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Abstract

As an alternative to a more or less standard derivation procedure for design wave heights in relatively shallow water, two improvements of the procedure are suggested which lead to less conservative results. These improvements are based on observations of shallow water effects on both the decay of total wave energy density and on the extreme waves in the wave height distribution. Existing semi-empirical formulations to account for these effects are adopted and somewhat further evaluated here. The implications of introducing the improved procedures for the resulting design wave loading on a slender cylinder are indicated.

Keywords

Shallow water waves; extreme waves; nonlinear waves; wave height distribution; wave kinematics; wave loading; wave forces.

1. Introduction

In the field of ocean and coastal engineering the use of wave generation and wave propagation models for the derivation of integral wave field parameters such as H_s and T_p is widely known and applied for design purposes. Relatively reliable results may be acquired as long as shallow water effects as refraction due to bottom or current variations and depth limited breaking are negligible. It is somewhat less known that also progress has been made with the modelling of reliable dissipative source terms due to depth induced wave breaking, that is when we are interested in the above integral parameters (Battjes and Stive, 1985; Dingemans et al.,1984).

With respect to the distribution of extreme wave and crest heights on shallow water however, there is still a lack of reliable model formulations. The extreme values are of great importance in the design of offshore structures. Because of the said lack a usual procedure is to apply the Rayleigh distribution until the theoretical limiting height of a regular wave on a horizontal bottom is exceeded. On the basis of measurements in the field and especially in the laboratory it has appeared that this does not give reliable results. A semi-empirical formulation, which models the expected deviation from the theoretical Rayleigh distribution in a heuristic manner, yields more reliable results. This formulation is described in Section 2.

So as to indicate the consequences of the above modelling suggestions for the design loads on offshore structures, firstly in Sections 3 and 4 the formulations are described for the wave-induced velocity field and the resulting horizontal loading, as caused by the extreme waves. Based on these formulations we show in Section 5 for a characteristic situation which differences can occur between a standard approach and two alternative approaches to derive design loads which account for the above modelling suggestions with respect to shallow water effects.

2. Extreme shallow water wave statistics

In relatively deep water it appears that the heights of individual waves follow a Rayleigh distribution. The eventual deviations from this distribution due to non-linearities and finite bandwidth are relatively small (Longuet-Higgins, 1980), i.e. judged against the deviations which may occur for extreme waves in the tail of the wave height distribution on relatively shallow water. For the lower wave heights in the distribution this is less clear; e.g. Thornton and Guza's (1983) interpretation of the NSTS data leads to negligible (non-significant) deviations up to the $H_{1/10}$.

The deviations in the tail of the wave height distribution in relatively shallow water are well-known. They are ascribed to the height (or steepness) limiting effects of a finite waterdepth d. The practical upper boundary which usually is adopted is the theoretical maximum for a steady solitary wave on a horizontal bottom $H/d = 0.833$ (Williams, 1981). The validity of this choice is amongst others determined by the degree in which individual waves in a natural wave field on shallow water behave as steady solitary waves. In the limit for $d \rightarrow 0$ this is a realistic assumption, but it appears from e.g. laboratory measurements that in the transition zone from nearly breaking to saturated breaking there is no proof for a more general validity of the limiting situation. This is illustrated in Figure 1 where, as a function of the relative water depth kd, extreme wave height values from two laboratory programs are plotted against an adapted Miche criterion. The measurements concern random wave decay in flumes on 1 in 40 and 1 in 100 slopes; they are described in Stive (1985, 1986). The Miche criterion for the maximum wave heights realizes a smooth transition between the limit deep water wave steepness and the maximum solitary wave height on shallow water. It is adapted (according to Battjes and Janssen, 1978) in order to create a shallow water limit different from 0.88:

$$\frac{H_{max}}{d} = \frac{0.88}{kd} \tanh \left(\gamma \frac{kd}{0.88} \right) , \qquad (1)$$

in which $\gamma = 0.833$ and k is the wave number.

Inspection of Figure 1 indicates that the data scatter is relatively large, and that even qualitatively the criterion does not seem to be a good approximation. Apparently, the relative water depth is not the dominant parameter in the physical process.

