the introduction of the favourable residual stresses. A qualitative discussion on the fatigue behaviour of ARALL follows in chapter 3.

The aim of the present investigation is to establish a comprehensive characterisation of the fatigue behaviour of ARALL. A large amount of experimental work had to be performed. The fatigue crack initiation and propagation behaviour of ARALL was investigated. Fatigue tests with constant-amplitude and flight-simulation fatigue loads were carried out. Some static tests were performed also. Further experiments were performed to analyze the individual fatigue mechanisms in ARALL. The experimental programme and the results are presented in chapter 4. A derivation of analytical concepts for the quantitative description of the fatigue mechanisms of ARALL is presented in chapter 5. Mechanical aspects of fracture in ARALL are considered as well.

In chapter 6 some of these analytical evaluation techniques are applied, to explain the delamination growth behaviour as observed in the experiments.

The development of a computer programme, employing the experimental data on the basic fatigue mechanisms of ARALL, is described in chapter 7. The combination of the analytical and empirical results within the computer programme led to a new calculation tool applicable to fatigue crack growth of different types of ARALL materials. The programme is developed for constant-amplitude fatigue loading. It was not yet tried to develop a programme for a quantitative calculation of crack growth rates of ARALL under flight simulation fatigue loading, because the material behaviour under such kinds of fatigue loads is already very complicated for monolithic material. Actually, for homogeneous materials a reliable prediction model for variable-amplitude loading is not yet available. However, a qualitative explanation of the experimental trends in ARALL under flight simulation loading was quite well possible on the basis of the present model.

The numerical, analytical and experimental investigations are discussed in chapter 8 and some recommendations for building up optimum types of ARALL are given. Finally the investigation is summarized in chapter 9.
2. ARALL as a structural material: a general description

2.1 Some aspects regarding the requirements for structural materials

It is useful to consider some of the requirements regarding aircraft materials first, before discussing the properties of ARALL in more detail. Because various components of the aircraft structure are loaded in a different way the failure modes can also be different. If components are predominantly loaded in compression they are usually designed for stability as a limiting criterion. A high elastic modulus and a high yield stress are profitable. Geometry aspects are related to stiffener shape and pitch and to the thickness of the skin and stiffeners.

If cyclic loading in tension is predominant components have to be designed against fatigue. The fatigue resistance (crack initiation and crack growth) and fracture toughness are significant material properties. Geometry aspects are now associated with avoiding or reducing stress concentrations and eccentricities.

An example of the first category of structures is the upper skin of an aircraft wing, which is predominantly loaded into compression due to the wing bending moment. Examples for the second category are the lower wing skin and the fuselage pressure cabin.

The lower wing skin is loaded by a cyclic stress in tension due to bending of the wing as caused by gust loads and manoeuvres. The skin of the fuselage pressure cabin is carrying a biaxial cyclic tensile stress during each flight. ARALL is a highly fatigue resistant material with a high tensile strength. Especially the superior fatigue properties are a great advantage for fatigue critical sheet structures like the pressure cabin and the lower wing skin.

Official airworthiness requirements [4] prescribe a fail safe design of aircraft structures, unless it is established that this is impractical for a particular structural component. In the latter case a safe life design can be applied, if it can be shown that the structure is able to withstand the expected fatigue loading without detectable cracks during the total service life. This analysis has to be supported by experimental evidence and appropriate safe life scatter factors have to be applied.

If a damage tolerant material like ARALL is applied, the fail safe requirements can more easily be fulfilled, because fatigue cracks will grow very slowly. Moreover, certain aircraft elements, previously considered to be safe life items, can now be considered to be fail safe. Swift [5] in a paper on damage tolerance is referring to a "single load path - damage tolerant" structural component. In the USAF damage tolerant requirements the "slow crack growth option" was introduced. A typical example is the lug type joint, which is known for a relatively short crack growth life if the material is a monolithic high strength alloy. However, this is no longer true if the lug is made from ARALL (see section 2.5).

The mechanical properties of a material may be affected by the environment. Corrosive environments are known to increase the crack growth rates in metallic structures and the
properties of composites are decreased by the presence of moisture. Preliminary investigations show that a corrosive environment does not considerably change the fatigue behaviour of ARALL [6]. Verbruggen showed that moisture affects the adhesion between aramid fibres and the adhesive, however the influence is not dramatic [7]. Verbruggen also showed that the absorption rate of moisture in ARALL is low [8], because the aluminium sheets are a barrier for the diffusion of moisture (see also [9]).

2.2 The production of ARALL

ARALL consists of three different types of materials:

1. High strength aluminium alloy sheet material.
2. Aramid fibres.
3. A structural adhesive.

Two laminating orders are shown in figure 1. The volume content of the aluminium alloy is about 65% (this is about 80% in weight) and it represents the major contribution to the mechanical properties in terms of strength and isotropic stiffness. The main function of the fibres is to provide an efficient crack bridging. In this connection the high tensile strength and the fatigue resistance of the aramid fibres are important. The fibres further contribute to the tensile strength and to the stiffness of the laminate. The adhesive has to provide a reliable connection between the fibres and the aluminium sheets. The quality of the adhesive is important for the efficiency of the crack bridging mechanism of the fibres. A more detailed discussion on the significance of the adhesive is presented in chapter 3.

ARALL is a laminated material built up from a number of thin aluminium alloy sheets with a thickness of 0.3 to 0.5 mm and intermediate thin fibre-adhesive layers. The aluminium sheets are pretreated for a bonding process (cleaning, pickling, anodizing and application of a primer). The aluminium sheets are bonded together, by the fibre-adhesive layers. These layers consist of an unidirectional prepreg with about 50% fibres and 50% partially cured adhesive. A prepreg of this kind, which contains Twaron HM aramid fibres and the AF-163-2 structural adhesive, has been developed by the 3M company especially for its application to ARALL. The thickness of the prepreg is about 0.2 mm.

An other possibility for providing a fibre-adhesive layer, is the application of an unidirectional aramid fabric with 90% of the fibres in the loading direction. The fabric is embedded in two thin films of a structural adhesive (right hand picture of figure 1).

The prepreg and the fabric adhesive films combination system are both used in the present investigation. The latter system is commonly constructed from a Twaron HM fabric and two BSL-312-UL adhesive films. The weight of the fabric is about 160 grams per square meter. This combination yields a layer thickness of about 0.25 mm and the fibre volume content is about 45% (40% if only the fibres in the loading direction are considered.). The stacking sequence for these two methods for the built up of ARALL is presented in figure 1.
After laminating the individual layers, the laminate is hot cured under pressure. The curing temperature for the adhesives, as used in the present investigation, is 120°C. A usual curing pressure for adhesives is 2 bar (0.2 MPa). However, for ARALL higher pressures are applied. Especially if the system with the individual adhesive systems and the unidirectional fabric is applied, pressures between 6 and 10 bar become necessary. At lower pressures the impregnation of the fabric by the adhesive will not be sufficient.

In case of the aramid adhesive prepreg such a high pressure is not required. During the curing process a vacuum technique may be applied, if the curing specifications of the adhesive allow for a vacuum. For the application of the vacuum technique, the whole stack of the individual layers is put into a bag, from which the air is pumped out. The bag is then put into an autoclave or under a press, where the curing process is performed. Using this technique, air traps are removed from the laminate. The curing time is one hour for the AF-163-2 adhesive and thirty minutes for the BSL-312-UL adhesive. Figure 2 shows a micrograph of a laminate, which contains the combination of the BSL-312-UL adhesive film and the Twaron HM aramid fabric.

As already mentioned before, the fatigue properties of ARALL can be further improved by the introduction of favourable residual stresses. This may be performed by two different methods.

1. **Prestraining:**
   
   The complete laminate is stretched into the fibre direction, in such a way that a plastic deformation of about 0.7% is reached in the aluminium layers. The fibres then still behave elastically. After the (elastic) unloading of the laminate, residual tensile stresses are left behind in the elastic fibres. The aluminium sheets are then loaded in compression (equilibrium condition).

2. **Prestressing:**
   
   An external tensile load is applied at the fibres, during the curing of the laminate. After the curing is finished, the temperature decreased, the pressure is removed and the fibres are unloaded. A part of the tensile stress remains in the fibres and, consequently, a compressive residual stress is present in the aluminium part.

Technologically the first technique is easier than the second one. However, a small reduction of the mechanical properties of the aluminium alloy occurs as a consequence of the plastic deformation. This is avoided if the second technique is chosen. ALCOA so far applied the prestraining procedure only. ARALL material tested in the present investigation was either produced in the Materials Laboratory of the Department of Aerospace Engineering (Delft University of Technology), or at the Fatigue Department of the Institute of Materials Research (DFVLR, Cologne). If a favourable residual stress system was introduced, it was done by prestraining if the ARALL was produced in Delft, and by prestressing if it was done in Cologne. A summary of the individual steps for the production of ARALL is presented in the following flow chart.
2.3 Some mechanical properties of ARALL

2.3.1 The static properties of unnotched ARALL

ARALL shows both good fatigue properties and a high static strength. Structures which are fatigue critical if conventional metallic materials are used, often will not remain fatigue critical if ARALL is used. An example is the lower wing skin, where a weight saving between 20% and 30% can be achieved, if ARALL is applied. Further weight savings are limited by the static compression strength of ARALL [10].

As a consequence the static properties of ARALL are also important. In aircraft especially two classes of aluminium alloys are used, the Al-7000 series for high strength applications and the Al-2000 series where better fatigue crack growth properties are required. The static properties of the Al-2000 are lower than those of the Al-7000 series. Typical representatives of the Al-7000 series are the 7075-T6 alloy and the 7475-T76 alloy, while 2024-T3 is a typical example for the Al-2000 series.
Because of the superior crack growth properties of the hybrid laminate ARALL, the relatively poor crack growth properties of the metallic component, if the Al-7000 series is used, are no longer a serious disadvantage. However, the static strength of ARALL is significantly increased by using 7075-T6 in ARALL.

The static tensile strength of ARALL as compared to monolithic material, is further increased by the high static tensile strength of the aramid fibres (about 2700 MPa). The compression strength of aramid fibres is only moderate (about one fifth of the tensile strength), and the compression strength of ARALL is somewhat lower if compared to the aluminium alloys. However, the compression yield stress of ARALL with the 7075-T6 alloy may still be higher than that of monolithic 2024-T3 sheet material.

The tensile and the compressive yield stress of ARALL depend strongly on the residual stress level. The presence of residual stresses contributes to the asymmetric tension-compression behaviour of ARALL.

Some representative static strength properties of ARALL are given in figure 3. The exact values will depend upon the type of ARALL (ratios of aluminium, fibres and adhesive).

The fibre-adhesive layer represents about 35% of the volume of ARALL. The density of this layer is about half the density of aluminium, consequently the density of ARALL is about 15% lower as compared to monolithic aluminium. That increases the specific strength values of ARALL by about 18% as compared to aluminium.

The two main materials of ARALL are essentially different; an aluminium alloy and aramid fibres. The fibre-adhesive combination represents a highly anisotropic component, whereas the aluminium component is isotropic. Due to the high amount of aluminium in ARALL, the resulting anisotropy remains moderate (as far as the static properties are concerned). Isotropy is favourable for such components where stresses may occur in different directions. This aspect will be discussed in more detail in section 2.5.

2.3.2 Static strength of notched ARALL

In aircraft components stress concentrations at geometrical notches cannot be avoided. The stress at the notch root is higher than the nominal stress in the component. For linear elastic materials the stress in the notch root is increased by a factor $K$, as compared to the nominal stress. A typical stress concentration factor is $K = 3$ for a circular hole in an infinite plate. In structures the $K$ values usually vary between 2 and 4. For a perfectly linear elastic material behaviour, the strength of a component is reduced by the $K$ value, due to the presence of a notch.

For metals which deform plastically before fracture occurs, the reduction in strength is considerably less. Composites, which behave nearly entirely linear elastic, are much more notch sensitive. However, also for composites some stress redistribution may be possible due to deformations in the sheet thickness direction, due to delaminations and due to secondary cracking near to the notch root. As a result the reduction of the strength of a com-
posite, due to the presence of notches, may be less severe than calculated with $K_c$. The same may be true also for ARALL, but for ARALL the favourable influence of possible plastic deformation in the aluminium component remains dominant. That is the reason why the static notch sensitivity of ARALL can be significantly better than for CFRP (Carbon Fibre Reinforced Plastics) composites. It is however slightly worse than for pure metals. The notch sensitivity of ARALL is dependent on various factors, which cannot be discussed here in all detail.

Early investigations were performed on ARALL specimens containing a circular hole with a diameter of 6.3 mm [11]. The stress concentration factor was $K_c = 2.4$. The experiments showed no significant notch sensitivity for these conditions. More recent investigations [10] on ARALL specimens containing notches with a size of several centimeters and $K_c$ values up to 4 showed that ARALL may be considerably more notch sensitive than monolithic aluminium alloys, especially at higher $K_c$ values. The discrepancy between the two investigations cannot be explained by the differences between the $K_c$ values only. The notch sensitivity of composites is known to be influenced by the size of the notches, also if the $K_c$ values are the same [12]. This aspect will also apply to ARALL. Altogether it may be concluded that ARALL from a static point of view will be somewhat notch sensitive for the stress concentration factors and the notch sizes occurring in aircraft structures. The designer should be aware of this aspect.

2.4 Technological properties of ARALL

For the manufacturing of aircraft also other material properties than the strength are important. ARALL is produced as laminated sheet material. Aircraft structures have to be manufactured from ARALL sheets. Conventional composites are formed to a large extent into the final shape during the material production stage. Some milling may be possible later on, however plastic bending of cured composites is usually impossible (except for some fibre reinforced thermoplastics). Metallic constructions are usually made by forging, casting, bending, cutting and milling techniques. ARALL with its high metal content lies between these technological boundaries.

Although milling of pure aramid composites is difficult, ARALL can be milled using normal workshop techniques. It has been shown that cold plastic bending of ARALL is also possible within certain limits [10]. The production of stiffeners has successfully been accomplished. Structural joints of ARALL components can be made by bonding, riveting or bolting, by similar techniques as employed for monolithic alloys [13].

Summarizing, it can be stated that several technological properties of ARALL are similar to those of monolithic aluminium sheets. Hence, to a large extent similar production processes can be used for aircraft manufacturing, if ARALL is applied as a structural material instead of monolithic aluminium alloy sheets. A consequence of this similarity is that the intro-
duction of ARALL as a structural material does not require great investments by the aircraft manufacturer.

2.5 Potential applications of ARALL

Typical examples of aircraft components where the application of ARALL can be profitable are:

The lower wing skin:
Fatigue of wing skins occurs due to cyclic bending of the wing. Bending causes a predominant principle stress in the spanwise direction. In such a case ARALL can be favourably applied, if the fibres are applied in the span direction.
The cyclic bending of the wings of transport aircraft is mainly caused by gust loads and ground-air-ground cycles, and a variable amplitude fatigue loading is the result of these loads. The standard flight simulation programme TWIST simulates the fatigue loading of a lower wing skin. Previous investigations showed that about 30% weight savings are possible if conventional metals are replaced by ARALL in the lower wing skin [2,10,11]. In section 4.4 of the present investigation it is demonstrated that a longer fatigue life and an increased safety against fatigue failure may additionally be achieved.

The pressure cabin of the fuselage:
Due to the cyclic pressurizing of fuselages, biaxial fatigue of the fuselage skin occurs. The fatigue load perpendicular to the longitudinal fuselage axis is twice the fatigue load into the direction of the axis. Other types of loading are induced due to bending of the fuselage. In this case also the fatigue resistant material ARALL is an attractive candidate. The fibres are orientated perpendicular to the fuselage axis. The application of ARALL as a material for the fuselage was investigated by Schijve Jr. [14], and again a considerable weight saving appeared to result from the application of ARALL.

Lugs:
Lugs are quite frequently used in aircraft constructions, because they allow easy assembly and they form rotation points in the structure.
Due to the high stress concentration factors of lugs, combined with fretting at the notch root under cyclic loads, rapid fatigue crack initiation is possible. The dimensions of lugs are limited and the crack growth trajectory is rather small. Consequently, it is difficult to obtain a fail safe design for lugs, unless a very low design stress, or a "slow crack growth material" like ARALL is adopted.
The complicated stress field around a lug hole, due to the load transfer at the pin hole interface, requires a material with nearly isotropic properties [15]. The fatigue cracks in lugs always start at the edges of the holes and propagate perpendicular to the loading direction. Consequently, the fibres should be applied into the loading direction (see figure 4).
Lugs represent only a very small part of the weight of an aircraft and weight saving aspects may be a secondary reason for the application of ARALL. The improvement of the safety against fatigue failure might well be a good primary reason.

**Crack stoppers:**
The damage tolerant properties of structures are often achieved or improved by the application of crack stoppers. Strips of material connected to the skin can take over a part of the external loading if fatigue cracks are present in the skin. The crack growth rate will be reduced as soon as the crack grows under the uncracked crack stoppers. It also delays the ultimate failure. This has recently been confirmed in a exploratory test programme [16]. Besides that, intact crack stoppers provide a considerable contribution to the residual strength of cracked panels. The load of intact crack stoppers, bridging a crack in a panel, can be very high, and rapid fatigue failure of a crack stopper should then be expected. A considerable improvement will be obtained if ARALL is used as a material for the crack stoppers. The residual fatigue life and the residual strength of the cracked panel will become higher and the safety of the structure will increase.
3. Qualitative description of the fatigue crack growth behaviour of ARALL

3.1 Some basic fatigue mechanisms

The crack growth rate in ARALL is significantly controlled by the efficiency of the crack bridging mechanism of the fibres. This efficiency is dependent upon the design parameters of the laminate, like layer thickness, stiffness, etc., and upon the applied fatigue loading. All these factors become important in view of two basic mechanisms, which will be briefly discussed below. As explained before ARALL consists of thin aluminium alloy layers and intermediate fibre-adhesive layers. The fibres are more or less concentrated in the centre of the fibre-adhesive layer. As a consequence the central part is "fibre dominated", and the two outer layers are "adhesive dominated". The fibre-adhesive layer will therefore be modelled as a central fibre layer with two outer layers of the adhesive. The two basic mechanisms mentioned before and the model are indicated schematically in figure 5.

1. Delamination.
   The cyclic crack closing fibre stresses, due to crack bridging, are partly transferred into the aluminium across the adhesive. This causes delamination in the adhesive behind the crack tip due to cyclic shear loading of the adhesive interface between the fibres and the aluminium sheets. In the delamination area, there is no connection between the fibres and the aluminium sheets. More delamination implies that the length over which the fibres are elongated due to crack opening is increased. Consequently, the specific strain of the fibres and the resulting crack bridging stresses will decrease. The larger the delaminated area, the lower the crack bridging stresses and the higher the crack growth rate.

2. Shear deformation in the adhesive between the fibres and the aluminium at the delamination boundary.
   Since the crack closing fibre stresses are transferred into the aluminium sheets through the adhesive at the delamination boundary, local shear deformations occur in the adhesive. These shear deformations reduce the crack bridging efficiency of the fibres, because they allow for some crack opening displacement in the metal. Consequently the crack growth rates are increased.

3.2 Consequences of a perfect crack bridging

The superior fatigue crack growth properties of ARALL are due to bridging of the crack by unbroken aramid fibres. The efficiency of the crack bridging is dependent on the two mechanisms: delamination and adhesive shear deformation. Some more detailed comments will be made below. First it will be considered what would happen if both mechanisms did not occur.
An ARALL sheet specimen with a starter notch and a crack is considered (see figure 6). It is assumed that delamination is absent and that the adhesive exhibits an infinite shear modulus. As a result the crack bridging of the aramid fibres will be perfect. The crack flanks cannot open. Consequently the stress intensity factor at the crack tip is zero, and fatigue crack growth will not occur. The load transmission "through" the crack occurs by the fibres only. Due to the stress concentration caused by the notch, the fibre loads will become high especially at the end of the starter notch. The high fibre loads at the notch root could lead to fibre failure starting from the notch root and the crack would be "zipped" open up to the crack tip. That implies a complete loss of the crack bridging effect. In other words, a perfectly stiff adhesive between the different layers and a zero delamination are not an optimum condition for the ARALL laminate. In reality this situation does not occur. There is some adhesive shear deformation which allows some crack opening. Further, some delamination will occur around the crack flanks, especially at locations where the fibre stresses are high. As a consequence a redistribution of the fibre stresses along the crack flanks occurs. This is a second reason why the crack flanks can open over a small distance. The stress intensity factor $K$ will not be zero, and some fatigue crack growth in the aluminium sheets can occur. In other words: the crack bridging efficiency with respect to a reduction of the $K$ value is diminished. Because the redistribution of the fibre stress along the crack implies a much lower peak value at the notch root, fibre failure behind the crack tip can be avoided.

However, the amount of adhesive shear deformation and delamination should remain within certain limits. Otherwise, the crack bridging efficiency would become too small. Fortunately for ARALL these aspects are not critical. Experimental evidence has shown that the delamination and the adhesive deformation were sufficient to prevent fibre failure. On the other hand, it is found that the crack bridging efficiency is good enough in all well designed types of ARALL with an adequate fibre volume content and thin aluminium alloy sheets.

### 3.3 Interaction between delamination and crack growth

When fatigue crack growth occurs in ARALL, it is accompanied by delamination growth. Usually delamination areas are found which exhibit a more or less elliptical shape. The situation for an ARALL sheet specimen containing a crack, which started from a notch, is schematically shown in figure 7. The ratio between the delamination distance and the crack length is an important factor for the crack bridging efficiency of the fibres. This ratio is dependent on the type of ARALL and the type of fatigue loading. If the delamination area is large, the strain in the fibres behind the crack tip is low and thus the crack bridging fibre stresses are also low. As a consequence, the stress intensity will be higher and the crack growth becomes faster.

The delamination to crack length ratio at a certain crack length is the result of the delamination growth rate and the crack growth rate during the previous fatigue cycles. Both, the
fatigue crack growth and the delamination growth represent continuous processes which influence each other. If, for some reason, the delamination distance becomes larger than normal, then the crack bridging fibre stresses become smaller. As a consequence the crack growth rate increases and the delamination rate decreases, because it depends on the cyclic stresses in the crack bridging fibres and the number of cycles. In this way the delamination distance to crack length ratio decreases and the situation approaches the normal stabilized situation again. It is a kind of self-balancing process, which also works in the opposite direction if the delamination distance is too small (see the discussion on perfect crack bridging in the previous section). It implies that a stable behaviour of the delamination to crack length ratio should occur in ARALL. The exact quantitative values of this ratio are influenced by all parameters, which influence the crack growth rate or the delamination behaviour.

In chapter 7 a computer programme is presented, by means of which the crack growth and the delamination growth can be calculated from basic material properties.

Similar arguments to understand the self controlled delamination, are valid to explain why constant crack bridging fibre stresses will build up along the entire crack flanks. If locally higher crack bridging stresses occur, the delamination rate at that location will become faster, thus increasing the delaminated length of those fibres and thus decreasing the local fibre stress to the average level. This behaviour causes a tendency to constant crack bridging stresses. The situation with constant crack bridging fibre stresses behind the crack tip is favourable, because the contribution of all crack bridging fibres to the reduction of the crack opening is the same. As a consequence there is no preferred starting point for a "zipping" mechanism as described before. Furthermore, the calculations of the stress intensity factor in the aluminium sheets become less complicated if the fibre stresses along the crack flanks are constant. These calculations will be presented in chapter 5.

3.4 Effect of adhesive shear deformation

The effect of the adhesive shear deformation on the stress intensity factor will be discussed here in a qualitative way. For reasons of simplicity a cracked ARALL sheet without delamination and without starter notch will be considered. According to the previous discussions, the stress intensity factor in the case of an infinitely stiff bond between the layers will be zero and the entire load is transferred through the fibres. However, in reality, due to local adhesive shear deformation near the crack flanks, some crack opening occurs. This situation is presented schematically in figure 8, at a location away from the crack tip. The crack has slightly opened and the stress intensity is not zero any more. The crack opening is proportional to the crack bridging stress if a linear elastic material behaviour is assumed. Away from the crack tip, the crack opening displacement COD is primarily constrained by the deformation field as shown in figure 8. However, the situation close to the crack tip is predominantly controlled by the stress intensity field only. The COD tends to zero if the
crack tip is approached. The crack opening displacement and the crack bridging stresses remain in a proportional relationship, also close to the crack tip where the COD decreases. Consequently, the crack bridging stresses decrease down to zero if the crack tip is approached. All together, the crack bridging stresses increase from zero at the crack tip, up to an amount corresponding to the total load transfer away from the crack tip. This distribution of the crack bridging stresses along the crack flanks is schematically presented in figure 9.

If delamination does occur, the crack opening displacements will be larger and the crack bridging stresses will be lower. The displacements due to adhesive shear deformation will be lower too. However, the qualitative shape of the crack bridging stress distribution in figure 9 will not be changed.

For small cracks the picture of figure 9 is no longer applicable. The COD of a small crack is very small and the crack opening restraint is largely a matter of the in-plane stiffness of the ARALL sheet. Crack bridging stresses will be relatively unimportant. As a result the crack growth rates will be similar to those of monolithic aluminium alloys.

In cases where starter notches are present, the situation after delamination is more complicated, but basically the general behaviour is the same.

3.5 Influence of material parameters on the fatigue crack growth

Fibres:

A first view point is that stiffer fibre layers enhance the crack bridging stresses. This can be achieved by using more or stiffer fibres. Aramid fibres with different Young’s moduli are available. Usually the stiffest type of fibres is applied (Twaron HM or Kevlar 49 fibres with a Young’s modulus of about 125,000 N/mm²).

Fibres with lower Young’s moduli show higher fracture strains (for the same strength). This would be beneficial for the fracture toughness of ARALL in case of a through crack [10] (no intact fibres in the wake of the crack). However this advantage is obtained at the cost of a reduction of the crack bridging efficiency.

The fracture strain of the Twaron HM fibres is about 2% and aramid with a higher stiffness does not exist. Carbon fibres are another type of high performance fibres. They are nearly two times stiffer than aramid fibres. However, the application of these fibres in ARALL turns out to be inappropriate, because the combination of carbon fibres and aluminium may lead to galvanic corrosion. Furthermore, the fracture strain of conventional carbon fibres, which is slightly higher than 1% may be too small for a successful application of the pre-straining process, which contributes considerably to the fatigue performance of ARALL. The fracture toughness may also be too low if carbon fibres are applied instead of aramid.

Some of these problems may be overcome by modern “high strain” carbon fibres, which became available just recently. These carbon fibres have a high Young’s modulus, combined
with a fracture strain of about 2%, leading to an extremely high strength. However, the problem of galvanic corrosion still remains.

Fibre volume content in relation to metal sheet thickness:

A further increase of the stiffness of the fibre layers, as compared to the aluminium layers, can be achieved by an increase of the amount of fibres relative to the aluminium. However, an increase of the fibre volume content reduces the compression strength of ARALL because of the moderate compression strength of aramid fibres. Previous investigations [2,10,11] have shown that an overall optimum for ARALL is reached if the fibre content in ARALL is sufficiently high to exclude the possibility of fibre fracture behind the crack tip under the anticipated loads. This is the case if the fibre-adhesive combination represents about 35 volume percent of ARALL. The fibre volume content of the fibre-adhesive layer should be about 50%. Summarizing it can be stated that the combined requirements for good mechanical properties of ARALL, including stiffness and sufficient fracture strain, emphasizes high fibre strength as being of prime importance. This strength requirement is further supported by the consideration that the fibres have to remain intact behind the crack during the entire fatigue life. At present, aramid fibres belong to a small group of materials exhibiting an extremely high specific strength and they are especially suitable for the present application.

Residual stresses:

Besides the increase of the stiffness of the fibre layers, further improvements in the fatigue properties of ARALL can be achieved by increasing the residual compressive stresses in the aluminium sheets. However, if these compressive stresses are increased above a certain limit, the compression strength of ARALL is reduced because the compression yield stress in the aluminium sheets is reached earlier than the compression limit of the aramid fibres. If the magnitude of the residual stress level is chosen such that both the aluminium sheets and the aramid fibres reach their compression limit stress at the same amount of external load, the material is "compression optimized ARALL". This problem has been treated experimentally and analytically by van Hengel [17]. It turned out that the amount of prestraining should be between 0.5 and 0.7%. Obviously, prestraining improves the ARALL properties with the penalty of higher material expenses.

Sheet thickness:

If the fibre to aluminium and adhesive ratios and the residual stress level are not modified, a further improvement of the ARALL properties can still be achieved by using thinner layers and thus more layers to obtain the same total ARALL thickness. An explanation is, that the adhesive deformation field as it is shown schematically in figure 8 remains similar, independently from the thickness of the single layers. The adhesive shear stress and deformation at the location of the crack is only dependent on the stiffness and thickness ratios of the three individual materials. Consequently, the crack opening displacement COD is proportional to
the thickness of the adhesive layer. A proportional reduction of the thickness of all layers implies a proportional reduction of COD. The stress intensity factor is related to the COD value, and it is reduced too. The fibre-adhesive layers in ARALL are standardized to some extent and their thickness is fixed. The effect described above, also occurs if the thickness is reduced in the aluminium sheets only. However, this implies an additional increase of the fibre aluminium ratio.

A further important consequence of a decrease of the thickness of the layers is that more fibre layers are applied for the same total laminate thickness. A direct consequence is that the crack bridging fibre stresses are transferred through a higher number of adhesive layers and the loading per layer is reduced. A decreased delamination rate is the result, thus improving the crack growth rates again.

The material expenses are increased if the thickness of the individual layers is decreased and the number of layers is increased. In view of these aspects a thickness of the individual layers of 0.3 to 0.5 mm for the aluminium sheets and of 0.2 to 0.3 mm for the fibre-adhesive layers appears to be the best overall compromise.

**Aluminium alloys:**

The crack growth rate in ARALL is still related to the correlation between the crack growth rate in non-reinforced aluminium sheets and the cyclic stress intensity factor. Consequently, the application of an aluminium alloy with a good fatigue crack growth resistance will be favourable for ARALL. However, the influence of the fatigue crack growth resistance of the aluminium alloy sheets is less important than it is for monolithic sheets. Increased crack growth rates imply that the number of cycles which is available for delamination growth is reduced. Consequently, smaller delamination to crack length ratios and an increased crack bridging efficiency are caused, and the effect of the lower fatigue crack growth resistance of the aluminium sheets is neutralized to some extent. After all the overall importance of the crack growth rate in ARALL is no longer so large because the crack growth rates are very small anyhow, and often ARALL components will not be fatigue critical. Consequently, arguments of strength will be more important in view of the material choice for ARALL, as compared to monolithic alloys.

The present considerations qualitatively show that the various design parameters of the ARALL composition are interrelated.

The present investigation is primarily dealing with the analysis of the fatigue related aspects. Experimental results of the present investigation are presented in chapter 4, analytical techniques in chapter 5 and quantitative considerations are presented in the chapters 6 and 7.
4. Experiments

4.1 Introduction and survey of the test programme

The behaviour of ARALL was qualitatively described in chapters 2 and 3, partly based on previous investigations and for another part on experimental and analytical results of the present investigation. The number of material variables of ARALL is large but the variables do affect a limited number of mechanical aspects only. These aspects are:

- Crack growth in the aluminium sheets.
- Crack bridging by fibres.
- Delamination in the fibre-adhesive layer.
- Shear deformation in the adhesive near the delamination boundary.

These aspects have a significant effect on the crack growth resistance of ARALL.

Several experimental programmes were carried out for two main purposes:

1. To obtain a qualitatively better understanding of the significance of several ARALL variables on fatigue crack growth.
2. To arrive at quantitative data required for modelling a fatigue crack growth prediction procedure.

The main topics of seven different test programmes are:

Section 4.2: static tensile properties

4.3: fatigue crack growth under constant amplitude (CA) loading
4.4: fatigue crack growth under flight-simulation (TWIST) loading
4.5: growth of delamination in the fibre-adhesive layers
4.6: shear deformation in the adhesive layers of specimens with cracks (static, cyclic, sustained loading)
4.7: crack opening displacements (COD)
4.8: initiation and growth of small cracks (CA-loading and TWIST)

A survey of the test programme is presented in the table on the next page.

In most programmes the problems studied are related to macrocracks (several millimeters and larger). However, small cracks (microcracks) and crack initiation were investigated in one test programme (section 4.8).

In most test series comparisons are made between ARALL, monolithic aluminium sheet materials and laminated sheets (sheets bonded together, but without any fibre reinforcement). Preliminary conclusions based on the experimental results are summarized for each test programme at the end of each section.
<table>
<thead>
<tr>
<th>Type of loading</th>
<th>Type of specimen</th>
<th>Purpose of test</th>
</tr>
</thead>
<tbody>
<tr>
<td>monotonic</td>
<td>tensile</td>
<td>determination of basic material properties</td>
</tr>
<tr>
<td>C.A. fatigue</td>
<td>centre notched sheets</td>
<td>crack growth measurements</td>
</tr>
<tr>
<td>TWIST flight simulation fatigue</td>
<td>centre notched sheets</td>
<td>crack growth measurements</td>
</tr>
<tr>
<td>C.A. and block programme fatigue</td>
<td>artificial cracks in the Al sheets</td>
<td>determination of the delamination properties</td>
</tr>
<tr>
<td>monotonic cyclic creep</td>
<td>thick adherend</td>
<td>determination of adhesive properties</td>
</tr>
<tr>
<td>monotonic cyclic creep</td>
<td>artificial cracks in the Al sheets</td>
<td>measurement of the COD due to adhesive deformation</td>
</tr>
<tr>
<td>C.A. and TWIST fatigue</td>
<td>double edge notched sheet specimens</td>
<td>initiation and growth of small cracks</td>
</tr>
</tbody>
</table>

Survey of the different types of tests

4.2 Tensile tests

The tensile properties of a material are important as basic design parameters for a structure. Therefore tensile tests on ARALL are presented here. The level of the residual stresses influences the tensile properties of ARALL. Specimens with different residual stress levels were tested. Additional tensile tests on the individual layers (aluminium sheets and fibre-adhesive layers) were performed. The results of the latter tests yield important input data for the calculations in the chapters 5, 6 and 7.

Summarizing, the three types of tensile tests are:

2. Aluminium alloy sheets.
3. ARALL

Fibre-adhesive layers

Two types of fibre-adhesive layers were tested. One is referred to as fabric-adhesive layer (right part of figure 1). It consists of a unidirectional fabric and two thin adhesive films (BSL-312-UL in the present tests). The nominal thickness is 0.25mm. The other type is referred to as prepreg (left part of figure 1). The prepreg is a special development by the 3M company for ARALL. It consists of about 50% unidirectional aramid fibres and 50%
AF-163-2 adhesive. The nominal thickness is 0.2 mm. The tests on the fibre-adhesive layers were performed on 20 mm wide strips with a length of 250 mm. The (main) fibre orientation coincided with the loading direction. The specimens were cured in a press. The curing process was performed between two release films. The individual specimens were cut from large cured sheets. The prepreg was cured at 120°C for one hour. The fabric-adhesive specimens were cured at 120°C for 30 minutes.

Tension tests on unidirectional composites can lead to some problems. The clamping at the ends of the specimens implies a local stress concentration and the clamping force may cause some damage in the specimens. Application of dog bone specimens does not substantially improve the situation, because the 'shoulders' of the specimens tend to split off from the specimen, due to longitudinal splitting. In the present tests the ends of the specimens were covered by a thin layer of a cured epoxy resin, as an attempt to protect them against clamping damage.

The tensile tests on both fibre-adhesive layers were performed on a mechanical spindle testing machine. The displacement rate was 0.02 mm/second at the clamping. Four tests were performed on both types of layers. For strain measurements, strain gages were bonded on the specimen surface.

Test results are presented in figures 10 and 11. The tensile behaviour is approximately linear. There is a very small increase of Young's modulus if the stress level is increased. At the upper end of the stress-strain curve, some sudden drops of the stress can be observed, especially for the prepreg material and at higher load levels. The drops are due to the local failure of some fibres, which is followed by a longitudinal split along the broken fibres. The strength of the specimen is then decreased to a lower value than the theoretical strength, because the effective cross section of the specimen is reduced.

Young's modulus E is usually measured at the lower end of the stress-strain curve. For the prepreg the modulus is:

\[ E_p = 62,000 \text{N/mm}^2 \]

For the fabric-adhesive combination it is:

\[ E_f = 48,600 \text{N/mm}^2 \]

Test results are summarized in the table on top of the next page.

The contribution of the adhesive to the strength and stiffness of the tensile specimens may be neglected, because the strength and stiffness values of the adhesive are very small as compared to these properties of aramid fibres. Consequently, the ratio of Young's modulus of the fibre adhesive combination (\( E_p \)) and of Young's modulus of the fibres alone (\( E_f \)) should be approximately equal to the fibre volume content. The Young's modulus of the fibres alone is 125,000 N/mm² [18,19]. The table shows that the ratios of the Young's moduli are indeed close to the designed value of the fibre volume content. The same behaviour should be expected for the strength ratio. However, with a fibre fracture strength \( S_{\text{fract}} \) of 3000 MPa [18,19] the strength ratio is considerably lower than the fibre volume
Survey of some test results

content $\nu_r$. The difference is probably caused by clamping damage. This view is supported by the fracture strain obtained in the tests, which was about 1.5% instead of the theoretical fibre fracture strain of 2.3% [18,19].

In the table the stiffness per mm width ($F_w$) of the fibre-adhesive layers is also given. The stiffness is calculated from $F_w = t_w E_w$ with $t_w$ as the thickness of the fibre-adhesive layers. The $F_w$ values are used for calculations in the next chapters. Both types of fibre-adhesive layers show about the same stiffness.

**Aluminium alloy sheets**

Figure 12 shows the monotonic stress-strain curve of 0.5 mm bare 7075-T6 sheet material (which was produced by ALCOA). The loading direction was in the rolling direction. The specimen was made according to the German Industrial Standards (specification 20×80 DIN 50114) as shown in figure 13. The material properties obtained are:

<table>
<thead>
<tr>
<th>$E_w$ (N/mm²)</th>
<th>$S_{0.2}$ (MPa)</th>
<th>$S_{fr}$ (MPa)</th>
<th>$\epsilon_{fract}$ (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>72000</td>
<td>530</td>
<td>580</td>
<td>13.5</td>
</tr>
</tbody>
</table>

**ARALL**

In figure 14 stress-strain curves are shown for the ARALL material used for the crack initiation tests (section 4.8). The ARALL type presented in this figure consists of two sheets of 0.5 mm bare 7075-T6 sheets and one layer of the fabric-adhesive combination.
Three different levels of residual stresses were used. In one case the specimens were tested with the residual stress as obtained after a normal curing cycle. In the two other cases a pre-stress was applied on the fibres during the curing. The residual stress in the aluminium sheets ($S_{\alpha\alpha}$) was measured by an etching technique developed by van Hengel [17]. One metal sheet is removed by chemically milling, which also removes the residual stress in this sheet. As a result the remainder of the material will be curved. The curvature is measured and the residual stress in the aluminium sheets can than be calculated from the radius of curvature. The residual stress in the aramid layers ($S_{\alpha\alpha}$) follows from $S_{\alpha\alpha}$ and the thickness ratio of the different layers (equilibrium). A calculation procedure for the residual stress calculations is given in [17].

The three different residual stress levels for the present specimens obtained in this way are:

\[
S_{\alpha\alpha} = +20\,\text{MPa and } S_{\alpha\alpha} = -80\,\text{MPa} \\
S_{\alpha\alpha} = -30\,\text{MPa and } S_{\alpha\alpha} = +120\,\text{MPa} \\
S_{\alpha\alpha} = -50\,\text{MPa and } S_{\alpha\alpha} = +200\,\text{MPa}
\]

Again 4 tensile tests were performed for each residual stress level. The tensile specimen as manufactured from ARALL is presented in figure 15. In order to minimize notch effects at the radii of the shoulders of the specimen, large radii were applied at these locations. Since ARALL is only slightly anisotropic it is possible to use 'dog bone' type tensile specimens, without getting longitudinal splitting along the fibres at the shoulders. Because clamping problems did not occur in these specimens, the theoretical fibre fracture strain of about 2.3% can be reached. Static fracture of ARALL is initiated by fibre fracture. If high residual tensile stresses are present in the aramid layers of the specimens, the fibres will fail at a nominal specimen strain lower than 2.3%, because some fibre pre-strain is already present in the unloaded specimen. The opposite is the case for specimens containing the residual curing stresses. These tendencies can be observed in figure 14. The fracture stress remains about the same for the different residual stress levels, but the fracture strains are affected by the residual stress levels. The tensile yield stress of ARALL is increased if compressive residual stresses are present in the aluminium layers of the ARALL specimens.

In some tests the aluminium layers remained intact after a complete failure of the fibres and the aluminium could be further strained after fibre failure. This straining process of the aluminium sheets after fibre fracture was accompanied by a delamination from the broken fibres.

**ARALL as compared to non-reinforced laminates**

In section 4.3 constant amplitude crack growth tests will be described. Those tests were performed on laminates with four sheets of bare 7475-T76 aluminium with a thickness of 0.45 mm each. Three fabric-adhesive layers are present. The tensile properties of the laminates (without fibres) are:
<table>
<thead>
<tr>
<th>$E_u$ (N/mm²)</th>
<th>$S_{0.2}$ (MPa)</th>
<th>$S_u$ (MPa)</th>
<th>$\epsilon_{fracture}$ (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>72000</td>
<td>480</td>
<td>550</td>
<td>8</td>
</tr>
</tbody>
</table>

The tensile properties of ARALL determined on specimens with residual curing stresses are:

<table>
<thead>
<tr>
<th>$E_u$ (N/mm²)</th>
<th>$S_{0.2}$ (MPa)</th>
<th>$S_u$ (MPa)</th>
<th>$\epsilon_{fracture}$ (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>65000</td>
<td>460</td>
<td>730</td>
<td>2.3</td>
</tr>
</tbody>
</table>

The nominal thickness of a fabric-adhesive layer is 0.25 mm. Due to scatter in the curing process, the actual thickness varied, the average value was about 0.3 mm. However, the values of Young’s modulus, $S_{0.2}$ and $S_u$ are based on the nominal thickness of the fibre-adhesive layers (4 × 0.45mm Al + 3 × 0.25mm Ar = 2.55mm). The shape of the stress-strain curve is nearly the same as shown in figure 14.

The material which has been used for the TWIST flight simulation crack growth tests (section 4.4) is very similar to the material described above and the tensile properties are similar as well.

4.3 Constant-amplitude fatigue crack growth tests

For the constant amplitude fatigue crack growth tests, centre notched specimens were used. Some tests were also performed on non-reinforced material. The aluminium sheet material used for the ARALL specimens was bare 7475-T76. The thickness was 0.45mm (a small batch of this material was kindly supplied by Dr. Falkenstein, VAW, Bonn). Unidirectional aramid fabrics and BSL-312-UL adhesive films were used. The production method was already described in section 2.2. The laminates were produced by the Department of Aerospace Engineering of the Delft University of Technology (Netherlands). The ARALL sheets consisted of four metal sheets and three layers containing aramid fabric and BSL-312-UL adhesive films. The nominal thickness of one fabric-adhesive layer is 0.25 mm. However, the actual thickness was close to 0.3mm. The specimen dimensions and the built up of the laminate are shown in figure 16. Micrographs of the material are presented in figure 2.

The residual stresses were introduced using the prestraining technique. For one test series the residual stress level in the aluminium sheets was $S_{\tau,u} = -95$MPa and for the other one it was $S_{\tau,u} = -115$MPa. The prestraining procedure was performed on long strips (70mm width) of ARALL. The specimens were cut from these strips, after the prestraining had been
finished, in order to keep differences in the residual stress levels for different specimens of one test series as low as possible.

Two specimens without fibre reinforcement were produced as well from the same 7475-T76 sheet material. Four metallic sheets and three BSL-312-UL adhesive films were applied in these specimens. The specimen dimensions and the laminate built up are shown in figure 17. Additionally six constant-amplitude fatigue crack growth tests were performed on single 0.5 mm thick 7075-T6 sheets as produced by ALCOA. The width of these specimens was 120 mm.

Aluminium sheet material.

Because of the very small thickness of these specimens, anti buckling guides were applied along both sides of the crack. Different combinations for the stress ratio $R$ ($R = S_{\text{max}}/S_{\text{min}}$) and the maximum stress $S_{\text{max}}$ were chosen for the tests:

- **$R=0.1$**
  - $S_{\text{max}} = 30$ MPa
  - $S_{\text{max}} = 60$ MPa
  - $S_{\text{max}} = 110$ MPa

- **$R=0.5$**
  - $S_{\text{max}} = 60$ MPa
  - $S_{\text{max}} = 150$ MPa

A saw cut with a total length of 2 mm was applied as a starter notch (this is denoted as $s = 1$ mm, where $s$ is the half length of the saw cut). A thin steel wire covered with diamond powder was used as a sawing device. An extremely thin slit could be achieved in this way, thus decreasing the number of cycles to crack initiation and the crack configuration becomes more symmetrical. Crack initiation for the lower load level had to be forced at the beginning of the tests, by the use of a higher stress level for the maximum load and compressive stresses for the minimum load (compression loading appears to be an effective tool for a rapid and symmetric crack initiation at low values of the maximum load level). After crack initiation load shedding to the actual test conditions was performed. The first half millimeter of subsequent crack growth was not used for the determination of the crack growth rates, in order to avoid sequence effects. The results are presented in figure 18 as a $da/dN - \Delta K$ plot. $\Delta K$ is the cyclic stress intensity factor ($\Delta K = K_{\text{max}} - K_{\text{min}}$). The stress intensity factor is calculated with

$$K = C S \sqrt{\pi a}$$

where $C$ is a correction factor for geometry effects. The crack growth data in figure 18 (double logarithmic plot) can be represented by a linear relation, which implies that the results can be described by the Paris law [20]:
\[
da/dN = c \cdot \Delta K^n
\]  \hspace{1cm} (4.3.1)

Figure 19 shows the crack growth rates as a function of the effective stress intensity factor \( \Delta K_{\text{eff}} \). The calculation of \( \Delta K_{\text{eff}} \) is performed by using Elber's equation [21].

\[
\Delta K_{\text{eff}} = (0.5 + 0.4 R) \Delta K
\]  \hspace{1cm} (4.3.2)

Figure 19 shows that the influence of the stress ratio \( R \) can be correctly accounted for by equation (4.3.2). and equation (4.3.1) can be written in a more general form:

\[
da/dN = c \Delta K_{\text{eff}}^n
\]  \hspace{1cm} (4.3.3)

with \( c = 6.04 \times 10^{-7} \) and \( n = 3.215 \). The effective stress intensity factor apparently is a unique parameter for the correlation of the present crack growth rates.

Aluminium sheet laminate (non-reinforced)

Crack propagation tests on laminated 7475-T76 sheets were performed with a stress ratio of \( R = 0.1 \) and a maximum stress \( S_{\text{max}} = 30 \text{ MPa} \) for one test and a maximum stress \( S_{\text{max}} = 100 \text{ MPa} \) for the other one. Saw cuts of 3mm (total length) were applied as starter notches \( (s=1.5\text{mm}) \). The initiation procedure was the same as described before. The results of the tests are presented in figure 20 as a \( da/dN-\Delta K \) plot.

The data in figure 18, 19 and 20 were generated at different stress levels. The results of the different stress levels overlap. As far as the results overlap the agreement is very good. The line of the \( R = 0.1 \) test on the single sheets is added in figure 20. The crack growth properties of the single 0.5mm 7075-T6 sheets are similar to those of the 7475-T76 laminates. The crack growth rates of figure 20 can be described with equation (4.3.3) and \( c = 9.7 \times 10^{-7} \) and \( n = 2.93 \).

ARALL

In the crack growth tests on the ARALL specimens different combinations of cyclic stresses and residual stress levels were applied. Again a saw cut with a half length of 's' millimeter was used as a starter notch. The length of the saw cut was also varied. An overview of the experiments is presented in the table on the next page.

\( S_{\text{u, max}} \) is the maximum stress as applied on the ARALL laminate. The nominal laminate thickness of \( t_s = 4 \times 0.45\text{mm} \) (Al) + \( 3 \times 0.25\text{mm} \) (Ar) = 2.55mm was used for the stress calculations. Two tests were performed for each configuration and condition, except for two cases, where the scatter exceeded a factor of two. In those cases an additional third test was performed.

In figures 21 to 25, the crack growth rates are plotted as a function of the half crack length \( a \) (including the half length of the starter notch). Average test results are presented only. The figures also show the results of the crack growth rate calculations, obtained with a
<table>
<thead>
<tr>
<th>$S_{\text{a, max}}$ (MPa)</th>
<th>$R$</th>
<th>$S_{\text{a}}$ (MPa)</th>
<th>$s$ (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>130</td>
<td>0.1</td>
<td>-95</td>
<td>2</td>
</tr>
<tr>
<td>130</td>
<td>0.1</td>
<td>-95</td>
<td>5</td>
</tr>
<tr>
<td>130</td>
<td>0.1</td>
<td>-115</td>
<td>2</td>
</tr>
<tr>
<td>130</td>
<td>0.1</td>
<td>-115</td>
<td>5</td>
</tr>
<tr>
<td>217</td>
<td>0.1</td>
<td>-115</td>
<td>2</td>
</tr>
<tr>
<td>217</td>
<td>0.1</td>
<td>-115</td>
<td>5</td>
</tr>
<tr>
<td>61</td>
<td>0.1</td>
<td>-115</td>
<td>3</td>
</tr>
<tr>
<td>61</td>
<td>0.1</td>
<td>-115</td>
<td>5</td>
</tr>
<tr>
<td>260</td>
<td>0.67</td>
<td>-95</td>
<td>5</td>
</tr>
</tbody>
</table>

The test conditions of the CA crack growth tests on ARALL computer programme described in chapter 7. As an example one curve is presented where the half crack length 'a' is plotted versus the number of fatigue cycles $N$ (see figure 26).

In figures 21 to 26 some typical features can be observed. The crack growth rate of ARALL can decrease with increasing crack length. This is due to the increasing amount of crack bridging fibres with increasing crack length. This effect is depending on the size of the starter crack notch, which was briefly explained already in section 3.1. Figure 25 shows that the crack growth rate of ARALL can also drop to zero. Crack arrest may especially be expected if the absolute value of the residual compressive stresses in the aluminium sheets of ARALL is higher than the stresses due to the external fatigue load.

4.4 TWIST flight simulation crack growth tests

4.4.1 The TWIST load spectrum

Fatigue loads on aircraft structures often are variable-amplitude loading patterns with load spectra depending on the structural component and the type of aircraft. The growth of fatigue cracks is considerably affected by the presence of overloads and underloads [22,23,24]. The fatigue crack growth rate in metals is usually delayed, after the occurrence of an overload, during a large number of cycles. Underloads cause a short increase of the subsequent crack growth rate and they may cause a considerable reduction of the beneficial effect of previous peak loads. Accurate crack growth calculations for variable amplitude conditions, based on constant amplitude crack growth data, are still difficult.

The variable-amplitude fatigue patterns on aircraft structures may exhibit overloads and underloads of different magnitudes. These load peaks interact in a complicated manner. The physical backgrounds of this behaviour are known; it is currently accepted that crack closure
is the governing mechanism [25,26]. However, an accurate calculation of the crack opening stress levels for different individual cycles is difficult.

The crack growth rate behaviour of ARALL under variable-amplitude loading appears to be even more complicated, because the delamination growth represents an additional important damage mechanism. The behaviour of the delamination growth mechanism under variable-amplitude fatigue loading is a new problem and hardly any knowledge is presently available.

Some delamination growth tests with simple variable-amplitude block loadings will be presented in section 4.5. The results indicate that significant sequence effects occur. The presence of overloads caused a significant increase of the fatigue delamination growth rates. This is in contrast to the behaviour of metals, where a considerable reduction of the fatigue crack growth rates may occur after overloads.

In this section the results of an experimental study of the fatigue crack growth behaviour of ARALL under standard flight simulation loading will be presented. Some standardized fatigue loading spectra are available for laboratory flight simulation testing. For ARALL which is a candidate material for the application to the lower wing skin of aircraft, the TWIST load spectrum is relevant for fatigue tests (TWIST = Transport WIng STandard).

The TWIST load history simulates the fatigue loading due to gust loads and ground to air cycles, of the lower skin of the wing root of a short to medium haul transport aircraft [27]. Ten different flight types, representing different flight severities were defined. The ten different flight types are usually referred to by the capitals 'A' to 'J', where 'A' refers to the most severe flight (extremely stormy weather) and 'J' to the least severe flight (nice weather). Ten different gust amplitudes with the Roman numbers I to X are used. The flights are separated by ground to air cycles.

The magnitude of the different amplitudes $S_i$ is related to the mean stress in flight $S_{\mu}$. During the ground to air cycles a compressive stress, $S_{\mu}$ occurs. This stress is: $S_{\mu} = -0.5 S_{\mu}$. A cumulative frequency diagram of the TWIST load spectrum is shown in figure 27. More detailed information is presented in table 1. The various flights occur in a (pseudo) random sequence. In each flight the gust amplitudes occur in a random sequence with a restraint, i.e. each positive gust amplitude is followed by a negative one and vice versa. The TWIST programme includes 4000 flights. For tests with longer fatigue lives, this block of 4000 flights is repeated. One block includes only one flight of type 'A', where the severest gust load with $S_{\mu,\text{max}} = 1.6 S_{\mu}$ occurs only once. The lower amplitudes occur in increasing numbers (see table 1).

For metallic materials, the crack growth is delayed significantly by the most severe upward gust loads and therefore they have a positive influence on the fatigue life. However, it may be expected that not every aircraft will experience such heavy loads. Hence, an unconservative indication of the fatigue behaviour in service may occur.

In order to obtain more conservative results, truncation of the spectrum is usually adopted. It means that the high load amplitudes are decreased to a certain lower value. Truncation
is also applied to most of the tests in the present investigation. The truncation level is: \( S_{\text{cut}} = 1.3 S_m \). The ground to air cycles are not truncated. The fatigue crack growth rate under truncated TWIST loading is compared to the crack growth rate under non truncated loading \( (S_{\text{cut}} = 1.6 S_m) \).

