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Design of the Wing of a Regional Airliner
in Composite Material

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Abstract

Nowadays, virtually all of the primary structure of larger civil aircraft is made of aluminium alloys. However, the everlasting search for lighter structures is leading to the application of different materials. It is not unforeseeable that the next generation of larger aircraft will have a fuselage of Glare and a wing structure of composites.

Composite materials have the advantage, in comparison to conventional aluminium alloys, that they have a low density, and a high strength and stiffness. There are also possible advantages in production and durability. Of course, there are also disadvantages. The damage tolerance of composites is lower than that of metals. Usually, damage tolerance is the restricting factor when designing with composites. This means that in a first phase strain limitations are used instead of stress limitations, which are used when designing aluminium alloy structures.

The section of the wing designed in this report is that of the 'Garteur G-70' aircraft. It is a transport aircraft for 70 passengers. Hopefully, the results of this report will give a good indication how composite wings of all larger aircraft should be designed.

Two designs of the torsion box of the wing are made. One that consists of skin panels stiffened by I-stringers, spars with a corrugated web, and a rib with a truss web structure. The other consists of sandwich skin panels, spars with a corrugated web, and a rib, which consists of a simple flat plate web with stiffeners and 'lightning holes'.

The calculations are based on netting theory. Although, the accuracy is not perfect, it does lead to results quickly. The calculations are done analytically, which means that sometimes certain constraints are put into the design. For example, the directions of the fibres are limited to $0^\circ$, $+/-45^\circ$, and $90^\circ$.

The composite materials in both of the designs are made of carbon fibres supported by an epoxy resin. The two types of carbon fibres in the designs are IM6 fibres and HMS-4 fibres.

The path that is followed to find the designs is as follows:

- A literature survey is done to see were the problems lie when designing with composite materials, and to use the knowledge of other people to limit the possible types of structure.
- The mechanical properties of different composite materials are compared, again to limit the possibilities.
- The loads on the Garteur G-70 are calculated to find the most critical loads.
- With netting theory an initial design is made, i.e. types of structure are compared and the options, which lead to the lightest structure, are chosen. Of course, the limitations of composite materials are kept in mind.

The designs that are made are initial designs. This means that they are a beginning point for a second design loop.
Samenvatting

Tegenwoordig worden vrijwel alle primaire constructies van grotere passagiers vliegtuigen van aluminium legeringen gemaakt. Maar de zoek naar lichtere constructies begint te leiden tot de toepassing van andere materialen. Het is niet onwaarschijnlijk dat de vliegtuigen van de volgende generatie rompen van Glare hebben en vleugels van composieten.

Composiete materialen hebben het voordeel, t.o.v. convensionele aluminium legeringen, dat ze een lage dichtheid, en een hoge sterkte en stijfheid hebben. Er zijn mogelijk ook voordelen op het gebied van produktie en levensduur. Natuurlijk zijn er ook nadeelen. De weerstand tegen inslag van composieten is lager dan die van metalen. Meestal is dit de kritische factor wanneer er met composieten ontworpen wordt. Daardoor wordt in eerste instantie een maximale rek gebruikt i.p.v. een maximaal spanningsniveau zoals wanneer er een constructie van aluminium legeringen wordt ontworpen.

Het gedeelte van de vleugel dat in dit verslag ontworpen is, is dat van de ‘Garteur G-70’. Dit is een passagiers vliegtuig voor 70 personen. Hopelijk geven de resultaten van dit verslag een goede indicatie van hoe composiete vleugels van alle grotere vliegtuigen ontworpen moeten worden.

Twee ontwerpen van de torsiedoos van de vleugel zijn gemaakt. Een bestaat uit huidpanelen met I-verstijvers, liggers met een gecorrigeerde lijfplaat, en een vakwerk rib. De ander bestaat uit sandwich huidpanelen, liggers met een gecorrigeerde lijfplaat, en een rib bestaande uit een vlakke plaat met verstijvers en verlichtingsgaten.

De berekeningen zijn gebaseerd op ‘netting’ theorie. Hoewel de nauwkeurigheid niet precies is, leidt dit snel tot resultaten. De berekeningen zijn analytisch, hierdoor moet er soms beperkingen aan het ontwerp worden opgelegd. Bijvoorbeeld, de richtingen van de vezels zijn beperkt tot $0^\circ$, $+/-45^\circ$, and $90^\circ$.

De composieten in beide ontwerpen bestaan uit koolstof vezels ondersteund door epoxy resin. De twee type koolstof vezels in de ontwerpen zijn IM6 vezels en HMS-4 vezels.

Het pad dat gevolgd is om tot de ontwerpen te komen is als volgt:

- Een literatuur onderzoek is gedaan om te zien waar de problemen liggen wanneer er met composieten ontworpen wordt en om de kennis van andere mensen te gebruiken om het aantal mogelijke ontwerpen te beperken.
- De mechanische eigenschappen van verschillende composiete materialen zijn vergeleken weer om het aantal mogelijkheden te beperken.
- De belastingen op de Garteur G-70 zijn berekend om de kritische belastingen te bepalen.
- M.b.v. ‘netting’ theorie zijn initiële ontwerpen gemaakt, m.a.w. type constructies zijn vergeleken en de opties die leiden tot de lichtste constructie zijn gekozen. Natuurlijk is er rekening gehouden met de beperkingen van composieten.

De ontwerpen die gemaakt zijn, zijn initiële ontwerpen. D.w.z. dat ze een beginpunt zijn voor een tweede ontwerpslag.
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Afstudeeropdracht  'Design of the Wing of a Regional Airliner in Composite Material'
Notations:

\( A \)  
aspect ratio

\( a \)  
enclosed area of the torsion box

\( b \)  
length sandwich panel

\( c \)  
distance between points of load calculations

\( C \)  
acceleration

\( CL \)  
distance between aerodynamic centre and centre of gravity

\( B \)  
width torsion box

\( h \)  
wing span

\( b_{\text{web}1} \)  
width of the flat sections

\( C \)  
coefficient

\( c_l \)  
coefficient

\( c_l \)  
chord

\( c_g \)  
mean geometric chord

\( c_{11},c_{22} \)  
stiffness coefficients

\( D \)  
force that the webs have to carry

\( D_{0}, D_{11}, D_{22} \)  
bending stiffness of the flat sections

\( D_{*1}, D_{*2} \)  
bending stiffness of the corrugated panel as a whole

\( d \)  
distance between struts joints

\( d_{b} \)  
bolt diameter

\( E \)  
modulus

\( E_{\text{prep}} \)  
modulus of the prepreg

\( E_{\text{long}} \)  
modulus of the panel in the direction with the long length

\( E_{\text{short}} \)  
modulus of the panel in the direction with the short length

\( e \)  
edge distance

\( G \)  
shearmodulus

\( H \)  
average height of the torsion box halfway down the wing span

\( H_{t} \)  
height of the webs halfway down the wing span

\( h_{t} \)  
height of the stiffeners

\( h_{w} \)  
height of the “plus” shaped strut

\( I_{yy} \)  
second moment of inertia

\( I_{\rho} \)  
moment of inertia

\( I_{1} \)  
second moment of inertia

\( K_{g} \)  
gust elevation factor

\( k \)  
radius of gyration

\( k_s \)  
buckling factor

\( L_{w\text{-exact}} \)  
exact lift of the wings

\( L_{w\text{-used}} \)  
ift of the wings used in calculations

\( L \)  
rib pitch

\( l \)  
efficiency factor

\( l_{l} \)  
distance between tailplane and centre of gravity

\( M \)  
Mach number

\( M_{b} \)  
bending moment

\( M_{t} \)  
torsion moment

\( M_{r} \)  
rolling moment

\( MTOW \)  
maximum take off weight

\( M2FW \)  
maximum zero fuel weight

\( m \)  
mass

\( N_{x,y,b} \)  
shear flow

\( n \)  
load factor
\( P \)  
shear force

\( P_i \)  
shear force in the front or the rear spar

\( p \)  
angular velocity by full deflection of the ailerons

\( p' \)  
required angular velocity

\( Q_{yy} \)  
static moment of inertia

\( q \)  
shear flow

\( q_{tu} \)  
local shear buckling loads, with reduction factor

\( q_o \)  
overall shear buckling load

\( R \)  
relative is thickness of the +/-45° fibre layers

maximum undercarriage reaction

\( r \)  
radius

\( r_{seg} \)  
distance between fuselage and engine

\( r_i \)  
radius of the corrugation

\( S \)  
shear loading

\( s \)  
wring area

\( t \)  
thickness

\( T_{1,2} \)  
loading in on the plate in 1,2 direction

\( U_{rea} \)  
design gust velocity (standard atmosphere)

\( V \)  
velocity

\( V_{max} \)  
maximum shear force

\( V/d \)  
volume per unit width

\( V_s \)  
shear force

\( V_{1,2,3} \)  
thickness of plate 1,2,3

\( v \)  
velocity

\( v_r \)  
fibre volume coefficient

\( v_r \sigma_f \)  
strength of the prepgs

\( U_{de} \)  
maximum gust speed when flaps are extended

\( W \)  
weight (per unit width per unit length)

\( w \)  
width near bolt

\( x \)  
distance to the centreline of the wing

\( y \)  
distance to neutral axis

\( \alpha \)  
angle between rib and line perpendicular to the skin

\( \alpha_t \)  
taper angle

\( \beta \)  
corrugation angle between flat sections

Prantl’s compressibility correction

\( \delta \)  
distance between centre fuselage and centre of gravity

\( e \)  
strain

\( \eta \)  
non-dimensional spanwise station

\( \eta_t \)  
tangens modulus coefficient

\( \varphi \)  
corrugation angle

\( \varphi'' \)  
rolling acceleration

\( \lambda \)  
taper ratio

\( \Lambda_{0.25}, \Lambda_{0.5} \)  
sweep angle at 0.25% or 50% of the chord

\( \mu_g \)  
(non-dimensional) mass parameter

\( \theta \)  
angle between struts and rib flange

\( \theta_{1,2,3} \)  
angle between each of the layers and the direction of the force \( T_i \)

\( \rho \)  
(mass) density

\( \rho_0 \)  
radius of gyration

\( \sigma \)  
stress

\( \sigma_f \)  
strength of the fibres

\( \tau \)  
shear stress
\( \nu \)  

poison's ratio

subscripts

\( a \)  

approach
additional
regarding the ailerons
allowable

\( a=b \)  

when width and length of the panel are same

\( a=0.5b \)  

when the width is half the length of the panel

\( a=0.25b \)  

when the width is a quarter of the length of the panel

\( b \)  

gust design

\( \text{basic} \)  

faces of sandwich panels together

\( \text{bearing} \)  

bearing

\( \text{bolts} \)  

bolts

\( \text{btm} \)  

bottom skin panel

\( c \)  

compressive
core

\( \text{con} \)  

connections

\( \text{cor} \)  

regarding a design with a corrugated web with flat sections

\( \text{cr} \)  

cruising
crushing

\( d \)  

diving

\( \text{eng} \)  

regarding the engines

\( \text{epoxy} \)  

regarding only the epoxy

\( eq \)  

equivalent

\( \text{extra} \)  

extra required

\( f \)  

flange
regarding only the fibres
when the aircraft is in flap configuration
for aircraft with fuel

\( \text{fa} \)  

face

\( \text{fibres} \)  

regarding only the fibres

\( f\)  

flexural

\( \text{flanges} \)  

rib flanges

\( \text{flaps} \)  

when the aircraft is in flap configuration

\( \text{fuel} \)  

for aircraft with fuel

\( \text{fusel} \)  

regarding the fuselage
due to gust
gdue to gust

\( \text{HMS-4} \)  

regarding a structure reinforced with HMS-4 fibres

\( \text{hat} \)  

regarding skin panels with hat-stringers

\( \text{hot/wet} \)  

in hot and wet conditions

\( I \)  

regarding skin panels with I-stringers

\( \text{IM6} \)  

regarding a structure reinforced with IM6 fibres

\( \text{I-stringer-panel} \)  

concerning a structure with I-stringer skin panels

\( i \)  

regarding front or rear web

\( l \)  

local

\( \text{MAC} \)  

mean geometric chord

\( \text{Mac} \)  

Moment around the aerodynamic centre

\( M=0.22 \)  

at a Mach number of 0.22

\( \text{max} \)  

maximum

\( \text{max-stain}=0.0037 \)  

if there is a maximum allowable strain (of 0.0037)
man | manoeuvre
---|---
manoeuvre | manoeuvre
min | minimum
neg | negative
o | overall
P-100 | regarding a structure reinforced with P-100 fibres
plate | simple flat plate
pos | positive
s | skin
| stalig
| stiffener
sandwich | concerning a structure with sandwich skin panels
soft-skin | structure with a soft skin configuration
stif | regarding a stiffened web design
strut | regarding the struts
Thornel75 | regarding a structure reinforced with Thornel 75 fibres
t | tensile
top | top skin panel
tot | total
titanium | concerning a titanium
rib | concerning the rib
unstif | regarding an unstiffened web design
w | wrinkling
| wing
| web
| strut
web | web
wing | regarding the wing
z | distance underneath hat stringer
α | per angle of attack
1 | regarding front web
2 | regarding rear web
0° | regarding the layers with 0° fibres
+/- 45° | regarding the layers with +/- 45° fibres
90° | regarding the layers with 90° fibres
Chapter 1  Introduction

Generally, people choose air travel over other forms of transportation because their desire for shorter traveling times justifies the cost. Also, in many cases, air travel is the only choice, so that if the cost is excessive the potential traveler does not make the trip. To open new markets and to expand current markets, the cost of this service must remain low enough to maintain that justification. Thus, greater fuel efficiency and reduced operational costs must be vigorously pursued.

Development of the structure of airframes and engines continues to be a key element in determining the economic success of aircraft. Structural weight is the single largest item in the empty weight of an aircraft and is, therefore, a major factor in the original acquisition and operating cost. One kilogram added to structural weight requires additional wing lift area to lift the aircraft (all other flight variables being held constant), additional thrust to overcome the associated incremental drag, and additional fuel to provide the same range. All of these additions result in further increases in structure. This vicious circle converges, in typical aircraft designs, to gross weight increases from 2 to 10 times the 1 kilogram empty weight increase that the cycle began with.

The aircraft industry started using fibreglass components in the early 1960s but they were only used for tertiary structures such as fairings or non-structural doors. It was in the mid to late 1960s that companies started examining composites for use on structural components; this coincides with the introduction of carbon fibre which probably had some bearing on the policy change. It was realized that the fibre reinforcement would allow designers to tailor the strength of a component in the direction most needed by strategic orientation of the fibres. Together with the fact that these composites have a lower density, the use of composite materials had the potential of reducing the weight of components.

At this stage it should be stated, for clarification, what a composite actually is. In the case of aircraft these composites mainly are fibre reinforced plastics. This means that the composites consist of fibres and a material, which keeps these fibres together, called the matrix. Usually a composite structure consists of laminates built up of individual layers with the same fibre orientation.

The slow rate at which composites are being adopted indicates that there must be drawbacks to using composites as well. The high cost of the raw materials is a major problem. The mechanical properties of a composite in hot and moist conditions can be greatly reduced. Furthermore, composite structures are sensitive to impact of foreign objects and, therefore, usually cannot reach their full potential. Thus, it cannot directly be stated that a primary structure made of fibre reinforced plastics will lead to the most cost-effective structure. Another reason for not using composites as of yet is that the topic is relatively new. Problems arise in the design phase that never arised before when designing with conventional materials like aluminium alloys.

Although, composite materials can be found in a wide variety of structures, it hardly is used in the main section of the wing of larger transport aircraft (i.e. the torsion box). The weight of an aircraft of that type might be reduced considerably if composites were used in such large part of the structure. The availability of composites also opens the door to structures that always were efficient but had to be disregarded because they were difficult to manufacture from conventional materials.
As composites could well be the future materials of the torsion wing box of large transport aircraft, research into this field seems justified. The ultimate goal, of course, is to create a wing box that is made of composites. In this report only an initial design is made. The project should, therefore, just be seen as a kick off. To achieve this initial design a study has been made to see what sort of similar structures exist. Furthermore, analytical calculations were done to find an initial design for a torsion box of a 70 passenger aircraft, which is able to withstand all the loads the aircraft will be exposed to.

This report is divided into 5 main chapters. In chapter 2 a survey has been done to see what type of structures can be created from composites. These structures also give an indication of what type of structures probably are the most efficient. The different sort of available fibres and matrices with their mechanical properties are presented as well at this stage. Problems regarding production also are assessed in this chapter because an efficient structure that cannot be made is useless. Furthermore, unique problems for composite structures are addressed. Chapter 3 gives the mechanical properties of the composites that will be used throughout this report. All of the loads that an aircraft of this size has to withstand are calculated in the chapter 4. In chapter 5 the different structures are assessed by analytic calculations. This leads to an initial design for a section of the wing box. In chapter 6 the results are discussed. It also states the problems that still have addressed to get to a final design and it gives an indication of how they can be solved.
Chapter 2 Composites in Wing Structures and its Problems

When researching the possibility of using composites more extensively in the design and construction of wings; it is essential to review development in this field so far. Earlier use and actual designs can be researched, and can form the basis for future development. Therefore, the first stage of every design process should be collecting information on the subject. This information can be found in books and papers. When dealing with a subject like the use of composites in aircraft structures one runs into several problems. Because the subject is relatively new a lot of problems have not been fully investigated yet. Thus, there is no information. Another problem is that the people who have the information, i.e. the aircraft manufactures, do not disclose it. This way they try to keep a competitive edge on their rivals. Two main groups of information can be considered:

- Indispensable information without which one cannot even start making a design, e.g. the mechanical properties of the materials.
- Current information which leads to efficient results without having to assess all of the possibilities, e.g. efficient structures found in aircraft made of composites.

This chapter is split up into different sections, dealing with various aspects relating to the use of composites in aircraft wings. Initially the type of aircraft already using composites are put together. Of course, it is important to know what the advantages and disadvantages are of using composites against conventional materials. These are stated in the second section. The third part addresses the methods of construction and design more fully. To finish of this chapter various "snags" that occur when using composites are reviewed.

2.1 Aircraft already using Composites in the Wing Structure

This paragraph presents the types of composite wing structures that actually are designed and build, in particular the structure of the torsion box. It also states the production process, and to what extent the torsion box is built up of individual panels, or as one integral structure, in so far as relevant information can be found in reference books.

As composite wing structures for transport jets are hardly used, the information has to be puzzled together from all kinds of aircraft and different types of structures. The available reference material is limited; it mainly consists of information relating to wing structure of smaller planes, fighter aircraft, and tailplanes of larger aircraft.

Set out below is the existing data:

All-composite utility aircraft

- Lear Fan 2100: The wing structure is a continuous three-spar wing box structure from tip to tip. The wing skins are made of fabric with tape build-ups over the spar channels to carry the bending loads. The entire wing structure is bonded. Fasteners reinforce the bonded skin-to-spar attachment in areas of high shear loads. Internal T-shaped stiffeners help to stabilise the
wing skins against buckling. The spars have a honeycomb web;
- Starship: The wing panels are laid up on female tools by laying down the outer skin first then a honeycomb core and the inner skin. The core thickness is reduced with steps of 0.125 inch over the wing span. The three spars are joined to the wing skins through corner clips and H-clips with continuous fibres through the intersections to provide load path continuity. Grooves are routed in the inner face and the corner clips and H clips are fitted up and adhesive is applied and the structure is clamped in place;
- Avtek 400 (with two box spars), Voyager, Speed Canard & kit planes: The shells consist of sandwich structures with no bonds on the outer surface.

**Fighter aircraft**

- The Harrier AV-8B/GR5: Graphite/epoxy is used in the wing box and substructure. The upper and lower wing skins are one-piece solid laminate panels. Sine-wave spars are extensively used, consisting of roll-formed webs with attached caps;
- F-18: Graphite/epoxy wing skin;
- X-29A: Graphite/epoxy forward swept wing;
- V-22 V/STOL: The wing consists of integrally stiffened composite laminate cover panels, with composite ribs and titanium spars. It has co-cured I-stiffened stringers;
- A-6: Carbon/epoxy wing box. The cover skins are fabricated as one-piece panel using unidirectional tape. It has composite intermediate ribs and spars;
- Jaguar: The wing box is a bolted multi-spar and rib configuration. The front and the rear spars are composite channel sections. Five intermediate spars are sine wave composite beams. The wing skins are predominantly ±45° lay-up (soft skin approach). Sandwiched between the wing skins and the spar flanges are booms of predominantly 0°, lay-up;
- Tornado: The main box has composite skins with full-depth aluminium honeycomb;
- Lavi, Rafale & Gripen: Use composites on wings;
- B-2: Largest all-composite airframe structure ever built;
- AFT: Extensive use of composite materials;
- European Fighter Aircraft: The wing layout consists of a lower skin co-bonded with spars and an upper skin that is mechanically permanently fastened. All 0°, ±45° and 90° plies are carbon/epoxy material. There are no planks in the skin (one-piece skin) and there are two fuel tanks with notched spars.

**Similar structures on transport aircraft**

**Airbus:**
A310/A320 fin torsion box: For the production of the skin panels a modular concept is used, see figure 2.1.1. The process consists of four stages:

- Carbon fabric prepreg is wrapped around three-part aluminium modules: this will constitute the web of the stringers and the rib shear tie.
- Carbon prepreg is laid into a mould and will constitute the external skin and stringer bottom flange.
- The aluminium modules are assembled into the mould.
- Unidirectional tapes are laid on the stringers to constitute their upper flanges. During the cure, pressure is applied to the skin and the stringers flanges in an autoclave and on the stringer webs by thermal expansion of the modules.
The torsion box of the A320 tail plane has the same structural set-up as the fin torsion box and, therefore, they both have the same structural set-up as a metal would have.

McDonnell-Douglas:
The DC-10 vertical fin box consists of honeycomb sandwich skins with a four spar and thirteen rib substructure. The skins consist of fabric graphite/epoxy face sheets oriented at 45° over a Nomex core; two plies on the outer surface and one ply on the inner surface. The spars and ribs all use graphite fabric/epoxy sine-wave webs in most areas. Spar and rib caps are built into the skin sandwich by gridwork of unidirectional tapes in a quasi-isotropic pattern to locally replace honeycomb.

Lockheed:
The TriStar vertical fin box structure is made of Thornel 300 and Narmco 5208 carbon/epoxy unidirectional (UD) tapes. The structure consists of two skins, two spars and eleven ribs. The skins are single stage cure assemblies of the panels with hat section stiffeners. Curing is effected on a contoured tool in an autoclave, the stiffeners being co-cured upon inflatable rubber mandrels. The spars are moulded in a steel matched-die tool using a thermal elastomeric process. Caps, webs, stiffeners and rib attachment angles are integrally consolidated and moulded in an autoclave, the resin cure being completed by oven post-cure. The composite rib designs consist of three solid web ribs and eight truss ribs with moulded caps and aluminium diagonals.

Boeing:
The Boeing 737 graphite/epoxy horizontal stabiliser is a two spar, eight rib box construction. The cover skin is a solid laminate; I-stiffened skin made from fabric and UD tape. The cover skin and its integral stiffeners are co-cured. The front and rear spars are solid laminates that are made in two channel sections, which are subsequently bonded back to back. The ribs are of a honeycomb sandwich construction in the vertical web portion only and the upper and lower flanges are solid laminates. The cover panels are mechanically fastened to the substructure. The Boeing 777 has a carbon/epoxy empennage with co-cured I stiffeners.

Transport aircraft

The ATR 72 has outside the engine nacelles carbon fibre reinforced plastics (CFRP) front and rear spars, light alloy ribs, and self-stiffening CFRP skin panels (it does not state if this is a sandwich panel or a stiffened panel).

C-130 Hercules: A composite centre wing box of the C-130 Hercules transport aircraft is being developed since 1984. The composite structure will, however, be geometrically the same as existing metal structural box. Therefore, it will have two spars and ribs at the same position as the metal one. It is of sufficient size to address fully many engineering and technology issues, which must be resolved before a composite wing box, can be fitted with confidence to a large transport aircraft.

Ref.[2.1.1], ref.[2.1.2], ref.[2.1.3], and ref.[2.1.4]
2.2 Advantages and Disadvantages of using a Composite Wing Structure

It is obvious that if one wants to design a structure with new materials, in this case composite materials, the pros and cons have to be established. If the cons outweigh the pros then, of course, the project should directly be abandoned.

The advantages of using composite materials as opposed to aluminium alloys may be considered in order of relative importance. The last two items are of minor importance in the case of transport aircraft:

1. Tailored directional mechanical properties;
2. Weight saving over aluminium alloys through high specific stiffness and specific strengths;
3. Excellent fatigue resistance;
4. Reduced number of assemblies and reduced fastener count when co-cure or consolidation is used;
5. Non-corroding in salt environments;
6. Aeroelastic tailoring is a possibility;
7. Tapered sections and compound contours are easily accomplished;
8. Reduced machining (mainly for smaller components);
9. Thermal expansion close to zero reduces thermal problems (important in outer space applications);
10. Modified radar response compared to metal structures (important for fighter aircraft).

Comment to:

Ad 1: One of the main reasons for using composite materials is that the designer can give the material different mechanical properties in different directions, the material is then called anisotrope. As the structure of the wing of the aircraft is heavily loaded in certain directions, mainly a bending moment and a shear force along the wing span, and hardly loaded in other directions, this is a great advantage. The reinforcing fibres give the composite its strength and stiffness. If a structure is heavily loaded in say the X-direction then most of the fibres will be placed in the same X-direction. This way the material is very efficiently used in contrary to an isotropic material like aluminium, that has the same strength and stiffness in every direction. An advantage for small aircraft is that certain sections of the aircraft carry certain parts of the loads. The skin only carries the torsion loads and the spar (caps) carry the heavy bending and shear loads. For larger aircraft that have a geometrically large torsion box this structure can not be used.

Ad 2: The other main reason for using composite materials is that it has better mechanical properties than conventional materials. This actually should be taken together with the fact that a tailored design can be made. One is useless without the other. A comparison between the important mechanical properties of a unidirectional (UD) composite reinforced with carbon fibres and aluminium alloys can be found in table 2.2.1. Carbon fibres are used because as will be stated at a later stage that this is the best reinforcement for this type of aircraft.

Ad 3: Composites have an excellent fatigue resistance as can be seen from figures 2.2.1 through 2.2.4. Figures 2.2.1, 2.2.2, and 2.2.4 show that if strains in a composite structure
(reinforced with carbon fibres), throughout the life time of the aircraft, are kept below 0.55%, fatigue problems will never occur. The strain in the skin panels is always (far) below this value for reasons concerning impact damage. The ribs and the spars are protected by the structure around them. If higher strains are allowed in the ribs and spars fatigue will have to be considered. Hot and wet conditions can also deteriorate the mechanical properties considerably; the strains, therefore, will also be low in the structures that are not likely to be damaged. In general it can be stated that fatigue will not be an issue when composites are used. However, it should be checked from time to time during the design phase, that fatigue really does not lead to failure. Figures 2.2.5 through 2.2.7 are given to complete the picture; they show potential problems regarding fatigue of composites.

Ad 4: If sections can be made in one process, expensive manual labour is heavily reduced. Local reinforcements are needed when mechanical joints are necessary because the holes needed for these connections are stress raisers. A composite with a thermoset resin needs to be cured: The final structure, which is still wet and flexible at this stage, is put into an autoclave. In this autoclave the temperature and the pressure are raised. After curing the structure is hard (set) and is ready for final assembly. If sections that normally would be constructed separately, are cured together it is called co-curing. A good example is that of the connection between the ribs and the skins. The rib flanges can be co-cured with the skin panels in the case of a composite structure. Neither joints nor holes have to be made in the skin panels. When a large aluminium structure is constructed this is unavoidable.

Ad 5: Unlike conventional aluminium structures composites are non-corroding in salt environments. Aluminium structures have to be treated with epoxy primers to protect it during the production process and are painted again before they are taken in use.

Ad 6: Civil transport aircraft do not use aeroelastic tailoring at the present. The only aircraft that uses aeroelastic tailoring is the X-29A fighter aircraft, which has forward swept wing. It is, therefore, questionable if it is such an advantage. In this report aeroelastic tailoring is disregarded. Thus, the laminates will all have symmetric lay-up. There is no coupling between bending and in-plane deformation of the middle surface, these problems can, therefore, be addressed separately.

Ad 7: Composites are usually flexible at the beginning of the production process. Composite skin panels can directly be curved by laying them on curved tools. Aluminium skins have to be curved as they usually come in as flat plates at the beginning of the production process.

Ad 8: Small sections of the structure that are highly complex of shape but not important in the way of load carrying capability, like fairings for example, can be easily made of composite materials. Using aluminium for these structures leads to an intensive production process, because of its highly curved shapes. This is big advantage but it has no bearing on the torsion box structure.

Ad 9 & 10: As stated above these advantages do not apply to transport jets (except maybe for the nose of the aircraft).

By contrast the disadvantages are listed below, also by order of importance.

i- High cost of the raw materials;
ii- A lower impact resistance and the lack of crack growth data;
iii- Resin material is sensitive to humidity and temperature change;
iv- Expensive autoclaves are needed;
v- Delamination of the composite is can occur especially at the ends and the joints;
vi- Carbon fibres corrode aluminium when in contact with each other.

Comment to:

Ad i: A major problem when constructing with composites is the high price of the raw materials. Firstly the fibres must be manufactured, which by itself is an expensive process, and then they have to be impregnated with resin. It is possible to buy fibres and then add the resin during the production phase. However, it is also possible to buy prepregs (thin layers of fibres pre-impregnated with a resin). Figure 2.2.8 gives the price range of these prepregs per kilogram. For prepregs with high strength fibres the cost is approximately 30 £/kg to 60 £/kg and for prepregs with high modulus fibres the price is approximately 90 £/kg. The price of aluminium alloys used in wing structures is estimated at 2.5 £/kg*. Although, the weight of a structure consisting of composites usually is lighter, these figures indicate that the cost of the materials will be a lot higher for a composite structure. The (operating) costs, of course, do not only consist of the raw materials, but it can be stated that the other costs for a composite structure have to be dramatically lower than for an structure of conventional materials, to make up for this difference in the price of the raw materials. Table 2.2.2 gives the cost of the different materials.

Ad ii: Composites can delaminate after impact. These delaminations cannot be detected from the outside of the structure. This is called “Barely visible damage” (BVD). This BVD must not reduce the static strength below the ultimate load. Due to the absence of a reliable damage growth prediction concept this only leaves a no-growth concept. BVD gives the matrix damage through the laminate. The structure will have a substantial reduction in load bearing capability, especially in compression. In practice this means that the strains, in the parts of the structure that can be hit by foreign objects, are kept low. A value of 0.37% or 0.42% should be used for a structure in compression and slightly higher value is allowed for structures in tension. Thus, the strength of a composite structure is based upon (barely visible) damage state unlike a metal structure, which is designed with fatigue phenomena and detectable crack length in mind. Visible damage can also occur. Impact damage, which is visible, does allow the strength to reduced to 1.2 times the load encountered in normal flight. Visible damage has to be repaired before the next flight.

A common form of impact damage can be caused by hailstones. Certification tests carried out upon the A 310 composite fin show that with the aircraft on the ground a skin thickness of 0.8 mm will withstand a 0.05 m hail strike (see figure 2.2.9). Airbus impact criteria norm states that all components must withstand damage up to an energy of 35 joules such as the free fall energy of a 0.05 m stone and at higher energies there shall be no growth or evidence of damage assessed by full scale tests. There is no major difference between sandwich and monolithic constructions for the same skin thickness.

Ad iii: Certification requirements of airframe structures are ultimately identical whatever material is used. Composite materials are influenced by the environment. Therefore, they have variable properties. Every composite has a certain equilibrium of moisture uptake (with a maximum of about 1% in the case of laminate with an epoxy resin) related to the temperature and humidity of the surrounding air. The aircraft will have to be air worthy under all conditions. Tests are, therefore, done in 'hot-wet' and 'cold-dry' conditions.

* According to Prof. Vogelesang, L.B., Faculty of Aerospace Engineering, Delft
The matrix will become softer and more yieldable due to the intake of moisture. The compressive strength will be reduced but the amount of delamination after impact will decrease. The decrease of properties due to 'hot-wet' conditions can be found in table 2.2.3 and table 2.2.4.

Tests were done on laminate consisting of IM6 reinforcing fibres and a toughened epoxy CYCOM 1808. In table 2.2.2 it can be seen that in hot and wet conditions (high rate of humidity) the laminate loses 59.4% of its strength in compression, at a temperature of 93°C, in comparison to its ultimate tensile strength. The 93°C is the maximum operating temperature for a transport aircraft. From table 2.2.3 it also follows that the high temperatures hardly have any effect on the modulus of the laminate. Table 2.2.4 shows that if the laminate is in tension that hot and wet conditions have no influence. This is not very surprising as the fibres carry virtually all of the loads. Thus, the softening of the resin does not have any effect on the strength of the laminate like in the case of compression. At this stage only thermoset resins are considered because of the lack in specific data of laminates with thermoplastic resins.

Ad iv: After the structure is constructed into its final shape all the layers of the composite are still flexible and separate. They have to be made into one solid structure. In the case of a composite with a thermoset matrix, this is done by elevating the temperature and pressure, so that the layers can chemically bond. This happens in an autoclave.

The main elements of an autoclave system are: A vessel to contain pressure; sources to heat the gas stream and circulate it uniformly within the vessel; a subsystem to pressurise the gas stream; a subsystem to apply vacuum to the parts covered by a vacuum bag; a subsystem to control operating parameters; and a sub system to load the moulds into the autoclave.

It is quite obvious that this is an expensive piece of machinery. For the wing of an aircraft this autoclave will also have to be very large. If an aluminium structure is manufactured then the elements firstly will have to get their required thicknesses, the skins then have to be curved into the right shape, and finally the separate elements have to be mechanically fastened. It will not, however, need an expensive autoclave at all. Thus, a composite structure can reduce the production process tremendously but high investments have to be made.

Ad v: At the edge of a composite the laminate acts less like a plate. Here the laminate actually can be seen as separate layers. Moisture can easily get in-between the layers and lead to delamination. There is also the additional problem of local inter-laminar stresses. Near the edge of a laminate the layers want to have different strains, which induce these inter-laminar stresses. This will be explained more fully in sub paragraph 2.4.2. However, generally speaking it can be said that edges of composites need extra attention.

Ad vi: Carbon fibres corrode aluminium. Therefore, a composite structure reinforced with carbon fibres cannot be directly connected to an aluminium structure. This problem is easily solved by applying layers reinforced with glass fibres instead of carbon fibres in-between the connections.

Ref.[2.2.1], ref.[2.2.2], ref.[2.2.3], ref.[2.2.4], ref.[2.2.5], ref. [2.2.6], and ref.[2.2.7]
2.3 Materials and Methods of Construction and Design

2.3.1 The Materials

It is important to know what types of material are available. Firstly, it is stated again that composites consist of reinforcing fibres bonded together by resin material.

Reinforcements

The reinforcing fibres are made of carbon because they will perform the best in a torsion box. The drawbacks of other fibre material to carbon fibres:

- Aramid fibres (Kevlar): Aramid fibres have an excellent toughness. For primary structures the modulus, however, is too low and it has a low compressive strength;
- Boron fibres: Boron fibres have a high stiffness and were used in primary structures but nowadays they are not used in high volume because of their relatively high cost and large fibre diameter. The fibres are extremely stiff and brittle; difficult to work with and limit the minimum radius around which they can be wrapped. Another problem is the ply thickness, which is determined by the filament diameter and desired fibre volume fraction;
- Glass fibres: The advantage of lower cost of glass fibre prepreg materials compared to carbon fibre prepreg materials is offset by lower specific strength and the much lower specific modulus. In highly loaded structures it is not feasible to use glass fibres.

It still has to be mentioned that there are ceramic fibres like Quartz, Silicon Carbide fibres and Alumina but these are for components that are exposed to extremely high temperatures. Polyethylene seems very promising. It has a better impact resistance than glass or carbon and the specific strength is high. Its melting point is low though (150°C).

The conclusion is that at the present carbon fibres are the only feasible reinforcement for a torsion box structure of the wing for a large transport aircraft. A comparison of the mechanical properties of glass fibre, boron fibre, kevlar and carbon fibre can be found in table 2.3.1.

Matrices

The main groups of matrices are thermosets and thermoplastic polymers.

- Thermosets: Epoxy resins are the major composite materials for low-temperature composite applications. Other thermosets are: Polyester and Vinyl ester resins for low temperature applications and Phenolic resins, Polymides and Bismaleimide resins for high temperature applications. The thermoset resins, which can be used at higher temperatures, have lower mechanical properties, though.
- Thermoplastic polymers: Thermoplastic polymers with reasonable temperature resistance included PEI, PES and PEEK. PAI is moulded as a thermoplastic but is then post-cured in the final composite to improve the temperature resistance. Excellent strain capabilities, high moisture resistance, unlimited shelf life and short processing cycles are the main advantages for aeronautical applications. Disadvantages are the high processing temperatures, less chemical solvent resistance than thermosets, the lack of processing experience and
drapeability.