Further analysis of the measurements has shown that there is a dominant parameter existing, i.e. the ratio total local energy density over depth, here represented by H_s / d, where H_s has been defined as $H_s \equiv 4 \sqrt{m_0}$, with m_0 the total water surface elevation variance. This is a measure for the breaking wave intensity (Battjes and Janssen, 1978). As the intensity of wave breaking increases the deviations from the Rayleigh distribution should increase. This is illustrated in Figure 2 for some characteristic extreme wave heights. It appears that the dependency of the parameter is such that other eventual dependencies are obscured, and relatively little scatter results at least up to $H_s / d = 0.6$ which is in the very shallow water range. The reasons for this behaviour are not really well understood, but our suggestion is that there are probably effects of wave grouping on the characteristics of extreme wave heights.

Qualitatively this dependency has already been accounted for by Glukhovskiy (1966) and it has been verified to a limited North Sea data set by Bouws (1979). The method essentially comes down to a Weibull formulation with the above ratio as a parameter, in the deep water limit reducing to the Rayleigh distribution. Here we present a quantitatively somewhat modified formulation, which accounts for consistency between the parameters (private communication Battjes, 1986):

$$P(H) = P\{\, \underline{H} > H \mid \bar{H},\, d \,\} = \exp\left[- A \left(H / \bar{H} \right)^{\kappa} \right] , \qquad (2)$$

with:

$$\kappa = 2 \left(1 - \tilde{d} \right)^{-1} , \qquad (3)$$

$$\tilde{d} = \frac{\bar{H}}{d} , \qquad (4)$$

where $\bar{H}$ denotes the mean wave height.

For the Weibull distribution we have the following expression for the n-th moment M_n:

$$M_n = \bar{H}^n A^{-n/\kappa} \; \Gamma\left(\frac{n}{\kappa} + 1 \right) , \qquad (5)$$

where $\Gamma(x)$ denotes the gamma function. Since M_1 has to be equal to $\bar{H}$, the following relation exists between A and κ:

$$A^{1/\kappa} = \Gamma\left(\frac{1}{\kappa} + 1 \right) .$$

The root mean square wave height H_{rms} becomes:

$$\frac{H_{rms}}{\bar{H}} = \frac{\sqrt{M_2}}{M_1} = \frac{\left[\Gamma\left(\frac{2}{\kappa} + 1 \right) \right]^{1/2}}{\Gamma\left(\frac{1}{\kappa} + 1 \right)} , \qquad (6)$$

and the wave height H_N with an exceedance probability of $P(H_N) = 1/N$ is equal to:

$$\frac{H_N}{\bar{H}} = \left(\frac{\ln\,(N)}{-A} \right)^{1/\kappa} . \qquad (7)$$

The significant wave height $H_{1/3}$, defined as the mean value of the waves higher than H_3, is related to the average wave height through:

$$\frac{H_{1/3}}{\bar{H}} = 3 \, \frac{\Gamma\left[\frac{1}{\kappa} + 1,\; A \left(H_3 / \bar{H} \right)^{\kappa} \right]}{\Gamma\left(\frac{1}{\kappa} + 1 \right)} , \qquad (8)$$

with $\Gamma(a,x)$ the incomplete gamma function. Some characteristic values of several ratio's as a function of the relative average wave height ratio $\bar{H} / d$ are given in the following table.

$\frac{\overline{H}}{d}$	$\frac{H_{1/3}}{d}$	$\frac{H_{1/3}}{H_{rms}}$	$\frac{H_{100}}{\sqrt{2}\, H_{rms}}$	$\frac{H_{1000}}{\sqrt{2}\, H_{rms}}$
0.00	0.00000	1.41573	1.51743	1.85846
0.10	0.15408	1.39153	1.43354	1.72049
0.20	0.29665	1.36366	1.34964	1.58729
0.30	0.42744	1.33202	1.26598	1.45901
0.40	0.54623	1.29654	1.18279	1.33578
0.50	0.65284	1.25714	1.10033	1.21772
0.60	0.74712	1.21377	1.01884	1.10491
0.70	0.82899	1.16638	0.93856	0.99741

In the above we have adopted Glukhovskiy's suggestion for a parameterization of κ, but disregarded his suggestion for a parameterization of A, for consistency reasons. Although further empirical optimisation would be possible, we suggest that for the moment this paramerization follows the data up to $H_s / d = 0.6$ with reasonable accuracy (see Figure 2). For $H_s / d > 0.6$ the data are probably influenced by a positive correlation of the extreme wave heights with increased set-up due to surfbeat.

3. Design wave kinematics

There has been a steady progress in the development of wave theories for periodic long-crested waves travelling over a horizontal bottom. At the moment, it is possible to obtain highly accurate solutions of the wave kinematics for engineering purposes, assuming potential flow theory is valid, see Teles da Silva & Peregrine (1988), and the reviews of Fenton (1989) and Sobey et al. (1987). The validity ranges of several theories are given by Fenton (1989).