4.4.2 The test specimens

The tests were performed on centre notched sheet specimens. Monolithic bare Al 7075-T6 specimens with a thickness of 2 mm were used, as well as non-reinforced laminated specimens. The latter ones are built up from four 0.45 mm thick bare Al-7075-T6 sheets and three single BSL-312-UL adhesive films (thickness 0.08mm). Furthermore, different types of ARALL were tested. The influence of variations of the thickness of the individual sheets, and the influence of the pre-strain level was investigated.

The thin aluminium sheets for the laminated material were obtained by etching 1 mm thick sheets down to a lower thickness. The etching technique was chosen because thin bare Al 7075-T6 sheets were not commercially available at the time of the experiments. The etching as well as the production of the laminates was carried out at the Delft University of Technology. A survey of the various specimens is given in the table below.

<table>
<thead>
<tr>
<th></th>
<th>( t_0 ) (mm)</th>
<th>( w \times l ) (mm × mm)</th>
<th>type</th>
<th>( S_{\text{cut}} ) (MPa)</th>
<th>laminate built up</th>
<th>adhesive type</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>2</td>
<td>100 × 300</td>
<td>monolithic</td>
<td>—</td>
<td>—</td>
<td>—</td>
</tr>
<tr>
<td>B</td>
<td>4 × 0.45 Al</td>
<td>100 × 300</td>
<td>non-reinforced laminate</td>
<td>—</td>
<td>see fig.17</td>
<td>BSL-312-UL</td>
</tr>
<tr>
<td>C</td>
<td>4 × 0.45 Al + 3 × 0.25 Ar</td>
<td>70 × 250</td>
<td>ARALL</td>
<td>-95 -120 -145</td>
<td>see fig.28a &amp; fig.16</td>
<td>BSL-312-UL</td>
</tr>
<tr>
<td>D</td>
<td>3 × 0.9 Al + 2 × 0.5 Ar</td>
<td>70 × 250</td>
<td>ARALL</td>
<td>-95 -120</td>
<td>see fig. 28b</td>
<td>BSL-312-UL</td>
</tr>
<tr>
<td>E</td>
<td>3 × 0.9 Al + 2 × 0.6 Ar</td>
<td>70 × 250</td>
<td>ARALL</td>
<td>-95 -120</td>
<td>see fig. 28c</td>
<td>FM 123-5</td>
</tr>
</tbody>
</table>

A survey of the test specimens for flight simulation testing
The non-reinforced laminate (type B) and ARALL (type C) are very similar to the laminate types which were considered before in section 4.3 where the constant-amplitude tests were described. The micrographs in figure 2 are also representative for the present ARALL type C. The construction of ARALL type C, D and E is shown in figure 28. Micrographs of the types D and E are given in figures 29 and 30 respectively. The laminates of type C, D and E were designed in such a way that the relative amounts of aluminium, fibres and adhesive were about the same for all three types of specimens. The adhesives used for the laminates type D and E are different. For laminate type E the four BSL-312-UL films are replaced by two FM 123-5 adhesive films (as produced by American Cyanamid). The FM 123-5 adhesive contains a light nylon carrier. The BSL-312-UL adhesive does not contain a carrier.

4.4.3 The experimental programme

The fatigue tests with the TWIST load history were performed on a computer controlled closed loop servo-hydraulic fatigue testing machine with a testing capacity of 100 kilo Newton. The data acquisition was performed by the computer. Manual checks of the various load levels were additionally performed.

Two different types of starter notches were used, a saw cut with a length of $2s = 2.6\text{mm}$ and a circular hole with a diameter $D = 7\text{mm}$.

Different levels of the mean stress in flight were applied for the crack growth tests. A mean stress level of $S_{\sigma} = 70\text{MPa}$ is often adopted for laboratory flight simulation testing on conventional aluminium alloy specimens (see [23]). This stress level is typical for conventional transport aircraft wing structures in service, although it is somewhat high for some types of aircraft.

The $S_{\sigma}$ values which were applied on the monolithic sheets and the non-reinforced laminates in the present investigation were 70 MPa and 100 MPa. An increase of the stress level from 70 MPa to 100 MPa corresponds to a weight saving of almost 30%. For ARALL $S_{\sigma} = 87\text{MPa}$ was chosen. The density of aluminium alloys is about 2.8 grams/cm$^3$. For ARALL the density is only about 2.4 grams/cm$^3$. The specific stress level is defined as stress divided by density. The specific stress in ARALL loaded with 87 MPa is the same as in a monolithic aluminium sheet loaded with 100 MPa. The specific stress level is the most important parameter for the structural efficiency, and the test results of ARALL loaded with $S_{\sigma} = 87\text{MPa}$ are directly comparable to the results on monolithic sheets loaded loaded with $S_{\sigma} = 100\text{MPa}$. A survey of the different experiments is presented in the table on the next page.

Some additional tests were performed on ARALL with a stress level of $S_{\sigma,\sigma} = 61\text{MPa}$ (which is equivalent to 70 MPa on monolithic aluminium). However, in these tests the crack length remained extremely short, and no relevant crack growth rates could be observed. This is the reason why the results with $S_{\sigma,\sigma} = 87\text{MPa}$ are presented only.
<table>
<thead>
<tr>
<th>Type of notch</th>
<th>Type of material</th>
<th>$S_{e,0}$ (MPa)</th>
<th>$S_a$ (MPa)</th>
<th>Truncation level $S_{a,\max}/S_{a,min}$</th>
<th>Specimen width (mm)</th>
<th>Number of tests</th>
</tr>
</thead>
<tbody>
<tr>
<td>2s=2.6mm</td>
<td>monolithic</td>
<td>—</td>
<td>70</td>
<td>1.3</td>
<td>100</td>
<td>2</td>
</tr>
<tr>
<td></td>
<td>(A)</td>
<td>—</td>
<td>100</td>
<td>1.3</td>
<td></td>
<td>2</td>
</tr>
<tr>
<td></td>
<td>unreinforced</td>
<td>—</td>
<td>70</td>
<td>1.3</td>
<td>100</td>
<td>2</td>
</tr>
<tr>
<td></td>
<td>laminate (B)</td>
<td>—</td>
<td>100</td>
<td>1.6</td>
<td></td>
<td>2</td>
</tr>
<tr>
<td></td>
<td>ARALL (C)</td>
<td>-120</td>
<td>87</td>
<td>1.3</td>
<td>70</td>
<td>2</td>
</tr>
<tr>
<td></td>
<td></td>
<td>-145</td>
<td></td>
<td></td>
<td></td>
<td>1</td>
</tr>
<tr>
<td>D=7mm</td>
<td>ARALL (C)</td>
<td>-95</td>
<td>87</td>
<td>1.3</td>
<td>70</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td></td>
<td>-120</td>
<td></td>
<td></td>
<td></td>
<td>2</td>
</tr>
<tr>
<td></td>
<td></td>
<td>-145</td>
<td></td>
<td></td>
<td></td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>ARALL (D)</td>
<td>-95</td>
<td>1.3</td>
<td>1.3</td>
<td>1.6</td>
<td>2</td>
</tr>
<tr>
<td></td>
<td></td>
<td>-120</td>
<td></td>
<td></td>
<td></td>
<td>2</td>
</tr>
<tr>
<td></td>
<td>ARALL (E)</td>
<td>-95</td>
<td>1.3</td>
<td>1.3</td>
<td>1.6</td>
<td>2</td>
</tr>
<tr>
<td></td>
<td></td>
<td>-120</td>
<td></td>
<td></td>
<td></td>
<td>2</td>
</tr>
</tbody>
</table>

Survey of the different flight-simulation experiments

4.4.4 The experimental results

4.4.4.1 Results for non reinforced material

The results of the tests on monolithic aluminium and unreinforced laminates are shown in figure 31 where the crack length is given as a function of the number of flights. Arrows indicate the positions of severe flights (A) which are followed by some delayed crack growth. Figure 32 gives the results in the form of a $da/dN - K_{ef}$ diagram for mean stress levels of 70 and 100 MPa ($S_{a,\max} = 1.3 S_{a,min}$). $K_{ef}$ is calculated with
\[ K_{mf} = C S_{mf} \sqrt{\pi a} \]

The \( da/dN \) values were obtained from "averaged" crack growth curves, which eliminates the irregular crack growth due to the severe flights (figure 31). In this way the 'scatter' due to such irregularities is excluded in the diagrams.

Figure 32 clearly illustrates the influence of the sheet thickness on the crack growth rates. The growth rate in the 2mm monolithic material is systematically higher than for the laminates built up from 0.45mm sheet material. A different sheet thickness leads to different stress states at the crack tip (plane stress - plane strain). This difference is especially pronounced if peak loads are present in the fatigue loading [1].

An influence of the magnitude of the \( S_{ur} \) stress level is also found in the \( da/dN - K_{ur} \) diagram for the laminated material. This behaviour is not unusual for variable amplitude tests [25].

The correlation between a characteristic \( K \) value (\( K_{ur} \) in this case) and the crack growth rates is obviously no longer unique in flight simulation tests. This is in contradiction to constant-amplitude fatigue loading where generally a unique correlation between \( \Delta K \) and the crack growth rates is found. The difference may be explained by the greater relevance of the plastic wake zone under variable amplitude loading conditions.

Since the effect cannot be properly described by using a stress intensity concept only, Schijve [28] proposed a correction for the influence of the plastic wake zone, which considers the gradient of the \( K \) value as a function of the crack length (\( dK/da \)).

An other contribution to the higher crack growth rates at higher load levels may be the higher compressive stress in the net section which then occurs. In a previous publication [29] the present author et al. demonstrated that the ratio of the compression stress and the material strength is a more appropriate parameter for the influence of compression loads than parameters which stem from stresses or stress intensities only (\( R \) values or characteristic \( K \)-values). When the crack is completely closed under compressive stresses, \( K \) is no longer an adequate parameter for a consideration of the influence of the compressive stresses. Short cracks combined with high stress levels are more severely loaded during compression than long cracks with low stress levels, also if the \( K \)-values during the tensile part of the load excursions are identical.

The good correlation for the results on the monolithic sheets suggest that the considerations about the \( dK/da \) parameter and the compression stress might be important for the plane stress case only.

4.4.4.2 Results for ARALL, influence of the notch and the residual stresses

In figure 33 the crack growth curves of ARALL with different levels of the residual stress and different notch sizes are compared to the crack growth curves of unreinforced aluminium laminates and monolithic aluminium sheets. The comparison in figure 33 is performed on the basis of a same specific stress level for the different types of material (same mean stress in flight / density). It implies that: \( S_{ur} = 100\text{MPa} \) for the monolithic material corresponds to \( S_{ur} = 87\text{MPa} \) for ARALL.
Figure 33 shows that ARALL exhibits extremely good fatigue properties as compared to non-reinforced metallic materials. The figure further shows the favourable effect of the residual stress on the crack growth rates. Figure 33 also shows the significance of the size of the starter notch for the crack growth rates during the entire fatigue life. An effect of the notch size and shape is common for a flight simulation loading [23]. However, the extreme differences which are found in figure 33 are mainly due to the greater amount of crack bridging fibres which are present as the notch is smaller.

Figure 34 shows an example of a crack growth curve (a-N) for ARALL. Due to the high load level, the high stress concentration factor, and the poor surface quality in the bore of the hole (no reaming or polishing had been performed) crack initiation occurred rapidly at the notch root. However, due to the slow crack growth the total fatigue life was long. After 60,000 flights the fatigue test was terminated, because the crack growth rates became so small that a fatigue life of more than 200,000 flights could be expected, and 60,000 flights represent already a long fatigue life for current aircraft.

4.4.4.3 The influence of the thickness of the individual layers

Figure 35 shows the influence of the sheet thicknesses for ARALL. In all specimens the ratios of metal, fibres and adhesive were approximately the same. For both types of ARALL (type C and D) the influence of the residual stresses showed the same tendency.

The crack growth rates of ARALL built up from 0.9mm aluminium sheets were considerably higher than those of ARALL made from 0.45 mm aluminium sheets. A reason for this trend is the higher sensitivity to delamination for the "thick-layer-laminates". Figure 36 shows that different amounts of delamination do occur. A larger delaminated area causes a decreasing crack bridging fibre efficiency, an increase in the stress intensity and higher crack growth rates in the aluminium. The dependence of the delamination rates on the layer thickness was already discussed in chapter 3, and the behaviour is confirmed by the delamination tests in section 4.5. It will be further analysed in section 5.3, 5.5 and chapter 6.

Another reason for the higher crack growth rates in ARALL with an increasing thickness of the aluminium sheets is that the displacements between the crack flanks, due to adhesive deformation, become larger for ARALL with thicker layers (see chapter 3). A similar behaviour was found by Roderick [30], for a boron-aluminium sandwich material. The significance of the displacements due to a shear deformation of the adhesive is experimentally shown in more detail in section 4.7.

An additional explanation for the differences in the crack growth rates may be found in the different stress-strain conditions at the crack tip, (plane stress vs. plane strain) these differences are especially pronounced for fatigue loadings which include overloads [1].
4.4.4.4 Influence of the truncation level and of the adhesive type

The crack growth results of ARALL type D and E (thick layers) were identical. A change of the adhesive from four BSL-312-UL films to two FM 123/5 films did not cause differences in the crack growth rates. Obviously both adhesives behaved in a similar way. The results for the ARALL types D and E are presented in figure 37, for two truncation levels of the TWIST spectrum $S_{\text{max}} = 1.6 S_{m}$ and $S_{\text{max}} = 1.3 S_{m}$. In both cases two levels of residual stresses were applied. The figure shows that truncation has a large influence on the crack growth rates of prestrained ARALL. The influence of truncation increases as the level of the residual compressive stresses in the aluminium sheets is increased. For a residual stress level of $S_{m} = -120 \text{MPa}$ in the aluminium sheets, the crack growth rate for the untruncated load sequence is up to a factor of hundred lower than the crack growth rate for the truncated sequence.

In figure 38 the influence of truncation is shown for the monolithic aluminium sheets. It can be seen that the influence of truncation is only by a factor of about two.

The extremely high influence of truncation on prestrained ARALL (which does not occur to that extent for unreinforced material and for non prestrained ARALL [2]) can be explained by the change of the ratios of the $K$ values at the overloads and the other load levels. Figure 39 explains this effect for the example of a simple block programme with overloads. When favourable residual stresses occur the crack flanks are pressed together and a certain load has to be applied to raise the stress intensity factor above zero. This load is higher if the compressive stress in the aluminium sheets is higher, and the remaining upper part of the load sequence, which causes a stress intensity factor which is smaller. The constant shift of the stress intensity factors associated to the various load levels implies a higher percentage of reduction for the stress intensity factor at the lower load levels. In other words: the relative magnitude of the overload stress intensity factors as compared to the stress intensity factors at lower load levels is larger. Consequently, the associated crack growth delays and the effect of truncation will be larger. This effect is demonstrated in chapter 8, with a calculation of stress intensity factors for some load levels of the TWIST spectrum.

4.4.5 Some concluding remarks

The results of the TWIST crack growth tests with ARALL showed that the crack growth resistance of ARALL to flight simulation loading was extremely high.

The results indicate that an application of prestrained ARALL for a lower wing skin allows weight savings of up to 30 %, as compared to the non-reinforced material. Additionally a significant improvement of the fatigue life is achieved. The best results were obtained with ARALL in the optimized condition (thin layers). However, for the non-optimized condition (thicker layers) good properties were observed as well for this type of loading, provided that the residual compressive stresses in the aluminium sheets are sufficiently high.
4.5 Delamination growth tests

4.5.1 The test set up

The fatigue crack growth in ARALL is significantly influenced by the amount of delamination which occurs during the fatigue process in the adhesive. Quantitative data about the delamination rate as a function of the cyclic crack bridging stress were obtained, by using specially prepared delamination test specimens. The specimens with a width of 20mm and a length of 300mm consisted of two aluminium layers and one fibre-adhesive layer in between (see figure 40). The specimens were provided with an "artificial" crack (complete separation of the aluminium sheets across the full specimen width). The artificial crack was produced by spark erosion of the aluminium layers, or by the lamination of four different aluminium strips, together with one fibre-adhesive layer.

No load transfer can occur across the artificial cracks. The entire load from the aluminium sheets is transferred through the adhesive into the fibres at the location of the artificial crack. At a later stage of the tests, the load transfer occurs near the end of the delaminated area which has developed until then. The crack bridging stress at the delamination boundary is exactly known for this typical delamination specimen, because it is directly related to the load on the specimen. Consequently, the experimentally determined delamination growth rates can be quantitatively correlated to the cyclic crack bridging stress. The delamination rate remains constant during a test, because the crack bridging stress is independent of the area which has already delaminated.

Three to five tests with different fatigue load levels could be performed on each specimen. Each test was run until a delamination growth of about 10 mm had occurred. After this growth the delamination growth rate \(\frac{db}{dN}\) was calculated. In general about three tests were performed for each loading condition and specimen type. The experimental \(\frac{db}{dN}\) values given in this chapter are average values of multiple test results.

The delamination growth was measured by a photo-elastic technique. One of the specimen surfaces was prepared by sand paper to achieve a good light reflection and a clean surface for the subsequent bonding of a birefringent foil (a plastic foil with photo-elastic properties) to this surface. The bonding was performed with a transparent epoxy adhesive. The delamination front could be observed during the tests by the application of a polar filter, which was mounted between the specimen and a lamp. The birefringent foil faced the polar filter. The polarizing direction of the filter was orientated at an angle of 45° to the loading direction of the specimen. The test set up is shown in figure 41. The delamination fronts of the specimen appear at the specimen surface in the form of fringes in the birefringent coating. These fringes are caused by the strain gradient occurring at the transition from the loaded aluminium sheets, which are not delaminated, to the delaminated parts of the sheets, which are no longer loaded. Figure 42 shows a photograph of a specimen with such fringes at the delamination front. At the back side of the specimen, a paper grid was attached for reading the location of the fringes.
The testing method also allows a determination of the residual stress in the non-delaminated area of the aluminium sheets. If a compressive residual stress is present in the non-delaminated aluminium sheets, an external tensile load can be applied at the specimen which just compensates this residual stress. Then the stress in the non-delaminated area of the aluminium sheets is zero. The stress in the delaminated area of the aluminium sheets is always zero. As a consequence, no load transfer occurs through the adhesive at the delamination front, and fringes are absent. The external load level at this point is a direct measure of the magnitude of the residual stresses in the specimen.

It was expected that delamination growth increments of about 10 mm should be required, to assure a sufficiently high accuracy, by the photo-elastic technique. Since very small delamination growth rates occurred at low load levels, many fatigue cycles had to be applied for a delamination growth increment of about 10 mm. Therefore an ink infiltration technique was applied for some tests at low load levels. The ink infiltration marked the location of the delamination front at the beginning of such a test. After the end of the test, the aluminium sheets were peeled off, and the delamination growth increment of that test becomes visible on the delaminated surface. This method requires a minimum delamination growth of about one or two millimeters for a measurement of a sufficient accuracy. Some tests were still extremely time consuming (up to 3 × 10^7 cycles had to be applied in some tests).

The delamination tests were performed at different load levels. Two limitations had to be considered: The lowest load level at which delamination growth rates could be obtained within an acceptable testing time, and secondly on the other hand the highest load level limited by the occurrence of fibre fracture. The maximum load level which could be applied was 150 Newton per millimeter width for the specimens containing a fabric, and 165 N per mm for the specimens containing the prepgs. These load levels corresponded to stresses which were respectively 5% and 15% lower than the fracture stresses in the tensile tests on the fibre-adhesive layers (see section 4.2).

The computer controlled testing machine compensated variations of the fatigue load level due to the decreasing stiffness of the specimens during increasing delamination. Manual checks on the load levels were additionally performed.

The frequency in the delamination tests varied between 5 Hz and 30 Hz. 5Hz was applied at the higher cyclic load levels, and 30 Hz was applied at the lower levels, in order to save testing time. In a special test it was shown that no significant frequency effect occurred at lower load levels in the frequency range between 5 Hz and 30 Hz. At high load levels a low frequency had to be applied in order to prevent a heating up of the test specimens.

Two types of delamination tests were performed:

- Constant-amplitude fatigue tests.
- Block-programme fatigue tests.
In the block programme test a block consists of one larger cycle \((N_i = 1)\) and a fixed number \((N_t)\) of smaller cycles. Blocks with overloads and blocks with underloads were applied. Such blocks are continuously repeated during the tests. The frequency in the block programme tests was 10 Hz.

The crack growth rates in metals under variable-amplitude loading can be strongly influenced by interaction effects. The reasons for the non-linear damage accumulation in metals are rather well understood. However, for delamination the behaviour under variable-amplitude loading is largely unknown. The block programme tests can be seen as a first step towards an evaluation of the delamination mechanism under variable amplitude fatigue loads.

4.5.2 The specimen types

Different types of specimens were used, see the table below.

<table>
<thead>
<tr>
<th>(t_{ul}) (mm)</th>
<th>fibre-adhesive system</th>
<th>(t_u)</th>
<th>residual stress (MPa)</th>
<th>type of fibre</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>nominal</td>
<td>actual</td>
<td>(S_{u,ul})</td>
</tr>
<tr>
<td>2×0.9</td>
<td>4×BSL + 2×fabric</td>
<td>0.5</td>
<td>0.56</td>
<td>25 *</td>
</tr>
<tr>
<td>2×0.45</td>
<td>2×BSL + fabric</td>
<td>0.25</td>
<td>0.25</td>
<td>0</td>
</tr>
<tr>
<td>2×0.50</td>
<td>2×BSL + fabric</td>
<td>0.25</td>
<td>0.25</td>
<td>0</td>
</tr>
<tr>
<td>2×0.50</td>
<td>AF prepreg</td>
<td>0.20</td>
<td>0.30</td>
<td>15 *</td>
</tr>
<tr>
<td></td>
<td>2×AF + fabric</td>
<td>0.25</td>
<td>0.25</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>0.25</td>
<td>0.40</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

* no prestressing applied

Survey of the different delamination specimens

The first column shows the thickness of the two aluminium alloy sheets. The second column shows the type of fibre-adhesive layer. BSL and AF correspond to BSL-312-UL and AF 163-2 adhesive, respectively. The AF 163-2 film which was used here in combination with fabrics, is a special product of the 3M company. The film was produced without a carrier.
It has a weight of 80g/m². This adhesive film is comparable to the BSL-312-UL film (100g/m²). It is a convenient film for the production of fibre-adhesive layers in a laboratory. The third column gives the nominal thickness of the fibre adhesive combination layer. This thickness should be obtained for a normal curing cycle. However, some specimens showed a slightly deviating thickness. The actual thickness is given in the fourth column.
The residual stress levels in the aluminium and aramid layers (\(S_{a}\) and \(S_{b}\)) respectively are found in the following columns. The positive stresses in the aluminium layers are due to the curing process. The negative stresses were obtained by the prestressing technique. This technique was also applied for the laminates with zero residual stresses.
The residual stresses were determined by the etching technique described before (see section 4.2). The residual stresses in the fibre-adhesive layers are based on the nominal thickness of the layers. Without prestressing the curing cycle leads to lower residual stresses in the laminates with the AF 163-2 adhesive, as compared to laminates with the BSL-312-UL adhesive. It can be explained by the lower temperature at which crosslinking starts in the AF 163-2 adhesive. The heating rate during curing was low, and solidification of the AF 163-2 adhesive started before the curing temperature of 120°C was reached. However, the BSL-312-UL adhesive remained as a liquid up to the temperature of 120°C and the solidification occurred later on. As a result, the differential expansion (contraction for the fibres) is larger.

For all specimen series, except one, TWARON HM fibres were applied. Teyin HM-50 fibres were used for one specimen series, in order to evaluate the influence of Young's modulus of the fibres. Young's modulus of the Teyin-HM fibres is 73,000N/mm² [31] as compared to 125,000N/mm² for Twaron-HM [19]. Unfortunately, the thickness and the weight of the Teyin fabric were higher than for the Twaron fabric, due to a different weaving process of the fabric manufacturer. The fabric properties are summarized in the table below.

<table>
<thead>
<tr>
<th>fibre type</th>
<th>fibre modulus (MPa)</th>
<th>fibre density g/cm³</th>
<th>fabric weight g/m²</th>
</tr>
</thead>
<tbody>
<tr>
<td>Twaron HM</td>
<td>125,000</td>
<td>1.44</td>
<td>160</td>
</tr>
<tr>
<td>Teyin-HM-50</td>
<td>74,000</td>
<td>1.39</td>
<td>190</td>
</tr>
</tbody>
</table>

In summary it was found that the Teyin fabric contains 1.24 times more fibres than the Twaron fabric (in volume). The actual thickness of the Teyin fibre adhesive layer was 0.4mm, as a consequence of the over-thickness of the fabric and some adhesive flow towards the centre of the specimens.
All specimens within one series were produced as a single sheet from which the specimens were cut. Tests on specimens which stem from the edges of the sheets were performed but the results are not presented, because higher delamination rates occurred in these specimens. The higher delamination rates are caused by a lower adhesive content which is due to
bleeding out of the adhesive during the curing (this does not occur in ARALL sheets with prepregs). As a result lower \( t_u \) values occurred, as low as 0.2mm for specimens with a fabric. The delamination rates of edge specimens were about 5 to 10 times higher than for regular specimens. A considerable scatter occurred and quantitative conclusions should not be drawn from the results of the edge specimens. Qualitatively, it can be concluded that a reduction of the adhesive content leads to an increase of the delamination rates.

4.5.3 Results of the constant amplitude fatigue delamination growth tests

The constant-amplitude delamination tests were performed at different amplitudes and stress ratios. The results are presented in the figures 43 to 51 to compare results of different test series. The delamination rate is plotted as a function of the stress in the delaminated fibre layers \( S_h \) (based on the nominal thickness \( t_u \) of the fibre-adhesive layer). In some figures the delamination rate is shown as a function of the load per mm width. A double logarithmic scale is used in all figures.

Some results from tests with very high \( R \) ratios are not presented in the figures. However, all results are given in table 2.

The results in figures 43 to 51 suggest that the delamination rate follows a typical 'Paris type' behaviour [20]. The trend of the results can be represented quite well by a straight line in the double logarithmic plot. Although the figures show some scatter, significant influences can be observed concerning the different parameters of the built up of the laminates and the different stress ratios. A discussion of the results is presented in section 4.5.5.

4.5.4 Results of the block programme fatigue delamination growth tests

The results for specimens with the BSL Twaron fabric layers are presented in figure 52. In the specimens residual curing stresses are present. Four different numbers of the small cycles were used, viz. 4, 20, 100 and 500. The experimental results (averages of four similar tests) are compared to the results of a linear delamination growth calculation, based on the constant-amplitude results. Mean values of the delamination growth rates are linearly calculated according to the Miner rule [32]:

\[
\overline{db}/dN = \frac{(db/dN_i) N_i + (db/dN_f) N_f}{N_1 + N_2}
\]  

(4.5.1)

where \( \overline{db}/dN \) is the average delamination rate due to the applied block loading and \( db/dN_i \) is the constant amplitude delamination growth rate for the small cycles. \( (N_i) \). The corresponding values for the large cycles are \( db/dN_f \) and \( N_f \) \( (= 1) \).

Figure 52 shows considerable differences between the experimental \( \overline{db}/dN \) values and the \( db/dN \) values from a linear calculation. The experimental \( \overline{db}/dN \) values turn out to be larger than the theoretical values! The peak loads considerably accelerate the delamination. Already one peak load per 500 small cycles gives a large effect. This trend is opposite to observations
on fatigue crack growth in monolithic metals, where peak loads usually cause a considerable retardation of the crack growth (see [22,26,29]).

Figure 53 shows the results of similar tests with specimens containing fibre-adhesive layers of a prepreg. A residual stress system due to curing is also present in these specimens. Two types of block programmes were used. The large cycles were applied as overloads (overload block) or as underloads (underload block). The ratio of number of the small and of the large cycles is 10. Two or three tests were performed for both types of block programmes.

The overload block in figure 53 shows the same trends as shown in figure 52. The peak loads cause an experimental delamination growth rate which is significantly higher than the theoretical value. The same trend is also found for the underload block in figure 53. However, for this loading type, the difference between the theoretical value and the experimental results is much smaller. The linear damage calculation indicates the underload programme to be much more damaging than the overload programme. The test results show the opposite trend.

Another test series was carried out to verify the unexpected results of figure 53. A larger difference between the high and the low cycles was chosen, and the amount of small cycles was increased up to fifty. The specimens were prestressed, to such an extent that no residual stresses were present after curing. The number of tests for each load level (constant-amplitude and block programme) was increased to about seven.

Figure 54 shows the average values of the test results. The trends for both types of block loadings are identical in the figures 53 and 54. Unfortunately, the scatter of the latter test series was larger than in figure 53. Nevertheless, the experimental difference between the overload and underload blocks is similar as in figure 53, although it is small. The differences between theoretical and experimental values is larger in figure 54 than in figure 53.

4.5.5 Discussion of the delamination growth results

4.5.5.1 Discussion of the constant amplitude tests

Effect of the load level

Various types of specimens and stress ratios ($R$) were tested. In general, the results follow a straight line on a double log scale. The delamination rate can then be described by the Paris type equation [20]:

$$\frac{db}{dN} = q \Delta S^n_{th}$$  \hspace{1cm} (4.5.2)

with constants $q$ and $m$.

Such a power law behaviour is not only typical for the crack growth behaviour of metals, it is also frequently found for delamination in composites and adhesive bonded joints e.g. [30,33,34].
Effect of the layer thickness

Figure 43 shows that a twofold increase of the thickness of all individual layers causes a
tenfold increase of the delamination rate. This may be due to the higher load which has to
be transferred per adhesive interface between the fibres and the aluminium if the thickness
of the individual layers is increased.

Figure 46 shows the influence of a change in the thickness of the aluminium sheets only.
The specimens for both test series were prestressed to obtain a zero residual stress level.
The delamination rate became smaller if the thickness of the aluminium sheets decreased.
A similar trend was already observed in figure 43. However, in figure 46 the fibre-adhesive
layer is not modified, but the aluminium to fibre ratio is changed. It is important to note
that the cyclic stress levels in the aluminium sheets are higher for the specimens with the
thinner aluminium sheets (same $\Delta S_0$). The difference in figure 46 becomes considerably
larger if the cyclic stresses in the aluminium sheets were taken as a basis for comparison,
as shown in figure 47. The reduction by a factor 2 of the aluminium sheet thickness causes
an increase by a factor 1.7 for the stress in the aluminium sheets. This latter way of com-
parison yields a good estimation of the delamination quality of an ARALL type, because the
stress level in the aluminium sheets is an important loading parameter for ARALL.

Effect of the stress ratio and the residual stress

Figure 43 shows the influence of the stress ratio $R$. If the stress ratio is increased from 0
to 0.5, the delamination ratio increases also. The figure indicates that the influence of the
stress ratio was smaller at lower stress levels. However, some remarks should be made
regarding this observation. The delamination rate is dependent on the shear stress in the
adhesive and on the $R$ ratio of the shear stress (experimental verification of the latter
assumption is presented in a later part of this discussion). Due to the residual curing stresses
in the specimens, a shear stress is already present in the adhesive at the delamination front,
in the unloaded specimen. That means that a static shear stress is superimposed on the cyclic
shear stress caused by the external load. This increases the stress ratio of the shear stresses
in the adhesive at the delamination front. $R$ is the stress ratio of the external loads, whereas
$R_{sp}$ denotes the shear stress ratio at the delamination front. The constant shift of the shear
stress will increase $R_{sp}$ and the increase will be larger for lower load levels. Moreover, the
increase of $R_{sp}$ will be more significant for lower $R$ values. The results in figure 43 for
$R = 0.1$ and $R = 0.5$ agree with these arguments.

Figure 44 shows results for one cyclic load range (constant $\Delta S_0$) and different maximum
loads. As a consequence $R$ will increase for increasing $S_{\text{eq}}$. The results show that the
delamination rate significantly increases as well.
Figure 45 shows a similar trend for results of another test series on specimens, which were
prestressed in such a way that the residual stresses were zero. In this case $R_{sp}$ and $R$ are
identical.
The delamination growth rates of a laminate with a relatively high compressive residual stress in the aluminium sheets are presented in figure 49. The residual stress in the aluminium sheets was: $S_{r,a} = -43$ MPa.

The adhesive shear stress at the delamination front is zero if the load level per mm width is:

$$ P = -t_a(1 + F_a/F_d) S_{r,a} = 50N/mm $$

(The derivation of this equation will be presented later in section 5.) The predicted load level was confirmed by photo-elastic measurements. The fringes at the location of the delamination front, in the birefringent coating disappeared at a load level per mm width of 50N/mm. It implies that $R_{av}$ will be zero, for external fatigue loads if the minimum load is 50N/mm. Fatigue tests of this kind were performed and the results are presented as the lower right data set in figure 49. The delamination rates are plotted as a function of the maximum load per mm width. The cyclic load for the data with $R_{av} = 0$ can be obtained by subtracting 50N/mm from the maximum load level. This has been done in figure 55 for a comparison to the data with the results with $R = R_{av} = 0.1$ and $S_{r,a} = 0$ from figure 46. The laminate types were the same for both cases. For a fully correct comparison, the apparent stress ratio should be zero for both test series. However, the difference in delamination rates for $R = 0$ and $R = 0.1$ is small (see figure 45), and it does not affect the agreement in figure 55.

The agreement in figure 55 is quite good. It indicates that the concept of the apparent stress ratio $R_{av}$ is correct. That means that the delamination rate is actually dependent on the stresses in the adhesive at the delamination front, independently of the stresses in the aluminium sheets and in the fibres.

Figure 49 also shows results for $R = 0$ ($P_{\text{av}} = 0$). In this case $R_{av}$ varies. In figure 50 these results are compared to results for $R = 0.1$ and $S_{r,a} = 0$. It turns out again that the influence of the residual stress on the delamination growth rates is large.

**Analytical description of the delamination results**

For quantitative calculations of delamination growth rates, (required for the computer programme described in section 7) a generalized description of the influence of the mean stress on the delamination growth rate is required. In the following, an analytic expression will be derived for the "driving force" for delamination, as a function of the mean stress. The stress ratio turned out to be inconvenient for the regression analysis of the results, which was applied for the derivation of an expression for the mean stress effect. Therefore a new parameter $Q$ is defined with:

$$ Q = \frac{\tau_{ad,m}}{\Delta \tau_{ad}} \left( = \frac{\text{mean}}{\text{range}} \right) \quad (4.5.3) $$
where $\tau_{ad}$ is the adhesive shear stress at the delamination front. The additional subscript $m$ denotes the mean stress and $\Delta$ denotes the cyclic stress range.

It will turn out that $Q$ is a more convenient parameter for the description of the mean stress effect than $R_{sp}$.

From the definitions of $Q$ and $R_{sp}$

$$R_{sp} = \frac{\tau_{ad, min}}{\tau_{ad, max}} \quad \text{and} \quad Q = \frac{\tau_{ad, m}}{\Delta \tau_{ad}} = \frac{\tau_{ad, min} + \tau_{ad, max}}{2 (\tau_{ad, max} - \tau_{ad, min})}$$

it follows that

$$Q = \frac{(1 + R_{sp})}{2 (1 - R_{sp})} \quad (4.5.4)$$

Before the derivation of an empirical expression for the "driving force" for delamination as a function of $Q$ in a later part of this section, some general comments regarding this "driving force" shall be made first.

Until now, the cyclic shear stress in the adhesive was considered to be controlling the delamination rate. However, due to secondary bending in the aluminium sheets at the delamination front, peeling stresses occur at positive $\tau_{ad}$ values. This is schematically indicated in figure 40. At negative $\tau$ values, peeling stresses do not occur.

From other investigations (on other types of specimens) it is known that the fibre-adhesive layers in ARALL are sensitive to peel stresses [7,10,35]. However, in the present investigation, the peel stresses due to secondary bending do not appear to be important. This conclusion is based on delamination tests where "anti buckling guides" were used at the delaminated area. Due to prevention of the opening mode, peel stresses could not develop.

Yet, this had no significant influence on the delamination growth rate. A second indication of the insignificance of the peel stress in the present specimens, came from the tests on the prestressed specimens in figure 49 (result for $P = 0$). For these tests a considerable part of the load excursions was in the range with negative shear stresses (especially at lower load levels, $R_{sp} = -2.5$), where the opening mode which is necessary for peeling did not occur (the opening mode does not occur at a load level, which is lower than 50N/mm). If the peeling process would be important, the delamination rate should significantly drop under the negative shear stresses, especially at the lower load levels. However, such a drop could not be observed.

As a summary of the discussion presented so far, it can be stated that the delamination rate depends on:

1. The type of laminate.
2. The cyclic shear stress in the adhesive at the delamination front, $\Delta \tau_{ad}$.
3. The mean value of the shear stress in the adhesive at the delamination front, $\tau_{ad, m}$. 

If a linear material behaviour is assumed, the cyclic shear stress in the adhesive is proportional to the cyclic stress in the delaminated fibres. Consequently, the delamination growth rate for a certain laminate can be described as a function of $\Delta S_{\text{a}}$ and $Q$ only.

The delamination rates of the specimens with 0.5 mm aluminium sheets in figure 46 (BSL fabric combination, $R = R_e = 0.1$) can be described by:

$$\frac{db}{dN} = q \Delta S^m_{\text{a}}$$

(4.5.2)

with $q = 5.3 \times 10^{-3}$ and $m = 9.65$ (for $R_e = 0.1$ and $Q = 0.61$).

If it is assumed that the constant $m$, which indicates the sensitivity to the stress level, is valid for other $R_e$ (and $Q$) values, the constant $q$ should be a function of $Q$. The effect of the mean stress can then be described as:

$$\frac{db}{dN} = q \{ f(Q) \cdot \Delta S_{\text{a}} \}^m$$

(4.5.5)

With the new $q$ (constant for all stress levels and stress ratios) the function $f(Q)$ accounts for the mean stress effect. It is chosen in such a way that $f(Q) = 1$ for $Q = 0$. It follows from the test result $\frac{db}{dN} = 4.8 \times 10^{-3}$mm/cycle at $\Delta S_{\text{a}} = 400$MPa and $Q = 0$ that $q = 3.7 \times 10^{-31}$. Substitution of the constants in equation (4.5.5) leads to:

$$f(Q) = \left\{ \frac{\frac{db}{dN}}{3.7 \times 10^{-31} \Delta S_{\text{a}}^{0.65}} \right\}^{1/9.65}$$

The function $f(Q)$ can now be obtained from the experimental results of specimens with one BSL-fabric layer (figures 43, 45, 46, 49) by plotting calculated $f(Q)$ values as a function of $Q$. The results are presented in figure 56 for the various $Q$ values derived with equation (4.5.4) (see table 2).

The highest stress ratios were obtained on the specimens with two BSL-fabric layers. However, these specimens showed 10 times faster delamination rates than the specimens with one fabric layer. After a correction for these higher delamination rates (which are not due to the mean stress effect), $f(Q)$ may also be derived from these specimens with:

$$f(Q) = \left\{ \frac{0.1 \times \frac{db}{dN}}{3.7 \times 10^{-31} \Delta S_{\text{a}}^{0.65}} \right\}^{1/9.65}$$

The results for the specimens with two layers are also presented in figure 56. There is scatter in the figure, but it shows a systematic trend which is described by the linear relation:

$$f(Q) = 1 + 0.37 Q$$

(4.5.6)

Equation (4.5.5) can now be written in the following more general form:

$$\frac{db}{dN} = 3.7 \times 10^{-31} \cdot \{(1 + 0.37 Q) \Delta S_{\text{a}}\}^{9.65}$$

(4.5.7)
Values of $R_m$ are also plotted on the horizontal axis of figure 56. $Q$ is a more convenient parameter than $R_m$, because a simple linear relation is found. It should be recognized that the relation is an empirical one.

**Effect of the fibre-adhesive layer**

As already mentioned the two fibre adhesive combination systems, AF 163-2 Twaron prepreg and the BSL-312-UL Twaron fabric combination were used most frequently in the present investigation. The delamination properties of these two systems are compared in figure 48. Because, the thickness of the two fibre-adhesive layers is different, the cyclic load per mm specimen width is taken as a basis for the comparison. The figure shows that the prepreg system exhibits better delamination properties. If the cyclic stress in the delaminated fibre-adhesive layers is taken as a basis for comparison, the difference in delamination properties becomes considerably larger.

It should be noted that the thickness of the aluminium layers and the residual stress levels of the two types of specimens are not fully identical. However, the differences are small and their influence on the delamination rates will be small as well. It should be concluded that the differences between the two laminate types in figures 48 are predominantly due to the different quality of the fibre-adhesive layers.

In figure 51, four data sets are plotted, where the main difference between the data sets is due to the different kinds of fibre adhesive combination layers. Small differences in stress ratios and residual stress levels are present. However, the associated differences in $Q$ values are small, and will not significantly influence the trends in the figure.

The two upper data sets apply to fibre-adhesive layers which were most frequently used in the present investigation. As could already be noticed from figure 48, the prepreg system gives better results.

A comparison of the two lower data sets in figure 51 shows a higher delamination growth rate for the Teyin fabric with the lower Young's modulus of the fibres. This trend may actually be more pronounced, because here it is compensated to some extent by the higher adhesive content of the specimens with the Teyin fibres (see section 4.5.2).

An interesting comparison can be made between the results of specimens with Twaron HM fabric and the AF-163-2 adhesive and the BSL-312-UL adhesive respectively (the lowest and the highest data sets in figure 51). The specimens with the BSL-312-UL adhesive show delamination rates, which are about 500 times higher than those of the laminates with the AF-163-2 adhesive. That means that the fatigue behaviour of the AF-163-2 adhesive is considerably better than that of the BSL-312-UL adhesive.

Another interesting comparison can be made between the two specimen types with the AF adhesive and Twaron HM fibres. For one type of specimens, prepregs were used, and for the other one an adhesive film fabric combination. It turns out that the delamination rates for the specimens with the prepreg material are about 70 times higher than those of the
specimens with the fabric-adhesive layers. The differences between the content of fibres and adhesive were small for both types of specimens. From the results it can be concluded that the delamination behaviour of ARALL may be improved if the fibres are applied in the form of fabrics, rather than as unidirectional laminates. However, the ARALL material with the fabrics exhibits a slightly increased weight and a reduction of about 10% in the strength of the crack bridging fibres.

The above delamination results indicate that:

1. Application of the BSL-312-UL adhesive is unfavourable as compared to application of the AF-163-2 adhesive.
2. Application of the fibres in the form of a unidirectional laminate is unfavourable as compared to application of the fibres in the form of a fabric.

Consequently, the combination of a unidirectional fibre orientation and the BSL adhesive will yield a laminate with a high delamination sensitivity. Such a laminate was applied in an early investigation [2]. These laminates indeed showed 'static delamination' during residual strength testing of cracked ARALL. Delamination under static loading was not observed in the present investigation, obviously, because the present laminates were less sensitive to delamination, due to the application of fabrics, or the AF adhesive.

An explanation for the favourable influence of a fabric on the delamination behaviour can be found if the delamination surfaces are observed. The delamination surfaces are rather smooth for the specimens with the prepregs. The surfaces are rougher for the specimens containing a fabric. The fabric left a "print" on the adhesive at the aluminium side of the surface (see figure 57). From fatigue crack growth studies on metals it is known that a rough fracture surface is associated with an irregular crack tip geometry, which causes a higher crack growth resistance, and consequently a lower crack growth rate [36,37,38]. The situation for the delamination growth may be analogous. Figure 58 shows the fibre side of the delamination surface for a specimen which contains the Twaron HM fabric in combination with the BSL-312-UL adhesive. The front of the delamination area due to fatigue can be seen after the non delaminated parts of the aluminium sheets are peeled off. The delamination front shows a 'zigzag' profile. The delamination always stopped at the thin rovings, which were orientated perpendicular to the loading direction. These rovings apparently act as 'delamination stoppers', and the overall delamination growth rate is reduced. A further discussion on the delamination mechanism will be given in section 8.

4.5.5.2 Discussion on the block-programme tests

The major trend of the fatigue delamination growth rates in the block programme tests (figures 52-54) at a first glance appears to be opposite to the trend usually found in fatigue tests on metals. The application of an overload on metals usually increases the remaining
fatigue life and decreases the fatigue damage rate. Accelerations as a consequence of underloads are not uncommon for crack initiation and propagation e.g. [39].

Retardations of the fatigue damage process in the crack initiation stage (often this stage is actually spent during growth of very small cracks, which are not observed) is caused by a stress redistribution due to plastic deformation at the notch. In the fatigue crack growth stage, crack closure leads to a crack growth deceleration. However, if these two mechanical causes for a decrease of the fatigue damage rate do not occur, or if they are accounted for, accelerations may be observed. It was shown in [40] for unnotched specimens (where stress redistribution does not occur) that peak loads may accelerate the subsequent damage rate in the crack initiation stage. Other indications for accelerations could be found in fractographic studies [41], where it was found that for certain cycles within block programmes, a crack growth rate occurred, which was considerably higher than that crack growth rate which was expected according to the instantaneous effective stress intensity ΔK_eff.

Accelerations of the crack growth rates due to overloads were also found in [42], where fatigue cracks were forced to grow in the tearing mode (Mode III). For this type of loading, the crack closure mechanism is not important either, and a deceleration due to crack closure does not occur.

For the present delamination growth tests, similar conditions are present. The usual mechanical causes for decelerations in metals are not relevant for the delamination mechanism. Crack closure does not occur, since the delamination grows in the shear mode (Mode II).

The above arguments suggest that accelerated delamination rates are not necessarily unexpected. A detailed mechanism how and why the acceleration does occur requires further study.

4.6 Measurements of the adhesive properties, using thick adherend specimens

4.6.1 Description of the testing procedure

In chapter 3 it was already discussed that the adhesive shear deformation has an important influence on the crack opening displacement and on the crack growth rate in ARALL. The basic shear stress-strain properties of adhesives are usually obtained in tests on the so called thick adherend specimen. In this specimen a nearly uniform shear stress distribution along the bond line occurs. Figure 59 shows the thick adherend specimen.

Two experimental techniques were applied to measure the adhesive shear deformations. First, the measurements were performed with an inductive shear strain gauge shown in figure 60. It turned out, however, that the very small shear deformations at the onset of the test were not accurately recorded by this clip on gauge (probably due to some slip at the fixation points of the gauge on the specimen). For that reason the adhesive shear modulus was also determined from measurements with a Moiré interferometry method. (The measurements using the clip on gauge were performed by Homan [43] and those with the Moiré interferometry
by Fortyr [44]). The determination of $G_{sd}$ values with the Moiré method was performed as follows:

The specimens were loaded to a low shear stress of $\tau = 4\text{MPa}$. Then some adjustments for secondary rotations of the specimens were performed, and the Moiré interference pattern was photographed. The adjustment procedure took about 15 minutes. So a (very small) creep component may be included in the $G_{sd}$ values. The Moiré method is not described here in further detail, because there is sufficient information in the literature [44-47]. The Moiré measurements confirmed that the shear strain distribution was indeed rather uniform along the length of the overlap.

A correction had to be applied to the clip on measurements, to account for deformations of that part of the aluminium adherends between the pins of the gauge. The correction was derived from a test on a dummy specimen of the monolithic aluminium alloy. The corrections were confirmed later on by the results of the Moiré method.

Three different types of tests were performed:

- Monotonous tensile test until final fracture.
- Static creep tests.
- Cyclic (creep) tests.

The loading rates in the tests were varied in order to get some insight into the time dependent deformation properties of the adhesive.

Four different types of "adhesive layers" were investigated.

1. BSL-312-UL adhesive without fibres (2 adhesive films per bond line, thickness of the bond line about 0.15mm).
2. BSL-312-UL adhesive with fibres (2 adhesive films and one Twaron HM aramid fabric, thickness 0.25mm).
3. AF-163-2 adhesive without fibres (one adhesive film without a carrier, thickness about 0.20mm).
4. AF-163-2 prepreg containing 50% Twaron HM fibres (fibres in loading direction, thickness 0.2mm).

The adhesive deformations could be divided into three parts, which are:

1. A linear (elastic) part at lower strain levels.
2. A visco-elastic (time dependent) part.
3. A plastic (non reversible) part.

In the elastic region the adhesive stiffness can be described with the adhesive shear modulus $G_{sd}$, which is defined by $G_{sd} = \tau/\gamma$ with $\tau$ as the shear stress and $\gamma$ as the shear strain. The deformation in the elastic region remains small and $\gamma = \nu/t_{sd}$ or:

$$G_{sd} = \tau t_{sd}/\nu \quad (4.6.1)$$
where \( t_{lm} \) is the thickness of the bond line and \( v \) is the shear displacement across the adhesive layer.

4.6.2 Monotonous tensile tests

The thick adherend specimens were loaded in displacement controlled tests until fracture occurred (except for the tests with the Moiré measurements, where the loading remained in the elastic area). Three different loading rates were applied, which led to displacement rates of:

- 0.8\( \mu \text{m/sec} \)
- 20\( \mu \text{m/sec} \)
- 500\( \mu \text{m/sec} \)

The displacement rates apply to the bolt holes at the end of the specimen. Due to the elastic straining of the aluminium the displacement rate at the bond line was lower. Because time dependent effects were investigated qualitatively, no attempt was made to calculate strain rates from the loading rates. In figure 61 an example is given of the shear deformation curves at different displacement rates for the AF-163-2 adhesive without fibres. The corresponding curves for the same adhesive with fibres are shown in figure 62. The effect of the fibres on the BSL-312-U1 adhesive was similar as for the AF-163-2 adhesive.

An example of a Moiré interference pattern is shown in figure 63. The fringes are lines with the same displacement (in the loading direction). Then the specimens were unloaded, adjusted for secondary rotations again, and a new interference pattern was photographed in order to check if some creep deformation had occurred. Some minor creep displacements were found for the two specimen types containing the AF-163-2 adhesive. However, these displacements were very small.

The main results of the static tests are summarized in the table on the next page.
The shear stress at a plastic deformation \( \gamma = 1\% \) is chosen as the adhesive yield stress \( \tau_p \). The values of tan \( \gamma_{fr} \), refer to the fracture shear strain.
The shear modulus could not be accurately measured by the clip on gauge. However, the measurements with the Moiré technique are considered to be accurate. The present \( G_{sn} \) values for the adhesives without fibres are in reasonably good agreement with results from the literature [48,49].

The \( G_{sn} \) values for specimens with and without fibres show a reasonable correlation with the fibre volume content \( v_f \) or the adhesive volume content \( v_{an} \), \( \left(v_{an} = 1 - v_f\right)\). The shear modulus of the fibres is considerably larger than that of the adhesive. If an infinite shear modulus is assumed for the fibres, the shear modulus of a bond line with adhesive and fibres \( (G_{sl}) \) is calculated from the shear modulus of the pure adhesive with the law of mixtures according to:
<table>
<thead>
<tr>
<th>type of bondline</th>
<th>$G_{\text{ad}}$ N/mm²</th>
<th>$G_{\text{ad}}$ N/mm²</th>
<th>$\tau_f$ N/mm²</th>
<th>$\tau_{\text{fr}}$ N/mm²</th>
<th>$\tan \gamma_{\text{max}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>BSL-312-UL</td>
<td>750 *</td>
<td>680</td>
<td>40</td>
<td>52</td>
<td>1.7</td>
</tr>
<tr>
<td>BSL-312-UL + fabric</td>
<td>1200 *</td>
<td>1030</td>
<td>42</td>
<td>42</td>
<td>0.3</td>
</tr>
<tr>
<td>AF-163-2</td>
<td>550 *</td>
<td>640</td>
<td>38</td>
<td>57</td>
<td>1.7</td>
</tr>
<tr>
<td>AF-163-2 prepreg</td>
<td>1100 *</td>
<td>1030</td>
<td>25</td>
<td>30</td>
<td>0.2</td>
</tr>
</tbody>
</table>

* estimated values, much scatter

Shear stress-strain properties of different materials

$$G_{\text{ad,fr}} = \frac{G_{\text{ad}}}{\nu_m} = \frac{G_{\text{ad}}}{1 - \nu_f}$$

For the fabric adhesive combination $\nu_f = 44\%$ (including the transverse fibres), and for the prepreg $\nu_f = 50\%$. Calculations of $G_{\text{ad,fr}}$ with these values lead to an overestimation of about 20% for both adhesives. The overestimation will be due to ignoring the shear compliance of the fibres (infinite shear modulus).

The most significant qualitative tendencies of the static tests are summarized below.

1. If the strain rate was decreased (see figure 61):
   - the adhesive yield stress became lower.
   - the adhesive fracture stress became lower.
   - the adhesive fracture strain became higher.

2. If fibres were present in the adhesive this caused (see figure 62):
   - a considerable decrease in the fracture strain.
   - a lower fracture stress.
   - a lower yield stress.
   - an increase of the stiffness of the adhesive layer.
   - The shear stiffness decreases with decreasing strain rate. (This trend is common for plastics. However, in the present tests it was only significant for specimens with fibres.)

4.6.3 Static creep tests

The static creep behaviour was determined at a shear stress level of $\tau_{\text{max}} = 20\,\text{MPa}$ during 20 or 30 minutes. Then the load was dropped and creep was observed during a subsequent recovering time at zero load. The displacements were recorded with the gauge. Additional
Moiré measurements were performed of the creep displacement after 20 hours sustained load at \( \tau = 4 \text{MPa} \) and about 15 minutes recovering at zero load. In the figures 64 and 65 the creep curves for the BSL-312 and the AF-163-2 adhesives (without fibres) are presented. During the hold time at maximum load some creep occurred. The creep rate at the beginning of the hold time was higher than at the end. After unloading some reversed deformation occurred during the recovering time. The 'recovering deformation rate' decreased with increasing time (Kelvin material [50]). However, a total recovery of the previous creep deformation was not found (visco-plasticity occurred). This is in agreement with results in the literature for other plastics [51]. Some of the remaining deformation may be due to slip in the gauge measuring system.