For special applications carbon and metals can be used. These are not used in wing structures.

Thermoplastic resins seem to be a realistic choice for the matrix, mainly because of the allowable strain after impact is higher and closer to the allowable strain of the carbon fibres. The processing experience is not yet at a stage that it is actually being used in transport aircraft. The lack of information is another drawback. A comparison of the mechanical properties of thermoplastics and epoxy can be found in the tables 2.3.2, 2.3.3, 2.3.4 and 2.3.5.

As carbon fibres will be used, the different types of carbon fibres are given:

It must be stated that the names carbon (preferred by European users) and graphite (preferred by U.S. users) are used interchangeable, but there is a difference. Carbon fibres have 93-95% carbon and graphite fibres have more than 95% carbon. Carbon fibres are classified into three categories:

- Polyacrylonitrile (PAN) precursors are the basis for the majority of carbon fibres available today. PAN-derived fibres offer the highest strength and best balance of mechanical properties in composites. They are usually selected for their high strength and efficient retention of properties.
- Pitch precursors based on petroleum asphalt, coal tar, and polyvinyl chloride can also be used to produce carbon fibre. Pitch-bases fibres are not as strong as the PAN fibres, but the ease with which they can be processed to a higher modulus makes them attractive for stiffness-critical applications. They are relatively low in cost. Their most significant drawback is nonuniformity from batch to batch.
- Rayon precursors which are derived from cellulosic materials, were one of the earliest precursors used to make carbon fibres. They do not have the high mechanical properties available in PAN and pitch-bases fibres.

The mechanical properties are frequently found in the names. H(T)S, HM and UHM stand for high, or ultra high, tensile strength fibre or modulus fibre. A list of available carbon fibres can be found in tables 2.3.6 and 2.3.7.

2.3.2 The Structures and Production Methods

It is important to know what types of structure are feasible. The possible structures are given for the individual components of the torsion box. This results from paragraph 2.1. This section, however, also regards the production process and the special limitations it places on the design with regard to lay-up or shape.

Skin panel structures found in practice:

These can be divided into two main groups sandwich panels and skin panel with stringers (see figure 2.3.1). The type of stringers is usually I-stringers. The fin of the TriStar, however, has hat-stringers.
Rib structures found in practice:

The TriStar has three solid web ribs and eight truss ribs with composite moulded caps and aluminium diagonals in the fin torsion box. The ATR 72 has aluminium ribs. The Boeing 737 has got ribs that are of honeycomb sandwich construction in the vertical web portion only and the upper and lower flanges are solid laminates. The DC 10 has mostly composite ribs with sine-webs. Generally speaking it can be said that all sort of rib structures are used. The main weight reduction is gained in the amount of ribs used in a composite structure.

Spar structures found in practice:

The spars in the larger aircraft usually have stiffened webs. Corrugated webs are also found. Smaller aircraft sometimes use a sandwich panel configuration for their spar structures. The V-22V has titanium spars.

The production of skin panels:

The skins are usually either cured on their own or co-cured together with the stringers. If a reasonable number of wing skins have be manufactured automation is worthwhile. In that case tape-laying machines are beneficial. The machine dispenses tape from a supply roll, places it directly onto the underlying mold or tape-covered surface, applies shoe pressure to seat and debulk the tape and at the end of each movement across or along the mold, cuts the tape to the required length and angle. It also changes the angle from layer to layer so as to provide the advantages of cross-ply structure. Local reinforcement tapes can also be applied. It uses preimpregnated unidirectional tape. The stringers and rib attachments for co-curing will have to be placed on the skin before it is placed in the autoclave. I- and J-sections as well as sections like hat shaped stringers can be co-cured. However, if hat shaped stringers are used, hat shaped rib attachments also have to be used if the panel is co-cured. Hat shaped sections need a rubber bladder, or something similar, that can be retracted from the structure after the cure.

The angles of the plies of the skins are unlimited. However, there are a couple of practical limitations. Too many changes in ply angles throughout the thickness of the skins can cost a lot of extra manufacturing time. Ply angle changes larger than 60° are not used for strength reasons. In practice fibre orientations are usually 0°, ±45° and 90°.

For sandwich panels both thermosets and thermoplastics can be used. For thermosets the prepregs are stacked on a clean mandrel then the core is put in place, with or without an adhesive layer and then the other facing is stacked. A vacuum bag is placed and then every thing is placed in an autoclave. Press cure can also be used. For thermoplastics the same procedure can be followed although it is better to consolidate the facing in an earlier phase. The drapeability is a problem. If co-curing is not possible, an adhesive layer is applied between the pre-cured laminates and the core; this assembly is then heated to cure the adhesive layer.

Extra attention should be given to the edges of the sandwich panels: The edge closeout area of sandwich panels usually require the use of edge doublers for heavily loaded panels. The edges must be strengthened to withstand fastener, screw, or bolt loading, as well as to protect the panel against damage during installation and/or repair and maintenance. When co-curing the panel edge closeouts, they are cured in place at the same time as the face-sheets and core is being cured together. With secondary bonding it is usually possible to attach face-sheets and
edge-closeouts to the core in a single bonding cycle. In either case, adequate cure pressure must be applied to the edge member to assure a good bond to the core and face-sheets. When designing edge-closeouts for either honeycomb or foam panels the following factors should be considered. The core has to be carved at about 30° or less to minimise angular bond pressure loading. Z or channel shaped edge closeouts are made as separate members prior to bonding and the inside core face must be bonded to the Z's vertical face to transfer the shear load without bending the skins. Some structures can be found figure 2.3.2.

In the case of the Starship a different solution has been used. Corner clips are used to connect the honeycomb skin and the front spar. The thickness of the skin sandwich structure is locally hardly reduced. The clips are a woven product with continuous fibres through the intersections providing load path continuity.

Before going any further some attention must be given to the core materials of the sandwich panels. They have either a honeycomb configuration or they are continuous.

- Honeycomb: This is produced from various aluminium alloys; glass fabric/resin composite; meta-aramide paper (Nomex)/phenolic resin and Kraft or other cellulose papers with or without resin impregnation. The honeycomb can have different shapes like hexagonal, rectangular and flexible. This also influences the compressive strength of the core.

- Plastic foams: These are produced with either 'closed' or 'open' pores and as either performed rigid sheets or as fluids for injection into cavities. Most thermoplastics and some thermoset resins can be foamed but the materials commonly used in aircraft components are PVC and polymethylacrylimide. Polystyrene and polyurethane are also available.

- Syntactic core materials: These consist of resins filled with microspheres to reduce the density. Various materials are formed into microspheres, including glass, ceramics, plastics and carbon.

Mechanical properties of some core materials can be found in tables 2.3.8 through 2.3.11.

The production of spars & ribs:

There are enough examples of sine-webs, corrugated webs and stiffened webs to say with confidence that these set-ups do not pose any problems. Elastometric forming is used to produce the sine or corrugated form (see figure 2.3.3). Cure or consolidation pressure is applied by thermal expansion of rubber. The web laminates are first placed over silicone rubber shell mold which are placed on strong backs that are removed before the cure. Another option is to press a heated composite, with a thermoplastic matrix, into the desired shape. The corrugated angle is limited to about 30° due to stretching of the material. Box shaped spars can also be manufactured. Pultrusion could be used to manufacture composite struts of truss ribs. In the pultrusion process reinforcing fibres, pulled from a series of creels, are pull through a resin bath and preformed to the shape of the profile. The composite material is then placed in a heated steel die, which has been machined to the final shape, to be manufactured.

In conclusion it can be stated that the ply angle lay-up does not have any limitations. Because of the curved shape of the skins it is difficult to manufacture composites with thermoplastic matrices. Although it is not impossible to use thermoplastics, especially when making sandwich structures, composites with thermoset matrices are preferred because of the ease in which panels can be co-cured. Open shaped stringers do pose extra problems. Spars and ribs can easily be shaped with thermoplastic and thermoset resins to almost any desired form. The angle of corrugation cannot be much greater than 30°.
The dimensions of the laminate can be varied along the span. With a tape laying machine it is thought that as long as the top layer is no smaller than the layers underneath, so that the laminate can always be cut to size, the thickness can vary. A more realistic solution is to first separately tape lay and cut the layers with the biggest wing span and then the one with the second biggest span etc. In a second phase they can be stacked, thus, creating a stepped design. It does not seem feasible that the lay-up can vary when tape laying is used.

Ref.[2.3.1], ref.[2.3.2], ref.[2.3.3], and ref.[2.3.4]

2.4 Various Issues that have to be Addressed when Making a Composite Structure

2.4.1 Problems Near the Edge of a Laminate

In the region of the edges of the panels and cut outs inter-laminar shear stresses are found. It must be emphasised that the inter-laminar shear stresses are confined to a region known as the bending boundary. This region of disturbance is restricted to a width, which is approximately equal to the laminate thickness. In this region bending stress couples and transverse shear resultants also exist. Outside of this region a state of membrane stress and deformation exists and inter-laminar shear stresses are zero. Hence, inspection of inter-laminar stress can be concentrated in the bending boundary layer regions near to any load or structural discontinuity, such as an edge or change of section. A practical solution to keep the through-the-thickness stresses down is to use repeated sub-laminates. The highly spliced or dispersed laminate is stronger and less likely to delaminate than a lay-up with a large grouping of plies with the same angle. Inter-laminar stresses, which can cause delamination, are not explicitly analysed in laminated plate theory. Adjacent plies should be oriented (when possible) with no more than 60° between them, the smaller the difference the better it is. Inter-laminar stresses are a problem near the free edge of a laminate. A good solution is a laminate spirally stacked like a staircase, see figure 2.4.1; the inter-laminar stress is low throughout the laminate in that case. If a 16-ply laminate is needed with the same angle a choice could have to be made between a [0/45/90/-45]_2, or a [0/45/90/-45]_3. The first laminate should be chosen because the inter-laminar normal stress is half of that of the second laminate. When the laminate exists of a lot of layers and stacked in the right way the effect of the stiffness of the separate layers is very small.

Fasteners:

Thus, the area near the joints needs extra attention. When a joint is designed in a composite structure the effect of inter-laminar stresses should not be forgotten, however, there are practical design guidelines for mechanical joints. These are:

- Use larger fastener edge distances than with aluminium design, such as c/d > 3;
- Use a minimum of 40% of ±45° plies;
- Use a minimum of 10% of 90° plies;
- Use a fastener spacing of four times the fastener diameter: w/d = 4;
- Use small values of d/t like 1-2;
- Use washer under collar or wide bearing head fasteners;
- Use tension protruding heads when possible;
- Use two row joints when possible;
- Use materials such as titanium, corrosion-resistant steels, nickel and cobalt alloys to withstand corrosive effects;
- Use for simplicity quasi-isotropic patterns (0/± 45/90) or (0/45/90/-45) which are close to the optimum;
- Do not use mixed fastener types, like permanent fasteners and screws;
- Beware that there is a great difference between single-lap and double-lap joints. Eccentricity in the load path for single-lap joints leads to non-uniform bearing stresses in the laminate.

At this stage it is also interesting to see what types of fasteners there actually are. The main group is metallic fasteners, they are:

- Compatible materials like titanium, inconel and A286 steel;
- Alloys which can be used with caution like stainless steels, monel and PH steels;
- Non-compatible alloys may be used if reliable coatings or environmental conditions permit their use.

Generally speaking one should avoid using aluminium and carbon fibres together as the carbon corrodes the aluminium.

Fastener configurations:
For primary structures, a shallow head and large diameter shear fasteners will be more efficient in thin laminates where thickness is less than one diameter. However, the standard tension head fastener is more efficient for thick laminate joints where t/d is greater than 1.0. A sleeve fastener system should be considered in critical joint applications. There are special bolts for composite structures because specific problem could occur. Examples are HUCK-COMP and HUCK-TITE fasteners made out of titanium. There are also blind fasteners, used when there is only access from one side. Examples are COMPOSI-LOK II and COMP-TITE blind fasteners, they have titanium threaded sleeve and stainless stem, back side sleeve and washer, that does not crush the composite during installation. Others are VISU-LOK, COMPOSI-BOLT and HUCK-TIMATIC.

Fasteners can also be made of composite materials. These are not usable for heavily loaded structures.

*Adhesively bonded joints:*

Although, attractive from the theoretical viewpoint, thicker bonds are not practical because of the impossibility of making them without intolerable levels of flaws or porosity. The inability to vary adhesive thickness means that inter-laminar failure of the adherend, peel of the adhesive or failure due to the shear strength of the adhesive arise mainly because of the influence of adherend thickness. In theory joints in composites should be adhesively bonded. For big structures adhesive bonding is not used, though. For thicknesses greater than 6.35 mm stepped or scarf joints should be considered. Theoretically, merely by making the angle small enough, it should be possible to make the joint. In practice, particularly for wide joints, it will be impossible to make the required knife-edge and the finite tip thickness could cause strength reductions of 25%.
Cut-outs:

If cut-outs lead to problems the question arises how access to the interior of the wing is provided and does the design of small cut-outs appear to be the different to the cut-outs in metal structures? The structures of transport aircraft which are similar to that of the wing, like fin torsion box, all seem to have their cut-outs for inspection and maintenance in the front and rear spars. The Harrier, however, does have cut outs in the top and bottom skin which externally are identical to those of metal structures. A cut out design can be found in figure 2.4.2. A door or hatch malfunction in a composite structure should not lead to a loss of aircraft, in this situation the structure must be able to carry the limit load. Inter-laminar stress problem should not be forgotten when dealing with cut-outs.

Stitching:

There are ways of improving the out of plane properties. Unconventional lay-ups, three-dimensional weaving or stitching can be used. It is, however, rarely found in practice because of its high cost.

One of the drawbacks of stitching is that the number of plies should not exceed a maximum of eight, in order to maintain sufficient conformability and permeability for impregnating resins. The advantage of three-dimensional forms is that the out-of-plane properties are improved. These properties are improved considerably by the reinforcement in Z-direction. The laminate will create boundaries of high shear stiffness when it is stitched-through, due to the fibres in the out of plane dimension. The boundaries reduce the inter-laminar stress and act as delamination stopper. For example, the inter laminar stress-strength can be increased by a factor 3 and the inter-laminar fracture toughness by a factor 10. The stitching must be close spaced before it will be active, so it will be expensive to manufacture.

Three-dimensional panel stiffeners also exist, in orthogrid and isogrid panels (see figure 2.4.3). The reinforcing three-dimensional grids are composed of layers of composite material and syntactic material. The UD composites are continuous through the nodes and the low density syntactic is discontinuous through the nodes. The advantages are structural continuity in frame and stringers, damage tolerant and it permits automated fabrication at low cost. The drawbacks are that it is difficult to splice, reinforcement around cut-outs is difficult and needs special design, careful design is needed to bond skin and grids. Furthermore, it is difficult to inspect and the end result is a blade panel.

2.4.2 Miscellaneous Problems

There are special problems for carbon fibre, requiring protection or other measures.

The aircraft has to be protected against the environment. Of course, these problems mainly exist on the outside surface, however the interior also has to be protected against chemicals like fuel.

Protection against fuel:

In tests it becomes apparent that unprotected composites are seriously affected by fuel
exposure. Tests have been done on short-beam shear and ± 45° tension specimens. The most degrading environment on the T300-5209 material was a fuel-water combination. The specimens lost about 11% of tensile strength. The short-beam lost 40% of its shear strength. Another problem besides the loss in strength is that the composite also absorbs fuel. Polysulphides are used for sealing integral fuel tanks because they have an excellent jet fuel resistance and are flexible and have good joint movement capability. Sometimes bladders are used in aircraft wings to carry fuel.

**Protection against Ultraviolet Radiation:**

Ultraviolet (UV) radiation affects the surface layer of epoxy. The fibres will become completely exposed. Polyurethane aircraft paint offers substantial protection against ultraviolet radiation. Carbon fibres themselves, unlike aramid fibres, are not affected by UV radiation.

**Protection against rain and sand erosion and marine environment:**

Erosion of composites can result from exposure to rain, dust and sand. This is mainly a problem for the aircraft leading edge components. Lightning strike protection is usually provides enough erosion protection. Polyurethane also gives protection. The forward facing edges of laminates, which are exposed to the airstream, should be protected with an anti-peel ply. Composites are able to withstand the marine environments, i.e. high humidity and highly corrosive salty moisture, better than metals. This environment could attack secondary adhesive bondlines and mechanically fastened joints and weaken the composite strength. In table 2.2.3 it already could be seen that hot and wet conditions reduce the strength of composites considerably.

**Protection against lightning strikes:**

A composite reinforced with carbon fibres is significantly less conductive than aluminium. Unless protected, composites suffer more damage due to lightning strikes than aluminium structures. The aircraft can be divided into three lightning zones:

- Zone 1: Surface of the vehicle for which there is a high probability of direct lightning flash attachment or exit.
- Zone 2: Surface of the vehicle across which there is a high probability of a lightning flash being swept by the airflow from a zone 1 point direct flash attachments.
- Zone 3: Zone 3 includes all of the vehicle areas other than those covered by zone 1 and zone 2 regions. In zone 3 there is a low probability of any direct attachment of lightning flash arc, but zone 3 areas may carry substantial amounts of electrical current by direct conduction between some pairs of direct or swept-stroke attachment points in other zones.

The different lightning strike zones can be found in figure 2.4.4. The torsion box of the wing is a 'zone 3' region except straight behind the engines where it is a 'zone 2' region. Lightning effects can be divided into direct effects and indirect effects. Direct effects cause physical damage to the aircraft and/or electrical systems due to the direct attachment of the lightning channel. Indirect effect are voltage and/or current transients induced by lightning in electrical wiring which can produce upset and/or damage to the components within the
electrical systems. The indirect effects are of not important in the case of the torsion box of the wing as the electrical wiring is either behind or in front of it.

There are a couple of issues to keep in mind. The protective system should always permit the dissipation and flow of static electricity to a substructure ground or toward static discharges (pigtails). Although, carbon fibre laminates have some conductivity, puncture danger exists for skin thickness less than 3.8mm. More importantly, external fastener heads attract lightning. This can result in combustion between fasteners. The sparking in a fuel tank can be fatal. If the fasteners on the inside of the structure have a polysulfide topcoat sparking is not possible (additional cover caps can be placed over the fasteners on the inside of the structure). Wing fuel tank skins provide adequate protection against ignition of fuel vapours for swept stroke areas, if the skin thicknesses are electrically equivalent to an aluminium thickness of 2mm. Fuel vapour should not be able to escape at the fuel tank access doors. This can be achieved by applying a gasket on the fuel side.

The lightning protective methods for carbon composite materials are:

- Woven wire mesh on the outer surface. Aluminium wire diameter up to 0.2mm located on 3.175mm centres;
- Aluminium flame or arc spray (0.15 to 0.2mm thick);
- Nickel coated fibres;
- Aluminium interwoven wires (0.1mm wires and 8 wires/inch);
- Conductive paints;
- Conducting polymers;
- Aluminium bar diverter with use of fasteners.

Although, the aluminium bar with diverter is the last on the list it is the best solution for 'zone3' regions. They transfer lightning currents across these regions and are able to withstand the first return stroke. This is also the solution used by the major aerospace companies like Lockheed and Airbus for similar structures. If aluminium-foil strips are placed at right angles to the airstream on a composite-skin external surface, accompanied by polyurethane paint over the fasteners it provides as much lightning protection for much less weight than a mesh wire.

So in conclusion it can be stated that the torsion box of the wing, excluding the area near the engines, has enough protection against fuel, rain and sand erosion, marine environment exposure and lightning if the following measures are taken:

- Usage of polysulfide sealant on the inside of the wing in the fuel tank regions and just over the fasteners in the wing section without fuel tanks;
- Usage of polyurethane paint on the exterior of the skins and fasteners;
- Usage of aluminium bar diverters and glass fibre for protection against galvanic erosion between the carbon layers and the aluminium bars;
- Anti-peel ply at the front of the exterior skins, if necessary.

Ref.[2.4.1], ref.[2.4.2], ref.[2.4.3], ref.[2.4.4], ref.[2.4.5], and ref.[2.4.6]
References:


[2.2.3] Rijn, L.P.V.M. van, “Design & Composites”, College diktaat LR 27 C, 1994, chapters 6, 7, and 10


<table>
<thead>
<tr>
<th>Material</th>
<th>Modulus of elasticity $E$ [Gpa]</th>
<th>Yield stress $\sigma_y$ [Mpa]</th>
<th>Density $\rho$ [Kg/m$^3$]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Aluminium alloy 2014-T6</td>
<td>73</td>
<td>410</td>
<td>2800</td>
</tr>
<tr>
<td>Aluminium alloy 7075-T6</td>
<td>72</td>
<td>480</td>
<td>2800</td>
</tr>
<tr>
<td>UD Composite*</td>
<td>178</td>
<td>2880</td>
<td>1548</td>
</tr>
</tbody>
</table>

* UD Composite consists of 60% IM6 Carbon fibres and 40% Epoxy resin

Table 2.2.1 Indication of the difference of mechanical properties of conventional materials and composites (ref.[2.2.1], figure 3.9, and ref.[2.2.8], page 743-745)

<table>
<thead>
<tr>
<th>Prepregs with high strength fibres</th>
<th>Prepregs with high modulus fibres</th>
<th>Aluminium Alloys</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cost in £/kg</td>
<td>30-60</td>
<td>90</td>
</tr>
</tbody>
</table>

Table 2.2.2 Price comparisons of raw materials
<table>
<thead>
<tr>
<th>Property</th>
<th>CYCOM 1808/IM6</th>
<th>CYCOM 1808/IM6 (with interlayer)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Unidirectional Properties</td>
<td></td>
</tr>
<tr>
<td>Tensile strength, ksi</td>
<td>325</td>
<td>295</td>
</tr>
<tr>
<td>Tensile modulus, Ksi</td>
<td>22.4</td>
<td>20.6</td>
</tr>
<tr>
<td>Compressive strength, ksi</td>
<td></td>
<td></td>
</tr>
<tr>
<td>23°C/dry</td>
<td>200</td>
<td>180</td>
</tr>
<tr>
<td>93°C/dry</td>
<td>184</td>
<td>160</td>
</tr>
<tr>
<td>132°C/dry</td>
<td>173</td>
<td>152</td>
</tr>
<tr>
<td>93°C/wet</td>
<td>132</td>
<td>143</td>
</tr>
<tr>
<td>132°C/wet</td>
<td>69</td>
<td>94</td>
</tr>
<tr>
<td>Compressive modulus</td>
<td></td>
<td></td>
</tr>
<tr>
<td>23°C/dry, Ksi</td>
<td>22.0</td>
<td>20.0</td>
</tr>
<tr>
<td>93°C/dry, Ksi</td>
<td>21.5</td>
<td>-</td>
</tr>
<tr>
<td>Quasi-Isotropic Properties</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Compressive strength, ksi</td>
<td></td>
<td></td>
</tr>
<tr>
<td>23°C/dry</td>
<td>81</td>
<td>95</td>
</tr>
<tr>
<td>93°C/dry</td>
<td>89</td>
<td>-</td>
</tr>
<tr>
<td>132°C/dry</td>
<td>85</td>
<td>62</td>
</tr>
<tr>
<td>93°C/wet</td>
<td>78</td>
<td>63</td>
</tr>
<tr>
<td>132°C/wet</td>
<td>47</td>
<td>50</td>
</tr>
<tr>
<td>Compressive strength after impact</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1500 in.-lb/in., ksi</td>
<td>30</td>
<td>53</td>
</tr>
</tbody>
</table>

Table 2.2.3 The effect of hot and wet conditions on composites (ref[2.2.5], table 1.7)

<table>
<thead>
<tr>
<th>Design conditions</th>
<th>Planar element</th>
<th>Non-primary single-riveted joint</th>
</tr>
</thead>
<tbody>
<tr>
<td>T°(C)</td>
<td>Humidity (%)</td>
<td>Tension (kg mm⁻²)</td>
</tr>
<tr>
<td>20</td>
<td>Dry (0.4%)</td>
<td>100</td>
</tr>
<tr>
<td></td>
<td>Moist (1%)</td>
<td>100</td>
</tr>
<tr>
<td>120</td>
<td>Dry (0.4%)</td>
<td>100</td>
</tr>
<tr>
<td></td>
<td>Moist (1%)</td>
<td>100</td>
</tr>
<tr>
<td>−40</td>
<td>Dry (0.4%)</td>
<td>100</td>
</tr>
<tr>
<td></td>
<td>Moist (1%)</td>
<td>100</td>
</tr>
</tbody>
</table>

Table 2.2.3 The effect of hot and wet conditions on composites in tension (ref[2.2.6], table 1.1)
<table>
<thead>
<tr>
<th>Fiber</th>
<th>Tensile strength/density</th>
<th>Tensile modulus/density</th>
</tr>
</thead>
<tbody>
<tr>
<td>AS-4</td>
<td>2.23 8.86 1.29 0.508</td>
<td></td>
</tr>
<tr>
<td>BM-4</td>
<td>2.76 10.9 1.70 0.669</td>
<td></td>
</tr>
<tr>
<td>E-glass</td>
<td>1.33 5.24 0.28 0.11</td>
<td></td>
</tr>
<tr>
<td>S-glass</td>
<td>1.73 6.81 0.32 0.13</td>
<td></td>
</tr>
<tr>
<td>Kevlar 49</td>
<td>2.50 9.84 0.90 0.35</td>
<td></td>
</tr>
<tr>
<td>Boron</td>
<td>1.30 5.91 1.60 0.63</td>
<td></td>
</tr>
<tr>
<td>P-75</td>
<td>1.04 4.09 2.58 1.02</td>
<td></td>
</tr>
</tbody>
</table>

(a) Data derived in epoxy matrix

Table 2.3.1 Comparison of different fibre types (ref.[2.3.1], table 3.7)

<table>
<thead>
<tr>
<th>Matrix</th>
<th>Flexural Strength (MPa)</th>
<th>Short Beam Shear (MPa)</th>
<th>Compression Strength (MPa)</th>
<th>G_{IC} (KJ/m²)</th>
<th>Compression After Impact^b</th>
<th>Stress (MPa)</th>
<th>Strain (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>J-polymer</td>
<td>1450</td>
<td>104</td>
<td>1040</td>
<td>1.1</td>
<td>345</td>
<td>0.75</td>
<td></td>
</tr>
<tr>
<td>PEKK</td>
<td>1620</td>
<td>117</td>
<td>1390</td>
<td>1.0</td>
<td>274</td>
<td>0.60</td>
<td></td>
</tr>
<tr>
<td>K-polymer</td>
<td>1590</td>
<td>98</td>
<td>1000</td>
<td>1.7</td>
<td>274</td>
<td>0.65</td>
<td></td>
</tr>
<tr>
<td>N-polymer</td>
<td>1530</td>
<td>120</td>
<td>—</td>
<td>—</td>
<td>—</td>
<td>—</td>
<td></td>
</tr>
<tr>
<td>PEEK</td>
<td>1500</td>
<td>117</td>
<td>1040</td>
<td>1.6</td>
<td>310</td>
<td>0.70</td>
<td></td>
</tr>
<tr>
<td>HTX</td>
<td>1770</td>
<td>—</td>
<td>1130</td>
<td>2.2</td>
<td>301</td>
<td>0.63</td>
<td></td>
</tr>
<tr>
<td>PPS^c</td>
<td>1360</td>
<td>69</td>
<td>635</td>
<td>0.9</td>
<td>179</td>
<td>0.52</td>
<td></td>
</tr>
<tr>
<td>1610</td>
<td>—</td>
<td>908</td>
<td>—</td>
<td>—</td>
<td>—</td>
<td>—</td>
<td></td>
</tr>
<tr>
<td>PAS-2^c</td>
<td>1650</td>
<td>—</td>
<td>897</td>
<td>—</td>
<td>—</td>
<td>—</td>
<td></td>
</tr>
<tr>
<td>1920</td>
<td>—</td>
<td>977</td>
<td>—</td>
<td>—</td>
<td>—</td>
<td>—</td>
<td></td>
</tr>
<tr>
<td>PEI</td>
<td>—</td>
<td>94</td>
<td>824</td>
<td>1.2</td>
<td>—</td>
<td>—</td>
<td></td>
</tr>
<tr>
<td>PES</td>
<td>—</td>
<td>78</td>
<td>—</td>
<td>—</td>
<td>248</td>
<td>—</td>
<td></td>
</tr>
<tr>
<td>PAI</td>
<td>2070</td>
<td>110</td>
<td>1380</td>
<td>1.6</td>
<td>345</td>
<td>0.90</td>
<td></td>
</tr>
<tr>
<td>LARC-TPI</td>
<td>95</td>
<td>—</td>
<td>—</td>
<td>0.8</td>
<td>—</td>
<td>—</td>
<td></td>
</tr>
<tr>
<td>Epoxy^d</td>
<td>1794</td>
<td>121</td>
<td>1518</td>
<td>0.2</td>
<td>145</td>
<td>0.34</td>
<td></td>
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<tr>
<td>BMI^d</td>
<td>1490</td>
<td>94</td>
<td>1345</td>
<td>0.3</td>
<td>—</td>
<td>—</td>
<td></td>
</tr>
</tbody>
</table>

^a Unidirectional laminates with nominal 60% fiber volume (AS-4 or equivalent).

^b With quasi-isotropic specimen and 67 J/cm impact energy.

^c Flexural/compression strengths

^d Untoughened thermoset systems.

Table 2.3.2 Mechanical properties of thermoplastic composites reinforced with carbon fibre (ref.[2.3.1], table 3.22)
<table>
<thead>
<tr>
<th>Polymer</th>
<th>( T_g(\text{oC})^a ) (Dry)</th>
<th>( T_m(\text{oC})^b )</th>
<th>( T_p(\text{oC})^c )</th>
<th>Crystallinity^d (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>J-polymer</td>
<td>156</td>
<td>None</td>
<td>300</td>
<td>None</td>
</tr>
<tr>
<td>PEKK</td>
<td>156</td>
<td>338</td>
<td>370</td>
<td>26</td>
</tr>
<tr>
<td>K-polymer</td>
<td>250</td>
<td>None</td>
<td>350</td>
<td>None</td>
</tr>
<tr>
<td>N-polymer</td>
<td>350</td>
<td>None</td>
<td>375</td>
<td>None</td>
</tr>
<tr>
<td>PEEK (150G)</td>
<td>144</td>
<td>340</td>
<td>380</td>
<td>33</td>
</tr>
<tr>
<td>HTX</td>
<td>205</td>
<td>386</td>
<td>420</td>
<td>20</td>
</tr>
<tr>
<td>PPS</td>
<td>85</td>
<td>285</td>
<td>343</td>
<td>None</td>
</tr>
<tr>
<td>PAS-2</td>
<td>215</td>
<td>None</td>
<td>330</td>
<td>None</td>
</tr>
<tr>
<td>PEI</td>
<td>217</td>
<td>None</td>
<td>343</td>
<td>None</td>
</tr>
<tr>
<td>PES</td>
<td>260</td>
<td>None</td>
<td>330</td>
<td>None</td>
</tr>
<tr>
<td>PAI</td>
<td>288</td>
<td>None</td>
<td>350</td>
<td>None</td>
</tr>
<tr>
<td>LARC-TPI</td>
<td>264</td>
<td>None</td>
<td>350</td>
<td>None</td>
</tr>
<tr>
<td>Epoxy^1</td>
<td>206</td>
<td>None</td>
<td>177</td>
<td>None</td>
</tr>
<tr>
<td>BMI^1</td>
<td>258</td>
<td>None</td>
<td>180/227</td>
<td>None</td>
</tr>
</tbody>
</table>

^aGlass transition temperature (dry).
^bMelt temperature.
^cNominal processing temperature.
^dX-ray diffraction.
^eAt processing temperature, 10 sec ' shear rate.
^fUntoughened thermoset systems.

Table 2.3.3 Thermal, morphological and rheological properties of neat resins (ref[2.3.1], table 3.23)
<table>
<thead>
<tr>
<th>Polymer</th>
<th>Density (g/cm$^3$)</th>
<th>Modulus a (GPa)</th>
<th>Strength b (MPa)</th>
<th>Elongation c (%)</th>
<th>Fracture Energy d (KJ/m$^2$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>J-polymer</td>
<td>1.16</td>
<td>3.17</td>
<td>103</td>
<td>25</td>
<td>2.0</td>
</tr>
<tr>
<td>PEKK</td>
<td>1.30</td>
<td>4.50</td>
<td>102</td>
<td>4</td>
<td>1.0</td>
</tr>
<tr>
<td>K-polymer</td>
<td>1.31</td>
<td>3.76</td>
<td>102</td>
<td>14</td>
<td>1.8</td>
</tr>
<tr>
<td>N-polymer</td>
<td>1.44</td>
<td>4.17</td>
<td>110</td>
<td>6</td>
<td>2.5</td>
</tr>
<tr>
<td>PEEK (150G)</td>
<td>1.30</td>
<td>3.79</td>
<td>103</td>
<td>11</td>
<td>2.0</td>
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<tr>
<td>HTX</td>
<td>—</td>
<td>2.48</td>
<td>86</td>
<td>13</td>
<td>—</td>
</tr>
<tr>
<td>PPS</td>
<td>1.36</td>
<td>3.91</td>
<td>80</td>
<td>3</td>
<td>—</td>
</tr>
<tr>
<td>PAS-2</td>
<td>1.40</td>
<td>3.28</td>
<td>101</td>
<td>8</td>
<td>—</td>
</tr>
<tr>
<td>PEI</td>
<td>1.27</td>
<td>2.96</td>
<td>104</td>
<td>60</td>
<td>2.5</td>
</tr>
<tr>
<td>PES</td>
<td>—</td>
<td>2.41</td>
<td>76</td>
<td>7</td>
<td>—</td>
</tr>
<tr>
<td>PAI</td>
<td>1.38</td>
<td>3.30</td>
<td>136</td>
<td>25</td>
<td>3.4</td>
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<td>—</td>
<td>3.73</td>
<td>119</td>
<td>5</td>
<td>1.8</td>
</tr>
<tr>
<td>Epoxy e</td>
<td>—</td>
<td>4.28</td>
<td>83</td>
<td>1</td>
<td>0.1</td>
</tr>
<tr>
<td>BMI e</td>
<td>—</td>
<td>3.71</td>
<td>—</td>
<td>—</td>
<td>0.2</td>
</tr>
</tbody>
</table>

a Tensile or flexural modulus.
b Tensile strength.
c Tensile elongation.
d $G_{1c}$ from compact tension test ASTM-E399.
e Untoughened thermoset.

Table 2.3.4 Mechanical and fracture toughness properties of neat resins (ref.[2.3.4], table 3.24)

<table>
<thead>
<tr>
<th>Property</th>
<th>PA6/6</th>
<th>PP</th>
<th>PEI</th>
<th>PES</th>
<th>PPS</th>
<th>PEEK</th>
</tr>
</thead>
<tbody>
<tr>
<td>Density (g/cm$^3$)</td>
<td>1.14</td>
<td>0.9</td>
<td>1.27</td>
<td>1.37</td>
<td>1.34</td>
<td>1.32</td>
</tr>
<tr>
<td>Tensile strength (MPa)</td>
<td>81</td>
<td>25</td>
<td>104</td>
<td>84</td>
<td>74</td>
<td>100</td>
</tr>
<tr>
<td>Flex. strength (MPa)</td>
<td>103</td>
<td>33</td>
<td>145</td>
<td>129</td>
<td>96</td>
<td>110</td>
</tr>
<tr>
<td>Flex. modulus (GPa)</td>
<td>2.8</td>
<td>1.1</td>
<td>3.3</td>
<td>2.5</td>
<td>4.1</td>
<td>3.8</td>
</tr>
<tr>
<td>HDT (°C at 1.82 MPa)</td>
<td>95</td>
<td>54</td>
<td>180</td>
<td>180</td>
<td>125</td>
<td>165</td>
</tr>
<tr>
<td>CTE $\alpha$ (10$^{-6}$K$^{-1}$)</td>
<td>8.1</td>
<td>8</td>
<td>6.1</td>
<td>5.5</td>
<td>5.4</td>
<td>4.7</td>
</tr>
<tr>
<td>Cost (1986)p/cc</td>
<td>0.25</td>
<td>0.1</td>
<td>1.24</td>
<td>1.03</td>
<td>1.04</td>
<td>5.28</td>
</tr>
</tbody>
</table>

* Coefficient of thermal expansion.