However, in nature waves are almost always short-crested, irregular, superimposed on turbulent shear flows, influenced by the air flow over the waves and the sea bed topography, and also the waves may be breaking. On many of these physical effects still a lot of research has to be done, in order to be able to incorporate them into sound engineering design procedures.

Subsequentially, we will use the design wave concept for predicting wave kinematics and forces. In deep water the design wave height will be much smaller than the limiting wave height, so low order Stokes's wave theories are sufficient to describe the design wave kinematics. In shallow water, however, the design wave will be a near-breaking wave, so a higher order theory applicable at shallow water should be used. Here we use the Fourier approximation method of Rienecker & Fenton (1981), which can describe the wave kinematics of periodic waves superimposed on a uniform current, up to the limiting wave height. A simplified version of the method, including the FORTRAN code, can be found in Fenton (1988).

Comparisons of the computed fluid velocities under the wave crest with measurements of Le Méhauté et al. (1968) were already presented by Rienecker & Fenton (1981). In Figure 3, a comparison is made with measured velocities under the wave crests at the breaking point on an 1 : 30 slope (Iwagaki & Sakai; 1976). Even though the waves are unsteady, the agreement is quite satisfactory.

4. Design wave forces

Presently, one still has to rely on empirical methods for the prediction of wave forces on cylinders at high Reynolds numbers and outside the diffraction regime. The inline forces on fixed vertical cylinders in long-crested waves can be described reasonably well by Morison's equation (Bearman et al., 1985; Bearman, 1988). However, for other cylinder orientations the agreement between wave forces as predicted by Morison's equation and measurements is quite poor (Bearman et al., 1985). For horizontal cylinders, improved formulations for forces at low Keulegan-Carpenter numbers have been derived by Chaplin (1988).

For cylinders in shallow water, a description of the forces in the splash zone and forces due to breaking waves are important. Results of field measurements on forces on vertical cylinders near the free surface can be found in Dean et al. (1981).

No results are yet available of laboratory experiments at sufficient large scale. It is essential, in order be able to extrapolate laboratory results to prototype conditions, to perform tests at postcritical Reynolds numbers, and at sufficient large scale for letting wave impact forces become independent of surface tension effects on air entrainment in the breaking waves. For laboratory experiments at subcritical Reynolds numbers and in deep water, Kjeldsen et al. (1986) found that inline forces on a vertical cylinder in breaking waves can be higher than the forces due to higher non-breaking waves. Force coefficients for a vertical cylinder due to non-breaking deep water waves in the splash zone and at subcritical Reynolds numbers are presented by Tørum (1989).

For the computation of wave forces, we have used Morison's equation, without taking free surface effects into account. As an example, a comparison is made between a measurement (Tørum, 1989) and computed results using Morison's equation. The experimental conditions were:

Mean water depth	3.0	m
Regular wave height	0.609	m
Wave period	2.012	s
Cylinder diameter	0.06	m

The velocity field and velocity partial time derivative field, as computed with the Fourier approximation method of Rienecker & Fenton (1981), are presented in Figure 4. An inertia coefficient $C_M = 1.60$ and drag coefficient $C_D = 1.10$, corresponding to the values found below trough level for this test by Tørum (1989), were used to compute the inline forces. Apart from very near the free surface, a reasonable agreement is found with the measurements, see Figure 5.

5. Results and discussion

In Sections 1 and 2 we have described two effects which are not commonly accounted for when shallow water wave design conditions have to be derived. The first effect concerns the decay of the total wave energy density due to depth limited breaking as a random wave field shoals towards shallower water. This may be accounted for by the type of model as described by Battjes & Stive (1985), which they show to result in a reliable variation of the total wave variance, $H_{rms} \cong \sqrt{(8\ m_0)}$. The second effect concerns the implications of shallow water effects on the wave height distribution of a random wave field as it propagates towards shallower depths, as explained in Section 2.

So as to illustrate the implications of incorporating the above effects in a design procedure for wave loading, we evaluate two alternative methods next to a more or less standard design method. The characteristic design case is as follows. Assume that on a water depth of 30 m a "significant" wave height $H_s \equiv 4\sqrt{m_0}$ and an extreme design wave height

H_{1000} and corresponding wave periods are known: say H_s = 7.60 m, H_{1000} = 14.13 m, T = 12.0 s. Say the design procedure requires design wave heights on shallower water depths, e.g. 20 m and 10 m.