The Moiré measurements showed a very small displacement after the 20 hours creep tests for the specimens with AF-163-2 adhesive only. No creep was observed for the specimens with the BSL-312 in these experiments.

From the static creep tests (figure 64 and 65) the following qualitative conclusions can be drawn:

- Creep occurred at a shear stress of 20 MPa.
- A partial 'recovery' of the creep deformation occurred after unloading of the specimen.
- At a shear stress of 20 MPa, the time dependent deformations were found to be significantly smaller than the elastic deformations.

From the Moiré measurements it was observed that:

- The AF-163-2 adhesive showed more creep than the BSL-312 adhesive.
- For the AF-163-2 adhesive some minor creep occurred also at a shear stress of 4 MPa.

4.6.4 Cyclic creep tests

The cyclic creep tests were performed with a cyclic load at a stress ratio \( R = 0 \). The shape of the load cycle was a sine wave. Frequencies of 0.04Hz, 1Hz and 5Hz and different maximum stress levels were applied. Again specimens with and without fibres in the adhesive layer were tested. For the tests with 0.04Hz and 1Hz an x-y recorder was used. In the tests with the higher frequency of 5Hz a transient recorder (Krenz TRM-2000) was also used. The transient recorder stores the measuring signal, and applies it more slowly to the x-y recorder.

In figures 66 and 67 the adhesive deformation for a maximum shear stress level of 40 Mpa \((=\tau_c)\) and for different frequencies is shown for the BSL 312 and the AF-163-2 adhesive respectively. The deformation is plotted as a function of the number of cycles. The tendencies are the same for both adhesives. The largest deformations occurred at the lowest frequency. This is due to the longer period of time available for creep.
Figures 68 and 69 show the creep deformation behaviour of specimens with the AF 163-2 adhesive without and with fibres respectively, both for static and for cyclic creep tests. The maximum shear stress of 20 MPa is lower than in figure 67. The deformation at maximum stress is plotted. The creep deformation of the specimens without fibres was smaller in the cyclic tests than in the static tests. This result can be explained by the lower 'average' mean stress in the cyclic tests as compared to the static tests. However, for the specimens containing fibres the tendencies were opposite. Under cyclic conditions a higher deformation rate occurred than in the static tests. An explanation for the latter trend may be that the presence of the fibres in the adhesive caused an increased fatigue sensitivity of the bondline, resulting in larger displacements.

The results of the cyclic tests may be summarized as follows:
- An increase of the frequency decreases the creep deformation rate.
- The introduction of fibres into the bond line increases the creep deformation rate.

4.7 Crack opening measurements on specimens containing an artificial crack

4.7.1 Description of the tests

In section 3.4 it was indicated that the crack opening due to adhesive shear deformation is an important factor. Figure 8 shows how the crack opening displacement (COD) is caused by the adhesive deformation. For the schematic situation in this figure an analytic stress analysis is possible, see chapter 5. In this section experimental results are presented, for specimens similar to the delamination specimen shown in figure 40. The artificial cracks in the present specimens were always introduced by spark erosion, and the width \(e^r\) of the material removed was about 0.3 mm (see figure 70). The spark erosion process was performed with a minimum energy per electric discharge in an oily fluid to minimize heating up of the adhesive. The different laminate types investigated are presented in the table on the next page. The specimens were already described in section 4.5.2.

The COD measurements were carried out with a clip on gauge. The gauge lengths (distance between two sharp steel pins) were 1.8, 2.0, 2.5 or 5mm. The clip on gauge was placed across the artificial crack in the specimen. Identical measurements were also performed on undisturbed parts of several specimens, away from the artificial cracks. The latter measurements provided a check on the accuracy of the measurement technique, because the stress strain behaviour in the undisturbed region is linear elastic, and well known.

For the specimens with very thin aluminium sheets, produced by an etching process from thicker specimens the measurements away from the artificial crack were also used for an accurate calculation of the actual thickness of the aluminium sheets.

Two different types of tests were performed:
<table>
<thead>
<tr>
<th>specimen type</th>
<th>$t_A$ (mm)</th>
<th>$t_w$ (mm)</th>
<th>adhesive</th>
<th>fibre reinforcement system</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>2×0.5</td>
<td>0.2</td>
<td>AF-163-2</td>
<td>Twaron prepreg</td>
</tr>
<tr>
<td>II</td>
<td>2×0.262</td>
<td>0.2</td>
<td>AF-163-2</td>
<td>Twaron prepreg</td>
</tr>
<tr>
<td>III</td>
<td>2×0.5</td>
<td>0.25</td>
<td>2× BSL-312-Ul</td>
<td>Twaron film fabric</td>
</tr>
<tr>
<td>IV</td>
<td>2×0.286</td>
<td>0.25</td>
<td>2× BSL-312-Ul</td>
<td>Twaron film fabric</td>
</tr>
<tr>
<td>V</td>
<td>2×0.5</td>
<td>0.25</td>
<td>2× AF-162-2</td>
<td>Twaron film fabric</td>
</tr>
<tr>
<td>VI</td>
<td>2×0.5</td>
<td>0.4 *</td>
<td>2× AF-162-2</td>
<td>Teyin film fabric</td>
</tr>
</tbody>
</table>

* nominal 0.25mm

Survey of the specimens for COD measurements

1. Static tensile tests including unloading and some more cycles.
2. (Cyclic) creep tests.

All specimens were prestressed in such a way that the residual stress level was zero. The presence of a residual stress level was checked by etching away one of the aluminium sheets. None of the specimen types showed a curvature after etching. The specimen types II and IV were produced from the specimen types I and III respectively, by etching away half of the thickness of the aluminium sheets. All specimen types except II were also investigated in the delamination tests (see section 4.5).

The gauge results include the deformation of the aluminium parts of the specimens within the gauge length and the elongation of the aramid fibres at the location of the artificial crack. A correction must be applied, to obtain actual COD values. The relevant dimensions and displacements are defined in figure 71. The crack opening displacement due to shear deformations is indicated as COD. However, the clip on gauge measures an extension $\Delta l$ which has to be corrected for fibre elongation ($\Delta e_f$) and the elongation in the aluminium sheets $2\Delta v_A$. It implies that

$$COD = \Delta l - \Delta e_f - 2\Delta v_A$$

The analytical expressions for the two corrections are derived in appendix D

4.7.2 Static tests

All static tests were performed on a mechanical spindle machine, at an elongation rate of 2 mm per minute. The maximum load applied was 2.7 kN for specimens with a single fibre
layer, and 5.4 kN for the specimens of type VIII which contained two fibre layers. The maximum load was reached in about 13 seconds. The gauge length $\Delta l$ for the static tests was 5mm. Identical clip on gauges were mounted at both sides of the artificial crack, in order to obtain a symmetric measuring system.

The test results of the various specimens shall be considered first. They are presented in the figures 72 to 76. The relevant material parameters are presented in the figures, $\tau_p$ is the adhesive yield stress. The figures show the $\Delta l$ values for two cycles, and the corrected COD values together with COD values from analytical calculations in section 5 and appendix D (COD$_{ep}$ and COD$_{ak}$ respectively). The two COD values are presented for the (first) uploading cycle only. The following trends are observed:

1. The figures show that the first part of the curves is linear elastic. At higher load levels plastic deformation does occur. After unloading the main part of these plastic deformations remains. In the second load cycle considerably less plasticity is observed. However, a hysteresis loop is still present.

2. After unloading, reversed creep occurred at zero stress. This is indicated as an example in figure 72 with an arrow on the horizontal axis.

3. A comparison shows that the crack opening behaviour of specimens with the AF-163-2 Twaron prepreg system (figure 72) is similar to that of the specimens with the BSL-312-UL Twaron fabric system (figure 74). There are only slight differences in stiffness. For the specimens with the AF-163-2 adhesive, the hysteresis loops are wider.

4. A comparison of the figures 72 and 74 on one side and the figures 73 and 75 on the other side shows the influence of the thickness of the aluminium sheets on COD. The effect is negligible if the comparison is made on the basis of the load. However, a lower COD occurs for a lower sheet thickness, if the nominal stress in the aluminium layers $S_u$ is taken as a basis for comparison, instead of the external load (The stress in the aluminium layers $S_u$ is an important design parameter for ARALL).

5. The slope of the specimens containing Teyin fibres (figure 76) is lower than the slopes in the previous figures where Twaron fibres were used.

The experimental trends can be explained:

The linear part of the curves is associated with linear elastic adhesive behaviour. At higher load levels, adhesive plasticity occurs, and the slope decreases. The hysteresis loops are caused by plasticity and creep of the adhesive. The amount of plasticity during uploading is larger than the cyclic plasticity. This is due to a stress redistribution during plasticity at uploading. Figure 77 shows the shear stress distributions as calculated with a one dimensional model and perfectly-elastic plastic adhesive behaviour (see chapter 5 and appendix D). A small reversed plastic zone occurs in the adhesive, the cyclic plasticity is related to the width of the hysteresis loops.
The reversed creep after unloading is primary caused by the reversed sign of the adhesive shear stress (see figure 77), which occurs as a consequence of stress redistribution. An additional reason is the "recovering" creep of the adhesive itself, as shown in the previous section.

The wider hysteresis loops of the specimens with the AF-163-2 adhesive, as compared to those with BSL-312-UL, are related to the lower yield stress and the higher creep sensitivity of the AF-163-2 adhesive (see section 4.6).

Lower COD values occur for a lower aluminium sheet thickness because a lower part of the external load is transferred from the fibres to the aluminium and the adhesive shear stress and COD is lower. There is also a slight tendency that the hysteresis loops become narrower for lower sheet thickness. This is also related to less plasticity as a consequence of lower load transfer.

The reduced slope of the curves in figure 76 as compared to the other figures is explained with the lower Young's modulus of the Teyin fibres, which is only 58% of the modulus of Twaron HM, and with the larger thickness of the fibre-adhesive layer. The Teyin fabric for this specimen type was not a regular industrial product. It was especially produced in a small quantity for research purposes. The weight turned out to be about 190 g/m² instead of 160 g/m² for the Twaron fabric. Due to the higher fibre content of the Teyin fabric, the stiffness of the fibre layers \( F_e \) was about 20% higher than the stiffness which was aimed at. The larger thickness of the fabric, and some flow of the adhesive towards the centre of the specimen during curing, caused an actual thickness of 0.4mm of the fabric adhesive layer. The nominal thickness of such a layer is 0.25mm. A micrograph of the fibre adhesive combination layer of a specimen with the Teyin fabric is shown in figure 78.

Taking into account the weight of the Teyin fabric and Young's modulus of the fibres, the stiffness \( F_e \) of the fibre adhesive combination layer was \( F_e = 8400 \text{ N/mm} \). The thickness \( t_e \) was assumed to be one fifth of \( t_e \), as it was assumed also for the other specimens, and the resultant shear stiffness of the 'adhesive layer' is \( F_e = 8000 \text{ N/mm}^2 \). These values were introduced into the equations for the corrections on \( \Delta t \) and the determination of COD in chapter 5 and appendix D. The agreement of the experimental and theoretical COD values in figure 76 indicates that the model of chapter 5 accounts correctly for the effect of the fibre Young's modulus and the thickness of the fibre-adhesive layer.

4.7.3 Cyclic creep tests

The adhesive shear deformation in ARAIL under fatigue loading does occur as a cyclic process for long periods. In order to get some further insight into the cyclic and time dependent behaviour of the adhesive at the delamination front, cyclic COD measurements on artificial cracks were performed. These tests were carried out by Homan [43] on a servo hydraulic testing machine. Because there were not enough specimens available, the tests could not all be performed on "virgin" specimens. In some specimens, some plastic
deformation was already present in the adhesive at the beginning of the test. However, it was found that the creep behaviour was nearly unaffected by a short previous load history. The effect of the load history was nearly wiped out provided that sufficient time had elapsed to allow creep recovery. Only during uploading at the first load cycle, an effect of pre-loading on plasticity was observed.

A sine wave loading was applied and two frequencies were adopted (0.04 and 1.0 Hz). Additional hold times were introduced in some tests, both at maximum and at minimum load. The time dependent effects were not systematically investigated for all types of specimens. However, the qualitative trends of the results were similar for all types.

The test results from some cyclic tests are presented in figures 79 and 80. The correction on the displacement \(( \Delta e' + 2 \Delta v_u)\) after equations in appendix D are also presented for the first uploading cycle (\(\Delta v_u\) is slightly different for unloading). The hysteresis loops moved into the tensile direction. The largest shift of hysteresis loops occurred always between the first and the second cycle. A plastic zone in the adhesive was formed, and after unloading a smaller reversed plastic zone is present (see figure 77). The shifts in the hysteresis loops during the following cycles (when the plastic zone was already present), occurred mainly due to the creep deformation of the adhesive. It has to be pointed out, however, that pure plastic deformation and pure creep deformation cannot easily be separated. After the tests were completed, a considerable part of the resulting deformation was recovered due to "reversed creep".

Figure 81 shows three curves obtained in three successive tests on the same specimen. A period of 20 minutes or more was allowed for recovery of the creep deformation between each test. During each test five cycles were applied, but not all of these cycles could be presented in figure 81, because the last cycles led to curves which were very close to the preceding ones. The first test showed a significant amount of adhesive plasticity during the first half cycle. The shifts during the following cycles were due to creep.

Apart from plasticity in the first half cycle, the behaviour observed in the second test was similar to that of the first test. For the third test, the frequency was increased from 0.04 Hz up to 1.0 Hz. The slope of the curves was not significantly affected, but the width of the hysteresis loops slightly decreased at the higher frequency.

In three additional tests on the same specimen hold periods at maximum load and minimum load were introduced. The first plot in figure 82 (fourth test) shows the behaviour at a frequency of 1 Hz and a hold period of 5 seconds (1 Hz refers to the uploading and unloading rate). During the hold time at maximum stress creep occurred. After unloading about the same amount of displacement occurred as recovery creep. (Again the main "driving force" for the recovery creep will be due to the adhesive shear stress redistribution, as presented in figure 77.)

The increase in the total amount of creep is only small, if the hold period is increased from 5 seconds up to 20 seconds (second plot in figure 82). The explanation is that the creep rates
during the holding periods, reduced considerably as a function of time. Due to the stress redistribution as a consequence of the creep, the shear stress near the crack flanks decreases with increasing creep displacements.

The third plot in figure 82 shows that the amount of creep during the hold times decreased when the uploading and unloading rates were reduced. Noticeable creep occurred already before the load extremes were reached, thus decreasing the "driving force" for creep during the hold times.

Similar trends were found for specimens with the BSL-312-UL adhesive (see figure 83). For this adhesive the amount of creep was somewhat smaller and the hysteresis loops were slightly narrower. Major differences were not found between the plots in figure 83 and in the figures 81 and 82. All specimens showed a rapid saturation of creep in the tensile direction (crack opening) and a recovering of creep after unloading.

The saturation and recovering behaviour can also be observed in figure 84, where an extremely long hold time (3500 seconds) was applied. The creep rate was large at the beginning of the hold time, and decreased rapidly with increasing hold time. After unloading a nearly complete reversion of the creep deformation occurred, also at a decreasing creep rate.

The test result presented in figure 84 was obtained on a specimen which was frequently preloaded. Some fatigue delamination may have been present at the beginning of the test, and therefore the measured COD values may be somewhat higher than expected. However, the relative displacements due to creep are not influenced by the location of the delamination front.

Figure 85 shows a time displacement diagram for a cyclic test with hold times at maximum and minimum load. The different creep components and creep rates can again be observed. Figure 86 shows the results of a similar test at a stress ratio of $R = 0.5$. It is surprising at first to recognize that the reversed creep also occurred at a minimum load which was still in the tensile direction. This can be explained by the stress redistribution due to adhesive plasticity at maximum load, which leads to reversion of the shear stress near the crack flank ($y = 0$) under the tensile load at the minimum of the load cycle (see figure 77).

4.7.4 Some concluding remarks

It was shown that the analytically predicted COD (equations derived in section 5 and appendix D) agreed to the experimental results. At this stage, some conclusions can already be drawn directly from the experimental results.

1. The displacement as measured across the artificial crack was considerably larger than the displacement if no crack is present, which indicates that significant adhesive shear deformations did occur. These deformations should be relevant indeed for the crack growth rate in ARALL.
2. The major part of the total displacement is elastic. However, plastic and creep deformations also occurred at higher load levels.
3. A considerable part of the displacements due to creep were recovered after unloading.
4. With increasing time a rapid saturation of the creep displacements occurred.
5. At the same stress level $S$, in the aluminium, the COD values were considerably smaller when the aluminium thickness was smaller.
6. The specimens with the AF-163-2 adhesive showed more creep than the specimens with the BSL-312-UL adhesive.

4.8 The growth of small cracks in ARALL

4.8.1 Description of the tests

In the previous sections cracks of several millimeters were considered and the mechanical behaviour of the adhesive was investigated for such large cracks. This approach appears to be obvious because the high macrocrack growth resistance of ARALL is the most prominent advantage of the material. In several cases the macrocrack growth may become so slow that by far the major part of the fatigue life of a structural component can be spent in the macrostrained stage. Examples were shown in the TWIST flight simulation tests on optimised prestrained ARALL in section 4.4.

Before a crack in an ARALL component has reached a size of a few millimeters and more, there is a "crack initiation life". It should be expected that crack bridging will be less effective in this phase of the life, especially as long as the crack size is of the same magnitude as the thickness of the individual aluminium sheets, or smaller. The results in figures 21 to 25 and 33 indicate that initial crack growth rates may be significantly higher than for larger cracks. Of course, residual compressive stresses can be very effective in reducing the growth rate of very small cracks, already before the crack bridging mechanism becomes effective. Anyhow, it was considered most worthwhile to study the behaviour of very small cracks, because the "crack initiation period" is technically relevant.

In this section the behaviour of small cracks in ARALL is studied and it is compared to the behaviour of unreinforced laminates and monolithic materials. Various constant-amplitude and TWIST flight simulation tests were performed on double edge notched specimens shown in figure 87. The $K$, factor for this specimen was determined by finite elements calculations (by W. Ott, presented in [52]) and from Moiré interferometry measurements by Berger [47]. $K$, was 3.3. Due to the anisotropy of ARALL the $K$, values are about 5% higher [47]. This difference was assumed to be of minor importance for the fatigue results.

The edge notch configuration was chosen because it allows microscopic observation at the notch root during the fatigue tests, and because there was some experience available at the DFVLR from an earlier investigation by Foth [52,53,54] on this specimen. In order to optimize the conditions for the observation of small cracks, the notch roots and the side
surfaces of the specimens close to the notches were mechanically polished. A survey of the different tests and materials is given in the table on the below.

<table>
<thead>
<tr>
<th>bondline type</th>
<th>aluminium sheets</th>
<th>material type</th>
<th>residual stress (MPa)</th>
<th>type of test</th>
</tr>
</thead>
<tbody>
<tr>
<td>ARALL</td>
<td>2×0.5mm Al 7075-T6 bare + Twaron HM fabric</td>
<td>0.25mm BSL-312-UL prepreg</td>
<td>$S_{r,w} = + 20$ $S_{r,w} = - 30$ $S_{r,w} = - 50$ $S_{r,w} = 0$</td>
<td>C.A. C.A. C.A. TWIST</td>
</tr>
<tr>
<td>non reinforced laminates</td>
<td>2×0.5mm Al 7075-T6 bare</td>
<td>0.08mm BSL-312-UL</td>
<td>——</td>
<td>C.A.</td>
</tr>
<tr>
<td>bare single sheets</td>
<td>0.5mm Al 7075-T6 bare</td>
<td>——</td>
<td>——</td>
<td>C.A.</td>
</tr>
<tr>
<td>bare single sheets</td>
<td>0.5mm Al 7075-T6 bare &amp; Cr-acid anod.</td>
<td>——</td>
<td>——</td>
<td>C.A.</td>
</tr>
<tr>
<td>Alclad single sheets</td>
<td>1.0mm Al 7075-T6 Alclad</td>
<td>——</td>
<td>——</td>
<td>C.A. TWIST</td>
</tr>
</tbody>
</table>

Survey of the different tests on double edge notched specimens.

The static properties of ARALL and the single 0.5mm sheets were already presented in section 4.2.

4.8.2 Constant-amplitude fatigue tests

The various types of specimens were compared at different load levels. These levels were chosen in such a way that the nett section stress in the aluminium sheets of the ARALL
specimens were identical to those of the unreinforced aluminium specimens. Four stress levels were used, see the table below.

<table>
<thead>
<tr>
<th>level</th>
<th>$S_{l,\text{max}}$ (MPa)</th>
<th>$R$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>110</td>
<td>0</td>
</tr>
<tr>
<td>2</td>
<td>140</td>
<td>0.1</td>
</tr>
<tr>
<td>3</td>
<td>250</td>
<td>0.1</td>
</tr>
<tr>
<td>4</td>
<td>360</td>
<td>0.1</td>
</tr>
</tbody>
</table>

Stress levels for the constant-amplitude tests

The maximum stress at the notch root for level 2 was $K_r \times 140\text{MPa} = 462\text{MPa}$. This value is below the yield stress and an elastic behaviour may be assumed throughout the whole test. The same is also valid for the lower level 1.

Under cyclic loading at level 3 the yield stress at the notch root was exceeded during the first half cycle (uploading) and a stress redistribution at the notch root occurred. The cyclic stress at the notch root $K_r (S_{\text{max}} - S_{\text{min}})$ was $225 \times 3.3 = 742\text{MPa}$. This value is considerably less than twice the yield stress of the aluminium alloy and it may be assumed that the cyclic material behaviour will be mainly elastic. However, for load level 4 the cyclic stress at the notch root must have led to cyclic plasticity.

First phenomenological observations on the crack initiation behaviour are presented (quantitative results are presented later). The behaviour was dependent on the material type and the load level. Typical crack patterns for the initiation phase of the various materials under the four load levels are presented in the figures 88 to 94. For most specimens the cracks were usually detected at a crack length in the notch root of 0.05mm or less. However, cracks were more difficult to detect in the 1 mm Aiclad specimens, because of slip lines in the cladding layer. It was hard to see the difference between the slip lines and microcracks, but cracks larger than 0.1 mm were usually detected.

In general, macrocrack growth rates show limited scatter. The scatter as observed in the present investigation on microcrack growth rates was larger, but systematic differences in the microcrack growth rates could quite well be distinguished. However, the scatter in the number of cycles until the first crack was observed, was dependent on the load level. For the lowest load level, the scatter was very high. A factor of about five difference was found for the number of cycles at which the first crack was observed. The differences in cycle numbers up to occurrence of the first cracks in the various material types are assumed to be statistically insignificant. At load level 2 the scatter of crack initiation was still high. However, at load level 3 and 4 the scatter remained limited for all fatigue stages.
Some comments on typical aspects of the crack initiation behaviour of the various materials are discussed below.

Single sheets, bare, not anodized

For the two low loading levels (1 and 2) initiation took place at one location only. The crack usually started and grew as a quarter circular corner crack (figure 88). The fatigue life up to the first detection of a crack was longer than the subsequent crack growth life (quantitative results are presented later). At load level 3 the main part of the fatigue life was spent during the growth of short cracks. The fatigue life before detectable cracks and the last part of the life with cracks larger than 2mm were relatively short. The main part of the fatigue life was spent during the growth of microcracks. Initiation of two quarter circular corner cracks, and a coalescence of these cracks were frequently observed. Sometimes initiation of semi-circular cracks occurred (see figure 89). Multiple crack initiation of quarter circular corner cracks and semi-circular cracks occurred at the highest load level. Coalescence of several cracks was frequently observed. However, not all cracks took part in the coalescence process and the final failure (see figure 90). Nearly the whole part of the fatigue life was spent during the growth of small cracks.

Single sheets, bare, anodized

The trends are similar to those of the non-anodized sheets, earlier crack initiation was observed, especially at lower load levels. The earlier crack initiation is caused by the presence of the oxide layer.

Al clad sheets, not anodized

For the two lower load levels the cracks always started as corner cracks in the cladding layer of the specimens. Crack initiation occurred considerably earlier as observed for the bare specimens. However, the differences decreased at higher load levels. The trends for the crack geometry under the higher load levels were similar as for the bare material.

Non-reinforced laminates

At the lower load levels (1 and 2) crack initiation always occurred at the oxide layer near the bond line (see figure 91). The cracks still grew as quarter circular corner cracks. The shape of such a crack could be made visible by ink infiltration during the test. After drying and completion of the test, the crack shape can be observed and photographed (see figure 95). The oxide layers from which the cracks start are applied by anodizing which is required as a pretreatment for bonding of aluminium sheet material. It reduces the crack initiation life. The oxide layer at the outside surfaces of the specimens was removed in order to provide sufficient surface quality for microscopic crack length measurements. Crack initiation in both sheets of the laminates is necessary before a through crack can be formed. Until that moment, the first cracked sheet is bridged by the intact second sheet and the crack growth rate is reduced. A similar behaviour was previously described by Schijve et al. [1].
The second crack in the other sheet often initiated close to the first crack. The second crack then grew considerably faster than the first one and after the two cracks had reached a length of about two millimeters they grew further like one through crack. A schematic example of crack geometries at various cycle numbers is presented in figure 96. The number of cycles between initiation of the first and the second crack was only moderate, but an improvement of the fatigue behaviour is obvious.

At load level 3 the number of cracks contributing to the final crack increased (see figure 92). However the major effect was still associated to the corner cracks initiated at the oxide layers. The number of cycles between initiation of the first and the second crack became negligible. At load level 4 the number of cracks contributing to final failure further increased (see figure 93) and again almost the complete fatigue life was spent during the growth of microcracks.

ARALL

The trends regarding the types and numbers of cracks were similar as for the non-reinforced laminates. However, some typical features were observed. It was already mentioned that the non-reinforced adhesive layer presents a barrier for crack growth from one sheet into another. The fibre-adhesive layer appeared to be a more effective barrier. It appeared at the lower load levels that crack initiation occurred independently in both sheets. A length of more than 2mm was frequently observed for the first crack, until a crack in the second sheet initiated (see figure 94). Figure 97 demonstrates that initiation of the second crack often occurred at another level than the first crack, indicating that initiation was hardly promoted by the presence of the first crack. Obviously, the "weakest" point of the notch root area is more important for the initiation process in the second sheet and both sheets initiate nearly independently. The growth rates of the first crack in ARALL and in non-reinforced laminates were about similar. However, the observed growth rate of the second crack was much smaller for ARALL than the growth rate of the second crack in the unreinforced material. The influence of the favourable decoupling effect decreased with increasing load level and it had disappeared at level 4, where initiation took place very early anyhow.

An additional favourable effect was observed for ARALL with compressive residual stresses in the aluminium sheets at the lowest load level.

An indication for the slow crack growth rates in ARALL, already rather easy in the fatigue life can be deduced from figure 98 where it is shown that a second crack was initiated and reached a considerable size, until the first crack dominated. In the unreinforced material such a behaviour was not observed.

Ultimate failure was not reached at the two lowest stress levels for the ARALL specimens, because the aramid fibres always remained intact, even when the aluminium layers had sometimes completely failed.

Average values of some typical stages of fatigue damage in the specimens are compared in the following tables.
For load level 1 the results are:

<table>
<thead>
<tr>
<th>material</th>
<th>number of kilocycles</th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>first crack</td>
<td>crack in 2nd sheet</td>
<td>failure</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$a=0.1\text{mm}$</td>
<td>$a=0.1\text{mm}$</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>through</td>
<td>through</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.5mm sheet anodized</td>
<td>81.5</td>
<td>89.3</td>
<td>—</td>
<td>112.6</td>
</tr>
<tr>
<td>1.0 mm Alclad</td>
<td>82.0</td>
<td>96.6</td>
<td>—</td>
<td>115.6</td>
</tr>
<tr>
<td>2×0.5mm laminate</td>
<td>47.0</td>
<td>62.0</td>
<td>70.5</td>
<td>72.0</td>
</tr>
<tr>
<td>ARALL $S_{u,w} = +20\text{MPa}$</td>
<td>38.0</td>
<td>45.0</td>
<td>58.7</td>
<td>64.8</td>
</tr>
<tr>
<td>ARALL $S_{u,w} = -50\text{MPa}$</td>
<td>57.5</td>
<td>79.5</td>
<td>103.8</td>
<td>114.1</td>
</tr>
</tbody>
</table>

For load level 2 the results are:

<table>
<thead>
<tr>
<th>material</th>
<th>number of kilocycles</th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>first crack</td>
<td>crack in 2nd sheet</td>
<td>failure</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$a=0.1\text{mm}$</td>
<td>$a=0.1\text{mm}$</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>through</td>
<td>through</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.5mm sheet bare</td>
<td>47.1</td>
<td>51.0</td>
<td>—</td>
<td>56.5</td>
</tr>
<tr>
<td>0.5 mm sheet anodized</td>
<td>18.0</td>
<td>21.8</td>
<td>—</td>
<td>27.26</td>
</tr>
<tr>
<td>2×0.5mm laminate</td>
<td>28.0</td>
<td>33.5</td>
<td>34.7</td>
<td>36.6</td>
</tr>
<tr>
<td>ARALL</td>
<td>25.5</td>
<td>32.5</td>
<td>34.0</td>
<td>36.8</td>
</tr>
</tbody>
</table>

average results for all levels of $S_{u,w}$
For load level 3 the results are:

<table>
<thead>
<tr>
<th>material</th>
<th>first crack</th>
<th>number of kilocycles</th>
<th>crack in 2nd sheet</th>
<th>failure</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>a=0.1mm</td>
<td>through</td>
<td>a=0.1mm</td>
<td>through</td>
</tr>
<tr>
<td>0.5mm sheet bare</td>
<td>4.7</td>
<td>5.7</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.5 mm sheet anodized</td>
<td>3.1</td>
<td>3.88</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2x0.5mm laminate</td>
<td>2.75</td>
<td>3.86</td>
<td>2.90</td>
<td>3.72</td>
</tr>
<tr>
<td>ARALL</td>
<td>2.00</td>
<td>3.00</td>
<td>2.58</td>
<td>3.20</td>
</tr>
</tbody>
</table>

average for all levels of $S_{cr}$

The results for load level 4 will be presented later.

4.8.3 Crack growth rate in the notch root

In figure 99 the short crack growth behaviour of the different materials is compared for the lowest load level ($S_{\text{max}} = 110$ MPa and $R = 0$). The curves have been shifted in such a way that they coincide at $a = 0.1$. Each curve in figure 99 represents the average results of two or three tests. The average values of the number of cycles as a function of the crack length was determined separately for the first and the second crack, and the number of cycles between the initiation of the first and the second crack is also an average value. The figure shows that for this low load level and stress ratio, the ARALL type with the highest residual compressive stresses in the aluminum sheets showed the best fatigue properties.

A similar plot is presented in figure 100 for load level 2 ($S_{\text{max}} = 140$MPa and $R=0.1$ ). The figure presents only one curve for ARALL, because effects of the residual stress ($S_{cr} = +20, -30$ and $-50$ MPa) could not be found. The curve for ARALL is the average for all tests. The difference in the sensitivity to the residual stress levels for load level 1 and 2 can be explained as follows: For the tests at $S_{\text{max}} = 110$ MPa and $R = 0$ (load level 1) the cracks are entirely closed as long as the stress in the notch root, due to the external load, is less than the residual compressive stress. Due to this closure the lowest part of the fatigue cycle is not effective and the crack growth rates are lower. The magnitude of the residual stress is significant

For the tests with $S_{\text{max}} = 140$ MPa and $R = 0.1$ (load level 2) the stress at the notch root at the minimum of the external load was $3.3 \times 14$ MPa = 46 MPa. So even with a residual
stress level of -50 MPa crack closure at the notch root will hardly occur (apart from the Elber type of crack closure which occurs due to the plasticity in the wake of the crack [21]).

The results from the tests with $S_{\text{max}} = 250$ MPa and $R = 0.1$ are presented in figure 101. In these tests often more than one crack was initiated in each sheet. In most cases two of these cracks coalesced and formed the final (macro)crack. Those cracks which did not contribute to the final crack are not considered in figure 101. If more than one crack contributes to the final crack, the geometry of the cracks becomes different for different specimens and it is no longer feasible to define average crack growth curves. For that reason only one representative test result is presented in figure 101 for each type of specimen. Figure 101 also illustrates that the number of cycles between the initiation of a first and of a second crack is small as compared to the number of cycles which is needed for the cracks to penetrate the whole sheet.

In the tests with $S_{\text{max}} = 360$ MPa and $R = 0.1$ many cracks coalesced and formed the final crack. In this case no representative crack growth curves could be drawn. The cracks initiated not only at the corners of the sheets but also all over the notch root very early in the course of the fatigue life. Some stages of the fatigue life of the different specimens are compared for different types of materials in the table below. The sum of the crack lengths is presented for all cracks which contribute to the final failure. For the extremely high stress level and the very short fatigue lives the results for all three materials are rather similar.

<table>
<thead>
<tr>
<th>$S_{\text{max}} = 360$ MPa</th>
<th>$N_{5\text{%}}$</th>
<th>$N_{50\text{%}}$</th>
<th>$N_{99\text{%}}$</th>
<th>$N_{\text{drough}}$</th>
<th>$N_{\text{sheet}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>single sheet 0.5mm</td>
<td>398 *</td>
<td>556</td>
<td>673</td>
<td>993</td>
<td>1020</td>
</tr>
<tr>
<td>laminate (2 sheets)</td>
<td>106</td>
<td>312</td>
<td>444</td>
<td>930</td>
<td>986</td>
</tr>
<tr>
<td>ARALL</td>
<td>126</td>
<td>351</td>
<td>563</td>
<td>1000</td>
<td>1194</td>
</tr>
</tbody>
</table>

* non annodized

Comparison of the "damage development" in different materials at the maximum stress level

In order to give an impression of the growth and the location of the cracks at such a high load level, a characteristic specimen with a non reinforced adhesive layer is shown in figure 102.

Anomalies of the crack growth rates, like crack arrest or unexpectedly high crack growth rates, were reported previously for short cracks e.g. [55], especially if the crack size is smaller or of the same magnitude as the grain size (the so-called short crack problem). In order to find out if such anomalies are relevant for the present results, micrographs of the material were made (see figure 103). It can be seen in figure 103 that the material exhibited fine grains with a pan cake shape. The thickness of the "pan cakes" is about 0.01 to 0.04
mm which is small as compared to the crack sizes considered here. That means that microstructural influences should not play a significant role. However, the increased scatter of the short cracks as compared to long cracks may be related to microstructural effects e.g. the crossing of a grain boundary by the crack. The absence of a short crack problem is confirmed by figure 104 where the experimental growth rates for the small cracks were compared to predictions based on large crack data (through cracks) of the same sheet material. The large crack data were presented in the form of da/dN vs. ΔK plots in the Figures 18 and 19. The stress intensity factors for the quarter circular cracks were then calculated using a modified solution by Newman [56] which is given in appendix G.

The modification of the Newman solution was performed in a similar way, as it was suggested in [52]. The simple form of the $K$-solution, which is valid only for the present specimen is:

$$K = \frac{2}{\pi} F_T S_{peak} \sqrt{\pi a}$$  \hspace{1cm} (4.8.1)$$

with $F_T$ as a geometrical correction factor for a quarter circular crack at a notch in a specimen with a finite sheet thickness.

$$F_T = 1.134 - 0.603 (a/q) + 38.0 (a/q)^2 - 139 (a/q)^3$$  \hspace{1cm} (4.8.2)

(for $t/q = 0.138$)

where $K$ is the stress intensity factor at the crack tip at the notch root, $a$ the crack length at the notch root, $t$ the specimen thickness (in this case 0.5 mm), $q$ the notch root radius (in this case 3.62 mm) and $S_{peak}$ the stress in the notch root.

The agreement between the theoretical and the experimental results, shown in figure 104, is good.

4.8.4 Crack growth rate at the side surfaces of the specimens

In the previous sections the main interest was focussed on the crack behaviour at the notch root. Now the crack growth behaviour as observed at the outer sheet surfaces of the specimen will be considered. After sufficient crack growth a crack will become a through crack (apart from the unbroken aramid fibres), which grows through the notch stress field towards the centre line of the specimens. Figure 105 shows the crack growth behaviour as a function of the crack length at the side surfaces of the specimens for $S_u = 140$ and $R = 0.1$. Four different materials were considered:

- A (single) bare Al 7075-T6 sheet of 0.5 mm thickness.
- A single clad Al 7075-T6 sheet of 1 mm thickness.
- An unreinforced laminate, 2 x 0.5mm bare Al 7075-T6.
- ARALL specimens of 2 x 0.5mm Al + 0.2mm Ar, with a residual stress of $S_{res} = -30$ MPa.
As a basis for comparison, the calculated behaviour of a straight crack after [57] is also presented in figure 105 (see appendix G equations (G-8) and (G-9)). The trends for the various materials in figure 105 are different and they will be discussed separately.

**Single sheets, 0.5 mm and 1.0 mm**

For the single layer specimens with a corner crack, a monotonous increasing crack growth rate was observed at one side (point A in figure 105). At the other side surface the crack became visible at a later stage when it grew through the thickness (point B). A high crack growth rate was measured for such a break through crack. Lateron, the crack growth rate of the break through crack decreased as the difference in the crack lengths of both side surfaces of the specimens decreased. Finally the crack lengths at both specimen sides became the same. For the specimens with a thickness of 1 mm this occurred at a higher crack length than for the specimens with a thickness of 0.5 mm.

**Non-reinforced laminates**

The behaviour of the non reinforced specimens with \( t_r = 2 \times 0.5 \text{ mm} \) was more complicated. Because the cracks started at the anodization layers close to the bond lines, they reached the side surfaces of the specimen some time after they were initiated as corner cracks. All cracks at the side surface were break through cracks. These break through cracks propagated comparatively fast at first, but thereafter the crack growth rates decreased as the crack fronts became more straight. The adhesive layer as a crack barrier became additionally effective. The sheet where the first crack started to grow showed lower crack growth rates than the sheets where the second crack began to propagate. It was shown before (figure 89), that the effects are associated with a difference in lengths of both cracks, which may be more than 1 millimeter. More than three millimeters of crack growth occurred until the cracks in both sheets showed about the same length.

**ARALL**

For ARALL a similar behaviour is found. At first a strongly decreasing crack growth rate occurred immediately after the break through of the corner cracks. However, the crack growth rates at both specimen sides were smaller than those of the unreinforced laminates. In a later stage an approximately constant crack growth rate occurred. The lengths and growth rates of the cracks at both surfaces remained different for a long time. There is a strong tendency for the cracks, to grow independently. This is due to the effective uncoupling of the aluminium sheets caused by the fibre reinforced adhesive layer. Figure 106 shows the results obtained at load level 2, for all three residual stress levels in ARALL as compared to the result of a single sheet. The ARALL curves in figure 106 represent the average crack growth rate from cracks in two sheets. Figure 107 shows results at load level 1 for ARALL at two residual stress levels, as compared to the results of a single sheet. The influence of the residual stress level became more significant than in the tests with
$S_o = 140\text{MPa}$ and $R = 0.1$ (figure 106). The overall crack growth rates decreased, and the decrease was large for the laminates with the high residual compressive stress.

The same trend was already observed for the small corner cracks as discussed before (figure 99). However, the influence becomes larger as the crack length increases. This can be explained by the increasing crack bridging efficiency.

4.8.5 Flight simulation tests

Flight simulation tests were performed on two types of ARALL. For one type the BSL-312-UL Twaron fabric combination was used and for the other one the AF-163-2 aramid prepreg. Both ARALL types were prestressed in such a way that the residual stress level after curing became zero. For comparison, the conventional structural material Alclad 7075-T6 with a thickness of 1 mm was also tested. The TWIST spectrum was not truncated. The mean stress level as calculated for the (uncracked) net section was $S_{\text{net}} = 75.7 \text{MPa}$.

For each material 2 or 3 tests were performed. Because every specimen has two notches, 4 or 6 sets of crack growth curves per configuration were determined. The results are presented in the figures 108 till 110.

During the most severe type of flight, which occurred only once in a block of 4000 flights (flight A), there was so much crack extension, that it became visible as a "step" in the crack growth curves. These "steps" could not be observed as long as the cracks were still at the notch root. The averages of crack growth curves were determined in the same way as it was done for the constant-amplitude tests at the lower stress levels. A complication with this type of presentation is the magnitude and the position of the steps in the presentation of average results. For the presentation in the figures, the average magnitude of the steps was first determined as a function of the crack length, then these steps were added to the crack growth curves in a representative way.

Some characteristic fatigue stages, as they can also be observed in the figures 108 to 110, are compared in the table below.
<table>
<thead>
<tr>
<th>material</th>
<th>number of flights</th>
<th>number of flights</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>first crack</td>
<td>crack in 2nd sheet</td>
</tr>
<tr>
<td></td>
<td>$a=0.1\text{mm}$</td>
<td>$a=0.1\text{mm}$</td>
</tr>
<tr>
<td></td>
<td>through</td>
<td>through</td>
</tr>
<tr>
<td>1.0mm Alclad</td>
<td>13,000</td>
<td>16,000</td>
</tr>
<tr>
<td>ARALL AF-163 prepreg</td>
<td>41,000</td>
<td>44,000</td>
</tr>
<tr>
<td>ARALL BSL fabric-film</td>
<td>51,000</td>
<td>60,000</td>
</tr>
<tr>
<td></td>
<td>63,500</td>
<td>69,000</td>
</tr>
</tbody>
</table>

It is observed that the crack initiation in ARALL occurred considerably later than in the 1 mm Alclad material. Altogether the ARALL specimens showed the typical initiation and crack growth behaviour as it was found under constant-amplitude loading at the lower stress levels. This occurred in spite of some high load cycles present in the TWIST spectrum, which caused considerable plasticity. The crack growth rates for ARALL are lower than those of the Alclad specimens over the entire range of crack lengths, and again the difference increases with increasing crack length.

In ARALL the cracks started at the oxide layers and in the Alclad specimens at the soft cladding layers. However, in contrast to the constant-amplitude test results the crack initiation of the oxide layers of ARALL occurred about 3 times later than the crack initiation at the cladding layer of the monolithic material. Another remarkable difference between the tests with TWIST loading and the constant-amplitude tests was that the crack growth rates at the side surface of the notch at first decreased with increasing crack length, for the Alclad sheet as well as for ARALL. The decrease in the crack growth rates with increasing crack length was more pronounced for ARALL. The crack growth rates in the Alclad sheets increased again in a later stage. ARALL showed decreasing crack growth rates during the whole tests (see figures 109 and 110).

The initial decrease of the crack growth rates for an increasing crack length during the TWIST tests may be due to an increasing cumulation of retardations caused by the most severe flights, while the increase of $K$ values with increasing crack length is still moderate due to the stress gradient at the notch. This explanation is supported by Figure 108, which shows that the decrease of the crack growth rate did not become clearly visible until the first severe flight occurred after penetration of the crack to the side surface of the specimen. The effect of decreasing crack growth was present in monolithic material and in ARALL. For ARALL the effect was enhanced by an increasing crack bridging effect by the unbroken fibres.
For ARALL the crack growth rates became very small at a higher crack length. The specimens were then removed from the fatigue testing machine between flight numbers 100,000 and 130,000. In none of these specimens the cracks had exceeded a length of 15 mm. The table above shows that the initiation behaviour of ARALL with the BSL-312-UL aramid fabric system was slightly better than that with the AF 163-2 aramid prepreg. However, it is still questionable, whether the difference is really significant, because there is no obvious reason for a different life to initiate a first crack in the two types of ARALL. At larger crack lengths the crack growth rates became about the same.

4.8.6 Concluding remarks

The ARALL type used for the small crack investigations was not an optimal type. Optimized ARALL should contain a higher amount of fibres. This could be achieved by using aluminium sheets which are thinner than 0.5 mm, or by increasing the number of layers, which implies increasing the ratio of fibre layers to aluminium layers (this ratio is 1:2 only for the simple laminate applied here). However, thinner layers were not yet available in a sufficient quantity and the application of more layers would hinder the crack observations at the side surface of the internal layers. For an optimized ARALL the behaviour of crack initiation and the individual growth of cracks in the first part of the fatigue life might well be more pronounced.

With respect to the crack initiation and the growth stage of short cracks, the main advantage of ARALL, as compared to monolithic materials, is due to the efficient crack uncoupling effect (crack divider) of the aramid reinforced adhesive layer. This layer prevents the cracks from "jumping" to the neighbouring sheet. The effect is more effective at low stress levels, where the crack initiation occurs late and where the scatter of the crack initiation life is relatively high. A compressive residual stress introduced by prestressing or prestraining also has a favourable influence on the initiation and growth of short cracks, although this effect becomes smaller at high load levels and stress ratios. At higher load levels a small improvement of the fatigue crack initiation behaviour of ARALL as compared to non-reinforced material could still be observed, which might be the result of restraint on (cyclic) plastic deformation in the metal sheets at the notch root, due to the elastic behaviour of the aramid fibres.

The crack initiation behaviour of ARALL is unfavourably affected by the anodizing process, which however, is necessary as a pretreatment for the adhesive bonding. The oxide layer has a similar negative influence as the cladding layer on conventional aluminium sheets. Under TWIST flight loading the oxide layer appeared to be less harmful than the cladding layer.
4.9 Summary of the more relevant observations

In this chapter the results of extensive test series were presented. The tests were carried out to provide empirical data and observations on the behaviour of ARALL under fatigue loading. The results can be used to support the modelling of a fatigue crack growth mechanism for ARALL as presented in chapter 5. At the same time the results elucidate the effects of several material variables of ARALL. Most experiments were carried out on ARALL specimens. The main variables were related to the types of fibres and adhesives, the sheet thickness and the residual stress system. In several cases comparisons were made with laminated specimens without fibres and with single sheet specimens. A summary of test results is given below.

Tensile tests on fibre-adhesive layers, aluminium sheets and ARALL supplied basic strength and stiffness properties.

Constant-amplitude crack growth tests were performed on non reinforced aluminium sheets and laminates, which provided the relation between \( \frac{da}{dN}, \Delta K \) and \( R \). Typical growth properties observed on ARALL were: a decreasing crack growth rate with increasing crack length, a favourable effect of the residual compressive stress obtained by prestraining and an influence of the size of the starter notch on the crack growth rate during the entire fatigue life. Small notches are favourable.

TWIST flight-simulation tests basically showed the same typical crack growth effects on ARALL (residual stress, notch size). Some additional observations were made. ARALL built up from thinner layers showed better crack growth properties. The effect of truncation in prestrained ARALL is larger than in non-reinforced material, and increases with increasing level of the residual stress. The low crack growth rates at high load levels indicated that ARALL offers the possibility of about 30% weight saving and an increased fatigue life.

Delamination tests provided relations between delamination growth rates and crack bridging stresses for a variety of ARALL types. The effect of sheet thickness, type of fibre-adhesive layer, stress ratio and residual stress was investigated. Delamination appeared to be mainly depending on the shear stress in the adhesive at the delamination front. Simple tests with block programmes demonstrated that the effect of periodic overloads and underloads increases the delamination growth rate.

Mechanical properties of the adhesive were investigated on thick adherend specimens. The adhesive shear modulus and strength were measured and some effects related to creep of the adhesive were demonstrated.

Crack opening measurements on ARALL specimens with an artificial crack (delamination specimen) were performed. Some different ARALL types were investigated. In contrast to the results of the delamination tests, only small differences were found for different fibre-adhesive layers. The main effects are related to differences in sheet thickness, fibre
Young's modulus and amount of adhesive. It was shown that the crack opening behaviour is affected by time dependent behaviour of the adhesive. However, the effects are moderate, reversible and decrease rapidly with increasing time.

The initiation and growth of small cracks in ARALL was investigated on double edge notched specimens. The most important observation was that the fibre-adhesive layer is a very effective barrier for the growth of cracks from one sheet into an adjacent sheet. However, the improvement of ARALL as compared to monolithic sheets is much smaller in the initiation phase, as compared to the very large benefits for large cracks.
5. Mechanical modelling of cracked ARALL

5.1 Introduction

The mechanical behaviour of ARALL was qualitatively discussed and experimentally investigated in chapter 3 and 4. The fatigue crack growth behaviour of ARALL is a rather complex phenomenon because of such aspects as fibre bridging, delamination and adhesive shear deformations. As a consequence, a theoretical model for calculations on fatigue crack growth requires some suppositions on mechanistic aspects and the behaviour of the ARALL material components. Problems involved are discussed in this chapter and proposals for the model behaviour are made. It will be shown in chapter 7 that the extraordinary fatigue crack growth in ARALL can be satisfactorily described on the basis of calculations made with the model proposed.

An elementary assumption is that the empirical relationship between the fatigue crack growth rate in the aluminium sheets and the cyclic stress intensity factor $\Delta K$ still holds for the cracked aluminium sheets of ARALL. Consequently, an important matter of concern is, that the stress intensity factor at the crack tip of cracked aluminium sheets must be calculated. The influence of the fatigue load, the crack length, delamination, adhesive shear deformation, specimen geometry, residual stress level and other design parameters of ARALL must be included in the calculations. The delamination is changing during the fatigue life and an additional parameter is required to which the delamination growth rate can be related. Two parameters are considered, which may be representative as a "driving force" for the delamination growth rate:

1. The energy release rate for delamination.
2. The theoretical elastic shear stress in the adhesive at the delamination boundary.

In the first sections of this chapter some general expressions for the mechanical behaviour of ARALL will be derived. The calculations of the stress intensity factor $K_{\alpha}$ in the aluminium sheets are presented in section 5.6.

The calculations of $K_{\alpha}$ for a crack and the strain energy release rate for delamination in an ARALL specimen is a very complex problem, because of the different layers, delamination and adhesive shear deformation in the adhesive layers. A finite-element calculation would require a complex 3-D model with a very fine mesh near the crack tip and the delamination front. This seems to be an impossible approach, the more since calculations should be carried out for several values of the crack length, delamination distance and the large number of material variables associated with ARALL (e.g. numbers of layers, thicknesses, stiffnesses). As a consequence the present analysis is primarily analytical, although it was clear from the beginning that exact solutions can not be obtained. Several assumptions are necessary to arrive at a model of the cracked and locally delaminated ARALL specimen, which can still be handled mathematically. The risk is that such a model is of a qualitative nature
only. Although this might already be helpful, it is tried to obtain a model which from a quantitative point of view, has some credible accuracy. This can only be achieved if the model is physically sound, i.e. if it accounts for the more relevant deformation and failure mechanisms in a way which seems to be physically correct. Moreover, it is tried to "borrow" solutions from similar problems. Finally, mathematical solutions should satisfy more trivial limit cases, for which answers are available. Nevertheless, it cannot be denied that the number of assumptions, which has to be made, is fairly large. However, in several cases they could be based on empirical information, while a final substantiation is given by the predicted results in comparison to crack growth in several tests.

5.2 The stress in the individual layers

An external load on an ARALL sheet causes different stresses in the aluminium and the fibre layers. These stresses can be calculated from the load level and the stiffness properties of the layers. First, the material behaviour in the elastic region is considered. The stress in the laminate as a whole is denoted as $S_{ln}$. It is calculated from the external load with:

$$S_{ln} = \frac{P}{w t_{ln}}$$  \hspace{1cm} (5.2.1)

where $P$ is the external load, $w$ is the specimen width and $t_{ln}$ is the total thickness of the laminate. $t_{ln}$ is the sum of the thickness of all individual layers (see figure 111).

$$t_{ln} = t_{Al} + t_{ar}$$  \hspace{1cm} (5.2.2)

where $t_{Al}$ is the sum of the thicknesses of the aluminium sheets together, and $t_{ar}$ is the sum of the thicknesses of all fibre adhesive layers.

The strain of the laminate can be calculated with Hooke’s law.

$$\varepsilon_{ln} = \frac{S_{ln}}{E_{ln}}$$  \hspace{1cm} (5.2.3)

$E_{ln}$ is Young’s modulus of the laminate. With the Young’s moduli of the aluminium layers ($E_{Al}$) and the fibre adhesive layers ($E_{ar}$), two stiffness parameters $F_{Al}$ and $F_{ar}$ are defined:

$$F_{Al} = t_{Al} E_{Al}$$  \hspace{1cm} (5.2.4)

$$F_{ar} = t_{ar} E_{ar}$$  \hspace{1cm} (5.2.5)

$E_{ln}$ can be calculated with the law of mixtures.

$$E_{ln} = \frac{F_{Al} + F_{ar}}{t_{Al} + t_{ar}} = \frac{F_{ln}}{t_{ln}}$$  \hspace{1cm} (5.2.6)

The effects of the different Poisson’s ratios of the aluminium and the fibre adhesive layers are neglected in equation (5.2.6). The difference between the Poisson’s ratios of the two
layers is small (stress in fibre direction), and due to the low transverse stiffness of the fibre adhesive layers their Poisson’s ratio has only a minor effect on the longitudinal stiffness anyhow.