Table 2.3.5 Unreinforced thermoplastics-physical and mechanical Properties (ref.[2.3.2], table 3.5)
<table>
<thead>
<tr>
<th>Trade name</th>
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Table 2.3.8 Comparison of honeycomb core materials (ref.[2.3.2], table 3.6)

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</tr>
<tr>
<td>2024</td>
<td>38.1</td>
<td>44.8</td>
<td>1.72</td>
</tr>
<tr>
<td>Commercial Alm</td>
<td>76.2</td>
<td>83.3</td>
<td>4.2</td>
</tr>
<tr>
<td>Glass/phenolic (hex)</td>
<td>—</td>
<td>56</td>
<td>3.45</td>
</tr>
<tr>
<td>(rect)</td>
<td>—</td>
<td>72.1</td>
<td>4.31</td>
</tr>
<tr>
<td>Glass/polyester (hex)</td>
<td>—</td>
<td>64.1</td>
<td>3.86</td>
</tr>
<tr>
<td>Glass/polymide (± in)</td>
<td>—</td>
<td>64.1</td>
<td>3.03</td>
</tr>
<tr>
<td>Meta-aramid (hex)</td>
<td>30.8</td>
<td>24</td>
<td>0.65</td>
</tr>
<tr>
<td>(rect)</td>
<td>30.8</td>
<td>48.1</td>
<td>2.65</td>
</tr>
<tr>
<td>Kraft paper</td>
<td>—</td>
<td>80.1</td>
<td>2.76</td>
</tr>
<tr>
<td>Balsa wood (endgrain)</td>
<td>—</td>
<td>96.1</td>
<td>3.45</td>
</tr>
</tbody>
</table>

NOTE: (hex) = hexagonal cell; (rect) = rectangular cell
### Table 2.3.9 5052 Aluminium alloy honeycomb – construction and strength (ref.[2.3.2], table 3.7)

<table>
<thead>
<tr>
<th>Cell size inch. mm.</th>
<th>Web thickness thou. µm</th>
<th>Nom. density kg/m³</th>
<th>Stab. compr. strength N/mm²</th>
</tr>
</thead>
<tbody>
<tr>
<td>1&quot;</td>
<td>3.2 1.5 38.1</td>
<td>97.7</td>
<td>6.27</td>
</tr>
<tr>
<td>1¾&quot;</td>
<td>3.2 2 50.8</td>
<td>129.7</td>
<td>10.1</td>
</tr>
<tr>
<td>1¾&quot;</td>
<td>6.4 1.5 38.1</td>
<td>54.5</td>
<td>2.34</td>
</tr>
<tr>
<td>1¾&quot;</td>
<td>6.4 2 50.8</td>
<td>68.1</td>
<td>3.48</td>
</tr>
<tr>
<td>1¾&quot;</td>
<td>9.5 2 50.8</td>
<td>48.0</td>
<td>1.86</td>
</tr>
</tbody>
</table>

### Table 2.3.10 Density and strength of plastics foam cores (ref.[2.3.2], table 3.8)

<table>
<thead>
<tr>
<th>Material</th>
<th>Density kg/mm³</th>
<th>Compressive Strength N/mm²</th>
</tr>
</thead>
<tbody>
<tr>
<td>PVC (Airex)</td>
<td>80</td>
<td>0.65</td>
</tr>
<tr>
<td>PVC (Plastitell)</td>
<td>40</td>
<td>0.35</td>
</tr>
<tr>
<td>Polymethacrylimide</td>
<td>30</td>
<td>0.4</td>
</tr>
<tr>
<td></td>
<td>50</td>
<td>0.9</td>
</tr>
<tr>
<td></td>
<td>70</td>
<td>1.5</td>
</tr>
<tr>
<td>Polyurethane</td>
<td>64</td>
<td>0.38</td>
</tr>
<tr>
<td>Polystyrene (Dow)</td>
<td>32</td>
<td>0.24</td>
</tr>
</tbody>
</table>

### Table 2.3.11 Micromeshes (ref.[2.3.2], table 3.9)

<table>
<thead>
<tr>
<th>Trade name</th>
<th>Supplier</th>
<th>Composition</th>
<th>Bulk density kg/mm³</th>
</tr>
</thead>
<tbody>
<tr>
<td>Atmospheres</td>
<td>A.M.L. Interntl.</td>
<td>Glass</td>
<td>400</td>
</tr>
<tr>
<td>Eccospheres EP</td>
<td>Emerson &amp; Cuming</td>
<td>Epoxy</td>
<td>120; 192; 250</td>
</tr>
<tr>
<td>Eccospheres FA-A</td>
<td></td>
<td>Ceramic</td>
<td>400</td>
</tr>
<tr>
<td>Eccospheres 1G101</td>
<td></td>
<td>Glass</td>
<td>311</td>
</tr>
<tr>
<td>Extendospheres</td>
<td></td>
<td>Aluminosilicate</td>
<td>400</td>
</tr>
<tr>
<td>Expancell</td>
<td>Expancel</td>
<td>Metallised</td>
<td>415–480</td>
</tr>
<tr>
<td>Fillite</td>
<td>Fillite</td>
<td>Silicate</td>
<td>340</td>
</tr>
<tr>
<td>Q-Cell</td>
<td>Phil. Quartz</td>
<td>Silicate</td>
<td>105–115</td>
</tr>
</tbody>
</table>
Figure 2.1.1 Modular concept principle (ref.[2.1.1], figure 14.1.11)
Figure 2.2.1  Tensile fatigue curves for U.D. CFRP (ref.[2.2.1], figure 5.20)

Figure 2.2.2  Tensile fatigue curves for CFRP, plain and notched (ref.[2.2.1], figure 5.21)
Figure 2.2.3 Fatigue stress vs. cycles with R=0 (ref.[2.2.3], figure 10.1)

Figure 2.2.4 Fatigue strain vs. cycles with R=0 (ref.[2.2.3], figure 10.2)
Figure 2.2.5  Tension compression, R=−1, is a heavier loading than reversed tension, R=0 (ref.[2.2.3], figure 10.5)

Figure 2.2.6  The effect of different R values (ref.[2.2.3], figure 10.6)
Figure 2.2.7 Adding $+/-45^\circ$ to U.D. fibres decreases the fatigue stress (ref. [2.2.3], figure 10.1)

Figure 2.2.8 Typical costs for aerospace prepreg materials (ref. [2.2.1], figure 7.6)
Figure 2.2.9 Hail strike resistance parameter (ref.[2.2.1], figure 14.1.23)

Figure 2.3.1 A sandwich panel and a skin panel with stringers
Figure 2.3.2 Edges of sandwich panels
(a) Elastomeric Tooling

(b) Schematic of spar parts

Figure 2.3.3 Sine-wave spar fabricated by elastomeric tooling
Figure 2.4.1 Inter-laminar tensile stress at a free edge for six different combinations of stacking sequence (ref. [2.4.2], figure 8.20)

Figure 2.4.2 A cut out design for a composite structure
Figure 2.4.3  Isogrid panel concept (ref.[2.4.3], figure 7.5.3)
Figure 2.4.4  Lightning strike zones (ref.[2.4.2], figure 14.1.24)
Chapter 3  The Mechanical Properties of the Composites

Before starting any calculations it is imperative to know the mechanical properties of the materials that will be used. Of course, it is hard to get the latest data as better composite materials are created daily. The information is usually generalised and not in the specific form necessary to use in the formulae. A simple example is that during manufacturing of the prepregs the fibres, and therefore the prepreg, already loses strength. Another problem is that the information of prepregs consisting of a certain fibre type and certain epoxy is available, but when different fibres are used with the same epoxy the information is unavailable. In this case an approximation of the mechanical properties will have to be made. A better solution would be to test the prepregs to create actual data.

3.1  The Mechanical Properties that will be used in the Calculations

3.1.1  Mechanical Properties for Composites in Compression

The basic building stones to create the mechanical properties are tables 3.1.1 and 2.2.3. From table 3.1.1, taken from ref.[3.1.1], directly a selection can be made of which type of fibres could lead to an optimal solution. From the fibres with a high strength and a relatively low modulus the IM6 fibre performs the best. It has the highest tensile strength, the highest tensile modulus and the lowest density. For high modulus fibres HMS-4 still has a reasonably high strength. Fibres with a very high modulus and a low strength will not be assessed. One important aspect that will not be addressed here but could lead to choice of a different fibre type is, of course, the cost.

Table 2.2.3 can also be found in ref.[3.1.2] together with the vital information that states that the fibre volume coefficient \( \nu = 0.6 \). This means that the prepreg, in the first column of table 2.2.3, consists of 60 % IM6 fibres and 40 % CYCOM 1808. CYCOM 1808 is a “second generation” toughened epoxy system. The first generation toughened epoxy systems, like the second-generation systems, also improved the impact damage resistance but the hot/wet performance was reduced to an unacceptable level. In the case of compression these two criteria are the most critical. For transport aircraft the maximum temperature in service is 93°C.

In ref.[3.1.3] it is stated that the maximum allowable strain \( \varepsilon_c \) after impact is between 0.35% and 0.375%. In which case if “barely visible damage” has occurred it will not lead to failure. In this report a value of \( \varepsilon_c = 0.0037 \) will be used. According to ref.[3.1.3] a strain of 0.0042 can be used when a “soft skin” approach is used. In this so-called “soft skin” approach the fibres are rearranged, i.e. the lay-up is not the same throughout the skin panel. Stringers have most of the UD carbon fibres and are co-cured on the skin. These UD fibres are preferably placed in flanges to increase overall bucking resistance. The high strength compressive loaded stringers are protected by the skin. The skin consists of more +/- 45° material than the stringers. The lower modulus of the skin is linear with the lower stress in the skin compared to the stringers. The lower stress in the more delaminated skin reduces delamination growth shortly before the ultimate residual strength is reached.
Knowing that 1 ksi = 6.895 Mpa, table 2.2.3 can be can be converted into single layer properties. Together with table 3.1.1 the difference in maximum fibre strength \( \sigma_{f,\text{fibres}} \) and useful fibre strength can also found, this leads to an efficiency factor \( \eta \). The tensile strength of the prepreg reinforced with IM6 fibres \( \sigma_{IM6} \) is:

\[
\sigma_{IM6} = \eta \cdot \nu_f \cdot \sigma_{f,\text{fibres}} = 2240.9 \text{ Mpa}
\]

\( \eta = 0.778 \) (3.1.1.1)

The efficiency factor \( \eta = 0.778 \) is difference between table 3.1.1 and table 2.2.3. This efficiency factor like the efficiency factors \( \eta_{\text{hot/wet}} \) and \( \eta_{\text{max-strain}} \) will also be used for the prepregs with HMS-4 fibres. Before going any further with figuring out what the allowable stresses are, the modulus of the prepreg must be assessed. The difference between the tensile and the compressive situation will be neglected to reduce the variables in the calculations in the following chapters. An efficiency factor \( \eta_m \) is be used again. The modulus \( E_{IM6} \) of the prepreg is:

\[
E_{IM6} = \eta_m \cdot \nu_f \cdot E_{f,\text{fibres}} = 154.45 \text{ Gpa}
\]

\( \eta_m = 0.864 \) (3.1.1.2)

Now the maximum allowable stresses, \( \sigma_{\text{hot/wet,IM6}} \) in hot/wet conditions and \( \sigma_{\text{max-strain,IM6}} \) for impact damage tolerance, of the prepreg consisting of IM6 fibres and the toughened epoxy can be found:

\[
\sigma_{\text{c,hot/wet,IM6}} = \eta_{\text{c,hot/wet}} \cdot \nu_f \cdot \sigma_{IM6} = 910.14 \text{ Mpa}
\]

\( \eta_{\text{c,hot/wet}} = 0.316 \)

\[
\sigma_{\text{c,max-strain,IM6}} = \varepsilon_c \cdot E_{IM6} = \eta_{\text{c,max-strain}} \cdot \nu_f \cdot \sigma_{IM6} = 571.46 \text{ Mpa}
\]

\( \eta_{\text{c,max-strain}} = 0.864 \)

\[
\sigma_{\text{c,soft-skin,IM6}} = \varepsilon_{\text{soft-skin}} \cdot E_{IM6} = 648.68 \text{ Mpa}
\]

(3.1.1.3)

Thus, for this prepreg impact tolerance is the most critical.

An estimation for the mechanical properties of a prepreg consisting of the same epoxy resin but reinforced with HMS-4 fibres can be made at this stage, simply by using the efficiency factors and table 3.1.1. The subscript HMS-4 means that the mechanical properties of the following are for a prepreg with HMS-4 fibres instead of IM6 fibres. The mechanical properties are:

\[
\sigma_{\text{l,HMS-4}} = \eta \cdot \nu_f \cdot \sigma_{\text{l,HMS-4, fibres}} = 2677.8 \text{ Mpa}
\]

\[
E_{\text{HMS-4}} = \eta_m \cdot \nu_f \cdot E_{f,\text{fibres}} = 175.22 \text{ Gpa}
\]

\[
\sigma_{\text{c,hot/wet,HMS-4}} = \eta_{\text{c,hot/wet}} \cdot \nu_f \cdot \sigma_{\text{HMS-4, fibres}} = 587.76 \text{ Mpa}
\]

\[
\sigma_{\text{c,max-strain,HMS-4}} = \eta_{\text{c,max-strain}} \cdot \nu_f \cdot \varepsilon_c \cdot E_{\text{HMS-4, fibres}} = 648.31 \text{ Mpa}
\]

(3.1.1.4)

In this case the hot/wet conditions lead to the most critical stress. It also is directly apparent that using a soft skin approach, i.e. using a maximum strain of 0.0042 instead of \( \varepsilon_c = 0.0037 \), has no use.
Prepregs with very high modulus fibres like P-100 and Thornel 75 will not be used because of their low strength. If a skin panel is in compression this low strength will lead to an equivalent thickness (i.e. the thickness of the skin panel taking the stringers into account) that is a lot larger than that of a panel with IM6 fibres or HMS-4 fibres.

3.1.2 Mechanical Properties for Composites in Tension

There is not much information to be found on composites in tension with regards to, for example, damage tolerance. In most of the wing structure compression is critical. However, in the case of the bottom skin it is important to see how it performs in tension. In section 2.2.2 it has already been stated that hot and wet conditions hardly have any influence on the mechanical properties of a composite in tension. In the situation of impact damage it is known that a laminate in tension performs slightly better than a laminate in compression. In ref.[3.1.1] a maximum allowable strain value of 0.48% is stated. In this paper a conservative value of $\varepsilon_t = 0.0047$ will be used, i.e. 0.001 higher than the compression value.

The following allowable strain values are found:

$$\sigma_{i,\text{max-strain,IM6}} = \varepsilon_t \cdot E_{IM6} = 725.92 \text{Mpa}$$
$$\sigma_{i,\text{max-strain,HMS-4}} = \varepsilon_t \cdot E_{HMS-4} = 823.53 \text{Mpa}$$

(3.1.2.1)

From these values it directly becomes apparent that a laminate reinforced with HMS-4 fibres should be used in the bottom skin panel.

Because the hot and wet conditions hardly influence the tensile properties, a fibre with a very high modulus and a low strength, like Thornel 75 or P-100, might be a good choice. The tensile strength of a UD laminate with, for example, Thornel 75 fibres could be higher than $\sigma_{\text{max-strain,HMS-4}}$. This will not be assessed any further in this report due to the lack of specific data.

3.1.3 Densities of the Prepregs

To calculate the density, at the assumed 0.6 fibre volume fraction, of the prepregs the following can be used:

$$\rho = 0.6 \rho_{\text{fibres}} + 0.4 \rho_{\text{epoxy}}$$

(3.1.3.1)

In ref.[3.1.2] it is stated that the density of the epoxy is about 1.26 kg/mm$^3$. The densities of the fibres can be found in table 3.1.1. This leads to the next densities for the different prepregs:

$$\rho_{\text{IM6}} = 1.548 \cdot 10^{-6} \text{kg/mm}^3$$
$$\rho_{\text{HMS-4}} = 1.572 \cdot 10^{-6} \text{kg/mm}^3$$

(3.1.3.2)
Thus, the density of a laminate with HMS-4 fibres is only 1.5% higher than that of a laminate reinforced with IM6 fibres. It should therefore, not be forgotten that a structure with a smaller (equivalent) thickness reinforced with HMS-4 fibres, need not lead to the lightest structure.

3.1.4 Core Materials of Sandwich Panels

If sandwich panels are, other types of failure should be considered as well. One of them is wrinkling. Wrinkling is a deformation of the sandwich perpendicular to its plane; eventually the core can shear off (see figure 3.1.1). Ref.[3.1.3] states that for wrinkling stress \( \sigma_w \) the following applies:

\[
\sigma_w = 0.82(E_s \cdot E_c \cdot G_c)^{0.333}
\]  \hspace{1cm} (3.1.4.1)

in which \( E_s \) is the modulus of the skin in plate direction, \( E_c \) is the modulus of the core material, and \( G_c \) is the shear modulus of the core material. Table 3.1.2 gives the properties of solid foam core materials.

If a honeycomb core is used then dimpling or intercell buckling should be considered (see figure 3.1.1).

3.2 The Results of the Mechanical Properties of Composites

To finish this chapter a few final remarks will be made. In the case of the skin panels IM6 fibres will probably just be used in the top skin and only if a soft skin approach can be taken. In the ribs and the spars they also could be used because delamination due to impact is unlikely. If allowable strains are higher in the spars and ribs, fatigue should not be forgotten. The other reinforcing fibre type that will be used in the calculations is HMS-4. The results are combined in table 3.2.1. These two fibre types are the best in their class. Very high modulus fibres will not be used although these might lead to an efficient structure of the bottom skin panel.

Core properties can be found in table 3.1.2.
References:


Table 3.1.1 Typical mechanical property values of commercially available carbon fibres

<table>
<thead>
<tr>
<th>Product name</th>
<th>Manufacturer</th>
<th>Precursor type</th>
<th>Density, g/cm³</th>
<th>Tensile strength, GPa</th>
<th>Tensile modulus, GPa</th>
</tr>
</thead>
<tbody>
<tr>
<td>AS-4</td>
<td>Hercules, Inc.</td>
<td>PAN</td>
<td>1.78</td>
<td>4.0</td>
<td>0.580</td>
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<tr>
<td>AS-6</td>
<td>Hercules, Inc.</td>
<td>PAN</td>
<td>1.82</td>
<td>4.5</td>
<td>0.652</td>
</tr>
<tr>
<td>IM-6</td>
<td>Hercules, Inc.</td>
<td>PAN</td>
<td>1.74</td>
<td>4.8</td>
<td>0.696</td>
</tr>
<tr>
<td>T300</td>
<td>Union Carbo/Toray</td>
<td>PAN</td>
<td>1.73</td>
<td>3.31</td>
<td>0.480</td>
</tr>
<tr>
<td>T500</td>
<td>Union Carbo/Toray</td>
<td>PAN</td>
<td>1.78</td>
<td>3.65</td>
<td>0.530</td>
</tr>
<tr>
<td>T700</td>
<td>Toray</td>
<td>PAN</td>
<td>1.80</td>
<td>4.48</td>
<td>0.650</td>
</tr>
<tr>
<td>T40</td>
<td>Toray</td>
<td>PAN</td>
<td>1.74</td>
<td>4.50</td>
<td>0.652</td>
</tr>
<tr>
<td>Celanese</td>
<td>Celanese/Toray</td>
<td>PAN</td>
<td>1.77</td>
<td>3.53</td>
<td>0.515</td>
</tr>
<tr>
<td>Celanese ST</td>
<td>Celanese/Toray</td>
<td>PAN</td>
<td>1.78</td>
<td>4.34</td>
<td>0.630</td>
</tr>
<tr>
<td>XAS</td>
<td>Grafil/Hyvol</td>
<td>PAN</td>
<td>1.84</td>
<td>3.45</td>
<td>0.500</td>
</tr>
<tr>
<td>HMS-4</td>
<td>Hercules, Inc.</td>
<td>PAN</td>
<td>1.78</td>
<td>3.10</td>
<td>0.450</td>
</tr>
<tr>
<td>PAN 30</td>
<td>Toray</td>
<td>PAN</td>
<td>1.81</td>
<td>2.41</td>
<td>0.355</td>
</tr>
<tr>
<td>HMS 33</td>
<td>Grafil/Hyvol</td>
<td>PAN</td>
<td>1.91</td>
<td>1.52</td>
<td>0.220</td>
</tr>
<tr>
<td>G-50</td>
<td>Celanese/Tolto</td>
<td>PAN</td>
<td>1.78</td>
<td>2.48</td>
<td>0.360</td>
</tr>
<tr>
<td>GY-70</td>
<td>Celanese</td>
<td>PAN</td>
<td>1.90</td>
<td>1.52</td>
<td>0.220</td>
</tr>
<tr>
<td>P-35</td>
<td>Union Carboide</td>
<td>Pitch</td>
<td>2.0</td>
<td>1.73</td>
<td>0.250</td>
</tr>
<tr>
<td>P-75</td>
<td>Union Carboide</td>
<td>Pitch</td>
<td>2.0</td>
<td>2.07</td>
<td>0.300</td>
</tr>
<tr>
<td>P-10U</td>
<td>Union Carboide</td>
<td>Pitch</td>
<td>2.15</td>
<td>2.24</td>
<td>0.335</td>
</tr>
<tr>
<td>HMG-50</td>
<td>HaccuOCTF</td>
<td>Rayon</td>
<td>1.9</td>
<td>2.07</td>
<td>0.300</td>
</tr>
<tr>
<td>Therrno 75</td>
<td>Union Carboide</td>
<td>Rayon</td>
<td>1.9</td>
<td>2.52</td>
<td>0.365</td>
</tr>
</tbody>
</table>

Table 3.1.2 Values of solid core foams

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Acrylic foam &quot;Rohacell&quot;</td>
<td>30</td>
<td>36</td>
<td>14</td>
<td>16.8</td>
</tr>
<tr>
<td></td>
<td>50</td>
<td>70</td>
<td>21</td>
<td>29.4</td>
</tr>
<tr>
<td></td>
<td>70</td>
<td>92</td>
<td>30</td>
<td>39.4</td>
</tr>
<tr>
<td></td>
<td>110</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>PVC foam, formable, &quot;Airex&quot;</td>
<td>30</td>
<td>40</td>
<td>55</td>
<td>75</td>
</tr>
<tr>
<td>PVC, croisslinked, 60%</td>
<td>45</td>
<td>27.5</td>
<td>10.3</td>
<td>6.29</td>
</tr>
<tr>
<td>PVC, 40% diisocyanate</td>
<td>75</td>
<td>34.5</td>
<td>12.4</td>
<td>5.70</td>
</tr>
<tr>
<td>&quot;Klegecel&quot;</td>
<td>100</td>
<td>44.8</td>
<td>15.0</td>
<td>6.70</td>
</tr>
<tr>
<td></td>
<td>250</td>
<td>106.8</td>
<td>20.7</td>
<td>8.83</td>
</tr>
<tr>
<td></td>
<td>IM6/CYCOM 1808</td>
<td>HMS-4/CYCOM 1808</td>
<td></td>
<td></td>
</tr>
<tr>
<td>------------------------------</td>
<td>----------------</td>
<td>------------------</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Modulus $E$</td>
<td>154.45 Gpa</td>
<td>175.22 Gpa</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Hot/Wet Compressive Strength</td>
<td>910.14 Mpa</td>
<td>587.76 Mpa</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Damage Impact Compressive</td>
<td>571.46 Mpa</td>
<td>648.31 Mpa</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Strength $\sigma_{\text{max-strain}}^{\text{impact}}$</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Damage Impact Compressive</td>
<td>648.68 Mpa</td>
<td>-</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Strength with a Soft Skin $\sigma_{\text{soft,skin}}$</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Damage Impact Tensile Strength</td>
<td>725.92 Mpa</td>
<td>823.53 Mpa</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\sigma_{\text{max-strain}}^{\text{impact}}$</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Density $\rho$</td>
<td>$1.548 \times 10^4$ kg/mm$^3$</td>
<td>$1.572 \times 10^4$ kg/mm$^3$</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 3.2.1 U.D. mechanical properties of the laminates that will be used in chapter 5

Figure 3.1.1 Several buckling possibilities typical for sandwich panels
Chapter 4  The Loads

In this chapter an estimation of the loads on the aircraft will be made. The loads thought to be the most critical, are the only loads that are calculated. These are, however, still very rough. Exact results are not that important in this project as it is of more interest to compare alternative structures. The results do have to have a certain accuracy because certain structures perform better under different conditions.

Aviation authorities have made rules to which the designed aircraft must comply. These airworthiness requirements make sure that the airplane can withstand all the loads encountered during its life span. For transport aircraft these rules are FAR 25 (or JAR 25).

The base-case in this project is the GARTEUR 70 aircraft. This is a 70 passenger, regional jet. Although, the GARTEUR 70 project was cancelled enough information can be found to create the loads for an aircraft of similar size. The GARTEUR 70 would have had a design payload of 70 passengers, including their luggage, plus freight. The maximum design range was 2500km with maximum payload, and it had a design cruising Mach number of $M = 0.70$ at an altitude of 30000ft. The aspect ratio varies, but for the following calculations an aspect ratio of $A = 11$ is used as the most information is available for this aspect ratio. A $CL_{\text{max}}$ of 2.9 can be obtained in the landing configuration. The aircraft and its wing can be found in figure 4.1 together with some of the most important dimensions.

This chapter consists of three sections:

- The creating of the manoeuvre and gust envelopes;
- Calculation of the basic elements needed to create the loads;
- Combining the basic elements to create the full load cases.

The results of the calculations can be found in tables 4.1 and 4.2, and in figure 4.4.1 at the end of this chapter.

4.1  The Manoeuvre and Gust Envelopes

In both cases the $MZF\alpha$ (i.e. the maximum zero fuel weight) has been used, as this generally will give the highest loads. All calculations are recalculated to sea level, and to equivalent airspeeds.

4.1.1  Manoeuvre Envelope

The maximum load factor, according to FAR 25, becomes:
\[ n = 2.1 + \frac{10890}{MTOW + 4540} \]  \hspace{1cm} (4.1.1.1)

in which \( MTOW \) is the maximum take off weight.

With \( n \) limited to:

\[ 2.5 \leq n \leq 3.8 \]  \hspace{1cm} (4.1.1.2)

This means that in this case the maximum load factor will be:

\[ n = 2.5 \]  \hspace{1cm} (4.1.1.3)

The maximum negative value of \( n = -1 \). The form of the curve has been taken the same as that of the positive curve, \( n < 1 \), but mirrored of course. With a \( MZFW \) of 301590N, a maximum lift coefficient \( CL_{max} \) of 1.878, as can be found in the next section, and knowing that the maximum Mach number \( M_a \) is approximately 0.065 higher than \( M_{cr} \), the following speeds were found:

\[
\begin{align*}
v_s &= 53.11 \frac{m}{s} \\
v_{cr} &= 121.67 \frac{m}{s} \\
v_d &= 132.97 \frac{m}{s} \\
v_s &= 83.96 \frac{m}{s}
\end{align*}
\]  \hspace{1cm} (4.1.1.4)

The subscript \( s \) stand for staling, \( cr \) for cruising, \( d \) for diving, and \( a \) for approach.

When the flap are extended a maximum \( n \) of 2, and a \( CL_{max} \) of 2.9 are used to create the additional lines. The manœuvre envelope can be found in figure 4.1.1.

4.1.2 Gust Envelope

The most critical height for gusts is 6100m as the gust speeds are constant until this height and then decrease with height. The design gust speeds are:

\[
\begin{align*}
66 \text{fps}, & \text{ at } v_b \\
50 \text{fps}, & \text{ at } v_{cr} \\
25 \text{fps}, & \text{ at } v_d
\end{align*}
\]  \hspace{1cm} (4.1.2.1)

The loads can be calculated with the following:
\[ CL_{\alpha,\nu} = \frac{1}{\beta} \frac{2\pi}{2 + 1 + 2\lambda} \]

\[ \mu_g = \frac{2m}{\rho \cdot c \cdot S \cdot CL_{\alpha}} = 52.43 \]

\[ K_g = \frac{0.88\mu_g}{5.3 + \mu_g} = 0.7992 \]

\[ n = 1 \pm \frac{\rho_0 \cdot U_{\infty} \cdot V_{eq} \cdot CL_{\alpha}}{2mg} K_g \]

(4.1.2.2)

The highest load factors are:

\[ n_{\text{max}} = 2.85 \]

\[ n_{\text{min}} = -1. \]

(4.1.2.3)

The gust envelope can be found in figure 4.1.2

4.2 The basic elements

In this section the basic elements which are needed to determine the full load cases will be created. Mostly these will be calculated for a unit load. These basic building stones can then be multiplied by the appropriate load factors and added together to produce the full load cases. Often the basic elements represent realistic values but some elements are only useful when multiplied by the appropriate factors.

The loads will vary along the wing span. This is the X-direction. In the figures and tables, the half-wing is divided into ten sections of equal span length. Therefore, the loads will be calculated at eleven cross sections.

4.2.1 Symmetric Air Load

The symmetric air load distribution can be made with Diederich's method as can be found in reference 4.2.1. The lift distribution consists of two parts, the basic lift distribution and the additional lift distribution. In the drawings of the GARTEUR 70 no twist could be found. This seems unrealistic but as the effect is small, twist usually gives a higher lift on the outer wing and a lower lift on the inner wing, the basic lift distribution will be disregarded. The lift distribution will become:

\[ c_1 \frac{C}{C_g} = L_0 \cdot CL \]

(4.2.1.1)
With:

\[ L_x = \frac{c_l \cdot c}{C_L \cdot c_s} \]  \hspace{1cm} (4.2.1.2)

in which \( c_s \) is the mean geometric chord and the capital letters are used for the lift and the lift coefficient of the wing as a whole and the small letters for 2-dimensional lift and lift coefficient.

If thought sweep angle \( \Delta \alpha = 0^\circ \), then:

\[ L_x = C_1 \frac{c}{c_s} + (C_2 + C_3) \frac{4}{\pi} \sqrt{1 - \eta^2} \]  \hspace{1cm} (4.2.1.3)

From figure 4.2.1 it follows that \( C_1 = 0.52 \), \( C_2 = 0.1 \) and \( C_3 = 0.38 \). If the influence of the tail is disregarded at this stage, the lift of the wing will have to carry the weight of the plane. A new formula can be created for the local lift \( l \) along the span:

\[ l = \left[ C_1 + (C_2 + C_3) \frac{c_s}{c} \frac{4}{\pi} \sqrt{1 - \eta^2} \right] \frac{W}{S} \cdot c \]  \hspace{1cm} (4.2.1.4)

With \( W/S = 301580/93 \). Knowing that \( \eta \) is the non-dimensional spanwise station (i.e. \( x/(b/2) \)) and that the chord \( c \) changes linear from the root to the tip, from 4.154m to 1.662m, the lift distribution can be created. This distribution can be found in figure 4.2.2.

In section 4.3.1 the effect of the tail will be assessed.

To find \( CL_{max} \) ref.[4.2.1] is used. This leads to the creation of table 4.2.1 and figure 4.2.3. With the help of equation 4.2.1.1 the \( c_l \) along the wing span can be calculated. The \( c_l \) still depends upon \( CL \). For the wing airfoils of the GRATEUR 70 a maximum value of \( c_l \) of approximately 2 can be found (see figures 4.2.4 and figure 4.2.5). With figure 4.2.3 the \( CL_{max} \) now can be found because the maximum lift coefficient of the wing may be estimated from the assumption that this coefficient is reached when the local section lift coefficient at any position along the span is equal to the local \( CL_{max} \) for the corresponding section.

To create the shear force, the bending moment and the torsion moment along the span the assumption has been made that between the eleven points the lift is linear. The change in shear force will become:

\[ D(1) = D(2) + \left( \frac{q_1}{2} + \frac{q_2}{2} \right) \cdot c \]  \hspace{1cm} (4.2.1.5)

The calculation method of the moments and the meaning of \( q_1 \), \( q_2 \) and \( c \) can be found in figure 4.2.6. The sweep angle is \( 8^\circ \) and the trailing edge has a \( 0^\circ \) angle.
4.2.2 Wing Structure Inertial Relief

The mass of the wing will also produce a load. Firstly the effect of using composite materials must be taken into account. A reduction of 20% compared to a metal structure is thought to be realistic. As the weight distribution is unknown a few assumptions have been made. The mass is thought to be on 50% of the chord and the mass distribution is directly related to the area of the cross section. The mass of the wing is:

\[ m_w = 0.8 \cdot (2162.82 + 267.2) = 1944.02 \text{kg} \quad (4.2.2.1) \]

The mass at span wise co-ordinate \( x \) (measured from the aircraft centreline) will be:

\[
m(x) = \rho \cdot A(x) = \rho \cdot c(x) \cdot t(x) = \rho \left( 0.15 \cdot c(x)^2 - \frac{0.02}{15.99} \cdot x \cdot c(x)^2 \right) = \rho \left( -3.038 \cdot 10^{-5} \cdot x^3 + 52.26 \cdot 10^{-3} \cdot x^2 - 0.2158 \cdot x + 2.588 \right) \quad (4.2.2.2)
\]

The mass density \( \rho \) of the wing can then be calculated:

\[
m_w = \int_0^{15.99} m(x) dx = 1944.02 \quad (4.2.2.3)
\]

and therefore:

\[
\rho = 94.944 \quad (4.2.2.4)
\]

The estimated mass distribution will be:

\[
m(x) = \left( -2.884 \cdot 10^{-3} \cdot x^3 + 0.4996 \cdot x^2 - 20.49 \cdot x + 245.7 \right) \text{kg/m} \quad (4.2.2.5)
\]

To make this into a force the mass must be multiplied by \( g \). It is of course a downward force. Now the shear force, bending moment and torsion moment can be calculated along the span (\( x \) measured from the centre of the aircraft):

\[
D = \int_x^{15.99} W(x) dx = \left( 19070 - 2410.8 \cdot x + 100.50 \cdot x^2 - 1.6338 \cdot x^3 + 7.0778 \cdot 10^{-3} \cdot x^4 \right) \text{Nm}
\]

\[
M_b = \int_x^{15.99} D(x) dx = \left( 108455. - 19070. \cdot x + 1204.4 \cdot x^2 - 33.50 \cdot x^3 \right) + 0.40846 \cdot x^4 - 1.0025 \cdot 10^{-3} \cdot x^5 \text{Nm}
\]

\[
M_t = M_b(x) \cdot \tan \phi_{\alpha_3} + D(x) \frac{c(x)}{4} = \left( 29930. - 5026.8 \cdot x + 310.82 \cdot x^2 - 8.7398 \cdot x^3 \right) + 0.10919 \cdot x^4 - 4.0768 \cdot 10^{-4} \cdot x^5 \text{Nm}
\]
The influence of the engine must also be taken into account, so therefore the loads change inside \( x = 4.797 \text{m} \). The extra loads are:

\[
D = -15828.\,N \\
M_b = -15828 \cdot (4.797 - x)\,Nm \\
M_t = 15828 \cdot (1.7082 - 0.14052 \cdot x) = (27037 - 2224.2 \cdot x)\,Nm
\] (4.2.2.7)

4.2.3 The Flap Load

Unlike the other calculated basic elements the flap load calculation itself does not give a realistic result. However, when multiplied by the right factors for the full load case, as will be shown in paragraph 4.3, the calculation will make more sense.

The extra lift as in the case of the aileron will be placed at 25% of the chord of the flap. For this calculation all the lift will be produced by the flaps, as a distributed force \( L_f \). As this is a unit load case, \( L_f \) will be used. The lift \( L_f \) is assumed to be constant (instead of \( c_l \)) in the spanwise direction. This is a rough estimation because it simplifies the calculations a lot.

The \( L_f \) can be found with:

\[
\int_{0}^{15} (c_l \cdot c \cdot q) \, dx = L_f \cdot b_c = 0.5 \cdot mg
\] (4.2.3.1)

Therefore \( L_f = 15514.5 \text{N/m} \) because \( b_c = 9.731 \text{ m} \).

Knowing that the flaps have no effect outside \( x = 11.529 \text{m} \), the shear force, the bending moment and the torsion moment can now be calculated:

\[
D(x) = \int_{x}^{11.529} L_f (x) \, dx = (178866 - 15514 \cdot x)\,N \\
M_b(x) = \int_{x}^{11.529} D(x) \, dx = (1031094 - 178866 \cdot x + 7758.5 \cdot x^2)\,Nm \\
M_t(x) = \int_{x}^{11.529} L_f \cdot [\alpha(\xi) + (\xi - x) \cdot \tan 8^\circ] \, d\xi = (385229 - 50000 \cdot x + 1438.6 \cdot x^2)\,Nm
\] (4.2.3.2)

The flaps end at the fuselage wing intersection and therefore the shear force will not change anymore, and loads will be:
\[ D(x) = 150972N \]

\[ M_s(x) = (1007400 - 150972 \cdot x)Nm \]

\[ M_s(x) = 150972 \cdot (1.798 - x) \cdot \tan 8^\circ + M_s(1.798) = (338128 - 21718 \cdot x)Nm \]

(4.2.3.3)

### 4.2.4 Air Load during Stationary Rolling

The FAR specify that the aircraft must be able to sustain an angular velocity of \( p \) at \( v_o \) and \( v_{cr} \) at load factors 0 and \( 2n/3 \). Furthermore, it must do the same for \( v_o \) but with an angular velocity of \( p/3 \). Whereby \( p \) is the angular velocity achieved at \( v_o \) by full deflection of the ailerons and the manoeuvre load factor, which is 2.5.