Method 1, the conservative design method, would realisticly follow the reasoning to apply the same design wave on the shallower depths as on the 30 m depth, as long as the theoretical limit for a regular wave on a horizontal bottom is not exceeded.

Method 2, accounting for total wave energy decay due to depth-induced breaking, would first apply the energy decay model to derive the variation of total wave energy, and therewith derive a characteristic wave height, such as $H_s \equiv 4 \sqrt{(m_0)}$ on the shallower depths. Based on this wave height design wave heights would be derived assuming a Rayleigh distribution. Here, again as long as the theoretical limit for a regular wave on a horizontal bottom is not exceeded.

Finally, method 3, accounting for wave decay as in method 2 but also for the shallow water effects on the distribution of the extreme waves, would derive the design wave height from the decayed significant wave height and the applying the modified Rayleigh distribution as described in Section 2.

The resulting wave heights which follow from these three methods are collected in the table below.

	H_s [m] at a water depth of			H_{design} [m] at a water depth of		
	30 m	20 m	10 m	30 m	20 m	10 m
Method 1	7.60	7.60	7.26*	14.13	13.67*	7.26*
Method 2	7.60	6.90	4.05	14.13	12.83	7.26*
Method 3	7.60	6.90	4.05	12.43	10.73	6.10

The wave heights marked with an asterisk (*) are limited by the maximum obtainable regular wave height at that particular water depth, as computed with the Fourier approximation method of Rienecker & Fenton (1981).

So as to illustrate the implications of the alternative design wave heights for the design wave loading, a loading calculation was done according to the method described in Sections 3 and 4. The results for the maximum total horizontal force and maximum total bed overturning moment on a slender cylinder extending over the total water column are given in the table below.

	F_{max} [kN] at a water depth of			M_{max} [kNm] at a water depth of		
	30 m	20 m	10 m	30 m	20 m	10 m
Method 1	223	298	100	5806	6664	1141
Method 2	223	254	100	5806	5070	1141
Method 3	163	160	73	3943	2847	681

From the results it clearly follows that the design wave heights and wave loading derived according to method 3 are substantially lower than that according to method 1 and 2. This indicates that a revision of shallow water design procedures may lead to important savings. A more sound basis of this revision though should be based on further research.

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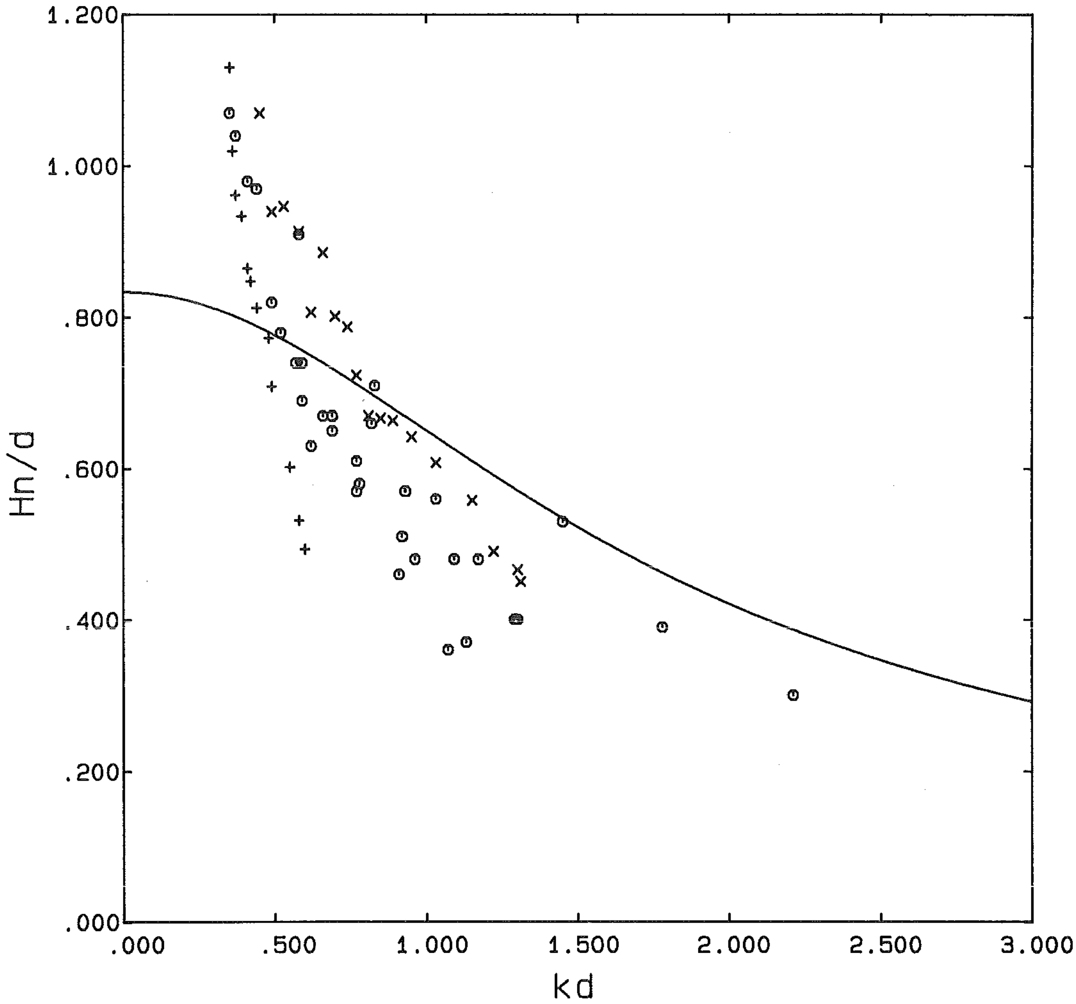