The different layers of ARALL are fully interconnected and the longitudinal strain is the same in all layers. (Only at the specimen ends some differences can occur, associated with interlaminar shear stress). Equation (5.2.3) can then be written as:

$$\epsilon = \epsilon_{AI} = \epsilon_{ar} = \epsilon_{la} = \frac{S_{la}}{E_{la}} = \frac{S_{AI}}{E_{AI}} = \frac{S_{ar}}{E_{ar}} \Rightarrow$$

$$S_{AI} = \frac{E_{AI}}{E_{la}} S_{la}$$  \hspace{1cm} (5.2.7)

and

$$S_{ar} = \frac{E_{ar}}{E_{la}} S_{la}$$  \hspace{1cm} (5.2.8)

Combining equations (5.2.6) to (5.2.8) yields:

$$S_{AI} = \frac{E_{AI}}{F_{la}} t_{la} S_{la}$$  \hspace{1cm} (5.2.9)

$$S_{ar} = \frac{E_{ar}}{F_{la}} t_{la} S_{la}$$  \hspace{1cm} (5.2.10)

An other useful expression follows directly from equilibrium:

$$S_{la} = \frac{t_{AI} S_{AI} + t_{ar} S_{ar}}{t_{la}}$$  \hspace{1cm} (5.2.11)

The equations (5.2.1) to (5.2.11) describe the stresses due to the external loading. A relation between the residual stresses in the aluminium sheets and the fibre-adhesive layers (denoted as $S_{r, AI}$ and $S_{r, ar}$ respectively) follows from equilibrium.

$$t_{AI} S_{r, AI} + t_{ar} S_{r, ar} = 0$$  \hspace{1cm} (5.2.12)

Under elastic conditions, the total stress in the layers is found by superposition of the stress due to external loading and the residual stress.

$$S_{r, AI} = S_{AI} + S_{r, AI}$$  \hspace{1cm} (5.2.13)

$$S_{r, ar} = S_{ar} + S_{r, ar}$$  \hspace{1cm} (5.2.14)

The subscript ‘t’ refers to the total stress in the layer concerned. Analogous to equation (5.2.11) the total stress in the laminate can also be written as:
\[ S_{ln} = \frac{t_{AI} S_{AI} + t_{sr} S_{sr}}{t_{ln}} = S_{ln} \] (5.2.15)

Because the residual stress system is an equilibrium system \( S_{ln} = S_n \), which also follows from equations (5.2.11) to (5.2.14). Equation (5.2.15) is still valid if plastic deformation occurs. However, in that case the actual stresses \( S_{sr} \) and \( S_{sr} \) may not be calculated with equations (5.2.13) and (5.2.14).

In figure 112 the experimental behaviour of an ARALL tensile specimen is compared to the theoretical result. The agreement is good. In the plastic region the "stiffness" of the aluminium is considerably reduced and the slope of the tensile curve of ARALL is mainly determined by the stiffness of the fibres.

5.3 The shear stress distribution in the adhesive at the delamination boundary

The stress intensity factor in ARALL is strongly influenced by the amount of delamination and by the additional crack opening displacement due to adhesive shear deformation. Both mechanisms are closely related to the adhesive shear stress at the delamination boundary.

Obviously, the adhesive stiffness is important for the calculation of the adhesive shear stress. Secondly, the distribution of the fibres in the fibre-adhesive layer is also important. The micrographs in figures 2, 29, 30, 70 and 78 reveal that the fibres are not uniformly distributed, they are more or less concentrated in the middle of the fibre-adhesive layer. Figure 113 schematically shows the model adopted for the fibre adhesive layer (see also figure 8) with a fibre layer in the middle, where the entire tensile stiffness of the fibres is concentrated. The tensile stiffness of the adhesive is low and will be neglected. The shear deformation is assumed to be concentrated in the two adhesive layers at both sides of the fibre layer. The thickness of the central layer can be calculated from the fibre volume content \( v \) of the fibre adhesive layer.

The thickness of the two 'adhesive' layers with shear deformation can also be calculated with the adhesive volume content of the fibre adhesive layer. However, there is also some adhesive present in the middle of the fibre adhesive layer. The adhesive in the middle is not loaded in shear, due to the symmetry. Consequently, there is less adhesive left for shear deformation in the two outer layers and the thickness of the two interfacial "adhesive" layers will be smaller.

The number of fibre aluminium interfaces is denoted as \( j \) which implies that the number of fibre-adhesive layers is equal to \( j/2 \). The thickness \( t_i \) of one central fibre layer follows from the fibre volume fraction as:

\[ t_i = \frac{2 v f_t s r}{j} \] (5.3.1)
The thickness of one interfacial adhesive layer is:

\[ t_{ad} = \frac{\eta \nu_m t_{sw}}{j} \]  \hspace{1cm} (5.3.2)

with \( \nu_m \) as the adhesive volume content and \( \eta \) (\( \leq 1 \)) is a correction factor because some adhesive is in the fibre layer. Then \( \nu_t t_{sw}/j \) would be the expected thickness of an individual adhesive layer. An estimation of \( \eta \) from the experimental results of section 4.6.1 yields a value of \( \eta = 0.8 \). This implies that the thickness of an adhesive layer is about 20 percent of the thickness of a fibre-adhesive layer.

The tensile stiffness of all fibre layers together is equal to \( F_{sw} \), previously defined as \( F_{sw} = t_{sw} E_{sw} \). It now can also be written as:

\[ F_{sw} = \frac{j}{2} t_{sw} E_{sw} \]

Substitution of equation (5.3.1) yields \( F_{sw} = \nu_t t_{sw} E_{sw} \). Ignoring the stiffness of the adhesive \( E_a = \nu_t E_{sw} \). Substitution shows that both stiffness definitions are the same.

For the layered model in figure 113 the shear stress distribution in the adhesive can be calculated. Such calculations for a one dimensional model are extensively described in literature [9,58]. Assumptions made are: elastic material behaviour, in the aluminium sheets tensile deformations only, finite shear stiffness and zero tensile stiffness for the adhesive layers. The result for the situation in figure 113 for a very long overlap length \( l \) is relevant for ARALL. It is given by the following equation:

\[ \tau_y = S_{A1} \frac{t_{A1}}{2} \sqrt{2 F_{sw} \left( \frac{1}{F_{A1}} + \frac{1}{F_{sw}} \right) \cdot e^{-y/\sqrt{2 F_{sw} \left( \frac{1}{F_{A1}} + \frac{1}{F_{sw}} \right)}}} \]  \hspace{1cm} (5.3.3)

where \( \tau_y \) represents the adhesive shear stress at a distance \( y \) from the delamination front. \( F_{sw} \) is the shear stiffness of one "adhesive" layer, defined with the adhesive shear modulus \( G_{ad} \), using the following equation:

\[ F_{ad} = G_{ad} t_{ad} \]  \hspace{1cm} (5.3.4)

Equation (5.3.3) was derived for a one dimensional model. Stress gradients in the adhesive in the thickness (z) direction are not considered, thus ignoring that the delamination front introduces a singularity with infinite stresses at this point according to a more precise linear elastic theory.

Equation (5.3.3) applies for a specimen with two aluminium layers and one fibre-adhesive layer (figure 113). If there are more aluminium sheets the equation has to be adjusted because \( t_{ad}, F_{sw} \) and \( F_{sw} \) apply to total thicknesses whereas \( F_{sw} \) applies to a single adhesive layer. Introducing again \( j \) as the number of adhesive layers (and \( j/2 \) as the number of fibre-adhesive layers) equation (5.3.3) becomes.
\[ \tau_y = S_{AI} \frac{t_{AI}}{j} \sqrt{j F_{sd} \left( \frac{1}{F_{AI}} + \frac{1}{F_{ar}} \right)} \cdot e^{-y} \sqrt{j F_{sd} \left( \frac{1}{F_{AI}} + \frac{1}{F_{ar}} \right)} \]  
(5.3.5)

The adhesive shear stress has its maximum at the delamination front (\(y=0\)). The shear stress at this location \(\tau_{ad}\) is:

\[ \tau_{ad} = S_{AI} \frac{t_{AI}}{j} \sqrt{j F_{sd} \left( \frac{1}{F_{AI}} + \frac{1}{F_{ar}} \right)} \]  
(5.3.6)

A quantitative example for a shear stress distribution is presented in figure 114. The relevant parameters are presented in the figure. Introduction of these values in equation (5.3.5) yields:

\[ \tau_y = 111.3 e^{-1.74y} \text{MPa} \]

This theoretical shear stress plotted in figure 114 decreases rapidly with increasing distance from the delamination front. It has decreased to about 3% of the maximum value at a distance of \(y = 2\text{mm}\). The maximum stress of 111 MPa is much larger than the adhesive yield stress and plastic deformation of the adhesive must be expected. A discussion on adhesive plasticity is presented in section 5.4.

The equations above were derived for laminates without residual stresses. The adhesive shear stress for a delamination specimen containing residual stresses is different. The stresses transferred from the aluminium sheets to the aramid fibres by the adhesive layers should be considered. It implies that \(S_{ad}\) in equation (5.3.6) has to be replaced by the stress difference between the delaminated and the non delaminated areas of the aluminium sheets. Consequently, equation (5.3.6) should be written as:

\[ \tau_{ad} = \frac{t_{AI}}{j} \left( S_{AI} + S_{\tau,Al} \right) \sqrt{j F_{sd} \left( \frac{1}{F_{AI}} + \frac{1}{F_{ar}} \right)} \]  
(5.3.7)

In section 4.5, the delamination rates were presented as a function of the stress \(S_{th}\) in the delaminated fibres, and it was demonstrated that the delamination growth rate is closely related to the shear stress in the adhesive. From equilibrium it follows that:

\[ S_{th} = \frac{t_{th}}{t_{ar}} S_{th} \]  
(5.3.8)

and with equations (5.2.7) and the definitions of the stiffness parameters, equation (5.3.6) can be rewritten as:

\[ \tau_{ad} = t_{ar} S_{th} \sqrt{\frac{F_{sd} F_{AI}}{j F_{ar} \left( F_{AI} + F_{ar} \right)}} \]  
(5.3.9)

This equation shows how \(\tau_{ad}\) is depending on the stiffness parameters and the thickness and the stress of the fibre layers. Equation (5.3.9) is valid for a laminate without residual
stresses. The load transfer over the adhesive is equal to \( t_{\text{ad}} (S_n - S_{\text{ad}}) \). For a laminate with residual stresses the load transfer is equal to \( t_{\text{ad}} (S_n - S_{\text{ad}} - S_{\text{res}}) \), which implies that the adhesive shear stress is zero if \( S_n = S_{\text{ad}} + S_{\text{res}} \). Expressing \( S_{\text{ad}} \) in \( S_n \) using equation (5.2.8) and than expressing \( S_n \) in \( S_{\text{ad}} \) with equation (5.3.8) yields that the shear stress is zero if \( S_{\text{res}} = (1 - F_{\text{ad}} / F_{\text{Al}}) S_n \). With \( F_{\text{ad}} = F_{\text{Al}} + F_{\text{w}} \) it follows after some rewriting that:

\[
\tau_{\text{ad}} = 0 \text{ if } S_{\text{ad}} = \left( 1 + \frac{F_{\text{w}}}{F_{\text{Al}}} \right) S_{\text{res}}
\]

The adhesive shear stress for delamination specimens with residual stresses can now be obtained by subtraction of this \( S_{\text{ad}} \) value from the \( S_n \) value in equation (5.3.9). The result is:

\[
\tau_{\text{ad}} = t_{\text{ad}} \left( S_n - \left( 1 + \frac{F_{\text{w}}}{F_{\text{Al}}} \right) S_{\text{res}} \right) \cdot \sqrt{\frac{F_{\text{ad}} F_{\text{Al}}}{j F_{\text{w}} (F_{\text{Al}} + F_{\text{w}})}}
\]

(5.3.10)

5.4 Crack opening in ARALL due to adhesive shear deformation

5.4.1 Calculations for linear elastic material behaviour

It was pointed out before, that the crack opening in ARALL as a consequence of adhesive shear deformation, is an important factor for fatigue crack growth in this material. Now analytical methods are described for the calculation of the crack opening displacement COD in delamination specimens, as caused by the adhesive shear deformation (see figures 8 and 113). The relative displacements between the aluminium and the fibre layers at the delamination boundary are not influenced by the amount of delamination. For zero delamination the COD is directly related to the relative displacements. First, linear-elastic material behaviour is assumed. Plasticity of the adhesive will be considered later.

The adhesive shear deformation is given by:

\[
\gamma = \frac{\tau_{\text{ad}}}{G_{\text{ad}}}
\]

(5.4.1)

For small values of \( \gamma \) the displacement is given by the product \( \gamma t_{\text{ad}} \). The COD value is twice as large.

\[
\text{COD} = 2 \gamma t_{\text{ad}} = 2 \frac{t_{\text{ad}} \tau_{\text{ad}}}{G_{\text{ad}}} = 2 \frac{\tau_{\text{ad}}}{F_{\text{ad}}}
\]

(5.4.2)

Substitution of equation (5.3.6) in equation (5.4.2) yields:

\[
\text{COD} = 2 S_{\text{Al}} t_{\text{Al}} \sqrt{\frac{1}{j F_{\text{ad}}} \left( \frac{1}{F_{\text{Al}}} + \frac{1}{F_{\text{w}}} \right)}
\]

(5.4.3)

Substitution of equation (5.2.7) yields
\[
COD = 2 \frac{E_{Al}}{E_{ia}} S_{\text{ad}} t_{Al} \sqrt{\frac{1}{j F_{ad}} \left( \frac{1}{F_{Al}} + \frac{1}{F_{ar}} \right)}
\] (5.4.4)

5.4.2 The influence of adhesive plasticity on the crack opening displacement

It was shown in section 5.3 that the theoretical adhesive shear stress can exceed the adhesive yield stress. An estimation of the influence of adhesive plasticity on the COD will be made. The method is described here, but detailed derivations of the equations are presented in appendix D. According to the model in figure 77 the theoretical elastic stress distribution exceeds the adhesive yield strength \( \tau_p \). The adhesive is assumed to be an elastic-perfectly plastic material. The area above the adhesive yield strength is truncated and a stress redistribution occurs, leading to higher stresses in the elastic region. Because the total load transfer from fibres to aluminium remains unchanged, the area of the truncated region of the stress distribution must be equal to the area which is added at larger \( y \) values.

The adhesive plastic zone \( y_p \), and the crack opening displacement are derived in appendix D on the basis of the behaviour described above. The results are:

\[
y_p = \frac{S_{\text{ad}} t_{Al}}{j \tau_p} - \frac{1}{\sqrt{j F_{ad} \left( \frac{1}{F_{Al}} + \frac{1}{F_{ar}} \right)}}
\] (5.4.5)

and

\[
COD = \frac{\tau_p}{F_{ad}} + \frac{S_{\text{ad}}^2 t_{Al}^2 \left( \frac{1}{F_{ar}} + \frac{1}{F_{Al}} \right)}{j \tau_p} \quad \text{(only for } \tau_{ad} \geq \tau_p \text{)}
\] (5.4.6)

If equation (5.4.5) yields negative results for \( y_p \), there is no plastic zone, the adhesive behaves elastically and equation (5.4.6) is no longer valid. The crack opening displacement is then given by equation (5.4.3).

An example of the influence of adhesive plasticity on the COD, both as calculated and as measured is presented in figure 72. The adhesive yield stress \( \tau_p \) is defined at 1% remaining plastic deformation (\( \tan \gamma = 0.01 \)) as measured in section 4.6, i.e. \( \tau_p = 38 \text{ N/mm}^2 \). The specimen in figure 73 is identical to the specimen for which the theoretical linear elastic stress distribution is presented in figure 114. The shear stress considerably exceeds the adhesive yield stress.
5.4.3 Corrections on the experimental measurements of the COD due to adhesive deformation

In section 4.7 the COD measurements on specimens with an artificial crack were presented. Figure 71 shows the displacements near the artificial crack (spark eroded slit). Because it was not possible to measure the crack opening directly at the crack flanks, and because of the presence of the spark eroded slit with a width \( e^* \), corrections on the measurements are necessary. A correction \( \Delta e^* \) for the elongation of the aramid fibres within the slit and a correction \( 2\Delta v_{AI} \) for the elongation of the aluminium between the gauge pins is applied. The corrections are derived in appendix D. The results are:

\[
\Delta e^* = S_{AI} \left( \frac{t_{AI}}{F_{ar}} + \frac{1}{E_{AI}} \right) e^* 
\]

(5.4.7)

For \( \Delta v_{AI} \) three cases have to be considered.

1. There is no plastic zone in the adhesive (elastic material behaviour).

\[
\Delta v_{AI} = \frac{S_{AI}}{E_{AI}} \cdot \frac{e^{-y_p} \sqrt{\frac{1}{F_{AI}} + \frac{1}{F_{al}}} - 1 \cdot y_p}{\sqrt{\frac{1}{F_{AI}} + \frac{1}{F_{al}}}} 
\]

(5.4.8)

2. The adhesive plastic zone is smaller than the distance between the gauge pins \( (y_s \leq y_p) \):

\[
\Delta v_{AI} = \frac{j\tau_p}{F_{AI}} \cdot \left\{ y_p y_s^2 - \frac{y_p^2}{2} + \frac{(y_p - y_s)}{B} + e^{-(y_p - y_s)} - 1 \right\} 
\]

(5.4.9)

3. The adhesive plastic zone extends beyond the distance between the gauge pins \( (y_s > y_p) \).

\[
\Delta v_{AI} = \frac{j\tau_p y_s^2}{2 F_{AI}} 
\]

(5.4.10)

The corrected experimental COD value is given by:

\[
COD = \Delta L - \Delta e^* - 2 \Delta v_{AI} 
\]

(5.4.11)

where \( \Delta L \) is the as measured elongation between the gauge pins.

In the figures 72 to 76 experimental COD and \( \Delta L \) values are presented, and compared to the calculated COD values. The agreement of theoretical and experimental results is acceptable. The figures indicate that plasticity may not be neglected, although the main displacement component of the COD is caused by elastic deformations.
5.5 The energy release rate for delamination

For the predictions of the delamination growth rate in ARALL specimens, a loading parameter for the "delamination driving force" is required. The loading parameter should preferably show a unique correlation to the delamination growth rates, independently of the type of ARALL, in a similar way as the cyclic stress intensity factor is a unique parameter for the fatigue crack growth rate in metals.

Due to the small thickness of the adhesive layers and adhesive plasticity and due to the presence of the aramid fibres and the aluminium layers, there is no straightforward solution to define a linear elastic stress field around the delamination front, in terms of stress intensity factors in the adhesive. An alternative fracture mechanics method is to use the energy release rate for delamination $G_i$ as a loading parameter. The energy release rate is derived in appendix C from the energy balance of a delamination specimen.

For static problems in cracked brittle materials, the energy balance is directly relevant to fracture. Griffith postulated that fracture can only occur if the energy release rate exceeds the surface energy of the material [59]. For situations where more energy conversion mechanisms do occur, like the energy dissipated by plastic deformation near to the fracture surface, the energy release rate can still be considered as a "driving force" for the fracture mechanism. However, instead of a surface energy, another specific energy $G_i$ has to be introduced, and it is assumed that fracture will occur if $G$ exceeds $G_i$. This approach has been used for static problems.

For subcritical crack growth, like fatigue crack growth, the situation is different, because the energy release rate $G$ does not exceed the specific energy $G_i$ necessary for static crack growth. In fatigue crack extension per cycle is usually correlated by the stress intensity factor $K$. However, $K$ is directly related to the energy release rate $G = (K^2/E)$. In other words the growth rate is equally well related to $G$. In a similar way, the energy release rate for delamination $G_i$ may be seen as a loading parameter for the adhesive at the delamination front, and the cyclic variations of $G_i$ may be seen as a controlling parameter for fatigue delamination growth.

Because cyclic $K$ values can not easily be defined for the delamination front, the alternative method of cyclic $G_i$ values will be considered. In appendix C the following expression is derived for $G_i$ for delamination in the delamination specimen.

$$ G_i = \frac{t_{a_r} F_{AI}}{2 j E_{ar} F_{AI} + F_{ar}} \left[ S_{a_r} - \left(1 + \frac{F_{ar}}{F_{AI}}\right) S_{r,a} \right]^2 $$ (5.5.1)

The residual stress level is also incorporated in equation (5.5.1). $S_{r,a}$ is the residual stress in the fibre-adhesive layer and $S_a$ is the stress in the delaminated fibre-adhesive layer. For $S_{r,a} = 0$ the equation agrees with literature solutions [30,60,61], which were derived for similar geometries without residual stresses.
In view of equation (5.5.1), it is not surprising that the experimental results in figure 55 indicate that the effect of the residual stress on the delamination rates can be described by a reduction of $S_{th}$ with $(1 + F_r/F_A) S_{th}$. In other words, the delamination rates can indeed be correlated to

$$\left\{ S_{th} - \left( 1 + \frac{F_r}{F_A} \right) S_{th} \right\}$$

It was shown before (section 5.3) that this term is proportional to the load transfer from the fibre layers to the aluminium sheets. Obviously, the energy release rate is also closely related to this load transfer.

For a laminate without residual stresses, another useful expression is derived from equation (5.5.1) and the equations of section 5.2:

$$G_d = \frac{F_A}{2 j F_r F_{in}} (t_{in} S_{th})^2$$

(5.5.2)

5.6 The stress intensity factor in an ARALL crack growth specimen

5.6.1 Introduction

In previous sections it was mentioned that the stress intensity factor of a crack in ARALL is influenced by a large number of parameters. The influence of these parameters mainly occurs by their effect on two mechanisms, which are:

1. Delamination.
2. Adhesive shear deformation.

In this section, a quantitative analytic method is derived for the calculation of the stress intensity factor in ARALL. The method is based on the effect of the above mentioned two mechanisms.

The derivation is performed in two steps. First, (in section 5.6.2) the stress intensity due to the delamination only, is derived. Secondly, (section 5.6.3) the stress intensity factor due to adhesive shear deformation is derived. The final result is obtained by a combination of the two stress intensity factors.

5.6.2 The influence of the delamination on the stress intensity factor in ARALL

5.6.2.1 No residual stresses

The stress intensity factor in ARALL is reduced as a consequence of crack bridging by unbroken fibres. The crack bridging stresses are treated as external forces on the laminate. The resulting stress intensity factor $K$ is found by superposition of the stress intensity factors
due to the actual external loading and due to the crack bridging stresses. The sign of the latter
contribution is negative, and the final result is a decrease of $K$ due to the crack bridging
stresses.

The first problem in this approach is the determination of the crack bridging stresses. The
problem is simplified by the assumption that the magnitude of the crack bridging stresses is
the same at every location along the crack flanks (except at the starter notch, where the fibres
are cut). This assumption looks crude at a first glance. However, it is supported by two
arguments.

1. The delamination increases more rapidly at locations where the crack bridging stresses
are relatively high, thus decreasing these stresses at that location (see also chapter 3).
2. The elongation of the fibres is closely related to the crack opening displacement $COD$
(see figure 115). Because the $COD$ contour is an ellipse (according to LEFM in an
infinite sheet), and because the observed delamination areas usually are nearly elliptic
as well, the strain in the delaminated fibres will be constant, and as a consequence the
same applies to the crack bridging stress.

Figure 116 schematically shows the "load flow" in ARALL around the crack in the alu-
muminium sheets.

Infinite sheet without a saw cut, calculation of the crack bridging stress

The infinite sheet with a crack but without a saw cut is the most simple case. This case is
solved first, as an introduction of the applied method. The problem to be solved is outlined
in figure 117. The fibre stress, which is assumed to be constant, can not be calculated
analytically, but a close approximation can be obtained. Two related problems, which allow
an exact solution are outlined in figure 118 and 119. In the first model (figure 118) the
aluminium sheets in the elliptical hole are left out. It implies that the in-plane stiffness is
underestimated. In the second model (figure 119) the fibres in the delaminated area con-
tribute to the in-plane stiffness. Now the in-plane stiffness is overestimated. A second
approximation will be made in the second model, namely that the crack bridging stresses
act on the crack flanks instead of on the boundary of the delaminated area.

The vertical opening ($v = \text{half the opening}$) of an elliptical hole or a crack can be written
as:

$$v = C \frac{S}{E} b \sqrt{1 - (x/a)^2}$$

where $C$ is a simple function of $a/b$, and $S$ is the relevant stress. For the first model sub-
stitution of the appropriate values of $C$, $S$ and $E$ (see also equation (A-15) of appendix A),
and ignoring anisotropy leads to:

$$v = \left\{ \left(1 + \frac{2a}{b} \right) \frac{S_{la}}{E_{la}} - 2 \frac{a}{b} \frac{S_{br}}{E_{la}} \right\} b \sqrt{1 - (x/a)^2}$$

(5.6.1)
\( S_w \) is the crack bridging stress based on the total laminate thickness \( t_w \).

For the fibres with a length \( 2b \sqrt{1 - \left(\frac{x}{a}\right)^2} \) half the extension is:

\[
v = \frac{S_{br}}{E_{fr} t_{fr}} b \sqrt{1 - \left(\frac{x}{a}\right)^2}
\]  (5.6.2)

The boundary condition (figure 117) requires that both \( v \)-values in equations (5.6.1) and (5.6.2) are equal. \( S_w \) can then be solved, which leads to:

\[
S_{br} = \left\{ \frac{1 + 2a/b}{F_{fr}/F_{fr} + 2a/b} \right\} S_{ln} \]  (5.6.3)

For the second model (half) the crack opening is:

\[
v = \left\{ 2a \frac{S_{ln}}{E_{ln}} - 2a \frac{S_{br}}{E_{ln}} \right\} \sqrt{1 - \left(\frac{x}{a}\right)^2}
\]  (5.6.4)

and (half) the fibre extension follows again from equation (5.6.2). Equating the two values now leads to:

\[
S_{br} = \left\{ \frac{2a/b}{F_{fr}/F_{fr} + 2a/b} \right\} S_{ln}
\]  (5.6.5)

A comparison between the two solutions shows:

\[
\frac{S_{br} \text{ of the 1st model}}{S_{br} \text{ of the 2nd model}} = 1 + \frac{b}{2a}
\]

For usual values of \( b/a \) between 0.1 and 0.3 the ratio varies from 1.05 to 1.15 which is still surprisingly close to 1.

In order to improve the accuracy of the second model the boundary condition could be changed to:

fiber extension = displacement of the edge of the delaminated area.

Another improvement would be to apply \( S_w \) at the edge of the delaminated area. Unfortunately there is no longer a linear relation between the displacements and \( (1 - (a/x)^2)^{1/2} \). In other words such a model would not be compatible with the constant \( S_w \) assumption. However, the effect of adopting either model 1 or model 2 can be estimated by considering \( v \) at \( x = 0 \). Ignoring anisotropy again, \( v \) for a non-bridged crack can be derived from the Westergaard stress functions [62], which are also presented in the book of Tada [63] for the bi-axial stress field shown in figure 120.

\[
Z(z) = \frac{S_{ln} z}{\sqrt{z^2 - a^2}}
\]  (5.6.6a)
\[ \overline{Z}(z) = S_{in} \sqrt{z^2 - a^2} \]  \hspace{1cm} (5.6.6b)

with
\[ z = x + iy \]

For plane stress the displacement \( \nu \) in the \( y \) direction is given in [63] as:
\[ 2 \frac{G \nu}{E_{in}} = 2 \left\{ 1 - \nu / (1 + \nu) \right\} \text{Im}\overline{Z} - \nu \text{Re}\overline{Z} \]  \hspace{1cm} (5.6.7)

After adding the term \( \nu S_{in} y / E_{in} \), the displacement for a uni-axial stress field is obtained, and with
\[ G = \frac{E_{in}}{2(1 + \nu)}, \quad \nu = \frac{1}{3}, \quad x = 0, \quad y = b \]

it follows:
\[ \nu = \frac{S_{in}}{E_{in}} \left( 2 \sqrt{b^2 + a^2} - \frac{4 b^2}{3 \sqrt{b^2 + a^2}} + \frac{b}{3} \right) \quad \text{(for } x = 0) \]  \hspace{1cm} (5.6.8)

In figure 121 the solution from the Westergaard stress functions (equation 5.6.8) is compared to the displacement of the edge of a non reinforced hole (first model) and to the displacement of the edge of a crack crack (second model) at \( x = 0 \). The figure shows that all models approach \( 2 \nu = 4 a S_{in} / E_{in} \) for \( b / a \rightarrow 0 \), which is the well known expression for the crack opening displacement COD. The figure also shows that the elliptic hole model yields higher \( \nu \) values than the model with the crack. However, for \( b / a \) ratios lower than 0.5, which are relevant for ARALL, the differences remain limited and COD is the main displacement component. Similar calculations for \( a > x > 0 \) are rather lengthy and will not be presented here. The results show the same trend, which is a rather small difference between the displacements obtained with the two models if \( b / a < 0.5 \). As said before, the second model overestimates the in-plane stiffness and will underestimate \( S_{in} \). The first model will do the opposite. Because the first model is considered to be closer to the physical reality and in view of the small differences between the \( S_{in} \) values of the two models, the first model will be adopted for the next step.

**Calculation of the stress intensity factor**

If the crack bridging stresses would act on the crack flanks instead of the delamination boundary, the stress intensity factor in ARALL \( K_{in} \) would be:
\[ K_{in} = (S_{in} - S_{dr}) \sqrt{\pi a} \]

However, the crack bridging stresses are less effective because they act at the delamination boundary, instead of the crack flanks. A correction factor \( C_{in} ( < 1) \) for this effect was derived in appendix B. It accounts for the reduced influence of \( S_{in} \) on the stress intensity
factor, as a function of the axis ratio $b/a$ of the delamination ellipse. The table below presents some calculation results for $C_{us}$, as a function of $b/a$.

<table>
<thead>
<tr>
<th>$b/a$</th>
<th>$C_{us}$ ( = $K_iS \sqrt{\pi , a}$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0.9994 (exact = 1)</td>
</tr>
<tr>
<td>0.1</td>
<td>0.9642</td>
</tr>
<tr>
<td>0.2</td>
<td>0.9258</td>
</tr>
<tr>
<td>0.3</td>
<td>0.8873</td>
</tr>
<tr>
<td>0.4</td>
<td>0.8500</td>
</tr>
<tr>
<td>0.5</td>
<td>0.8144</td>
</tr>
<tr>
<td>0.6</td>
<td>0.7808</td>
</tr>
<tr>
<td>0.8</td>
<td>0.7197</td>
</tr>
<tr>
<td>1.0</td>
<td>0.6662</td>
</tr>
</tbody>
</table>

Curve fitting yielded the following expression for $C_{us}$.

$$C_{bl/a} = \frac{K}{S \sqrt{\pi \, a}} = \frac{5}{3(1 + b/a)^2} - \frac{2}{3(1 + b/a)^2}$$ (5.6.9)

The stress intensity factor is now given by:

$$K_u = (S_{li} - C_{bl/a} S_{li}) \sqrt{\pi \, a}$$ (5.6.10)

Substitution of equation (5.6.3) in (5.6.10) yields:

$$K_u = \left\{ 1 - C_{bl/a} \left( \frac{1 + 2 \, a/b}{F_{li}/F_{ir} + 2 \, a/b} \right) \right\} S_{li} \sqrt{\pi \, a}$$ (5.6.11)

The stress intensity factor $K_u$ must be written in the form of the stress intensity factor $K_u$ in the aluminium only, before it can be used for a prediction of the crack growth rate. Two assumptions have to be made. First, the anisotropy of ARALL is neglected. This assumption is analysed in appendix A, where the error is estimated to be in the order of 10%. Second, the crack tip deformation field following from the stress intensity factor of the laminate ($K_u$) is supposed to be also characteristic for the aluminium sheets. This assumption implies that the stress intensity factors are related by the elastic moduli:

$$\frac{K_{Al}}{K_{us}} = \frac{E_{Al}}{E_{us}}$$ (5.6.12)

with $E_{Al}/E_{us} = S_{Al}/S_{us}$ (equation 5.2.7). The result is:

$$K_{Al} = \left\{ 1 - C_{bl/a} \left( \frac{1 + 2 \, a/b}{F_{li}/F_{ir} + 2 \, a/b} \right) \right\} S_{Al} \sqrt{\pi \, a}$$ (5.6.13)
Another method for the approximation of $K_a$.

The above method is rather straightforward, and the accuracy of the solution for the stress intensity factor will be acceptable. However, an extension of the method to cover the effects of the starter notch (fibres cut) is difficult. Therefore, an alternative analysis is discussed below. The conditions of an infinite sheet and the absence of a starter notch are considered again as a first step. The result will be compared to equation (5.6.11). The effects of the starter notch and the finite width will be introduced later.

It was shown in figure 121 that $COD_{x=0}$ (for small values of $b/a$) represents the main portion of $2 \nu_{x=0}$. Ignoring the other part:

$$2 \nu_{x=0} = COD_{x=0} \quad (5.6.14)$$

$COD_{x=0}$ can be expressed as a function of $K$ and $a$:

$$COD_{x=0} = \frac{4 K_{ja}}{E_{ja}} \sqrt{a/\pi} \quad (5.6.15)$$

Substitution of equation (5.6.10) in (5.6.15) yields with (5.6.14)

$$COD_{x=0} = 2 \nu_{x=0} = \frac{4 (S_{ja} - C_{b/a} S_{ja}) a}{E_{ja}} \quad (5.6.16)$$

The displacement of the entire crack edge, assuming an elliptical crack opening are then approximated with:

$$\nu = \frac{2 (S_{ja} - C_{b/a} S_{ja}) a}{E_{ja}} \sqrt{1 - (x/a)^2} \quad (5.6.17)$$

which is an expression for $\nu$ of a bridged crack, similar to the second model resulting in equation (5.6.4). The difference is the introduction of $C_{ab}$ which approximates the effect of the crack bridging stress acting on the edge of the delamination boundary instead of the crack flanks. This improvement is achieved by an implicit assumption enclosed in equation (5.6.15) namely an elliptical displacement distribution also at the delamination boundary. Furthermore, the effect of $C_{ab}$ on the displacements is assumed to be the same as on $K$. Equating of the two values of $\nu$ according to equation (5.6.17) and (5.6.2) yields:

$$S_{br} = \frac{1}{\left( \frac{b}{2} \frac{F_{ja}}{F_{ar}} + C_{b/a} \right)} S_{ja} \quad (5.6.18)$$

So far equation (5.6.18) may be considered as an improved version of the second model.

The two models and equation (5.6.18) are compared in the table on the next page, for $F_r = 36450N/mm$ and $F_a = 166050N/mm$. These values are representative for the ARALL type used for the constant-amplitude crack growth tests in section 4.2.
<table>
<thead>
<tr>
<th>$b/a$</th>
<th>first model</th>
<th>second model</th>
<th>improved second model</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>0.05</td>
<td>0.92</td>
<td>0.90</td>
<td>0.91</td>
</tr>
<tr>
<td>0.1</td>
<td>0.86</td>
<td>0.81</td>
<td>0.84</td>
</tr>
<tr>
<td>0.2</td>
<td>0.76</td>
<td>0.69</td>
<td>0.72</td>
</tr>
<tr>
<td>0.3</td>
<td>0.68</td>
<td>0.59</td>
<td>0.64</td>
</tr>
<tr>
<td>0.4</td>
<td>0.63</td>
<td>0.52</td>
<td>0.57</td>
</tr>
<tr>
<td>0.5</td>
<td>0.58</td>
<td>0.47</td>
<td>0.51</td>
</tr>
<tr>
<td>0.7</td>
<td>0.52</td>
<td>0.39</td>
<td>0.43</td>
</tr>
<tr>
<td>1.0</td>
<td>0.46</td>
<td>0.31</td>
<td>0.34</td>
</tr>
</tbody>
</table>

The improved second model (equation 5.6.8) is an intermediate solution between the first and the second model, as it should be expected for a more accurate model. For small $b/a$ values, the improved second model is closer to the first model. In view of the considerations above and the assumption that the first model is more accurate than the second model, it may be concluded that the accuracy of the improved second model is quite acceptable especially for $b/a < 0.3$.

The stress intensity is now obtained by substitution of equation (5.6.18) in (5.6.10):

\[
K_{is} = \left\{ 1 - \frac{C_{bi/a}}{\frac{b}{2a}} \left( \frac{F_{in}}{F_{wa}} + C_{bi/a} \right) \right\} S_{in} \sqrt{\pi a} \quad (5.6.19)
\]

Defining a correction factor $C_d$ for the effect of crack bridging and delamination, equation (5.6.19) can be written as:

\[
K_{is} = C_d S_{in} \sqrt{\pi a} \quad (5.6.20)
\]

Equation (5.6.11) and (5.6.19) are compared in the table below.
<table>
<thead>
<tr>
<th>$b/a$</th>
<th>0</th>
<th>0.05</th>
<th>0.1</th>
<th>0.2</th>
<th>0.3</th>
<th>0.4</th>
<th>0.5</th>
</tr>
</thead>
<tbody>
<tr>
<td>$C_t$ (5.6.11)</td>
<td>0</td>
<td>0.09</td>
<td>0.18</td>
<td>0.30</td>
<td>0.39</td>
<td>0.47</td>
<td>0.52</td>
</tr>
<tr>
<td>$C_u$ (5.6.19)</td>
<td>0</td>
<td>0.10</td>
<td>0.19</td>
<td>0.33</td>
<td>0.44</td>
<td>0.52</td>
<td>0.58</td>
</tr>
<tr>
<td>(5.6.11)/(5.6.19)</td>
<td>0.93</td>
<td>0.92</td>
<td>0.92</td>
<td>0.91</td>
<td>0.91</td>
<td>0.90</td>
<td>0.90</td>
</tr>
</tbody>
</table>

Again, the differences are rather small.

The influence of the saw cut and the finite width

Because the saw cut (half length $s$ figure 122a) also interrupts the fibres, crack bridging stresses are present over a part of the crack only ($a - s$). Some complications have to be considered.

1. The crack bridging stress ($S_b$) will differ from the preceding case.
2. Also here the fibre stress does not act on the crack flanks due to delamination (figure 122b).
3. Width corrections should be approximated.

Without delamination the stress intensity factor for an infinite sheet and constant $S_b$ is exactly known:

$$K_{in} = S_{in} \sqrt{\pi a} - 2 S_{br} \sqrt{\frac{a}{\pi}} \arccos\left(\frac{s}{a}\right)$$

However, the crack bridging stress acts at the delamination boundary (see figure 122b). The absence of a crack bridging stress at the saw cut will affect $C_{br}$. The effect decreases at decreasing $s/a$ ratios, and it will be ignored. The stress intensity factor is then approximated with:

$$K_{in} = S_{in} \sqrt{\pi a} - 2 C_{br} S_{br} \sqrt{\frac{a}{\pi}} \arccos\left(\frac{s}{a}\right) \quad (5.6.21)$$

The effect of the finite width will be different for the stress intensity associated with $S_u$ and $S_b$. The main reason for the difference is the presence of the saw cut. As a first approximation the difference is ignored. The effect of the finite width is accounted for with the finite width correction factor $C_{fw}$ for a non bridged crack. A finite width correction factor is adopted from [63]:

$$C_{fw1} = \left\{ 1 - 0.1 \left(\frac{a}{W}\right)^2 + 0.96 \left(\frac{a}{W}\right)^4 \right\} \cdot \sqrt{\sec\left(\frac{\pi a}{W}\right)} \quad (5.6.22)$$

Application of this finite width correction factor on both terms of equation (5.6.21) yields:

$$K_{in} = C_{fw1} S_{in} \sqrt{\pi a} - 2 C_{fw1} C_{br} S_{br} \sqrt{\frac{a}{\pi}} \arccos\left(\frac{s}{a}\right) \quad (5.6.23)$$
An improved expression, accounting also for the effect of the saw cut on the finite width correction associated with the crack bridging stress can be estimated with the book of Tada et al. [63] (p. 7.8). After application of the superposition principle and some mathematical analysis, it can be derived that a more accurate finite width correction is obtained if the term 

\[ C_{fw1} \arccos\left(\frac{s}{a}\right) \]

in equation (5.6.23) is replaced by:

\[ C_{fw1} \arccos\left\{ \frac{\sin(\pi s/w)}{\sin(\pi a/w)} \right\} \]

The improved finite width correction factor will not be applied until the final result is reached, in order to restrict the length of the intermediate equations.

The presence of the saw cut also influences the the crack bridging stresses. Secondly, the in-plane stiffness of the laminate is influenced by the finite width. Consequently, a new expression for \( S_{\nu} \) must be derived, which accounts for these two additional effects. It is assumed that both influences are sufficiently accounted for, if their effect on COD is considered. Still assuming an elliptical crack contour, COD may be derived by substitution of equation (5.6.23) in (5.6.15). However, in this way the finite width correction factor \( C_{fw1} \) for the stress intensity factor is implicitly applied on COD. In appendix F solutions by Tada [63] and Koiter [64] were compared, and it was shown that the finite width correction factor for COD is somewhat smaller than for \( K \). The finite width correction factor on COD is denoted as \( C_{fw} \). It is described by the following equation:

\[ C_{fw2} = -0.071 - 1.07(a/w) + 0.676 (a/w)^2 - 0.72 (a/w)^3 + 0.32 (a/w)^4 - 0.5355 (w/a) \ln(1 - 2a/w) \quad (5.6.24) \]

A correction of equation (5.6.15) for the different finite width corrections is approximated by adding the factor: \( C_{fw2}/C_{fw1} \). The result is:

\[ COD_{x=0} = \frac{4 C_{fw2} K_{la} \sqrt{\frac{a}{\pi}}}{C_{fw1} E_{la}} \quad (5.6.25) \]

Still assuming an elliptical displacement contour, it can be written:

\[ v = \frac{2 C_{fw2} K_{la} \sqrt{\frac{a}{\pi}}}{C_{fw1} E_{la}} \sqrt{1 - \left(\frac{x}{a}\right)^2} \quad (5.6.26) \]

Equating of this expression with equation (5.6.2) yields with the definition of \( F_{\nu} \):

\[ S_{\nu} = 2 \frac{C_{fw2}}{C_{fw1}} \frac{F_{\nu} a}{F_{la} b} \frac{K_{la}}{\sqrt{\pi a}} \quad (5.6.27) \]
\( K_a \) can now be solved from equation (5.6.23) and (5.6.27) by elimination of \( S_w \). The result is:

\[
K_a = \left\{ \frac{C_{fw1}}{1 + \frac{4}{\pi} \frac{C_{fw2}}{C_{w_a}} \frac{a}{b} \frac{F_{ar}}{F_{in}} \arccos\left( \frac{s}{a} \right)} \right\} S_{in} \sqrt{\pi a} \tag{5.6.28}
\]

With equation (5.6.12) it follows:

\[
K_{AI} = \left\{ \frac{C_{fw1}}{1 + \frac{4}{\pi} \frac{C_{fw2}}{C_{w_a}} \frac{a}{b} \frac{F_{ar}}{F_{in}} \arccos\left( \frac{s}{a} \right)} \right\} S_{AI} \sqrt{\pi a} \tag{5.6.29}
\]

The symbol \( b \) denotes the delamination at \( x = 0 \). For a laminate with a saw cut, this distance is fictitious. The delamination \( b_i \) at the tip of the saw cut (\( x = s \)) is derived for an elliptical delamination area with:

\[
b_i = b \sqrt{1 - \left( \frac{s}{a} \right)^2} \tag{5.6.30}
\]

Substitution in equation (5.6.29) yields:

\[
K_{AI} = \left\{ \frac{C_{fw1}}{1 + \frac{4}{\pi} \frac{C_{fw2}}{C_{w_a}} \frac{\sqrt{a^2 - s^2} b_i}{b} \frac{F_{ar}}{F_{in}} \arccos\left( \frac{s}{a} \right)} \right\} S_{AI} \sqrt{\pi a} \tag{5.6.31}
\]

The presence of the saw cut will also affect \( C_{w_a} \). An approximation of \( C_{w_a} \) is obtained by substitution of equation (5.6.30) in (5.6.9)

\[
C_{w_a} = \frac{5}{3(1 + b_i \sqrt{a^2 - s^2})} - \frac{2}{3(1 + b_i \sqrt{a^2 - s^2})^2} \tag{5.6.36}
\]

With equation (5.6.12), (5.6.27) and (5.6.30) a new expression for \( S_w \) is found:

\[
S_{br} = 2 \frac{C_{fw2}}{C_{fw1}} \frac{F_{ar}}{E_{AI} I_{in}} \frac{\sqrt{a^2 - s^2}}{b_i} \frac{K_{AI}}{\sqrt{\pi a}} \tag{5.6.33}
\]

At this stage the improved finite width correction is applied, which was derived before. The \( \arccos(s/a) \) term in equation (5.6.31) is replaced by \( \arccos\{ \sin(\pi s/w)/\sin(\pi a/w) \} \). The result is written as:

\[
K_{AI} = C_d S_{AI} \sqrt{\pi a} \tag{5.6.34}
\]

with
\[ C_d = \frac{C_{Fw1}}{1 + \frac{4}{\pi} C_{Fw2} C_{u/a} \frac{\sqrt{a^2 - s^2}}{b_s} \frac{F_{sw}}{F_{sw}} \arccos \left( \frac{\sin \pi s/w}{\sin \pi a/w} \right)} \]

(5.6.35)

Some values of \( C_d \) are presented in table 3. The results are based on the laminate properties of the specimens used for the constant-amplitude crack growth tests (section 4.3). The stress intensity factors for extreme cases are considered in more detail below.

1. The first case is the stress intensity for zero delamination, \( b_s = 0 \):
   It is easily recognized from equation (5.6.35) that \( C_d = 0 \) and hence \( K_a = 0 \). This is indeed the trivial solution, because the crack bridging fibres do not allow any crack opening.

2. An infinite delamination: \( b_s = \infty \) and \( C_v = 0 \). It is found that \( K_a = C_{rv} S_{u} \sqrt{\pi a} \) or \( K_a = C_{rv} S_{u} \sqrt{\pi a} \). This solution may represent a slight overestimation because there is no connection left between fibres and aluminium for \(-a < x < a\). The stiffness of the cracked area is lower than the stiffness of the uncracked area, and the actual stress intensity will also be lower than \( S C_{rv} \sqrt{\pi a} \). However, the error will not be large. Moreover, the case of infinite delamination is hardly relevant for ARALL anyhow.

3. There is no fatigue crack if \( a = s \): The saw cut is a crack like notch and there are no crack bridging stresses. Equation (5.6.35) indeed yields \( C_d = C_{Fw1} \) and thus the trivial solution: \( K_a = C_{rv} S_{u} \sqrt{\pi a} \)

5.6.2.2 The influence of the residual stresses.

An infinite sheet without starter notch

Again it is convenient, to study first the simple case of an infinite sheet without a saw cut. A sheet with a residual stress \( S_{rv} \), is loaded by an external stress \( S_u \) of such a magnitude that the total stress \( S_{u} = S_{rv} \) is zero. According to the equations of section 5.2, \( S_{rv} = 0 \) if:

\[ S_{sw} = - \frac{E_{sw}}{E_{sw}} S_{rv} \]

(5.6.36)

It is evident that this stress level is important, because the stress intensity factor in the aluminium sheets is zero if \( S_{rv} = 0 \), independent of the amount of delamination. The entire aluminium sheets are unloaded and the crack flanks are in contact without stress. There is no load transfer from fibres to aluminium at the delamination boundary. The external load is completely carried by the fibres, and the total fibre stress \( S_{sw} \) is the same in the delaminated and the non-delaminated area. From equilibrium it can be derived that

\[ S_{uw} = - \frac{F_{sw}}{F_{sw} E_{sw}} S_{sw} \]

(5.6.37)
If the $S_a$ -value at which $K = 0$ in equation (5.6.36) is denoted as $S_{a,0}$, the $K_a$ -value for larger $S_a$ -values can be obtained by superposition of two external loads, namely $S_a - S_{a,0}$ and $S_{a,0}$. For the latter one $K_a = 0$ and consequently according to equation (5.6.19) the $K_a$ -value (now denoted as $K_{l,al}$ ) becomes:

$$K_{l,al} = \left\{ 1 - \frac{b}{2a} \left( \frac{F_{la}}{F_{ar}} + C_{bl/a} \right) \right\} \frac{(S_{la} - S_{la,0}) \sqrt{\pi a}}{a}$$  \hspace{1cm} (5.6.38)

In a similar way:

$$K_{l,al} = \left\{ 1 - \frac{C_{bl/a}}{2a} \left( \frac{F_{la}}{F_{ar}} + C_{bl/a} \right) \right\} \frac{(S_{al} - S_{al,0}) \sqrt{\pi a}}{a}$$  \hspace{1cm} (5.6.39)

where $S_{al,0}$ is the stress in the aluminium sheets associated with the external load, at which $K_{al} = 0$. $K_{l,al}$ is the stress intensity factor in the aluminium sheets considering external loading, residual stress and delamination. In equation (5.6.39) is $S_{al,0} = -S_{r,al}$. It will be shown in the next part that $S_{al,0}$ is smaller if a saw cut is present.

The effect of a saw cut

The saw cut is complicating the residual stress effect on $K_a$. For the analysis a specimen without a cut ($s=0$), loaded by $S_a$ until $K=0$, is again considered first. This stress level is given by equation (5.6.36), and the corresponding fibre stress by equation (5.6.37). Now a saw cut is made, which implies that the fibre stresses are fully released. As a result the release will contribute as a kind of an external load on the crack flanks. Correcting $S_{al}$ to the full laminate thickness, a stress $-S_{r,al}E_a/E_{al}$ will act on the crack flanks between $x = -s$ and $x = s$. The corresponding $K$-value will be:

$$K_{l,al} = C_d \left[ -S_{r,al} \frac{E_{la}}{E_{al}} \left( 1 - \frac{2}{\pi} \arccos \frac{s}{a} \right) \right] \sqrt{\pi a}$$

This stress intensity factor can be nullified again, by adding a second additional external stress on the laminate which is equal to

$$S_a = \left[ \frac{E_{la}}{E_{al}} \left( 1 - \frac{2}{\pi} \arccos \frac{s}{a} \right) S_{r,al} \right]$$

Adding this $S_a$ increment to the initial one (equation 5.6.36) to obtain $K_{l,al} = 0$, the total external stress $S_{a,0}$ for a zero stress intensity factor is found to be:

$$S_{a,0} = \frac{2E_{la}}{\pi E_{al}} \arccos(s/a) S_{r,al}$$  \hspace{1cm} (5.6.39)

and similar:
\[ S_{A,0} = -\frac{2}{\pi} \arccos(s/a) \cdot S_{r,Al} \quad (5.6.40) \]

The latter equation shows directly that for a laminate with a saw cut \( K_{Al} \neq 0 \) if \( S_{Al} = -S_{r,Al} \). The two extreme boundary conditions are now briefly considered.

1. For \( s = 0 \) the trivial boundary condition \( S_{A,0} = -S_{r,Al} \) arises
2. For \( s = a \) it follows that \( S_{A,0} = 0 \), which is indeed the other boundary condition, because the effect of the residual stress is zero if there are no intact fibres behind the crack tip.

For the case that \( S_{Al} = S_{A,0} \), the stress intensity factor is zero. The crack flanks at the crack tip are in contact, without the occurrence of contact stresses. There is no crack opening near the crack tip, and no restraint on crack opening occurs by crack bridging fibres. This explains why the correction factor \( C_d \) is absent in the equations (5.6.39) and (5.6.40).

It is evident that crack opening at the "tip" of the saw cut will start immediately at the onset of external loading, independently of the presence of a residual compressive stress in the aluminium sheets. This crack opening will cause a crack bridging stress in front of the tip of the saw cut, also if the crack tip is still closed. This means that the assumption of a constant crack bridging stress introduces some inaccuracies. However, the effect is assumed to be small, and will be neglected.

Application of the improved finite width correction derived before, on equation (5.6.40) yields:

\[ S_{Al,0} = -\frac{2}{\pi} \arccos \left\{ \frac{\sin(\pi s/w)}{\sin(\pi a/w)} \right\} \cdot S_{r,Al} \quad (5.6.41) \]

The stress intensity factor \( K_{Al} \) in the aluminium sheets, as caused by the residual stress and the external load can now be written as:

\[ K_{L,Al} = C_d (S_{Al} - S_{Al,0}) \sqrt{\pi a} \quad (5.6.42) \]

### 5.6.2.3 The crack bridging stress.

The magnitude of the crack bridging stresses is important for the calculation of the delamination growth rates and for the stress intensity factor as affected by adhesive shear deformation. For both aspects the load transfer from the fibres to the aluminium is more important, than the actual stresses in the delaminated fibres. The adhesive shear deformation and the energy release rate for delamination are directly related to the load transfer. That means that only that part of \( S_{r} \) which is associated with load transfer has to be considered for the derivation of adhesive shear deformation and delamination. It was already noted before that no crack tip opening occurs if \( K_{L,Al} = 0 \) and \( S_{Al} = -S_{Al,0} \). Consequently, no load transfer from fibres to aluminium occurs then. Consequently, no further correction for the residual stress or the presence of a saw cut is necessary, if equation (5.6.27) is written as a function of \( K_{Al} \) instead of \( K_{Al} \), and \( K_{Al} \) is then replaced by \( K_{L,Al} \). The result is:
\[ S_w = 2 \frac{C_{fw2}}{C_{fw1}} \frac{F_w}{t_w E_{Al}} a \frac{K_{IAl}}{\sqrt{\pi a}} \]

Substitution of equation (5.6.30) and (5.6.42) yields:

\[ S_w = 2 \frac{C_{fw2}}{C_{fw1}} \frac{F_w}{t_w E_{Al}} \frac{\sqrt{a^2 - s^2}}{b_s} C_d (S_{Al} - S_{Al,0}) \]  
(5.6.47)

Because \( S_w \) and \( S_r \) are both calculated over the total thickness of ARALL, the energy release rate and the crack opening displacement due to adhesive shear deformation can both be obtained by replacing \( S_r \) in the equations (5.4.4) and (5.5.2) by \( S_w \) according to equation (5.6.47). However, one comment still has to be made: The equation for \( G_a \) was derived for a delamination specimen, where the stress in the delaminated aluminium sheets is always zero. This is not the case for the crack growth specimen. However, a calculation of the stress in the y-direction (at \( x = 0 \)), from the Westergaard stress functions shows that the stress remains very low up to \( b/a \) ratios hardly relevant for ARALL (see figure 123), and this effect can be ignored. The same will apply to the adhesive shear stress. Replacing \( S_r \) by \( S_w \) in equation (5.5.2) and substitution of equation (5.6.47) yields after some rewriting:

\[ G_a = 2 \frac{F_w}{F_{Al} F_{la}} \left\{ t_w \frac{C_{fw2}}{C_{fw1}} \frac{E_w}{E_{Al}} \frac{\sqrt{a^2 - s^2}}{b_s} C_d (S_{Al} - S_{Al,0}) \right\}^2 \]  
(5.6.44)

5.6.3 The influence of the adhesive shear deformation on the stress intensity factor.

The crack opening due to adhesive shear deformation causes an additional increment to the stress intensity factor. It was shown in section 4.7 that plasticity and visco elastic behaviour in the adhesive is relevant for the crack opening displacement. However, the influence on the cyclic displacements remains moderate. Consequently, a linear elastic approach for the calculation of the stress intensity factor will yield an acceptably accurate result.