The maximum aileron deflection angles are not known and therefore a different approach is taken. A value for the required rolling velocity is given by the handling requirement:

\[ \frac{p' \cdot b}{2v} = 0.07 \]

(4.2.4.1)

In which full aileron deflection is assumed. The \( p' \) will be taken equal to \( p \). So at the maximum deflection the ailerons will produce just enough rolling velocity.

The aerodynamic damping moment can be calculated with the moment produced by the ailerons, as the condition is stationary.

Now \( p' \) becomes:

\[ p' = 0.07 \cdot \frac{2 \cdot \dot{v_o}}{b} = (0.3676) \text{rad/s} \]

(4.2.4.2)

The aileron rolling moment is:

\[ M_a = -L_a \cdot b_a \cdot d_a = (-61.38 \cdot L_o)Nm \]

(4.2.4.3)

The \( b_a \) and \( d_a \) are the length and the distance of the centre of the aileron to the centre of the aircraft. They are 4.461m and 13.760m. Distributed force \( L_o \) again is taken constant along the aileron span, and can be found by comparing the two moments. The aerodynamic moment is:

\[ l(x) = \frac{P_o}{2} \cdot \dot{v}_o \cdot c_{l_a} \cdot \frac{p' \cdot x}{v_{cr}} \cdot c(x) = 167.1 \cdot x \cdot c(x) = 694.13 \cdot x - 26.034 \cdot x^3 \]

(4.2.4.4)

As \( c_{l_a} \) is approximately 6.1. For the rolling moment this means:
\[ M_y = \int_0^{15.99} x \cdot l(x) dx = (520481) Nm \]  

(4.2.4.5)

The tail compensates the (lack of) effect of the fuselage. A value of 7480.7 N/m follows for \( L_s \). The loads on the wing can now be calculated and are:

\[ 15.99m \geq x \geq 11.529m \]
\[ q = \left( 694.13 \cdot x - 26.034 \cdot x^2 - 8479.6 \right) N/m \]
\[ D = \left( -82328 \cdot x - 347.07 \cdot x^2 + 8.678 \cdot x^3 \right) N \]
\[ M_b = \left( -563545 \cdot x - 82328 \cdot x^2 + 4239.8 \cdot x^3 + 115.69 \cdot x^4 - 2.1965 \cdot x^4 \right) Nm \]
\[ M_r = \left\{ \begin{array}{c}
(73151.7 - 7485.0 \cdot x + 16.258 \cdot x^3 - 0.3049 \cdot x^4) \right\} Nm
\end{array} \]
\[ (238029.7 - 30222.2 \cdot x - 232.59 \cdot x^3 + 1191.7 \cdot x^5) \right\} Nm \]  

(4.2.4.6)

\[ 11.529m \geq x \geq 0m \]
\[ q = \left( 694.13 \cdot x - 26.034 \cdot x^2 \right) N/m \]
\[ D = \left( 15433.3 - 347.07 \cdot x^2 + 8.678 \cdot x^3 \right) N \]
\[ M_b = \left( -15433.3 \cdot x + 115.69 \cdot x^3 - 2.1965 \cdot x^4 \right) Nm \]
\[ M_r = \left\{ \begin{array}{c}
(73151.7 - 7485.0 \cdot x + 16.258 \cdot x^3 - 0.3049 \cdot x^4) - 17082 \right\} Nm
\end{array} \]  

(4.2.4.7)

4.2.5 Rotational Inertia

The regulations also require that the aircraft must be able to withstand an asymmetric gust of 100% of the maximum load on one wing and 80% on the other. For calculation reasons it is better to see this as a symmetric load with an intensity of 90% of the maximum value, and a 10% antisymmetric gust. The antisymmetric part will also create a rolling moment of 0.2 times the bending moment due to the gust load, at the plane of symmetry of the aircraft.

To calculate the inertia loads the moment of inertia of the airplane and the rolling acceleration must be known. An estimation of the moment of inertia is:

\[ I_\varphi = I_{\text{wing}} + I_{\text{fusel}} + I_{\text{eng}} = 2 \cdot \int m(x) \cdot x^2 dx + (m_{\text{fusel}} - m_{\text{wing}} - 2 \cdot m_{\text{eng}}) \cdot \rho + 2 \cdot m_{\text{eng}} \cdot r_{\text{eng}} \]  

(4.2.5.1)

In the formula \( m(x) = 245.8 - 20.49x + 0.4997x^2 - 2.884.10^{-3}x^3 \), \( m_{\text{eng}} = 2529 \text{ kg} \), \( m_{\text{eng}} = 1613 \text{ kg} \) and \( \rho = 1.7252 + \delta^2 \) in which 1.723m is the diameter of the fuselage and \( \delta \) is the distance between the centre of the fuselage and the centre of gravity, which is 0.2477m, and therefore \( \rho = 3.037 \), and finally \( r_{\text{eng}} = 4.798 \text{ m} \). The second moment of inertia becomes:
\[ I_\phi = (338976) \text{kgm}^2 \]  \hspace{1cm} (4.2.5.2)

Knowing that 0.2 times the bending moment at the root wing is equal to 1039853Nm and using the just calculated \( I_\phi \), the rolling acceleration will be:

\[ \ddot{\phi} = \frac{M}{I_\phi} = (3.068) \text{rad/s}^2 \]  \hspace{1cm} (4.2.5.3)

The loads are calculated for the down moving wing. However, for the up moving wing the loads are equal to the other wing but with an opposite sign. The vertical force needed to accelerate the mass at a distance \( x \) from the root is:

\[ F_n(x) = m \cdot a = (245.75 - 20.489 \cdot x + 0.49966 \cdot x^2 - 2.8843 \cdot 10^{-3} \cdot x^3) \cdot (3.068 \cdot x) \]  \hspace{1cm} (4.2.5.4)

The shear force, the bending moment and the torsion moment become:

\[ 15.99m \geq x \geq 4.797m \]

\[ D = \int_{x}^{15.99} F_n(x) dx = (33924. - 376.98 \cdot x^2 - 20.953 \cdot x^3 - 0.3832 \cdot x^4 + 1.7698 \cdot 10^{-3} \cdot x^5)N \]

\[ M_b = \int_{x}^{15.99} D(x) dx = \left( \frac{295964. - 33924 \cdot x + 125.66 \cdot x^3 - 5.2383 \cdot x^4 + 0.07664 \cdot x^5 - 0.29497 \cdot x^6}{4} \right)Nm \]

\[ M_t = M_b \cdot \tan 5.33^\circ + D \cdot \frac{c}{4} = \left( \frac{87511 - 5412.8 \cdot x - 391.49 \cdot x^2 + 48.174 \cdot x^3 + 1.9248 \cdot x^4}{-1.7879 \cdot 10^{-3} \cdot x^3 - 5.5027 \cdot 10^{-3} \cdot x^6} \right)Nm \]  \hspace{1cm} (4.2.5.5)

\[ 4.797m \geq x \geq 0m \]

\[ D = (57670. - 376.98 \cdot x^2 - 20.953 \cdot x^3 - 0.3832 \cdot x^4 + 1.7698 \cdot 10^{-3} \cdot x^5)N \]

\[ M_b = (409873. - 57670. \cdot x + 125.66 \cdot x^3 - 5.2383 \cdot x^4 + 0.07664 \cdot x^5 - 0.29497 \cdot x^6)Nm \]

\[ M_t = \left( \frac{87511 - 5412.8 \cdot x - 391.49 \cdot x^2 + 48.174 \cdot x^3 + 1.9248 \cdot x^4}{+1.7879 \cdot 10^{-3} \cdot x^3 - 5.5027 \cdot 10^{-3} \cdot x^6 + 10912.} \right)Nm \]  \hspace{1cm} (4.2.5.6)
4.2.6 The Touchdown Loads

The maximum undercarriage reaction \( R \) is:

\[
R = \lambda (\text{static} - \text{load})
\]

Whereby \( \lambda \) is in-between 1.5 and 2.5 for a civil aircraft. For the following calculations \( \lambda = 2 \) has been used. The relief of the undercarriage and the loads on the inner wing are disregarded. The wing will be accelerated by \( g + a = g + 2g \) during touchdown. This results in inertia loads. In this case the most critical will be when the wing is full with fuel.

The shear force will consist of:

\[
D = \int_{x}^{15.99} \left( m_w + m_f + m_{eq} \right) (a + g) \, dx
\]

(4.2.6.1)

The mass distribution of the fuel, \( m_f \), is unknown. It can be calculated in the same way as the rest of the wing. The fuel tank is in-between the front and rear spar, which are at 20% and 65% of the chord. The average height is 0.9 times the maximum height of the cross section. The tank starts at the wing-fuselage intersection, 1.725m, and ends at a distance of \( x = 10.2m \). So then again it follows:

\[
\rho \int_{1.725}^{10.2} \left( 1.0472 - 0.0840 \cdot x + 1.68 \cdot 10^{-3} \cdot x^2 \right) \, dx = (1386.44 + 1167.05)
\]

(4.2.6.2)

So the relative density \( \rho = 489.39 \) and the mass distribution of the fuel will be \( m_f = 512.5 - 41.15x + 0.822x^2 \). The shear force, the bending moment and the torsion moment become:

\[
15.99m \geq x \geq 10.2m
\]

\[
D = -3 \cdot g \cdot \left( 3253.6 - 245.75 \cdot x + 5.1223 \cdot x^2 - 0.16655 \cdot x^3 + 0.72108 \cdot 10^{-3} \cdot x^4 \right) N
\]

\[
M_b = -3 \cdot g \cdot \left\{ 25016 - 3253.6 \cdot x + 122.88 \cdot x^2 - 1.7074 \cdot x^3 + 0.041638 \cdot x^4 - 1.4422 \cdot 10^{-4} \cdot x^5 \right\} Nm
\]

\[
M_t = 3 \cdot g \cdot \left\{ M_b \tan 5.33^\circ + \frac{D \cdot c}{4} \right\}
\]

(4.2.6.3)
\[10.2m \geq x \geq 4.797m\]
\[D = -3 \cdot g \cdot \left\{ \left( \frac{3253.6 - 245.75 \cdot x + 5.1223 \cdot x^2 - 0.16655 \cdot x^3 + 0.72108 \cdot 10^{-3} \cdot x^4}{3377.7 - 512.49 \cdot x + 20.574 \cdot x^2 - 0.27406 \cdot x^3} \right) + \right\}_N\]
\[M_b = -3 \cdot g \cdot \left\{ \frac{39344.6631.3 \cdot x + 379.13 \cdot x^2 - 8.5654 \cdot x^3}{0.11015 \cdot x^4 - 1.4422 \cdot 10^{-4} \cdot x^4} \right\}_Nm\]
\[M_i = 3 \cdot g \cdot \left\{ M_b \cdot \tan 53.33^\circ + D \cdot \frac{c}{4} \right\}_Nm\]  
(4.2.6.4)

\[4.797m \geq x \geq 1.725m\]
\[D = -3 \cdot g \cdot \left\{ \left( \frac{3253.6 - 245.75 \cdot x + 5.1223 \cdot x^2 - 0.16655 \cdot x^3 + 0.72108 \cdot 10^{-3} \cdot x^4}{3377.7 - 512.49 \cdot x + 20.574 \cdot x^2 - 0.27406 \cdot x^3} \right) + \right\}_N\]
\[M_b = -3 \cdot g \cdot \left\{ \frac{39344.6631.3 \cdot x + 379.13 \cdot x^2 - 8.5654 \cdot x^3}{0.11015 \cdot x^4 - 1.4422 \cdot 10^{-4} \cdot x^4} \right\}_Nm\]
\[M_i = 3 \cdot g \cdot \left\{ M_b \cdot \tan 53.33^\circ + D \cdot \frac{c}{4} + 3556.6 \right\}_Nm\]  
(4.2.6.5)

4.2.7 Aerodynamic Moment

From the data sheets of the profiles a \(c_{max}\) of approximately -0.072 at cruising speed can be found. The aerodynamic moment does not vary a lot but it must be stated that in reality it changes with the velocity, the number of Reynolds and the angle of attack. However, for this calculation the aerodynamic moment is assumed constant.

4.3 The Full Load Cases

In this section the basic load cases are combined and multiplied with the correct load factors to create the full load cases, which are:

- Gust upwards \((-0.85 \text{~g})\);
- Gust downwards \((-0.85 \text{~g})\);
- Manoeuvre loads \((2.5 \text{~g} \& -1 \text{~g})\);
- Manoeuvre loads, full flaps \((2.5 \text{~g} \& -1 \text{~g})\);
- Gust loads, full flaps \((0 \text{~g}, \text{up wing})\);
- Stationary rolling \((0 \text{~g}, \text{down wing})\);
- Stationary rolling (1.67 g, up wing);
- Stationary rolling (1.67 g, down wing);
- Asymmetric gust (2.28 g - 3.068 rad/s² rolling acceleration, up wing);
- Asymmetric gust (2.28 g + 3.068 rad/s² rolling acceleration, down wing);
- Landing impact;
- Taxiing;

4.3.1 Symmetric Gust & Manoeuvre Loads

At an altitude of 6100 m and a speed of \( v_w = 121.67 \) m/s (standard atmosphere) the maximum gust loads will occur, these are:

- \( n_{\text{gust, max}} = 2.85 \);
- \( n_{\text{gust, max}} = -0.85 \);

The maximum load factors during manoeuvres is:

- \( n_{\text{man, max}} = 2.5 \);
- \( n_{\text{man, max}} = -1 \);

In full load cases the effect of the tail load has not been taken into account. The full load cases are, therefore, calculated as follows:

Maximum; gust upwards: \( 2.85\times(\text{symmetric air load} + \text{wing structure initial relief}) \)
Minimum; manoeuvre: \( -1\times(\text{symmetric air load} + \text{wing structure initial relief}) \)

The maximum manoeuvre load of 2.5 g is smaller than the 2.87 g of the gust load, therefore, the 2.5 g is not critical, and not considered. A similar situation occurs with the negative load factors but in that case the manoeuvre load of -1g is the greatest.

The effect of the tailplane has been neglected because this simplifies the calculations drastically. The influence of the tailplane can be seen in the following:

\[
L_{w,exact} = \frac{l_i \cdot n \cdot mg}{l_i + a} + \frac{\rho}{2} \cdot V^2 \cdot S \cdot C_{\text{Mac}} \cdot c_{\text{MAC}} \frac{C_{\text{Mac}}}{l_i + a}
\]

\[
L_{w,used} = n \cdot mg
\]

(4.3.1.1)

in which \( l_i \) is the distance from the tail to the centre of gravity of the plane (c.g.) and \( a \) the distance from the aerodynamic centre of the wing to the c.g. The \( c_{\text{MAC}} \) is the mean aerodynamic chord.

In numbers: \( mg = 301563N, l_i = 14.44m, a = 0.44m, C_{\text{Mac}} = -0.072, c_{\text{MAC}} = 3.2m, V = 121.67m/s, S = 93m², \) and \( \rho = 1.225kg/m³ \).

If the load factor is 2.85; \( L_{w, exact} \) is 859456N and \( L_{w, exact} \) consists of 0.97*859456N and -13050N. This means that the lift of the wing that is used for the load calculations is actually approximately 4½ % higher than the exact lift. Therefore, this calculation is on the safe side. For the minimum load factor of -1 the \( L_{w, exact} \) will be 0.97*301563N plus -13050N instead of -301563N, the exact lift is 305566N which is 1.4 % higher than the lift used in the calculations. So an extra factor of 1.02 only in the -1g case will be used.

From the gust envelope it directly becomes clear that the cruising speed is the critical speed. If the aircraft has a lower velocity the difference of \( L_{w, exact} \) and \( L_{w, used} \) also is reduced. The \( MZFW \) is the critical weight because then the wing structure inertial relief is at a minimum.
4.3.2 Gust & Manoeuvre Load with Flaps Extended

In FAR 25 it is stated that there is more than one full load case when flying with flaps extended. This depends upon whether the flaps are used at only the take-off and landing, or in the en-route configuration as well.

Here only the situation of flying at \( v_t \) is regarded. This is the design speed with fully extended flaps, which is used for landing. The maximum load factor is one of the next three:

- \( n_{\text{man}} = (2.0) \);
- \( n \) of a 25 fps gust perpendicular to the flight path;
- \( n \) of a head-on gust of 25 fps in level flight.

The \( MZFW \) is used again. The design speed, when flaps are extended, can be calculated as follows:

\[
v_{s,f} = \left( \frac{2 \cdot n \cdot m g}{C L_{\text{max}} \cdot \rho \cdot S} \right)^{0.5} = 42.7 \text{ m/s}
\]

(4.3.2.1)

With: \( n = 1 \) and \( C L_{\text{max}} = 2.9 \). For the flap design speed \( v_f \) can be stated:

\[
v_f = 1.8 \cdot v_{s,f} = 76.86 \text{ m/s}
\]

(4.3.2.2)

The load factor produced by a 25 fps (7.62 m/s) gust perpendicular to the flight path can now be calculated, knowing that \( C L_{\alpha=0.22} = 5.2 \):

\[
\mu_g = \frac{2 \cdot m}{\rho \cdot c \cdot S \cdot C L_{\alpha}} = 35.58 \text{ m/s}
\]

\[
K_g = 0.766
\]

\[
n = 1 + K_g \cdot C L_{\alpha} \cdot U_{\infty} \cdot \rho \cdot \frac{v_f}{2} \cdot \frac{W}{S} = 1.44
\]

(4.3.2.3)

The head on gust of 25 fps in a first approximation induces loads only by increasing the speed and leaving \( C L \) constant. Thus, the load factor will be:

\[
n = \frac{C L \cdot \rho \cdot (v + 7.62)^2 \cdot S}{C L \cdot \rho \cdot (v)^2 \cdot S} = 1.21
\]

(4.3.2.4)

It now also directly can be seen that in normal cruising situation the head on gust is not critical. If \( v_{\infty} \) is 121.67 m/s and \( v_g \) is 15.2 m/s, \( n \) will be 1.27.

The two lift distributions, that of the clean wing and that of the fictitious 1g flap only, now have to be combined and scaled to a load factor of 2.0.
The full load cases are composed of the clean wing air load, the flap lift load and the wing mass inertia. Suppose that the clean wing carries \( \zeta \) times the weight, and the flaps \((1-\zeta)\) times the weight of the aircraft. The \( \zeta \) can then be defined by supposing that the clean wing produces the same load whether the flaps are extended or not. The increase in \( CL \) is only dependent upon the flap influence. Now the load carrying capabilities of the clean wing and that of the flaps can compared. The \( \zeta \) will become:

\[
\zeta = \frac{CL_{\text{max}}}{CL_{\text{max, f}}} = \frac{1.878}{2.9} = 0.648
\]

(4.3.2.5)

It must be stated that if the aircraft is not flying at the optimal angle of attack, the \( CL_{\text{max}} \) will be reduced and the flaps will have to carry more of the load.

The resulting load distribution becomes:

Full load: \( n_f \{ \zeta \cdot \text{clean wing load} + (1-\zeta) \cdot \text{flap load - mass inertia} \} \), with \( n_f = 2.0 \)

4.3.3 Stationary Rolling Cases

Two rolling cases are considered, with 0 \( g \) and with 1.67 \( g \). For the 0 \( g \) situation, only the pitching moment at zero lift and the rolling moment are required. The load case at 1.67 \( g \) also has an extra load of 1.67*(symmetric air load - wing inertia). For every load case there are two calculations, the up and the down moving wing.

4.3.4 Asymmetric Gust

It is not directly clear in which situation the loads are the highest. If the wing is light the angular acceleration is higher, and thus the inertia loads. The higher mass of the wing with fuel will also yield higher loads even though the acceleration is lower. An estimation of the effect of adding fuel can be seen as follows:

\[
\delta l_{\text{fuel}} = 2 \cdot \int_{1.725}^{10.2} x^2 \cdot m_f(x) \, dx = 174606 \cdot \text{kgm}^3
\]

(4.3.4.1)

and

\[
\ddot{\phi} = \frac{M}{I_{\text{mefw}} + I_{\text{fuel}}} = 2.025 \text{rad/s}^2
\]

(4.3.4.2)

This is only an approximation because with maximum fuel, the aircraft can not carry a full payload. This effect is small though because the payload will be near the centre of the aircraft. The vertical force needed to accelerate the mass at a distance \( x \) of the root, keeping in mind that the engine is also on the wing, becomes:

\[
F_w(x) = m \cdot a = m \cdot 2.025 \cdot x
\]

(4.3.4.3)
The shear force only changes for values of $x$ in-between 10.2m and 1.725m, and will be:

$$10.2m \geq x \geq 4.797m$$  
$$D = \left\{ \frac{\left[ 22391 \cdot x^2 - 248.82 \cdot x^3 + 13.83 \cdot x^4 - 0.25293 \cdot x^5 + 1.1681 \cdot 10^{-3} \cdot x^6 \right]}{\left( 29022 \cdot -519.9 \cdot x^2 + 27.77 \cdot x^3 - 0.41623 \cdot x^4 \right)} \right\} N$$  
(4.3.4.4)

$$4.797m \geq x \geq 1.725m$$  
$$D = \left\{ \frac{\left[ 22391 \cdot x^2 - 248.82 \cdot x^3 + 13.83 \cdot x^4 - 0.25293 \cdot x^5 + 1.1681 \cdot 10^{-3} \cdot x^6 \right]}{\left( 29022 \cdot -519.9 \cdot x^2 + 27.77 \cdot x^3 - 0.41623 \cdot x^4 \right) + 15673.3} \right\} N$$  
(4.3.4.5)

The maximum increase is about 50 %. This is smaller than the reduction of loads due to the effect of the wing structure inertial relief. The Empty wing situation will still be the most critical.

The full load case consists of:

Asymmetric gust = 1.0*(symm gust air load, with 2.85g)
(Up wing) + 0.9*(wing inertia relief, with 2.85g)
- (rotational inertia loads)
+ (pitching moment at zero lift)

Asymmetric gust = 0.8*(symm gust air load, with 2.85g)
(Down wing) + 0.9*(wing inertia relief, with 2.85g)
+ (rotational inertia loads)
+ (pitching moment at zero lift)

4.3.5 Landing and Taxiing Loads

The main loads during touchdown are the inertia loads of the wing. It is thought that the aircraft lands on both its two landing gears at the same time and therefore there is no rolling inertia. The effects of relief near the fuselage are not taken into account. The calculations can, therefore, only be used for the outer 90 % of the wing. As the inner wing will not be researched in this project this will give more than sufficient information. Gravity and impact together produce a 3g inertia force downwards. The only relief of the wing the remaining lift and the pitching moment at zero lift. This is considered to be equal to the weight. The most critical weight is the operational empty weight plus maximum fuel weight.

For taxiing the regulations specify some ground handling loads. A 1.2 g inertia load is the only one concerning the wing loads. There is no lift, and the wing contains fuel.
4.3.6 The Drag Load

The drag load is disregarded because loads due to drag are far smaller than those due to lift etc. and the form of the wing is better suited to carry these loads.

4.4 The Results and Conclusions of the Load Calculations

The results of the calculations of the basic elements and the full loads are stated in tables 4.1 and 4.2. The full loads along the wing span are also plotted in figure 4.4.1.

These calculations have been made with the help of ref.[4.4.1], ref.[4.4.2], and ref.[4.4.3]. Ref.[4.2.1] was used to create the lift distribution. For the dimensions and the information on the GARTEUR 70 aircraft ref.[4.4.4] and ref.[4.4.5] were used.

A few conclusions can be drawn when looking at table 4.2 and figure 4.4.1. As the calculations in the next chapter are only done for a cross section halfway down the wing span, the results of \(x/(b/2)\) are assessed at this stage. The highest shear force and the bending moment that the aircraft wing will encounter in normal flight are those due to gust or manoeuvres. The initial design will be made to withstand the loads under these conditions. It should, however, not be forgotten that actually the highest torsion moment occurs due to gust or manoeuvres when the flaps are extended. Therefore, it must be checked that the structure that has been designed can also carry these (and all the other) loads.

The maximum negative load should not be forgotten either because it is important to know what compressive loads the bottom skin has to carry.

The loads in this chapter are the limit loads. The structure has to withstand ultimate loads. The limit loads all have to be multiplied by a safety factor of 1.5 before they can be used in chapter 5.
References:


[4.2.2] NASA TP-1498, 1979 & NASA TP-1486, 1980, "Geometric and Aerodynamic data of the NASA MS(1)-0313 & MS(1)-0317"

[4.4.1] Bladel, P.G. van, "Calculations of the Loads on the composite Wing of a 105 Passenger Transport Aircraft", Delft University of Technology, Faculty of Aerospace Engineering 595, 1988


[4.4.3] Riks, E., "Inleiding vliegtuigbelastingen", College diktaat LR2-21, 1996


[4.4.5] Auto-CAD drawings of the GARTEUR70, Delft University of Technology
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Table 4.1 The Basic Load Elements
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Table 4.2.1 Calculation of $CL_{\text{max}}$
Figure 4.1 Three-view drawing of the baseline design
Figure 4.2.1 The constants C1, C2, and C3 (ref.[4.2.1], figure E-5)

4.2.2 The lift distribution along the wing span
Figure 4.2.3 Assessing the maximum lift coefficient

Figure 4.2.4 Maximum lift coefficient of the MS(1)-0313 Airfoil (ref.[4.2.2])
Figure 4.2.4 Maximum lift coefficient of the MS(1)-0317 Airfoil (ref.[4.2.2])

\[ R = 2.0 \times 10^5 \]

\[ 4.0 \]

\[ 6.0 \]

\[ 9.0 \]

\[ 12.0 \]

\[ \alpha, \text{ deg} \]

\[ c_l \]

\[ a \]

\[ c_l \]

\[ b \]

\[ c \]

\[ c_1 \]

\[ c_2 \]

\[ c_3 \]

\[ c_4 \]

\[ q_1 \]

\[ q_2 \]

\[ q_3 \]

\[ q_4 \]

\[ M_b = \left( \frac{q_1}{2} + q_2 \right) s \]

\[ M_c = \left( \frac{q_1}{2} + q_2 \right) a \]

\[ M_t = -\left( q_3 + q_4 \right) \frac{(b+c_2-c_1)s}{3} - \frac{q_3+q_4}{2} s c_1 \]

Figure 4.2.6 The calculation method of the moments of forces
Figure 4.4.1 The full load cases
Figure 4.4.1 continued:

Maximum bending moment due to gust or maneuvers, when flaps extended

Maximum shear force due to gust or maneuvers when flaps extended

Maximum torsion moment due to gust or maneuvers when flaps extended
Figure 4.4.1 continued:

**Bending moment of the down wing due to stationary rolling, 0g**

**Shear force on the down wing due to stationary rolling, 0g**

**Torsion moment of the down wing due to stationary rolling, 0g**
Figure 4.4.1 continued:

Maximum Bending Moment Down Wing due to Stationary Rolling, 1.67g

Maximum Shear Force Down Wing due to Stationary Rolling, 1.67g

Maximum Torsion Moment Down Wing due to Stationary Rolling, 1.67g
Figure 4.4.1 continued:

Maximum Bending Moment Up Wing due to Stationary Rolling, 1.67g

Maximum Shear Force Up Wing due to Stationary Rolling, 1.67g

Maximum Torsion Moment Up Wing due to Stationary Rolling, 1.67g
Figure 4.4.1 continued:

Maximum Bending Moment due to Asymmetric Gust, Down Wing

Maximum Shear Force due to Asymmetric Gust, Down Wing

Maximum Torsion Moment due to Asymmetric Gust, Down Wing
Figure 4.4.1 continued:

Maximum Bending Moment due to Asymmetric Gust, Up Wing

Maximum Shear Force due to Asymmetric Gust, Up Wing

Maximum Torsion Moment due to Asymmetric Gust, Up Wing
Figure 4.4.1 continued:

Maximum bending moment during touchdown

Maximum Shear Force during Touchdown

Maximum Torsion Moment during Touchdown
Figure 4.4.1 continued:

Maximum Bending Moment during Taxiing

Maximum Shear Force during Taxiing

Maximum Torsion Moment during Taxiing
Chapter 5  Comparison of Wing Structures

In this chapter comparison of wing structures, with regard to the torsion box, will be made, as well as an indication of what type of fibres should be used for certain parts of the structure. The structure of the spars, skin panels, and ribs will be assessed. It is thought at this stage that a conventional two spar wing box will be used. To do these preliminary calculations the wing box is thought to be a rectangular box with constant dimensions, although for the spars the exact height will be used, and shear relief will be taken into account. Figure 5.1 shows the schematic wing box structure. The section that will be considered is halfway down the wing span, so $x/(b/2) = 0.5$. The dimensions are taken from the CAD-drawing of the GARTEUR70. They can be found in table 5.1. The loads following from the load calculations in chapter 4 are:

\[
P = 1.5 \cdot 225170.1N

M_t = 1.5 \cdot 60388.3Nm

M_s = 1.5 \cdot 492726Nm

M_{b,reg} = 1.5 \cdot 172886Nm
\]  
(5.1)

The factor 1.5 is the safety factor. The symbols are the same as in chapter 4. These are the maximum loads during gust or manoeuvres, and are the most critical in flight. In a second phase it must be checked that the structure is able to withstand all of the load situations.

5.1  Comparison of Spar Structures

In the following paragraph it will become clear that buckling usually is the most critical factor for the wing spars. Firstly, however, the minimum thickness necessary for the strength of the webs will be assessed. This leads to a minimum allowable thickness of the webs. In the second sub-paragraph calculations regarding buckling of the webs will be made. The results are, of course, not allowed to lead to thicknesses smaller than the thickness necessary for strength.

The goal of this section is to compare different type of structures; the calculations are simplified in two ways:

- It is supposed that all the webs are long panels that are simply supported at all the edges. It is, definitely in the case of overall buckling, questionable if this actually is completely fair. The spars are quite rigidly connected to the skins. However, for all of the structure types the same assumption is made.
- Secondly “netting theory” is used, like in the rest of this chapter. The reasons for doing this is that it leads to reasonably good results, the calculations are a lot simpler, and that there is not much to be found in the literature in the case of stiffened panels when the “classical laminate theory” is used.

As the calculations in this chapter are done with netting theory a basic idea of how this works should be given.

In ref.[5.1.1] it is stated that netting theory is by definition a three fibre layer system. Three unidirectional layers of fibres under different angles can carry all the applied loads. The fibres
carry all the loads, and the matrix only prevents the individual fibres from buckling. Netting theory is based on a force equilibrium between the three fibre layers. It is assumed that there is no shear between the fibres in a single layer. However, shear is transferred between the fibre layers, from one layer into the next layer. In other words, there is no shear in a plane of a layer, only perpendicular to the layer. The force equilibrium dictates for a three-layer system:

\[
\begin{align*}
V_1 \cdot \sigma_1 + V_2 \cdot \sigma_2 + V_3 \cdot \sigma_3 &= T_1 + T_2 \\
V_1 \cdot \sigma_1 \cdot \cos 2\theta_1 + V_2 \cdot \sigma_2 \cdot \cos 2\theta_2 + V_3 \cdot \sigma_3 \cdot \cos 2\theta_3 &= T_1 - T_2 \\
V_1 \cdot \sigma_1 \cdot \sin 2\theta_1 + V_2 \cdot \sigma_2 \cdot \sin 2\theta_2 + V_3 \cdot \sigma_3 \cdot \sin 2\theta_3 &= 2S
\end{align*}
\]

(5.1.1)

in which \( T_{1,2} \) is the loading in on the plate in 1,2 direction, \( S \) the shear loading, \( V_{1,2,3} \) the thickness of plate 1,2,3, and \( \theta_{1,2,3} \) the angle between each of the layers and the direction of the force \( T_1 \). Like with all the existing composite wing structures, fibre angles \( \theta_{1,2,3} \) will only be \( 0^\circ, +/- 45^\circ, \) and \( 90^\circ \). The angles will be defined for each structure separately.

The assumption that the panels are long and simply supported at the edges leads to a buckling factor \( k_c = 32.45 \). In reality this factor is not a constant factor but depends, for example, on the length and the width of the panel.

In the first phase the composite will consist of prepps consisting of a toughened epoxy and reinforcing IM6 fibres. At the end of this paragraph the effect of using a different fibre type will be assessed.

5.1.1 The Required Thickness of the Spar Webs Regarding Strength

Two aspects have to be thought of before the strength calculations can be done. Firstly the shear relief has to be taken into account, and secondly an indication must be made of how much of the shear force \( P_i \) will be carried by the front web, \( i=1 \), and how much by the rear web, \( i=2 \). Subscript \( i \) stand for the front or the rear spar in this entire paragraph. For the total force \( D \) that the webs have to carry figure 5.1.1 can be used together with:

\[
D = P - 2 \cdot \frac{M_b}{H_1} \cdot \tan \alpha_1
\]

(5.1.1.1)

From the figure it follows that taper angle \( \alpha_1 \) is 0.66\(^\circ\), and \( H_1 \) is the height of the front web at \( x/b = 0.5 \), is 378.49nm. Therefore:

\[
D = 292690 \cdot N
\]

(5.1.1.2)

To give an indication of how much force each spar carries figure 5.1.2 is used. This is a highly simplified way of doing it, but at this stage there does not seem to be another way. From figure 5.1.2 the next equation can be made:

\[
1308.6 \cdot P_i = 1163.2 \cdot D - M_i
\]

(5.1.1.3)

So keeping in mind that the forces together are equal to the total shear force, the forces are:
\[ P_1 = 190948.2N \]
\[ P_2 = 101742.2N \]  
(5.1.1.4)

The shear forces in the webs are carried by the +/- 45° layers with a thickness \( t_{+/-45^\circ} \). The definition of the fibre angles for the spar structure can be found in figure 5.1.3. The shear flow \( S_i \), whereby subscript \( i \) again stands for either the front spar, \( i=1 \), or the rear spar, \( i=2 \), and \( H_i \) is the height of each web, is:

\[ S_i = \frac{P_i}{H_i} \]  
(5.1.1.5)

The required thickness can be calculated with netting theory, and is:

\[ t_{+/-45^\circ, i} = \frac{2 \cdot S_i}{\nu_f \cdot \sigma_f} \]  
(5.1.1.6)

The fibre volume coefficient \( \nu_f=0.6 \), and \( \sigma_f \) is strength of the fibres. The results from chapter 3 can be used. The factor \( \nu_f \cdot \sigma_f \) is the strength of the prepreg. The strength of the prepreg reinforced with IM6 fibres is 910.14Mpa. This is the strength of the prepreg under hot/wet conditions. Maximum allowable strain for damage tolerance is disregarded because it thought that leading (or trailing) edge, and the top and bottom skin of the wing protect the spars enough. The height of the front spar is 378.45mm, and the height of the rear spar is 311.83mm. The shear flow is, therefore, 504.55Mpa in the front spar, and 326.27Mpa for the rear spar.

The required thicknesses of the +/- 45° layers of the front spar, and rear spar are:

\[ t_{+/-45^\circ,1,IM6} = 1.1087mm \]  
(5.1.1.7)
\[ t_{+/-45^\circ,2,IM6} = 0.71697mm \]

The web, however, also has to carry a crushing load. In the paragraph on rib structures it can be seen that these loads are very low. Therefore, they can be disregarded in the case of the spars. This is partly because the ribs carry most of these crushing loads. If the web had to carry these crushing loads 90° layers would have to be applied to the web.

### 5.1.2 Issues Regarding Buckling of the Spar Webs

There are a lot of ways to solve the problem of the buckling of webs. The web will buckle if the shear or compressive loads are too high. The structural lay outs that will be assessed in this section can be seen in figure 5.1.4. They are a corrugated web consisting of flat sections, an unstiffened web (not in the figure), a panel with stiffeners, and a corrugated web consisting of circular arcs. A sandwich panel structure will also be assessed here, although, in practice, it does not get used in larger aircraft (see section 2.2). There are two modes of buckling, overall buckling and local buckling. Overall buckling sees the panel as a whole, in which the reinforcements or corrugation only increases the bending stiffness of the panel. In the local buckling mode the deformation is confined to the area in
between the stiffeners or the separate faces. The most efficient structure usually is created when local, and overall buckling occur at the same time. This is, therefore, the basis for the following buckling calculations.

In these calculations a buckling coefficient of 32.45 is used. In the local buckling formula a reduction factor of 0.85 is added to prevent simultaneous buckling modes. This is done because local buckling causes loss of stiffness, which causes reduction in flexural stiffness, and small imperfections also cause loss of stiffness.

Only shear buckling is taken into account in the case of the spars.

### 5.1.2.1 The Corrugated Web with Flat Sections

With the formulae used here, and stating that the web has a constant thickness, there is no sense in giving the flat sections different widths because then the widest will always buckle first due to local buckling. The calculations are based on ref.[5.1.1], ref.[5.1.2] was also used to clarify some of the elements. The dimensions can be found in figure 5.1.5.