Figure 1 Dimensionless extreme wave height values H_N/d versus relative water depth kd;

+ : H_{700}/d on a 1:40 slope, $(H_{rms}/\lambda)_0 = 0.01$ (Stive, 1985);

× : H_{800}/d on a 1:40 slope, $(H_{rms}/\lambda)_0 = 0.04$ (Stive, 1985);

⊙ : H_{1000}/d on a 1:100 slope, several tests series with $0.03 < (H_{rms}/\lambda)_0 < 0.06$ (Stive, 1986);

—— : equation (1).

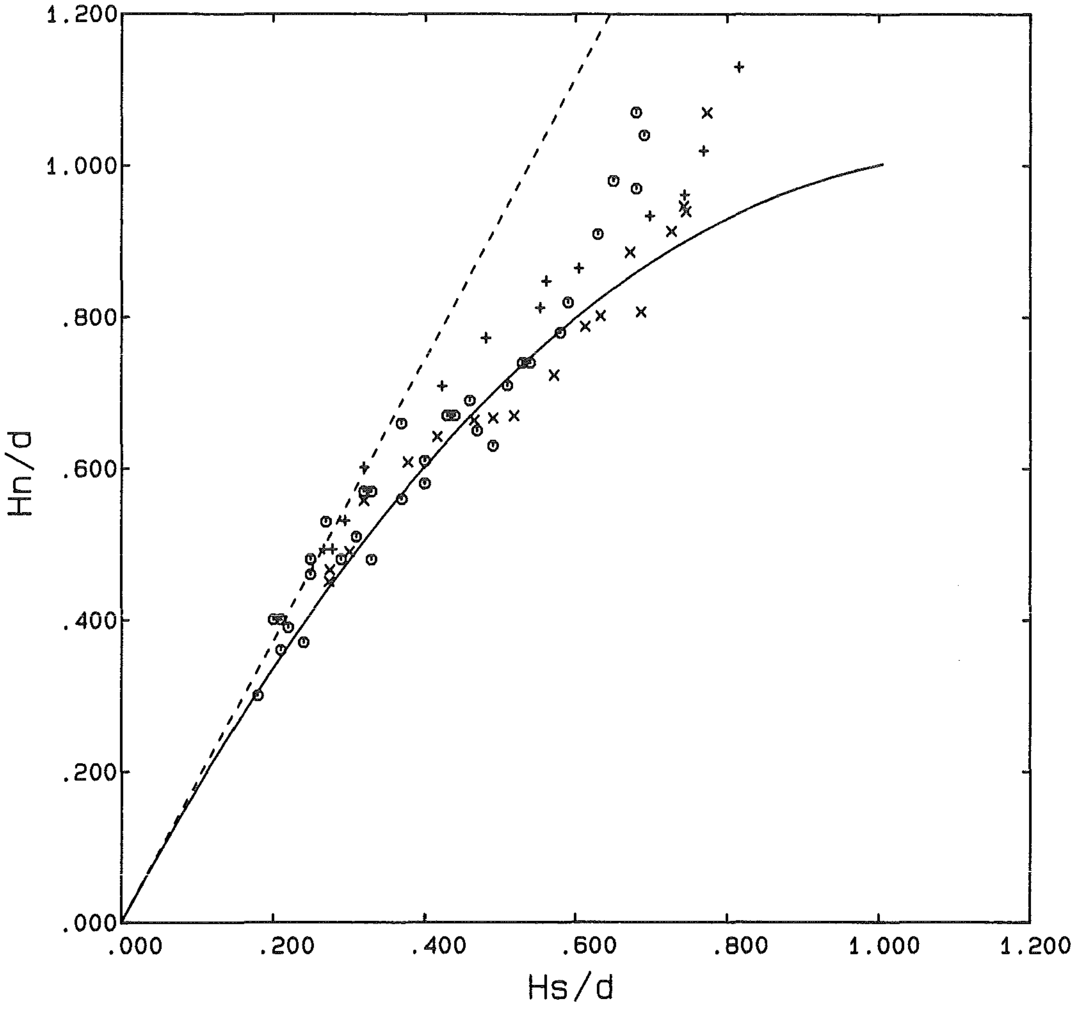