First, the situation for a crack without delamination and without a starter notch (saw cut) is considered. An infinite sheet is assumed and the residual stresses are zero. In section 5.4.1 a delamination specimen was analysed, i.e. a specimen with a crack in the aluminium layers over the full width. For this case \( S_w = S_r \) and the crack opening displacement was written as:

\[ COD = 2 \frac{E_{Al}}{E_{la}} S_{la} t_{Al} \sqrt{\frac{1}{j F_{Al} F_{ad}}} + \frac{1}{j F_w F_{ad}} \]  
(5.4.4)

The equation is an upper asymptote for the crack opening displacement of a crack growth specimen with a central crack in the aluminium layers. The COD value according to equation (5.4.4) can not be exceeded when the crack length increases. For very small cracks, where the in-plane stiffness is more important, another asymptote can be recognized:
\[
COD = \frac{4 S_{AI} a}{E_{AI}} = \frac{4 S_{in} a}{E_{in}}
\]

If the COD value according to the latter equation is considerably lower than the COD value according to equation (5.4.4), significant crack bridging stresses can not arise.

The two asymptotes are presented in figure 124a. For intermediate crack lengths the actual COD is below the two asymptotes. The corresponding crack opening contours are also presented in figure 124b. For long cracks it is obvious that the crack opening near the crack tip is lower than the value according to equation (5.4.4), because the crack flanks are "connected" at the crack tip. The crack bridging stress is proportional to the adhesive shear stress and the corresponding COD values. Consequently, the crack bridging stress is reduced near the crack tip. The crack behaves as if it is bridged by linear elastic springs between the crack flanks. The stress intensity is directly related to the decrease of the crack bridging stress near the crack tip. For \( S_a = S_n \) and without a reduction of the crack bridging stress near the crack tip, the stress intensity factor would be zero. It is also evident that the area with a reduced crack bridging stress near the crack tip is not dependent on the crack length, if the horizontal asymptote for the COD value is reached. The same will be true for the stress intensity factor caused by the reduction of the crack bridging stresses near the crack tip. Consequently, a horizontal asymptote will also be reached for the stress intensity factor.

For small cracks where \( COD = \frac{4 S_{AI} a}{E_{AI}} = \frac{4 S_{in} a}{E_{in}} \) the stress intensity factor is described with \( K = S_n \sqrt{\pi a} \) as an other asymptote (see figure 125).

An explicit solution for the stress intensity factors is not directly evident. However, a solution for a geometry with similar characteristics is available. The new geometry is a row of parallel cracks in an infinite plate (see figure 126). The distance between the cracks is denoted as \( 2h \). The geometry shows the same asymptotic behaviour for the crack opening shape of long cracks and thus for the stress intensity factor and the COD values.

The crack opening displacement for parallel cracks is found from continuity:

\[ 2h + COD = 2h (1 + \epsilon) \quad \Rightarrow \]

\[ COD = 2 \frac{S_{AI}}{E_{AI}} h = 2 \frac{S_{in}}{E_{in}} h \quad (5.6.45) \]

A representative \( h \) value for this analogy is found, when the COD values for the bridged cracks and the parallel cracks are the same. Substitution of equation (5.6.49) in (5.4.4) yields:

\[ h = F_{ad} \sqrt{\frac{1}{j F_{ad} F_{ad}} + \frac{1}{j F_{ad} F_{in}}} \quad (5.6.46) \]

The value \( h \) is only dependent on the laminate construction parameters. It may be considered as a material property, describing the sensitivity of ARALL for crack growth due to adhesive
shear deformation. A typical value for ARALL is: \( h \approx 2.5 \text{mm} \). The stress intensity for the horizontal asymptote (parallel cracks) is found in [63] or [65]:

\[
K_{(h)} = S_{AI} \sqrt{h}
\]

After correction for the residual stresses the equation changes to:

\[
K_{(h)} = (S_{AI} - S_{AI0}) \sqrt{h}
\]  \hspace{1cm} (5.6.47)

It is demonstrated in appendix H that the value for the horizontal asymptote is identical to the value which arises for a model where the crack is bridged by linear elastic springs and the result is judged to be close to the present problem.

For intermediate crack lengths an interpolation is required. Benthem and Koiter [65] derived a tenth order polynomial, which describes the entire region of \( a/h \) values for a row of parallel cracks. After adding a factor \( \sqrt{\pi} \) which is often omitted in older publications (according older conventions is \( K = S \sqrt{a} \) for an infinite sheet, instead of the modern notation \( K = S \sqrt{\pi \ a} \)), the polynomial is:

\[
K = S \cdot \left\{ \left( 1 + \frac{1}{2} d + \frac{3}{8} d^2 + \frac{5}{16} d^3 + \frac{35}{128} d^4 + \frac{63}{256} d^5 + \frac{231}{1024} d^6 \right) + \sqrt{\pi} \cdot (15.342 d^7 - 47.020 d^8 + 45.279 d^9 - 14.255 d^{10}) \right\} \cdot \sqrt{a \ d}
\]  \hspace{1cm} (5.6.48)

with

\[
d = \frac{h}{a + h}
\]  \hspace{1cm} (5.6.49)

Equation (5.6.48) shows a 1.1\% overshoot of the horizontal asymptote. The overshoot led Benthem and Koiter to assume an accuracy of about 2\% for their approach.

An equation which is close to the result of Benthem and Koiter, but which is shorter and more convenient for the use with modern electronic calculators is:

\[
K = S \sqrt{h} \cdot \tanh(\pi a/h)
\]  \hspace{1cm} (5.6.50)

Equation (5.6.50) is close to (5.6.48) (maximum difference 1.8\% and usually much smaller), but does not show an overshoot. The accuracy of both equations may be similar.

The stress intensity factor is related to the decreased crack bridging stresses near the crack tip. Figure 127 shows how this crack bridging stress distribution may be changed if delamination occurs. The overall level of the crack bridging stresses is changed from \( S_{\nu} \) to a value \( S_{\nu} \), which is usually lower than \( S_{\nu} \). However, at those locations near the crack tip, where the level of the crack bridging stress is already lower than \( S_{\nu} \) without the influence of the delamination, further delamination will remain very limited, because the crack bridging stresses are the driving force for the delamination. (The delamination tends to equalize the crack bridging stresses.). Consequently, the crack bridging stress closely behind the crack
tip is not decreased further by the occurrence of delamination, and the original stress distribution on the crack flanks is truncated down to $S_n$. Micrographic investigations by Roebroeks [66] showed that delamination indeed started at some distance behind the crack tip. An estimation of the effect of delamination on the stress intensity factor due to adhesive shear deformation is performed in appendix I. The estimation is based on the truncated stress distribution of figure 127. The result is presented as a correction factor $C_{ad,d}$ on the stress intensity factor due to adhesive deformation.

$$ C_{ad,d} = \frac{S_{br}}{S_{ln}} + \left( 1 - \frac{S_{br}}{S_{ln}} \right) \cdot \ln \left( 1 - \frac{S_{br}}{S_{ln}} \right) \quad (5.6.51) $$

For laminates with a saw cut, but with very little delamination, the $S_r$ -values according to equation (5.6.47) may become higher than $S_n$. This looks "unnatural" at a first glance, but is consistent with the assumption of constant crack bridging stresses which lead to equation (5.6.31). For zero delamination is $K = 0$, also if a saw cut is present, this is only possible if $S_r > S_n$ (the "unnatural" behaviour can also be understood from figure 6 if it is assumed that the peak in the crack bridging stress at the tip of the saw cut reduces due to some very minor delamination. A redistribution of the crack bridging stress will occur, yielding higher stresses at larger $x$ values). However, $S_r$ values larger than $S_n$ are rare, because they will rapidly be reduced by delamination and a sufficiently accurate set of expressions is:

$$ C_{ad,d} = \frac{S_{br}}{S_{ln}} + \left( 1 - \frac{S_{br}}{S_{ln}} \right) \cdot \ln \left( 1 - \frac{S_{br}}{S_{ln}} \right) \quad (for \: S_{br} < S_{ln}) \quad (5.6.52a) $$

$$ C_{ad,d} = \frac{S_{br}}{S_{ln}} \quad (for \: S_{br} \geq S_{ln}) \quad (5.6.52b) $$

The equations (5.6.52) apply to laminates without residual stresses. It is easily understood, that the residual stresses may be accounted for, by replacing $S_n$ with $S_n - S_{ln,0}$. The influence of the residual stress on $S_r$ is already included in equation (5.6.43), and the equations (5.6.52) may be modified to

$$ C_{ad,d} = \frac{S_{br}}{S_{ln} - S_{ln,0}} + \left( 1 - \frac{S_{br}}{S_{ln} - S_{ln,0}} \right) \cdot \ln \left( 1 - \frac{S_{br}}{S_{ln} - S_{ln,0}} \right) \quad (for \: S_{br} < S_{ln} - S_{ln,0}) \quad (5.6.53a) $$

$$ C_{ad,d} = \frac{S_{br}}{S_{ln} - S_{ln,0}} \quad (for \: S_{br} \geq S_{ln} - S_{ln,0}) \quad (5.6.53b) $$

The correction factor $C_{ad,d}$ was derived for long cracks where $K = K_{ln}$. An extension of the correction factor for smaller cracks is difficult, but it is not really needed, because only a limited part of the entire crack growth life is involved. A similar aspect arises if the influence of the saw cut on $K$ is considered. This effect is described by the correction factor $C_r$. Some boundary conditions for the derivation of $C_r$ can be recognized:
1. Crack bridging fibres and thus a region with reduced crack bridging stresses, are not present if \( a = s \). Consequently, the stress intensity factor due to adhesive deformation is also zero and \( C_s = 0 \).

2. For small \( s/a \) ratios, the influence of the saw cut is damped out and \( C_s = C_{bs} \) (thus roughly approximating the effect of the axis ratio of the delamination ellipse).

3. The \textit{increase} of the stress intensity factor due to adhesive shear deformation cannot be larger than the \textit{decrease} of the stress intensity factor as a consequence of crack bridging derived without accounting for adhesive shear deformation. In other words, adhesive shear deformation may at most annihilate the effect of crack bridging, but it can never increase the stress intensity factor beyond that.

The above boundary conditions are fulfilled if equation (5.6.50) is written as:

\[
K_{ad} = C_s C_{ad,d} (S_{Al} - S_{Al,0}) \sqrt{h \cdot \tanh(\pi a/h)}
\]

(5.6.54)

with

\[
C_s = \frac{\arccos(1 - \frac{\pi h + 8 a - 8 s}{\pi h + 8 a})}{\arccos(1 - \frac{\pi h}{\pi h + 8 a})} \cdot C_{bs}
\]

(5.6.55)

The derivation of equation (5.6.55) as an interpolation between the boundary conditions is lengthy and not of great interest. Therefore it is not presented here. However, it is not difficult to recognize that it fulfils the three boundary conditions. The replacement of \( S_{Al} \) by \( S_{Al} - S_{Al,0} \) yields the correction for the residual stresses. Equation (5.6.37) fulfills the boundary conditions for long and short cracks. However, the result for intermediate crack lengths may be inaccurate if a saw cut is present. Anyhow this possible inaccuracy does only occur for a small range of intermediate crack lengths.

So far the influence of the finite specimen width on the stress intensity factor due to adhesive shear deformation was not considered. A correction factor for this influence is derived in appendix E. These derivations are based on finite element calculations, which were performed by W. Ott. The finite width correction factor on \( K_{ad} \) is found in equation (E-10) in appendix E. However, the influence remains negligible up to \( a/w \) values which are very close to 0.5, and the application of a correction is not necessary.

Summarizing: The stress intensity \( K_{ad} \) due to adhesive shear deformation is calculated with:

\[
K_{ad} = C_s C_{ad,d} (S_{Al} - S_{Al,0}) \sqrt{h \cdot \tanh(\pi a/h)}
\]

(5.6.54)

where \( C_s \) and \( C_{ad,d} \) are described by:
\[
C_s = \frac{\arccos\left(1 - \frac{\pi h - \pi h s/a}{\pi h + 8 a - 8 s}\right)}{\arccos\left(1 - \frac{\pi h}{\pi h + 8 a}\right)} \cdot C_{b/s}
\] (5.6.55)

and

\[
C_{ad,d} = \frac{S_{br}}{S_{ln} - S_{ln,0}} + \left(1 - \frac{S_{br}}{S_{ln} - S_{ln,0}}\right) \cdot \ln\left(1 - \frac{S_{br}}{S_{ln} - S_{ln,0}}\right)
\]
(for \(S_{br} < S_{ln} - S_{ln,0}\)) (5.6.53a)

\[
C_{ad,d} = \frac{S_{br}}{S_{ln} - S_{ln,0}}
\]
(for \(S_{br} \geq S_{ln} - S_{ln,0}\)) (5.6.53b)

The final stress intensity factor in the aluminium sheets \(K_{fn}\) is found by superposition.

\[
K_{fn} = K_{L,Al} + K_{ad}
\] (5.6.56)

The cyclic stress intensity factor is:

\[
\Delta K_{fn} = K_{fn,\text{max}} - K_{fn,\text{min}}
\] (5.6.57)

And the apparent stress (intensity) ratio \(R_{ap}\) is given by:

\[
R_{ap} = \frac{K_{fn,\text{min}}}{K_{fn,\text{max}}}
\] (5.6.58)

5.7 Discussion of the analytical results

It was already discussed at the beginning of this chapter that the applied analytical techniques require assumptions and that exact solutions may not be expected. As a consequence of the complexity of the problem, the obtained set of equations became lengthy in spite of the approximative character. A very complex aspect of the problem is related to the effect of the saw cut on the stress intensity factor and the crack bridging stress. At some places in the derivations, a small improvement of the accuracy might have been possible. However, the length of the derivations and the resulting equations would be increased progressively. A consequence would also be that it would be even more difficult to recognize the effects of single parameters from the results. It was also discussed before that the alternative method namely finite element calculations has disadvantages. Some additional problems as they would arise during finite element calculations can be recognized now. Finite element calculations require well defined information about the geometry. For ARALL an important geometrical aspect is the shape of the delamination area. Because this shape is determined by local crack bridging stresses, an assumed elliptical delamination area will also
be approximative. Exact information about the shape of the delamination area would require experimental observation. The present approach overcomes this problem by the assumption that the delamination equalizes the crack bridging stresses. The resulting precise shape of the delamination area need not be known. The same applies for the effect that delamination starts at a small distance behind the crack tip. The associated geometry is difficult to describe. However, the associated stress distribution can reasonably be approximated.

The proposed solutions for the stress intensity factors are based on linear elastic material behaviour. Plasticity effects are associated with the behaviour of the adhesive at the delamination front and the behaviour of the aluminium at the crack tip. The effect of adhesive plasticity is related to a region with decreased crack bridging stresses behind the crack tip. The decrease of the crack bridging stresses will be more significant as a consequence of the increased COD due to adhesive plasticity. The stress intensity factor is increased as well. On the other hand, the plasticity at the crack tip in the aluminium sheets causes an increased crack opening displacement as compared to an elastic model for the metallic component. In fracture mechanics the increased crack opening displacement due to crack tip plasticity is often referred to as crack tip opening displacement. Its effect is mainly restricted to the region closely behind the crack tip. A consequence of the increased crack tip opening displacement is an increased strain of the crack bridging fibres in the crack tip region. The resulting increased crack bridging stresses cause a reduction of the stress intensity factor. Apparently, plasticity in the adhesive and crack tip plasticity in the aluminium have opposite effects on the stress intensity factor. Although the opposite influences need not cancel one another it helps to explain why linear elastic calculations can still lead to rather accurate results. Moreover, the effect of plasticity on cyclic displacements is considerably smaller in both cases, and it may be assumed indeed that plasticity effects can be ignored.

The set of analytical expressions in this chapter offers a flexible tool for calculations on different types of ARALL and different fatigue loads. Some trends as they can be obtained from the equations are summarized below.

The stress intensity factor in ARALL can be reduced by an

1. increase of the fibre Young’s modulus
2. decrease of the Young’s modulus of the metallic sheets.
3. decrease of the thickness of the metallic sheets.
4. increase of the fibre aluminium ratio
5. increase of the adhesive shear modulus
6. increase of the resistance of the adhesive to delamination
7. increase of the compressive residual stress $S_{r,ai}$. 
6. Application of some analytical results to the delamination growth tests

6.1 The energy release rate and the adhesive shear stress as loading parameters for the delamination rate

In the previous sections two parameters have been described to characterize the severity of the loading state in the adhesive at the delamination boundary:

1. The theoretical elastic shear stress $\tau_{el}$ at the delamination front, according to a one dimensional mechanical model.
2. The energy release rate for delamination $G_v$.

Both parameters are "field parameters", they do not explicitly describe the stress distribution at the crack tip. The use of field parameters is adopted in fracture mechanics for prediction purposes. Prediction methods are based on the so-called similarity principle. As an example, similar stress intensity factors imply a similarity of stress distributions at crack tips, even within the plastic zone (provided that the plastic zone is much smaller than the surrounding elastic K-field and the crack length). The two field parameters considered here, indeed imply a considerable amount of similarity. However, it is less perfect than for example for stress intensity factors. Especially if the thickness of the (fibre)-adhesive layer is changed the similarity is reduced. The energy release rate concept may be expected to perform correctly for a perfectly brittle adhesive. However, in a real adhesive, energy dissipation is possible as a consequence of adhesive plasticity. An increase of the thickness of the adhesive will increase the amount of energy dissipated, and thus change the energy balance. Yet, the energy release rate is not depending on the properties of the adhesive layer, and deviations have to be expected.

The adhesive shear stress depends on the properties of the adhesive. However, it must be noted that this concept is based on a rigorous modelling of the laminate, which required somewhat arbitrary assumptions, like the one dimensional approach ignoring stress gradients in the $z$ direction. Consequently, also the adhesive shear stress concept needs some further consideration.

In this chapter, the energy release rate for delamination and the adhesive shear stress concept are compared, and their merits as parameters for the correlation to delamination growth rates are judged on the basis of the experimental data from section 4.5. The field parameters are compared below.

The maximum shear stress $\tau_{ad}$ at the delamination front of a delamination specimen is:

$$\tau_{ad} = \tau_{at} \left\{S_{th} - \left(1 + \frac{F_{ar}}{F_{Al}}\right)S_{r,ar}\right\} \cdot \sqrt{\frac{F_{ad}F_{Al}}{jF_{ar}(F_{Al} + F_{ar})}}$$ \hspace{1cm} (5.3.10)

The energy release rate for delamination as derived in appendix C (equation C-32) is:
\[ G_d = \frac{F_{Al}}{F_{ar}(F_{Al} + F_{ar})} \cdot \frac{\tau_{ad}^2}{2j} \cdot \left\{ S_{in} - \left(1 + \frac{F_{ar}}{F_{Al}}\right) S_{\tau_{ad}} \right\}^2 \]  

(6.1.1)

It follows from both equations that

\[ \tau_{ad}^2 = 2 F_{ad} G_d \]  

(6.1.2)

Equation (6.1.2) shows that the parameters \( \tau_{ad} \) and \( G_d \) are closely related. The only basic difference is that the parameter \( F_{ad} \) is not present in the equation for \( G_d \). Both parameters \( \tau_{ad} \) and \( G_d \) account for the stiffness of the aluminium layers and the fibre layers in the same way. The shear stress criterion additionally accounts for possible effects of the shear stiffness \( F_{ad} \) of the adhesive layers.

The influence of \( F_{ar} \) was not investigated separately. However, some qualitative conclusions may be drawn from tests on delamination specimens cut from the edges of the cured ARALL sheets. In section 4.5 it was already noted that those tests were performed, but they were not used for the determination of empirical delamination laws. The delamination specimens from the edges of the cured plates, showed considerable higher delamination rates than regular delamination specimens. An explanation is the lower amount of adhesive, due to bleeding out of adhesive at the sheet edges. Consequently, the stiffness of the adhesive layer \( F_{ad} \) is higher, (lower \( \tau_{ad} \Rightarrow \) higher \( F_{ad} \) and \( \tau_{ad} \)) and higher delamination rates are expected according to the shear stress criterion. The energy release rate criterion implies that the delamination rate should not be influenced by bleeding out of the adhesive. So far the adhesive shear stress criterion seems to be a more appropriate tool for a correlation of the load and the delamination rate than the energy release rate criterion. However, as it will be shown later, a possible overestimation of the influence of \( F_{ar} \) may be an inherent feature of the shear stress criterion.

The influence of the parameters \( F_{al} \) and \( F_{ar} \) are accounted for in the same way by both concepts, and an evaluation of the influence of a change of these parameters is performed for the energy release rate concept only. The results for the shear stress concept are identical.

The effect of a change in the thickness of the aluminium sheets

The influence of the thickness of the aluminium sheets on the delamination rates is presented in figure 46. For both types of specimens \( F_{ar} = 12150 \) N/mm. The stiffness of the aluminium sheets is: \( F_{al} = 72000 \) N/mm for the laminates with \( 2 \times 0.5 \) mm aluminium sheets, and \( F_{al} = 36000 \) N/mm for the laminates with \( 2 \times 0.25 \) mm aluminium sheets. Substitution of these values in (6.1.1) for \( S_{\tau_{ad}} = 0 \) will show that \( G_d \) is 1.14 times lower for the laminate with the thinner aluminium sheets. In section 4.5 it was shown that the delamination rate

\[ \frac{db}{dn} + \Delta \frac{S_{in}^9}{G_d} \]

Because in the present case (since \( S_{\tau_{ad}} = 0 \)) \( G_d + S_{in}^2 \rightarrow \frac{db}{dn} + G_d^{4.82} \)
A 1.14 times lower $G_i$ then implies a 1.9 times slower delamination rate. The experimental difference in figure 46 is a factor of about 2.3. Considering the scatter in the experimental results, the agreement between the experimental and the theoretical influence of changing $t_e$ from 1.0 to 0.5 mm is good.

The effect of a change of the residual stress

The equations (5.3.10) and (6.1.1) show that the effect of the residual stress can be described by a reduction of $S_n$ with $(1 + F_u/F_u)$ $S_n$, for both concepts. A series of delamination tests with and without residual stresses was described in section 4.5. Figure 55 showed that delamination growth data can indeed be correlated to a fibre stress reduced with $(1 + F_u/F_u)$ $S_n$, (which is 50 N/mm in figure 55) so both concepts account correctly for residual stress effects.

The effect of a simultaneous change of stiffness parameters

The two lower data sets in figure 51 represent the influence of a change of the stiffness of the fibre layers. Fabrics with Teyin fibres are compared to fabrics with Twaron fibres. Unfortunately, the thickness of the layers was not identical. The thickness of the fibre adhesive layer with the Teyin fabric is higher, due to a higher fibre content and due to adhesive accumulation in the measuring area of the specimen. Consequently, the $F_u$ value is lower for this specimen type. The stiffness values for the laminates are:

1. Specimens with Twaron fabric:
   - $t_e = 0.25$ mm
   - $F_u = 12150$ N/mm
   - $F_u = 72000$ N/mm
   - $F_u = 640/(0.25/5) = 12800$ N/mm

2. Specimens with Teyin fabric:
   - $t_e = 0.40$ mm
   - $F_u = 8400$N/mm (see section 4.7.2 and figure 76)
   - $F_u = 72000$N/mm
   - $F_u = 640/(0.40/5) = 8000$N/mm (see section 4.7.2 and figure 76)

The influence of $F_u$ is not considered by the energy release rate concept. This concept predicts a factor of $1.5^{46} = 7.1$ faster delamination growth for the specimens containing the Teyin fibres. If the shear stress concept is applied, the factor is 0.7, so a lower delamination rate for the Teyin specimens is predicted. The experimental results indicate a factor of about 2.5, which implies that the energy release rate criterion overestimates the experimental trend. The shear stress criterion predicts a trend opposite to the experimental difference. Apparently, both concepts fail to account correctly for the combined influence of a change in $F_u$ and $F_u$. A discussion on the discrepancies is presented later.

The results presented in figure 43 (for $R=0.1$) show the effect of a proportional increase of the different layers on the delamination rate. It is evident that the adhesive shear stress
is not changed by such a proportional change of the layers. Equation (5.3.10) shows indeed that doubling of $t_{ul}$, $F_{A_l}$ and $F_{ul}$ and a reduction of $F_{ad}$ with a factor 0.5 lead to the same $\tau_{ad}$. Consequently, the shear stress criterion predicts no change in delamination rates for the same stress $S_{ul}$. On the other hand, the energy release rate per interface is increased by a factor 2 if the thickness of the individual layers is doubled. This would imply an increase by a factor of $2^{43} \approx 30$ for the delamination rates. Again the experimental results show a tendency which is between the two theoretical results. An increase with a factor of 10 is found for the double thickness of the individual layers.

A detailed discussion of the observed trends is presented in section 6.4. So far it is sufficient to note that all cases of deviations from theoretical results are associated with modifications in the fibre-adhesive layer. That might imply that an empirical law for the delamination rates should be formulated for each type of fibre-adhesive layer.

6.2 An equation for the delamination rate, on the basis of the adhesive shear stress

As discussed above the theoretical adhesive shear stress at the delamination front can be considered as a loading parameter to correlate the delamination rates. An empirical law for the delamination rate as a function of the cyclic crack bridging stress and the mean stress was proposed in equation (4.5.7). Taking the adhesive shear stress as a basis, the equation is of the following type:

$$\frac{db}{dN} = q (\Delta \tau_{ad} + 0.37 \tau_{ad,m})^m$$  \hspace{1cm} (6.2.1)

A preliminary evaluation yielded $m = 9.65$ (see equation (4.5.7)). The values of $(\Delta \tau_{ad} + 0.37 \tau_{ad,m})$ are calculated and presented in table 2. In figure 128 the delamination rates are plotted as a function of $(\Delta \tau_{ad} + 0.37 \tau_{ad,m})$ for all specimens containing a single Twaron fabric combined with two BSL-312-UL films. The result can be represented by the following equation:

$$\frac{db}{dN} = 3.26 \times 10^{-26} \cdot (\Delta \tau_{ad} + 0.37 \tau_{ad,m})^{11}$$  \hspace{1cm} (6.2.2)

Equation (6.2.2) is valid for all stress ratios, residual stress levels and aluminium sheet thicknesses. The Paris exponent of 11.0 in equation (6.2.2) is different from the value of 9.65 which was found before. The small difference is due to data scatter. The amount of scatter in figure 128 is not significantly larger than in the original graphs (figures 42 till 45). Therefore it may be concluded that equation (6.2.2) correctly accounts for the different influences.

Figure 128 also shows the results if the same concept is applied to specimens containing the Twaron AF-163-2 prepreg. The observed scatter is acceptable, and the delamination rate for this fibre adhesive layer can be described by:
\[ \frac{db}{dN} = 3.1 \times 10^{-25} \cdot (\Delta \tau + 0.37 \tau_{ad,m})^{10} \]  
(6.2.3)

Application of the equations (6.2.2) and (6.2.3) requires, that the value of \( F_{ad} \) is determined in the same way as it was done for the present results (e.g. \( t_{ad} = 0.4t_{ad}/j \)) and the same adhesive shear modulus must be applied. The latter restriction would be a disadvantage if the influence of temperature and humidity has to be considered. A possible reduction of \( G_{ad} \) would imply that the adhesive shear stress and the delamination rates would be reduced. So far there has been no experimental evidence that such predicted reductions are realistic. Regarding the latter complications, the application of the energy release rate concept looks more appropriate, because it is independent of \( F_{ad} \). Possible influences of the environment could then be accounted for with a modification of the variables \( q \) and \( m \) in order to fit the equations to experimental results.

6.3 An equation for the delamination rate, on the basis of the energy release rate

Equation (6.1.2) shows that the energy release rate \( G_{e} \) is related to the adhesive shear stress \( \tau_{ad} \) and it is possible to convert the equations (6.6.2) and (6.6.3) to functions of the energy release rate. A complication is that the energy release rate is positive, independently of the sign of the adhesive shear stress. Consequently it is not directly possible to distinguish between the energy release rates belonging to positive or negative adhesive shear stresses (at minimum load). On the other hand, the stress intensity factor \( K_{f} \) in AARALL does not become negative because the crack flanks are already closed if \( K_{f} = 0 \). \( S_{e} \) and \( G_{e} \) are related to \( K_{f} \). Consequently, negative \( R_{e} \) values are not relevant for AARALL crack growth specimens \((Q \geq 0.5)\) and the equations for the delamination growth rates in an AARALL crack growth specimen may be given as a function of \( G_{d_{\text{max}}} \) and \( G_{d_{\text{min}}} \) only. It can be derived from equation (6.1.1) that \( \Delta \tau + 0.37 \tau_{ad,m} \) is proportional to \( \sqrt{G_{d_{\text{max}}}} - 0.69 \sqrt{G_{d_{\text{min}}}} \). The delamination rates are plotted against this function in figure 129. The trends are described with the following equations:

\[ \frac{db}{dN} = 5.5 \times 10^{-1} \cdot \left( \sqrt{G_{d_{\text{max}}}} - 0.69 \sqrt{G_{d_{\text{min}}}} \right)^{11} \]  
(6.3.1)

for the Twaron fabric BSL-312-UL combination layer (one fabric two adhesive films).

And:

\[ \frac{db}{dN} = 4.7 \times 10^{-2} \cdot \left( \sqrt{G_{d_{\text{max}}}} - 0.69 \sqrt{G_{d_{\text{min}}}} \right)^{10} \]  
(6.3.2)

for the Twaron AF-163-2 prepreg.

The sign problems (there are no negative energy release rates) for the energy release rate were already noted before. They occur for \( Q < 0.5 \). The delamination rates for \( Q < 0.5 \) are therefore not included in figure 129. However, the results for \( Q < 0.5 \) are calculated by adopting the sign of the corresponding shear stress for the square root terms. Thus
adopting the trend belonging to the shear stress criterion. The results which were obtained this way are presented in table 2 between quotation marks.

6.4 Evaluation of the comparisons

The two test series, which showed limitations of the two concepts, were associated with changes in the fibre-adhesive layer ($F_{ad}$ and $F_e$). However, it was also shown that as long as the type of fibre-adhesive layer is kept unchanged, both concepts account correctly for the influences caused by geometrical effects and the residual stress. It was discussed at the beginning of this chapter, that limits of both concepts are most probably associated with effects related to changes of the thickness of the adhesive layer. A thick adhesive layer allows more energy dissipation as a consequence of adhesive plasticity. Consequently, an increase of the delamination growth resistance may be expected. This explains the results of the test series where the layer thicknesses were doubled, and where the increase of the delamination rates was smaller than predicted on the basis of an increased $G_c$ value. The overestimation of the delamination rates in the Teyin specimens according to the energy release rate can also be explained by the inherent neglect of the increased delamination resistance as a consequence of the increased thickness of the fibre-adhesive layer in these specimens.

The deviations of the shear stress criterion indicate the opposite trend. A thick adhesive layer shows higher delamination rates than predicted, It should be recognized that the adhesive shear stress concept does not consider the singularity of the "delamination crack", and deviations should be expected if the local (fibre-adhesive layer) geometry is changed. The differences between predictions and experimental results indicate that the shear stress criterion overestimates the effect of the increased delamination resistance for an increased thickness of the adhesive layer.

The effect of the fibre stiffness $F_e$ could not be studied separately, because in the specimens with the lower $F_e$ of the Teyin fibres a lower $F_{ad}$ also occurred. Further, the type of fibres were different which may result in a different adhesion strength. However, there are some arguments indicating that the differences between observed and predicted trends might be associated to the effect of $F_{ad}$ only. Both fibres are types of aramid and similar adhesion properties might be assumed, and both types show the typical moderate interfacial bonding strength to the adhesive. In spite of the moderate interfacial bonding strength, scanning electron microscope fractographs show a considerable amount of cohesive delamination (see figure 130), indicating that interfacial debonding is not a main delamination mechanism in the present tests anyhow. Probably, the elastic properties of the fibres and the adhesive represented by $F_e$ and $F_{ad}$ are more important. The differences between experimental and theoretical trends might then be explained by the overestimation of the effect of $F_{ad}$ with the shear stress criterion and the neglect of this effect with the energy release rate criterion.
The experimental trend was indeed inbetween of the two predicted trends. However, more investigations are recommended to establish the effect of \( F_w \) and \( F_a \) separately.

Fortunately, it will be the usual case for ARALL that the type of fibre adhesive combination layer is not changed, because a choice out of a limited number of standardized types of these layers will be made (possibly only one type of fibre adhesive layer will remain relevant, e.g. the prepreg). If the delamination properties of such a layer are once determined, both concepts may be used to correlate the delamination rates in laminates containing that layer. An advantage of the energy release rate concept is that it does not require information about the stiffness of the adhesive layer \( F_w \). Therefore this concept will be adopted for the computer programme described in chapter 7.
7. An iterative programme to calculate crack growth rates in ARALL

7.1 Description of the programme

In chapter 5 a set of algebraic equations was presented, for the calculation of the stress intensity factor in the cracked aluminium sheets of ARALL and for calculating the crack bridging stress. The stress intensity factor is used to calculate the fatigue crack growth rates in ARALL based on empirical correlations between crack growth rates and cyclic stress intensity factors, obtained on the non reinforced material, in section 4.3. The energy release rate for delamination can be derived from the crack bridging stress. An empirical relation was derived in section 6.3, correlating the delamination rates to the energy release rate. The calculations of crack growth and delamination growth can be performed, if the geometry, i.e. the actual crack length and the delamination distance is known. However, the geometry is dependent on the crack and delamination growth during previous fatigue cycles, and the crack and delamination growth as they occur during the fatigue history have to be determined first. That means that there is a need for a complete calculation of the crack and delamination growth rates from the beginning, during the entire fatigue life of ARALL. A computer programme for such calculations is described in the following.

The programme is based on an iterative routine, where small increments of the crack and delamination growth are subsequently calculated (see figure 131). The geometry in each iteration step is established on the basis of the crack and delamination growth increments, accumulated during previous iteration steps. The crack growth increments in each iteration step are chosen to be very small (0.05mm, or even much smaller in the first iteration steps). In this way a simulation of the natural continuous crack and delamination growth behaviour in ARALL is achieved. In view of the complications associated with overload effects on crack growth and delamination growth, the programme was developed for constant amplitude fatigue loads only.

The programme requires the following input data:

1. The fatigue load data.
2. The characteristic laminate construction parameters of ARALL and the specimen dimensions.
3. Two sets of empirical equations describing the crack growth in the metal component as a function of the cyclic stress intensity factor, and delamination growth as a function of the crack bridging stress or a related parameter.

The flow diagram of the programme is given in figure 132. Some of the programme steps will be explained later in more detail. The equations used will be summarized. The value $V$ in the diagram controls the cycle number $\Delta N$ for each iteration step in such a way that $\Delta a = 1/V$. The $da/dN$ value of the last iteration step is used in the following step for the calculation of the cycle number $\Delta N$. To initialize the programme the first $\Delta N$ is chosen to
be equal to 1. The results of the calculations are printed into a register after every tenth iteration. The variable $I$ indicates the number of iteration steps, performed after the last printing.

The most important equations used in a subsequent order for calculating the stress intensity factors and the energy release rate are (the calculations are performed for the maximum and the minimum loads):

For the energy release rate equation (5.6.44). The delamination growth follows then from equation (6.3.1) or (6.3.2), depending on the type of fibre adhesive layer.

The stress intensity factor is derived with the equations (5.6.42), (5.6.54) and (5.6.56), the apparent stress ratio with (5.6.58), the effective cyclic stress intensity factor and the crack growth rate result for the aluminium sheet material from equation (4.3.2) and (4.3.3). The constants $c$ and $n$ are derived on the basis of the results given in figures 18 to 20. One comment has to be made, millimeters are used as a dimension for crack length and sheet thickness. Consequently, the resulting dimension for the stress intensity factors yielding from the equations in chapter 5.6 are N/mm$^2$. The constant $c$ refers to the dimension MPa $\sqrt{m}$ which is usually adopted. The relation between the dimensions is:

$$31.6 \text{ N/mm}^2 = 1 \text{ MPa } \sqrt{\text{m}} .$$

Control routines for the iteration steps are shown in the lower part of the flow diagram (figure 132). The process is controlled in such a way that the cycle numbers $\Delta N$ cause very small crack growth increments in the beginning. During each iteration step the crack growth increments are then increased by about 10%. The increase is continued until the crack growth increments $\Delta a$ are 0.05mm. The changes of $\Delta a$ are controlled by $V$ which was initially equal to 5000, and is reduced with 10% in each iteration step until it reaches the value 20. The reason for increasing the crack growth increments so slowly is that during the first iteration steps a too high delamination propagation can be calculated, if the cycle number is chosen too high. For the same reason a small initial delamination distance is chosen.

The reason of the convergence complications in the beginning of the iteration procedure, is that the influence of the adhesive shear deformation on the crack bridging fibre forces is neglected (its influence on the stress intensity is considered implicitly with the correction factor $C_0$). The neglect of the adhesive shear deformation leads to unrealistically high crack bridging forces if the crack length $a$ is only slightly larger than $s$ and if the delamination distance is zero. (The crack bridging stresses for this case would be so high that $K$ becomes 0 and this would yield infinite values of $S_w$ for $a \rightarrow s$). A consequence of high calculated $S_w$ values is an extremely high delamination growth increment $\Delta b$, during the first iteration step. This results in an overestimation of the subsequent crack growth rates, because all the delamination occurs in the first step. This problem can successfully be overcome by the method mentioned above. As an additional measure, a small initial fatigue crack was assumed, $a = (1 + 10^{-5}) s$. The initial delamination distance and the small number of cycles
during the first iteration steps reduce the amount of delamination per step, and the programme can gradually approach an adequate equilibrium between delamination and crack growth. A realistic $S_n$ value is achieved within a small amount of crack growth. The delamination distance obtained is slightly larger than that distance which would be calculated, if the direct influence of adhesive shear deformation on $S_n$ were considered. The initial errors introduced by the above procedure are small as compared to the "steady state" results which are obtained at a later stage of the programme. The accuracy of the final results is not affected by the conditions at the beginning of the programme.

Except for those initial complications which were easily overcome, the programme operates in a convergent way. The convergence was further investigated by the application of large iteration steps. The "steady state" results were hardly affected by the application of crack growth increments which were as large as 1 mm. However, there is no reason why such large increments should be applied, because the computer time remains very low anyhow, because of the algebraic character of the calculations.

The influence of an assumed large initial delamination $b_i$ was also investigated. In this case the programme predicted lower delamination rates and higher crack growth rates until the correct steady state of delamination and crack length ratio was reached. Then "normal" delamination and crack growth rates are calculated. This behaviour does not only show the excellent convergence of the programme, but also reflects the natural "convergence" of the behaviour of ARALL if for some reason the steady state balance of the crack growth and delamination growth is disturbed.

7.2 Application of the programme

7.2.1 Application to present results

Figures 21 to 25 show experimental results of constant amplitude fatigue crack growth tests on ARALL in comparison to the calculated behaviour. The mechanical parameters used for the calculations are summarized below.

\[ t_w = 4 \times 0.45 \text{ mm} = 1.80 \text{ mm} \]
\[ t_s = 3 \times 0.3 \text{ mm} = 0.9 \text{ mm (actual)} \]
\[ w = 70 \text{ mm} \]
\[ j = 6 \]
\[ \eta \cdot v_n = 0.4 \quad (\Rightarrow t_0 = 0.06 \text{ mm}) \]
\[ G_{\text{ef}} = 680 \text{ N/mm}^2 \]
\[ E_A = 72000 \text{ N/mm}^2 \]
\[ E_\nu = 40500 \text{ N/mm}^2 \]
\[ c = 3.91 \times 10^{-11} \quad (\text{refers to } t_{\nu,\nu}) \]
\[ n = 2.93 \]
\[ q = 0.55 \]
\[ m = 11 \]

The material constants for delamination $c$ and $n$ refer to $\Delta K_{\nu}$ (in N/mm$^{1/2}$) and the constants $q$ and $m$ for delamination refer to the energy release rate function $\sqrt{G_{\text{ef,max}}} = 0.69\sqrt{G_{\text{ef,min}}}$.
The prediction capability of the programme can be judged from figures 21 to 25. The calculation is within an acceptable accuracy. Only in figure 25 higher deviations can be observed. These data stem from specimens in which a high residual stress was present as compared to the stress due to external loading. The deviations result from absolute errors in the equations for the stress intensity factors, related to the residual stress and the external loading. Due to the opposite sign of the stress intensities caused by the external loading and by the residual stresses, the resulting $K$ value becomes small and therefore the relative errors become large. Consequently, inaccurate results are possible. On the other hand, it can be expected that the crack length where crack arrest occurs is calculated satisfactorily. Figure 25 shows that this is indeed the case. Similar arguments may explain the only moderate prediction accuracy in figure 22 for the specimen with a saw cut of 2mm. The residual stress level was nearly as high (but opposite) as the stress due to external loading.

The present programme was developed for application to centre cracked specimens. Since the mechanical situation for double edge notched specimens is not much different, the programme was also applied to predict the crack growth rates for this type of specimens. These specimens were primarily used for investigating the behaviour of small cracks, but after some crack growth the cracks become through cracks. The depth of the edge notches was taken as half the length of the starter notch $s$ ($s=11.13$mm). The nett section stress and the stress ratio were:

<table>
<thead>
<tr>
<th>$S_{\text{ nett}}$ (MPa)</th>
<th>140</th>
<th>110</th>
</tr>
</thead>
<tbody>
<tr>
<td>$R$</td>
<td>0.1</td>
<td>0</td>
</tr>
</tbody>
</table>

The material and specimen parameters were the following:

- $t_w = 1.0$ mm
- $t_w = 0.25$ mm
- $w = 73$ mm
- $j = 2$
- $\eta \cdot v_w = 0.4$ ($\Rightarrow t_w = 0.05$ mm)
- $G_m = 680$ N/mm$^2$
- $E_m = 72000$ N/mm$^2$
- $E_w = 48600$ N/mm$^2$
- $c = 9.09 \times 10^{-12}$
- $n = 3.22$
- $q = 0.55$
- $m = 11$

Specimens with three residual stress levels were investigated: $S_{\text{ res}} = +20$, -30, or -50MPa.

The predicted and the experimental results are compared in figures 133 and 134. The agreement in the figures is acceptable and the trends are well predicted. The initiation behaviour at blunt notches cannot be considered by the present programme, where the starter notches are assumed to be sharp "crack like" saw cuts. Moreover, in the first part of the crack growth period the crack is growing as a part through crack Therefore the first parts of the prediction curves are omitted in the figures 133 and 134.
7.2.2 Application to results from the literature

Some early results on the crack growth behaviour of material similar to ARALL, were published by Koch [67,68]. Aluminium laminates reinforced by an unidirectional carbon prepreg, or a square fabric aramid prepreg (the same amount of fibres in the length and width direction) were tested. These laminates did not represent a properly optimized material. However, the fatigue mechanism is similar. It is expected that the present programme is also able to predict the behaviour of these laminates reasonably well. In Koch’s material, conventional prepreg was used. The delamination properties of the prepreg layers may be different from those of the fibre-adhesive layers used in the present investigation. However, since the delamination parameters of the specimens used by Koch are not known, it was assumed that they are similar to those derived in the present investigation for ARALL having layers with a Twaron HM aramid fabric and BSL-312-UL adhesive. A similar assumption was made regarding the adhesive shear deformation. The parameters of the Paris equation for the crack growth behaviour were derived from Koch’s test results on non-reinforced materials. The $E_v$ values for the laminates were estimated from the amount of fibres arranged into the loading direction and from the fibre Young’s modulus. The stress levels defined by Koch are stress levels for the complete laminate ($S_{\text{c}}$).

The loading was: $S_{\text{max}} = 91\text{Mpa}$ for one series and $S_{\text{max}} = 76\text{MPa}$ for the other one. The stress ratio was $R = 0.55$ for both load levels. The other parameters are presented in the table on the next page.

The experimental results from Koch, and the calculated results are compared in the figures 135 and 136. The data from [67] are adopted here. Calculations were also performed for the non-reinforced material. In this case an extremely low value $E_v = 1$ was taken as the Young’s modulus for the fibre adhesive layer, which means that only negligible crack bridging stresses are present and that the behaviour of a non-reinforced material will be approached (setting $E_v = 0$ leads to an error statement due to division by zero). This result is a "trivial" case for the present programme.

Figure 135 and 136 show an acceptable agreement between the trends of the predictions and the test results. Deviations may be explained by the fact that the values of certain basic material properties had to be estimated. The experimental results of laminates containing the aramid fabric show a sharp increase of the crack growth rates at larger crack lengths. This increase can be due to fibre fracture, because Koch’s laminates did not contain enough fibres to resist the high crack bridging forces which do occur at the higher crack lengths. The present programme does not include a special fibre fracture criterion, and consequently such a sharp increase of the crack growth rates was not predicted.
<table>
<thead>
<tr>
<th></th>
<th>aramid reinforced</th>
<th>carbon reinforced</th>
<th>non-reinforced</th>
</tr>
</thead>
<tbody>
<tr>
<td>$t_{\text{f}}$</td>
<td>3mm</td>
<td>3mm</td>
<td>3mm</td>
</tr>
<tr>
<td>$t_{\text{w}}$</td>
<td>0.5mm</td>
<td>0.5mm</td>
<td>0.5mm</td>
</tr>
<tr>
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<td>65,000</td>
<td>65,000</td>
</tr>
<tr>
<td>$E_{\text{r}}$</td>
<td>27,000</td>
<td>130,000</td>
<td>1</td>
</tr>
<tr>
<td>$\eta$</td>
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<td>0.8</td>
<td>0.8</td>
</tr>
<tr>
<td>$G_{\text{ad}}$</td>
<td>680</td>
<td>680</td>
<td>680</td>
</tr>
<tr>
<td>$j$</td>
<td>4</td>
<td>4</td>
<td>4</td>
</tr>
<tr>
<td>$c$</td>
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<td>$6.1 \times 10^{-13}$</td>
<td>$6.1 \times 10^{-13}$</td>
</tr>
<tr>
<td>$n$</td>
<td>3.67</td>
<td>3.67</td>
<td>3.67</td>
</tr>
<tr>
<td>$q$</td>
<td>0.55</td>
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<td>0.55</td>
</tr>
<tr>
<td>$m$</td>
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<td>11</td>
<td>11</td>
</tr>
<tr>
<td>$s$</td>
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<td>6.5</td>
<td>6.5</td>
</tr>
<tr>
<td>$w$</td>
<td>60</td>
<td>60</td>
<td>60</td>
</tr>
<tr>
<td>$S_{\text{rad}}$</td>
<td>10</td>
<td>40</td>
<td>0</td>
</tr>
</tbody>
</table>

* Corrected for the cladding layers

7.3 Numerical variable analysis

The optimization of ARALL appears to come to a final definition of the material [69]. It was recognized quite early that thin individual layers should be applied and that the amount of fibres must be high enough to prevent fibre fracture [2,3,11]. The significance of residual stresses was also recognized. The early optimization of ARALL was mainly based on a qualitative theoretical understanding and on experience obtained in fatigue tests.

The present programme allows more detailed considerations, and the influence of different parameters on the fatigue crack growth can be determined quantitatively. The influence of some parameters is demonstrated in this section. A comprehensive study of the different parameters will not be presented. However, it will be shown that the fatigue response of ARALL on a change of design parameters is strongly influenced by the type of loading.

First, the influence of the following modifications is considered:
1. The decrease of the adhesive shear modulus by a factor of 2 (new $G_a$ value = half old $G_a$).
2. The increase of the delamination sensitivity by a factor of 2 (new $q$ = twice old $q$).
3. The increase in the crack growth sensitivity of the aluminium component by a factor of 2 (new $c$ = twice old $c$).

The data of the fatigue test presented in figure 23 are taken as a basis for the analysis. A saw cut of 10 mm ($s = 5\text{mm}$) is adopted. In figure 137 the calculated influences of the individual modifications are compared to the original result. The same comparison is presented in figure 138 for a lower load level (based on the result presented in figure 22). It can be seen in the figures 137 and 138 that the influence of a change of one parameter is relatively small, i.e. smaller than a factor 2. The fatigue response of ARALL to such a change is reduced by synergistic effects of the other parameters, which remain the same. However, an exception is the response of ARALL on a change in the crack growth properties of the aluminium sheets.

Figure 137 and 138 further show that the influence of the adhesive shear deformation is more significant at lower load levels. The explanation is that the amount of delamination is small at low load levels, and the stress intensity factor is caused by adhesive shear deformation to a larger extent.

The influence of the load level on the delamination ($b/a$) ratio is explained by the high Paris exponent for delamination as compared to the Paris exponent for crack growth. The influence of the load level on the crack growth rate is presented in figure 139a. Figure 139b shows the corresponding theoretical values of $b$ as a function of $a$. In figure 139b two pictures are given, which could be taken by peeling off one of the aluminium sheets.

The crack growth properties of ARALL can be improved by an increase of fibre to aluminium ratio. It was already discussed in chapter 3 that such an increase reduces the compression strength of ARALL, and that the anisotropy is increased. There are two other ways to further improve the fatigue properties of ARALL, without changing the ratios of fibres, aluminium and adhesive:

1. Increasing the level of the compressive residual stresses in the aluminium sheets (e.g. more prestraining).
2. Decreasing the thickness of all individual layers (increase of the interface number $j$). The bulk properties of ARALL remain unchanged, if this decrease is proportional for all layers.

The second method implies a reduction of the thickness of the fibre-adhesive layer too. It was shown in section 6 that this will cause a small overestimation of the improvement of the delamination behaviour. However, the effect on the crack growth is small and will be ignored.
The effect of the two measures are again demonstrated on the ARALL type of the figures 21 till 25. Figure 140 shows how the modifications change the fatigue crack growth in ARALL specimens which are loaded with $S_{min} = 130$ MPa and $R = 0.1$. Figure 141 shows the fatigue response for ARALL loaded with $S_{min} = 260$ MPa and $R = 0.67$. In both figures the influence of an increase of the residual compressive stresses from -95 MPa to -115 MPa is compared to the influence of an increase of the number of interfaces $j$ by a factor two. It can be seen that the effect of the number of interfaces (is effect of layer thickness) is about the same for both load types. The effect of the residual stress level is only significant for the loading type with a low $R$ ratio. The residual stress level is mainly relevant, if at least a part of the cyclic stress range is shifted into the compressive region.

7.4 Discussion of the computer programme

It was shown in the previous sections that the programme allows the calculation of crack growth rates in ARALL with an acceptable accuracy. The programme is suitable for various types of ARALL, under constant amplitude fatigue loads. The calculations are performed on the basis of the properties of the constituent materials of ARALL. The material characteristics are related to the stiffness properties of fibres, adhesive and aluminium, the crack growth behaviour of the aluminium sheets and the delamination properties of the fibre-adhesive layer. The latter two are associated with the fatigue crack growth resistance of the sheet material and the delamination resistance in the fibre-adhesive layer. These properties have to be determined for the respective material, and can then be used for any combination in ARALL. It should be noted that apart from the experimental determination of the crack growth and delamination growth properties of the constituent materials, no further fatigue tests are necessary for crack growth predictions on ARALL. The programme is not a so called "relative" concept, it calculates the crack growth directly from basic material data. The programme is developed for centre cracked sheets, but it was shown that it can also be used for similar geometries, like a double edge notched specimen.

The applicability of the programme is still restricted to constant-amplitude fatigue. The reason for this restriction is that an extension to variable-amplitude loading would require a description in general terms of the crack and delamination growth response to variable amplitude fatigue. Approximative descriptions may be possible for the crack growth, because a number of models on this topic were already proposed in literature. However, so far there is not enough information available on the effect of overloads on delamination growth.

The acceptable accuracy of the programme indicates that the approximative equations (chapter 5) on which it is based, are sufficiently accurate. The equations yield explicit results for stress intensity factors and the crack bridging stress, that means that the programme could easily be extended to a calculation of the residual strength in the course of the crack growth life. This is not done here, because the present tests did not yield enough
residual strength data for a check on such calculations. The aim of the present work was focussed on fatigue, rather than on residual strength behaviour.

As a final comment on the fatigue behaviour of ARALL, it is noted again, that the effect of measures to further improve the fatigue behaviour of ARALL may be highly dependent on the type of fatigue load. An example is the effect of prestraining which is very effective at low $R$ ratios (or more general, if low load levels are present, like in the TWIST programme), but which is hardly effective at high $R$ ratios. The present programme is able to detect the effect of such measures for a wide variability of load conditions. Consequently, the programme will also be helpful for further developments of ARALL, and for the planning of experimental investigations.
8. Discussion

8.1 Some general remarks

It was shown in the previous chapters that ARALL is a material with an extremely good resistance to fatigue crack growth. This resistance is caused by bridging of the fatigue crack by unbroken aramid fibres. The efficiency of this crack bridging depends on the shear and delamination behaviour of the adhesive. The tensile strength of ARALL is also enhanced by the presence of the fibres.

The mechanical behaviour of ARALL appeared to be accessible to quantitative modelling and calculations. Analytical expressions were derived, which describe the fatigue behaviour of ARALL as a whole as well as the behaviour of the separate mechanisms involved. The expressions were used for crack growth calculations in an iterative computer programme. In general, the correlations between experiments and calculations were good, showing that the fatigue mechanisms of ARALL are quite well understood. In some cases only a moderate prediction quality was found, but the trends were predicted well in all cases. Many aspects of the present investigation were discussed before. The experimental test series were discussed at the end of each section in chapter 4, where the tests were described. Also the other chapters were concluded with a discussion. In this chapter some additional aspects will be discussed.

8.2 Delamination around a growing fatigue crack

The constant-amplitude fatigue tests presented in figures 21 to 25 clearly show the resistance of ARALL to fatigue crack growth. Typical features of ARALL can be recognized, such as: decreasing crack growth rates with increasing crack length, and crack lengths which may cover the entire specimen width, without complete failure of the specimen. Some more insight in the fatigue behaviour of ARALL can be obtained by considering the calculated delamination, which should also give realistic results. Figure 139b shows the calculated delamination distance (2b) as a function of the calculated crack length for a high and a low fatigue load respectively. In the figure a picture of the delamination is given, which could be taken after peeling of one of the aluminium sheets. Several observations can now be made.