For overall buckling the following can be stated:

\[
q_{o,i} = k_s \frac{\sqrt{D_{11} \cdot D_{22}}}{H_i^2}
\]

\[
D_{11} = \frac{t_i^3}{12} E_{\text{short}} \frac{1 + \cos \beta}{2}
\]

\[
D_{22} = \frac{1}{3} t_i b_{\text{web,i}}^2 E_{\text{short}} \frac{\sin^2 \beta}{1 + \cos \beta}
\]  

(5.1.2.1)

in which \(q_{o,i}\) is the overall shear buckling load of the front or the rear spar, \(D_{11}\) and \(D_{22}\) the bending stiffness of the corrugated panel as a whole, \(\beta\) the corrugation angle, and \(b_{\text{web,i}}\) the width of the flat sections.

For local buckling the following equations have been used, keeping in mind that the “long” and the “short” direction have interchanged as can be seen in figure 5.1.6:

\[
q_{l,i} = 0.85 \cdot k_s \frac{\sqrt{D_{11} \cdot D_{22}}}{b_{\text{web,i}}^2}
\]

\[
D_{11} = \frac{t_i^3}{12} E_{\text{short}}
\]

\[
D_{22} = \frac{t_i^3}{12} E_{\text{long}}
\]  

(5.1.2.2)

in which \(q_{l,i}\) is the local shear buckling load, with reduction factor, and \(D_{11}\) and \(D_{22}\) are the bending stiffness of the flat sections.

To get an idea of how big the moduli \(E_{\text{long}}\) and \(E_{\text{short}}\) appendix 1 and are ref.[5.1.3] is used. It is based on netting theory. It leads to the next results:

\[
E_{\text{short}} = c_{11} = E_f \cdot v_f \left( l - R \cos^2(0^\circ) + R \cos^2(45^\circ) \right) = E_f \cdot v_f (1 - 0.75 R)
\]

\[
E_{\text{long}} = c_{22} = E_f \cdot v_f \left( l - R \sin^2(0^\circ) + R \sin^2(45^\circ) \right) = E_f \cdot v_f \cdot 0.25 R
\]  

(5.1.2.3)
In these equations $R$ is thickness of the +/-45° fibre layers divided by the total thickness of the laminate, $E_{xy}$ is the same as the modulus of the prepreg. It is 291.95Gpa for the epoxy-IM6 prepreg. The coefficient $c_{11}$ and $c_{22}$ do not have any meaning here but do clarify the step from ref.[5.1.3] to these formulae. It must be remembered that the other fibres in the spars have a 90° angle. This is important because if the other fibres had an angle of 0° then $E_{\text{long}}$ would be $c_{11}$, and $E_{\text{short}}$ would be $c_{22}$.

Now if overall buckling, and local buckling (with the reduction factor) occur simultaneously the most efficient structure is found. Of course, the buckling loads are equal to the shear flow in paragraph 5.1.1. For production reasons an angle $\beta$ between 30° and 35° is preferred because the fibres will stretch when the structure is shaped. The optimal angle $\beta$ is then 35°. If larger angles are allowed, then a more efficient structure would be found. Smaller angles than 30° give an unreliable local buckling behaviour.

An optimisation, i.e. minimisation of $t$, can be made with the help of a simple spread sheet program because there are not very many variables. This is also partly because the results are not very sensitive to changes of $R$. One can choose an arbitrary $R$, say 0.5, optimise the structure and then change $R$ to find the absolute optimum. This leads to an $R$-value of 0.62 for the front and the rear spar. With this result the required thicknesses $t_{\text{cor,IM6}}$ and the required width $b_{\text{cor,web,IM6}}$ can be calculated for a corrugated web with flat sections, reinforced with IM6 fibres. They are:

$$t_{\text{cor,IM6}} = 1.8702\text{mm}$$
$$b_{\text{web,cor,IM6}} = 33.55\text{mm}$$
$$t_{\text{cor,2,IM6}} = 1.3762\text{mm}$$
$$b_{\text{web,cor,2,IM6}} = 26.34\text{mm}$$

(5.1.2.4)

The equivalent thicknesses of the corrugated web with flat section and IM6 fibres $t_{\text{eq,cor,IM6}}$ are:

$$t_{\text{eq,cor,1,IM6}} = t_{\text{cor,1,IM6}} \cdot \frac{2}{1 + \cos \beta} = 2.0561\text{mm}$$

$$t_{\text{eq,cor,2,IM6}} = t_{\text{cor,2,IM6}} \cdot \frac{2}{1 + \cos \beta} = 1.5130\text{mm}$$

(5.1.2.5)

5.1.2.2 Spars with an Unstiffened Web

Before going on with the stiffened web it is useful to see what happens if the web is unstiffened. Its bending stiffness will solely come from the thickness of the web. It is, therefore, only governed by one set of equations. They are similar to the local buckling equations for infinitely long, simply supported panel. However, the “long” and “short” directions, of course, interchange when comparing them with the local buckling formulae of the corrugated web (5.1.2.2) of the last sub-paragraph. No reduction factor is necessary either, if there is only one sort of buckling. The buckling width $b_{\text{web},l}$ in this case is equal to $h$. As there is one buckling shear flow it will be called $q_l$. The formulae for buckling of the unstiffened web are:
\[ q_i = k_i \frac{\sqrt{D_{11} \cdot D_{22}^3}}{H_i^2} \]

\[ D_{11} = \frac{t^3}{12} E_{\text{long}} \]

\[ D_{22} = \frac{t^3}{12} E_{\text{short}} \]  \hspace{1cm} (5.1.2.6)

This leads to the next equation:

\[ q_i = k_i \frac{1}{H_i^2} \cdot \frac{t^3}{12} E_f \cdot v_f \cdot (0.25R)^{0.25} \cdot (1 - 0.75R)^{0.75} \]  \hspace{1cm} (5.1.2.7)

If an \( R \)-value of 0.62 is used as in the last paragraph this leads to a minimum thickness for the front web of the unstiffened web, reinforced with IM6 fibres, \( t_{\text{unstiff,IM6}} \) of 9.0234mm. The absolute minimum of \( t_{\text{equiv,IM6}} \) when +/- 45° fibres are used (theoretically using +/- 60° fibres is better), is found for both of the webs when \( R \) is 0.333. The results are:

\[ t_{\text{equiv,1 IM6}} = t_{\text{equiv,1 IM6}} = 8.7320\text{mm} \]

\[ t_{\text{equiv,2 IM6}} = t_{\text{equiv,2 IM6}} = 6.6370\text{mm} \]  \hspace{1cm} (5.1.2.8)

So the equivalent thickness of the unstiffened web with IM6 fibres \( t_{\text{equiv,IM6}} \) for both of the spars, is much larger than that of the corrugated web (5.1.2.5). Thus, the unstiffened web will be disregarded from now on.

### 5.1.2.3 Spars with a Stiffened Web

An indication of an optimum stiffened web design is made in the first part of this section. This, however, leads to a very small stiffener pitch, which for production reasons is unfavourable. In the second part another approach will be taken, i.e. a structure with a larger stiffener pitch. It will be checked if this gives a useful solution.

The structure of the spar now consists of a flat web with stiffeners on each side of web. A sketch with the dimensions can be found in figure 5.1.7. In the first case the fibre orientation is the same in all the elements. The local buckling formulae are the same as in the sub-paragraph on the corrugated web (5.1.2.1). The overall buckling formula is also the same but the bending stiffness elements, \( D^*_{11} \) and \( D^*_{22} \), change, so:

\[ q_{i,j} = k_i \frac{\sqrt{D_{11}^* \cdot D_{22}^*}}{H_i^2} \]

\[ D_{11}^* = \frac{t_{\text{stiff},i}^3}{12} E_{\text{long}} \]

\[ D_{22}^* = \frac{1}{12} \cdot t_{\text{stiff},j} (2 \cdot h_{i,j})^3 \cdot E_{\text{short}} / b_{\text{web},j} \]  \hspace{1cm} (5.1.2.9)

in which subscript stiff is used to indicate that these results concern a stiffened web design.
If a thickness of 1.8702mm, for the front spar, as in the case of the corrugated web is taken, a clear comparison can be made. Another advantage is that $b_{web,1}$ remains 33.55mm because the local buckling formulae are the same. The thickness of the web, and the thickness of the stiffeners is the taken the same. The height of the stiffeners $h_{s,1}$ is the only unknown in the equations. This then leads to a height of the stiffeners on the front web of:

$$h_{s,1} = 13.840mm$$  \hspace{1cm} (5.1.2.10)

Now a comparison can be made with the corrugated web (5.1.2.5). Only looking at the thickness, and the cross area the of the web leads to an equivalent thickness of:

$$t_{eq,surf,1,IM6} = t_{surf,1,IM6} \cdot \frac{3 \cdot h_{s,1} + b_{web,1}}{b_{web,1}} = 4.1847mm$$  \hspace{1cm} (5.1.2.11)

So the corrugated web is far more efficient. What happens, though, if the thickness of the web is changed? If the web is thickness of the web is 2mm, the equivalent thickness is 4.344mm. If the thickness of the web is 1.6mm, a equivalent thickness of 4.126mm is found. This is still must heavier that the corrugated web.

For the rear spar the results are:

$$h_{s,2} = 11.645mm$$
$$t_{surf,2,IM6} = 1.3762mm$$

$$t_{eq,surf,2,IM6} = t_{surf,2,IM6} \cdot \frac{3 \cdot h_{s,2} + b_{web,2}}{b_{web,2}} = 3.2014mm$$  \hspace{1cm} (5.1.2.12)

Although, this is not yet the optimal solution, it can be seen that for the rear spar the stiffened web is not a sensible solution.

The last thing that will be checked in this section is what happens when the stiffeners have a different fibre orientation. Suppose that the stiffeners are made of 90%, 90° and 10%, +/- 45° fibres, then the only thing that changes is the $R$ value in $D^{s,2}$ of the overall buckling formula. In the case of the front spar the height of the stiffeners $h^{s,1}$ are reduced to 11.531mm, so the equivalent thickness is:

$$t_{eq,surf,1,IM6} = t_{surf,1,IM6} \cdot \frac{3 \cdot h^{s,1} + b_{web,2}}{b_{web,2}} = 3.7986mm$$  \hspace{1cm} (5.1.2.13)

Which is not enough of an improvement.

If the stiffener pitch is enlarged to an acceptable level, for example, the stiffener pitch is half of the height of the web, the assumption that the panel can be seen as an infinitely long panel cannot be made anymore. In this case another approach must be taken. With the help of ref.[5.1.4], which is a figure plotting the buckling behaviour of a composite plate loaded in shear, the necessary thickness of the panel can be found, that is needed to prevent buckling. $R$ is assumed to remain at 0.62 and if $a=b$, $a=0.5b$ or $a=0.25b$ the effect of changing the stiffener pitch can be seen. Ref.[5.1.4] is governed by three equations. The first two can be used in
combination with the figure to solve the last one. The only remaining variable in the last equation is thickness. The formulae are:

\[
\frac{D_0}{(D_{11} \cdot D_{22})^{0.5}} = 1.614
\]

\[
a \cdot \left( \frac{D_{22}}{D_{11}} \right)^{0.25} = 0.7336 \cdot \frac{a}{b}
\]

\[
\frac{N_{xyb} \cdot a \cdot b}{(D_{11} \cdot D_{22})^{0.5}}
\]

(5.1.2.14)

in which \(N_{xyb}\) is the same as the shear flow \(q_n\), \(a\) and \(b\) are the length and the width of the panel, and \(D_0\), \(D_{11}\), and \(D_{22}\) can be found with the help of ref. [5.1.3] and are:

\[D_0 = D_\frac{t_3}{4} = E_r \cdot v_f \cdot (0.25R)\]

\[D_{22} = \frac{t_3}{12} = E_r \cdot v_f \cdot (1 - 0.75R)\]

\[D_{22} = \frac{t_3}{12} = E_r \cdot v_f \cdot (0.25R)\]

(5.1.2.15)

In the case of the front web this leads to the next results:

\[\left( \frac{N_{xyb} \cdot a \cdot b}{(D_{11} \cdot D_{22})^{0.5}} \right)_{a=b} \approx 126\]

\[t_{1, stf, a=b} = 5.3687 \text{mm}\]

\[\left( \frac{N_{xyb} \cdot a \cdot b}{(D_{11} \cdot D_{22})^{0.5}} \right)_{a=0.5b} \approx 198\]

\[t_{1, stf, a=0.5b} = 3.6630 \text{mm}\]

\[\left( \frac{N_{xyb} \cdot a \cdot b}{(D_{11} \cdot D_{22})^{0.5}} \right)_{a=0.25b} \approx 350\]

\[t_{1, stf, a=0.25b} = 2.4060 \text{mm}\]

\[t_{2, stf, a=0.25b} = 2.0806 \text{mm}\]

(5.1.2.16)

The subscripts \(a=b\), \(a=0.5b\), and \(a=0.25b\) stand for the situations whereby obviously \(a=b\), \(a=0.5b\), and \(a=0.25b\).

A couple of conclusions can be drawn from the results. A smaller the stiffener pitch will lead to a smaller skin thickness, necessary to resist buckling. The main conclusion, though, is that the thickness of the stiffened web is already larger than the equivalent thickness of the corrugated web. The stiffeners needed to resist overall buckling will raise the weight considerably. The design of the stiffeners is left for what it is as the stiffened web design of the spar will not lead to an efficient design.
It should not be forgotten that practical considerations, like the attachment to the spar caps and the ribs, could influence the choice of the type of spar structure.

In the case of the spars with a stiffened web a very small stiffener pitch leads to the optimal solution. The small stiffener pitch is for production reasons unfavourable. The optimum design of a spar with a stiffened web has an equivalent thickness that is a lot larger than that of the spar with a corrugated web with flat sections. For these reasons the stiffened web design for the spars does not seem sensible, and will also be disregarded from now on.

5.1.2.4 The Corrugated Web with Circular Arc Sections

In figure 5.1.8 a different corrugated web is given together with its dimensions. It is made up of more or less half arcs that are connected in the centre line of the web. For overall buckling the same formula (5.1.2.1) is used but with different stiffness coefficients:

\[
q_{el} = k_s \frac{4 \sqrt{D_1 \cdot D_2}}{H_i} \cdot \frac{3}{12} \cdot \frac{\sin \varphi}{\varphi} \cdot (0.25R)E_f \cdot v_f \\
D_1 = I_{1,i} \cdot (1 - 0.75R)E_f \cdot v_f \\
D_2 = I_{2,i} \cdot (1 - 0.75R)E_f \cdot v_f \tag{5.1.2.17}
\]

These formulae can be found in ref.[5.1.5]. The formulae had to be adapted because the equations in ref.[5.1.5] are for isotropic materials. Subscript arc is used to indicate the fact that this concerns a corrugated web with circular arc sections. The second moment of inertia \(I_1\) can be calculated with:

\[
I_{1,i} = \rho_\alpha^2 \cdot t_{eq,arc,i} \\
\rho_\alpha = r_i \cdot \left(0.5 - 0.75 \left(\frac{\sin 2\varphi}{\varphi} + \cos^2 \varphi\right)\right)^{0.5} \\
t_{eq,arc,i} = \left(\frac{\varphi}{\sin \varphi}\right) \cdot t_{arc,i} \tag{5.1.2.18}
\]

in which \(r_i\) is the radius of the corrugation, \(\varphi\) the corrugation angle, and \(\rho_\alpha\) the radius of gyration. In ref.[5.1.5] it is stated that for the most efficient metal construction \(\varphi\) is 80°. In the equations above it can be seen that the shape, i.e. \(\varphi\), is independent of the fibre lay-up. So for \(\varphi\) an angle of 80° is taken.

Local buckling, of a metal structure, can be calculated from the semi-empirical equation:

\[
q_{el} = 1.55 \cdot \eta_T^{0.5} \cdot E \cdot \left(\frac{t_{arc,i}}{2r_i}\right)^{1.5} \cdot 0.85 \tag{5.1.2.19}
\]

For the composite structure the Young's modulus \(E\) will be \(E_T(1 - 0.75R)^{0.25}(0.25R)^{0.75}\), and the tangents modulus factor \(\eta_T\) is equal to 1. The factor 0.85 was added again to not let local and
overall buckling coincide. The shear flow in the web is known. If $q_{cl}=q_{ol}$ is used again, and $R$ is equal to 0.778, the minimum weight with enough $+/45^\circ$ fibres, needed for the strength of the web, is found. The equations (5.1.2.18 and 5.1.2.19) lead to the next results, for the front spar:

$$t_{arc,1,IM6} = 1.4268 \text{mm}$$
$$r_1 = 14.840 \text{mm}$$  (5.1.2.20)

A comparison with the corrugated web with flat sections (which has an equivalent thickness of 2.0561 mm) from paragraph 5.1.2.1 can be made:

$$t_{eq,arc,1,IM6} = t_{arc,1,IM6} \cdot \frac{\varphi}{\sin \varphi} = 2.0229 \text{mm}$$  (5.1.2.21)

So the corrugated web with circular arc sections is the lightest structure, the difference is small, though.

For the rear spar the minimum thickness, $R=0.778$, and the equivalent thickness are:

$$t_{arc,2,IM6} = 0.93767 \text{mm}$$
$$t_{eq,arc,2,IM6} = 1.3294 \text{mm}$$  (5.1.2.22)

The equivalent thickness of the rear spar is smaller than that of the spar with the corrugated web with flat sections (1.5130 mm).

The results of the corrugated web with circular arc sections are not very different to those of the corrugated web with flat sections. It even performs slightly better.

**5.1.3 The Effect of Using a Fibre with a Higher Modulus, and a Lower Strength**

It is, of course, possible to use a different type of fibre. In this section a similar calculation for a laminate reinforced with HMS-4 fibres will be made. Only the two types of corrugated webs will be assessed.

In chapter 3 an approximation for the strength and the modulus of a unidirectional prepreg reinforced with HMS-4 fibres was made. Its modulus is 175.18 GPa, and its strength is 587.80 Mpa. Without doing any calculations one can see that the web will need more $+/45^\circ$ fibres in the laminate than when a structure with IM6 fibres is made. It will, however, have a better buckling resistance. The necessary thickness of the $+/45^\circ$ fibre layers for strength is:

$$t_{+,-/45^\circ, HMS-4} = \frac{2 \cdot S_i}{\sigma_{HMS-4, hot/wet}}$$  (5.1.3.1)

The shear flow is 504.55 Mpa in the front spar, and 326.27 Mpa for the rear spar. The following thicknesses for the front and the rear spars were found:
\[ t_{1,+,/-45°,\text{IM6}-4} = 1.7168\text{mm} \]
\[ t_{2,+,/-45°,\text{IM6}-4} = 1.1102\text{mm} \]  
(5.1.3.2)

Unlike in the structure with the prepreg with IM6 fibres, strength here is critical, and one must make sure there are enough +/-45° fibres, and there is at least one layer of 90° fibres, so:

\[ R \cdot t_i \geq t_{1,+,/-45°} \]
\[ (1 - R) \cdot t_i \geq 0.125\text{mm} \]  
(5.1.3.3)

Using the same formulae as in sub-paragraph 5.1.2.1 the following results were found for the front spar:

\[ t_{\text{cor},1,\text{HMS}-4} = 1.9485\text{mm} \]
\[ R = 0.881 \]
\[ b_{\text{web},1} = 38.06\text{mm} \]
\[ t_{\text{eq,cor},1,\text{HMS}-4} = 2.1422\text{mm} \]  
(5.1.3.4)

and for the rear spar the results are:

\[ t_{\text{cor},2,\text{HMS}-4} = 1.4028\text{mm} \]
\[ R = 0.792 \]
\[ b_{\text{web},2} = 28.41\text{mm} \]
\[ t_{\text{eq,cor},2,\text{HMS}-4} = 1.5428\text{mm} \]  
(5.1.3.5)

Of course, a corrugated web with circular arc section is also a feasible option. When the formulae from sub-paragraph 5.1.2.4 are used for a structure reinforced with HMS-4 fibres the following dimensions lead to the smallest equivalent skin thickness for the front spar:

\[ t_{\text{arc},1,\text{HMS}-4} = 1.8480\text{mm} \]
\[ R = 0.932 \]
\[ t_{\text{eq,arc},1,\text{HMS}-4} = 2.3129\text{mm} \]
\[ \varphi_1 = 65.4° \]  
(5.1.3.6)

For the rear spar they are:

\[ t_{\text{arc},2,\text{HMS}-4} = 1.2313\text{mm} \]
\[ R = 0.899 \]
\[ t_{\text{eq,arc},2,\text{HMS}-4} = 1.5221\text{mm} \]
\[ \varphi_2 = 62.8° \]  
(5.1.3.7)

The corrugation angle \( \varphi_1 \) does not have an angle of 80 degrees anymore due to the minimum thickness of the web needed for strength reasons.
So the webs with the HMS-4 fibres are marginally thicker, it also has a slightly higher density. If a fibre volume fraction of 0.6 is used the density of the prepreg with HMS-4 fibres is 1.572g/cm³, and 1.548g/cm³ for the prepreg with IM6 fibres (see chapter 3). When one uses prepregs with IM6 fibres this leads to the lightest structures.

There is one important aspect that has to be checked and that is the maximum strain in the different type of spar structures. To check the strain in the +/-45° fibres layers one can use:

\[
\sigma_f \cdot v_f = \varepsilon_{1,+/-45°} \cdot E_f \cdot v_f = \frac{2 \cdot S_t}{t_{+/-45°,t}} \tag{5.1.3.8}
\]

in which \( \varepsilon_{1,+/-45°} \) is the maximum strain in the webs of the front or the rear spar. \( E_f \cdot v_f \) depends on the type of reinforcing fibres the value of which can be found in chapter 3. The maximum strains in the different type of corrugated webs are:

\[
\begin{align*}
\varepsilon_{\text{corr},1,+/-45°,IM6} &= 0.00563 \\
\varepsilon_{\text{arc},1,+/-45°,IM6} &= 0.00588 \\
\varepsilon_{\text{corr},2,+/-45°,IM6} &= 0.00495 \\
\varepsilon_{\text{arc},2,+/-45°,IM6} &= 0.00578 \\
\varepsilon_{\text{corr},1,+/-45°,HMS-4} &= \varepsilon_{\text{arc},1,+/-45°,HMS-4} = \varepsilon_{\text{corr},2,+/-45°,HMS-4} = \varepsilon_{\text{arc},2,+/-45°,HMS-4} = 0.00335
\end{align*}
\tag{5.1.3.9}
\]

The strain in the webs with IM6 fibres is low enough so that fatigue should not be a problem (see figure 5.1.10). Impact is disregarded because the leading or trailing edge, and the top and the bottom skins protect the spars. The strains are much lower in the structures with HMS-4 fibres. One could, therefore, chose using HMS-4 fibres as the reinforcement because, although, the weight is slightly higher, the strain levels are very safe.

Usage of a low strength fibre, with a high modulus will lead to unacceptable high thicknesses necessary for strength.

5.1.3.1 A Sandwich Structure

If the structure is made from a prepreg consisting of HMS-4 fibres a sandwich panel structure is also an option. A sandwich panel can give the panel enough buckling resistance, without the high weight in comparison to the unstiffened panel. Equation 5.1.2.6 is used again, but the bending stiffness coefficients \( D_{11} \) and \( D_{22} \) change. The sandwich panel gets its bending stiffness from the distance in-between the faces, i.e. the core thickness \( t_c \), and the thickness of the faces \( t_f \).

The following equations apply:
\[ q_i = k_s \sqrt[3]{\frac{D_{11} \cdot D_{22}}{H_i^2}} \]

\[ D_{11} = 2t_{fa} \left( \frac{t_{fa} + t_e}{2} \right)^2 \cdot E_{\text{long}} \]

\[ D_{22} = 2t_{fa} \left( \frac{t_{fa} + t_e}{2} \right)^2 \cdot E_{\text{short}} \]  \hspace{1cm} (5.1.3.10)

These can be combined, which leads to:

\[ q_i = k_s \cdot \frac{1}{H_i^2} \cdot 2t_{fa} \left( \frac{t_{fa} + t_{c,i}}{2} \right)^2 \cdot E_f \cdot v_f \cdot (0.25R)^{0.25} \cdot (1 - 0.75R)^{0.75} \]  \hspace{1cm} (5.1.3.11)

The 2 faces have a thickness \( t_{fa} \) of:

\[ t_{fa,i} = \frac{t_{fa,i-45^\circ,\text{HMS-4}}}{2} + t_{i,\text{min,90^\circ}} \]  \hspace{1cm} (5.1.3.12)

Which in the case of the front spar is:

\[ t_{fa,1} = 0.8584 + 0.125 = 0.9834mm \]  \hspace{1cm} (5.1.3.13)

Thus, \( R=0.873 \). With equation 5.1.3.11 the required thickness of the core thickness can be calculated. Which is:

\[ t_{c,1} = 8.4140mm \]  \hspace{1cm} (5.1.3.14)

The question, however, remains is why a sandwich panel with HMS-4 fibres can be constructed and a structure with IM6 fibres cannot. This has got to do with the wrinkling stress of the sandwich panel. Wrinkling is a deformation of the sandwich perpendicular to its plane; the core could shear off. Chapter 3 states that for wrinkling stress \( \sigma_w \) the following applies:

\[ \sigma_w = 0.82(s\cdot E_i \cdot G_i)^{0.333} \]  \hspace{1cm} (5.1.3.15)

in which \( E_i \) is the modulus of the skin in plate direction, \( E_c \) is the modulus of the core material, and \( G_c \) is the shear modulus of the core material. Table 3.1.2 gives the properties of solid foam core materials. Acrylic foam "Rohacell" with the highest density leads to the maximum wrinkling stress. \( E_i = (1-R) E_{\text{HMS-4}} \) and therefore the wrinkling stress of a sandwich with acrylic foam in this case will be:

\[ \sigma_w = 609.88\text{MPa} \]  \hspace{1cm} (5.1.3.16)

This is higher than the stress that occurs in a structure made with HMS-4 fibres. Therefore, the sandwich panel will not wrinkle. If IM6 fibres are used the wrinkling stress will be lower due to
the lower modulus of the IM6 fibre. The allowable stress of 910.14Mpa cannot be achieved because the structure will wrinkle at a much lower stress rate.

To see how a sandwich structure performs in comparison to a corrugated web the thickness of the faces and the core must be accounted for. The density of the core is much lower, of course (see table 3.1.2). Therefore, the follow will be used for the equivalent thickness of the front spar:

$$t_{eq, sandwich,1, IM6-4} = 2t_f + t_c \frac{\rho_c}{\rho_f} = 2.4483\text{mm}$$  \hspace{1cm} (5.1.3.17)

Thus, the equivalent thickness is higher than that of a corrugated web. Applying more +/- 45° fibres leads to an even larger equivalent thickness. The minimum equivalent thickness of the rear spar is:

$$t_{eq, sandwich,2, IM6-4} = 1.7266\text{mm}$$  \hspace{1cm} (5.1.3.18)

Of course, a honeycomb core also is a also possibility, in which case intercell buckling has to be addressed. A sandwich structure with a honeycomb core is not assessed at this stage.

It can be stated that a sandwich panel performs quite well, but a corrugated web performs better.

5.1.4 Concluding Remarks on Spar Structures

From this paragraph it becomes apparent that with the relatively low loading intensities of this aircraft, definitely a corrugated web should be used. The corrugated web with flat sections performs virtually the same as a corrugated web with circular arc sections.

For the type of reinforcement a choice can be made between IM-6, and HMS-4 fibres. They are the best in their class of fibres. There is not much difference in efficiency if the maximum corrugation angle is 35°. The web with the IM-6 fibres does perform the best. The strains in the web are high, though.

For an isotropic corrugated web the best angle between the web sections is 45°, if the sections have the same width. For a composite web this does not have to be the most optimal angle but it is certain that a greater angle than 35° would lead to a lighter structure. In that case failure due to buckling, and lack of strength could coincide, in the case of the structure with IM6 fibres.

The exact way of calculating the buckling coefficients can be found in ref.[5.1.2]. Usage of the 32.45 factor, i.e. the factor for a plate simply supported at the edges, and with a infinite length, seems allowable for local buckling, but questionable for overall buckling.

Discrete values for the ply layers, usually 0.125mm, have at this stage also not been taken into account. The discrete values of the layers might change the lay up.

The results of this paragraph can be found in table 5.1.1 and 5.1.2.

100
5.2 Comparison of Skin Panels

Different types of structures can be used to give the skin panels the necessary buckling resistance. The main two groups are sandwich panels, and panels with stringers.

First the top skin panel, stiffened with stringers, will be looked at. This is done because the rib pitch depends on buckling of the top skin panel. An efficiency formula will be made by combining the local, and global buckling equations together with the strength limitations of the panel. This equation divides the problem up into two separate parts, that of an optimum lay-up, and that of an optimum cross section of the panel. A maximum rib pitch for minimum equivalent thickness of structure can found with this new formula. It will then be used to make an initial design for a panel with hat-shaped stringers, reinforced with HMS-4 fibres, and a design for a panel stiffened with I-stringers, with IM6 fibres.

The panel with stringers will then be compared to a sandwich panel. There is only one main type of buckling in the case of sandwich panels (although, wrinkling should not be forgotten). This means that both failure due to lack of strength, and due to lack buckling resistance can coincide. This way the thickness of the core of the sandwich will be calculated. The core itself also will increase the weight of the panel. In the case of the stringer panels extra layers of fibres will have to be applied in certain directions to make the structure efficient in buckling. Therefore, it cannot directly be said which type of structure will be lighter.

5.2.1 The Design of the Top Skin Panel Stiffened by Stringers

The design conditions are flexural buckling of the panel, local buckling of the skin, and the compressive strength of the material. The skin will have layers of +/- 45°, and 0° orientated fibres (see figure 5.2.1). If the skin and the stringers are of the same composition and lay-up the stress in the skin and the stiffeners is the same, and the following analysis, taken from ref.[5.2.1], can be made:

\[
\sigma = \frac{P}{t_{eq}} = \sigma_{fl} = \sigma_{l} \\
\sigma_{l} = \frac{2 \cdot \pi^2}{b^2 \cdot t_s} \left( \sqrt{D_1 \cdot D_2 + D_3} \right) \\
\sigma_{fl} = \frac{\pi^2 \cdot E_x \cdot k^2}{L^2} = c_1 \cdot E_x \left( \frac{b}{L} \right)^2 \\
t_{eq} = c_2 \cdot t_s
\]  

(5.2.1.1)

in which \( p \) is the loading intensity, \( \sigma \) is the strength of the structure, \( \sigma_{fl} \) is the flexural buckling strength, and \( \sigma_{l} \) is the local buckling strength. The thickness of the skin is \( t_s \), and \( t_{eq} \) is the equivalent thickness of the panel. \( E_x \) is the modulus in the direction along the wing span. \( L \) is the rib pitch, \( b \) is the distance between the stringers, and \( k \) is the radius of gyration. \( D_1, D_2, \) and \( D_3 \) are the bending stiffness of the panel, and \( c_1 \) and \( c_2 \) are coefficients, defined above.

To state that the stringers and the skin have the same composition and lay-up is useful to get an efficient formula, but it is not entirely practical. In reality the skin panel with stringers will have a different lay-up throughout its structure. Combining the equations 5.2.1.1 leads to an optimal solution that can be presented in the form of the following new formula:
\[
\sigma = \left( \frac{\pi^2}{3} \right)^{0.25} \left( \frac{c_1}{c_2^2} \right)^{0.25} \left[ E_x \cdot 0.5 \sqrt{c_{11} \cdot c_{22}} + 3c_{12} \right]^{0.25} \cdot \sqrt{\frac{P}{L}}
\]

(5.2.1.2)

The factors \( c_{11}, c_{22}, \) and \( c_{12} \) are bending coefficients, the exact meaning of which is stated in appendix 1 and ref. [5.1.3].

Using the results of chapter 4 it can be seen that the only variable is \( L \). The factored loading intensity is:

\[
p = \frac{M_b}{H \cdot B} = \frac{739089.3 \cdot 10^{-6}}{1308.6 \cdot 386.92} = 1459.72 \frac{N}{mm}
\]

(5.2.1.3)

The \( M_b, H, \) and the \( B \) are the bending moment, the average distance between the top and the bottom skin, and the distance between the spars halfway down the wing \( \alpha/b/2 = 0.5 \).

The optimum rib pitch can now be calculated. Interestingly enough the factor \( c_1/c_2^2 \) is independent of the material of the structure. It only depends on the shape of cross-section, and has the same optimum as that of an isotropic panel. The factor \( E_x \cdot 0.5 \left( 3c_{12} + \sqrt{c_{11} \cdot c_{22}} \right) \) is independent of the shape of the panel. These two problems can be solved individually. First the optimum thickness of the layers with +/- 45° fibres relative to the entire thickness, \( i.e., R \), must be found. The next statement can be made:

\[
E_x \cdot 0.5 \left( \sqrt{c_{11} \cdot c_{22}} + 3c_{12} \right) = E_f \cdot v_f \left( 1 - R \right) \cdot 0.5 \cdot E_f \cdot v_f \left( \sqrt{\left( 1 - 0.75R \right)(0.25R)} + 0.75R \right)
\]

(5.2.1.4)

If \( R \) is equal to 0.4 a maximum value is found. Using \( E_fv_f \) is 154.45Gpa for a laminate reinforced with IM-6 fibres, and 175.22Gpa when HMS-4 fibres are used (see chapter 3). The material optima are:

\[
\left[ E_f,IM-6 \cdot v_f \left( 1 - R \right) \cdot 0.5 \cdot E_f,IM-6 \cdot v_f \left( \sqrt{\left( 1 - 0.75R \right)(0.25R)} + 0.75R \right) \right]^{0.25} = 252.40 \sqrt{Mpa}
\]

\[
\left[ E_f,HMS-4 \cdot v_f \left( 1 - R \right) \cdot 0.5 \cdot E_f,HMS-4 \cdot v_f \left( \sqrt{\left( 1 - 0.75R \right)(0.25R)} + 0.75R \right) \right]^{0.25} = 268.52 \sqrt{Mpa}
\]

(5.2.1.5)

With the use of appendix 2, taken from ref. [5.2.2], the optimum for the shape of a hat, and that of a J-stringer can be found. These two structure types are compared because these are the two types of structure that are used in practice (as well as the J-stringer but appendix 2 does not have these results). The panel types can be found in figure 5.2.2. The results for the J-stringer stiffened panels are:

\[
c_1 = \frac{\pi^2}{b^2} = \pi^2 \left( \frac{\gamma - \beta^2}{\alpha} \right)
\]

\[
c_2 = \frac{t_{2i}}{t_i} = \alpha
\]

\[
\alpha = 2.2759
\]

\[
\beta = 1.0232
\]

\[
\gamma = 0.82459
\]

\[
\left( \frac{c_1}{c_2} \right)^{0.25} = 0.65174
\]

(5.2.1.6)
For the with hat-stringers stiffened panel the optimum is:

\[ c_1 = \frac{\pi^2 \cdot k^2}{b^2} = \pi^2 \left( \frac{\gamma - B^2}{\alpha} \right) \]

\[ c_2 = \frac{t_{eq}}{t_s} = \frac{\alpha}{2} \]

\[ \alpha = 3.9862 \]

\[ \beta = 1.0232 \]

\[ \gamma = 0.82459 \]

\[ \left( \frac{c_1}{c_2} \right)^{0.25} = 0.76929 \]

(5.2.1.7)

The coefficients \( \alpha, \beta, \) and \( \gamma \) are the same as in appendix 2.

It is very useful to give an indication of how a panel with I-stringers would perform. An educated guess is made for the dimensions of the I-stringer stiffened panel. The first assumption is that all the elements of the panel have the same thickness. The height of the web is equal to the distance between the stringers. It is expected that if the thickness, the lay-up, and the stress are the same, the elements would buckle at the same time. The two flanges have a width of 0.3 times the height of the web. This length is chosen as according to ref.[5.2.3]. This width gives enough torsional stiffness. The following results are found:

\[ c_1 = \frac{\pi^2 \cdot k^2}{b^2} = \pi^2 \left( \frac{\gamma - B^2}{\alpha} \right) \]

\[ c_2 = \frac{t_{eq}}{t_s} = \alpha \]

\[ \alpha = 2.6 \]

\[ \beta = 0.8 \]

\[ \gamma = 0.63333 \]

\[ \left( \frac{c_1}{c_2} \right)^{0.25} = 0.68285 \]

(5.2.1.8)

Thus, as expected the panel with the hat stringers performs the best. It has the highest \((c_1/c_2)^{0.25}\) coefficient.

The I-stringer panel performs better that the panel with J-stringers, so the J-stringer panel will be disregarded from now on. The I-stringer panel is not disregarded because open section panels have disadvantages in production.