Figure 2 Dimensionless extreme wave heights H_N/d versus dimensionless total wave energy H_s/d, with $H_s=4\sqrt{m_0}$; legend for data points see Figure 1;

– – – : Rayleigh distribution for H_{1000}/d;

——— : modified Glukhovskiy distribution for H_{1000}/d.

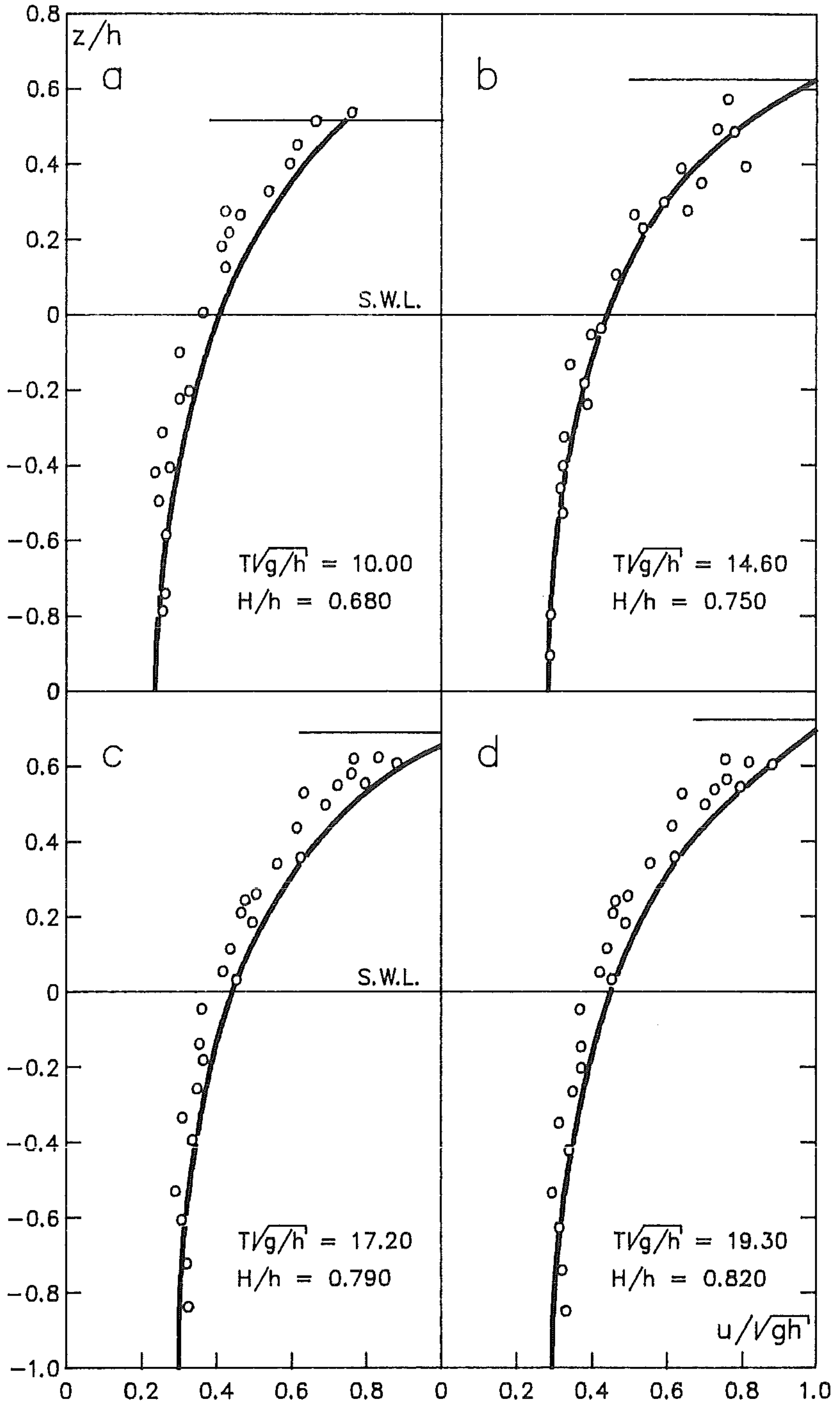

Figure 3 Maximum horizontal velocity profiles, at the point of breaking on a 1:30 slope;