- The calculated delamination growth rate is very high in the beginning of the crack growth in the aluminium sheets. This should not be surprising because initially there is no fatigue crack and the starter notch induces a high stress concentration (figure 6). Although the fatigue crack initiation will start immediately, the crack bridging stress will still be very high and the delamination will be enhanced.
- The delamination distance (2b) for the higher load level is about 1.5 times the calculated value, while the delamination at the right hand side and the left hand side are
different. Apparently scatter does occur. The difference between predicted and measured delamination is not too disturbing. It should be recalled that the exponent of the delamination growth equation (10 in eq. 6.3.1) is very high, which implies that small deviations of the crack bridging stress will cause substantially different delaminations. It should also be recalled that the delamination rate does not have a large effect on the fatigue crack growth (see section 7.3).

- The shape of the delamination deviates from a real elliptical contour, especially for the lower load level in figure 139b. In this case the load on the specimen caused stresses which exceeded only slightly the absolute value of the residual stresses.

In spite of the above differences between the predicted and observed delamination pattern, it is believed that after the initially rapid delamination the situation converges to constant fibre stresses along the fatigue crack. Evidently, it does not always lead to a nice elliptical shape. However, from the analysis in chapter 5 it can be understood that other shapes will probably have a small effect on $K_{\infty}$.

8.3 Flight simulation tests

The crack growth experiments under the TWIST flight simulation showed the same trends as the constant-amplitude experiments. The typical influence of the residual stress and the size of the notch were observed (figure 33). The tests showed that the crack growth rate in pre-strained ARALL drops to very low values at stress levels which would be present in ARALL parts where 30 % weight saving, as compared to monolithic alloys is realized. The crack growth rates in ARALL under constant amplitude fatigue can be calculated from basic material properties. This calculation cannot (yet) be performed for TWIST flight simulation testing. The reason is that the complicated sequence effects on the crack growth and the delamination growth have to be predicted. Although, models exist which are able to account reasonably well for sequence effects on crack growth in aluminium sheet material, there is no model which can describe the influence of overloads and underloads on the delamination rate. As shown in section 4.5 the sequence effects on delamination are quite different from those on fatigue crack growth in metals. Actually, even the physical backgrounds of the sequence effects on delamination growth are not yet known.

In spite of these difficulties, the analytical methods derived in the previous chapters, can be helpful for a qualitative explanation of the large truncation effect on crack growth in pre-strained ARALL (figure 37). Some comments on this topic were already made in section 4.4. (figure 39). A more specific explanation is given below. The slow crack growth for a high truncation level should be associated with high $K$ values occurring during the most severe flights. In order to see how these $K$ values change during crack growth, calculations were made with the computer programme described before. The programme assumes constant amplitude loading. Calculations were made for $S_{\text{max}}$ values occurring during the
flight-simulation loading for $S_{um} = 0 \ (R = 0)$. Since the delamination behaviour under flight-simulation behaviour is unknown, the coefficient in the delamination equation (6.3.1) was calibrated to match the observed delamination in the tests, see figure 36, where $b = 9$mm at $a = 18$mm. The calculated results are presented in figure 142. The graph shows that the $K$ value belonging to the maximum truncation level ($2.6 \ S_{um}$) increases with crack length. For lower truncation levels the $K$ value is smaller and the increase during crack growth is also smaller or even negative. In other words, the overload ratio increases with increasing crack length. This effect will be larger for a higher residual stress. This is in agreement with the test results in figure 37.

The increasing $K$ value is also confirmed by figure 143, where the development of the plastic zone is made visible by a simple illumination technique. The picture shows an increasing plastic zone size with increasing crack length for a specimen loaded by a non truncated load spectrum. The specimen showed a strongly decreasing crack growth rate.

The influence of the residual stress on the truncation effect can also be explained in other terms. A "constant" crack opening stress intensity factor depending only on the maximum stress intensity factor at the most severe flight is assumed (ignoring compression effects). A further assumption is that the crack opening stress intensity factor may be derived with an equation like Elber's equation. Now the stress intensity factor where crack opening occurs can easily be calculated (see also a proposal by Schijve [25]). It can be observed in figure 142 that the load level at which crack opening occurs, increases with increasing crack length. Consequently, the damaging part of the load sequence is reduced with increasing crack length. This effect will be more pronounced for higher residual (compressive) stress levels.

8.4 Delamination growth tests

A preliminary discussion on the results of the delamination tests was presented in section 4.5.5. In chapter 6 the results were analysed by adopting mechanical parameters in order to correlate the delamination growth rate to the cyclic adhesive shear stress and the strain energy release rate. The empirical correlations obtained apply to specific combinations of type of fibre, fabric or unidirectional fibres and type of adhesive. Extrapolation to other combinations is not allowed. In this respect a remarkable difference was found between a fabric-adhesive combination and a unidirectional prepreg. For the fabric the delamination rate was about 50 to 100 times slower (figure 51). The difference was explained by considering the roughness of the fracture surface and the associated erratic shape of the delamination front. The small fibre bundles perpendicular to the direction of delamination growth act as "delamination stoppers". Such a behaviour might be utilized for further improvement of the crack growth behaviour of ARALL. However, there is a small drawback because a fabric shows about 10% less tensile strength than the unidirectional prepreg in the present investigation. Further, the weight of a fibre adhesive layer containing a fabric is nearly 15% higher than the weight of a prepreg. On the other hand, it might well be possible to
develop a prepreg containing a fabric with a different weave pattern, in such a way that the favourable delamination properties are obtained with a smaller weight and strength penalty. Another approach was explored at the Delft University of Technology [70] by forcing thin fibrils out of the surface of the aramid fibres. Such fibrils will bridge the delamination crack and increase the total adhesion surface, thus improving the bonding strength.

As pointed out in section 4.5.5.2 the delamination under simple block programme tests with periodic overloads and underloads have revealed unfavourable interaction effects, i.e. accelerations did occur (figure 52-54). A satisfactory explanation cannot be given at the moment. Even the question whether the acceleration is a consequence of either a larger $\frac{db}{dN}$ during the low amplitude cycles or on the contrary during the overload cycles can not be answered as yet. It was hoped that fractographic work as carried out by Roebroeks [66] might solve the question. Some exploratory work was done by taking SEM pictures of the delaminated area. Although, "striation" type markings were observed in the fibre prints (see figure 144). A clear answer could not be obtained. Further study should be recommended. For instance, the present interaction effects were observed on delamination specimens. It would be of interest to see whether similar effects on delamination do occur in crack growth specimens with a central crack, and to which extent that might affect the fatigue crack growth rate. The present modelling of fatigue crack growth in ARALL leads to the expectation that the effect on the crack growth rate in the aluminium sheets may be rather moderate. Some more delamination sensitivity does not have a significant effect on the stress intensity factor as discussed before.

A last comment on delamination deals with the type of adhesive. The sensitivity to delamination can be very different for different adhesives. In figure 51 the upper and the lower data set represent the difference in fatigue response of the AF-163-2 and the BSL-312-UL adhesive (the same aramid fabric was present in both cases). The delamination rate for the AF-163-2 adhesive is nearly a factor of 500 lower than for the BSL-312-UL adhesive. Moreover, the amount of the AF-163-2 adhesive was approximately 80 % of the amount of the BSL-312-UL adhesive. Obviously the fatigue quality of the AF-163-2 adhesive is considerably better than of the BSL-312-UL adhesive.

8.5 Initiation and growth of small cracks at notches in ARALL

For metallic components a relatively large part of the life can be spent in the crack initiation phase, if that phase is defined as the life until visible cracks or cracks that can be detected under service conditions with normal inspection procedures. The situation is essentially different for ARALL because large cracks grow extremely slowly. Moreover, a small crack in ARALL in the initiation phase does not easily become a macrocrack for some reasons. First, microcracks are initiated as part through cracks. Because of the very low thickness of the aluminium sheets the crack length can still be small when the crack has grown through
the thickness of one sheet, and then meets the fibre-adhesive layers as a crack growth barrier. Generally, cracks initiated in one sheet do not easily propagate into an adjacent sheet.
Second, the crack bridging forces will become active if the crack size is in the order of the magnitude of the aluminium thickness, which is still small indeed.
The third argument applies to prestrained ARALL only. If there are compressive residual stresses in the aluminium sheets even a microcrack will grow more slowly.
The initiation of the very first microcracks will not benefit from the above arguments. On the contrary, initiation will be promoted by the anodic coatings of the aluminium sheets, which have to be applied as a pretreatment of adhesive bonding. The contribution of the anodic coatings to crack initiation was clearly borne out in the present tests on specimens with side notches (section 4.8). For constant-amplitude loading the effect was similar to the effect of cladding layers on conventional monolithic sheets. For flight simulation loading the tests showed the anodized layer to be less harmful.

Since ARALL in view of its good fatigue resistance does allow considerable weight savings, it implies that higher allowable stresses are possible in fatigue critical components of the aircraft structures. It should be recognized that minute cracks, still invisible, can be present during a substantial part of the service life. This is most probably also true for civil aircraft built from the classical aluminium alloys, and in this respect it is difficult to see why ARALL should be an exception, especially because of its high crack growth resistance after initiation.
9. Conclusions

The fatigue crack growth behaviour of small and large cracks in ARALL has been investigated, under constant amplitude and flight simulation loading. In additional test series several related properties were studied, which includes static properties, delamination in the fibre-adhesive layer under cyclic loads and shear deformations in the fibre-adhesive layer under static and cyclic loads. The additional test series were necessary for a better understanding of the ARALL behaviour under cyclic loads, and for providing material data to be used in a model, which has been developed for the prediction of fatigue crack growth in ARALL. In the experimental programme several variables were studied, such as the thickness of the aluminium sheet layers and the fibre-adhesive layers, different types of adhesives and fibres, and residual stress systems introduced by prestraining or prestressing.

The results of the experimental work has led to a number of conclusions.

1. The excellent crack growth resistance of ARALL, both under constant-amplitude and variable-amplitude loading, was amply confirmed. The resistance is considerably enhanced by a favourable residual stress system. i.e. compressive stress in the aluminium alloy layers and tensile stress in the aramid fibres. Thinner individual layers also lead to better properties.

2. The favourable residual stress system is very effective under constant-amplitude loading at low \( R \)-ratios. At high \( R \)-ratios the effect is less pronounced.

3. Truncation of the TWIST load spectrum has a significant effect on fatigue crack growth in ARALL. A high truncation level considerably reduced the crack growth rate.

4. The initiation of small cracks in the side notched specimens \( (K_c = 3.3) \) occurs relatively early in the fatigue life. Before such cracks become a through crack the growth is considerably hampered by the fibre-adhesive layer. A favourable residual stress system gives another contribution to the reduced growth rates of small cracks. Both effects become less at higher stress levels.

5. The delamination growth rate observed in delamination tests, and crack opening displacements due to adhesive shear deformation, are directly dependent on the load transfer from the fibres to the aluminium alloy layers. The delamination growth rate and the adhesive shear stress at the delamination front can be correlated with a simple exponential equation (Paris equation). The same is true for the delamination growth rate and the strain energy release rate associated with delamination. The stress ratio \( R \) (or the mean stress) does affect the delamination growth rate. A simple empirical correction has been obtained for this effect.
6. A non-linear delamination growth accumulation occurred under variable-amplitude fatigue loading. Increased delamination growth rates were observed if compared to linear predictions.

7. A fibre-adhesive layer containing an aramid fabric was considerably more resistant to delamination than a uni-directional fibre-adhesive layer.

8. The AF-163-5 adhesive showed much better fatigue delamination properties than the BSL-312-UL adhesive.

The analytical study to arrive at a prediction model for fatigue crack growth in ARALL under constant-amplitude loading, has led to the following conclusions.

9. The high crack growth resistance of ARALL is primarily a result of crack bridging by aramid fibres, and the large effect of crack bridging on the stress intensity factor (K) of the fatigue cracks in the aluminium sheets. Two relevant mechanisms for the calculation of K are the delamination in the fibre-adhesive layer and the shear deformation in the adhesive between the fibres and the aluminium alloy layers.

10. The fatigue crack growth mechanism in ARALL is rather complex, due to the hybrid composition of ARALL and the simultaneous crack growth and delamination. A model has been postulated, which includes the relevant physical aspects of the crack growth mechanism. Simplifying assumptions were necessary to arrive at analytical equations to describe the effects of various properties involved. It turned out that the model can predict several trends not only qualitatively, but also quantitatively with a satisfactory accuracy. This applies to the effects of the thickness and elastic properties of the various layers, the residual stress system and the delamination resistance. It also indicates that the model is based on a sound understanding of the fatigue behaviour of the material.

11. Because the model is capable to account for the main variables of ARALL as a family of materials, the model can be useful if the development of new types of similar materials is considered.
10. Acknowledgement

The contributions of many persons or institutions were valuable for the present work. However, the author only can address his gratitude explicitly to some of them:

The "Deutsche Forschungsgemeinschaft" (DFG) is acknowledged for its financial support of the research on ARALL at the DFVLR.

Prof. W. Bunk is acknowledged for creating the conditions at the DFVLR Institute for Materials Research, which were necessary for the completion of this investigation. The author wants to express his gratitude to Dr. H. Nowack for his help and encouraging discussions, and for being a tolerant boss, especially during the last phase of this thesis. Mr. K. H. Trautmann is acknowledged for the practical way in which he did make many technical things work.

As a representative of the technicians of the Fatigue Departement in the Institute for Materials Research at the DFVLR, Mr. H. J. Strunk is thanked for the careful and accurate way in which he performed a considerable part of the experimental work for this thesis.

Mr. J. Homan is thanked for his work during his stay at the DFVLR in Cologne. As representatives of the scientific colleagues which contributed to the present investigation, Dr. W. Ott is acknowledged for his finite element calculations, and Dr. K. Schulte for his discussions.

Gratitude is also debt to Mrs. C. Kotautschek who typed significant parts of this thesis. Mrs. H. Hanigk and Mrs. R. Fox are thanked for drawing numerous figures for this thesis. Mr. H. Brings is mentioned as a representative of the workshop of the Institute of materials research, for his skillful way of preparing many types of ARALL specimens.

The development of the numerical part of the present work and the lay out of this thesis have significantly been simplified by the help of the advisory group of the computer centre at the DFVLR in Cologne. As a representative of this group Mrs. H. Hahne is thanked for the patient and humorous way in which she helped solving software problems.

Dr. Falkenstein of VAW Bonn is thanked for his supply of thin aluminium alloy sheets in an early stage of the development of ARALL when thin ALCOA sheets were not yet available.

Ir. L. B. Vogele Sang from the Delft University of Technology is acknowledged for his kindness of supplying "ingredients" for ARALL, and for some ready ARALL sheets.

The author especially wants to recognize Prof. J. Schijve The quality of this thesis is improved considerably by his clear comments, advises, discussions, and his tactful critics.
11. References

   Fatigue properties of adhesive bonded laminated sheet material of aluminium alloys.

2. R. Marissen

3. J. Schijve, L.B. Vogelesang, R. Marissen
   U.S. patent numbers 4,500,589 and 4,489,123.
   European patent number 0056289.

4. J.A.R. Section 1 J.A.R.25.571. Also, Code of Federal Regulations: Aeronautics and
   20590.

5. T. Swift
   Verification of methods for damage tolerance evaluation of aircraft structures to FAA
   Requirements. Proceedings of the twelfth ICAF Symposium. Held at Toulouse (France),
   May 1983.

6. R.J.H. Wanhill, J.J. de Luccia, L.B. Vogelesang
   Environmental fatigue of aluminium alloy structural joints. 7th International Light
   pp.92-93.

7. M.L.C.E. Verbruggen
   Investigation on fracture of fibre/adhesive bondlines. Delft University of Technology,

8. M.L.C.E. Verbruggen
   Moisture absorption of ARALL. Delft University of Technology, Dept. of Aerospace

9. M.L.C.E. Verbruggen
   Aramid reinforced aluminium laminates: ARALL adhesion problems and environmental

10. L.B. Vogelesang, J.W. Gunink
    ARALL, a material for the next generation of aircraft. A state of the art. Delft Uni-
11. R. Marissen, L.B. Vogelesang
   Development of a new hybrid material: ARALL. Paper presented at the Intercontinental

12. M.E. Waddoups, J.R. Eisenmann, B.E. Kaminski
   Macroscopic fracture mechanics of advanced composite materials. J. Composite Mate-

13. J.W. Gunnink, A. Rothwell
   An assessment of the static strength of flush head aluminium alloy rivets in two grades
   of 'ARALL' sheet. Delft University of Technology, Dept. of Aerospace Engineering.

14. W. Schijve
   Investigation of the structural design in ARALL of the Airbus A-320 fuselage. (in
   Dutch) Thesis Dept. of Aerospace Engineering. Delft University of Technology, August
   1983.

15. Th. de Jong, A. Beukers
   Stresses around a pin loaded hole in an elastically orthotropic or isotropic plate. Delft

16. R. de Kruyff
   ARALL as a crack stopper material. Thesis Dept. of Aerospace Engineering. Delft
   University of Technology, October 1985.

17. C.G. van Hengel
   ARALL static properties: An engineering theory. Delft University of Technology,
   Dept. of Aerospace Engineering. Memorandum, November 1980. Also thesis (in
   Dutch).

18. Kevlar aramid. The fibre that let you re-think strength and weight. Du Pont Circular,
    USA.

19. Twaron. The aramid fibre for high performance composites. ENKA Industrial Fibres
    Circular, Wuppertal, Germany, 1984.

20. P.C. Paris
    The growth of fatigue cracks due to variations in load. Ph.D. Thesis, Lehigh University,
    Bethlehem, Pa, USA, 1962.

21. W. Elber
22. J. Schijve, F.A. Jacobs, P.J. Tromp

23. J. Schijve

24. K. Schulte

25. J. Schijve
Prediction methods for fatigue crack growth in aircraft material. ASTM STP 700, pp. 3-34, 1980.


27. J.B. de Jonge, D. Schütz, H. Lowak, J. Schijve

28. J. Schijve

29. R. Marissen, K.H. Trautmann, H. Nowack

30. G. Roderick

31. Technical information on HM-50 prepared by Teyin.

32. M.A. Miner
33. W.S. Johnson, S. Mall

34. C. Bathias, A. Laksmi

35. M.L.C.E. Verbruggen

36. S. Suresh

37. J. Schijve

38. M. Peters, K. Welpmann, H. Döker.

39. H. Nowack

40. D. Hanschmann


42. H. Nayeb-Hashemi, F.A. Mc Cintock, R.O. Ritchie.
43. J. Homan

44. B. Fortyr
Moiré interferometry applied on two fracture mechanics problems (in German). Thesis No. 805763, University of Technology Cologne, FRG., September 1985.

45. D. Post

46. D. Post, R. Czarnek, C.W. Smith

47. C. Berger


49. 3M circular

50. W. Flügge

51. G.W. Ehrenstein

52. J. Foth

53. J. Foth, R. Marissen, H.Nowack, G. Lütjering
54. J. Foth, R. Marissen, H. Nowack, G. Lütjering
   Fatigue crack initiation and microcrack propagation in notched and unnotched aluminium

55. R.G. de Lange

56. J.C. Newman Jr, I.S. Raju

57. H. J. Jergéus

58. L.J. Hart Smith

59. A.A. Griffith

60. F. Flaschner, S. Kenig, I.G. Zewi, H. Dodiuk

61. T.R. Brussat, S.T. Chiu, S. Mostovoy

62. H.M. Westergaard


64. W.T. Koiter

65. J.B. Bentheim, W.T. Koiter
66. G.H.J.J. Roebroeks

67. G. Koch

68. G. Koch

69. ALCOA Aerospace Technical Fact Sheet

70. G.H.J.J. Roebroeks, W.H.M. van Dreumel
12. Tables

Table 1:
TWIST: Flight types and numbers of load cycles per flight and per block.

<table>
<thead>
<tr>
<th>block</th>
<th>flight type</th>
<th>( \frac{S_{n,\text{max}}}{S_{mf}} )</th>
<th>number of cycles at the 10 amplitude levels</th>
<th>total cycle number per flight</th>
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<td>II</td>
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<td>1</td>
</tr>
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<td>1</td>
<td>1</td>
</tr>
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<td>1</td>
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<td>D</td>
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</table>

One block is 4000 flights
Table 2:
The delamination growth rates in the constant amplitude tests.
\[ t_{\text{cut}} = 0.2 \cdot t_{\text{cut,act}} \cdot G_{\text{bsl}} = 680 \text{ N/mm}^2; \quad G_{\text{AF-161-2}} = 640 \text{ N/mm}^2; \]
\[ f(G) = \sqrt{G_{\text{bsl}}} - 0.69 \sqrt{G_{\text{bsl}}} \]

<p>| $\Delta S_n$ nominal (MPa) | $R$ | $Q$ | $\Delta \tau + 0.37|\tau_m|$ (MPa) | $f(g)$ (N/mm)$^{1/2}$ | $\frac{db}{dN}$ (mm/cycle) |
|---------------------------|-----|-----|--------------------------|----------------|-----------------|
| BSL Twaron-HM fabric, $t_{\text{cut}} = 0.9 \text{ mm}$, $t_{\text{cut,act}} = 0.3 \text{ mm}$, $t_{\text{cut,com}} = 0.25 \text{ mm}$, $S_{\text{cut}} = -90 \text{ MPa}$ |
| 270 | 0.1 | 1.01 | 58.1 | 0.323 | $1.2 \times 10^{-6}$ |
| 288 | 0.1 | 0.98 | 61.5 | 0.341 | $7.4 \times 10^{-6}$ |
| 324 | 0.1 | 0.94 | 68.4 | 0.380 | $1.9 \times 10^{-5}$ |
| 360 | 0.1 | 0.91 | 75.4 | 0.419 | $3.4 \times 10^{-5}$ |
| 450 | 0.1 | 0.85 | 92.7 | 0.514 | $2.1 \times 10^{-4}$ |
| 540 | 0.1 | 0.81 | 110.0 | 0.610 | $1.4 \times 10^{-3}$ |
| 270 | 0.33 | 1.40 | 64.2 | 0.357 | $6.4 \times 10^{-7}$ |
| 288 | 0.45 | 1.69 | 73.3 | 0.407 | $2.2 \times 10^{-5}$ |
| 270 | 0.46 | 1.75 | 69.7 | 0.387 | $3.8 \times 10^{-6}$ |
| 200 | 0.6 | 2.53 | 60.7 | 0.337 | $1.5 \times 10^{-6}$ |
| 200 | 0.5 | 2.03 | 54.9 | 0.305 | $7.0 \times 10^{-7}$ |
| 400 | 0.2 | 1.02 | 86.3 | 0.479 | $1.3 \times 10^{-4}$ |
| BSL Twaron-HM fabric, $t_{\text{cut}} = 1.8 \text{ mm}$, $t_{\text{cut,act}} = 0.56 \text{ mm}$, $t_{\text{cut,com}} = 0.50 \text{ mm}$, $S_{\text{cut}} = -90 \text{ MPa}$ |
| 180 | 0.1 | 1.21 | 42.3 | 0.321 | $3.7 \times 10^{-7}$ |
| 225 | 0.1 | 1.09 | 51.2 | 0.388 | $3.5 \times 10^{-6}$ |
| 270 | 0.1 | 1.01 | 60.2 | 0.456 | $2.8 \times 10^{-5}$ |
| 360 | 0.1 | 0.91 | 78.0 | 0.592 | $3.6 \times 10^{-4}$ |
| 450 | 0.1 | 0.85 | 95.9 | 0.727 | $2.4 \times 10^{-3}$ |
| 540 | 0.1 | 0.81 | 113.8 | 0.863 | $1.5 \times 10^{-2}$ |
| 180 | 0.4 | 1.76 | 48.2 | 0.366 | $3.4 \times 10^{-7}$ |
| 180 | 0.55 | 2.32 | 54.3 | 0.412 | $7.2 \times 10^{-6}$ |
| 180 | 0.64 | 2.87 | 60.2 | 0.457 | $2.3 \times 10^{-5}$ |
| 180 | 0.7 | 3.43 | 66.2 | 0.503 | $6.3 \times 10^{-5}$ |
| 154 | 0.5 | 2.19 | 45.2 | 0.343 | $1.1 \times 10^{-7}$ |
| 182 | 0.5 | 2.09 | 52.3 | 0.397 | $6.6 \times 10^{-7}$ |
| 200 | 0.5 | 2.03 | 56.8 | 0.431 | $1.7 \times 10^{-6}$ |
| 225 | 0.5 | 1.98 | 63.2 | 0.480 | $2.6 \times 10^{-5}$ |
| 250 | 0.5 | 1.93 | 69.5 | 0.527 | $1.2 \times 10^{-4}$ |</p>
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<td>0.477</td>
<td>5.5 × 10^{-5}</td>
</tr>
</tbody>
</table>

**BSL Twaron-HM fabric, t_{u} = 1.0 mm, t_{s,act} = 0.25 mm, t_{s,nom} = 0.25 mm, S_{s,u} = 0**

<table>
<thead>
<tr>
<th>t_{u}</th>
<th>0.1</th>
<th>0.61</th>
<th>61.5</th>
<th>0.312</th>
<th>6.4 × 10^{-6}</th>
</tr>
</thead>
<tbody>
<tr>
<td>270</td>
<td>0.1</td>
<td>0.61</td>
<td>57.3</td>
<td>0.290</td>
<td>2.1 × 10^{-6}</td>
</tr>
<tr>
<td>288</td>
<td>0.1</td>
<td>0.61</td>
<td>61.1</td>
<td>0.309</td>
<td>2.0 × 10^{-6}</td>
</tr>
<tr>
<td>324</td>
<td>0.1</td>
<td>0.61</td>
<td>68.7</td>
<td>0.348</td>
<td>7.5 × 10^{-6}</td>
</tr>
<tr>
<td>360</td>
<td>0.1</td>
<td>0.61</td>
<td>76.3</td>
<td>0.387</td>
<td>5.4 × 10^{-5}</td>
</tr>
<tr>
<td>396</td>
<td>0.1</td>
<td>0.61</td>
<td>84.0</td>
<td>0.425</td>
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</tr>
<tr>
<td>450</td>
<td>0.1</td>
<td>0.61</td>
<td>95.4</td>
<td>0.483</td>
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</tr>
<tr>
<td>504</td>
<td>0.1</td>
<td>0.61</td>
<td>106.9</td>
<td>0.541</td>
<td>6.3 × 10^{-4}</td>
</tr>
<tr>
<td>522</td>
<td>0.1</td>
<td>0.61</td>
<td>110.7</td>
<td>0.561</td>
<td>8.1 × 10^{-4}</td>
</tr>
<tr>
<td>540</td>
<td>0.1</td>
<td>0.61</td>
<td>114.5</td>
<td>0.580</td>
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<tr>
<td>300</td>
<td>0.25</td>
<td>0.83</td>
<td>67.8</td>
<td>0.344</td>
<td>1.4 × 10^{-5}</td>
</tr>
<tr>
<td>300</td>
<td>0.33</td>
<td>1.0</td>
<td>71.1</td>
<td>0.360</td>
<td>2.0 × 10^{-5}</td>
</tr>
<tr>
<td>300</td>
<td>0.4</td>
<td>1.17</td>
<td>74.5</td>
<td>0.377</td>
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<tr>
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<td>1.5</td>
<td>80.7</td>
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**BSL Twaron-HM fabric, t_{u} = 0.5 mm, t_{s,act} = 0.25 mm, t_{s,nom} = 0.25 mm, S_{s,u} = 0**

<table>
<thead>
<tr>
<th>t_{u}</th>
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<th>0.61</th>
<th>64.2</th>
<th>0.325</th>
<th>1.4 × 10^{-6}</th>
</tr>
</thead>
<tbody>
<tr>
<td>324</td>
<td>0.1</td>
<td>0.61</td>
<td>71.4</td>
<td>0.362</td>
<td>2.6 × 10^{-5}</td>
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<tr>
<td>360</td>
<td>0.1</td>
<td>0.61</td>
<td>74.9</td>
<td>0.380</td>
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</tr>
<tr>
<td>378</td>
<td>0.1</td>
<td>0.61</td>
<td>78.5</td>
<td>0.398</td>
<td>4.4 × 10^{-5}</td>
</tr>
<tr>
<td>396</td>
<td>0.1</td>
<td>0.61</td>
<td>85.6</td>
<td>0.434</td>
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</tr>
<tr>
<td>432</td>
<td>0.1</td>
<td>0.61</td>
<td>89.2</td>
<td>0.452</td>
<td>1.5 × 10^{-4}</td>
</tr>
<tr>
<td>450</td>
<td>0.1</td>
<td>0.61</td>
<td>92.8</td>
<td>0.470</td>
<td>1.3 × 10^{-4}</td>
</tr>
<tr>
<td>468</td>
<td>0.1</td>
<td>0.61</td>
<td>96.3</td>
<td>0.488</td>
<td>1.3 × 10^{-4}</td>
</tr>
<tr>
<td>504</td>
<td>0.1</td>
<td>0.61</td>
<td>99.9</td>
<td>0.506</td>
<td>2.2 × 10^{-4}</td>
</tr>
<tr>
<td>522</td>
<td>0.1</td>
<td>0.61</td>
<td>103.5</td>
<td>0.524</td>
<td>5.0 × 10^{-3}</td>
</tr>
<tr>
<td>540</td>
<td>0.1</td>
<td>0.61</td>
<td>107.0</td>
<td>0.542</td>
<td>8.0 × 10^{-4}</td>
</tr>
</tbody>
</table>

**BSL Twaron-HM fabric, t_{u} = 1.0 mm, t_{s,act} = 0.25 mm, t_{s,nom} = 0.25 mm, S_{s,u} = +175 MPa**

<table>
<thead>
<tr>
<th>t_{u}</th>
<th>0.0</th>
<th>-0.21</th>
<th>52.2</th>
<th>&quot;0.265&quot;</th>
<th>1.2 × 10^{-7}</th>
</tr>
</thead>
<tbody>
<tr>
<td>280</td>
<td>0.0</td>
<td>-0.13</td>
<td>58.0</td>
<td>&quot;0.294&quot;</td>
<td>3.2 × 10^{-7}</td>
</tr>
<tr>
<td>320</td>
<td>0.0</td>
<td>0.0</td>
<td>69.2</td>
<td>&quot;0.351&quot;</td>
<td>4.8 × 10^{-6}</td>
</tr>
<tr>
<td>400</td>
<td>0.0</td>
<td>0.05</td>
<td>77.5</td>
<td>&quot;0.393&quot;</td>
<td>1.3 × 10^{-5}</td>
</tr>
<tr>
<td>$t_w$</td>
<td>$t_{cr,me}$</td>
<td>$S_{cr,me}$</td>
<td>$S_{cr,mn}$</td>
<td>$\sigma_{cr}$</td>
<td></td>
</tr>
<tr>
<td>-----</td>
<td>-------------</td>
<td>-------------</td>
<td>-------------</td>
<td>-------------</td>
<td></td>
</tr>
<tr>
<td>460</td>
<td>0.0</td>
<td>0.07</td>
<td>81.6</td>
<td>&quot;0.414&quot;</td>
<td>$3.2 \times 10^{-5}$</td>
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<tr>
<td>500</td>
<td>0.0</td>
<td>0.10</td>
<td>89.7</td>
<td>&quot;0.455&quot;</td>
<td>$1.1 \times 10^{-4}$</td>
</tr>
<tr>
<td>540</td>
<td>0.0</td>
<td>0.13</td>
<td>97.9</td>
<td>&quot;0.496&quot;</td>
<td>$1.3 \times 10^{-4}$</td>
</tr>
<tr>
<td>560</td>
<td>0.0</td>
<td>0.14</td>
<td>101.9</td>
<td>&quot;0.516&quot;</td>
<td>$2.5 \times 10^{-4}$</td>
</tr>
<tr>
<td>600</td>
<td>0.0</td>
<td>0.17</td>
<td>110.3</td>
<td>&quot;0.559&quot;</td>
<td>$4.6 \times 10^{-4}$</td>
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<tr>
<td>260</td>
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<td>0.5</td>
<td>53.3</td>
<td>0.270</td>
<td>$6.7 \times 10^{-7}$</td>
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<tr>
<td>300</td>
<td>0.4</td>
<td>0.5</td>
<td>61.5</td>
<td>0.312</td>
<td>$1.1 \times 10^{-6}$</td>
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<tr>
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<td>0.5</td>
<td>65.6</td>
<td>0.332</td>
<td>$6.1 \times 10^{-6}$</td>
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<tr>
<td>360</td>
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<td>73.8</td>
<td>0.374</td>
<td>$3.3 \times 10^{-5}$</td>
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<tr>
<td>380</td>
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<td>0.5</td>
<td>77.9</td>
<td>0.395</td>
<td>$6.3 \times 10^{-5}$</td>
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<tr>
<td>400</td>
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<td>0.5</td>
<td>82.0</td>
<td>0.415</td>
<td>$8.6 \times 10^{-5}$</td>
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<tr>
<td>400</td>
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<td>0.25</td>
<td>75.6</td>
<td>&quot;0.383&quot;</td>
<td>$1.5 \times 10^{-5}$</td>
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Prepreg, $t_w = 1.0$ mm, $t_{cr,me} = 0.22$ mm, $t_{cr,mn} = 0.20$ mm, $S_{cr,m} = -75$ MPa

<table>
<thead>
<tr>
<th>$t_w$</th>
<th>$t_{cr,me}$</th>
<th>$S_{cr,me}$</th>
<th>$S_{cr,mn}$</th>
<th>$\sigma_{cr}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>360</td>
<td>0.1</td>
<td>0.86</td>
<td>67.2</td>
<td>0.329</td>
</tr>
<tr>
<td>405</td>
<td>0.1</td>
<td>0.83</td>
<td>74.8</td>
<td>0.367</td>
</tr>
<tr>
<td>450</td>
<td>0.1</td>
<td>0.81</td>
<td>82.7</td>
<td>0.405</td>
</tr>
<tr>
<td>475</td>
<td>0.1</td>
<td>0.80</td>
<td>86.7</td>
<td>0.425</td>
</tr>
<tr>
<td>495</td>
<td>0.1</td>
<td>0.79</td>
<td>90.5</td>
<td>0.443</td>
</tr>
<tr>
<td>540</td>
<td>0.1</td>
<td>0.77</td>
<td>98.2</td>
<td>0.481</td>
</tr>
<tr>
<td>565</td>
<td>0.1</td>
<td>0.77</td>
<td>102.3</td>
<td>0.501</td>
</tr>
<tr>
<td>608</td>
<td>0.1</td>
<td>0.76</td>
<td>110.2</td>
<td>0.540</td>
</tr>
<tr>
<td>653</td>
<td>0.1</td>
<td>0.75</td>
<td>118.0</td>
<td>0.578</td>
</tr>
<tr>
<td>675</td>
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<td>0.74</td>
<td>121.7</td>
<td>0.596</td>
</tr>
<tr>
<td>742</td>
<td>0.1</td>
<td>0.73</td>
<td>133.3</td>
<td>0.653</td>
</tr>
<tr>
<td>647</td>
<td>0.07</td>
<td>0.71</td>
<td>115.6</td>
<td>0.566</td>
</tr>
<tr>
<td>472</td>
<td>0.32</td>
<td>1.16</td>
<td>95.5</td>
<td>0.468</td>
</tr>
<tr>
<td>500</td>
<td>0.33</td>
<td>1.18</td>
<td>101.6</td>
<td>0.498</td>
</tr>
<tr>
<td>375</td>
<td>0.5</td>
<td>1.73</td>
<td>87.0</td>
<td>0.427</td>
</tr>
<tr>
<td>735</td>
<td>0.07</td>
<td>0.69</td>
<td>130.5</td>
<td>0.639</td>
</tr>
</tbody>
</table>

Prepreg, $t_w = 1.0$ mm, $t_{cr,me} = 0.20$ mm, $t_{cr,mn} = 0.20$ mm, $S_{cr,m} = 0$

<table>
<thead>
<tr>
<th>$t_w$</th>
<th>$t_{cr,me}$</th>
<th>$S_{cr,me}$</th>
<th>$S_{cr,mn}$</th>
<th>$\sigma_{cr}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>500</td>
<td>0.0</td>
<td>0.5</td>
<td>87.9</td>
<td>0.411</td>
</tr>
<tr>
<td>750</td>
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<td>0.5</td>
<td>131.9</td>
<td>0.616</td>
</tr>
<tr>
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<td>0.33</td>
<td>1.0</td>
<td>101.6</td>
<td>0.475</td>
</tr>
</tbody>
</table>

AF Twaron-HM fabric, $t_w = 1.0$ mm, $t_{cr,me} = 0.25$ mm, $t_{cr,mn} = 0.25$ mm, $S_{cr,m} = 0$

<table>
<thead>
<tr>
<th>$t_w$</th>
<th>$t_{cr,me}$</th>
<th>$S_{cr,me}$</th>
<th>$S_{cr,mn}$</th>
<th>$\sigma_{cr}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>480</td>
<td>0.0</td>
<td>0.5</td>
<td>95.5</td>
<td>0.498</td>
</tr>
<tr>
<td></td>
<td>0.0</td>
<td>0.5</td>
<td></td>
<td></td>
</tr>
<tr>
<td>---</td>
<td>-----</td>
<td>-----</td>
<td>----</td>
<td>----</td>
</tr>
<tr>
<td>500</td>
<td>0.0</td>
<td>0.5</td>
<td>99.5</td>
<td>0.519</td>
</tr>
<tr>
<td>520</td>
<td>0.0</td>
<td>0.5</td>
<td>103.4</td>
<td>0.540</td>
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<tr>
<td>540</td>
<td>0.0</td>
<td>0.5</td>
<td>107.4</td>
<td>0.561</td>
</tr>
<tr>
<td>560</td>
<td>0.0</td>
<td>0.5</td>
<td>111.4</td>
<td>0.582</td>
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<tr>
<td>580</td>
<td>0.0</td>
<td>0.5</td>
<td>115.4</td>
<td>0.602</td>
</tr>
<tr>
<td>600</td>
<td>0.0</td>
<td>0.5</td>
<td>119.4</td>
<td>0.623</td>
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</table>

AF Teyin-50 fabric, $t_w = 1.0\text{mm}$, $t_{w,air} = 0.40\text{mm}$, $t_{w,nom} = 0.25\text{mm}$, $S_{w} = 0$

<table>
<thead>
<tr>
<th></th>
<th>0.0</th>
<th>0.5</th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>460</td>
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<td>0.5</td>
<td>88.9</td>
<td>0.588</td>
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<tr>
<td>480</td>
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<td>0.5</td>
<td>92.8</td>
<td>0.613</td>
<td>$2.6 \times 10^{-6}$</td>
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<tr>
<td>500</td>
<td>0.0</td>
<td>0.5</td>
<td>96.8</td>
<td>0.639</td>
<td>$8.5 \times 10^{-6}$</td>
</tr>
<tr>
<td>520</td>
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<td>0.5</td>
<td>100.6</td>
<td>0.664</td>
<td>$9.8 \times 10^{-6}$</td>
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<tr>
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<td>0.5</td>
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<td>0.690</td>
<td>$1.1 \times 10^{-5}$</td>
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<tr>
<td>560</td>
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<tr>
<td>580</td>
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<td>0.5</td>
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<td>0.741</td>
<td>$3.4 \times 10^{-5}$</td>
</tr>
<tr>
<td>600</td>
<td>0.0</td>
<td>0.5</td>
<td>116.1</td>
<td>767</td>
<td>$5.7 \times 10^{-5}$</td>
</tr>
</tbody>
</table>
Table 3:
Some values of $C_a$ for an ARALL sheet with $F_u = 129600$ N/mm and $F_o = 36450$ N/mm.

<table>
<thead>
<tr>
<th>$a/s$</th>
<th>0.1</th>
<th>0.2</th>
<th>0.3</th>
<th>0.4</th>
<th>0.5</th>
<th>0.7</th>
<th>1.0</th>
<th>2.0</th>
<th>$\infty$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\infty$</td>
<td>0.19</td>
<td>0.33</td>
<td>0.43</td>
<td>0.52</td>
<td>0.58</td>
<td>0.68</td>
<td>0.77</td>
<td>0.90</td>
<td>1</td>
</tr>
<tr>
<td>10</td>
<td>0.20</td>
<td>0.35</td>
<td>0.45</td>
<td>0.54</td>
<td>0.60</td>
<td>0.70</td>
<td>0.79</td>
<td>0.91</td>
<td>1</td>
</tr>
<tr>
<td>7</td>
<td>0.21</td>
<td>0.35</td>
<td>0.46</td>
<td>0.54</td>
<td>0.61</td>
<td>0.70</td>
<td>0.79</td>
<td>0.91</td>
<td>1</td>
</tr>
<tr>
<td>5</td>
<td>0.22</td>
<td>0.37</td>
<td>0.47</td>
<td>0.56</td>
<td>0.62</td>
<td>0.71</td>
<td>0.80</td>
<td>0.92</td>
<td>1</td>
</tr>
<tr>
<td>3</td>
<td>0.24</td>
<td>0.40</td>
<td>0.51</td>
<td>0.59</td>
<td>0.66</td>
<td>0.75</td>
<td>0.83</td>
<td>0.93</td>
<td>1</td>
</tr>
<tr>
<td>2</td>
<td>0.29</td>
<td>0.46</td>
<td>0.58</td>
<td>0.66</td>
<td>0.71</td>
<td>0.79</td>
<td>0.86</td>
<td>0.95</td>
<td>1</td>
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<tr>
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<td>0.56</td>
<td>0.69</td>
<td>0.74</td>
<td>0.79</td>
<td>0.85</td>
<td>0.91</td>
<td>0.97</td>
<td>1</td>
</tr>
<tr>
<td>1.2</td>
<td>0.54</td>
<td>0.72</td>
<td>0.81</td>
<td>0.86</td>
<td>0.89</td>
<td>0.93</td>
<td>0.96</td>
<td>0.99</td>
<td>1</td>
</tr>
<tr>
<td>1.1</td>
<td>0.69</td>
<td>0.83</td>
<td>0.89</td>
<td>0.92</td>
<td>0.94</td>
<td>0.96</td>
<td>0.98</td>
<td>0.99</td>
<td>1</td>
</tr>
<tr>
<td>1.05</td>
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<td>0.91</td>
<td>0.94</td>
<td>0.96</td>
<td>0.97</td>
<td>0.98</td>
<td>0.99</td>
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<td>0.98</td>
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20x80 DIN 50114 Tensile test specimen

Dimension in millimeters

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ARALL fatigue crack growth specimen

Laminate built up

- 0.45mm Al 7475-T76
- BSL-312-UL adhesive
- unidirectional aramid fabric

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non-reinforced fatigue crack growth specimen

Laminate built up

- 0.45mm Al 7475-T76
- BSL-312-UL adhesive

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Without residual stress

No truncation

\[ K_{ol} \quad K_{max} \]

Truncation

moderate reduction of \( K_{ol}/K_{max} \)

With residual stress

No truncation

\[ K_{ol} \quad K_{max} \]

effect of residual stress

K=0

Truncation

larger reduction of \( K_{ol}/K_{max} \)

K=0

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specimen type I
2x0.5mm Al
1x0.2mm ar
AF 163-2 Twaron pre,preg
gauge length 5mm
e* = 0.3mm
E_{al} = 72000 N/mm^2
F_{ar} = 12400 N/mm
G_{ad} = 640 N/mm^2
t_{ad} = 0.04 mm
\tau_p = 38 MPa

reversed creep after unloading
Figure 73. Load displacement diagram for an artificial crack.
Figure 74. Load displacement diagram for an artificial crack.
Figure 75. Load displacement diagram for an artificial crack.
specimen type VI
2 x 0.5 mm Al
1 x 0.4 mm ar
AF 163-2 Teyin fabric
gauge length 5 mm
e*=0.3 mm
$E_{al} = 72000 \text{ N/mm}^2$
$F_{ar} = 8400 \text{ N/mm}$
$G_{ad} = 640 \text{ N/mm}^2$
$t_{ad} = 0.08 \text{ mm}$
$\tau = 38 \text{ MPa}$

Figure 76. Load displacement diagram for an artificial crack.
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Figure 82. Three tests on the same specimen with a minimum of 20 minutes between the tests.
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$K_t = 3.3$

$S_{\text{net, max}} = 110 \text{MPa}$  \hspace{1cm} R = 0

$S_{0.2} = 525 \text{MPa}$

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**Prediction using large crack data**

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**Test (short crack)**

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ARALL:
2×0.5 mm Al 7075-T6+
0.25 mm fabric
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1 Portion of the external load which is carried by the aramid fibres

2 Portion of the load which is transferred from the aluminium to the fibres, at the delamination boundary

3 Portion of the load which remains in the aluminium, passes around the crack tip and causes the stress intensity factor

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\[ Z(z) = \frac{S z}{\sqrt{z^2 - a^2}} \]

\[ \bar{Z}(z) = S \sqrt{z^2 - a^2} \]

\[ z = x + iy \]

Figure 120. The Westergaard equations for a bi-axial stress field.

\[ \nu = 1/3 \]

\[ x = 0 \]

\[ \frac{2 \nu E}{a S} \]

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for long cracks is:

\[ \varepsilon = \frac{S_{1a}}{E_{1a}} = \frac{\text{COD}}{2h} \text{ (continuity)} \]

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Stress distribution for zero delamination.
The crack bridging stress near the crack tip is reduced due to adhesive shear compliance.

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energy release rate function $\sqrt{G_{d,\text{max}}} - 0.69\sqrt{G_{d,\text{min}}}$

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Appendix A. The influence of the anisotropy on the stress intensity factor in ARALL.

A.1 Introduction

Equations for the stress intensity factor in ARALL are presented in chapter 5. In these equations the anisotropy of ARALL is ignored. The anisotropy of ARALL is caused by the aramid-adhesive layer, with its low transverse stiffness (x-direction) and low shear stiffness. In the present appendix the anisotropy effect on crack opening displacements will be considered. COD-values will be adopted as an indication of the anisotropy effect.

In chapter 5 the anisotropy of ARALL was ignored and it was assumed that the stiffness properties of the fibre-adhesive layers in the longitudinal direction are also representative for its stiffness in other directions. As a consequence the following equations should be valid:

\[ E_{la,x} = E_{la,y} = E_{la} \]

\[ G_{la} = \frac{E_{la}}{2(1 + \nu_{la})} \quad (\text{with } \nu_{la} = \nu_{AI}) \]

\( E \) refers to the Youngs modulus, and the subscripts \( la \) and \( AI \) refer to the total laminate and the aluminium layers respectively. \( G \) is the shear modulus and \( \nu \) is the Poisson ratio. With the above assumptions, which neglect the anisotropy, solutions for isotropic materials may be used, like:

\[ COD = \frac{4aS}{E} \quad (A - 1) \]

The isotropy also implies (as explained in section 5.6):

\[ \frac{K_{AI}}{K_{la}} = \frac{S_{AI}}{S_{la}} = \frac{E_{AI}}{E_{la}} = \frac{t_{AI}E_{la}}{F_{AI} + F_{ar}} \quad (A - 2) \]

where the subscript \( ar \) refers to the the aramid-adhesive layers. \( S \) denotes the stress and \( a \) is the crack length. \( F_{ar} \) and \( F_{ar} \) are stiffness parameters.

\[ F_{AI} = t_{AI}E_{AI} \quad \text{and} \quad F_{ar} = t_{ar}E_{ar} \]

where \( t_{ar} \) denotes the total thickness of all aluminium layers and \( t_{ar} \) the total thickness of all aramid-adhesive layers. \( t_{la} \) is the total ARALL thickness.

In the next section, the influence of the anisotropy will be discussed qualitatively and in section A3 the errors in the above equations will be studied quantitatively.
A.2 Some remarks about the stress intensity factor in anisotropic materials

For an isotropic aluminium sheet, the stress field in the crack tip region (assuming elastic behaviour) is described by:

\[ S_y = \frac{K_{AI}}{\sqrt{2\pi r}} \cdot f_y(\phi) \quad (A - 3) \]

Similarly for the anisotropic ARALL:

\[ S_y = \frac{K_{AI}}{\sqrt{2\pi r}} \cdot g_y(\phi) \quad (A - 4) \]

where \( \phi \) and \( r \) are polar coordinates, with the crack tip at \( r = 0 \)

For both cases the stress intensity factor is given by:

\[ K = S \sqrt{\pi a} \quad \text{(infinite sheet)} \quad (A - 5) \]

Unfortunately, the functions \( f_y(\phi) \) and \( g_y(\phi) \) are not identical (see e.g. [A1]) and there is no direct way to calculate \( K_{AI} \) from \( K_{AI} \). The function \( g_y(\phi) \) depends on the anisotropy of the material. It can no longer be stated that equal \( K \)-variations in the aluminium sheets of ARALL and in pure aluminium alloy sheets will give the same crack growth rate (similarity concept). The growth rates in both materials will still be a function of the deformations around the crack, but it is not obvious how the correlation can be obtained. A discussion on this fundamental fatigue problem is beyond the scope of the present investigation. As an arbitrary choice it will be assumed, that stresses and displacements in the \( y \) direction are of major importance for fatigue crack growth.

An additional complication is that geometrical corrections on the stress intensity factor are also different for isotropic and anisotropic materials (e.g. finite width correction). These differences are assumed to be small for the present problems, and they will be ignored.

A.3 The influence of the anisotropy on the crack opening displacement

Expressions for the crack opening displacement will be adopted to evaluate the anisotropy effect on the stress intensity. For this purpose a 'through crack' in ARALL will be considered (no fibres intact behind the crack tips). De Jong [A2] derived a solution for this geometry.

For an elliptical hole with a long axis of \( 2a \) and a short axis of \( 2b \), in an infinite anisotropic plate, the displacements in the \( y \) direction of the hole edges at \( x = 0 \), \( \nu_{(x=0,y=b)} \) are given by:
\[ v = \frac{a S_{ln}}{E_y} \cdot \left\{ \frac{\beta}{a} + \sqrt{\frac{2(c_1 + c_2)}{c_1}} \right\} \quad (A - 6) \]

where Young's modulus in the \( y \) direction is denoted as \( E_y \), thus:

\[ E_y = E_{ln} \quad (A - 7) \]

Furthermore:

\[ c_1 = \sqrt{\frac{E_y}{E_x}} \quad (A - 8) \]

\[ c_2 = \frac{E_y}{2G_{12}} - \nu_{12} \quad (A - 9) \]

where \( E_x \) is Young's modulus in the \( x \) direction. Assuming that the low transverse stiffness of the fibre-adhesive layers is zero, the result is:

\[ E_x = \frac{t_{AI}}{t_{ln}} E_{AI} \quad (A - 10) \]

assuming that the low shear stiffness of the fibre-adhesive layers is zero too:

\[ G_{12} = \frac{t_{AI}}{t_{ln}} G_{AI} = \frac{t_{AI}}{t_{ln}} \cdot \frac{E_{AI}}{2(1 + \nu_{AI})} \quad (A - 11) \]

\( \nu_{12} \) for the fibre-adhesive layers is not much different from \( \nu_{AI} \), so \( \nu_{12} \) for ARALL will be very close to \( \nu_{AI} \) (law of mixtures).

\[ \nu_{12} = \nu_{AI} \quad (A - 12) \]

With the equations (A-7), (A-10), (A-11) and (A-12) the equations (A-8) and (A-9) can be written as:

\[ c_1 = \sqrt{\frac{t_{ln}}{t_{AI}}} \frac{E_{ln}}{E_{AI}} = \sqrt{\frac{F_{ln}}{F_{AI}}} \quad (A - 13) \]

\[ c_2 = (1 + \nu_{AI}) \cdot \frac{F_{ln}}{F_{AI}} - \nu_{AI} \quad (A - 14) \]

Substitution in equation (A-6) yields:

\[ v = \frac{a S_{ln}}{E_{ln}} \cdot \left\{ \frac{\beta}{a} + \sqrt{2 + 2\sqrt{\frac{F_{ln}}{F_{AI}}} + 2\nu_{AI} \left( \sqrt{\frac{F_{ln}}{F_{AI}}} - \sqrt{\frac{F_{AI}}{F_{ln}}} \right) } \right\} \quad (A - 15) \]
For a crack $\beta = 0$, $a = a$, and with $COD = 2\nu$:

$$
COD = \frac{2aS_{in}}{E_{in}} \cdot \sqrt{2 \left\{ 1 + \sqrt{\frac{F_{in}}{F_{Al}}} + \nu_{Al}\left(\sqrt{\frac{F_{in}}{F_{Al}}} - \sqrt{\frac{F_{Al}}{F_{in}}}\right)\right\}} 
$$

(A - 16)

The crack opening displacement for an isotropic plate is obtained if $F_{in} = F_{Al}$, and it is easily recognized that equation (A-16) then reduces to the well known solution:

$$
COD = \frac{4aS}{E}
$$

For the ARALL type used for the constant-amplitude fatigue crack growth tests in section 4.3 the stiffness values are:

$F_{Al} = 129600$ N/mm

$F_{\nu} = 37200$ N/mm

and

$F_{in} = F_{Al} + F_{\nu} = 166800$ N/mm

for aluminium $\nu_{Al} = 0.33$.

Introduction of these values in equation (A-16) yields:

$$
COD = \frac{4.44aS}{E}
$$

This is about 10% higher than the result for the case with assumed isotropic conditions. Because the crack contour in an infinite anisotropic sheet is an ellipse as well [A2], the same percentage will apply to the displacements closely behind the crack tip. These displacements are assumed to be of major importance for the fatigue crack growth. It appears that the effect of the anisotropy is quite small.

References.


A2. Th. de Jong
    Discussion at the Technical University of Delft.
Appendix B. The influence of the axis ratio of an elliptical delamination area on the efficiency of the crack bridging stresses

In chapter 5 a calculation is presented, in which the stress intensity factor of a crack in ARALL is determined, considering also the crack bridging stress. The most simple approach to the effect of the crack bridging stress on the stress intensity factor is to assume that this stress acts on the crack edges (see figure B1a). If the crack bridging force per unit of width and per unit of aluminium sheet thickness is represented by the stress $S$, the well known solution is:

$$K = S \sqrt{\pi a} \quad \text{(infinite sheet)}$$

This simple approach has been applied in chapter 5. However, the crack bridging stress $S$ is not acting on the crack edges, but on the elliptical boundary of the delamination area (see figure B1b). The influence of this more realistic case on the stress intensity factor is calculated in this appendix.