The allowable stress for the panels must be ascertained. A maximum strain of 0.0042 is taken for impact reasons, in the case of the structure with IM6 fibres. It must be remembered that this is a value for a "soft skin" approach, though. For the structure with HMS-4 fibres the hot/wet situation is the most critical. First the equivalent thickness of the panels must be found. With the help of the mechanical properties found in chapter 3, and knowing that an \( R \)-value of 0.4 leads to an optimum the following applies:
\[ t_{\varphi, IM6} = \frac{P}{\sigma_{IM6, max-strain}} = \frac{1459.72}{648.68} = 2.2503 \text{mm} \]

\[ t_{eq, IM6} = \frac{t_{\varphi, IM6}}{(1 - R)} = 3.7505 \text{mm} \]

\[ t_{\varphi, HMS-4} = \frac{P}{\sigma_{HMS-4, hot/wet}} = \frac{1459.72}{587.76} = 2.4835 \text{mm} \]

\[ t_{eq, HMS-4} = \frac{t_{\varphi, HMS-4}}{(1 - R)} = 4.1392 \text{mm} \]  

(5.2.1.9)

Now the buckling stress can be calculated with the help of equation 5.2.1.1. The buckling stress for a panel reinforced with IM6 fibres \( \sigma_{IM6} \), and that of a panel reinforced with HMS-4 fibres \( \sigma_{HMS-4} \) is:

\[ \sigma_{IM6} = \frac{P}{t_{eq, IM6}} = 389.21 \text{MPa} \]

\[ \sigma_{HMS-4} = \frac{P}{t_{eq, HMS-4}} = 352.58 \text{MPa} \]  

(5.2.1.10)

The results of equations 5.2.1.3, 5.2.1.5, 5.2.1.7, 5.2.1.8, and 5.2.1.10 can be combined, and with the use of formula 5.2.1.2 the following optimal rib pitches are found:

\[ L_{1, HMS-4} = 715.75 \text{mm} \]

\[ L_{Hat, HMS-4} = 908.44 \text{mm} \]

\[ L_{1, IM6} = 519.17 \text{mm} \]

\[ L_{Hat, IM-6} = 658.93 \text{mm} \]  

(5.2.1.11)

In the next paragraph it can be seen that a large rib pitch is beneficial. So the panel with hat-stringer reinforced with HMS-4 performs the best. Another advantage is that the maximum strain is 0.0037, and therefore the structure is more damage tolerant than the structure with IM-6 fibres. A soft skin approach is not even necessary here. Open section stringers do pose extra problems with production. Of course, the thickness of the panel is also important to its weight. With help of chapter 3 the weight per unit width per unit length \( (W) \) of the panels can be calculated:

\[ W_{HMS-4} = t_{eq, HMS-4} \cdot \rho_{HMS-4} = 6.5068 \cdot 10^{-6} \text{kg/mm}^2 \]

\[ W_{IM-6} = t_{eq, IM6} \cdot \rho_{IM6} = 5.8058 \cdot 10^{-6} \text{kg/mm}^2 \]  

(5.2.1.12)

The weight of the panel itself is lighter when IM-6 fibres are used. For the lightest structure the combination of ribs with the panel must be compared.

Finally the dimensions of the two designated designs are given. The two designs are:

- A panel with hat-stringers, reinforced with HMS-4 fibres;
- A panel with I-stringers, reinforced with IM6 fibres.
The first design leads to the maximum rib pitch, and the second design is thought to be the most realistic. The panel itself has the least weight, and the I-shaped stringers lead to a relatively easy production process.

The dimensions of the panel with hat-stringers can be calculated with the local buckling equation and appendix 2. They are for the panel with HMS-4 fibres:

\[ b = b_z = b_w = 36.507\text{mm} \]
\[ b_f = 10.952\text{mm} \]
\[ t_s = 2.0768\text{mm} \]
\[ t_w = t_f = 1.2500\text{mm} \] \hspace{1cm} (5.2.1.13)

in which \( b \) is the width of the different elements and \( t \) the thickness (see figure 5.2.3). Subscript \( w \) is used for the web elements, and \( f \) for the flanges. The \( b \) without subscript is the distance from one flange to the flange of the next stringer. The \( b \) with subscript \( z \) is the distance between the flanges inside the stringer. The thickness of the skin is \( t_s \).

For an I-stringer panel with IM6 fibres, see figure 5.2.4, the results are:

\[ b = b_z = b_w = 39.152\text{mm} \]
\[ b_f = 11.746\text{mm} \]
\[ t_s = t_w = t_f = 1.4425\text{mm} \] \hspace{1cm} (5.2.1.14)

This is, of course, not yet the final design. The fibres must be rearranged to make the panel into a soft skin design, i.e. the stress in the skin must be reduced.

The skin panels also have to withstand a shear flow, resulting from the torsion moment. At this stage this is seen as a separate problem. The shear flow halfway down the wing span is:

\[ S = \frac{M_t}{2 \cdot H \cdot B} = \frac{60388.26 \cdot 10^3 \cdot 1.5}{2 \cdot 386.92 \cdot 1308.6} = 89.450 \frac{N}{\text{mm}} \] \hspace{1cm} (5.2.1.15)

The necessary thickness of +/- 45° layers needed for torsion is:

\[ t_{+/45°} \cdot \sigma = 2S \]
\[ t_{+/45°,HMS-4} = \frac{2S}{\sigma_{HMS-4}} = 0.30437\text{mm} \]
\[ t_{+/45°,IM6} = \frac{2S}{\sigma_{IM6}} = 0.27579\text{mm} \] \hspace{1cm} (5.2.1.16)

The panel with hat-shaped stringers has a thickness of the +/- 45° fibre layers of 0.4 times the skin thickness of 2.0768mm. This is 0.83072mm, and therefore the structure already had enough +/- 45° fibres in the skin.

In the case of the I-stringer panel the thickness of the +/- 45° fibre layers is 40 percent of 1.4425mm, which is 0.57700mm. So this is also enough.
The stress in the skin of the panel with I-stringers and IM6 fibres is too high. A soft skin is necessary. This can be achieved by rearranging the fibres in the structure. By raising the $R$-value in the skin the Young’s modulus of the skin will be reduced. The lower modulus in the skin is linear with the lower stress in the skin compared to the stringers. This is what happens if the $R$-value in the skin is raised to 0.5:

$$E_{sk} = E_f \cdot \nu_f (1 - R_s) = 77224 \text{Mpa}$$
$$E_s = E_f \cdot \nu_f (1 - R) = 92669 \text{Mpa}$$
$$\sigma_s = 389.21 \text{Mpa}$$
$$\sigma_{sk} = \sigma_s \cdot \frac{E_{sk}}{E_s} = 324.34 \text{Mpa}$$

There are a lot of ways to achieve an $R$-value of 0.5 in the skin. The simplest method is by exchanging $0^\circ$ fibres in the skin with +/- $45^\circ$ fibres in the flanges. The flexural buckling strength of the panel remains virtually unchanged. It will even improve a little. The dimensions become:

$$t_s = t_w = t_f = 1.4425 \text{mm}$$
$$R_s = 0.5$$
$$R_w = 0.4$$
$$R_f = 0.23333$$

The stress in the flanges will be higher:

$$\sigma_{sk} = \sigma_s \cdot \frac{E_{sk}}{E_s} = 389.21 \cdot \frac{(1 - R_f)E_f \cdot \nu_f}{(1 - R)E_f \cdot \nu_f} = 467.38 \text{Mpa}$$

It is thought that the skin protects these flanges enough against impact. Therefore, a higher stress is acceptable in the flanges. When designing with composites the normal procedure is to leave all the $0^\circ$ fibres out of the web of the stringers, and put them in the flanges where they are the most effective in the case of flexural buckling. The centre of the web is close to the neutral axis of the panel. Therefore, the $0^\circ$ fibres in the web have hardly any effect on the flexural buckling resistance of the panel. The stress in the flanges will, however, increase again. The web(s) only have to be able to resist local buckling. However, this leads to a completely new way of designing, which can be found in ref. [5.2.3].

To finalise the design, the minimum thickness of the fibre layers, of 0.125mm, must be taken into account. The thickness of the $0^\circ$ fibre layers in web of the I-stringer would probably be 0.875mm, i.e. 7 layers of 0.125mm. The thickness of $0^\circ$ fibre layers in the flanges would be 1.125mm. The thickness in the +/- $45^\circ$ fibre layers in the I-stringer could be 0.5mm (4 layers). So $R_s=0.364$ is slightly lower than 0.4, and $R_f=0.308$. The skin could have 6 +/- $45^\circ$ fibre layers and 6 $0^\circ$ fibre layers. The structure would still have to have an “equivalent” thickness of $0^\circ$ fibre layers of 2.2503mm that was required, see expression 5.2.1.9. If the height of the stringer web remains the same the stringer pitch becomes 45.518mm and the width of the flanges will be 15.112mm. The equivalent thickness of the panel would become 3.7620mm, instead of 3.7505mm. Thus, it is hardly increased.

The length of the rib pitch will have to be re-calculated because this is not the optimum design any more.
5.2.2 The Design of a Sandwich Panel for the Top Skin

In this paragraph the effect of using a sandwich panel will be assessed. The thickness of the core of the sandwich panel will be calculated. The faces of the sandwich structure only have a thickness that is required for strength of the laminates. There will be less +/- 45° fibres in the laminate, when comparing it to the panels with stringers. The core itself will have some weight. The weight of the sandwich panel will be compared to that of panel with I-stringers.

The smaller the rib pitch, the lighter the structure because less core material is needed to prevent buckling. It has to be investigated if the rib-skin connection can be co-cured together with the skin. Then holes for joints, which are stress raisers, would not have to be made on the outer surface of the wing. If mechanical fasteners are needed, a small rib pitch is not preferable. A small rib pitch also leads to more work during the production phase of the plane.

The comparison with the I-stringer panel of the last paragraph is done by giving the sandwich panel the same rib pitch as that of the I-stringer panel, i.e. 517.17mm. The minimum layer thickness of 0.125mm will directly be taken into account for the sandwich panel unlike for the I-stringer panel. As the wing box is wide, and there are not a lot of +/- 45° fibres needed to prevent torsion, a design with spar caps that have most of the UD material and skin panels with mainly +/- 45° fibre material, is disregarded. As a soft skin approach is not possible in this case IM6 fibres will not be used but HMS-4 fibres.

The next buckling formula from ref.[5.2.4] can be used:

\[
p = \frac{D_{11}}{a \cdot b} \left\{ \pi^2 \left( \frac{b}{a} \right) + \frac{D_{22}}{D_{11}} \left( \frac{a}{b} \right)^3 + \frac{2D_{12} + 4D_{66}}{D_{11}} \left( \frac{a}{b} \right) \right\}
\]

(5.2.2.1)

For the bending coefficients ref.[5.1.3] can be used, in the case of netting theory. The second moment of inertia thought to be only affected by the faces of the sandwich panel. Subscript c relates to the core of the sandwich, and \(f_a\) to both of the faces separately. They have the same thickness. So then these expressions are found:

\[
D_{11} = \frac{E_s}{1-\nu_y\nu_x} \cdot 2t_{f_a} \left( \frac{t_{f_a} + t_c}{2} \right)^2 = c_{11} \cdot 2t_{f_a} \left( \frac{t_{f_a} + t_c}{2} \right)^2
\]

\[
D_{22} = c_{22} \cdot 2t_{f_a} \left( \frac{t_{f_a} + t_c}{2} \right)^2
\]

\[
2D_{12} + 4D_{66} = 2 \cdot 3 \cdot c_{12} \cdot 2t_{f_a} \left( \frac{t_{f_a} + t_c}{2} \right)^2
\]

(5.2.2.2)

The minimum thickness 2\(t_{f_a}\) of the faces together is the same as that of the thicknesses found in the last sub-paragraph. The thickness of the 0° fibre layers has to be at least 2.2520mm so 19 layers are used, i.e. 2.375mm, and 0.5mm of the +/- 45° fibre layers. A thickness of 0.5mm is used instead of 0.27667mm because 0.5mm is the minimum thickness of the +/- 45° fibre layers that can lead to a symmetric lay-up of the laminate. This leads to and R-value of 0.174.

If expressions 5.2.2.1, and 5.2.2.2 are combined one formula can be made from which the core thickness can be calculated:
\[ p = \frac{1}{a \cdot b} E_f \cdot v_f (1 - 0.75R) \cdot 2t_{fa} \left( \frac{t_{fa} + t_e}{2} \right)^2 \left[ \pi^2 \left( \frac{b}{a} \right) + \frac{0.25R}{1 - 0.75R} \left( \frac{a}{b} \right)^3 + \frac{6 \cdot 0.25R}{1 - 0.75R} \left( \frac{a}{b} \right) \right] \] (5.2.2.3)

The loading intensity is still 1459.72 Mpa. The width of the panel \( b = 1308.6 \text{mm} \) and \( a \) is the length of the panel. In this case the length of the panel is the rib pitch which is 519.17 mm. This leads to a core thickness of:

\[ t_e = 17.644 \text{mm} \] (5.2.2.4)

The weight of the I-stringer panel and the sandwich can be compared. If the core material as in section 5.1.3.1 the density is 0.070 \( 10^{-6} \text{kg/mm}^3 \) (see chapter 3). The density depends upon what type of core material is used. The weight per unit width per unit length is:

\[ W_{\text{sandwich}} = 2t_{fa} \cdot \rho_{fa} + t_e \cdot \rho_e = 2.875 \cdot 1.572 \cdot 10^{-6} + 17.644 \cdot 0.070 \cdot 10^{-6} = 5.7545 \cdot 10^{-6} \text{kg/mm}^2 \]

\[ W_{\text{I-stringer-panel}} = t_{eq} \cdot \rho_{D\&6} = 3.7505 \cdot 1.548 \cdot 10^{-6} = 5.8058 \cdot 10^{-6} \text{kg/mm}^2 \] (5.2.2.5)

The sandwich panel is lighter compared to the theoretical optimum of the I-stringer panel. The sandwich could be heavier in the connection between the skin and the ribs. Therefore, the structure with the sandwich panel does not necessarily have to be the lightest.

It should, however, be checked that the sandwich panel does not wrinkle. The same procedure and core material as in section 5.1.3.1 is used. The stress in the structure and the wrinkling stress \( \sigma_w \) can be calculated as follows:

\[ \sigma_e = \frac{P}{2t_{fa}} = 507.73 \text{Mpa} \]

\[ E_s = (1 - R) \cdot E_{HBM-4} = 143872 \text{Mpa} \]

\[ \sigma_w = 0.82(E_s \cdot E_e \cdot G_c)^{0.333} = 598.75 \text{Mpa} \] (5.2.2.6)

It is obvious that wrinkling will not occur.

5.2.3 The Design of the Bottom Skin Panel Stiffened by Stringers

For the bottom skin panels the same route as for the top skin panels will be taken. An I-stringer panel will be designed in this sub-paragraph, in the next that of a sandwich panel is designed. The differences, compared to the design of the top skin are:

- There is a higher allowable strain for the bottom skin, i.e. at least 0.0047 instead of 0.0037 or 0.0042;
- The bottom skin only has to be able to withstand a compressive loading intensity of 512.18N/mm instead of 1459.72N/mm.

As the maximum strain is the same for both the fibre types it is not sensible to use IM6 fibres. A laminate with HMS-4 fibres will have a higher strength because of its higher modulus.

The required thickness of the 0° fibres in the bottom skin is found with the help of chapter 3 and using a maximum loading intensity, in tension, of 1459.72N/mm. The $R$-value is left at 0.4, this means that the thicknesses of the bottom skin are:

$$t_{or} = \frac{P}{\sigma_1} = \frac{P}{\epsilon_1 \cdot E_{x,\text{3045}}} = 1.7729\text{mm}$$

$$t_{eq} = \frac{t_{or}}{(1 - R)} = 2.9549\text{mm} \quad (5.2.3.1)$$

The equivalent thickness has to be distributed throughout the I-stringer panel to give it the necessary buckling resistance. Two approaches are taken. The first is to use appendix 2 to find the optimum design. These results are then used to calculated the maximum stringer pitch that will not lead to local buckling. The rib pitch has to be the same as that of the top skin panel. Because of the lower compressive loading intensity in the bottom skin this will mean that it has a too high a flexural buckling resistance. In the second approach the stringers are made smaller to reduce the flexural buckling strength. The stringer pitch will be larger. Less stringers are preferable for the production. The equivalent thickness, however, is not reduced. So there is no weight reduction.

The compressive loading intensity $p_c=512.18\text{N/mm}$. Now with the use of the equivalent thickness the compressive strength of the panel, needed to resist buckling, can be calculated:

$$\sigma_c = \frac{p_c}{t_{eq}} = 175.70\text{MPa} \quad (5.2.3.2)$$

With expressions 5.2.1.1 and 5.2.1.8 the skin thickness and the stringer pitch can be calculated:

$$\alpha = 2.6$$

$$t_s = \frac{t_{eq}}{\alpha} = 1.1365\text{mm}$$

$$\sigma_i = \sigma_c = \frac{2 \cdot \pi^2}{b^2 \cdot t_{\text{skin}}} \left(\sqrt{(1 - 0.72R)0.25R + 0.75R}\right)$$

$$b = 37.080\text{mm} \quad (5.2.3.3)$$

The flexural buckling strength of the panel can be calculated with expression 5.2.1.1 and 5.2.1.8, knowing that the rib pitch is 519.17mm. It is:
\[ c_i = \frac{\pi^2 \cdot k^2}{b^2} = \pi^2 \left( \frac{\gamma - \beta^2}{\alpha} \right) \]

\[ \alpha = 2.6 \]
\[ \beta = 0.8 \]
\[ \gamma = 0.63333 \]

\[ \sigma_{\beta} = \frac{\pi^2 \cdot E_s \cdot k^2}{L^2} = c_i \cdot E_s \left( \frac{b}{L} \right)^2 = 788.00 \text{MPa} \]

(5.2.3.4)

The flexural buckling strength is a lot higher than the required compressive strength.

In this second section the results from above are manipulated to reduce the flexural buckling strength to the necessary compressive strength.

If the height of the stringers is reduced the distance of all of the fibres to the neutral axis is smaller. The flexural resistance is less. It is thought that if the width to thickness ratio of all of the elements of the panel remain the same, the elements will have the same local buckling strength. So the following is used:

\[ \frac{b}{t_s} = \frac{b_w}{t_w} = \text{const.} \]
\[ b_f = 0.3b_w \]
\[ t_f = t_w \]

(5.2.3.5)

If \( b/b \) is reduced from 1 to 0.303, then the flexural strength is the same as the required compressive strength. The dimensions of all of the elements are:

\[ t_s = 1.9900 \text{mm} \]
\[ b = 64.930 \text{mm} \]
\[ t_w = t_f = 0.60298 \text{mm} \]
\[ b_w = 19.674 \text{mm} \]
\[ b_f = 6.5580 \text{mm} \]

(5.2.3.6)

In this case the second approach will be disregarded for three reasons:

- The weight of the panel depends on the equivalent thickness of the panel and therefore, the weight is the same for both of the approaches.
- The thickness of the first approach can be made up of 4 layers of prepregs with +/- 45° fibres with a thickness of 0.125mm, and 6 with 0° fibres. The thickness of the panel elements is 1.25mm instead of 1.1365mm. If, in this case, 2 0° fibre layers in the skin are exchanged for 2 +/- 45° fibre layers in the flanges, the stringer pitch is increased to 49.95mm. The flanges, however, have to be enlarged to 17.885mm, so the equivalent thickness becomes 3.0731mm. In the second approach the discrete thickness of the prepregs lead to problems because the stringer elements are very thin.
- Higher stringer are preferable for the connection between the skin panels and the ribs, this will become apparent in the paragraph on rib structures.
The stringer pitch (and the stringer height) will become slightly larger because the discrete thickness of the prepregs increases the thickness of the panel.

5.2.4 The Design of a Sandwich Panel for the Bottom Skin

The design of the bottom skin, when using a sandwich panel, is identical to that of the top skin. The differences between the top and the bottom skin are the same as in the last sub-paragraph, i.e. a different maximum strain value in tension, and a different compressive loading intensity than in the case of the top skin.

The thickness of the layers with 0° fibres has to be at least 1.7729mm (see equation 5.2.3.1). Therefore, 15 layers of 0.125mm are used. The necessary thickness of the layers with +/-45° fibres, to prevent failure due to torsion, can be found in expression 5.2.1.16. The discrete thickness is 0.5mm. So the R-value is 0.211. The thickness of each of the faces is:

\[ t_{fa} = \frac{t_{op} + t_{+/-45}}{2} = 1.1875\text{mm} \] (5.2.4.1)

Of course, this is a theoretical value because of the number of layers. The top face in reality will have to have 9 layers and the bottom face 8. This is disregarded to simplify the calculations. The following results are probably slightly too low because the neutral axis in reality is not in the middle. This effect is thought to be small, and if it is taken into account the formulae would have to be changed. A slightly thicker core will not increase the weight of a panel a lot because the density of core materials is low.

With the use of equation 5.2.2.1, and taking a rib pitch of 519.17mm again, the core thickness can be calculated:

\[ t_c = 11.450\text{mm} \] (5.2.4.2)

Now the weight of the different type of panels can be compared again, for a rib pitch of 519.17mm.

With the use of expression 5.2.2.5 the following results are found:

\[ W_{\text{sandwich}} = 4.5350 \cdot 10^{-6} \text{kg/mm}^2 \]

\[ W_{\text{f-stringer panel}} = 4.6450 \cdot 10^{-6} \text{kg/mm}^2 \] (5.2.4.3)

Thus, the sandwich panel is slightly lighter than the stiffened panel. There is not much difference in weight, though. Of course, the effect of the rib-skin connections must be assessed again, to see which of the two structures is the best.

Again it will be checked that the sandwich panel does not wrinkle. The same core material as in section 5.1.3.1 is used. The stress in the structure and the wrinkling stress are:

\[ \sigma_c = \frac{P_c}{2t_{fa}} = 215.65\text{MPa} \]

\[ E_s = (1 - R) \cdot E_{\text{HMS-4}} = 137427\text{MPa} \]

\[ \sigma_w = 0.82(E_s \cdot E_c \cdot G_c)^{0.333} = 589.67\text{MPa} \] (5.2.4.4)
Therefore, wrinkling will not occur. If, however, the same core material is used with half of its
density, the wrinkling stress is reduced to 334.74 Mpa. Wrinkling will still not occur. The weight of the
sandwich panel is then reduced to 4.0770 kg/mm$^2$.

5.2.5 The Effect of the Rib Pitch on the Weight of the Panels

With the help of sub-paragraphs 5.2.1 through 5.2.4 an indication can be made of what happens to the
weight of the skin panels if the rib pitch is varied. The weight of the top and bottom skins will be
added together.
The stiffened panel will not be any lighter if the rib pitch made smaller than 519.17 mm. If the
equivalent thickness would be reduced the stress in the fibres would be too high. If the rib pitch is
enlarged the panel will gain weight because there are not enough fibres to resist buckling. Changing
the shape of the I-stringer panel or the $R$-value will not help because they were already optimised. Of
course, the optimum rib pitch, i.e. strength and buckling limitations coincide, is different for the top
and the bottom skin.
The weight of the sandwich panel will always increase when the rib pitch is enlarged. The weight of
the panel consists of a basic (constant) weight of the faces, and the weight of the core, which increases
if the rib pitch is enlarged.

To see what the effect of the rib pitch has on the weight of the skin panels together it is better to firstly
address the top skin and the bottom skin separately. The weight of the top skin, if the rib pitch $L$ is
larger than 519.17 mm, can be calculated with the use of formula 5.2.1.2, and expressions 5.2.1.5,
5.2.1.8, and 5.2.1.10, which leads to:

$$
\sigma = \left( \frac{\pi^2}{3} \right)^{0.25} \left( \frac{c_1}{c_2} \right)^{0.25} \left\{ E_{x,m} \cdot 0.5 \left( \sqrt{c_{11} \cdot c_{22} + 3c_{12}} \right) \right\}^{0.25} \cdot \frac{P}{\sqrt{L}}
$$

$$
= 1.3468 \cdot 0.68285 \cdot 252.40 \cdot \frac{P}{\sqrt{L}}
$$

$$
\sigma_{m} = \frac{P}{t_{eq, top}} = 389.21 \text{ Mpa}
$$

$$
p = 1459.72 \text{ N/mm}
$$

Therefore, equivalent thickness of the top skin panel is:

$$
t_{eq, top} = 0.16460 \cdot \sqrt{L}
$$

Thus, the weight of the top skin panel stiffened with stringers is:

$$
L \leq 519.17 \text{ mm}
$$

$$
W_{\text{stringer--panel, top}} = 6.2524 \text{ kg/mm}^2
$$

$$
L \geq 519.17 \text{ mm}
$$

$$
W_{\text{stringer--panel, top}} = \rho_{m} \cdot t_{eq, top} = \left( 0.25480 \cdot 10^{-6} \cdot \sqrt{L} \right) \text{ kg/mm}^2
$$

(5.2.5.3)
For the bottom skin the same expressions are used but with the exception that loading intensity \( p_e = 512.18 \text{N/mm} \), and that the panel is reinforced with HMS-4 fibres. The optimum rib pitch is change. It is equal to 1039.6mm. The equivalent thickness and the weight of the bottom panel become:

\[
L \leq 1039.6\text{mm} \\
W_{1\text{-stringer-panel,bom}} = 4.6450 \frac{kg}{mm^2} \\
L \geq 1039.6\text{mm} \\
t_{eq,bom} = 0.091644 \cdot \sqrt{L} \\
W_{1\text{-stringer-panel,bom}} = \rho_{HMS-4} \cdot t_{eq,bom} = \left(0.14406 \cdot 10^{-6} \cdot \sqrt{L}\right)kg/\text{mm}^2
\]  

(5.2.5.4)

The results of equations 5.2.5.3 and 5.2.5.4 can be combined to see what the effect of the rib pitch is on the weight of the wing with 1-stringers. This effect can be seen in figure 5.2.5.

The same can be done for the sandwich panel structure. To calculate the weight of the faces of the top and the bottom skin the thicknesses found in sub-paragraph 5.2.2 and 5.2.4, i.e. 2.875mm and 2.375mm, can be used. The density of the HMS-4 laminate is 1.572 10^4 kg/mm^3. Together this leads to a constant basic weight, independent of the rib pitch, of:

\[
W_{\text{sandwich, basic}} = (2t_{f0, top} + 2t_{f0, bom}) \cdot \rho_{HMS-4} = 8.2530 \cdot 10^{-6} kg/\text{mm}^2
\]  

(5.2.5.5)

A sandwich panel gets its buckling resistance from its second moment of inertia, mainly created by the distance between the faces. The larger the rib pitch, the further the faces have to apart. So for a larger rib pitch a thicker, and heavier, core is needed. If extra fibres are placed in the faces a thinner core would suffice, this does not reduce the weight of the panel. The density of the core material is 0.070 10^4 kg/mm^3 for the top skin panel and 0.030 10^4 kg/mm^3 for the bottom skin panel. With formula 5.2.2.3 the core thickness that is necessary to resist buckling can be calculated for the top and the bottom skin. For the top skin the thickness of the faces 2t_{f0} is 2.875mm, with an R-value of 0.174; the loading intensity is 1459.72N/mm. The results for the bottom skin are: 2t_{f0}=2.375mm, R=0.211, and \( p=p_e=512.18 \text{N/mm} \). A new formula can be made from formula 5.2.2.3:

\[
t_{c} = 2 \cdot \left[ \frac{p \cdot a \cdot b}{E_f \cdot \nu_f (1-0.75R) \cdot 2t_{f0} \cdot \left\{ \pi \left( \frac{b}{a} \right)^2 + \frac{0.25R}{1-0.75R} \left( \frac{a}{b} \right)^3 + \frac{6 \cdot 0.25R}{1-0.75R} \left( \frac{a}{b} \right) \right\}} \right]^{-0.5} - t_{f0}
\]  

(5.2.5.6)

in which \( E_f \nu_f \) is the modulus of the with HMS-4 fibres reinforced faces, b=1308.6mm is the width of the panel, and a is the length of the panel which, of course, is equal to the rib pitch.

The total weight of the top and the bottom skin together is:

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\[ W_{\text{sandwich}} = W_{\text{sandwich,basic}} + (l_{c,\text{top}} + l_{c,\text{bot}}) \cdot \rho_c = \]
\[ 8.2530 \cdot 10^{-6} + \\
0.070 \cdot 10^{-6} \cdot \left( \frac{4.1771}{\pi^2 \left( \frac{1308.6}{a} \right) + 0.050029 \left( \frac{a}{1308.6} \right)^3 + 0.30017 \left( \frac{a}{1308.6} \right)} \right)^{0.5} - 1.4375 + \\
0.030 \cdot 10^{-6} \cdot \left( \frac{2.7668}{\pi^2 \left( \frac{1308.6}{a} \right) + 0.062667 \left( \frac{a}{1308.6} \right)^3 + 0.37600 \left( \frac{a}{1308.6} \right)} \right)^{0.5} - 1.1875 \]

\[ \text{kg/} mm^2 \]

(5.2.5.7)

This equation can be used to see what the effect of the rib pitch is (see figure 5.2.6). The comparison of the weight of the I-stringer panels and the sandwich panels can be found in table 5.2.1 and figure 5.2.7.

5.2.6 The Maximum Shear Flow

The skin panels also have to be able to withstand a shear flow, resulting from the torsion moment. The highest torsion moment occurs during gust or manoeuvres when the flaps are extended. It is 96894Nm. The shear flow halfway down the wing span is:

\[ S_{\text{max}} = \frac{M_t}{2 \cdot H \cdot B} = \frac{96894 \cdot 10^3 \cdot 1.5}{2 \cdot 386.92 \cdot 1308.6} = 143.53 \text{N/mm} \]  

(5.2.6.1)

The necessary thickness of +/- 45° layers needed for torsion is:

\[ t_{\pm 45^\circ} \cdot \sigma = 2S \]

\[ t_{\pm 45^\circ, \text{HMS-4}} = \frac{2S}{\sigma_{\text{HMS-4}}} = 0.48838 \text{mm} \]

\[ t_{\pm 45^\circ, \text{BMW-6}} = \frac{2S}{\sigma_{\text{BMW-6, stran}=0.0037}} = 0.44277 \text{mm} \]  

(5.2.6.2)

The minimum thickness of the +/- 45° fibre layers in all of the structures is 0.5mm. Therefore, the structures will never fail due to torsion loads.
5.2.7 Concluding Remarks on Skin Panel Structures

From this paragraph it becomes apparent that there is not much difference in weight if either a panel with I-stringers or a sandwich panel is used. Both of the panels have the same amount of 0° fibres. The stringer panel has more +/- 45° fibres than the sandwich panel. The sandwich panel does, of course, have the extra weight of the core. The dimensions of the skin panel structures halfway down the wing span can be seen in table 5.2.2.

The difference between the situation when the discrete thickness of the prepregs is taken into account and the situation when it is not taken into account is virtually negligible.

In the case of the stringer panels, it becomes apparent that by rearranging the fibres in the structure the stress in the separate elements of the panel, is not the same any more. An advantage of this is that the stringer pitch can be enlarged with hardly any weight increase. The flexural buckling resistance of the panel is even increased slightly by the rearranging of the fibres.

For both of the structures it does not seem sensible to use IM6 fibres in the bottom skin panel. If a stringer panel is used IM6 fibres should be used because a “soft skin” design leads to the lightest structure. The sandwich panel will have to have HMS-4 fibres. The core material of the bottom panel is a lot lighter because the maximum compressive load is considerably smaller. The wrinkling stress is also lower; therefore, a core material with a lower density can be used.

There are a couple of issues that will have to be addressed in the future.

Sandwich panels are affected by eccentricities. If an eccentricity is present, a compressive load will introduce a bending moment that could lead to premature failure of the panel. Transverse shear effects are another problem, especially when the core thickness is large. When a sandwich panel is in compression one of the faces would be compressed more than the other face because it is below the neutral axis of the sandwich panel, i.e. it is closer to the centre of curvature. However, the core material is very flexible so it is more likely that the faces have the same strain, and the core will have a relatively large transverse shear strain.

The shear flow in the structures will definitely be different than that what was assumed in paragraphs 5.1 and 5.2.

Chapter 6 will address these issues.
5.3 Comparison of Rib Structures

In this chapter a comparison will be made of different types of rib structure, namely truss structures (with plus-shaped struts, or round tube struts), and a simple flat plate structure. It is thought that the only loads that these ribs have to carry is a crushing load due to the bending of the wing, aerodynamic loads, and fuel loads. The top skin panel will have a tendency to move downwards, and the bottom skin panel a tendency to move upwards. To keep the skin panels apart the ribs have to be able to resist this “crushing load”. The main reason for placing ribs is to give the skin panels a finite length for global buckling resistance. Another reason for placing ribs is to let external forces flow into the skin, e.g. near the flap or engine attachments. These special ribs are not considered here. The aerodynamic loads, and fuel loads are thought to be too small to have any influence on these calculations.

The weight of the ribs depends on the rib pitch. The buckling strength of the skin panels also depends on the rib pitch, therefore, it seems that both problems should be connected. However, the weight of ribs is very low in comparison to the weight of the skin panels. So, optimisation of the skin panels is more important. The two different types of skin panels lead to different problems regarding the ribs.

Firstly, only the ribs of the structure with I-stringer skin panels are assessed. If the rib pitch is made smaller than the ideal pitch, the weight of the skin panel itself will not be reduced because it already has the minimum equivalent thickness required for the strength of the panel. Should the rib pitch be enlarged then, of course, the panel cannot have the minimum equivalent thickness anymore, and therefore the skin panel itself will become heavier. If the rib pitch is reduced the loss in weight of the ribs will have very little effect on the weight of the total structure for two reasons. More ribs will have to be placed but more importantly the ribs are only a couple of percent of the torsion box structures, even if the spars are excluded. From the calculations it will become apparent that a smaller rib pitch will even increase the weight of the wing box. So it is thought that the best, and easiest way to create the optimum wing structure is to optimise the skin panels, and then see what for ribs are needed. In the first section a design of a rib is made, for a rib pitch of 519.17mm. Further along in the paragraph it will be checked if this actually is the lightest structure.

In the second half of this paragraph an initial design of a rib, for a structure with sandwich skin panels, is made. It will become apparent that the joints between the ribs and the skin panels have a great influence on the weight of the structure. A simple flat plate with stiffeners will be used for the rib. The rib will have weight saving holes. The calculations are done for a rib pitch of 519.17mm, although, this does not have to be the optimum rib anymore. Therefore, the effect of the rib pitch will be assessed as well.

Finally, the weight of the two structures, i.e. the structure with I-stringer skin panels and with sandwich skin panels, will be compared.
5.3.1 Truss Ribs for the Wing Box with I-Stringer Skin Panels

The first calculations are made for a structure with skin panels that have I-shaped stringers. In that case the connection between the ribs and the skin can be co-cured, see paragraph 2.2. The structure will not have to have any holes in the top and bottom skin panels near the ribs, needed for mechanical fasteners. These holes would be stress raisers. The skin would have to be thickened up near the mechanical fasteners. The weight of the structure would, therefore, increase.

The most used structure is a truss web. On the inside of the wing, connections will have to be made with the struts. In this section a design of a truss web is made. This design consists of the struts, the skin-rib connections, and the joints between these elements.

The crushing load on the ribs can be found with help of ref.[5.3.1] and figure 5.3.1. In this case the following expressions can be used:

\[ p_{cr} = p \cdot (\alpha_{top} + \alpha_{bon}) \]

\[ \alpha_{top} = \frac{p \cdot L}{H \cdot E_{x, IM 6} \cdot t_{eq,top}} \]

\[ \alpha_{bon} = \frac{p \cdot L}{H \cdot E_{x, IM 6 - 4} \cdot t_{eq,bon}} \]

\[ p_{cr} = \frac{p^2 \cdot L}{H} \left( \frac{1}{E_{x, IM 6} \cdot t_{eq,top}} + \frac{1}{E_{x, IM 6 - 4} \cdot t_{eq,bon}} \right) \] (5.3.1.1)

The largest loading intensity on the skin panels is \( p = 1459.72 \text{N/mm} \). The rib pitch \( L \) is equal to 519.17mm, this rib pitch should lead to the lightest structure. The average height of the rib \( H = 386.92 \text{mm} \). The equivalent thicknesses and the modulus of the skin can be found in the last paragraph and chapter 3. So the crushing load is:

\[ p_{cr} = 17.432 \frac{N}{\text{mm}} \] (5.3.1.2)

Now that the load the structure must carry is known a design can be made. Two types of truss structures will be looked at. The most efficient structure has round tube struts. However, most of the calculations are based on a structure with “plus” shaped struts. These structures can be seen in figure 5.3.2. Production influences the choice of the structure; especially the end fittings of each strut could give problems. Figure 5.3.2 also gives definition of the 0° fibre direction.