⊙ : measurements by Iwagaki and Sakay (1976);

—— : computation with Rienecker & Fenton Fourier approximation method.

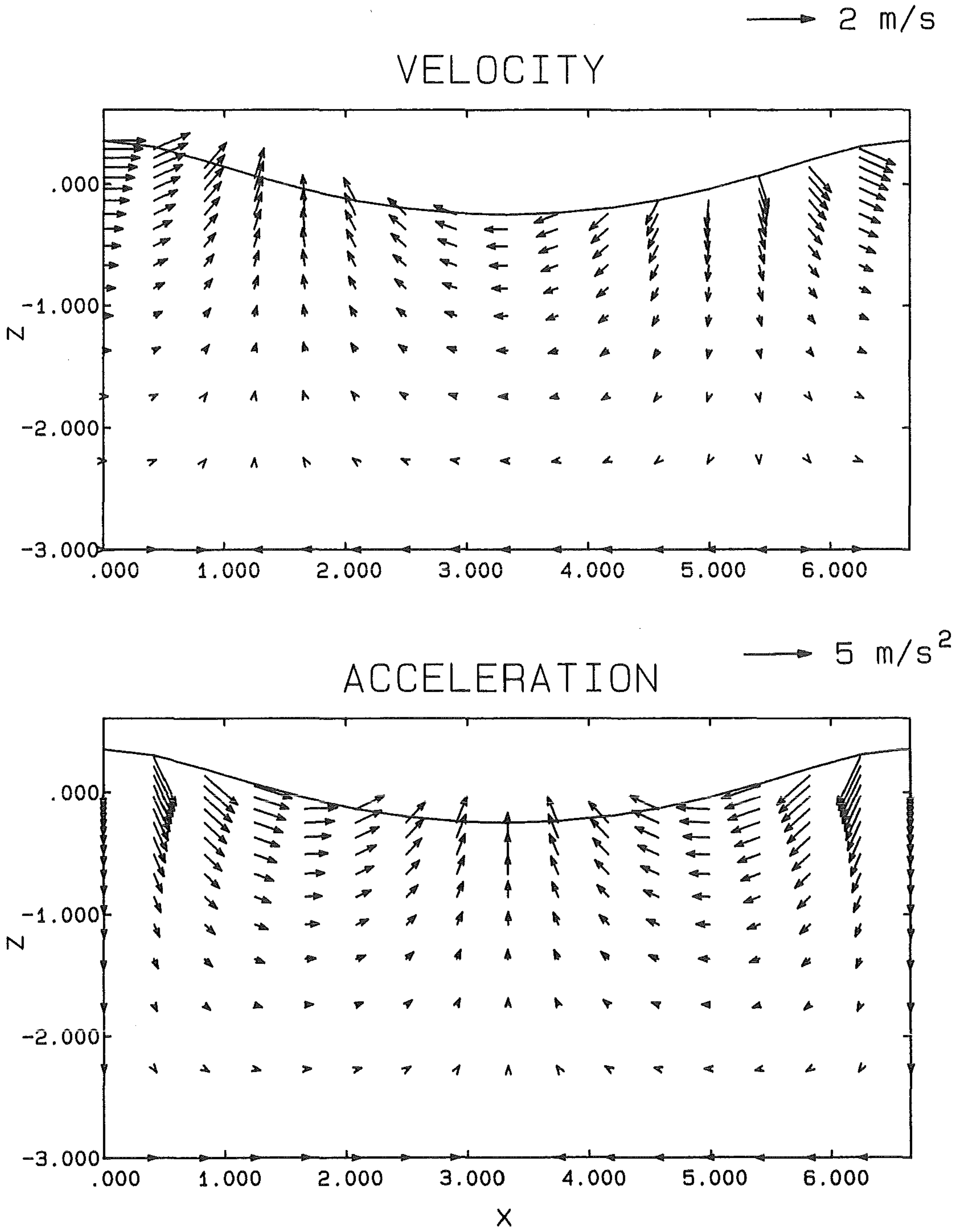

Figure 4 Velocity and acceleration field under a wave of period T = 2.012 s and height H = 0.609 m in a mean water depth of d = 3.0 m, as computed with the Fourier approximation method of Rienecker & Fenton (1981).