A solution for the stress intensity factor belonging to figure B1b can not be found in the literature. However, it can be derived by integration of the available solution for symmetric forces, as shown in figure B1c. The solution is presented in the handbook by Tada et al [B1], by adopting complex parameters $z_0$ and $\bar{z}_0$ (see figure B2):

$$z_0 = x_0 + iy_0 \quad \text{and} \quad \bar{z}_0 = x_0 - iy_0.$$  

The solution is:

$$K = \frac{P}{\sqrt{\pi a}} \cdot \left(1 - a y_0 \frac{\partial}{\partial y_0}\right) \cdot \left(\frac{a}{\sqrt{a^2 - z_0^2}} + \frac{a}{\sqrt{a^2 - \bar{z}_0^2}}\right) \quad (B-1)$$

where $P = S \cdot dx$ (figure B1c).

For the case of plane stress, which is considered here:

$$a = (1 + \nu)/2 \quad (B-2)$$

where $\nu$ is the Poisson ratio. For aluminium alloys is $\nu = 0.33$

After differentiation of equation (B-1) it can be written as:

$$K = P \sqrt{\frac{a}{\pi}} \cdot \left[\left\{(a^2 - z_0^2)^{-1/2} + (a^2 - \bar{z}_0^2)^{-1/2}\right\} - \right.$$

$$\left.ia y_0 \left\{z_0(a^2 - z_0^2)^{-3/2} - \bar{z}_0(a^2 - \bar{z}_0^2)^{-3/2}\right\}\right] \quad (B-3)$$

Equation (B-3) has to be integrated over the elliptical contour of figure B1c. Due to the symmetric situation in figure B2, an integration between:
\[ x_0 = 0 \text{ and } x_0 = a, \quad \text{with } y_0 = b \sqrt{1 - (x_0/a)^2} \]

is sufficient.

The integration must be performed numerically. It can be performed directly from equation (B-3), if a Fortran computer programme is used, because Fortran is able to handle complex variables. However, equation (B-3) can also be written in a form with real variables only, followed by a numerical integration. Both methods were used. The results are identical.

The latter approach is described below.

Equation (B-3) can be written in a form with real variables as follows:

\[ K = P \sqrt{\frac{2a}{\pi}} \cdot \left\{ \frac{B}{\sqrt{A}} + y_0^2(1 + \nu) \cdot \left( \frac{BC}{A^{3/2}} - \frac{\sqrt{A} + C}{2BA} \right) \right\} \quad (B-4) \]

where:

\[ A = (a^2 - x_0^2 + y_0^2)^2 + 4x_0^2y_0^2 \quad (B-5) \]

\[ B = \sqrt{a^2 - x_0^2 + y_0^2 + \sqrt{A}} \quad (B-6) \]

\[ C = a^2 + x_0^2 + y_0^2 \quad (B-7) \]

Equation (B-4) is equivalent to an equation which was presented in [B2], which, however, contains a misprint. After a correction (replacing a factor \( \beta \) by \( \beta' \)) and adding the factor \( \sqrt{\pi} \) which is often omitted in older papers, the equation of [B2] is:

\[ K = 2P \sqrt{a/\pi} \cdot \left[ \frac{(3 + \nu)}{2} \cdot I_1 - (1 + \nu) \cdot I_2 \right] \quad (B-8) \]

with:

\[ I_1 = \frac{\beta}{\sqrt{(y_0^2 + a^2 - x_0^2)^2 + 4x_0^2y_0^2}} \quad (B-9) \]

\[ I_2 = \frac{\{(a^2 + x_0^2)y_0^2 + (a^2 - x_0^2)^2\} \cdot \beta^2 + x_0^2y_0^2(\nu_0 - a^2 + x_0^2)}{2\beta \cdot \{(y_0^2 + a^2 - x_0^2)^2 + 4x_0^2y_0^2\}^{3/2}} \quad (B-10) \]

\[ \beta = \frac{1}{\sqrt{2}} \cdot \sqrt{(y_0^2 + a^2 - x_0^2) + \sqrt{(y_0^2 + a^2 - x_0^2)^2 + 4x_0^2y_0^2}} \quad (B-11) \]

Equation (B-4) and (B-8) are equivalent. Both are valid for the plane stress state. The equations do not contain complex variables and a normal numerical integration process can be performed.
Integration of equation (B-4) with \( P = Sdx \) was performed numerically for several \( b/a \) ratios. The integration is performed from \( x = 0 \) to \( x = a \) after substitution of:

\[
x_0 = x \quad \text{and} \quad y_0 = y = b \sqrt{1 - (x/a)^2}
\]

The results are presented in table B1 and figure B3. The curve in figure B3 is accurately approximated by the following relation:

\[
C_{b/a} = \frac{K}{S \sqrt{\pi a}} = \frac{5}{3 (1 + b/a)} - \frac{2}{3 (1 + b/a)^2} \tag{B - 12}
\]

Equation (B-12) turns out to be accurate within 0.1 % for any \( b/a \) value (also for a \( b/a \) value approaching infinity, which is not presented in table B1).

References.


    Fracture mechanics research at Lehigh University, 1960-61 D6-7960.

Tables and figures

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Table B1. Some results of the calculations of the influence of the axis ratio \( b/a \) on the stress intensity factor.
Figure B1. Stress on an elliptical contour around a crack, compared to a stress on the crack flanks.
Figure B2. Stress intensity factor in an infinite sheet containing a crack. The sheet is loaded by four forces which are ordered symmetrically to the $x$ and the $y$ axis.

\[ K = P \sqrt{\frac{2a}{\pi}} \left\{ \frac{B}{\sqrt{A}} + \frac{y_0^2(1+v)}{2} \left( \frac{2BC}{A^{3/2}} \frac{\sqrt{A+C}}{BA} \right) \right\} \]

\[ A = (a^2 - x_0^2 + y_0^2)^2 + 4x_0^2y_0^2 \]

\[ B = \sqrt{a^2 - x_0^2 + y_0^2 + \sqrt{a}} \]

\[ C = a^2 + x_0^2 + y_0^2 \]

Figure B3. The influence of the axis ratio $b/a$ of the delamination ellipse on the stress intensity factor due to the crack bridging fibre stress $S$. 
Appendix C. Calculation of the energy release rate for delamination

C.1 Energy release rate if residual stresses are not present

The energy release rate for delamination is calculated from the energy balance of a delamination specimen (cracked over the full width).

First a laminate without residual stresses is considered. Figure C1 shows the situation after some delamination. For delamination over a distance \( b \) the elastic energies of the sheets and the fibre layers have changed, due to a change of the stress in those layers after delamination. Also the external energy is changing due to a displacement of the external load \( P \).

Although it is possible to calculate the energy release rate from the change in the elastic energy only, here the total energy balance is given.

\[
U - F + W = 0 \quad \Rightarrow \quad W = F - U \quad \quad (C - 1)
\]

\( U \) is the increase of the elastic energy in the various layers during a delamination over distance \( b \).

\( F \) is the work applied to the system during the displacement \( \Delta y \) of the external force \( P \). \( W \) is the 'surface' energy which is consumed during delamination over a distance \( b \).

Considering the energy per unit of specimen width:

\[
F = P \Delta y \quad \quad (C - 2)
\]

\[
\Delta y = b \Delta \varepsilon^* \quad \quad (C - 3)
\]

where \( \Delta \varepsilon^* \) is the strain difference between the delaminated and the non-delaminated area of the aramid-adhesive layer.

\[
\Delta \varepsilon^* = (S_{fa} - S_{wa}) / E_{ar} \quad \quad (C - 4)
\]

where \( S_{fa} \) is the stress in the delaminated area of the fibre-adhesive layer and \( S_{wa} \) is the stress in the non-delaminated area.

From equations (C-2) to (C-4), it follows that:

\[
F = P b (S_{fa} - S_{wa}) / E_{ar} \quad \quad (C - 5)
\]

If \( U_1 \) is the increase of elastic energy in the aramid-adhesive layer and \( U_2 \) the decrease of elastic energy in the aluminium layers, during delamination over distance \( b \), the total change (increase) of the elastic energy is:

\[
U = U_1 - U_2 \quad \quad (C - 6)
\]

The elastic energy of a body which is loaded uni-axially is given by \((1/2) \cdot S' / E \) per unit of volume. As a result the decrease of the elastic energy in the aluminium layers during delamination over a distance \( b \) is (per unit of specimen width):
\[ U_2 = \frac{1}{2} t_{AI} b S_{AI}^2 / E_{AI} \]  

(C - 7)

In a similar way the increase of the elastic energy in the fibre-adhesive is:

\[ U_1 = \frac{1}{2} t_{ar} b (S_{sa}^2 - S_{ar}^2) / E_{ar} \]  

(C - 8)

The stresses \( S_{ar} \) and \( S_{AI} \) will be expressed as a function of \( S_n \) because \( S_n \) can be calculated directly from the external load for every type of laminate.

In view of equilibrium:

\[ S_{AI} t_{AI} + S_{ar} t_{ar} = S_n t_{ar} \]  

(C - 9)

The condition of equal strain in all non-delaminated layers implies:

\[ \epsilon_{AI} = \epsilon_{ar} = S_{AI} / E_{AI} = S_{ar} / E_{ar} \]  

(C - 10)

With the stiffness definitions \( F_{AI} = t_{AI} E_{AI} \) and \( F_{ar} = t_{ar} E_{ar} \), it follows from equation (C-9) and (C-10) that:

\[ S_{ar} = \frac{F_{ar}}{F_{AI} + F_{ar}} \cdot S_n \]  

(C - 11)

and

\[ S_{AI} = \frac{t_{ar}}{t_{AI}} \cdot \frac{F_{AI}}{F_{AI} + F_{ar}} \cdot S_n \]  

(C - 12)

The external load \( P \) can be expressed as a function of \( S_n \) (per unit of specimen width):

\[ P = S_n t_{ar} \]  

(C - 13)

With equations (C-11) until (C-13) the energy terms \( F \), \( U_1 \) and \( U_2 \) in equations (C-5), (C-7) and (C-8) can now be expressed as a function of \( S_n \). The result is:

\[ F = \frac{t_{ar} b}{E_{ar}} \cdot \left( \frac{F_{AI}}{F_{AI} + F_{ar}} \right) \cdot S_n^2 \]  

(C - 14)

\[ U_1 = \frac{t_{ar} b}{2 E_{ar}} \cdot \left\{ 1 - \frac{F_{ar}^2}{(F_{AI} + F_{ar})^2} \right\} \cdot S_n^2 \]  

(C - 15)

\[ U_2 = \frac{t_{AI} b}{2 E_{al}} \cdot \left\{ \frac{E_{AI} t_{ar}}{F_{AI} + F_{ar}} \right\}^2 \cdot S_n^2 \]  

(C - 16)

With equations (C-1) and (C-6) it follows:

\[ W = F - U_1 + U_2 \]  

(C - 17)
Introducing equations (C-14), (C-15) and (C-16) in equation (C-17) and together with the definitions of $F_{al}$ and $F_{ar}$ the result obtained is:

$$W = \frac{b}{2} \cdot \frac{t_{ar}}{E_{ar}} \cdot \frac{F_{al}}{F_{al} + F_{ar}} \cdot S_{fa}^2$$  \hspace{1cm} (C - 18)

Equation (C-18) presents the energy which is generated during delamination over distance $b$, which has been used as 'surface energy' for that delamination.

The total energy release rate $G'$ follows from differentiation:

$$G' = \frac{dW}{db}$$  \hspace{1cm} (C - 19)

It then follows from equations (C-18) and (C-19) that:

$$G' = \frac{t_{ar}}{2 E_{ar}} \cdot \left( \frac{F_{al}}{F_{al} + F_{ar}} \right) \cdot S_{fa}^2$$  \hspace{1cm} (C - 20)

Equation (C-20) describes the total energy release rate for two fibre-aluminium interfaces. The energy release rate per fibre-aluminium interfaces is given by $G'/2$, or in general by $G'/j$, if $j$ is the number of fibre-aluminium interfaces (usually $j$ is twice the number of fibre-adhesive layers) The energy release rate for delamination, per fibre-aluminium interface $G_j$ is given by

$$G_j = \frac{t_{ar}}{2 j E_{ar}} \cdot \left( \frac{F_{al}}{F_{al} + F_{ar}} \right) \cdot S_{fa}^2$$  \hspace{1cm} (C - 21)

C.2 Energy release rate if residual stresses are present

Equation (C-21) describes the energy release rate for delamination in a laminate without residual stresses. A derivation of the energy rate for laminates with residual stresses introduced by prestraining of ARALL is given below.

First a partly delaminated specimen is considered in which residual stresses are present. (Figure C2).

Three different cases are presented in figure C2:

Case I: Unloaded specimen. Equilibrium of residual stresses:

$$S_{r,al} t_{Al} + S_{r,ar} t_{ar} = 0 \quad \Rightarrow \quad S_{r,al} = - \frac{t_{ar}}{t_{al}} \cdot S_{r,ar}$$  \hspace{1cm} (C - 22)

Case II: Specimen loaded, no delamination. Equilibrium of external loads:

$$S_{Al} t_{Al} + S_{ar} t_{ar} = P$$  \hspace{1cm} (C - 23)

Case III: Specimen loaded, delamination length $b$
\[ P = S_{fa} t_{ar} \quad (C - 13) \]

With equations (C-11) and (C-12), \( S_{ar} \) and \( S_{Al} \) can be expressed in \( S_{fa} \)
With equation (C-22) the displacement \( \Delta y \) in case III is:
\[ \Delta y = b \cdot \frac{(S_{fa} - (S_{ar} + S_{r,ar}))}{E_{ar}} \quad (C - 24) \]

The work done by \( P \) as a result of the delamination is found with equation (C-13) and (C-24):
\[ F = P \Delta y = \frac{b t_{ar} S_{fa}}{E_{ar}} \cdot (S_{fa} - S_{ar} - S_{r,ar}) \quad (C - 25) \]

The increase of the strain energy in the fibre layer after delamination is (case III - case II):
\[ U_1 = \frac{b t_{ar}}{2 E_{ar}} \cdot \left\{ \frac{S_{fa}^2}{2} - (S_{ar} + S_{r,ar})^2 \right\} \quad (C - 26) \]

Similar for the decrease of the strain energy in the aluminium sheets (case II - case III):
\[ U_2 = \frac{b t_{Al}}{2 E_{al}} \cdot (S_{Al} + S_{r,al})^2 \quad (C - 27) \]

The total energy \( W \) which is generated during delamination, and which is consumed as 'surface' energy is found by substitution of equation (C-6), (C-25), (C-26) and (C-27) in equation (C-1):
\[ W = F - U = \frac{b t_{ar} S_{fa}^2}{2 E_{ar}} \cdot [\ldots] \quad (C - 28) \]

with:
\[ [\ldots] = 1 - 2 \left( \frac{S_{ar} + S_{r,ar}}{S_{fa}} \right) + \left( \frac{S_{ar} + S_{r,ar}}{S_{fa}} \right)^2 + \frac{t_{Al} E_{ar}}{t_{ar} E_{Al}} \cdot \left( \frac{S_{Al} + S_{r,al}}{S_{fa}} \right)^2 \quad (C - 29) \]

The corresponding energy release rate is:
\[ G^* = \frac{dW}{db} = \frac{t_{ar} S_{fa}^2}{2 E_{ar}} \cdot [\ldots] \quad (C - 30) \]

Substitution of equation (C-11), (C-12) and (C-22) into equation (C-29), and the result in equation (C-30) lead to:
\[ G^* = \frac{F_{Al}}{F_{ar} (F_{Al} + F_{ar})} \cdot \frac{t_{ar}^2}{2 j} \cdot \left[ S_{fa} - S_{r,ar} \cdot \frac{F_{ar} + F_{Al}}{F_{Al}} \right]^2 \quad (C - 31) \]

and after accounting for the number of fibre-aluminium interfaces:
\[ G_d = \frac{F_{Al}}{F_{ar}(F_{Al} + F_{ar})} \cdot \frac{t_{ar}^2}{2j} \cdot \left[ S_{fa} - S_{cr,ar} \cdot \frac{F_{ar} + F_{Al}}{F_{Al}} \right]^2 \]  \hfill (C - 32)

If \( S_{ar} = 0 \) equation (C-32) reduces to equation (C-21).

The energy release rate is zero if:

\[ S_{fa} = S_{cr,ar} \cdot \frac{F_{ar} + F_{Al}}{F_{Al}} \]

Substitution of this result in equation (C-12) and adding equation (C-22) leads to: \( S_l + S_{cr} = 0 \). It implies that the aluminium sheets are just unloaded (from a residual compression stress to a total stress equal to zero) when \( S_n \) is large enough to obtain \( G_d = 0 \). For larger values of \( S_n \) the energy release rate necessary for delamination is dependent on the difference between the actual \( S_n \) value and the \( S_n \) value where \( G_d = 0 \).

In section 5.3 it was pointed out that the term between the brackets in equation (C-32) is proportional to the load transfer from fibres to aluminium, and it is evident that the energy release rate depends directly on this load transfer.

The load transfer from fibres to aluminium can be characterized by a stress \( S_t \) (based on the thickness of the fibre-adhesive layer).

If there are no residual stresses:

\[ S_t = S_{fa} - S_{ar} \]  \hfill (C - 33)

Combination of equation (C-11) and (C-33) yields:

\[ S_t = \left( \frac{F_{Al}}{F_{Al} + F_{ar}} \right) \cdot S_{fa} \]  \hfill (C - 34)

Combination of equation (C-34) and (C-21) yields:

\[ G_d = \frac{(F_{Al} + F_{ar})}{2j F_{Al} F_{ar}} \cdot (t_{ar} S_t)^2 \]  \hfill (C - 35)

Because \( S_t \) is defined as that part of the fibre stress which is transferred to the aluminium layers, equation (C-35) is valid for every ARALL specimen, independently of the crack bridging efficiency, or the residual stress level. Equation (C-35) is more convenient for the calculation of \( G_d \) in ARALL crack growth specimens, although the stress \( S_t \) is not so easily calculated as \( S_n \) for the delamination specimen.
Figures

Figure C1. The change of stress levels after delamination.

Figure C2. Stresses in the different layers due to external loading and due to the residual stress system, before and after delamination.
Appendix D. The influence of adhesive plasticity on the COD, and the deformation field near the crack flanks

D.1 Introduction

The crack opening in ARALL due to adhesive shear deformation is one significant mechanism, which influences the stress intensity factor and the crack growth rate. Consequently, crack growth predictions for ARALL require analytical expressions for the crack opening displacement due to adhesive shear deformation. Some expressions which were derived for linear elastic material behaviour were presented in section 5. It was also demonstrated in chapter 5, that the yield stress of the adhesive may be exceeded considerably, if stresses according to a linear elastic model are present. Therefore an estimation will be made of the influence of adhesive plasticity, assuming a perfectly-elastic plastic adhesive behaviour. This estimation is presented in section D2.

The crack opening displacement was measured, on specimens with an artificial crack (described in section 4.7). A clip on gauge was used as a measuring device. It was not possible to apply the gauge directly at the crack flanks, and some corrections on these measurements are necessary. These corrections are presented in section D3. The method for the derivation of the corrections was proposed by Homan [D1].

D.2 The crack opening displacement COD

In section 5, an equation for the adhesive shear stress is presented (5.3.5). This equation can be written as:

$$\tau_y = A e^{-By} \quad (\text{D} - 1)$$

where:

$$A = S_{Al} t_{Al} \sqrt{\frac{F_{sd}}{j} \left(\frac{1}{F_{Al}} + \frac{1}{F_{sr}}\right)} \quad (\text{D} - 2)$$

and:

$$B = \sqrt{j F_{sd} \left(\frac{1}{F_{Al}} + \frac{1}{F_{sr}}\right)} \quad (\text{D} - 3)$$

where $j$ is the number of interfaces between the different layers, $F_{Al} = t_{sl} E_{sl}$, $F_{sr} = t_s E_s$ and $F_{sr} = G_d/\gamma$. The calculations of the COD in chapter 5 were based on these equations, assuming linear elastic behaviour.

The model of the specimen to be analysed is shown in figure D1.
Figure D2 shows the theoretical elastic shear stress distribution, according to equation (D-1). The maximum shear stress exceeds the yield stress of the adhesive. The area above the adhesive yield stress \( \tau_p \) is truncated, and a stress redistribution occurs. The area above \( \tau_p \) is compensated by increasing stresses in the elastic region. This is modelled by a shift of the elastic curve to the right.

At a distance \( y = y_n \) (as measured from the delamination front) the theoretical elastic shear stress is equal to the adhesive yield stress. The distance \( y_n \) follows from equation (D-1)

\[
y_n = -\frac{1}{B} \cdot \ln \frac{\tau_p}{A} \quad (D - 4)
\]

For negative results of \( y_n \), the yield stress is not exceeded and \( y_n = 0 \).

The area which is added to the elastic stress distribution is given by the product of \( \tau_p \) and \( y_p^* \), where \( y_p^* \) is the shifting distance (see figure D2). This area must be equal to the truncated area above the adhesive yield stress:

\[
\tau_p y_p^* = \int_0^{y_n} (\tau_y - \tau_p) dy = \int_0^{y_n} \left(Ae^{-by} - \tau_p\right) dy \quad (D - 5)
\]

Combining equation (D-4) and (D-5) yields:

\[
y_p^* = \frac{1}{B} \cdot \left\{ \left(\frac{A}{\tau_p} - 1\right) + \ln \left(\frac{\tau_p}{A}\right) \right\} \quad (D - 6)
\]

The size of the plastic zone in the adhesive \( y_p \) is equal to the sum of \( y_n \) and \( y_p^* \), and with the equations (D-4) and (D-6), it is found:

\[
y_p = \frac{1}{B} \cdot \left(\frac{A}{\tau_p} - 1\right) \quad (\text{for } \frac{A}{\tau_p} \geq 1) \quad (D - 7)
\]

If \( A/\tau_p < 1 \), the yield stress is not exceeded, and there is no plastic zone.

With equation (D-2) and (D-3), equation (D-7) is written as:

\[
y_p = \frac{S_{Al} l_{Al}}{j \tau_p} - \frac{1}{\sqrt{\frac{1}{F_{Ad}} \left(\frac{1}{F_{Ad}} + \frac{1}{F_{Al}}\right)}} \quad (D - 8)
\]

At the edge of the plastic zone (\( y = y_p \) in figure D1) the relative displacement \( v_{o-\tau_p} \) between the fibre layer and the aluminium sheets is easily obtained from \( \tau = \tau_p \) as:

\[
v_{y = y_p} = \frac{\tau_p}{G_{Ad}} \cdot l_{sd} = \frac{\tau_p}{F_{Ad}} \quad (D - 9)
\]
The relative displacement \( v_y \) at \( y = 0 \) is larger because of an increasing strain differences between the aluminium sheets and the fibre layers.

\[
v_y = \frac{\tau_p}{F_{ad}} \int_0^{y_p} (\varepsilon_{as,y} - \varepsilon_{AI,y}) dy
\]

\( (D - 10) \)

The strains are obtained from the corresponding stresses. Because the shear stress in the adhesive plastic zone is constant (\( \tau_p \)) it is easily seen that \( S_{AI} \) increases linearly from zero at \( y = 0 \) to:

\[
S_{AI,y} = \frac{j \tau_p y}{t_{AI}} \Rightarrow \varepsilon_{AI,y} = \frac{j \tau_p y}{F_{AI}} \quad (for \ 0 \leq y \leq y_p)
\]

\( (D - 11) \)

The stress in the fibre layers follows from the equilibrium:

\[
t_{AI} S_{AI,y} + t_{ar} S_{ar,y} = t_{ar} S_{ar,(y=0)}
\]

\( (D - 12) \)

\( S_{ar,(y=0)} \) is related to the remote stress in the aluminium layers ( \( S_{AI} \) ) by (see section 5.2):

\[
S_{ar,(y=0)} = S_{AI} \cdot \frac{F_{AI} + F_{ar}}{t_{ar} E_{AI}}
\]

\( (D - 13) \)

Converting stresses into strains, and substitution of the strains into equation (D-10), and integration lead to:

\[
v_y = \frac{\tau_p}{F_{ad}} + \frac{S_{AI}(F_{AI} + F_{ar}) y_p}{E_{AI} F_{ar}} - \frac{j \tau_p}{2} \cdot \left( \frac{1}{F_{AI}} + \frac{1}{F_{ar}} \right) \cdot y_p^2
\]

\( (D - 14) \)

Substitution of equation (D-8) in equation (D-14) yields, together with \( COD = 2 \ v_y \):

\[
COD = \frac{\tau_p}{F_{ad}} + \frac{S_{AI}^2 \varepsilon_{AI} \left( \frac{1}{F_{ar}} + \frac{1}{F_{AI}} \right)}{j \tau_p}
\]

\( (D - 15) \)

D.3 Deformation of the aluminium sheets near the crack flanks

Experimental results of the crack opening measurements were presented in section 4.7. The measurements were performed on specimens with an artificial crack. The width of the artificial crack is defined as \( e' \), see figure D1 and D3.

The gauge which was used for the measurements could not be applied directly at the crack flanks. The pins of the gauge contacted the aluminium sheets at some distance from the crack flanks. Consequently a deviation between the experimental result and the actual COD value is present. This deviation is caused by two displacement components:
1. The elongation of the aramid fibres in the artificial crack.
2. The displacement, due to the strain of the aluminium sheets, within the distance between the pins of the gauge.

The first component $\Delta e^*$ is given by:

$$\Delta e^* = \frac{e^*}{E_{ar}} = \frac{S_{Al}}{t_a} \cdot \left( \frac{1}{F_{ar}} + \frac{1}{F_{Al}} \right) \cdot e^*$$  \hspace{1cm} (D - 16)

The second displacement component $\Delta v_{Al}$ is given by:

$$\Delta v_{Al} = \int_0^{y_s} \epsilon_{Al,y} \, dy$$  \hspace{1cm} (D - 17)

where $y_s$ is the distance in the $y$ direction between the crack flanks and the pins of the gauge (figure D1). The strain $\epsilon_{Al,y}$ is a function of $y$, which depends also on the amount of adhesive plasticity.

Three situations will be considered:

1. There is no plastic zone.
2. The plastic zone is smaller than the distance $y_s$.
3. The plastic zone is larger than the distance $y_s$.

For the first case, with the boundary condition $S_{Al,y} = 0$ for $y = 0$, $S_{Al,y}$ can be written as:

$$S_{Al,y} = \int_0^y j \tau_y \, dy$$  \hspace{1cm} (D - 18)

After substitution of equation (D-1) in (D-18), and with $\epsilon = S/E$ it follows:

$$\epsilon_{Al,y} = \int_0^y \frac{j A e^{-By}}{F_{Al}} \, dy = \frac{j A}{B F_{Al}} (1 - e^{-By})$$  \hspace{1cm} (D - 19)

Substitution of equation (D-2), (D-3) and (D-19) in (D-17) yields:

$$\Delta v_{Al} = \frac{S_{Al}}{E_{Al}} \cdot \left\{ \frac{e^{-y_s} \sqrt{j F_{ar} \left( \frac{1}{F_{Al}} + \frac{1}{F_{ar}} \right)}}{\sqrt{j F_{ar} \left( \frac{1}{F_{Al}} + \frac{1}{F_{ar}} \right)}} - 1 + y_s \right\}$$  \hspace{1cm} (no plastic zone)  \hspace{1cm} (D - 20)

The total correction on the as measured COD value ($\Delta l$) is given by the sum of $\Delta e^*$ and $2 \Delta v_{Al}$.

The experimental crack opening displacement COD is then given by:
\[ COD = \Delta l - \Delta e^* - 2 \Delta v_{AI} \]  \hfill (D - 21)

where \( COD_{an} \) is the as measured \( COD \) value.

For the second case, where a small adhesive plastic zone occurs until \( y = y_r \), with \( y_r \leq y_p \), equation (D-18) is replaced by:

\[ S_{AI,y} = \int_0^y \frac{j \tau_p y}{t_{AI}} dy = \frac{j \tau_p y}{t_{AI}} \quad (0 \leq y \leq y_p) \]  \hfill (D - 22a)

and

\[ S_{AI,y} = \frac{j \tau_p y_p}{t_{AI}} + \int_{y_p}^y \frac{j \tau_y}{t_{AI}} dy \quad (y_p \leq y \leq y_B) \]  \hfill (D - 22b)

with \( \tau_y = \tau_p e^{-(y-y_p)/B} \)

After integration the results for the plastic and elastic zone become:

\[ S_{AI,y} = \frac{j \tau_p y}{t_{AI}} \quad (0 \leq y \leq y_p) \]  \hfill (D - 23a)

and

\[ S_{AI,y} = \frac{j \tau_p}{t_{AI}} \cdot \left\{ y_p + \frac{1 - e^{-B(y-y_p)}}{B} \right\} \quad (y_p \leq y \leq y_B) \]  \hfill (D - 23b)

with:

\[ \epsilon_{AI,y} = \frac{S_{AI,y}}{E_{AI,y}} \]

and integration according to equation (D-17) in two steps (from \( y = 0 \) to \( y = y_r \), and \( y = y_r \) to \( y = y_B \)) it follows:

\[ \Delta v_{AI} = \frac{j \tau_p}{F_{AI}} \cdot \left\{ y_p y_k - \frac{y_p^2}{2} + \frac{(y_k - y_p)}{B} + \frac{e^{-(y_k-y_p)/B} - 1}{B^2} \right\} \quad (y_p \leq y_B) \]  \hfill (D - 24)

The experimental \( COD \) value may now be derived, by substitution of equation (D-16) and (D-24) in equation (D-21).

For the third case, where the plastic zone exceeds the gauge length, it is easily recognized that:
\[ \Delta v_{AI} = \frac{j \tau_p y^2}{2 F_{AI}} \quad (y \leq y_p) \tag{D - 25} \]

The experimental COD values for the three cases are derived from the respective \( \Delta l \) measurements with the equations (D-16), (D-20), (D-21) and (D-25).

In figure D4 an example is given, where the COD values and the values of \( 2 \Delta v_{AI} \), according to the present calculations are plotted as a function of \( S_{AI} \). The specimen contains two aluminium layers with a thickness of 0.5mm each ( \( t_w = 1mm \) ), and an AF-163-2 Twaron HM prepreg.

It can be seen in figure D4 that the influence of plasticity on the COD is considerable at higher load levels. The influence of the plasticity on \( \Delta v_{AI} \) remains moderate.

References.

D1. J. Homan

Figures

Figure D1. Definition of the symbols for the present specimen.

Figure D2. Shear stress distribution at the delamination front.
Figure D3. Schematic illustration of the deformations and displacements.
Figure D4. Example of the influence of adhesive plasticity on the crack opening displacement COD and the correction factor $\Delta v_{al}$ and on the adhesive plastic zone size $y_p$. 

2Δv_{al} for $y_0=2.5\text{mm}$
2Δv_{al} for $y_g=1.25\text{mm}$
as measured COD values ($\Delta e^*$ ignored)

100 MPa
$Y_p \cdot 10^{-2}$ (eq. D-9)

F_{al} = 72,000 N/mm
F_{ar} = 12,400 N/mm
F_{ad} = 16,000 N/mm$^3$
$\tau_p = 38\text{ MPa}$

2x0.5mm Al+0.2 mm Ar Pre-preg

corrections $2\Delta v_{al}$

theoretical COD

0 $10^{-2}$ $2 \cdot 10^{-2}$ $3 \cdot 10^{-2}$ mm
Appendix E. Finite element calculation of the stress intensity factor in a finite width strip containing closely spaced parallel cracks

E.1 Introduction

In chapter 5 it is stated that the stress intensity factor concept for a fibre bridged crack, as a consequence of the adhesive shear deformation, is comparable to that of a row of parallel cracks with the same crack opening displacement.

Solutions for rows of parallel cracks are available from the literature [E1,E2,E3] for infinite specimen width. Isida [E3] presents solutions for parallel cracks in a finite width strip. However, not for cracks which are spaced very closely, as compared to the specimen width.

A geometry with very closely spaced parallel cracks \((h/w \leq 0.05)\) is especially relevant to ARALL. In this appendix a solution is presented for the finite width correction on the stress intensity factor for very closely spaced parallel cracks. The calculation of \(K\) was based on wellknown fracture mechanics relations, which are

\[
K = \sqrt{EG} \quad (E-1)
\]

where \(G\) is the strain energy release rate. \(G\) was derived from the compliance by using the relation

\[
G = -\frac{P^2}{2t} \cdot \left\{ \frac{\partial(v/P)}{\partial a} \right\} \quad (E-2)
\]

where \(P\) is the load and \(v\) is the displacement.

The solution for the compliance is based on finite element calculations. The F.E. calculations were performed by W. Ott for the following data.

- \(w = 40\)mm
- \(2h = 2\)mm
- \(\nu = 0.02\)mm
- \(t = 1\)mm
- \(E = 70,000 \text{ N/mm}^2\)
- \(\nu = 0.3\)

where \(w\) is the specimen width, \(h\) is the half distance between the cracks, \(\nu\) is the elongation (displacement) over a distance \(2h\), \(t\) is the specimen thickness, and \(\nu\) is the Poisson’s ratio.

The calculations were performed with a two dimensional programme assuming plane stress conditions and linear elastic material behaviour (see figure 1).
E.2 Finite element calculations

Figure E2 shows the modelling of the problem for the finite element calculations. For reasons of convenience the ligament length \( f = w/2 - a \) is taken as a parameter for the crack length. Isoparametric elements were used. The elements contain eight nodal points. A linear strain gradient within the elements is present. Modified crack tip elements which simulate the singularity are provided by a shift of the nodal points, as proposed by Henshell and Shaw [E1] (see figure E3). The shift is an easy technique to provide a change in the stress distribution in the elements, in such a way that the \( 1/\sqrt{r} \) behaviour is simulated (\( r \) is distance from the crack tip).

In table E1 some calculation results are presented for two mesh densities, and for meshes with and without modified crack tip elements. It can be seen in table E1 that the application of modified crack tip elements causes a very rapid convergence of the results, even if a coarse mesh would be applied. For shorter ligament lengths, the element size near the crack tip region was reduced. Table E2 shows the calculation results for different ligament lengths and mesh densities. The agreement of the results for different mesh densities suggests an accuracy of 0.05\% for the finest meshes.

The table gives also results of calculations where the ligament length contains only two and one 'singularity' element respectively (at \( f = 0.5\text{mm} \) and at \( f = 0.25\text{mm} \)). These results are hardly different from results with considerably finer meshes. The agreement indicates that calculations for very small ligament lengths, containing only one or two 'singularity elements' may be used too.

An additional calculation was performed for a ligament length \( f = 0.0625\text{mm} \) and a width of 5mm. The result was identical to the result for a width of 20mm and it is concluded that all present results are valid for \( a \geq 5h \). This is also revealed by the distribution of the reaction stresses which is presented in figure E4. The parts of the plate which are remote from the crack tip are practically unloaded.

E.3 Derivation of the energy release rate and the stress intensity factor

Because linear elastic material behaviour was assumed the finite element results can be presented in a more generalized form:

\[
P = \eta h \frac{t v E}{2 h} \tag{E - 3}
\]

Where \( \eta h \) may be regarded as the width of a hypothetical tensile specimen with the same compliance as the considered geometry. \( \eta \) is a dimensionless curve fitting parameter which accounts for the specimen geometry. \( \eta \) is a function of \( f/h \) only (for \( a > 5h \)). Equation (E-3) may also be written as:
\[ \eta = \frac{2P}{t \nu E} \quad (E - 4) \]

Because fixed displacement conditions were chosen, equation (E-2) may be written as:

\[ G = \frac{-\nu}{2t} \cdot \left( \frac{dP}{da} \right) \quad (E - 5) \]

Substitution of equation (E-3) in (E-5) yields:

\[ G = -\frac{\nu^2 E}{4} \cdot \frac{d\eta}{da} \quad (E - 6) \]

With \( \frac{d\eta}{da} = -\frac{d\eta}{df} \) and with equation (E-1) and (E-6) it follows:

\[ K = \frac{\nu E}{2} \cdot \sqrt{\frac{d\eta}{df}} \quad (E - 7) \]

The stress intensity factor may be calculated with equation (E-7), after the derivation of \( \eta \) as a function of \( f \) and \( h \). Such a function is obtained by a curve fitting process on the basis of the finite element solutions. Table E3 shows values of \( \eta \) as a function of \( f/h \). The values are derived from the finite element results with equation (E-4). The curve fitting result is:

\[ \eta = \frac{0.435}{\ln\left(1 + \frac{0.435h}{f}\right)} - 0.033(1 - e^{4.7\frac{f}{h}}) \quad (E - 8) \]

The values of \( \eta \) according to equation (E-8) are also given in table E3 to illustrate the high accuracy of the curve fitting. After differentiation of equation (E-8), substitution in equation (E-7) and some rewriting, the stress intensity factor is given by:

\[ K = C_{ad} \cdot \frac{\nu E}{2h} \cdot \sqrt{h} \quad (E - 9) \]

where

\[ C_{ad} = \sqrt{\frac{1}{\left\{ \left(\frac{f}{0.435h}\right)^2 + \frac{f}{0.435h} \right\} \ln^2(1 + \frac{0.435h}{f}) - 0.155e^{-4.7\frac{f}{h}}} \quad (E - 10) \]

Some values of \( C_{ad} \) are presented in table E4. The term \( \nu E/2h \) in equation (E-9) tends to the net section stress for large \( f/h \) values and \( C_{ad} \) tends to one. So the common solution \( K = S\sqrt{h} \) arises for large \( f/h \) values. Table E4 shows that \( C_{ad} \) remains close to 1 up to very small ligament lengths. It indicates that the influence of the finite width may be neglected for calculations on ARALL.
E.4 Error estimation

The accuracy of the present method is mainly dependent on the accuracy of the curve fitting process. The deviations of equation (E-8) from the FE results may cause some inaccuracy. Because equation (E-8) is differentiated, before it enters the equation for the stress intensity factor, the accuracy of the differential form is more important than the accuracy of the original form. The accuracy of the differential form may be estimated from table E5, it is observed that it is well within 3%. Due to the application of the square root, the accuracy of the final result as presented by equation (E-13) will be about 1.5%.

E.5 The behaviour at small ligament lengths

Although the influence of the finite width appears to be negligible for ARALL, the present solution is of some general importance. Therefore, it may be interesting to consider the quality of the present solution for $f/h$ ratios which are below the ratios of the present investigation. The present geometry shows a similarity to the solution in the book of Tada [E1, p.9.2] (see figure E5). The geometry yields identical $K$ values as the present geometry if the ligament length $f$ approaches zero, because the ratio $K/P$ is only dependant on the geometry and because the present geometry fulfills the essential condition of no rotation. The solution of Tada is:

$$
\frac{K}{P} = \frac{1.464}{\sqrt{f}} \quad (E-11)
$$

It will now be demonstrated that the $K/P$ ratio according to the present approach tends to equation (E-11) too, for $f/h$ ratios approaching zero. For $f/h \to 0$, the curve fitting parameter $\eta$ tends to

$$
\eta = \frac{0.435}{\ln\left(1 + \frac{0.435h}{f}\right)} \quad (E-12)
$$

Substitution of equation (E-12) in (E-3) and division by $t$ yields per unit of thickness:

$$
P = \frac{0.435 \nu E}{2 \ln\left(1 + \frac{0.435h}{f}\right)} \quad (E-13)
$$

Differentiation of equation (E-12) and substitution in (E-7) yields:

$$
K = \frac{0.435 \nu E}{2 \ln(1 + 0.435 h/t)} \cdot \frac{1}{\sqrt{f(fh + 0.435)}} \quad (E-14)
$$

Division of equation (E-14) by (E-13) yields
\[ \frac{K}{P} = \frac{1}{\sqrt[5]{f(\theta h + 0.435)}} \]  \hspace{1cm} (E - 15)

For \( \theta h \to 0 \) the result is \( K/P = 1.52 \) The difference with the exact solution of equation (E-11) is smaller than 4\%, so the present solution is also acceptably accurate for very small ligament lengths.

References.


E2. J.P. Benthem, W.P. Koiter  

E3. M. Isida  

E4. R. D. Henshell, K.G. Shaw  

Tables and figures

| External force \( P \) (N) at \( \theta = 10mm \) |
|-----------------|-----------------|-----------------|-----------------|
| normal elements | "singularity" elements |
| mesh type 1 | mesh type 2 | mesh type 1 | mesh type 2 |
| 7156.67 | 7140.50 | 7122.45 | 7124.25 |

| mesh type 1 is 80 square elements of 0.5\( \times \)0.5 mm  |
| mesh type 2 is 320 square elements of 0.25\( \times \)0.25 mm  |

Table E1. The influence of the size of the elements and the type of elements at the crack tip on the load to obtain a displacement \( \nu = 0.02 \) mm.
<table>
<thead>
<tr>
<th>ligament length $f$ (mm)</th>
<th>external force $P$ (N) (&quot;crack tip elements&quot;)</th>
<th>element length near crack tip reduced with a factor of two</th>
<th>element length near crack tip reduced with a factor of eight</th>
</tr>
</thead>
<tbody>
<tr>
<td>11</td>
<td>320 elements (0.25 mm $\times$ 0.25 mm)</td>
<td>7824.25</td>
<td></td>
</tr>
<tr>
<td>10</td>
<td></td>
<td>7124.25</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td></td>
<td>3624.25</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td></td>
<td>2224.25</td>
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</tr>
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<td></td>
<td>1524.23</td>
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</tr>
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<td></td>
<td>1174.09</td>
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</tr>
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<td>822.69</td>
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<td>644.94</td>
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<td>0.5</td>
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<td>465.02 *</td>
<td>464.79</td>
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<td></td>
<td>283.46 **</td>
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<td>166.14</td>
</tr>
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<td></td>
<td></td>
<td>139.88 *</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>109.28 **</td>
</tr>
</tbody>
</table>

* ligament length is two elements
** ligament length is only one (singularity) element

Table E2. Finite element calculation results for different ligament lengths and different mesh types.
<table>
<thead>
<tr>
<th>$\delta h$</th>
<th>$\eta$</th>
<th>(E-8)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>0.03125</td>
<td>0.1561</td>
<td>0.1564</td>
</tr>
<tr>
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<td>0.1998</td>
<td>0.2013</td>
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<tr>
<td>0.09375</td>
<td>0.2374</td>
<td>0.2397</td>
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<tr>
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<td>0.2723</td>
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<tr>
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<td>0.3061</td>
<td>0.3097</td>
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<td>0.375</td>
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</tr>
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<td>10</td>
<td>10.1775</td>
<td>10.1830</td>
</tr>
<tr>
<td>20</td>
<td>20.1775 *</td>
<td>20.1837</td>
</tr>
<tr>
<td>50</td>
<td>50.1775 *</td>
<td>50.1842</td>
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</tbody>
</table>

* extrapolation

Table E3. Empirical values of $\eta$ as compared to values resulting from curve fitting.
<table>
<thead>
<tr>
<th>$f/h$</th>
<th>$C_\omega$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0001</td>
<td>7.861</td>
</tr>
<tr>
<td>0.0003</td>
<td>5.214</td>
</tr>
<tr>
<td>0.0003</td>
<td>5.214</td>
</tr>
<tr>
<td>0.001</td>
<td>3.405</td>
</tr>
<tr>
<td>0.003</td>
<td>2.376</td>
</tr>
<tr>
<td>0.01</td>
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<td>0.03</td>
<td>1.293</td>
</tr>
<tr>
<td>0.1</td>
<td>1.077</td>
</tr>
<tr>
<td>0.2</td>
<td>1.028</td>
</tr>
<tr>
<td>0.4</td>
<td>1.011</td>
</tr>
<tr>
<td>1.0</td>
<td>1.005</td>
</tr>
<tr>
<td>2.0</td>
<td>1.002</td>
</tr>
<tr>
<td>4.0</td>
<td>1.000</td>
</tr>
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</table>

Table E4. The finite width correction factor $C_\omega$, for some values of the ligament length $f$. 
<table>
<thead>
<tr>
<th>f-range</th>
<th>range η(β,η)/range η</th>
</tr>
</thead>
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<tr>
<td>0</td>
<td>0.03125</td>
</tr>
<tr>
<td>0.03125 - 0.0625</td>
<td>0.0625 - 0.09375</td>
</tr>
<tr>
<td>0.09375 - 0.125</td>
<td>0.125 - 0.15625</td>
</tr>
<tr>
<td>0.15625 - 0.1875</td>
<td>0.1875 - 0.25</td>
</tr>
<tr>
<td>0.25</td>
<td>0.375 - 0.5</td>
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</tr>
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<td>0.5</td>
<td>0.75 - 1</td>
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Table E5. Estimation of the accuracy of equation (E-8) after differentiation.
Figure E1. A row of narrow spaced parallel cracks in a finite width strip and the finite element calculation geometry.
Figure E2. Finite element model for closely spaced parallel cracks in a finite width strip.

Figure E3. Modification of the elements at the crack tip.
Figure E4. Reaction stress distribution on the fixed ends of the specimen, for three ligament lengths.
Figure E5. Two geometries showing the same stress intensity factor for very small ligament lengths.
Appendix F. The influence of the finite width on the crack opening displacement, and the shape of the crack opening contour

The calculations of the crack bridging stresses in chapter 5 are based on the crack opening displacement. For an infinite sheet the crack opening displacement at the crack centre is given by:

\[
COD = \frac{4Sa}{E} \quad (F - 1)
\]

The crack contour is an ellipse, and the crack opening displacement at a distance \(x\) from the crack centre is:

\[
COD_x = \frac{4Sa \sqrt{1 - (x/a)^2}}{E} \quad (F - 2)
\]

For sheets with finite width, the COD is larger and the crack contour is no longer elliptical. The problem of the contour is treated here starting from a solution by Koiter [F1] for a row of collinear cracks (see figure F1). When one crack in such a row of collinear cracks is considered, the situation is very similar to a crack in a finite width strip. A difference occurs at the edges of the strip, which are unloaded, whereas stresses in the \(x\) direction occur for the case of a row of collinear cracks. Yet those stresses do hardly influence the displacements in the \(y\) direction near the crack, and they are often neglected.

Koiter presents an equation for the crack opening displacement of collinear cracks:

\[
COD_x = \frac{4Sw}{\pi E} \cdot \ln \left[ \frac{\cos(\pi x/w) + \sqrt{\cos^2(\pi x/w) - \cos^2(\pi a/w)}}{\cos(\pi a/w)} \right] \quad (F - 3)
\]

The crack contour as described by this equation is not an ellipse. It can be shown that for an infinite width \((w \to \infty)\) equation (F-3) reduces to equation (F-2).

The crack opening displacement at the centre of the crack will be considered first. Substitution of \(x = 0\) in equation (F-3) yields:

\[
COD_{x=0} = \frac{4Sw}{\pi E} \cdot \ln \left[ \frac{1 + \sin(\pi a/w)}{\cos(\pi a/w)} \right] \quad (F - 4)
\]

The wellknown finite width correction factor for the stress intensity is, assuming the collinear cracks case to be relevant:

\[
\frac{K}{K_{w=\infty}} = \sqrt{\frac{\tan(\pi a/w)}{\pi a/w}} \quad (F - 5)
\]
The finite width correction factor on COD for \( x = 0 \), is found from the equations (F-4) and (F-1), still assuming the collinear cracks result to be applicable.

\[
\frac{\text{COD}}{\text{COD}_{w=\infty}} = \frac{w}{\pi a} \cdot \ln \left\{ \frac{1 + \sin(\pi a/w)}{\cos(\pi a/w)} \right\}
\]  \hspace{1cm} (F - 6)

The two finite width correction factors according to equations (F-5) and (F-6) are presented in table F1. The equations are exact for the case of a row of collinear cracks.

For the case of a finite width strip, deviations from the equations (F-5) and (F-6) will occur. A solution for the stress intensity factor in an infinite strip is presented in the book of Tada [F2].

\[
\frac{K}{K_{w=\infty}} = \left\{ 1 - 0.1(\frac{a}{w})^2 + 0.96(\frac{a}{w})^4 \right\} \cdot \sqrt{\sec(\pi \cdot \frac{a}{w})}
\]  \hspace{1cm} (F - 7)

Equation (F-7) is claimed to be accurate within 0.1% for any \( a/w \) ratio.

The differences between the two finite width correction factors for the stress intensity according to the equations (F-5) and (F-7) are shown in table F2. The differences occur due to the stresses in the \( x \) direction, which are present in a sheet with collinear cracks (figure F1). The differences are small for small and intermediate \( 2a/w \) values, but they become rather large at \( 2a/w \) values which are close to one. A similar difference may be expected for the finite width correction factors for the crack opening displacement.

Tada presents an equation for the crack opening displacement at the centre of a crack in a finite width strip:

\[
\frac{\text{COD}}{\text{COD}_{w=\infty}} = -0.071 - 1.07(a/w) + 0.676(a/w)^2 - 0.72(a/w)^3
\]

\[+ 0.32(a/w)^4 - 0.5355(w/a) \ln(1 - 2 \cdot a/w)\]  \hspace{1cm} (F - 8)

Equation (F-8) is claimed to be accurate within 0.6% for any \( a/w \).

The finite width corrections on COD according to equation (F-6) and (F-8) are compared in table F3. The table also includes the comparison between finite width corrections on \( K \) (equations F-5 and F-7, data from table F2). The last column of table F3 shows the remarkable fact that the ratios of the correction factors for collinear cracks and for a finite width strip are very much similar for COD and \( K \), even up to high \( a/w \) values.

\( K \) represents the crack tip region (stress intensity). Apparently, there it a good deal of similarity between the width effects on COD, and on \( K \) for a row of collinear cracks and for a finite width strip. It then may be suggested that the same will apply for COD at intermediate locations along the crack flanks. Consequently, it is concluded that equation (F-3) predicts the relative shape of the crack contour well, also for a crack in a finite width strip. The absolute value of the crack opening displacement will be different for collinear
cracks and a crack in a finite width strip. A correction factor can now be derived from the above similarity.

In table F4 the shape of the crack contour according to equation (F-3) is compared to the shape of an elliptical crack opening with the same minor and major axis. It implies that the COD at the centre of the crack is taken as a reference for the comparison. The values in table F4 are then obtained as:

\[
\frac{[COD_{x=0}]}{[COD_{x=0}]} \text{ according to Eq. (F-3)}
\]

\[
\frac{[COD_{x=0}]}{[COD_{x=0}]} \text{ according to Eq. (F-2)}
\]

Table F4 shows that the shape of a the collinear cracks is no longer elliptical. The contour has a more flattened shape (see also figure F2). That will also apply to a crack in a finite width strip.

However, table F4 also shows that the differences remain rather small up to large \(a/w\) and \(x/a\) values, and the assumption of an elliptical contour for the calculations on ARALL will be sufficiently accurate. Based on the similarity between the finite width case and the collinear crack case a combination of equation (F-2) and (F-8) is then justified, which gives:

\[
COD_x = \frac{4S\sqrt{a^2 - x^2}}{E} \cdot \left\{ -0.071 - 1.07(a/w) + 0.676(a/w)^2 - 0.72(a/w)^3 + 0.32(a/w)^4 - 0.5355(w/a) \ln(1 - 2a/w) \right\}
\]

\[(F - 9)\]

If a more accurate equation is required the COD, equation (F-3) could be adopted in a similar way. It would lead to:

\[
COD_x = \frac{COD_x \text{ according to eq. (F - 3)}}{COD_{x=0} \text{ according to eq. (F - 4)}} \cdot COD_{x=0} \text{ according to eq. (F - 8)}
\]

References.

F1. W.T. Koiter

### Tables and figures

<table>
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<tr>
<th>$2a/w$</th>
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Table F1. Comparison of the finite width correction factors for the crack opening displacement at the centre of the crack, and for the stress intensity factor. Calculated on the basis of a row of collinear cracks.
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Table F2. Comparison of different finite width corrections for the stress intensity factors.
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Table F4. The crack opening displacement along the crack flanks for a finite width sheet, as compared to a crack with an elliptical crack opening (infinite sheet).
Figure F1. A row of collinear cracks.

Figure F2. Deviation of the crack opening contour from the elliptic shape, in a finite width strip.
Appendix G. The stress intensity factor of small cracks at notches

G.1 Introduction

In chapter 4 fatigue tests are described, which were performed on double edge notched specimens. Aspects to be investigated are the crack initiation behaviour, the growth of very small cracks (0.05-0.5 mm) in the notch root, and the growth of the (small) cracks at the sides of the specimen, when the cracks have grown through the thickness. The behaviour of small cracks is explained in terms of stress intensity factors. In this appendix solutions for the stress intensity factors of cracks at notches are presented (for isotropic homogeneous material). For cracks in the notch root a three dimensional approach is essential. For through the thickness cracks a two dimensional approach is sufficient. This last approach is presented first.

G.2 The stress intensity factor of a through the thickness crack at a notch

The presently investigated specimen is similar to a specimen which was investigated by Foth [G1,G2]. He proposed a calculation scheme, which is based on finite element calculations of the notch stress field without a crack. The finite element calculations were performed by W. Ott (see specimen geometry and results in figure G1). The K-calculation based on finite element calculations will be compared to equations proposed by Schijve [G3] and Jergéus [G4].