Unlike with the other parts of the structure it is beneficial to directly take the minimum thickness of the laminate into consideration because of the low load, i.e. thickness. The minimum thickness \( t_w = 0.5 \text{ mm} \). It consists of a woven +/- 45° layer of 0.25mm, i.e. 2*0.125mm, and on each side a 0° layer of 0.125mm. This is the only way that a symmetric lay up can be achieved for \( t_w = 0.5 \text{ mm} \).
5.3.1.1 The “Plus” Shaped Struts

The dimensions of the plus shaped strut can be seen in figure 5.3.3, and a sketch of structure with its load can be found in figure 5.3.4. As can be seen from the figure the distance \( d \) is defined as the distance between the rib connections. For flexural buckling of the struts the now familiar equation is used:

\[
\sigma_f = \frac{\pi^2 \cdot E \cdot k^2}{L_{strut}^2}
\]  

\( (5.3.1.3) \)

With:

\[
k^2 = \frac{I}{A} = \frac{V^2 \cdot t_w \cdot h_w^3}{2 \cdot t_w \cdot h_w^2}
\]  

\( (5.3.1.4) \)

To minimise the weight it is the best to look at the volume per unit width \( V/d \). This is dependent upon the angle \( \theta \) between the skin panel and the strut. This angle can be optimised, knowing that the length of the strut elements \( L_{strut} \) is the same as \( H \sin \theta \). For the buckling stress and the volume per unit width the next two statements can be made:

\[
\sigma_f = \sqrt{\frac{V}{2A}} \cdot \pi^2 \cdot E_f \cdot v_f (1 - R) \cdot h_w^2 \cdot \frac{\sin \theta}{H^2} = \frac{0.5 \cdot d \cdot p_e}{\cos \theta \cdot 2 \cdot t_w \cdot h_w}
\]

\[
\frac{V}{d} = \frac{2B \cdot 2 \cdot t_w \cdot h_w}{2B \cdot \cos \theta} = \frac{22.535}{\cos \theta \cdot \sin \theta}
\]  

\( (5.3.1.5) \)

The weight is minimised if the angle \( \theta \) is 45°. If HMS-4 fibres are used in the struts the necessary height \( h_w \) of the “plus” is:

\[
h_w = 31.869mm
\]  

\( (5.3.1.6) \)

and therefore the relative weight is:

\[
\frac{V}{d} = 45.070mm^2
\]  

\( (5.3.1.7) \)

Of course, the plus shape strut will buckle under an angle \( \psi \), see figure 5.3.5, whereby the second moment of inertia is at its minimum. A quick calculation is made to see what happens to the second moment of inertia under an angle of 45°. It becomes apparent that (because plus shape is doubly symmetric) this has no influence:

\[
I_{45^\circ} = 2 \cdot \sqrt{\frac{1}{12}} \cdot \frac{t_w}{0.5 \cdot \sqrt{2}} \cdot h_w^3 \cdot (0.5 \cdot \sqrt{2})^3 = \frac{1}{12} \cdot t_w \cdot h_w^3 = I_{\psi}
\]  

\( (5.3.1.8) \)

For the strength of the strut the following equations can be found:
\[ V = \frac{A}{d \cos \theta} \]
\[ A = \frac{[S]}{\sigma_{allow}} = \frac{1}{\sin \theta} \cdot \frac{P_\sigma \cdot h}{\sigma_{allow}} \]  

Thus:

\[ \frac{V}{d} = \frac{1}{\sin \theta \cos \theta} \cdot \frac{P_\sigma \cdot h}{\sigma_{allow}} \]  

The allowable stress \( \sigma_{allow} \) of this type of structure, for strength reasons, then becomes 299.30Mpa, which is low. With the use of \( E_{excess} = 87590 \)Mpa the maximum strain will be 0.34%. Therefore, it is apparent that this structure has enough strength, and it only has to be designed to withstand flexural buckling of the members. Local buckling and torsional buckling is not taken into account, but it should not lead to problems because of the \( R \)-value of 0.5, and the small height of the "plus".

5.3.1.2 The Structure with Round Tube Struts

The calculations for the tube shaped strut are the same as that of the plus shaped strut. The only difference lies in the fact that the second moment of inertia and the cross section area are different. For this round section the results are:

\[ I = \pi \cdot r^3 \cdot t \]
\[ A = 2 \cdot \pi \cdot r \cdot t \]
\[ k^2 = 0.5 \cdot r^2 \]  

in which \( r \) is the radius of the thin walled tube and \( t \) its thickness.

For flexural buckling of this strut similar equations as for the plus shaped strut are found, these are:

\[ \sigma_f = \sqrt{\frac{2}{2} \cdot r^2 \cdot \pi^2 \cdot E_f \cdot \nu_f (1 - R) \cdot \sin^2 \theta}{H^2} = \frac{0.5 \cdot d \cdot P_\sigma}{\cos \theta \cdot 2 \cdot r \cdot \pi \cdot t_w} \]  

The volume per unit width is calculated with:

\[ V = \frac{2 \cdot \pi \cdot r \cdot t_w}{\cos \theta} = \frac{2 \cdot \pi \cdot t_w \cdot 6.5742}{\sin \theta \cdot \cos \theta} \]  

Again the optimal angle is \( 45^\circ \), and with the minimum thickness of 0.5mm this leads to a relative weight of:

\[ \frac{V}{d} = 41.307 mm^2 \]
Thus, the round tube strut is slightly lighter than the plus shape structure.

5.3.1.3 The Rib Flanges

A truss rib structure is likely to be heavy and expensive in the joints. The weight of the structure usually has a direct bearing on the (operational) costs. It is obvious, though, that if the structure has more connections the manual labour costs will be higher. These will not be assessed here.

Figures 5.3.6 and 5.3.7 are used for the design of the skin-rib connections. These joints will be called the rib flanges. Figure 5.3.6 shows that the flanges can be seen as a fixed-end beam loaded by a distributed force \( p_{cr} \). The beam has fixed-ends because the force on either side of the nodes, is the same. The figure also gives the reactions at the supports of the beam. The supports are the struts. Figure 5.3.7 gives the shape of the cross section of the flange with its dimensions.

For the stress calculations it is important to know what the maximum bending moment \( M_{b,\text{max}} \), and the maximum shear force \( V_{\text{max}} \) is. They both have a maximum value at the nodes as follows:

\[
M_{b,\text{max}} = \frac{p_{cr} \cdot d^2}{12}
\]
\[
V_{\text{max}} = \frac{p_{cr} \cdot d}{2}
\]

The tensile stress in the cross section of the beam is the highest at the furthest distance from the neutral axis. This is the bottom of the T-shaped structure. The T-shape structure has a constant thickness \( t_r \). The vertical section \( b_w \), and the horizontal section have the same length and are equal to the height of the stringers of the skin panels. For the top skin \( b_w = 39.152 \text{mm} \), and for the bottom skin \( b_w = 37.080 \text{mm} \). The neutral axis is at \( 0.25b_w \) from the top of the T-shape. The allowable strain is the same as in the skin panels. The same fibre type is used in the flanges as in the skin to avoid problems with different stresses. The thickness of the layers with \( 0^\circ \) fibres in the top layer is:

\[
\sigma_{\text{top}} = \frac{M_{x,\text{max}} \cdot y}{I} = \frac{p_{cr} \cdot d^2}{12} \cdot \frac{0.75b_w}{5} = \frac{E_{\text{soft-skin}} \cdot E_{\text{I/M6}}}{b_w^2 t_{r,0^\circ,\text{top}}} = 648.69 \text{MPa}
\]

\[
t_{r,0^\circ,\text{top}} = 3.1494 \text{mm}
\]

The shear stress has a maximum value at the neutral axis, because static moment \( Q \) is the largest at neutral axis. So the following is used to find the minimum thickness of the +/- 45\(^\circ\) fibre layers:
\[ \tau = \frac{V \cdot Q}{I \cdot t} \]

\[ S = \frac{V \cdot Q}{I} = \frac{P_{cr} \cdot d}{2} \left( \frac{3}{4} t_{r,+1-45^\circ,lep} b_w \cdot \frac{3}{8} b_w \right) \]

\[ t_{r,+1-45^\circ,lep} = \frac{2S}{\sigma} = \frac{2S}{E_{soft-skin} \cdot E_{D/6}} \]

\[ t_{r,+1-45^\circ,lep} = 0.71704 \text{mm} \]  \hspace{1cm} (5.3.1.17)

Because the height of rib flanges constitutes about 20% of the total height of the rib, a smaller thickness would lead to a much lighter rib structure. The most effective way of reducing the thickness of the rib flanges is to reduce \( d \). The weight of the struts will rise, though. The effect of halving the distance \( d \) will now be assessed. From expressions 5.3.1.16 and 5.3.1.17 it can directly be concluded that if the \( d \) is halved, \( t_{r,+1-45^\circ,lep} \) is halved and \( t_{r,0,lep} \) is reduced by 75%. The results for the top rib flange together with the results for the bottom rib flange (which are calculated in the same manner) are:

\[ t_{r,0,lep} = 0.78735 \text{mm} \approx 7 \cdot 0.125 = 0.875 \text{mm} \]

\[ t_{r,0,lep} = 0.87850 \text{mm} \approx 8 \cdot 0.125 = 1.0 \text{mm} \]

\[ t_{r,+1-45^\circ,lep} = 0.35852 \text{mm} \approx 4 \cdot 0.125 = 0.5 \text{mm} \]

\[ t_{r,+1-45^\circ,lep} = 0.37890 \text{mm} \approx 4 \cdot 0.125 = 0.5 \text{mm} \]  \hspace{1cm} (5.3.1.18)

The volume per unit width of the rib flanges is reduced from approximately 732 mm² to 218.91 mm² (exact value). The volume per unit width of the struts will increase if \( d \) is reduced. With the help of equation 5.3.1.5 it can be calculated for the structure with "plus" shaped struts. The new volume per unit width is 52.042 mm² (exact value).

A further decrease of distance \( d \) is not made for several reasons:

- The thicknesses are close to their minimum;
- If \( d \) is reduced \( \theta \) increases. If \( \theta > 75^\circ \) the volume per unit width of the struts will increase drastically;
- If distance \( d \) is reduced further the reduction of the thickness of the flanges will not be large;
- More connections are needed. The connections are expensive, e.g. titanium bolts, and the production cost are higher because more bolts have to be placed and holes must be made.

To find the optimum structure too many aspects have to be looked at. Reducing the distance \( d \) to 385.92 mm (equal to \( H \)) does seem justified because the weight of the rib is approximately halved.

The volume per unit width of the rib flanges and the struts, when \( d=H \), is:
\[
\left( \frac{V}{d} \right)_{\text{flange, top}} = 107.67 \text{mm}^2
\]
\[
\left( \frac{V}{d} \right)_{\text{flange, lam}} = 111.24 \text{mm}^2
\]
\[
\left( \frac{V}{d} \right)_{\text{struts}} = 52.042 \text{mm}^2
\]

(5.3.1.19)

5.3.1.4 The Connection between the Rib Flanges and the Struts

The following calculations are based on ref.[5.3.2]. From figure 5.3.8 it can be seen that if the laminate consists of 40% to 60% of +/- 45° fibres a bearing strength of 1000Mpa can be achieved. For both the width/bolt diameter ratio (w/d_b) and the edge distance/bolt diameter ratio (e/d_b) a value of 4 is taken. From figures 5.3.9 and 5.3.10 it can be seen that in that case a bearing strength of 1000Mpa can be achieved if R=0.5. The rib flanges, therefore, only have to be thickened up in the direct neighbourhood of the bolt connection (see figure 5.3.11). In these figures d is used for the bolt diameter, in this section d_b will be used to prevent confusion with distance d.

A value of e/d_b = 4 has to be achieved above and below the bolt connection. Therefore, twice the edge distance is equal to the height of the rib flange b_w. Now the maximum allowable bolt diameter can be found. In the case of the top rib flange this is:

\[
d_{b, \text{max}} = \frac{b_w}{2 \cdot 4} = 4.8940 \text{mm}
\]

(5.3.1.20)

The width w also has to be 4 times the bolt diameter d_b. Therefore, w=19.576mm.

Now that the diameter of the holes of the bolts is known, the necessary thickness of the panel can be calculated by using:

\[
\sigma_{\text{bearing}} = \frac{P_{\text{max}}}{d_b \cdot t_{\text{con}}}
\]

(5.3.1.21)

The force P_{\text{max}} can be found in figure 5.3.4. If the distance d=386.92 then P_{\text{max}}=6366.0N. For the bearing strength \sigma_{\text{bearing}} a value of 1000Mpa is used. The thickness t_{\text{con}} required to transfer the load from the rib flange to the struts is:

\[
t_{\text{con}} = 1.3008 \text{mm}
\]

\[
t_{\text{con}} = t_{90°} + t_{+, -45°} \approx 6 \cdot 0.125 + 6 \cdot 0.125 = 1.5 \text{mm}
\]

(5.3.1.22)

It must be remembered that this consists of 90° and +/- 45° fibres. The rib flange itself only has 0° and +/- 45° fibres. It does, however, have 0.5mm of +/- 45° fibre layers which do not have to be applied additionally to the connection area. The extra thickness needed near the connections, therefore, is 1.5mm minus the 0.5mm which are already there.
In reality the structure of the rib flanges near the bolt connections consists of 30% 0° fibre layers, 30% 45° fibre layers, and 40% 90° fibre layers. Until now the 0° fibre layers were disregarded in the calculations. However, from figure 5.3.12 it can be seen that the bearing stress is in-between 145 and 140 ksi, which is about 980Mpa. The thickness is 2.375mm instead of 1.5mm in the case of the top flange. From the large increase in thickness it directly can be seen that the design is on the safe side. In ref[5.3.2] it is said that the best bearing strength is achieved when \( d/\ell \) is in-between 1 and 2. Values of \( d/\ell \) lower than 1 can lead to shearing off of the bolts, due to the linear decrease of bearing force with bolt diameter and the quadratic decrease of shear area in the fastener. Here \( d/\ell = 2.0606 \).

Of course, the loads cannot be directly transferred from a 90° direction to a 0° direction. A extra factor of 1.5 is used for a gradual transfer in loading direction (see figure 5.3.11). The results for extra thicknesses of the bottom rib flange are the same. The weight of thickening up the flanges is:

\[
\begin{align*}
V \quad & = 4 \cdot \frac{b_w}{8} \\
\frac{V}{d} \quad & = \frac{1.5 \cdot t_{\text{con,extra}} \cdot \left( b_{w,\text{top}} \cdot 4 \cdot \frac{b_{w,\text{top}}}{8} \right)}{d} = 2.9713 \text{mm}^2 \\
\frac{V}{d} \quad & = \frac{1.5 \cdot t_{\text{con,extra}} \cdot \left( b_{w,\text{bon}} \cdot 4 \cdot \frac{b_{w,\text{bon}}}{8} \right)}{d} = 2.6651 \text{mm}^2 \quad (5.3.1.23)
\end{align*}
\]

The bolts themselves also have some weight. For an approximation of the weight of the bolts it is thought that the length of the bolt is twice the thickness of the flange near the joints. The diameter of the bolt is known. A factor 2.5 is used to compensate for the weight of the head of the bolt, the nut, etc. A comparison in volume per unit width is not completely fair because the bolts are made of different materials, e.g. titanium. The volume per unit width of the bolts is:

\[
\frac{V}{d} \quad = \frac{2.5 \cdot \left( 2t_{\text{con}} + t_{\ell} \right)_{\text{top}} \cdot \frac{\pi}{4} \left( \frac{b_{w,\text{top}}}{8} \right)^2 + \left( 2t_{\text{con}} + t_{\ell} \right)_{\text{bon}} \cdot \frac{\pi}{4} \left( \frac{b_{w,\text{bon}}}{8} \right)^2}{d} = 0.90708 \text{mm}^2 \quad (5.3.1.24)
\]

From expressions 5.3.1.19, 5.3.1.23 and 5.3.1.24 it becomes apparent that the weight of the joints hardly has any influence on the weight of the ribs. The weight of the rib structure can be calculated. The density of the elements is not the same, so it will have to be taken into account. For the density of titanium \( \rho_{\text{titanium}} \) a value of 4.5 \( 10^6 \) is given in ref[5.3.3]. The total weight is:

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\[ W_{\text{rib}} = \left( \left( \frac{V}{d} \right)_{\text{flange, top}} + \left( \frac{V}{d} \right)_{\text{con, top}} \right) \cdot \rho_{\text{IM6}} + \left( \left( \frac{V}{d} \right)_{\text{flange, btm}} + \left( \frac{V}{d} \right)_{\text{con, btm}} + \left( \frac{V}{d} \right)_{\text{strut}} \right) \cdot \rho_{\text{HMS-4}} + \left( \frac{V}{d} \right)_{\text{bolts}} \cdot \rho_{\text{Titanium}} = 436.22 \cdot 10^{-6} \text{ kg/mm} \]  

(5.3.1.25)

5.3.1.5 The Effect of the Rib Pitch on the Weight of the Ribs

One of the main parts of the volume per unit width of the ribs depends directly upon \( h_w \). This is the part of the struts. The crushing load depends directly upon the rib pitch but the height \( h_w \) only depends upon the crushing load to the power one third, see expression 5.3.1.5. To see that a maximum rib pitch leads to maximum efficiency the \( V/d \) per unit length \( L \) (that is the rib pitch) should be looked at:

\[ \left( \frac{V}{d} \right)_{\text{strut}} \propto \frac{L^{0.333}}{L} = \frac{1}{L^{0.667}} \]  

(5.3.1.26)

The other main part of the rib are the rib flanges. From equations 5.3.1.16 and 5.3.1.17 it can be seen that the thickness, and therefore the weight, changes linear with the crushing load. As stated just above the crushing load depends directly upon the rib pitch. Thus, for the influence of the rib pitch the following applies:

\[ \frac{V}{d} \propto K_1 + K_2 \frac{1}{L^{0.667}} \]  

(5.3.1.27)

in which \( K_1 \) and \( K_2 \) are two constants. So if the rib pitch is larger the structure will be (slightly) lighter. From expressions 5.3.1.19, 5.3.1.23 and 5.3.1.24 it can be seen that the weight of the struts is only about 20% of the total weight of the rib and rib flanges about 80%. An approximation for \( K_1 \) and \( K_2 \) can be made by stating that when \( L = 519.17 \text{mm} \) weight of the struts is 20% of the total weight of the rib, and rib flanges 80%. The structure still has not got a constant density because IM6 and HMS-4 are both used. The density of the both of the laminates is, however, vitally the same. The joints are disregarded. \( K_1 \) and \( K_2 \) then are:

\[ K_1 = 0.43360 \text{mm} \]
\[ K_2 = 6.9730 \text{mm}^{2/3} \]  

(5.3.1.29)

If the density of the composites \( \rho_{\text{comp}} \) is thought to be \( 1.56 \cdot 10^6 \), then the weight of the ribs, in figure 5.3.13, is:

\[ W_{\text{rib}} = \rho_{\text{comp}} \cdot \frac{V}{d} = \rho_{\text{comp}} \cdot \left( K_1 + K_2 \frac{1}{L^{0.667}} \right) = \left( 0.6742 + \frac{10.878}{L^{0.667}} \right) \cdot 10^{-6} \text{ kg/mm}^2 \]  

(5.3.1.30)

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The structure has a minimum weight if the rib pitch \( L = 519.17 \text{mm} \).
This can be seen in Table 5.3.1 and Figure 5.3.13. Figure 5.3.13 shows the weight of the stringer panels (taken from Figure 5.2.5) and the weight of the ribs in relation to the rib pitch. Table 5.3.1 states the weights of the I-stringer panels individually, the panels together, the ribs, and the total weight of the structure without the spars. In Table 5.3.1 it can be more clearly seen than in Figure 5.3.13, that of rib pitch of 519.17mm gives the lightest structure. From both Figure 5.3.13 and Table 5.3.1 it can be seen that the rib pitch has little effect on the weight of the ribs.

If the rib pitch is larger than 519.17mm then the structure definitely gains weight. If the rib pitch is smaller than 519.17mm the structure is only slightly heavier. A smaller rib pitch, however, leads to higher production costs.

It must be remembered that equation 5.3.1.27 only applies when the thickness of the rib flanges is larger than the minimum allowable thickness of 0.5mm. The weight for a rib pitch smaller than 200mm will rise rapidly, and the results should be disregarded.

Why use “plus” shaped struts? It is thought that the “plus” section makes for simpler attachments. End fittings as can be seen in Figure 5.3.14 could be easily glued to the end of all of the struts. The weight of the structure would hardly be raised by these elements. These attachments as well as the struts can be mass produced. A solution for the connections of the round tube struts is not so obvious.

### 5.3.1.6 A Rib Consisting of a Simple Flat Plate

To conclude this section, the weight of a simple plate acting as a rib will be assessed. It also is connected on the inside of the wing box, therefore, the same rib flanges are used. Lighter rib flanges could be made but more connections would be necessary. The minimum thickness of 0.5mm is used. The volume per unit width is equal to the height of the rib \( H = 386.92 \text{mm} \) times the thickness \( t_{\text{plate}} \) of the rib. The volume per unit width of the flat plate is:

\[
\left( \frac{v}{d} \right)_{\text{plate}} = t_{\text{plate}} \cdot H = 193.46 \text{mm}^2
\]  

\[ (5.3.1.31) \]

This simple flat plate already has a volume per unit width which is a lot higher than that of the struts, see equation 5.3.1.19. This simple plate still has to be reinforced, or corrugated, to prevent buckling.

### 5.3.2 The Rib Structure for a Sandwich Panel

In the case of a rib for a structure with sandwich skin panels a connection must be made with the skin structure. It seems, therefore, that a truss web should not be used because in that case two sets of connection must be made in the rib flanges. One to join the rib flange to the skin, and one to join it to the struts. It is probably best to make a very simple rib design. A simple flat plate with stiffeners will be assessed, although, a rib with a corrugated web might also lead to an efficient structure. The web of the rib and the connections to the skin panels will be assessed in this section.
5.3.2.1 The Web of the Rib Structure

As the amount of 0° fibres in the sandwich skin panels is the same as that in the I-stringer panel the crushing load is the same as in section 5.3.1, i.e. $P_{cr}=17.432\text{N/mm}$. Expression 5.3.1.31 states that a rib with consisting of a simple flat plate has a volume per unit width of:

$$
\left( \frac{V}{d} \right)_{\text{plate}} = t_{\text{plate}} \cdot H = 193.46\text{mm}^2
$$

(5.3.2.1)

The minimum thickness of 0.5mm is used and the height of the rib $H=386.92\text{mm}$.

This panel cannot resist buckling as of yet. Stiffeners will be co-cured to give the panel the out of plane stiffness to prevent flexural buckling. The following buckling equation can be used again:

$$
\sigma_{fl} = \frac{\pi^2 \cdot E_{0^\circ} \cdot k^2}{H^2}
$$

(5.3.2.2)

in which $E_{0^\circ}$ is the modulus in the direction of the 0° fibre. The fibre orientations can be found in figure 5.3.15. The dimensions of the panel can be found in figure 5.3.16. For the radius of gyration $k$ the following can be stated:

$$
k^2 = \frac{I}{A} = \frac{t_s (2h_s)^3}{12b + 3.1t_s h_s}
$$

(5.3.2.3)

in which $b$ is the stiffener pitch and $h_s$ is height of the stiffeners. As $t$ is the minimum allowable thickness of the rib. The stiffener thickness $t_s$ cannot be thinner than $t$, thus, $t = t_s$ will be used.

In that case equation 5.3.2.2 can be rewritten as:

$$
\sigma_{fl} = \frac{\pi^2 \cdot E_{0^\circ} \cdot 8.12 \cdot h_s^3}{b + 3h_s} = \frac{P_{cr}}{t_{eq}} = \frac{P_{cr}}{(b + 3h_s) \cdot t}
$$

(5.3.2.4)

For the minimum thickness of 0.5mm the $R$-value is 0.5. $E_{0^\circ}$ then becomes $87090\text{Mpa}$ for a laminate with HMS-4 fibres. From equation 5.3.2.3 it can be see that the a large $b$-value leads to a relatively small $h_s$-value. This means that a maximum $b$-value gives a minimum equivalent thickness and, therefore, the lightest rib structure.

However, “lightning holes” should be used to save weight. This “lightning holes” are circular, see figure 5.3.15. Therefore, the choice of stiffener pitch is limited. Usually a rib with “lightning holes” has an array of 2 or 3 holes underneath each other (see figure 5.3.15 again). If a 2 hole configuration is used a larger stiffener pitch can be used. The stiffener pitch can be estimated at half of the height of the rib $H$.

With expression 5.3.2.3 the dimensions of the rib are found. They are:

$$
t = t_s = 0.5\text{mm}$$

$$
b = \frac{H}{2} = 193.46\text{mm}
$$

$$
h_s = 11.987\text{mm}
$$

(5.3.2.5)
It is thought that if approximately half of the $0^\circ$ fibres go past the holes, the weight of the flat plate is reduced by 25%. The equivalent thickness will be:

$$t_{eq} = 0.75 \cdot t + \frac{3h}{b} \cdot t_s = 0.46794\,mm$$  \hspace{1cm} (5.3.2.6)

It should now be checked that the stress in the fibres is not too high. Figure 5.3.17 shows that if a laminate with a hole is in tension and the lay-up is quasi-isotropic then a stress concentration factor of 3 can be used. In actual fact this stress concentration factor only applies in the neighbourhood of the hole, but in this case it will be used in the whole of the structure. The stress in the $0^\circ$ fibres will be:

$$\sigma_{0^\circ} = 3 \cdot \frac{P_e}{(1-R) \cdot t} = 209.18\,Mpa$$  \hspace{1cm} (5.3.2.7)

From chapter 3 it can be seen that this is a relatively low stress for carbon fibres. Thus, the rib structure with "lightning holes" has enough strength.

The volume per unit width of the web of the rib structure now is:

$$\left(\frac{V}{d}\right)_{rib,web} = t_{eq} \cdot H = 181.06\,mm^2$$  \hspace{1cm} (5.3.2.8)

### 5.3.2.2 The Connection of the Ribs to the Skin Panels

The main problem with using sandwich skin panels is that holes for mechanical fasteners have to be made in the skin structure. Unlike in the rib structure itself, the skin panels are highly loaded. To keep the stress low enough a lot of material must be added, because the structure must be quasi-isotropic and then still there is a stress concentration near the holes. The structure near the joints can be seen in figure 5.3.18.

In section 5.2.2 it is stated that the thickness of the $0^\circ$ fibre layers in the top skin is 2.375mm and the total thickness of the laminate is 2.875mm. To make this structure more or less quasi-isotropic 3 prepregs of $90^\circ$ fibre layers, with a thickness of 0.125mm, are added and 8 prepregs of $+/-45^\circ$ fibre layers. The stress concentration factor of 3 is used again. The total thickness of the skin panel near the joints $t_{tot,top}$ and the extra thickness $t_{extra,top}$ that has to be applied are:

$$t_{tot,top} = 3 \cdot (2.875 + 3 \cdot 0.125 + 12 \cdot 0.125) = 14.250\,mm$$

$$t_{extra,top} = t_{tot,top} - 2.875\,mm = 11.375\,mm$$  \hspace{1cm} (5.3.2.9)

So there is a tremendous increase in thickness due to the necessity of joint holes.

The rib flanges have to be able to transfer the crushing load from the rib into the skin. In this case strength requirements are not critical but the minimum bolts diameter is. Ref.[5.3.4] states that the minimum diameter of a bolt is in-between 4mm and 5mm. If a bolt diameter $d_b$
of 5mm is used and a value \( d_{y}/t \) of 2 as stated in section 5.3.1 is used, then the rib flange will have a thickness \( t_{\text{flange, top}} \) of 2.5mm.

To calculate the amount of bolts that is required, i.e. the distance \( b \) between the bolts, the following is used:

\[
\sigma_{\text{bearing}} = \frac{P}{d_b \cdot t_{\text{flange, top}}} = \frac{p_{cr} \cdot b}{d_b \cdot t_{\text{flange, top}}} \quad (5.3.2.10)
\]

The bearing stress of a joint that is not clamped can be found in figure 5.3.19. It is approximately 550 Mpa. Therefore, the maximum allowable distance in-between the bolts is:

\[
b = 358.52\, \text{mm} \quad (5.3.2.11)
\]

So only approximately 4 bolts are required to transfer the crushing load from the rib into the top skin.

For the rib attachment to the bottom skin panel the results are:

\[
t_{\text{tot, bom}} = 3 \cdot (2.375 + 3 \cdot 0.125 + 8 \cdot 0.125) = 11.250\, \text{mm}
\]

\[
t_{\text{extra, bom}} = t_{\text{tot, bom}} - 2.375\, \text{mm} = 8.875\, \text{mm}
\]

\[
t_{\text{flange, bom}} = 2.5\, \text{mm}
\]

\[
d_b = 5\, \text{mm}
\]

\[
b = 358.52\, \text{mm} \quad (5.3.2.12)
\]

For the width of the area near the bolts \( e/d_b = 4 \) (on each side of the joint) can be can be used again, which leads to a width \( e_{\text{tot}} \) of:

\[
e_{\text{tot}} = 2 \cdot e = 40\, \text{mm} \quad (5.3.2.13)
\]

Of course, the skins and the rib flanges only have to be thickened up in the neighbourhood of the bolts. However, in this case the thicknesses will probably be the same all across the rib flange, for the rib flanges themselves and the skin panels. This is due to 3 reasons:

- It required a lot of labour to create the change in thickness of the laminate (see figure 5.3.20)
- There is a big difference in required thickness near the joint and the thickness away from the joint. As the slope can only be approximately 10° (see figure 5.3.20), the transitional area will be large.
- In practice far more joints will be used because if the distance between the joints is large, the skin and the rib flanges will slide over each other and get damaged due to friction.

The weight of the bolts themselves will be neglected because, as can be see in section 5.3.1, these hardly have any influence on the weight of the panel.

The total extra volume per unit width \( V/d_{\text{rib}} \) due to the rib and its connection is:
\[
\left( \frac{V}{d} \right)_{\text{rib}} = \left( \frac{V}{d} \right)_{\text{rib, web}} + \left( \frac{V}{d} \right)_{\text{flange, top}} + \left( \frac{V}{d} \right)_{\text{extra, top}} + \left( \frac{V}{d} \right)_{\text{flange, beam}} + \left( \frac{V}{d} \right)_{\text{extra, beam}} = \]
\[
H \cdot t_{eq} + e_{tot} \cdot (t_{\text{flange, top}} + t_{\text{extra, top}} + t_{\text{flange, beam}} + t_{\text{extra, beam}}) = 1191.1 \text{mm}^2
\] (5.3.2.14)

Thus, if expressions 5.3.2.14 and 5.3.2.8 are compared it can be seen that the joints in this case have a tremendous influence on the weight of the structure if a sandwich panel is used.

5.3.2.3 The Effect of the Rib Pitch upon the Weight of the Rib Structure

From section 5.3.2.2 it becomes apparent that \( V/d_{\text{rib}} \) hardly influenced by the crushing load and, therefore, the rib pitch. Only the height of the stiffener is affected by the rib pitch. The volume per unit width can be written as:

\[
\left( \frac{V}{d} \right)_{\text{rib}} = 1155.1 \text{mm}^2 + 0.069266 \text{mm}^{-1} \cdot L^3
\] (5.3.2.15)

in which \( L \) is the rib pitch. To see what the influence the weight of the rib has on the weight of the total structure the density of the composite, with HMS-4 fibres, and the amount of ribs in the structure have to be taken into account. Therefore, the weight of the ribs is written as:

\[
W_{\text{rib}} = \rho_{\text{HMS-4}} \cdot \frac{V}{d} = \left( \frac{1815.8}{L} + 0.10889 \cdot L^2 \right) \cdot 10^{-6} \text{kg/mm}^2
\] (5.3.2.16)

Figure 5.3.21 displays the weight of the skin panels and the ribs separately and combined. These results can also be found in table 5.3.2. It directly becomes apparent that the weight of the rib has a big influence on the weight of the structure, unlike in the case of the structure with I-stringer skin panels.

It would be better if rib could be bonded to the skins. If a structure like can be seen in figure 5.3.22 is used, the thickness of the laminate will not have to be increased in the neighbourhood of the joints. This would reduce the weight of the structure considerably. However, bonded joints can only carry a low load. The inner face of the of the sandwich is not continuous, thus the bond will be highly loaded. A way to get around this problem is to put all of the 0° fibres in the outer (continuous) face of the sandwich panel. The load in the inner skin will be low in that case.

This problem will not be assessed any further at this stage. It can, however, be stated that if a structure with sandwich skin panels is going to be used the ribs probably would have to be bonded to the skins, in order to create an efficient structure.

5.3.3 Concluding Remarks on Rib Structures

From this paragraph it becomes apparent that a truss rib should be used if the structure has skin panels with I-stringers, and a simple, flat plate rib if the structure has sandwich skin
panels. Due to the inefficiency of the joints, a structure with sandwich skin panels it will be heavier than a structure with I-stringer skin panels. This can be seen in tables 5.3.1 and 5.3.2, and figure 5.3.23.

If the ribs in a structure with sandwich skin panels could be bonded to the skin panels, an efficient might be created.

The struts of the truss rib are “plus” shaped because in that case a relatively simple joint can be created.

Furthermore it is now clear that the rib pitch should be approximately 520mm for the structure with I-stringer skin panels. For the structure with the sandwich panels the optimum rib pitch is slightly higher.