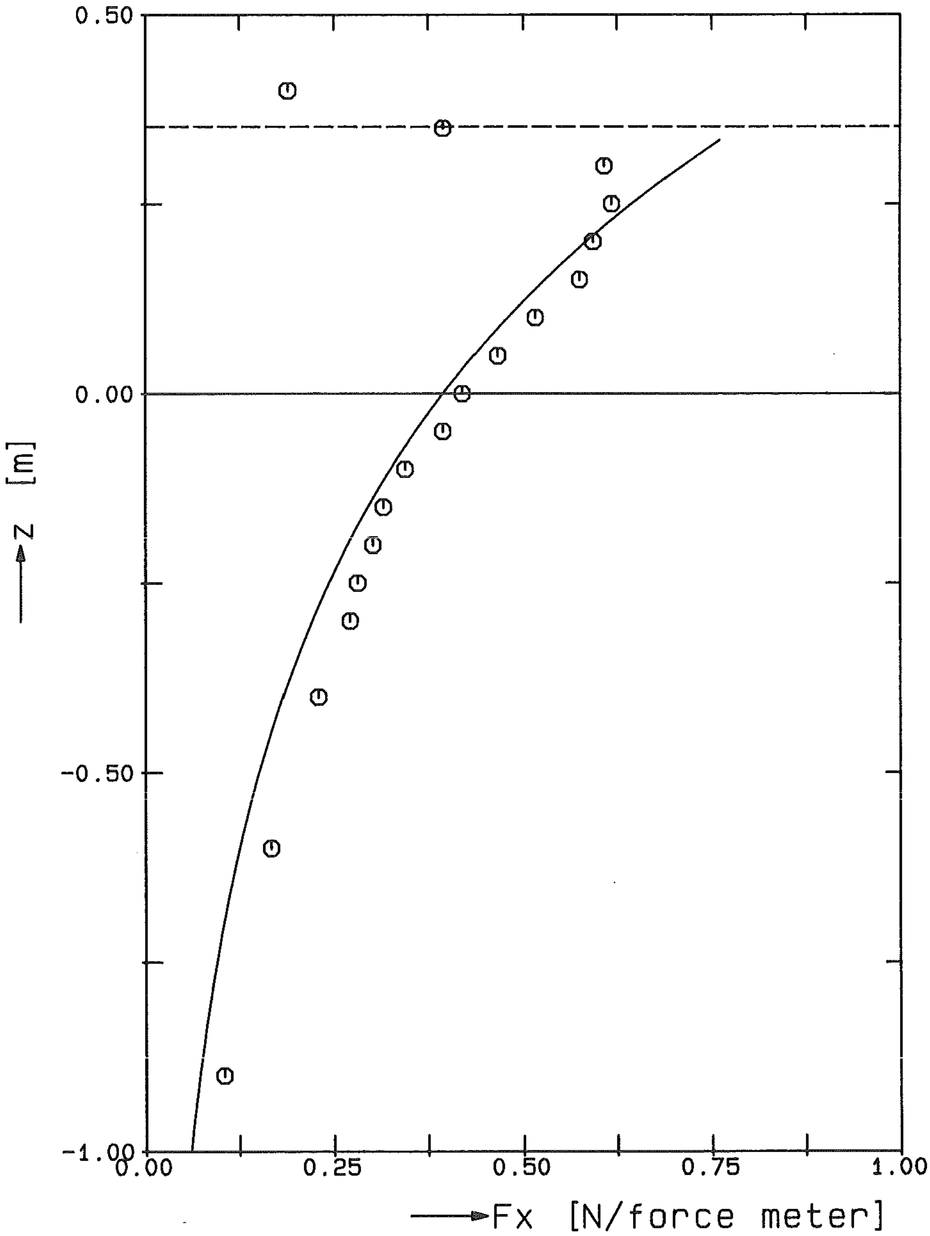

Figure 5 Inline force F_x on a vertical cylinder as a function of the vertical coordinate z; cylinder diameter 0.06 m, wave conditions as in Figure 4; wave phase relative to wave crest $\theta = 348^0$;

⊙ : measurements by Tørum (1989);

—— : Morison equation with velocities and accelerations as in figure 4.