Figure G2a illustrates the wellknown principle to obtain the stress intensity factor of a crack by adopting the stress field for an uncracked situation. The stress distribution for the uncracked situation is applied to the crack flanks of an externally unloaded sheet. The stress intensity factor is then obtained by integrating the $dK$ increments of all $Sdx$ increments acting on the crack flanks. This procedure yields exact results for the stress intensity factor, if a semi-infinite sheet is considered. Figure G2b shows a similar approach for a notch edge crack, which avoids difficult calculations for the influence of the notch geometry. Due to the omission of a part of the sheet the result is no longer exact. Yet reasonably accurate results will still be obtained if the crack length is much smaller than the notch root radius $\rho$. For the application of this method, the notch stress field has to be known. For the present specimen, finite element results are available. The maximum stress $S_{\text{max}}$ at the notch root is given by $S_{\text{max}} = K_r \cdot S_\text{net}$, where $K_r$ is the elastic stress concentration factor and $S_\text{net}$ is the nett section stress. For the present specimen is: $K_r = 3.3$. Calculated on the gross section, the stress concentration factor $K_{\text{gross}} = 4.7$. The stress at distance $x$ from the notch root is given by the following polynomial, which is developed on the basis of FE results.
\[ \frac{S_x}{S_{peak}} = 1 - 0.616x + 0.356x^2 - 0.133x^3 + 0.022x^4 \quad (0 < x < 2\text{mm}) \quad (G - 1) \]

The stress distribution over the full width of the specimen is presented in figure G1 and in table G1. With equation (G-1) the stress intensity factor can be calculated by integration. For this integration the solution presented by Tada [G5, p. 8.3] is used.

\[ K = \int_{0}^{a} \frac{2F(x/a)S_x}{\sqrt{\pi a} \{1 - (x/a)^2\}} \, dx \quad (G - 2) \]

Tada presents \( F(x/a) \) in the form of a graph only. A polynomial has been developed for this graph:

\[ F(x/a) = 1.3 - 0.027(x/a) - 0.369(x/a)^2 - 0.086(x/a)^3 + 0.182(x/a)^4 \quad (G - 3) \]

After substitution of equation (G-3) and equation (G-1) in equation (G-2) the stress intensity \( K \) can be calculated as a function of \( S_{peak} \). The integration is performed numerically. The results of the integration are presented in Figure G3 and Table G2.

A well-known and simple approximation for a notch edge crack is obtained by adding the notch depth to the crack length (see also [G6]). The crack is then considered as a single edge crack.

Using this approach, the stress intensity factor of the crack is calculated with:

\[ K = S_{gross} Y \sqrt{\pi (a + l)} \quad (G - 4) \]

where \( l \) is the depth of the notch and \( Y \) represents a geometry factor. The approximation is quite good if the crack length \( a \) is not small as compared to the notch root radius \( q \) (\( a > q \)), while it is asymptotically correct for increasing values of \( a \). This was clearly confirmed by the work of Nisitani [G7]. For the present specimen the geometry factor \( Y \) is found in the book of Tada [G5] (p. 2.6. and 2.7).

\[ Y = \left[ 1 + 0.122 \cos^4 \{\pi (a + l)/w\} \right] \cdot \sqrt{\frac{w}{\pi (a + l)}} \cdot \tan\{\pi (a + l)/w\} \quad (G - 5) \]

The result of this approach is also presented in Figure G3 and Table G2. It is easily recognized that this approach yields poor results for small \( a \) values. However, at higher crack lengths the result becomes more accurate.

The stress field near the notch may be described by two parameters, the maximum stress \( S_{peak} \) at the notch root, and the notch root radius. If these parameters are the same, the stress fields near the notch are approximately similar for different specimen geometries. The same argument is valid, when the stress intensity of a small crack at the notch root is considered. Consequently, it can be stated that the stress intensity factor of a small crack in a notch stress field is only dependent on the maximum stress at the notch root (\( S_{peak} = K S_{net} \)), the notch
root radius, and the crack length. Considering the long crack solution, it is even possible to develop an interpolation solution which describes the whole regions of crack length. Schijve [G3] and Jergéus [G4] derived equations based on the considerations mentioned above.

The solution by Schijve is:

\[ K = C S_{\text{peak}} \sqrt{\pi a} \]  \hspace{1cm} (G - 6)

with:

\[ C = 1.1215 - 3.12(a/\ell) + 5.16(a/\ell)^{1.5} - 3.73(a/\ell)^2 + 1.14(a/\ell)^{2.5} \]  \hspace{1cm} (G - 7)

The result of this approach is also plotted in Figure G3. Schijve checked equation (G-7) with data from calculations by Newman for a centre notched sheet and found equation (G-7) to be valid up to \( a/\ell \) values of one. For edge notched semi-infinite specimens Schijve checked the equation with calculations by Nisitani and found an excellent agreement up to \( a/\ell \) values of 0.4. The results for the present specimen suggest also a good accuracy for higher \( a/\ell \) values, since the "long crack" curve is approached quite well. Obviously the influence of the finite widths of the specimen is insignificant.

The negligible effect of the specimen width is confirmed by calculations of Nisitani [G7] on the peak stress at the notch root of an elliptical edge notch in a semi-infinite sheet. Taking one semi-axis \( a \) equal to the depth of the present notch and calculating the other \( \beta \) (short axis) from \( \ell = \beta^2/a \) the result is \( a/\beta = 11.25 \text{mm} / 6.38 \text{mm} = 1.76 \) Nisitani presented tabular results for different axis ratios. These results are presented below.

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The results do not deviate much from the classical result for an infinite sheet: \( K_c = 1 + 2 a/\beta \). For the semi-elliptical edge notch the results also indicate a nearly linear increase of \( K_{c_{\text{gross}}} \) with \( a/\beta \). A linear interpolation for \( a/\beta = 1.76 \), gives \( K_{c_{\text{gross}}} = 4.70 \). This \( K_{c_{\text{gross}}} \) value is identical to the value obtained from the finite element calculations, which confirm the negligible effect of the specimen width on the stress at the notch root.

Another interpolation for the stress intensity factors of cracks in a notch field is presented by Jergéus [G4], who considered cracks at edge notches. His equation is:

\[ K = 1.1215 S_{\text{gross}} \sqrt{\pi a_{\text{eff}}} \]  \hspace{1cm} (G - 8)

with
\[ a_{\text{eff}} = a + l \left( 1 - e^{-4(1+\sqrt{\frac{a}{l}})} \right) \]  

\[(G - 9)\]

The stress intensity factors according to the equations (G-2), (G-6), (G-8) and (G-4) are compared in table G2 and figure G3. The results from (G-2), (G-6) and (G-8) agree for very small crack lengths because they approach the asymptotic solution for \( a \to 0 \). However, equation (G-2) exceeds both other solutions if the crack length increases. At \( a=0.5\text{mm} \) the difference with (G-8) is about 7% and at \( a=1\text{mm} \) even the long crack solution is exceeded. The equations (G-6) and (G-8) approach the long crack quite well. The equation of Jergéus (G-8) shows the best fit to the long crack data at higher crack length. For the largest crack lengths small differences occur due to the finite width influence, which is incorporated in the long crack curve (equation G-4), but not in the equation of Jergéus (G-8). For the crack lengths to be considered here the differences due to the finite width are small.

Comparing the four curves in Figure G3 it is recognized that at a crack length of 1.5 mm (\( a/l = 0.4 \)) the long crack solution is also reached by the equations (G-6) and (G-8). The results after Schijve and after Jergéus follow the long crack curve in an acceptable way for longer crack lengths. The results calculated from the notch stress field according to the finite element results show an "overshoot" as compared to the long crack curve. The method appears to be valid for small \( a/l \) values only. An advantage for the approach by Jergéus is that the whole range of crack lengths which is relevant to the present investigation is covered with one equation.

G.3 Quarter circular corner cracks in a notch stress field

In the very early crack growth stage, the cracks are not yet through the thickness cracks. In the present investigation the cracks usually started as corner cracks at the notch root. Crack growth observations showed that the cracks had grown as quarter circular corner cracks. Solutions for the stress intensity factors of such cracks will be derived here. The approach for through the thickness cracks was two dimensional. However, corner cracks obviously imply three dimensional aspects. For both cases the stress field at the notch (stress gradient) is important. Data for corner cracks at circular holes are presented in the literature [G8] as tabular data and as curve fitting equations. These data apply to notch radius to thickness ratios of 0.5 and 1, which are much smaller than for the present specimens (\( q/t = 7.2 \)) and to a lower stress concentration factor \( K \) (\( K_{\text{eq}} = 3 \), here \( K_{\text{eq}} = 4.7 \)). Therefore another approach is considered first. The influences of the finite sheet thickness, and of the stress gradient are considered separately.

The stress intensity factor of an embedded circular crack in an infinite solid, loaded by a stress perpendicular to the crack plane, is given by:

\[ K = \frac{2}{\pi} S \sqrt{\pi a} \]  

\[(G - 10)\]
Newman and Raju [G8] calculated stress intensity factors for quarter elliptical corner cracks with the finite element method. From the results they derived analytical expressions using curve fitting techniques. The expressions are given in the form of correction functions on equation (G-10).

\[ K = S \sqrt{\pi a/Q} \cdot F \left( \frac{a_1}{a_2}, \frac{a_1}{t}, \varphi \right) \quad (G - 11) \]

where \( t \) is the specimen thickness, \( a_1 \) and \( a_2 \) are the semi-axis lengths of the quarter elliptic crack, and \( \varphi \) is the location angle, see figure G4. For quarter circular cracks \( \varphi \) is equal to the angle with the axis "a_2". \( Q \) ( \( \sqrt{Q} \) is the so-called shape factor) is given by the following equations:

\[ Q = 1 + 1.464 \left( \frac{a_1}{a_2} \right)^{1.65} \quad \text{for} \quad \frac{a_1}{a_2} \leq 1 \quad (G - 12a) \]

\[ Q = 1 + 1.464 \left( \frac{a_2}{a_1} \right)^{1.65} \quad \text{for} \quad \frac{a_1}{a_2} > 1 \quad (G - 12b) \]

For quarter circular cracks \( a_1/a_2 = 1 \) and the equations given by Newman and Raju in [G8] considerably simplify to:

\[ K = \frac{2}{\pi} S \sqrt{\pi a} \cdot F \left( \frac{a}{t}, \varphi \right) \quad (G - 13) \]

with:

\[ F \left( \frac{a}{t}, \varphi \right) = g_1 g_2 \left\{ 1.05 + 0.375 \left( \frac{a}{t} \right)^2 - 0.25 \left( \frac{a}{t} \right)^4 \right\} \quad (G - 14) \]

where:

\[ g_1 = 1 + \left\{ 0.08 + 0.4 \left( \frac{a}{t} \right)^2 \right\} \cdot (1 - \sin \varphi)^3 \quad (G - 15) \]

\[ g_2 = 1 + \left\{ 0.08 + 0.15 \left( \frac{a}{t} \right)^2 \right\} \cdot (1 - \cos \varphi)^3 \quad (G - 16) \]

Some results for the free surface crack tip (A and B in figure G4) are presented below. The results for \( a/t = 0 \) are relevant to very small cracks.

| Location (fig. G5) | \( F(a/c, a/t, \varphi) \) for \( a/c = 1 \) |
|-------------------|-----------------|-----------------|-----------------|
|                   | \( a/t = 0 \)   | \( a/t = 0.2 \) | tab. FEM data   |
|                   | (G-14)          | (G-14)          |                 |
| A (\( \varphi = \pi/2 \)) | 1.134           | 1.156           | 1.159           |
| B (\( \varphi = 0 \))      | 1.134           | 1.167           | 1.162           |
The results for \( a/t = 0.2 \) indicate that the curve fitting equations agree quite well with the tabular data from which they were derived. Newman and Raju claim an agreement within 5%. Obviously, the accuracy is better for the crack tip at points A and B. Newman and Raju assume that the tabular FE results are about 1.5% below the exact solutions. This accuracy is now briefly considered for an embedded circular crack at mid thickness with \( a/t = 0.2 \) (\( a = \) half crack length, \( t = \) half specimen width). The tabular data in [G8] give \( F_{A,B} = 0.986 \). The exact solution for infinite thickness gives \( F_{A,B} = 1 \) (equation G-10). The FE result is 1.4% lower. According to the finite thickness effect the result should be higher in stead of lower. So the FE result is at least 1.4% below the exact solution.

Figure G4 shows how the Raju and Newman corner crack model will be compared to the present notched specimen. The comparison ignores the stress gradient effect (it will be added at a later stage). In figure G5 the results for \( F_A \) and \( F_B \) are presented as a function of \( a/t \). Tabular data and curve fitting results (equations G-14 to G-16) are shown both. The limited accuracy of the curve fitting for these cracks is obvious.

So far the influence of the stress gradient at the notch is not considered and an additional correction factor \( T_A \) and \( T_B \) for the influence of the stress gradient on the stress intensity factor must be derived. Analogous to the method which is applied for the two dimensional crack, it is recognized that \( S_{\text{root}} \) at the notch root is relevant. For the small crack an additional correction factor for the stress gradient must then be determined. Again different approaches are investigated. At first a simple two dimensional approach is considered. It is assumed that the correction for the two dimensional case, as determined with equations (G-2) and (G-6) may also be applied for the quarter circular corner crack at the locations A and B. The correction factors are easily derived from the results in Table G2 by dividing the \( K/S_{\text{root}} \) values of this table by the corresponding values of \( 1.1215\sqrt{\pi a} \). Or according to the equation by Schijve (G-6)

\[
T_{A,B} = C/1.1215 \quad (G - 18)
\]

where \( T \) represents the the correction factor for the stress gradient, while the indices A and B refer to the location of the crack tip. The results are presented in Table G3.

Foth [G1] also calculated the influence of the stress gradient separately for the locations A and B in a three dimensional model. He again adopted the expression for the stress distribution (according to equation (G-1)). Analogous to the two dimensional calculation described in section G2 he derived correction factors \( T_A \) and \( T_B \) for a "replacing" geometry where the crack surfaces are loaded by the stress distribution which is present at the notch in the uncracked condition. The replacing configuration is a penny shaped crack in an infinite solid. The location of the notch root with the high peak stress is "positioned" in a symmetry plane. Then, similar to the procedure which is described for the two dimensional case, the local stresses are treated as stresses on the crack flanks. In order to obtain symmetry to the "notch root plane", these stresses are assumed to be present at both sides of this plane (see Figure G6).
The stress intensity factor at any point of the crack front can be obtained by integrating the contribution of all elements of the crack surface. The solution for $K$ of a load point $(r, \varphi)$ is given by Tada [G5,p.24.2] The integration has been performed numerically for points A and B (see the equations in figure G7). Dividing the stress intensity by $K = 2S_{\text{peak}} \sqrt{a/\pi}$ (solution for uniform stress distribution) yields the values $T_A$ and $T_B$ respectively. The results are presented in Table G3 and figure G8.

The results for $T_A$ and $T_B$ can be approximated by:

$$T_A = 1 - 0.392 \frac{a}{Q} + 0.337 \left(\frac{a}{Q}\right)^2 - 0.175 \left(\frac{a}{Q}\right)^3$$  \hspace{1cm} (G \text{-} 19a)

$$T_B = 1 - 1.534 \frac{a}{Q} + 2.148 \left(\frac{a}{Q}\right)^2 - 1.373 \left(\frac{a}{Q}\right)^3$$  \hspace{1cm} (G \text{-} 19b)

The equations (G-19) describe the calculated $T_A$ and $T_B$ values for $a/Q$ values up to $a/Q = 0.5$. However, it was found for the two dimensional case that the integration method may suffer from an overestimation of the $K$ values of about 15% at $a/Q = 0.5$. For the present problem smaller values are relevant ($a/Q = 0.1$) and analogous to the two dimensional case the error may be within 6%.

At this stage, the calculation of the stress intensity factor is possible using the previous equations. For reasons of convenience the functions $F_A$ and $F_B$ are combined with $T_A$ and $T_B$ for the present specimen:

$$F_{TA} = F_A T_A \quad \text{and} \quad F_{TB} = F_B T_B$$  \hspace{1cm} (G \text{-} 20)

It is assumed that the correction factors $F_{TA}$ and $F_{TB}$ account for the combined influence of the stress gradient at the notch and the finite thickness. The resulting stress intensities are then calculated with

$$K_A = \frac{2}{\pi} F_{TA} S_{\text{peak}} \sqrt{\pi a}$$  \hspace{1cm} (G \text{-} 21a)

$$K_B = \frac{2}{\pi} F_{TB} S_{\text{peak}} \sqrt{\pi a}$$  \hspace{1cm} (G \text{-} 21b)

For the present specimen $t=0.5 \text{mm}$ and $d=3.6 \text{mm}$ ($t/Q = 0.138$). The values of $F_{TA}$ and $F_{TB}$ are presented in figure G9, as a function of the crack length. It can be seen in figure G9 that the difference between $F_{TA}$ and $F_{TB}$ does not exceed 5%. High values of $F$ are compensated by lower values of $T$. Considering that $F_{TA}$ and $F_{TB}$ are almost equal, is not surprising that the cracks grow in a quarter circular shape indeed, because the differences between $K_A$ and $K_B$ are rather small.

In order to enable an easy calculation of $K_A$, the function $F_{TA}$ is approximated by the following equation:

$$F_{TA} = 1.134 - 0.603 \frac{a}{Q} + 38.0 \left(\frac{a}{Q}\right)^2 - 139 \left(\frac{a}{Q}\right)^3 \quad \text{(for} \ t/Q = 0.138 \text{)}$$  \hspace{1cm} (G \text{-} 22)
Equation (G-22) is only valid for the present specimen thickness to notch root radius ratio.

In the previous calculations it was assumed that the correction factor for the influence of the stress gradient may be applied separately from the correction factor for the finite thickness, and that the correction factor for the combined influence of the notch stress field and the finite thickness can be obtained by a multiplication of the individual correction factors. This assumption causes an additional inaccuracy of the result. It is difficult to estimate quantitatively this inaccuracy. Because the individual correction functions are not very large, it may be assumed that the overall result yields a reasonable estimation of the stress intensity factors of the corner cracks in the notch of the present specimen. At location A (in the notch root) the stress intensity for the present specimen is found by substitution of equation (G-22) in (G-21). After some rewriting it follows:

\[ K_A = \{0.72 - 0.38(a/q) + 24(a/q)^2 - 89(a/q)^3\}S_{peak}\sqrt{\pi a} \quad \text{(for } u/q = 0.138) \]  \hspace{1cm} (G - 23)

References.

G1. J. Foth  

G2. J. Foth, R. Marissen, H. Nowack, G. Lütjering  

G3. J. Schijve  


G6. D. Broek  
G7. H. Nisitani

G8. J.C. Newman Jr, I.S. Raju
Tables and figures

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Table G1. Stress distribution in the double edge notched specimen, according to the finite element calculations.
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Table G2. Stress intensity factors of cracks at a notch, as calculated with different equations.
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Table G3. Correction factors for the stress gradient, on the stress intensity factor of a corner crack at the notches of the presently investigated specimen.
Figure G1. Geometry of the double edge notched specimen and the stress distribution in the notched area.
Figure G2. Similar edge crack situations with similar stress intensity factors.
Figure G3. The stress intensity factor as a function of the crack length.

Figure G4. A corner crack in a notched specimen, compared to an infinite plate.
Figure G5. Correction factors for the free edges and the finite thickness.
Figure G6. Stress distribution on the projected crack area, to be used in figure G7.

\[
K_A = \frac{2\pi a}{\int_0^\pi \int_0^r S(\varphi, r) r \sqrt{a^2 - r^2} \, d\varphi \, dr}{\int_0^\pi \int_0^r \frac{a^2 + r^2 - 2ar \cos(\varphi - \pi/2)}{a^2 + r^2 - 2ar \cos(\varphi)} \, d\varphi \, dr}
\]

\[
K_B = \frac{2\pi a}{\int_0^\pi \int_0^r S(\varphi, r) r \sqrt{a^2 - r^2} \, d\varphi \, dr}{\int_0^\pi \int_0^r \frac{a^2 + r^2 - 2ar \cos(\varphi)}{a^2 + r^2 - 2ar \cos(\varphi)} \, d\varphi \, dr}
\]

Figure G7. Penny shaped crack in an infinite solid loaded in each quadrant with a crack flank stress distribution \(S(\varphi, r)\) as shown in figure G6.
Figure G8. Correction factors accounting for the stress gradient effect on the stress intensity factor of a quarter circular corner crack at a notch.

Figure G9. The correction functions for the influence of the free edges, the stress gradient and the finite thickness, on the stress intensity factor of a circular corner crack at the notch of the present specimen.
Appendix H. The stress intensity factor of a long crack bridged by springs on the crack flanks

In chapter 5 it was argued that the stress intensity factor of a fibre bridged crack, without delamination, but with adhesive shear deformation, may be calculated, using the analogy of a row of parallel cracks in an unreinforced sheet. The stress intensity factor for a row of parallel cracks increases with increasing crack length up to a certain asymptotic value. A further increase of the crack length does not cause an increase of the stress intensity factor \( K \) or the crack opening displacement \( COD \). Qualitatively, the behaviour of a bridged crack is the same. It was assumed in section 5 that the stress intensity factors for both cases are the same, if similar crack opening displacement profiles occur. As a characteristic indication the crack opening displacement \( COD \) away from the crack tip was adopted, and the stress intensity for the fibre bridged crack was calculated for the assumption that the stress intensity factor of a bridged crack is equal to that of parallel cracks if the \( COD \) values away from the crack tip are the same. Actually, \( K \) is related to \( COD \) values near to the crack tip and it is not directly obvious that the \( COD \) values away from the crack tip may be used as an indication for the stress intensity factor. This assumption will be checked here for long cracks, where the \( COD \) and the \( K \) values have reached their upper asymptotic values.

The load transfer from the fibres to the aluminium sheets implies adhesive shear stresses and deformations very near to the crack flanks. The load transfer is proportional to the crack opening displacement, if linear elastic adhesive behaviour is assumed. In this appendix it is assumed that the stress intensity due to adhesive shear deformation may be calculated for the situation of an externally loaded homogeneous sheet, containing a single crack which is bridged by linear elastic springs on the crack flanks, see figure H1. Because the load transfer is assumed to be proportional to the local \( COD \) all springs have the same stiffness. The model ignores that the shear stresses of the crack bridging fibres do not act on the crack flanks, but on the sheet surface over a small distance to the crack flanks. Adhesive plasticity is also neglected.

The stress intensity factor will be calculated by using the energy release rate approach. In appendix C and E, the energy release rates are derived from the complete energy balance. However, the energy release rate can also be derived from the elastic energy only if a fixed grip situation is assumed \([H1]\).

\[
G = - \left( \frac{dU}{da} \right) \quad \text{ (fixed grips)} \quad (H - 1)
\]

where \( G \) is the energy release rate, and \( U \) is the elastic energy per unit of thickness, \( a \) is the crack length. The stress in the springs remote from the crack tip is equal to \( S \), where \( S \) is the remote stress on the sheet.
The increment of the elastic energy in the springs \((dU_t)\) for an incremental crack extension \((da)\) is per unit of thickness and per crack tip can be written as:

\[
dU_t = \frac{COD \cdot S}{2} \cdot da \tag{H - 2}
\]

\(COD\) is the crack opening displacement away from the crack tip. The increment of elastic energy per unit of thickness in the aluminium sheets \(dU_{Al}\) decreases.

\[
dU_{Al} = -COD \cdot S \cdot da \tag{H - 3}
\]

With \(dU = dU_t + dU_{Al}\) it follows that:

\[
dU = -\frac{COD \cdot S}{2} \tag{H - 4}
\]

In chapter 5 a \(COD\) equation for parallel cracks has been obtained:

\[
COD = \frac{2S}{E} \frac{h}{h} \tag{H - 5}
\]

where \(E\) is Young’s modulus of the homogeneous sheet. This \(COD\) value for parallel cracks is equal to \(COD\) for fibre bridged cracks if \(h\) is (see section 5.6):

\[
h = F_{Al} \cdot \sqrt{\frac{1}{jF_{ad}F_{Al}}} + \frac{1}{jF_{ad}F_{ar}} \tag{H - 6}
\]

Consequently, the \(COD\) value of a fibre bridged crack may also be expressed by equation (H-5) if \(h\) is given by equation (H-6). Substitution of equation (H-5) in (H-4) yields:

\[
dU = -\frac{hS^2}{E} \cdot da \tag{H - 7}
\]

And with equation (H-1):

\[
G = \frac{h \cdot S^2}{E} \tag{H - 8}
\]

The stress intensity factor \(K\) is calculated with:

\[
K = \sqrt{E} \cdot \frac{h}{E} \tag{H - 9}
\]

The final result is:

\[
K = \frac{S\sqrt{h}}{} \tag{H - 10}
\]

This is the same result as it was found for parallel cracks (see section 5.6). The stress intensity factor of a spring bridged crack appears to be the same as for a row of spring bridged crack, if the \(COD\) values away from the crack tip are the same. Obviously, the crack opening contour for a spring bridged crack and for parallel cracks is very similar. The
similarity of a spring bridged and a fibre bridged crack is evident, and it can be assumed that the stress intensity factor of a fibre bridged crack may be derived from the analogy of a row of parallel cracks too.

References

H1. D. Broek

Figures

Figure H1 Quarter part of the symmetrical situation of a crack loaded by an external stress and by elastic springs on the crack flanks.
Appendix I. The influence of delamination on the stress intensity factor due to adhesive shear deformation

1.1 Description of the calculation model

As pointed out in chapter 3 and 5 the crack bridging forces become smaller if delamination occurs and also as a result of shear deformation of the adhesive between the fibres and the aluminium layers. Both effects are treated separately in chapter 5. The final stress intensity is obtained by superposition of both contributions. The stress intensity factor due to delamination is derived for zero adhesive shear deformation. A problem is that the effect on the \( K \) value caused by the shear deformations is also depending on the amount of delamination. The effect of adhesive shear deformation is related to the magnitude of the crack bridging forces and as a consequence to the amount of delamination. The corresponding effect on \( K \) is the main topic of this appendix. The shear deformation effect on \( K \) and on the crack opening contour will be considered first for the case of "no delamination": (section I2), followed by a section (I3) on the additional delamination effect. Below the method applied for the present considerations will be discussed first.

It was discussed in section 3 and 5 that delamination occurs preferably at locations along the crack flanks with high crack bridging forces. As a result the crack bridging forces are reduced at those locations and a uniform distribution of the crack bridging forces is approached. On the other hand it was shown for the case of "no delamination" that the crack bridging forces in a limited region near the crack tip are reduced as a consequence of adhesive shear deformation. The stress intensity factor due to adhesive shear deformation is closely related to the reduction of crack bridging stress near the crack tip. Apparently there are two opposite tendencies: the approach to a uniform crack bridging stress distribution as a consequence of delamination on the one hand, and the effect of reduced crack bridging near the crack tip as a consequence of adhesive shear deformation on the other hand. The two tendencies yield complicated interactions with a significant effect on the stress intensity factor. However, a typical feature of the fatigue behaviour of ARALL offers an approach which allows a reasonable estimation of the resulting stress distribution, without a detailed consideration of geometrical aspects. As said before delamination occurs preferably at locations with a high crack bridging stress. Consequently, a reduction of the crack bridging stresses occurs predominantly at these locations. It is assumed for the present approach that the delamination does not extend into the region near the crack tip where the crack bridging stress is reduced already as a consequence of adhesive shear deformation, and the stress distribution near the crack tip is not affected by delamination. Figure 11 and 12 show how the crack bridging stress distribution changes at increasing delamination. It is observed that the effect of delamination may be described with a truncation of the original stress distribution to a lower maximum value of the crack bridging stress. The present approach is based on this truncation model. Aspects of a possible stress redistribution or some minor delamination in the "assumed non delaminated area" are ignored. Roebroeks
[11] indeed found from scanning electron microscope investigations, that delamination lags behind the crack tip. He concluded that this would cause a high crack bridging stress near the crack tip. In view of the present modelling it is concluded that the lack of delamination closely behind the crack tip is a consequence of a small crack bridging stress.

The calculations of stress intensity factors in chapter 5 are divided in two parts. In the first part, the influence of delamination only is considered and a uniform stress distribution is adopted. In the second part the influence of adhesive shear deformation only is considered, the stress intensity factor for this case is related to the reduced crack bridging stress near the crack tip, which is ignored in the first part. Figure I3 shows the consequences of the "truncation model" for the stress distribution near the crack tip, according to the two step modelling scheme. Increasing delamination causes a reduction of the crack bridging stress level. The extent of the area with a reduced crack bridging stress, which was ignored first, is reduced too. The same applies to the magnitude of the ignored reduction. Consequently, the associated stress intensity factor (= stress intensity due to adhesive shear deformation) is reduced too. The stress intensity factor belonging to the area with reduced crack bridging stresses will be calculated as a function of the crack bridging stress level in the uniform part of the stress distribution, thus expressing the effect of delamination as a function of the crack bridging stress (in the uniform part of the distribution, remote from the crack tip) and the stress as caused by the external loading.

The calculation of the stress intensity factor due to delamination only (chapter 5) is based on the crack bridging stress $S_v$ (based on the full laminate thickness). The adhesive shear deformation is directly related to the load transfer (at the delamination boundary) from the aluminium sheets to the fibres. This load transfer depends on $S_v$ (also based on the thickness of the aluminium sheets only). It is shown in chapter 5 how $S_v$ depends on the delamination distance and other geometrical aspects. For cracks without delamination and without a starter notch $S_v$ is equal to $S_v$. Because the adhesive shear deformation is directly related to $S_v$, the ratio $S_v/S_v$ will be used as a parameter for the effect of delamination.

Figure I4 shows how the situation of an externally loaded sheet with a crack bridging stress distribution may be transformed to a situation of an unloaded sheet with a "crack opening" stress distribution with the same stress intensity factor, by the application of the superposition principal. Figure I5 shows how a specimen with a "truncated" stress distribution may be decomposed in two stress systems. The stress intensity factor belonging to the first system is accounted for by the calculations in which the effect of adhesive shear deformation was ignored (chapter 5). The second stress system with "crack opening" stresses $S_v$ near the crack tips is related to the present problem. The stress intensity factor belonging to the "crack opening" stress distribution can be calculated by integration. However, the shape of the distribution has to be known first. The shape of the "crack opening" stress distribution is determined from the shape of the original crack bridging stress distribution, which is derived in the next section.
1.2 The crack opening contour and the corresponding stress distribution for a specimen without delamination

It was demonstrated in chapter 5 that the influence of adhesive shear deformation can be described with the analogy of a row of parallel cracks with a distance $2h$. The use of this analogy avoided explicit considerations on the shape of the crack opening contour and the corresponding shape of the crack bridging stress distribution. As explained before this shape is important to the present problem and it will be derived below, starting from the boundary conditions which result from the choice of the analogy.

The present considerations are made for a crack with a very large $a/h$ ratio. The reason is that it may than be assumed that the crack opening stress distributions at both crack tips do not interact and only the near crack tip region where the "crack opening" stress is present, is essential to the present problem. The limitation that only large $a/h$ ratios are considered, simplifies the calculations in the following part considerably. (Typical for ARALL is $h = 2$ mm, and large $a/h$ ratios are present during the main part of the crack growth life.)

An explicit description of the original crack bridging stress distribution can not be derived. However, several boundary conditions can be given for this stress distribution. If the crack bridging stress at a distance $x$ from the crack centre is denoted as $S_x$, the boundary conditions may be formulated in the following way:

1. $S_{x=0} = S_a$ ($a \gg h$)
2. $S_x$ is continuously decreasing down to zero, if $x$ increases up to $a$ ($S_{x=a} = 0$).
3. $S_x$ is proportional to $COD_x$.
4. For $x=0$ is $COD_x = 2h S_{a}\sqrt{E_a} = 2h S_a / E_a$ ($a \gg h$) where $h$ is given by (see section 5.6):

$$h = F_{ad} \sqrt{\frac{1}{j F_{ad} F_{sl}} + \frac{1}{j F_{ad} F_{sr}}}$$

5. $COD_x$ is proportional to $\sqrt{a-x}$ at locations close to the crack tip, where the displacement field due to stress intensity factor is dominant.

6. The stress intensity factor is given by $K = K_{th} = S_a \sqrt{h}$ (see section 5.6).

7. The stress intensity factor $K$ obtained by integration of the "crack opening stresses" must also yield $K = S_a \sqrt{h}$ as a result, because the model with the parallel cracks, as well as the model with "crack opening stresses" are both considered to yield correct solutions for the stress intensity factor caused by adhesive shear deformation.

For long cracks ($a \gg h$) where the interaction between the crack opening stress distributions at the two crack flanks is negligible, the stress intensity factor may be calculated by integration of the contributions of $S_{op,x} \, dx$ between $x = a$ and $x = -a$, or by inte-
The integration of \( S_{\alpha, \beta} \cdot dx' \) between \( x' = 0 \) and \( x' = -2a \), where \( x' \) is the distance from the considered crack tip. (Because \( a \gg h \) the boundary condition \( x' = -2a \) may be replaced by \( x' = -\infty \), since the contributions at high values of \( x' \) are negligible, and the integration process is simplified.)

The above mentioned boundary conditions do not allow a wide range of shapes of the stress distribution. If a feasible function for \( S_{\alpha} \) can be found, which fulfills the boundary conditions, the function will be very similar to the actual stress distribution.

The following function is chosen for the crack opening displacement \( COD_x \) as a function of \( x' \).

\[
COD_x = \frac{2h}{E_{in}} \cdot \left( 1 - e^{-2\sqrt{\frac{2x'}{\pi h}}} \right)
\]  

(I - 1)

Because \( x' = a - x \) it is easily recognized that the second boundary is fulfilled.

Because \( (a \gg h) \) the boundary conditions 1 and 4 imply very large values of \( x'/h \). For \( x' \to \infty \) equation (I-1) gives:

\[
\lim_{x' \to \infty} COD_x = \frac{2h}{E_{in}} \cdot \frac{S_{in}}{E_{in}}
\]

which fulfills the boundary conditions 1 and 4.

For \( x' \) close to zero, equation (I-1) may be written as:

\[
\lim_{x' \to 0} COD_x = \frac{2h}{E_{in}} \cdot \left\{ 1 - \left( 1 - 2\sqrt{\frac{2x'}{\pi h}} \right) \right\} = 4 \frac{S_{in}}{E_{in}} \cdot \frac{\sqrt{h}}{E_{in}} \cdot \sqrt{\frac{2x'}{\pi}} = 4 \frac{K_{th}}{E} \cdot \sqrt{\frac{2x'}{\pi}}
\]

The last expression indeed describes the crack opening displacement as a function of the distance from the crack tip, as it is known from fracture mechanics for the state of plane stress and the boundary conditions 5 and 6 are also fulfilled.

Condition 7 may be checked by using the conditions 3 and 4.

\[
S_x = S_{in} \cdot \frac{COD_x}{\left( \frac{2h}{E_{in}} \right)} = \frac{E_{in}}{2h} \cdot COD_x
\]

(I - 2)

Substitution of equation (I-1) in (I-2) yields:

\[
S_x = \left( 1 - e^{-2\sqrt{\frac{2x'}{\pi h}}} \right) \cdot S_{in}
\]

(I - 3)
And the "crack opening" stress $S_{op,x} = (S_a - S_c)$ is given by:

$$S_{op,x} = S_a e^{-2\sqrt{\frac{2x}{\pi h}}} \quad (I-4)$$

For $a \gg h$ the solution on page 3.6 in the book of Tada [12] may be used for the calculation of the corresponding stress intensity factor and with $K_a/K_a = E_a/E_a = S_a/S_a$:

$$K = \frac{E_A}{E_a} \sqrt{\frac{2}{\pi}} \cdot \int_0^{\infty} \frac{S_{op,x}}{\sqrt{x^*}} \, dx \quad (I-5)$$

Substitution of equation (I-3) and (I-4) in (I-5) yields:

$$K = \frac{E_A}{E_a} S_a \sqrt{\frac{2}{\pi}} \cdot \int_0^{\infty} \frac{-2\sqrt{\frac{2x}{\pi h}}}{\sqrt{x^*}} \, dx^* = S_a \sqrt{h} \quad (I-6)$$

As a result condition 7 is also fulfilled and equation (I-1) may well be assumed to represent a satisfactory description of the crack opening contour. Consequently equation (I-4) describes the "crack opening" stress distribution sufficiently accurate.

I.3 Derivation of the crack opening stresses and the associated stress intensity factor, after delamination

The change of the crack opening stress distribution after delamination is shown qualitatively in figure 16. Figure 17 shows a plot of $S_{op,x}/S_a$ according to equation (I-4). It can be observed that the crack opening stress distribution (according to the truncation model) after delamination is given by:

$$S_{op,x} = S_{br} - \left(1 - e^{-2\sqrt{\frac{2x}{\pi h}}}\right)S_{ln} \quad (0 \leq x^* \leq x_d^*) \quad (I-7)$$

$x_d^*$ is distance from the crack tip where no delamination occurred. It is also the boundary of the stress distribution. After some elementary rewriting it can be derived that:

$$x_d^* = \frac{\pi h}{8} \ln \left(\frac{S_{ln}}{S_{ln} - S_{br}}\right) \quad (I-8)$$

The corresponding stress intensity factor is:

$$K_{ad} = \frac{E_A}{E_a} \sqrt{\frac{2}{\pi}} \cdot \int_0^{x_d^*} \frac{S_{op,x}}{\sqrt{x^*}} \, dx^* \quad (I-9)$$

Substitution of equation (I-7) in (I-9) yields:
\[
K_{ad} = S_d \sqrt{\frac{2}{\pi}} \cdot \int_{0}^{x_d^*} \frac{(S_{br}/S_{in} - 1 + e^{-2\sqrt{\frac{2x^*}{\pi h}}})}{\sqrt{x^*}} dx^* \quad (I-10)
\]

Dividing equation (I-10) by the \( K_{ad} \) value for \( S_p = S_n \) (\( K = K_{in} \)) yields a correction factor \( C_{ad,d} \) for the influence of the delamination on the stress intensity factor due to adhesive shear deformation.

\[
C_{ad,d} = \sqrt{\frac{2}{\pi h}} \cdot \int_{0}^{x_d^*} \frac{(S_{br}/S_{in} - 1 + e^{-2\sqrt{\frac{2x^*}{\pi h}}})}{\sqrt{x^*}} dx^* \quad (I-11)
\]

The first part of the integral expression is easily solved, and after substitution of equation (I-8), equation (I-11) is written as:

\[
C_{ad,d} = \left( \frac{S_{br}}{S_{in}} - 1 \right) \cdot \ln\left( \frac{S_{in}}{S_{in} - S_{br}} \right) + \sqrt{\frac{2}{\pi h}} \cdot \int_{0}^{x_d^*} e^{-2\sqrt{\frac{2x^*}{\pi h}}} \frac{dx^*}{\sqrt{x^*}}
\]

Substitution of: \( 8x^*/(\pi h) = z^2 \) and \( dx^* = \pi h/4 \ z \) and substitution of (I-8) yields:

\[
C_{ad,d} = \frac{S_{br}}{S_{in}} + \left( 1 - \frac{S_{br}}{S_{in}} \right) \cdot \ln\left( 1 - \frac{S_{br}}{S_{in}} \right) \quad (I-12)
\]

Some values of \( C_{ad,d} \) are presented in table I1. It shows that \( C_{ad,d} \) decreases rapidly if \( S_p/S_n \) decreases.

An increase of the stress intensity factor due to delamination is compensated to some extent by a decrease of the stress intensity factor due to adhesive shear deformation. In other words, if the \( K \) value due to delamination alone (\( K_d \) in section 5.6) is increased due to increasing delamination, the stress intensity factor \( K_{ad} \) decreases. However, the final stress intensity factor being the sum of \( K_d \) and \( K_{ad} \) increases.

References.

II. G.H.J.J. Roebroeks
H. Tada, P.C. Paris, G.R. Irwin

Tables and figures

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<tr>
<th>$S_{Dr}/S_{ln}$</th>
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Table II. Some values of $C_{md,d}$ as a function of $S_{Dr}/S_{ln}$. 
Figure II. The influence of adhesive shear deformation and delamination on the distribution of the crack bridging stress.
Figure 12. The influence of delamination on the crack bridging stress distribution related to adhesive shear deformation.
uniform distribution
if no adhesive shear deformation occurs

dered crack bridging stress due to adhesive shear deformation

no delamination

area with delamination
and adhesive shear deformation

area without delamination
but with adhesive deformation

area with reduced crack bridging stress, as compared to the uniform distribution.

this area is closely related to the stress intensity factor due to adhesive shear deformation

Figure 13. Consequences of the "truncation model" for the crack bridging stress distribution near the crack tip.
**K - equivalence principle**

\[ K = 0 \quad \Rightarrow \quad K = CS \sqrt{\pi a} \quad \Rightarrow \quad K = -CSV \sqrt{\pi a} \]

- **No crack**
- **Crack is not a stress raiser**

**Application**

Figure 14. Superposition principal applied on the present stress field.
Figure 15. Disassembling of the stress system in two sub-systems, from which the first was already considered for by calculations without adhesive shear deformation.
Figure 16. Modification of the "crack opening" stress distribution on the crack flanks, according to the "truncation model" for the effect of delamination.

\[
\frac{S_{op,x*}}{S_{la}} = e^{-\sqrt{\frac{8x*}{\pi h}}} - 1 + \frac{S_{br}}{S_{la}}
\]

\[
x^* = \frac{\pi h}{8} \ln\left(\frac{S_{la}}{S_{la} - S_{br}}\right)
\]

Figure 17. Distribution of the "opening stresses" on the crack flanks near the crack tip, before and after truncation.
Samenvatting

Vermoeïngsscheurgroei in ARALL, een hybride aluminium-aramide composiet materiaal. Scheurgroeimechanismen en kwantitatieve berekeningen van de scheurgroeisnelheid.

Vliegtuigen moeten zo licht mogelijk worden uitgevoerd in verband met de vliegtuigprestaties en het vervoersrendement. In de praktijk betekent dit dat zo hoog mogelijke ontwerpspanningen in het materiaal van de constructie worden toegelaten. In belangrijke onderdelen van een vliegtuig (de trekhuid van de vleugel, de huid van de drukcabine) is bij gebruik van klassieke vliegtuigmateriaal niet de statische sterkte maatgevend, maar de weerstand tegen scheurinitiatie en scheurgroei. Onderzoek van de Technische Universiteit Delft heeft aangetoond dat vooral de weerstand tegen scheurgroei aanzienlijk verhoogd kan worden door plaatmateriaal niet uit te voeren als massieve plaat, maar als op elkaar gelijmde dunne platen. De scheurgroei-eigenschappen kunnen nog weer aanzienlijk verder worden verbeterd door in de lijmlagen vezels aan te brengen in de richting van de belasting. Door het optimaliseren van de samenstelling van het gelijmde pakket en de vezels in de lijmlaag werd een essentieel nieuw hybride materiaal ontwikkeld: ARALL (ARamid ALuminium Laminates). Het bleek een zeer hoge weerstand tegen vermoeïngsscheuren te hebben. Een volgende stap in de ontwikkeling van ARALL was het voorstrekken van ARALL platen tot even in het plastisch gebied. Daardoor worden de aanvankelijke ongunstige inwendige spanningen (trek in de dunne aluminium platen en druk in de vezels) van teken omgekeerd. Groeisnelheden van vermoeïngsscheuren kunnen dan een factor 100 en meer lager komen te liggen dan bij de klassieke aluminium legeringen. Bij vermoeïngskritieke vliegtuigonderdelen zijn gewichtsbesparingen van 30% mogelijk.

Bij de groei van vermoeïngsscheuren in ARALL blijven de vezels intakt, en overbruggen de scheuren in de aluminium platen. Omdat de vezels het open gaan van de scheur verhinderen of beperken, wordt de spanningsintensiteit bij de scheurtip aanzienlijk verlaagd. Dat verklaart de zeer lage groeisnelheid. Als gevolg van de scheuroverbrugging treedt in de vezel-lijmlagen afschuiving van de lijm op, alsmede een beperkte delaminatie tussen de vezels en de lijm. Hierdoor wordt het scheurgroeimechanisme in ARALL een complex verschijnsel. Het onderzoek in dit proefschrift is gericht op het begrijpen van dit verschijnsel, en op het opstellen van een mechanisch scheurgroeimodel voor het voorspellen van de groei van vermoeïngsscheuren in ARALL.

Bij het scheurgroeimechanisme van ARALL spelen twee begrippen een centrale rol:

1. **Delaminatie**.

   De scheursluitende vezelkrachten worden via de lijmlagen in het aluminium overgebracht. Dit veroorzaakt lokaal achter de scheurtip vermoeïng van de lijm, waardoor een gedeeltelijk losliggende lijmnaad naast de scheurflanken ontstaat. De omvang van het
delaminatiegebied beïnvloedt de scheursluitkrachten. Hoe groter het delaminatie gebied, hoe geringer de scheursluitkrachten en hoe sneller de scheurgroei.

2. *Afschuifvervormingen in de lijm tussen de verschillende lagen.*

Omdat de scheursluitkrachten via schuifspanningen in de lijm naar het metaal worden doorgeleid, treden lokale afschuifvervormingen in de lijm op. Deze vervormingen laten een zekere scheuropening in het metaal toe, waardoor het rendement van de scheursluiting verminderd wordt.

De delaminatie en de afschuifvorming zijn van vele factoren afhankelijk, zoals de stijfheid van de vezels, het aluminium en de lijm, de dikte van de diverse lagen, de inwendige spanningen, enz. Een omvangrijk proevenprogramma is uitgevoerd om enerzijds de betekenis van die factoren te onderzoeken, en anderzijds de karakteristieke materiaalgegevens te verzamelen, die nodig zijn om met het gepostuleerde scheurgroeimodel voorspellingen te kunnen maken. Het experimentele programma omvat:

- Trekproeven op ARALL en op de afzonderlijke bestanddelen, nl. metaal en de vezel-lijmlaag.
- Scheurgroeiproeven met constante-amplitude belasting op onversterkt metaal en op ARALL met als veranderlijk de inwendige spanningen, de vermoeingsbelasting en de grootte van de kerf, die het uitgangspunt van de vermoeiingsscheur vormt.
- Scheurgroeiproeven onder een TWIST vluchtsimulatiebelasting. De resultaten voor ARALL zijn vergeleken met het gedrag van onversterkt materiaal. Onderzocht zijn de invloeden van de inwendige spanningen, de grootte van de uitgangskerf, de dikte van de individuele lagen en de mate waarin truncatie ("afsnijden") van de hoogste pieklasten de scheurgroei versnelt.
- Het scheurinitiatie gedrag van ARALL is onderzocht aan proefstukken met twee symmetrische randkerven, zowel bij een constante-amplitude als bij een TWIST vluchtsimulatie. Hierbij zijn de hoogte van de vermoeiingsbelasting en de inwendige spanningen als experimentele parameters gevarieerd.
- De verschillende mechanismen die maatgevend zijn voor het gedrag van ARALL, zijn ook afzonderlijk onderzocht. Metingen betreffende het afschuifgedrag van de lijm zijn uitgevoerd met behulp van "thick adherend" proefstukken. Verder zijn metingen verricht aan de scheuropening ten gevolge van lijmafschuiving in ARALL. Hiervoor zijn proefstukken gebruikt met een "kusmatige" scheur in het aluminium over de gehele proefstukbreedte. Delaminatie is afwezig bij de kunstmatige scheur, en de externe belasting wordt alleen door de vezels naast de scheur gedragen. De lokale scheursluitende spanning is dan precies bekend, en kan met de scheuropeningsmetingen worden gecorreleerd. Soortgelijke proefstukken zijn gebruikt om het delaminatie gedrag afzonderlijk te onderzoeken als functie van de scheursluitende belasting. Daarbij is ook de invloed van incidentele piekbelastingen onderzocht.
Op basis van de resultaten van de proeven zijn analytische betrekkingen geformuleerd, die de correlatie beschrijven van de scheuropening en van de delaminatie met diverse belastings- en laminaatparameters. Voorts zijn vergelijkingen ontwikkeld, waarmee voor een vermoeiingsscheur in ARALL de scheursluitende vezelspanning en de spanningsintensiteitsfactor in de aluminium lagen berekend kunnen worden, met als veranderlijken de laminaatparameters, de mechanische belasting en grootte van het delaminatiegebied. Met behulp van de experimenteel bepaalde elementaire delaminatie- en scheurgroeigegevens, maken deze vergelijkingen het mogelijk de scheurgroei en de delaminatie-uitbreiding in ARALL op incrementele basis te berekenen. Hiervoor is een computerprogramma opgesteld, waarmee de scheurgroeisnelheid voor verschillende soorten ARALL onder constante-amplitude vermoeiingsbelasting berekend kan worden. De resultaten van de berekeningen vertonen in het algemeen een goede overeenkomst met de experimentele scheurgroeiresultaten. Dit bevestigt dat het vermoeiingsgedrag van ARALL met het scheurgroeimodel goed wordt beschreven.

Een aantal karakteristieke kenmerken van vermoeiing in ARALL, die uit de proeven, de analyse en de modelvorming naar voren zijn gekomen, kunnen als volgt worden samengevat:

1. Bij ARALL wordt het scheurinitiatie gedrag vooral bepaald door de eigenschappen van het metaal, omdat scheursluitende spanningen pas optreden als er scheuren zijn. Wel kan de initiatieperiode gunstig worden beïnvloed door inwendige spanningen, die in ARALL door plastic sterkken worden verkregen.

2. Al vroeg in het scheurgroeiestadium, als de scheurtjes nog erg klein zijn, heeft de aanwezigheid van de vezelversterkte lijmvlaag een gunstige invloed. De laag werkt als een barrière voor het "overspringen" van scheurtjes naar een naburige metaalplaat. Hierdoor duurt het langer tot een scheur zich over de gehele dikte van een ARALL plaat heeft uitgebreid.

3. Bij massieve platen is de grootte en vorm van de uitgangskerf niet meer belangrijk, zodra de scheur in vergelijking tot de kerf een relatief grote lengte heeft. De kerf kan dan beschouwd worden als een deel van de scheur. Bij ARALL is de situatie geheel anders, omdat er ter plaatse van de kerf geen vezels zijn die de scheurflanken verbinden. In een kleine kerf zijn minder vezels onderbroken, waardoor tijdens de gehele scheurgroeievenstuur de scheursluitende belasting groter is, en de scheurgroei langzamer verloopt dan bij een grote kerf.

4. De efficientie van de scheursluiting wordt vooral beïnvloed door de mate van delaminatie in de lijm en door de afschuiving van de lijm aan het delaminatiefront. Beide mechanismen worden gunstig beïnvloed door:
   - het toepassen van stijve vezels,
   - het toepassen van zeer dunne individuele lagen,
   - een hoge verhouding van vezels tot metaal.
De lijmnaad afschuiving wordt minder wanneer:
   - een stijve lijm gebruikt wordt,
   - weinig lijm gebruikt wordt (hoog vezel-percentage).
Van beide laatste maatregelen moet echter aangenomen worden, dat ze de delamina-tiesnelheid doen toenemen.

5. De delaminatiesnelheid wordt mede bepaald door de vermoeiingsweerstand van de lijm. Twee lijmsoorten zijn uitvoerig onderzocht, waarbij een aanzienlijk verschil tussen de lijmsoorten gevonden is. Verder kan het delaminatie gedrag gunstig beïnvloed worden door het toepassen van de vezels in de vorm van een "unidirectioneel" weefsel in plaats van een prepreg. Dit veroorzaakt echter weer een klein verlies in de treksterkte van de vezelversterkte lijmlag.

6. De invloed van gunstige inwendige spanningen, als middel om de vermoeiingsweerstand van ARALL verder te verbeteren, wordt vooral belangrijk wanneer daardoor een gedeelte van de vermoeiingsspanningen in het metaal naar het drukgebied verschoven wordt. Zolang drukspanningen aanwezig zijn, wordt de scheur helemaal niet geopend en zal ook niet groeien. Ook bij vermoeiingsbelastingen met variabele-amplitude zoals TWIST, treedt een verschuiving van een gedeelte van de spanningen naar het drukgebied op, en daarom is de mogelijke verbetering van de scheurgroei als gevolg van gunstige inwendige spanningen zeer groot. De verschuiving van het vermoeingspectrum naar beneden doet de relatieve grootte van de piekbelastingen toenemen en daarmee neemt ook de gunstige vertragende invloed van de piekbelastingen op de scheurgroei toe. Dit verklaart de zeer grote invloed van het truncatieniveau bij ARALL met gunstige inwendige spanningen.

7. De verhouding delaminatie tot scheurlengte in ARALL bij variabele amplitude belastingen is veelal groter dan bij constante amplitude belastingen. Dit komt omdat piekbelastingen vertragend werken op de scheurgroei in het aluminium, maar versnellend op de delaminatie groei. Daar beide invloeden moeilijk in algemene termen kwantificeerbaar zijn, is geen poging ondernomen om met analytische methoden het scheurgroeigedrag van ARALL ook bij TWIST vluchtsimulatie proeven kwantitatief te voorspellen. Een kwalitatieve verklaring van het vermoeiingsgedrag van ARALL is echter ook goed mogelijk voor de TWIST vermoeiingsbelasting.
Curriculum vitae


Het afstudeer onderzoek onder leiding van prof.dr.ir. J. Schijve was gewijd aan het vermoeiingsgedrag van met aramid-vezels versterkte aluminium-plaat laminaten. Het onderzoek omvatte het optimeren van dit materiaal met betrekking tot de hoeveelheid vezels, de vezelrichting, de dikte van de aluminium platen, de lijmkeuze en het introduceren van gunstige inwendige spanningen door voorstrekken. Tevens werden enkele analytische betrekkingen geformuleerd die de vermoeiingseigenschappen van dit (later als ARALL gepatenteerde) hybride materiaal kwalitatief beschreven.

Na het afstuderen in april 1980 trad de auteur in dienst bij de "Deutsche Forschungs und Versuchsanstalt für Luft und Raumfahrt e.V." (DFVLR) in Keulen. Hier werkte hij in de vermoeiingsafdeling van het "Institut für Werkstoff-Forschung" verder aan ARALL en aan andere materialen. De resultaten van die onderzoeken hebben geleid tot dit proefschrift en tot circa 30 publicaties en enkele voordrachten.

Naast zijn functie als onderzoeker was hij betrokken bij de begeleiding en de examinering van studenten die bij de DFVLR in de praktijk afstudeerden.

Sinds april 1988 is de auteur wetenschappelijk medewerker in het laboratorium voor compositesmaterialen van DSM in Geleen.