The dimensions of the two rib designs can be found in table 5.3.3.
References:


[5.1.4] ESDU, data sheet 80023, figure 5


<table>
<thead>
<tr>
<th>Height of the front web</th>
<th>Height of the rear web</th>
<th>Average height of the torsion box</th>
<th>Width of the torsion box</th>
</tr>
</thead>
<tbody>
<tr>
<td>378.45mm</td>
<td>311.83mm</td>
<td>386.92mm</td>
<td>1308.6mm</td>
</tr>
</tbody>
</table>

Table 5.1  The dimensions of the torsion box halfway down the wing span, i.e. \( x/(b/2) = 0.5 \)

<table>
<thead>
<tr>
<th></th>
<th>Corrugated web with flat sections</th>
<th>Unstiffened web</th>
<th>Stiffened web</th>
<th>Corrugated web with circular arc sections</th>
<th>Sandwich panel</th>
</tr>
</thead>
<tbody>
<tr>
<td>Prepreg with IM6 fibres front spar</td>
<td>2.0561mm</td>
<td>8.7320mm</td>
<td>4.1847mm</td>
<td>2.0229mm</td>
<td>-</td>
</tr>
<tr>
<td>Prepreg with IM6 fibres rear spar</td>
<td>1.5130mm</td>
<td>6.6370mm</td>
<td>3.2014mm</td>
<td>1.3294mm</td>
<td>-</td>
</tr>
<tr>
<td>Prepreg with HMS-4 fibres front spar</td>
<td>2.1422mm</td>
<td>-</td>
<td>-</td>
<td>2.3129mm</td>
<td>2.4483mm</td>
</tr>
<tr>
<td>Prepreg with HMS-4 fibres rear spar</td>
<td>1.5428mm</td>
<td>-</td>
<td>-</td>
<td>1.5221mm</td>
<td>1.7266mm</td>
</tr>
</tbody>
</table>

Table 5.1.1  The equivalent thicknesses of different structure types

<table>
<thead>
<tr>
<th></th>
<th>Corrugated web with flat sections</th>
<th>Corrugated web with circular arc sections</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( t )</td>
<td>( R )</td>
</tr>
<tr>
<td>Prepreg with IM6 fibres front spar</td>
<td>1.8702mm</td>
<td>0.62</td>
</tr>
<tr>
<td>Prepreg with IM6 fibres rear spar</td>
<td>1.3762mm</td>
<td>0.62</td>
</tr>
<tr>
<td>Prepreg with HMS-4 fibres front spar</td>
<td>1.9485mm</td>
<td>0.881</td>
</tr>
<tr>
<td>Prepreg with HMS-4 fibres rear spar</td>
<td>1.4028mm</td>
<td>0.792</td>
</tr>
</tbody>
</table>

Table 5.1.2  The dimensions of the corrugated webs
<table>
<thead>
<tr>
<th>Rib pitch</th>
<th>Weight top skin</th>
<th>Weight bottom skin</th>
<th>Total weight skin panels</th>
<th>Sandwich panels</th>
<th>Weight top skin</th>
<th>Weight bottom skin</th>
<th>Total weight skin panels</th>
</tr>
</thead>
<tbody>
<tr>
<td>in [mm]</td>
<td>in [kg/m²]</td>
<td>in [kg/m²]</td>
<td>in [kg/m²]</td>
<td>in [kg/m²]</td>
<td>in [kg/m²]</td>
<td>in [kg/m²]</td>
<td>in [kg/m²]</td>
</tr>
<tr>
<td>130.86</td>
<td>5.8058</td>
<td>4.6450</td>
<td>10.451</td>
<td>4.7555</td>
<td>3.7934</td>
<td>8.5489</td>
<td></td>
</tr>
<tr>
<td>261.72</td>
<td>5.8058</td>
<td>4.6450</td>
<td>10.451</td>
<td>5.0918</td>
<td>3.8889</td>
<td>8.9807</td>
<td></td>
</tr>
<tr>
<td>523.44</td>
<td>5.8314</td>
<td>4.6450</td>
<td>10.476</td>
<td>5.7623</td>
<td>4.0790</td>
<td>9.8413</td>
<td></td>
</tr>
<tr>
<td>654.3</td>
<td>6.5196</td>
<td>4.6450</td>
<td>11.165</td>
<td>6.0956</td>
<td>4.1734</td>
<td>10.269</td>
<td></td>
</tr>
<tr>
<td>785.16</td>
<td>7.1419</td>
<td>4.6450</td>
<td>11.787</td>
<td>6.4274</td>
<td>4.2672</td>
<td>10.695</td>
<td></td>
</tr>
<tr>
<td>916.02</td>
<td>7.7142</td>
<td>4.6450</td>
<td>12.359</td>
<td>6.7569</td>
<td>4.3603</td>
<td>11.117</td>
<td></td>
</tr>
<tr>
<td>1177.74</td>
<td>8.7470</td>
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<td>13.691</td>
<td>7.4076</td>
<td>4.5434</td>
<td>11.951</td>
<td></td>
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<tr>
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<td>9.2202</td>
<td>5.2113</td>
<td>14.431</td>
<td>7.7276</td>
<td>4.6331</td>
<td>12.361</td>
<td></td>
</tr>
</tbody>
</table>

Values at optimum rib pitch

519.17 | 5.8075 | 4.6450 | 10.453 | 5.7514 | 4.0759 | 9.8273

Table 5.2.1 The effect of the rib pitch on the weight of the sandwich and the stringer panels

<table>
<thead>
<tr>
<th>The I-stringer panels</th>
</tr>
</thead>
<tbody>
<tr>
<td>( t_{skin} )</td>
</tr>
<tr>
<td>[mm]</td>
</tr>
<tr>
<td>Top skin</td>
</tr>
<tr>
<td>Bottom skin</td>
</tr>
</tbody>
</table>

The sandwich panels

<table>
<thead>
<tr>
<th>2( t_{faces} )</th>
<th>( t_{core} )</th>
<th>( R_{foam} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>[mm]</td>
<td>[mm]</td>
<td>[mm]</td>
</tr>
<tr>
<td>Top skin</td>
<td>2.875</td>
<td>17.644</td>
</tr>
<tr>
<td>Bottom skin</td>
<td>2.375</td>
<td>11.450</td>
</tr>
</tbody>
</table>

Table 5.2.2 The dimensions of the sandwich panels and the I-stringer panels halfway down the wing span, i.e. \( x/(b/2)=0.5 \), if the rib pitch 519.17mm

133
<table>
<thead>
<tr>
<th>Rib pitch [mm]</th>
<th>Top skin panel $10^6$[kg/mm$^2$]</th>
<th>Bottom skin panel $10^6$[kg/mm$^2$]</th>
<th>Total top and bottom skin panels $10^6$[kg/mm$^2$]</th>
<th>Rib $10^6$[kg/mm$^2$]</th>
<th>Total rib and skin panels $10^6$[kg/mm$^2$]</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>5.8058</td>
<td>4.645</td>
<td>10.4508</td>
<td>11.5522</td>
<td>22.003</td>
</tr>
<tr>
<td>130.86</td>
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<td>4.645</td>
<td>10.4508</td>
<td>1.4315</td>
<td>11.882</td>
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<td>11.1646</td>
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<td>11.855</td>
</tr>
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<td>7.1419</td>
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<td>11.7869</td>
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<td>12.475</td>
</tr>
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<td>4.645</td>
<td>12.3591</td>
<td>0.68608</td>
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</tr>
<tr>
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<td>8.2467</td>
<td>4.645</td>
<td>12.8917</td>
<td>0.68459</td>
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<td>4.9438</td>
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<td>0.68444</td>
<td>14.374</td>
</tr>
<tr>
<td>1308.6</td>
<td>9.2201</td>
<td>5.2113</td>
<td>14.4314</td>
<td>0.68251</td>
<td>15.114</td>
</tr>
</tbody>
</table>

Values at optimum the rib pitch

| 519.17         | 5.8075                        | 4.645                             | 10.4525                                  | 0.69515         | 11.148                                   |

Table 5.3.1 The weight of the top skin, the bottom skin, and the ribs at different rib pitches for a structure with I-stringer skin panels

<table>
<thead>
<tr>
<th>Rib pitch [mm]</th>
<th>Weight top skin $10^6$[kg/mm$^2$]</th>
<th>Weight bottom skin $10^6$[kg/mm$^2$]</th>
<th>Total weight skin panels $10^6$[kg/mm$^2$]</th>
<th>Weight rib $10^6$[kg/mm$^2$]</th>
<th>Weight rib and skin panels $10^6$[kg/mm$^2$]</th>
</tr>
</thead>
<tbody>
<tr>
<td>130.86</td>
<td>4.755511</td>
<td>3.793435</td>
<td>8.548946</td>
<td>13.88052</td>
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<td>261.72</td>
<td>5.091837</td>
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<td>6.96476</td>
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<tr>
<td>523.44</td>
<td>5.762272</td>
<td>4.078999</td>
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<td>2.94769</td>
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<td>4.360271</td>
<td>11.11719</td>
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<tr>
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<tr>
<td>1177.7</td>
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<td>4.543402</td>
<td>11.95097</td>
<td>2.101894</td>
<td>14.05286</td>
</tr>
<tr>
<td>1308.6</td>
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<td>4.633103</td>
<td>12.36066</td>
<td>2.079191</td>
<td>14.43985</td>
</tr>
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</table>

Table 5.3.2 The weight of the top skin, the bottom skin, and the ribs at different rib pitches for a structure with sandwich skin panels
<table>
<thead>
<tr>
<th>Truss rib</th>
</tr>
</thead>
<tbody>
<tr>
<td>Struts</td>
</tr>
<tr>
<td><em>h</em></td>
</tr>
<tr>
<td>26.021mm</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Rib flanges</th>
</tr>
</thead>
<tbody>
<tr>
<td><em>l</em>&lt;sub&gt;top&lt;/sub&gt;</td>
</tr>
<tr>
<td>1.375mm</td>
</tr>
</tbody>
</table>

Near the joints

| *l*<sub>extra, 90°</sub> | *l*<sub>extra, +/-45°</sub> | *d*<sub>0</sub> | *w*  |
| 0.75mm     | 0.25mm | 4.894mm | 19.576mm |

<table>
<thead>
<tr>
<th>Flat plate rib</th>
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<tr>
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<table>
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<td><em>R</em>&lt;sub&gt;hbm&lt;/sub&gt;</td>
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<table>
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<th>Extra laminate skin panels</th>
</tr>
</thead>
<tbody>
<tr>
<td><em>l</em>&lt;sub&gt;extra, 0°&lt;/sub&gt;&lt;sup&gt;top&lt;/sup&gt;</td>
</tr>
<tr>
<td>4.75mm</td>
</tr>
</tbody>
</table>

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the J-stringer

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\[ \frac{p_{cr} \cdot d^2}{12} \quad \frac{p_{cr} \cdot d^2}{12} \]

\[ \frac{1}{2} p_{cr} \cdot d \quad \frac{1}{2} p_{cr} \cdot d \]

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Chapter 6  The Discussion and Recommendations

In considering the practical application of the use of composites in wing structure several aspects deserve attention. Unfortunately only an initial design is made in this report and, therefore, the picture is not complete yet. It is at this stage not yet fair to make a comparison with an aluminium alloy structure. For example, it can be seen in Chapter 5 that if mechanical fasteners are needed a composite structure gains a lot of weight. Without doing any calculations it can already be stated that the connection between the spars and the skin panels will be heavy. Therefore, it would be better to completely work out the design of the composite wing structure before comparing it to a conventional aluminium structure.

This report is more in the nature of a feasibility study. It is, of course, just as important to know if the use of composites in wing structures is actually viable. Whether or not the widespread use of composites is viable depends largely upon the cost of a structure made of composites. That cost then has to be compared to the cost of a conventional aluminium structure.

When considering this aspect, there are four broad components that should receive attention; the first two are the capital outlay required for tooling up the project and the general production costs.

Limited research has shown that the capital outlay will be higher because of expensive machinery like expensive autoclaves and tape laying machines. However, general production costs in the long term can show a saving against conventional methods, which require more manpower in terms of mechanical staff.

There are, however, two cost aspect that are more important. The main advantage of using a composite structure is that it will lead to a lighter structure. A lighter structure will lead to lower operational costs. The aircraft will be more competitive. The main disadvantage of using a structure made of composites is that the cost of the raw material is a lot higher than that of aluminium alloys.

This chapter is split up into 4 section. The materials, the loads, the individual components, and the extra problems when designing with composites will be assessed.

6.1 The Materials

Carbon fibres are used in the structures of this report. This is due to the fact that glass fibres have a lower Young’s modulus of elasticity, and because of the low allowable strains this leads to a low allowable stress in the structure; kevlar fibres have no compressive strength; and boron fibres have a large diameter and are difficult to work with.

Two types of carbon fibre are used: namely HMS-4 fibres and IM6 fibres. These are more or less the best in their class. IM7 fibres perform slightly than IM6 fibres, however, IM6 fibres were used because exact data was found on a combination of IM6 fibres with an epoxy resin. The lack of specific data is a problem when designing with composites. In this report values have been puzzled together. In reality it would be better to create data from tests. A unidirectional composite with HMS-4 fibres has a higher modulus but lower strength than a unidirectional composite with IM6 fibres. The choice of the type of reinforcing fibre depends
upon the type of structure, in which it will be used. Of course, the cost of the fibre can influence the choice as well.

An epoxy resin has been used. In virtually all of the structures made of composites, the matrix is an epoxy resin. The use of a toughened epoxy resin leads to a better impact tolerance. However, they usually also reduce the hot and wet performance of the composite. Thermoplastic matrix materials also might be used more in the future, but again the lack of data prevented its use in this report. A common fibre volume coefficient of 0.6 was used.

The strength of a prepregs depends either upon the hot and wet conditions or upon the maximum allowable strain after impact, in the case of a structure in compression. If a structure is in tension a maximum strain (again for impact reasons) is the only criteria. The strain in the structure is usually so low that fatigue will never be a problem.

6.2 The Loads

In chapter 4 it can be seen that the load calculations lead to the conclusion that gust or manoeuvres lead to highest shear force and bending moment. The torsion moment on the inner wing section, i.e. the area where the flaps are placed, is the most critical when the flaps extended and the load factor is increased due to gust or manoeuvres. For the outer wing asymmetric gust on the down wing leads to the highest torsion moment.

Because the spars and the skin panels require +/- 45° fibre layers to prevent buckling it is best to design the torsion box structure just to be able to withstand the shear force and bending moment due to gust or manoeuvres. In a second phase it then should be checked whether or not the structure has enough torsional strength and stiffness.

6.3 The Individual Elements of the Structure

Before starting the discussion on the spar structure one aspect has to be underlined. The shear flow distribution in the spars and the skin panels in chapter 5 is not realistic. In the first design loop, however, ones hands are tied. The shear flow in the structure depends upon the material distribution in the structure itself, which before the design is made, of course, is unknown. Two approaches can be taken at this stage. The shear flow can be analytically re-calculated with the help of the following formulae:

\[
q_1 = \frac{V_x \cdot Q_y}{I_{yy}}
\]

\[
2q_2 \cdot A = M_t
\]

(6.3.1)

in which \(V_x\) is the shear force, \(M_t\) the torsion moment, \(A\) the enclosed area of the torsion box, \(I_{yy}\) the second moment of inertia, and \(Q_{yy}\) the static moment of inertia.
The other approach would be to see how the structure performs in a finite element program. This approach seems better (and probably easier as well) because the initial design is checked and it directly can be adjusted. Just by adding (+/- 45°) fibre layers in certain areas, the stress can be kept low enough in the whole of the structure.

With the help of chapter 4, it is also possible to use the approach taken in chapter 5 to calculate the initial dimensions of the cross sections all the way down the wing span. Chapter 4 gives the loads along the wing span. In a second phase a finite element model could then be made of the whole of the wing torsion box structure. It should, however, be remembered that extra attention has to be given to the areas near the heavy ribs, i.e. the ribs to which the flaps, engines, and ailerons are connected.

In chapter 5 the calculations are based on netting theory, for simplification. This means that the fibres carry all of the loads.

6.3.1 The Spar Structure

The spar structures consist of +/- 45° fibre layers and 90° fibre layers. The minimum thickness of +/- 45° fibre layers can be calculated from the acting shear force and the strength of fibres. The 90° fibre layers in the structure are mainly there to make it into a three fibre layer system. It will also carry some of the crushing load. This load is extremely low, though.

Besides that the laminate has to have enough strength the spar structures must also be able to withstand buckling. A spar can get its buckling resistance from either the thickness of its web (or in the case of a sandwich structure, the distance between the faces), corrugating the web, or placing stiffeners on the web.

Both the stiffened and especially the unstiffened web are inefficient, i.e. heavy. A very small stiffener pitch leads to the lightest structure of panel with stiffeners. A very small stiffener pitch is unacceptable from the manufacturing point of view.

A corrugated web should be used, for the spars of this 70 passenger transport aircraft because the spars are relatively lightly loaded.

A sandwich structure does not perform too badly. However, it does not get used in practice on larger aircraft. Maybe this is because problems occur when designing the connections between the ribs and the spars. The loads of the ribs have to be transferred into both faces of the sandwich spar structure.

If a spar with a corrugated web is used the thickness of the front and the rear spar is in-between 1mm and 2mm.

The corrugation angle for a web with flat sections is 35°. Although, a larger angle would give the spar a higher buckling resistance, it is thought that 35° is the maximum angle for production reasons. A further investigation to see if this is actually true would be useful. To get to the final design the exact buckling coefficients should be used. These can be found in ref.[6.3.1].

The other possibility is to use a corrugated web with circular arc sections. There is hardly any weight difference between the types of corrugated web. It has to be stated that calculations in chapter 5 are based on a semi-empirical (local) buckling formula for isotropic materials. To get to the final design tests on a composite web will have to be done to see if the semi-empirical formula also applies for a composite design.
One of the spar elements that has not been assessed are the spar caps. The spar caps of an aircraft for 70 passengers should just be design to transfer the loads from the skin panels into the spars, additional $0^\circ$ fibres should not be added. The $0^\circ$ fibres have to remain as much as possible in the skin panels to prevent buckling of these skin panels.

A good reason for using a corrugated web with flat section, instead of circular arc sections, is that the corrugations could end just before the spar to spar cap connection. This way a simple spar cap design can be made. This type of structure cannot be made if a corrugated web with circular arc sections is used.

About the type of fibres that should be used a couple of remarks can be made. A structure with IM6 fibres is slightly lighter than a structure with HMS-4 fibres. The strain is higher in the structure with IM6 fibres, though. The strain is low enough so that fatigue will not occur, but it is too high for a damaged structure. It is, however, thought that the spar structure will not be damaged because it is protected by the structure around it. Still, it might be better to use HMS-4 fibres because, although, the structure is slightly heavier, it is a lot safer.

6.3.2 The Skin Panels

There are two types of skin panels: Sandwich panels and panels with stringers. Firstly, the amount of $0^\circ$ fibre layers (and $+/-45^\circ$ fibre layers) are calculated that are needed so that the panel will not fail due to either impact damage or in hot and wet conditions. These thicknesses can then be used to find an optimum design to prevent buckling. The compressive load in the bottom skin panel is, of course, a lot lower than in the top skin. It is important, though, for the buckling resistance of the skin panel. Therefore, unlike the top skin panel the bottom skin panel has two different types of critical loads, i.e. a high tensile load and a low compression load. It must be stated that the thicknesses of the skins are remarkable small, i.e. in-between 1mm and 2mm.

The panels with stringers in a composite structure are mainly I-stringer skin panels. Hat stringer skin panels also exist. The reason for using I-stringer panels is that in that case the rib flanges can be co-cured, which means that there will be no stress raiser in the skin panels. Skin panels with hat stringer have a better buckling resistance. This means that if the weight of a skin panel with hat stringers is the same as that of an I-stringer skin panel a larger rib pitch can be used. This will not have much influence on the total weight of the structure but it will reduce manual labour (costs). However, the connection between the ribs and the skin panels with hat stringers lead to problems. Therefore, an I-stringer skin panel should be used.

In chapter 5 only a section of the stringer skin panel is design, i.e. a stringer on a skin section with the width of the stringer pitch. If, though, enough of these skin panel sections are placed next to each other a design of the whole of the skin panel is can be made. Of course, the top and the bottom skin panels are slightly curved. The design could be made even better if this (local) height of the torsion box is taken into account. The height of the torsion box affects the overall buckling behaviour of the skin panel.

In the top stringer skin panel IM6 are used because then a “soft skin” design can be used, which leads to smaller thicknesses. In this so-called “soft skin” design more $+/-45^\circ$ fibre layers are used in the skin than in the rest of the skin panel. This way the stress is kept low in the skin, which is exposed to external impact. In the case of the bottom skin panel HMS-4 fibres are
used because the allowable strain is thought to be the same for both of the fibre types. HMS-4 fibres have a higher modulus and, therefore, in this case a higher allowable stress.

The stringer pitch is still rather small. The design might have to be adjusted for production reasons. The design made in chapter 5 is the theoretical optimum. By changing the lay-up, i.e. put more $+$/$-$ 45° fibres in the skin and less in the stringers, the stringer pitch can be enlarged. To give a better indication of what the effect of rearranging the fibres is, ref.[6.3.2] can be used.

If sandwich skin panel structures are used on a 70 passenger aircraft, the following approach leads to the lightest structure for the sandwich panels. The thickness of the faces together have a thickness that is required to keep the stress in the fibres low enough. The required buckling resistance then determines the thickness of the core. Of course, wrinkling should not be forgotten. In the design of chapter 5 a lot of issues concerning sandwich panels are neglected. Ref.[6.3.2] can be used to find more reliable results.

6.3.3 The Rib Structures

The design in chapter 5 only accounts for crushing loads. In the future the effect of the aerodynamic loads and the fuel loads have to be assessed. The crushing loads are so low that all of the ribs structures have a minimum allowable thickness of 0.5mm, which consists of two 0° fibre layers and one $+$/$-$ 45° cross ply layer.

The main difference the design of the structure with I-stringer skin panels and the structure with sandwich skin panels is found in the design of the ribs. A clever concept is thought of for a structure with I-stringer skin panels. In that type of structure the rib flanges can be co-cured to the skin panels. Therefore, there are no stress raisers in the highly loaded skin panels but just in the lowly loaded rib flanges. If a truss rib is used the struts have to be connected to the rib flanges with bolts. However only 4 or 5 bolts are required in the top and the bottom rib flange. Locally the rib flange can be thickened up to transfer the load for the struts into the rib flanges and skin panels. Therefore, these joints have hardly any influence weight of the structure.

The ribs in a structure with sandwich skin panels have to be mechanically fastened to the skin panels, unless it can be bonded. Joint holes have to be made for these fasteners. These are stress raisers, and now they are in the structures with the highest loads. A stress concentration factor of 3 can be used if the structure is quasi-isotropic. Therefore, a lot layers will (locally) have to be added to the skin panels. The weight of the extra plies needed near the joints of outweighs the weight of the "face" rib. A simple flat plate with stiffeners and lighting holes is used so that, unlike in the case of a truss structure, the rib face and the rib flanges are one structure. Thus, additional stress raisers and extra manual labour is avoided.

The composite struts in the truss rib are "plus" shaped. The connections between the struts and the rib flanges can then be more easily be made than when round tube struts are used. It is questionable if composite struts should be used. The minimum allowable thickness of an aluminium strut is approximately the same as that of a composite strut. The weight of the struts will, therefore, not be much higher. As the struts are only a very small percentage of the total weight of the structure it is probably better to use aluminium struts. The aluminium alloy will be a lot cheaper than the composite material. If aluminium struts are used then it must be
remembered that they are not allowed to make direct contact with the carbon fibres. Glass fibre layers will have to be used near these connections.

The struts in chapter 5 are made of HMS-4 fibres. However, if different fibres are used the weight of the struts is hardly influenced. The choice of fibres has virtually no influence on the total weight of the structure. Thus, the cheapest available fibres should be used.
The rib flanges should have the same type of fibres as the skin panels. During the co-curing no problems will occur due to different coefficient of thermal expansion of different fibre types.

The connections between the spars and the ribs have not been design yet. They are actually a part of the rib but in the case of a composite truss rib structure, it is better to co-cure these to the spars. In a second phase they then can be mechanically fastened to the struts and the rib flanges, in the corners of the torsion box.
In the case of a simple flat plate rib structure this part, like with the rib flanges, will probably be an integral part of the rib. The ribs will then be bolted to the spars and the skin panels in a second phase.

6.4 Extra Problems Concerning Composite Structure

The last step before certification of the aircraft is the structural testing. These tests can be divided into four groups. Of course, it is beneficial to do some of the testing in an earlier stage (for example, see section 6.1). These tests, for a composite structure, consist of testing of:

- Coupons: Modulus, static strength in different environments, damage sensitivity, etc.
- Elements: Joints, structural details, etc.
- Components: Stiffened panels, full-scale sections, etc.
- Full scale (in all possible environments): Static, fatigue, damage tolerance.

For the production of a composite wing of a 70 passenger aircraft a lot of machinery must be used. The investment capital will be high. However, a great number of aircraft of that size, will have to be sold, in any case, for it to be viable. These high investments should easily pay themselves back over the “lifecycle” of the aircraft, due to the lower manual labour costs. If the wing structure of a new type of aircraft is constructed out of aluminium alloys, a lot of production equipment and knowledge could be used from the prior generation of aircraft. These are, of course, economical questions, and lie far beyond the scope of this report. At some stage a cost analysis should be made, though.
It must be underlined that the structures that were designed in chapter 5 are, from the manufacturing point of view, all are feasible.

As stated in chapter 2 and follows from the calculations in chapter 5, the joints between the elements lead to problems. Locally, a lot of plies have to be added to keep the stresses low enough. A well-chosen stacking sequence can, however, keep the inter-laminar stresses low.

Inspection hatches still have to be made in the structure. This is rather unfortunate because composite structures perform badly when they are holed. Although, it is the least favourable location, because it is mainly in tension, it seems improbable that the hatches can be situated anywhere else than in the bottom skin panel. It is the easiest section of the wing to access.
The aircraft has to be protected against the environment, which include fuel, UV radiation, water and sand, and lightning strikes. For the protection against fuel a polysulfide sealant on the inside of the fuel tank can be used. Polyurethane paint gives the aircraft protection against erosion and UV radiation. An aluminium bar diverter on top of the connection between the spars and the skin panels will make certain that no electricity, from lighting strikes, will go into the skin panels of the torsion box. However, directly behind the engines extra protective measures must be taken because a lighting strike can be swept over this area. All in all, it can be stated that the problems surrounding the protection of composites against the environment are easily solved.
References:


Chapter 7  The Conclusions

A lot of work still has to be done before a final composite wing torsion box structure is designed, which is ready for production. However, it can be stated that a composite wing box structure is feasible. Whether or not it is viable cannot be stated as of yet. Comparing the design made in this report to that of an aluminium wing box structure is not fair because the design is not close enough to the final design. When the design is final, a full cost analysis of the composite and the aluminium structure should be made to see if a composite structure is more cost efficient than an aluminium structure.

The strength of composite laminates is governed by the maximum allowable strains after impact and their performance in hot and wet conditions. Because the allowable strains are so low, fatigue is no problem.

If the ribs of a composite structure cannot be bonded to the skin panels a structure with I-stringer skin panels should be used. This kind of structure will be light in comparison to a structure with sandwich skin panels. The production process for a structure with I-stringers skin panels is relatively easy. Spar structures should always have a corrugated web. In that case the spars will be the lightest. If the wing box structure has I-stringer skin panels, the ribs must be truss ribs. It is very doubtful whether or not composite struts are viable. It is probably better to have aluminium struts in this kind of structure. This is exactly the structural set up of most of the large composite torsion box structure.

The spars have to have corrugated webs, either with circular arc sections or with flat sections. This will lead to the lightest spar structures. For the construction of the spars it seems that a corrugated web with flat sections is better.

The optimum rib pitch of a composite structure with I-stringer skin panels is found at the distance whereby minimum (equivalent) thickness required for the strength of the skin panels, must be enlarged for the panel to resist buckling. Although, a smaller rib pitch is not much less efficient, it leads to more manual labour (costs).

Joints in composite structures are a major issue. They need a lot of attention, and should as much as possible be avoided.

The problems specific to structures constructed out of composites are usually easily solved by some protective measures, which do not lead to an increase in weight, and in comparison to an aluminium structure do not lead to extra work.

It is worth pursuing this path of research in order to promote better and operationally cheaper aircraft in the next century.
Appendices
Appendix 1  The Buckling Coefficients

The following analysis is taken from ref.[A 1.1]. It is based on netting theory. In this report the only section of interest is that of the buckling coefficients of a composite plate. The analysis is as follows:

Considering first a typical fibre in a direction $\alpha$, the strain $\varepsilon$ produced in the fibre by strains $\varepsilon_x$, $\varepsilon_y$ and $\gamma_{xy}$ in the composite is

$$\varepsilon = \varepsilon_x \cos^2 \alpha + \varepsilon_y \sin^2 \alpha + \gamma_{xy} \sin \alpha \cos \alpha$$

and the corresponding stress

$$\sigma = E_f \varepsilon$$

where $E_f$ is the elastic modulus of the fibre. Resolving the fibre stress $\sigma$ into average stress components $\sigma_x$, $\sigma_y$ and $\tau_{xy}$ in the composite, for a volume fraction of fibres $m/2$ ($m$ is the volume fraction of fibres in the $+\alpha$ and the $-\alpha$ direction together), we find

$$\sigma_x = \frac{m}{2} \sigma \cos^2 \alpha = \frac{m}{2} E_f \varepsilon \cos^2 \alpha$$

$$\sigma_y = \frac{m}{2} \sigma \sin^2 \alpha = \frac{m}{2} E_f \varepsilon \sin^2 \alpha$$

$$\tau_{xy} = \frac{m}{2} \sigma \sin \alpha \cos \alpha = \frac{m}{2} E_f \varepsilon \sin \alpha \cos \alpha$$

and substitution for $\varepsilon$ gives equations in terms of three strain components $\varepsilon_x$, $\varepsilon_y$ and $\gamma_{xy}$. Therefore, for a 4-fibre system ($m/2$ is the volume fraction of other fibres in each of the $+\beta$ and the $-\beta$ direction), we may write

$$\sigma_x = c_{11} \varepsilon_x + c_{12} \varepsilon_y + c_{16} \gamma_{xy}$$

$$\sigma_y = c_{21} \varepsilon_x + c_{22} \varepsilon_y + c_{26} \gamma_{xy}$$

$$\tau_{xy} = c_{61} \varepsilon_x + c_{62} \varepsilon_y + c_{66} \gamma_{xy}$$

(retaining here the usual notation of three-dimensional elasticity) where

$$\begin{align*}
c_{11} &= E_f \left(m \cos^4 \alpha + n \cos^4 \beta\right) \\
c_{22} &= E_f \left(m \sin^4 \alpha + n \sin^4 \beta\right) \\
c_{12} &= c_{21} = c_{66} = E_f \left(m \cos^2 \alpha \sin^2 \alpha + n \cos^2 \beta \sin^2 \beta\right) \\
c_{16} &= c_{61} = 0, \\
c_{26} &= c_{62} = 0
\end{align*}$$

(A1.1)  

(A1.2)

The zero coefficients are the results of a symmetrical fibre arrangement.
The effective elastic constants $E_x$, $E_y$, $\nu_x$ and $\nu_y$ of the composite are defined by

$$
\varepsilon_x = \frac{\sigma_x - \nu_y \frac{\sigma_y}{E_y}}{E_x},
$$

$$
\varepsilon_y = \frac{\sigma_y - \nu_x \frac{\sigma_x}{E_x}}{E_y},
$$

and by deleting the zero coefficients from eqn. (4.1.1) and solving for $\varepsilon_x$ and $\varepsilon_y$, we obtain

$$
E_x = \frac{c_{12}c_{22} - c_{12}^2}{c_{22}}, \quad E_y = \frac{c_{11}c_{22} - c_{12}^2}{c_{11}}
$$

$$
\nu_x = \frac{c_{12}}{c_{22}}, \quad \nu_y = \frac{c_{12}}{c_{11}}
$$

Also, for the shear modulus we have

$$
G = \frac{\tau_{xy}}{\gamma_{xy}} = c_{12}
$$

The usual expressions for the flexural rigidity of an orthotropic plate are

$$
D_1 = \frac{E_t t^3}{12(1 - \nu_x \nu_y)}, \quad D_2 = \frac{E_y t^3}{12(1 - \nu_x \nu_y)}
$$

which on substitution is reduced to

$$
D_1 = c_{11} \frac{t^3}{12}, \quad D_2 = c_{22} \frac{t^3}{12}
$$

For the torsional rigidity

$$
D_{12} = \frac{Gt^3}{6} = c_{12} \frac{t^3}{6}
$$

and therefore

$$
D_3 = \frac{1}{2} (\nu_x D_2 + \nu_y D_1) + D_{12} = c_{12} \frac{t^3}{4}
$$
It is convenient to define

\[ m = (1 - R)v_f, \quad n = Rv_f \]

in which case eqn. (A 1.2) can be rewritten as

\[
\begin{align*}
    c_{11} &= E_f v_f \left\{ (1 - R)\cos^4 \alpha + R \cos^4 \beta \right\}, \\
    c_{22} &= E_f v_f \left\{ (1 - R)\sin^4 \alpha + R \sin^4 \beta \right\}, \\
    c_{12} &= c_{21} = E_f v_f \left\{ (1 - R)\cos^2 \alpha \sin^2 \alpha + R \cos^2 \beta \sin^2 \beta \right\}, \\
    c_{16} &= c_{61} = 0, \\
    c_{26} &= c_{62} = 0
\end{align*}
\]

Reference:

Appendix 2  Optimum Dimensions for Stringer Panels

This appendix is based on ref.[A 2.1]. It is a paper on wing box optimisation under combined shear and bending. It was written in this time that composite structure did not exist yet. However, if the optimisation of a composite skin panel with stringers can be divided into two section, i.e. optimisation of the lay-up and optimisation of the cross-section, it can be used for a composite structure as well as for an isotropic structure. Only the result of ref.[A 2.1] that lead to feasible structures are taken are displayed in this appendix.

The following relations hold for all wide column configurations except as noted in the table.

\[
\frac{N_s}{L\cdot E} = \varepsilon \cdot \left(\frac{t_{eq}}{L}\right)^2
\]

\[
t_s = \frac{t_{eq}(1 + r_{bs})}{\alpha}
\]

\[
b_s = \left[\frac{N_s \cdot L^2}{\pi^2 \cdot E \cdot t_{eq} \cdot \frac{\alpha}{\gamma - \frac{\beta^2}{\alpha}}}\right]^{0.5}
\]

\[
k = b_s \cdot \left[\frac{\gamma - \frac{\beta^2}{\alpha}}{\alpha}\right]^{0.5}
\]

\[
b_i = r_{bi} \cdot b_s
\]

\[
t_i = r_{ti} \cdot t_s
\]

<table>
<thead>
<tr>
<th>Configuration</th>
<th>Optimum values</th>
<th>Auxiliary relations</th>
<th>Dimensionless geometric expressions</th>
</tr>
</thead>
<tbody>
<tr>
<td>[Diagram]</td>
<td>$\varepsilon_{max} = 0.656$</td>
<td>$r_{bs} = 0$</td>
<td>$\alpha = 1 + r_{bw} r_{tw}$</td>
</tr>
<tr>
<td></td>
<td>$r_{bw} = 0.65$</td>
<td>$\gamma = 0.5 r_{bw}^2 r_{tw}$</td>
<td>$\beta = 0.33 r_{bw}^3 r_{tw}$</td>
</tr>
<tr>
<td>Integrally Stiffened</td>
<td>$r_{tw} = 2.25$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>[Diagram]</td>
<td>$\varepsilon_{max} = 0.911$</td>
<td>$r_{bf} = 0.3 r_{bw}$</td>
<td>$\alpha = 1 + 1.6 r_{bw} r_{tw}$</td>
</tr>
<tr>
<td></td>
<td>$r_{bw} = 0.87$</td>
<td>$r_{ty} = r_{tw}$</td>
<td>$\beta = 0.8 r_{bw}^2 r_{tw}$</td>
</tr>
<tr>
<td></td>
<td>$r_{tw} = 1.06$</td>
<td>$r_{bs} = 0$</td>
<td>$\gamma = 0.63 r_{bw}^3 r_{tw}$</td>
</tr>
<tr>
<td>Zee Stiffened</td>
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<td></td>
</tr>
</tbody>
</table>

Table A 2.1
<table>
<thead>
<tr>
<th>Configuration</th>
<th>Optimum values</th>
<th>Auxiliary relations</th>
<th>Dimensionless geometric expressions</th>
</tr>
</thead>
<tbody>
<tr>
<td>Integral Zee</td>
<td>$\varepsilon_{\text{max}}=1.03$</td>
<td>$r_p=0.3 r_w$</td>
<td>$\alpha=1+1.3 r_{bw} r_w$</td>
</tr>
<tr>
<td></td>
<td>$r_{pw}=1$</td>
<td>$r_d=r_w$</td>
<td>$\beta=0.8 r_{bw}^2 r_w$</td>
</tr>
<tr>
<td></td>
<td>$r_w=1$</td>
<td>$r_{by}=0$</td>
<td>$\gamma=0.633 r_{bw}^3 r_w$</td>
</tr>
<tr>
<td>Integral Tee</td>
<td>$\varepsilon_{\text{max}}=1$</td>
<td>$r_p=0.6 r_w$</td>
<td>$\alpha=1+1.6 r_{bw} r_w$</td>
</tr>
<tr>
<td></td>
<td>$r_{pw}=0.8$</td>
<td>$r_d=r_w$</td>
<td>$\beta=1.1 r_{bw}^2 r_w$</td>
</tr>
<tr>
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<td>$r_w=0.7$</td>
<td>$r_{by}=0$</td>
<td>$\gamma=0.933 r_{bw}^3 r_w$</td>
</tr>
<tr>
<td>&quot;J&quot; Stiffened</td>
<td>$\varepsilon_{\text{max}}=0.793$</td>
<td>$r_p=0.3 r_w$</td>
<td>$\alpha=1+1.9 r_{bw} r_w$</td>
</tr>
<tr>
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<td>$r_{pw}=0.85$</td>
<td>$r_d=r_w$</td>
<td>$\beta=0.8 r_{bw}^2 r_w$</td>
</tr>
<tr>
<td></td>
<td>$r_w=0.79$</td>
<td>$r_{by}=0$</td>
<td>$\gamma=0.633 r_{bw}^3 r_w$</td>
</tr>
<tr>
<td>Straight Y-Tee Stiffened</td>
<td>$\varepsilon_{\text{max}}=1.23$</td>
<td>$r_{bh}=0.938 r_w$</td>
<td>$\alpha=1+ r_{bw}^+ + 5.17 r_{bw} r_w$</td>
</tr>
<tr>
<td></td>
<td>$r_{bw}=0.9$</td>
<td>$r_{bh}=1.06 r_w$</td>
<td>$\beta=4.83 r_{bw}^2 r_w$</td>
</tr>
<tr>
<td></td>
<td>$r_w=0.9$</td>
<td>$r_d=r_w$</td>
<td>$\gamma=7.07 r_{bw}^3 r_w$</td>
</tr>
<tr>
<td>Trap. Corr. Semisandwich</td>
<td>$\varepsilon_{\text{max}}=0.685$</td>
<td>$r_{ac}=(4/K_1)^{0.5} r_{bd}$</td>
<td>$\alpha=1+$</td>
</tr>
<tr>
<td></td>
<td>$m=b_f/ b_d = 0.18$</td>
<td>$r_{bd}=0.348 r_w$</td>
<td>$K_1^{0.5} (m +\cos\phi)^2$</td>
</tr>
<tr>
<td></td>
<td>$\phi=70^\circ$</td>
<td>$r_{ac}=2.125 r_w$</td>
<td>$\beta= (m+1)\sin\phi$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$r_{bd}=1.04 r_{bd}$</td>
<td>$4K_1^{0.5} (m +\cos\phi)^3$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$K_1$ (ref. [A 2.2])</td>
<td>$\gamma= (m+0.667)\sin^2\phi$</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>$8K_1^{0.5} (m +\cos\phi)^6$</td>
</tr>
<tr>
<td>Hat Section Stiffened</td>
<td>$\varepsilon_{\text{max}}=0.928$</td>
<td>$r_{by}=r_w b_f/ b_w$</td>
<td>$\alpha=2+$</td>
</tr>
<tr>
<td></td>
<td>$r_{bw}=1$</td>
<td>$r_{bw}=2(1-r_{by})$</td>
<td>$r_w (1+2r_{bw}+r_{by})$</td>
</tr>
<tr>
<td></td>
<td>$b_f/ b_w =0.3$</td>
<td>$r_d=r_w$</td>
<td>$\beta= r_{bw} r_w (1-r_{by}+r_{bw})$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$r_{by}=1$</td>
<td>$\gamma= r_{bw}^2 r_w (1-r_{by}+r_{bw})$</td>
</tr>
</tbody>
</table>
References:


Appendix 3  Sketch of the Two Structures